"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu. V.

"On the Possibility of a Unique Method in Certain Problems of "Additive" and "Distributive" Prime Number Theory," Dok.An, 49, No.1, 1945. -1945-.

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, YU.V.

Linnik, U. V. A new proof of the Goldbach-Vinogradow theorem. Rec. Math. [Mat. Sbornik] N.S. 19(61), 3–8 (1946). (Russian. English summary)

The author gives a new proof of Vinogradov's theorem that every large odd integer can be expressed as a sum of three odd primes. The author's previous proof of the theorem [C. R. (Doklady) Acad. Sci. URSS (N.S.) 49, 3–7 (1945); these Rev. 7, 507] was based on deep results about the zeros of L-series in the critical strip, involving an application of the Eratosthenes-Brun method. The present paper derives Vinogradov's theorem by classical arguments from the author's theorem [Bull. Acad. Sci. URSS. Sér. Math. [Izvestia Akad. Nauk SSSR] 10, 35–46 (1946); these Rev. 8, 11]: if χ is an Abelian character mod q, $L(s,\chi)$ the corresponding Dirichlet L-series, $\frac{1}{2}\leqq\beta<1$, $T\geqq q^{50}$, then $L(s,\chi)$ has in the rectangle $\beta\leqq\sigma\leqq 1$, $|t|\leqq T$, at most

$$O(q^{2\beta-1}T^{10(1-\beta)/(3-2\beta)}\log^{10}T+q^{20})$$

zeros. This theorem is a straightforward generalization of Titchmarsh's theorem on the number of zeros of $\zeta(s)$ [Proc. London Math. Soc. (2) 30, 319–321 (1929)].

H. A. Heilbronn (Bristol).

Source: Mathematical Reviews, Vol 8 No. 4

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

the approximation to the
independent variables.
[Izvestia Akad. Nauk
English summary)
for the error term in the
mutually independent
vanishing expectations,
absolute moments α_j. Let
Liapunov ratio is then
are that the σ_j^2 are
and $L_n \to 0$. Moreover,
of x_j truncated at the
tend to 1 uniformly in ϵ.
to 0 uniformly in $j \leq n$,
$F_n(x)$ is the distribution
normalized Gaussian dis-
interval $|x| < M$
$e^{-x^2/2}(1+\epsilon_n)$
the best since for the
is possible to exhibit a
the inequality in (1) is
shown that under the
a sequence of equidis-
the left hand in (1) is
to unsymmetric vari-
negative part have same
to σ_j^2. The proof is based
uses a technique said to
in additive number
trigonometrical sums.
Feller (Ithaca, N. Y.).

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, YU. V.

V., and Rényi, A. A. On certain hypotheses in
of Dirichlet characters. Izvestiya Akad. Nauk
Mat. 11, 539–546 (1947). (Russian)
and author's name appears as Rényi in non-
lications.] The authors prove the following
$<\eta\leqq 1$, $0\leqq\vartheta\leqq 1$, $k>1$, $x>1$, $a(m)$ is a multi-
thmetical function, $|a(m)|\leqq 1$, and if $f(y)$ is a
th period 1, then

$$|a(n)f(n\vartheta)|\leqq c\max\left|\sum_{m=1}^{n} m^{-1}a(m)f(m\vartheta)\right|,$$

constant depending only on k, where the sum
is taken over all $n\leqq\eta x$ whose prime divisors do
$x^{1/k}$, and where the max is taken over the range
$\leqq n\leqq x$. With the help of this lemma, the authors
if $\chi(n)$ is a nonprincipal Dirichlet character
if $\chi(n)=1$ for $1\leqq n\leqq x^{1/k}$, then

$$\sum_{n=1}^{n}\chi(n)=o(D^{\frac{1}{2}}\log D)$$

to infinity uniformly in x. If $\chi(n)$ is real and
for $1\leqq n\leqq x^{1/k}$, it is also proved that (*) holds
Davenport's theorem

$$\sum_{n=1}^{n}\mu(n)e^{2\pi i n\theta}=O(x\log^{-k}x)$$

θ for fixed $k>0$ [Quart. J. Math., Oxford Ser.
(1937)]. *H. Heilbronn* (Bristol).

LINNIK, YU. V.

USSR/Mathematics
Statistics

Mar 1947

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"The Accuracy of the Approximation of the Gaussian Distribution by Sums of Independent Random Variables," U. V. Linnik, 3 pp

"CR Acad Sci" Vol LV, No 7, 1947.

More precise results are claimed for the solution proposed, with the variables subject to more restrictive conditions than in the work of Berry and Eseon, plus an explanation of the Bernoullian scheme.

8T44

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

"On Expressing the L-Series by the Zeta Function,"
Yu. V. Linnik, 3 pp

"Dok Akad Nauk SSSR, Nova Ser" Vol LVII, No 5

Short discussion of simplest substitutions and transformations, as of Mellin, necessary in subject problem. Submitted by Academician I. M. Vinogradov.

[redacted] 58157

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, YU. V.

1

Linnik, Yu. V. On nonstationary Markov chains. Doklady Akad. Nauk SSSR (N.S.) 60, 21–24 (1948). (Russian)

Suppose that for every n the random variables $x_{1,n}, x_{2,n}, \cdots, x_{n,n}$ form a Markov chain. The variable $x_{j,n}$ can assume $k_{j,n}$ different values $a_{1,n}^{(j)}, a_{2,n}^{(j)}, \cdots$. It is supposed that the $a_{i,n}^{(j)}$ are uniformly bounded and that $2 \leqq k_{j,n} \leqq$ constant. For fixed j, n, let $\xi_{j,n}$ be the arithmetic mean of $a_{i,n}^{(j)}$ and $\sigma_{j,n}^2$ the mean square deviation of $a_{i,n}^{(j)}$ from $\xi_{j,n}$. It is supposed that $\sigma_{j,n}$ is bounded away from zero. Suppose finally that the transition probabilities $p_{i,s,n}^{(j)}$ satisfy the inequality (*) $p_{i,s,n}^{(j)} \geqq n^{-\frac{1}{2}+\epsilon}$ for some $\epsilon > 0$. Then $S_n = x_{1,n} + \cdots + x_{k_{j,n},n}$ satisfies the central limit theorem. On the other hand, this is not necessarily the case if the exponent in (*) is replaced by $\frac{1}{2}$. The proof is accomplished by an adaptation of the method of subsequences used for independent variables. The paper continues recent work of Sapogov [same Doklady 58, 193–196, 1905–1908 (1947); these Rev. 9, 491, 361].

W. Feller (Ithaca, N. Y.).

Doklady Akademii Nauk SSSR Vol. 60 No. 1

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, YU. V

Gel'fond, A. O., and Linnik, Yu. V. On Thue's method in the problem of effectiveness in quadratic fields. Doklady Akad. Nauk SSSR (N.S.) 61, 773-776 (1948). (Russian)

Thue's method in the theory of Diophantine equations leads to upper bounds for the number of solutions, but in general not to bounds for these solutions themselves. The implications of this fact in the problem of obtaining all imaginary quadratic fields of class number 1 are discussed. Further, a generalized form of the Thue-Siegel theorem is given without proof; it gives the same exponent as Dyson's recent improvement of this theorem [Acta Math. 79, 225-240 (1947); these Rev. 9, 412]. *K. Mahler.*

Source: Mathematical Reviews, Vol 10 No. 6

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, YU.V. i SAPOGOV, N.A.

28176

Ob intyegralbnom i lokalbnom zakonakh dlya mnogomyernoy nyeodnorodnoy tsyepi markova - V. ogl. 2 - Y avt: a.n. Sapogov. Doklady akad. nauk UzSSR, 1949 No. 6, s. 7-10-Ryezyumye na usbyek-Yaz.- Bibliogr: 5 nazv.

SO. LETOPIS NO. 34

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, YU. V. i SAPOGOV, N. A.

37146. LINNIK, YU. V. i SAPOGOV, N. A. Mnogomernye integralnyy i lokal'nyy zakony dlya neodnorodnykh tsepey markova. Izvestiya akad. Nauk SSSR, seriya matem., 1949, No. 6 s. 533-66. — Bibliogr: 16 Nazv.

SO: Letopis' Zhurnal'nykh Statey, Vol 7, 1949

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

Source: Mathematical Reviews, Vol. 10 No. 6

Linnik, Yu. V. On the theory of nonuniform Markov chains. Izvestiya Akad. Nauk SSSR. Ser. Mat. 13, 65–94 (1949). (Russian)

Consider an infinite sequence of sequences of chance variables, the nth sequence being $X(1, n), X(2, n), \cdots, X(n, n)$; let $X(h, n)$ take the $k(h, n)$ values

$$a(1, h, n), \cdots, a(k(h, n), h, n).$$

It is assumed that (1) $|a(i, h, n)| < K_0$ for all i, h, n;

$$(2) \qquad [k(h, n)]^{-1} \sum_{i=1}^{k(h,n)} [a(i, h, n) - \xi(h, n)]^2 > c_0,$$

where $\xi(h, n) = [k(h, n)]^{-1} \sum_{i=1}^{k(h,n)} a(i, h, n)$; (3) $2 \leqq k(h, n) \leqq K_1$ for all n and $h \leqq n$; (4) for all n, $X(1, n), \cdots, X(n, n)$ form a simple Markov chain. Define

$$P(i, l, h, n) = P\{X(h, n) = a(l, h, n) \mid X(h-1, n) = a(i, h-1, n)\},$$

$$S(n) = \sum_{i=1}^{n} X(i, n), \quad A(n) = E(S(n)),$$

and $B(n)$ = variance of $S(n)$. The author proves that the distribution of $[B(n)]^{-\frac{1}{2}}[S(n) - A(n)]$ approaches the normal distribution (with mean zero and variance one) if for some $c_0 > 0$, $P(i, l, h, n) \geqq n^{-c_0}$. The theorem is no longer true if we are given only that $P(i, l, h, n) \geqq n^{-1}$. This problem was studied earlier by S. N. Bernstein and N. A. Sapogov, who obtained a weaker result. The author improves on their methods in a long and elaborate proof.

J. Wolfowitz (New York, N. Y.).

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

4
6
9
0

Linnik, Yu. V., and Sapogov, N. A. Multiple integrals and local laws for inhomogeneous Markov chains. Izvestiya Akad. Nauk SSSR. Ser. Mat. 13, 533–566 (1949). (Russian)

In the first part of the paper Sapogov proves the central limit theorem for a sequence of k-dimensional vectors whose values depend on the states of a discrete Markov chain. The number of states is supposed to be finite, but may vary with time. [illegible] Markov chain with $k+1$ states [illegible] and stationary transition probabilities. For $1 \leqq \nu \leqq k$ let $N_\nu(n)$ be the number of passages during the first n trials through ϵ_ν. It is supposed that the chain belongs to the type A of Doeblin's classification which means essentially that the variance of each $N_\nu(n)$ tends to infinity together with n. It is shown that the joint distribution of $N_\nu(n)$ is asymptotically normal. The proof is exceedingly complicated and depends on intricate estimates.

W. Feller (Princeton, N. J.)

Source: Mathematical Reviews, 1950 Vol 11 No. 8

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

Linnik, Yu. V.

Linnik, Yu. V. **An elementary method for a problem of the theory of prime numbers.** Uspehi Matem. Nauk (N.S.) 5, no. 2(36), 198 (1950). (Russian)

The author suggests a possible method for proving elementarily Siegel's theorem on the class-number of positive definite binary quadratic forms [Acta Arith. 1, 83-86 (1935)]. He has since constructed a proof on these lines [Izvestiya Akad. Nauk SSSR. Ser. Mat. 14, 327-342 (1950); these Rev. 12, 482]. *H. Davenport* (London).

Source: Mathematical Reviews, Vol. 12 No. 8

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

Yu. V.

2

Linnik, Yu. V. An elementary proof of Siegel's theorem based on the method of I. M. Vinogradov. (With an appendix of a short analytical proof.) Izvestiya Akad. Nauk SSSR. Ser. Mat. 14, 327–342 (1950). (Russian)

The author gives two proofs of Siegel's inequality [Acta Arith. 1, 83–86 (1935)] that $L_d(1) > \gamma(\epsilon)|d|^{-\epsilon}$ for every positive ϵ, and positive γ depending on ϵ only, where $L_d(s)$ is the Dirichlet series whose coefficients are the Kronecker symbol (d/n). The proofs are not very illuminating, though the first one is elementary in the technical sense.

H. Heilbronn (Bristol).

Source: Mathematical Reviews, Vol 12 No. 7

SOVW

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, YU. V.

[illegible] where a, σ, and λ are constants. Let [illegible] represent an observation on $(X_1, X_2, \ldots, X_{n-1})$. The author constructs confidence intervals for λ and studies their asymptotic behavior under the four possible assumptions that both a and σ^2 are known, a is unknown, etc.

W. Feller (Princeton, N. J.).

Source: Mathematical Reviews, Vol 12 No. 7

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, YU. V.

Čudakov, N. G., and Linnik, Yu. V. On a class of completely multiplicative functions. Doklady Akad. Nauk SSSR (N.S.) 74, 193–196 (1950). (Russian)

A generalized character $h(n)$ in the sense of the paper reviewed above is completely determined by the values of $h(p)$, where p runs over the primes. The set of primes p for which $h(p)\neq 0$ is called the basis of $h(n)$. It is easy to construct generalized characters for which the basis consists of exactly one prime. This paper is devoted to proving that there are no generalized characters with a finite basis consisting of more than one prime. The proof uses quantitative work of Vinogradov on uniform distribution [cf. Trav. Inst. Math. Stekloff 23 (1947); these Rev. 10, 509] and work of Gelfond [Bull. Acad. Sci. URSS, Sér. Math. [Izvestiya Akad. Nauk SSSR] 1939, 507–518; these Rev. 1, 295] on the rational approximations to $\log a/\log b$, where a and b are positive algebraic numbers such that $\log a/\log b$ is irrational and thus transcendental. It is unknown whether or not there exist any generalized characters with an infinite basis other than the nonprincipal Dirichlet characters.

P. T. Bateman (Urbana, Ill.).

Source: Mathematical Reviews, Vol 12 No. 10

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu. V.

SOV: MATHEMATICAL REVIEW (unclassified)

*Linnik, Yu. V. **Prime numbers and powers of two.** Trudy Mat. Inst. Steklov., v. 38, pp. 152–169. Izdat. Akad. Nauk SSSR, Moscow, 1951. (Russian) 20 rubles.

The author proves that the number of representations of an integer N as a sum of two primes and k powers of 2 is greater than

$$N\left(\frac{\log N}{\log 2}\right)^{k-1}(c_1-c_2(1-\eta)^{k-2})$$

where c_1, c_2 and $\eta<1$ are absolute constants and $k\geqq 3$. It follows at once that there exists a constant $k\geqq 3$ such that every large integer N is representable in the form

$$N=p_1+p_2+2^{x_1}+\cdots+2^{x_k},$$

where p_1 and p_2 are primes and $x_1, \cdots, x_k$ are positive integers.

The proof is based on the investigation of the integral

$$\int_0^1 e(2\pi iN\alpha)S^2(\alpha)T^k(\alpha)d\alpha,$$

where

$$S(\alpha)=\sum_{p<N} e(-pN^{-1}-2\pi ip\alpha),$$

$$T(\alpha)=\sum_{m=1}^{\infty} e(-2^mN^{-1}-2\pi i2^m\alpha).$$

On the "major arcs" one can evaluate the integral

$$\int e(2\pi iN\alpha)S^2(\alpha)d\alpha,$$

whereas on the "minor arcs" one uses the inequality

$$\int_0^1 |S(\alpha)|^2|T(\alpha)|^2d\alpha=O(N)$$

by Romanov [Math. Ann. **109**, 668–678 (1934)] and a detailed ingenious study of the set of values of α where $T(\alpha)\geqq(1-\alpha)T(0)$. There is one serious misprint. The last displayed formula in the enunciation of the theorem on p. 154 should read

$$\sum_{(N_1)} > c_3N\left(\frac{\log N}{\log 2}\right)^{k-1}.$$

H. Heilbronn (Bristol).

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

Linnik, Yu. V.

2

Linnik, Yu. V. A remark on products of three primes. Doklady Akad. Nauk SSSR (N.S.) 72, 9–10 (1950). (Russian)

The author sketches a proof of the following result. If x is large, then in the interval $[x, x+x^{1}\exp\{(\log x)^{0.99}\}]$ there exist integers which are products of exactly three prime factors.

P. T. Bateman (Urbana, Ill.).

Source: Mathematical Reviews, Vol 11 No. 9

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

Linnik, Yu. V.

Linnik, Yu. V. Some conditional theorems concerning binary problems with prime numbers. Doklady Akad. Nauk SSSR (N.S.) 77, 15–18 (1951). (Russian)

Assuming the Riemann hypothesis for $\zeta(0)$ the author shows that for every large integer N and $\epsilon>0$ primes p and p' can be found such that $|N-p-p'|<(\log N)^{3+\epsilon}$. The proof is based on the formula

$$\int_0^{(\log N)^{-3-\epsilon}} \Big(\sum_{\nu} \log p\, e^{-2\pi(p\alpha-p/N)^2} e^{2\pi i N\alpha}\Big)^2 d\alpha = \tfrac{1}{2}e^{-1}N$$

[illegible]

[illegible]

[illegible]

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, YU. V.

PA 233T92

USSR/Mathematics - Number Theory Nov/Dec 52

"Certain Conditional Theorems Concerning Goldbach's Binary Problem," Yu. V. Linnik

"Iz Ak Nauk SSSR, Ser Matemat" Vol 16, No 6, pp 503-520

Work possesses a conditional character. It discusses deductions which can be made from Riemann's hypothesis and various hypotheses on denseness in the direction of Goldbach's binary problem (i.e. $p+p' = 2N$). Development of previous work (cf. "Primes and Powers of Evens," "Trudy Matemat Inst Ak Nauk imeni Steklov" Vol 38, 1951). Submitted by Acad I. M. Vinogradov.

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, YU. V.

PA 227T50

USSR/Mathematics - Statistics, [illegible]
Distribution Law

"Linear Statistics and the Normal Law [illegible] tribution," Yu.V. Linnik

"Dok Ak Nauk SSSR" Vol 83, No 3, [illegible]

Considers r independent observations [illegible] tain chance quantity X: $x_1,..,x_r$ ([illegible] selection from vol r); and the linear [illegible] tions of observations (e.g., by the [illegible] least squares). Two such functions [illegible] statistics are: $L_1(x) = a_1x_1 + .. + a_rx_r$ [illegible] $L_2(x) = b_1x_1 + .. + b_rx_r$, which are assum[illegible]

[illegible]

become zero for certain values of [illegible] and b_i. Introduces the entire (integ[illegible] function of a complex variable z: [illegible] $/a_1/z + .. + /a_r/z - /b_1/z - .. - /b_r$ [illegible] monstrates a number of theorems [illegible] these expressions. Submitted by Acad [illegible] Kolmogorov 24 Jan 52.

[illegible]

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu. V.

2
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(1)

Linnik, Yu. V. Prime numbers and powers of one and the same number. Doklady Akad. Nauk SSSR (N.S.) 85, 953–954 (1952). (Russian)

Preliminary announcement of the results in the paper reviewed above.

10-28 54 LL

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

USSR/Mathematics - Modern Algebra, 11 Aug 52
Admissible Subgroups

"Free Operator Groups," S. T. Zavalo

"DAN SSSR" Vol 85, No 5, pp 949-951

States that the problem of the construction of admissible subgroups is extremely difficult. Gives a complete description of the construction of all admissible subgroups of a free operator group with a group of operators; however, for the case of free operator groups with free associative system of operators, a class of admissible subgroups which are free operator groups is indicated. Submitted by Acad A. N. Kolmogorov 21 Jun 52.

239T87

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu.V.

Linear forms and statistical criteria, part 1. Ukr.mat.zhur. 5 no.2:207-243 '53. (MLRA 6:6)

(Forms (Mathematics)) (Mathematical statistics)

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

GNEDENKO, B.V. [author]; LINNIK, Yu.V. [reviewer].

"Course in the theory of probabilities." Usp.mat.nauk 8 no.3:206-209 My-Je '53. (MLRA 6:7)

(Probabilities) (Gnedenko, B.V.)

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu.V.; MALYSHEV, A.V.

Application of the arithmetic of quaternions in the theory of ternary quadratic forms and to the decomposition of numbers into cubes. Usp.mat. nauk 8 no.5:3-71 S-O '53. (MLRA 6:10)

(Quaternions) (Forms quadratic) (Numbers, Theory of)

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, YU. V.

Mathematical Reviews
Vol. 15 No. 2
Feb. 1954
Mechanics

Linnik, Yu. V., and Novoselov, V. S. Random disturbances of the regular precession of a gyroscope. Akad. Nauk SSSR. Prikl. Mat. Meh. 17, 361–368 (1953). (Russian)

Consider a system

$$(1)\quad \frac{dx_i}{dt} = X_i[x_j, A_k(t), t] + S_i[x_j, A_k(t), t] \quad (i, j = 1, \cdots, n)$$

where the functions $A_k(t)$ $(k = 1, \cdots, l)$ characterize an l-dimensional random process. Let $a_k(t)$ denote the mathematical expectation of $A_k(t)$ and let $A_k(t) = a_k(t) + b_k(t)$, so that the mathematical expectation of the random functions $b_k(t)$ is zero. Further, let $S = \{S_i[x_j, A_k(t), t]\}$ denote an n-dimensional random vector-function which for the given values of the arguments characterizes an n-dimensional random process. The initial data of the system (1) are assumed to be random and given by a distribution with probability density

$$P[\xi_1^0 \le x_1^0 \le \xi_1^0 + d\xi_1^0] = f(\xi_1^0, \cdots, \xi_n^0)\, d\xi_1^0 \cdots d\xi_n^0,$$

[illegible]

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$$x_i^0 = y_i^0, \quad A_k(t) = a_k(t), \quad S\{S_i[x_j, A_k(t), t]\} = 0$$

holds. Consider a finite time interval and assume that the probable values of $\max|b_k(t)/a_k(t)|$ and $\max|(x_i - y_i)/y_i|$ are small in this interval. The random process characterized by the functions $S_i[x_j, A_k(t), t]$ may be considered as a collection of random surfaces having the property that the probability is unity that these surfaces have bounded partial derivatives $\partial S_i/\partial x_j$ and $\partial S_i/\partial A_k$. Under these assumptions the system (1) can be linearized and reduced to the form

$$\frac{dz_i}{dt} = \sum_{j=1}^{n} X_{ij}(t) z_j + F_i(t) \quad (i = 1, \cdots, n), \tag{2}$$

where

$$z_i = x_i - y_i, \quad F_i(t) = \varphi_i(t) + S_i(t), \quad S_i(t) = S_i[y_j(t), a_k(t), t].$$

The functions $\varphi_i(t)$ and $X_{ij}(t)$ can be easily evaluated and the system (2) solved with given initial values z_i^0.

Introduce one column matrices $B(t)$, $S(t)$ and z^0 with the elements $b_k(t)$, $S_i(t)$ and z_i^0 respectively, and assume that they represent statistically independent processes, $B(t)$ and $S(t)$ being, in addition, stationary. In order to secure continuity of the processes their correlation matrices are assumed to be continuous. Further, the processes $B(t)$, $S(t)$ and z^0 are assumed to have Gaussian distributions. Then $z = \{z_i(t)\}$ is also a Gaussian process with correlation matrix R, and probability density

$$f(\xi_1, \cdots, \xi_n) = [2^n \pi^n D(R)]^{-1/2} \times \exp\left[(-1/2D(R)) \sum_{i,j=1}^{n} D_{ij}\xi_i\xi_j\right], \tag{3}$$

where $D(R)$ is the determinant of the matrix R and D_{ij} are the algebraic complements of its elements.

The second part of the paper is concerned with application of the results obtained to the motion of a gyroscope. Let θ, ψ, φ be the angles of nutation, precession and proper rotation of a gyroscope, respectively. Furthermore, let A and C be the moments of inertia of a gyroscope, m its mass and l the distance of its center of gravity from the fixed point. Then the Lagrangian equations of motion can be put in the form

$$\ddot{\theta} = \dot{\psi}^2 \sin\theta \cos\theta - (C/A)(\dot{\varphi} + \dot{\psi}\cos\theta)\dot{\psi}\sin\theta + (mgl/A)\sin\theta + M_\theta/A,$$

$$\ddot{\psi} = [C(\dot{\varphi} + \dot{\psi}\cos\theta)\cos\theta - 2A\dot{\psi}\dot{\theta}\cos\theta]/A\sin\theta + M_\psi/A\sin^2\theta,$$

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$$d/dt(\dot{\bar{\varphi}} + \dot{\psi}\cos\theta) = M_\varphi/C,$$

where the moments M_θ, M_ψ and M_φ are assumed to be random functions of the variables θ, $\dot{\theta}$, $\dot{\varphi}$, t for the given initial conditions [illegible]

[illegible] matrices of the processes [illegible] and [illegible] then the correlation matrix R of the process $z = \{z_i(t)\}$ can be exhibited. Finally the density of the distribution of the deviations of the motions of a gyroscope from the regular precession can be evaluated by formula (3). *E. Leimanis*

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu. V.

Linnik, Yu. V. Addition of prime numbers with powers of one and the same number. Mat. Sbornik N.S. 32(74), 3–60 (1953). (Russian)

The author proves that every large even integer can be written as a sum of two primes and k powers of 2, where k is an absolute constant. In a previous paper [Trudy Mat. Inst. Steklov. 38, 152–169 (1951); these Rev. 14, 355] the author proved this theorem assuming the generalised Riemann hypothesis. If 2 is replaced by an integer $g>2$, the proof also applies, k becoming a function of g. The author also states that any large integer, written in the binary scale, can be changed into a Goldbach number by altering a bounded number of digits only. *H. Heilbronn.*

Mathematical Reviews
Vol. 15 No. 1
Jan. 1954
Analysis

7-13-54
LL

This is a continuation of a previous paper [same Doklady (N.S.) 83, 353–355 (1952); these Rev. 14, 60] and we refer to the previous review. A simple sufficient condition that (B) implies (A) is given. A theorem is given where the normal law in (A) is replaced by a convolution of symmetrical stable laws, followed by two theorems on characteristic functions, of independent interest. The second asserts that if a ch. f. is of the form $e^{P(u)}$ in $-\delta \leq u \leq \delta$ where P is a polynomial, then it is so for all real u. The maximum of a sequence of independent random variables is the "dual" of their sum, in the sense that they correspond to [illegible] cation of [illegible]'s and [illegible]. Two theorems of the dual type are stated. A more detailed review must await publication of proofs of these announcements. *K. L. Chung.*

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Translation 9003417-V

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LINNIK, Yu. V.

Mathematical Reviews
May 1954

(2) Linnik, Yu. V., and Malyšev, A. V. On integral points on a sphere. Doklady Akad. Nauk SSSR (N.S.) 89, 209-211 (1953). (Russian)

Suppose q is a given odd prime number and [illegible] positive number [illegible]

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

Category Linnik, Yu. V

Linnik, Yu. V. Some applications of Lobačevskiĭ's geometry to the theory of binary quadratic forms. Doklady Akad. Nauk SSSR (N.S.) 93, 973–974 (1953). (Russian)

The author announces some results on the distribution of the reduced positive definite binary quadratic forms $a\xi^2+2b\xi\eta+c\eta^2$ of large determinant $b^2-ac=-D<0$. On putting $2x=c+a$, $y=b$, $4z=c-a$, the reduced forms are represented by points (x, y, z) on the hyperboloid $x^2-y^2-z^2=D>0$ with y integral and x and z halves of integers of the same parity, in the region A defined by $2|y|\leqq x-z\leqq x+z$. Results are given concerning the number of such points in certain sub-regions of A. The method of proof is said to be similar to that indicated in a previous paper [Linnik and Malyšev, same Doklady (N.S.) 89, 209–211 (1953); these Rev. 15, 406], but to depend on the existence, in certain algebras of generalized quaternions with indefinite norm, of a ring of integers with a Euclidean algorithm. One of the results stated is as follows. Suppose $q\geqq 3$ is a prime, and $(-D|q)=+1$. Then there exist forms (a, b, c) satisfying

(over)

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[illegible]

[illegible]

where $a_1, \ldots, a_4$ are constants consistent with the conditions of reduction, provided $D \geq D_0(q, a_1, a_2, a_3, a_4)$. Moreover, the number of such forms, divided by the total number $h(-D)$, is greater than a positive constant depending on q, a_1, a_2, a_3, a_4.

H. Davenport (London).

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu. V

5

Linnik, Yu. V., and Šanin, N. A. Andrei Andreevič Markov (on his fiftieth birthday). Uspehi Matem. Nauk (N.S.) 9, no. 1(59), 145–149 (1 plate) (1954). (Russian)

1

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

USSR/Mathematics - Bibliography FD-1188

Card 1/1 Pub. 118-29/30

Author : Petrov, V. V., and Linnik, Yu. V. (reviewers)

Title : Review of the book 'Teoriya veroyatnostey' [Theory of probabilities], V. A. Unkovskiy, Navy Press, 1953, 320 pp, 10.90 rubles

Periodical : Usp. mat. nauk, 9, No 3(61), 278-282, Jul-Sep 1954

Abstract : The content of the book consists of the following: principal concepts and theorems in probability theory; Bayes formulas; binormal distribution; random quantities and their characteristics; law of large numbers in the Chebyshev form; properties of the normal distribution and of certain other distribution laws; concept of the Lyapunov limit theorem; the laws governing the distribution of functions of random quantities; probability distributions in the plane and in space; elements of mathematical statistics including A. N. Kolmogorov's criteria of agreement [soglasiya] and chi-square (without proof); method of least squares. The reviewer states that the book cannot be recommended as a textbook for any class of reader because of the numerous errors of printing (more than 100), besides errors of a technical nature.

Institution :

Submitted :

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, YU. V.

Linnik, Yu. V. Application of the theory of Markov chains to the arithmetic of quaternions. Uspehi Mat. Nauk (N.S.) 9, no. 4(62), 203–210 (1954). (Russian) — F/W

Let r be a positive odd integer and let there be σ_r distinct integral quaternions of norm r. The author considers products of the type $B = R_1 \cdots R_s$ where the R_i are (not necessarily distinct) quaternions of norm r. The construction of such B which are primitive (do not contain a rational factor) follows a Markoff chain (in the sense of probability theory) since if B is primitive the R_{s+1} such that BR_{s+1} is primitive depend on R_s. In particular, let $s \geq 0$ be fixed, R^* be some fixed quaternion of norm r, W be the number of primitive $B = R_1 \cdots R_s$ and W^* the number of primitive $B = R_1 \cdots R_s$ in which R^* occurs at most $(1-\varepsilon)s/\sigma_r$ times; then $W^*/W = \varepsilon r^{-\eta}$, where ε, η depend only on s. From this can be deduced the following result of Malyšev which has application to the problem of the distribution of integral points on a sphere $x^2+y^2+z^2=m$ = integer (*). Let $m \to \infty$ through the m for which (*) and $l_0^2+m \equiv 0 \pmod r$ are soluble. Let L_α $(\alpha = 1, \ldots, t(m))$ be the integral vectors such that $L_\alpha^2 = -m$. Then the number $t^*(m)$ of α such that $l_0 + L_\alpha$ is divisible on the left by a given R^* of norm r satisfies $t^*(m)/t(m) \to \sigma_r^{-1}$ as $m \to \infty$ [Yu. V. Linnik and A. V. Malyšev, Uspehi Mat. Nauk (N.S.) 8, no. 5(57), 3–71 (1953); 10, no. 1(63), 243–244 (1955); MR 16, 450].

J. W. S. Cassels (Cambridge, England).

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu.V. (Leningrad); Khusu, A.P. (Leningrad)

Mathematical statistical account of surface-uneveness contours following polishing. Inzh. sbor. 20:154-159 '54. (MLRA 8:7)
(Surfaces (Technology)) (Grinding and polishing)

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu.V. ; KHUDOV, A.P., starshiy nauchnyy sotrudnik

Statistical characteristics of surface profilograms. [Izd.] LONITOMASH no.34:223-229 '54. (MLRA 8:10)

1. Chlen-korrespondent Akademii nauk SSSR (for Linnik). 2. Leningradskiy gosudarstvennyy universitet imeni A.A.Zhdanova.
(Surfaces (Technology))

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu. V.

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu. V.

USSR/Mathematics

Card : 1/1

Authors : Linnik, Yu. V., Memb. Corres. of Acad. of Sc. USSR

Title : Asymptotic distribution of integral points on a sphere

Periodical : Dokl. AN SSSR, 96, Ed. 5, 909 - 912, June 1954

Abstract : Some preliminary results regarding the asymptotic distribution of integral points on a sphere were obtained by A. V. Malyshev and the author. The hypothesis concerning the existence of a uniform asymptotic distribution is basically confirmed in this report. Nine references.

Institution : The A. A. Zhdanov State University, Leningrad.

Submitted : March 5, 1954

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, YU. V.

USSR/ Mathematics - Binary quadratic forms

Card 1/1 Pub. 127 - 1/12

Authors : Linnik, Yu. V.

Title : An asymptotic distribution of reduced binary quadratic forms in connection with the Lobachevskian geometry (II)

Periodical : Vest. Len. un. ser. mat. fiz. khim. 5, 3-32, May 1955

Abstract : In connection with an interpretation of the Lobachevskian geometry, a series of lemmas and theorems is presented. By their proof, the asymptotic distribution of reduced binary quadratic forms used for presentation of points on a plane (Lobachevskian) is conformed. The proof and conformation are accomplished in view of the Cayley theorem with application of the matrix method. Twenty-six references: 9 German, 9 English, and 8 USSR (1889-1954).

Institution : ....

Submitted : October 14, 1954

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu. V.

Linnik, Yu. V. The asymptotic distribution of reduced binary quadratic forms in relation to the geometries of Lobačevskiĭ. I, II, III. Vestnik Leningrad. Univ. 10 (1955), no. 2, 3–23; no. 5, 3–32; no. 8, 15–27. (Russian)

Let Δ consist of those points (a, b, c) in three-dimensional Euclidean space such that $|2b| < a < c$ or $0 \leq 2b < a = c$ or $0 < 2b = a \leq c$. Let Σ be a convex cone with center at the origin lying entirely either in that portion of Δ for which $c \leq Ka$ or in that portion for which $c \geq Ka$, where K is a given positive number > 1. Let $A(\Delta)$ be the volume of that portion of Δ lying inside the hyperboloid $ac - b^2 = 1$ and similarly for $A(\Sigma)$. (Thus if we regard the positive sheet of the hyperboloid $ac - b^2 = 1$ as a realization of the Lobatchevskian plane in the usual way, $A(\Sigma)$ is a constant times the Lobatchevskian area of that portion of the hyperboloid which lies in Σ.) If D is an odd positive integer let $h(-D)$ be the number of lattice points (a, b, c) on the hyperboloid $ac - b^2 = D$ such that a, $2b$, and c have g.c.d. 1 and (a, b, c) lies in Δ, and let $h_\Sigma(-D)$ denote the number of these lattice points which also lie in Σ. Thus, if to each triple (a, b, c) we make correspond the binary quadratic form $ax^2 + 2bxy + cy^2$, then $h(-D)$ is the number of reduced properly primitive binary quadratic forms of determinant $-D$. (Still another interpretation is in terms of 2 by 2 matrices L such that $L^2 = -DI$, where I is the 2 by 2 identity matrix.) By the Heilbronn-Siegel Theorem $h(-D)$ tends to infinity with D. The author proves that

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1/3

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Linnik, Yu. V.

if D goes to infinity through the odd integers such that $(-D|p)=1$ for some fixed odd prime number p, then

$$\lim_{D\to\infty} h_\Sigma(-D)/h(-D)=\Lambda(\Sigma)/\Lambda(\Delta).$$

This is an improvement of a result announced earlier [Dokl. Akad. Nauk SSSR (N.S.) 93 (1953), 973; MR 15, 856] to the effect that if K is sufficiently large, then $\liminf_{D\to\infty} h_\Sigma(-D)/h(-D)>0$ (with the same set of values for D). The arguments employed are related to those used in proving a similar result concerning the asymptotic distribution of lattice points on spheres [ibid. 96 (1954), 909–912; MR 16, 451]. A certain generalized quaternion algebra is an important tool. {The reviewer believes that in § 43 there is a minor error in a volume calculation which the author makes in applying the main theorem to a special situation.} *P. T. Bateman.*

Linnik, Yu. V. An new arithmetic application of the geometry of Lobačevskiĭ. Dopovidi Akad. Nauk Ukrain. RSR 1955, 112–114. (Ukrainian. Russian summary)

A summary of the paper reviewed above.

P. T. Bateman (Princeton, N. J.).

2/3

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

Linnik, Yu. V. Markov chains in the analytical arithmetic of quaternions and matrices. Vestnik Leningrad. Univ. 11 (1956), no. 13, 63–68. (Russian)

A proof using less recherché machinery from the theory of probability of the author's result about primitive integral quaternions B which can be expressed in the form $R_1R_2\cdots R_s$, where each R_j is an integral quaternion of fixed norm r [Uspehi Mat. Nauk (N.S.) 9 (1954), no. 4(62), 203–210; MR 16, 1002]. The present proof is also valid if B is a primitive integral 2 by 2 matrix and the R_j are taken from a fixed set of representatives of the classes of integral matrices of determinant r under right association. [Cf. the paper reviewed second above.]

J. W. S. Cassels (Cambridge, England).

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu.V.

Problem for characteristic functions of probability distributions. Usp.mat.nauk. 10 no.1:137-138 '55 (MLRA 8:6)
(Distribution(Probability theory))

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu. V.

Linnik, Yu. V., and Rodosskii, K. A. Nikolai Grigor'evic Chebyshev. (On his fiftieth birthday.) Uspehi Mat. Nauk (N.S.) 10, no. 3(65), 213-219 (1 plate) (1955). Uspehi.
A list of Chebyshev's published papers is included.

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu. V.

SUBJECT USSR/MATHEMATICS/Theory of probability CARD 1/1 PG - 307
AUTHOR ZINGER A.A., LINNIK Ju.V.
TITLE On an analytic generalization of the Cramér theorem and its appliaction.
PERIODICAL Vestnik Leningradsk. Univ. 10, No.11, 51-56 (1955)
reviewed 10/1956

The authors prove the following generalization of a well known theorem of H. Cramér (Random variables and probability distributions, Cambridge Tracts 36, (1937)) : If $f_1(t), f_2(t), \ldots f_s(t)$ are characteristic functions, $a_1, a_2, \ldots, a_s$ positive numbers and

$$f_1^{a_1}(t) f_2^{a_2}(t) \ldots f_s^{a_s}(t) = e^{i\gamma t - \frac{t^2}{2}} \tag{1}$$

is valid for $-\infty < t < +\infty$, where γ is a real number, then $f_j(t)$ is the characteristic function of a normal distribution ($j=1,2,\ldots,s$). This theorem is applied to give a new and simple proof of a theorem of V.P.Skitovic (Izvestija Akad. Nauk 18, (1954) 952) according to which if $x_1, x_2, \ldots, x_n$ are independent random variables, a_k and b_k ($k=1,2,\ldots,n$) real constants, further $y_1 = \sum_{k=1}^{n} a_k x_k$ and $y_2 = \sum_{k=1}^{n} b_k x_k$ are also independent, then, for those values of k for which $a_k b_k \neq 0$, x_k is normally distributed

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

Some New Data on Independent Statistics. 124

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

HALD, Anders, 1913- ; VOROB'YEV, N.N. [translator]; PETROV, V.V. [translator]; KHUSU, A.P. [translator]; LINNIK, Yu.V., redaktor

[Statistical theory with engineering applications. Translated from the English] Matematicheskaia statistika s tekhnicheskimi prilozheniiami. Perevod s angliiskogo N.N. Vorob'eva, V.V. Petrova i A.P.

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu.V.

Determining the probability distribution by distribution of statistics [with summary in English]. Teor.veroiat.i ee prim. 1 no.4:466-478 '56. (MLRA 10:5)

(Probabilities)

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

KISLITSYN, S.S.; LINNIK, Yu.V.

"A course in the theory of probabilities." B.V. Gnedenko. Reviewed by S.S. Kislitsyn, IU.V. Linnik. Ukr.mat.zhur. 8 no.2: 231-232 '56. (MLRA 9:8)

(Probabilities)
(Gnedenko, Boris Vladimirovich, 1912-)

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu.V.

On polynomial statistics in connection with the analytic theory of differential equations. Vest.Len.un 11 no.1:35-48 '56.(MLRA 9:5)
(Differential equations) (Mathematical statistics)
(Distrubution (Probability theory)

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu.V.

SUBJECT USSR/MATHEMATICS/Number theory CARD 1/1 PG - 388
AUTHOR KUBILJUS I.P., LINNIK Ju.V.
TITLE An elementary theorem of the prime number theory.
PERIODICAL Uspechi mat. Nauk 11, 2, 191-192 (1956)
reviewed 11/1956

By a very simple argument the authors show that there are infinitely many pairs of prime numbers p_1, p_2 such that $p_1 p_2 = a^2 + b^2$ and $0 < b < \log p_1 p_2$, where a and b are integers.

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK Yu.V.

SUBJECT USSR/MATHEMATICS/Statistics CARD 1/2 PG - 634
AUTHOR LINNIK Yu.V.
TITLE A problem of the differential algebra arising from mathematical statistics.
PERIODICAL Uspechi mat.Nauk 11, 3, 169-170 (1956)
reviewed 3/1957

Let $x_1, x_2, \ldots, x_n$ denote independent and identically distributed random variables having the distribution function $F(x)$ and the characteristic function $y(t)$. Let us put $S = x_1 + x_2 + \ldots + x_n$ and denote by Q a polynomial of the variables $x_1, x_2, \ldots, x_n$:

$$(1) \qquad Q = \sum b_{k_1 k_2 \ldots k_n} x_1^{k_1} x_2^{k_2} \ldots x_n^{k_n} .$$

The condition that Q and S are independent leads to the following differential equation for $y(t)$:

$$(2) \qquad D(t) = D(0)\,(y(t))^n,$$

where

$$(3) \qquad D(t) = \sum b_{k_1 k_2 \ldots k_n} (-i)^{k_1 + \ldots + k_n} y^{(k_1)}(t)\, y^{(k_2)}(t) \ldots y^{(k_n)}(t).$$

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Uspechi mat.Nauk 11, 3, 169-170 (1956) CARD 2/2 PG - 634

Thus the statistical problem of determining the distribution functions F(x) which have the property that S and Q are independent, leads to the investigation of all solutions of the differential equation (2), i.e. to a problem in differential algebra (see H.Ritt, Differential algebra, N.Y. (1943)).

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu.V.; MARKUSHEVICH, A.I.

Aleksandr Osipovich Gel'fond (on the 50th anniversary of his birth). Usp.mat.nauk 11 no.5:239-248 S-O '56. (MLRA 10:2)
(Gel'fond, Aleksandr Osipovich, 1906-)
(Bibliography--Mathematics)

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu.V.

Markov's chains in analytical arithmetics of quaternions and matrices. Vest. Len. un. 11 no.13:63-68 '56. (MLRA 9:10)

(Probabilities)

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu.V.

Theory of probabilities in practice. Nauka i zhizn' 23 no.10:11-13 O '56. (MLRA 9:11)

1. Chlen-korrespondent Akademii nauk SSSR.
(Probabilities)

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

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[illegible]

Let $P(x_1, \cdots, x_n)$ be a polynomial in the random variables $x_1, \cdots, x_n$, which have independent one-dimensional distributions. One terms P invariant in the mean with respect to $T(x_1, \cdots, x_n)$ if the conditional expectation of P for fixed T is independent of the value of T. The author states sufficient conditions that, when such a polynomial is invariant in the mean with respect to $T=x_1+\cdots+x_n$, one can conclude that the one-dimensional distributions are normal. He shows how this theorem can be deduced from a theorem on analyticity of solutions of an algebraic differential equation.

W. Kaplan (Ann Arbor, Mich.).

Corres Mbr AS USSR
Leningrad Branch, Math Inst im V.A. Steklov

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, YU. V.

1-FW

Linnik, Yu. V. The asymptotic geometry of the Gaussian genera; an analogue of the ergodic theorem. Dokl. Akad. Nauk SSSR (N.S.) 108 (1956), 1018–1021. (Russian)

In a previous paper [Vestnik Leningrad. Univ. 10 (1955), no. 2, 3–23, no. 5, 3–32, no. 8, 15–27; MR 18, 193] the author obtained results on the distribution, as $D \to \infty$, of the integers (a, b, c) which correspond to reduced positive definite forms $ax^2 + 2bxy + cy^2$ of odd determinant [illegible]

[illegible]

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, YU. V.

$$H_R(\Sigma_0, D) \sim \frac{9}{2\pi} A(\Sigma_0) 2^{\beta(p)} h(-D)(1+\eta(p, D)),$$

3

1-F\W

where $A(\Sigma_0)$ is the area of Σ_0 in the sense of Lobatchevsky, $h(-D)$ is the total number of reduced forms of determinant $-D$, and $\eta(p, D) \to 0$ as $D \to \infty$ for fixed p. The detailed proof is not given, but it is indicated that it depends on an "ergodic theorem" to the effect that, for almost all forms $\varphi(x, y)$, the forms $\varphi^\nu(x, y)$ obtained by applying a particular linear transformation ν times, are uniformly distributed for a long sequence of ν, all this to be interpreted asymptotically as $D \to \infty$. *H. Davenport.*

3/2

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

Linnik, Yu. V. More on the analogues of the ergodic theorems for the imaginary quadratic field. Dokl. Akad. Nauk SSSR (N.S.) 109 (1956), 694-696. (Russian)

3

I-F\W

Let $\mathfrak{H}$ be the group of properly primitive positive definite binary quadratic forms $(a\ b\ c)=ax^2+2bxy+cy^2$ of odd determinant $D=ac-b^2$. Each such form corresponds to a 'fundamental point' $\mathfrak{a}=(a_0, b_0, c_0)$ $(a_0=a/\sqrt{D}$, etc.) on the hyperboloid $a_0c_0-b_0^2=1$ $(a_0>0)$, and the reduced forms correspond to points inside a triangular region Δ_0 whose Lobatchevskian area $\mu(\Delta_0)$ is taken to be 1. Let p be an odd prime such that $(-D/p)=1$ and let $\mathfrak{p}=(a_0', b_0', c_0')$ denote the fundamental point corresponding to either of the forms $(p, \pm\xi, n)$ of $\mathfrak{H}$. By Gaussian composition we obtain a third fundamental point $\mathfrak{a}\mathfrak{p}=(a_0'', b_0'', c_0'')$, and so a transformation $\mathfrak{T}$ is defined which transforms the class of forms $\mathfrak{a}$ into the class $\mathfrak{a}\mathfrak{p}$; powers of this transformation can then be defined. Let Ω be a simply-connected subset of Δ_0 bounded by piece-wise smooth arcs and of Lobatchevskian area $\mu(\Omega)$ and let $f_\Omega(P)=1$ if $P\in\Omega$ and $f_\Omega(P)=0$ if $P\notin\Omega$. The author states, without proof, the following theorem concerning the behaviour of the ergodic mean value of $f_\Omega(\mathfrak{a}\mathfrak{p}^r)$:

Linnik, Yu. V.

3

I-F\W

for $r\geq c_0 \log D$, as $D\to\infty$, for all classes $\mathfrak{a}$ with the possible exception of $o(h(-D))$ classes. Here $h(-D)$ is the order of $\mathfrak{H}$. In this result $\mathfrak{T}$ can be replaced by $\mathfrak{T}^r$ and the positive constant c_0 will then depend on r. Various consequences of this theorem are [illegible] ular, if $\mathfrak{G}$ is a subgroup of $\mathfrak{H}$ of bounded index g and r is chosen so that $\mathfrak{p}^r\in\mathfrak{G}$, then the number of points in any coset of $\mathfrak{G}$ which are mapped into Ω by the transformation $\mathfrak{T}^r$ is

$$\frac{h(-D)}{g}\mu(\Omega)(1+o(1))$$

as $D\to\infty$. If $\mathfrak{G}=\mathfrak{H}$ this yields an earlier result of the author's [Vestnik Leningrad. Univ. 10 (1955), no. 2, 3-23, no. 5, 3-32, no. 8, 15-27; MR 18, 193].

R. A. Rankin (Glasgow).

2/2

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu.V. (Leningrad)

Factorizing the composition of Gauss' and Poisson's laws (with summary in English). Teor.veroiat.i ee prim. 2 no.1:34-59 '57. (MIRA 10:7)

(Probabilities)

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[illegible]

[illegible]

[illegible]

[illegible]

[illegible] Nr 3, pp. [illegible] ([illegible])

ABSTRACT: We consider the problem of a least square estimation of the matrix of "elements"

$$A = \begin{Vmatrix} a_1 \\ \vdots \\ a_n \end{Vmatrix}$$

by means of the equations:

$$Y = X^{(o)} + XA,$$

where $X^{(o)} = X^{(o)}_{N1} = \begin{Vmatrix} x_{01} \\ x_{02} \\ \vdots \\ x_{0N} \end{Vmatrix}$; $X = X_{Nn} = \| x_{rj} \|$ are

Card 1/6

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52-3-4/9

Some Remarks on Least Squares in Connection with Direct and Inverse Location Problems.

[illegible]

52-3-4/9

Some Remarks on Least Squares in Connection with Direct and Inverse Location Problems.

N is greater than n, and that the rank X is equal to n. In such a case it is possible to obtain an evaluation of the elements of A by the method of least squares. If the evaluation is denoted by $\tilde{A}$ we have

$$\tilde{A} = C^{-1} X^T P (L - X^{(0)}), \qquad \text{(Eq.1.3)}$$

where

$$C = C_{nn} = X^T P X \qquad \text{(Eq.1.4)}$$

and C is an indefinite symmetric matrix. In Ref.2 Kolmogorov introduces the construction of confidence intervals for the several estimators of $a_1, a_2, \ldots, a_n$ and proves that to obtain fuller information it is essential to use in the construction of the corresponding statistics all the elements of the matrix $\sigma^2 C^{-1}$, and not only its diagonal elements which occur in the construction of the above-mentioned confidence intervals. Such considerations lead to the construction of confidence domains for the estimators of all or some of the elements

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52-3-4/9

Some Remarks on Least Squares in Connection with Direct and Inverse Location Problems.

Theorem. Let $G = G_{mn}$ be a known $m \times n$ matrix with range m; $H = \begin{Vmatrix} h_1 \\ \vdots \\ h_m \end{Vmatrix} = GA$; $Z - \begin{Vmatrix} z_1 \\ \vdots \\ z_m \end{Vmatrix}$ the general coordinate vector; $C = X^T PX$; $K = GC^{-1}G^T$;

$\tilde{A} = \begin{Vmatrix} \tilde{a}_1 \\ \vdots \\ a_n \end{Vmatrix}$ the matrix of least square estimates of A;

$$\tilde{V} = X^{(o)} - X\tilde{A} - L; \quad \tilde{H} - G\tilde{A}$$

Then K is non-singular, and the confidence ellipsoid Θ_{γ_0}:

$$(Z - \tilde{H})^T . K^{-1} . (Z - \tilde{H}) = \gamma_0 \left[P^{\tilde{v}\tilde{v}} \right]$$

Card 5/6 covers the point $Z = H$ with the probability p_0, where

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[illegible]

$F_{m,N-n}(\gamma_0) = p_0$, $F_{m,N-n}(x)$, being Fisher's distribution with m and N - n degrees of freedom. The extreme cases are: $d = E$; $H = A$; $K = C^{-1}$; $K = 0$ and $d = \|00, \ldots, 1, 0, \ldots 0\|$ (1 in the i-th place); $K = dC^{-1}d^T = (C^{-1})_{ii}$, $\exists_{\gamma_0}$ of the type

$$\frac{(z - \tilde{a}_i)^2}{(C^{-1})_{ii}} = \gamma_0 [\tilde{p}^{\tilde{v}\tilde{v}}]$$

which is tantamount to the construction of the well known separate confidence intervals for a_i involving Student's distribution. An application to geodetic location problems on a plane is given. In this case $n = 2$, and the equation $F_{2,N-2}(\gamma_0) = p_0$ is solvable in elementary functions.

There are 2 figures and 7 references, 6 of which are Slavic.

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Card 6/6

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SARYMSAKOV, T.A.; LINNIK, Yu.V.

Nikolai Pavlovich Romanov; on the occasion of his 50th birthday. Usp.mat.nauk 12 no.3:251-253 My-Je '57. (MIRA 10:10)
(Romanov, Nikolai Pavlovich, 1907-)

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

ZINGER, A.A.; LINNIK, Yu.V.

One class of differential equations and its application to certain problems of the regression theory (with summary in English). Vest. LGU 12 no.7:121-130 '57. (MLRA 10:6)

(Differential equations)
(Distribution (Probability theory))

AUTHOR: LINNIK, Yu.V. (Leningrad) 39-2-6/7

TITLE: Asymptotic-Geometric and Ergodic Properties of the Set of Integral Points on the Sphere (Asimptotiko-geometricheskiye i ergoticheskiye svoystva mnozhestva tselykh tochek na sfere)

PERIODICAL: Matematicheskiy Sbornik, 1957, Vol. 43, Nr.2, pp.257-276 (USSR)

ABSTRACT: The author considers the integral points of the sphere $K_3(m)$: $x^2+y^2+z^2 = m$, $m \equiv 1,2 \pmod 4$ or $m \equiv 3 \pmod 8$. $K_3(m)$ is projected from the center onto $K_3(1)$. Let Γ_0 be a closed convex domain on $K_3(1)$ with a piecewise smooth contour, let Γ be the projection of Γ_0 onto $K_3(m)$. Let $H(\Gamma)$ be the number of integral points on $K_3(m)$ inside of Γ; $H_0(\Gamma)$ - number of primitive points. Let $H_0(K_3(m)) = H_0(m)$ and $H(K_3(m)) = H(m)$. The author proves a theorem which is a certain geometric completion of the result of Siegel $\ln H_0(m) \sim \frac{1}{2} \ln n$.

Theorem: Let $q \geqslant 3$ be a prime number such that $\left(\frac{-m}{q}\right) = +1$. Let $\omega(\Gamma)$ be the solid angle under which Γ can be seen from the center. For $m \to \infty$ then we have

Card 1/2

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

AUTHOR LINNIK, Yu.V., Corresponding Member of the Academy of Science of the U.S.S.R. 20-5-7/67

TITLE On the "defining" Statistics; a Generalization of the Problem of the Moments.
(Ob "opredelyayushchikh" statistikakh; odno obobshcheniye problemy momentov - Russian)

PERIODICAL Doklady Akademii Nauk SSSR, 1957 Vol 113, Nr 5, pp 974-976, (U.S.S.R.)
Received 6/1957 Revised 7/1957

ABSTRACT X is assumed to be a onedimensional change quantity with the law of distribution $F(x) = P(X < x)$, and $\xi(x_1, x_2, \ldots x_m)$ is assumed to be corresponding, repeated selection (of a chance vector with independent components distributed according to the law $F(x)$). Indirect observations are carried out on X, which furnish the values of a certain continuous statistic $Q(\vec{\xi})$. The following problem may then be set up: In what cases do observations as to the statistic $Q(\vec{\xi})$ permit the setting up of the law $F(x)$. The author here investigates the following analytical problem: If the law of distribution of the statistic $F_Q(x) = P(Q < x)$ is exactly known, the law $F(x)$ has to be determined uniquely. If it is possible to do so at a given statistic $Q(\vec{\xi})$ in the class of the laws R, the statistic $Q(\vec{\xi})$ is denoted as defined in the class R of the laws $F(x)$. The investigation of the defining statistics and the classes of laws corresponding to them is connected with the classical problem of the moments

Card 1/2

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20-5-7/67

On the "defining" Statistics; a Generalization of the Problem of the Moments.

looked upon as a generalization of this problem. The auther here investigaties homogeneous statistics Q. Three theorems and a corresponding lemma are given. As a proof of this theorem some nonlinear integro-functional equations are to be studied (not carried out here). When investigating the uniqueness of the solutions of these integral equations, the integral equations within the demain of the slight increases of the solutions are linearized approximatively. (No illustration).

ASSOCIATION Leningrad Department of the Mathematical Institute "STEKLOV, V.A." of the Academy of Science of the U.S.S.R.

PRESENTED BY

SUBMITTED 20 Jan 1956

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

20-5/62

AUTHOR LINNIK, Yu.V., Corresponding Member of the Academy of Science of the USSR.

TITLE On the Composition of the Probability Theorems By Gauss and Puasson/read Poisson/.
(O kompositsii veroyatnostnykh zakonov Gaussa i Puassona.- Russian)

PERIODICAL Doklady Akademii Nauk SSSR 1957, Vol 114, Nr 1, pp 21-24 (USSR).

ABSTRACT The paper under review contains a brief discussion of the proof of the following theorem which represents a generalisation of the well-known by H. Cramer and D.A. Raykov.
Theorem: The composition of the theorems by Gauss and Puasson /read Poisson/ can be decomposed only into the similar compositions, with the sum of the dispersions of the Gauss terms of sum being equal to the dispersion of the Gauss main component. The same is true also for the Puasson/read Poisson/ terms of sum.
First of all, the paper under review elaborates on the formulation of this theorem. Then a presupposition is given for the proof of this theorem. This proof is not based on the theorems by H.Cramer and D.A. Rykov; for the time being, it is very complicated and it bases itself on many facts of the theory of the whole functions as well as of the functions, summable in accordance with Lebesque,

CARD 1/2

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CARD 2/2

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu. V.

AUTHOR: LINNIK, Yu.V.
Corresponding Member AN USSR

TITLE: Some Theorems on the Decomposition of Unboundedly Divisible Laws (Nekotoryye teoremy o razlozhenii bezgranichno delimykh zakonov) 20-4-7/51

PERIODICAL: Doklady Akademii Nauk SSSR, 1957, Vol.116, Nr.4, pp.549-551 (USSR)

ABSTRACT: Let X be a random variable with the unboundedly divisible law F(x). For its characteristic function $\varphi(t)$ holds:

$$(1) \quad \ln \varphi(t) = \beta it - \gamma t^2 + \int_{-\infty}^{0} \left(e^{itx} - \frac{itx}{1+x^2}\right) dN(x) + \int_{0}^{\infty} \left(e^{itx} - \frac{itx}{1+x^2}\right) dN(x),$$

$$(2) \quad \ln \varphi(t) = \beta_1 it - \gamma t^2 + \sum_{m=1}^{N} \lambda_m \left(e^{it\mu_m} - 1\right),$$

where $N \leq \infty$, $\lambda_m > 0$, $\mu_m > 0$, $\sum \lambda_m$ converges. The characteristic function of the single Poisson component Y_m is $\exp\left(\lambda_m\left(e^{it\mu_m} - 1\right)\right)$, $\mu_m = \frac{D(Y_m)}{E(Y_m)}$. The spectrum is called rational if μ_m / μ_1 is rational for all m,1.

Theorem 1: Let F be an unboundedly divisible law with a bounded, positive, rational finite or countable Poisson spectrum and with $\gamma > 0$. In order that F has only unboundedly divisible components it is necessary and sufficient that the frequencies μ_m in (2) are identical with the decreasing number sequence

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$$(3) \qquad \mu, \frac{\mu}{k_1}, \frac{\mu}{k_1 k_2}, \frac{\mu}{k_1 k_2 k_3}, \ldots, \frac{\mu}{k_1 k_2 \ldots k_s}, \ldots,$$

where $\mu > 0$, k_i arbitrary (also equal) integers. Thus in the [illegible]

where $P_3(it)$ is a polynomial of at most third degree and $\phi(u)$ is a real function on $[0,a]$ summable in the square.
A further theorem gives the form of the characteristic functions in the case of a positive, bounded spectrum being rational at the right of b.
The proofs base on the theorem of Paley Wiener on the representation of entire functions of exponential type which on one axis belong to L_2, and on certain special functions due to Vinogradov

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[illegible]
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Card 4/4

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK Yu. V.

AUTHOR: LINNIK Yu.V., Corresponding Member, Acad.of Sci. USSR 20-5-5/48

TITLE: On the Decomposition of Unboundedly Divisible Laws (O razlozhenii bezgranichno delimykh zakonov)

PERIODICAL: Doklady Akad.Nauk SSSR, 1957,Vol.116,Nr.5,pp.735-738 (USSR)

ABSTRACT: The present paper is an extension of the results obtained in the preceding paper (Doklady Akad.Nauk,Ser.Mat.,1957,Vol.116,Nr.4). The notations are the same ones.
Theorem: In order that an unboundedly divisible law F with a Gaussian component ($\gamma > 0$) can be decomposed only into unboundedly divisible components it is necessary that its Poisson spectrum is finite or countable. Here the Poisson frequencies μ_m and ν_n in

$$(1) \quad \ln\varphi(t) = \beta i t - \gamma t^2 + \sum_{m=1}^{\infty} \lambda_m \left(e^{it\mu_m} - 1 - \frac{it\mu_m}{1+\mu_m^2}\right) + \sum_{n=1}^{\infty} \lambda_{-n} \left(e^{-it\nu_n} - 1 + \frac{it\nu_n}{1+\nu_n^2}\right)$$

have to be identical with the number sequences

$$(2) \quad [illegible]$$

and

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respectively. Here k_i and l_j are any integers > 1. If the spectra (2) and (3) are bounded, then the conditions are sufficient too. The theorem is generalized to the analytic direction as follows:
Theorem 2: For a sequence of real numbers $\{t_k\}$, $t_k \to 0$ tending to zero let

$$(\varphi_1(t_k))^{\alpha_1} \cdots (\varphi_s(t_k))^{\alpha_s} = \varphi(t_k),$$

where $\alpha_j > 0$, $\varphi_j(t)$ are characteristic functions of random variables, $\varphi(t)$ are characteristic functions of an unboundedly divisible law of the type (1) with a bounded Poisson spectrum. Then every $\varphi_j(t)$ $(j=1,2,\ldots,s)$ is of the type (1). Its Poisson spectrum is contained in (1). Two Soviet and 1 foreign references.

ASSOCIATION: Leningrad Sect. Inst.of Math.imeni V.A.Steklov,Acad. Sc.USSR (Leningradskoye otdeleniye matematicheskogo instituta im. V.A.Steklova A.N.SSSR)

SUBMITTED: April 4, 1957

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Card 2/2

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

LINNIK, Yu. V.

Linnik, Yu. V., and A.P. Khusu (Leningrad). Some Considerations Concerning the Statistical Analysis of the Roughness of Ground Profiles p. 144

Interchangeability, Accuracy and Measuring Methods in Machine Building, Moscow, Mashgiz, 1958, 251 pp. (Sbornik Nauchno-tekh. obshch. mashinostroitel'noy promyshlennosti, Leningradskoye oblast pravleniya, kn. 47).

This collection of articles deals with the topics discussed at the 3rd Leningrad Sci. and Engineering Conference on Interchangeability, accuracy and Inspection Methods in Machine-building and Instrument-making, held 18-22 Mar 1957.

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[illegible] and astronomic and geodetic measurements and observation.

COVERAGE: The book deals with the theory of the method of least squares based on new ideas and new achievements in mathematical statistics, with emphasis on the mathematical and statistical measuring of the data obtainable by this method. Computational procedures of the method of least squares are presented in

Card 1/10

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

Method (Cont.)

SOV/2126

numerous examples. Starting with the introduction of the necessary fundamentals from algebra, probability theory, and mathematical statistics, the author later discusses the theory of data processing of direct and indirect (conditional and unconditional) observations, including theorems of J. Neyman and F. David concerning the estimation of linear forms of basic parameters. Two chapters of the book are devoted to the theory of adjustments with the aid of elements and correlates. Certain cases of the processing of geodesic data are discussed and the theory of intersections is presented where application of confidence ellipses is somewhat of an innovation. Parabolic interpolation according to Chebyshev, certain studies of A. Wald concerning the adjustment of a series of points along a straight line, and certain additional data on the method of least squares including Gauss' formula, the theorem of A.N. Kolmogorov, A.A. Petrov, and Yu.M. Smirnov, and the Cauchy method of data processing, are the final items considered in this book. It should be noted that throughout the book the constructions of confidence intervals for the estimation of the

Card 2/10

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Method (Cont.) SOV/2126

[illegible] of measurement are given and the presentation of the theory is made in matrix form. The author expresses his gratitude to A. P. [illegible] for her help in preparing the book, especially for writing [illegible] chapter VI [illegible] computing [illegible]

[illegible]

[illegible]

[illegible]

Ch. 1. The Theory Information From Algebra

1. Vectors
2. Linear equations. Matrices 22
3. Certain theorems on determinants. Gram's determinant 23 31

Card 3/10

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

Method (Cont.)

SOV/2126

4. Symmetric matrices. Quadratic forms. Orthogonal matrices 35

Ch. II. Necessary Information From Probability Theory
1. Random values 39
2. Normal random vector 45
3. Linear functions of an n-dimensional normal vector 51
4. Reduction of a normal vector to simplest form. Correlation ellipsoid and ellipsoid of constant dispersion 61
5. Comparison of various normal distributions 65
6. Distributions found in mathematical statistics of random values associated with the normal distribution 67
7. Approximately normal distributions; their role in probability theory 75

Ch. III. Necessary Information From Mathematical Statistics
1. Sampling. Statistics
2. Estimation of parameters
3. [illegible]

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

2. Estimation by means of confidence intervals 94
3. Estimation of [illegible] of equiaccurate measurements 96
4. Examples 100
5. Observations which stand out sharply 102
6. Making Abbe's criterion more precise 108
7. Grouped direct equiaccurate measurements 110
8. Example 113

Ch. V. Direct Nonequiaccurate Observations 117
1. Statement of problem
2. Accurate estimation of a and σ^2 120
3. Estimation of a and σ^2 by confidence intervals 121
4. Examples 125
129

Card 5/10

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

Ch. VI. Indirect (Skew) Unconditional Measurements

The theorems of J. Neyman and F. David 183
3. Estimation of linear forms 187
4. Summary of formulas and the rule of estimating a linear function of the parameters 189
5. Particular cases encountered in practice. The problem on linear regression 190
6. Examples 196

Ch. VIII. Indirect (Skew) Conditional Measurements (Adjustment by Elements)
1. Statement of problem 201
2. Adjustment by elements according to the method of least squares 204
3. Rules of adjustment by elements 207

Ch. IX. Adjustment by Correlates
1. Statement of problem 210

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

Method (Cont.)

SOV/2126

2. Computing estimates by correlates 211
3. Proof of minimum 215
4. Statistical behavior of correlates and estimates 218
5. Various expressions for [$\tilde{p}v\ \tilde{v}$] and its statistical behavior 220
6. Estimation of γ_1 and Δ by means of confidence intervals 225
7. Estimation of a linear function from measurable parameters during skew observations 226
8. Comparison of equations by means of elements and correlates 228
9. Summary of formulas. Rules of adjustment by correlates 228
10. Examples 230

Ch. X. Certain Cases of Processing Observations in Geodesy
1. Adjustment of a single leveling traverse 237
2. Adjustment of leveling traverses based on a bench mark 240
3. Measurement of horizontal angles by the Gauss - Schreiber method 246

Card 8/10

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

Method (Cont.)

SOV 2126

Ch. XI. Estimation of the Results of Intersections and Restrictions
1. Intersection from more than two points. Confidence regions 255
2. Intersection from two points with repeated observations 263
3. Resection from many points. Confidence regions 269
4. Confidence regions in the Potenot problem during multiple measurements 271

Ch. XII. Parabolic Interpolation by the Method of Least Squares
1. Statement of problem 272
2. Normal equations. Orthogonal Chebyshev polynomials 273
3. Verification of the hypothesis on the presence of parabolic regression of a given order. Examples 279

Ch. XIII. Certain Studies by A. Wald. Straight Line of Orthogonal Regression and Its Application
1. Statement of problem. Consistent estimates 287
2. Confidence intervals 292
3. Grouping of observations 297
4. Line of orthogonal regression (gradient) and its application 298

Card 9/10

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

Method (Cont.)

SOV/2126

Ch. XIV. Additional Information on the Method of Least Squares
1. Confidence ellipsoids 300
2. Dependent observations 305
3. Role of the normal law in the theory of the method of least squares 307
4. Nonnormal vector of errors. One formula of Gauss. The theorem of A.N. Kolmogorov, A.A. Petrov, and Yu.M. Smirnov 311
5. Cauchy's method of processing observations 317

Bibliography [illegible]

Appendices [illegible]

AVAILABLE: Library of Congress [illegible]

Card 10/10 [illegible]

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

AUTHOR: Linnik, Yu. V. (Leningrad)

52-III-1-1/9

TITLE: General Theorems on the Factorization of Infinitely Divisible Laws. I. Formulation. Three Fundamental Lemmas. Necessary Conditions.
(Obshchiye teoremy o razlozhenii bezgranichno delimykh zakonov. I. Formulirovki. Tri osnovnyye lemmy. Neobkhodimyye usloviya.)

PERIODICAL: Teoriya veroyatnostey i yeye primeneniya, 1958, Vol.III, Nr.1, pp.3-40. (USSR).

ABSTRACT: About 20 years ago the well-known work of G. Kramer (Cramer)(Ref.1) on the factorization of the normal law initiated a series of investigations on possible factorizations for some forms of infinitely divisible laws. Papers on this topic were written by P. Levi (Lévy) (Refs.2-3), A.Ya. Khinchin (Ref.4), D.A. Raykov (Refs.5-6), R.A. Fisher and D. Dyuge (Dugué) (Ref.7). But these investigations were concerned either with the normal law or with compositions of a finite number of laws of Poisson type. [illegible]

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law is given by

$$\ln \varphi(t) = \beta i t - \gamma t^2 + \int_{-\infty}^{0} \left(e^{itu} - 1 - \frac{itu}{1+u^2}\right) dG_-(u) +$$

$$+ \int_{0}^{\infty} \left(e^{itu} - 1 - \frac{itu}{1+u^2}\right) dG_+(u), \qquad \text{Eq.0.1}$$

where the real branch of $\ln u$ is taken for positive w; β and $\gamma \geqslant 0$ are real constants; $G_-(u)$ and $G_+(u)$ are non-vanishing functions such that $G_-(-\infty) = G_+(\infty) = 0$

Card 2/12

and $\int_{-a}^{0} u^2 dG_-(u) + \int_{0}^{a} u^2 dG_+(u) < \infty$ for any finite $a > 0$.

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

General Theorems on the Factorization of Infinitely Divisible Laws. I. 52-III-1-1/9

$$\beta = \beta_1+\beta_2; \quad \gamma = \gamma_1+\gamma_2; \quad G_- = G_-^{(1)}+G_-^{(2)}; \quad G_+ = G_+^{(1)}+G_+^{(2)}. \qquad \text{(Eq.0.3)}$$

Whereas the components F_1 and F_2 in the factorization

$$F = F_1 * F_2 \qquad \text{(Eq.0.4)}$$

for $F \in I$ need not belong to I for all forms of the characteristic function of the type (0.1). There arises the question of describing all divisible laws F which permit a factorization only into infinitely divisible components. The class of all such laws is denoted by I_0; $I_0 \subset I$. If in Eq.0.1 $\gamma \neq 0$, then it may be said that the corresponding law F has a Gaussian component. The functions $G_-(u)$ and $G_+(u)$ are the negative and positive parts of the Poissonian spectrum of the law F. The concepts of bounded, continuous, countable or finite Poissonian spectra are introduced, in particular in the case of the finite P spectrum wave:

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APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

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General Theorems on the Factorization of Infinitely Divisible Laws: I.

where $\lambda_m \geq 0$, $\lambda_{-n} \geq 0$, and the series

$$\sum_{m=1}^{\infty} \frac{\lambda_m \mu_m^2}{1 + \mu_m^2} \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{\lambda_{-n} \nu_n^2}{1 + \nu_n^2} \qquad \text{(Eq.0.9)}$$

converge while

$$\sum_{\mu_m < \varepsilon} \lambda_m \mu_m^2 + \sum_{\nu_n < \varepsilon} \lambda_{-n} \nu_n^2 \longrightarrow 0 \qquad \text{(Eq.0.10)}$$

as $\varepsilon \longrightarrow 0$. An interesting class of infinitely divisible lots is composed of lots with a bounded P spectrum. Their characteristic function has the form

$$\ln\varphi(t) = \beta it - \gamma t^2 + \int_{-b}^{0} (e^{itu} - 1 - itu)\, dG_1(u) + \int_{0}^{a} (e^{itu} - 1 - itu) \times$$

Card 6/12

$$\times\, dG_2(u), \qquad \text{(Eq.0.11)}$$

APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2"

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

where $a \geqslant 0$, $b \geqslant 0$ are bounded numbers; $G_1(u)$ and $G_2(u)$ are non-vanishing functions which are bounded in the segment $[-\varepsilon, \varepsilon]$ for any $\varepsilon > 0$ such that the sum

$$\int^{\varepsilon} u^2 dG_1(u) + \int^{\varepsilon} u^2 dG_2(u) \qquad (\text{Ma}, 0, 1)$$

[illegible]

[illegible] we note that the separate elements of this set

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have been indicated in several papers. For example, the Gaussian law $G \in I_0$ (Cramer, Ref.1), the Poissonian law $P \in I_0$ (Raykov, Refs. 5,6); and the composition of the Gaussian and Poissonian laws belongs to I_0 (Linnik, Refs.11,12).

Theorem 1. For an infinitely divisible law with a Gaussian component $\gamma > 0$ to decompose only into infinitely divisible components it is necessary that its Poissonian spectrum be finite or countable. Moreover, the corresponding Poissonian frequencies μ_m and ν_n must coincide with the following series of numbers:

For μ_m,

[illegible]

[illegible] ponent is essential. If the Poissonian spectrum is not bounded, it is not known whether the necessary condition is also sufficient. It is proved only that if the high frequency energy is sufficiently small (theorem 4), it is sufficient if

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General Theorems on the Factorization of Infinitely Divisible Laws. I.

$$\log\log\frac{1}{\lambda_m} > c\mu_m^{1+a}\ ;\quad \log\log\frac{1}{\lambda_{-n}} > c\nu_n^{1+a}$$

for some $c > 0$, $a > 0$ and sufficiently large μ_m and ν_n.

Theorem 2. If the Poissonian spectrum of the law is bounded so that $dG_-(u) = 0$ $(u < -b)$; $dG_+(u) = 0$ $(u > a)$, then all its components have char-

[illegible]

[illegible] components have characteristic function of the form

$$\log\varphi(t) = P_3(it) + t^4\int_{-b}^{a} e^{itu}\Phi(u)du + \sum_{n=1}^{q}(\alpha_n+\beta_n it)\ \times$$

$$\times\ (\exp(it\frac{n}{q}\mu) - 1),$$

where α_n, β_n are real numbers (β_n necessarily positive), $\Phi(u) \in L^2(-b, a)$; $P_3(it)$ a polynomial of degree ≤ 3.

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A "stability theorem" (theorem 5) is also proved: Let I_1 be the set of all infinitely divisible laws with bounded Poissonian spectra satisfying the necessary condition of the theorem 1 (the Gaussian component may be absent). Let

"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

52-III-1-1/9

General Theorems on the Factorization of Infinitely Divisible Laws. I.

$F \in I_1$ and K_F be the set of all infinitely divisible laws with the Poissonian spectra contained in the spectrum of F. Let $\{F_j\}$ be the sequence of laws such that:

$$\sup_x \left| F_j(x) - F(x) \right| = \varepsilon_j \to 0, \text{ as } j \to \infty$$

and $F_j = F_{1j} * F_{2j}$ (composition). Then

$$\inf_{F^{(1)} \in K_F} \sup_x \left| F_{1j}(x) - F^{(1)}(x) \right| = \delta_j \to 0.$$

There are 15 references, of which 1 is English, 1 German, 4 French and 9 Soviet.

SUBMITTED: September 5, 1957.

AVAILABLE: Library of Congress.

Card 12/12 1. Lemmas-Theory 2. Factor analysis 3. Algebraic functions 4. Logarithmic functions

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

AUTHOR: [illegible]

TITLE: More on the generalizations of H. Cramer's [illegible] (Yeshcheob obobshcheniyakh teoremy G. Kramera)

PERIODICAL: Vestnik Leningradskogo Universiteta, Seriya Matematiki, Mekhaniki i Astronomii, 1958, Nr 1(1), pp. 39-44 (USSR)

ABSTRACT: Let the class B consist of all functions $V(x)$ which are of bounded variation for $-\infty < x < \infty$ and for which it is $\int_{-\infty}^{\infty} d\,V(x) = 1$. Let $G(x)$ be a normal integral law with mean value 0 and dispersion 1. The authors consider decompositions $G(x) = \int_{-\infty}^{\infty} V_1(x-z)\,d\,V_2(z) = V_1(x) * V_2(x)$ where $*$ is the composition sign. Let $\mathrm{Var}\, V(x) \Big|_a^b$ denote the total variation of $V(x)$ on $[a,b]$.

Theorem: Let $G(x) = V_1(x) * V_2(x)$, $V_i(x) \in B$ $(i = 1,2)$,

$$\mathrm{Var}\, V_i(x) \Big|_y^{\infty} = O(\exp(-y^{1+\gamma})),\quad \mathrm{Var}\, V_i(x) \Big|_{-\infty}^{-y} = O(\exp(-y^{1+\gamma})),$$

Card 1/3

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More on the Generalizations of H. Cramer's Theorem 43-1-3/10

$y > 0$, $\lambda > 0$. Then $V_1(x)$ and $V_2(x)$ are normal probability distributions. On the other side there exist $W_1(x) \in B$, so that $G(x) = W_1(x) + W_2(x)$ and $\mathrm{Var}\, W_1(x) \int_0^{\infty} \sigma \exp(-\lambda \ln(\lambda+1))$, $\mathrm{Var}\, W_1(x) \int_0^{\infty} \sigma \exp(-\lambda \ln(\lambda+1))$, [illegible] is a probability distribution [illegible]

[illegible]

$a_1, b_1 \neq 0$ are normal probability distributions.

Card 2/3

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More on the Generalizations of H. Cramer's Theorem 43-1-3/10

1 Soviet and 8 foreign references are quoted.

SUBMITTED: 24 April 1957

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1. Integral functions 2. Cramers theory 3. Probability distributions

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"APPROVED FOR RELEASE: 07/12/2001 CIA-RDP86-00513R000930010015-2

AUTHOR: Linnik, Yu.V., Corresponding Member of the Academy of Sciences of the USSR 20-120-5-8/67

TITLE: The Dispersion of the Divisors and Quadratic Forms in Arithmetic Series and Some Binary Additive Problems (Dispersiya deliteley i kvadratichnykh form v progressiyakh i nekotoryye binarnyye additivnyye zadachi)

PERIODICAL: Doklady Akademii nauk SSSR, 1958, Vol 120, Nr 5, pp 960-962 (USSR)

ABSTRACT: Let $\tau(n) = \tau_2(n) = \sum_{x_1 x_2 = n} 1$ be the number of divisors of n;

[illegible]

it holds:

$$(1) \quad \sum_{\substack{m \equiv 1 (\mathrm{mod}\ D) \\ m \leq n}} \tau(m) = n \ln n + \frac{\varphi(D)}{D^2} \varrho(n, D) + O(n^{1-\alpha_1}),$$

$$\varrho(n, D) = 1 - \frac{1}{\ln n} \left(1 - 2C_0 + 2 \sum_{p/D} \frac{\ln p}{p-1}\right),$$

$$(2) \quad \sum_{\substack{Q \equiv n (\mathrm{mod}\ D) \\ Q \leq n}} 1 = \frac{2\pi n}{\sqrt{|\Delta|}\, D} \sigma(n, D) + O(n^{1-\alpha_1}).$$

Here $\sigma(n,D)$ is defined as follows: Let $(n,D) = d_1$; $D = D_1 d_1$; $d_1 = d_{11} d_{12}$, where d_{11} consists of the prime divisors p/D_1 and $(d_{12}, D_1) = 1$; let further $X_d(m) = (d/m)$. Then

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