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EDL Project #2002 -- Analysis Requirement

The basis for this analysis requirement is shown in reference (1), as the appendix.

The non-linear sinusoidal transmittance distribution, *

$$f(x) = To(1 - m \cos \omega_o x)^{-\gamma} \qquad (1)$$

is subjected to an operator which forms the second spatial derivative and recombines a percentage of it with the original signed. This produces

$$T(x) = To\Big\{(1 - m \cos [\omega_o x])^{-\gamma} -$$

$$- cm\gamma\omega_o^2 \cos(\omega_o x)\left[1 - m \cos(\omega_o x)\right]^{-(\gamma + 1)} +$$

$$- cm^2\omega_o^2\gamma(\gamma + 1)\sin^2(\omega_o x)\left[1 - m \cos(\omega_o x)\right]^{-(\gamma + 2)}\Big\}. \qquad (2)^{**}$$

In an effort to ascertain the modulation and related harmonic strength, this will become the object in a coherent system, exactly in the manner outlined in reference (2). To compare theory with experiment it will be necessary to take the Fourier transform of equation (2). Assume a fluid gate with correct index match (and hence no phase term). The transform will be

$$f(\omega) = \int_{-\infty}^{\infty} \sqrt{T(x)}\, e^{-i\omega x}\, dx \,. \qquad (3)$$

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* Radian spatial frequency, ω, is used in these equations by preference and will introduce no ambiguities.

** Note changes in sign from reference (1).

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This first requires the square root of equation (2). From the previous work in reference (2) and from additional experimental work, the form of $\sqrt{T(x)}$ must be

$$\sqrt{T(x)} = \sqrt{To}\sum_{m=o}^{\infty} B_n(c,\gamma,m)\cos n\omega_o x \ , \qquad (4)$$

a linear combination of the cosines of multiple angles. It can be expected that $B_i(c,\gamma,m)$ will be a series, hopefully in closed form, which is readily evaluated for given values of c, γ, and m. Once $\sqrt{T(x)}$ is in the form of equation (4), only the expansions for B_o, B_1, and B_2 are significant. See Note #1.

The series for B_1/B_o is determined by dividing the one by the other. This is then reverted as a power series in m, as

$$m = g\left(\frac{B_1}{B_o},\ c,\gamma\right), \qquad (5)$$

from which modulation can be determined from a physical measurement of $\left(\frac{B_o}{B_1}\right)^2$, c and γ.

The series from B_2/B_1 is next determined by the same process, but is not reverted. This permits calculation of the theoretical second harmonic strength from m,γ, and c for comparison with its measured value.

The Specific Problems

1. Expand equation (2) and extract the square root, winding up with a series in the form of equation (4) or equivalent. The following ranges of the parameter can be expected to appertain:

$\frac{1}{2} \leq \gamma \leq 3.0$

$0 < m \leq 1.0$ (closer to 1.0)

$0 \leq c \leq ?$

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Make no small-variable assumption involving these parameters! The analysis must be general. See notes # 2 and 3.

2. a. Determine the specific series for B_o, B_1, B_2, from which derive the series for B_1/B_o. Revert this series to give a polynomial in B_1/B_o, as

$$m = g\left(\frac{B_1}{B_o}, c, \gamma\right)$$

$\left(\frac{B_1}{B_o}\right)^2$ is measurable.

b. Determine the series for B_2/B_1, but do not revert.

See reference (3) (formula #50.) for specific equations needed in reversion.

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References

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Notes:

1. When c = o, equation (2) reduces to equation (19) of reference (2), and resultant expansion, for the coefficients should be identical.

2. As a preliminary look into the problem, a truncated binomial expansion of equation (2) was effected, winding-up with a series of the form

$$T(x) = T_o \sum_{n=o}^{\infty} A_n \cos^n(\omega_o x).$$

The root may now be extracted by application of formula 51.2 of reference (3)(the coefficients for which require extending above the cited fourth order), using $\cos\omega_o x$ as the variable. This power function in $\cos\omega_o x$ can now be reduced to a linear combination of cosines of multiple angles, through application of the formulas 404.22 - 404.27 (and subsequent) of reference (3). Reduction to this form certainly gave the equivalent to equation (4), although incompletely because of the truncation. There is doubt about its practical usefulness -- computation of the coefficients promises to be exceedingly lengthy. However, the method is correct and it will produce the desired answer -- in theory.

3. The value of γ will vary experimentally, and its effect on the B's must be computed for each case separately. The value of c and how it will eventually be determined is not known at this time -- nor its probable upper limit.

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