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Compression Wave Surrounding an Expanding Sphere

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The method of describing the propagation of finite amplitude waves based on the introduction of coordinat "accompanying" the wave system is used for the consideration of the radiation of a compression wave by sphere that is expanding at a constant rate in a perfect medium. To estimate the degree of accuracy of this method, the results are compared with the exact solution of G. I. Taylor. [Proc. Roy. Soc. (London) A186, 273-292 (1946)].

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An effective method of consideration of the propagation of finite-amplitude waves was developed recently.[1,2] As an illustration of this method, the radiation of a compression wave by an expanding sphere is considered in this paper. The rate of expansion U was taken to be constant. The comparison of this solution with the data of G. I. Taylor,[3] which were obtained by numerical integration of the exact equations, gives an estimate of the accuracy of the method.

Assuming the motion to be spherically symmetric

[1] I. S. Mendousse, J. Acoust. Soc. Am. 25, 51-54 (1953).
[2] R. V. Khokhlov and S. I. Soluyan, Vestn. Mosk. Gos. Univ. 3, 52-61 (1961).
[3] G. I. Taylor, Proc. Roy. Soc. (London) A186, 273-292 (1946).

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$$\frac{\partial^2 \varphi}{\partial r^2} \quad \frac{\partial^2 \varphi}{\partial t^2} \quad \frac{2}{r}\frac{\partial \varphi}{\partial r} \quad \frac{\partial \varphi}{\partial r}\frac{\partial^2 \varphi}{\partial r \partial t} \quad \frac{\partial \varphi}{\partial t}\frac{\partial^2 \varphi}{\partial r^2}$$

$$+(\gamma-1)\frac{2}{r}\frac{\partial \varphi}{\partial r}\frac{\partial \varphi}{\partial t}, \quad (1)$$

where $\gamma = C_0/C_*$ and c_0 is the velocity of sound. After introducing the new variable u via the substitution $\varphi = u/r$, transforming from the variables r, t to the variables $y = t-(r/C_0)$, $s = Mr$, $M = U/C_0$, following the method of Ref. 4, and integrating with respect to y, one can obtain

[4] K. A. Naugolnykh, S. I. Soluyan R. V. Khokhlov, Soviet Phys.—Acoust. 9, 42-46 (1963).

$$\frac{\partial u}{\partial r} = \frac{g}{2C_0 r}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{C_0^2 r^2}\frac{\partial u}{\partial y} + \frac{u^2}{2C_0 r^3} + f(r). \quad (2)$$

Here, and $f(r)$ is an arbitrary function.

Equation (2) can be integrated in principle by using standard techniques. However, for simplification, we introduce a further approximation. That is, bearing in mind that the nonlinear effects are most essential on the outer boundary of the wave, we retain only one nonlinear term in Eq., (2), which is dominant in the region of small values of y:

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\alpha}{2C_0}\left(\frac{\partial u}{\partial y}\right)^2 + f(r). \quad (3)$$

Then, as an approximate boundary condition on the surface of the sphere ($r = R$), we require that the solution of Eq. (3) coincide with the solution of this problem in the linear approximation, which can be written as follows:

$$u(r,y) = (U^2 y^2)/(1-M^2), \; r = R.$$

Now one can obtain the following solution with the help of the method of characteristics:

$$y = \frac{r}{C_0}\left[-1+\left(1+\frac{V}{C_0}\frac{1-M^2}{M^2}\right)^{\frac{1}{2}}\right]\left\{\frac{(\gamma+1)M^2}{1-M^2}\ln\frac{M}{1-M}\right.$$

$$\left.\times\left[-1+\left(1+\frac{V}{C_0}\frac{1-M^2}{M^2}\right)^{\frac{1}{2}}\right]+1\right\}, \quad (4)$$

where $V = \partial\varphi/\partial r$ is the hydrodynamic velocity.

One can see from Eq. (4) that the hydrodynamic velocity V is a multivalued function of y in region $y \approx 0$ (Fig. 1), which implies the existence of a shock

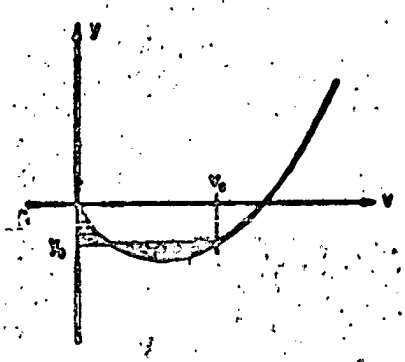

FIG. 1. *Author must supply caption.*

wave.[4] The position y_0 and the strength V_0 of the shock wave can be determined from the condition, which geometrically expresses the equality of the dotted areas in Fig. 1.[5] The calculation gives, in agreement

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with Lighthill[6] and Whitham,[7]

[6] L. D. Landau and E. M. Lifshits, *Mechanics of Continuous Media* (CTTI, Moskow, 1954); English transl. *Fluid Mechanics* (Addison-Wesley Publ. Co., Inc., Reading, Mass., 1959).
[7] M. J. Lighthill, Quart. J. Mech. Appl. Math. 1, 309 (1948).
[7] G. B. Whitham, J. Fluid Mech. 1, 303 (1956).

$$y_0 = -\frac{aV_0 r}{2}, \quad \frac{V_0}{C_0} = 2M^2 \frac{e^{-1}}{1+M} e^{-(1-M^2/M^2)(1/\gamma+1)}. \quad (5)$$

The graphs of the velocity distributions, which correspond to the solutions (4) and (5), are presented in Fig. 2 (curves 1,2,4) for $M = 0.203$, 0.523, and 0.638.

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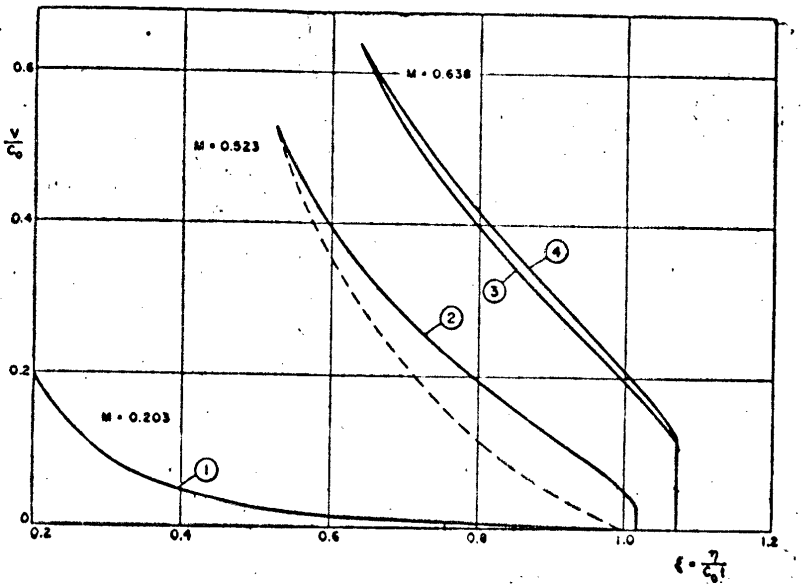

FIG. 2. *Author must supply caption.*

Along the abscissa $\xi = r/c_0 t$ is plotted and $\xi = M$ corresponds to the position of the sphere. The curves 1,2,3 on these Figures represent the exact solution of Taylor[2] for the same values of M, respectively. The solutions in the approximation of linear acoustics are represented by curve 1 and by a dotted line for $M = 0.203$ and $M = 0.523$, respectively. To the same approximation as before one can obtain the following expression for the pressure distribution in the compression wave:

$$\frac{p}{p_0} = 1 + \gamma\left\{\frac{2M^2}{1-M^2}\left[-1 + \left(1 + \frac{V}{C_0}\frac{1-M^2}{M^2}\right)^{\frac{1}{2}}\right] - \frac{1}{2}\left(\frac{V}{C_0}\right)^2\right\}, \quad (6)$$

where p_0 is the equilibrium pressure. The plots of the Eq. (6) are presented in Fig. 3 (curves 1,4,5) for

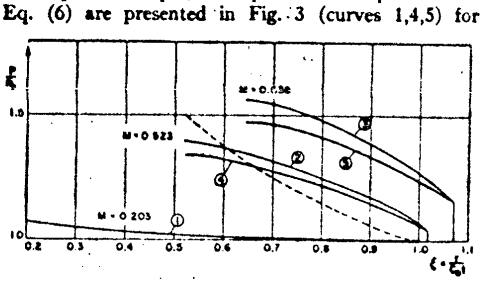

FIG. 3. *Author must supply caption*

$M = 0.203$, 0.523, and 0.638, respectively. The agreement between the exact solutions of Taylor (curves 1,2,3) and the approximate ones grows worse with decrease in ξ as a result of neglecting and second and third nonlinear terms in Eq. (2). The solution in the linear approximation is represented by curve 1 and by the dotted line for $M = 0.203$ and 0.523.

In conclusion, the author is grateful to Professor R. T. Beyer and Professor P. J. Westervelt for their stimulating discussions and help in performing this work.

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SPHERICAL COMPRESSION WAVE

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