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QUANTUM HYDRODYNAMICS

K. P. Gurov

The following paper examines the problem of the construction of general macroscopic equations of hydrodynamics for "normal" fluids, derived from the quantum-mechanical equations of the molecular system. Fundamental attention is given to such constructions and to the analysis of the physical premises in the expanding method, which, particularly, appears as a quantum analogy corresponding to the Bogolyubov's method in classical physics. Considering the definite results, it appears that the original quantum-mechanical picture finds its reflection in various formulas for the coefficients of hydrodynamic equations.

I. STATING THE PROBLEM

The problem of the derivation of hydrodynamic equations of classical physics, coming from the molecular-kinetic picture, has lately received much attention, as is shown, for example, in the monographs of Chapman [1] and N. N. Bogolyubov [2], and the work of Kirkwood [3] and others. A number of authors have also noticed that an analogous problem can be made by quantum-theoretical examinations.

Operator examinations of quantum hydrodynamics have already been formulated by L. D. Landau [4]. Landau formed, analogously to hydrodynamic equations of classical physics, differential equations for the operators of the densities and velocities and, to complete these equations, he created a corresponding condition of relationships for all denoted operators. However, since Landau's equations were not formed from generally understood and exact quantum-mechanical equations, there is an interest in further research on these equations for the purpose of creating steps to their approximations and limits of their applicability.

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In the present paper we assign ourselves the problem of constructing general macroscopic hydrodynamic equations which are derived from the quantum-mechanical equations for the system of molecular reactions. In this way we hope to obtain hydrodynamic equations for "normal" (undegenerate) fluids. Here also, in a few words, we will mention the method which leads to the calculation, from the quantum-mechanical molecular picture, of basic kinematic coefficients (viscosity, etc.). We will leave the detailed investigation as a subject of future work.

The development in the present paper of the method of solving the basic problem presented here appears as a quantum-mechanical generalization of Bogolyubov's method. The first attempt in this direction was made in our previous dissertation /5/; however, that problem was only the solution for one part of the case while at this time we have no limitations.

Bogolyubov's characteristic method will be given below. We will notice only that this method is very similar to methods of variations of derived constants. The nature of the latter in BoBolyubov's method of "parametric conditions," i.e., physical dimensions, which after being introduced in the given problem, should determine the form of molecular description in the static equilibrium. As in Bogolyubov's paper in which the author assigned himself the problem of constructing the classical hydrodynamics of ordinary fluids, the following parametric conditions were chosen: density, macrovelocity, and heat energy. In static equilibrium these three dimensions (two scalar and one vector) appear constant and fully determine the corresponding equilibrium condition (parametric condition).

In cases other than static equilibrium, the dimensions of the denoted type comply as before with the parameter condition, but are regarded as slowly changing functions of time and space coordinates.

Such an analogy with "the methods of variation of derived constants" is conserved even in quantum-mechanical generalization. Having in view the examination only of "normal" fluids for the changing parameter conditions there can be selected five scalar dimensions as in the classical case. (If we were to set ourselves the problem of studying superfluidity -- fluidity with "degeneration" -- then, in the nature of parameter conditions, we should have taken two scalar and two vector dimensions (i.e., eight scalars) since in this case there are two macrovelocities.)

The method of generalization infers that instead of the chain of distribution functions determining the molecular picture in classical physics, we include a corresponding chain of distribution operators complying with quantum-mechanical equations and arrangements. The determination of these operators was given in the papers of N. N. Bogolyubov and K. P. Gurov /6/ and K. Gurov /7/. The corresponding mathematical system was developed and equations of quantum-mechanical arrangement to which these operators comply were established.

Obviously, by this means there can be established a stage for the basis of obtaining general hydrodynamic equations in those cases where the molecular model can be described by quantum-mechanics.

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II. GENERAL CONCEPTIONS AND COMPUTATIONS

We select as molecular models spinless particles of the system N, reacting according to a central (binary) law and occupying the volume V. According to the quantum-mechanical description of such a model [7] there is assigned, with the aid of the distribution operators (matrices) F_s (s = 1, 2, ...):

$$F_s(1, 2, \dots, s) = V^s \underset{(s+1, \dots, N)}{\mathrm{Spur}}\, D(1, \dots, N), \quad s \ll \qquad (2, 1)$$

or in the matrix form (in coordinate representation)

$$= V^s \int_V \dots \int_V D(q_1, \dots, q_s, q_{s+1}, \dots q_N; q_1', \dots q_s', q_{s+1}, \dots q_N)\, dq_{s+1} \dots dq_N. \qquad (2, 2)$$

Here D is the known density matrix of Dirac; q_j is understood as the combination of three cartesian coordinates in configuration of the area j of particles $[q_j = (q_j^1, q_j^2, q_j^3)\ ;\ dq_j = dq^1 dq^2 dq^3]$

Investigation of F_s leads to the asymptotic approximation

($N \to \infty$, $V \to \infty$, but $V/N = \zeta =$ const).

The general characteristics of F_s:

1. Hermitian

$$\{F_s\} q q' = \{F_s^*\} q' q \qquad (2, 3)$$

2. Normalization

$$F_s = \lim_{V \to \infty} \frac{1}{V} \underset{(s+1)}{\mathrm{Spur}}\, F_{s+1}\ ;\ \lim_{V \to \infty} \frac{1}{V} \mathrm{spur}\, F_1 = 1. \qquad (2, 4)$$

3. Symmetry

$$P_{ij}\, F_s(1, \dots, s) = F_s(i, \dots, s)\, P_{ij} = F_s(1, \dots, s), \qquad (2, 5)$$

where P_{ij} is the operator of the stated i and j coordinates acting on the change of q, if it stands on the left of F_s, and on the change of q' if it stands to the right.

4. Asymptotic equation for F_s:

$$\frac{\partial F_s}{\partial t} = [H_s, F_s] + \frac{1}{V} \underset{(s+1)}{\mathrm{spur}} \left[\sum_{1 \le j \le s} \Psi(j, s+1), F_{s+1} \right], \qquad (2, 6)$$

where H_s is the Hamiltonian operator of the closed system s reacting with the particles, and Ψ (j, s+1) is the operator of the potential energy of the binary reaction of the j and s+1 particles.

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5. Operators (matrices) F_s, complying to the conditions of the "decreasing correlation," appear as quantum modifications of the classical arrangement which states that at a sufficient withdrawal of s molecules from each other the distribution function F_s can be approximated with the aid of the formation of the function F_1.

The condition of the decreasing correlation has a higher value for the full solution of the chain of deductively connected operators of the equation (2, 6).

6. For any physical dimension having the characteristic operator form

$$\sum_{1 \leq j_1 < \cdots < j_s \leq N} L(j_1, \cdots, j_s),$$

the average value can be calculated by the asymptotic formula

$$L = \frac{1}{s!} \nu^s \cdot \text{spur } L F_s. \qquad (2, 7)$$

Now, following Landau, we introduce the operators of the density, current, and energy parametrically connected with the term x of the general area:

$$\{\sigma(x)\}_{qq'} = \frac{m}{2} \sum_{1 \leq j \leq N} \{\delta(q_j - x) + \delta(q'_j - x)\}, \qquad (2,8)$$

$$\{J^\alpha(x)\}_{qq'} = \frac{1}{2} \sum_{1 \leq j \leq N} \{p^\alpha_{q_j} \delta(q_j - x) + \delta(q'_j - x) p^\alpha_{q'_j}\} \qquad (2,9)$$

$$\{E(x)\}_{qq'} = \frac{1}{2} \sum_{1 \leq j \leq N} \left\{\left(p^2_{q_j} + \sum_{1 < k \leq N} \Psi_{q_j q_k}\right) \delta(q_j - x) + \delta(q'_j - x)\left(p^2_{q'_j} + \sum_{j < k \leq N} \Psi_{q'_j q'_k}\right)\right\} \qquad (2,10)$$

(p is the impulse operator.)

From this, accounting for the well-known form of the operators p and Ψ (the last potential of the binary reaction we will designate by Φ (|q|); where |q| is the interval between the two reacting particles) according to the formula (2, 7) we get the average dimensions:

$$\sigma(x) = \frac{m}{v} F_1(x, x), \qquad (2, 11)$$

$$J^\alpha(x) = -\frac{i\hbar}{2v} \left\{\left(\frac{\partial}{\partial q'^\alpha_1} - \frac{\partial}{\partial q^\alpha_1}\right) F_1(q_1, q'_1)\right\}_{q'_1 = q_1 = x}, \qquad (2,12)$$

$$E_{(x)} = -\frac{\hbar^2}{2mv} \{(\Delta_{q_1} - \Delta_{q'_1}) F_1(q_1, q'_1)\}_{q'_1 = q_1 = x} + \frac{1}{2v^2} \int \Phi(|x - q_2|) F_2(x, q_2; x, q_2) dq_2 \qquad (2,13)$$

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Having in mind the discussions in section I, and accounting for the general physical treatment as operators (2, 9) - (2, 10) and average dimensions (2, 11) - (2, 13), we can take the following dimensions in the nature of "parameter conditions" (introducing new symbols at the same time):

$$\rho(x) = \overline{\sigma(x)}\ ;\ \rho(x)\, u^{\alpha}(x) = \overline{J^{\alpha}(x)}\ ; \qquad (2,\ 14)$$

$$\rho(x)\ \theta(x) = \overline{E(x)} - \frac{1}{2}\rho(x) u^2(x)$$

III. CONVERTING F_s TO A FORM WHICH TAKES INTO ACCOUNT APPARENT SPACE HETEROGENEITY

In an above-mentioned paper [5] it was noticed that cases of "uniform-space distribution" are characterized by the invariance of the operators (matrix) F_s to space transfer, so that the mathematical arrangement becomes

$$F_s(q_1+q_0, \dots, q_s+q_0; q_1'+q_0, \dots, q_s'+q_0) \equiv \qquad (3,1)$$
$$\equiv F_s(q_1, \dots, q_s; q_1', \dots, q_s').$$

From the arrangement (3, 1) for the uniform-space distribution it follows that

$$F_1(q_1, q_1') = f_1(q_1 + q_1'). \qquad (3,\ 2)$$

In the general case of "nonuniform space distribution," by conversion of F_1 coordinates, it can have the form

$$F_1(q_1, q_1') = f_1\left(\frac{q_1+q_1'}{2}; q_1 - q_1'\right), \qquad (3,\ 3)$$

from which it is clear that the uniform-space case is characterized by the independence of f_1, from the first argument.

Further, we have similarly

$$F_s(q_1, \dots, q_s; q_1', \dots, q_s') \equiv \qquad (3,\ 4)$$
$$\equiv F_s\left(\frac{q_1+q_1'}{2} + \frac{q_1-q_1'}{2}, \frac{q_1+q_1'}{2} + q_2 - \frac{q_2+q_1'}{2}, \dots; \frac{q_1+q_1'}{2} + \frac{q_1'+q_1}{2}, \frac{q_2+q_1'}{2} + q_1' - \frac{q_1+q_1'}{2}, \dots\right).$$

It is not difficult to notice that, in the general case of space non-uniformity, the change of F_s during the space transfers is connected also with the change of the first item in all the arguments. With the change of $\frac{1}{2}(q_1+q_1')$, i.e., in the general case $s > 1$, space nonuniformity is characterized by the presence of the dependence of F_s on the argument $\frac{1}{2}(q_1+q_1')$, so that it can be written

$$F_s = f_s\left(\frac{q_1+q_1'}{2}; q_1, \dots, q_s; q_1', \dots, q_s'\right),$$

in which

$$f_s\left(\frac{q_1+q_1'}{2}, q_1+q_0; \dots, q_s+q_0; q_1'+q_0, \dots, q_s'+q_0\right) \equiv$$
$$\equiv f_s\left(\frac{q_1+q_1'}{2}, q_1, \dots, q_s; q_1', \dots, q_s'\right).$$

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It is convenient to introduce the small parameter μ. The method of introducing this parameter and the physical expediency of such an introduction is easy to understand if the ideas of N. N. Bogolyubov for the classical case are followed and applied to the peculiarities of our problem.

It is not difficult to be convinced, in taking into account (3, 1), that the functions u^{α}, ρ, θ, which were not too well determined by equations (2, 11) - (2, 14) in the case of space uniformity, appear constant in time and space dimensions. Our following macroscopic treatment of these functions predetermines the factual neutralization of the right-hand parts of the formula (2, 11) - (2, 14) along the space volume. These dimensions are large in comparison with the "radius of action" r_0 of the molecular forces, and the time interval is long in comparison to the "unit of molecular time" $r_0 M/|P|_{cp}$ where $|P|_{cp}$ is the mathematical expectation of the absolute value of the molecular impulse.

The very selection of the method of "variation constants" specifies that we will be dealing with functions which slowly change in time and space and with the calculation of only that which is indicated. This will indicate that u^{α}, ρ, θ appear as functions which slowly change in relation to the indicated areas of the "molecular scale."

In the light of the above indications, it is necessary to examine the right-hand parts of the formulas (2, 11) - (2, 13), which leads us to a complete solution for the form of F_s. The solution of the equation (2, 6) for F_s, which satisfies the conditions in (2, 11) - (2, 14), should differ only a little from the corresponding space-uniformity solution, which is natural for the equation (2, 6).

The last mathematical conclusion will correspond to the introduction of the small parameter μ, which can be written

$$F_s = f_s(\mu(q_1+q_1')/2, q_1, \ldots, q_s', \mu)$$

with the idea that, as $\mu \to 0$, the solution of F_s goes over to the corresponding space-uniformity solution. It immediately follows that the parameter conditions in the general case appear as functions of t and μq.

Thus, introducing the symbol $\xi = \mu(q_1+q_1')/2$, we get

$$F_s = f_s(\xi, q_1, \ldots, q_s') \qquad (3, 5)$$

in which

$$f_s(\xi, q_1+q_0, \ldots, q_s'+q_0) \equiv f_s(\xi, q_1, \ldots q_s') \qquad (3, 6)$$

The "Hermitian" of the operators in such a margin is conserved: as to the arrangement of the symmetry (2, 5) we now have

$$P_{jk} f_s = f_s P_{jk} = f_s \quad (j, k > 1) \qquad (3, 7)$$

but

$$(P_{1j} f_s)_{\xi} = \mu(q_j + q_1')/2 = (f_s)_{\xi} = \mu(q_1+q_j')/2 = (f_s P_{j1})_{\xi} = \mu(q_1+q_j')/2 \qquad (3, 8)$$

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The operators of the arrangement can also be presented in the following form:

$$P_{1j}\ f_s = e^{\frac{\mu}{2}(q_1^\alpha - q_j^\alpha)\frac{\partial}{\partial \xi^\alpha}} f_s\ ;$$

$$f_s P_{1j} = e^{\frac{\mu}{2}(q_1'^\alpha - q_j'^\alpha)\frac{\partial}{\partial \xi^\alpha}} f_s$$

$$\text{and } P_{1j}\ f_s\ P_{1j} = e^{\frac{\mu}{2}(q_1^\alpha + q_1'^\alpha - q_j^\alpha - q_j'^\alpha)\frac{\partial}{\partial \xi^\alpha}}\ f_s \qquad (3,9)$$

since, for example,

$$P_{1j}\ f_s(\xi; q_1, \dots, q_s') = f_s\left(\xi + \mu\frac{q_1 - q_j}{2}; q_1, \dots, q_s'\right) =$$

$$= f_s(\xi; q_1, \dots, q_s') + \mu\left(\frac{q_1^\alpha - q_j^\alpha}{2}\frac{\partial}{\partial \xi^\alpha}\right) f_s(\xi; q_1, \dots, q_s') +$$

$$+ \frac{1}{2}\mu^2\left(\frac{q_1^\alpha - q_j^\alpha}{2}\frac{\partial}{\partial \xi^\alpha}\right)\left(\frac{q_1^\beta - q_j^\beta}{2}\frac{\partial}{\partial \xi^\beta}\right) f_s(\xi; q_1, \dots, q_s') + \dots \qquad (3,10)$$

From (3,6) and (2,6) it is not difficult to construct an equation for f_s which we shall write out here in a developed form (in which the operations along q and ξ are already accounted for as independent):

$$\frac{\partial f_s}{\partial t} = \frac{ik}{2m}\mu\frac{\partial}{\partial \xi^\alpha}\left(\frac{\partial}{\partial q_1^\alpha} - \frac{\partial}{\partial q_1'^\alpha}\right) f_s + \frac{i\hbar}{2m}\sum_{1\le j\le s}(\Delta_{q_j} - \Delta_{q_j'}) f_s +$$

$$+ \frac{1}{i\hbar}\sum_{1\le j<k\le s}\left\{\Phi(|q_j - q_k|) - \Phi(|q_j' - q_k'|)\right\} f_s + \qquad (3,11)$$

$$+ \frac{1}{i\hbar}\int \sum_{1\le j\le s}\left\{\Phi(|q_j - q_{s+1}|) - \Phi(|q_j' - q_{s+1}'|)\right\}\left\{f_{s+1}\right\}_{q_{s+1}' = q_{s+1}} dq_{s+1}$$

In order to continue we need a term f_1 in the described impulses. In our other works [5-7] it was shown that, in the absence of degeneration for cases of space uniformity F_1, in the described impulses, it is expressed through the general function of the impulse distributions $w(p)$, namely

$$F_1(p_1, p_1') = f(p_1, p_1') = (2\pi\hbar)^3 w(p_1)\,\delta(p_1 - p_1') \qquad (3,12)$$

from which

$$F_1(q_1, q_1') = \int w(p)\, e^{\frac{i}{\hbar}p^\alpha(q_1^\alpha - q_1'^\alpha)}\, dp_1 =$$

$$= f_1(q_1 - q_1') = f_1(q) \qquad (3,13)$$

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(Henceforth, for brevity we will designate $f_1(q_1-q_1')$ by $f_1(q)$.)

In these works in was noticed that it could be done in the reverse order as well i.e., given $F_1(q_1, q_1') = f_1(q_1)$ we can obtain (3, 12) if we only calculate the arrangement in the absence of degeneration: $f_1(q) \to 0$ when $|q| \to \infty$.

Actually

$$S_1(p_1, p_1') = \frac{1}{(2\pi\hbar)^3} \iint f(q_1 - q_1') e^{-\frac{i}{\hbar}(q_1^\alpha p_1^\alpha - q_1'^\alpha p_1'^\alpha)} dq_1 dq_1' =$$

$$= \frac{1}{(2\pi\hbar)^3} \iint f(q_1 - q_1') e^{-\frac{i}{\hbar}\{p_1^\alpha (q_1^\alpha - q_1'^\alpha) + q_1'^\alpha (p^\alpha - p_1'^\alpha)\}} dq_1 dq_1' =$$

$$= (2\pi\hbar)^3 \, w(p_1) \delta(p_1 - p_1') \qquad (3, 14)$$

where specifically

$$w(p) = \frac{1}{(2\pi\hbar)^3} \int F(q) e^{-(i/\hbar) q^\alpha p^\alpha} dq \qquad (3, 15)$$

Since $f(q)$ has "Hermitian" characteristics, i.e., $f_1^*(q) = f_1(-q)$ then $w(p)$ is the material function. All these conclusions, as can be easily shown, remain in full force and are valid in cases of space nonuniformity if ξ_1 is regarded as a parameter. It is necessary only in formulas (3, 12) - (3, 15) to change $f_1(q)$ and $w(p)$ to $f_1(\xi_1, q)$ and $w(\xi_1, p)$.

In concluding this section it still remains for us to express (2,11) - (2, 13) with the aid of $f_1(q)$ and $w(p)$ which is easily done by calculating (3, 5) and (3, 15) (here $\xi_1 = \frac{1}{2}(q_1 + q_1')$, $q_1' = q_1$):

$$\rho(\xi_1) = \frac{m}{v} f_1(\xi_1, 0) = \frac{m}{v} \int w(\xi_1, p) dp; \qquad (3, 16)$$

$$\rho(\xi_1) u^\alpha(\xi_1) = -\frac{i\hbar}{v} \left\{\frac{\partial}{\partial q^\alpha} f_1(\xi_1, q)\right\}_{q=0} = \frac{1}{v} \int p^\alpha w(\xi_1, p) dp; \qquad (3, 17)$$

$$\rho(\xi_1)\theta(\xi_1) + \frac{\rho(\xi_1) u^2(\xi_1)}{2} = -\frac{\hbar^2}{6mv} \mu^2 \Delta_{\xi_1} f_1(\xi_1, 0) - \frac{\hbar^2}{2mv} \{\Delta_q f(\xi_1, q)\}_{q=0} +$$

$$+ \frac{1}{2v^2} \int \Phi(|q|) h_2(\xi_1, q) dq \qquad (3, 18)$$

where

$$h_2(\xi_1, q) = h_2(\xi_1, q_1 - q_2) = f_2(\xi_1, q_1, q_2, q_1, q_2)^4, \qquad (3, 19)$$

in which

$$-\frac{\hbar^2}{2mv} \{\Delta_q f(\xi_1, q)\}_{q=0} = \frac{1}{2mv} \int p^2 w(\xi_1, p) dp. \qquad (3, 20)$$

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A study of the right-hand parts of the formulas (3, 17) - (3, 20) shows that if we eliminate the first component in (3, 19) which has a definite quantum derivation (as we shall see, in our valid approach to this), then we will come to a full analogy with the classical theory in the recorded form. This leads to important conclusions: First, if ξ is to be regarded as a parameter, then $w(\xi, \rho)$ can be treated in the classical way, as a "function of the density of the distributed impulses in the space point ξ^{α}". Second, the analogy in the recorded form and indications of the treatment of $w(\xi, \rho)$ fully indicate in use the correctness of the selection of $\rho, \rho u^{\alpha}$ and θ in the nature of parameter conditions used in Bogolyubov's method, the composition of which, with alterations for our problem, we shall now study.

IV. BOGOLYUBOV'S METHOD

In conformance with the examination of the problem here, Bogolyubov's method can be formulated in the following way.

The above mentioned dimensions $\rho, \rho u^{\alpha}$ and $\rho\theta$ introduced in the equilibrium case appear as constant dimensions which are taken as "parameter conditions," i.e., the form of operator chains (matrix) f_s in our case is fully distinguished by these parameters. At deviations from the equilibrium case we find as before that these five dimensions remain in parameter condition, i.e., in every moment of time t, the form of the chain f_s fully distinguishes the task of $\rho, \rho u^{\alpha}$ and $\rho\theta$ for that very moment of time at which these five dimensions are now regarded as slowly changing functions of time and space. The form of the last function is determined by the solution of five differential equations -- hydrodynamic equations -- determining the behavior of these five dimensions.

The basic problem includes the construction of these equations in a clear way. We will notice at the outset that the derivatives of $\rho, \rho u^{\alpha}$ and $\rho\theta$ in time should automatically return to zero when the equilibrium condition occurs; formal arrangement of this appears as $\mu \to 0$. Because of this, we will present the desired equations in the forms:

$$d\rho/dt = \mu A_1 + \mu^2 A_2 + \cdots \qquad (4, 1)$$

$$d\rho u^{\alpha}/dt = \mu B_1^{\alpha} + \mu^2 B_2^{\alpha} + \cdots \qquad (4, 2)$$

$$d\rho\theta/dt = \mu C_1 + \mu^2 C_2 + \cdots \qquad (4, 3)$$

where the apparent forms of $A_1, A_2, \ldots, B_1^{\alpha}, B_2^{\alpha}, \ldots, C_1, C_2, \ldots$ must yet be found.

We will return now to the chain f_s which, for any moment of time, is fully determined by the five parameters for that moment of time. This means that among the solutions of the equations (3, 12) for f_s, we shall choose only those solutions in which the dependence of f_s on time is not direct but indirect -- through the form of dependence of the functions $\rho, \rho u^{\alpha}, \rho\theta$.

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There is a deep physical meaning in such a selection. First of all, we will notice that, since we consider that parameter conditions appear as slowly changing functions of space and time, it follows from (4, 1) - (4, 3) that μ must be made a small dimension. Further, once the dependence of f_s on time is determined only by the form of the dependence on the time function $\rho, \rho u^\alpha, \rho\theta$, then from this the expressions for the derivatives ($s = 1, 2 \ldots$) will be linearly dependent on the right-hand parts of the equations (4, 1) - (4, 3) in which the solutions for $\rho, \rho u^\alpha$ and $\rho\theta$ are substituted from these equations, i. e., the series of derivative dimensions in a definite calculation will be determined by a series of μ dimensions which, as we showed, should be small. Thus it can be said the change of f_s in time is synchronized with the change of the five parameters; for a small μ, the same thing occurs for space changes.

From this we conclude that "smoothing out" of f_s, i.e., the neutralization, is done in a determined way. This is physically very regular. Indeed, f_s serves in the description of the molecular picture, and at the same time with the aid of f_s we determined $\rho, \rho u^\alpha$ and $\rho\theta$ (formulas 3, 16 - 3, 19). Because of this the change to macroscopic description automatically needs such a neutralization. The gist of the matter is that all neutralizations are already included in the recording of f_s in the form of $f_s(\mu(q+q')/2, \ldots)$ with the understanding that μ is small -- this can be seen in our discussion in the introduction of μ in the preceding section.

Thus, we have come to the macroscopic treatment of ρ, u, θ. We can now call these dimensions, analogous to the classical theory, that is, as densities of average velocities and thermal energy in a space point ξ. The equations (4,1) - (4,3) can be treated as actual macroscopic equations of hydrodynamics which should agree fully with classical theory.

We can now formulate a method of getting a clear form of $A_1, A_2, \ldots, G^\alpha, B_2^\alpha, \ldots, C_1, C_2 \ldots$ Since the magnitude of μ is small, we regard it as a disordered parameter theory and we look for the solution of f_s in:

$$f_s = g^{(0)}(\xi, q, \ldots, q_s'; \rho, u^\alpha, \theta) + \mu g_s^{(1)}(\xi, q, \ldots, q_s'; \rho, u^\alpha, \theta) + \ldots, \quad (4,4)$$

where the entry of ρ, u^α, θ in the arguments signifies the indirect dependence of f_s on t through the form of these parameters. The expression (4,4) should be satisfied by equation (3,11), but in it $\partial/\partial t$ should be presented as $\partial/\partial t = \mu D_1 + \mu^2 D_2 + \ldots$, D_1 is understood as the operation of indirect differentiation with a base A_1, instead of $\partial\rho/\partial t$, in which it is considered that in A_1 there enter the quantities ρ, u^α, θ in the solution of the equations (4,1) - (4,3) etc.

Each of the equations (3,11) are thus decomposed into a series of equations:

$$[H_s, g_s^{(0)}] + \frac{1}{v} \sum_{1 \le j \le s} \operatorname{spur}_{(s+1)} [\psi_{j,s+1}, g_{s+1}^{(0)}] = 0, \quad (4,5)$$

$$[H_s, g_s^{(1)}] + \frac{1}{v} \sum_{1 \le j \le s} \operatorname{spur}_{(s+1)} [\psi_{j,s+1}, g_{s+1}^{(1)}] + \frac{i\hbar}{2m} \frac{\partial}{\partial \xi^\alpha} \left(\frac{\partial}{\partial q_1^\alpha} - \frac{\partial}{\partial q_1'^\alpha} \right) g_s^{(0)} - D_1 g_s^{(0)} = 0, \quad (4,6)$$

$$[H_s, g_s^{(2)}] + \frac{1}{v} \sum_{1 \le j \le s} \operatorname{spur}_{(s+1)} [\psi_{j,s+1}, g_{s+1}^{(2)}] + \frac{i\hbar}{2m} \frac{\partial}{\partial \xi^\alpha} \left(\frac{\partial}{\partial q_1^\alpha} - \frac{\partial}{\partial q_1'^\alpha} \right) g_s^{(1)} - D_2 g_s^{(0)} - D_1 g_s^{(1)} = 0 \quad (4,7)$$

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Here H_s signifies the Hamiltonian operator of the closed system S of the reacting particles, which does not correspond to the operation along ξ_1.

These equations have a complete arrangement. We introduce them in the construction of the possible unidentical breakdown of f_S into a series (4,4). These arrangements can be obtained simply by the substitution of (4,4) in (3,16) - (3,20):

$$\rho(\xi_1) = \frac{m}{V} g_1^{(0)}(\xi_1, 0) = \frac{m}{V} \int w^{(0)}(\xi_1, p)\, dp, \quad (4,8)$$

$$\rho(\xi_1) u^{\alpha}(\xi_1) = -\frac{i\hbar}{V} \left\{ \frac{\partial}{\partial q^{\alpha}} g_1^{(0)}(\xi_1, q) \right\}_{q=0} = \frac{1}{V} \int p^{\alpha} w^{(0)}(\xi_1, p)\, dp, \quad (4,9)$$

$$\rho(\xi_1)\Theta(\xi_1) + \tfrac{1}{2}\rho(\xi_1) u^2(\xi_1) = \frac{-\hbar^2}{2mV} \left\{ \Delta_q g_1^{(0)}(\xi_1, q) \right\} + \frac{1}{2V^2} \int \Phi(|q|) h^{(0)}(q)\, dq = \frac{1}{2mV} \int p^2 w^{(0)}(\xi_1, p)\, dp + \frac{1}{2V^2} \int \Phi(|q|) h^{(0)}(q)\, dq, \quad (4,10)$$

$$g_1^{(k)}(\xi_1, 0) = \frac{m}{V} \int w^{(k)}(\xi_1, p)\, dp = 0 \quad (k = 1, 2, \cdots), \quad (4,11)$$

$$-\frac{i\hbar}{V} \left\{ \frac{\partial}{\partial q^{\alpha}} g_1^{(k)}(\xi_1, q) \right\}_{q=0} = \frac{1}{V} \int p^{\alpha} w^{(k)}(\xi_1, p)\, dp = 0, \quad (4,12)$$

$$\frac{1}{2mV} \int p^2 w^{(k)}(\xi_1, p)\, dp + \frac{1}{2V^2} \int \Phi(|q|) h^{(k)}(q)\, dq = 0. \quad (4,13)$$

If we formally present $d\rho/dt$, $d\rho u^{\alpha}/dt$ and $d\Theta/dt$, derived from (3,16) - (3,18), then we immediately can notice that the apparent form of A_1, B_1, C_1 is determined by the solution of equation (4,5) with the calculation of additional conditions. If we could solve the equations (4,1) - (4,3) in the first approximation then by the same token we could fully determine the apparent form of functions, obtained in the result of the operation $\partial g_s^{(0)}/\partial \xi$ and $D_1 g_s^{(0)}$. However the general (not detailed) form of A_2, B_2, C_2 can be found without solving the equation (4,6), with the additional conditions of (4,11) - (4,13), at which the general qualitative analysis of the operations just mentioned can be used. Such is Bogolyubov's scheme.

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V. CALCULATION OF THE COEFFICIENTS OF SEPARATION

At first, as we mentioned previously, we will undertake the formal construction from (3,16) - (3,18):

$$\frac{d\rho(\mathcal{E}_1)}{dt} = \frac{d}{dt}\left\{\frac{m}{v} f_1(\mathcal{E}_1, q)\right\}_{q=0} = \left\{\frac{m}{v}\frac{df_1(\mathcal{E}_1, q)}{dt}\right\}_{q=0}, \quad (5\ 1)$$

$$\frac{d\rho(\mathcal{E}_1)u^\alpha(\mathcal{E}_1)}{dt} = \frac{d}{dt}\left\{\frac{i\hbar}{2v}\left(\frac{d}{dq_1^\alpha} - \frac{d}{dq_1^\alpha}\right) f(\mathcal{E}_1, q_1 - q_1')\right\}_{q_1'=q_1} =$$

$$= \left\{\frac{i\hbar}{2v}\left(\frac{d}{dq_1^\alpha} - \frac{d}{dq_1^\alpha}\right)\frac{df(\mathcal{E}_1, q_1 - q_1')}{dt}\right\}_{q_1'=q_1}, \quad (5,2)$$

$$\frac{d\rho(\mathcal{E}_1)\theta(\mathcal{E}_1)}{dt} = \frac{d}{dt}\left\{\left[-\frac{\hbar^2}{2mv}\Delta_q f_1(\mathcal{E}_1, q)\right]_{q=0} + \right.$$

$$\left. + \frac{1}{v^2}\int \Phi(|q_1 - q_2|) f_2(\mathcal{E}_1, q_1, q_2, q_1, q_2)\, dq_2\right\} - \frac{1}{2}\frac{d}{dt}\rho u^2 =$$

$$= \left\{\frac{-\hbar^2}{2mv}\Delta_q \frac{df_1(\mathcal{E}_1, q)}{dt}\right\}_{q=0} +$$

$$+ \frac{1}{v}\int \Phi(|q_1 - q_2|)\left(\frac{df_2(\mathcal{E}_1, q_1, q_2, q_1', q_2')}{dt}\right)_{\substack{q_1'=q_1 \\ q_2'=q_2}} dq_2 - \frac{1}{2}\frac{d\rho u^2}{dt} \quad (5,3)$$

(The last term is also expressed by df_1/dt.)

We compute further from (3,11). Then for (5,1) by way of several conversions we get:

$$\frac{d\rho(\mathcal{E}_1)}{dt} = \mu\frac{i\hbar m}{v}\frac{d}{d\mathcal{E}_1^\beta}\left\{\frac{d}{dq^\beta} f_1(\mathcal{E}_1, q)\right\}_{q=0} = -\mu\frac{d}{d\mathcal{E}^\beta}\left\{\rho(\mathcal{E}_1)u^\beta(\mathcal{E}_1)\right\} \quad (5,4)$$

The conversions for (5,2) and (5,3) are more complex. In them we use quantum characteristics of symmetry f_1^*, at which it is obvious that in the first approximations the equations of the quantum effect do not play any role. All this is of definite interest and through our conversions for (5,2) we are led also to the conversions for (5,3), which are basically analogous but even more cumbrous, and so for them we will write out here only the definite results:

Thus for (5,2) we get from (3,11):

$$\frac{d\rho u^\alpha}{dt} = -\frac{\mu\hbar^2}{2mv}\left\{\left(\frac{d}{dq_1'^\alpha} - \frac{d}{dq_1^\alpha}\right)\frac{d}{d\mathcal{E}^\beta}\left(\frac{d}{dq^\beta} - \frac{d}{dq_1'^\beta}\right) f_1(\mathcal{E}_1, q_1 - q_1')\right\}_{q_1'=q_1} +$$

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$$+\frac{1}{2v^2}\int\left\{\left(\frac{\partial^2}{\partial q_1'^{\alpha}}-\frac{\partial}{\partial q_1'^{\alpha}}\right)\left[\Phi(|q_1-q_2|)-\Phi(|q_1'-q_2|)\right]f_2(\xi,q_1,q_2,q_1',q_2')\right\}_{q_1'=q_1}dq_2=$$

$$=-\frac{\hbar^2}{mv}\frac{\partial}{\partial\xi^{\beta}}\left\{\frac{\partial^2}{\partial q^{\alpha}\partial q^{\beta}}f_1(\xi,q)\right\}_{q=0}-\frac{1}{v}\int\Phi'_{|q|}(|q|)\frac{q^{\alpha}}{|q|}h(\xi,q)\,dq; \qquad (5,5)$$

Here $h(\xi,q)=h(\xi,q_1-q_2)=f_2(\xi,q_1,q_2,q_1',q_2')$, $\Phi'_{|q|}(|q|)$ signifies the derivative for $|q|$, which we obtained, accounting for the fact that

$$\partial\Phi(|q_1-q_2|)/\partial q_1^{\alpha}=\Phi'_{|q|}(|q|)(q^{\alpha}/|q|).$$

Further, from (3,13) we find:

$$\mu\frac{\hbar^2}{mv}\frac{\partial}{\partial\xi^{\beta}}\left\{\frac{\partial^2}{\partial q^{\alpha}\partial q^{\beta}}f_1(\xi,q)\right\}_{q=0}=-\frac{1}{mv}\frac{\partial}{\partial\xi^{\beta}}\int p^{\alpha}p^{\beta}w(\xi,p)\,dp$$

$$=-\mu\frac{\partial}{\partial\xi^{\beta}}\left\{\rho u^{\alpha}u^{\beta}+\frac{1}{mv}\int(p^{\alpha}-mu^{\alpha})(p^{\beta}-mu^{\beta})w(\xi,p)\,dp\right\}. \qquad (5,6)$$

Whereupon

$$\int\Phi'_{|q|}(|q|)\frac{q^{\alpha}}{|q|}h(\xi,q)\,dq=$$

$$=\frac{1}{2}\int\Phi'_{|q|}(|q|)\frac{q^{\alpha}}{|q|}\left\{h(\xi,q)-h(\xi,-q)\right\}dq. \qquad (5,7)$$

But in compliance with the symmetry arrangement (3,9) we have:

$$h(\xi,q)-h(\xi,-q)=-\mu q^{\beta}\frac{\partial}{\partial\xi^{\beta}}h(\xi,q)-$$

$$-\frac{1}{2}\mu^2 q^{\beta}q^{\gamma}\frac{\partial^2}{\partial\xi^{\beta}\partial\xi^{\gamma}}h(\xi,q)-\cdots, \qquad (5,8)$$

from which

$$\int\Phi'_{|q|}(|q|)\frac{q^{\alpha}}{|q|}h(\xi,q)\,dq=$$

$$=-\mu\frac{1}{2}\frac{\partial}{\partial\xi}\int\Phi'_{|q|}(|q|)\frac{q^{\alpha}q^{\beta}}{|q|}h(\xi,q)\,dq+\mu^3\ldots, \qquad (5,9)$$

since

$$\int\Phi'_{|q|}(|q|)\frac{q^{\alpha}q^{\beta}q^{\gamma}}{|q|}h(\xi,q)\,dq=$$

$$=\frac{1}{2}\int\Phi'_{|q|}(|q|)\frac{q^{\alpha}q^{\beta}q^{\gamma}}{|q|}\left\{h(\xi,q)-h(\xi,-q)\right\}dq\sim\mu.$$

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Thus

$$\frac{\partial \rho u^\alpha}{\partial t} = -\mu \frac{\partial}{\partial \xi^\beta}\left\{(\rho u^\alpha u^\beta) + T_{\alpha\beta}\right\} + \mu^3 \ldots, \qquad (5,10)$$

where

$$T_{\alpha\beta} = \frac{1}{m\nu}\int (p^\alpha - m u^\alpha)(p^\beta - m u^\beta)\, w(\xi, p)\, dp - \\ - \frac{1}{2\nu^2}\int \Phi'_{|q|}(|q|)\frac{q^\alpha q^\beta}{|q|} h(\xi, q)\, dq \qquad (5,11)$$

Which is the symmetrical tensor of the second degree. A basically similar conversion for (5,4) leads to the following result:

$$\frac{\partial \rho\theta}{dt} = -\mu\left\{\frac{\partial}{\partial \xi^\alpha}(u^\alpha \rho\theta) + T_{\alpha\beta}\frac{\partial u^\beta}{\partial \xi^\alpha} + \frac{\partial S^\alpha}{\partial \xi^\alpha}\right\} + \mu^3 \ldots, \qquad (5,12)$$

where $T_{\alpha\beta}$ is determined through (5,11) and

$$S^\alpha = \frac{1}{m\nu}\int \frac{(p - m u)^2}{2m}(p^\alpha - m u^\alpha)\, w(\xi, p)\, dp + \\ + \frac{1}{2\nu^2}\int \Phi(|q|) L_1^\alpha(\xi, q)\, dq - \\ - \frac{1}{4\nu^2}\int \Phi'_{|q|}(|q|)\frac{q^\alpha q^\beta}{|q|}\left(L_1^\beta(\xi, q) + L_2^\beta(\xi, q)\right) dq, \qquad (5,13)$$

$$L_j^\alpha(\xi, q) = \frac{i\hbar}{2m}\left\{\left(\frac{\partial}{\partial q_j'} - \frac{\partial}{\partial q_j}\right) f_2(\xi, q_1, q_2, q_1', q_2')\right\}_{\substack{q_1' = q_1 \\ q_2' = q_2}} - u^\alpha h(\xi, q). \qquad (5,14)$$

Thus we will notice once again that quantum effects in (5,10) and (5,12), not being analogous to the classical theory, are responsible for only the terms of the series no lower than the third degree of μ.

The comparisons of (5,4), (5,10), and (5,12) with (4,1), (4,2), and (4,3) respectively permit the immediate writing of:

$$A_1 = -\frac{\partial}{\partial \xi^\beta}\left(\rho(\xi) u^\beta(\xi)\right); \qquad (5,15)$$

$$B_1^\alpha = -\frac{\partial}{\partial \xi^\beta}\left\{\rho u^\alpha u^\beta + T^{(0)}_{\alpha\beta}\right\}; \qquad (5,16)$$

$$C_1 = -\frac{\partial}{\partial \xi^\alpha}(u^\alpha \rho\theta) + T^{(0)}_{\alpha\beta}\frac{\partial u^\beta}{\partial \xi^\alpha} + \frac{\partial S^\alpha}{\partial \xi^\alpha}; \qquad (5,17)$$

$$A_k = 0, \; k = 2, 3, \ldots;$$

$$B_2^\alpha = -\frac{\partial T^{(1)}_{\alpha\beta}}{\partial \xi^\beta}, \quad C_2 = T^{(1)}_{\alpha\beta}\frac{\partial u^\beta}{\partial \xi^\alpha} + \frac{\partial S^\alpha}{\partial \xi^\alpha}, \qquad (5,18)$$

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where $T_{\alpha\beta}^{(k)}$ is determined by the expression (5,11) in which w and h are placed instead of $w^{(k)}$ and $h^{(k)}$; S_{α}^{β} is similarly determined. Thus, we have come to the problem of finding $w^{(0)}$ and $h^{(0)}$ and fully determining the hydrodynamic equations of the first approximation (ideal fluid), of finding the ways of solving for $w^{(1)}$ and $h^{(1)}$ and, consequently, the possibility of constructing hydrodynamic equations of the second approximation (viscous fluidity).

VI. OBTAINING THE HYDRODYNAMIC EQUATIONS OF THE FIRST APPROXIMATION

As is known from the general theory of equilibrium conditions, in cases of the absence of motion of the system as a whole, i.e., the absence of macrovelocity, and in the case of space uniformity, the function of the distributed impulses will be w even $(|p|)$ and, due to our basic assumptions, will be dependent only on two "parameter conditions" - p and θ. If the system moves with the velocity u, then the function of distribution will be w even $(|p-mu|)$; this remains true even in the parametric dependence of w on ξ.

Thus, our function $w^{(0)}$ when $u=0$ appears as $w^{(0)}=w^{even}(|p|;\rho(\xi),\theta(\xi))$, and when $u\neq 0$, $w^{(0)}=w^{even}(|p-mu(\xi)|;\rho(\xi);\theta(\xi))$. From (3,13) we can determine $g_1^{(0)}$.

We will introduce a separate value for $u=0$

$$g_1^{(0)}(q_1-q_1')=g_1^{(0)}(q)=r_1(q)$$

Then

$$r_1(q)=\int w^{even}(|p|;\rho(\xi),\theta(\xi))e^{(i/\hbar)q^{\alpha}p^{\alpha}}dp. \qquad (6,1)$$

From (6,1) it is not difficult to see that

$$r_1(q)=r_1(-q). \qquad (6,2)$$

In the general case ($u\neq 0$)

$$g_1^{(0)}(q)=\int w^{even}(|p-mu|;\rho(\xi),\theta(\xi))e^{(i/\hbar)q^{\alpha}p^{\alpha}}dp=$$
$$=e^{(i/\hbar)mq^{\alpha}u^{\alpha}(\xi)}r_1(q;\rho(\xi),\theta(\xi)). \qquad (6,3)$$

Analysis of the (4,5) equation shows that they appear as equations for equilibrium conditions. Our problem appears now as the solving of the (4,5) equation with additional arrangements (4,8) - (4,10). It is not difficult to show that the solution of equation (4,5) in satisfying the arrangements (4,8)-(4,10), will appear as (6,3)

$$g_s^{(0)}(q_1,\dots,q_s;q_1',\dots,q_s')=$$
$$=e^{\frac{im}{\hbar}\sum_{1\leq j\leq s}(q_j'q_j-q_jq_j')u^{\alpha}(\xi)}r_s(q_1,\dots,q_s';\rho(\xi),\theta(\xi)) \qquad (6,4)$$

in which r_s has similar characteristics as in (6,2).

Actually, if $r_s(q_1,\dots,q_s)$ is solved by (4,5), thus satisfying the arrangements (4,8), (4,10), and $u\neq 0$, then $g_s(q_1,\dots,q_s)$ will also be solved by these solutions, and because of a similar solution, it should be:

$$r_s(q_1,\dots,q_s)=r_s(q_1',\dots,q_s). \qquad (6,5)$$

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The use of these solutions to carry on further transformations of the apparent form of $T^{(0)}_{\alpha\beta}$ and S^{α}_{β} does not present any special difficulty; it is only necessary to account at all times for the evenness or unevenness of the integral functions analyzing the expression. For $T^{(0)}_{\alpha\beta}$ we get:

$$T^{(0)}_{\alpha\beta} = \delta_{\alpha\beta}\left\{\frac{1}{3mv}\int |p-mu|^2 w^{even}(|p-mu|;\rho,\theta)\,dp - \right.$$

$$\left. -\frac{1}{6v^2}\int \Phi'_{|q|}(|q|)|q|r_2(q,\rho,\theta)\,dq\right\} = \delta_{\alpha\beta}P(\rho,\theta), \quad (6,6)$$

where δ is Kronecker's symbol, and the function P according to the known "virial" theory is the pressure.. As for S^{α}, it is not difficult to show that $S^{\alpha}_{\beta}=0$.

Actually, when $w^{e}=w^{even}$ $(|p - mu|)$, it is easy to notice that the integral expression of the first component in the right-hand side of (5,3) will be an odd-numbered function and, actually, the integral is equal to zero. Further, from (5,14) and the calculation of (6,4) and (6,5) we get

$$\underset{(j=1,2)}{L^{\alpha}_{j}} = \frac{i\hbar}{2m}\left\{\left(\frac{\partial}{\partial q^{\alpha}_{j}} - \frac{\partial}{\partial q^{\alpha}_{3}}\right) g^{(0)}_{2}\right\}_{\substack{q'_1=q_1\\ q'_2=q_2}} - u^{\alpha}\left\{g^{0}_{2}\right\}_{\substack{q'_1=q_1\\ q'_2=q_2}} = 0, \quad (6,7)$$

and, consequently, the second and third components in (5,3) will also be equal to zero, whence $S^{\alpha}_{0}=0$.

Thus, we get the hydrodynamic equations of the first approximation

$$\frac{\partial\rho}{\partial t} + \mu\frac{\partial(\rho u^{\alpha})}{\partial\xi^{\alpha}} = 0, \quad (6,8)$$

$$\frac{\partial\rho u^{\alpha}}{\partial t} + \mu\frac{\partial(\rho u^{\alpha}u^{\beta})}{\partial\xi^{\beta}} = -\mu\frac{\partial P(\rho,\theta)}{\partial\xi^{\alpha}}, \quad (6,9)$$

$$\frac{\partial\rho\theta}{\partial t} + \mu\frac{\partial(\rho u^{\alpha}\theta)}{\partial\xi^{\alpha}} = -\mu P(\rho,\theta)\frac{\partial u^{\alpha}}{\partial\xi^{\alpha}}, \quad (6,10)$$

i.e., we obtained the general hydrodynamic equations of ideal fluids, from which we have completed the problem of this paper. The analysis of the equations (6,8) - (6,10) shows that quantum-mechanical (and not the classical) description, coming from the molecular model, appears in the computation of the function P. Although the formula (6,6) determines P, it is similar to the classical, however, and the change of the classical function of distribution is similar to the quantum. The apparent form of these analogies is connected with the further development of the molecular model. Thus, for example, in the determination of w^{even} we may have the known function of distribution of Bose.

It remains for us to consider the general perspectives of calculating the kinetic coefficients in this way.

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Equation (4,6) is distinguished from (4,5) only by the presence of a nonuniform term. In the calculation of our results it is easy to be convinced that this nonuniform term will appear as a linear combination of primary derivatives for ξ^{α} from the functions ρ, u^{β} and θ; coefficients of these derivatives remain unknown. It follows from this that, in accordance with the general theory of nonuniform linear equations, the solution of (4,6), i.e., $g_s^{(1)}$, will depend linearly on the denoted higher derivatives. If we investigate $T_{\alpha\beta}^{(1)}$ and $S_{(1)}^{\alpha}$ where the solution of $g_s^{(1)}$ is substituted in satisfying the additional arrangements of (4,11) - (4,13), then we will arrive at expressions, by way of the determined mathematical operations, which are similar to the classical. Similar comparisons will show us the true physical interpretation of the coefficients of derivatives, and clear expressions of these coefficients through g_s, will give us a solution for the calculation of the kinetic coefficients (viscosity and thermal conductivity) coming from the quantum-mechanical molecular picture.

The immediate realization of the scheme put forth here requires greater development of the molecular picture, i.e., the selection of some concrete model. In particular, such a selection will fully determine $w^{even}(|p|)$ and also r_1. The determination of r_s needs the computation of a series of additional considerations; for example, the computation "of a condition of decreasing correlation" for the matrix F_s, etc.

The solution of the indicated problems will serve as a subject for future study.

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