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[Note: In the original work, vectors were indicated by black-faced type, which cannot be used here. Therefore the context mainly, or sometimes arrows over the symbols, will have to show which symbols signify vector quantities]

PRODUCTION DURING ELECTRON PAIR RADIOACTIVE DECAY

B. V. Tisan

1. According to Dirac's theory an electron in the negative state may, when sufficiently disturbed, transfer to the state positive energy, thus forming the so-called "pair". In this work, we employ to the first approximation of the perturbation to study the probability of pair production.

We first consider a plane electromagnetic wave, sometimes called a pseudo-quantum

$$A = A_0 e^{i(kr-\omega t)}, \quad \Phi = \Phi_0 e^{i(kr-\omega t)} \tag{1}$$

where A_0 are respectively vector and scalar potentials with A_0 and Φ_0 constant. ω and k are frequency and wave vector. The energy and momentum of an electron will be designated by E_-, p_- and for a position by E_+, p_+. Relativistic units will be used throughout. Thus, in order to change from the usual formulas to those used here, and the elementary charge e equal to α, where α is constant the fine structure. The corresponding eigenfunctions are plane waves; consequently, to a finite volume v we can write

$$\psi_- = \frac{1}{\sqrt{V}} u_- e^{i(p_- r - E_- t)} \tag{2a}$$

$$\psi_+ = \frac{1}{\sqrt{V}} u_+ e^{-i(p_+ r - E_+ t)} . \tag{2b}$$

Since both the eigenfunctions and the perturbation are plane waves, we obtain the laws of conservation of energy and momentum:

$$\left.\begin{aligned} E_+ + E_- &= \omega \\ p_+ + p_- &= k \end{aligned}\right\} \tag{3}$$

from which it follows directly that pair production is possible only when the following condition holds

$$\omega^2 - k^2 = 2 + 2E_+E_- - 2(p_+p_-) \geq 2(1 + E_+E_- - p_+p_-) \geq 4. \tag{4}$$

we set $\hbar = c = m = 1$

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The last inequality follows from the fact that $E_+E_- = p_+p_-$ has a minimum at $E_+ = E_-$ equal to 1.

Since $\omega^2 - k^2 = 0$ for a light, obviously a light quantum cannot by itself produce a pair. Similarly, a free electron does not satisfy condition (4), which can be shown as follows: If we designate the energy and momentum of a free electron in the initial and final states as E, p and E', p', then the perturbation caused by it is expressed according to Möller in the form of (1), where

$$\omega = E - E', \quad k = p - p';$$

hence

$$\omega^2 - k^2 = 2 - 2E'E + 2(p'p) \leq 2(1 - E'E + p'p) \leq 0.$$

Actually, still a second particle, which could take up the excess momentum, is necessary for pair production. Since in this case two particles enter into interaction with an electron of negative energy, the perturbation theory to the second approximation must be employed here. Pair production when gamma- and beta-particles pass through matter has often been calculated in this way (H. Bethe and W. Heitler, Proc. Roy. Soc. (A) 146, 83, 1934; L. Landau and E. Lifschitz, Sow. Phys. 6, 244, 1934).

The production pair within the field of a decaying nucleus during beta- and gamma-decay can be calculated by the theory perturbation to the first approximation. First consider gamma-decay.

The field of a gamma-quantum is represented as a diverging spherical wave. If we express this field as a Fourier integral, then part of the plane waves obtained will satisfy condition (4), as we shall later discuss in detail. In this process, the nucleus, strictly speaking, is important not because of its interaction with the electron of negative energy, but only as a singular point of the spherical wave, which physically

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corresponds to transfer of momentum to the nucleus. Consequently, the probability of pair production in this case will not depend upon the atomic number Z, but will be proportional to the square of the elementary charge, i.e., proportional to the constant of the fine structure α alpha.

Moreover, there naturally exists a second order effect proportional to αZ^2, in exactly the same way that this was obtained from Heitler's formula for the passage of gamma-rays through matter.

Similar considerations apply to beta-decay, but here the perturbing field itself is caused by an electron; consequently, the first order effect is proportional to α^2, and the second order effect, to αZ^2.

This "inner conversion" of gamma- and beta-rays is actually observed (it should be distinguished from the natural inner conversion of gamma-rays, which consists of the ionisation of the emitting atom by gamma-quanta).

Further, we shall consider pair production under the action of a potential (1), which, generally speaking, has no direct physical meaning for the present and enters the discussion only as a component, the expression of the Fourier integral expression.

(In section 2 we have used the results obtained for gamma-decay and in section 3 for beta-decay.)

And thus, we now study the perturbing potential (1), which acts so that the electron transfers from the state (2) with negative energy, into the state (2) with positive energy. First obtain the relations (3) for energy and momentum, and then the probability of pair production per unit volume per unit time in the solid angle $d\Omega$:

$$d\sigma_{h\omega} = \frac{2\pi d\Omega \Delta E}{(2\pi)^3} \sum_{1,2} |V|^2 \qquad (5)$$

(Wahrscheinlichkeit)

[Note: distinguish w, probability, and omega ω, angular velocity or circular frequency.]

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where V is the matrix element of the perturbing energy:

$$V = K^{1/2}\left[(u_-^* \vec{A}_0 \vec{\alpha} u_+) - \Phi_0 (u_-^* u_+)\right]. \qquad (6)$$

The sum is distributed over the spin of the electron in the initial and final states.

Further,

$$|V|^2 = \alpha\left[\sum_{i,k=1} A_{i0} A_{k0} (u_-^* \alpha_i u_+)(u_+^* \alpha_k u_-) - 2\sum_{i=1}^{3} A_{i0}\Phi_0 (u_-^* \alpha_i u_+)(u_+^* u_-) + \Phi_0^2 (u_-^* u_+)(u_+^* u_-)\right].$$

We now introduce the well-known form of Casimir's expression:

$$\sum_{1,2} u_+^* u_- = \frac{-(\vec{\alpha}\vec{p}_+) + \beta - E_+}{-2E_+} \qquad (7a)$$

$$\sum_{1,2} u_-^* u_- = \frac{(\vec{\alpha}\vec{p}_-) + \beta + E_-}{2E_-} \qquad (7b)$$

Inasmuch as $E_+ > 0$ and $E_- > 0$, the operator (7a) when applied to a function with negative energy will give unity, and when applied to a function with positive energy will give zero, while the reverse will hold for the operator (7b). $\sum_{1,2}|V|^2$ is then calculated from the condition of closure through the calculation of the trace (spur), and we obtain:

$$\sum_{1,2}|V|^2 = \frac{\alpha}{E_+E_-}\left\{A_0^2\left[E_+E_- - (\vec{p}_+\vec{p}_-) + 1\right] - \Phi_0^2\left[E_+E_- + (\vec{p}_+\vec{p}_-) - 1\right] + 2(\vec{A}_0\vec{p}_+)(\vec{A}_0\vec{p}_-) - 2\Phi_0\left[E_+(\vec{A}_0\vec{p}_-) + E_-(\vec{A}_0\vec{p}_+)\right]\right\}.$$

We consider, in particular, the boundary case of low energy and set in the zero approximation $E_+ = E_- = 1$, $p_+ = p_- = 0$; we obtain then:

$$\sum_{1,2}|V|^2 = 2\alpha A_0^2 \qquad (8)$$

Consequently, for low energies (that is, kinetic energy of the pair low in comparison with their potential energy), pair production depends in the zero approximation only upon the vector, and not upon the scalar potential. We will need this result later.

In order to obtain the complete probability of pair production, we still must integrate with respect to all directions of the electron's momentum.

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~~tron~~ (by ~~the~~ relation (3) the direction of the positron is already determined through k and p_-). This integration, however, is extremely difficult, since according to ~~the~~ relation (3) the energy and momentum of the positron and electron depend in a complex way upon the angle between their directions of momenta for a constant ω and k.

This difficulty can be easily avoided if we remember that the probability per unit space per unit time is a relativistic invariant. In addition, we can select a system in which the pseudoquantum is at rest; i.e., a system which moves with a velocity $k/\omega < 1$ relative to the original system; the pseudoquantum is then represented as a periodic process with a wave vector equal to zero.

We now consider a process in this coordinate system where the case becomes especially simple. By an Eich-transformation we can eliminate the scalar potential, and by setting the z axis along the direction of the vector potential, we obtain for the matrix element of the perturbing potential

$$V = \alpha^{1/2} A_z e^{i\omega t} (u_-^* \alpha_+ u_+) \qquad (9)$$

Here, the condition of equating the divergence of potential to zero is observed.

Instead of (3), we have now

$$\left.\begin{aligned} E_+ + E_- &= \omega \\ p_+ + p_- &= 0 \end{aligned}\right\} \qquad (10)$$

from which follows: $E_+ = E_- = \omega/2$; thus the electron and positron have equal energies and equal but opposite momenta.

As before, $dw_{k,\omega}$ is given by formula (5), but now V is taken from (9). We now obtain

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$$\sum_{1,2} |V|^2 = \alpha |A_z|^2 \sum_{1,2} (u_-^* \alpha_z u_+)(u_+^* \alpha_z u_-)$$

$$= \alpha |A_z|^2 \, \mathrm{Spur} \left\{ \alpha_z \frac{-(\vec{p}_+\alpha) + \beta - E_+}{-2E_+} \alpha \frac{(\vec{p}_-\alpha) + \beta + E_-}{2E_-} \right\}$$

$$= \frac{\alpha |A_z|^2}{E_+ E_-} \left[E_+ E_- + 1 - (\vec{p}_+ \vec{p}_-) + 2 p_{+z} p_{-z} \right] \left(1 - \frac{\nu_0^2}{\omega^2} \right).$$

and with the help of (10) we arrive at $\sum |V|^2 = 2\alpha |A_z|^2$

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We substitute this value in (5), where p^2_{-z} is now the only quantity depending upon direction.

Then, taking into consideration

$$p = \sqrt{E^2 - 1} = \sqrt{\frac{\omega^2}{4} - 1}$$

we can integrate directly and finally obtain

$$w_{k\omega} = \frac{\alpha}{6\pi} |A_0|^2 \sqrt{1 - \frac{4}{\omega^2}(\omega^2 + 2)} \qquad (12)$$

Since this expression is a relativistic invariant, for an arbitrary coordinate system we can set

$$w_{k\omega} = \frac{\alpha}{6\pi} \sqrt{1 - \frac{4}{\omega^2 - k^2}(\omega^2 - k^2 + 2)}\,(A_0^2 - \Phi_0^2) \qquad (12')$$

where $A_0^2 - \Phi_0^2$ is determined uniquely from the fact that the dependence of A and Φ upon p and t is given by (1) and div $A + \frac{\partial \Phi}{\partial t} = 0$.

2. We can now determine the probability of pair production in gamma decay by Formula (12').

We will consider only dipole radiation, since the calculation of quadrupole radiation is completely analogous and the results differ only slightly.

The potential of the dipole wave we will write in the well-known form:

$$A_x = A_y = 0, \quad A_z = iB\frac{e^{ikr}}{r}, \quad \Phi = \frac{k}{\omega}\frac{\partial}{\partial z}\left(\frac{e^{ikr}}{r}\right) \qquad (13)$$

The time fact $\exp(-i\omega t)$ is discarded.

Obviously, (13) satisifies the continuity condition:

$$\frac{\partial A_z}{\partial z} - i\omega\Phi = 0$$

B is the glving coefficient, determined by the condition that the radiation must be one light quantum per second.

The Poynting vector is : $\frac{1}{4\pi r^2} B c \omega^2 (x^2 + y^2)$

Integrating over a sphere of radius r and equating to the energy of a light quantum we obtain :

$$B = \frac{\pi^2}{c\omega} \qquad (14)$$

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We now express the potential (13) as a Fourier [Transform] and apply formula (12) to each component. To obtain the expression, we must calculate integrals of the following type: $\int u e^{-ikr} dv$, $\int \frac{\partial u}{\partial z} e^{-ikr} dv$ (15)

where u [illegible]. This is accomplished as follows: u is an integral of the differential equation

$$\Delta u + \omega^2 u = -4\pi\delta(r)$$

Multiplying by exp (-ikr) and integrating over the space, we obtain

$$\int e^{-ikr} \Delta u \, dv + \omega^2 \int u e^{-ikr} dv = -4\pi$$

Using Green's formula, we can write

$$\int u e^{-ikr} dv = \frac{4\pi}{k^2 - \omega^2}$$

[illegible] k, integrating by parts [illegible]

$$\int \frac{\partial u}{\partial z} e^{-ikr} dv = \frac{4\pi i k_z}{k^2 - \omega^2}$$

With the help of these formulas, we [illegible] the following expressions for the coefficients [illegible] (16)

$$A_{zk} = \frac{1}{(2\pi)^{3/2}} \int A_z e^{-ikr} dv = \frac{4\pi i B}{(2\pi)^{3/2}(k^2 - \omega^2)},$$

$$\Phi_k = \frac{1}{(2\pi)^{3/2}} \int \Phi e^{-ikr} dv = \frac{4\pi i B k_z}{(2\pi)^{3/2}(k^2 - \omega^2)\omega};$$

and, taking (14) into consideration we have

$$A_{zk}^2 - \Phi_k^2 = \frac{3}{\pi\omega(\omega^2 - k^2)^2}\left(1 - \frac{k_z^2}{\omega^2}\right). \quad (19)$$

If we substitute (19) and (12), we obtain [illegible], i.e., the probability for the individual [illegible] Fourier [illegible]. The complete probability is obtained by integration of this [illegible]

$$w_\omega = \int w_{k\omega} dk = \frac{\alpha}{2\pi\omega} \int \sqrt{1 - \frac{4}{\omega^2 - k^2}} \cdot \frac{\omega^2 - k^2 + 2}{(\omega^2 - k^2)^2}\left(1 - \frac{k_z^2}{\omega^2}\right) dk$$

$$= \frac{2\alpha}{\pi\omega} \int \sqrt{1 - \frac{4}{\omega^2 - k^2}} \cdot \frac{\omega^2 - k^2 + 2}{(\omega^2 - k^2)^2}\left(1 - \frac{k^2}{3\omega^2}\right) k^2 dk.$$

We calculate this integral asymptotically for large ω. We take $\omega^2 - k^2 \gg 1$. Then $\sqrt{1 - \frac{4}{\omega^2 - k^2}} \approx 1$, and we obtain

$$w_\omega \approx \frac{2\alpha}{3\pi\omega^3} \int_0^\omega \frac{3\omega^2 - k^2}{\omega^2 - k^2} k^2 dk = \frac{2\alpha}{3\pi\omega^3} \int_0^\omega \left(k^2 - 2\omega^2 + \frac{\omega^3}{\omega - k} + \frac{\omega^3}{\omega + k}\right) dk$$

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This integral diverges logarithmically. Therefore, precise knowledge of the limits of integration is not important, and the divergence of the third term for large k's is only apparent, since the preceding formulas show that the expression under the integral goes to zero for these values of k. Consequently, we obtain:

$$\omega \cong \frac{2\alpha}{3\omega}\left(\log 2\omega - \frac{5}{3}\right)$$

$$\omega \cong \frac{2\alpha}{3\pi}\left(\log \frac{2\hbar\omega}{mc^2} - \frac{5}{3}\right)$$

or, in the ordinary units:

$$\omega \cong \frac{2}{3} \qquad (20)$$

Formula (20) coincides with that of Oppenheimer and Nedelsky (Phys. Rev. [illegible], 1933) Rose and Uhlenbeck (Phys. Rev. 48, 211, 1935).

In these works, however, there is a 4/5 instead of [illegible] in our formula (20). It is easy to see that formula (20) [illegible] which becomes clear even from Fig. 7 of Rose and Uhlenbeck, which was obtained by graphical integration and clearly conforms asymptotically with (20).

The question of in what region is our approximation applicable will be discussed in more detail in section 4. For the meantime, we simply note that we have disregarded the nuclear field and have considered the electron pair as free, which corresponds to the Born approximation of Rose and Uhlenbeck.

3. We now consider secondary effects in beta-decay. These we understand to be those effects which occur when the beta-particle is close to the emitting nucleus.

One effect is, for example, inner stopping radiation, in which a beta-particle in the field of a decaying nucleus transfers into a state with less energy and emits a quantum. This effect was recently calculated by J. K. Knipp and Uhlenbeck (Physica, III, 425, 1936).

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Another similar effect ~~of the same type~~ is ~~the~~ annihilation ~~of the~~ positron in the shell of the atom in the case of positron decay, which has been calculated by G. Rumer (Sow. Phys. 9, 317, 1936) and Charles Moller (Sow. Phys. - in ~~press~~ publication).

A third effect is pair production, which is calculated type in this work and by Möller (report read by Dr. Möller) using ~~by~~ another method.

Finally, a fourth effect is the ionisation of the emitting atom by beta-particles, which has not ~~not~~ yet been calculated ~~at the present~~.

Two different methods are used for the solution of all these problems. According to one of them, the emergence of the particle and the secondary effect are considered as a single process; ~~and~~ consequently, Fermi's theory of beta-decay in the second approximation is used, an additional term ~~matrix~~ being added to the differential equation, which corresponds to the interaction of the emerging beta-particle with the radiation field (inner stopping radiation) or with an electron in the negative state (pair production).

In the case of inner stopping radiation, for example, ~~we have~~ the processes the following scheme ~~diagram of the process~~.

neutron → proton + electron + neutrino →

→ proton + electron + quantum + neutrino.

The schemes ~~diagrams~~ for other processes can be written in a similar fashion.

The other method is more simple and considers the beta-particle as to possess ~~given with~~ some distribution ~~of~~ energy P(E), which can be taken from ~~using~~ Fermi's theory; it will be ~~and~~ not considered in detail here. Moreover, it is ~~not~~ unnecessary to consider the unobservable neutrino, ~~and~~ the indefiniteness of the Fermi or Konopinski-Uhlenbeck term ~~matrix~~ of interaction ~~vanishes~~ in the function P(E) ~~P(E)~~; also, ~~we~~ we need concern ourselves only with the secondary effect which interests us. On the other hand, however, we must in this case describe the particle as a diverging Dirac spherical wave with ~~which has a~~ singularity at ~~the~~ zero;

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Such an eigenfunction is, strictly speaking, inadmissible.

Knipp and Uhlenbeck studied inner stopping radiation by both methods and obtained consistent results. Annihilation of positrons with radiation was calculated by Rumer using the second method and by Møller using the first method. The difference between their results will be considered later. Now consider the applicability of the second, simpler method to this problem.

We start from the Dirac equation in the form

$$E\psi = [(\vec{\alpha}\vec{p}) + \beta]\psi \qquad (21).$$

We choose the following form for the Dirac matrices

$$\alpha_x = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \qquad \alpha_y = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix},$$

$$\alpha_z = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \text{[sic!]}, \qquad \beta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

Thus, we disregard the nuclear field and attempt to find the solution of equation (21) in the form of spherical waves (limiting ourselves to a spherical wave) with angular momentum $j = 1/2$ which corresponds to "permitted" transitions. This gives a solution of two types with complete angular momentum $j = 1/2$; for the first, the orbital momentum $l = 0$, and for the second, $l = 1$. In easily-understood terminology, we call them respectively the s- and p-electrons.

By simple calculations from the Dirac equation in polar coordinates, we obtain for eigenfunctions:

I. $j = \frac{1}{2}$, $l = 0$, s-electron:

$$\psi = N_0 \begin{cases} u, \\ 0, \\ -\frac{i}{E+1}\frac{\partial u}{\partial z} \\ -\frac{i}{E+1}\left(\frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}\right) \end{cases} \qquad (22a)$$

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$$\psi = N_0 \begin{cases} 0, \\ u, \\ -\frac{i}{E+1}\left(\frac{\partial u}{\partial x} + i\frac{\partial u}{\partial y}\right) \\ \frac{i}{E+1}\frac{\partial u}{\partial z} \end{cases} \tag{22b}$$

II. $j = \frac{1}{2}$, $\ell = 1$, p-electron:

$$\psi = N_1 \begin{cases} \frac{-i}{E-1}\frac{\partial u}{\partial z} \\ \frac{-i}{E-1}\left(\frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}\right) \\ u, \\ 0 \end{cases} \tag{23a}$$

$$\psi = N_1 \begin{cases} \frac{-i}{E+1}\left(\frac{\partial u}{\partial x} + i\frac{\partial u}{\partial y}\right) \\ \frac{i}{E+1}\frac{\partial u}{\partial z} \\ 0, \\ u, \end{cases} \tag{23b}$$

when there is a spin of $\frac{1}{2}$ or $-\frac{1}{2}$ in the z direction. Moreover,

$$u = \frac{1}{r}e^{ipr}, \quad p = \sqrt{E^2 - 1}$$

N_0 and N_1 are normal. [illegible] coefficients which are determined from the condition that the current is equal to one electron per second. For the current in case I, we obtain

$$j_x = (\varphi^* \alpha_x + \psi) = N_0^2 \frac{2px}{(E+1)r^3}, \ldots\ldots$$

The [illegible] which disappear more rapidly than $1/r^2$ for great distances are discarded. Similar expressions are obtained for j_y and j_z. The total current through a spherical surface of radius

$$N_0^2 \frac{8\pi p}{E+1} = 1$$

from which $N_0 = \sqrt{\frac{E+1}{8\pi p}}$ (24a)

In a similar way, we obtain $N_1 = \sqrt{\frac{E-1}{8\pi p}}$ (24b)

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If the beta-particle interacts with the radiation field or with any electron, it can transfer into another state, which is described by a plane wave. To this transition corresponds a certain current and charge, which in their turn determine through D'Alembert's equation certain vector and scalar potentials. This gives directly the radiation or the perturbing potential of the second electron. This is exactly what we now wish to determine.

The eigenfunction of an electron in the final state is a plane wave:

$$\psi' = \frac{1}{\sqrt{V}} u' e^{ip'r} \tag{25}$$

where u' is the spin tensor ("spinor"), which depends upon the direction of spin:

$$\sqrt{\frac{E'-1}{2E'}}\left(\frac{p_z'}{E'-1}, \frac{p_x'-ip_y'}{E'-1}, 1, 0\right) \tag{25a}$$

or

$$\sqrt{\frac{E'-1}{2E'}}\left(\frac{p_x'+ip_y'}{E'-1}, \frac{-p_z'}{E'-1}, 0, 1\right) \tag{25b}$$

We obtain, consequently, the current and charge; it is impossible, of course, to combine the fine structure constant with the Dirac matrices a_x, a_y, a_z:

$$\vec{j} e^{-i\omega t} = \alpha^{\frac{1}{2}}(\psi^* \vec{\alpha} \psi) e^{-i\omega t}$$

$$\rho e^{-i\omega z} = \alpha^{1/2}(\psi'^* \psi) e^{-i\omega t} \tag{26}$$

where we have substituted $E - E' = \omega$. The potential has the same time dependency $\exp(-i\omega t)$; therefore the time factor can be separated, and we write the D'Alembert equation in a form independent of time

$$\Delta \vec{A} + \omega^2 \vec{A} = -4\pi\alpha^{\frac{1}{2}}(\psi'^* \vec{\alpha} \psi)$$

$$\Delta \Phi + \omega^2 \Phi = -4\pi\alpha^{1/2}(\psi'^* \psi) \tag{27a}$$

the current and charge (27) for a certain closed surface including the zero point obviously does not satisfy the continuity equation:

$$\operatorname{div} j e^{-i\omega t} + \frac{\partial}{\partial t} \rho e^{-i\omega t} = 0$$

or

$$\operatorname{div} j - i\omega\rho = 0 \tag{28}$$

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Physically, the basis for this is immediately clear. We are discussing a diverging electron wave, but we are not considering the decrease in nuclear charge. For Dirac current and charge, the continuity equation follows from the Dirac equation; this derivation, however, ceases to be valid for the zero point since here ϕ goes to infinity. But this is without physical meaning, and the potential following from this in its turn does not satisfy the condition

$$\operatorname{div} A - i\omega\Phi = 0 \qquad (28a)$$

which is, however, necessary. Consequently, we must introduce into equation (27b) a term corresponding to the charge which is always uniquely possible with an accuracy down to a trivial constant in order to satisfy the continuity equation.

For this purpose, we must use a special form of the functions ϕ, ϕ'. Moreover, we will distinguish four cases, i.e., s- and p-electrons, and spin in the initial and final state either alike or opposite.

We first take the case of p-electron and alike spins. The current in (27a) then reduces simply to

$$j_x = C_1\left[\frac{p_x' + ip_y'}{E'-1} + \frac{p\frac{x}{r} - ip\frac{y}{r} + i\frac{x-iy}{r^2}}{E-1}\right]\frac{e^{i(pr-\vec{p}'\vec{r})}}{r}$$

$$j_y = C_1\left[\frac{ip_x' + p_y'}{E'-1} + \frac{ip\frac{x}{r} + p\frac{y}{r} - \frac{x-iy}{r^2}}{E-1}\right]\frac{e^{i(pr-\vec{p}'\vec{r})}}{r} \qquad (29)$$

$$j_z = C_1\left[\frac{p_z'}{E'-1} + \frac{p\frac{z}{r} + i\frac{z}{r^2}}{E-1}\right]\frac{e^{i(pr-\vec{p}'\vec{r})}}{r}$$

where $C_1 = \sqrt{\frac{E-1}{8\pi\rho}}\sqrt{\frac{E'-1}{2E'\nu}}$

The radial current will be

$$j_r = \frac{1}{r}\vec{j}\vec{r} = C_1\left[\frac{\vec{p}'\vec{r}}{(E'-1)r} + \frac{i(xp_y' - yp_x')}{(E'-1)r} + \frac{p}{E-1} + \frac{i}{(E-1)r}\right]\frac{e^{i(pr-\vec{p}'\vec{r})}}{r}$$

Integrating with respect to a spherical surface of radius r

$$\int j_r\,dv = C_1 4\pi i e^{ipr}\left[\frac{1}{E'-1}\left(\cos p'r - \frac{\sin p'r}{p'r}\right) + \frac{p}{(E-1)p'}\sin pa + \frac{1}{E-1}\frac{\sin p'r}{p'r}\right]$$

If $r \to 0$, this expression tends to [illegible]. Because of this, the additional charge is determined from (28) as $\rho' = \frac{4\pi}{(E-1)\omega}\delta(r)C_1$ from which it is seen that ρ' is the Dirac function.

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By a similar calculation, we obtain the additional charge for the case of an p-electron:

$$\rho' = \frac{\pi p'}{(E-1)(E'-1)\omega}\,\delta(r)\,C_0,$$

where $p' = \sqrt{E'^2-1}$ and $C_0 = \sqrt{\frac{E+1}{8\pi p}} \cdot \sqrt{\frac{E'-1}{2E'v}}$

Further, we find that the additional charge for the case of oppositely directed spins, both for [illegible] and p-electrons, goes to zero.

In the case of high energies $E \gg 1$, $E' \gg 1$, we have in both cases

$$\rho = \frac{\delta(r)}{\sqrt{\pi E \omega}}$$

Consequently, for high energies, the D'Alembert equation in correct form with an additional member is:

$$\Delta A + \omega^2 A = -4\pi\alpha^{\frac{1}{2}}(\psi'^{*}\vec{\alpha}\psi) \qquad (30a)$$

$$\Delta\Phi + \omega^2\Phi = -4\pi\alpha^{\frac{1}{2}}\left[(\psi'^{*}\psi) + \frac{\delta(r)}{\sqrt{\pi}\,E\omega}\right] \qquad (30b)$$

Thus, we see that the additional term appears only in the scalar potential and not in the vector potential. Consequently, all effects depending only upon vector potential can be calculated simply without the additional term. In particular, Knipp and Uhlenbeck, and also Rumer, were able to obtain the correct results because they calculated the radiation only for the vector potential. However, this was accidental to a certain degree; if one calculates the radiation from the vector potential according to the rules of electrodynamics, one assumes that the correct scalar potential satisfying condition (28a) corresponds to this. But this holds true only when the additional term is present.

In comparing the two methods mentioned, one more circumstance, which is best illustrated by an example of positron decay, must be taken into consideration. In the language of the Fermi theory (calculated by Möller), this effect is represented as follows: the proton decays into a neutron, positron, and neutrino, and the positron is annihilated with the release of radiation with one of the electrons of the atomic shell. Finally, the nuclear charge

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is decreased by 1, and then one electron (of the shell) is annihilated, and a quantum appears. However, the same effect may take place in such a way that the nucleus absorbs an electron from the K-level with radiation of a quantum and absorption of a neutrino. This process, however, becomes impossible, if, following Rumer, we apply the method of spherical waves to the positron. In this case, the least emitted quantum is $2mc^2$, while for absorption of an electron, the energy is only mc^2 plus the energy of the K-electron. As Möller calculated by us for heavy nuclei, the absorption effect may be considerable.

It might be thought that we are dealing with a similar phenomenon in the case of pair production, which, owing to the symmetry of the theory, is possible both for electron and positron decay; consequently, by applying the method of spherical waves, we would obtain an incomplete effect. This is not so, however, because the free energy liberated in the absorption of a K-electron is less than $2mc^2$, and is thus insufficient for pair production.

4. Finally, we calculate pair production in beta-decay by the method of spherical waves.

Apparently we must determine the Fourier coefficients in the Fourier potential integral from equation (30) and then substitute them in (12) and integrate with respect to k and ϑ, as in the case of gamma-decay. This would involve an error, however, for we must take into consideration not only the field of the beta-electron, but also the static weight of its final state and, in addition, the exchange with an electron in the negative state.

We adopt the so-called Born approximation and will consider all three particles in the final state as free. Consequently, the a geofunctions will be plane waves of the form (2). According to the well-known formula in the theory of perturbations of the first order, the probability

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of pair production is

$$dw = \frac{2\pi v\, dp'\, dp_+\, dp_-\, \delta(E - E' - E_+ - E_-)}{(2\pi)^3} \sum_{1,2} |V|^2 \quad (31)$$

Here E and E' are the energies of the beta-particle in the initial and final states; E_- and E_+ are the energies of the electron and positron; the corresponding designations for the momenta are $dp' = dp_x' dp_y' dp_z'$, etc.; v is the volume; and V is the matrix element of the perturbing energy.

It is possible to calculate from this formula the distribution of pairs (more correctly, not pairs, but two electrons and one positron) with respect to direction for the most general case. However, this formula is very complex and uninteresting in practice, and integration with respect to directions can be carried out only for the limiting cases of low and high energies. We will obtain a real simplification if from the very beginning we limit ourselves to these boundary cases.

a) We discuss first the limiting case of low energies. The minimum energy necessary for pair production is 3 ($3mc^2$ in the usual units). Consequently, for small energies we have $E = 3$, $E_+ = 1$, $E_- = 1$, $E' = 1$, all momenta small in comparison with unity. As we have seen in section 1, pair production in this approximation depends only upon the vector potential, and not upon the scalar potential. We can therefore limit ourselves to the differential equation

$$\Delta A + {}^2A = -4\pi\alpha'^{\frac{1}{2}}(\psi'^{*}\vec{\alpha}\psi) = -\frac{4\pi\alpha^{\frac{1}{2}}}{\sqrt{V}} \cdot (u^{*}\vec{\alpha}\psi)e^{-i\vec{p}\vec{r}} \quad (30a) \quad [p\,707]$$

in which the additional term does not enter.

We again write $E_+ + E_- = \omega$, $\vec{p}_+ + \vec{p}_- = \vec{k}$.

Owing to the delta-function in (31) $E - E' = E_+ + E_- = \omega$.

We multiply equation (30b) scalarly by

$$\alpha^{\frac{1}{2}}(\psi_-^{*}\vec{\alpha}\psi_+) = \frac{\alpha^{\frac{1}{2}}}{V}(u_-^{*}\vec{\alpha}u_+)e^{-i\vec{k}\vec{r}}$$

and integrate over the space. The left side of the equation then is

$$\frac{\alpha^{\frac{1}{2}}}{V}(u_-^{*}\vec{\alpha}u_+)\int e^{-i\vec{k}\vec{r}}(\Delta A + {}^2A)dv = \frac{\alpha^{\frac{1}{2}}}{V}(u_-^{*}\vec{\alpha}u_+)(\omega^2 - k^2)\int e^{-i\vec{k}\vec{r}}\vec{A}\,dv$$

$$= \alpha^{\frac{1}{2}}(\omega^2 - k^2)\int(\psi_-^{*}\vec{\alpha}\vec{A}\psi_+)dv = (\omega^2 - k^2)V,$$

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(32)

where V is the matrix element of perturbation. From the right side, we obtain

$$-4\pi\frac{\alpha}{V^{3/2}}(u_-^*\vec{\alpha}u_+)\int(u'^*\vec{\alpha}\psi)e^{-i(\vec{p}+\vec{k})r}\,dv$$

$$=-\frac{4\pi\alpha}{V^{3/2}}(u_-'^*\vec{\alpha}u_+)(u'^*\vec{\alpha}v). \qquad (33)$$

if we substitute the following spin tensor for v

$$v=\int\psi e^{-i(\vec{k}+\vec{p}')r}\,dv \qquad (34)$$

then with the help of (16), (17), (22), and (23), we obtain the following expressions for the e-electron, if we designate k+p = q in dependence upon the direction of spin:

$$\left.\begin{aligned} v&=\frac{4\pi N_0}{q^2-p^2}\left(1,0,\frac{q_z}{E+1},\frac{q_x-iq_y}{E+1}\right)\\ v&=\frac{4\pi N_0}{q^2-p^2}\left(0,1,\frac{q_x+iq_y}{E+1},\frac{-q_z}{E+1}\right)\end{aligned}\right\} \qquad (35)$$

and for the e-electron:

$$\left.\begin{aligned} v&=\frac{4\pi N_1}{q^2-p^2}\left(\frac{q_z}{E-1},\frac{q_x-iq_y}{E-1},1,0\right)\\ v&=\frac{4\pi N_1}{q^2-p^2}\left(\frac{q_x+iq_y}{E-1},\frac{-q_z}{E-1},0,1\right)\end{aligned}\right\} \qquad (36)$$

From (30), (32), (33), and (34), we obtain

$$V=\frac{4\pi\alpha}{V^{3/2}}\cdot\frac{1}{\omega^2-k^2}(u_-^*\vec{\alpha}u_+)(u'^*\vec{\alpha}v) \qquad (37)$$

or, written differently

$$V=\frac{4\pi\alpha}{V^{3/2}}\frac{1}{(E_++E_-)^2-(\vec{p}_++\vec{p}_-)^2}(u_-^*\vec{\alpha}u_+)(u'^*\vec{\alpha}v) \qquad (38)$$

We consider now the exchange between the beta-electron and the electron in the negative state, by making the functions of the electrons antisymmetric.

Without consideration for exchange, the eigenfunctions in the initial and final states are respectively $\phi(1)\,\phi_+(2)$ and $\phi'(1)\,\phi_-(2)$, where the numbers in parentheses represent the first and second electrons. The antisymmetric eigenfunctions for the initial and final states will be:

$$\left.\begin{aligned}\psi&=\frac{1}{\sqrt{2}}\left(\psi(1)\psi_+(2)-\psi(2)\psi_+(1)\right)\\ \psi&=\frac{1}{\sqrt{2}}\left(\psi'(1)\psi_-(2)-\psi'(2)\psi_-(1)\right)\end{aligned}\right\} \qquad (39)$$

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$$\frac{\sum_{i,k}(u_-^*\vec{\alpha}_i u_+)(u_+^*\vec{\alpha}_k u')(u'^*\vec{\alpha}_i v)(v^*\vec{\alpha}_k u)}{[(E_+ + E_-)^2 - (\vec{p}_+ + \vec{p}_-)^2][(E_+ + E')^2 - (\vec{p}_+ + \vec{p})^2]} +$$

The matrix element of perturbation will now be:

$$V = \frac{4\pi\alpha}{V^{3/2}}\left\{\frac{(u_-^*\vec{\alpha}u_+)(u'^*\vec{\alpha}v)}{[(E_+ + E_-)^2 - (\vec{p}_+ + \vec{p}_-)^2]} \frac{(u'^*\vec{\alpha}u_+)(u_-^*\vec{\alpha}v)}{[E_+ + E')^2 - (\vec{p}_+ + \vec{p})^2]}\right\} \quad (40)$$

and squared will be:

$$|V|^2 = \frac{16\pi^2\alpha^2}{V^3}\left\{\frac{\sum_{i,k}(u_-^*\vec{\alpha}_i u_+)(u_+^*\vec{\alpha}_k u_-)\sum_{i,k}(u'^*\vec{\alpha}_i u')(v^*\vec{\alpha}_k u')}{[(E_+ + E_-)^2 - (\vec{p}_+ + \vec{p}_-)^2]^2}\right. \quad (41)$$

plus the same terms with u_- and u' transposed.

We must now sum $|V|^2$ with respect to the spins of three particles in the final state. It is convenient to also sum with respect to the spin of a beta-particle in the initial state and divide by 2. For the amplitude of the plane wave, we can take Casimir's well-known equation

$$\sum_{1,2} u^* u' = \frac{\vec{\alpha}\vec{p} + \beta + E'}{2E'} \quad (42)$$

The spin tensor v is not the amplitude of the plane wave, but is constructed similarly, and differs only in that q is not a momentum belonging to E. By direct calculation, we find for the s-electron:

$$\sum_{1,2} v^* v = \left(\frac{4\pi N_0}{q^2 - p^2}\right)^2 \frac{1}{E+1}\left[(\vec{\alpha}\vec{p}) + \frac{(E+1)^2 + q^2}{2(E+1)} + \beta\frac{(E+1)^2 - q^2}{2(E+1)}\right] \quad (43)$$

and for the p-electron

$$\sum_{1,2} v^* v = \left(\frac{4\pi N_1}{q^2 - p^2}\right)\frac{1}{E-1}\left[(\vec{\alpha}\vec{p}) + \frac{q^2 + (E-1)^2}{2(E-1)} + \beta\frac{q^2 - (E-1)^2}{2(E-1)}\right] \quad (44)$$

If we use these expressions, we can calculate $\sum_{i,j}|V|^2$ in the usual way by calculating the traces of the matrix products. The result becomes particularly simple if we make use of the fact that the energy is low and we simply set $E = 3,\ E_+ = E_- = E' = 1,\ p = \sqrt{E^2 - 1} = \sqrt{8}$

$$\vec{p}' = \vec{p}_+ = \vec{p}_- = \vec{k} = \vec{q} = 0 \quad (45)$$

For the p-electron, we then arrive at

$$\sum_{1,2}|V|^2 = \frac{(2\pi)^3\alpha^2}{V^3 8^{5/2}} \quad (46)$$

the s-electron giving zero in this approximation. Since beta-rays always consist of a mixture of s- and p-electrons, we can disregard the action of s-electrons at low energies.

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If we substitute (46) into (31) and set $d\vec{p}' = 4\pi p' E' dE'$,

we obtain $$dw = \frac{9a^2 p' p_+ p_- \, dE' dE_- \, \delta(E - E' - E_+ - E_-)}{(2\pi)^2 \delta^{3/2}} \quad . \qquad (47)$$

Now we must integrate with respect to all energies in the region defined by (45).

We assume $p' = \sqrt{E'^2 - 1} \cong \sqrt{2(E'-1)}$ and call $E'-1 = \varepsilon_1$, $E_+ - 1 = \varepsilon_2$, $E_- - 1 = \varepsilon_3$

From (46), we then have $$w = \int dw = \frac{9\alpha^2}{32\pi^2} J \qquad (48)$$

with $$J = \int_{\varepsilon_1, \varepsilon_2, \varepsilon_3 > 0} \sqrt{\varepsilon_1 \varepsilon_2 \varepsilon_3} \, d\varepsilon_1 d\varepsilon_2 d\varepsilon_3 \, \delta(E - 3 - \varepsilon_1 - \varepsilon_2 - \varepsilon_3) \quad . \qquad (49)$$

If we set $\varepsilon_1 = x^2$, $\varepsilon_2 = y^2$, $\varepsilon_3 = z^2$, $E-3 = a^2$, then

$$J = \int x^2 y^2 z^2 dx dy dz \cdot \delta(x^2 + y^2 + z^2 - a^2) \qquad (50)$$

The integral must be taken with respect to the entire spherical surface. From the well-known formula $\delta(x^2 + y^2 + z^2 - a^2) = \delta(r^2 - a^2) = \frac{1}{2a}\delta(r - a)$

we obtain $J = \frac{a^2}{2} \iint \cos^2\theta \sin^2\theta \cos^2\phi \sin^2\phi \sin\theta \, d\theta d\phi = \frac{2\pi}{3 \cdot 5 \cdot 7} (E-3)^{3/2}$

Substituting in (47), we finally obtain

$$w \cong \frac{3\alpha^2}{560\pi} (E-3)^{7/2} \qquad (51)$$

or in the ordinary units $$w \cong \frac{3\alpha^2}{560\pi} \left(\frac{E - 3mc^2}{mc^2}\right)^{7/2} . \qquad (51a)$$

b) We now move to the limiting case of high energies: $E \gg 1$. We take and later prove that the essential region for complete probability of pair production is the energy region $$1 \ll \omega \ll E; \qquad (52)$$

~~consequently,~~ that is, the region where the complete energy of the pair is great in comparison with the potential energy but small in comparison with the energy of the particle.

Thus we obtain $$E \cong E - 1 \cong p \cong E - \omega \cong E' \cong p' \quad . \qquad (53)$$

It follows directly from this that exchange may be disregarded for high energies, since the process in which the beta-particle yields little energy. What is important here is Then transposition of electrons will correspond to high energy yields. Evaluation of the exchange terms confirms this result.

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We now rewrite formula (31) with the help of k and ω in the form

$$dp'dp_+dp_-\delta(E-E'-E_+-E_-) = 4\pi E'^2 d\Omega_+ p_+ E_+ d\omega dk = 4\pi E^2 d\Omega_+ p_+ d\omega dk$$

But, according to section 1

~~and consequently~~ $w_{k\omega} = \int \frac{2\pi d\Omega_+ p_+ E_+}{(2\pi)^3} \sum_{1,2} |V|^2$

and consequently $dw = \frac{2\nu^3 E\, 2 d\omega d\vec{k}}{(2\pi)^5} w_{k\omega}$. (54)

However, from (12), we can take

$$dw = \frac{2\nu^3 E^2 d\omega dk}{(2\pi)^5} \frac{\alpha}{6\pi} \sqrt{1-\frac{4}{\omega^2-k^2}}\,(\omega^2-k^2+2)\,(A^2_{k\omega}-\Phi^2_{k\omega}).$$

Thus, we now need only determine the Fourier coefficients $A_{k\omega}$ $\Phi_{k\omega}$ from equation (30). For this purpose, we multiply (30) by $\frac{1}{\nu}\exp(-ikr)$ and integrate

$$\vec{A}_{\vec{k}\omega} = -\frac{4\pi\alpha^{1/2}}{\nu^{3/2}(\omega^2-k^2)}\int (u'^*\vec{\alpha}\psi)\, e^{-i(\vec{k}+\vec{p}')\vec{r}}\, dv$$

$$\Phi_{\vec{k}\omega} = -\frac{4\pi\alpha^{1/2}}{\nu^{3/2}(\omega^2-k^2)}\left[\int (u'^*\psi)\, e^{-i(\vec{k}+\vec{p}')\vec{r}}\, dv + \frac{1}{\sqrt{\pi E\omega}}\right]$$

or according to (34)

$$\vec{A}_{\vec{k}\omega} = \frac{4\pi\alpha^{1/2}}{\nu^{3/2}(\omega^2-k^2)}(u'^*\vec{\alpha}v)$$

$$\Phi_{\vec{k}\omega} = -\frac{4\pi\alpha^{1/2}}{\nu^{3/2}(\omega^2-k^2)}\left[(u'^*v)+\frac{1}{\sqrt{\pi E\omega}}\right]$$

If u' and v are substituted from (25) and (35) and we transfer to high energies according to (53), we obtain for both the s- and p-electrons:

$$\vec{A}_{\vec{k}\omega} = \frac{4\pi^{3/2}\alpha^{1/2}}{\nu^{3/2}(\omega^2-k^2)E}\,\frac{\vec{p}'}{(\vec{k}\vec{p}')-E\omega} \qquad (55)$$

$$\Phi_{\vec{k}\omega} = \frac{4\pi^{3/2}\alpha^{1/2}}{\nu^{3/2}(\omega^2-k^2)E}\left[\frac{E}{(\vec{k}\vec{p}')-E\omega}+\frac{1}{\omega}\right]$$

We set the z-axis along the direction p':

$$\vec{A}^2_{\vec{k}\omega} - \Phi^2_{\vec{k}\omega} = \frac{16\pi^3\alpha}{\nu^3(\omega^2-k^2)E^2}\left[\frac{p'^2-E^2}{(k_z p'-\omega E)^2} - \frac{2E}{\omega(k_z p'-E)} - \frac{1}{\omega^2}\right] \qquad (56)$$

The first ~~[illegible]~~ term in the parentheses is ~~obviously~~ of the order of magnitudes disregarded here; it would be a natural ~~[illegible]~~ term, however, if it were not an additional ~~[illegible]~~ term in Φ. Using (53), we write

$$\vec{A}^2_{\vec{k}\omega} - \Phi^2_{\vec{k}\omega} = -\frac{16\pi^3\alpha}{\nu^3(\omega^2-k^2)E^2}\left(\frac{2}{\omega}\cdot\frac{1}{k_z-\omega}+\frac{1}{\omega^2}\right).$$

Substituting in (54), we have:

$$w = \int dw = -\frac{\alpha^2}{6\pi^3}\iint\sqrt{1-\frac{4}{\omega^2-k^2}}\,\frac{\omega^2-k^2+2}{(\omega^2-k^2)^2}\left(\frac{2}{\omega(k_z-\omega)}+\frac{1}{\omega^2}\right)d\vec{k}\,d\omega.$$

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The first integration with respect to the angles in the $\vec{k}$ space is

$$w = -\frac{2\alpha^2}{3\pi^2}\int_2^E\int_0^\omega \sqrt{1-\frac{4}{\omega^2-k^2}}\cdot\frac{\omega^2-k^2+2}{(\omega^2-k^2)^2}\left(\frac{1}{k}\log\frac{\omega-k}{\omega+k}-\frac{1}{2}\right)k^2\,dk\,d\omega \qquad (57)$$

We introduce the new variable $y^2 = \omega^2-k^2$ and consider the ~~[illegible]~~ of integration $1 \leq y \leq \omega$.

Then $k = \omega - \frac{y^2}{2}$, $kdk = -ydy$.

Consequently, $w = -\frac{2\alpha^2}{3\pi^2}\int_1^\omega\left(\log\frac{y^2}{\omega^2}+1\right)\frac{dy}{y}\cdot\frac{d\omega}{\omega}$ $\quad \frac{2\alpha^2}{9\pi^2}(\log E)^3$

In view of (57), we can write 1 instead of the root:

$$w \cong -\frac{2}{3}\frac{\alpha^2}{\pi^2}\int_2^E\int_0^\omega \log\frac{y^2}{\omega^2}\cdot\frac{dy}{y}\cdot\frac{d\omega}{\omega} \cong -\frac{4\alpha^2}{3\pi^2}\iint\log\frac{y}{\omega}\cdot\frac{dy}{y}\cdot\frac{d\omega}{\omega} \cong \qquad (58)$$

or, in the usual units: $w = \frac{2\alpha^2}{9\pi^2}\left(\log\frac{E}{mc^2}\right)^3$

In conclusion, we note that it would have possible to arrive at the same result by Williams method, by making use of the fact that an electron with energy E is equivalent to a set of light quanta:

$$N(\omega)d\omega = \frac{2\alpha}{\pi}\log\frac{E}{\omega}\cdot\frac{d\omega}{\omega} \qquad (59)$$

and applying formula (20) to the light quanta. Of course, at first glance the applicability of this simple method here is quite unapparent. However, since we have proved the correctness of this method, it can be used and by using also the Rose Wienbeck formula for the energy distribution of positrons ~~with respect to energy~~ we can multiply it by (59) and thus obtain the energy distribution of pairs ~~with respect to energies~~: $dw(E_+E_-) =$

$$\frac{2\alpha^2}{\pi^2}\frac{(E_+^2+E_-^2)}{(E_++E_-)}\log\frac{E_++E_-}{E_++E_-}\log\frac{E}{E_++E_-}\,dE_+dE_- \qquad (60)$$

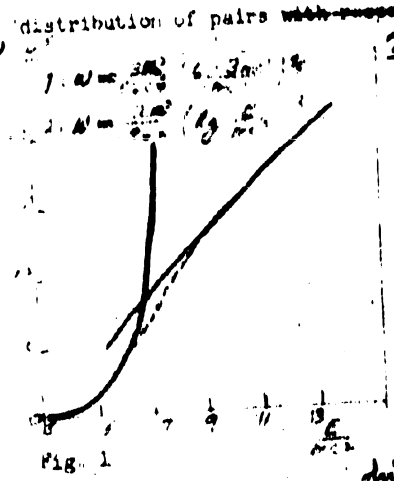

Fig. 1

The logarithmic coefficients reduce to zero, one at $\omega = E_+ + E_- = E$ and the other for small ωs (approximately $E_+ + E_- = 2$). This confirms the correctness of our considering only the region of integration given by (52).

~~For~~ Fig. 1, the curves for (51) and (58) are represented graphically, and ~~it is seen~~ quite good interpolation obviously exists for intermediate energies.

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We see that for ordinary energies of beta-particles, the probability of pair production is of the order of 10^{-6}, while Alikhanov found this probability to be of the order of 10^{-4}.

We now discuss briefly several factors which we have disregarded.

1. We disregarded the nuclear field and have considered the electrons as free, so that we do not have a good approximation for low energies. Disregarding this field, however, only increases the effect.

2. We have calculated only the effect of the first approximation, proportional to α^2. In addition, there is a second order effect, proportional to $\alpha^4 Z^2$. It is highly improbable, however, that this would produce substantial corrections. This can be seen from the similar circumstances in pair production during gamma-decay. Figs. 7 and 8 of Rose and Uhlenbeck show that the Born approximation, which is similar to ours, gives good correspondence with Hulme and Jager's accurate results, ~~the accurate results~~ but ~~[illegible]~~ slightly lower.

In conclusion, we wish to thank Professor L. D. Landau for suggesting the subject and for his constant attention during our work. Ukrainian Physico-Technical Institute, Kharkov

— END —

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