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CHAPTER 21

INTEGRATION OF THE WEATHER EQUATIONS

I THE SYSTEM OF THE WEATHER EQUATIONS

In chapter 8 it was shown that success had not been achieved in formulating a complete system of weather equations, taking into account the simultaneous effects of all atmospheric processes. An approximate schematism of the processes involved was possible from the very beginning. But the most that could be accomplished was the formulation of a closed system of hydromechanical, thermodynamical and radiation equations, accounting for the transference of the water vapor, (in the absence of evaporation and condensation, as well as in the absence from the atmosphere of condensation products and other radio-active admixtures such as carbon dioxide, ozone, and dust).

However, this system of equations is not directly applicable to the atmosphere, since it does not take into account the molecular viscosity, the molecular thermoconductivity and the molecular diffusion. In the atmosphere the processes of internal friction, thermoconductivity, and diffusion take place. Let us write out consecutively all the equations of the weather-equation system. This system, above all, comprises three equations of average atmospheric motion taking place in the force fields of Coriolis, and gravitation:

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\left[\omega_z v - \omega_y w\right] + \frac{1}{\rho}\left[\frac{\partial}{\partial x}\left(A'\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(A'\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(A\frac{\partial u}{\partial z}\right)\right], \quad (1)$$

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$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + 2\left[\omega_x w - \omega_z u\right] + \frac{1}{\rho}\left[\frac{\partial}{\partial x}\left(A'\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(A'\frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial z}\left(A\frac{\partial v}{\partial z}\right)\right], \qquad (2)$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + 2\left[\omega_y u - \omega_z v\right] + \frac{1}{\rho}\left[\frac{\partial}{\partial x}\left(A'\frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial y}\left(A'\frac{\partial w}{\partial y}\right) + \frac{\partial}{\partial z}\left(A\frac{\partial w}{\partial z}\right)\right]. \qquad (3)$$

Then we have the equation of continuity in its unchanged form:

$$\frac{d\rho}{dt} + \rho \operatorname{div} v = 0 \qquad (4)$$

The Clapeyron's equation introduces the following change of variables:

$$\frac{p}{\rho} = RT \qquad (5)$$

The equation of heat inflow accounts for the inflow of heat determined by the turbulent thermoconductivity and radiation, neglecting as an infinitesimally small quantity ~~the~~ of heat due to the dissipation of mechanical energy:

$$\frac{\partial}{\partial x}\left(K'\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K'\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right) + \rho\int_0^\infty a_\nu\, d\nu \int I_\nu\, d\nu \int_{\pi} I_\nu\, d\omega = \rho c_v \frac{dT}{dt} + p \operatorname{div} v - 4\pi\int_0^\infty \eta_\nu\, d\nu \qquad (6)$$

The transfer of ~~the~~ radiant energy is given by the equation

$$\frac{1}{\rho}\frac{\partial I_\nu}{\partial s} = \eta_\nu + \frac{\alpha_\nu}{4\pi}\int I_\nu(P, r')\,\gamma_\nu(t, P, r')\, d\omega' - (a_\nu + \sigma_\nu) I_\nu. \qquad (7)$$

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Radiation is defined by the equation:

$$\eta_\nu = \alpha_\nu \cdot \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{KT}} - 1} \qquad (8)$$

which is derived by means of generalization of ~~the~~ Kirchhoff's and Planck's laws.

In the equations (6), (7) and (8) the coefficient of absorption of the radiant energy is a function of the specific humidity S, ~~where~~ $\alpha = \alpha'_\lambda \times S$.

The transfer of water vapor is described by the equation of the turbulent diffusion

$$\frac{\partial s}{\partial t} + u\frac{\partial s}{\partial x} + v\frac{\partial s}{\partial y} + w\frac{\partial s}{\partial z} = \frac{\partial}{\partial x}\left(D'\frac{\partial s}{\partial x}\right) + \frac{\partial}{\partial y}\left(D'\frac{\partial s}{\partial y}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial s}{\partial z}\right) \qquad (9)$$

The system of the weather equations (1) - (9) contains 9 unknown functions: u, v, w, p, ρ, T, I, η_ν, S. Beside the physical parameters ω_x, ω_y, ω_z, R, Cp, Co, h, ν, C, K which are constants or quasi-constants, these equations contain the following variable physical parameters: A', A, K', D', D, σ_ν, γ_ν, α'_ν.

This system of the weather equation is so complex, that no solution is possible without preliminary simplifications.

In the derivation of the simplified weather equation system, the following two, different in principle, methods of approach, must be borne in mind. First, we can schematize the subject physical processes, and thus ~~arrive at the~~ simplify ~~fied~~ weather equation system. With this approach to the problem the simplified equations do not lose their rigor; there only arises the question of the degree of reality of such simplified processes.

Secondly, the system of the weather equations can be simplified

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without the process schematization, but assuming that not all of the terms comprising a given equation are of equal significance, in view of which we can neglect and discard certain terms. With this approach, it might happen that the same quantity will be neglected in one equation and retained in another. The so-~~called~~ obtained simplified equations are not rigorous, but approximate, and caution must be used with this type of simplification. It must be remembered that the equations to be simplified are differential equations, which, must be integrated after simplification. To start with, we shall discuss several examples of simplification of the first type.

(1) ~~Assuming~~ Thus assume that the air is perfectly dry, then $s = 0$, and the equation of the water vapor transfer vanishes. Assuming further, that the radiation and absorption processes are determined exclusively by the water vapor, we conclude, that the subject medium (dry air) does not radiate or absorb radiant energy, but only scatters it. Then $\alpha = \eta_\nu = 0$, in which case the radiation terms will vanish from the heat inflow equation, and the hydro-mechanical and the thermodynamic equation group will become independent of the radiation processes. Planck's and Kirchhoff's equations will vanish from the group of the radiation equations, and the equation of the radiant energy transfer will become:

$$\frac{1}{\rho}\frac{\partial I_\nu}{\partial s} = \frac{\sigma_\nu}{4\pi}\int I_\nu(P, r')\,\gamma_\nu(t, P, r', r)\,d\omega' - \sigma_\nu I_\nu \qquad (7')$$

This equation contains a single unknown function, which must be ~~obtained~~ determined beforehand from a system of hydromechanical and thermodynamic equations. This function is the density ρ of the scattering substance.

Knowing ρ, we obtain an integral-differential equation which ~~defines~~ determines I_ν.

Let us note that the separation of the weather equation system into two independent systems, for in the case of pure scattering, could have been foretold, since the process of scattering radiant energy is

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not accompanied by heat transfer. In itself the process of scattering radiant energy is of no interest in reference to the problems of dynamic meterology. Therefore, equation (7') is nowhere treated in dynamic meteorology. This example only shows that the system of weather equations of dry air is made up of unchanged equations (1)-(5), whereas the equation of heat inflow becomes:

$$\frac{\partial}{\partial x}\left(K'\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(K'\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right)=\rho c_v\frac{dT}{dt}+ \tag{6'}$$

$$+\rho \operatorname{div} v$$

while the other equations are discarded.

(2) We shall also discuss one important ~~and particular~~ case in practice. Assume that the subject medium absorbs and radiates energy with no scattering, i.e. that $\sigma_\nu=0$. With this condition all the equations of the weather equations system will have their previous form; only the equation of the radiant energy transfer will take the form of:

$$\frac{1}{\rho}\frac{\partial I_\nu}{\partial s}=\eta_\nu-\alpha_\nu I_\nu \tag{7''}$$

For practical applications, a further simplification is made in the works of dynamic meteorology, which consists in treating the subject medium as a "gray" body. This means that the processes of radiation and absorption of the radiant energy which take place in the subject medium, are independent of the frequency of radiation. This simplification permits us to eliminate the independent variable ν by integrating the equation of the radiant energy transfer (7'') with respect to ν and by the introduction of the new functions:

$$\eta=\int_0^\infty \eta_\nu\, d\nu;\quad I=\int_0^\infty I_\nu\, d\nu \tag{10}$$

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upon which the equation of the radiant energy transfer becomes:

$$\frac{1}{\rho}\frac{\partial I}{\partial s} = \eta - \alpha I. \qquad (7'''')$$

where α is the coefficient of absorption, having the same value for the entire spectrum. With this simplification the unified Kirchhof-Planck equation, can be replaced by the Stefan-Boltzmann equation. Actually, integrating equation (8) with respect to ν and, taking into account ~~(10)~~, we get

$$\eta = \alpha \int_0^\infty \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1} d\nu \qquad (11)$$

The last integral by radiation theory is equal to $\frac{\sigma T^4}{\pi}$; consequently we get:

$$\eta = \alpha \cdot \frac{\sigma}{\pi} \cdot T^4 \qquad (8')$$

where σ is the ~~Stefan~~-Boltzmann' constant.

Usually we do not stop with these simplifications. In problems of dynamic meteorology it is considered that the flow of radiant energy is vertical, and downward-flowing energy is considered as being composed of the "gray" shortwave solar radiation S and the "gray" longwave ~~earth and~~ atmospheric radiation B.

Thus, the following amount of heat will be received by a unit volume of moist air: $\alpha\rho_w$ (A + B) due to the absorption of ~~the~~ long-wave radiation, and $\alpha\beta\rho_w S$ due to the absorption of ~~the~~ shortwave solar radiation. The same unit volume of air will lose heat by radiation, the quantity of heat lost due to upward and downward radiation being equal to $2\alpha\rho_w f E$, where α is the absorption coefficient of the "gray" longwave radiation, $\alpha\beta$ is the absorption coefficient of the ~~black~~ gray short-wave radiation, f is the radiation selectivity factor of the water vapor, E is ~~the~~ black-body radiation, which by the Stefan-Boltzmann's law is

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equal to σT^4. Employing the stated assumptions, the heat inflow equation can be written as:

$$\rho c_v \frac{dT}{dt} + p \operatorname{div} v = \frac{\partial}{\partial x}\left(K' \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K' \frac{\partial T}{\partial y}\right) + \tag{6''}$$

$$+ \frac{\partial}{\partial z}\left(K \frac{\partial T}{\partial z}\right) + \alpha \rho_w (A + B) + \alpha \beta \rho_w S - 2\alpha \rho_w \gamma E$$

The equation of the radiant energy transfer breaks up into 3 equations corresponding to the "gray" radiation flow

$$\frac{\partial A}{\partial z} = \alpha \rho_w (A - \gamma E); \quad \frac{\partial B}{\partial z} = \alpha \rho_w (\gamma E - B); \quad \frac{\partial S}{\partial z} = -\alpha \beta \rho_w S \tag{7IV}$$

These simplified equations are entirely rigorous, if it is assumed that the subject radiation process is real. However, here arises a number of serious doubts. Actually, the radiant energy is not only absorbed and radiated, but scattered as well. Moreover, the flow of radiant energy is not vertical and isotropic. Such schematization of the radiation processes of the atmosphere involves ~~an approximation~~ which ~~is compensated for by a~~ great simplifi~~cation~~ ~~of~~ the weather equations system. Comparison of the theoretical results with observations will show ~~that~~ such simplifications of the atmospheric radiation processes, ~~as~~ applied to the problems of dynamic meteorology are permissible.

(3) We assume now that the turbulent t~~ransfer~~ takes place only in the vertical direction. Then the terms determined ~~by the~~ horizontal turbulent ~~transfer~~ vanish, and the weather equations system becomes:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \tag{1}$$

$$+ 2(\omega_z v - \omega_y w) + \frac{\partial}{\partial z}\left(K \frac{\partial u}{\partial z}\right),$$

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$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + 2(\omega_x w - 2\omega_z u) + \frac{\partial}{\partial z}\left(K\frac{\partial v}{\partial z}\right), \qquad (2)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + 2(\omega_y u - \omega_x v) + \frac{\partial}{\partial z}\left(K\frac{\partial w}{\partial z}\right), \qquad (3)$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \qquad (4)$$

$$\frac{p}{\rho} = RT \qquad (5)$$

$$\rho c_v \frac{dT}{dt} + p\,\mathrm{div}\,v = \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right) + \alpha\rho_w(A + B) + \alpha\beta\rho_w S - 2\alpha\rho_w f E \qquad (6)$$

$$\frac{\partial A}{\partial z} = \alpha\rho_w(A - fE); \quad \frac{\partial B}{\partial z} = \alpha\rho_w(fE - B); \qquad (7)$$

$$\frac{\partial S}{\partial z} = -\alpha\beta\rho_w S; \quad E = \sigma T^4 \qquad (8)$$

$$\frac{\partial s}{\partial t} + u\frac{\partial s}{\partial x} + v\frac{\partial s}{\partial y} + w\frac{\partial s}{\partial z} = \frac{\partial}{\partial z}\left(K\frac{\partial s}{\partial z}\right).$$

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These are the simplified weather equations applicable to the problems of dynamic meteorology. However, as was seen above, further simplifications of the weather equations are possible, based not on the schematization of the subject processes, but upon the evaluating of the order of ~~magnitude of~~ terms comprising the equations. In this chapter we will discuss some of the simplest integration problems of the approximate weather equations which were solved by the Soviet meteorologists.

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II. THE DISTRIBUTION OF TEMPERATURE WITHIN THE EARTH'S ATMOSPHERE AS DETERMINED BY THE RADIANT AND THE TURBULENT HEAT TRANSFER.

The theory of radiant equilibrium, which accounts satisfactorily for the stratospheric isothermism, results in exaggerated tropospheric temperature gradients. As was already shown by Friedman, the considerable deviation of the observed gradients from the values predicted on the basis of the theory of radiant equilibrium, can be explained by vertical motions.

Kibel, when solving the problem of temperature distribution of the earth's atmosphere, determined by the radiant and turbulent heat transfer, obtained good correlation between the computed and observed temperatures.

Kibel starts with the heat inflow equation, which he takes in the following form:

$$\varepsilon_1 + \varepsilon_2 = c_p \rho \frac{dT}{dt} - \frac{dP}{dt}, \qquad (1)$$

Where ε_1 is the heat input (per unit volume, per unit time) due to radiation, ε_2 is the heat input due to the turbulent thermo-conductivity. No consideration is given to heat input due to condensation and evaporation. In solving the static problem of the annual mean temperature distribution, Kibel makes a series of simplifications. In this case we can abstract the velocity field and consider the atmosphere as being quiescent. Then the ordinary derivative $\frac{dT}{dt}$ can be replaced by a partial derivative $\frac{\partial T}{\partial t}$ which in the static case is equal to zero; consequently, $\frac{dT}{dt} = 0$. Besides, we can consider the pressure as being independent of time, whence $\frac{dP}{dt} = 0$. The heat input by radiation, Kibel computes by the simplified scheme, assuming the existence of two vertical currents of radiant energy:

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(1) The upward current composed of the "gray" short wave solar radiation S and the "gray" long wave atmospheric radiation A;

(2) The downward current composed of the "gray" long-wave earth and atmospheric radiation B.

Thus, we have,

$$\mathcal{E}_1 = \alpha \rho_w (A + B) + \alpha \beta \rho_w S - 2\alpha \rho_w f E \qquad (2)$$

where α is the absorption coefficient of the "gray" long wave radiation; $\alpha\beta$ is the absorption coefficient of the "gray" short wave radiation; f is the absorption selectivity factor of water vapor; ρ_w is the density of the water vapor--the substance which absorbs and emits radiant energy; E is the black body radiation, according to Stefan-Boltzmann.

$$E = \sigma T^4 \qquad (3)$$

The heat input due to the turbulent ~~thermo~~-conductivity is given by the formula

$$\mathcal{E}_2 = \frac{\partial}{\partial x}\left(K' \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K' \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K \frac{\partial T}{\partial z}\right) \qquad (3a)$$

Accounting only for the vertical ~~inter~~mixing, Kibel simplifies this equation, setting:

$$\mathcal{E}_2 = \frac{\partial}{\partial z}\left(K \frac{\partial T}{\partial z}\right) \qquad (4)$$

With all these assumptions, Kibel writes the heat input equation as follows:

$$\frac{d}{dz}\left(K \frac{dT}{dz}\right) + \alpha \rho_w (A + B - 2 f E + \beta S) = 0 \qquad (5)$$

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Thus, equation (5) contains five unknown functions: T, E, A, B, and S, of which we are only interested in T; the other functions are of an auxiliary nature. In order to complete this system of equations with five unknown functions, Kibel associates equation (3) with the equation (5) as well as three other equations which define the transfer of radiant energy in the atmosphere:

$$\frac{dA}{dz} = \alpha \rho_w (A - fE),$$

$$\frac{dB}{dz} = \alpha \rho_w (fE - B), \qquad (6)$$

$$\frac{dS}{dz} = \alpha \beta \rho_w \cdot S$$

Furthermore, Kibel formulates the following boundary conditions. The upper atmospheric boundary receives a portion W of the shortwave solar radiation, since the other portion of it is reflected by the atmosphere, earth's surface, and the clouds. Consequently, there is taken into account the average reflection factor of the earth.

Besides, it is assumed that at its upper boundary the atmosphere radiates out into the cosmic space as much energy as it receives. Therefore, the upper boundary conditions can be stated as:

$$A = 0; \quad S = W; \quad B = W \text{ (when } z = \infty). \qquad (7)$$

For the boundary condition at the earth's surface, Kibel takes the heat balance equation

$$-K\frac{dT}{dz} + K^*\frac{dT^*}{dz} = A + S - B \text{ (when } z = 0), \qquad (8)$$

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where for the upward heat flow we can take

$$B = q\,E \quad (\text{when } z = 0), \tag{9}$$

where q is the factor accounting for the fact that the earth's surface does not radiate as a perfect black body.

However, the boundary condition (8) introduces a new unknown function--soil temperature T* (the coefficient K* of soil thermo-conductivity is regarded as a given physical parameter). The introduction of T* requires the setting up of a new equation for its determination. But Kibel feels that, for the given problem of the annual mean temperature distribution, he can neglect heat ~~input due to~~ soil, which results in the simplification of the boundary condition (8), as follows

$$-K\frac{dT}{dz} = A + S - B \quad (\text{when } z = 0) \tag{10}$$

Furthermore, in place of the Z coordinate, Kibel introduces a new independent variable--"the optical thickness" τ of "gray" radiation which is given by the following formulas:

$$\tau = \int_z^{\infty} \alpha \rho_w \, dz \quad \text{and} \quad d\tau = -\alpha \rho_w \, dz \tag{11}$$

Kibel also eliminates the temperature T from the subject equations, expressing it as E. Since

$$\frac{dE}{d\tau} = 4\sigma T^3 \frac{dT}{d\tau} = \frac{-4\sigma T^3}{\alpha \rho_w} \cdot \frac{dT}{dz} \tag{11a}$$

equation (5) can be written as follows:

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$$\frac{d}{d\tau}\left(\frac{K\alpha\rho_w}{4\sigma T^3}\frac{dE}{d\tau}\right)+A+B-2fE+BS=0 \tag{12}$$

Similarly, equations (6) in terms of the new variables are as follows:

$$\left.\begin{aligned} dA &= fE-A \\ \frac{dB}{d\tau} &= B-fE \end{aligned}\right\} \tag{13}$$

$$\frac{dS}{d\tau} = -BS \tag{14}$$

The boundary conditions are taken as follows:

$$A=0;\quad S=W;\quad B=W\ (\text{when } \tau=0) \tag{15}$$

$$\frac{K\alpha\rho_w}{4\sigma T^3}\cdot\frac{dE}{d\tau} = A+S-qE\left(\text{when } \tau=\tau_0=\int_0^\infty \alpha\rho_w\,dz\right). \tag{16}$$

Equation (14) can be integrated directly. By means of the boundary condition (15) we find

$$S=W\cdot e^{-\beta\tau} \tag{17}$$

Thus, we have obtained 3 equations (12) and (13) which contain 3 unknown functions A, B, E. To solve these equations, Kibel introduces ~~non-~~dimensionless functions a, y, z which are defined by the equalities

$$B+A=W\cdot a;\quad B-A=W\cdot y;\quad fE=W\cdot z. \tag{18}$$

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By means of the relationships (13) we can easily obtain the formulas expressing the two new auxiliary functions in terms of the third one, for example, the Y-function. Since

$$\frac{d}{d\tau}(A+B) = B - A \tag{19}$$

then

$$\frac{da}{d\tau} = y \tag{20}$$

Integrating (20) we get

$$a(\tau) = a(0) + \int_0^\tau y\, d\tau \tag{20a}$$

or

$$a = 1 + \int_0^\tau y\, d\tau \tag{21}$$

since

$$a(0) = \frac{B(0) + A(0)}{W} = 1 \tag{21a}$$

Similarly from (13) and (18) it follows that

$$\varepsilon = \frac{f \cdot E}{W} = \frac{1}{2}\left\{\left(\frac{A+B}{W} - \frac{d}{d\tau}\left(\frac{B-A}{W}\right)\right)\right\} = \frac{1}{2}\left\{a - \frac{dy}{d\tau}\right\} \tag{21b}$$

or, by (21) we get

$$\varepsilon = \frac{1}{2}\left\{1 + \int_0^\tau y\, d\tau - \frac{dy}{d\tau}\right\} \tag{22}$$

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To determine the unknown y we set up a differential equation as follows. Since by (13) we have

$$A + B - 2fE = \frac{d}{d\tau}(B - A), \tag{23}$$

then by means of (14) and (23), the equation (12) can be written as

$$\frac{d}{d\tau}\left(\frac{K\alpha\rho_w}{4\sigma T^3}\cdot\frac{dE}{d\tau}\right) + \frac{d}{d\tau}(B - A) - \frac{dS}{d\tau} = 0 \tag{24}$$

Integrating, we get:

$$\frac{K\alpha\rho_w}{4\sigma T^3}\cdot\frac{dE}{d\tau} + B - A = W\cdot e^{-\beta\tau} \tag{25}$$

But

$$\frac{dE}{d\tau} = \frac{W}{f}\cdot\frac{d\varepsilon}{d\tau} = \frac{W}{2f}\left(y - \frac{d^2\nu}{d\tau^2}\right) \tag{25a}$$

and therefore,

$$B - A = yW \tag{26}$$

Consequently, the y-defining equation is as follows:

$$\frac{d^2y}{d\tau^2} - \left(1 + \frac{4\sigma T^3}{K\alpha\rho_w}\cdot 2f\right)y = -\frac{4\sigma T^3}{K\alpha\rho_w}\cdot 2f\cdot e^{-\beta\tau} \tag{27}$$

The boundary conditions of the Y-functions are obviously:

$$Y = 1 \quad (\text{at } \tau = 0) \tag{28}$$

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$$y + \frac{q}{f}\frac{dy}{d\tau} = \left(\frac{q}{f} - 1\right)\left(1 + \int_0^{\tau} y\, d\tau\right) \quad (\text{when } \tau = \tau_0). \tag{29}$$

Furthermore, Kibel believes that while each of the quantities T, ρ_w, K, α vary with height, the combination $\sigma T^3/K\alpha\rho_w$ remains practically constant. Setting

$$\frac{4\sigma T^3 f}{K\alpha\rho_w} = \frac{m^2-1}{2} = \text{const} \tag{30}$$

there is obtained an ordinary second-order differential equation with constant coefficients:

$$\frac{d^2y}{d\tau^2} - m^2 y = (m^2 - 1)\, e^{-\beta\tau} \tag{31}$$

which is easily integrated. The general solution of this equation is as follows:

$$y = C_1 e^{m\tau} + C_2 e^{-m\tau} + \frac{m^2-1}{m^2-\beta^2}\, e^{-\beta\tau} \tag{32}$$

The arbitrary constants C_1 and C_2 are determined from the boundary conditions (28) and (29). With this Kibel points out that further simplifications facilitating calculation are possible. Since $\tau_0 \gg 1$, the terms containing $e^{-m\tau_0}$ can be neglected, since they are small compared with unity. Then we have:

$$C_1 \approx \frac{m^2-1}{m^2-\beta^2}\, \frac{\left[(q-1)\frac{1-e^{-\beta\tau}}{\beta} - (1-q\beta)e^{-\beta\tau_0}\right] + (q-1)\left(1 + \frac{1-\beta^2}{m^2-\beta^2}\,\frac{?}{m}\right)}{1 + m\cdot q + \frac{1-q}{m}} \tag{33}$$

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$$C_2 = \frac{1-\beta^2}{m^2-\beta^2} \tag{34}$$

Finally Kibel obtains the following solution of the subject problem:

$$\varepsilon = \frac{1}{2}\,\frac{m^2-1}{m^2-\beta^2}\left\{\frac{m^2-\beta^2}{m^2-1} + \frac{1-\beta^2}{m^2-\beta^2}\cdot\frac{1}{m} + \beta e^{-\beta\tau} + \frac{1-e^{-\beta\tau}}{\beta} + \frac{1-\beta^2}{m}\cdot e^{-m\tau} - \frac{m^2-\beta^2}{m}\cdot C_1 e^{m\tau}\right\} \tag{35}$$

Thus, Kibel arrives at a result, which represents ~~the unification~~ a generalization of Emden's conclusions in the theory of radiant equilibrium. Emden's solution represents a particular instance of Kibel's solution, corresponding to the ~~case of the lack~~ absence of turbulence, when K=0. Kibel made calculations for the 42 degrees latitude, taking M=1.75; q/f=1.15; W=0.138 calories per square centimeter minute. To describe the distribution of water vapor with height, Kibel uses the following empirical formula:

$$\rho_w = \rho_{w_0}\cdot 10^{-\frac{z}{\ell}} \tag{36}$$

where

$$\ell = 1\cdot 10^{5} \text{ centimeters} \tag{37}$$

then

$$\tau = \int_z^{\infty} \frac{\alpha\,\rho_{w_0} T_0 \cdot 10^{-\frac{z}{\ell}}}{T}\,dz \tag{38}$$

To calculate T, he made use of a ~~quantity under the integral sign~~ integral value, which, ~~according to~~ given by Emden, ~~is~~ as ~~follows~~:

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$$\frac{T}{T_\infty} = (1+\tau)^{\frac{1}{4}} \tag{39}$$

Thus, calculations were made in ~~accordance with~~ from the formula:

$$\frac{4}{5}\left[(1+\tau)^{\frac{5}{4}} - 1\right] = \alpha \rho_{w_0} \frac{T_0}{T_\infty} \frac{\ell}{\ln 10} \cdot 10^{-\frac{z}{\ell}} \tag{40}$$

where it was assumed that:

$$\begin{aligned} \rho_{w_0} &= 6.9 \cdot 10^{-6} \\ \alpha &= 7.25 \ \mathrm{cm}^2/\mathrm{g} \\ T_0 : T_\infty &= 250 : 220 \end{aligned} \tag{41}$$

then $\tau_0 = 12.6$. For $WE = \sigma f T^4$ it follows that:

$$\sigma f T^4 = 0.047\left[6.80 - 4.80 \cdot e^{-0.2\tau} + 0.57 \cdot e^{-1.75\tau} - 0.36^{1.75(\tau - 12.6)}\right] \tag{42}$$

A distribution of temperature with height was obtained, which is given in Table 56. The same table also gives the average temperatures, computed over a three year period (1938, 1939, 1940) on the basis of the radiosonde ~~reception~~ ascents at station Omaha, U.S.A. $\varphi \approx 60°$.

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TABLE 56

Temperature Distribution with Height

~~Zkm~~ Z (km)	0.3	0.5	1.0	2.0	3	4	5	6	7	8	9	10	11	12	13
τ	12.6	12.0	10.5	8.4	6.5	5.1	4.0	3.2	2.5	1.9	1.45	1.1	0.8	0.6	0.4
T computed	8	9	9	4.5	-0.5	-6.5	-13.0	-19.5	-26.5	-33.5	-40.0	-45.0	-49.0	-52.0	-53.0
T observed	8.1	9.1	8.9	5.2	-0.1	-6.1	-12.2	-19.5	-26.4	-33.7	-40.8	-47.2	-52.3	-55.7	-55.7

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As can be seen from the above table, there was close agreement between the observed and theoretical values. Despite its close agreement with the ~~experiment~~ observed values, Kibel's theory was criticized by N. R. Malkin.

According to Kibel, the coefficients of the equation (27) are constant. Actually, these coefficients increase limitlessly with height, since ρ_w, the water vapor density (which appears in the denominators of the coefficients), approaches zero ~~value~~ with an increase in height. Malkin points out that in the layer extending from 1 to 10 kilometers, ρ_w decreases by 160 times. According to Malkin, the close agreement between Kibel's theory and experiment is purely accidental. As can be seen from the above cited example, the obtained results depend primarily on the ~~accepted~~ values ~~of~~ for certain physical parameters, which are precise values are not ~~accurately~~ known.

Malkin solves the same problem more rigorously (by means of integral equations), considering the coefficients of equation (31) as variable. Unfortunately Malkin does not give a single numerical example to illustrate his results. Therefore, the question of error due to Kibel's simplifying assumptions ~~about the~~ of constancy ~~of the~~ coefficients ~~of~~ in equation (31) ~~were~~ was not clarified.

In connection with Malkin's criticism, it must be pointed out that the correlation of Kibel's theory with the results of observations can be explained if we regard ρ_w, not as representing water vapor density, but the general density of all the substances playing an active part in the radiation processes (water vapor, plus ozone, plus carbon dioxide.)

III. THE MEAN-ANNUAL TEMPERATURE DISTRIBUTION IN THE EARTH'S ATMOSPHERE

Kibel's work in connection with the average temperature distribution with height considerably advanced the theoretical solution of

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this problem. Kibel's calculated example indicates the close agreement of the theoretically determined average temperature-versus-height distribution with the observed results. However, this close correlation between the theoretical and empirical results does not hold at all geographical latitudes. This is explained by the fact that in addition to the vertical turbulent exchange, the temperature-versus-height distribution is materially affected by the large-scale horizontal exchange. This horizontal exchange is quite appreciable at low and high geographical latitudes, its role being insignificant at the moderate latitudes. We recall that Kibel compared the results of his computations with observations made at 40 degrees latitude, at which the effect of the horizontal exchange is a minimum.

Therefore, in solving the problem of the mean-annual temperature-versus-height distribution anywhere on earth, we must take into account not only ~~the~~ radiation and ~~the~~ vertical turbulent ~~thermo~~-heat conductivity, but the large-scale horizontal exchange as well.

The first attempt at the theoretical solution of the problem of the mean-annual temperature distribution in the earth's atmosphere.--taking into account the influences of the continents, oceans, and the large-scale horizontal exchange-- was made by Ye. N. Blinov.[2] She starts with the same heat ~~input~~ inflow equation used by Kibel:

$$c_p \rho \frac{dT}{dt} - \frac{dp}{dt} = \varepsilon_1 + \varepsilon_2 \qquad (1)$$

where ε_1 is the heat input due to radiation, ε_2 the heat input due to the turbulent ~~thermo~~heat conductivity. The heat ~~input~~ inflow due to ~~the~~ change of state of water ~~phase variation~~ is neglected.

The Blinov[2] expression for heat ~~input~~ inflow by radiation is identical ~~to~~ with that of Kibel's, namely:

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$$\varepsilon_1 = \alpha \rho_w (A+B) + \alpha \beta \rho_w S - 2 q \rho_w E \tag{2}$$

Since she makes the same assumptions as Kibel in regard to the nature of the atmospheric currents of radiant energy.

Taking the earth as a sphere of radius a_0, Blinov introduces spherical coordinates: r - distance from the center of the earth; latitude complement, ψ longitude of a point, going eastward from the initial meridian. In spherical coordinates the heat ~~input~~ transfer equation due to turbulence is as follows:

$$\varepsilon_2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 K \frac{\partial T}{\partial r}\right) + \frac{1}{r \sin\theta}\left[\frac{\partial}{\partial \theta}\left(\frac{\sin\theta}{r} K' \frac{\partial T}{\partial \theta}\right) + \frac{\partial}{\partial \varphi}\left(\frac{K'}{r \sin\theta}\frac{\partial T}{\partial \varphi}\right)\right], \tag{3}$$

where K is the coefficient of turbulent ~~thermo~~heat conductivity ~~determined by the~~ accounting for vertical exchange, and K' is the coefficient of turbulent ~~thermo~~heat conductivity ~~determined by the~~ accounting for large-scale horizontal exchange. Instead of the coordinate r, we introduce the coordinate $z = r - a_0$, after which equation (3) becomes:

$$\varepsilon_2 = \frac{\partial}{\partial z}\left(K \frac{\partial T}{\partial z}\right) + \frac{1}{a_0^2 \sin\theta}\left[\frac{\partial}{\partial \theta}\left(\sin\theta \cdot K \frac{\partial T}{\partial \theta}\right) + \frac{\partial}{\partial \varphi}\left(\frac{K}{\sin\theta} \cdot \frac{\partial T}{\partial \varphi}\right)\right]. \tag{4}$$

In solving this problem of the mean-annual temperature distribution, Blinov treats it as a static problem. Besides, the average velocities are taken by her to be identically equal to zero. This presupposes that no ~~regular~~ orderly heat transfer takes place, all of the horizontal heat transfer being the result of the large-scale horizontal turbulence. Then the equations of the problem become:

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$$\alpha \rho_w (A + B + \beta S - 2E) + \frac{\partial}{\partial z}\left(K \frac{\partial T}{\partial z}\right) + \frac{1}{a^2 \sin\theta}\left[\frac{\partial}{\partial \theta}\left(K' \sin\theta \frac{\partial T}{\partial \theta}\right) + \frac{\partial}{\partial \varphi}\left(\frac{K'}{\sin\theta} \frac{\partial T}{\partial \varphi}\right)\right] = 0 \tag{5}$$

$$\frac{\partial A}{\partial z} = \alpha \rho_w (A - E), \tag{6}$$

$$\frac{\partial B}{\partial z} = \alpha \rho_w (E - B) \tag{7}$$

$$\frac{\partial S}{\partial z} = \alpha \beta \rho_w S \tag{8}$$

$$E = f \sigma T^4 \tag{9}$$

The ~~required~~ function we are interested in is that of the temperature T; all the other functions are of auxiliary importance. The boundary conditions are as follows:

(1) The upper atmospheric boundary receives only ~~the~~ shortwave solar radiation, therefore

$$A = 0 \text{ with } z \to \infty \tag{10}$$

(2) Not all of the shortwave solar radiation, which impinges on the upper atmospheric boundary, enters the atmosphere. A part of it enters the cosmic space as a result of reflection from the earth's surface, air molecules, and the clouds. This circumstance is accounted for by Blinov[3] in the following boundary condition:

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$$S = [1 - \Gamma(\theta, \varphi)] \cdot w(\theta) = W(\theta, \varphi)$$

(with $z \to \infty$) (11)

where Γ is the earth's reflection factor (albedo), determined mainly by the nature of the underlying surface, as well as by cloudiness; $W(\varphi)$ is the average quantity of solar radiation delivered per unit of time to a unit surface of the upper atmospheric boundary. This quantity only depends on the latitude φ of the point.

(3) It is assumed, that at the upper atmospheric boundary, the temperature levels off along the horizontal, as a result of which

$$\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial \varphi} = 0 \qquad \text{(with } Z = \infty\text{)} \qquad (12)$$

(4) Besides, it is considered that, ~~in general~~, the total amount of ~~the~~ shortwave incoming solar radiation is equal to the total amount of ~~the~~ longwave radiation leaving the atmosphere, i.e.

$$\iint_{(S)} B\,ds = \iint_{(S)} W\,ds \qquad \text{(with } Z = \infty\text{)} \qquad (13)$$

(5) At the surface of the earth there obtains the condition of heat balance:

$$-K\frac{\partial T}{\partial z} + K^*\frac{\partial T^*}{\partial z} = A + S - B \qquad \text{(with } Z = 0\text{)} \qquad (14)$$

where $K^*\frac{\partial T^*}{\partial Z}$ is the heat contributed by the soil. However, Blinov thinks that we can neglect this latter heat ~~input~~ inflow in our problem of the earth's atmosphere.

(6) At last, we have

$$B = qE = qf\sigma T^4 \qquad \text{(with } Z = 0\text{)} \qquad (15)$$

where $q \cdot f$ is the grayishness soil factor, the coefficient (close to unity) which indicates the less-than-black-body radiation capacity of the soil. Introducing for Z the new independent variable, the optical thickness X, we have:

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$$X = \frac{1}{\tau_0} \int_z^\infty \alpha \rho_w \, dz,$$

$$\text{, where } \tau_0 = \int_0^\infty \alpha \rho_w \, dz \qquad (16)$$

and assuming like Kibel that $A + B = a$; $B - A = y$, Blinov transforms the heat ~~input~~ inflow equation as follows:

$$a + \beta S - 2E + \frac{1}{\tau_0} \frac{\partial}{\partial X}\left(\frac{2}{m^2-1} \frac{\partial E}{\partial X}\right) +$$

$$+ \frac{1}{\sin\theta}\left[\frac{\partial}{\partial \theta}\left(M \sin\theta \frac{\partial E}{\partial \theta} + \frac{\partial}{\partial \varphi}\left(\frac{M}{\sin\theta} \frac{\partial E}{\partial \varphi}\right)\right]\right. = 0 \qquad (17)$$

where

$$\frac{2}{m^2-1} = \frac{K \alpha \rho_w}{4 f \sigma T^3}; \quad M = \frac{K'}{4 f \sigma a_0^2 \alpha \rho_w} \qquad (18)$$

The quantities m and M Blinov considers to be practically constant. From (17) it follows that :

$$a = 2E - \beta S - \frac{2}{\tau_0 (m^2-1)} \frac{\partial^2 E}{\partial X^2} -$$

$$- M\left[\frac{1}{\sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial E}{\partial \theta}\right) + \frac{1}{\sin^2\theta} \frac{\partial^2 E}{\partial \varphi^2}\right] \qquad (19)$$

As was shown in paragraph II, it follows from the equations (6) and (7) that:

$$\frac{da}{dX} = \tau_0 y \qquad (20)$$

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$$\frac{\partial y}{\partial x} = \tau_0 (a - 2E) \tag{21}$$

and from (8) and (11) we get:

$$S = W e^{-\tau_0 \beta x} \tag{22}$$

Therefore, from the equations (19) and (20) we get for y:

$$y = \beta^2 S + \frac{2}{\tau_0}\frac{\partial E}{\partial x} - \frac{2}{\tau_0^3 (m^2-1)}\frac{\partial^3 E}{\partial x^3} - \frac{M}{\tau_0}\frac{\partial}{\partial x}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial E}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2 E}{\partial\varphi^2}\right)\right] \tag{23}$$

Substituting further into (21) for $\overset{d}{x}$ and $\overset{y}{y}$ their equivalents from (19) and (23), and, transforming further, we get an equation, which contains one unknown function, E:

$$\frac{2}{m^2-1}\frac{\partial^4 E}{\partial x^4} - \frac{2 m^2 \tau_0^2}{m^2-1}\frac{\partial^2 E}{\partial x^2} + \frac{M \tau_0^2}{\sin\theta}\left[\frac{\partial^3}{\partial x^2 \partial\theta}\left(\sin\theta\frac{\partial E}{\partial\theta}\right) + \frac{\partial^3}{\partial x^2 \partial\varphi}\left(\frac{1}{\sin\theta}\frac{\partial E}{\partial\varphi}\right)\right] - \tau_0^2\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial E}{\partial\theta}\right) - \tau^2\frac{\partial}{\partial\varphi}\left(\frac{1}{\sin\theta}\frac{\partial E}{\partial\varphi}\right) = \beta(1-\beta^2)\tau_0^4 e^{-\tau_0\beta x} W. \tag{24}$$

The boundary conditions for the new functions, with the new independent variable x, are as follows:

$$a - y = 0 \quad \text{when} \quad x = 0 \tag{25}$$

$$\frac{\partial E}{\partial\theta} = \frac{\partial E}{\partial\varphi} = 0 \quad \text{when} \quad x = 0 \tag{26}$$

$$\iint_S (a - y)\, dS = 2\iint_S W\, dS \quad \text{with} \quad x = 0 \tag{27}$$

$$y + \frac{2}{(m^2-1)\tau_0}\cdot\frac{\partial E}{\partial x} = 5 \quad \text{when} \quad x = 1 \tag{28}$$

$$y + \frac{1}{\tau_0}\frac{\partial y}{\partial x} - 2(q-1)E = 0 \quad \text{when} \quad x = 1 \tag{29}$$

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Equation (24) shows that ~~taking into account the~~ consideration of large-scale horizontal transfer complicates the problem considerably. Actually, with ~~the~~ vertical exchange only, the problem of the temperature distribution versus height ~~boiled down~~ reduced to the performance of two integrations, and the solution of one second-order ordinary differential equation. Taking into account ~~the~~ horizontal exchange, we have to solve a fourth-order partial differential equation.

We note that the majority of the problems dealing with spherical surfaces, can be solved comparatively simply by means of the well-known ~~class of the~~ so-called spherical functions, which play the same role as the trigonometric functions in problems involving circles. Various functions on the surface of a sphere, can be represented in the form of an infinite series of spherical functions, analogous to the infinite-series trigonometric representation of ~~other~~ functions. This circumstance was employed by Blinov in the integration of ~~the~~ equation (24).

The function W (θ, φ), which expresses the quantity of solar radiation entering the atmosphere anywhere on the earth, can be represented by an infinite series of spherical functions as follows:

$$W(\theta,\varphi) = \sum_{n=0}^{\infty}\left[W_n^0 P_n(\cos\theta) + \sum_{h=1}^{n}\left(W_n^h \cos h\varphi + W_n'^h \sin h\varphi\right) P_n^h(\cos\theta)\right] \tag{30}$$

where W_n^0, W_n^h, $W_n'^h$, are the corresponding constants, P_n (Cos θ) are the so-called Legendre polynomials, given by:

$$P_n(y) = \frac{1}{2^n \cdot n!}\,\frac{d^n (y^2-1)^n}{dy^n} \tag{31}$$

and satisfying the equation

$$(1-y)^2 \frac{d^2 P_n(y)}{dy^2} - 2y\frac{d P_n(y)}{dy} + n(n+1) P_n(y) = 0 \tag{32}$$

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and P_n^h (y) are the spherical functions given by the formula

$$P_n^h(y) = (1-y^2)^{h/2} \frac{d^h P_n^h(y)}{dy^h} \tag{33}$$

and satisfying the equations

$$\frac{d}{dy}\left[(1-y^2)\frac{dP_n^h(y)}{dy}\right] + \left(\lambda_n - \frac{h^2}{1-y^2}\right) P_n^h(y) = 0 \tag{34}$$

where λ_n is the parameter.

Blinov[3] looks for the solution of equation (24) in the form of an infinite series of spherical functions

$$E(x,\theta,\phi) = \sum_{h=0}^{\infty} \left\{ E_n^0(x) P_n(\cos\theta) + \right.$$
$$\left. + \sum_{h=1}^{n} \left[E_n^h(x)\cos h\phi + E'^h_n(x)\sin h\phi \right] P_n^h(\cos\theta) \right\} \tag{35}$$

We substitute the series (30) and (35) into the equation (34). Taking into account the equations satisfied by the spherical functions, and ~~equal-~~ ~~ing~~ selling the coefficients of ~~the~~ identical spherical functions equal, we get the following equations for the determination of the coefficients of the series (35)

$$\frac{2}{m^2-1}\frac{d^4 E_n^h}{dx^4} - \tau_0^2\left[\frac{2m^2}{m^2-1} + M\cdot n(n+1)\right]\frac{d^2 E_n^h}{dx^2} +$$
$$+ M^4\tau_0\cdot n(n+1)E_n^h = B(1-B^2)\cdot\tau_0^4\cdot e^{-\tau_0 B x}\cdot W_n^h \tag{36}$$

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where n takes on all ~~the~~ values from 0 to ∞, and h takes on all the values from 0 to n. To determine E_n^h we get an equation similar to (36), putting W_m^h in place of W_m^n where h varies from 1 to n.

The solution of equation (36) for all E_n^h (except E_0^0) has the form

$$E_n^h = (C_1)_n^h e^{K_n x} + (C_2)_n^h e^{\tilde{K}_n x} + (C_3)_n^h e^{-K_n x} + (C_4)_n^h e^{-\tilde{K}_n x} + A_n^h e^{-\tau_0 \beta x}, \tag{37}$$

where $(C_1)_n^h, (C_2)_n^h, (C_3)_n^h, (C_4)_n^h$ has arbitrary constants determined from the boundary conditions (25)-(29) and K_n, $\tilde{K}_n$ are given by the formulas

$$\left.\begin{aligned} K_n &= \frac{\tau_0 \cdot m}{\sqrt{2}} \sqrt{1 + \frac{D_n}{m^2} + \sqrt{1 + \frac{2}{m^4}(m^2 - 2) D_n + \frac{(D_n)^2}{m^4}}} \\ \tilde{K}_n &= \frac{\tau_0 \cdot m}{\sqrt{2}} \sqrt{1 + \frac{D_n}{m^2} - \sqrt{1 + \frac{2}{m^4}(m^2 - 2) D_n + \frac{(D_n)^2}{m^4}}} \end{aligned}\right\} \tag{38}$$

where

$$D_n = \frac{M(m^2 - 1) \cdot n(n+1)}{2} \tag{39}$$

At last

$$A_n^h = \frac{1}{2} \frac{\beta (1-\beta)^2 (m^2 - 1) W_2^h}{\beta^2 (\beta^2 - m^2) + D_n (1 - \beta^2)} \tag{40}$$

Blinov[2] assumed the following numerical values for the physical parameters: $m = 1.75$; $\tau = 12.6$; $\beta = 0.2$; $q/f = 1.15$; in computing the quantity

$$\frac{M(m^2 - 1)}{2} = \frac{K'}{K} \frac{1}{(\alpha \rho_w a_0)^2} \tag{40a}$$

She sets: $K'/K = 10^6$, and in the case of α and ρ_w she takes their average ~~gaseous~~ values (for surface conditions): $\alpha = 7.27$ square centimeters per gram,

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$\rho_w = 6.2 \times 10^{-6}$ grams per cubic centimeter. Then $M\,(m^2-1)/2 = 0.00128$.

Besides, we can with great accuracy write down:

$$k_n \approx \tau_0 \cdot m; \quad \tilde{k}_n \approx \frac{\tau_0 \sqrt{D_n}}{m} \tag{40b}$$

In the determination of the arbitrary constants for all the E_n^h and $E_n'^h$, with the exception of E_0^0, we take: $E_n^h = E_n'^h = 0$ with $x=0$, which corresponds to the boundary condition (26). Thus, the arbitrary constants are given as follows:

$$(C_1)_n^h \approx A_n^h \left(Q + \frac{m}{\beta}\right) \cdot e^{-k_n - \tau_0 \beta}$$

$$(C_2)_n^h \approx \frac{A_n^h}{2\Lambda_n} \left[\left(\frac{1}{m-1} + \frac{1}{1-\beta}\right) e^{-\tilde{k}_n} + \right.$$

$$\left. + Q \left(\frac{\sqrt{D_n}}{m} + \frac{m}{m-1}\right) \frac{\sqrt{D_n}}{m^2} e^{-\tau_0 \beta} \right.$$

(41)

$$(C_3)_n^h \approx \frac{A_n^h}{\Lambda_n} \left[\frac{\sqrt{D_n}}{m} \cdot e^{\tilde{k}_n} - \right.$$

$$\left. - \left(\frac{\beta}{1-\beta} + \frac{\sqrt{D_n}}{m}\right) \operatorname{ch} \tilde{k}_n - \frac{D_n}{m^3} Q e^{-\tau_0 \beta} \right];$$

$$(C_4)_n^h \approx \frac{A_n^h}{\Lambda_n} \left[\left(\frac{1}{m-1} + \frac{1}{1-\beta}\right) e^{\tilde{k}_n} + \right.$$

$$\left. + Q \left(\frac{\sqrt{D_n}}{m} - \frac{m}{m-1}\right) \frac{\sqrt{D_n}}{m^2} e^{-\tau_0 \beta} \right]$$

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where

$$Q = \frac{\frac{1}{1+\beta} - q}{\frac{1}{m-1} + q} - \frac{m}{\beta}; \quad \Lambda_n = \left(\frac{\sqrt{D_n}}{m} - \frac{m}{m-1}\right) ch\tilde{\kappa}_n - \frac{\sqrt{D_n}}{m} e^{\tilde{\kappa}_n} \tag{42}$$

To determine E_0^0 we use the boundary condition (27). Besides, from (36) it follows that E_0^0 satisfies the equation

$$\frac{2}{m^2-1}\frac{d^4 E_0^0}{dx^4} - \frac{2m^2\tau_0^2}{m^2-1}\frac{d^2 E_0^0}{dx^2} = \beta(1-\beta)\tau_0^4 e^{-\tau_0 \beta x} W_0^0 \tag{43}$$

Equation (43) had been already solved by Kibel, whose solution is used to get E_0^0. Having determined E_n^0 and E_n^k by means of formulas (37) a solution for E can be obtained in the form of series (35) and, by ~~giving~~ assigning various values to X, ϑ, ψ, we can ~~to~~ compute the distribution of E for the height, longitude and latitude of a given point. Knowing the distribution $E = f\sigma T^4$, it is easy to get the temperature distribution (supplement 15).

Blinov[3] made a computation of the mean-annual distribution of the zonal air temperature according to the height and meridian. The earth reflection coefficient (albedo) is taken to be constant and equal to 0.7. The function W which represents the amount of solar radiation entering the atmosphere, is a function of ϑ only, and can be ~~represented as~~ expanded into a series ~~of the~~ in Legendre polynomials:

$$W(\theta) = \sum_{n=0}^{\infty} W_n^0 P_n(\cos\theta) \tag{44}$$

Then the solution of the problem is sought in the form of a series ~~of the~~ in Legendre polynomials:

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$$E(x,\theta) = \sum_{n=0}^{\infty} E_n^0(x) \cdot P_n(\cos\theta) \tag{45}$$

It is known that the earth's mean-annual ~~expense~~ irradiation is symmetrical about the equator. Therefore, $W(\theta)$ must be an even function of θ, i.e. $W(\theta) = W(-\theta)$. It is also necessary that all the coefficients of the uneven Legendre polynomials be identically equal to zero, i.e. that $W_1^0 = W_3^0 = W_5^0 = \ldots = 0$.

Blinov takes the $W(\theta)$ values from a book by Milankovich "Mathematical Climatology and the Astronomical Theory of Climate Variations". Using Milankovich's data, Blinov obtains the following values for the coefficients of the series (44): $W_0^0 = 0.1498$, $W_2^0 = -.0709$; $W_4^0 = 0.0057$. The remaining coefficients W_n^0 for $n \geq 0$ became negligibly small. Computing K_n, $\tilde{K}_n$, A_n^0, $(C_1)_n^0$, $(C_2)_n^0$, $(C_3)_n^0$, $(C_4)_n^0$ for $n = 2$ and 4 according to formulas (38)-(42), Blinov obtains the following expressions for E_2^0 and E_4^0 according to formula (37):

$$E_2^0(x) = 10^{-2}\left[-0.12\, e^{22(x-1)} - 2.97 \cdot e^{0.61x} + 1.11\, e^{-22x} - 10.48\, e^{-0.61x} + 12.29\, e^{-2.52x}\right], \tag{46}$$

$$E_4^0 = 10^{-3}\left[-0.11\, e^{22(x-1)} - 1.06\, e^{1.12x} + 0.79\, e^{-22x} - 11.26\, e^{-1.12x} + 11.47\, e^{-2.52x}\right],$$

and for E_0^0

$$E_0^0 = 0.051\left[6.73 - 4.80\, e^{-2.52x} + 0.55\, e^{-22x} - 0.26\, e^{22(x-1)}\right] \tag{47}$$

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The final expression for the function E (x, θ) is as follows:

$$E(x,\theta) = E_0^0(x) + E_2^0(x)\cdot P_2(\cos\theta) + E_4^0(x)\cdot P_4(\cos\theta). \qquad (48)$$

On the basis of formulas (46)-(48), Blinov computed the mean-annual distribution of the air temperature with height and meridian. In ~~going~~ converting from the optical thickness x to the height z she uses the following relationship:

$$(1+\tau_0 x)^{5/4} - 1 = \left[(1+\tau_0)^{5/4} - 1\right]\cdot 10^{-\frac{z}{\ell}} \qquad (49)$$

($\ell = 8.10^5$ centimeters)

Table 57 ~~has~~ shows the results of these computations.

(See page 82)

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TABLE 57

THE MEAN ANNUAL DISTRIBUTION OF TEMPERATURE WITH HEIGHT AND MERIDIAN

HEIGHT Z (kilometers)	LATITUDE 90	80	70	60	50	40	30	20	10	0
0	253	255	261	269	277	284	291	295	298	299
0.5	57	59	64	71	79	86	92	96	99	99.5
1	56	58	63	70	77	84	90	94	97	98
2	52	54	59	65	73	80	85	90	92	93
3	47	49	53	60	67	74	79	83	83	86
4	41	44	48	54	61	67	73	77	79	80
5	36	38	42	48	54	60	65	69	72	72
6	33	35	38	43	49	55	59	63	64	65
7	26	28	31	36	41	46	51	55	57	58
8	21	22	26	30	35	40	44	47	49	50
9	18	20	22	27	31	35	39	42	43	44
10	16	17	19	23	27	30	33	35	37	38
11	15	16	18	21	24	27	30	32	33	34
12	15	16	17	20	22	25	27	29	30	30
13	15	16	17	19	21	24	25	27	28	28
14	16	17	18	19	21	23	24	25	26	27
15	17	17	18	19	21	22	23	24	24	25
16	18	18	18	19	21	21	22	22	23	23

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The theoretical distribution of zonal temperature determined by Blinov is in very good agreement with empirical results.

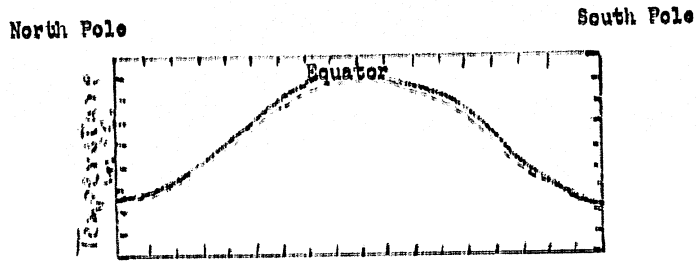
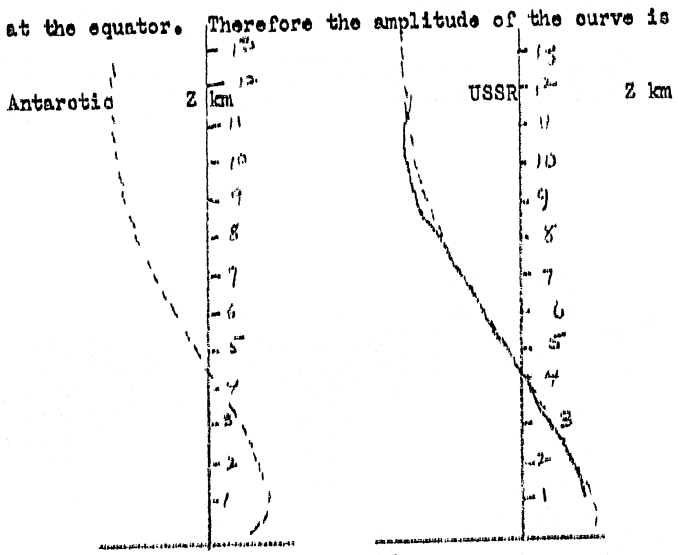


Figure 158. The Distribution of Zonal Temperature along the Meridian according to Observed Data (Solid Line) and Blinov's Computations (Dashed Line).

In figure 158 the solid line shows the temperature distribution at sea level along the meridian according to the empirical data by Hanna and Zuring. The dashed line shows the theoretical distribution of temperature at sea level based on the first line of Table 57. From this graph it is seen that in the Southern hemisphere the two curves practically coincide, while in the Northern hemisphere the discrepancy does not exceed 2°C. It is ~~remarkable~~ notable that the theoretical and empirical temperatures coincide at the poles (-20 degrees Centigrade) and at the equator. Therefore the amplitude of the curve is accurate.



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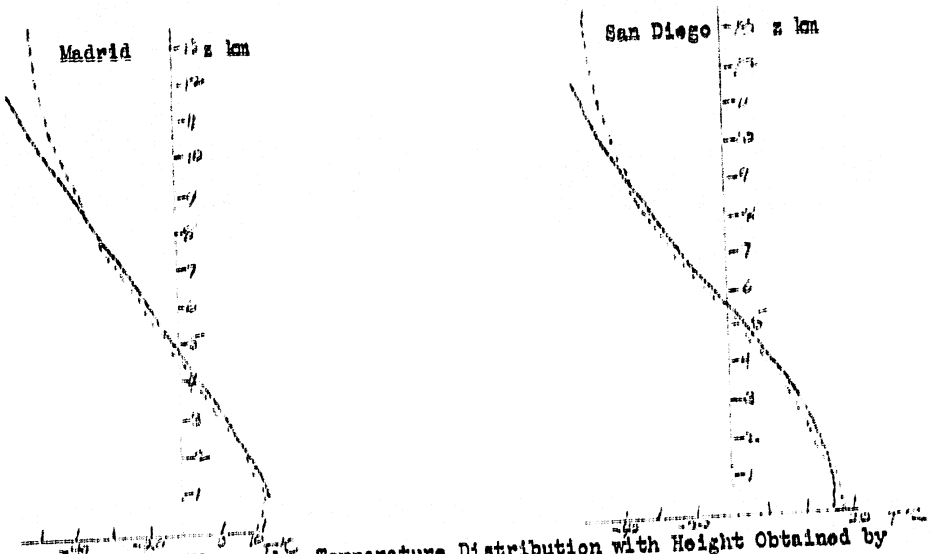
(Figure 159, a, b. (facing page) Temperature Distribution with Height Obtained by Observation (Solid Line) and Blinov's Computations (Dashed Line)



(Figure 159, c, d, Temperature Distribution with Height Obtained by Observation (Solid Line) and Blinov's Computations (Dashed Line)

Figure 159 a, b, c, d, shows temperature distribution with height for the different latitudes. Figure 159 a, corresponds to the 78° latitude. The solid curve represents the results of ~~Byrd's~~ Byrd's Antarctic Expedition taken at Little America ($\varphi = 78°34'S$; $\psi = 163°55'W$).

Figure 159 b, is the USSR data for 55 - 60 degrees North and for 30 - 39 degrees East. Figure 59 c, is the Madrid data, and figure 59 d, is the result of the radiosonde ~~reception~~ ascent at San Diego 32°42'N; 117°9'W.

From all of these figures it is seen that through out the atmosphere up to 9 - 10 kilometers above the surface of the earth, the calculated curves practically coincide with the empirical curves. Only at great heights (particularly at low latitudes) do the discrepancies between theoretical and observed results become significant, ~~showing that~~ the lower stratosphere is calculated to be warmer than is actually the case.

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IV. DOVODNITSYN'S ANALYSIS OF THE DIURNAL TEMPERATURE VARIATION IN THE MIXING LAYER

The question of the diurnal temperature variation in the mixing layer was analyzed by A. Dorodnitsyn, who considered the heat input due to radiant transfer as a given function of time. The temperature of the air is determined by the turbulent thermoconductivity, radiation, as well as by the advective heat transfer. Emphasizing especially the heat input determined by the turbulent transfer of air-masses, Dorodnitsyn takes the equation of the problem in the following form:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(K \frac{\partial T}{\partial z}\right) + \rho c_p \varphi(z, t) \tag{1}$$

or

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) + \varphi(z, t), \tag{2}$$

where $\lambda = K/\rho c_p$ and $\varphi(z, t)$ is a heat input function due to radiation and advection.

Advection is, of course, determined by height and time. However, according to Dorodnitsyn, the variation of temperature with time due to advection is very slow, and therefore can be neglected in the problem of the diurnal variation. Furthermore, Dorodnitsyn considers that the heat input due to radiation in the lower layer (one kilometers) is small compared with the heat input due to the turbulent heat transfer. This assumption is corroborated by the fact that the diurnal variation of temperature is observed only at the lower (kilometer) layer. Were the effect of the radiant energy as significant as that of the turbulent heat transfer, the diurnal variation of temperature would be observed at all heights, which actually is not the case.

In estimating the effect of all the various factors on the transfer of heat, Dorodnitsyn points out that at the lower kilometer layer

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the heat input due to radiation constitutes only ~~several~~ a few percent or a fraction of one percent of the heat input due to the turbulent ~~thermo~~heat conductivity. Thus, Dorodnitsyn considers that the function ϕ depends on z only and not on t, in view of the negligible effect of ~~the~~ radiant energy, and the long period of advection. The solution of the ~~thermo~~heat conductivity equation

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) + \Phi(z) \tag{3}$$

is sought by Dorodnitsyn in the following form

$$T(z,t) = T_0(z) + \tau(z,t) \tag{4}$$

where T_0 is the mean diurnal temperature value. Whence we get:

$$\frac{\partial T}{\partial t} = \frac{\partial \tau}{\partial t} \; ; \; \frac{\partial T}{\partial z} = \frac{dT_0}{dz} + \frac{\partial \tau}{\partial z} \tag{5}$$

Consequently, the equation (3) is transformed into

$$\frac{\partial \tau}{\partial t} = \frac{d}{dz}\left(\lambda \frac{dT_0}{dz}\right) + \frac{\partial}{\partial z}\left(\lambda \frac{d\tau}{dz}\right) + \Phi(z); \tag{6}$$

since $\frac{d}{dz}\left(\lambda \frac{dT_0}{dz}\right)$ and $\Phi(z)$ are functions of height only, ~~and therefore,~~ equation (6) breaks up into two independent equations:

$$\frac{d}{dz}\left(\lambda \frac{dT_0}{dz}\right) + \Phi(z) = 0 \tag{7}$$

$$\frac{\partial \tau}{\partial t} = \frac{\partial}{\partial z}\left(\lambda \frac{\partial \tau}{\partial z}\right) \tag{8}$$

Dorodnitsyn does not undertake the solution of equation (7), since he believes that the value of the mean diurnal temperature can be determined, for example, by the Kibel method (chapter 20). He only treats the equation (8), and assumes the following law of λ versus height variation.

$$\lambda = \lambda_0 (1 + \varepsilon - e^{-mz}) \tag{9}$$

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The quantity ε is introduced so that at $z = 0$, λ will be ~~in~~ of the order of the molecular temperature conductivity. Let us note that this assumption corresponds to the hypothesis that the earth's surface is ~~flat~~ fairly flat and devoid of vegetation. In connection with the equation (8) Dorodnitsyn also discusses the process of soil ~~thermo~~ heat conduction, taking as constant the coefficient λ^* of the soil ~~thermo~~ heat conductivity. Therefore, according to Dorodnitsyn, the problem ~~comes down~~ reduces to the integration of the following system of equations:

$$\frac{\partial \tau}{\partial t} = \frac{\partial}{\partial z}\left(\lambda \frac{\partial \tau}{\partial z}\right) \quad \text{when } z > 0$$

$$\frac{\partial \tau^*}{\partial t} = \frac{\partial}{\partial z}\left(\lambda^* \frac{\partial \tau}{\partial z}\right) \quad \text{when } z < 0 \tag{10}$$

where τ is the deviation of the soil temperature from its mean diurnal value. No initial conditions are attached to this problem, since the periodic temperature variation is sought. The boundary conditions are formulated as follows: at infinity all of the periodic temperature variations cease. Therefore

$$\tau(z,t) \longrightarrow 0 \quad \text{when } z \longrightarrow +\infty \tag{11}$$

$$\tau^*(z,t) \longrightarrow 0 \quad \text{when } z \longrightarrow -\infty \tag{12}$$

Furthermore, Dorodnitsyn assumes continuous variation of temperature at the earth's surface, i.e.

$$\tau(0,t) = \tau^*(0,t) \tag{13}$$

Finally, Dorodnitsyn formulates the condition of heat balance at the surface of the earth. Without radiation transfer this boundary condition would be as follows:

$$K\frac{\partial T}{\partial z} = K^*\frac{\partial T^*}{\partial z} \qquad K\frac{\partial \tau}{\partial z} = K^*\frac{\partial \tau^*}{\partial z} \quad \text{when } z = 0$$

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However, with radiation and the incoming flow of ~~the~~ radiant energy, the boundary condition expressing ~~of~~ the energy balance must be taken as follows:

$$-K^* \frac{\partial T^*}{\partial z} = -K \frac{\partial T}{\partial z} - \sigma T^4 - (1-\Gamma) W(t) \qquad (14)$$

where Γ is the reflection factor of the earth.

Since the equations to be integrated involve τ and not T, the boundary condition (14) must be expressed in terms of τ.

Let

$$\begin{aligned} T^*(z,t) &= T_0^*(z) + \tau^*(z,t) \\ T(z,t) &= T_0(z) + \tau(z,t) \\ W(t) &= W_0 + W_1(t) \end{aligned} \qquad (15)$$

where W_0 is the mean diurnal heat input from the sun and ~~the~~ atmosphere, then the boundary condition (14) becomes:

$$-K^* \frac{\partial T_0^*}{\partial z} - K^* \frac{\partial \tau^*}{\partial z} = -K \frac{\partial T_0}{\partial z} - K \frac{\partial \tau}{\partial z} + \sigma \left[T_0(0) + \tau(0,t)\right]^4 -$$

$$- (1-\Gamma) W_0 - (1-\Gamma) W_1(t) \quad \text{(when } z=0) \qquad (16)$$

If, there exists also a mean diurnal balance, then we have

$$-K^* \frac{\partial T_0^*}{\partial z} = -K^* \frac{\partial T_0}{\partial z} + \sigma T_0^4(0) - (1-\Gamma) W_0 \quad \text{when } z=0 \qquad (17)$$

~~Satisfying~~ Combining (16) and (17) we get:

$$(1-\Gamma) W_1(t) = \sigma \left[T_0(z) + \tau(0,t)\right]^4 - \sigma T_0^4(0) - K \frac{\partial \tau}{\partial z} +$$

$$+ K^* \frac{\partial \tau^*}{\partial z} \quad \text{when } z=0 \qquad (18)$$

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This fourth boundary condition introduces a new function W_1 (t), which, according to Dorodnitsyn is a given function.

The boundary condition (18) is non-linear with respect to τ, which complicates the solution of the given problem. To circumvent this difficulty, Dorodnitsyn linearizes this boundary condition in the following manner. It is obvious, that

$$\sigma\left[T_0(0)+\tau(0,t)\right]^4-\sigma T_0^4(0)=4\sigma T_0^3\tau+\dots \qquad (19)$$

Letting

$$4\sigma T_0^3=\mu=const. \qquad (20)$$

and neglecting the terms containing τ of second degree and higher, Dorodnitsyn believes that his simplification results in an error not in excess of 10 percent if instead of the righthand side of (19) we simply take $\mu\tau$. Since the radiation term is small compared to the ~~thermo~~heat conductivity term, according to Dorodnitsyn the resultant error connected with the boundary condition will not exceed 3 percent. Consequently, the boundary condition (18) is supplanted by:

$$(1-T)W_1(t)=\mu\tau(0,t)-K\frac{\partial\tau}{\partial z}+K^*\frac{\partial\tau^*}{\partial z} \quad \text{when } z=0 \qquad (21)$$

Thus, the problem of the diurnal temperature variation ~~comes~~ reduces ~~down~~ to the integration of the system of equations (10) with the boundary conditions (11), (12), (13) and (21).

The solution of these equations is sought in the form of the following infinite series:

$$\left.\begin{aligned}\tau(z,t)&=\sum_{\nu=1}^{\infty}\left[\tau_{1,\nu}(z)\cos\nu\omega t+\tau_{2,\nu}(z)\sin\nu\omega t\right]\\ \tau^*(z,t)&=\sum_{\nu=1}^{\infty}\left[\tau^*_{1,\nu}(z)\cos\nu\omega t+\tau_{2,\nu}(z)\sin\nu\omega t\right]\end{aligned}\right\} \qquad (22)$$

The given function W_1 (t) is also represented in the form of an infinite series:

$$W_1(t)=\sum_{\nu=1}^{\infty}\left[a_\nu\cos\nu\omega t+b_\nu\sin\nu\omega t\right] \qquad (23)$$

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Substituting the series (22) and (23) into the equations of the problem, and equating coefficients, we get a closed system of equations defining $\tau_{1,\nu}$, $\tau_{2,\nu}$, $\tau^*_{1,\nu}$, $\tau^*_{2,\nu}$.

Derednitsyn does not analyze the problem of convergence of the obtained solution, but only looks for a particular solution. Substituting (22) into (10) we get:

$$-\nu\omega\tau_{1,\nu}(z)\sin\nu\omega t + \nu\omega\tau_{2,\nu}(z)\cos\nu\omega t =$$
$$= \lambda_0 \frac{d}{dz}\left[(1+\varepsilon - e^{-mz})\frac{d\tau_{1,\nu}}{dz}\right]\cos\nu\omega t +$$
$$+ \lambda_0 \frac{d}{dz}\left[(1+\varepsilon - e^{-mz})\frac{d\tau_{2,\nu}}{dz}\right]\sin\omega t \tag{24}$$

Since the relationship (24) is satisfied for any value of t, ~~then~~ we ~~get~~ obtain:

$$\left.\begin{aligned} \lambda_0 \frac{d}{dz}\left[(1+\varepsilon - e^{-mz})\frac{d\tau_{1,\nu}}{dz}\right] &= \nu\omega\tau_{2,\nu}(z) \\ \lambda_0 \frac{d}{dz}\left[(1+\varepsilon - e^{-mz})\frac{d\tau_{2,\nu}}{dz}\right] &= -\nu\omega\tau_{1,\nu}(z) \end{aligned}\right\} \tag{25}$$

Similarly

$$\left.\begin{aligned} \lambda^* \frac{d^2\tau^*_{1,\nu}}{dz^2} &= \nu\omega\tau^*_{2,\nu}(z) \\ \lambda^* \frac{d^2\tau^*_{2,\nu}}{dz^2} &= -\nu\omega\tau^*_{1,\nu}(z) \end{aligned}\right\} \tag{26}$$

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For the boundary conditions we get:

$$\left.\begin{aligned}
&\tau_{1,\nu}(\infty) = \tau_{2,\nu}(\infty) \equiv 0 \\
&\tau^*_{1,\nu}(-\infty) \equiv \tau^*_{2,\nu}(-\infty) = 0 \\
&\tau_{1,\nu}(0) \equiv \tau^*_{1,\nu}(0) \\
&\tau_{2,\nu}(0) \equiv \tau^*_{2,\nu}(0) \\
&(1-\Gamma)a_\nu = \nu\omega\tau_{1,\nu}(0) - K\frac{d\tau_{1,\nu}}{dz} + K^*\frac{d\tau^*_{1,\nu}}{dz} \quad \text{when } z=0 \\
&(1-\Gamma)b_\nu = \mu\tau_{2,\nu}(0) - K\frac{d\tau_{2,\nu}}{dz} + K^*\frac{d\tau^*_{2,\nu}}{dz} \quad \text{when } z=0
\end{aligned}\right\} \qquad (27)$$

Introducing new complex functions:

$$\tau_\nu = \tau_{1,\nu} + i\tau_{2,\nu} \; ; \; \tau^*_\nu = \tau_{1,\nu} + i\tau^*_{2,\nu} \qquad (28)$$

multiplying the second equation of (25) by i and adding it to the first equation, we get

$$\lambda_s \frac{d}{dz}\left[(1-\varepsilon-e^{-\nu z})\frac{d\tau_\nu}{dz}\right] + i\nu\omega\tau_\nu = 0 \qquad (29)$$

Similarly the equation (26) is transformed as follows:

$$\lambda^* \frac{d^2\tau^*_\nu}{dz^2} + i\nu\omega\tau^*_\nu = 0 \qquad (30)$$

We first solve equation (30) whose characteristic equation is

$$r^2 + \frac{i\nu\omega}{\lambda^*} = 0 \qquad (30A)$$

whence

$$r = \pm\sqrt{-\frac{i\nu\omega}{\lambda^*}} = \pm\frac{1-i}{\sqrt{2}}\sqrt{\frac{\nu\omega}{\lambda^*}} \qquad (30A\ 1)$$

Consequently the two particular solutions of the equation (30) are

$$\frac{1-i}{e^{\sqrt{2}}}\sqrt{\frac{\nu\omega}{\lambda^*}}\cdot z \; ; \; e^{-\frac{1-i}{\sqrt{2}}\sqrt{\frac{\nu\omega}{\lambda^*}}\cdot z} \qquad (30B)$$

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The second ~~one~~ of these particular solutions must be discarded, since it tends to infinity at $z = -\infty$. Therefore we have:

$$\tau_1 = (C_1 + iD_1) \cdot e^{\frac{1+\varepsilon}{\varepsilon}\sqrt{\frac{\varepsilon}{1+\varepsilon}}\, z} \tag{31}$$

In (31), separating the real part from the imaginary part, we ~~get~~ $\tau_{1,1}$ and $\tau_{2,1}$.

The equation (29) is considerably more complex, since it has variable coefficients. To begin with, we determine the type of this equation. Differentiating (29) we get:

$$[illegible] \tag{31-a}$$

Furthermore, Dorodnitsyn changes ~~variables~~ the independent variable according to the formula

$$x = 1 - (1+\varepsilon)\, e^{mz} \tag{32}$$

Since

$$\frac{d\tau_1}{dz} = \frac{d\tau_1}{dx} \cdot \frac{dx}{dz}; \quad \frac{d^2\tau_1}{dz^2} = \frac{d^2\tau_1}{dx^2}\left(\frac{dx}{dz}\right)^2 + \frac{d\tau_1}{dx} \cdot \frac{d^2x}{dz^2} \tag{32-a}$$

and

$$\frac{dx}{dz} = (1+\varepsilon) \cdot m e^{mz} \frac{d\tau_1}{dx}$$

$$\frac{d^2\tau_1}{dz^2} = (1+\varepsilon)^2 \cdot m^2 e^{2mz} \frac{d^2\tau_1}{dx^2} - (1+\varepsilon) m^2 e^{mz} \cdot \frac{d\tau_1}{dx} \tag{32-b}$$

(32-c)

we have

$$\frac{dx}{dz} = -(1+\varepsilon) m e^{mz}; \quad \frac{d^2x}{dz^2} = -(1+\varepsilon) m^2 e^{mz}$$

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and the equation (29) is transformed into

$$\lambda_0(1+\varepsilon-e^{-mz})(1+\varepsilon)^2 m^2 e^{2mz}\frac{d^2\tau_\nu}{dx^2}-\left[\lambda_0(1+\varepsilon)m^2+\right.$$
$$\left.+\lambda_0(1+\varepsilon-e^{-mz})(1+\varepsilon)m^2e^{mz}\right]\frac{d\tau_\nu}{dx}+i\nu\omega\tau_\nu=0 \qquad (33)$$

We express the coefficient of the first term of equation (33) by ~~means~~ of X. From (32) we have:

$$(1+\varepsilon)e^{mz}=1-X \qquad (33\text{-a})$$

Consequently we get:

$$\lambda_0(1+\varepsilon-e^{mz})(1+\varepsilon)^2\cdot m^2e^{2mz}=$$
$$\lambda_0(1+\varepsilon)\cdot m^2(1+\varepsilon)\cdot e^{mz}\left[(1+\varepsilon)e^{mz}-1\right]=\lambda_0(1+\varepsilon)m^2(1-X)\cdot X \qquad (33\text{-b})$$

From (33) we obtain:

$$-\lambda_0(1+\varepsilon)m^2(1-X)X\frac{d^2\tau_\nu}{dx^2}-\left[\lambda_0(1+\varepsilon)m^2+(1-X)\right]\frac{d\tau_\nu}{dx}+i\nu\omega\tau_\nu=0 \qquad (33\text{-c})$$

or

$$X(1-X)\frac{d^2\tau_\nu}{dx^2}+(1-X)\frac{d\tau_\nu}{dx}-\frac{i\nu\omega}{\lambda_0(1+\varepsilon)m^2}\tau_\nu=0 \qquad (34)$$

Letting

$$\delta^2=\frac{\omega}{\lambda_0(1+\varepsilon)m^2} \qquad (34\text{-a})$$

we get

$$X(1-X)\frac{d^2\tau_\nu}{dx^2}+(1-X)\frac{d\tau_\nu}{dx}-i\nu\delta^2\tau_\nu=0 \qquad (35)$$

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and letting

$$i\nu\delta^2 = -a^2 \tag{35-b}$$

whence we have

$$a = \sqrt{-i\nu\delta^2} = \frac{(1-i)}{\sqrt{2}} \cdot \delta\sqrt{\nu} = \frac{1-i}{2} \cdot \sqrt{\frac{\omega}{\lambda_0(1+\varepsilon)m^2}} \cdot \sqrt{\nu} \tag{36}$$

and finally

$$x(1-x)\frac{d^2\tau_\nu}{dx^2} + (1-x)\frac{d\tau_\nu}{dx} + a^2\tau_\nu = 0 \tag{37}$$

This equation represents a particular case of the well-known equation of the hypergeometric series

$$x(1-x)\frac{d^2y}{dx^2} + \left[\gamma - (\alpha+\beta+1)x\right]\frac{dy}{dx} - \alpha\beta y = 0 \tag{38}$$

with

$$\beta = -\alpha \; ; \quad \gamma = 1$$

The equation (36) is not susceptible to elementary integration. The integral of this equation is given by the following infinite hypergeometric series:

$$y_1 = F(\alpha,\beta,\gamma,x) = 1 + \frac{\alpha\beta}{1\cdot\gamma}x + \frac{\alpha(\alpha+1)\beta(\beta+1)}{1\cdot 2\cdot\gamma\cdot(\gamma+1)}x^2 + \dots \tag{39}$$

However, the hypergeometric series (39) cannot be taken as the solution of our problem, since at $z \to \infty$, $x \to -\infty$ and, therefore, this solution does not satisfy the boundary condition at infinity, and a different solution must be sought. For this purpose Dorodnitsyn uses the same series but with different parameters:

$$y_2 = (-x)^\beta \cdot F(\alpha, \alpha-\gamma+1, \alpha-\beta+1, \tfrac{1}{x}) \tag{40}$$

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In our case $\beta = -\alpha$, $\gamma = 1$. Consequently, we have,

$$\tau_\nu = (A_\nu + iB_\nu)\cdot\left(-\frac{1}{x}\right)^{\alpha}\cdot F\left(\alpha, \alpha, 2\alpha+1, \frac{1}{x}\right) \tag{41}$$

where A_ν, B_ν are arbitrary constants determined by the boundary conditions, and F is a hypergeometric series.

Consequently, the required solution is defined by the parameter α. However, this series converges only in the region $|x| > 1$ whereas the boundary condition takes in $Z = 0$, i.e. for $X = -\varepsilon$, where ε is a ~~very small quantity~~. Therefore, the obtained solution will not converge in the region [illegible].

By means of the induction theory, Dorodnitsyn constructs an analytic continuation of the hypergeometric series $F\left(\alpha, \alpha, 2\alpha+1, \frac{1}{x}\right)$ in the region of $|x| < 1$, and finds the solution of the equation (37) in the form of

$$\tau_\nu = (A_\nu + iB_\nu)(\Theta_{1,\nu} + i\Theta_{2,\nu}) \tag{42}$$

The previously obtained solution (31) can be taken as

$$\tau_\nu^* = (C_\nu + iD_\nu)(\Theta_{1,\nu}^* + i\Theta_{2,\nu}^*) \tag{43}$$

Upon separating the real and imaginary parts in (42) and (43), we have

$$\left.\begin{aligned}
\tau_{1,\nu} &= A_\nu\Theta_{1,\nu} - B_\nu\Theta_{2,\nu} \\
\tau_{2,\nu} &= A_\nu\Theta_{2\nu} + B_\nu\Theta_{1,\nu} \\
\tau_{1,\nu}^* &= C_\nu\Theta_{1,\nu}^* - D_\nu\Theta_{2,\nu}^* \\
\tau_{2,\nu}^* &= C_\nu\Theta_{2,\nu}^* + D_\nu\Theta_{1,\nu}^*
\end{aligned}\right\} \tag{44}$$

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Substituting (44) into the boundary conditions rel~~ationships~~, and transforming, we get equations in terms of A_ν and B_ν :

$$\left.\begin{aligned}(1-\Gamma)\,a_\nu &= A_\nu \gamma_{1,\nu} - B_\nu \gamma_{2,\nu}\\ (1-\Gamma)\,b_\nu &= A_\nu \gamma_{2,\nu} + B_\nu \gamma_{1,\nu}\end{aligned}\right\} \qquad (45)$$

where

$$\left.\begin{aligned}\gamma_{1,\nu} &= \mu\theta_{1,\nu} - K\theta'_{1,\nu} + K^*(\theta_{1,\nu} + \theta_{2,\nu})\cdot\sqrt{\frac{\mu\nu}{2\lambda^*}}\\ \gamma_{2,\nu} &= \mu\theta_{2,\nu} - K\theta'_{2,\nu} - K^*(\theta_{1,\nu} + \theta_{2,\nu})\cdot\sqrt{\frac{\mu\nu}{2\lambda^*}}\end{aligned}\right\} \qquad (46)$$

whence

$$\left.\begin{aligned}A_\nu &= (1-\Gamma)\,\frac{a_\nu\gamma_{1,\nu} + b_\nu\gamma_{2,\nu}}{\gamma_{1,\nu}^2 + \gamma_{2,\nu}^2}\\ B_\nu &= (1-\Gamma)\,\frac{b_\nu\gamma_{1,\nu} - a_\nu\gamma_{2,\nu}}{\gamma_{1,\nu}^2 + \gamma_{2,\nu}^2}\end{aligned}\right\} \qquad (47)$$

To illustrate the obtained solution, Dorodnitsyn computed the theoretical diurnal temperature variation at the city of Slautsk on a clear, cloudless day of 13 July 1938. The W values were obtained actinographically. The following values of parameters were used:

$\lambda_0 = 8.1$ meters per second, $K = 0.5 \times 10^{-5}$ large ~~kilogram~~ calories per meter per second per degree

$M = 0.02\ M^{-1}$, $K^* = 0.9 \times 10^{-6}$ meters per second

$K^* = 3 \times 10^{-4}$ large Kilogram calories per meter per second per degree

$\Gamma = 0.1$

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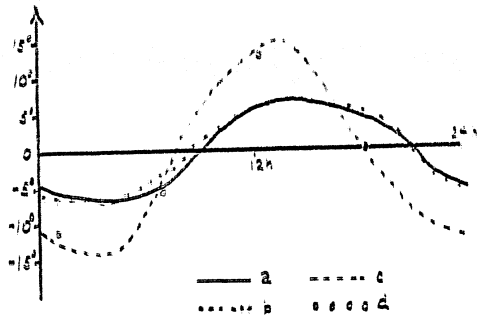
As is seen from figure 160, the results of this calculation agree closely with the observed results.

V. THE THEORETICAL METHOD OF DETERMINING THE DIURNAL TEMPERATURE VARIATION

The question of the diurnal variation of the air temperature and of the underlying surfaces, taking into account the atmospheric radiant heat transfer, was analyzed theoretically by M. E. Shvets. By using the method of successive approximations, Shvets simultaneously solved the equation of the heat input and the equations of the radiant energy transfer, from very general assumptions.



(Figure 160. The Diurnal Variation of Temperature, by Dorodnitsyn: (a) Computed Temperature at the 2-meter Height; (b) Observed Temperature at the 2-meter Height; (c) Computed Temperature of the Soil; (d) Observed Temperature of the Soil.)

For the starting equation in the study of the periodic temperature variations, Shvets takes the heat input equation:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\lambda\frac{\partial T}{\partial z} + \frac{a\rho\omega}{c_p\cdot\rho}\left(A + B + \beta I - 2fE\right), \qquad (1)$$

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associating with it the equations of the radiant energy transfer:

$$\frac{\partial A}{\partial z} = a\rho_\omega (A - fE), \tag{2}$$

$$\frac{\partial B}{\partial z} = a\rho_\omega (fE - B), \tag{3}$$

$$\frac{\partial I}{\partial z} = -a\beta\rho_\omega \sec\theta, \tag{4}$$

where I is the shortwave solar radiation, and θ is the zenith angle of the sun.

Since there is a constant ~~thermal~~ exchange ~~taking place~~ between the air and the underlying surface, we must consider the equation of soil in addition to the above-~~cited~~ equations. ~~heat distribution.~~

$$\frac{\partial T^*}{\partial t} = \frac{\partial}{\partial z}\lambda^* \frac{\partial T^*}{\partial z} \qquad z < 0 \tag{5}$$

where the asterisk denotes the quantities which characterize the underlying surface (soil or water).

Finally, between the air and the underlying surface there ~~takes place a transfer of~~ moisture, in the form of water evaporation from the underlying surface, or in the form of the water vapor concentration at the same surface. Therefore, with the above equations we must also associate the equation of moisture transfer.

Letting Q denote the specific humidity and neglecting the horizontal turbulent moisture ~~transfer~~, we get the following equation of ~~the~~ turbulent moisture transfer.

$$\frac{\partial Q}{\partial t} = \frac{\partial}{\partial z}\left(D\frac{\partial Q}{\partial z}\right) \tag{6}$$

Formulating the boundary conditions, for the upper atmospheric boundary we have:

$$\begin{aligned} A &= 0 \quad \text{when } z = \infty \\ I &= W(t) \quad \text{when } z = \infty \end{aligned} \tag{7}$$

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At the underlying surface a ~~lack of~~ temperature discontinuity is assumed to ~~exist~~:

$$T(0,t) = T^*(0,t), \tag{8}$$

and also

$$B(0,t) = \delta E(0,t), \tag{9}$$

where δ is the "gray factor" to account for the ~~smaller~~ less than ~~the~~ black-body-radiation capacity of the underlying surface.

Besides, at the underlying surface the following heat balance condition must be fulfilled:

$$-k\frac{\partial T}{\partial z} - k^*\frac{\partial T^*}{\partial z} - LM = A - B - (1-\gamma)I, \tag{10}$$

where M is the quantity of water evaporating per unit surface, which is equal to the ~~velocity~~ of evaporation: $M = -D\frac{\partial Q}{\partial z}$, L is the latent heat of evaporation, γ is the reflection factor of the underlying surface.

Finally, it is necessary to formulate the boundary condition for Q at $z=0$, or for the ~~velocity~~ of evaporation. The rate of evaporation at the soil surface depends on many variable factors, all of which cannot be generally accounted for. However, if the evaporation takes place from ~~the~~ water surface, the boundary condition for the moisture is quite simple. It is natural to consider that at the water surface, the air is constantly and completely saturated with ~~the~~ water vapor:

$$Q(0,t) = Q_{max}, \tag{11}$$

In order to estimate the relative importance of evaporation in the heat balance, Shvets assumes complete saturation at the soil surface, observing, however, that the rate of water evaporation from

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dry soil is considerably smaller than that from a moist soil surface.

All elements of the atmospheric heat balance evidence a well-pronounced diurnal variation, the classic example of which is represented by the diurnal variation of ~~the~~ direct solar radiation. In order to arrive at the most general expression for the diurnal variation of solar radiation, Shvets proceeds in this way:

Neglecting, for simplicity, the variation of the earth-to-sun distance, ~~then~~ the amount of energy per unit of horizontal surface, per unit of time, and in the absence of the atmosphere, is given by the formula $I = I_0 \sin\theta$, where I_0 is the solar constant. By means of the well-known relationship in spherical trigonometry

$$\sin\theta = \sin\varphi \cdot \sin\delta + \cos\varphi \cdot \cos\delta \cdot \cos\omega t, \qquad (11\text{-a})$$

we get

$$I = I_0 (\sin\varphi \cdot \sin\delta + \cos\varphi \cdot \cos\delta \cdot \cos\omega t) \qquad (12)$$

where φ is the geographical latitude, δ is the angle of the solar declination, ω is the angular velocity of the earth. Considering δ to be constant over ~~the~~ 24-hour period and assuming the following relationships

$$\left.\begin{aligned} M &= I_0 \sin\varphi \cdot \sin\delta, \\ N &= I_0 \cos\varphi \cdot \cos\delta, \end{aligned}\right\} \qquad (13)$$

we get

$$I = M + N \cdot \cos\omega t, \qquad (14)$$

Formula (14) is only applicable for the ωt values for which $I \geq 0$, i.e. between the times of sunrise and sunset, when $M + N\cos\omega t \neq 0$ ~~[illegible]~~, where the negative root, $-\omega t_0$ corresponds to sunrise, and the positive root $+\omega t_0$ corresponds to sunset. At all other hours $I = 0$.

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Shvets expresses both portions of the curve ~~of the~~ diurnal variation of ~~the~~ direct solar radiation by means of a single continuous function, ~~represented by the~~ Fourier Series

$$I(t) = \sum_{n=0}^{n=\infty} J_n e^{-in\omega t} \tag{15}$$

in which he ~~deems~~ it sufficient to take only the first four terms. The expressions of the first four coefficients of the series (15) are:

$$\left.\begin{aligned}
J_0 &= \frac{N}{\pi}\left[\sin\omega t_1 - \omega t_1 \cos\omega t_1\right] \\
J_1 &= \frac{N}{\pi}\left[\omega t_1 - \frac{\sin 2\omega t_1}{2}\right] \\
J_2 &= \frac{N}{3\pi} \sin^3 \omega t_1 \\
J_3 &= \frac{N}{3\pi} \sin^3 \omega t_1 \cos\omega t_1
\end{aligned}\right\} \tag{16}$$

The decrease in the solar radiation caused by the atmosphere can be easily accounted for. Integrating equation (4) with the boundary condition (7) we get:

$$I(z,t) = W(t) \cdot e^{-\sec\theta \int_z^\infty \alpha \rho_\omega dz} \tag{17}$$

Consequently, with the atmosphere present, the earth's surface receives the following amount of solar radiation:

$$I(0,t) = W(t) \cdot e^{-\sec\theta \int_0^\infty \alpha \rho_\omega dz} \tag{18}$$

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or, if we introduce the coefficient of transparency, $P = e^{-\int \alpha \rho_a dz}$,

we have

$$I(0,t) = W(t) \cdot p \sec\theta = \sum_{-\infty}^{\infty} j_n e^{-i_n \omega t} \tag{19}$$

Shvets compiled detailed tables of the values of the first three coefficients of the series (19) for the variables φ, δ, and P.

Thus, the diurnal variation of heat ~~input~~ due to ~~the~~ shortwave solar radiation can be represented by the following Fourier Series:

$$I(z,t) = \bar{I}(z) + \sum j_n e^{-i_n \omega t} = \bar{I}(z) + J(z,t) \tag{20}$$

where $\bar{I}(z)$ is the mean diurnal value of the heat ~~input~~. It is natural to seek the solution of the system of equations (1)-(6) in the form

$$\left.\begin{aligned} T &= \bar{T}(z) + \tau(z,t); \quad T^* = \bar{T}^*_{(z)} + \tau^*(z,t); \\ E &= \bar{E}(z) + e(z,t), \\ A &= \bar{A}(z) + a(z,t); \quad B = \bar{B}(z) + \beta(z,t); \\ Q &= \bar{Q}(z) + q(z,t) \end{aligned}\right\} \tag{21}$$

Substituting the expressions (21) into the equations (1) to (6), we get two systems of equations. The first system, which defines the mean diurnal values of the required functions, is not analyzed by Shvets, since he assumes that these mean diurnal values can be determined by other methods, such as Kibel's method (Chapter 20). The second system defines the unknown deviations of the required functions from their mean diurnal values, and is as follows:

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$$\frac{\partial \tau}{\partial t} = \frac{\partial}{\partial z} \lambda \frac{\partial \tau}{\partial z} + \frac{\alpha \rho_w}{C_p \rho} (a + b + \beta j - 2fe), \qquad (1')$$

$$\frac{\partial a}{\partial z} = \alpha \rho_w (a - fe), \qquad (2')$$

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$$\frac{\partial b}{\partial z} = \alpha \rho_w (fe - b), \qquad (3')$$

$$\frac{\partial \tau^*}{\partial t} = \frac{\partial}{\partial z} \lambda^* \frac{\partial \tau^*}{\partial z}, \qquad (5')$$

$$\frac{\partial q}{\partial t} = \frac{\partial}{\partial z} D \frac{\partial q}{\partial z}. \qquad (6')$$

Shvets also makes corresponding changes in the boundary conditions. Since at the underlying surface there is assumed complete saturation, the pressure of the saturated-water vapor then depends only upon the temperature and is given by the Magnus' formula, then the boundary condition (11) can be taken as:

$$Q(0,t) = \bar{Q}(0) + q(0,t) = \frac{0.622 \times 6.10}{P_0} \cdot 10^{\frac{7.45(T-273)}{T-38}} \qquad (6\text{-a})$$

where P_0 is the air pressure at the earth's surface. Since $T = \bar{T} + \tau$, we have:

$$\bar{Q}(0) + q(0,t) \approx \frac{0.622 \times 6.10}{P_0} \cdot 10^{\frac{7.45(\bar{T}-273)}{\bar{T}-38}} \cdot 10^{\frac{7.45}{\bar{T}-38}\tau} =$$

$$= \bar{Q}_{max}(0)\left[1 + \frac{7.45}{\bar{T}-38} \ln 10 \cdot \tau(0,t)\right]. \qquad (6\text{-b})$$

Thus, the boundary condition for $q(0, t)$ with sufficient accuracy, can be taken as:

$$q(0,t) = \nu \cdot \tau(0,t), \qquad (11'\text{-a})$$

where

$$\nu = \frac{7.45 \cdot \ln 10}{\bar{T} - 38} \cdot \bar{Q}_{max}(0). \qquad (11\text{-a})$$

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The rest of the boundary conditions are obviously:

$$\left.\begin{aligned} a &= 0 \quad \text{at} \quad z = \infty, \\ j &= \omega(t) \quad \text{at} \quad z = \infty, \end{aligned}\right\} \tag{7'}$$

$$\tau = \tau^* \qquad z = 0, \tag{8'}$$

$$b = \delta\varepsilon \qquad z = 0, \tag{9'}$$

$$-K\frac{\partial\tau}{\partial z} + K^*\frac{\partial\tau^*}{\partial z} - LD\rho\frac{\partial g}{\partial z} = a - b + (1-\Gamma)j. \tag{10'}$$

Finally, the natural boundary condition is that of the final decay of all the temperature variations at infinity:

$$\left.\begin{aligned} \tau &= 0 \quad \text{when} \quad z = \infty \\ \tau^* &= 0 \quad \text{when} \quad z = -\infty \end{aligned}\right\} \tag{22}$$

Shvets considers $\frac{\alpha\rho_\omega}{c_p\rho} = \varepsilon$ to be a very small constant quantity, and therefore, he looks for the solution of the system (1')-(6') in the form of the infinite power series of the parameter ε

$$\left.\begin{aligned} \tau &= \tau_0(z,t) + \varepsilon\cdot\tau_1(z,t) + \cdots \\ \tau^* &= \tau_0^*(z,t) + \varepsilon\cdot\tau_1^*(z,t) + \cdots \\ a &= a_0(z,t) + \varepsilon\cdot a_1(z,t) + \cdots \\ b &= b_0(z,t) + \varepsilon\cdot b_1(z,t) + \cdots \\ g &= g_0(z,t) + \varepsilon\cdot g_1(z,t) + \cdots \end{aligned}\right\} \tag{23}$$

Substituting the series (23) into the equation system (1')-(6') and into the boundary conditions, and equating the coefficients of the terms of equal powers of ε, we get the following system of equations defining τ_0, τ_0^* etc:

$$\frac{\partial\tau_0}{\partial t} = \frac{\partial}{\partial z}\lambda\frac{\partial\tau_0}{\partial z}, \tag{24}$$

$$\frac{\partial a_0}{\partial z} = 0, \tag{25}$$

$$\frac{\partial b_0}{\partial z} = 0, \tag{26}$$

$$\frac{\partial\tau_0^*}{\partial t} = \frac{\partial}{\partial z}\lambda^*\frac{\partial\tau_0^*}{\partial z}, \tag{27}$$

$$\frac{\partial g_0}{\partial t} = \frac{\partial}{\partial z}D\frac{\partial g}{\partial z}, \tag{28}$$

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and the following boundary conditions:

$$-K\frac{\partial T_0}{\partial z} + K^*\frac{\partial T_0^*}{\partial z} - L\rho D\frac{\partial g_0}{\partial z} = a_0 - b_0 + (1-r)j \quad \text{when } z=0 \tag{29}$$

$$\tau_0 = \tau_0^* \quad \text{when } z=0, \tag{30}$$

$$b_0 = \delta e_0 \quad \text{when } z=0, \tag{31}$$

$$g_0 = \nu\tau_0 \quad \text{when } z=0, \tag{32}$$

$$\tau_0 = a_0 = g_0 = 0 \quad \text{when } z=\infty \tag{33}$$

$$j = \omega(t) \quad \text{when } z=\infty \tag{34}$$

$$\tau_0^* = 0 \quad \text{when } z=-\infty \tag{35}$$

Similarly for τ_1, τ_1^*, a_1, etc., we get:

$$\frac{\partial \tau_1}{\partial t} = \frac{\partial}{\partial z}\lambda\frac{\partial \tau_1}{\partial z} - (a_0 + b_0 - \beta j_1 - 2fe_0), \tag{36}$$

$$\frac{\partial \tau_1^*}{\partial t} = \frac{\partial}{\partial z}\lambda^*\frac{\partial \tau_1^*}{\partial z}, \tag{37}$$

$$\frac{\partial a_1}{\partial z} = a_0 - fe_0, \tag{38}$$

$$\frac{\partial b_1}{\partial z} = fe_0 - b_0, \tag{39}$$

$$\frac{\partial g_1}{\partial z} = \frac{\partial}{\partial z}D\frac{\partial g_1}{\partial z}, \tag{40}$$

$$-K\frac{\partial \tau_1}{\partial z} + K^*\frac{\partial \tau_1^*}{\partial z} - L\rho D\frac{\partial g_1}{\partial z} = a_1 - b_1 \quad \text{at } z=0 \tag{41}$$

$$\tau_1 = \tau_1^* \quad \text{at } z=0 \tag{42}$$

$$b_1 = \delta\mu\tau_1 \quad \text{at } z=0 \tag{43}$$

$$g_1 = \nu\tau_1 \quad \text{at } z=0 \tag{44}$$

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$$\tau_1 = a_1 = q_1 = 0 \qquad \text{at} \quad z = \infty \qquad (45)$$

$$\tau_1^* = 0 \qquad \text{at} \quad z = -\infty \qquad (46)$$

The solution of equations (24), (27), and (28) is sought in the form of the following infinite series

$$\left.\begin{aligned} \tau_0 &= \sum_{-\infty}^{+\infty} \tau_{0n}(z) \cdot e^{-in\omega t}, \\ \tau_0^* &= \sum_{-\infty}^{+\infty} \tau_{0n}^*(z) \cdot e^{-in\omega t}, \\ q_0 &= \sum_{-\infty}^{+\infty} q_{0n}(z) \cdot e^{-in\omega t}, \end{aligned}\right\} \qquad (47)$$

Substitution of (47) into equations (24),(27) and (28) results in the following equations for the determination of τ_{0n}, τ_{0n}^*, q_{0n}

$$\frac{d}{dz} \lambda \frac{d\tau_{0n}}{dz} + in\omega\tau_{0n} = 0, \qquad (48)$$

$$\frac{d}{dz} \lambda^* \frac{d\tau_{0n}^*}{dz} + in\omega\tau_{0n}^* = 0, \qquad (49)$$

$$\frac{d}{dz} D \frac{dq_{0n}}{dz} + in\omega q_{0n} = 0, \qquad (50)$$

Integrating (25) with the boundary condition (33), we get:

$$a_0 = 0. \qquad (51)$$

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Noting that

$$e_0 \approx 4\sigma T^3 \cdot \tau_0 = \mu \cdot \tau_0 , \tag{52}$$

and upon integrating (26) with the boundary condition (31) we get:

$$b_0 = \delta\mu \sum_{-\infty}^{+\infty} \tau_{0n}(0)\, e^{-in\omega t} \tag{53}$$

The boundary conditions for τ_{0n}, τ^*_{0n} and q_{0n} become then:

$$\delta\mu\tau_{0n} + K^* \frac{\partial \tau^*_{0n}}{\partial z} - K\frac{\partial \tau_{0n}}{\partial z} - z D\rho \frac{\partial q_{0n}}{\partial z} = (1-r) j_n \quad \text{at } z = 0 \tag{54}$$

$$\tau_{0n} = \tau^*_{0n} \quad \text{at } z = 0 \tag{55}$$

$$q_{0n} = \nu \tau_{0n} \quad \text{at } z = 0 \tag{56}$$

$$\tau_{0n} = q_{0n} = 0 \quad \text{at } z = \infty \tag{57}$$

$$\tau^*_{0n} = 0 \quad \text{at } z = -\infty \tag{58}$$

Shvets considers the coefficient of soil temperature conductivity λ^* as being constant with depth, and he also considers the coefficient of the turbulent temperature conductivity λ to be equal to the coefficient of turbulent diffusion, and assumes it to vary with height according to the relationships:

$$\lambda = \lambda_0 + cz \qquad \text{when } z \leqslant h,$$

$$\lambda = \lambda_0 + ch \approx ch = \text{const.} \qquad \text{when } z > h,$$

where λ_0 is the coefficient of molecular temperature conductivity.

The solution of equation (49) which satisfies the boundary

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condition (58) is as follows:

$$\tau_{0n}^{*} = B_n \cdot e^{\sqrt{-\frac{in\omega}{\lambda^{*}}} \cdot z}, \qquad (59)$$

where B_n is an arbitrary constant of integration.

Changing variables we have:

$$\frac{z}{c}\sqrt{in\omega\lambda} = x; \quad \frac{dx}{dz} = \frac{z}{c} \cdot in\omega x^{-1} \qquad (60)$$

and the equations (48) and (50) are transformed into:

$$\frac{d}{dx} x \frac{d\tau_{0n}}{dx} + x\tau_{0n} = 0, \qquad (61)$$

$$\frac{d}{dx} x \frac{dq_{0n}}{dx} + x q_{0n} = 0, \qquad (62)$$

The solutions of these equations are given by the Bessel's and Neyman's functions:

$$\left.\begin{aligned} \tau_{0n} &= R_n \cdot I_0(x) + S_n \cdot N_0(x) = Z_0(x), \\ q_{0n} &= \tilde{R}_n \cdot I_0(x) + \tilde{S}_n \cdot N_0(x) = \tilde{Z}_0(x), \end{aligned}\right\} \text{at} \quad z \geqslant h, \qquad (63)$$

where R_n, S_n, $\tilde{R}_n$, $\tilde{S}_n$ are the arbitrary constants of integration; I_0 is Bessel's function of the zero order, N_0 is Neyman's function of the zero order, Z_0 is a cylindrical function.

At $z \geqslant h$ we have

$$\left.\begin{aligned} \tau_{0n} &= A_n \cdot e^{-\sqrt{-\frac{in\omega}{\lambda_0 + ch}} \cdot z}, \\ q_{0n} &= \tilde{A}_n \cdot e^{-\sqrt{-\frac{in\omega}{\lambda_0 + ch}} \cdot z}, \end{aligned}\right\} \qquad (64)$$

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where A_n and $\tilde{A}_n$ are the arbitrary constants of integration. Besides the boundary conditions (41)-(46), the obtained solutions must also satisfy the following conjugate conditions:

$$\left.\begin{aligned} \tau_{0n}(h-0) &= \tau_{0,n}(h+0) \\ q_{0n}(h-0) &= q_{0,n}(h+0) \\ \frac{\partial \tau_{0,n}(h-0)}{\partial z} &= \frac{\partial \tau_{0,n}(h+0)}{\partial z} \\ \frac{\partial q_{0,n}(h-0)}{\partial z} &= \frac{\partial q_{0,n}(h+0)}{\partial z} \end{aligned}\right\} \qquad (65)$$

Satisfying the boundary condition (55) we have:

$$B_n = R_n \cdot I_n(x_0) + S_n \cdot N_0(x_0) \qquad (66)$$

We eliminate further from the boundary condition (54) the temperature τ^* of the underlying surface. According to (59) we have

$$K^* \frac{\partial \tau_{0n}^*}{\partial z} = c^* \rho^* \sqrt{-in\omega\lambda^*} \cdot B_n = c^* \rho^* \sqrt{-in\omega\lambda^*} \cdot \tau_{0,n}(0), \qquad (67)$$

Noting that according to (32) we have

$$\tilde{R}_n = \nu R_n \; ; \quad \tilde{S}_n = \nu S_n \; ; \qquad (68)$$

and therefore

$$q_{0n} = \nu \tau_{0n} \qquad (69)$$

holds for the entire space. Consequently, the heat balance equation becomes:

$$-\lambda \rho (c_p + L\nu) \frac{d\tau_{0n}}{dz} + (\rho^* c^* \sqrt{-in\omega\lambda^*} + \delta\mu)\tau_{0n} = (1-\Pi) j_n \qquad (70)$$

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Using this latest expression (70) of the heat balance equation, Shvets estimates the importance of the various factors affecting the input and the output terms of the heat balance, for example:

$\varepsilon_{\mu} = 1.2 \times 10^{-4}$ cal/deg.cm^2 seconds; $\rho^* = 1.5$ gm/cm^3

$c^* = 0.33$ cal/gm. degree; $\lambda^* = 7 \times 10^{-3}$ cm^2/seconds

$c_p = 0.24$ cal/gm. degree; $N = 1$ $\nu z = 0.3$ cal/gm. degree

(This corresponds to the water vapor pressure of 10 millimeters). With these values of the parameters, the amount of heat used to heat the soil, is about three times as great as the amount of heat lost in radiation. The quantity of heat used up in evaporation is less, or about the same as the heat required to heat the air due to turbulent thermoconductivity. Furthermore, for still water $\lambda^* = 1.8 \times 10^{-2}$ square centimeters per second; consequently, for water $\rho^* c^* \sqrt{\lambda^*} = 0.135$ and for the soil it is 0.19. Therefore, the diurnal variation of the air temperature over perfectly still water is about the same as that over dry soil. However, the turbulent mixing which takes place in moving water radically changes the relationship between the components of the heat balance. The coefficient of the turbulent temperature conductivity of water is several thousand times as great as the molecular coefficient. Therefore, in the case of an ocean, the term $\rho^* c^* \sqrt{-im\omega\lambda^*}\,\tau_{on}$ in the equation (70) is hundreds of times greater than the other left-hand terms of this equation. This means that the solar radiation is predominantly used up to heat the ocean, and only a negligible portion of it is used to heat the air.

Neglecting the negligibly small terms of the ocean surface heat balance equation, this equation can be written as follows:

$$\tau_{on} = \frac{(1 - \Gamma) j_n}{\sqrt{-im\omega\lambda^*}} \qquad (71)$$

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Substituting the τ_{on} expressions from (63) into the heat balance equation (70) we make use of the following well-known relationship in the theory of cylindrical functions:

$$\frac{dZ_0}{dx} = -Z_1 \qquad (71\text{-a})$$

Then, after obvious transformations, the equation of the heat balance becomes:

$$(1-\Gamma)\, j_n = R_n X_n + S_n Y_n , \qquad (72)$$

where

$$\left.\begin{aligned} X_n &= \left[\delta_\mu + c^4 \rho^4 \sqrt{\frac{n\omega\lambda_0}{2}}\,(1-i)\right] I_0(x_0) + \\ &\quad + \rho(c_p + \nu L)\cdot\sqrt{\frac{n\omega\lambda_0}{2}}\,(1+i)\, I_1(x_0), \\ Y_n &= \left[\delta_\mu + c^4 \rho^4 \sqrt{\frac{n\omega\lambda_0}{2}}\,(1-i)\right] N_0(x_0) + \\ &\quad + \rho(c_p + \nu L)\cdot\sqrt{\frac{n\omega\lambda_0}{2}}\,(1+i)\, N_1(x_0), \end{aligned}\right\} \qquad (73)$$

The conjugate conditions (65) result in the following two equations:

$$\begin{aligned} A_n e^{-\sqrt{-\frac{in\omega}{\lambda_0+ch}}\cdot h} &= R_n I_0(x_h) + S_n N_0(x_h), \\ -\sqrt{-\frac{in\omega}{\lambda_0+ch}}\cdot A_n \cdot e^{-\sqrt{-\frac{in\omega}{\lambda_0+ch}}\cdot h} &= \\ = -\sqrt{-\frac{in\omega}{\lambda_0+ch}}\left[R_n I_1(x_h) + S_n N_1(x_h)\right] & \end{aligned} \qquad (74)$$

Eliminating A_n from (74) we get:

$$R_n\left[I_0(x_h) - i I_1(x_h)\right] + S_n\left[N_0(x_h) - i N_1(x_h)\right] = 0 \qquad (75)$$

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Equations (72) and (75) permit us to determine the arbitrary constants of integration

$$\left.\begin{aligned} \frac{P}{(1-P)_0} &= \frac{M_0(\alpha_0) - M_1(\alpha_0)}{[L_0(\alpha_0) - L_1(\alpha_0)]\cdot Y_0 - [M_0(\alpha_0) - M_1(\alpha_0)]X_0} \\ \frac{Q_0}{(1-P)_0} &= \frac{L_0(\alpha_0) - L_1(\alpha_0)}{[L_0(\alpha_0) - L_1(\alpha_0)]\cdot Y_0 - [M_0(\alpha_0) - M_1(\alpha_0)]X_0} \end{aligned}\right\} \tag{76}$$

and for $z > h$ we have

$$\tau_n = \tau_n(h)\, e^{-\sqrt{-\frac{in\omega}{\lambda_0 + \alpha h}}\,(z-h)} \tag{77}$$

We direct our attention now to the determination of functions τ, a, b, which depend on the ~~atmospheric radiant transfer~~. Substituting the obtained ~~expressions of~~ τ_0, a_0, b_0 into the equations (36)-(40) and into the boundary conditions (41)-(46), we get:

$$\frac{\partial \tau}{\partial t} = \frac{\partial}{\partial z}\lambda \frac{\partial \tau}{\partial z} + \sum_{-\infty}^{+\infty} e^{-in\omega t}\left[\delta\mu\tau_{0n}(0) + \beta J_n(0) - 2\mu\tau_{0n}(z)\right] \tag{78}$$

$$\frac{\partial a_1}{\partial z} = -c_p \rho \delta \mu \sum_{-\infty}^{+\infty} \tau_{0n}(z)\, e^{-in\omega t}, \tag{79}$$

$$\frac{\partial b_1}{\partial z} = c_p \rho \mu \delta \sum_{-\infty}^{+\infty} \left[\tau_{0n}(z) - \tau_{0n}(0)\right] e^{-in\omega t}, \tag{80}$$

$$\frac{\partial \tau^*}{\partial t} = \frac{\partial}{\partial z}\lambda^* \frac{\partial \tau^*}{\partial z}, \tag{81}$$

$$\frac{\partial q_1}{\partial t} = \frac{\partial}{\partial z}\lambda \frac{\partial q_1}{\partial z}, \tag{82}$$

$$-K\frac{\partial \tau}{\partial z} + K^*\frac{\partial \tau^*}{\partial z} - L\rho\lambda\frac{\partial q_1}{\partial z} = a_1 - b_1 \tag{83}$$

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$$\left.\begin{aligned} p_1 &= \delta\mu\tau_1 , && (84)\\ \tau_1 &= \tau_1^* , && (85)\\ q_1 &= \nu\tau_1 , && (86)\end{aligned}\right\} \text{ for } Z = 0$$

$$\tau_1 = a_1 = q_1 = 0 \quad \text{when } Z = \infty, \qquad (87)$$

$$\tau^* = 0 \quad \text{when } Z = -\infty, \qquad (88)$$

Integrating (79) with the boundary condition (87), we get:

$$a_1 = -c_p \rho f_\mu \sum_{-\infty}^{+\infty} e^{-in\omega t} \int_{\infty}^{z} \tau_{0n}\, dz \qquad (88\text{-a})$$

But, according to (48) we have:

$$\int_{-\infty}^{z} \tau_{0n}\, dz = -\frac{1}{in\omega}\int_{\infty}^{z} \frac{\partial}{\partial z}\lambda \frac{\partial \tau_{0n}}{\partial z}\, dz = -\frac{\lambda}{in\omega}\frac{\partial \tau_{0n}}{\partial z} \qquad (88\text{-b})$$

Consequently, we get

$$a_1 = \frac{K f_\mu}{i\omega} \sum_{-\infty}^{+\infty} \frac{1}{n} \frac{\partial \tau_{0n}}{\partial z} \cdot e^{-in\omega t} \qquad (89)$$

Writing the boundary condition (83) as follows:

$$-K\frac{\partial \tau_1}{\partial z} + K^*\frac{\partial \tau^*}{\partial z} - L\rho\lambda\frac{\partial \tau_1}{\partial z} + \delta\mu\tau_1 = \frac{K f_\mu}{i\omega}\sum \frac{1}{n}\frac{\partial \tau_{0n}}{\partial z} \cdot e^{-in\omega t} \qquad (90)$$

we proceed to seek the solution of equations (78), (81), and (82) in the form of the infinite series

$$\left.\begin{aligned} \tau_1 &= \sum_{-\infty}^{+\infty} \tau_{1n}(z) \cdot e^{-in\omega t}\\ \tau_1^* &= \sum_{-\infty}^{+\infty} \tau_{1n}^*(z) \cdot e^{-in\omega t}\\ q_1 &= \sum_{-\infty}^{+\infty} q_{1n}(z) \cdot e^{-in\omega t}\end{aligned}\right\} \qquad (91)$$

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To determine τ_{1n}, τ^*_{1n}, and q_{1n} we get the following equations:

$$\frac{d}{dz}\lambda\frac{d\tau_{1n}}{dz} - in\omega\tau_{1n} = \delta\mu\tau_{0n}(0) + \beta J_n(0) - 2\mu f\tau_{0n}(z), \qquad (92)$$

$$\frac{d}{dz}\lambda^*\frac{d\tau^*_{1n}}{dz} + in\omega\tau^*_{1n} = 0 \qquad (93)$$

$$\frac{d}{dz}\lambda\frac{dq_{1n}}{dz} + in\omega q_{1n} = 0 \qquad (94)$$

$$-K\frac{\partial\tau_{1n}}{\partial z} + K^*\frac{\partial\tau^*_{1n}}{\partial z} + \delta\mu\tau_{1n} - L\rho\lambda\frac{\partial q_{1n}}{\partial z} = \frac{K f\mu}{in\omega}\frac{\partial\tau_{0n}}{\partial z} \quad z=0 \qquad (95)$$

$$\tau_{1n} = \tau^*_{1n} \qquad (96)$$

$$q_{1n} = \nu\tau_{1n} \qquad (97)$$

$$\tau_{1n} = q_{1n} = 0 \quad \text{when } z = \infty \qquad (98)$$

$$\tau^*_{1n} = 0 \quad \text{when } z = -\infty \qquad (99)$$

Changing variables in equation (92) by means of:

$$\frac{z}{c}\sqrt{in\omega\lambda} = x$$

~~Then~~ this equation becomes:

$$\frac{d}{dx}x\frac{d\tau_{1n}}{dx} + x\tau_{1n} = x\left[\delta\mu\tau_{0n}(x_0) + \beta J_n(x_0) - 2\mu f\tau_{1n}(x_0)\right]\frac{1}{in\omega} \qquad (100)$$

The particular solution of the non-homogeneous equation (100) is as follows

$$\tau_{1n} = \frac{1}{in\omega}\left[\delta\mu\tau_{0n}(x_0) + \beta J_n(x_0) + \mu f x\frac{\partial\tau_{0n}}{\partial x}\right] \qquad (101)$$

which can be verified by substitution. The general solution of equation (100) for $z < h$ has the form of

$$\tau_{1n} = R_{1n}I_0(x) + S_{1n}N_0(x) + \frac{1}{in\omega}\left[\delta\mu\tau_{0n}(x_0) + \beta J_n(x_0) + \mu f x\frac{d\tau_{0n}}{dx}\right] \qquad (102)$$

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and for $z > h$ it becomes

$$\tau_{1n} = A_{1n} e^{-\sqrt{-\frac{i n \omega}{\lambda_0 + \epsilon h}}\, z + \frac{1}{i n \omega}\left[\delta\mu\tau_{0n}(x_0) + \beta J_n(0) + \mu f z \frac{\partial \tau_{0n}}{\partial z}\right]} \qquad (103)$$

where A_{1n}, R_{1n}, S_{1n} are arbitrary constants. Equations (102) and (103) can be simplified by neglecting the right-hand ~~side~~ terms $\mu f x \frac{\partial \tau_n}{\partial x}$ and $\mu f z \frac{\partial \tau_{1n}}{\partial z}$, which are hundreds of times smaller than the other terms. To determine the arbitrary constants, account must be taken of the conjugate conditions of the solutions (102) and (103) at $z = h$, besides the boundary conditions. This condition, as already shown, results in the relationship

$$R_{1n}(I_0 - iI_1) + S_{1n}[N_0 - iN_1] = 0 \qquad (103\text{-a})$$

Shvets pointed out that the coefficients R_{1n}, S_{1n} are small compared to R_n and S_n. Therefore, we can neglect the terms containing R_{1n} and S_{1n}, and for the case of $z < h$, the solution becomes:

$$\tau_n = R_n\left[I_0(x) - i\epsilon\frac{\delta\mu}{\omega n} I_0(x_0)\right] + S_n\left[N_0(x) - i\epsilon\frac{\delta\mu}{\omega n} N_0(x_0)\right] - \frac{i\epsilon\beta}{\omega n} J_n(0), \qquad (104)$$

and for $z > h$ it becomes

$$\tau_n = \tau_n(h) e^{-\sqrt{-\frac{i n \omega}{\lambda_0 + \epsilon h}} - \frac{i\epsilon}{\omega n}\left[\delta\mu\tau_{0n}(0) + \beta J_n(0)\right]} \qquad (105)$$

Expressions (104) and (105) show that ~~thermal air emission~~ heat radiation of the air has a greater effect on the diurnal temperature variation in summer, when both the humidity ϵ and ~~the~~ solar radiation are ~~greater~~ high.

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Shvets' solution of the system of equations (105) facilitates solving the problem of diurnal variation ~~in~~ of effective ~~emission~~. Finally, Shvets was successful in formulating a theoretical explanation of the peculiar diurnal temperature variation over the ocean, ~~as is evidenced by~~ i.e., the increase of the amplitude of variation with height, whereas this amplitude decreases with height over land. This peculiarity is explainable by the effect of the radiant heat ~~transfer~~.

It was shown above, that over land, the effect of the radiant heat transfer on the diurnal air temperature variation is less than that of the turbulent heat transfer. Over the ocean the effects of these two processes are approximately equal.

To satisfy the conditions of conjugacy at $z = h$ as well as the conditions of equality of water and air temperatures at $z = 0$, we get the condition (57) and

$$\left(1-\varepsilon i\frac{\xi\mu}{\omega n}\right)R_n I_0(x_0)+S_n N_0(x_0)-\varepsilon\frac{i\beta m}{\omega n}=\frac{(1-\Gamma)j_m}{\sqrt{2\lambda^{*}n\omega}}(1+i). \tag{106}$$

whence we get:

$$\frac{R_m}{j_m}=\frac{\ell_m}{1-i\varepsilon\frac{\xi\mu}{\omega n}}\left\{\frac{1-\Gamma}{\sqrt{2\lambda^{*}n\omega}}(1+i)+\frac{\varepsilon\cdot i\beta}{\omega n}\right\}, \tag{107}$$

$$\frac{S_m}{j_m}=\frac{m_m}{1-i\varepsilon\frac{\xi\mu}{\omega n}}\left\{\frac{1-\Gamma}{\sqrt{2\lambda^{*}n\omega}}(1+i)+\frac{\varepsilon i\beta}{\omega n}\right\}, \tag{108}$$

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where

$$j = \frac{N_0(x_h) - iN_1(x_h)}{[I_0(x_h) - iI_1(x_h)]\,N_0(x_0) - [N_0(x_h) - iN_1(x_h)]\,I_0(x_0)} \tag{109}$$

$$m_m = \frac{I_0(x_h) - iI_1(x_h)}{[I_0(x_h) - iI_1(x_h)]\cdot N_0(x_0) - [N_0(x_h) - iN_1(x_h)]\cdot I_0(x_0)}$$

Shvets cites the following example: $j_1 = 7.3 \times 10^{-3}$; $j_2 = 3.7 \times 10^{-3}$ (which corresponds to 50 degree latitude, the month of March, and the coefficient of transparency 0.7); $\lambda^* = 200$; $\delta\mu = 1.2 \cdot 10^{-4}$; $\Gamma = 0.1$; $\beta = 0.34$; $\sigma = 2$; $c = 14$; $h = 4000$ centimeters. Then we have

$$T = 0.04 \cos \omega\tau + 0.04 \sin \omega\tau + 0.014 \cos 2\omega\tau + 0.014 \sin 2\omega\tau \tag{110}$$

and at the height of 20 meters:

$$T = -0.32 \sin \omega\tau - 0.08 \sin 2\omega\tau \tag{111}$$

In summarizing the results of his very thorough analysis of the diurnal variation of the air temperature, Shvets arrives at the following important conclusions.

The diurnal variation of the air temperature depends on numerous physical parameters and, ~~before all also~~, on the parameters which characterize the physical properties of the underlying surface. Shvets cites a number of tables which show conclusively the effect of this parameter on the separate harmonics ~~of~~ into the diurnal air temperature ~~variation~~.

The expenditure of heat in the evaporation of moisture from the soil surface has an appreciable effect on the heat balance. However, the effect of this factor is not easily accounted for, since the factor depends on the moistness and structure of the soil which vary quite irregularly with time and space.

Shvets' calculations indicated that the effect of this factor frequently does not have to be taken into account. Some justification of this conclusion Shvets sees in the mutual compensation between the amounts of heat expended for evaporation from the surface of ~~the moist~~ soils having different

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soil, and that corresponding to the ground reflection factor. For example, a very moist soil loses more heat than the dry soil. However, the ground reflection factor of the moist soil is considerably less than that of the dry soil. Therefore, the greater expenditure of heat for evaporation is accompanied by a greater input of heat by emission and vice versa.

Shvets' theory also discloses the effect of thermal air emission on the diurnal temperature variation. He gives detailed tables for the respective harmonics, which when added, result in a correction of the diurnal temperature variation so as to account for air emission. The effect of air emission increases as the air moisture and the solar heat input become greater.

Shvets' theoretical results permit us to determine the diurnal temperature variation at different heights. Figure 161 gives several comparisons between the observed and computed graphs of the diurnal temperature variation.

It should be emphasized that such a remarkable agreement between theory and practice was obtained by varying only the two parameters $\frac{K^*}{\sqrt{X^*}}$ and the ground reflection factor. The transfer coefficient at the height of 40 meters was taken to be equal to 60 grams per centimeter second.

However, the determination of the diurnal temperature variation is complicated by the irregular variations in cloudiness, which completely distorts the shape of the temperature curve. Therefore, the prognosis of the diurnal temperature variation should be preceded by a cloudiness prognosis.

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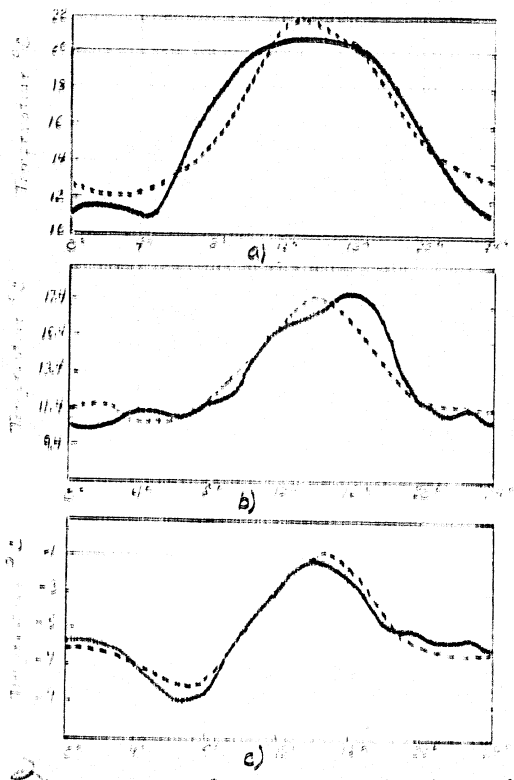

(Figure 161. Comparison of the Observed (Solid Line) and the Computed, according to Shvets, (dotted line) Diurnal Temperature Variation, at the Observatory "Vysokaya Dubrava" (N.L. 56); (a) 9 August 1941, (b) 15 September 1941 (c) 16 October 1941.

VI. ~~THE~~ TRANSFORMATION OF ~~THE~~ AIR MASS AS INFLUENCED BY THE UNDERLYING SURFACE

One of the most difficult and important problems of comtemporary meteorology is the problem of the transformation of ~~the~~ air-mass. The essence of this problem is as follows:

According to the modern meteorological theories, the air which exists for a period of time over the same underlying surface, gradually acquires certain properties, characteristic of the given underlying surface, geographical latitude and the season of the year. Of all these characteristic properties, the vertical temperature distribution is the most important one. If the air-mass stays over a given underlying sur-

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face for a considerable period of time, and is only moving slowly, then the process of transformation, or the process of acquiring new properties by the air mass is susceptible of a quantitative estimate. The first important contribution to theory of the air mass transformation is represented by the works of Kibel and Malkin.

Here we shall outline Kibel's work.

Just as in the investigation of temperature-versus-height distribution, Kibel starts with the heat ~~input~~ equation:

$$c_p \rho \frac{dT}{dt} - \frac{dp}{dt} = \frac{\partial}{\partial z}\left(K \frac{\partial T}{\partial z}\right) + \alpha \rho_w (A + B + \beta S - 2 f E). \quad (1)$$

In the problem of the vertical temperature distribution Kibel takes the left hand side of equation (1) to be equal to zero. In the problem of the air mass transformation, the term $c_p \rho \frac{\partial T}{\partial t}$ must obviously remain in the lefthand side of this equation. The equations of the problem then become:

$$c_p \rho \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(K \frac{\partial T}{\partial z}\right) + \alpha \rho_w (A + B + \beta S - 2 f E) \quad (2)$$

$$E = \sigma T^4$$

$$\frac{\partial A}{\partial z} = \alpha \rho_w (A - f E) \quad (3)$$

$$\frac{\partial B}{\partial z} = \alpha \rho_w (f E - B) \quad (4)$$

$$\frac{\partial S}{\partial z} = -\alpha \beta \rho_w S \quad (5)$$

and the boundary conditions are

$$-K \frac{\partial T}{\partial z} + K^* \frac{\partial T^*}{\partial z} = A + S - B;$$

$$B = q f E \quad \text{at } z = 0 \quad (6)$$

$$A = 0,$$

$$B = W,$$

$$S = W \text{ or } \frac{\partial T}{\partial t} = 0 \quad \text{as } z = \infty \quad (7)$$

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Besides, in the case of the soil temperature T^* (the new unknown function) the following equation holds:

$$c^* \rho^* \frac{\partial T^*}{\partial t} = K^* \frac{\partial^2 T^*}{\partial z^2} \tag{8}$$

where C^* is the specific heat of the soil, ρ^*, the soil density, K^* is coefficient of the soil thermoconductivity. The boundary conditions for T^* are:

$$\begin{aligned} &T^* = T \quad \text{for } z = 0 \\ &T^* \text{ is bounded for } z \to -\infty \end{aligned} \tag{9}$$

Instead of the height z we introduce, according to Kibel a new independent variable, the ~~derived~~ optical thickness χ:

$$\chi = \frac{1}{\tau_0} \int_z^\infty a \rho_w \, dz \tag{10}$$

where

$$\tau_0 = \int_0^\infty a \rho_w \, dz \tag{11}$$

and for t we introduce the non-dimensional time S:

$$S = \omega t \tag{12}$$

where $1/w$ is the characteristic time.

Then the equation (2) becomes:

$$\frac{c \rho \omega}{a \rho_w} \frac{\partial T}{\partial S} = \frac{1}{\tau_0^2} \frac{\partial}{\partial \chi} \left(K a \rho_w \frac{\partial T}{\partial \chi} \right) + \left(A + B + \beta S - 2 f E \right) \tag{13}$$

Noting that

$$\frac{\partial E}{\partial \chi} = 4 \sigma T^3 \frac{\partial T}{\partial \chi} \tag{13-a}$$

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and ~~going~~ changing from T to E we get:

$$\frac{c_p \rho_w}{4a\rho_w f T^3}\frac{\partial E}{\partial s} = \frac{1}{\tau_0^2}\frac{\partial}{\partial x}\left(\frac{Ka\rho_w}{4f\sigma T^3}\frac{\partial E}{\partial x}\right) + A + B + \beta S - 2fE \qquad (14)$$

Kikel considers the coefficients of the equation as constant and assumes that:

$$\frac{4f\sigma T^3}{Ka\rho_w} = \frac{m^2-1}{2} \; ; \; \frac{c_p \rho_w}{4a\rho_w f\sigma T^3} = N \qquad (15)$$

Then, equation (14) becomes:

$$N\frac{\partial E}{\partial s} = \frac{1}{\tau_0^2}\frac{\partial}{\partial x}\left(\frac{2}{m^2-1}\frac{\partial E}{\partial x}\right) + A + B + \beta S - 2fE \qquad (16)$$

Introducing the new variable functions a and y, defined by the equalities:

$$a = A + B; \quad y = B - A \qquad (17)$$

We solve (16) for a to get:

$$a = \frac{1}{\tau_0^2}\frac{\partial}{\partial x}\left(\frac{2}{m^2-1}\frac{\partial E}{\partial x}\right) + N\frac{\partial E}{\partial s} - \beta S + 2fE \qquad (18)$$

Noting that according to (4), $\frac{1}{\tau_0}\frac{da}{dx} = y$, and differentiating (18) with respect to x, we get, by means of (15):

$$y = \frac{1}{\tau_0^3}\frac{\partial^2}{\partial x^2}\left(\frac{2}{m^2-1}\frac{\partial E}{\partial x}\right) + \frac{2}{\tau_0}\frac{\partial E}{\partial x} + \frac{1}{\tau_0}\frac{\partial}{\partial x}\left(N\frac{\partial E}{\partial s}\right) + \beta^2 S \qquad (19)$$

Furthermore, we note that

$$\frac{1}{\tau_0}\frac{dy}{dx} = a - 2fE \qquad (20)$$

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Differentiating (19) with respect to X and taking into account (18), we get:

$$\frac{1}{\tau_0^2}\frac{\partial^2}{\partial x^2}\left(\frac{2}{m^2-1}\frac{\partial E}{\partial x}\right)-\frac{1}{\tau_0^2}\frac{\partial}{\partial x}\left(\frac{2}{m^2-1}\frac{\partial E}{\partial x}\right)-\frac{a}{\tau_0^2}\frac{\partial^2 E}{\partial x^2}-$$

$$-\frac{1}{\tau_0^2}\frac{\partial^2}{\partial x^2}\left(N\frac{\partial E}{\partial s}\right)+N\frac{\partial E}{\partial s}=\beta(1-\beta^2)s \qquad (21)$$

The boundary conditions then become:

$$y+\frac{2}{m^2-1}\frac{1}{\tau_0}\frac{\partial E}{\partial x}+K^*\frac{\partial T^*}{\partial z}=S \quad \text{at } x=1$$

$$a+y=2gfE \qquad (22)$$

$$a-y=0$$

$$a+y=2W \text{ or } \frac{\partial E}{\partial s}=0 \quad \text{at } x=0 \qquad (23)$$

Changing ~~Going~~ to numerical values, Kibel takes: $T = 280$ degrees $a = 7.25$ square centimeters per gram, $\rho\omega = 6.2 \times 10^{-6}$ grams per cubic centimeter, $\rho = 1.3 \times 10^{-3}$ grams per cubic centimeter. Then

$$N = 10^5\omega$$

If the characteristic time is taken as a 24 hour period $\left(\frac{1}{\omega}\approx 10^5 \text{ seconds}\right)$ ~~then~~ N is of the first order. Noting that $m^2\approx 3$, $\tau = 10$ Kibel, in estimating the order of the terms of ~~the~~ equation (21), arrives at the following highly important result: away from the boundaries ($X=0, X=1$) equation (21) can be replaced by a more simple approximate equation

$$N\frac{\partial E}{\partial s}\approx\beta(1-\beta^2)s \qquad (24)$$

and, only close to the boundaries, is it necessary to deal with the complete equation (21).

However, this conclusion is only justified in the case where the

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characteristic time is of the order of 24 hours. If the characteristic time is longer, the order of magnitude of the coefficients of the first few terms of equation (21) and the order of magnitude of the quantities N become equal. Then, equation (21) cannot be ~~superseded~~ replaced by ~~the~~ equation (24).

Kibel points out that in this case the boundary layer "swells up" so to speak. This is confirmed by the fact that ~~the~~ annual temperature variations ~~take place at a~~ extend to considerably ~~higher~~ greater height elevation than the diurnal. Temperature variations extend to great heights and thus express the fact of ~~the~~ air mass transformation.

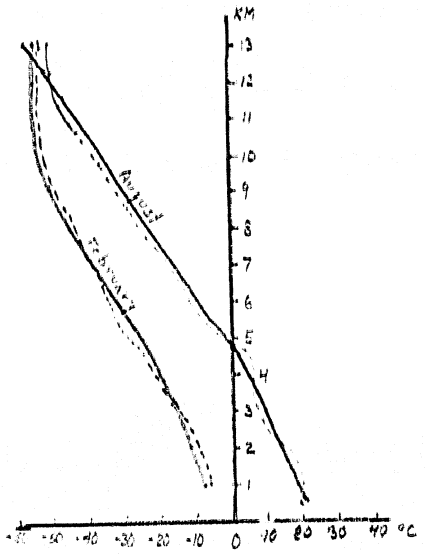

Figure 162. The Actual (Solid Line) ~~and Kibel Computed~~ Temperature versus Height Variation and That Computed by Kibel (Dotted Line)

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Kibel limited himself to the analysis of the annual temperature variation.

Since the equation (21) and boundary conditions are linear, ~~then~~ the solution (21) can be sought in the form of an infinite trigometric series such as:

$$E = E_0(x) + \sum_1^{\infty} \left[E_{1m}(x) \cos ms + E_{2m}(x) \sin ms \right] \qquad (25)$$

where E corresponds to the mean-annual temperature distribution with height.

Solar radiation arriving at the upper atmospheric boundary can also be taken in the form of an infinite series:

$$W = W_0 + \sum_1^{\infty} \left(W_{1m} \cos ms + W_{2m} \sin ms \right) \qquad (26)$$

Substituting (25) and (26) into (21) and equating the coefficients of ~~the equal power~~ terms (of equal degree), Kibel obtained the following approximate solution of equation (21) which satisfies the boundary conditions (22) and (23):

$$E_m(x) \approx \left\{ \frac{m-\beta}{1-\beta} \frac{1}{m} \frac{ch[\lambda_2(1-x)+\xi]}{ch(\lambda_2+\xi)} - \frac{m-1}{m(1-\beta)} \cdot \beta \cdot e^{-\lambda_1 x} - e^{-\beta \tau_0 x} \right\} \frac{1-\beta^2}{2\beta} \frac{m^2-1}{m^2-\beta^2} - \frac{W_m}{1 - \frac{1-\beta^2}{m^2-\beta^2} \frac{m^2-1}{2\beta^2} m N i} \qquad (27)$$

where

$$E_m = E_{1m} - i E_{2m}$$
$$W_m = W_{1m} - i W_{2m}$$
$$\lambda \approx m \tau_0$$
$$\lambda_2 \approx \sqrt{\frac{m^2-1}{2} \frac{N m i}{m^2}} \qquad (28)$$
$$\xi = \frac{1}{2} \ln \frac{1+M}{1-M}$$
$$M = \sqrt{\frac{m^2-1}{2m^2 N} \frac{\sqrt{\lambda^2 \omega}}{4 k \sigma T^{*3}}}$$

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For a numerical example Kibel takes the following parameters: $M = 1.75$, $\tau_0 = 12.6$; $\beta = 0.2$. Then $M \approx 0.45$; $N \approx 0.024$; $\lambda_1 = 22$; $\lambda_2 = 0.8\ (1 + i)$ $k = 0.3$. The W_m value Kibel took from Milankovich's ~~book~~ (atlas) for the 40 degree latitude assuming the ground reflection factor to be equal to 0.6.

Kibel determined that $W_1 = 0.07 \pm i$; $W_2 = W_3 = 0$.

He also computed the temperature distribution for February and August. Figure 162 shows the observed results and the results of Kibel's calculations, which show a very good agreement.

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