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Ministry of Higher Education of SSSR

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Lectures

on theory of automatical controls

Elements of the theory of intermittent control

INTRODUCTION

In the lecture "Introduction to the theory of automatical controls" has already been pointed out that the method of control by the performing device of the automatical control system, can be divided into constant control systems, relay control systems and the intermittent or impulse control systems.

Constant control systems have been considered in previous lectures.

The present lecture is dealing with the system of intermittent control.

In intermittent control systems, the control by the performing device is accomplished not constantly, but through equally separated (exactly or approximately) moments of time by rectangular impulses, parameters of which (hight, duration, symbol) depend on the significance deviation of the controled value (or, in general case, deviations of of/derivatives or intagrals thereof, etc.) in discrete equally separated moments of time. Physically this means, that in the intermittent control systems the chain of control is subjected to compulsory periodical disconnection (breaking off).

The principles of intermittent control at the present time are widely utilized in various systems of automatical control and direction.

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On one hand, this principle is utilized for control of comparatively slow flowing processess, for example, for temperature control in industrial furnesses, temperatures and pressures in boilers, and for control of whole series of technological processess. Here, side by side with simplification of the controlling apparatus, magnification of its sensitivity, the principle of intermittent control under predetermined conditions allows to obtain more favorable flow of the control process. In addition, application of the principle of intermittent control allows very simple realization of the multiple - projection control.

On the other hand, this principle lies on the foundation of function of great number of tele-control systems, tele-direction, tele-measuring systems in which known methods of tele-measuring are used and also in tracking systems of radar stations of special design.

Because of periodical compulsory disconnection of the control chain, which is characteristic phenomenon of the intermittent control systems, it is impossible to describe their processes with the same differential equasions, as it took place, for example, where the constant control systems were described.

Therefore, the analysis of the intermittent control systems requires different approach.

In this lecture the method of analysis of intermittent control systems is only briefly summarized, being based on utilization of descrete reform of Lhples, which is formally similar to the method of analysis of constant control systems.

I. Classification of intermittent control systems.

Structural sketch of intermittent control system is shown on Fig. 1.

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It shows following: 1. - regulated object; 2. - measuring device; 3. - master device; 4. - comparator; 5. - directing device; 6. - impulse element; 7. - executive device; 8. - inner communication.

The intermittent control system differs from the constant control system in that the impulse element is used in the former instead of an implifyer which isused in the later.

The input value of the impulse element appears as a signal, produced by the directing device, - directing signal, which determins the law of control.

The output value of the impulse element is expressed in the sequence of equally separated rectangular impulses. These impulses are formed by following parameters: hight, duration and symbol (polarity).

The impulse element transforms, generally speaking, constantly changing input value in the sequence of equally separated impulses into constant value T_p^1, parameters of which depends on the magnitude of the input variable in moments of time t = n Tp, called moments of incidence.

The value "Tp" is called "control interval" or "the period of alternation of impulses", and "n" represents a whole positive number.

In addition to this, the energy of impulse succession represents the output variable of the impulse element, normally greater than the energy of the input variable, thanks to the presence of additional source of energy in the impulse element. Thus, the impulse element is an amplifier.

Depending on, which parameter impulses are changing, the systems of intermittent control can be divided into three types.

To the first type belong systems with impulse element, which

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transforms input value into sequence of impulses, the hight of which are proportioned to the magnitude of its input value in moments of incidence (Fig. 2).

Duration of impulses, equal to $Y T_p$ ($Y = 1$), is called working interval, because during this interval of time the impulses will exert influence upon the executive device.

Schematic representation of the impulse element of the intermittent control system belonging to the first type is shown in Fig. 3. The impulse element is represented as a straight falling Lever 1, performing periodical movement and pressing the indicator 2 to one or the other point of rectilinear resistance 3, to which, as shown in Fig. 3, batteries 4 of equal voltage E_o are connected. The input value of the impulse element is represented by the shifting of the indicator relative to the middle point of rectilinean resistence. Output value is represented by impulses of constant duration voltage, but different in value (and symbol) depending on declination of the indicator from the middle point in the moment the falling lever is pressing on it.

To the second type belong systems with impulse element which transforms the variable in the sequence of impulses, duration of which is proportional to magnitude of its input value in the moment of incidence (Fig. 4). In this case the operational (working) interval $Y T_p$ is variable.

Schematic representation of the impulse element of intermittent control system belonging to the second type is shown in Fig. 5. The impulse element is represented as tapered falling lever 1, performing periodical movement and pressing the indicator 2 to the contact plate 3,

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insulated in the middle point, connected to batteries 4 of equal voltage E_o. The input value of the impulse element, as shown earlier, is represented by the shifting of indicator relative to the middle point. The output value is represented by impulses of voltage of constant hight, but different, thanks to the taper of the falling lever, in duration (and symbol) depending on declination of the indicator from the middle point in the moment the falling lever is pressing on it.

Finally, to the third type belong systems with impulse element, transforming input value into sequence of impulses of constant hight and duration, but with different symbols (polarities), depending on symbols of input value in the moments of incidence (Fig. 6).

Schematic representation of impul e element in systems of third type is shown on Fig. 7. This impulse element differs from impulse element in systems of second type only in the fact, that the tapered falling lever is substituted with the straight falling lever.

In systems of intermittent control of the first type the speed of servo-engine, representing executive device, is changed in each operating (working) interval proportional to the magnitude of input value of the impulse element in the moments of incidence. Depending on the speed in each operating (working) interval, the "run" of the servo engine is also changed (dotted curves in the Fig. 2 and 4).

In the systems of intermittent control of second type, the speed of the servo engine is constant; depending on the duration of impulses in each working interval the time is changed, which means migration of the servo engine (dotted curve in Fib. 4b).

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In the systems of intermittent control of third type, the speed and time of run of the servo engine are constant; only the symbol of speed is changed. In each working interval, the migration of the servo engine is constant.

The systems of intermittent control of the first and second type are representative of analogous linear system of intermittent control in that sense, that the influence upon the executive device is proportional to the magnitude of the directing signal. The systems of intermittent control of the third type are analogous to the relay systems of control.

In the above cited classification of intermittent control, systems were not included systems with interrupers or keys, in which the chain of control is periodically and constrainedly closed, and which during working interval does not differ in any way from systems of constant control.

The impulse element in these systems is represented by the key or interrupter. The output value of the impulse element represents impulses of non-rectangular form, "cut out" from input value (Fig. 8).

If the working interval $Y T_p$ is equal to the interval of control T_p (Y=1), which physically signifies removal of the interrupter or key, then these systems are transformed into systems of constant control.

But, if the working interval $Y T_p$ is suficiently small (Y=1), so that we could consider the impulses approximately rectangular, then this system is transformed into the system of intermittent control of the first type.

If the relay control system is supplied with a interrupter or key, then such system will differ from the intermittent control system in that

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the polarity of inpulse can change in the working interval by transition of the input value of the impulse element through zero (Fig. 9).

Standard systems not entered into classification are not considered any further.

As a result of the above description of impulse element's action, the working principle of intermittent control systems can be described in the following manner (Fig. 1).

The measuring element constantly measures controlled value. The measured controlled value is compared with the given value in the comparator, and their difference, that is, deviation of the regulated value will influence the directing device, in which the directing signal is formed. The directing signal will influence the impulse element. The impulse element reproduces succession of impulses, the hight and duration of which (depending on what system it is: of the first or second type) is proportional to the directing signal or polarity of which, depends on the polarity of the directing signal (in the systems of the third type) in equally separated with considerable accuracy) moments of time $t = n T_p$ ($n = 0, 1, 2, 3, 11$). These impulses will influence the executive device (servo-engine), which changes the position of the controlling device.

Thus, in the intermittent control systems, the transmission of the directing signal to the executive device occurs by means of modulation of directing signal impulses.

In the systems of the first type, the hight of impulses is modulated (pulse - amplitude modulation, PAM).

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In the systems of the second type, the duration or the width of impulses is modulated (width - pulse modulation or pulse - duration modulation).

In the systems of the third type, the polarity or the symbol of impulses is modulated.

Here, we will indicate some of the characteristics of the intermittent control systems, which are the direct result of their working principle.

1. If on the system of intermittent control acts periodical perturberance (change of load, tuning, etc.), half period of which is equal to the interval of control T_p, then it could occur that the regulator will not react to the change of the controlled value caused by this perturberance. Actually, in this case the input value of the impulse element (directing value) will be periodically (Fig. 10a). Let us suppose, that the moments of incidence correspond with moments of transmission of the output value through zero; then the output value of the impulse element will be indentical and equal to zero, and the influence upon the executive device will be absent, that is, physically the controlling system will always be disconnected.

Therefore, doubled interval of control 2 T_p must be smaller than the period of fluctuations of perturberance and own fluctuations of the linear part of the system. Fulfillment of this requirement is also necessary, so that the sequence of impulses will reflect with sufficient accuracy the essential characteristics of the directing signal (Fig. 10b).

2. If the intermittent control system should be influence by the perturberance of the type of singular impulse jump (for example, load

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drop, instantaneous change of tuning), then the controlled value under established conditions in the system with a static executive device (that is, executive device containing intergrating link /section/) will be constant in any given moment of time, and in systems with static executive device (at Y 1) will experience perturberances with aperiod equal to the interval of control T.

This is due to the fact, that in the established conditions in the first case, the rigid tie (connection) between the value of the directing signal and the position of the controlling device is absent, while in the second case it is present.

3. In the intermittent control systems of the second and third types with astatic executive device, the power of which can be fully exploited, because servo-engine works here in the condition of constant speed.

In intermittent control systems of the first type, the full exploitation of power occurs only under greatest magnitude of directing signals, because the speed of servo-engine during different control intervals - is different.

(Fig. 11)

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No.		Type of impulse element	Explanations
1	I	Double falling lever Measuring device	Measuring device 4, turns the arrows 2 and 3, connected with a spring 5 owing to periodical wearing away of the electromagnet 6, falling levers alternately press the arrows to the resistance 1, changing therewith the hight of impulse voltage in the chain $Y = 1$; $T_p = 10$ seconds.
2	II	Two-sided clamping circuit Constant signal (input) Impulses (output) Fixed impulses	The circuit is applied in impulse technology for transformation of the constant sound signal 1, into impulses 3, of changing hight. Fixed impulses 2 determine the period T_p.
3	II	Notching falling lever with contacts of mercury.	The measuring device 1 (galvanometer) turns the arrow 2. Periodically moving falling lever 3, pressing the arrow, inverts the mercury contacts. Duration of their closing is proportional to the angle of arrow declination 2, from the center position.

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No.		Type of impulse element	Explanations
4	II	Falling lever with contact washers.	The measuring device 1 turns the arrow. 2. Periodically moving falling lever 3, pressing the arrow against the lever 4, turns him, and with him the contact lever 5. Rotating contact sectors 6 form contact with contact lever, returning him to horisontal position. Duration of the contact is proportional to the angle of arrow 2 declination from the center position.
5	II	Falling lever with piston value	The measuring device turns the arrow 1, which periodically is pressed by the falling lever against the lever 2 or 2'. Owing to the turning of levers the piston shifts, opening a hole for passage of oil into the servo engine. Duration of opening of the hole is proportional to the angle of arrow 1 declination from the center position.
6	II	Differential relay (balanced relay)	The measuring device 1 turns the arrow with contacts 2, which close with toothed washers 3 and 3'. Duration of contact is proportional to the arrow declination from the center position.

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No.		Type of impulse element	Explanation
7	II	Impulse element of the regulator with CH 91 voltage.	The measuring device 1 (snap engine, instantaneous engine) rotates the contact system 2 and teeth of the rotating star 3, begin to touch the contacts 4 in the interval of time, proportionally to the angle of declination of the contact system from the vertical axis. When the contact system rotates at large angles, a contact disk 5 closes the circuit, therewith forming uninterrupted contact.
8	II	Photoelectric impulse element of Chugunoff.	The measuring device rotates a mirror 1 deflecting the light ray. This ray periodically reflected from the swinging mirror, hits the phototube 3, controlled together with contacts K_L and K_p with the angle of fire a thyratrons. In anode circuit last connected winding of the engine. Time of engine contact is proportional to the angle at declination of the mirror 1 from the center position.
9	II	Impulse element of electronic regulator which regulates proportion oil residue (mazut) - air EHRS - 60 From measuring device	The measuring device turns the lever 1. The end of this lever is lifted by the mould with radial out 2 and 3, and tilts the mercury contact 4. Duration of closing of the circuit is proportional to the angle of declination of the lever 1 from center position.

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No.		Type of impulse element	Explanations
10	II	Impulse element with fluctuating washer.	The measuring device (membrance) rotates the fluctuating disk 1 with the help of a differential. The disk's cames touch the rollers 2, which tilt the mercury contact 3. Duration of the contact closing is proportional to the angle of rotation of the fluctuating disk's axis.
11	II	Impulse element in the form of a system made of two relays To the executive device	The relay 1 switches itself on and off from the impulses of the tele-measuring, duration of which is proportional to the measured value, and the relay 2 is under influence of impulses of constant duration. The circuit of the executive device is closing only during differences in duration of these impulses and for the time proportional to this difference.
12	II	The straight falling lever with mercury contacts.	The measuring device 1 (galvanometer) turns the arrow 2. Periodically moving straight falling lever 3, pressing the arrow against the mercury contacts and therewith closes the circuit. Duration of the closing of the circuit is constant.

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5. The impulse element of the second type, exerts influence upon the non-linear element with symmetric characteristic, it removes the non-linear property of the latter. This follows the selfevident fact, that input value of the non-linear element, representing output value of the impulse element, is not changed in the working interval, and on duration of impulses the non-linearity does not have any effect. (Fig. 11)

In the industrial automation, for control of comparatively slow flowing processes (temperature, concentration, pressure, etc.) are most widely utilized the systems of intermittent control of second and partially of the third types.

In the measuring and radar technology, it isjust the opposite, in majority of cases the systems of intermittent control of the first type are utilized.

Practially applied impulse elements in systems of intermittent control differ from the above presented schematic representation explaining their principles of action and construction.

In tables (see pages 10 - 13) were presented few typical electro-mechanical, electronic and mechanical impulse elements and were given brief explanations. Typical systems of intermittent control are described in the lecture "Introduction to the theory of automatical control" page 54, 57 and also in the chapter III of the present lecture.

II. The foundation of the theory of intermittent control.

1. Equasions of the intermittent control systems.

Structural sketch of the intermittent control (Fig. 1) can be presented in more convenient form for further exposition (Fig. 12).

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To the linear or, as it is sometimes refered to as a constant part of the system, belong all elements of a system with the exception of the impulse element. The linear part of a system represents open condition (of a circuit) at the input of the impulse element of intermittent control system[1], not containing the impulse element.

We will assume that external perturberance Y_o (t) is applied to the input of the impulse element. This does not represent limitation, because perturberance applied at any given point on the linear part can be always adduced to the input of the impulse element.

If at any point in the system the external perturberance is acting, then this perturberance will cause a reaction at the input of the impulse element, which could be commuted by one or other known method. During this normally no difficulties arise, because we are considering a part of the linear system under open condition (of the circuit).

This reaction isthe perturberance, brought to the input of the impulse element.

The impulse element is characterised by the interval of control T_p, porosity Y, representing ratio of impulse duration Y T_p to the interval of control T_p, and coefficient of amplification K_1, representing the ratio of the output value of impulse element to the input value in the moment of incidence.

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[1] In the future, under the open condition (of the circuit) in the system of intermittent control, is always understood as the system having open condition (of the circuit) at the input of the impulse element

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In the systems of intermittent control of the first type (and third type)

$$Y = \text{constant}$$

In the systems of second type

$$Y = X \left| X \text{ input } (n\, T_p) \right| \text{, where } X \text{ - is constant value.}$$

The output value of the impulse element represents succession of impulses which act upon the linear part of the system.

The linear part of the system is characterised by its function of transmission W (p) which depends on the function of transmission of separate links K_i (p) and it is composed by known rules. Considering the linear part of a system with concentrated constants, the function of transmission W (p) can be represented in following manner:

(Formula No. 1)

where the P (p) and Q (p) - are polynominal by p, whereby degree of L_1 of the polynominal P (p) does not exceed the degree of L of the polynominal Q (p). The function of transmission W (p) is easy to find, by applying the transformation of Laplas to the differential equations of the linear part of the system or directly out of the differential equations after exchanging in them $\frac{d}{dt}$ with p.

The linear part of the system can be determined by its frequency or time characteristic.

The frequency characteristic W (jw), which defines the set reaction of the linear part of the system on the harmonic (rythmic) reaction with singular amplitude, can be obtained either by the function of transmission W (p), by exchanging in her p with jw, or be found by the known method with experimental data.

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The time characteristic h (t) represents the reaction of the linear part of the system on the influence in the form of a singular jump.

In that case, when the function of transmission W (p) has end (terminal) number of poles $p_1, p_2, \ldots\ldots p_i$, and these poles are different from each other and also not equal to zero, the time characteristic h (t) can be determinedby aknown formula of dissociation (separation).

(Formula No. 2)

The knowledge of the time characteristic of the linear part of the system allows, on the basis of the principle of superimposition, to find aprocess occuring at the output of the linear system, when the succession of impulses affects her.

In the future will be more convenient to consider the process as a function relative to time $\bar{t} = \frac{t}{T_p}$. By such change of time rate, the relative interval of control will be equal to one, the relative duration of impulse porosity Y, and the moment of incidence $\underline{t}$ will be equal to $\underline{n}$. Besides, h (t) can be determined by the formula, analogous to formula No. 2, if beforehand in the function of transmission W (p) the p will be exchanged by $\frac{q}{T_p}$, so that

(Formula No. 3)

where $q = T_p$. For the simplicity sake, we preserve the service designations of functions P and Q.

The influence of the impulse element upon the linear system is determined by the magnitude X input $(\bar{t})$ in the moment of incidence $t = n$. For closed (circuit) system this value is equal to the difference of the external reaction and output value of the linear part in the moment

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of incidence, that is

(Formula No. 4)

where the X [a] is understood as notching (echelon) function which coincides with x ($\bar{t}$) in moments $\bar{t}$ = n, and between them remaining constant[1].

For composition of equations of the intermittent control system, we will use the descrete transformation of Laplas, determined by the correlation.

(Formula No. 5)

where x [n] - notching (echelon) function, coinciding with the function x ($\bar{t}$) in points t = n (original), and X* (q) - image of the notching (echelon) function. Symbol D { } denotes operation of the descrete transformation of Laplas.

The descrete transformation of Laplas in relation to the intermittent control system, plays the same role, as the transformation of Laplas in relation to the system of constant control.

Subjecting the correlation (formula No. 4) to the descrete transformation of Laplas, we will receive the following equation:

(Formula No. 6)

which we may call equation of closing (the circuit).

The second equation, binding the image of input value of the impulse element X* ~~out~~IN put (q) = D { x ~~out~~IN put [n] } and output value of the linear part of the system X* output (q) = D { x output [n] } in the moment of incidence $\bar{t}$ = n, determines the behavior of the open (circuit) intermittent control

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[1] See appendix on the page _____ of the present lecture.

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system. This equation can be found as an image of notching (echelon) function corresponding to the reaction of the linear part on the succession of impulses in discrete moments of time $\bar{t} = n$.

If the linear part of the system is influenced by an impulse during moment of time $\bar{t} = m$, the relative duration of which is equal to Y, and the value (hight) to 1, then the output variable of an open (circuit) of the system isequal to (Fig. 13)

(Formula No. 7)

the r (t) represents the reaction of the impulse on the linear part of the system.

In the system of intermittent control of the first type the hight of an impulse is changed, but the relative duration of the impulse is constant.

In the systems of intermittent control of the second type is just the opposite, the relative duration of an impulse is changed, and the hight remains constant.

Let us consider first the intermittent control system of the first type.

If on the linear part of the system act succession of impulses in moments $\bar{t} = m = 0, 1, 2, 3, \ldots$ of different hight k_L x input (m), then on the basis of the principle of superimposition, the reaction of the linear part of the system will be equal to the sum of reactions from each impulse (Fig. 14). This fact can be analitically represented in the form

(Formula No. 8)

For the descrete moments of time $\bar{t} = n$, in which we are interested,

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we receive

(Formula)

Values, entering the later correlation, concern only the separate discrete moments of time. We can exchange them with notching (echelon) functions, magnitudes of which coincide with them in discrete point. Then we will receive

(Formula No. 9)

Here the notching (echelon) function $r[n - m]$ corresponds to $r(\bar{t} - m)$ when $\bar{t} = n$.

Please note, that when $m = n$

(Formula No. 10)

By subjecting both parts (Formula No. 9) to the descrete transformation of Laplas (that is, multiplying both parts thereof by e^{-qn} and summarizing from 0 to ∞) and applying the theory of convolution (theory of the 6th appendix) we will receive

(Formula)

Taking into account the designations of images (Formula No. 5) and designating

(Formula No. 11)

we get fundamental correlation between the images of notching (echelon) functions, corresponding to input and output variables of the open (circuit) system of intermittent control

(Formula No. 12)

The value W* (q), which represents relation of the image of output variable to the input variable, according to the analogy with the systems of constant control, we will call it "transmission function" of the open

1 See appendix on the page ——

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(circuit) system of intermittent control.

Transmission function W* (q), as it follows from (Formula No. 11) and the determination of descrete transformation of Laplas (Formula No. 5) it is equal to

(Formula No. 13)

or, if accepting, that normally

(Formula)

then we will obtain

(Formula No. 14)

Let us consider now the system of intermittent control of the second type.

In that case

(Formula No. 15)

where

(Formula No. 16)

The reaction of linear part of the system to the succession of impulses of constant value and variable duration in moments of time $\bar{t} = n$ on the basis of the principle of superimposition (Fig. 15) will be equal

(Formula No. 17)

where

(Formula No. 18)

Further we will suppose, that

(Formula No. 19)

which corresponds to small changes of the input value. Then $r(\bar{t} - m)$ we can represent in the following form, after expansion $h(t - m - y_m)$ in the order according to $\bar{Y}_m$ and retaining only the first degree of Y_m:

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(Formula No. 20)

Taking into account (Formula No. 16), when $\bar{t} = n$ we will have

(Formula)

But if $n = m$, then, evidently, as before

(Formula)

By substituting $r\,[n - m]$ in (Formula No. 17) and noting, that

(Formula)

we receive for the system of the second type

(Formula No. 21)

Comparing (Formula No. 21) with (Formula No. 9) we conclude, that under conditions of (Formula No. 19), the systems of the second type are equivalent to the systems of the first type with the coefficient of amplification K_u x and the reaction of the linear part of the system

(Formula)

From this follows, that all results for the systems of intermittent control of the second type can be obtained from the results for systems of intermittent control of the first type when $Y \ll 1$ and exchanging Y with X. In the future, we will take advantage of this circumstance.

For calculation of the transmission function $W^*(q)$, which can be determined by (Formula No. 14), we will first find

(Formula No. 22)

Because

(Formula No. 23

then

(Formula No. 24)

Substituting (when $t = n$) this magnitude in (Formula No. 4) and

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changing the order of summation after elementary calculation and application of the formula of the sum of geometric progression, we obtain

(Formula No. 25)

where

(Formula No. 26)

If one of the roots, for example q_L, is equal to zero, then the item, corresponding to Y = L can be exchanged with

(Formula No. 27)

If for the system of the second type, according to above mentioned, the expression (Formula No. 25) is preserved, then now when $Y \ll 1$

(Formula No. 28)

and

(Formula)

The condition $Y = X \left| X \text{ input } [n] \right| \ll 1$ which physically means, that the small changes of the directing signal are considered or that duration of switch-on of the executive device is disregarded.

Often the linear part can possess lagg t.

If element of lagg is subsequently connected with the linear part of the system, then the transmission function of such connection is equal to

(Formula No. 29)

where $\bar{t} = \frac{t}{T_p}$ - is relative lagg of time.

In this case the reaction of the linear part of the system will lagg in time t, that is

(Formula No. 30)

Considering this circumstance, which is analogousto the preceding one, we will obtain expression for W* (q); in this case when

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(Formula No. 31)

where C_y are determined by expressions of (Formula No. 26) or (Formula No. 28), or, when

(Formula No. 32)

(Formula No. 33)

Please note, if $q_L = 0$, then items, entering into (Formula No. 32), when $y = L$ after discovery of uncertainty will have the form of

(Formula No. 34)

If the element of lagg is connected in different way with the system, so that

(Formula No. 35)

then instead $W^*(q)$ in the equation of the systems it is suggested to substitute

(Formula No. 36)

where $W^*_{1t}(q)$ is determined by the (Formula No. 31), (Formula No. 32), and $W^*_2(q)$ - by the (Formula No. 25).

For the systems of the first type, depending on the value t the transmission function $W^*_{1t}(q)$ is determined by the (Formula No. 31) or (Formula No. 32) when $Y < 1$ or by (Formula No. 32) when $Y = 1$.

For the systems of the second type, when $Y \ll 1$, then $W^*_t(q)$ will be determined by the (Formula No. 31).

In the expressions of (Formula No. 35, 31, 32) q_v denote poles of the transmission function of the linear part of the system $W^*(q)$ (1), that is, the roots of the equation $Q(q) = 0$, which are supposed to differ from each other. In such cases, when the linear part of the system does not contain interval feedbacks, this poles are simply

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expressed through parameters of links of the linear part.

In other cases, for example, when the linear part is complex, it could be more convenient to use different expression of the function of transmission W* (q), which can be obtained from (Formula No. 13) after some transformations:

(Formula No. 37)

for the systems of the first type and

(Formula No. 38)

for the systems of second type.

Expressions (Formula No. 37) and (Formula No. 38) determine the function of transmission of open (circuit) system of impulse control directly through functions of transmission of the linear part. Those expressions are justified also by the presence of lagg and distributed parameters.

Thusly, for the systems of intermittent control of the third type equation of the open (circuit) system will have the form

(Formula No. 39)

For the system of intermittent control of the third type; the equation of the open (circuit) system will be represented similarly,

(Formula No. 40)

where Φ (X input) - relay characteristic (Fig. 16), and W* (q) is determined by the expression (Formula No. 25). This arises in that the impulse element system of the third type is equivalent tothe subsequent connection of relay element with characteristic form (Fig. 16) and the impulse element of the system of first type (Fig. 17).

(Fig. 16) (Fig. 17)

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Before we go into the composition of equations of closed (circuit) systems of control, let me note some characteristics of transmission functions of the open (circuit) system of intermittent control W* (q). As it is evident from above presented expressions, W* (q) represents the function of e^q.

Because $e^{q + 2\pi m j} = e^q$, then $W^*(q + 2\pi m j) = W^*(q)$, that is W* (q) represents the periodic function along the imaginary axis of the plane q.

Consequently, W* (q) has L poles e^{qv} and innumerable quantity of poles q, differing from qj on $\pm$ 2 s t m j.

Let us call the poles distributed along the belt $-\pi < \mathrm{Im}\, q \leq \pi$ (Fig. 18), basic. In the future we will be interested only in basic poles.

For composition of equations of closed (circuit) systems it is necessary to add to the equation of open (circuit) systems the equations of closing (Formula No. 6)

(Formula)

By excluding from these equations the X* input (q), we obtain equation of closed (circuit) system of intermittent control of the first and second type.

(Formula No. 41)

If the magnitude X* output (q) in (Formula No. 41) should be placed in (Formula No. 6), then we will obtain equation relative to input variable of the impulse element (signal of direction)

(Formula No. 42)

The equation relative to the image of any given value of linear

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part of a system, for example X^*_1 (q) is easy to find by determining the transmission function K^*_1 (q), corresponding to the section of an open (circuit) system from the impulse element to the unknown value by the (Formula No. 39), where X* input (q) which, however, is not arbitrary but determined by the (Formula No. 42). Thusly,

(Formula No. 43)

When K^*_1 (q) = W* (q) or K^*_1 (q) = 1 we arrive at preceding equations. The value

(Formula No. 44)

determines the transmission function of the closed (circuit) system of control.

The equation (Formula No. 44) which is analogous in form with equations of intermittent control system determines the process in descrete moments of time $\bar{t}$ = n - moments of incidence.

The equations of closed (circuit) system of intermittent control of the third type, we will obtain after substituting in (Formula No. 40) the magnitude of X input [n] from (Formula No. 4) or X* output (q) from (Formula No. 6)

(Formula No. 45)

or

(Formula No. 46).

The difference between the equations of the systems of intermittent control of the first and second type and the equations of the systems of intermittent control of the third type is that the latter's are non-linear.

For composition of equations for the systems of intermittent control,

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it is required:

1. To find transmission function of the linear part of the system and by substituting $p = \frac{q}{T_p}$ and reducing it to the measureless variable q. Then

(Formula)

2. With this transmission function W (q) to determine the transmission function of the open (circuit) system of intermittent control W* (q), using corresponding (Formula No. 25) or, when the lagg is present, the (Formula No. 31, 32). With analogous method, the K_1 (q) determines the K^*_1 (q).

3. Substitute W* (q) in corresponding to one or the other type of systems of equations (Formula No. 41, 42, 45, 46, or 43).

The presented equations describe processess in moments of incidence $\bar{t} = n$. Slight change in their form permits to describe processess in any given moment of time $\bar{t}$. We will not dwell on this here any more.

If the expressions of transmission functions of open (circuit) W* (q) or closed (circuit) K^*_3 (q) systems of intermittent control are substituted with $q = j\,\bar{w}$, where $\bar{w} = w\,T_p$ - relative frequency, then we will obtain corresponding frequency characteristics.

These frequency characteristics are fully determined by the change of relative frequency $\bar{w}$ in the interval $-\pi \leq \bar{\omega} < \pi$ or, as it will be seen, in the interval $0 \leq \bar{\omega} < \pi$.

The frequency characteristics play an important role as in the study of stability, and also in the study of quality of systems of intermittent control. Therefore, we will dwell on the methods of composition of frequency characteristics.

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For composition of frequency characteristics of the open (circuit) of intermittent control system $W^*(j\bar{w})$ we can avail ourselves of expressions in (Formula No. 37 or 38) and also (Formula No. 25).

In many cases it is more convenient to use (Formula No. 37 and 38). Assuming that in them the $q = j\bar{w}$, we will obtain after simple transformations, the expression for frequency characteristics,

(Formula No. 47)

or the system of the first type and

(Formula No. 48)

if the system is of the second type.

(Fig. 19)

The (Formula No. 47 and 48) determine the frequency characteristics of the open (circuit) of the intermittent control system $W^*(j\bar{w})$ through frequency characteristic of the linear part thereof $W(j w)$ by different magnitudes of frequency. Therefore, $W^*(j\bar{w})$ can be composed in following manner. Let us first consider the expression of frequency characteristic $W^*(j\bar{w})$ for the systems of the second type (formula No. 48). Let the frequency characteristic of the linear part $W(j\omega)$ have form, shown in the Fig. 19. The continuous curve corresponds with $\omega > 0$, and the dotted curve is symetric with $\omega < 0$. Giving oneself the magnitude of control interval T_p, let us plot on the frequency characteristic instead of the magnitude of ω, the magnitude $\bar{\omega}$, proceeding from the ratio $\bar{\omega} = \omega T_p$, the designation ω will be changing.

Giving oneself now magnitude $\omega = \omega' < \pi$, we plot on the frequency characteristic of the linear part $W(j\bar{\omega})$ points $\bar{\omega}_1$, $\bar{\omega}_1 - 2\pi$, $\bar{\omega}_1 - 4\pi$, ..., $\bar{\omega}_1 + 2\pi$, $\bar{\omega} + 4\pi$, ... etc.

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The sum of vectors derived from the principle of coordinates and brought to these points according to (Formula No. 48) determines the magnitude of unknown quantity of frequency characteristic $W^*(j\bar{\omega})$ when $\bar{\omega} = \bar{\omega}_1$ (Fig. 19b).

These vectors can be arranged not geometrically, but summarized separately the actual and imaginery parts (Fig. 20), displacement to $\pm 2k\pi$ along the axis of frequency, as it is shown in Fig. 21. The bald lines in Fig. 21a and b show magnitudes of $U^*(j\bar{\omega})$ and $U^*(j\bar{\omega})$. The value $W^*(j\bar{\omega})$ is found according to

(Formula)

As by sufficiently great $\bar{\omega}$ modulus of the frequency characteristic of linear part $|W(j\bar{\omega})|$ normally strives toward the zero, then in the sum (Formula No. 48) it is possible to limit oneself with small number of items, often equal to two.

(Formula No. 49)

In this case the composition of $W^*(j\bar{\omega})$ is especially simple (Fig. 22). In each point $\bar{\omega}_1 < \pi$ of the frequency characteristic of linear part of the system $W(j\bar{\omega})$ as from the principle of coordinates, a vector is laid $W[j(\bar{\omega}-2\pi)]$, as shown in Fig. 22.

For systems of intermittent control of the first type, the difference in composition of $W^*(j\bar{\omega})$, as it is evident from (Formula No. 47) consists in turning of corresponding vectors at angles $\frac{\omega + 2\pi k}{2}\gamma$ and multiplication of the modulus by _____, is determined according to corresponding tables $\frac{\sin \gamma}{\gamma}$.

Frequency characteristic of the closed (circuit) system of intermittent control $K^*_3(j\bar{\omega})$ is obtained from the transmission

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function K_3 (q) after exchange of the q by j $\bar{\omega}$.

(Fig. 21)

Introducing normalised frequency characteristic

(Formula)

where K - common multiplier not depending on the frequency, we will write the expression for the frequency characteristic of the closed (circuit) system, when K^*_1 (j $\bar{\omega}$) = 1 and K^*_1 (j $\bar{\omega}$) = W* (j $\bar{\omega}$):

(Formula No. 50)

(Formula No. 50.2)

As frequency characteristics K^*_3 (j $\bar{\omega}$) and W* (j $\bar{\omega}$) in this case are connected between each other with simple number, then it's easy according to frequency characteristic $\overline{W}$* (j $\bar{\omega}$) to construct geometricly the frequency characteristic of the closed (circuit) system K^*_3 (j $\bar{\omega}$), as it isproduced in the theory of constant control. As in Fig. 23, where frequency characteristic $\overline{W}$* (j $\bar{\omega}$), is represented, is evident, that for the given magnitude $\bar{\omega}$

(Formula)

The sum of these vectors is equal to

(Formula)

Thusly, when given magnitude $\bar{\omega}$

(Formula No. 51)

where ao and ab - are modulus, and O and φ - are phases of vectors $\vec{ao}$ and $\vec{ab}$. The modulus of the frequency characteristic $K^*_{3_1}$ (j $\bar{\omega}$) of closed (circuit) system of intermittent control is equal to the ratio of constant cutout ao to outout ab, and the phase is equal to the angle between the vectors $\vec{ao}$ and $\vec{ab}$. From this structure we can

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also obtain substantial and imaginary frequency characteristic.

As the

(Formula)

then

(Formula No. 52)

and

(Formula No. 53)

All these structures almost without changes remain with calculations K^*_{32} (j) according to reverse frequency characteristic

.

The frequency characteristics will be used in the future for the analysis of stability and quality of systems of intermittent control.

2. The stability of intermittent control systems.

Under stability of the system is understood the property of the system to return to stationary position after perturbance, which caused the change of this condition. In the case of the system of intermittent control this property is related to the descrete magnitudes of some values (for example, input value of the impulse element or output value of the linear part of the system) in the moment of incidence, described by the notching (echelon) function.

Let the notching (echelon) function $x = [n] = x_{st} [n]$ characterize stationary condition of the intermittent control system of the first and second type.[1] In particular, $x_{st} [n]$ can be constant. If for any reason

[1] In the future, if otherwise indicated, you will consider only these systems, related to linear system.

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the condition of the system should change, then after removal of this cause $x[n]$ will change and be equal to

(Formula)

The value $\delta x[n]$, denoting free fluctuation of the system, can be represented in the form.

(Formula)

where d_ν - are constant numbers, and $\bar{q}_\nu$ - are basic poles of transmission function of the closed (circuit) system, that is, basic roots of the equation.

(Formula No. 54)

If representing W* (q) in the form

(Formula)

when $\tilde{q}_\nu$ will be basic roots of the equation

(Formula)

which represents analogous characteristic of the equation.

It is evident, that the system will be stable, if $\delta x[n]$ with the growth (increase) of $\bar{t} = n$ tends toward zero, and for this, it is required and is sufficient that all basic roots of characteristic equation of transmission in closed (circuit) system have negative actual parts or, in other words, that they should lie in the left part of the band $-\pi < \mathrm{Im}\, q \leq \pi$ (Fig. 24).

The difference between the theory of constant control, and the theory of intermittent control is, that the role of the left part of the flat surface of roots, is played by the left part of the band $-\pi < \mathrm{Im}\, q \leq \pi$ $\mathrm{Re}\, q < 0$ (Fig. 24).

As the basic poles W* (q) coincide with poles W(q), then we come to

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the conclusion, that if the linear part of the intermittent control system is stable, then the open (circuit) system of the intermittent control system is also stable. This conclusion is physically self-evident. The criteria used for the analysis of the stability of the closed (circuit) of intermittent control system, allows us to by-pass the cumbersome operation of direct calculation of roots of the characteristic equation.

We will now bring the formulation of stability criteria, which are analogous to criteria of stability of constant control systems.[1*]

Analogous criteria Rauss - Gurvits.

Permit (Formula No. 55)

We will denote (Formula No. 56)

where $\binom{M}{K} = \frac{M!}{K!(M-K)!}$ - binominal coefficients.

In order for a system to be stable, it is required to fulfill following conditions

(Formula No. 57)

In open form these conditions look like this: for L = 1

(Formula)

and, consequently

(Formula No. 58)

For L=2

(Formula)

1* Proof of these criteria can be found in following books: H. Oldenburg and G. Sartorius, Dynamics of automatical control (GEI - State Electronic Publication) 1948; Ya. Z. Cypkin, Transitional and set processess in impulse circuits (chains) GEI - State Electronic Publication) 1951.

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and, consequently

(Formula No. 59)

For L = 3:

(Formula)

and consequently, after simplification

(Formula No. 60)

Analogous criteria of Michailov

Let substitute q = j into polynominal G* (q), where = T_p.

Then

(Formula No. 61)

(Formula No. 62)

actual and imaginary parts are G* (j $\bar{\omega}$). U* ($\bar{\omega}$) and V* ($\bar{\omega}$), and this means, G* (j $\bar{\omega}$) represents periodical functions of $\bar{\omega}$ and are fully determined by exchange of $\bar{\omega}$ in the interval $-\pi < \bar{\omega} \leq \pi$.

By exchanging $\bar{\omega}$ in this interval, the vector G* (j $\bar{\omega}$) describes with its end a curve, which is called characteristic curve, or "godograph". In order for the system of intermittent control to be stable, it is required and it is sufficient that by the beginning from the positive part of the real axis, then traversed 2 L quadrants counter-closkwise, where L degree of characteristic equation, or, which is equivalent if U* ($\bar{\omega}$) and V* ($\bar{\omega}$) are material between the interval $\pi < \bar{\omega} \leq \pi$? and alternating roots[1*] when $\bar{\omega}$ = 0 U* (o) > 0, V*' (0) > 0.

In Fig. 25 is shown an example of characteristic curves corresponding to stable systems for L = 1, 2, 3, 4: In Fig. 26 are shown characteristic

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1* That is, between two adjoining roots of the equations U* ($\bar{\omega}$) = 0 is located the root of the equation V* ($\bar{\omega}$) = 0.

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curves $G^*(j\bar{\omega})$, $U^*(\bar{\omega})$ and $V^*(\bar{\omega})$ for stable and unstable systems when $L = 5$.

From examination of characteristic curves result necessary conditions of stability:

(Formula)

Shown formulations of criteria are analogous to theknown criteria of Michailov for systems of constant control.

Analogous criteria of Nayquist

If substituting $q = j\bar{\omega}$ in the transmission function $W^*(q)$, where $\bar{\omega} = \omega T_p$ is relative frequency of garmonic fluctuations, then we will obtain frequency characteristic of open (circuit) of intermittent control system.

(Formula No. 63)

where $A^*(\bar{\omega})$ and $B^*(\bar{\omega})$ are actual and imaginary parts. During exchange of $\bar{\omega}$ from 0 to π the end vector $W^*(j\bar{\omega})$ describes a curve, which is called godograph of frequency characteristic.

Supposing, that open (circuit) system of intermittent control is stable, that is, that the linear part of the system is stable, we can formulate a criteria of stability in following manner:

The system of intermittent control is stable, if the godograph of frequency characteristic does not encompass the point 1; j 0.

In Fig. 27 are shown godographs of frequency characteristic, corresponding with stable (Fig. 27a) and unstable (Fig. 27b) systems of intermittent control.

If open (circuit) system of intermittent control is neutral, that is, transmission function contains S poles equal to zero, then formulation

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of stability criteria will remain without change; only during construction of frequency characteristic, it is necessary to supplement it with a curve of infinitely great radius, corresponding to an angle $S \frac{\pi}{2}$ (Fig. 28). The curve 1 corresponds to the stable system, the curve 2 to an unstable. (Fig. 27)

Finally, if the open (circuit) system of intermittent control is unstable, that is, the transmission function of the linear part of the system has S poles with positive actual part, then:

The system of intermittent control will be stable if the godograph of frequency characteristic encompasses point 1; j 0 in positive direction (that is, counter clock-wise) S/2 times. When S = 0 we arrive at preceding formulation.

Let us note, that for normalization of frequency characteristic, the role of point 1; j 0 is played by point $\frac{1}{K}$; j 0.

The frequency characteristic which is not normalized $W^*(j\bar{\omega})$ are normalized $\bar{W}^*(j\bar{\omega})$ are calculated analitically or graphicly according to formula shown above, proceeding from frequency characteristic of the linear part, as it was described above.

In conclusion of this section let usnote, that introduction of intermittent control can appear as a strong medium from stabilization of systems, if frequency characteristic of the linear part, beginning with certain frequency, goes through I and IV quadrants (Fig. 29). Such frequency characteristic possess, for example, systems with lagg, systems with distributed parameters, etc. This fact follows out of above indicated method of construction $W^*(j\bar{\omega})$ according $W(j\bar{\omega})$ (See Fig. 29). The interval of control at the same time is chosen out of

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condition

$$T_p \geq \frac{\pi}{\omega_0}$$

3. The process of control in systems of intermittent control.

Let us consider the equation of intermittent control system (of the first and second type)

(Formula)

This can be changed into

(Formula No. 64)

where H* (q) polynominal of the degree L_2, G* (q) polynominal of the degree L, corresponds to characteristic equation.

Let us suppose, that $x_0 [n]$ has a form of a single jump, that is

(Formula)

then the input value of impulse element will be determined by the expression*

(Formula No. 65)

where $\widetilde{q}_j$ are basic roots of characteristic equation, that is, basic poles of transmission function of closed (Circuit) system (basic roots of equation G* (q) = 0, and

(Formula)

This formula determines the process of control in moments of incidence.

If the system is stable, then actual parts of basic roots of characteristic equation G* (q) = 0 are negative, and all items in (Formula No. 65), containing e^{q+n}, with growth n tend to zero. The limit to which it tends x input $[n]$, will be equal to

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* See appendix on pages _____.

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(Formula No. 66)

This value characterises the static (set) declination of input value of the impulse element. The magnitude X_{input} [∞] differs from zero when $W^*(0) \neq \infty$ and transforms itself into zero, when $W^*(0) = \infty$. The last case has place in, for example, when executive device contains integral link.

Construction of the process according to (Formula No. 65) is connected with the necessity to calculate the roots of characteristic equation $G^*(q) = 0$, which at $L \geq 3$ becomes extremely cumbersome operation.

We will show an expression of a process, which is free from these shortcomings and which allows to construct a process under any given perturberance.

If the transmission function of the closed (circuit) system is presented in the form

(Formula)

then, separating it in order of degrees e^{-q}, it is possible to show

(Formula No. 67)

where coefficients G_k are determined out of reccurent ratio

(Formula No. 68)

Thus, the determination of control process in moments of incidence $t = n$ boils down to calculation of coefficients G_k.

The expression (Formula No. 67) determines the process of control [n]. If Xo has the form of a singular jump (skip) then Xo (for $\bar{t} = n$) at any form of $X_0 / [n - (L - L_2 + k)]$ is equal to one when $k \leq n - L + L_2$ and is equal to zero when $k > n - L + L_2$ and the magnitudes of the process are found by simple summation of coefficients G_k.

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At given magnitudes of parameters of a system, the calculation of a process according to the expression (Formula No. 67) does not offer any difficulties. It is finally possible to determine a process by grapho-analitical method with frequency characteristic.

For this purpose $W^*(j\bar{\omega})$ according to above indicated method is constructed substantial frequency characteristic $B^*(\bar{\omega})$.

Fig. 30 Fig. 31

If representing it in the form of a sum of typical trapezoidal characteristics (Fig. 30), then the unknown value G_k can be expressed in the form

(Formula No. 69)

Here the A_j is surface of the trapex. Remaining symbols are clearly seen in the Fig. 31. By using the tables $\frac{\sin y}{y}$, we determine G_k, and later, as indicated above, the sought process x input [n].

4. Indirect method of evaluation of aprocess in systems of intermittent control.

As in the theory of constant control, for selection of parameters of a system of automatical control, the important role is played by indirect method of evaluation of process control, which do not require the construction of the latter. We will briefly consider three such evaluations: degree of stability, degree of fluctuation and integral evaluations.

Degree of stability

The greater are, according to absolute value, the actual parts of poles q_j, the faster, the components of the process $e^{\tilde{q}_j n}$ with increase of n, fade. As in the case of constant control systems, the speed of

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fading of a process in a stable system can be characterized by absolute value of actual part nearest to the imaginary axis of complex surface q of the root of characteristic equation.

(Formula No. 70)

This value we will call degree of stability and express it by ξ (See Fig. 24), so that

(Formula No. 71)

The degree of stability ξ represents the relative value, because the processes are expressed in the function of relative time t = n.

The absolute value of the degree of stability, will evidently be equal to

(Formula No. 72)

To determine the degree of stability it is sufficient to substitute q by ξ in the transmission function of open (circuit) system of intermittent control W* (q), and thusly obtained transmission function is considered as a transmission function of some fictitious system, whose limit of stability corresponds to a line, which isequal to the degree of stability of studied system. Similarly, as in theory of constant control, the problem boils down to the study of stability of a fictitious system. The latter can be carried through on the basis of criteria of stability, indicated above.

The difference of the system of constant control consists in possessing always the end (final) degree of stability, while in the system of intermittent control the degree of stability can reach the infinity. The infinite degree of stability corresponds to that, that actual parts of equation roots G* (q) = 0, which we will clearly present in

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(Formula)

and they are equal to ∞. It is evident that conditions under which the infinite degree of stability is achieved, has the form

(Formula No. 73)

because in this case all equation roots

(Formula)

are equal to ∞.

If it is given transmission function of open (circuit) system

(Formula No. 74)

then the conditions (Formula No. 73) under which the infinite degree of stability is achieved, according to (Formula No. 70 and 74) they will assume the form of

(Formula No. 75)

By the infinite degree of stability, the process of control during perturberances in the form of a singular jump, ends in the final number of intervals of control.

Actually, if $a_0 = a_1 = a_2 = \ldots = a_{L-1} = 0$, then form the recurring ratio (Formula No. 68) we will obtain

(Formula)

and in this case according to (Formula No. 67) we will have

(Formula)

and

(Formula)

$x_{input} + [n]$ when $\underline{n} > \underline{L}$ does not depend on $\underline{n}$.

In particular, if $W^*(0) =$, the X input $[n] = 0$ when $n \geq L$.

From this we conclude, that in systems with infinite degree of

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stability, the process of control ends with the running out of final number of intervals.

The conditions in (Formula No. 73 and 75) we will call conditions of final duration of control process.

Parameters, satisfied by this conditions, can be called optional, from the view of infinite degree of stability. Naturally, not for all systems exist optional parameters.

Let us explain physically the meaning of infinite degree of stability.

Let us consider the controlled object, which is described with differential equation of the first order. In Fig. 32a this object is represented in the form of electrical stage RC. The voltage U_C in capacity is compared with standard voltage E_0, and their difference will exert influence upon the impulse element. The impulse element will influence the input stage RC with impulse, the hight and (duration) of which will depend on the difference $E_0 - U_c$. We could select parameters of the impulse element (k_i, y) in such a way, that the process of control would terminate during one interval t = 1, as it is shown in the Fig. 32b.(the curve 1). A process with different parameter values will not terminate when t = 1 (curves 2, 3).

Degree of fluctuation

As with the systems of intermittent control, we will call the degree of fluctuation 7 of stable system of intermittent control the absolute value of the relative of imaginary part of the root of characteristic equation nearest to the axis to the actual part, that is

(Formula No. 76)

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Geometrically, the η represents the tangent of an angle of inclination of a straight line, drawn from the beginning of coordinates through roots of characteristic equation nearest to the imaginary axis (Fig. 33).

Calculation of the degree of fluctuation can also be brought to the analysis of stability of some fictitious systems. Such as, for example, if substituting in $G^*(q)$ $q = \xi + j\bar{\omega} = \bar{\omega}\,[j - \eta]$, where $\eta = \frac{\bar{\omega}}{\xi}$ = constant, then applying the criteria of Michailov to

(Formula No. 77)

it is not difficult to determine, does the system possess the given degree of fluctuation or not, and select parameters, by which the degree of fluctuation is equal or not less then the given value. Let us note, that the degree of fluctuation relates to notching (echelon) function, that is, to descrete values of a process in moments of incidence.

Analogs of integral evaluations

Dynamic property of transmission process, that is, process of control, arising as a consequence of perturberance in the form of a singular jump, can be evaluated by values (Formula No. 78) which represent analogs of integral evaluation in the theory of intermittent control.

In such cases, when static declination $X_{input}\,[\infty] = 0$, which is often encountered in systems of intermittent control,

(Formula No. 79)

I_1 represents a surface, enclosed between the notching (echelon) function describing the process in the system of intermittent control $X_{input}\,[n]$, and its established value, that is, surface of declination of notching (echelon) function from its specific value (Fig. 34,1 & 35,1).

I_2 represents the surface of a square of this declination (Fig. 34,2 and 35,2).

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Let us note, that I_1 and I_2 are relative values. In order to obtain absolute values, they must be multiplied by T_p.

According to the theory of surface (see appendix)

(Formula No. 80)

It is evident, that I_1 can be useful for evaluation of not fluctuating processess.

Evaluation of I_2 is striped of this disadvantage.

We will bring expressions, which allow us to claculate directly I_2 according to coefficients of transmission function of the closed (circuit) system.

Let

(Formula)

where X^*_{input} (q) represents singular jump (in this case $X_{input}[\infty] = \frac{H^*(0)}{U^*(0)} = 0$),

(Formula)

characteristic equation and

(Formula)

then when $L = L_2 = 1$

(Formula No. 81)

when $L = L_2 = 2$

(Formula No. 82)

I_2 represents the function of parameters of intermittent control system. The values of parameters, by which I_2 reaches the minimum value, we will call optimal from the point of view of this evaluation. I_2 can also be determined graphycaly according to frequency characteristic of the closed (circuit) system.

It is possible to show, that

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(Formula No. 83)

that is, I_2 is equal to the surface of the square modulus $\frac{H_1^* (j\bar{\omega})}{Q^* (j\bar{\omega})}$.

5. Constant control as the limit (border) of intermittent control.

With the tendency of the interval of control toward zero ($T_p \rightarrow 0$) it is natural to expect, that the properties of the intermittent control system will approach to the properties of corresponding system of constant control. Considering the expressions (Formula No. 37 and 38) we can show, that

(Formula No. 84)

where $W^* (T_p P) = W^* (q)$ is transmission function of the open (circuit) system of intermittent control; $w (p)$ istransmission function of the linear part of the system; $\bar{K} = k_1 Y$ is for the systems of the first type; $\bar{K} = k_1 x$ is for the systems of the second type.

Actually, when the given value is $\bar{\omega} = \bar{\omega}_1 = \bar{\omega}_1 T_p$, the smaller the T_p, the greater the value of $\omega = \frac{\bar{\omega}_1}{T_p}$. If for $\omega > \omega_1$, the modulus of frequency characteristic of the linear part of the system is very small, and the values $| W (j\omega) |$ when $\omega > \omega_1$ could be ignored, then from the (Formula No. 37 and 38) follows, that

(Formula No. 85)

and, this means, that for transmission functions the relation in (Formula No. 84) is justifiable.

Thus, with the tendency of the interval of control toward zero, the transmission function of the open (circuit) system of intermittent control $W^* (q) = W^* (T_p P)$ as of the first and so of the second types tend toward the transmission function of the system of constant control, consisting of linear part and (instead of impulse element) linear amplifier with the coefficient of amplification $\bar{K}$.

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And because the form of equations for closed (circuit) systems of intermittent and constant control are indentical, then from this follows that at $T_p \to 0$ equations of systems of intermittent and constant control coincide. From this follows, that constant control represents the limit (border) of the intermittent control system.

On the strength of constant dependance of W* (T_p P) on T_p we can conclude, that with T_p, differing from the zero, but sufficiently small, so that for the greatest, according to absolute value or modulus $\widetilde{q}_{\vartheta}$, the inequality is accomplished.

(Formula No. 86)

properties of systems of intermittent control differ very little from properties of corresponding systems of constant control, that is from systems in which instead of impulse element, the linear amplifier is located.

If with the $\omega > \omega_c$ modulus of frequency characteristic is equal to zero, then with $T_p < \frac{\pi}{\omega_c}$ the peoperties of the system of intermittent control are identical with properties of systems of constant control. This conclusion is closely associated with the well known V. A. Kotelnikov theory of common action.

The above indicated conclusion represents the basis of application of intermittent control for control of objects with slow flowing processess. This principle allowed to accomplish great amplifications, simplify apparatus, whereby not changing the dynamic properties of process in comparison with constant control.

The above noted conclusion permits us to simply and graphicly explain the linearization of relay systems of automatic control by exposing them to external periodic fluctuation of high frequency.

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(Fig. 36)

Let us consider the contact arrangement (for example, of polarized relay), which controls the engine 5 (Fig. 36). Theposition of the moving contact 2 is determined by voltage, brought tothe winding 1 from the measuring element, which is proportional to the deviation of controled value from the given magnitude. Now we will add to rigidly connected contacts 3 and 4 fluctuating movement (for example, with the help of a small fist) according to sinusoidal or nearest to the triangle how with the amplitude, approximately equal to (somewhat smaller) half of the distance between the contacts 3 and 4. Then during declination of the controled value, which means position change for the contact 2, not exceeding the amplitude of given fluctuations, the engine will be directed by voltage impulses, duration of which is approximately proportional to the declination of the controled value.

The same effect can be obtained, if the contacts 3 and 4 are left motionless, and to the additional winding of relay 6 feed voltage with corresponding form and amplitude from on external source.

Imposition of external fluctuations by one or another method, thus transforms the relay system of automatic control into system of intermittent control of the second type.

If the frequency of external fluctuations is sufficiently high, the interval of control will be quite small ($T_p > \frac{\pi}{\omega}$); the system will behave as a linear system of constant control.

6. Systems of intermittent control of the third type.

As it was noted above, the systems of intermittent control of the third type are nonlinear systems, and which tend to auto-fluctuation.

In systems, which in their executive device contain integrating link,

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the amplitude of this auto-fluctuations decreases with the decrease of their periods. Therefore, considering the decrease of amplitude, it is advisable to select the parameters of a system in such a way, that the periods of auto-fluctuation be the shortest, that is, equal to $\tau_n = 2$ or $t_n = 2\,T_p$.

We will not discuss here the detailed analysis of auto-fluctuation, which is analogous in many instances to the analysis of normal relay systems, but will discuss only the condition, which assures auto-fluctuation of minimal period, and consequently, in astatic systems and optional amplitude. This condition has the form of

(Formula No. 87)

where $W^*(j\pi)$ is determined, for example, with the expression (Formula No. 47) when $\omega = \pi$.

Let us note, that with the tendency of T_p toward zero, the system of intermittent control of the third type tends toward normal relay system. This is evident from the remarks in the preceeding paragraph and the equivalent sketch of impulse element of the third type (Fig. 17).

III. Basic properties of typical systems of intermittent control.

We will discuss here briefly the basic properties and calculation graphs of some typical systems of intermittent control.

1. The system of automatic control of temperature.

A sketch of a system of automatical control of temperature with delayed feedback is shown in Fig. 37. The delayed feedback in its effect is equivalent to the resilient feedback, but in its construction it is simpler.

The measurement of the temperature is conducted with a thermometer

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of resistance 1, which represents one of the shoulders of the bridge 2. Into the diagonal of the bridge is connected the frame of a galvanometer 3 of the falling lever, which in the Fig. 37 is represented schematically. During the displacement of the indicator 4, which corresponds with the change of temperature of the object controled, one of the contacts 5 closes and brings into action the reversible motor of direct current 6, which in turn changes the position of the valve 7, which controls the flow of the fuel (or the heating agent).

At the same time, with the rotation of the reversible motor 6 one of the resistances 8 is heated. Thanks to this, the gas in that part of the U-shaped tube where this resistance is located expands, therewith lowering the level of mercury 9; and, consequently, the resistence, for example 10, connected with one of the shoulders of the bridge - increases. During this the level of mercury in the second part, by rising, decreases the resistance 11, connected to the adjacent shoulder of the bridge. Caused by this change unbalance of the bridge creates additional correcting impulse. By the end of the control process the balance (equilibrium) of the bridge always corresponds to the given magnitude of the controled value. The value of the nominal temperature, which must be maintained by the regulator, is established by the setter of the temperature 12. The direction of the reversible motor and the value is conducted by impulses, duration of which depends on the duration of closing of contacts, that is, the magnitude of the controled value in specific equally separated from each other moments of time.

In the system of the first type (sketch of which we are not showing) the duration will not be changed, but the value of impulses.

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