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Next 5 Page(s) In Document Denied

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BREMSSTRAHLUNG AND PAIR PRODUCTION IN CONDENSED MEDIA AT HIGH ENERGIES

A. B. Migdal

The effect of multiple scattering on bremsstrahlung and pair production is considered. The probability of these processes considerably decreases at energies $\gtrsim 10^{13}$ ev.

The calculations are carried out with the aid of the density matrix. The formulae thus obtained yield the probability of pair production and bremsstrahlung for arbitrary electron and photon energies.

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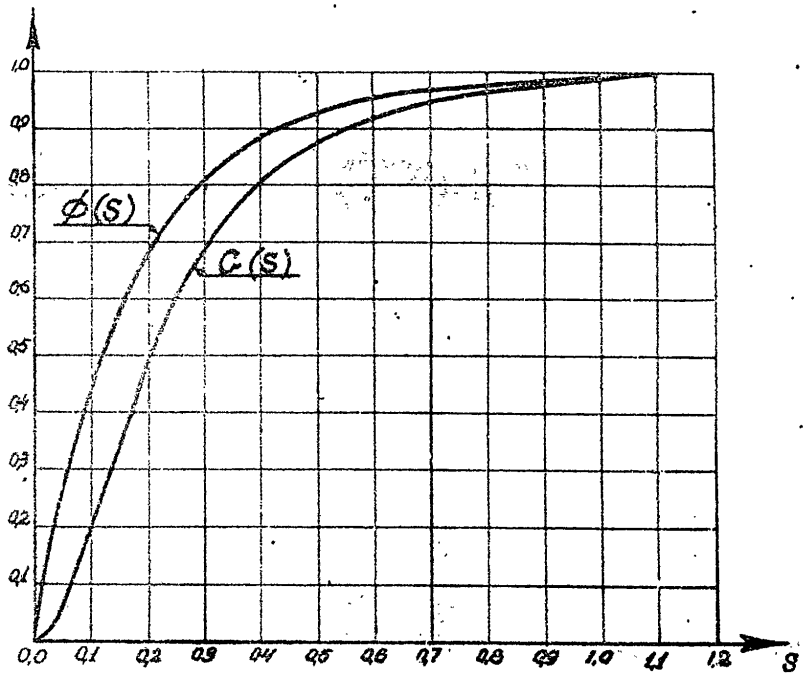

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1. Introduction.

At high energies when the directions of the particles participating in pair production and bremsstrahlung almost coincide large longitudinal distances begin to play an important role. Thus, if a photon of wavelength λ is emitted during bremsstrahlung a certain length $\ell \sim \frac{\lambda}{1 - v/c}$ is found to be essential, v - being the electron velocity. L. Landau and I. Pomeranchuk have shown that multiple scattering over this length leads to a significant decrease of the probability of the aforementioned processes[1,2].

An estimate of the cross sections for bremsstrahlung and pair production in the limiting case of ultra high energies $(\varepsilon \gg 10^{8}\,ev)$ is given in[2].

The entensity of emission of soft photons by electrons of arbitrary energy has been computed in[3]. In this paper the classical formula for intensity of emission by an electron moving along a given trajectory has been averaged over all possible trajectories. This procedure was carried out by means of the distribution function which was averaged over the positions of the atoms of the scattering medium and which satisfies the usual kinetic equation.

The aim of the present paper is the deduction of formulae for the probability of bremsstrahlung (formula 61) and pair production (Formula 63) per unit path in a condensed medium for arbitrary photon and electron energies. This is done by connecting the transition probability with

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1 L. Landau, I. Pomeranchuk, Dokl. Acad.Nauk S.S.S.R., v. 92, No. 3, p. 535, (1953) (In Russian);

2 L.Landau, I. Pomeranchuk, Dokl.Akad.Nauk S.S.S.R., v.92, No. 4, p. 735, (1953) (In Russian).

3 A. Migdal, Dokl. Akad.Nauk S.S.S.R., v. 96, No. 1, p.49, (1954) (In Russian).

50X1-HUM

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50X1-HUM

the density matrix and then using the equation for the density matrix averaged over the scattering atom coordinates deduced in [4,5]. At low energies formulas (61) and (63) transform into the Bethe-Heitler formula [6]; in the limiting case of ultra high energies the formulas confirm the estimation obtained in [2]. At photon energies much lower than that of the electron, formula (61) changes into the expression obtained in [3]. Finally, for every soft photons, when the deviation of the dielectric constant from unity is important, formula (56) of the present paper yields in the limiting case the same results as those presented in [7].

Formulae (61) and (63) can be used to construct a theory of shower production in condensed materials at energies exceeding 10^{13} ev.

II. Relation Between Transition Probability and Averaged Density Matrix.

The probabilities for bremsstrahlung and pair production must be averaged over all possible distributions of the atoms of the scattering material. We first express the radiation transition rate through the density matrix and then make use of the equation for the averaged density

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[4] A. Migdal, Dokl. Akad. Nauk, v. 105, No. 1, p. 77, (1955) (In Russian).

[5] A. Migdal, E. Polievktov-Nikoladze, Dokl. Akad. Nauk, v. 105, No. 2, p. 233, (1955) (In Russian).

[6] B. Rossi, K. Greisen "Cosmic Ray Theory," Rev. of Mod. Phys., v. 13, p. 240, (1941).

[7] Ter-Mikaelyan, Dokl. Akad. Nauk, v. 94, No. 6, p. 1033, (1954) (In Russian).

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50X1-HUM

matrix obtained in [illegible]

Restricting our treatment to the first approximation with respect to the electron - radiation field interaction and denoting the electron proper functions in the scattering medium by Ψ_s and the initial electron wave function by Ψ_0 we get

$$\dot{C}_s^{(1)} = \sum_{s'} (\Psi_s | e^{iHt} A e^{i\kappa t} e^{-iHt} | \Psi_{s'}) C_{s'}^0 = (\Psi_s | e^{iHt} A e^{i\kappa t} e^{-iHt} | \Psi_0)$$

where A is the radiative transition operator

$$A = \Sigma \vec{\varepsilon}_\nu e^{-i\vec{\kappa}\vec{r}} e \sqrt{\frac{2\pi}{\kappa}}$$

$\vec{\varepsilon}_\nu$ is the polarization vector, $\vec{\kappa}$ - the photon wave vector, H - the electron Hamiltonian which includes the potential of all the scatterers

$$H = H_0 + \sum_m V(\vec{r} - \vec{r}_m) \quad ; \qquad H\Psi_s = E_s \Psi_s$$

The $m = \hbar = c = 1$ unit system has been chosen. The electron and photon Ψ - functions have been normalised per unit volume.

The radiative transition rate is then given by

$$Q_s = \frac{d}{dt} |C_s|^2 = 2 \mathrm{Re}\, \dot{C}_s^0 C_s =$$

$$= 2 \mathrm{Re} \int_0^t (\Psi_0 | e^{iHt} A^+ e^{-iHt} | \Psi_s)(\Psi_s | e^{iHt} A e^{-iHt} | \Psi_0) e^{i\kappa(t-t_1)} dt_1 \quad (1)$$

We now determine the rate of transitions to all final states of the electron. For bremsstrahlung, which we first consider, one must sum over all values of s corresponding to positive electron energies. Indtoducing an energy sign operator $\mathcal{K} = \frac{H + |E(\vec{p})|}{2|E(\vec{p})|}$ where $\vec{p}$ is the electron momentum operator, and applying

50X1-HUM

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50X1-HUM

the $\sum_{s} \Psi_s^*(x)\, \Psi_s(x') = \delta(x - x')$

we obtain

$$Q = \sum_{E_s > 0} Q_s = 2\,\mathrm{Re}\int_0^t (\Psi_0 | e^{iHt_1} A^+ \mathcal{K} e^{iH(t-t_1)} A e^{-iHt} | \Psi_0)\, e^{i\varkappa(t-t_1)}\, dt_1 \qquad (2)$$

At high energies the operator $\mathcal{K}$ in (2), practically does not differ from the free motion electron operator $\mathcal{K}_0 = \frac{H_0 + |E_0^*|}{2|E_p^*|}$. Substituting $\mathcal{K}$ by $\mathcal{K}_0$ it will be seen that the coordinates of the scattering centers enter (2) only through a factors of the form $e^{\pm iHt}$.

We shall now show that it is possible to independently average over the scatterer coordinates entering the factors $e^{\pm iHt_1}$ and $e^{\pm iH(t-t_1)}$. Indeed, the nuclear coordinates in the first expression correspond to collisions which take place in a time interval $0 - t_1$ whereas in the second expression the coordinates correspond to scatterers which undergo collisions at a later period $t_1 - t$.

Suppose that a large number of collisions takes place during a time t_1; in this case after averaging over the first collisions the factors of the type $e^{\pm iHt_1}$ will practically cease to be dependent on the collisions taking place at times close to t_1 and this is just why independent averaging was found to be possible.

50X1-HUM

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50X1-HUM

We now write the integrand of (2) as the matrix element of the product of the operators in the representation of the electron free motion wave functions

$$\varphi_p^\lambda = u_p^\lambda e^{i\vec{p}\vec{r}}$$

Assuming $\Psi_0 = \varphi_{P_0}^{\lambda_0}$

($\vec{P}_0$ - is the initial electron momentum) and designating the average by the sign < > we obtain from (2)

$$\langle Q \rangle = 2\mathrm{Re}\int_0^t d\tau\, e^{i\kappa\tau} J \tag{3}$$

where

$$J = \langle (\Psi_0 | e^{iHt_1} A^+ \mathcal{K}_0 e^{iH\tau} A e^{-iH\tau} e^{-iHt_1} | \Psi_0) \rangle =$$

$$= \sum_{\substack{\vec{P}_1 \vec{P}_2 \\ \lambda_1 \lambda_2}} \langle (e^{iHt_1})^{\lambda_0\lambda_1}_{\vec{P}_0\vec{P}_1} (A^+ \mathcal{K}_0 e^{iH\tau} A e^{-iH\tau})^{\lambda_1\lambda_2}_{\vec{P}_1\vec{P}_2} (e^{-iHt_1})^{\lambda_2\lambda_0}_{\vec{P}_2\vec{P}_0} \rangle$$

and $\tau = t - t_1$. As the factors $e^{\pm iHt_1}$ and $e^{\pm iH\tau}$ are statistically independent we find

$$J = \sum_{\substack{\vec{P}_1 \vec{P}_2 \\ \lambda_1 \lambda_2}} \langle (e^{-iHt_1})^{\lambda_2\lambda_0}_{\vec{P}_2\vec{P}_0} (e^{iHt_1})^{\lambda_0\lambda_1}_{\vec{P}_0\vec{P}_1} \rangle \langle (A^+ \mathcal{K}_0 e^{iH\tau} A e^{-iH\tau})^{\lambda_1\lambda_2}_{\vec{P}_1\vec{P}_2} \rangle .$$

It should be noted that at high energies when small relative changes of the electron momentum are important, scattering does not change the spinor state, i e

$$(e^{\pm iHt})^{\lambda\lambda'}_{\vec{P}\vec{P}'} = \delta_{\lambda\lambda'} (e^{\pm iHt})^{\lambda\lambda}_{\vec{P}\vec{P}'}$$

with an error of the order $\frac{|\vec{P}' - \vec{P}|}{p}$ and in this case

50X1-HUM

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50X1-HUM

$$J = \sum_{\vec{p}\,\vec{p}_2} \langle (e^{-iHt_1})^{\lambda_0\lambda_0}_{\vec{p}_2\vec{p}_1} (e^{iHt_1})^{\lambda_0\lambda_0}_{\vec{p}_2\vec{p}_1} \rangle \langle (A^+ \mathcal{K}_0 e^{iH\tau} A e^{-iH\tau})^{\lambda_0\lambda_0}_{\vec{p}\vec{p}_2} \rangle \qquad (4)$$

The first factor in (4) as a function of the variables $\vec{p}_1, \vec{p}_2, t_1$ satisfies the same equation as the averaged density matrix

$$\langle \rho^{\lambda_0\lambda_0}_{\vec{p}_1\vec{p}_2} \rangle = \langle (e^{-iHt_1} \rho_0 e^{iHt})^{\lambda_0\lambda_0}_{\vec{p}_1\vec{p}_2} \rangle$$

It follows from this equation[4,5] that the difference $\vec{p}_2 - \vec{p}_1$ remains constant during scattering (this is a result of uniformity of the scattering medium). As the first factor in (4) equals

$$\delta_{\vec{p}_1,\vec{p}_0}\, \delta_{\vec{p}_2,\vec{p}_0} \qquad \text{for} \quad t_1 = 0$$

it may be written as follows

$$\langle (e^{-iHt_1})^{\lambda_0\lambda_0}_{\vec{p}_2\vec{p}_0} (e^{iHt_1})^{\lambda_0\lambda_0}_{\vec{p}_0\vec{p}_1} \rangle = f_0^{\lambda_0\lambda_0}(\vec{p}_1, t_1)\, \delta_{\vec{p}_2,\vec{p}_1} \qquad (5)$$

The second factor in (4) may be written as the sum of the operator products in the momentum representation. Using (5) and the expression for operator A we obtain

$$\langle (A^+ \mathcal{K}_0 e^{iH\tau} A e^{-iH\tau})^{\lambda_0\lambda_0}_{\vec{p}\vec{p}_1} \rangle = \frac{2\pi e^2}{K} \Big\langle \sum_{\substack{\vec{p}\lambda_1 \\ E^\lambda > 0}} (\alpha \vec{\mathcal{E}}_\nu)^{\lambda_0\lambda_1}_{\vec{p}_1, \vec{p}-\vec{k}} (e^{iH\tau})^{\lambda_1\lambda_1}_{\vec{p}-\vec{k}, \vec{p}-\frac{\vec{k}}{2}} (\alpha \vec{\mathcal{E}}_\nu)^{\lambda_1\lambda_0}_{\vec{p}-\frac{\vec{k}}{2}, \vec{p}+\frac{\vec{k}}{2}} (e^{-iH\tau})^{\lambda_1\lambda_0}_{\vec{p}+\frac{\vec{k}}{2}, \vec{p}_1} \Big\rangle$$

Here

$$(\alpha \vec{\mathcal{E}}_\nu)^{\mu_1\mu_2}_{\vec{g}_1\vec{g}_2} = (u^{\mu_1}_{\vec{g}_1}, \alpha \vec{\mathcal{E}}_\nu u^{\mu_2}_{\vec{g}_2}) \quad , \quad u^{\mu_{1,2}}_{\vec{g}_{1,2}} -$$

spinor functions.

Denote

$$(e^{-iH\tau})^{\lambda_1\lambda_1}_{\vec{p}-\frac{\vec{k}}{2}\,\vec{p}} (e^{iH\tau})^{\lambda_1\lambda_1}_{\vec{p}-\vec{k}\,\vec{p}-\frac{\vec{k}}{2}} = f_{\vec{k}}^{\lambda_1\lambda_1}(\vec{p}, \tau) \qquad (6)$$

50X1-HUM

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50X1-HUM

The equation coefficients and initial conditions (10) and (12) for $f_{\vec{k}}^{\lambda_0\lambda_1}(\vec{p},\tau)$ and $\tau_0^{\lambda_0\lambda_0}(\vec{p}_1,t_1)$ are independent of the spin orientation and therefore on summing over λ_0 λ_1 for a fixed energy sign one may drop the spinor indices in these functions.

Inserting (5) and (6) in (4), summing over the photon polarization and averaging over the initial state spins we get

$$J_1 = \frac{1}{2}\sum_{\substack{\lambda_0,\nu \\ E^{\lambda_0}>0}} J = \frac{\pi e^2}{k}\int \mathcal{L}(\vec{p}_1,\vec{p}) f_0(\vec{p}_1,t_1) f_{\vec{k}}(\vec{p},\tau)\frac{d\vec{p}_1}{(2\pi)^3}\frac{d\vec{p}}{(2\pi)^3} \qquad (7)$$

where

$$\mathcal{L}(\vec{p}_1,\vec{p}) = \sum_{\substack{\lambda_0\lambda_1,\nu \\ E^{\lambda_0},E^{\lambda_1}>0}} (\mathcal{L}\vec{\varepsilon}_\nu)^{\lambda_0\lambda_1}_{\vec{p}_0,\vec{p}_1-\vec{k}} (\mathcal{L}\vec{\varepsilon}_\nu)^{\lambda_1\lambda_0}_{\vec{p}-\frac{\vec{k}}{2},\vec{p}+\frac{\vec{k}}{2}} \qquad (8)$$

It may be noted that the quantity

$$\rho^{\lambda_0\lambda_1}_{\vec{p}+\frac{\vec{k}}{2},\vec{p}-\frac{\vec{k}}{2}} = \left(e^{-iH\tau}\right)^{\lambda_0\lambda_0}_{\vec{p}+\frac{\vec{k}}{2},\vec{p}_1}\left(e^{iH\tau}\right)^{\lambda_1\lambda_1}_{\vec{p}_1-\vec{k},\vec{p}-\frac{\vec{k}}{2}} = \left(e^{-iH\tau}\rho_0 e^{iH\tau}\right)^{\lambda_0\lambda_1}_{\vec{p}+\frac{\vec{k}}{2},\vec{p}-\frac{\vec{k}}{2}}$$

satisfies the equation $\frac{\partial\rho}{\partial\tau} = -i[H,\rho]$

Moreover, $Sp\,\rho = \sum_{g_1 g_2} \rho^{g_1 g_2}_{g_1 g_2} = \sum_{p\lambda_1}\left(e^{iH\tau}\right)^{\lambda_0\lambda_1}_{\vec{p}_1,\vec{p}}\left(e^{-iH\tau}\right)^{\lambda_1\lambda_0}_{\vec{p},\vec{p}_1} = 1$

i.e. ρ — is an element of the density matrix in the momentum representation. We shall call the quantity $f_{\vec{k}}(\vec{p},\tau)$ the averaged density matrix.

50X1-HUM

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50X1-HUM

The problem of averaging the transition rate reduces to determination of the averaged density matrix and to evaluation of sum (8) and integral (7).

III. Equation for Averaged Density Matrix

As was shown in[4] the averaged density matrix satisfies the equation

$$\frac{\partial f_{\vec{\kappa}}^{\lambda_0\lambda_1}(\vec{p},\tau)}{\partial \tau} + i\left(E^{\lambda_0}_{\vec{p}+\frac{\vec{\kappa}}{2}} - E^{\lambda_1}_{\vec{p}-\frac{\vec{\kappa}}{2}}\right) f_{\vec{\kappa}}^{\lambda_0\lambda_1}(\vec{p},\tau) =$$

$$= n\pi \int \frac{d\vec{p}'}{(2\pi)^3} \left| v_{\vec{p}'-\vec{p}} \right|^2 \left\{ \delta\left(E^{\lambda_0}_{\vec{p}'+\frac{\vec{\kappa}}{2}} - E^{\lambda_0}_{\vec{p}+\frac{\vec{\kappa}}{2}}\right) + \delta\left(E^{\lambda_1}_{\vec{p}'-\frac{\vec{\kappa}}{2}} - E^{\lambda_1}_{\vec{p}-\frac{\vec{\kappa}}{2}}\right) \right\} \left[f_{\vec{\kappa}}^{\lambda_0\lambda_1}(\vec{p}',\tau) - f_{\vec{\kappa}}^{\lambda_0\lambda_1}(\vec{p},\tau) \right] \qquad (9)$$

together with the initial condition which follows from the definition of $f_{\vec{\kappa}}(\vec{p},\tau)$

$$f_{\vec{\kappa}}^{\lambda_0\lambda_1}(\vec{p},\tau)\Big|_{\tau=0} = \delta_{\vec{p},\vec{p}-\frac{\vec{\kappa}}{2}} \qquad (10)$$

This equation differs from the classical kinetic equation in that the difference $E^{\lambda_0}_{\vec{p}+\frac{\vec{\kappa}}{2}} - E^{\lambda_1}_{\vec{p}-\frac{\vec{\kappa}}{2}}$ enters the left hand side instead of $\frac{\partial E}{\partial \vec{p}}\vec{\kappa}$ and the collision term is the half sum of the usual collision terms for the momentum $\vec{p}+\frac{\vec{\kappa}}{2}$ energy $E^{\lambda_0}_{\vec{p}+\frac{\vec{\kappa}}{2}}$ and for the momentum $\vec{p}-\frac{\vec{\kappa}}{2}$ and energy $E^{\lambda_1}_{\vec{p}-\frac{\vec{\kappa}}{2}}$

For $\kappa \ll p$ (9) changes into the classical

50X1-HUM

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50X1-HUM

kinetic equation for the $\vec{k}^{th}$ Fourier component of the distribution function. The function $f_0(\vec{p}, t)$ satisfies the equation

$$\frac{\partial f_0^{\lambda_0\lambda_0}(\vec{p}_1, t_1)}{\partial t_1} = 2\pi n \int \frac{d\vec{p}'}{(2\pi)^3} |V_{\vec{p}-\vec{p}'}|^2 \delta(E^{\lambda_0}_{\vec{p}} - E^{\lambda_0}_{\vec{p}'}) \left[f_0^{\lambda_0\lambda_0}(\vec{p}', t_1) - f_0^{\lambda_0\lambda_0}(\vec{p}_1, t_1) \right] \quad (11)$$

and initial condition

$$f_0^{\lambda_0\lambda_0}(\vec{p}_1, t_1)\Big|_{t=0} = \delta_{\vec{p}_1, \vec{p}_0} \quad (12)$$

From (11) and (12) we find

$$\int f_0^{\lambda_0\lambda_0}(\vec{p}_1, t_1) \frac{d\vec{p}_1}{(2\pi)^3} = 1 \quad (13)$$

It follows from (11) and (12) that $f_0^{\lambda_0\lambda_0}$ differs from zero only for $p_1 = p_0$ and therefore a function $v_0(\vec{\theta}, t_1)$ can be introduced, in accord with the formula

$$f_0^{\lambda_0\lambda_0}(\vec{p}_1, t_1) \frac{d\vec{p}_1}{(2\pi)^3} = \delta(p_1 - p_0)\, v_0(\vec{\theta}, t_1)\, dp_1\, d\vec{\theta}$$

$$v_0(\vec{\theta}, t_1)\Big|_{t_1=0} = \delta(\vec{\theta}) \quad (14)$$

where $\vec{\theta} = \frac{\vec{p}_1}{p_0} - \frac{\vec{p}_0}{p_0}$ is the vector of the angle between $\vec{p}_0$ and $\vec{p}_1$

From (13) we obtain

$$\int v_0(\vec{\theta}, t_1)\, d\vec{\theta} = 1 \quad (15)$$

50X1-HUM

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50X1-HUM

It is easy to see that $f_{\vec{k}}(\vec{p}, \tau)$ differs from zero only for values of p which are close to $g = p_0 - \frac{k}{2}$. For $\tau = 0$ this follows from (10):

$$D^2 = p_0^2 + \frac{k^2}{4} - p_0 k + p_0 k (1 - \cos \widehat{\vec{p}_1 \vec{k}}) = g^2 (1 - \frac{p_0 k}{g} \eta^2)$$

where η is a small angle between $\vec{p}_1$ and $\vec{k}$

We now introduce vectors of the angles between $\vec{p}$ and $\vec{k}$ and between $\vec{p}'$ and $\vec{k}$

$$\vec{\eta} = \frac{\vec{p}_\perp}{g} \qquad \vec{\eta}' = \frac{\vec{p}_\perp'}{g} \qquad g = p_0 - \frac{k}{2} \tag{15}$$

Here $\vec{p}_\perp$, $\vec{p}_\perp'$ designates the projection of $\vec{p}, \vec{p}'$ on the plane perpendicular to $\vec{k}$ The δ-functions on the right hand part of (9) may be rewritten as follows

$$\delta(E_{\vec{p}' \pm \frac{\vec{k}}{2}} - E_{\vec{p} \pm \frac{\vec{k}}{2}}) \cong \delta(p' - p \pm \frac{gk(\eta'^2 - \eta^2)}{2(g \pm \frac{k}{2})}) \cong \delta(p' - p)$$

Thus during collisions the modulus of $\vec{p}$ remains approximately constant, with an accuracy $\frac{\delta p}{p} \sim \eta^2$

One may use the approximate constancy of p to determine the function $v(\vec{\eta}, \tau)$ from the formula

$$f_{\vec{k}}(\vec{p}, \tau) \frac{d\vec{p}}{(2\pi)^3} = \delta(p - g)\, v(\vec{\eta}, \tau)\, dp\, d\vec{\eta} \tag{16}$$

From condition (10) we obtain

$$v(\vec{\eta}, \tau)\Big|_{\tau=0} = \delta(\vec{\eta} - \vec{\eta}_0), \quad \vec{\eta}_0 = \frac{(\vec{p}_1 - \frac{\vec{k}}{2})_\perp}{g} = \frac{\vec{p}_{1\perp}}{g} \tag{17}$$

50X1-HUM

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50X1-HUM

where $\vec{\eta}_0$ is the vector of the angle between $\vec{p}_0 - \frac{\vec{k}}{2}$ and $\vec{k}$. Vector $\vec{\eta}_0$ is related to vector $\vec{\vartheta}$ introduced above by the relation

$$\vec{\vartheta} = \frac{\vec{p}}{p} - \frac{\vec{p}_0}{p_0} = \frac{\vec{p}}{p_0} - \vec{n} + \vec{n} - \frac{\vec{p}_0}{p_0} = \frac{\vec{p}_\perp}{p_0} + \vec{\vartheta} = \frac{g}{p_0}\vec{\eta}_0 + \vec{\vartheta} \qquad (18)$$

The vector of the angle between $\vec{n} = \frac{\vec{k}}{k}$ and the initial direction of the electron has been denoted by $\vec{\vartheta}$

From definition (15) we obtain.

$$\vec{p} = (\vec{p}\vec{n})\vec{n} + \vec{p}_\perp \cong p_0\vec{n} + g\vec{\eta} \quad \vec{p} + \frac{\vec{k}}{2} \cong p_0\vec{n} + g\vec{\eta}, \quad \vec{p} - \frac{\vec{k}}{2} \cong (p_0 - k)\vec{n} + g\vec{\eta} \qquad (19)$$

The difference $E^{\lambda_0}_{\vec{p}+\frac{\vec{k}}{2}} - E^{\lambda_0}_{\vec{p}-\frac{\vec{k}}{2}}$ in (9) takes the form

$$E^{\lambda_0}_{\vec{p}+\frac{\vec{k}}{2}} - E^{\lambda_0}_{\vec{p}-\frac{\vec{k}}{2}} = \sqrt{1 + (p_0\vec{n} + g\vec{\eta})^2} - \sqrt{1 + [(p_0 - k)\vec{n} + g\vec{\eta}]^2} =$$

$$= k\left[1 - \frac{1}{2p_0(p_0 - k)} - \frac{g^2}{2p_0(p_0 - k)}\eta^2\right] \qquad (20)$$

Using (16) and integrating (9) over Δ we get

$$\frac{\partial v(\vec{\eta}, \tau)}{\partial \tau} + \left(a - b\frac{\eta^2}{2}\right) v(\vec{\eta}, \tau) =$$

$$= \frac{n g^2}{(2\pi)^2} \int |V_{g(\vec{\eta}' - \vec{\eta})}|^2 \{ v(\vec{\eta}', \tau) - v(\vec{\eta}, \tau) \} d\vec{\eta}' \qquad (21)$$

where

$$a = k\left(1 - \frac{1}{2p_0(p_0 - k)}\right), \quad b = \frac{g^2 k}{p_0(p_0 - k)}, \quad \vec{\eta}' = \frac{\vec{p}_\perp'}{g} \qquad (21')$$

For V_g we accept the expression

$$V_g = \frac{4\pi Z e^2}{g^2 + \varkappa^2} \qquad \varkappa \sim \frac{1}{a} \qquad (22)$$

50X1-HUM

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Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

50X1-HUM

. a is the Thomas-Fermi radius $a \sim \frac{3\gamma}{Z^{1/3}}$

Inserting in (21) we get

$$\frac{\partial v}{\partial \tau} + (a - b\frac{\eta^2}{2})v = \frac{4nZ^2e^4}{g^2}\int \frac{d\bar{\eta}}{[(\bar{\eta}'-\bar{\eta})^2+\theta_1^2]^{1/2}}[v(\bar{\eta}',\tau) - v(\bar{\eta},\tau)] \quad (23)$$

$$\theta_1 = \frac{\varkappa}{g}$$

Expanding $v(\bar{\eta}',\tau)$ into a power series of $\bar{\eta}'-\bar{\eta}$ we obtain from (23) the Fokker-Planck differential equation

$$\frac{\partial v}{\partial \tau} + (a - b\frac{\eta^2}{2})v = q\,\Delta_\eta v$$
$$q = \frac{2\pi n Z^2 e^4}{g^2}\ln\frac{\theta_2}{\theta_1} = \frac{3}{g^2} \quad (24)$$

The quantity θ_2 may be determined from the condition of applicability of the Fokker-Planck expansion. The first term of the series expansion is

$$\int_{\theta_1}^{\theta_2} \frac{\sin\theta}{\theta^4}\,\theta^2\frac{}{4}\Delta_\eta v = \frac{}{4}\ln\frac{\theta_2}{\theta_1}\Delta_\eta v$$

The next term is of the order

$$\int_{\theta_1}^{\theta_2} \frac{\theta d\theta}{\theta^4}\,\theta^4\frac{\partial^4 v}{\partial \eta^4} \sim \theta^2\frac{}{\eta^2}\Delta_\eta v$$

The quantities $\int_0^\infty v e^{i\kappa\tau}d\tau = \int_0^\infty v' d\tau$, $v' = e^{i\kappa\tau}v$ will participate in further calculations. The equation for v' can be obtained from equation (24) by replacing a by $a' = a - \kappa$

The significant values of η^2 are determined by the relation

$$(a - b\frac{\eta^2}{2})v \sim b\eta^2 v \sim q\,\Delta_\eta v \qquad \eta^2 \sim \sqrt{\frac{q}{b}} \quad (25)$$

50X1-HUM

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Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

50X1-HUM

This estimation will be confirmed below.

Thus, the condition for expansion of v into a series has the form

$$\vartheta_2^2 \sqrt{\frac{\varepsilon}{q}} \sim \ln \frac{\theta_2}{\theta_1}, \quad \vartheta_2 \sim \sqrt[4]{\frac{q}{\varepsilon}}\, L^{1/2}; \quad L = \ln \frac{\theta_2}{\theta_1} \tag{26}$$

At sufficiently high energies θ_2 may be of the same order of magnitude as the angle of diffraction by the nucleus which is equal to $\frac{1}{qR}$ and in this case the upper limit of integration with respect to $\vec{\eta} - \vec{\eta}'$ is determined by the quantity $\frac{1}{qR}$ Puting $R \cong 0.5\, r_0 Z^{1/3}$ we obtain for $\theta_2 > \frac{1}{qR}$

$$L = \ln \frac{37^2}{0.5 Z^{2/3}} = 2 \ln \frac{190}{Z^{1/3}} \tag{27}$$

IV. Summation Over Election Spin and Photon Polarization

It is not possible to carry out the summation over λ_0 and λ_1 in expression (8) in the usual manner as the momentum subscripts in the spinor functions are different.

Summation in (8) can be reduced to determination of the trace of two-row matrices. The spinor functions are taken in the form

$$u_g^{\mu} = \begin{Bmatrix} v_\mu \\ \frac{\vec{\sigma}\vec{g}}{E_g+1} v_\mu \end{Bmatrix} N_g; \quad v_1 = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}, \quad v_2 = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}; \quad N_g^2 = \frac{1}{1 + \frac{g^2}{(E_g+1)^2}} \cong \frac{1}{2} \tag{28}$$

50X1-HUM

Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

50X1-HUM

$\sigma_1, \sigma_2, \sigma_3$ are the Pauli matrices. The $3'$ axis is oriented along $\vec{n}$

Substitution in (8) gives

$$\mathcal{L}(\vec{p}, \vec{p}') = \frac{1}{4}\sum_{i=1,2} Sp\left\{\left[\frac{\sigma_i \vec{\sigma}(\vec{p}-\vec{k})}{E_{\vec{p}-\vec{k}}+1} + \frac{\vec{\sigma}\vec{p}\sigma_i}{E_p+1}\right]\left[\frac{\sigma_i \vec{\sigma}(\vec{p}'+\frac{\vec{k}}{2})}{E_{\vec{p}'+\frac{\vec{k}}{2}}+1} + \frac{\vec{\sigma}(\vec{p}'-\frac{\vec{k}}{2})\sigma_i}{E_{\vec{p}'-\frac{\vec{k}}{2}}+1}\right]\right\} \quad (29)$$

We introduce the notation

$$\vec{A} = \frac{\vec{p}}{E_p+1}; \quad \vec{B} = \frac{\vec{p}-\vec{k}}{E_{\vec{p}-\vec{k}}+1}; \quad \vec{C} = \frac{\vec{p}'+\frac{\vec{k}}{2}}{E_{\vec{p}'+\frac{\vec{k}}{2}}+1}; \quad \vec{D} = \frac{\vec{p}'-\frac{\vec{k}}{2}}{E_{\vec{p}'-\frac{\vec{k}}{2}}+1} \quad (30)$$

From (29) we then get

$$\mathcal{L}(\vec{p}, \vec{p}') = \frac{1}{4}\sum_{i=1,2} Sp(\sigma_i \vec{B}\vec{\sigma} + \vec{A}\vec{\sigma}\sigma_i)(\sigma_i \vec{\sigma}\vec{C} + \vec{D}\vec{\sigma}\sigma_i) =$$
$$= \vec{B}\vec{D} + \vec{A}\vec{C} - (\vec{B}\vec{n})(\vec{C}\vec{n}) - (\vec{A}\vec{n})(\vec{D}\vec{n}) \quad (31)$$

Each of the terms in (31) is close to unity. However, as the further calculations show the complete expression is of the order η^2 We now express (31) as a sum of small terms in each of which only the first term of the expansion in powers of $\frac{1}{p}$ is retained.

Each of the vectors $\vec{A}, \vec{B}, \vec{C}, \vec{D}$ can be expressed as the sum of two terms, one of these being parallel and the other perpendicular to $\vec{n}$. $\vec{A} = \vec{A}_1 + \vec{A}_2$; $\vec{A}_1 \parallel \vec{n}$, $\vec{A}_2 \perp \vec{n}$ and similarly for $\vec{B}, \vec{C}, \vec{D}$

From (31) we then obtain

$$\mathcal{L} = (D_1 - C_1)(B_1 - A_1) + \vec{B}_2\vec{D}_2 + \vec{A}_2\vec{C}_2 \quad (32)$$

50X1-HUM

Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

50X1-HUM

The magnitude of each term in (32) is the order

$$\sim \frac{1}{p_0^2} \quad \text{or} \quad \eta^2$$

Using (17), (19) and (30) we find with the required accuracy

$$A_1 = C_1 = 1 - \frac{1}{p_0} \qquad B_1 = D_1 = 1 - \frac{1}{p_0 - \kappa}$$

$$\vec{A}_2 = \frac{\vartheta \vec{\eta}_0}{p_0}, \quad \vec{B}_2 = \frac{\vartheta \vec{\eta}_0}{p_0 - \kappa}, \quad \vec{C}_2 = \frac{\vartheta \vec{\eta}}{p_0}, \quad \vec{D}_2 = \frac{\vartheta \vec{\eta}}{p_0 - \kappa} \tag{33}$$

Insertion in (32) gives

$$\mathcal{L} = \mathcal{K}_1 + \mathcal{K}_2 \vec{\eta} \vec{\eta}_0$$

$$\mathcal{K}_2 = \frac{\kappa^2}{p_0^2 (p_0 - \kappa)^2}, \quad \mathcal{K}_1 = \frac{\vartheta^2 [p_0^2 + (p_0 - \kappa)^2]}{p_0^2 (p_0 - \kappa)^2} \tag{34}$$

V. Probability of Bremsstrahlung

Let $W_2(p, \kappa)\, d\kappa$ designate the probability of emission per unit length of a photon having an energy lying between κ and $\kappa + d\kappa$ The initial election Ψ-function is normalized per unit volume or, for $c = 1$ per unit flux and thus from formulae (3) and (7) we obtain

$$W_2 = \frac{1}{2} \sum_{\substack{\lambda_0, \nu \\ E^{\lambda_0} > 0}} \int \langle Q \rangle d\vec{\vartheta} \frac{\kappa^2}{(2\pi)^3} = \frac{\kappa^2}{(2\pi)^3} 2 \mathrm{Re} \int_0^t d\tau e^{i\kappa\tau} \int \mathcal{J}_1 d\vec{\vartheta} \tag{35}$$

Inserting expressions for U_0, U and $\mathcal{L}$ defined by formulae (14), (16) and (34), into (7) we obtain

$$\int \mathcal{J}_1 d\vec{\vartheta} = \frac{\pi e^2}{\kappa} \int U_0(\vec{\theta}, t) U(\vec{\eta}, \tau) [\mathcal{K}_1 + \mathcal{K}_2 \vec{\eta} \vec{\eta}_0] d\vec{\theta}\, d\vec{\eta}\, d\vec{\vartheta}$$

Expressing $\vec{\theta}$ through $\vec{\eta}_0$ and $\vec{\vartheta}$ by formula (18) we get

$$\int \mathcal{J} d\vec{\vartheta} = \frac{\pi e^2}{\kappa} \int U_0\left(\frac{\vartheta}{p_0} \vec{\eta}_0 + \vec{\vartheta}, t\right) U(\vec{\eta}, \tau) (\mathcal{K}_1 + \mathcal{K}_2 \vec{\eta} \vec{\eta}_0) \frac{\vartheta^2}{p_0^2} d\vec{\eta}_0\, d\vec{\eta}\, d\vec{\vartheta}$$

50X1-HUM

Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

50X1-HUM

Using normalization condition (14') ρ_{0z} v_0 we obtain

$$W_z = \frac{e^2 q^2 \kappa}{(2\pi)^2 p_0^2} \operatorname{Re} \int_0^t d\tau e^{i\kappa\tau} \int (\mathcal{K}_1 + \mathcal{K}_2 \bar{\eta} \bar{\eta}_0) v(\bar{\eta}, \tau) d\bar{\eta} d\bar{\eta}_0 \qquad (36)$$

We denote the singificant value of τ in this integral by τ_0 From (24) and (25) we obtain the estimations.

$$\eta^4 \sim \frac{q}{b}, \quad \tau_0 \sim \frac{1}{b p^2} \sim \frac{1}{\sqrt{bq}} \qquad \eta^2 \sim q \tau_0 \qquad (37)$$

If the time of movement of the election in the medium, t is much greater than τ_0 the upper limit of the integral in (35) with respect to τ can be replaced by infinity and W_z ceases to depend on t.

Only the case $t \gg \tau_0$ or $l \gg l_\kappa$ will be considered below; l is the thickness of the scattering material and $l_\kappa = c\tau_0$

For a condensed medium $(n = 3 \cdot 10^{22} cm^{-3})$ one obtaine from (37).

$$l_\kappa \sim \frac{p_0}{\sqrt{\kappa}} \frac{1}{Z} 10^{-5} cm \qquad (38)$$

For $Z = 10$, $E_0 = 10^{16} ev$ $(p_0 = \frac{10^{16}}{5 \cdot 10^5} = 2 \cdot 10^{10})$, $\kappa = \frac{1}{2} p_0$ one obtains $l_\kappa \sim 0{,}2$ cm

For $t \gg \tau_0$ the angular destribution of the photons can easily be found. The width of the angular distribution

50X1-HUM

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Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

50X1-HUM

defined by the function $v_0(\vec{\theta}, t_1)$ is of the order $\overline{\theta^2} \sim q t_1$ However, $\eta^2 \sim \eta_0^2 \sim q\tau$ and therefore the function $v_0(\frac{q}{\rho_0}\vec{\eta}_0 + \vec{\vartheta}, t_1)$ can be substituted by $v_0(\vec{\vartheta}, t)$ and the photon angular distribution is given by the expression

$$W_2'(\rho_0, \kappa, \vec{\vartheta})d\vec{\vartheta} = v_0(\vec{\vartheta}, t)\, W_2(\rho_0, \kappa)d\vec{\vartheta} \qquad (39)$$

Thus, the photon angular distribution is the same as that for multiple seattered elections with an energy ρ_0

Denote

$$\int v(\vec{\eta}, \tau; \vec{\eta}_0)d\vec{\eta}_0 = h_1(\vec{\eta}, \tau); \quad \int v(\vec{\eta}, \tau; \vec{\eta}_0)\vec{\eta}_0 d\vec{\eta}_0 = \vec{R}(\vec{\eta}, \tau)$$

As the coefficients in equation (24) do not contain $\vec{\eta}_0$ the equations for h_1 and $\vec{R}$ will coincide with (24) and the initial conditions $h_1(\vec{\eta}, \tau)\big|_{\tau=0} = 1$; $\vec{R}(\vec{\eta}, \tau)\big|_{\tau=0} = \vec{\eta}$ directly obtain from (17).

The coefficients in equation (24) contain only η^2 and the solution can therefire be written in the form $h_1(\vec{\eta}, \tau) = h(z, \tau)$; $\vec{R}(\vec{\eta}, \tau) = \vec{\eta}\, g(z, \tau)$ where $z = \frac{\eta^2}{2}$; h, g satisfies the equations

$$\frac{\partial h}{\partial \tau} + i(a - bz)h = 2zq\left(\frac{\partial^2 h}{\partial z^2} + \frac{1}{z}\frac{\partial h}{\partial z}\right)$$
$$\frac{\partial g}{\partial \tau} + i(a - bz)g = 2zq\left(\frac{\partial^2 g}{\partial z^2} + \frac{2}{z}\frac{\partial g}{\partial z}\right) \qquad (40)$$

We introduce the functions

$$\varphi_1(z) = \int_0^\infty e^{i\kappa\tau} h(z, \tau)d\tau$$
$$\varphi_2(z) = z\int_0^\infty e^{i\kappa\tau} g(z, \tau)d\tau \qquad (41)$$

50X1-HUM

Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

50X1-HUM

Then according to (36)

$$W_2 = \frac{e^2 \kappa q^2}{2\pi \rho_0^2} \mathrm{Re} \left\{ \mathcal{K}_1 \int_0^\infty \varphi_1 dz + 2\mathcal{K}_2 \int_0^\infty \varphi_2 dz \right\} \tag{42}$$

The equations for φ_1 and φ_2 can be obtained by integrating (40) over τ and using the initial conditions for i and q

$$z\varphi_1'' + \varphi_1' + i(\alpha + \beta z)\varphi_1 = -\frac{1}{2q} \tag{43}$$

$$z\varphi_2'' + i(\alpha + \beta z)\varphi_2 = -\frac{z}{2q} \tag{44}$$

Where

$$\alpha = \frac{\kappa - a}{2q} = \frac{\kappa}{4\rho_0(\rho_0 - \kappa)q} > 0, \quad \beta = \frac{\kappa q^2}{2\rho_0(\rho_0 - \kappa)q} > 0 \tag{45}$$

Solution of equation (43) and (44) and computation of the integrals in (42) are carried out in the appendix.

The following results are obtained

$$\mathrm{Re} \int_0^\infty \varphi_1 dz = \frac{1}{12 q \alpha^2} G(s), \quad \mathrm{Re} \int_0^\infty \varphi_2 dz = \frac{1}{6\alpha\beta q} \Phi(s)$$

$$G(s) = 48 s^2 \left(\frac{\pi}{4} - \frac{1}{2} \int_0^\infty e^{-st} \frac{\sin st}{\mathrm{sh}\, \frac{t}{2}} dt \right) \tag{46}$$

$$\Phi(s) = 12 s^2 \int_0^\infty \mathrm{cth}\, \frac{x}{2}\, e^{-sx} \sin sx\, dx - 6\pi s^2$$

$$s = \frac{1}{3q} \sqrt{\frac{\kappa}{\rho_0(\rho_0 - \kappa)q}} \tag{47}$$

Function $\Phi(s)$ was introduced in[3].

The values of $\Phi(s)$ and $G(s)$ are presented in the

50X1-HUM

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Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

50X1-HUM

table. The plot of these functions is shown in fig.1.

s	$\Phi(s)$	$G(s)$
0	0	0
0,05	0,258	0,094
0,1	0,446	0,206
0,2	0,686	0,475
0,3	0,805	0,695
0,4	0,880	0,800
0,5	0,931	0,875
0,6	0,954	0,917
0,7	0,965	0,945
0,8	0,975	0,963
0,9	0,985	0,975
1,0	0,990	0,985
1,5	0,998	0,994
2	0,999	0,998

The asymptotic behavior of Φ and G is given by the formulae

$$\Phi \underset{s\to 0}{\to} 6s \quad , \quad \Phi \underset{s\to\infty}{\to} 1 - \frac{0{,}012}{s^4}$$
$$G \underset{s\to 0}{\to} 12\pi s^2 \quad , \quad G \underset{s\to\infty}{\to} 1 - \frac{0{,}022}{s^4} \qquad (48)$$

For $s \lesssim 1$ one obtains $\eta_0^2 \sim \eta^2 \sim \frac{Re\int \varphi_0 dz}{Re\int \varphi_1 dz} \sim \frac{\alpha}{\beta s} \sim \frac{1}{\sqrt{\beta}} = \sqrt{\frac{2q}{b}}$ which confirms the estimation of the significant values of η^2, η_0^2 given above.

Substitution of (24), (34), (45) and (46) into (42) yields

50X1-HUM

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Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

50X1-HUM

$$W_2 = \frac{e^2 \kappa \nu^2}{2\pi \rho_0^2}\left\{\mathcal{K}_1 \frac{G}{12 q \alpha^2} + 2\mathcal{K}_2 \frac{\Phi}{64 \beta q}\right\} =$$

$$= \frac{2e^2}{3\pi \rho_0^2 \kappa} B\left\{\kappa^2 G(s) + 2\left[\rho_0^2 + (\rho - \kappa)^2\right]\Phi(s)\right\}, \quad B = 2\pi Z^2 e^4 \ln \frac{\theta_2}{\theta_1} \qquad (49)$$

Fig 1

Estimation (26) for θ_2 can be expressed in a more convenient form

$$\theta_2 \sim \sqrt[4]{\frac{q}{6}}\, \nu^{1/2} = \sqrt[4]{\frac{1}{2^3}}\, \nu^{1/2} \sim \frac{L^{1/2}}{q s^{1/2}} \qquad (50)$$

Put $s \gtrsim 1$ then

$$W_2 = \frac{2e^2}{3\pi \rho_0^2 \kappa} B\left\{\kappa^2 + 2\left[\rho_0^2 + (\rho - \kappa)^2\right]\right\}$$

This expression differs from the familiar formula (see, for example [6]) only by a factor of the order of unity under the logarithm sign.

Uncertainty of the factor under the logarithm sign is a result of application of the Fokker-Planck method.

Solution of integral equation (23), a difficult task, should yield more precise formulae.

As functions Φ and G are close to unity for $s = 1$ a convenient formula can be obtained by defining the numerical factor under the logarithm sign in the following manner

$$L = \ln \frac{\theta_2}{\theta_1} = \ln \frac{190}{2^{1/2} s^{1/2}} \qquad (51)$$

In this case W_2 is defined for $s \leq 1$ and for $s = 1$ it coincides with the usual expression.

For $s \ll 1$ (48) and (49) yield

50X1-HUM

Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

50X1-HUM

$$W_2 = \frac{8e^2}{\pi p_0^2 \kappa} B s \left[p_0^2 + (p_0-\kappa)^2\right] = \frac{e^2}{\pi p_0^2} \frac{B}{\sqrt{\kappa p_0 (p_0-\kappa)}} \left[p_0^2 + (p_0-\kappa)^2\right] \qquad (52)$$

In this case the emission probability is proportional

$$W_2 = \frac{8e^2}{3\kappa} B \cos(s) \qquad (53)$$

At very small photon energies the deviation of the dielectric constant from unity must be taken into account.

The dielectric constant ε enters the initial formulae through normalization of operator A and also enters the integral over τ in (3), where $e^{i\kappa\tau}$ should be replaced by $e^{i\omega\tau}$, $\omega = \frac{\kappa}{\sqrt{\varepsilon}}$

By considering the frequencies $\omega \gg \omega_a = \sqrt{4\pi n Z e^2}$ we obtains

$$\varepsilon \cong 1 - \frac{\omega_a^2}{\omega^2}, \quad \omega = \frac{\kappa}{\sqrt{\varepsilon}} \cong \kappa\left(1 + \frac{1}{2}\frac{\omega_a^2}{\omega^2}\right) \qquad (54)$$

Account of ε in the normalization factor of A is equivalent to multiplication of W_2 by a factor which is close to unity and which can therefore be dropped.

Thus, the effect of the dielectric constant on the calculations is equivalent to substitution of the quantity α defined in (45) by α'

$$\alpha' = \frac{\omega - a}{2q} = \alpha + \frac{\omega - \kappa}{2q} \cong \alpha\left(1 + p_0^2 \frac{\omega_a^2}{\omega^2}\right) \qquad (55)$$

50X1-HUM

Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

50X1-HUM

Using (49) and (47) one obtains for small κ instead of (53) a more general formula

$$W_2 = \frac{8e^2}{3\pi\kappa} B' \Phi(s\gamma)\frac{1}{\gamma} \qquad \gamma = 1 + p_0^2\frac{\omega_0^2}{\omega^2} \qquad (56)$$

where B' differs from B in that under the logarithm sign s is replaced by $s\gamma$. At $s\gamma > 1$, $\gamma \gg 1$ one finds

$$W_2 = \frac{4}{3\pi} Z e^4 - \frac{\kappa}{p_0^2} \qquad (57)$$

which accords with the result obtained in [7] for this limiting case. Thus, radiation in a medium is not attended by any infrared catastrophe difficulties.

In order to introduce the shower unit of length it will be convenient to define a function $\xi(s)$ which takes into account the variation of L with energy

$$\xi(s) = 1 + \frac{\ln s}{\ln s_1}, \quad \xi(s) = 1, \quad \xi(s) = 2, \quad s_1^{1/2} = \frac{Z^{1/3}}{190} \qquad (58)$$
$$1 \geq s \geq s_1 \qquad s > 1 \qquad s < s_1$$

Here, s_1 is the value of s for which $L = 2\ln\frac{190}{Z^{1/3}}$

Formulae (51) and (58) yield

$$B = 2\pi n e^4 Z^2 \xi(s) \ln\frac{190}{Z^{1/3}} = \frac{\pi}{2}\cdot 137\,\frac{\xi(s)}{t_0'}$$

$$\frac{1}{t_0'} = \frac{\pi n e^4 Z^2}{137}\ln\frac{190}{Z^{1/3}}, \quad t_0' = t_0\frac{mc}{\hbar} = 2{,}59\cdot 10^{10}\, t_0 \qquad (59)$$

where t_0 is the shower unit expressed in centimeters.

From expression (47) for s one finds

50X1-HUM

Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

50X1-HUM

$$S = \frac{1}{8}\sqrt{\frac{2\kappa t_0'}{p_0(p_0-\kappa)\,137\pi\,\xi(s)}} = 1{,}37\cdot 10^{3}\sqrt{\frac{\kappa t_0}{p_0(p_0-\kappa)}} \qquad (60)$$

The probability of emission per shower unit length is

$$W_2 t_0' = \frac{\xi(s)}{3p_0^2\kappa}\left\{\kappa^2 G(s) + 2\left[p_0^2+(p_0-\kappa)^2\right]\varphi(s)\right\} \qquad (61)$$

In lead ($t_0 \cong 0{,}5\,cm$) for $\kappa = \frac{1}{2}p_0$ we obtain at $S=1$, $p_0 = 2\cdot 10^{6}\,t_0$, $E_0 = 5\cdot 10^{11}\,ev$; for $S=0{,}2$ which corresponds to a 30% deviation of (67) from the Betha-Heitler formula $E_0 = 1{,}25\cdot 10^{13}\,ev$, the value $S=S_1$ corresponds to an energy $E_0 \cong 2\cdot 10^{18}\,ev$

VI. Probability for Pair Production

Let $W_p(\kappa, p_0)$ denote the probability per unit length of production of a pair, the electron of which posseses an energy lying between p_0 and $p_0 + dp_0$ (W_p is summed over all possible positron states).

The probability for the inverse process $\tilde{W}_*$ may be found by summing over negative energy states in (8) and by changing the sign of E^{λ_1} in (9). The final formulae are obtained from those given above by substituting $p_0-\kappa$ by $\kappa-p_0$. Thus, for example the quantity $g = \frac{p_0+p_2-\kappa}{2}$ changes into $\tilde{g} = \frac{p_0-p_2+\kappa}{2} = \frac{\kappa}{2}$

Thus, the probability $\tilde{W}_*$ which differs from W_p only in statistical weight can be obtained from (49) by replacing $p_0-\kappa$ by $\kappa-p_0$

50X1-HUM

Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

Declassified in Part - Sanitized Copy Approved for Release 2014/06/20 : CIA-RDP80-00809A000100140020-8

50X1-HUM

$$W_p = \frac{p_0^2}{k^2}\tilde{W}_2 = \frac{2e^2}{3\pi k} B(\tilde{s})\left\{G(\tilde{s}) + 2\left[\frac{p_0^2}{k^2} + \left(1-\frac{p_0}{k}\right)^2\right]\phi(\tilde{s})\right\} \qquad (62)$$

Here $\tilde{s}$ differs from s only in that $p_0 - k$ has been replaced by $k - p_0$. The probability of pai production per shower unit of length is

$$W_p t_0' = \frac{\xi(\tilde{s})}{3k}\left\{G(\tilde{s}) + 2\left[\frac{p_0^2}{k^2} + \left(1-\frac{p_0}{k}\right)^2\right]\phi(\tilde{s})\right\} \qquad (63)$$

At $\tilde{s} = 1$ this expression transforms into the familiar formula for pair production[6]. For $\tilde{s} \ll 1$ we get

$$W_p t_0' = \frac{4\xi(\tilde{s})}{k}\left\{\frac{p_0^2}{k^2} + \left(1-\frac{p_0}{k}\right)^2\right\}\tilde{s} \qquad (64)$$

Formulae (61) and (63) are solutions of the bremsstrahlung and pair production problem for high energies in condensed media.

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The author expresses his appreciation to V.M.Galitsky for discussion of the results and to S.A. Heifets for carrying out the numerical calculations.

Appendix

Equations (43) and (44) can be solved by the Laplace method. Assuming $u(\lambda) = \int_0^\infty \varphi_1(z) e^{-\lambda z} dz$ one obtains from (43)

$$u' + \frac{\lambda - i\alpha}{\lambda^2 + i\beta} u = \frac{1}{2q\lambda(\lambda^2 + i\beta)} \qquad (1A)$$

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Hence

$$U(\lambda) = \frac{1}{2q}\,\frac{1}{(\lambda_1^2-\lambda^2)^{1/2}}\left(\frac{\lambda_1+\lambda}{\lambda_1-\lambda}\right)^{\mu}\int_{\lambda}^{\xi}\frac{d\xi}{\xi(\lambda^2-\xi^2)^{1/2}}\left(\frac{\lambda_1-\xi}{\lambda_1+\xi}\right)^{\mu} \qquad (2A)$$

Here $\lambda_1 = \sqrt{3}\,e^{-i\frac{\pi}{4}}$ $\quad \mu = \frac{\alpha}{2\sqrt{3}}\,e^{-i\frac{\pi}{4}}$ The arbitrary constant ξ_1 is determined from the condition of finity of function $\varphi_1(z)$ at $z \to \infty$ For this it is necessary that function $U(\lambda)$ does not possess any singularities in the right semi-plane, i e $\xi_1 = \lambda_1$

Expression (42) contains $\operatorname{Re}\int_0^\infty \varphi(z)\,dz = \operatorname*{Re}_{\lambda\to 0} U(\lambda)$

At $\lambda = 0$, $U(\lambda)$ has a logarithmic singularity and therefore $\lim_{\lambda\to 0} U(\lambda)$ depends on how λ approaches zero. It follows from the expression $\operatorname{Re} U(\lambda) = \int_0^\infty e^{-\lambda' z}(\operatorname{Re}\varphi_1 \cos\lambda' z - \operatorname{Im}\varphi_1 \sin\lambda' z)\,dz$, $\lambda = \lambda' + i\lambda''$

that it is sufficient for λ to approach zero along the real axis in order that the equality $\int_0^\infty \operatorname{Re}\varphi_1\,dz = \operatorname{Re} U(0)$ be satisfied.

Separating from integral (2A) the divergence at $\xi = 0$ we obtain

$$\operatorname*{Re}_{\substack{\lambda\to 0\\ \arg\lambda=0}} U(\lambda) = \operatorname*{Re}_{\substack{\lambda\to 0\\ \arg\lambda=0}} \frac{1}{2q}\,\frac{1}{\lambda_1}\left\{\int_{\lambda}^{\lambda}\frac{d\xi}{\xi}\left[\frac{1}{(\lambda_1^2-\xi^2)^{1/2}}\left(\frac{\lambda_1-\xi}{\lambda_1+\xi}\right)^{\mu}-\frac{1}{\lambda_1}\right]+\frac{1}{\lambda}\ln\frac{\lambda_1}{\lambda}\right\}$$

as $\operatorname*{Re}_{\substack{\lambda\to 0\\ \arg\lambda=0}} \frac{1}{\lambda_1^2}\ln\frac{\lambda_1^2}{\lambda} = \frac{\pi}{4\sqrt{3}}$

$$\operatorname*{Re}_{\substack{\lambda\to 0\\ \arg\lambda=0}} U(\lambda) = \frac{\pi}{8\sqrt{3}q} - \frac{1}{2q\sqrt{3}}\operatorname{Im}\int_0^1\frac{dx}{x}\left\{\frac{1}{(1-x^2)^{1/2}}\left(\frac{1-x}{1+x}\right)^{\mu}-1\right\}$$

Inserting $x = \operatorname{th}\frac{t}{2}$ and separating the imaginary part we find

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50X1-HUM

$$\int_0^\infty Re\,\varphi_1\,dz = \frac{1}{12 q \alpha^2} G(S) \qquad G(S) = 48 S^2\left(\frac{\pi}{4} - \frac{1}{2}\int_0^\infty e^{-st}\frac{\sin st}{sh\frac{t}{2}}dt\right) \qquad (3A)$$

$$S = \frac{\alpha}{2\sqrt{2\beta}} = \frac{1}{8g}\sqrt{\frac{k}{2(\rho_2-\rho_1)q}}$$

In order to solve equation (44) it will be convenient to introduce $f = \varphi_2 - \frac{i}{2q_1\beta}$

$$z f'' + i(\alpha + \beta z) f = \frac{\alpha}{2 q_1 \beta} \qquad (4A)$$

Inserting $v(\lambda) = \int e^{-\lambda z} f\,dz$ we obtain

$$v' + \frac{2\lambda - i\alpha}{\lambda^2 + i\beta} v = \frac{f(0) - \alpha/2q\beta i}{\lambda^2 + i\beta} \qquad (5A)$$

$\varphi_2(0) = 0$ and therefore $f(0) = -\frac{i}{2q_1\beta}$

The solution of equation (5A) is

$$v(\lambda) = -\frac{i}{2q_1\beta}\,\frac{1}{\lambda_1^2 - \lambda^2}\left(\frac{\lambda_1 + \lambda}{\lambda_1 - \lambda}\right)^{\mu}\int_0^{\lambda}\left(1 - i\frac{\alpha}{\xi}\right)\left(\frac{\lambda_1 - \xi}{\lambda_1 + \xi}\right)^{\mu} d\xi$$

$Re\,v(0)$ is calculated in exactly the same manner as $Re\,U(0)$

Performing once again the substitution $\xi/\lambda_1 = x$ and separating the diverging part from the integral we obtain

$$\underset{\substack{\lambda\to 0\\ \arg\lambda = 0}}{Re\,v(\lambda)} = \frac{1}{2q_1\lambda^2}\underset{\substack{\lambda\to 0\\ \arg\lambda = 0}}{Re}\left\{\lambda_1\int_0^1\left(\frac{1-x}{1+x}\right)^{\mu}dx - i\alpha\int_0^1\frac{dx}{x}\left[\left(\frac{1-x}{1+x}\right)^{\mu} - 1\right] - i\alpha\ln\frac{\lambda_1}{\lambda}\right\}$$

Inserting $x = th\frac{t}{2}$

and separating the real part we get

$$\underset{\substack{\lambda\to 0\\ \arg\lambda = 0}}{Re\,v(\lambda)} = \frac{1}{2q_1\lambda^2}\left\{-\frac{\pi\alpha}{4} + \alpha\int_0^\infty e^{-st}\frac{\sin st}{sh\,t}dt + \frac{1}{2}\sqrt{\frac{\beta}{2}}\int_0^\infty e^{-st}\frac{\cos st + \sin st}{sh\frac{t}{2}}dt\right\}$$

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Thus,

$$\int_0^\infty Re\, \varphi_2\, dz = \int_0^\infty Re\, t\, dz = \frac{}{6\alpha s q}\, \Phi(s) \qquad (6A)$$

where

$$\Phi = 3s\int_0^\infty e^{-st}\frac{\cos st + \sin st}{ch^2\frac{t}{2}}dt + 24s^2\int_0^\infty e^{-st}\frac{\sin st}{sh\, t}dt - 6\pi s^2 =$$

$$= 2s^2\int_0^\infty cth\frac{t}{2}\, e^{-st}\sin st\, dt - 6\pi s^2 \qquad (7A)$$

Functions $\Phi(s)$ and $G(s)$ can be expressed through the logarithmic derivatives of the Γ-function.[1)]

$$\Phi(s) = 2s^2\left[-Im\left[\Psi(s-is) + \Psi(s+1-is)\right] - \frac{\pi}{2}\right] =$$

$$= 6s - 6\pi s^2 + 24s^3\sum_{k=0}^{\infty}\frac{1}{(k+s+\frac{1}{2})^2+s^2}$$

$$G(s) = 48s^2\left[\frac{\pi}{4} + Im\,\Psi\left(s+\frac{1}{2}-is\right)\right] = \qquad (8A)$$

$$= 2\pi s^2 - 48s^3\sum_{k=0}^{\infty}\frac{1}{(k+s+\frac{1}{2})^2+s^2}$$

The formulae are useful for tabulations $\Phi(s)$ and $G(s)$

1) Relations (8A) were obtained by S.A. Heifetz.

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