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Application of a Laminar Medium For the Focussing of Waves

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"APPLICATION OF A LAMINAR MEDIUM FOR

THE FOCUSSING OF WAVES"

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[Note: the following report appeared in the regular technical physics section of the thrice-monthly journal Doklady Akademii Nauk SSSR, Volume 81, No. 4 (1 December 1951), pages 569-571. It had been submitted by academician B.A. Vvedenskiy, 3 November 1951.]

Let there be a medium with continuously decreasing, in the y-direction, index of refraction n (y) (see figure 1). A linear source of electromagnetic oscillations which coincides with the OZ-axis radiates cylindrical waves. It is required to select such a law of variation of the index of refraction n(y) that all "rays" emitted by this source should arrive at the plane $x = x_0$ after one and the same interval of time. This means that a wave having a cylindrical front close to the source by being propagated in such a medium is gradually deformed and arrives at the plane $x = x_0$ with a plane front. If at the right of the plane $x = x_0$ the medium is homogenous, then a portion from the straight line OZ to the indicated plane focusses parallel rays which are incident upon the plane $x = x_0$, in a line which coincides with the OZ axis. If we have $n = n(y)$ everywhere, then we obtain a picture of the propagation of rays like that indicated in figure 2.

Assuming a medium and a linear source of infinite extent along the OZ-axis, we are led to a plane (two-dimensional) problem, which will be discussed below. Therefore, in what follows we will speak of a point radiator and a straight line $x = x_0$, keeping in mind here that all results will also be applicable to a linear radiator and plane $x = x_0$.

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The equation of the family of trajectories of rays $y(x,z)$, where z is a point on the straight line $x = x_0$, through which point passes a ray of the family, is determined from Fermat's condition:

$$\int_0^{x_0} \frac{\sqrt{1 + y'^2}}{v(y)} \cdot dy = \text{minimum}, \qquad (1)$$

where $v(y) = c_0 / n(y)$ is the velocity of propagation in the medium.

It is easy to show that Euler's equation for this integral possesses the following form:

$$\frac{1}{v(y)} \cdot \frac{\partial}{\partial y} v(z) = \frac{\frac{\partial^2 x}{\partial y^2}}{\frac{\partial x}{\partial y} \left[1 + \left(\frac{\partial x}{\partial y} \right)^2 \right]}, \qquad (2)$$

where $x = x(y,z)$ is the equation of the family of trajectories.

Integrating it with respect to y, we obtain:

$$v(y) = \frac{\varphi(z) \frac{\partial x}{\partial y}}{\sqrt{1 + \left(\frac{\partial x}{\partial y} \right)^2}}. \qquad (3)$$

Taking into consideration that all rays must be parallel during intersecting of the straight line $x = x_0$, that is,

$$\left. \frac{\partial x}{\partial y} \right|_{y = z} = \infty,$$

we find $\varphi(z) = v(z)$. (4)

Integrating once more, we obtain

$$x = \int_0^y \frac{dy}{\sqrt{\left[\frac{v(z)}{v(y)} \right]^2 - 1}}. \qquad (5)$$

Setting $x_0 = 1$, which does not vitiate the generality, and taking into consideration the boundry condition at the straight line $x = 1$, we obtain the following integral equation for $v(y)$:

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$$\int_0^z \frac{dy}{\sqrt{\left[\frac{v(z)}{v(y)}\right]^2 - 1}} = 1 . \qquad (6)$$

General methods for solving such an integral equation are unknown

To employ differentiation with respect to the upper limit here is impossible, since the integrand function possesses a singularity at the point $y = z$. There remains a way of selecting the function $v(y)$ which satisfies equation (6). Here we can proceed from a consideration of a function which would give, during a double substitution of the limits, the necessary unity, and afterwards, by equating the derivative of this function with respect to y to the integrand, we can obtain the equation for $v(y)$. After rather long calculations we found, by such a method, that the solution of the integral equation (6) is the following function:

$$v(y) = v_0 \cdot \cosh \frac{\pi}{2} y, \qquad (7)$$

and thus we have solved the problem formulated above

The curve showing the dependence of $n = c_0/v$ upon y is given in figure 3.

The equation of the family of trajectories of rays is determined from equation (5), along with equation (7). We finally obtained:

$$y(x,z) = \frac{2}{\pi} \cdot \mathrm{Ar\,sh}\left[\mathrm{sh}\,\frac{\pi}{2} z \cdot \sin \frac{\pi}{2} x\right]$$

$$\left[y(x,z) = \frac{2}{\pi} \operatorname{arc}\left[\sinh \frac{\pi}{2} z \cdot \sin \frac{\pi}{2} x\right]\right] \qquad (8)$$

We note in conclusion that we can immediately transfer the results of the plane problem solved above to the case of the body (volumetric, three-dimensional) problem.

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At the origin O of the cylindrical system of coordinates is placed a source of spherical waves. If the indexes of refraction varies according to the following law:

$$n(r, \phi, z) = \frac{n_0}{\cosh \frac{\pi}{2} r}, \qquad (8)$$

then in the plane $z = 1$ the phase will be constant; that is, a spherical wave emitted by the source O, in being propagated in such a laminar medium will gradually changed its front: and, arriving at the plane $z = 1$, will be converted into a plane wave.

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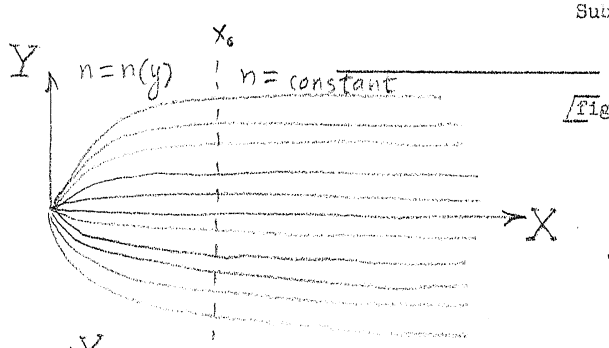

/figures 1,2,3, follow/

Figure 1.

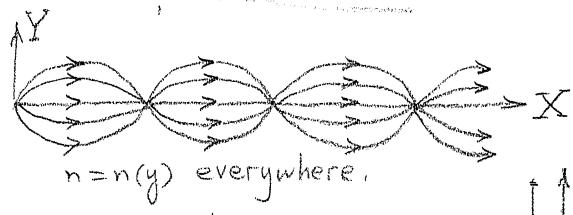

Figure 2.

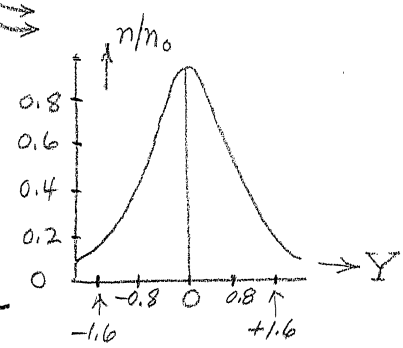

Figure 3

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