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SPHERICAL ELECTROMAGNETIC WAVES

V. Sorokin
Ivanovo State Pedagogical Inst
Submitted 25 June 1947

Simple expressions for spherical electromagnetic waves in a vacuum and multipolar electromagnetic fields were obtained by elementary methods with the aid of spherical vector functions.

In the theory of radiation, plane electromagnetic waves are generally considered. This is done even in problems with spherical symmetry (for example, in the investigation of atomic radiations) when it would be more natural to consider spherical waves. In all problems with spherical symmetry spherical waves, to which group belong photons with a definite impulse moment are more appropriate than plane waves. It is known, for example, that the rules of selection are obtained without any computations from the principle of conservation of angular momentum; nevertheless, the significance of the use of plane waves is not generally understood. If we attempted to do away with spherical waves wherever possible, then we would be forced to deal with the clumsy and nonsymmetrical formulas that would result.

These formulas are obviously very awkward [1,2] but this is due not to the existing conditions but to the unsuccessful methods of examination. Vector fields must be broken down into spherical vector functions from which simple and symmetrical expressions are obtained, the use of which is as easy as is the use of plane waves.

I. SPHERICAL VECTOR FUNCTIONS

Spherical vector functions can be constructed in essentially the same way as Laplace's spherical scalar functions. Let us examine the surface of the sphere of the vector field $\vec{\xi}$, i.e., a three-dimensional vector, at any given point on the surface of the sphere. We will rotate this vector field through an infinitely small angle $\delta\omega$, around the axis η, which passes through the center of the sphere which is assumed to be solid. At point r, where there had earlier been the vector $\vec{\xi}$, there is now the vector $\vec{\xi}'$, which until the rotation was located at the point $r' = r - \delta\omega(n r)$. It can be said that the vector

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$\vec{\xi}$ at the point r in increased [illegible] which obviously is made up of two parts: first, the vector $\vec{\xi}'$ which until the transposition was distinguished from $\vec{\xi}$ in magnitude by

$$\delta_1 \vec{\xi} = -(\delta r \nabla)\vec{\xi} = -\delta\omega(n[r\nabla])\vec{\xi},$$

second, in the transposition it is rotated and is increased by

$$\delta_2 \vec{\xi} = \delta\omega[n\vec{\xi}].$$

The complete change in point r will be

$$\delta\vec{\xi} = \delta_1\vec{\xi} + \delta_2\vec{\xi} = \delta\omega\{-(n[r\nabla])\vec{\xi} + [n\vec{\xi}]\}.$$

Infinitely small rotation around the axis n on a single angle corresponds to the operator I_n, determining the equation:

$$\delta\vec{\xi} = I_n\vec{\xi} = -(n[r\nabla])\vec{\xi} + [n\vec{\xi}], \qquad (1)$$

and the rotation about the axis X, Y, Z, will correspond to the operators

$$I_x\vec{\xi} = -[r\nabla]_x\vec{\xi} + [n_x\vec{\xi}] \qquad (2)$$

etc., where n_x is the transversal of the axis X.

We will construct the operator

$$I^2 = I_x^2 + I_y^2 + I_z^2, \qquad (3)$$

for which, with the aid of (2), there is obtained after simple conversion:

$$I^2\vec{\xi} = [r\nabla]^2\vec{\xi} - 2[[r\vec{\nabla}]\vec{\xi}] - 2\vec{\xi}, \qquad (4)$$

We find the proper function of this operator, i.e., vector function $\vec{\xi}$ for which the equation

$$I^2\vec{\xi} = \lambda\vec{\xi}, \qquad (5)$$

takes place, where λ is the natural value. We will need to have the vector $\vec{\xi}$ well defined and not to have any special features on the surface of the sphere. The natural values will be expressed by the form

$$\lambda = -l(l+1), \qquad (6)$$

since it will appear further (as can be predicted from the general considerations) that $l = 0, 1, 2, \ldots$ The solution of equation (5) for any l we will call the spherical vector function of the l series.

To construct the equations clearly, we will introduce spherical coordinates r, ϑ, φ and we will design both parts of equation (5) on coordinate lines. Taking for I^2, equation (4) and making the conversions, we get three equations for spherical components of the spherical vector functions. Designating Λ as the operator entering into Laplace's equation of spherical functions

$$\Lambda = \frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial}{\partial\vartheta}\right) + \frac{1}{\sin^2\vartheta}\frac{\partial^2}{\partial\varphi^2},$$

then our equations will be

$$\Lambda\xi_r = -l(l+1)\xi_r, \qquad (7)$$

$$\Lambda\xi_\vartheta - \frac{\xi_\vartheta}{\sin^2\vartheta} - \frac{2\cos\vartheta}{\sin^2\vartheta}\frac{\partial\xi_\varphi}{\partial\varphi} = -l(l+1)\xi_\vartheta, \qquad (8)$$

$$\Lambda\xi_\varphi - \frac{\xi_\varphi}{\sin^2\vartheta} + \frac{2\cos\vartheta}{\sin^2\vartheta}\frac{\partial\xi_\vartheta}{\partial\varphi} = -l(l+1)\xi_\varphi. \qquad (9)$$

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Obviously, with these equations one can determine whether to put both transversals making up the vector equal to zero, and for the radial component, whether to take the usual spherical function of the series Y_ℓ.

We then get on the surface of the sphere a radial vector field, which can be called a spherical vector function of the first type

$$\vec{\xi}^{(1)} = \frac{r}{r} Y_\ell \tag{10}$$

After this we can examine equations (8) and (9) separately. With the aid of the spherical functions we can easily see that we will get a solution if we get

$$\xi_r^{(2)} = 0, \quad \xi_\vartheta^{(2)} = \frac{\partial Y_\ell}{\partial \vartheta}, \quad \xi_\varphi^{(2)} = \frac{1}{\sin\vartheta}\frac{\partial Y_\ell}{\partial \varphi}$$

or in the vector form

$$\vec{\xi}^{(2)} = r \nabla Y_\ell \tag{11}$$

This vector field we shall call the spherical vector function of the second type.

It is a transversal and if we examine it as a two-dimensional field on the surface of the sphere, it is not vortical. Actually, the radial component of the vortex is

$$[\nabla \vec{\xi}^{(2)}]_r = [\Delta r \Delta Y]_r = 0$$

Finally, we must notice that equations (8) and (9) change into each other if ξ_ϑ is changed to ξ_φ, and ξ_φ to ξ_ϑ; then we can instantly obtain a third solution

$$\xi_r^{(3)} = 0, \quad \xi_\vartheta^{(3)} = -\frac{1}{\sin\vartheta}\frac{\partial Y_\ell}{\partial \varphi}, \quad \xi_\varphi^{(3)} = \frac{\partial Y_\ell}{\partial \vartheta}$$

or

$$\vec{\xi}^{(3)} = [r \nabla Y_\ell]. \tag{12}$$

This field is obtained from a field of the second type by rotating the latter around a given point $\pi/2$ times in a positive direction. Its divergence (on the surface of the sphere, as in space) is equal to zero. This will be the spherical vector function of the third type.

Since all these solutions are expressed through general spherical functions, it is clear that ℓ should be a positive integer of zero, as we observed earlier. Other solutions which satisfy our conditions, other than (10) and (12), do not have equations.

For a given ℓ there can be constructed $2\ell+1$ independent linear functions of each type; for this it is sufficient to take in place of Y_ℓ, for example, the function $Y_{\ell m}$. Thus when $\ell \neq 0$ there are obtained the functions:

$$\vec{\xi}_\ell^{(1)} = \frac{r}{r} Y_{\ell m}, \quad \vec{\xi}_{\ell m}^{(2)} = r \nabla Y_{\ell m}, \quad \vec{\xi}_{\ell m}^{(3)} = [r \nabla Y_{\ell m}] \tag{13}$$

At $\ell = 0$ both transversal functions become zero (or $Y_0 = 1/4\pi$), and we have only one function

$$\vec{\xi}_{00}^{(1)} = r/r. \tag{13'}$$

Functions of each type are converted during rotation into functions of the same type.

Actually, first, the radial field remains radial when rotating. Second, the conditions under which the vortex or origin is absent also do not change in rotation. It is also easy to see that if for any two-dimensional field on the surface of the sphere $\operatorname{div} \vec{\xi} = \operatorname{rot} \vec{\xi} = 0$ simultaneously (div and rot are both two-dimensional here) then this field is equal to zero. Consequently, the vector of the second type cannot be converted during rotation into a linear combination of vectors of the second and third types, etc.

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Sometimes it will be better to designate spherical vector functions by $\vec{\xi}$ (klm) since when $l \neq 0$, $l \neq 1$, $l \neq 0$ and when $l = 0$, $l = 1$.

Spherical vector functions are orthogonal to each other in that if (klm) $\neq$ (k'l'm') then the integral along the surface of the sphere from the scalar product is equal to zero

$$\int \vec{\xi}^{*}(klm)\, \vec{\xi}(k'l'm')^{*}\, d\Omega = 0 \qquad (14)$$

Actually, it is obvious that $k \neq k'$ since the vectors of various types are orthogonal at every point. If $k = k'$, then the proof is usually made through the self-combining of the operator (4).

In normalizing $\vec{\xi}$ it is sufficient to let the spherical functions Y_{lm} be normalized in such a way that

$$\int Y_{lm}\, Y^{*}_{l'm'}\, d\Omega = \delta_{ll'}\, \delta_{mm'}. \qquad (15)$$

Then

$$\int \vec{\xi}^{(1)}_{lm}\, \vec{\xi}^{(1)*}_{l'm'}\, d\Omega = \delta_{ll'}\, \delta_{mm'},$$
$$\int \vec{\xi}^{(2,3)}_{lm}\, \vec{\xi}^{(2,3)*}_{l'm'}\, d\Omega = l(l+1)\, \delta_{ll'}\, \delta_{mm'}, \qquad (16)$$

which can be easily calculated by using the expressions (11) and (12).

We will not demonstrate that the combination of all spherical vector functions $\vec{\xi}$(klm) form a complete system, i.e., that any proper vector function given on the sphere can be resolved on it.

II. SPACE ELECTROMAGNETIC FIELD

We will now examine an electromagnetic field without charges. We will consider that the field periodically changes with time so that all potentials contain the term $e^{-i\omega t}$. We will introduce the wave number $k = \omega/c$; then the equations for the potentials A and φ will be

$$(\nabla^2 + k^2) A = 0, \quad (\nabla^2 + k^2)\varphi = 0, \quad \nabla A - ik\varphi = 0 \qquad (17)$$

Let us resolve the potential φ along the spherical functions of Laplace, and the vector potential A along the spherical vector functions. These terms in the analyses, which contain functions of series l, will have the forms

$$\varphi = \varphi_l(r)\, Y_l, \qquad (18)$$

$$A = f_l(r)\frac{r}{r}\, Y_l + g_l(r)\, r\nabla Y_l + h_l(r)\left[r \nabla Y_l\right] \qquad (19)$$

(The index l will henceforth be omitted)

These expressions we shall place in equation (17) and complete the differentiation. We again obtain only spherical functions of the l series, so that because of their orthogonal character the equations of (17) decompose into a series of equations for each l separately. This means that there are electromagnetic fields of various degrees and a very general field is obtained by their superposition.

Completing the computation and gathering the terms with spherical functions of every degree, we get for the radial parts of the potentials of the equations:

$$f'' + \frac{2f'}{r} - \frac{2 + l(l+1)}{r^2} f + k^2 f + \frac{2\, l(l+1)}{r^2} g = 0. \qquad (20)$$

$$g'' + \frac{2g'}{r} - \frac{l(l+1)}{r^2} g + k^2 g + \frac{f}{r^2} = 0, \qquad (21)$$

$$h'' + \frac{2h'}{r} - \frac{l(l+1)}{r^2} h + k^2 h = 0, \qquad (22)$$

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$$\varphi'' + \frac{2\varphi'}{r} - \frac{l(l+1)}{r^2}\varphi + k^2\varphi = 0, \qquad (23)$$

$$f' + \frac{2f}{r} - \frac{2(l+1)}{r}g - ikq = 0 \qquad (24)$$

We will notice that when $k=0$ it is necessary (since then there are only functions of the first type) to compute $g=h=0$. But then the equations (21) and (24) will give $f=\varphi=0$ i.e., a wave of zero order does not exist.

From the equations obtained it is obvious that the solutions will be of two types: (1) "magnetic" waves for which $f=g=\varphi=0$ and (2) "electric" waves for which $h=0$.

To obtain clear expressions for potentials in both cases, we will signify by $q_l(r)$ the solution of the equation:

$$q'' + \frac{2q'}{r} - \frac{l(l+1)}{r^2}q + k^2 q = 0, \qquad (25)$$

When $r=0$ we have

$$q_l(r) = \sqrt{\frac{\pi}{2kr}}\, I_{l+\frac{1}{2}}(kr),$$

where I is the Bessel function. Then for magnetic waves we get from (22)

$$A = C q_l(r)[r\nabla Y_l]. \qquad (26)$$

For electrical waves it is necessary to take, as can easily be proven,

$$\begin{aligned} f &= A\frac{q}{r} + Bq', \\ g &= \frac{A}{l(l+1)}\left[q' + \frac{q}{r}\right] + B\frac{q}{r}, \\ \varphi &= ikBq, \end{aligned} \qquad (27)$$

where A, B, C, are arbitrary constants. In computing the electrical and magnetic fields E and H, it appears that the constant B does not enter into their expressions. A different selection of B signifies gradient conversion of potentials.

The general formulas (27) will be useful further on. Here we will make B = 0 and will write out the potentials and fields of spherical electromagnetic waves of series l in empty space. For magnetic waves we get

$$E = ikA = ikCq[r\nabla Y], \qquad (28)$$

$$H = -C\left[\frac{l(l+1)}{r}q\frac{r}{r}Y + \left(q' + \frac{q}{r}\right)r\nabla Y\right], \qquad (28')$$

and for electrical waves,

$$E = ikA = ikA\left[-\frac{q}{r}\frac{r}{r}Y + \frac{q' + (q/r)}{l(l+1)}r\nabla Y\right], \qquad (29)$$

$$H = -\frac{Ak^2}{l(l+1)}q[r\nabla Y]. \qquad (29')$$

Formulas (29) are obtained (correct up to the constant) from (28) by changing E to H and H to E.

III. RADIATION OF A SYSTEM OF CHARGES

We will now suppose that near the point $r=0$ the distribution of charges and currents periodically change with time. In these analyses the density of the

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charge ρ and of the current j which can be resolved along spherical functions and the terms of the series j_1, can be expressed as

$$\rho(r)Y;\ j_1(r)\frac{r}{r}Y+j_2(r)r\nabla Y+j_3(r)[r\times\nabla Y] \quad (30)$$

(the index ℓ will henceforth be omitted). From the equation of continuity we get

$$-i\omega\rho+j_1'+\frac{2j_1}{r}-\frac{\ell(\ell+1)}{r}j_2=0. \quad (31)$$

The equations for the potentials will now be

$$(\nabla^2+k^2)A=-\frac{4\pi}{c}j,\ (\nabla^2+k^2)\varphi=-4\pi\rho, \quad (32)$$
$$\nabla A-ik\varphi=0.$$

Correspondingly, we get for the radial parts of the potentials a system of nonuniform equations, which are distinguished from a uniform system such as (20)-(24) by the presence in the right-hand sides of the equations (20) - (23) of the corresponding terms

$$-\frac{4\pi}{c}j_1,\ -\frac{4\pi}{c}j_2,\ -\frac{4\pi}{c}j_3,\ -4\pi\rho.$$

Lorentz' conditions (24) remain as before.

We will designate $q(r)$ and $S(r)$ two independent linear solutions of the equation (25) which we shall select in the following way

$$S(r)=\sqrt{\frac{\pi}{2kr}}H^{(1)}_{\ell+1/2}(kr);\ q(r)=\sqrt{\frac{\pi}{2kr}}I_{\ell+1/2}(kr) \quad (33)$$

(H and I are Hankel's and Bessel's functions).

When $k\gg 1$ we obtain for them the asymptotic expressions:

$$S=\frac{1}{kr}e^{i(kr-\frac{\ell+1}{2}\pi)};\ q=\frac{1}{kr}\cos(kr-\frac{\ell+1}{2}\pi),$$

and when $kr\ll 1$, the expressions:

$$S=-i\frac{(2\ell)!}{2^{\ell}\ell!}(kr)^{-\ell-1};\ q=\frac{2^{\ell}\ell!}{(2\ell+1)!}(kr)^{\ell} \quad (34)$$

It is easy to see that at all values of r

$$Sq'-qS'=-\frac{i}{kr^2} \quad (35)$$

The uniform system of equations, which is obtained if the right-hand part is discarded in (32), can also be solved as shown in section I. However, we now need a general solution and Lorentz's equation will not be examined. With the aid of (27) we get.

$$f=A\frac{S}{r}+BS'+\tilde{A}\frac{q}{r}+\tilde{B}q'; \quad (36)$$
$$g=\frac{A}{\ell(\ell+1)}[S'+\frac{S}{r}]+B\frac{S}{r}+\frac{\tilde{A}}{\ell(\ell+1)}[q'+\frac{q}{r}]+\tilde{B}\frac{q}{r};$$
$$h=CS+\tilde{C}q;\ \varphi=DS+\tilde{D}q \quad (36')$$

The solution of a nonuniform system is obtained by the method of varying the constants. Performing the computation, we get:

$$A=\frac{4\pi i}{ck}\ell(\ell+1)\int_0^r[\frac{q}{r}j_1+(q'+\frac{q}{r})j_2]r^2dr;$$
$$B=\frac{4\pi i}{ck}\int_0^r[q'j_1+\ell(\ell+1)\frac{q}{r}j_2]r^2dr; \quad (37)$$
$$C=\frac{4\pi ik}{c}\int_0^r qj_3r^2dr;\ D=4\pi ik\int_0^r q\rho r^2dr.$$

The remaining four coefficients ($\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{D}$) are obtained if, in these

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expressions, q is changed to s and the integrals are taken from r to ∞.

They become zero in the regions occupied by charges and therefore we do not write them out. The constant integration in (37) was so selected that the potentials would have the continual appearance of expanding waves, and they would be definite at the beginning of the coordinates. These requirements determine the potentials identically, Lorentz's conditions are satisfied by the equation of continuity.

If the limiting conditions were established not for the potentials but for fields, then the expressions could be increased to f, g and φ, proportionally corresponding to S', S/r, ik and q', q/r, ikq from which the fields would not change. The formulas (36) and (37) give exact solutions for all outer spaces.

Let us examine the final case when the volume of the space occupied by the charges is small in comparison with the wave length. In the outer space the integrals in formulas (37) can be taken from 0 to ∞, and instead of q the expression (34) can be placed as an approach to them. After simple conversions in which the equation of continuity is used, we get, by making

$$\alpha = i\,\frac{2^{l}\,l!}{(2l+1)!}, \tag{38}$$

$$A = (l+1)B = -i\,\alpha(l+1)k^{l}\,4\pi\int^{\infty} p\,r^{l+2}\,dr;$$

$$C = \alpha k^{l+1}\frac{4\pi}{c}\int^{\infty} j_3\,r^{l+2}\,dr, \quad D = \alpha k^{l+1}\,4\pi\int^{\infty} p\,r^{l+2}\,dr.$$

We will designate the components lm as the electrical moment of the series l by D_{lm}:

$$D_{lm} = 4\pi\int^{\infty} p_{lm}\,r^{l+2}\,dr, \tag{39}$$

and the magnetic moment of series l by M_{lm}:

$$M_{lm} = \frac{4\pi}{c}\int^{\infty} j^{(3)}_{lm}\,r^{l+2}\,dr. \tag{39'}$$

Then the potentials in the outer space will be:

For electrical waves of series,

$$A_{lm} = \alpha k^{l} D_{lm}\left[S' + \frac{(l+1)S}{r}\right]\left[\frac{r}{l}Y_{lm} + \frac{r\nabla Y_{lm}}{l}\right]; \tag{40}$$

For magnetic waves,

$$\varphi_{lm} = \alpha k^{l+1} D_{lm}\,S\,Y_{lm}, \tag{40'}$$

$$A_{lm} = \alpha k^{l+1} M_{lm}\,S\,[r\nabla Y_{lm}]. \tag{41}$$

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