Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

25X1

Page Denied

Next 1 Page(s) In Document Denied

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

Joint Institute of Nuclear Research

Theoretical Physics Laboratory

N. N. Bogolubov

About New Method in the Theory of Superconductivity

III

1957

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

In the past great success was attained in solution of statistical physics problems using the method of summation of most important graphs.

In present article we will show, that in the theory of superconductivity it is possible to obtain by this method the same results, that were found in previous works (1, 2) with the aid of canonic transformation and principle of compensation of graphs with a "dangerous" energy denominator.

As was shown by V. V. Tolmachev and S. V. Tiablikov, it is possible to consider the Bardeen's Hamiltonian instead of Fröhlich's Hamiltonian, for they both are actually completely equivalent in accounting the influence of electron-phonon interaction on the electron dynamics near the Fermi surface. In this case it is much simpler to use Bardeen's Hamiltonian.

Therefore, for the sake of illustration and establishment of connection with the ideas presented in the work of Bardeen, Cooper and Schrieffer (3), we shall proceed from Bardeen's Hamiltonian:

$$H_B = \sum_{k,s} E(k)\, a^+_{ks} a_{ks} -$$

$$- \frac{\gamma}{V} \sum_{(k_1,k_2,k_1',k_2')} a^+_{k_1,-\frac{1}{2}} a^+_{k_2,\frac{1}{2}} a_{k_2',\frac{1}{2}} a_{k_1',-\frac{1}{2}} \Theta(k_1)\Theta(k_2)\Theta(k_1')\Theta(k_2')\Delta(k_1+k_2-k_1'-k_2')$$

where

$$\Theta(k) = \begin{cases} 1, & E(k_F) - \omega < E(k) < E(k_F) + \omega \\ 0, & |E(k) - E(k_F)| > \omega \end{cases}$$

$$\Delta(k) = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases}$$

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

and where $E(K)$ is a radially symmetrical function representing the energy of an electron with impulse K,

γ & V are the Bardeen's parameters.

In Fröhlich's model we should let [2].

$$\gamma = g^2 , \quad \omega = \frac{\tilde{\omega}}{2}$$

We will account the quantity N of whole number of electrons with the aid of chemical potential λ. For this we add $-\lambda N$ to H_B

Hence we get Hamiltonian

$$H = H_0 + H_{int}$$

$$H_0 = \sum_{K,s} \{E(K) - \lambda\} a^+_{K,s} a_{K,s} \qquad (1)$$

$$H_{int} = \frac{\gamma}{V} \sum a^+_{K_1,-\frac{1}{2}} a^+_{K_2,\frac{1}{2}} a_{K_2',\frac{1}{2}} a_{K_1',-\frac{1}{2}} \Theta(K_1)\Theta(K_2)\Theta(K_1')\Theta(K_2')\Delta(K_1+K_2-K_1'-K_2')$$

for which we shall consider the question about the summation of the most important graphs. In so far as the interaction is effective only in a small region of Fermi surface and only between the particles (electrons or holes) with oppositely oriented spins, we see that the graphs of the type illustrated in Fig. 1 will play an especially important role. These graphs were constructed from "indivisible complex" (see Fig. 2)

Fig. 1

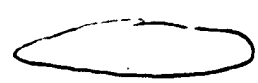

Fig. 2

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

consisting of a pair of particles with impulse $\pm K$ and spins $\pm\frac{1}{2}$. For their summation we will use the method of approximate secondary quantization i.e. we will construct a simplified Hamiltonian which will have graphs only of that class which we wish to sum up, but with the same contribution as in real Hamiltonian.

In so far as in the presently examined graphs the groups of the particle pair ($\pm K$, $\pm\frac{1}{2}$) do not break, it is natural to compare them with the quantum amplitudes b_k, b_k^+ with commutation relations:

$$[b_k, b_{k'}] = 0, \quad [b_k^+, b_{k'}^+] = 0, \quad [b_k^+, b_{k'}] = 0; \; k \neq k' \qquad (2)$$

Further, because several pairs with the same value of $\vec{k}$ do not exist, we must let

$$b_k^2 = 0 \qquad (3)$$

We should also note, that the group's self energy will be

$$2\{E(k) - \lambda\} b_k^+ b_k$$

and that the matrix element of Hamiltonian (1) for transition will be proportional $-\frac{\gamma}{V}$.

From these considerations we get the simplified Hamiltonian in form

$$H = H_0 + H_{int}$$

$$H_0 = \sum_k 2\{E(k) - \lambda\} b_k^+ b_k \qquad (4)$$

$$H_{int} = -\frac{\gamma}{V} \sum_{k \neq k'} b_k^+ b_{k'} \, \theta(k) \, \theta(k')$$

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

containing the operators b_k, b_k^+, which satisfy the relations (2), (3). Let us call these operators the Pauli-operators. Let us take an expression of any order

$$H_{int}(H_0-E)^{-1}H_{int}(H_0-E)^{-1}H_{int}$$

now, it is easy to verify directly that, the sum of contributions from the graphs of the type being examined for the Hamiltonian (I) will be equal to the sum of contributions of all the graphs for the simplified Hamiltonian (4). Hence the problem of summation of the special class of graphs for Hamiltonian (I) turns out to be equivalent to the problem of dynamic system model, characterized by Hamiltonian (4).

We start the construction of asymptotically exact solution of this latter problem, neglecting only those values that disappear in the process of the limiting transition $V \to \infty$ We shall distinguish the electron pairs from hole pairs. Therefore, we introduce new Pauli-operators by letting

$$\beta_k = b_k \quad , \quad E(k) > \lambda$$
$$\beta_k = b_k^+ \quad , \quad E(k) < \lambda$$

We get

$$H = U + 2\sum_k |E(k)-\lambda| \beta_k^+ \beta_k -$$
$$- \frac{\gamma}{V}\sum_{kk'} \theta(k)\theta(k')\{\theta_F(k)\beta_k^+ + \theta_F(k)\beta_k\}\{\theta_G(k')\beta_{k'} + \theta_F(k')\beta_{k'}^+\} \quad (5)$$

where

$$\theta_F(k) + \theta_G(k) = 1$$
$$\theta_F(k) = \begin{cases} 1, & E(k) < \lambda \\ 0, & E(k) > \lambda \end{cases}$$

$$U = 2\sum_k \{E(k)-\lambda\}\theta_F(k)$$

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

Let [illegible]

numbers [illegible]

[illegible]

[illegible]

exact [illegible]

Indeed we have

$$H = H' + H'' + U$$

$$H' = 2\sum_{k} |E(k) - \lambda| \beta_k^{+} \beta_k -$$

$$- \frac{J}{V} \sum_{k \neq k'} \theta(k)\theta(k') \{\theta_G(k)\beta_k^{+} + \theta_F(k)\beta_k\} \theta_G(k')\beta_{k'} -$$

$$- \frac{J}{V} \sum_{k \neq k'} \theta(k)\theta(k')\theta_F(k)\theta_F(k')\beta_k^{+}\beta_{k'}$$

$$H'' = - \frac{J}{V} \sum_{k \neq k'} \theta(k)\theta(k')\theta_F(k)\theta_F(k')\beta_k^{+}\beta_{k'}^{+}$$

But obviously

$$H \; [illegible]$$

On the other hand

$$\langle C^{*} | H'' |^{2} C \rangle = \langle C^{*} H''^{+} H'' C \rangle$$

$$= \frac{J^2}{V^2} \sum_{k} \theta(k)\theta(k')\theta_G(k)\theta_F(k') \langle \text{[illegible]} \rangle$$

$$V \to \infty$$

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

But in the process of [illegible]

be proportional to V, [illegible]

remains finite [illegible]

proper function of [illegible] we have

$$\bar{N} = \langle C^* N C \rangle = \sum_{\varepsilon(k) < \lambda} 2$$

By equating this expression [illegible] the whole number of electrons in the Fermi shell

$$\sum_{\varepsilon(k) < \varepsilon_F} 2$$

we see, that

$$\lambda = E_F = E(K_F)$$

Now, let us analyze the question of stability of condition C. Let us examine first the case when

$$\mathcal{J} < 0$$

To the double sum of (5) we add the members for which $k = k'$. Doing this we do not introduce any quantities that would contribute in transition to the limit $V \to \infty$. We note then, that H-U is essentially equal to the positive form. The value of U, consequently, will be minimum, and therefore the state C will be stable. A different situation will be in the case

$$\mathcal{J} > 0$$

Note, that in this case in state C all occupation numbers $n_k = \beta_k^* \beta$ are equal to zero; and in calculation of elementary excitation energy we can regard the auxiliary operators β, β^* as Bose-operators. It only remains then to diagonalize the quadratic form from operators β, β^* representing H-U (5). This diagonalization can be done, for example with the aid

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

of the [illegible]

For [illegible]

E we get the [illegible]

$$1 = \frac{\gamma}{V} \sum_{k} \theta(k) \left\{ \frac{[illegible]}{\epsilon_k - [illegible]} + \frac{[illegible]}{\epsilon_k [illegible]} \right\}$$

$$\epsilon_k = 2 | E(k) - E [illegible]$$

from where we find after simplifi[illegible]

$$1 = \frac{\gamma}{V} \sum_{(E_F < E(k) < E_F + \omega)} \frac{2\epsilon_k}{\epsilon_k - E^2}$$

or

$$1 = \frac{\rho}{2} \int_0^{\omega} k^2 \frac{dk}{dE} \frac{2z\,dz}{z^2 - E^2/2} \qquad (9)$$

where

$$\rho = \frac{\gamma}{(2\pi^2)} \left(k^2 \frac{dk}{dE} \right)_{k = k_F}$$

As is evident, this equation in the case under examination (9) always has negative root for E. Consequently, we get purely imaginary [illegible] for the energy E

$$E \sim i\,2\omega e^{-\frac{1}{\rho}}$$

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

Hence, the s[illegible] stable condit[illegible] amplitudes β_k, β_k^+ [illegible] but with [illegible] and [illegible]

$$\beta_k = U_k V_k (2 b_k^+ b_k - 1) + U_k^2 b_k - V_k^2 b_k^+$$

$$\beta_k^+ = U_k V_k (2 b_k^+ b_k - 1) - V_k^2 b_k + U_k b_k^+ \qquad (10)$$

Where U_k, V_k - are real numbers, bound by equality

$$U_k^2 + V_k^2 = 1 \qquad (11)$$

It is easy to observe, that amplitudes of (10) really satisfy all commutation relations of [illegible]-operator[illegible]

Reducing the transformation (10), we find

$$b_k = U_k V_k (1 - 2\beta_k^+ \beta_k) + U_k^2 \beta_k - V_k^2 \beta_k^+$$

$$b_k^+ = U_k V_k (1 - 2\beta_k^+ \beta_k) - V_k^2 \beta_k + U_k^2 \beta_k^+$$

$$b_k^+ b_k = V_k^2 + (U_k^2 - V_k^2)\beta_k^+ \beta_k + U_k V_k (\beta_k + \beta_k^+) \qquad (12)$$

Substituting these expressions into Hamiltonian (4), we find

$$H = U + \sum_k \Big\{ 2(E(k) - \lambda) U_k V_k - \qquad (13)$$

$$- \frac{g}{V} \theta(k) (U_k^2 - V_k^2) \sum_{k'} \theta(k') V_{k'} U_{k'} \Big\} (\beta_k + \beta_k^+)$$

$$+ \sum_k 2 E_e(k) \beta_k^+ \beta_k -$$

$$- \frac{g}{V} \sum_{k_1 \neq k_2} \{ U_{k_1}^2 \beta_{k_1}^+ - V_{k_1}^2 \beta_{k_1} - 2 U_{k_1} V_{k_1} \beta_{k_1}^+ \beta_{k_1} \}$$

$$\times \{ U_{k_2}^2 \beta_{k_2} - V_{k_2}^2 \beta_{k_2}^+ - 2 U_{k_2} V_{k_2} \beta_{k_2}^+ \beta_{k_2} \} \theta(k_1) \theta(k_2)$$

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

where $$U = \sum_{k} 2\{E(k) - \lambda\} V_k^2 - \frac{\gamma}{V} \sum_{k_1, k_2} \theta(k_1)\theta(k_2) U_{k_1} V_{k_1} U_{k_2} V_{k_2} \qquad (14)$$

$$E_e(k) = \{E(k) - \lambda\} U_k V_k + \theta(k) U_k V_k \frac{2\gamma}{V} \sum_{k'} \theta(k') U_{k'} V_{k'} \qquad (15)$$

In the expression (12) at the coefficients of β_k - β_k^* equal to zero. We get then the equation

$$2\left(E(k) - \lambda\right) U_k V_k - \frac{\gamma}{V} \theta(k)\left(U_k^2 - V_k^2\right) \sum_{k'} \theta(k') U_{k'} V_{k'} = 0 \qquad (16)$$

found in work by (2) with the aid of dangerous graph compensation principle. Noticing that with required here exactnes $\lambda = E(k_F)$ we get same as in (2)

$$U^2(k) = \frac{1}{2}\left\{1 + \frac{E(k) - E(k_F)}{\sqrt{\{E(k) - E(k_F)\}^2 + \theta(k) C^2}}\right\}$$

$$V^2(k) = \frac{1}{2}\left\{1 + \frac{E(k_F) - E(k)}{\sqrt{\{E(k) - E(k_F)\}^2 + \theta(k) C^2}}\right\}$$

$$C = 2\omega e^{-1/\lambda}$$

$$E_e(k) = \sqrt{\{E(k) - E(k_F)\}^2 + \theta(k) C^2} \qquad (17)$$

Hamiltonian (13) now can be presented in form of:

$$H = H_0 + H' + H'' + U$$

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

where $H_0 = \sum_k 2E_0(k)\beta_k^+\beta_k$

$$H' = -\frac{\gamma}{V}\sum_{k_1 \neq k_2}\theta(k_1)\theta(k_2)\{U_{k_1}^2\beta_{k_1}^+ - V_{k_1}^2\beta_{k_1}\}\{U_{k_2}^2\beta_{k_2} - V_{k_2}^2\beta_{k_2}^+\}$$

$$H'' = 2\frac{\gamma}{V}\sum_{k_1 \neq k_2}\theta(k_1)\theta(k_2)\{U_{k_2}^2\beta_{k_2} - V_{k_2}^2\beta_{k_2}^+ - 2U_{k_2}V_{k_2}\beta_{k_2}^+\beta_{k_2}\}U_{k_1}V_{k_1}\beta_{k_1}^+\beta_{k_1} + \tag{18}$$

$$+\frac{2\gamma}{V}\sum_{(k_1 \neq k_2)}\theta(k_1)\theta(k_2)\{U_{k_1}^2\beta_{k_1}^+ - V_{k_1}^2\beta_{k_1} - 2U_{k_1}V_{k_1}\beta_{k_1}^+\beta_{k_1}\}U_{k_2}V_{k_2}\beta_{k_2}^+\beta_{k_2}$$

Let us take the wave function C, for which all occupation numbers $\nu_k = \beta_k^+\beta_k$

are equal to zero. We will show, as previously, that with accuracy up to the values disappearing in the process of limiting transition $V \to \infty$, C is a proper function of Hamiltonian H, giving it a value of U. We have, really,

$$(H_0 + H'')C = 0$$

and

$$\langle C^*|H'|^2C\rangle = \frac{\gamma^2}{V^2}\sum_{k_1}\theta(k_1)\,\theta(k_2)\{U_{k_1}^4V_{k_2}^4 + U_{k_1}^2V_{k_1}^2U_{k_2}^2V_{k_2}^2\} < \underset{V\to\infty}{\text{const}},$$

Now let us examine elementary excitations. Because in state C all occupation nembers ν_k are equal to zero, in calculation of elementary excitation energy we can regard the Pauli-operators β_k, β_k^+ as Bose-operators and in hamiltonian expression (18) we can restrict ourselves to quadratic form $H_0 + H'$

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

Diagonalizing it by the [illegible] (previously 14) we are getting to the [illegible] equation

$$(E - 2E_e(K))\varphi_K = \theta(K)\, U_K \frac{\gamma}{V} \sum_{K'} \{U_{K'}^2 \varphi_{K'} - V_{K'}^2 \chi_{K'}\} \theta(K') -$$

$$- \theta(K)\, V_K^2 \frac{\gamma}{V} \sum_{K'} \{U_{K'}^2 \chi_{K'} - V_{K'}^2 \varphi_{K'}\} \theta(K') .$$

$$-(E + 2E_e(K))\chi_K = \theta(K)\, U_K^2 \frac{\gamma}{V} \sum_{K'} \{U_{K'}^2 \chi_{K'} - V_{K'}^2 \varphi_{K'}\} \theta(K') \qquad (19)$$

$$- \theta(K)\, V_K^2 \frac{\gamma}{V} \sum_{K'} \{U_{K'}^2 \varphi_{K'} - V_{K'}^2 \chi_{K'}\}$$

with normalizing condition

$$\sum_K \{ |\varphi_K|^2 - |\chi_K|^2 \} = 1 \qquad (20)$$

From here we get secular equation

$$\left\{ 1 + \frac{\gamma}{V} \sum_K \theta(K) \left[\frac{U_K^4}{2E_e(K) + E} + \frac{V_K^4}{2E_e(K) - E} \right] \right\} \times$$

$$\times \left\{ 1 + \frac{\gamma}{V} \sum_K \theta(K) \left[\frac{V_K^4}{2E_e(K) + E} + \frac{U_K^4}{2E_e(K) - E} \right] \right\} \qquad (21)$$

$$- \left\{ \frac{\gamma}{V} \sum_K \theta(K)\, U_K^2 V_K^2 \left[\frac{1}{2E_e(K) + E} + \frac{1}{2E_e(K) - E} \right] \right\}^2$$

It is not difficult to notice, that when

$$|E| < 2_{min} E_e(K) = 2E_e(K_F)$$

this equation has no solutions, for the subtrahend in (21) is smaller than minuend. In case

$$|E| > 2E_e(K_F)$$

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

there is a continuous spectrum

$$E = \pm 2E_e(k) + O\left(\frac{1}{V}\right)$$
$$O\left(\frac{1}{V}\right) \to 0 \quad \text{when} \quad V \to 0$$

As can be seen from (19) - sign does not agree with the normalizing condition (20).

Hence, all E's are positive (this could be foreseen by noticing directly that the quadratic form under examination is definitely positive) and are separated from zero by a gap

$$E = 2E_e(k) \geqslant 2E_e(k_F) = 2C = 4\omega e^{-1/\rho} \qquad (22)$$

Here again we get the same Bardeen's results, as in previous works (1, 2). As we can see, the graph summation method appears to be quite descriptive and it does permit the establishment of connection with the ideas presented in the work of Bardeen-Cooper-Schrieffer.

Nevertheless, in our opinion, the method of canonic transformation is more flexible and permits to obtain higher approximations quite easily. Besides it permits various generolizations; for example, in calculation of thermodynamic quantities.

In conclusion I would like to express my gratitude to: D. N. Zubarev, V. V. Tolmachev, C. V. Tiablikov and I. A. Tserkovnikov for valuable criticism and suggestions.

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3

References

1. N. N. Bogolubov, (being printed)

2. V. V. Tolmachev and C. V. Tiablikov (being printed)

3. ~~See original~~ J. Bardeen, L. N. Cooper, J. R. Schrieffer, Phys Rev 106, 162, (1957)

4. N. N. Bogolubov "Lectures on quantum statistics", Kiev, 1949 pp. [illegible]

Approved For Release 2008/12/04 : CIA-RDP80T00246A002900600001-3