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TECHNICAL NOTE

TARGET RESEARCH:

COMPUTER PROGRAM DOCUMENTATION

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## SECTION I

### INTRODUCTION

This technical note is divided into two main parts, each pertaining to target research through computer programs. The first part is devoted to the theory and application of a solution to background error present in the STFC (system transfer function calculations) program. The program COMBGEN (Modified Dirac Comb generator) is documented in the second main part of the report. COMBGEN facilitates the generation of realistic modified Dirac Comb traces for a system having a known modulation transfer function (MTF). Results of COMBGEN are input to the STFC program.

## SECTION II

## SOLUTION TO BACKGROUND ERROR

## A. THEORY

Conceptually, the optical transfer function of a photographic system is given by relating a known ground target,  $g(x)$ , to its corresponding image on film,  $h(x)$ . This relationship is given by

$$T(f) = \frac{F\{h(x)\}(f)}{F\{g(x)\}(f)} \quad (1)$$

where  $F$  denotes the Fourier transform operator and  $f$  is spatial frequency.

Since neither  $g$  nor  $h$  is available as a continuous function, we estimate  $T$  by

$$\hat{T}(f), \text{ given by } \hat{T}(f) = \frac{\sum_{l=-M}^M h(l \Delta x) \exp(-2\pi i f l \Delta x)}{\sum_{l=-M}^M g(l \Delta x) \exp(-2\pi i f l \Delta x)} \quad (2)$$

For convenience, the function  $h(x)$  has background zero. Let us denote, by  $h'(x)$ , the image in effective exposure as it is actually measured. Since  $h'(x)$  involves a background exposure  $E_B$ , in order to use Equation (2), we must set

$$h(x) = h'(x) - E_B \quad (3)$$

Let us now suppose that an error has been made in the calculation of  $E_B$ . Suppose we have estimated  $E_B$  by  $\hat{E}_B$  and

$$\hat{E}_B = E_B - \epsilon, \quad (4)$$

where  $\epsilon$  is the error which has been made. Let us denote by  $\bar{T}$  the new value of  $\hat{T}$ , which is due to this background error. We will show that it is possible to cause

$$\bar{T}(f) = \hat{T}(f) \quad (5)$$

at frequencies of interest.

If we denote by  $\hat{h}(x)$  the image function involving the background error, then

$$\bar{T}(f) = \frac{\sum_{l=-M}^M \hat{h}(l \Delta x) \exp(-2\pi i f l \Delta x)}{\sum_{l=-M}^M g(l \Delta x) \exp(-2\pi i f l \Delta x)} \quad (6)$$

Now,

$$\begin{aligned} \hat{h}(x) &= h'(x) - \hat{E}_B \\ &= h'(x) - (E_B - \epsilon) \\ &= h'(x) - E_B + \epsilon \\ &= h(x) + \epsilon \end{aligned}$$

Hence, we may rewrite Equation (6) as

$$\begin{aligned} \bar{T}(f) &= \frac{\sum_{l=-M}^M [h(l \Delta x) + \epsilon] \exp(-2\pi i f l \Delta x)}{\sum_{l=-M}^M g(l \Delta x) \exp(-2\pi i f l \Delta x)} \\ &= \frac{\sum_{l=-M}^M h(l \Delta x) \exp(-2\pi i f l \Delta x) + \epsilon \sum_{l=-M}^M \exp(-2\pi i f l \Delta x)}{\sum_{l=-M}^M g(l \Delta x) \exp(-2\pi i f l \Delta x)} \end{aligned} \quad (7)$$

If we let

$$G(f) = \sum_{l=-M}^M g(l \Delta x) \exp(-2\pi i f l \Delta x),$$

we may rewrite Equation (7) as

$$\bar{T}(f) = \hat{T}(f) + \frac{\epsilon}{G(f)} \sum_{l=-M}^M \exp(-2\pi i f l \Delta x).$$

Moreover, if we let  $\alpha = 2\pi f \Delta x$ , we may write

$$\begin{aligned} \bar{T}(f) &= \hat{T}(f) + [\epsilon/G(f)] \sum_{j=-M}^M \exp(-i j \alpha) \\ &= \hat{T}(f) + [\epsilon/G(f)] \exp(-i M \alpha) \sum_{j=0}^{2M} \exp(i j \alpha) \\ &= \hat{T}(f) + [\epsilon \exp(-i M \alpha)/G(f)] \frac{1 - \exp[i(2M+1)\alpha]}{1 - \exp(i \alpha)} \end{aligned}$$

$$\begin{aligned}
&= \hat{T}(f) + [e/G(f)] \frac{\exp(-iM)\alpha - \exp[i(M+1)\alpha]}{1 - \exp(i)\alpha} \\
&= \hat{T}(f) + [e/G(f)] \frac{\exp[-i(M+\frac{1}{2})\alpha] - \exp[i(M+\frac{1}{2})\alpha]}{\exp(-i\alpha)/2 - \exp(i\alpha)/2} \\
&= \hat{T}(f) + [e/G(f)] \frac{\sin(M+\frac{1}{2})\alpha}{\sin \frac{1}{2}\alpha} \\
&= \hat{T}(f) + [e/G(f)] \frac{\sin[(2M+1)\pi f \Delta x]}{\sin(\pi f \Delta x)} \quad (8)
\end{aligned}$$

Finally, observe that when  $f$  is a multiple of  $1/(2M+1)\Delta x$ , i.e.,

$$f = \frac{k}{(2M+1)\Delta x} \quad (9)$$

we may rewrite Equation (8) as

$$\bar{T}(f) = \hat{T}(f) + [e/G(f)] \sin k\pi \sin \frac{k\pi}{(2M+1)}$$

$$\bar{T}(f) = \hat{T}(f)$$

Notice in Equation (9) that  $k$  is a function of the record length  $L$ , where  $L$  is given by

$$L = (2M+1)\Delta x$$

## B. APPLICATION

Since the error introduced in the Fourier transform of the image due to an error in calculating the background is a function of the record length, we may virtually eliminate this error in the following way.

In the STFC program, Fourier transforms are calculated at multiples of the fundamental frequency of the target on the ground. This fundamental frequency is 0.05 lines/ft. Further, the distance between the first and last bars in the target is 120 feet. Utilizing these facts in Equation (9), we obtain

$$\frac{0.05}{\text{ft.}} = \frac{k}{L \text{ ft.}} \quad (10)$$

where  $L$  is the desired record length. Solving Equation (10) for  $L$ , we obtain

$$L = \frac{k}{0.05}$$

Since it is required that  $L > 120$ , the proper choice for  $L$  is

$$L = 140 \text{ feet} \quad (11)$$

Since this value of  $L$  represents a one-sixth increase in the target length, the STFC program adjusts the record length of the target trace by estimating the centers of the first and last bars ( $\bar{x}_1, \bar{x}_2$ ). Then the record length is adjusted by defining the left-hand endpoint of the trace,  $MY$ , to be

$$MY = \bar{x}_1 - \frac{(\bar{x}_2 - \bar{x}_1)}{12}$$

and the right-hand endpoint,  $LY$ , to be

$$LY = \bar{x}_2 + \frac{(\bar{x}_2 - \bar{x}_1)}{12}$$

It is possible that  $MY \leq 0$  or that  $LY$  exceeds the number of points in the trace. Such traces are rejected by the STFC program.

As a final check on the background correction outlined above, the STFC program was run with an iteration on various values for the background exposure. The MTF varied significantly with the change in background exposure. A plot of the MTF associated with each background level is given in Figure 1. Next, the record length correction was inserted and the program rerun. The calculated MTF was almost precisely the same, regardless of the background exposure, demonstrating that the background problem was virtually eliminated.

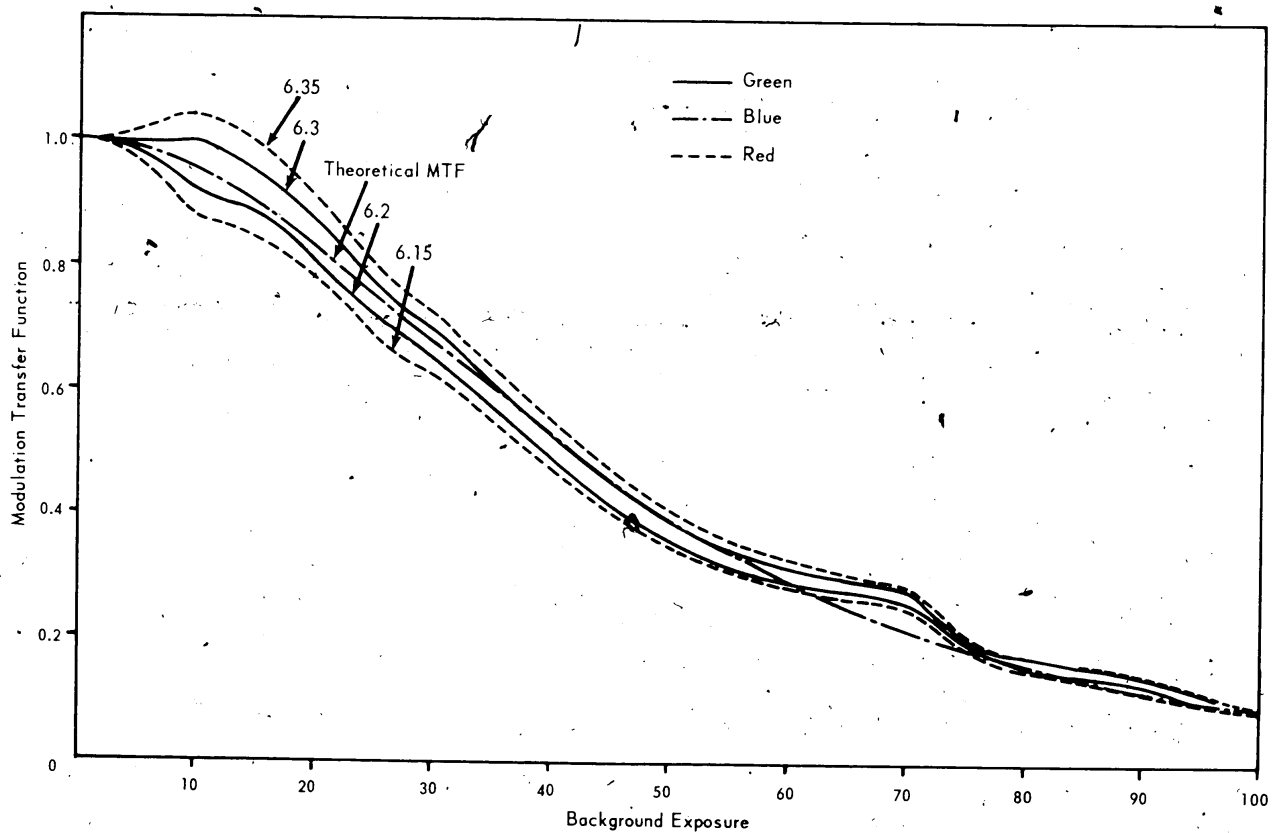


Figure 1. Theoretical MTF and Results of Looping on Background



## SECTION III

## COMBGEN PROGRAM DOCUMENTATION

The modified Dirac Comb target can be considered as the sum of 31 rectangular unit pulses of the form

$$\varphi(a, w, x), \quad (12)$$

where  $a$  is the center of the pulse and  $w$  is the width. Hence,

$$\varphi(a, w, x) = \begin{cases} 1 & \text{for } |x - a| \leq w/2 \\ 0 & \text{otherwise.} \end{cases}$$

If we denote the target by  $T(x)$ , we may write

$$T(x) = \sum_{i=1}^{31} \varphi(a_i, w, x). \quad (13)$$

If we denote the impulse response (spread function) of the system by  $S(x)$ , then the image of  $T(x)$ , denoted by  $g(x)$ , is given by

$$g(x) = T(x) * S(x), \quad (14)$$

where the asterisk denotes the convolution operation.

The particular spread function selected was

$$S(x) = 0.1 \operatorname{sech}(0.1\pi x).$$

Hence, the MTF of the system, denoted by  $F(f)$ , is given by

$$F(f) = \frac{|F\{S(x)\}(f)|}{|F\{S(x)\}(0)|} \quad (15)$$

$$F(f) = \operatorname{sech}(10\pi f),$$

where  $f$  is spatial frequency in lines/micron.

We may now rewrite Equation (14) as

$$\begin{aligned} g(x) &= T(x) * 0.1 \operatorname{sech}(0.1\pi x) \\ &= \sum_{i=1}^{31} \varphi(a_i, w, x) * 0.1 \operatorname{sech}(0.1\pi x) \end{aligned}$$

$$\begin{aligned}
&= \sum_{i=1}^{31} \int_{-\infty}^{\infty} \wp(a_i, w_i, \mu) .1 \operatorname{sech} [.1\pi(x-\mu)] du \\
&= \sum_{i=1}^{31} \int_{a_i - \frac{w_i}{2}}^{a_i + \frac{w_i}{2}} .1 \operatorname{sech} [.1\pi(\mu-x)] du \\
&= \sum_{i=1}^{31} \frac{2}{\pi} \arctan \{ \exp [.1\pi(\mu-x)] \} \Big|_{a_i - \frac{w_i}{2}}^{a_i + \frac{w_i}{2}} \\
&= 2/\pi \sum_{i=1}^{31} \left\{ \arctan \{ \exp (.05\pi w_i) \exp [.1\pi(a_i-x)] \} \right. \\
&\quad \left. - \arctan \{ \exp (-.05\pi w_i) \exp [.1\pi(a_i-x)] \} \right\} \quad (16)
\end{aligned}$$

The various values of  $a$  and  $w$  for the target on the ground are given in Table I.

The function  $g(x)$  is translated to the actual image of  $T(x)$  on film by correcting each

$a_i$  and  $w_i$  for the scale factor  $S_x$ :

$$\frac{a_i}{S_x} = \frac{304800 \mu}{\text{foot}},$$

$$\frac{w_i}{S_x} = \frac{304800 \mu}{\text{foot}}.$$

Further, the Micro-Analyzer sampling rate is simulated in  $g(x)$  by generating this function at sample spacing of  $\Delta x$  microns:

$$\begin{aligned}
g(n \Delta x) = 2/\pi \sum_{i=1}^{31} \left\{ \arctan \{ \exp (.05\pi w_i) \exp [.1\pi(a_i - n \Delta x)] \} \right. \\
\left. - \arctan \{ \exp (-.05\pi w_i) \exp [.1\pi(a_i - n \Delta x)] \} \right\} \quad (17)
\end{aligned}$$

Recall that the function  $T$  (and hence the function  $g$ ) was normalized to range from zero to one; i.e., the background of the target has a value of zero and a bar of the target has a value of one. Once the function  $g$  has been constructed by Equation (17), the actual exposure levels can be set by redefining  $g$  by

$$g(n \Delta x) = \alpha + \beta g(n \Delta x), \quad (18)$$

where  $\alpha$  is the background exposure and  $(\beta - \alpha)$  is the exposure level of a bar of the target.

In order to convert  $g$  from exposure to density, the COMBGEN program uses the film response function

$$H(x) = b + 2 \exp [-(x-3)^2], \quad (19)$$

TABLE I. FIELD TARGET SPECIFICATIONS

POSITION RELATIVE  
TO CENTER

WIDTH OF BAR

<i>a</i>	<i>w</i>
-60 ft.	8 in.
-57 ft. 6 in.	2 in.
-52 ft. 8 in.	6 in.
-47 ft. 6 in.	2 in.
-42 ft. 8 in.	6 in.
-40 ft.	8 in.
-37 ft. 6 in.	2 in.
-32 ft. 8 in.	6 in.
-27 ft. 6 in.	2 in.
-22 ft. 8 in.	6 in.
-20 ft.	8 in.
-17 ft. 6 in.	2 in.
-12 ft. 8 in.	6 in.
- 7 ft. 6 in.	2 in.
- 2 ft. 8 in.	6 in.
0 ft.	8 in.
2 ft. 6 in.	2 in.
7 ft. 4 in.	6 in.
12 ft. 6 in.	2 in.
17 ft. 4 in.	6 in.
20 ft.	8 in.
22 ft. 6 in.	2 in.
27 ft. 4 in.	6 in.
32 ft. 6 in.	2 in.
37 ft. 4 in.	6 in.
40 ft.	8 in.
42 ft. 6 in.	2 in.
47 ft. 4 in.	6 in.
52 ft. 6 in.	2 in.
57 ft. 4 in.	6 in.
60 ft.	8 in.

where  $b = 0.29$  and  $x$  is given in relative log exposure units.

Once  $g$  has been converted to density through Equation (19), realistic grain noise is added to  $g$  in the same way as in the general DATGEN<sup>1</sup> program.

Finally, the function  $g$  is punched on cards for use with the STFC program.

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