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FOCUSING OF MOLECULAR BEAMS BY NON-HOMOGENEOUS MAGNETIC FIELD.

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ABSTRACT. Several types of non-homogeneous magnetic fields, permitting accomplishment of double focusing of molecular beams are presented and the motion of molecules in these fields is discussed with consideration of the force of gravity. It is shown that by means of a plane non-homogeneous magnetic field, as presented in this article, the accomplishment of a lens for molecular rays is possible.

INTRODUCTION

Because of the development of mass-spectroscopy, beta-spectroscopy and electron optics, the problems of focusing electron and ion beams by electric and magnetic fields were thoroughly studied theoretically as well as experimentally. The possibility of focusing a molecular beam by a non-homogeneous axial field was first indicated by Korsunskiy [1]. In this work the focusing properties of an axial magnetic field with a gradient of the form $\partial H_y/\partial x = P/r^2$ was discussed.

The present work considers special properties of magnetic fields, axial as well as plane, securing double focusing of the molecular beam. The motion of molecules in these fields is studied with consideration of the action of gravity.

1. GENERAL EXPRESSION OF FORCE, ACTING ON A PARTICLE WITH A MAGNETIC MOMENT.

The force acting on a particle in a magnetic field is

$$\boxed{f = \nabla(\vec{\mu}\vec{H})} \qquad f = \nabla(\vec{\mu}\vec{H}) \tag{1}$$

Using the vector equality $\nabla(\vec{\mu}\vec{H}) = (\vec{\mu}\nabla)\vec{H} + [\vec{\mu}\,\mathrm{rot}\,\vec{H}]$ and considering that $\mathrm{rot}\,\vec{H} = 0$ (magnetostatic field), we obtain for the force f the expression [Note: The symbol μ stands for the Greek "mu"]

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$$\boxed{f = (\vec{\mu}\nabla)H} \qquad f = (\vec{\mu}\nabla)H \qquad (2)$$

In a cylindrical system of coordinates the components of the force f in an axial magnetic field will have the form

$$f_r = \mu_r \frac{\partial H_r}{\partial r} + \mu_z \frac{\partial H_r}{\partial z} \qquad f_z = \mu_r \frac{\partial H_z}{\partial r} + \mu_z \frac{\partial H_z}{\partial z} \qquad (3)$$

Using the conditions $\operatorname{rot} \vec{H} = 0$ and $\mu_r = \mu H_r / H$, and $\mu_z = \mu H_z / H$, for the components of the force f we obtain expressions:

$$f_r = \frac{\mu}{H}\left(H_r \frac{\partial H_r}{\partial r} + H_z \frac{\partial H_r}{\partial r}\right) \qquad f_z = \frac{\mu}{H}\left(H_r \frac{\partial H_z}{\partial z} + H_z \frac{\partial H_z}{\partial z}\right) \qquad (4)$$

In the cartesian system of coordinates in a plane magnetic field the force components of f will have the form:

$$f_x = \frac{\mu}{H}\left(H_x \frac{\partial H_x}{\partial x} + H_y \frac{\partial H_x}{\partial x}\right) \qquad f_y = \frac{\mu}{H}\left(H_x \frac{\partial H_y}{\partial y} + H_y \frac{\partial H_y}{\partial y}\right) \qquad f_z = 0 \qquad (5)$$

~~Double focusing of the molecular beam will take place in an axial magnetic field will take place under the conditions~~

Double focusing of the molecular beam in an axial magnetic field will occur, as it will be shown below, under the condition that the force components will be expressed in the following way

$$\boxed{f_r = \mu a r} \qquad f_r = \mu a r \qquad f_z = 0 \qquad (6)$$

where a is a constant characterising the magnetic field.

Double focusing in a plane field will occur under the condition:

$$\boxed{f_x = \mu a x} \rightarrow \quad f_x = \mu a x \qquad f_y = \mu a y \leftarrow \boxed{f_y = \mu a y} \qquad (7)$$

It is obvious that the magnetic field will focus ~~under the condition~~ provided that the value μ of the projection of the magnetic moment in the direction of the vector $\vec{H}$ will remain constant during the motion of the molecule in the nonhomogeneous field. It will be shown below that this condition is satisfied.

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[redacted]

The possibility to accomplish double focusing in an axial magnetic field is related to the finding of such a magnetic field, the component values of which substituted into equation (6) would result in expressions (6) for the components of the force acting on the molecule.

In the interpolar space of an electromagnet, Laplace's equation holds for the potentials

$$\Delta V = 0 \tag{8}$$

Let us write Laplace equation in spherical coordinates, considering the condition of axial symmetry of the field ($\partial^2 V/\partial \varphi^2 = 0$):

$$\frac{\partial}{\partial R}\left(R^2 \frac{\partial V}{\partial R}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) = 0. \tag{9}$$

Let us find now the axial magnetic fields, the potential of which will be expressed by the formula:

$$V = CR^n f(\theta) \tag{10}$$

In order to find the function $f(\theta)$ let us substitute the value of the potential from formula (10) into equation (9), which will result in a differential equation of the following form:

$$f''(\theta) + \operatorname{ctg}\theta\, f'(\theta) + (n+1)nf(\theta) = 0. \tag{11}$$

By exchanging the variables in equation (11) according to formula $t = \cos\theta$, we obtain:

$$(1 - t^2)F''(t) - 2tF'(t) + n(n+1)F(t) = 0 \tag{12}$$

where $f(\theta) = F(t) = f(\arccos t)$.

Equation (12) represents the differential equation of Legendre. The solution of this equation (12) has the form:

$$F(t) = \frac{1}{2^n n!}\frac{d^n}{dt^n}(t^2-1)^n \quad [\text{sic.}] \tag{13}$$

$$\boxed{F(t) = \frac{1}{2^n n!}\frac{d^n}{dt^n}(t^2-1)^n}$$

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Let us consider this [illegible]

$$F(t) = \frac{3}{2}t^2 - \frac{3}{2}t \quad \text{or} \quad [illegible]$$

Substituting the value $F(t)$ into formula (10), we obtain the value of the potential of the considered field in the form:

$$V = cz^3\left(\frac{3}{2}\cos^2\theta - \frac{3}{2}\cos\theta\right) = C\left(z^3 - \frac{3}{2}zr^2\right). \qquad (14)$$

Using this formula for the potential we shall find the values of the components of the field tension

$$H_z = a\left(\frac{1}{2}r^2 - z^2\right), \quad H_r = arz, \qquad (15)$$

where $a = 3C$.

It is easy to ~~tx~~ prove that the Laplace equation is satisfied by a field of more general type, the components of which are of the form:

$$H_z = H_0 + a\left(\frac{1}{2}r^2 - z^2\right), \quad H_r = arz. \qquad (16)$$

Let us now find the force acting on a particle with magnetic moment in a field, characterized by equations (16). For this purpose let us compute f_z and f_r according to formulas (4) using also formulas (16). The result of computation will be

$$f_r = \mu ar\left[1 + \frac{a^2z^4 - 2H_0az^2}{(H_0 + ar^3/2)^2}\right]^{-\frac{1}{2}} \qquad (17a)$$

$$f_z = \frac{-2\mu az\left(H_0 - \frac{1}{2}az^2\right)\left(H_0 + \frac{a}{2}r^2\right)^{-1}}{\left[1 + \frac{a^2z^4 - 2H_0az^2}{(H_0 + ar^3/2)^2}\right]^{\frac{1}{2}}} \qquad (17b)$$

In the median plane of the magnetic field ($z = 0$) the force components will be

$$\boxed{f_r = \mu ar} \qquad f_r = \mu ar, \quad f_z = 0 \qquad (18)$$

Hence for molecules moving in the median field plane, characterized by equations (16), the force will be focusing along the whole plane.

It is possible to prove that in any magnetic field with axial symmetry in some region of the median plane near the symmetry axis a focusing force will act on a particle with a magnetic moment. Indeed the component H_z may be expanded, in a region near the symmetry axis, in series following

$$H_z = [illegible] \qquad (19)$$

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where $a_k = \mathcal{P}_k(z)$.

Because $a_1 = \partial H_z(0,z)/\partial z$, and from the condition rot $H = 0$, we have that $\partial H_z/\partial r = \partial H_r/\partial z$, we then obtain from the condition of symmetry $H_r(0,z) \equiv 0$ the following:

$H_r(0,z) \equiv 0$

$$a_1 = 0 . \tag{20}$$

Now we may prove that all terms of the series (19) with odd indices are zero.

Let us write the Laplacian of the H_z component in cylindrical coordinates under condition of axial symmetry:

$$\frac{1}{r}\frac{\partial H_z}{\partial r} + \frac{\partial^2 H_z}{\partial r^2} + \frac{\partial^2 H_z}{\partial z^2} = 0 . \tag{21}$$

Let us substitute in the expression (21) the series (19) for H_z, then we shall obtain

$$\sum_0^\infty k\left[k a_k r^{k-2} + k(k-1) a_k r^{k-2} + a_k'' r^k\right] = 0 \tag{22}$$

The condition that the right side of expression (22) is identical to zero leads to the expression

$$k^2 a_{k0} = -a_k'' \tag{23}$$

But because $a_1 = 0$, it then results from expression (23) that all remaining coefficients of series (19) with odd indices are zero; hence the series for H_z may be written as follows

$$H_z = a_0 + a_2 r^2 + a_4 r^4 + \dots \tag{24}$$

If we shall limit ourselves to the region near the symmetry axis where $a_4 r^4 \ll a_2 r^2$, then H_z may be written in the following form:

$$H_z = a_0 + a_2 r^2 . \tag{25}$$

$= 2\mu a_2 r$

Keeping in mind that in the median plane $H_r = 0$ and $H_z = H$, we obtain, with consideration of formula (4),* $f_r = 2\mu a_2 r$, $f_z = 0$; it means that in the region of the median plane where relation (25) holds, the force has focusing effect.

As follows from equations (16), the equation of equipotential surfaces of the considered field will be

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$$V = \frac{az^3}{3} - \frac{a}{2} r^2 z + H_0 z = \text{const.} \qquad (26)$$

The equation of the lines of forces of the given field is

$$r^2 z^2 - \frac{r^4}{4} - \frac{H_0}{a} r^2 = \text{const}. \qquad (27)$$

Figure 1 represents the lines of forces and equipotential surfaces of the field, characterised by equations (26) and (27) in the case $H_0/a = -72$. The zero equipotential surface, besides the plane $z = 0$, is the hyperboloid of rotation, the equation of which is

$$\frac{z^2}{3} - \frac{r^2}{2} = \frac{H_0}{a} \qquad (28)$$

(see surface V on figure 1).

The considered field may be realised as well by means of the quadripole system I, II, III, IV, as by means of the tripole system I, II, V, and also by other means.

Let us now consider the motion of molecules in the plane $z = 0$ (plane xy) under mutual action of magnetic field forces and force of gravity. As will be shown in the following [redacted] the action of the force of gravity is [redacted], because for small gradients [redacted] of the magnetic field the value of gravity acting on the molecule is of a magnitude comparable to that of the magnetic force.

If the plane $z = 0$ is located vertically, then the differential equation of the motion of the molecule, the magnetic moment of which is directed against the field, will be

$$m\ddot{x} = -\mu a x, \qquad m\ddot{y} = -\mu a y - mg \qquad (29)$$

If the slit (or aperture) of the molecule source is located on the y-axis at a distance y_0 from the field center, the velocity of the released molecule is u, and it forms in the point of release an angle α with the y-axis, then the integration of equations (29) gives

$$x = (u \sin \alpha/\omega) \sin \omega t \qquad (30a)$$

$$y = (u \cos \alpha/\omega) \cdot \sin \omega t + (y_0 + g/\omega^2) \cos \omega t - (g/\omega^2) \qquad (30b)$$

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where

$$\boxed{\omega^2 = \mu a/m} \rightarrow \omega^2 = \mu a/m \quad . \qquad \boxed{t = \tau = \pi/\omega} \qquad (31)$$

Let us now assume in (30a) and (30b) $t = \tau = \pi/\omega$. At this instant the coordinates (x, y) of the molecule will be

$$\bar{x} = 0, \quad \bar{y} = -(y_0 + 2\varepsilon/\omega^2) = -(y_0 + 2m\varepsilon/\mu a) \qquad (32)$$

After a time

$$\tau = \pi/\omega = \pi\sqrt{m/\mu a}$$

all molecules possessing the same specific magnetic moment μ/m will [illegible] independently of their velocity and [illegible] initial angle [illegible] locate on the [illegible] a distance [illegible]

[illegible] a [illegible] loading of the bunch [illegible] according to μ [illegible] τ [illegible] initial [illegible] t [illegible] the [illegible] radius. Thus, [illegible] the [illegible] at [illegible], here $\omega_0\tau$ = [illegible] we have $\tau \sim$ [illegible] sec.

In the [illegible] section [illegible] we mentioned [illegible] the value μ remains [illegible] but [illegible] the [illegible] combination is not [illegible] the particle moving on its trajectory is under the action of a magnetic field, variable as well in magnitude as in direction. The value μ will remain constant under the following two conditions: 1) the magnetic moment of the particle follows the direction of ~~[illegible]~~ variation of the magnetic field during its motion along the trajectory — thus the moment will be directed against the field during the whole motion; and 2) the magnetic moment is independent of ~~[illegible]~~ the field strength.

The first condition is satisfied if $\tau \gg T_L$, where T_L is the period of Larmor precession. Because in fields of the order of 1000 Oe the period of Larmor precession equals 10^{-6} - 10^{-7} sec., the first condition for μ stability is cetainly satisfied. The dependence

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of μ on the value of field strength is expressed by the formula:

$$\mu = \mp \frac{x + 2m(2I+1)^{-1} g_J \mu_0}{2\left(1 + \frac{4m}{2I+1}x + x^2\right)^{1/2}} \qquad (34)$$

where $x = \mu_0 H g_J / \delta W$, μ_0 is the magneton, g_J the factor of Landé, I the spin of the nucleus, and δW is the splitting of the hyperfine structure.

In a concrete case for the atom of silver $J = \frac{1}{2}$, $I = \frac{1}{2}$, $g_J = 2$; the value m takes the values:

$$m = \begin{cases} 1, \ -1 & \text{for } F = 1 \\ 0 & \text{for } F = 0 \end{cases}$$

$\boxed{\mu = \pm \mu_0}$

At $m = \pm 1$, $2m/2I = 1 = \pm 1$ and $\mu = \pm\mu_0$. Thus in this case among the four bunches, into which the atomic bunch of silver is split in the [illegible] field, one bunch exists in which the magnet [illegible] themselves against the field and do not depend on the value of the field strength.

Similar circumstances occur in other atoms and therefore the assumption $\mu = \text{const}$ is entirely founded.

Let us now study some properties of particle trajectories in the considered field. [illegible] we obtain equations of particle trajectories in cartesian coordinates in the following form:

$$\frac{x^2}{A^2} - \frac{2x(y + g/\omega^2)}{AB}\cos\varphi + \frac{(y + g/\omega^2)^2}{B^2} = \sin^2\varphi \qquad (35)$$

where

$$\left\{ \begin{aligned} A &= \frac{u \cdot \sin\alpha}{\omega} \\ \operatorname{tg}\varphi &= \omega\left(y_0 + \frac{g^2}{\omega^2}\right) / u \cdot \cos\alpha \ . \end{aligned} \right\} \qquad (35a)$$

The equation (35) represents an ellipse. The coordinates of its center are

$$x_0 = 0 \ ; \quad y_0 = -g/\omega^2 = -g/\mu \qquad (36)$$

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If the molecule flies out perpendicular to the axis y (i.e. $\alpha = \pi/2$), then, as seen from the last equality (35a), also $\varphi = \pi/2$. In this case the equation of the ellipse in a canonical form will be

$$\frac{x^2}{A^2} + \frac{(y + g/\omega^2)}{B^2} = 1 \qquad (37)$$

where $A = u/\omega$, $B = y_0 + g/\omega^2$. This ellipse degenerates into a circle under the condition

$$u = \omega(y_0 + g/\omega^2) . \qquad (38)$$

Equation (36) makes it possible to determine in every concrete case the location and dimensions of the elliptic trajectory of the molecule. For this purpose we shall write the equation of the ellipse taking the symmetry axes of the ellipse as coordinates. This equation is

$$\frac{x'^2}{-u_0/\lambda} + \frac{y'^2}{-u_0/\lambda} = 1 \qquad (39)$$

where $u_0 = ac - b^2$ and λ_1 and λ_2 are the roots of the equation

$$\lambda^2 - (a + c)\lambda + (ac - b^2) = 0$$

In our case $u_0 = \sin^2\varphi$. Therefore the semiaxes of the ellipse equal respectively $\sin\varphi/\sqrt{\lambda_1}$ and $\sin\varphi/\sqrt{\lambda_2}$. The equations of the symmetry axes will be

$$(Ox' \text{ axis}) \quad y - y_0 = (x - x_0)b/(\lambda_1 - c) \qquad (40a)$$

$$(Oy' \text{ axis}) \quad y - y_0 = (x - x_0)b/(\lambda_2 - c) \qquad (40b)$$

An important property of the molecule trajectory is the equality of the 'angle of slide' η in the focused point M' (figure 2) to the 'exit angle' of the molecule from the source slit (aperture) in the point M_0.

We may deduce an interesting conclusion in the case where the source aperture (slit) will be set in a point having coordinates

$$x_0 = 0, \quad y_0 = -g/\omega^2$$

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In this case the parametric equations of the molecule trajectory will take the form:

$$x = \frac{u \cdot \sin\alpha}{\omega} \sin\omega t \; y + g/\omega^2 = \frac{u \cdot \cos\alpha}{\omega} \cdot \sin\omega t \,. \qquad (41)$$

The equation of the trajectory in cartesian coordinates will be

$$y = g/\omega^2 = x \cdot \mathrm{ctg}\,\alpha \qquad (42)$$

(42) shows that each atom emitted from point M oscillates along the direction of path. The period of these oscillations is $\tau = 2\pi/\omega = 2\pi\sqrt{m/\mu a}$.

As follows from the equations of motion, all molecules with given value of m/μ, emitted from the point M (figure 2), will assemble in the focused point M' independent of their velocity and initial angle. However this does not mean that really all molecules emitted by the source slit will assemble in the focused point. A matter of fact is that the focusing magnetic field [illegible] in a limited space, therefore only those particles will assemble in the focused point, the trajectories of which [illegible] this limited space. As follows from [illegible] of the molecule, the [illegible] it will escape from the origin of coordinates. [illegible] which satisfy the condition:

$$r_{max} \leqslant R \qquad (43)$$

where r_{max} is the maximum value for the radius vector of the orbit, and R the space radius containing the focusing magnetic field. The condition (43) is determined by u_{max}; i.e. by the maximum velocity that the molecule may acquire on its trajectory, without leaving the boundaries of the focusing field.

Making use of the laws of conservation of energy and inertia, we may approximately evaluate the magnitude of u_{max}. Because of the small value of potential energy of the molecule in the gravitational field in comparison with that in the magnetic field, we may write, on the basis of the laws of conservation,

$$\frac{mu^2}{2} + \frac{\mu}{2} a y_0^2 = \frac{1}{2} m v^2 + \frac{1}{2} \mu a r^2 \,, \qquad y_0 \cdot m u \cdot \sin\alpha = m v r_{max} \qquad (44)$$

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where v is the velocity of the molecule in the point where $r = r_{max}$ (figure 2).

Eliminating from equations (44) the value v, we obtain

$$u = \omega\sqrt{\frac{r_{max}^2 - y_0^2}{1-(y_0^2/r_{max}^2)\sin^2\alpha}} \quad \ldots\ldots\ldots \quad (45)$$

Assuming in formula (45) $r_{max} = R$, we obtain a formula for u_{max}:

$$u_{max} = \omega\sqrt{\frac{R^2 - y_0^2}{1-(y_0^2/R^2)\sin^2\alpha}} \quad . \quad (46)$$

As seen from formula (46) the value u_0 depends on the initial angle of emission and varies within the limits:

$$\omega\sqrt{R^2-y_0^2} < u_{max} < \omega R \quad \ldots\ldots\ldots \quad (47)$$

where the lower limit corresponds to angles $\alpha = 0, \pi$, and the upper to $\alpha = \pi/2$.

By making use of relation (4[illegible]) we may evaluate u_{max} for the silver atoms at $a = 200$ Oe/cm^2 and $R = 10$ cm, we have u_{max} [illegible] m/sec. Hence at above mentioned conditions [illegible] Oersteds. This circumstance [illegible] this method [illegible] equipment only to slow molecules, since the practical realization of this method rather difficult. Indeed [illegible] values a and R in order to obtain higher values for u_{max} is practically not realizable, because [illegible] we have $H_{max} = aR^2/2 = 10\,000$ Oe.

Until now we considered the motion of the molecule in the median plane, where a strictly focusing force is acting (15), and the component H_r equals zero. If the slit (aperture) emitting the molecular beam presents a line perpendicular to the median plane, then obviously the molecule trajectory will fly out from this plane. In this case the forces acting on the molecule will be expressed by the formulas (17).

For all points of the orbit, where $z \neq 0$, the radial force f_r will not be strictly focusing, while the degree of defocusing will be determined by the value $(a^2z^4 - 2H_0az^2)/(H_0 + az^2/2)^2$, contained in the denominator of expression (17a). Obviously, in order to avoid

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appreciable defocusing, it is necessary to limit the region, in which the molecule orbit lie, in the axial direction.

Besides, at $z \neq 0$, the direction of f_z is essential, because if it is directed toward the plane $z = 0$, it has a concentrating effect on the molecular beam, while at the inverse direction it disperses the beam. A detailed study of molecule motion in the region where $z \neq 0$ showed that the optimum conditions for the detection of the focusing action of axial magnetic field are created in the case where $H_0 < 0$ and $a > 0$.

However the evaluation of the number of molecules entering the focusing region, even in the case of optimum conditions of the axial field ($H_0 < 0$, $a > 0$), shows that the intensity of the molecular beam in the focusing location is so small that it requires a highly sensitive [redacted] detection of molecules for detection.

[The last and third section follows, with the figures]

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3. Focussing of a Molecular Beam in a Plane Magnetic Field.

We consider a plane magnetic field with complex potential of the form

$$f(z) = Cz^3 . \tag{48}$$

For n = 3 the equations of lines of force and equipotential are written in the following manner:

$$U = C(x^3 - 3xy^2) , \tag{49}$$

$$V = C(3x^2y - y^3) . \tag{49a}$$

The field's lines of force and equipotential lines (the components of the field are given by equations (49)) are represented in figure 3. Differentiating the equation (49a) with respect to the coordinates we find the expressions for the components of the field intensity:

$$H_x = -6Cxy$$
$$H_y = 3C(y^2 - x^2) . \tag{50}$$

Setting $a = -6C$, we obtain

$$H_x = axy$$
$$H_y = \tfrac{1}{2}a(x^2 - y^2) . \tag{51}$$

Employing the equations (5) we find the components of the force that is acting on the molecules with the magnetic moment in the magnetic field. They turn out to be equal to:

$$f_x = \mu ax$$
$$f_y = \mu ay \tag{52}$$
$$f_z = 0 .$$

Thus the force acting upon the molecules are focussing in all the surfaces of the plane magnetic field in the same time that the force is focussing, in the axial field, only in the surface z = 0.

If one places the axis of the electromagnet (z-axis) horizontally, then the differential equations of motion of the molecules with magnetic moment acting against the field are written, taking into consideration the force of gravity, in the following form:

$$\ddot{x} = -w^2x, \quad \ddot{y} = -w^2y - g, \quad \ddot{z} = 0. \tag{53}$$

Using the results obtained in Section 2 we can write the integrals of equation (53) in the following form:

$$x = w^{-1}u_0 \sin\theta_0 \cdot \sin wt ,$$
$$y = (y_0 + g/w^2)\cos wt + w^{-1}u_0 \cos\theta_0 \cdot \sin wt - g/w^2 ,$$
$$z = z_0 + u_z t . \tag{54}$$

where, as earlier, $w = (\mu a/m)^{\frac{1}{2}}$

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In equations (54) the quantity u is the projection of the total velocity c of a molecule in the xy-plane; u_z is the projection of the total velocity on the z-axis; α is the angle formed by the projection of velocity u with the y-axis; and it is assumed that the molecule has flown from the point with the coordinates $(0, y_0, z_0)$ (see figure 4).

The first two of the equations (54) represent the parametric equations of the trajectory of a molecule on the xy-plane. As was shown earlier, these equations are ellipses. Thus the trajectory of a molecule will be an elliptical spiral.

From equations (54) it follows that all molecules issuing from the point A (figure 4) and having the same value m/μ, independently of their angle of flight and velocity, will in time $\tau = \pi/w$ arrive at a point in the line BD parallel to the z-axis located from it at a distance equal to $y_0 + 2mg/\mu a$. It is easy to realize that a molecule will arrive at a farther point of the line BD the greater the component u_z of velocity since after time τ the displacement of an atom along the z-axis will increase linearly with the quantity u_z (see the preceding formula (54)). The molecules issuing from the point A and having another value of m/μ will arrive at another line parallel to the line BD.

If we consider another point emitting molecules and lying together with the point A on one and the same line parallel to the z-axis, then it is clear that the molecules issuing from this point in time τ will arrive at the same line BD. Thus the molecules issuing from the linear aperture parallel to the z-axis will collect in time τ at one and the same line.

If the aperture emitting the molecules has a width δy, then the region where the molecules are focussed will be a strip of width δy. If the aperture of the source of molecules placed in the yz-plane is not parallel to the z-axis, then this causes a widening of the region of focussing equal to the projection of the aperture on the xy-plane; that is, this widening will be

$$\delta y' = L \sin\beta \qquad (55)$$

where L is the length of the aperture, and β is the angle formed by the direction of the aperture with the direction of the z-axis. If the aperture of the source is displaced a distance δx from the yz-plane, then it is easy to show that the focal line is displaced to the other side from the yz-plane by the same quantity δx.

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However, if the aperture of the source does not lie in the yz-plane, then this fact will cause a widening of the focal line equal to: $\delta y^2 = L \sin\gamma \cdot \tan\theta$ (56) where γ is the angle formed by the aperture with the yz-plane, and θ is half the angle of divergence of the molecular beam in the vertical plane.

The previous considerations permit one to formulate two requirements guaranteeing the absence of fuzziness or diffuseness of the focal line: a) the aperture of the source of molecular beams must be parallel to the z-axis; b) the aperture of the source and the focal line must be located either in the yz-plane or in two parallel planes disposed symmetrically with respect to the yz-plane.

Just as in the case of the axial field, only part of the molecules issuing from the source is incident upon the focal line; in both cases this is caused by the fact that the focussing magnetic field is concentrated in a limited space. Actually, in the case considered by us the velocity the molecules arriving at the focal line are limited by two conditions: a) the projection of the velocity u of a molecule on the xy-plane cannot be greater the value u_{max} determined from formula (46) and dependent upon R, the radius of the operating space of the electromagnet, since in the opposite case the molecule before falling on the focal line falls on the wall of the vacuum chamber in which the motion of the molecules originates; b) the component u_z of velocity of a molecule cannot be greater than a certain u_{z_0}, since in the opposite case the molecule before falling upon on the focal line would go beyond the limits of the operating space of the electromagnet.

The quantity u_{z_0} is determined from the previous formula (54), in which z_0 is set equal to zero, since the length of the aperture of the source is small in comparison with L, the length of the operating space of the electromagnet. Obviously, if in the preceding formula (54) one sets, for $t = \tau$, $z = L$, then $u_z = u_{z_0}$ and consequently we have: $u_{z_0} = L/\tau = \omega L/\pi$. (57)

Keeping in mind the above-deduced conditions limiting the magnitude of the projections of velocity, one can express definite considerations concerning the allowable values for the vector of a molecule's total velocity, for which turn to figure 5. It is easy to see that only those molecules fall on the focal line whose termini of the velocity vector do not go beyond the limits of the cylinder's surface (height of the cylinder is u_{z_0}, and radius of the base is u_{max}). Designating the angle formed by the velocity of a molecule c with the z-axis by the letter

is meant to stand for omega "ω".

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ϕ, we can write the following expressions for the allowable values of the velocity

for $0 < \phi < \phi_k$: $\quad c_{allow} = u_{z_0}/\cos\phi$ (58a)

for $\phi_k < \phi \quad \pi/2$: $\quad c_{allow} = u_{z_0}/\sin\phi$ (58b)

where $\phi_k = \arctan(u_{max}/u_{z_0}) = \arctan(\pi u_{max}/wL)$. (58c)

Conditions (58a) and (58b) are imposed upon the molecules incident on any place of the focal line in the limits $0 < z < L$.

Upon the molecules incident on the portion of the focal line close to $z = L$ the following conditions are imposed: $\quad u_z = u_{z_0} = c\cdot\cos\phi = \text{const}$

$$u_{xy} = c\cdot\sin\phi \leq u_{max} \quad (59)$$

Consequently, the following condition is imposed upon the angle ϕ :

$$\tan\phi \leq u_{max}/u_{z_0} = \tan\phi_k \text{ ; that is, } \phi \leq \phi_k \quad (60)$$

Thus in a given place of the focal line molecules can fall for which the termini of the velocity vector pass through the base of the cone represented in figure 5.

Using formulas (46), (57), (58a) and (58b) and proceeding from the assumption that, in the molecular beam issuing from the source's aperture Maxwell's law of distribution of velocities holds, one can evaluate the number of molecules arriving per second both at the entire region of focussing and at any portion of it. These evaluations show that for not too large sizes of the electromagnet creating the focussing magnetic field the number of molecules arriving in the region of focussing is not large and is required for its sensitive detection of molecular beams.

Let us consider now the possibility of focussing a molecular beam by the magnetic field in the case where the point source of the beam is located outside of the magnetic field, for which let us turn to figure 6. Let there issue from the source of molecules located on the z-axis at the point O a divergent beam of rays with the velocity v which is incident upon the magnetic field concentrated in the region AA'B'B. In the given case we will consider the beam with a small angle of divergence α (paraxial rays) and will disregard the marginal distortions of the field near AA' and BB'.

Let us find the trajectory of one of the rays issuing from the point O, disregarding the action of the force of gravity. The coordinate y of the point C of the entrance of a molecule into the magnetic field and its component of velocity in the direction of the y-axis will be: $\quad y_1 = f\alpha \;, \quad \dot{y}_1 = v\alpha \;.$ (61)

Part of the trajectory CD of a molecule passes under the action of the force acting from the side of the magnetic field. According to formula (30b) the equation of

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[illegible]

$$\dot{y}_2 = v\alpha \cos(wL/v) - (f\alpha w) \sin(wL/v) \qquad [illegible]$$

The equations of motion of a molecule in the EE portion of the trajectory will be

$$y = y_2 + \dot{y}_2 t, \qquad z = vt \qquad (64)$$

or

$$y = \alpha(f + z)\cos(wL/v) + (v/w - wfz/v)\alpha \sin(wL/v). \quad (65)$$

If the point E is a point of intersection of the molecules' trajectory (velocity v) coming out within the limits of a small angle α, then it means that the sum of all the terms in expression (65) that contain factors in the quantity α must be equal to zero; that is, $(f + z)\cos(wL/v) + (\frac{L}{v} - \frac{wf}{v} z)\sin(wL/v) = 0$. Hence the distance z_1 of the point E from the boundary of the field EE' will be:

$$z_1 = f_1 = (f \cos wL/v + (v/w)\sin wL/v)(fw/v \sin wL/v - \cos wL/v)^{-1} \quad (66)$$

Thus the equation (66) gives the coordinate of the point of focusing of a monovirtic and paraxial beam of molecular rays passing through the field. From equation (66) one can conclude that

$$f_1 = \infty \quad \text{for } L = (v/w)\arctan(v/wf),$$

$$f_1 = f \quad \text{for } L = (2v/w)\arctan(v/wf), \qquad (67)$$

$$f_1 = 0 \quad \text{for } L = (v/w)\arctan(2wf/v)$$

For definite values of w, L, f one can calculate in formula (66) the value of of the velocity v, giving a definite focus distance f_1. It is obvious that a convergent beam can be obtained for velocities less than that calculated from the first equation in (67).

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Physicotechnical Institute,

Academy of Sciences Ukrainian SSR.

REFERENCE

[1] M. I. Korsunskiy. Zhurnal Eksperimental'noy i Teoreticheskoy Fiziki. Volume 16, 151 (1946).

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(figures follow)

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Figure 1.

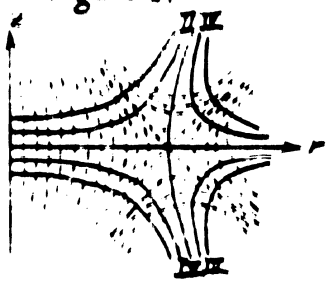


Figure 2

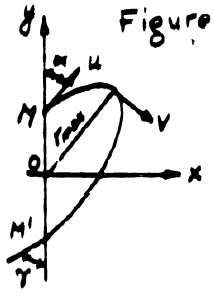


Figure 3.

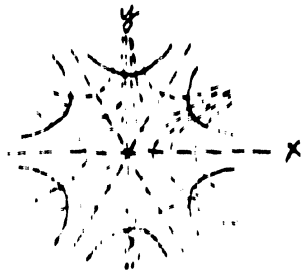
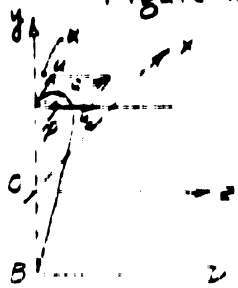


Figure 4



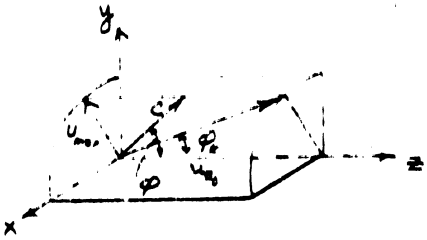


Figure 5

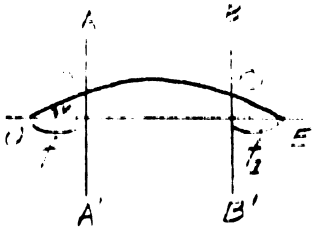


Figure 6.

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