3. BASIC CONCEPTS OF HYPERSONIC LIFTING VEHICLES

A hypersonic glide vehicle achieves range by exchanging its kinetic energy, plus a small amount of potential energy, for distance. The kinetic energy is used to overcome the aerodynamic drag forces encountered during the course of the flight. Thus the aeroballistic glide concept differs from the conventional notion of gliding flight in that, for the latter case, the main source of energy is the potential energy, and altitude is exchanged for distance rather than velocity.

The derivation of the equation for the aeroballistic glide range is based on the fundamental principle that the work done is equal to the change in energy. For a vehicle having some initial velocity and moving through a resisting medium, the work accomplished is equal to the resisting force times the distance covered. Hence, if the resisting force is known and the initial and final energy levels are specified, the distance may be determined. In the case of a glide vehicle the resisting force is the drag, and the initial and final energy is determined from the initial and final glide velocities. This neglects a contribution of potential energy which is small with respect to the kinetic energy. The differential equation expressing this energy relationship is

$$D ds = -dE \tag{1}$$

where D is the aerodynamic drag, s is the distance along the flight path, and E is the kinetic energy. Since the flight-path angle is small, the distance along the flight-path, s, may be assumed to be equal to the range, R, so that equation (1) becomes

$$R = -\int_{E_i}^{E_f} \frac{dE}{D} \qquad (2)$$

where the limits of integration, E_i and E_f, are the initial and final values of kinetic energy of the missile, respectively.

Since the range is assumed to be accomplished solely due to the change in kinetic energy, $KE = 1/2\ mV^2$, the differential, dE, in equation (1) is

$$dE = 1/2\ mdV^2 \qquad (3)$$

Also the drag, D, in equation (2) may be expressed as

$$D = \frac{L}{(L/D)} \qquad (4)$$

where L/D is the lift-drag ratio of the vehicle in glide. Now in equilibrium flight over a spherical earth, the aerodynamic lift, (L), must be equal to the weight of the vehicle minus the centrifugal relieving effect, or

$$L = W - \frac{W}{g} \frac{V^2}{(r_o+h)} \qquad (5)$$

$a = \frac{V^2}{R}$

$g = \frac{V_s^2}{(R_0+h)}$

where L is the lift

g local acceleration of gravity
W the weight
r_o the radius of the earth = 20.9 x 10^6 feet
h the altitude of the missile above the earth

Substituting equations (3), (4), and (5) into equation (2), the range equation becomes

$$R = -\int_{V_i}^{V_f} \frac{(L/D)\, dV^2}{2g\left[1 - \frac{V^2}{g(r_o+h)}\right]} \qquad (6)$$

This may readily be integrated, if (L/D) is assumed constant, to

$$R = (1/2)(L/D)(r_o+h)\ \ln \left[\frac{1 - \frac{V_f^2}{g(r_o+h)}}{1 - \frac{V_i^2}{g(r_o+h)}}\right] \qquad (7)$$

To gain a better insight into the effect of the primary parameters, (L/D) and V, on range, the equation may be further simplified by noting that the quantity $V^2/g(r_o+h)$ is less than 1.0 for the region of velocities of interest allowing the logarithm term to be expanded in a series.

$$R = \frac{(L/D)}{2g} \sum_{n=1}^{n=\infty} \frac{V_i^{2n} - V_f^{2n}}{ng^{n-1}\,(r_o+h)^{n-1}}$$

A first-order approximation to the glide range is thus obtained by neglecting all terms in the series except the first term which gives

$$R = \frac{(L/D)}{2g} \quad (V_i^2 - V_f^2) \tag{8}$$

This simplification amounts to omitting the effects of the earth's curvature, i.e., the effects of centrifugal lift. For the longer ranges, this results in some appreciable conservatism. Now by assuming the glide is maintained to near zero velocity, i.e., $V_f = 0$

$$R = 1/2 \ (L/D) \ \frac{V_i^2}{g} \tag{9}$$

This shows clearly the sensitivity of glide range to the initial velocity and lift-drag ratio.

The flight altitude for glide vehicles is shown in figure (1) as a function of velocity and the parameter W/SC_L. These results are obtained from the relation established by equating the aerodynamic lift on the missile to the fraction of the weight not sustained by centrifugal forces

$$L = W \left(1 - \frac{V^2}{V_s^2}\right) = C_L \frac{\rho V^2}{2} S \tag{10}$$

where V_s is orbital velocity, and from whence the density becomes

$$\rho = 2 \frac{W}{SC_L} \left[\frac{1}{V^2} - \frac{1}{V_s^2}\right] \tag{11}$$

The fraction of the weight supported by lift is shown as a function of velocity in figure (2).

GLIDE ALTITUDE FOR LIFTING RE-ENTRY VEHICLES

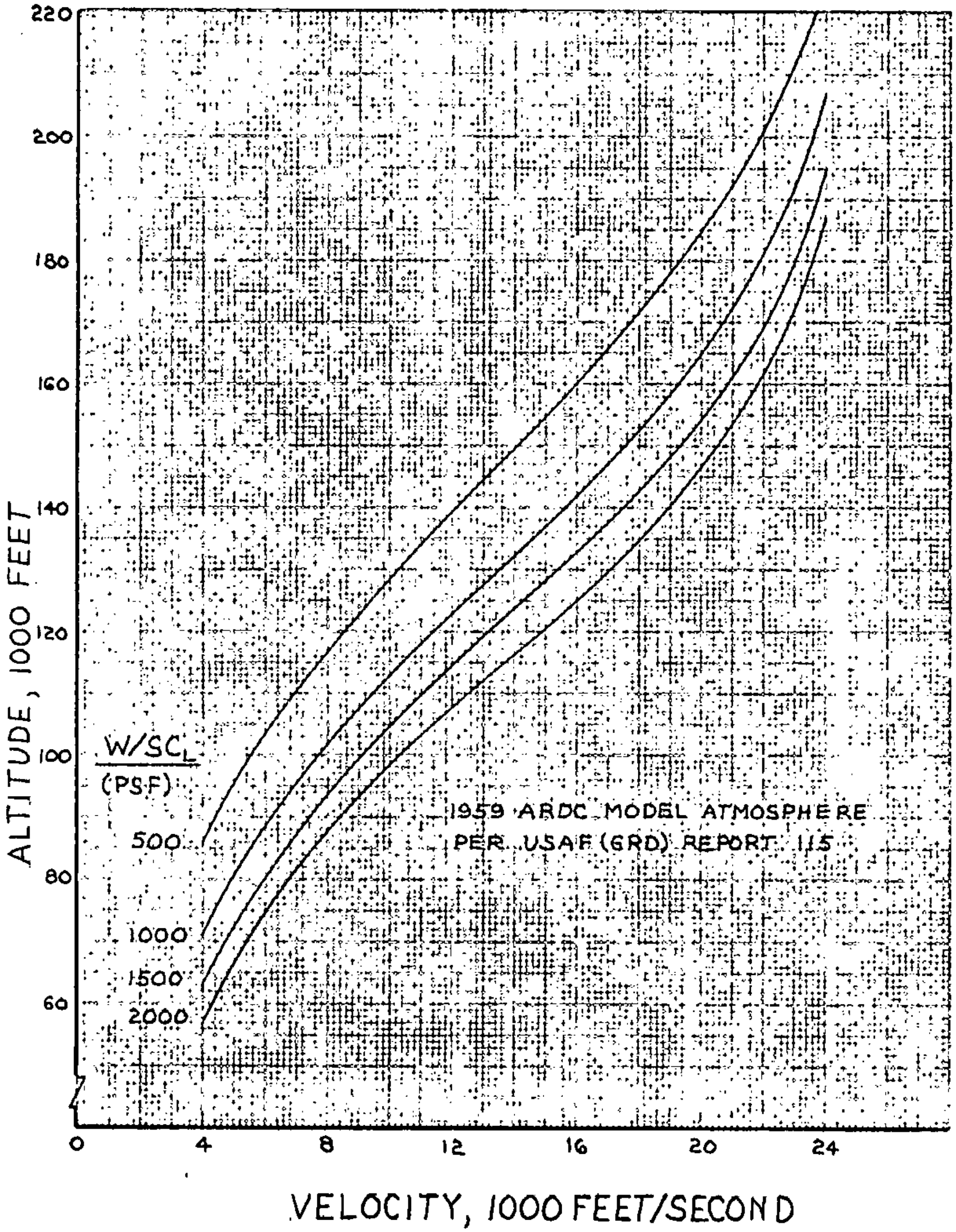


FIG. 1

HYPERSONIC GLIDE LIFT-TO-WEIGHT RATIO

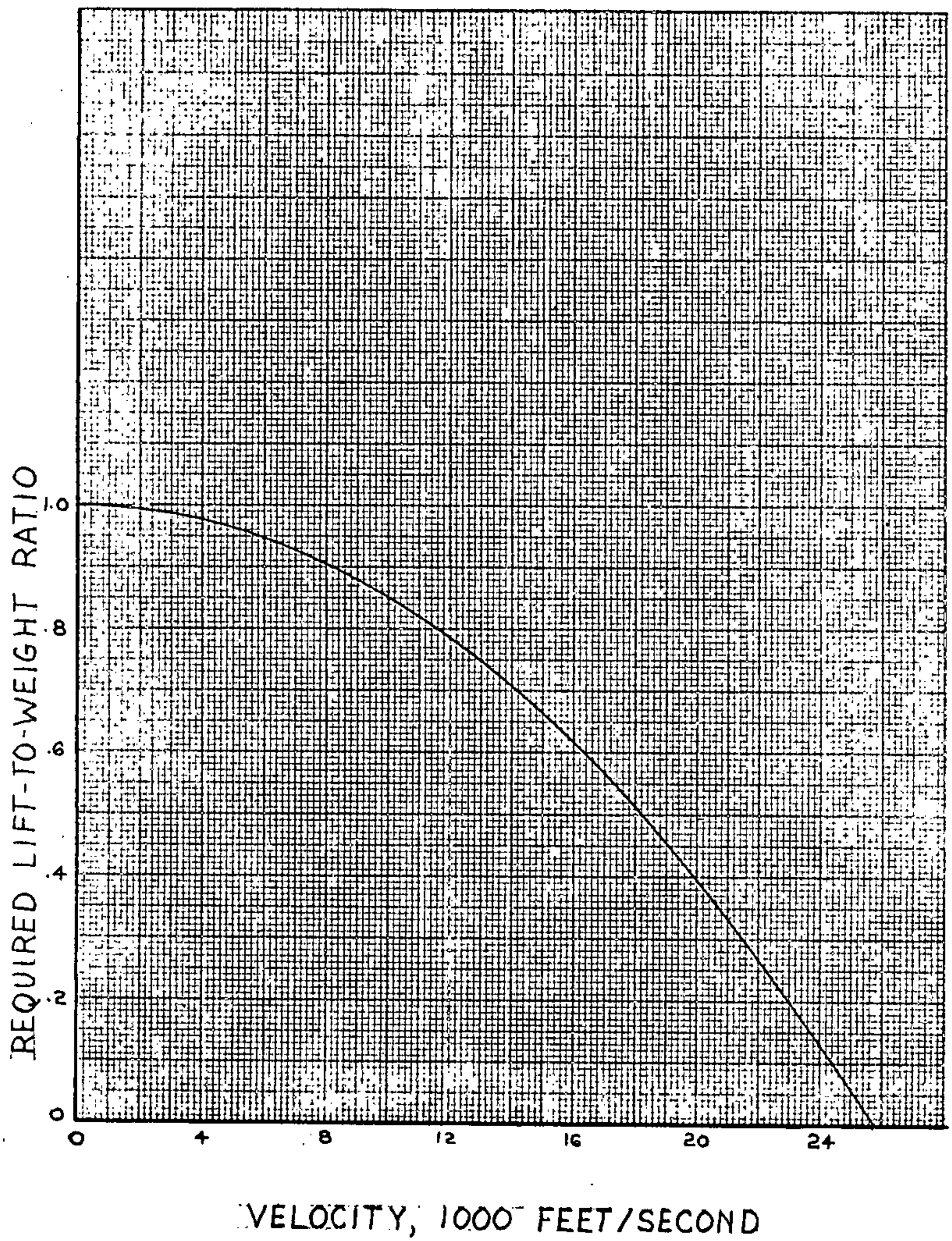


FIG. 2

$\frac{V}{R}$ = NORMAL A...

$\left(\frac{V}{R}\right) = \dot{\gamma}$

$\frac{mV^2}{R} = m\left(\frac{V}{R}\right)V = m\dot{\gamma}V = \frac{W}{g}\dot{\gamma}V$

HIGH SPEED DESCENTS

General calculations are complex but a few simplifying assumptions can yield results that can be used for approximate checking. First by assuming the gravity gradient, the planet rotation rate, and airplane bank angle to be zero, the equations of motion are:

$$\dot{V} = -\frac{D}{W}g - g\sin\gamma$$

$$\dot{\gamma} = \frac{g}{V}\left[\frac{L}{W} - \left(1 - \frac{V^2}{gR_o}\right)\cos\gamma\right]$$

$$\dot{h} = V\sin\gamma$$

At very high speeds the flight path angle, γ, is very low, and its rate of change is small, then,

$$\gamma = 0, \quad \cos\gamma = 1.0 \qquad \frac{L}{W} = 1 - \frac{V^2}{gR_o}$$

and

$$\dot{V} = V\frac{dV}{ds} = -\frac{D}{L}\frac{L}{W}g - \frac{g}{V}\dot{h}$$

$$V\,dV = -\frac{D}{L}\frac{L}{W}g\,ds - g\,dh$$

Now, we must somehow relate dV and dh. A variety of assumptions are ameniable to calculation and it makes little difference which we choose. Say,

$$q = \text{CONSTANT}$$
$$= K_1 V^2 e^{bh} \quad \text{(APPROX)}$$

thus

$$dh = -\frac{2}{b}\frac{dV}{V}$$

between $100K$ & $200K$ $\quad -\frac{1}{b} \approx 23{,}000$

Now,

$$V\,dV = -\frac{D}{L}\frac{L}{W}g\,ds - g\,(46000)\frac{dV}{V}$$

and then (Ro = EARTH RADIUS)

$$-\frac{R_o}{2}\frac{\left(-\frac{2V\,dV}{gR_o}\right)}{\left(1-\frac{V^2}{gR_o}\right)} + \frac{46000\; d\frac{V}{(gR_o)^{1/2}}}{\frac{V}{(gR_o)^{1/2}}\left(1-\frac{V^2}{gR_o}\right)} = -\frac{D}{L}\,ds$$

Integrating, we get for small $\frac{D}{L}$ variation,

$$\left\{-\frac{R_o}{2}\ln\left(1-\frac{V^2}{gR_o}\right) + \frac{46000}{2}\ln\left[\frac{\frac{V^2}{gR_o}}{1-\frac{V^2}{gR_o}}\right]\right\}_{V_1}^{V_2} = \left(-\frac{D}{L}\right)_{AVG} \text{RANGE}$$

Substituting the proper constants gives

$$\frac{D}{L}\ RANGE = 1721\ \ln\left[\frac{1-\frac{V_{END}^2}{gR_o}}{1-\frac{V_{START}^2}{gR_o}}\right] - 3.78\ \ln\left(\frac{V_{END}}{V_{START}}\right)^2$$

where $gR_o = (26{,}000)^2$

the second term is comparatively small so for many purposes

$$RANGE = \left(\frac{L}{D}\right)_{MAX}\ \ln\left[\frac{1-\left(\frac{V_{END}}{26000}\right)^2}{1-\left(\frac{V_{START}}{26000}\right)^2}\right] \times (1721)$$

A detailed calculation for specific case with Vstart = 21,824, VEND = 7550 and (L/D)max varying between 2.95 and 3.6 gives a range of 6630. Using (L/D)avg. = 3.28 the formula at the top gives 6460 while the lower, simpler one gives 6430.

Detailed performance calculations utilize IBM machine programs that do not include these simplifications to provide a more mission analysis.

BOOST PHASE SIMPLIFIED PERFORMANCE

The velocity increase due to rocket operation in a gravity free vacuum is

$$V_{END} - V_{START} = -g\, I_{SP} \ln \frac{W_{END}}{W_{START}}$$

Several approximate methods may be used to include the effects of gravity and atmosphere like

$$\Delta V = \left[-g\, I_{SP} \ln \frac{W_{END}}{W_{START}} \right] \times \text{EFFICIENCY}$$

where efficiencies of 85-90% are common or we can say

$$\Delta V = \left(-g\, I_{SP} \ln \frac{W_{END}}{W_{START}} \right) - \Delta V_{LOSS}$$

The loss or the efficiency is very dependent upon the flight path and for that matter on the desired end condition. A number of calculated trajectories using optimization techniques indicate that ΔV loss $\approx$ 3000 for the type of boost here considered indications are that as long as the end speed does not vary greater than ±2000 fps or end altitude by ±20,000, for this particular mission

$$V_{END} = V_{START} - g\, I_{SP} \ln \frac{W_{END}}{W_{START}} - 3000$$

ΔV_{LOSS}:

$$\text{DRAG} = \int_{t_0}^{t_{END}} \frac{C_D\, q\, S}{m}\, dt$$

$$\text{GRAVITY} = \int_{t_0}^{t_{END}} g \sin\gamma\, dt$$

MCDONNELL AIRCRAFT

WORK SHEET
FORTRAN DATA

REQUESTOR

DATE PAGE 1 OF 2

DECK NO. 1931 TITLE *High Speed Descending Turn*

IDENTIFICATION

1 5	8	17	26	35	44	53	62	71
1	XNOV 4.	M(1) 6.5	(2) 9.	ETC. 13.	C_L (1) 22.	(2) .1105	ETC. .1139	.1156
9	.0986							
17								
25								
33								
41	XNV 12.	M(1) 6.5	(2) 7.5	ETC. 9.	C_{D_0} (1) 12.	(2) 14.	ETC. 15.	26.
49	18.	19.	20.	21.	22.3	.0143	.0133	.0121
57	.0105	.0096	.0093	.0091	.0091	.0094	.0098	.0103
65	.0116							
73								
81	XXNOV 6.	M(1) 6.	(2) 8.	ETC. 9.	L'(1) 13.	(2) 16.	ETC. 22.3	1.835
89	1.87	1.9	2.085	2.25	2.66			
97								
105								
113	XXNV 2.	M(1) 0.	(2) 22.	ETC. 45.	PHI (1) 45.	(2)	ETC.	
121								
129								
137								
145	XNVX	V(1)	(2)	ETC.	h(1)	(2)	ETC.	
153								

MAC 20

MCDONNELL AIRCRAFT

APPLIED MATHEMATICS
WORK SHEET
FORTRAN DATA

REQUESTOR ______

DATE ______ PAGE 2 OF 2

DECK NO. ______ TITLE ______

IDENTIFICATION ______

1 5	8	17	26	35	44	53	62	71
161								
169								
177	WVEL₀ 21824.	Ho 205700.	μ₀ 106.	ψ₀ 335.	λ₀ 10.	ΔV 500.	VEND 7550.	GW 25450.
1 185	SW 1075.	TEST 0.	CONFIG 2460.3	V₁ 21500.				

MAC 2

 23 March 1965

HIGH SPEED DESCENDING TURN

ALT. FT.	MACH NO.	VEL. FPS	MU DEG.	LAMBDA DEG.	RANGE NAM	TIME SEC.	PSI DEG.	PHI DEG.
205700.	21.47	21824.	105.98	9.99	0.0	0.0	334.93	45.00
203672.	21.03	21500.	104.58	13.15	206.5	57.9	336.39	45.00
200776.	20.37	21000.	102.73	17.79	505.3	143.3	338.67	45.00
197975.	19.73	20500.	101.17	22.15	781.7	224.3	341.04	45.00
195242.	19.11	20000.	99.87	26.25	1038.7	301.4	343.53	45.00
192557.	18.50	19500.	98.82	30.13	1278.2	375.1	346.17	45.00
189914.	17.90	19000.	98.00	33.79	1502.2	445.8	348.94	45.00
187276.	17.31	18500.	97.43	37.24	1711.8	513.7	351.92	45.00
184632.	16.72	18000.	97.09	40.49	1907.9	579.0	355.19	45.00
181997.	16.15	17500.	97.02	43.55	2091.9	642.0	358.72	45.00
179346.	15.59	17000.	97.21	46.42	2264.6	702.8	2.56	45.00
176668.	15.03	16500.	97.68	49.12	2426.6	761.6	6.72	45.00
173954.	14.47	16000.	98.45	51.64	2578.5	818.4	11.24	45.00
171488.	14.02	15500.	99.53	53.99	2721.0	873.3	16.14	45.00
169051.	13.56	15000.	100.93	56.17	2854.5	926.5	21.46	45.00
166593.	13.11	14500.	102.66	58.16	2979.8	978.1	27.22	45.00
163925.	12.66	14000.	104.72	59.96	3097.6	1028.3	33.45	45.00
161161.	12.21	13500.	107.11	61.57	3208.4	1077.3	40.17	45.00
158354.	11.76	13000.	109.80	62.97	3312.8	1125.2	47.40	45.00
155499.	11.30	12500.	112.76	64.16	3410.9	1171.9	55.10	45.00
152651.	10.88	12000.	115.91	65.13	3503.1	1217.7	63.24	45.00
149943.	10.47	11500.	119.16	65.87	3589.6	1262.4	71.75	45.00
147189.	10.06	11000.	122.43	66.39	3670.7	1306.2	80.58	45.00
144384.	9.65	10500.	125.59	66.70	3746.8	1349.2	89.65	45.00
141517.	9.23	10000.	128.55	66.82	3818.1	1391.5	98.88	45.00
138546.	8.82	9500.	131.22	66.75	3884.7	1433.0	108.23	45.00
135449.	8.40	9000.	133.54	66.53	3946.7	1473.7	117.63	45.00
132260.	7.97	8500.	135.46	66.19	4004.2	1513.7	127.08	45.00
128967.	7.55	8000.	136.97	65.75	4057.5	1552.9	136.60	45.00
125900.	7.16	7550.	137.99	65.28	4101.8	1587.5	145.25	45.00

CONFIG = 2460.3

HIGHER C_L THAN $C_{L_{OPT.}}$

WIND TUNNEL RESULTS TO DATE

HYPERSONIC (M=14 TO 22)

- VALIDATES HYPERSONIC L/D

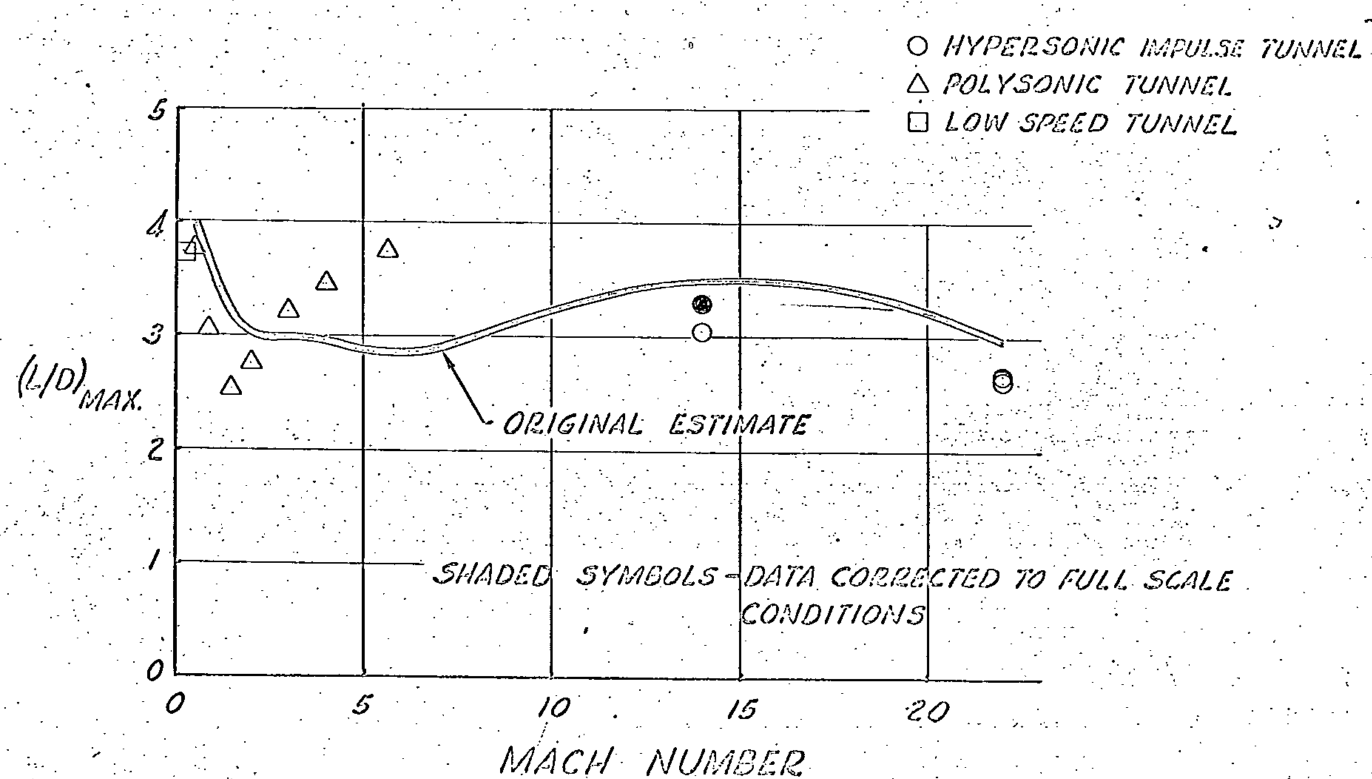
POLYSONIC (M=.5 TO 5.6)

- VALIDATES SUPERSONIC L/D
- SUBSONIC L/D LOW; BASE FAIRINGS INCREASE L/D UP TO 75%
- ELEVATORS AMPLE FOR LONGITUDINAL CONTROL IN GLIDE
- TIP EXTENSIONS INCREASE L/D AND LONGITUDINAL STABILITY
- TIP AILERONS PROVIDE INCREASED ROLL CONTROL
- TOED-IN ARRANGEMENT OF VERTICAL TAILS IMPROVES DIRECTIONAL STABILITY
- AMPLE RUDDER POWER AVAILABLE
- NEUTRAL POINT CONSTANT WITH MACH NUMBER

LOW SPEED (M=.26)

- DATA AGREES WITH POLYSONIC
- BASE FAIRING & VERTICAL TAIL ARRANGEMENT PROVIDES SELF-TRIM
- END PLATE EFFECT OF VERTICALS PAYS FOR THEIR DRAG
- GOOD LATERAL-DIRECTIONAL STABILITY & CONTROL

L/D SUBSTANTIATION



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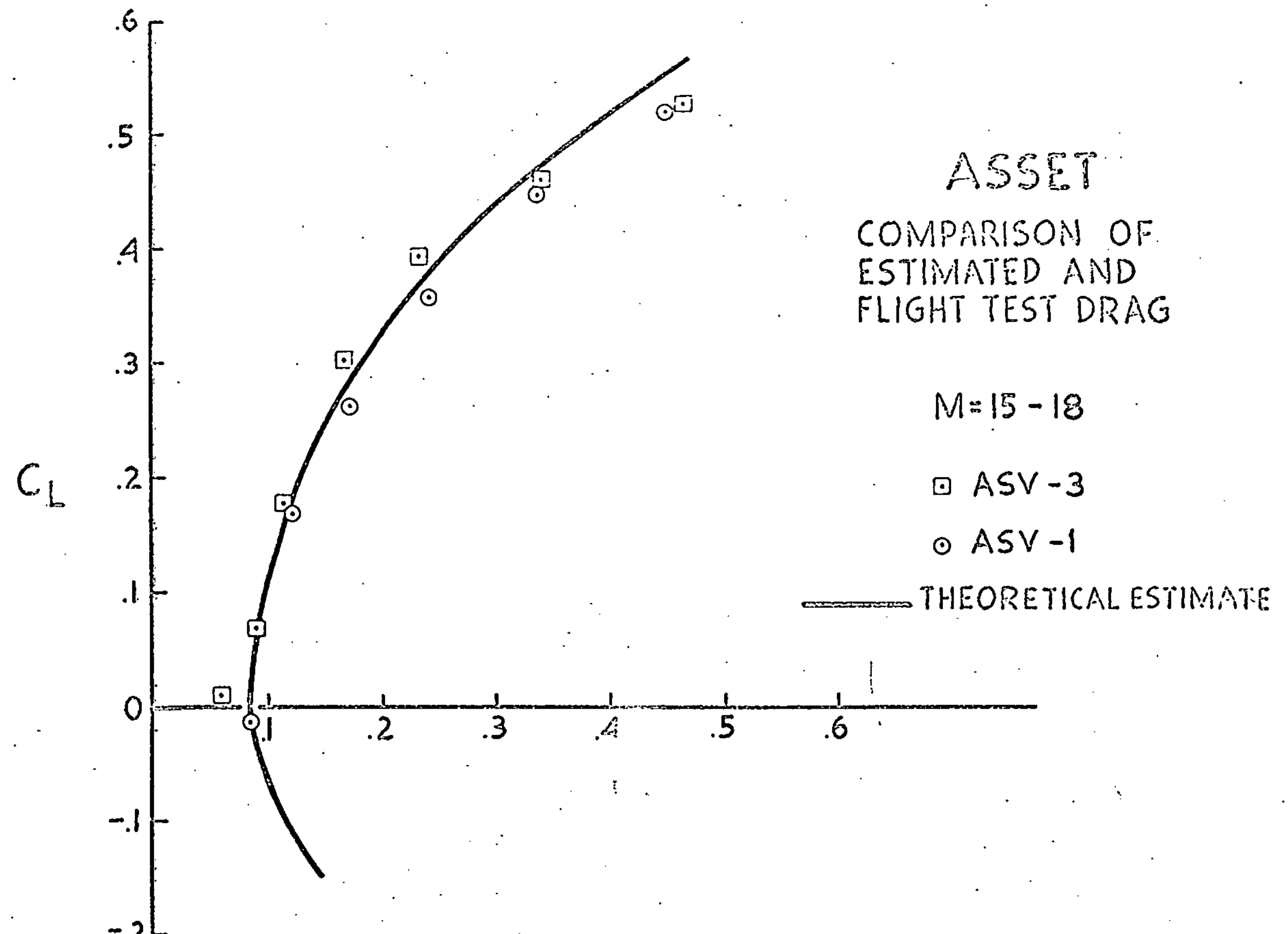
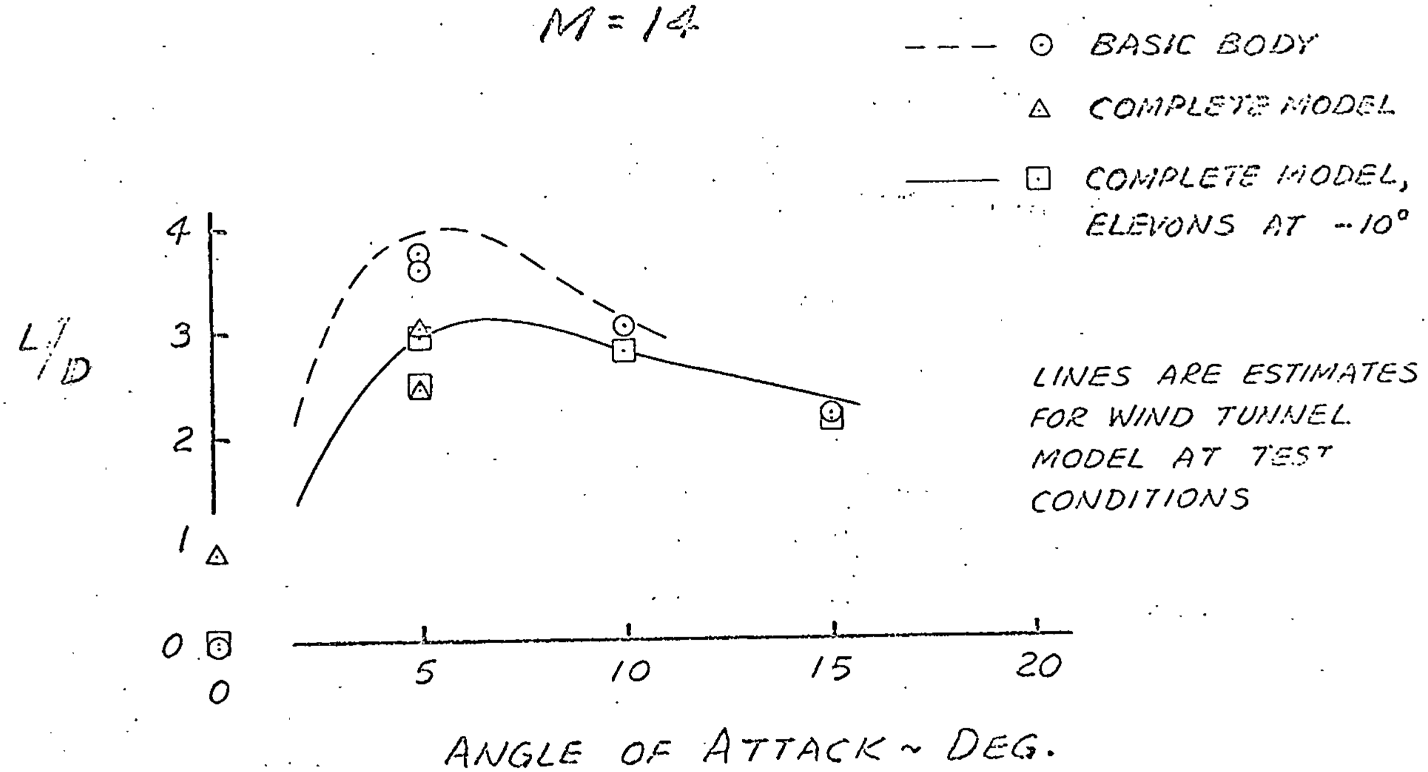

ASSET
COMPARISON OF ESTIMATED AND FLIGHT TEST DRAG
M= 15 - 18
ASV -3
ASV -1
THEORETICAL ESTIMATE
C_L
.6
.5
.4
.3
.2
.1
0
-.1
-.2
.1
.2
.3
.4
.5
.6

L/D VS. ANGLE OF ATTACK

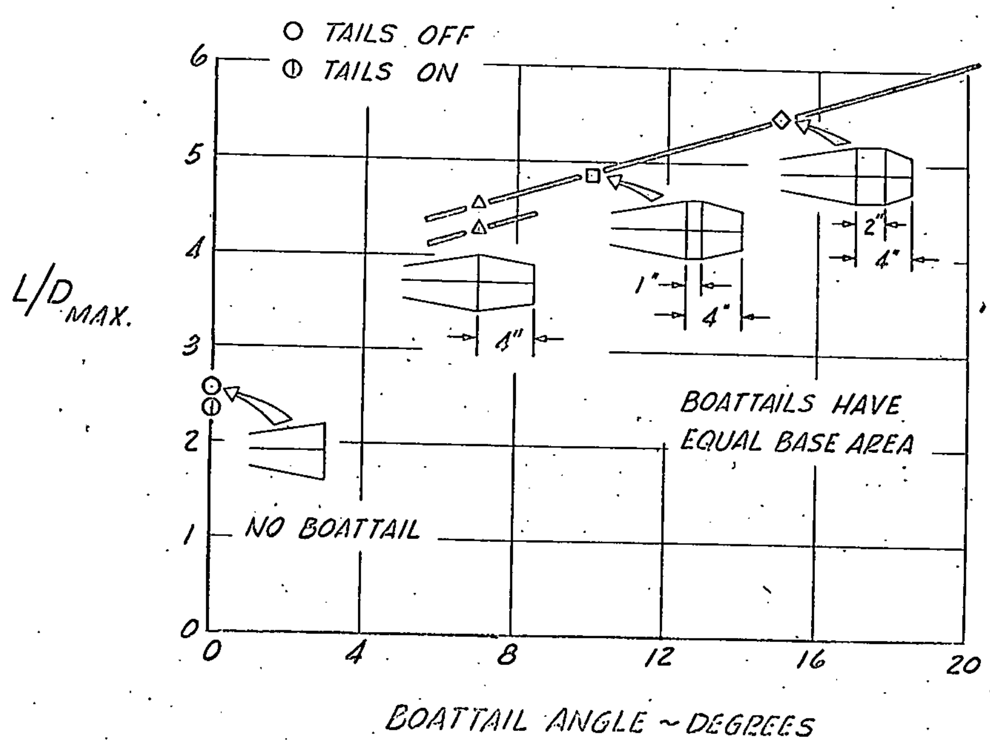
MAC HYPERVELOCITY IMPULSE TUNNEL

M = 14



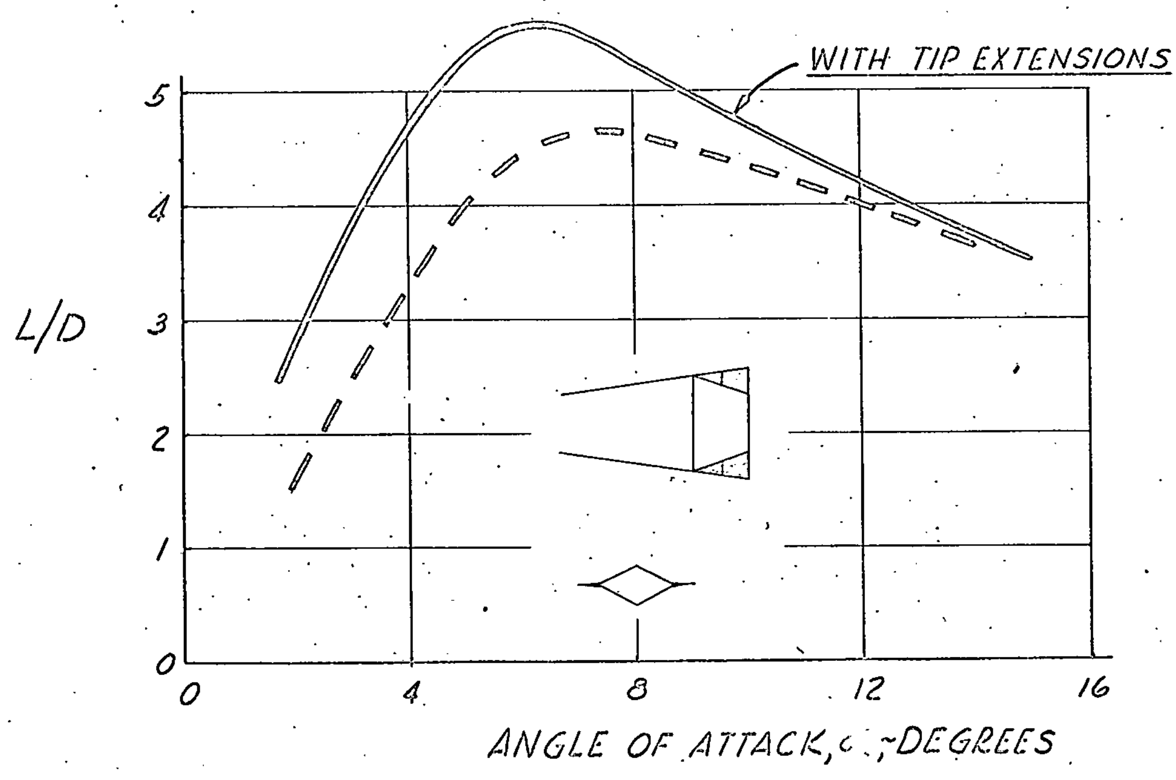
EFFECT OF BOATTAIL SHAPE

M = .5



EFFECT OF TIP EXTENSIONS ON L/D

M=.5



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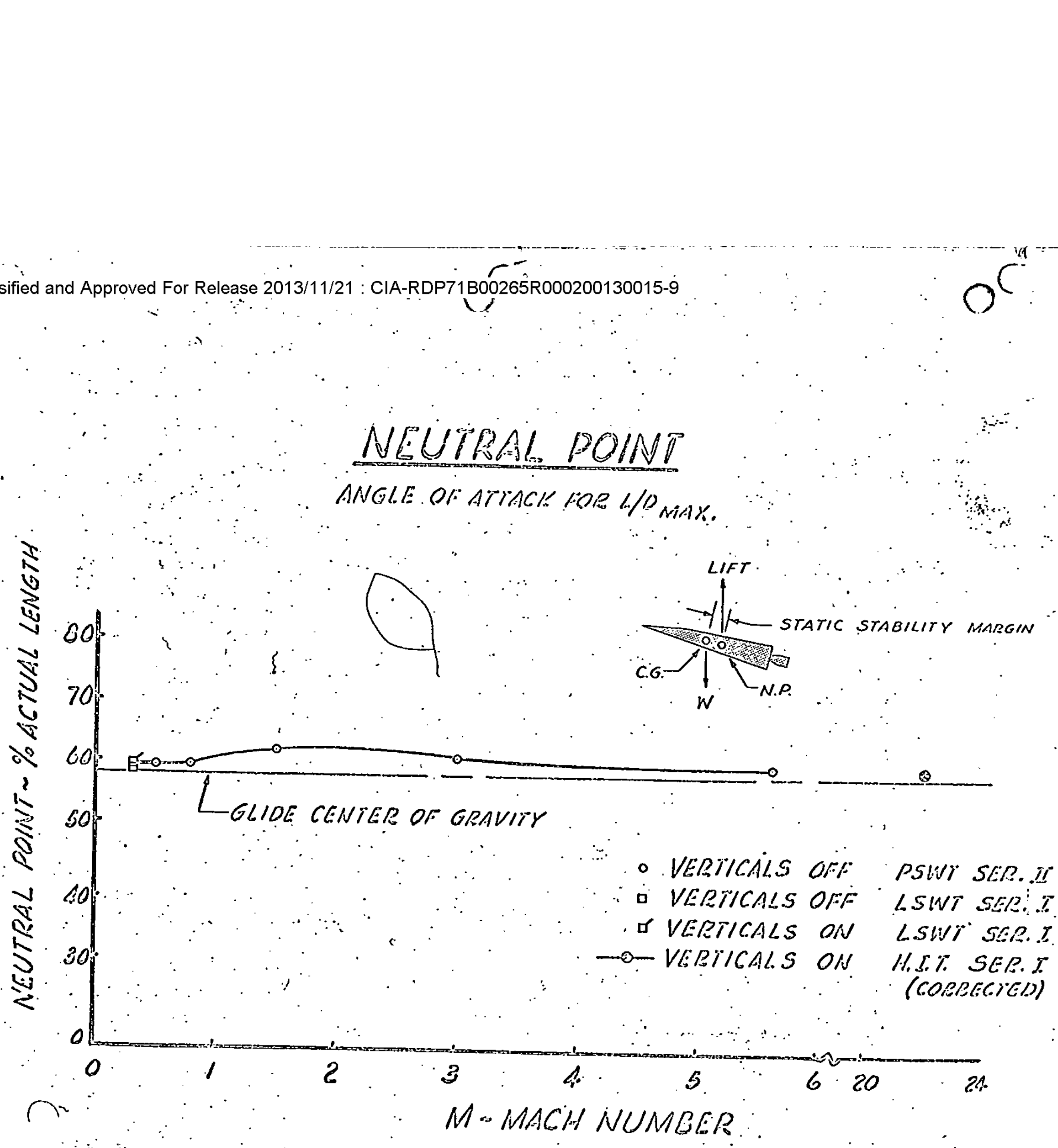
NEUTRAL POINT

ANGLE OF ATTACK FOR L/D_{MAX}.



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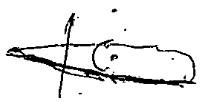
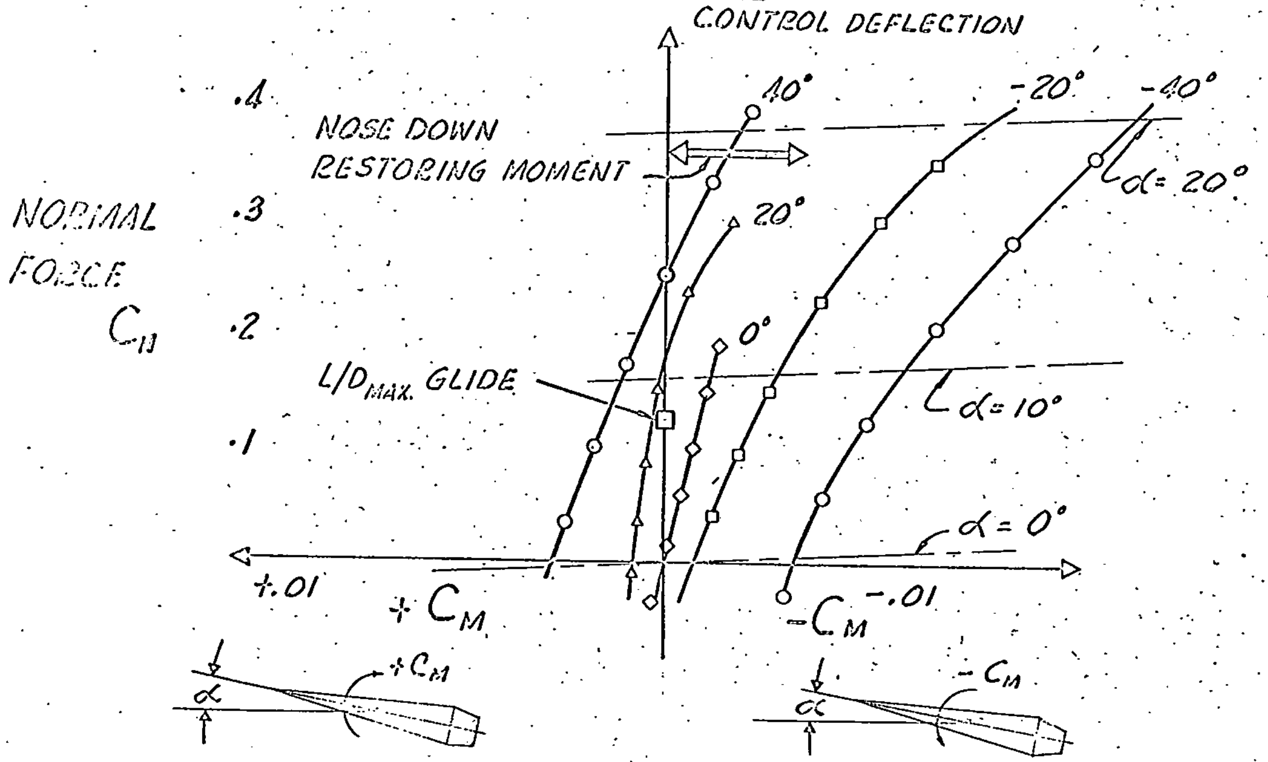


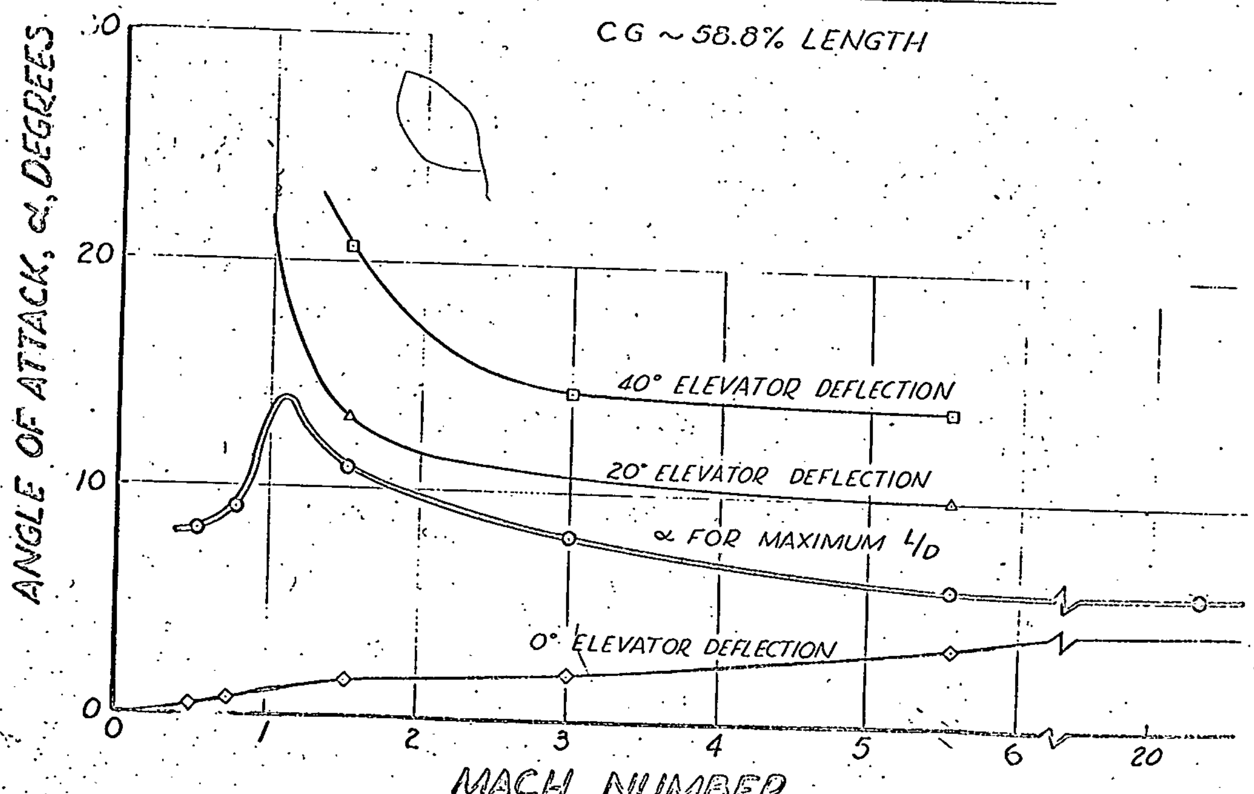
LONGITUDINAL STABILITY AND CONTROL

MACH 5.6



TRIM CAPABILITY

CG ~ 58.8% LENGTH



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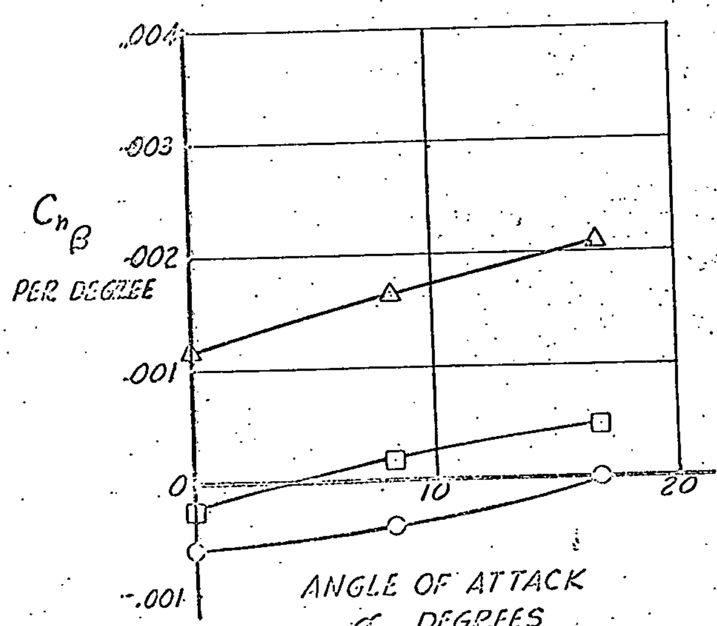
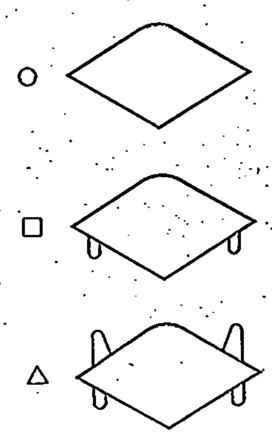
DIRECTIONAL STABILITY

PSWT ~SERIES I

EFFECTS OF OUTBOARD VERTICALS AND VENTRALS
M = 3.0



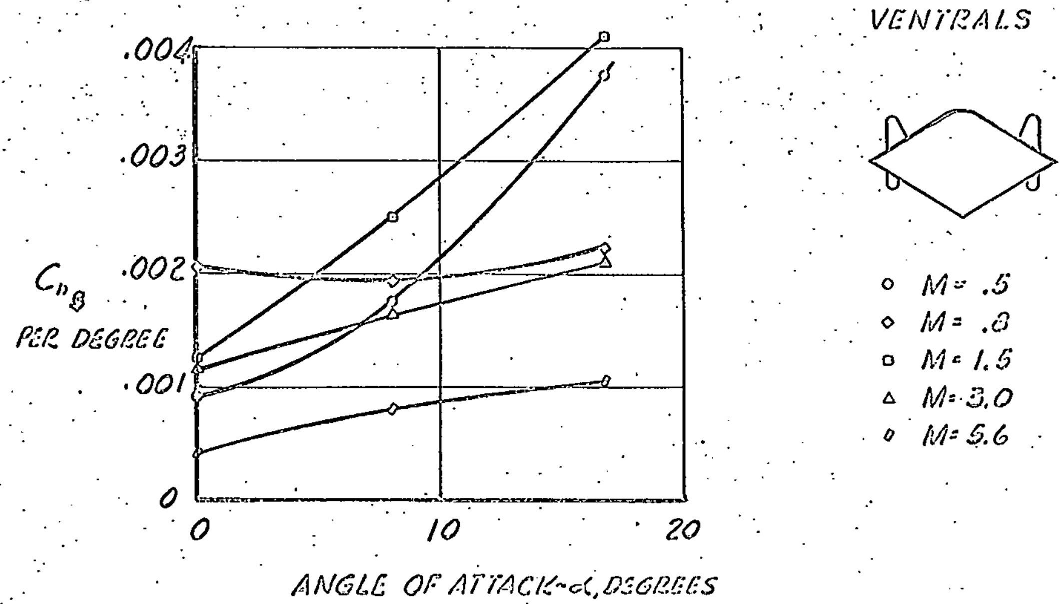


217

DIRECTIONAL STABILITY

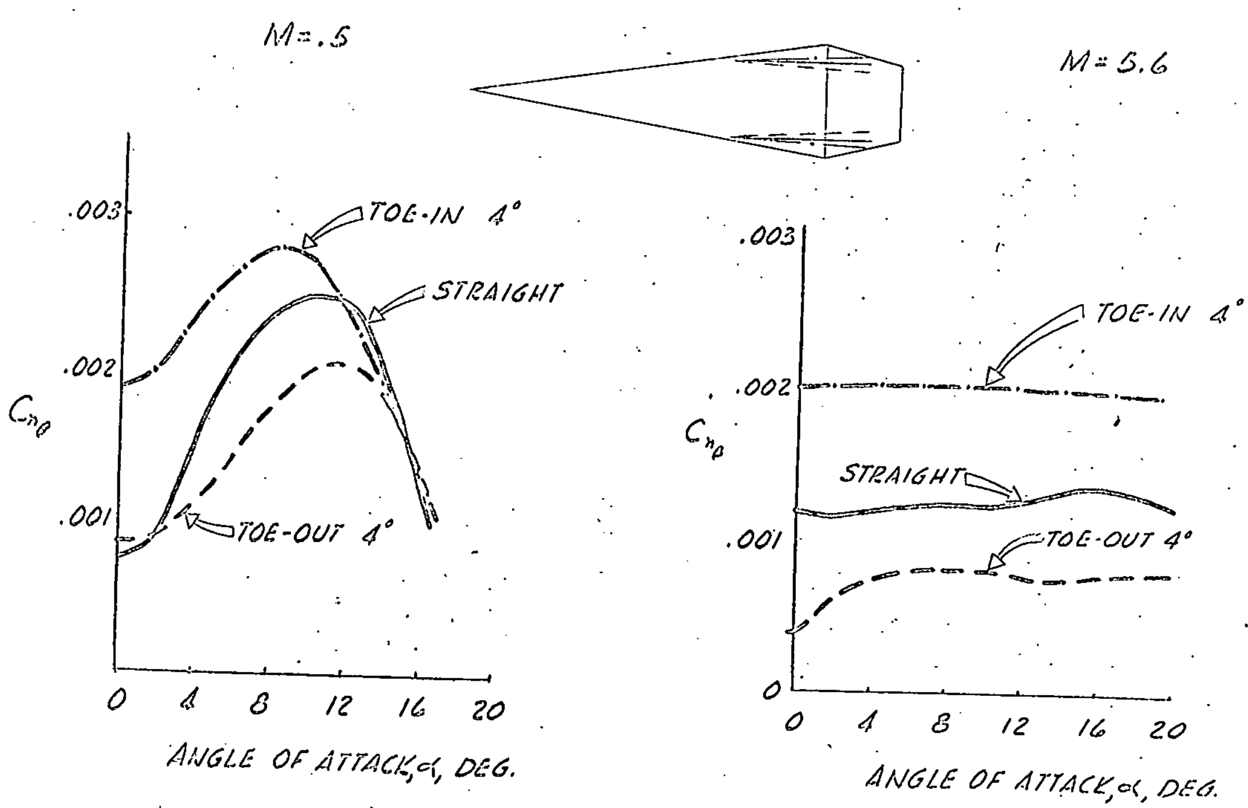
PSWT - SERIES I

EFFECT OF MACH NUMBER - OUTBOARD VERTICALS AND VENTRALS



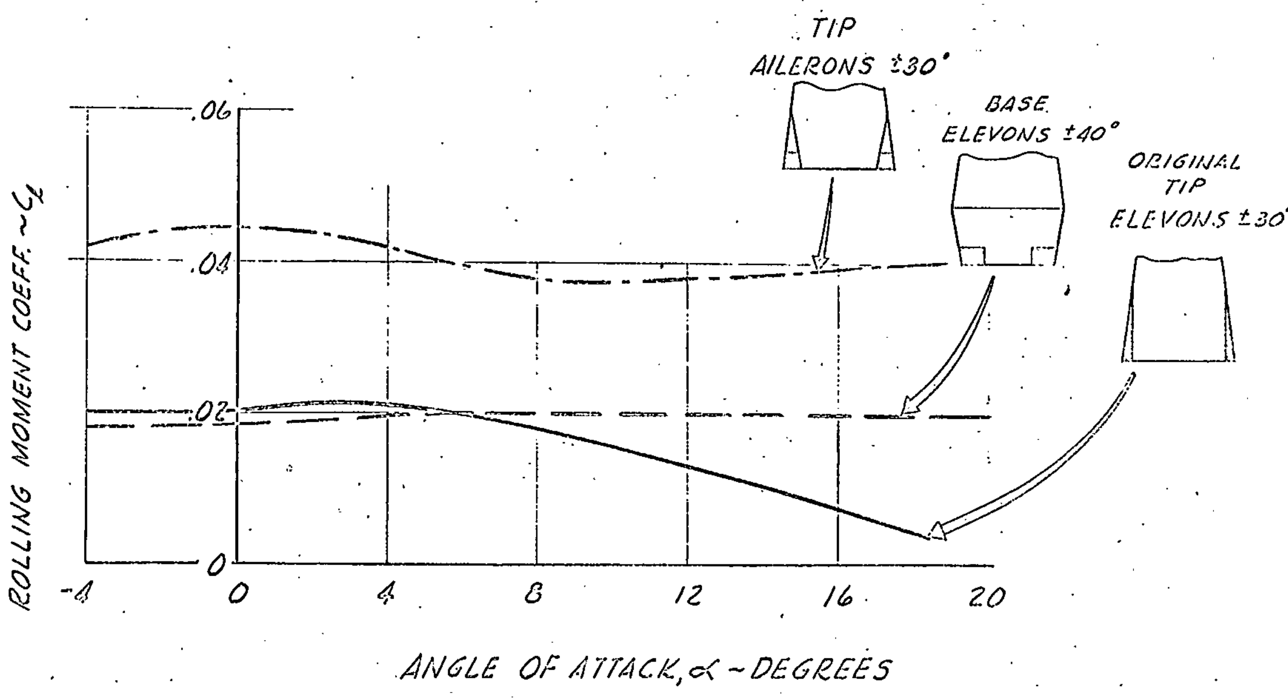
NM

EFFECT OF TOE-IN OF VERTICAL TAILS



ROLLING EFFECTIVENESS

M = .5



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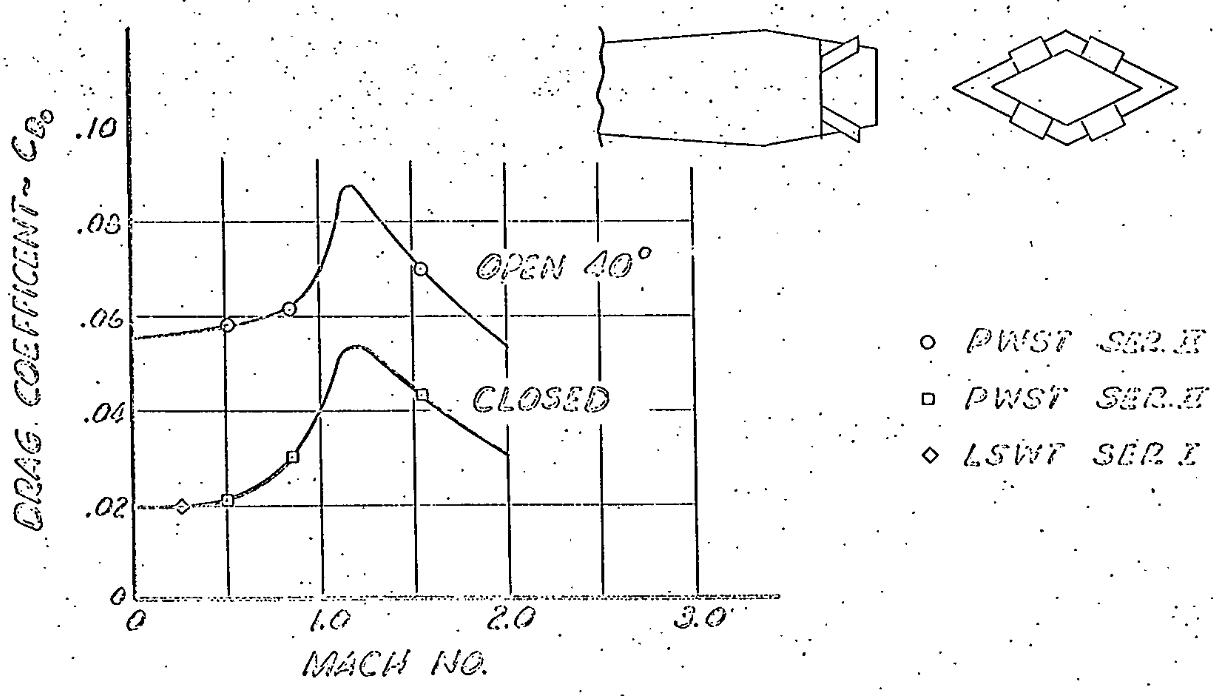
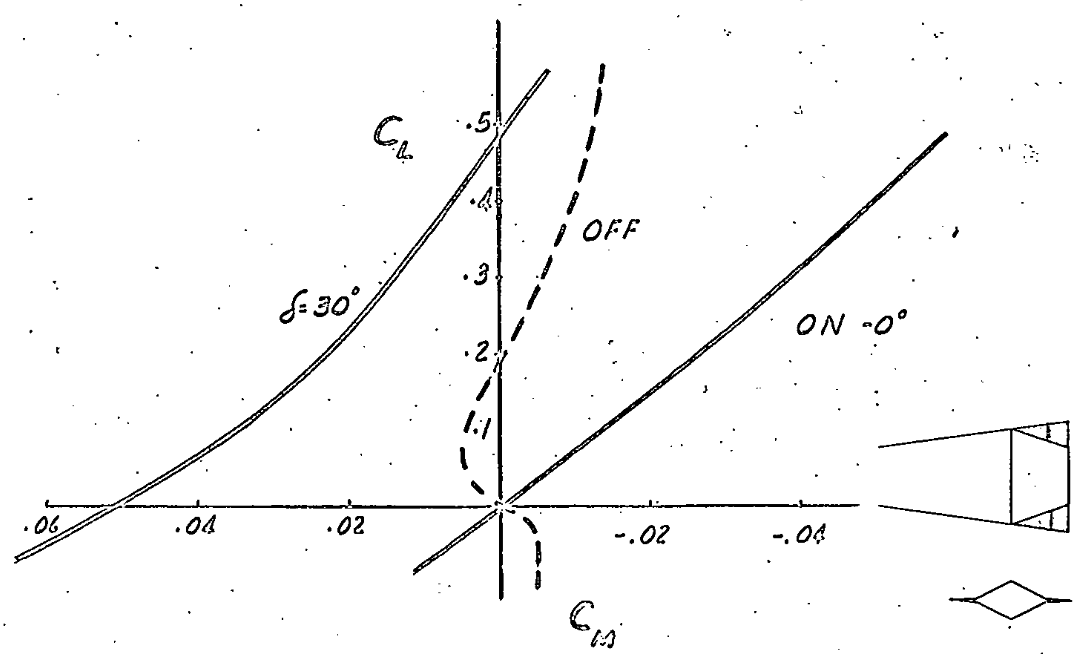
SPEED BRAKE EFFECTIVENESS



221

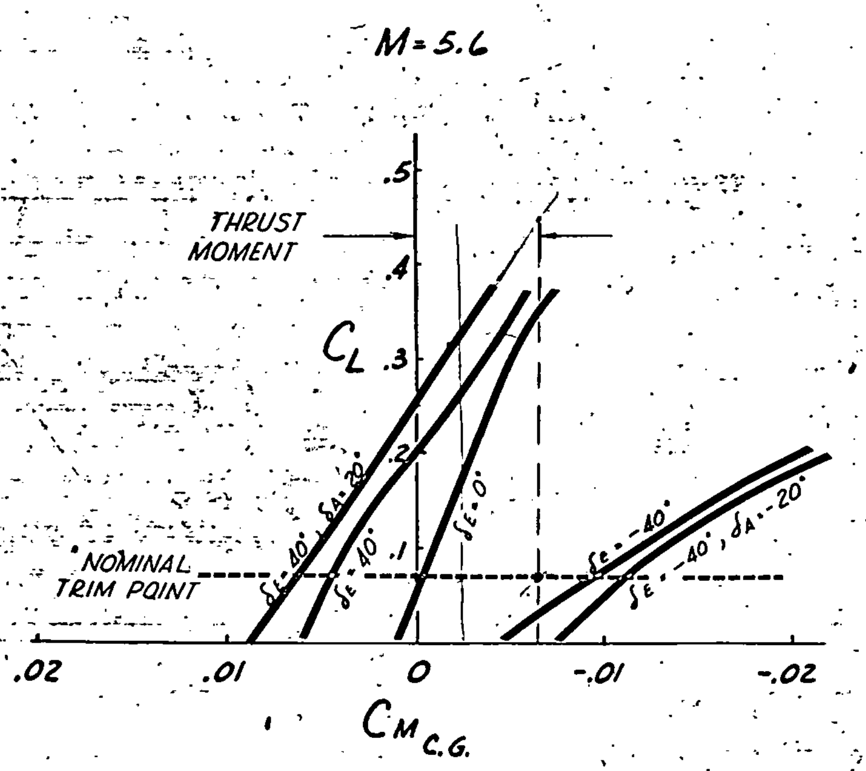
EFFECT OF TIP EXTENSIONS AND OUTBOARD ELEVATORS ON LONGITUDINAL STABILITY AND CONTROL

M = .5



609

STABILITY AND CONTROL DURING BOOST



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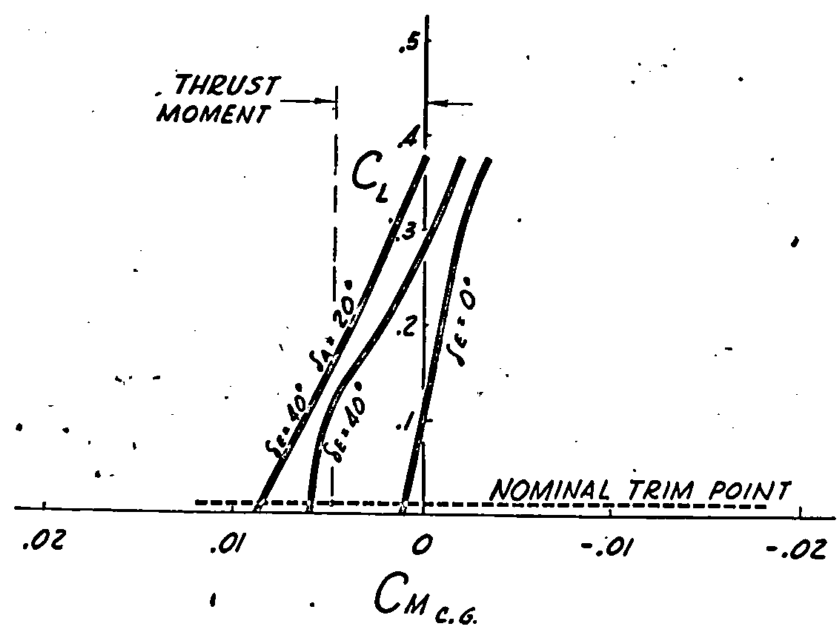
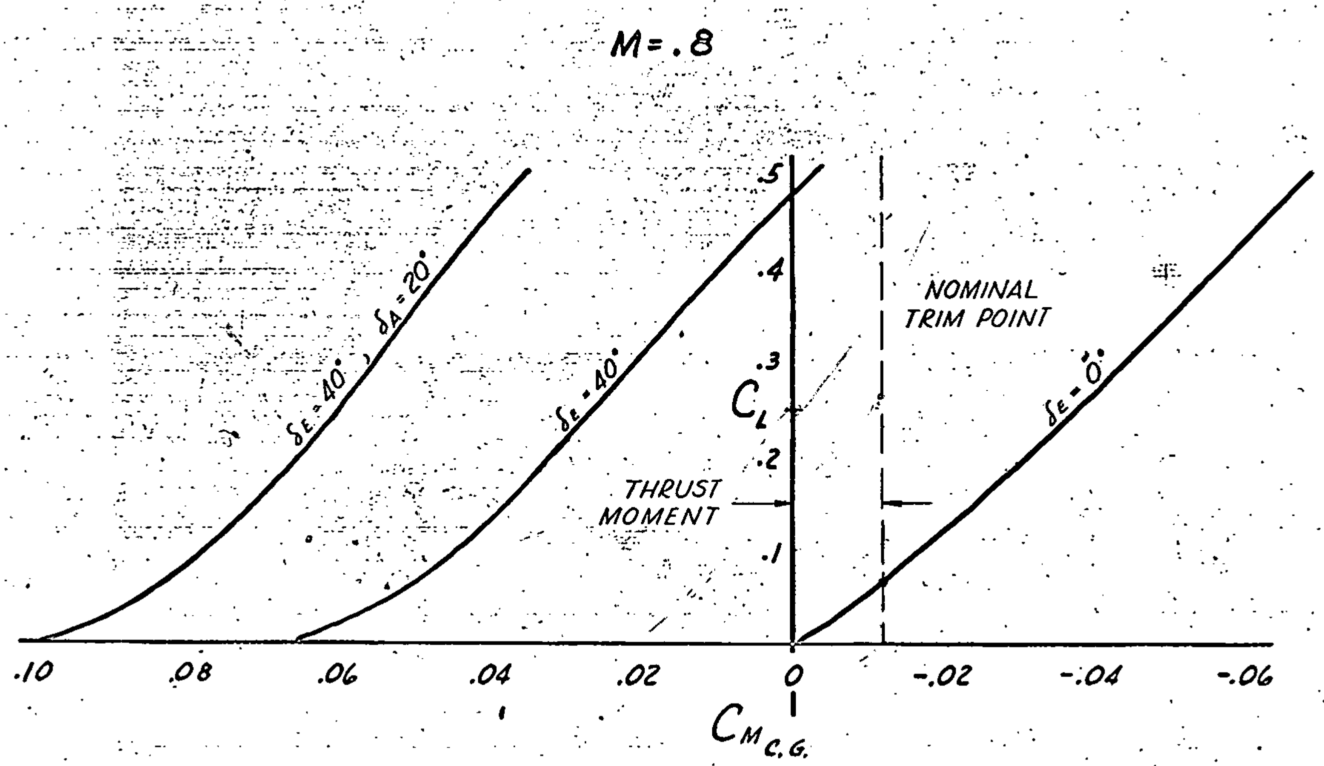
STABILITY AND CONTROL AT BURNOUT

M=21.0

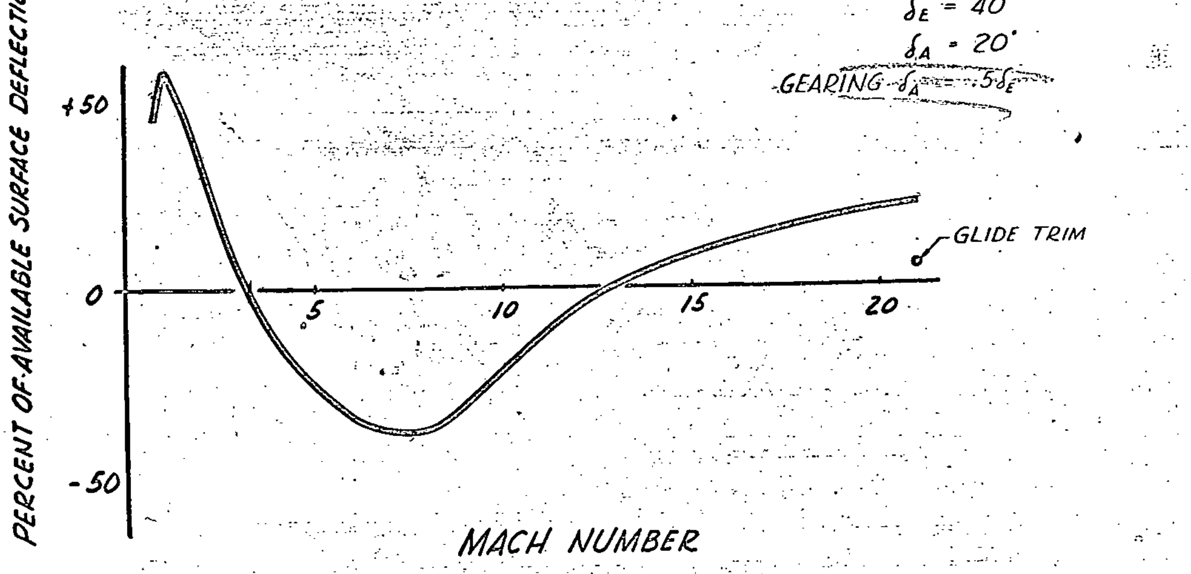


STABILITY AND CONTROL DURING LAUNCH



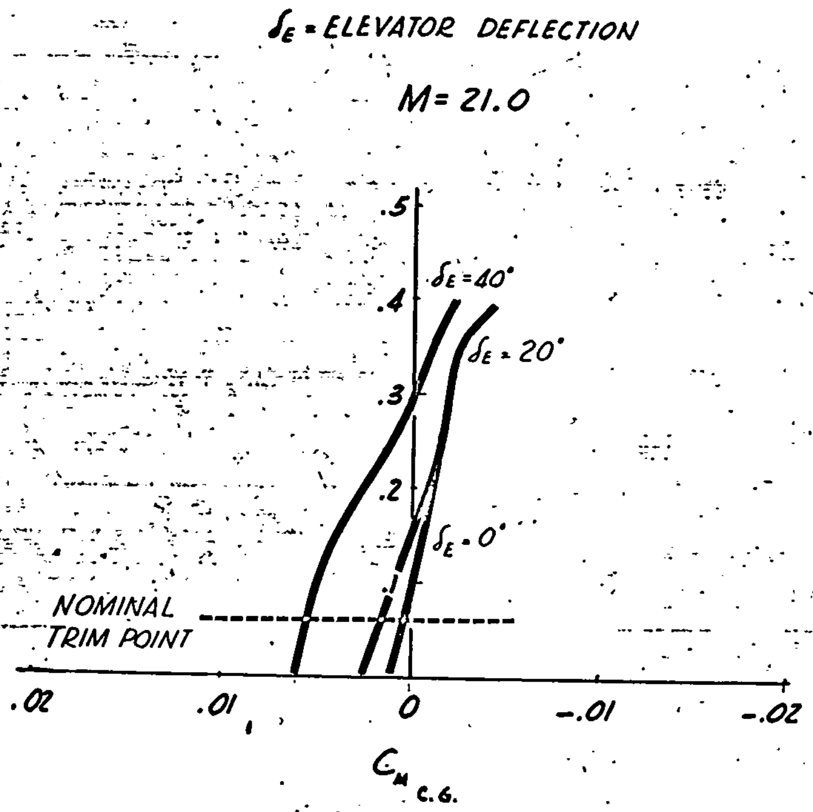
ELEVATOR REQUIRED FOR TRIM DURING BOOST

~~VARIABLE C.G.~~



C-5131

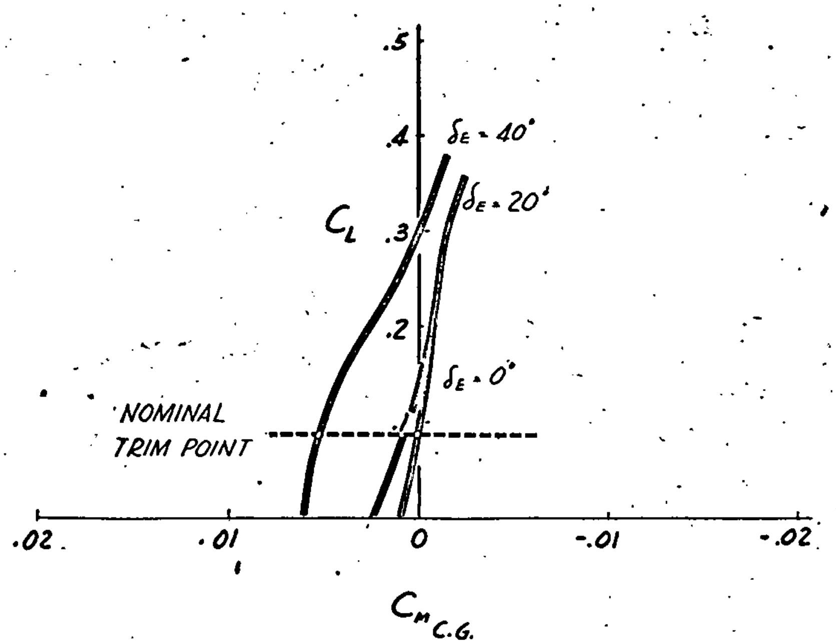
STABILITY AND CONTROL DURING GLIDE INSERTION



STABILITY AND CONTROL DURING GLIDE

δ_E = ELEVATOR DEFLECTION

M = 5.6



STABILITY AND CONTROL DURING LANDING