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FINAL REPORT

on the

FERRITE ANTENNAS FOR VERY LOW FREQUENCIES

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CONTENTS

SCOPE AND OBJECTIVES

The investigation described in this report seeks to appraise the relative merits of the ferrite cored loop as a very low frequency receiving antenna. The restrictions imposed upon the antenna, briefly, are that it operate at 25000 cps and have a 3 db passband of 2000 cps. The basis used for judging the relative merits of a ferrite antenna is a comparison of induced voltages and signal-to-noise ratios with an air core loop antenna of comparable size. Comparisons made are primarily analytical with experimental checks whenever feasible.

I. DISCUSSION OF RESULTS

Atmospherics vs Thermal Noise

The relative importance of atmospheric noise and internally generated thermal noise at 25 kc/s is examined with respect to magnetic loop receiving antennas. It is found that the ratio of atmospheric-to-thermal-noise volts at the antenna terminals increases as the 3/2 power of the maximum dimension* (Table I). For median atmospheric noise field strengths found in the U.S.A., the thermal noise in a tuned-circuit (singly tuned) loop antenna is found to be appreciable for loop diameters as large as 50 to 100 cms, depending upon the specific loop antenna. Noise in larger tuned loops is found to be predominately due to atmospheric noise sources. The amount of thermal noise generated in a singly tuned antenna circuit is inordinately high because sufficient noise producing resistance must be present to keep the passband 2000 cps wide. At a 25 kc/s center frequency, this means a circuit Q no greater than 12.5. The advantages of a non-resonant antenna circuit (or an overcoupled double-tuned antenna circuit) with high Q coils is therefore of interest and is discussed in Section VI. Using untuned loop antennas with Q's of about 250 (quite feasible with ferrite core loops at 25 kc/s), thermal noise is reduced so that it becomes negligible with respect to atmospheric noise for loop diameters of 30 to 60 cms and greater. The resulting improvement in signal-to-noise ratio is roughly $\frac{Q \text{ untuned}}{2\ Q \text{ tuned}}$ (Eq. 6-18). However, even with an untuned circuit, enough thermal noise is produced so that, in general, it must be taken into account - even at 25 kc/s.

Air Core Criterion and Ferrite Coil Length

In order to establish a criterion for evaluating the performance of ferrite cored loop antennas, it was decided to use a simple air core loop antenna as a reference. In Section III, antenna equations are presented which describe induced

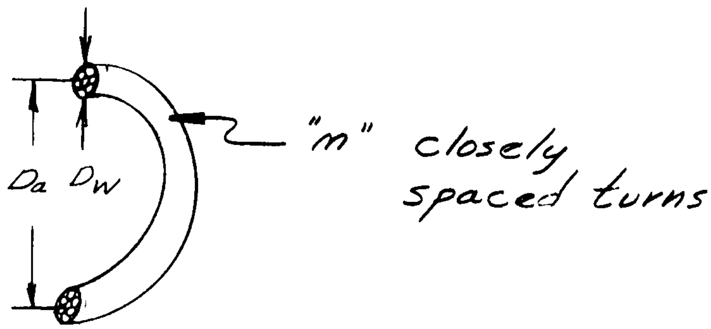


FIGURE 1-1

REFERENCE AIR CORE LOOP ANTENNA GEOMETRY

voltage and internal impedance for air core loops and ferrite cored loop antennas. Experimental data presented in this section together with calculations set forth in the Appendix indicate that, both from an induced voltage point of view and a signal-to-thermal-noise point of view, the coils wound on ferrite rod antennas should extend over the entire length of the rod.

Induced Voltage (or Effective Height)

Using the reference air core loop and ferrite rod antennas with full length windings as a basis for calculation, equations and graphs are obtained in Section V which show the relative merits of ferrite and air core loop antennas with respect to induced voltage and signal-to-noise ratios. These comparisons are made for a number of different ferrite rod characteristics; viz., rod length-to-diameter ratios of 10 to 100 and ferrite toroidal permeabilities of 50 to 3000. Included in these graphs (Figures 5-3 to 5-8) as a parameter is an F-factor which relates ferrite rod length to air core loop diameter; i.e., (ferrite rod length) $\approx$ (F) (air core loop diameter). Figure 5-8 shows that, when $F \approx 1$, $\mu_{toroidal} = 3000$ and $\ell/d = 20$, the best that can be expected from a ferrite rod antenna is that it produce 53% of the induced voltage produced in the air core loop; i.e., $\nu_L = 0.53$. (It seems reasonable to say that $\mu_{toroidal} = 3000$ is good upper limit for stable ferrite materials foreseeable at present.) This curve further shows that F must be approximately 1.5 in order to obtain $\nu_L = 1$; i.e., equal induced voltage from ferrite and air core antennas. The relatively poor showing of even the highest permeability ferrite antenna is due to the difficulty of improving the effective permeability of an open rod. Figure 5-9 clearly shows that the ν_L increases at a progressively lower rate as $\mu_{toroidal}$ increases.

Signal-to-Thermal-Noise Ratio

These same curves for ν_L also represent the relative values of the signal-to-thermal-noise ratios of ferrite-to-air core antenna for the case of equal inductance, equal Q antennas (see Eq. (5-16)). Hence, the same comments made above concerning the relative induced voltages apply to signal-to-thermal-noise ratios for singly-tuned antennas wound with the maximum allowable inductance. Equation (5-15), on the other hand, shows that the curves for ν_L are all multiplied by the factor $\frac{Q\ \text{ferrite}}{Q\ \text{air core}}$ when the Q"s of the ferrite and air core antennas are not equal. Practically speaking, unequal Q's require that the singly-tuned circuit be abandoned and an untuned or a doubly-tuned coupling circuit be used in its stead. Using the untuned circuit as an example, the performance of the ferrite antenna relative to the air core antenna will be improved if it is possible to reach appreciably higher Q loops with a ferrite core than with an air core. For the full length ferrite coil and the reference air core used in this report, a comparison of Q reduces to a comparison of the core losses introduced by the ferrite core and the proximity effects caused by the closely spaced turns of the air core antenna. Although the matter of maximum obtainable Q has not been examined in detail in this report, indications are that, in general,

$$\frac{\text{max. ferrite Q}}{\text{max. air core Q}} < 3^{*}$$

Thus the picture presented by the curves of ν_L are modified at most by a factor less than 2.

* It is noted that, for specific cases, this number can vary widely.

Conclusions

As a result of the preceding discussion, some conclusions are reached regarding the relative merits of the ferrite cored VLF receiving antenna; viz, in general, the ferrite cored antenna is comparable to an air core antenna only if its maximum dimension is permitted to be somewhat larger (1.2 - 1.8 times) than an air core antenna diameter. The practical value of replacing an air core antenna with a ferrite antenna depends upon the value that can be placed upon the difference in the geometry of the two antennas in a specific receiver.

Aside from the physical packaging of an antenna, the line geometry of ferrite antenna permits the reduction of stray capacity to the chassis and, hence, a reduction in E-field pickup. Furthermore, E-field shielding of a line geometry is somewhat simpler than shielding an air core loop.

The problem of winding high impedance antennas (determined by maximum permissible inductance) and yet maintaining the highest possible Q and the lowest distributed capacity is simpler for the coil geometry of the ferrite antenna than for the coil geometry of the air core antenna.

It is noted that the ferrite material introduces problems not found in air core antennas; e.g., ferrite is hard and brittle; extremes of temperature affect its characteristics; hum pickup is possible due to non-linear B-H curve; vibration stabilization can be destroyed by exposure to high level AC or DC flux fields. All these properties, however, are controlled sufficiently well in available ferrites to permit satisfactory operation in portable broadcast band receivers.

II. DISCUSSION OF NOISE at 25 kc/s

General

The minimum signal that can produce a useful output from a radio receiver is determined by the output signal-to-noise ratio. For any given signal amplitude, this ratio is set by the external noise entering the receiver via the antenna and by the internal noise generated in the antenna and low level circuits of the receiver. The antenna and associated circuitry, therefore, is a determining factor in setting the level of minimum signal reception. In this report, the relative merits of ferrite loop receiving antennas are investigated with respect to operation at a center frequency of 25 kcs and a 2 kc passband. It is of some importance, therefore, to briefly review sources of noise generation and single out those sources which are most objectionable at this operating frequency.

TABLE I

NOISE SOURCES

EXTERNALLY GENERATED	INTERNALLY GENERATED
1. Atmospheric (electrical storms)	1. Antenna ohmic resistance (thermal)
2. Cosmic (extra-terrestrial radiation)	2. Coupling circuit resistance (Thermal)
3. Man-made static	3. First amplifier or mixer (shot noise)
4. Precipitation static	
5. Radiation resistance (thermal)	

External Noise Sources at 25 kc

Considering first external generators, it is noted that cosmic noise plays no important role at 25 kc/s(1). Also, the radiation resistance of antennas to be considered in this report will be insignificantly small by comparison with other resistive components as a direct consequence of restricting the investigation to antennas which are very much smaller than a wavelength. Precipitation static is caused by the discharge of charged particles in the immediate vicinity of the antenna. Accumulation of the charged particles can be caused by raindrops, hailstones, snow or dust clouds. This type of noise is of particular importance in aircraft receiving antennas. For the purposes of this report, man-made static and precipitation static will be grouped together and called electrostatic disturbances. Thus, at 25 kc/s the main external noise sources are

1. Atmospheric (electrical storms)
2. Electrostatic disturbances

These two sources generate noise signals which are basically different. The former gives rise to a true radiation field in which the electric and magnetic field intensities are related by the characteristic impedance of free space which is a constant ($\approx 120\pi$ ohms). The latter on the other hand is a near-field phenomenon for which the E-field and H-field intensities are related by a factor which is a function of frequency and which is far greater than 120π ohms for frequencies below 1.5 megacycles. In effect, therefore, electrostatic disturbances in the region 25kc/s are primarily electric field noise signals.

Summing up these points, it is noted that atmospheric noise and desired transmitted signals both propagate via radiation fields and, in its passband, an omnidirectional antenna cannot distinguish one from the other. In this respect, then, one antenna is superior to another only if its directivity is higher. An ideal loop antenna (insensitive to E-field) has a figure-of-eight pattern in the horizontal plane compared to the uniform pattern of a vertical open antenna.

Electrostatic disturbances can be effectively discriminated against by antennas that respond only to magnetic fields. This attribute is one of the chief advantages of the magnetic loop type of low frequency receiving antenna.

Thus, in evaluating a ferrite cored loop antenna, directivity and insensitivity to electric field pickup are important factors to be considered. In this report, only simple loop antennas are considered; hence, directivity is considered only to the extent that E-field pickup changes the ideal figure-of-eight pattern.

Internal Noise Sources at 25 kc/s

It is generally accepted that vacuum tube noise generation (shot effect) at low frequencies (less than 10 mc/s) can be made negligibly small by comparison with other unavoidable noise generators by proper design; e.g., by making the grid circuit impedance level sufficiently high so that the vacuum tube equivalent noise resistor is small by comparison.

The coupling circuit and its resistance is associated with impedance level changing, and the extent to which it affects signal-to-noise ratio will be determined by such considerations as tuned vs. untuned antenna circuits and the need (or lack of it) to interpose a transmission line between the antenna and the first amplifier (or mixer) grid.

The antenna circuit itself is another and probably the most important source of internal noise. The ohmic resistance of the antenna is due in part to unavoidable losses such as copper losses (d.c. resistance, skin effect, and proximity effects) and for non-air core antennas core losses (hysteresis and eddy current). These losses can be minimized by the selection of materials and proper design. However, if the usual practice of tuning the antenna circuit is followed, it is found that a sizeable amount of additional resistance is required in order to obtain the desired bandwidth. It is possible, of course, to circumvent a simple single-tuned antenna circuit at a cost of additional components and, therefore, other loss elements.

III. ANTENNA EQUATIONS

Assumptions

1. Maximum antenna dimension $<<$ wavelength
2. Incident electromagnetic field is
   a) vertically polarized
   b) uniform and planar over antenna dimensions

3. Antenna oriented for maximum pickup

4. Antennas respond either to magnetic or electric field component but not both

5. Rationalized MKS units are used unless otherwise specified.

Air Core Loop

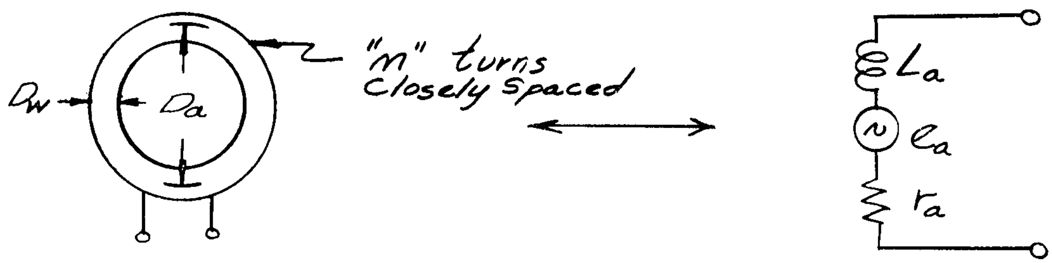


FIGURE 3-1

AIR CORE LOOP AND ITS EQUIVALENT CIRCUIT

$$e_a = \mu_o \, \omega \, A_a \, n_a \, H \tag{3-1}$$

where H = magnetic field intensity ⊥ to paper

$$L_a = \mu_o K n_a^2 \frac{D_a}{2} \quad (2) \tag{3-2}$$

$$\text{where } K = \ln\left(\frac{8\,D_a}{D_w}\right) - 2 \tag{3-2a}$$

Note: $6 < \frac{D_a}{D_w} < 1000$ corresponds to $2 < K < 7$

$$r_a = \frac{\omega L_a}{Q_a} \tag{3-3}$$

Equation (3-2) is a simplification of

$L = \frac{n\,\mu_w\,D_a}{8} + \mu_o K n_a^2 \frac{D_a}{2}$ where the inductance due to the internal flux $\left(\frac{n\,\mu_w\,D_a}{8}\right)$ is assumed negligible for wire permeabilities $\mu_w \approx \mu_o$, $n > 5$, and $\frac{D_a}{D_w} > 2$.

Ferrite Core Loop

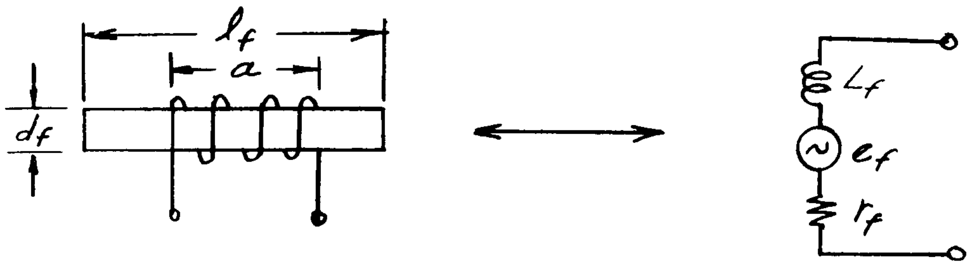


FIGURE 3-2

FERRITE CORE ANTENNA AND EQUIVALENT CIRCUIT

$$e_f = \mu_f \, \mu_o \, \omega \, A_f \, n_f \, H \; * \qquad (3\text{-}4)$$

where

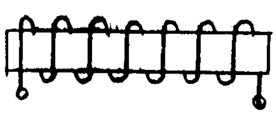
$$\mu_f \approx \mu_{rod} \left[1 - 8/27 \, (a/\ell_f)^2 \right] * \qquad (3\text{-}4a)$$

and H = magnetic field intensity (parallel to rod)

$$L_f = \left[\frac{\mu_{coil} \, \mu_o \, A_f \, n_f^{\,2}}{a} \right] \qquad (3\text{-}5)$$

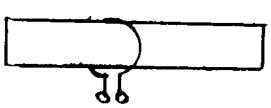
$$r_f = \frac{\omega L_f}{Q_f} \qquad (3\text{-}6)$$

Special cases:



$\frac{a_f}{\ell_f} = 1$

$$\left(\begin{array}{l} \mu_f \approx \dfrac{\mu_{rod}}{\sqrt{2}} \\ \mu_{coil} \approx \mu_{rod} \end{array} \right) \qquad (3\text{-}7)$$



$\frac{a_f}{\ell_f} = 0$

$$\left(\begin{array}{l} \mu_f \approx \mu_{rod} \\ \mu_{coil} \approx 1 \end{array} \right) \qquad (3\text{-}8)$$

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* See Appendix and empirical curve of Figure 3-5

In describing a single ferrite antenna, two different effective relative permeabilities are encountered; viz, μ_f associated with the induced voltage and μ_{coil} associated with the coil self-inductance. Perhaps the use of the term permeability is somewhat misleading since that term is often used to describe intrinsic (or toroidal) permeability which is an inherent property of the material and the magnetizing force employed. In the case of a ferrite rod antenna, these factors are used to describe how effectively a magnetic field is deformed and caused to increase the flux linking the turns of the antenna coil. Thus, effective relative permeability is a function of coil design and magnetic field configuration as well as core material and magnetizing force.

For example, consider μ_f. This factor describes the increase in flux linkages in the antenna coil due to external field deformation caused by the presence of the ferrite core over the flux linkages in the antenna coil when the ferrite is absent and the external field is uniform.

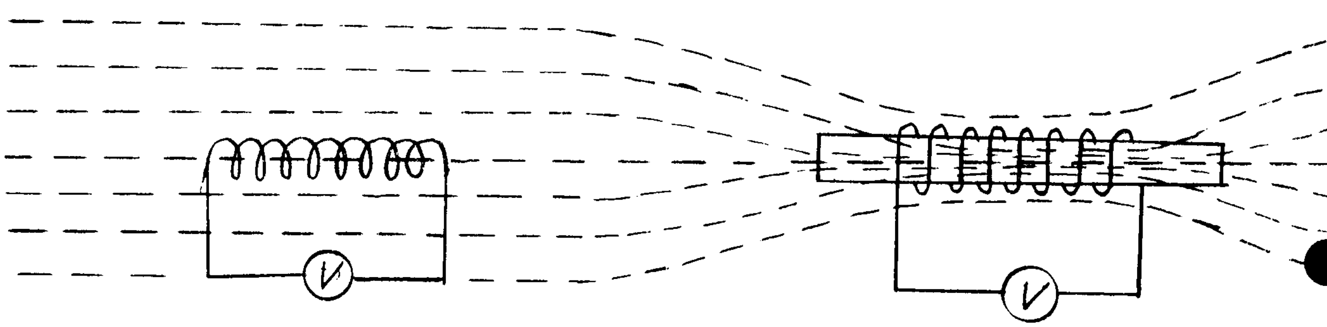


FIGURE 3-3

ILLUSTRATION OF μ_f

On the other hand, μ_{coil} describes the increase in flux linkages in the antenna coil due to the deformation of the non-uniform magnetic field created by current flowing in the antenna coil.

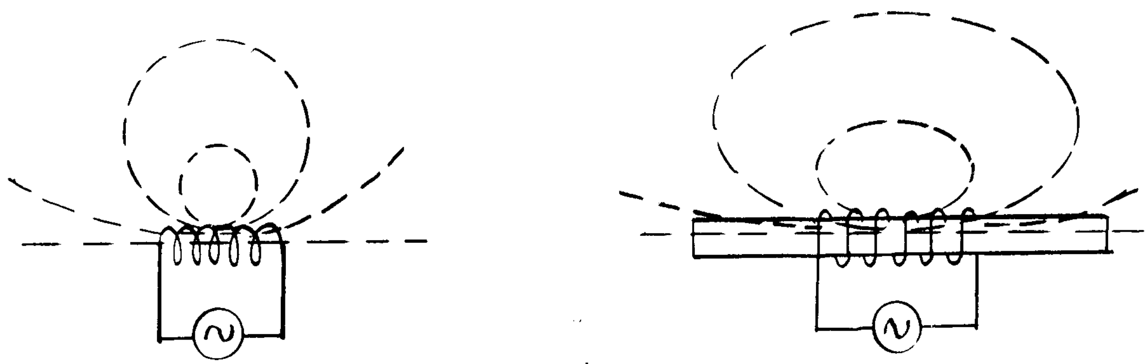


FIGURE 3-4

ILLUSTRATION OF μ_{coil}

Both of these factors describe increases of flux linkages in the same coil due to the same core, but they differ because two different magnetic field configurations are involved. That is to say, the resulting deformed fields differ from each other because the undeformed fields differed.

Figure 3-5 shows μ_f and μ_{coil} as obtained from four equal inductance antennas having identical cores but different winding lengths. Of particular interest in this figure is the induced voltage curve which shows that the full length coil antenna has an induced voltage which is 60% greater than that induced in a concentrated coil antenna.

The increase in induced voltage with coil length is attributable to the fact that more turns are required on the longer coils than the shorter coils in order to obtain the same value of self-inductance. In practice, maximum antenna self-inductance is limited by permissible (or unavoidable) shunt capacity. The highest induced voltage is, therefore, obtained when the antenna coil is wound so that it maximizes the ratio of turns-to-inductance.

It is also noted that, if coil Q can be assumed constant as a function of a/ℓ ratio, then equal inductance antennas generate equal thermal noise voltages. Consequently, induced voltage-to-thermal-noise ratio has the same functional dependence on a/ℓ as does the induced voltage alone. It is concluded, therefore, that

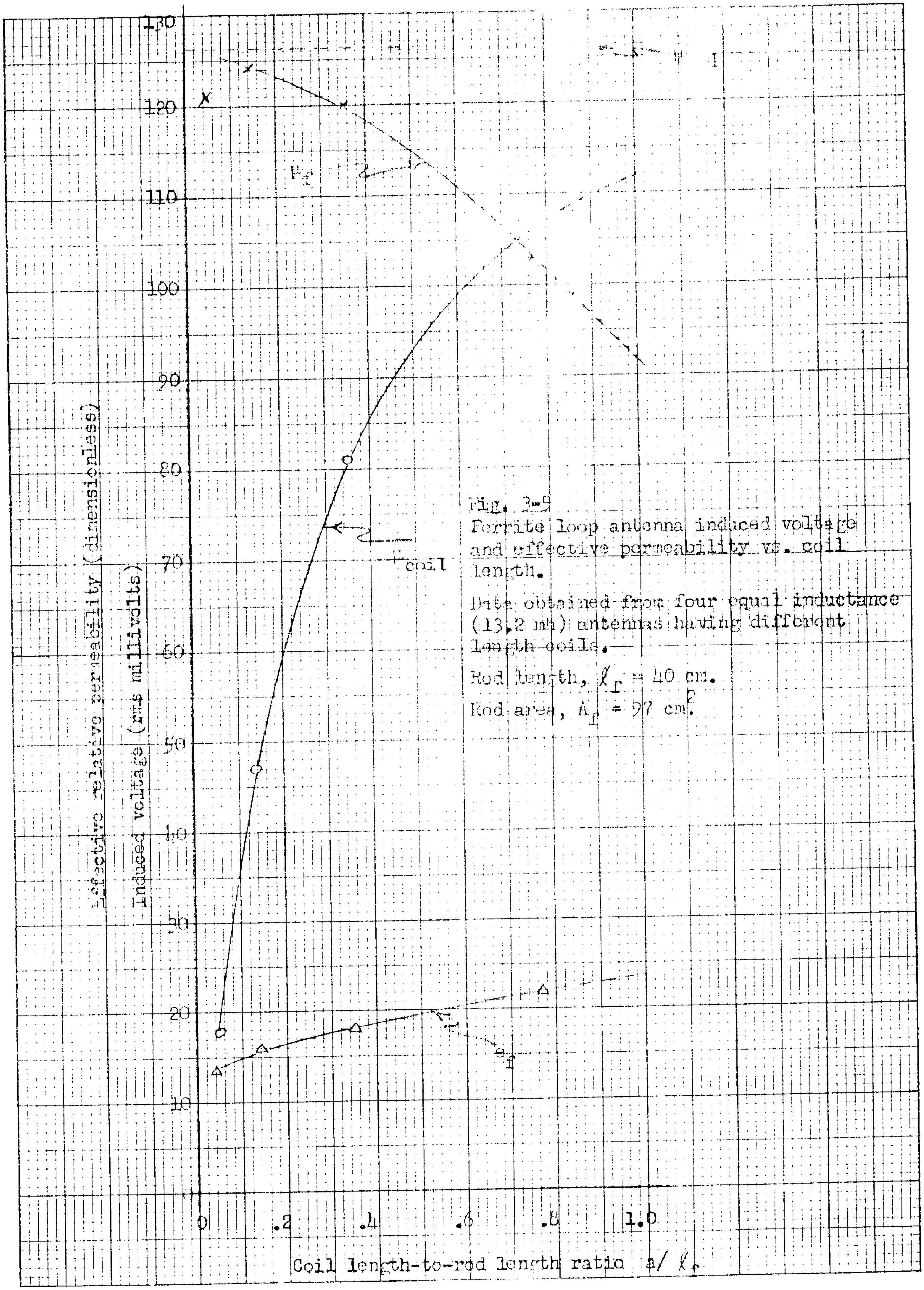


Fig. 3-9
Ferrite loop antenna induced voltage and effective permeability vs. coil length.

Data obtained from four equal inductance (13.2 mh) antennas having different length coils.

Rod length, ℓ_r = 40 cm.

Rod area, A_f = 97 cm^2.

full length windings are more advantageous from the standpoint of induced volts-to-thermal-noise ratio, e_f/e_n, provided coil Q can be kept constant while increasing a/ℓ ratio.

Since full length coils produce larger induced voltages, and since it is reasonable to expect that in many cases this also corresponds to larger induced voltage-to-thermal-noise voltage ratios, the ferrite antennas considered hereafter will have full length coils (unless otherwise specified). In passing, it is worthwhile to note that a distributed winding (full length coil) is, in general, simpler to wind with low distributed capacity than a concentrated coil; this, in turn, simplifies the problem of obtaining high impedance levels. As noted in Equation (3-7) for full length coils

$$\mu_f \approx \frac{\mu_{rod}}{\sqrt{2}} \qquad \text{and} \qquad \mu_{coil} \approx \mu_{rod}$$

These approximations are borne out fairly well by the empirical curves shown in Figure 3-5.

IV. COMPARISON OF ATMOSPHERIC NOISE AND THERMAL NOISE IN LOOP ANTENNAS AT VERY LOW FREQUENCIES

Introduction

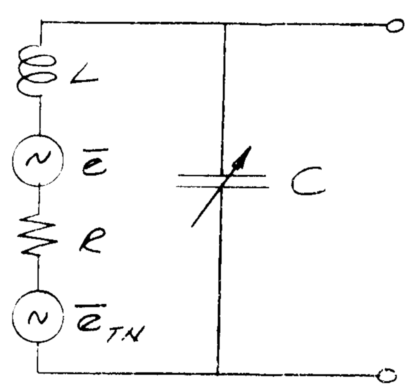
The degree of importance attached to thermal noise generation in the antenna circuit depends upon the relative level of thermal noise volts compared to the total noise in the system. At 25 kcs (and VLF in general) total receiver noise is largely thermal-noise plus external-noise sources such as atmospherics and electrostatic disturbances. The last noise source is predominately an electric field phenomena arising due to sparking in nearby electrical equipment, etc. (see external noise source discussion). The ideal loop antenna responds only to a magnetic field. Thus, if the loop antennas considered are idealized to the extent that E-field pick-up is assumed to be zero, then internal thermal noise and external atmospheric noise remain as the major sources of noise in a loop antenna receiver system.

A comparison of the relative magnitudes of noise voltages introduced by these two noise sources into a loop antenna will help determine whether it is necessary to consider thermal noise generation in the design, or evaluation, of a receiving loop antenna. Such a comparison is provided in the following work by estimating the ratio of voltage induced in a loop due to atmospheric-noise to thermal-noise generated in the loop due to the resistive component of the antenna impedance; i.e., $\bar{e}_{AN}/\bar{e}_{TN}$.

Assumptions Made in the Calculation of $\bar{e}_{AN}/\bar{e}_{TN}$

1. Zero E-field pickup in loop antenna.
2. First amplifier noise is negligible compared to thermal noise generated in antenna circuit.
3. Atmospheric noise field strength given by median day and night values for the latitudes of the U.S.A. (1) .
4. Antenna circuit must pass a band of frequencies 2000 cps wide centered at 25 kc/s with a maximum of 3 db variation of the band.
5. Loop antennas considered are:
    a) air core circular loop with closely spaced turns.
    b) ferrite core loops with full length coils.
6. Antenna circuit : a) resonated, b) not resonated.
7. In the case of the non-resonant antenna circuit, it is assumed that a 2000 cps passband is established by tuned circuits located in the receiver after the first amplifier stage.

Resonant Air Core Loop



$\bar{e}$ = voltage induced in loop due to external fields (rms)

$\bar{e}_{TN}$ = thermal noise voltage due to resistance R

FIGURE 4-1
EQUIVALENT CIRCUIT OF RESONANT AIR CORE LOOP

The induced voltage from Equation (3-1) is

$$\bar{e}_a = \mu_o \, \omega \, A_a \, n_a \, \bar{H} \tag{4-1}$$

where subscript "a" refers to air core loop and $\bar{H}$ = magnetic field intensity (rms). The electric and magnetic field intensities are related by

$$\frac{\bar{E}}{\bar{H}} = \sqrt{\frac{\mu_o}{\varepsilon_o}} \tag{4-2}$$

Equation (4-1) can be rewritten as

$$\bar{e}_a = \frac{2\pi \, A_a \, n_a \, \bar{E}}{\lambda} \tag{4-3}$$

where λ = wavelength = $\dfrac{1}{f\sqrt{\mu_o \, \varepsilon_o}}$

and λ = 12,000 meters for $f = 25$ kc

The induced voltage at f = 25 kc is given by

$$\bar{e}_a = \left[\frac{n_a \, A_a}{1910} \right] \bar{E} = (410)\, 10^{-6} \, n_a \, D_a^{\,2} \, \bar{E} \tag{4-4}$$

where D_a = loop diameter.

For a passband of 2000 cps at 25 kc center frequency, the loaded Q of the antenna circuit must be

$$\bar{Q} = \frac{\omega \, L_a}{R_a} = 12.5 \tag{4-5}$$

where R is composed of the internal resistance of the antenna plus additional resistance, if necessary, such that equation (4-5) is satisfied. In the absence of a signal, the induced voltage is entirely due to atmospheric noise pickup and is given by Equation (4-4) when an equivalent (rms) electric rield intensity, $\bar{E}_{AN}$, due to atmospheric noise is substituted for $\bar{E}$.

The effective thermal-noise voltage associated with R is given by

$$\bar{e}_{TN} = \sqrt{4K \, T \, \triangle f \, R} = \sqrt{\left(\frac{4kT \, \triangle f \, \omega}{\bar{Q}}\right) L} \tag{4-6}$$

where K = Boltzmann's constant = $(1.37)\ 10^{-23}\ \dfrac{\text{watt-sec}}{\text{deg.(Absol.)}}$

T = absolute temperature of R = 293° Abs.

Δf = passband ≈ 2000 cps

$\omega = 2\pi f_o \approx 2\pi(25000)$ rad/sec.

Equation (4-6) reduces to

$$\bar{e}_{TN} \approx 2.24\sqrt{L/Q} \approx 0.635\ \sqrt{L}\ \text{microvolts} \tag{4-7}$$

The inductance of an air core loop composed of n closely spaced turns is given by Equations (3-2) and (3-2a) which are repeated here:

$$L \approx k\,\mu_o\, n^2\, D/2$$

where $k = \ln(8D/d) - 2$

A reasonable range of values for D/d is

$$6 < D/d < 1000 \tag{4-8a}$$

The corresponding range of values of K is

$$2 < K < 7 \tag{4-8b}$$

For $K = 2$, inductance, call it L_1, becomes

$$\sqrt{L_1} = \sqrt{\mu_o\, n^2\, D} = (1.12)\ 10^{-3}\ n\sqrt{D} \tag{4-9}$$

For $K = 7$, inductance, call it L_2, becomes

$$\sqrt{L_2} = \sqrt{3.5\,\mu_o\, n^2\, D} = (2.1)\ 10^{-3}\ n\sqrt{D} \tag{4-10}$$

Hence, letting the subscripts 1 and 2 denote $K = 2$ and 7, respectively, the noise voltages become

$$\bar{e}_{TN_1} = (0.71)\ 10^{-3}\ n\ \sqrt{D}\ \text{microvolts} \tag{4-11a}$$

and

$$\bar{e}_{TN_2} \approx (1.30)\ 10^{-3}\ n\ \sqrt{D}\ \text{microvolts} \tag{4-11b}$$

Comparing atmospheric noise pickup with thermal noise generation, we obtain

$$\frac{\bar{e}_{AN}}{\bar{e}_{TN_1}} \approx 0.58\ D^{3/2}\quad \bar{E}_{AN} \tag{4-12a}$$

and

$$\frac{\bar{e}_{AN}}{\bar{e}_{TN_2}} = 0.30\, D^{3/2}\ \bar{E}_{AN} \qquad (4\text{-}12b)$$

For daytime atmospheric noise (for $\Delta f = 2000$, $f_o = 25$ kc):

$$\bar{E}_{AN} \approx 50 \text{ microvolts(rms)/meter (Median value USA)}^{(1)} \qquad (4\text{-}13a)$$

For nighttime atmospheric noise

$$\bar{E}_{AN} \approx 100 \text{ microvolts(rms)/meter}^{(1)} \qquad (4\text{-}13b)$$

From Equations (4-12) and (4-13), we obtain

	Daytime	Nighttime
$\frac{\bar{e}_{AN}}{\bar{e}_{TN_1}}$	$29\ D^{3/2}$	$58\ D^{3/2}$
$\frac{\bar{e}_{AN}}{\bar{e}_{TN_2}}$	$15\ D^{3/2}$	$30\ D^{3/2}$

where D is in meters.

Non-resonant Air Core Loop

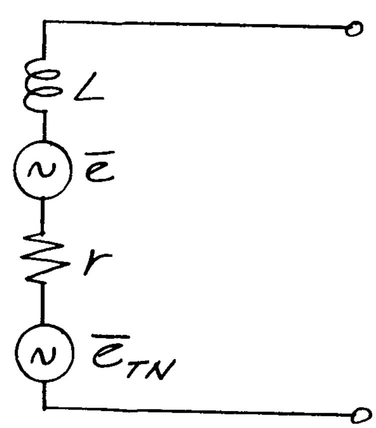


FIGURE 4-2

EQUIVALENT CIRCUIT OF NON-RESONANT AIR CORE LOOP

Atmospheric-noise volts to thermal-noise volts ratio for the non-resonant air core loop is the same as for the resonant air core loop with the exception that the circuit Q is no longer restricted to the low value of 12.5 in order to pass the required band of frequencies. The antenna coil Q is made as large as physically realizable (call it Q_a). The results of the resonant antenna are then directly applicable when multiplied by the factor $\sqrt{Q_a/2\overline{Q}}$ (see Section VI). Hence,

	Daytime	Nighttime
$\frac{\overline{e}_{AN}}{\overline{e}_{TN_1}}$	$5.8 \sqrt{Q_a}\ D^{3/2}$	Twice Daytime Values
$\frac{\overline{e}_{AN}}{\overline{e}_{TN_2}}$	$3.0 \sqrt{Q_a}\ D^{3/2}$	Twice Daytime Values

Resonant Ferrite Core Loop

The induced voltage for the ferrite core antenna is given by Equation (3-4) as

$$\overline{e}_f = \mu_f \mu_o \omega A_f n_f \overline{H} \qquad (4-14a)$$

or in terms of the E-field

$$\overline{e}_f = \sqrt{\mu_o \varepsilon_o}\ \mu_f \omega n_f \frac{\pi d_f^{\ 2}}{4} \overline{E} \qquad (4-14b)$$

where d_f = ferrite rod core diameter.

The thermal noise is given by Equation (4-7) as

$$\overline{e}_{TN} = 0.635 \sqrt{L_f} \text{ microvolts} \qquad (4-15)$$

Using Equation (3-5) for L_f and the full coil length approximations $\mu_f \approx \frac{\mu_{rod}}{\sqrt{2}}$ and $\mu_{coil} \approx \mu_{rod}$, we obtain the ratio $\frac{\overline{e}_{AN}}{\overline{e}_{TN}}$ to be

$$\frac{\overline{e}_{AN}}{\overline{e}_{TN}} = 0.46 \sqrt{\mu_{rod}}\ (d_f/\ell_f)\ \ell_f^{3/2}\ \overline{E}_{AN} \qquad (4-16)$$

where $\overline{E}_{AN}$ is given in (rms) microvolts/meter.

For purposes of comparison with air core loops, let

$$\ell_f = \ell = FD \tag{4-17}$$

where F = constant

and D = air core loop diameter

Substituting into Equation (4-16), we obtain

$$\frac{\bar{e}_{AN}}{\bar{e}_{TN}} = 0.46 \sqrt{F \mu_{rod}} \; (Fd/\ell) \, D^{3/2} \; \bar{E}_{AN} \tag{4-18}$$

where $\bar{E}_{AN}$ (rms) microvolts/meter

	Daytime	Nighttime
$\frac{\bar{e}_{AN}}{\bar{e}_{TN}}$	$23\sqrt{\mu_{rod}} \left(\frac{d_f}{\ell_f}\right) \ell_f^{3/2}$	Twice daytime values

or in terms of equivalent air core loops

$\frac{\bar{e}_{AN}}{\bar{e}_{TN}}$	$23\sqrt{F \mu_{rod}} \left(\frac{F_d}{\ell}\right) D^{3/2}$	Twice daytime values

Non-resonant Ferrite Core Loop

Multiply $\bar{e}_{AN}/\bar{e}_{TN}$ values of resonant case by $\sqrt{Q_f/2\bar{Q}}$ where Q_f = maximum obtainable value of Q for a ferrite core loop.

TABLE II

SUMMARY OF ATMOSPHERIC NOISE VOLTS-TO-THERMAL-NOISE VOLTS RATIOS LOOP ANTENNAS

ANTENNA TYPE	$\frac{\bar{e}_{AN}}{\bar{e}_{TN}}$ RATIO Daytime (Median Value of Atmos. Noise E-Field = 50 rms μ volts/meter
I. AIR CORE	
Resonant ($Q = \bar{Q} = 12.5$)	
K = 2	$29\ D^{3/2}$
K = 7	$15\ D^{3/2}$
Non-Resonant (Q = Max. air core Q)	
K = 2	$5.8\sqrt{Q}\ D^{3/2}$
K = 7	$3.0\sqrt{Q}\ D^{3/2}$
II-a FERRITE CORE	
Resonant ($Q = \bar{Q} = 12.5$)	$23\sqrt{\mu_{rod}}\,(d/\ell)\ \ell^{3/2}$
Non-Resonant (Q = Max. ferrite core "Q" = Q_{max})	$(4.6\sqrt{Q})\sqrt{\mu_{rod}}\,(d/\ell)\ \ell^{3/2}$

NOTE: As a means of comparing Air Core and Ferrite Core loops, write ℓ in terms of D, as follows:

Let ℓ = FD, where F = constant

ANTENNA TYPE	$\frac{\bar{e}_{AN}}{\bar{e}_{TN}}$ RATIO
II-b FERRITE CORE	
Resonant ($Q = \bar{Q} = 12.5$)	$23\sqrt{F\ \mu_{rod}}\,(Fd/\ell)\ D^{3/2}$
Non-Resonant ($Q = Q_{max}$)	$(4.6\sqrt{Q})\sqrt{F\ \mu_{rod}}\ (Fd/\ell)\ D^{3/2}$

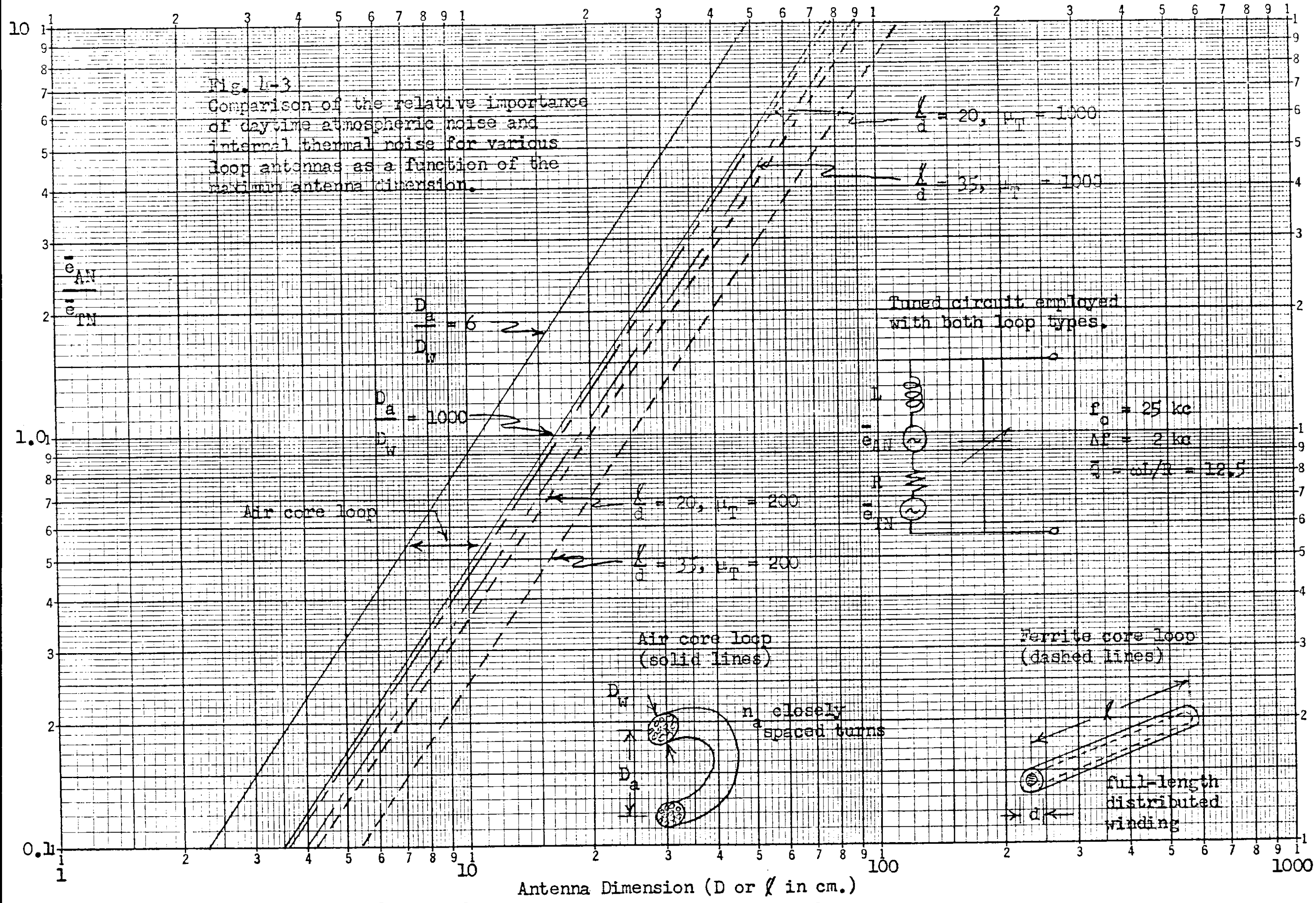


Fig. 4-3
Comparison of the relative importance of daytime atmospheric noise and internal thermal noise for various loop antennas as a function of the maximum antenna dimension.

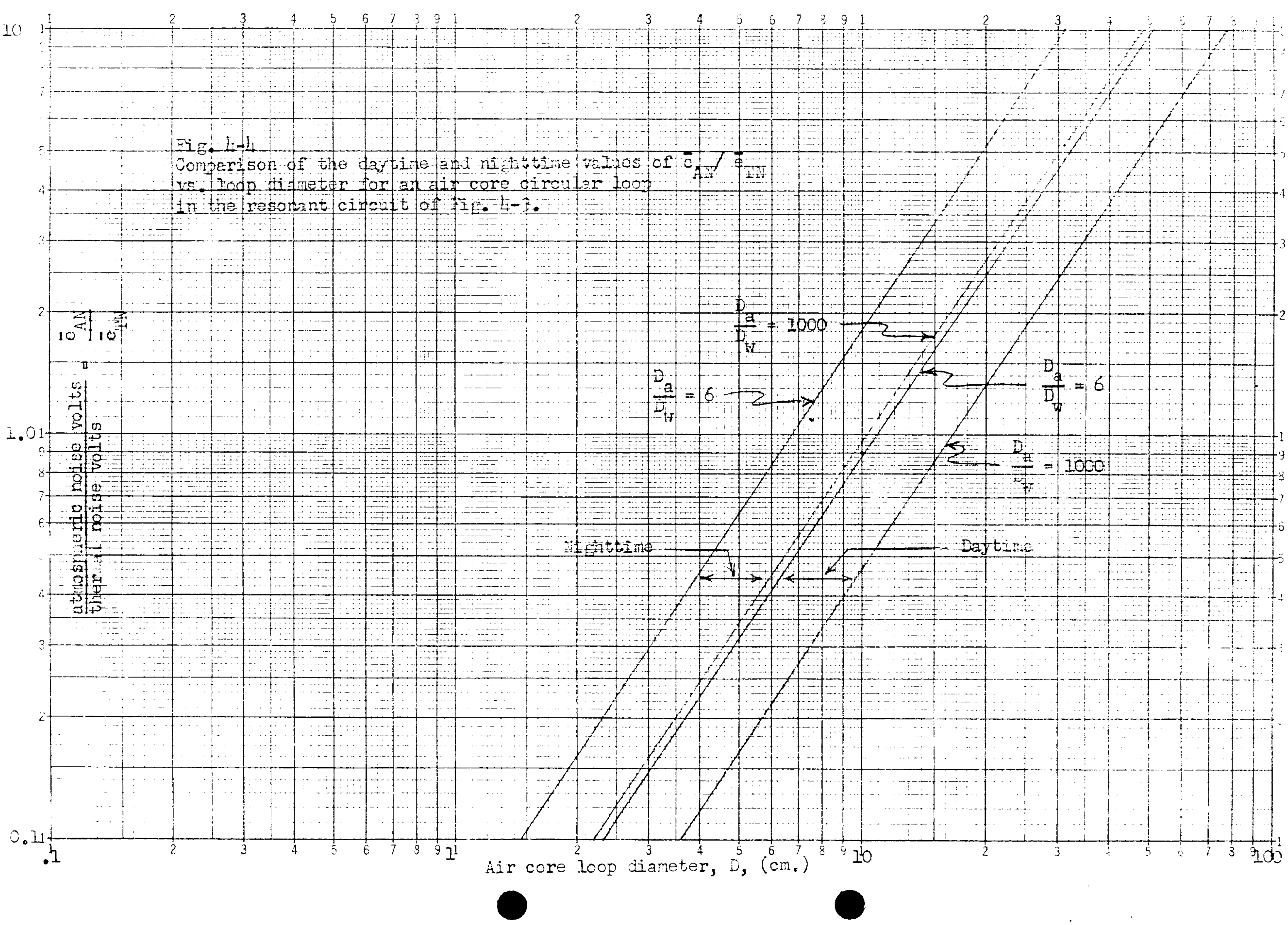


Fig. 4-4
Comparison of the daytime and nighttime values of $\bar{e}_{AN} / \bar{e}_{TN}$ vs. loop diameter for an air core circular loop in the resonant circuit of Fig. 4-3.

V. COMPARISON OF INDUCED VOLTAGES AND SIGNAL-TO-THERMAL-NOISE RATIOS IN FERRITE AND AIR CORE ANTENNAS

Referring to Equations (3-1) through (3-7), we obtain

Induced voltages:

Air Core Loop $$e_a = \mu_o \omega A_a n_a H \tag{5-1}$$

Ferrite Core Loop $$e_f = \frac{\mu_{rod}}{\sqrt{2}} \mu_o \omega A_f n_f H \tag{5-2}$$

Self Inductance and turns:

Air Core Loop $$L_a \approx K \mu_o n_a^2 D_a/2: \quad 2 < K < 7 \tag{5-3}$$

or $$n_a \approx \sqrt{\frac{2L_a}{K \mu_o D_a}} \tag{5-4}$$

Ferrite Core Loop $$L_f = \frac{\mu_{rod} \mu_o A_f n_f^2}{\ell_f} \tag{5-5}$$

or $$n_f = \sqrt{\frac{L_f \ell_f}{\mu_{rod} \mu_o A_f}} \tag{5-6}$$

The ratio of induced voltage for the two loops is given by

$$\frac{e_f}{e_a} = \frac{\mu_{rod}}{\sqrt{2}} \frac{n_f}{n_a} (d/D)^2 \tag{5-7a}$$

or equivalently in terms of self-inductances

$$\frac{e_f}{e_a} = \sqrt{\frac{K}{\pi} \mu_{rod}} \sqrt{\frac{L_f}{L_a}} \frac{d\sqrt{\ell}}{D^{3/2}} \tag{5-7b}$$

Once again for purposes of comparing air core and ferrite core loops, let

$\ell = FD$

Then for the case of <u>equal</u> <u>inductance</u> antennas

$$\left.\frac{e_f}{e_a}\right|_{L_f = L_a} \triangleq \nu_L = \sqrt{\frac{KF}{\pi} \mu_{rod}} \left(\frac{Fd}{\ell}\right) \tag{5-8}$$

whereas equal turns antennas yield

$$\left.\frac{e_f}{e_a}\right|_{n_f = n_a} \triangleq \nu_n = \frac{\mu_{rod}}{\sqrt{2}} \left(\frac{FD}{\ell}\right)^2 \tag{5-9}$$

NOTE: $\left[\left(\frac{\sqrt{2}\ KF}{\pi}\right) \nu_n\right]^{1/2} = \nu_L$

Now the ratio of thermal noise generated in the two loops is given by the ratio of their resistances, i.e.,

$$\frac{\bar{e}_{TN_a}}{\bar{e}_{TN_f}} = \sqrt{\frac{R_a}{R_f}} = \sqrt{\frac{Q_f}{Q_a} \ \frac{L_a}{L_f}} \tag{5-10}$$

Signal-to-thermal ratios are given by

$$(S/TN)_f = \frac{\bar{e}_f}{\bar{e}_{TN_f}} \tag{5-11}$$

$$(S/TN)_a = \frac{\bar{e}_a}{\bar{e}_{TN_a}} \tag{5-12}$$

Thus,

$$\frac{(S/TN)_f}{(S/TN)_a} = \frac{e_f}{e_a} \times \frac{\bar{e}_{TN_a}}{\bar{e}_{TN_f}} \tag{5-13}$$

Substituting (5-7b) and (5-10) into (5-13), we obtain

$$\frac{(S/TN)_f}{(S/TN)_a} = \left[\sqrt{\frac{K\ \mu_{rod}}{\pi}}\ \frac{d\sqrt{\ell}}{D^{3/2}}\right]\sqrt{\frac{Q_f}{Q_a}} \tag{5-14a}$$

or upon letting $\ell = FD$

$$\frac{(S/TN)_f}{(S/TN)_a} = \left[\sqrt{\frac{KF\ \mu_{rod}}{\pi}}\ \frac{Fd}{\ell}\right]\sqrt{\frac{Q_f}{Q_a}} \tag{5-14b}$$

The factor within the braces in Equation (5-14b) is identically ν_L, the ratio of induced voltage for equal inductance antennas, hence,

$$\frac{(S/TN)_f}{(S/TN)_a} = \nu_L \sqrt{\frac{Q_f}{Q_a}} \qquad \text{for } L_f = L_a \text{ and } L_f \neq L_a \qquad (5\text{-}15)$$

Equation (5-15) shows that, when the circuit Q is specified (e.g., for purposes of necessary bandwidth in a resonant antenna circuit, $Q_f = Q_a = \overline{Q}$ and the ratio of signal-to-thermal-noise ratios is equivalent to the ratio of induced voltages when antenna inductances are equal, i.e.,

$$\left.\frac{(S/TN)_f}{(S/TN)_a}\right|_{Q_f = Q_a} = \left.\frac{e_f}{e_a}\right|_{\substack{Q_f = Q_a \\ L_f = L_a}} \triangleq \nu_L \qquad (5\text{-}16)$$

Thus Equation (5-8) which specifies ν_L, the ratio of induced voltages for equal inductance antennas, provides a useful means of comparing air core and ferrite core loops. ν_L is plotted in the following figures. In these plots, an optimistic estimate of air core induced voltage is made by assuming $K = \ln\left(\frac{8 D_a}{D_w}\right) - 2 = 2$ which corresponds to $D_a/D_w = 7$.

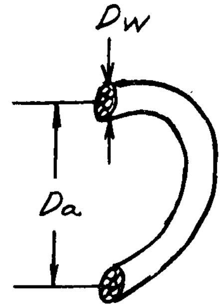


FIGURE 5-1
AIR CORE LOOP GEOMETRY

However, for values of K different from 2, as will probably be the case for large loops, $\nu_L\Big|_{K=2}$ can readily be corrected to correspond to an arbitrary value of K by multiplying by $\sqrt{K/2}$ since ν_L varies as the square root of K.

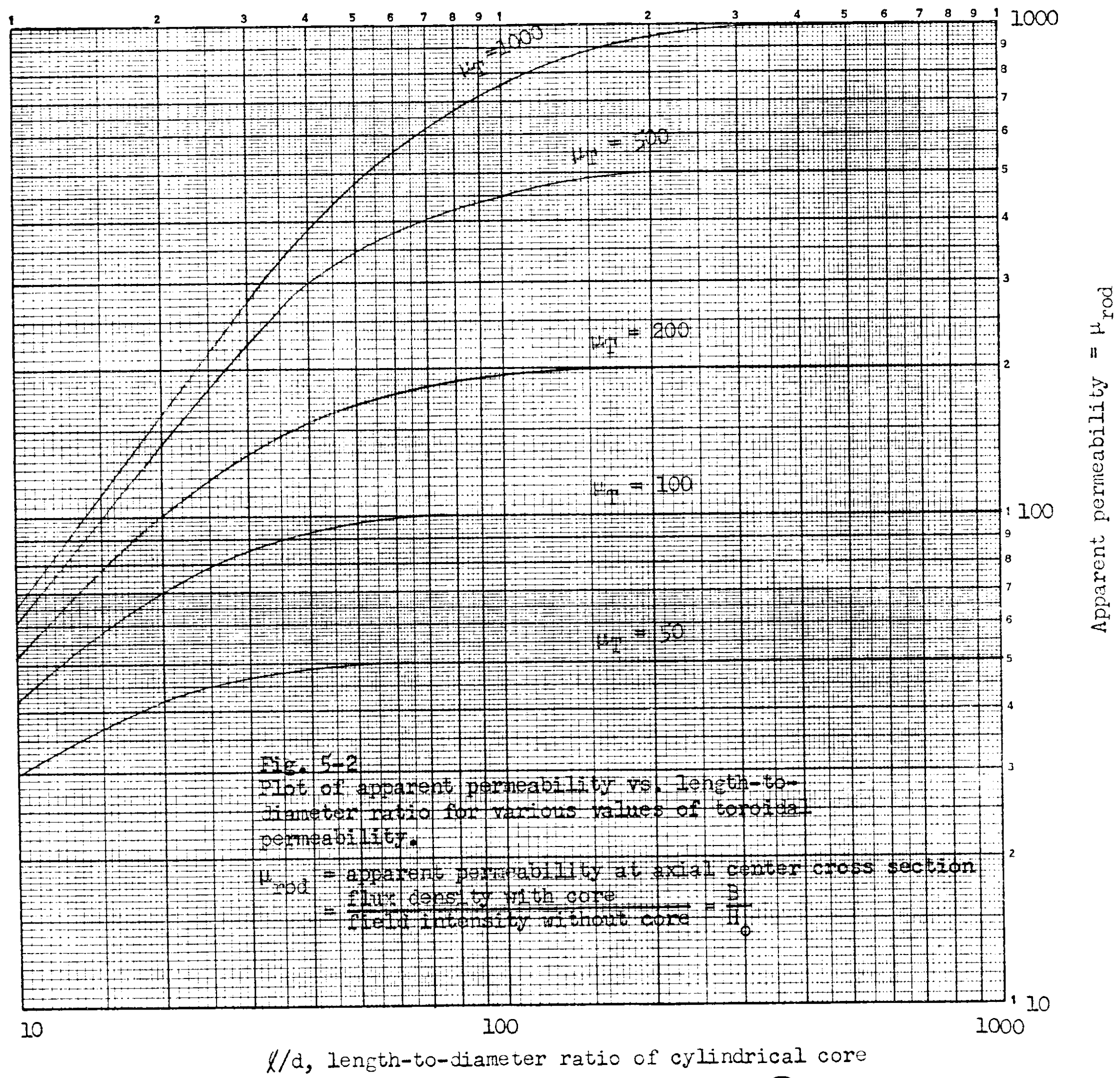


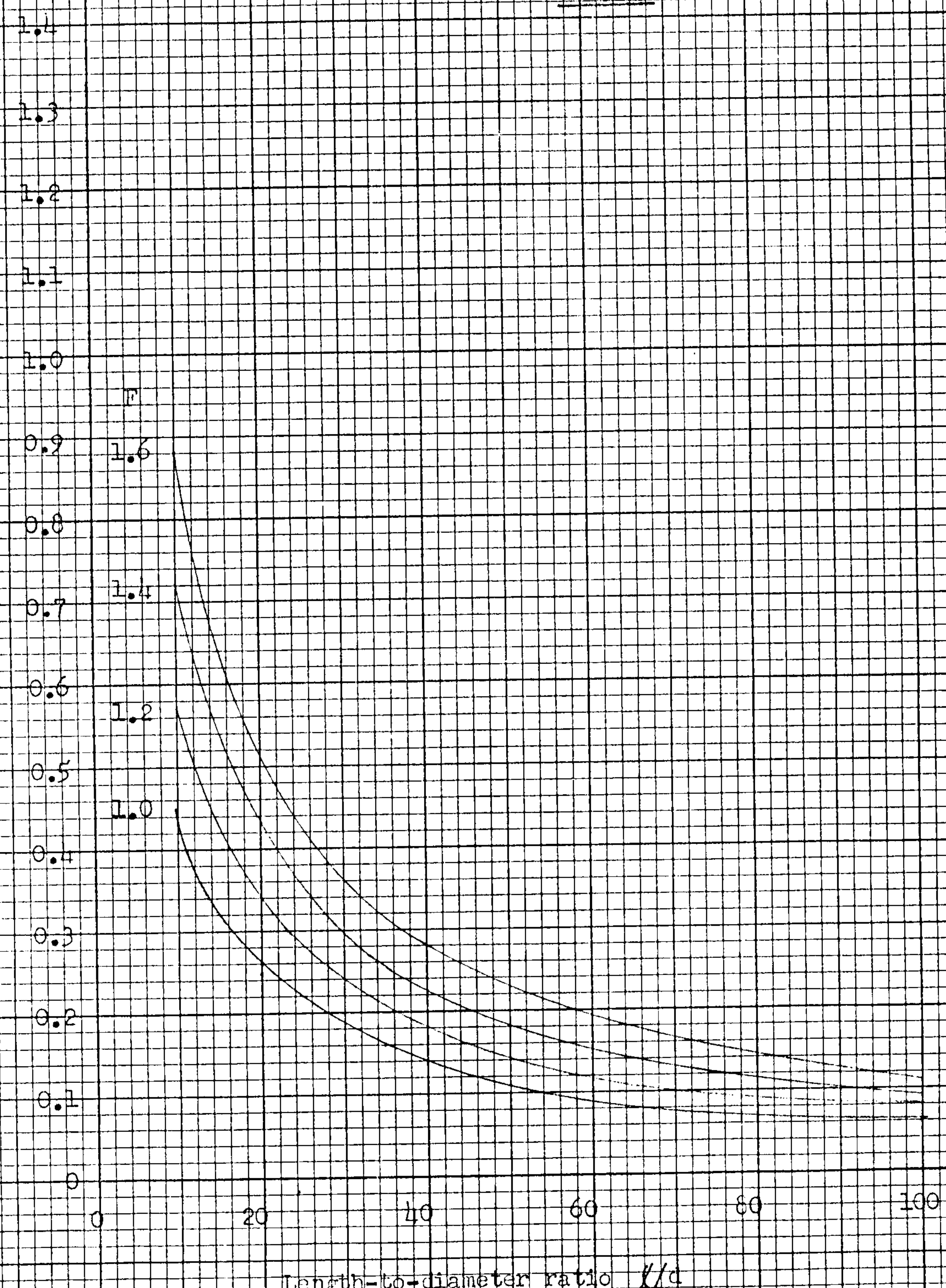
Fig. 5-2
Plot of apparent permeability vs. length-to-diameter ratio for various values of toroidal permeability.

μ_{rod} = apparent permeability at axial center cross section
$= \dfrac{\text{flux density with core}}{\text{field intensity without core}} = \dfrac{B}{H_o}$

Fig. 5-3
Plot of the ratio of induced voltage in a ferrite core antenna to the induced voltage in an equal-inductance air core antenna, (ν_L), as a function of length-to-diameter ratio of the ferrite core.

F = ratio of ferrite core length to air core diameter

Toroidal permeability $\mu_T = 50$



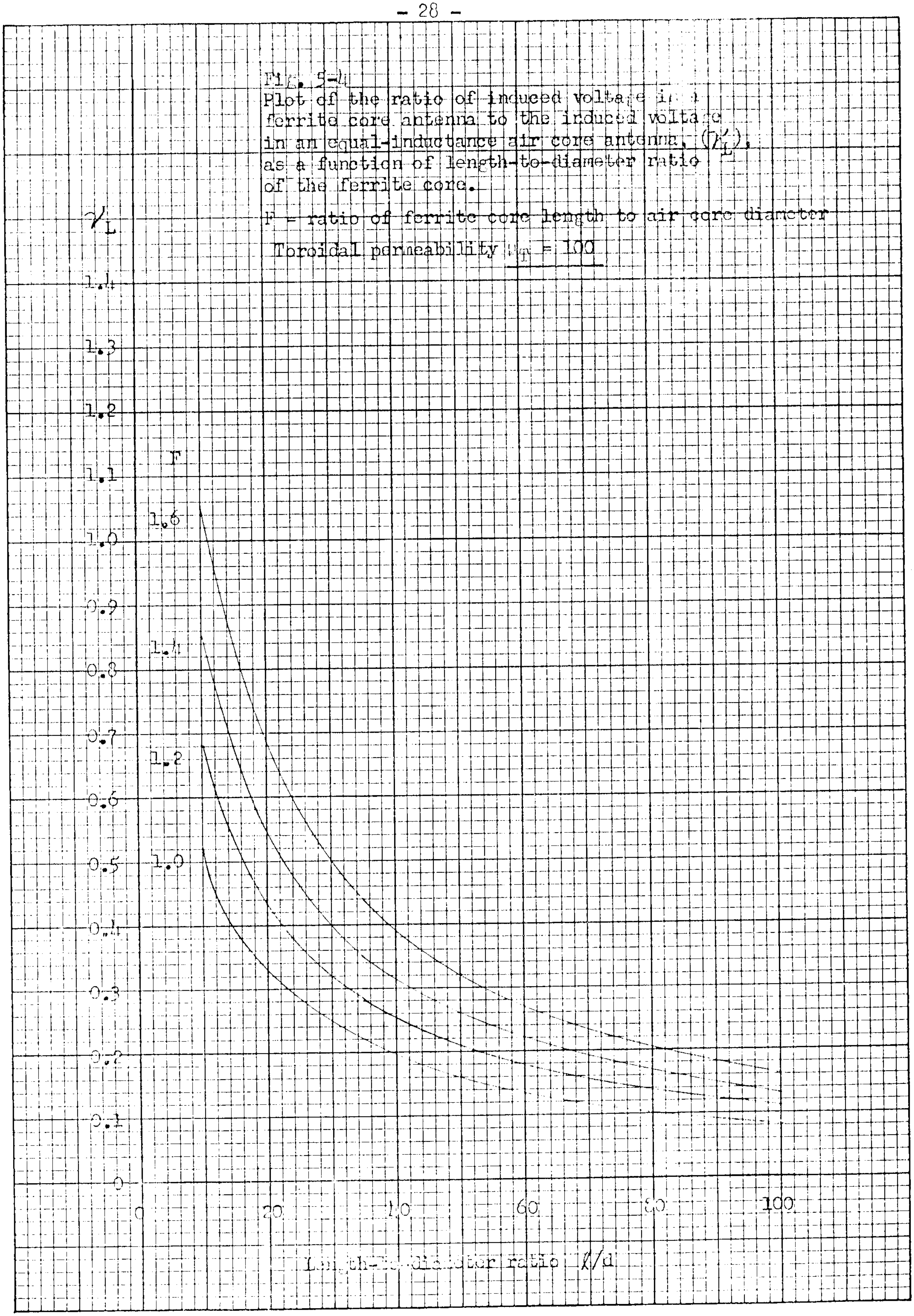


Fig. 5-4
Plot of the ratio of induced voltage in a ferrite core antenna to the induced voltage in an equal-inductance air core antenna, (γ_L'), as a function of length-to-diameter ratio of the ferrite core.

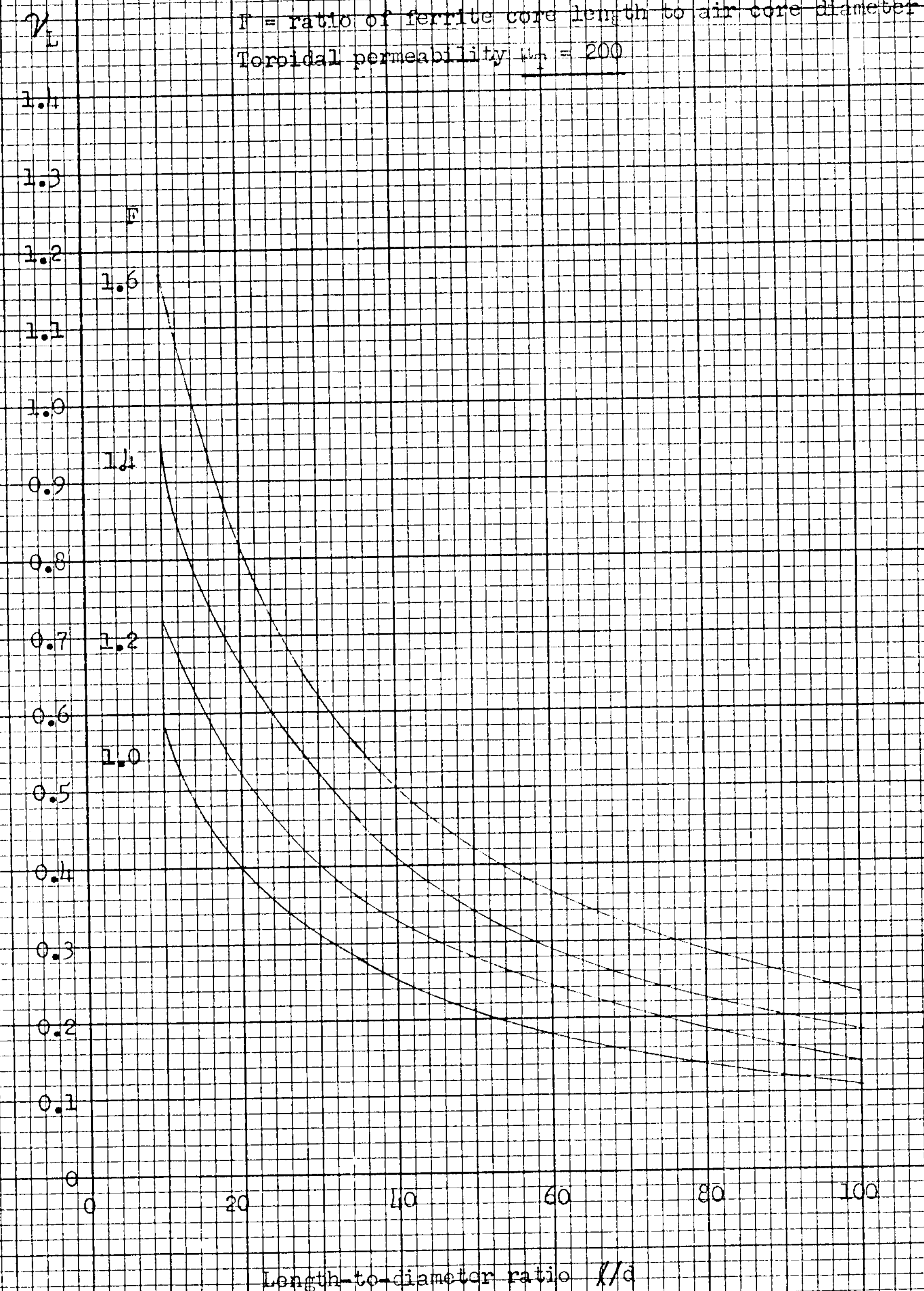
F = ratio of ferrite core length to air core diameter

Toroidal permeability μ_T = 100

Fig. 5-5
Plot of the ratio of induced voltage in a ferrite core antenna to the induced voltage in an equal-inductance air core antenna, (ν_L), as a function of length-to-diameter ratio of the ferrite core.

F = ratio of ferrite core length to air core diameter

Toroidal permeability $\mu_T = 200$



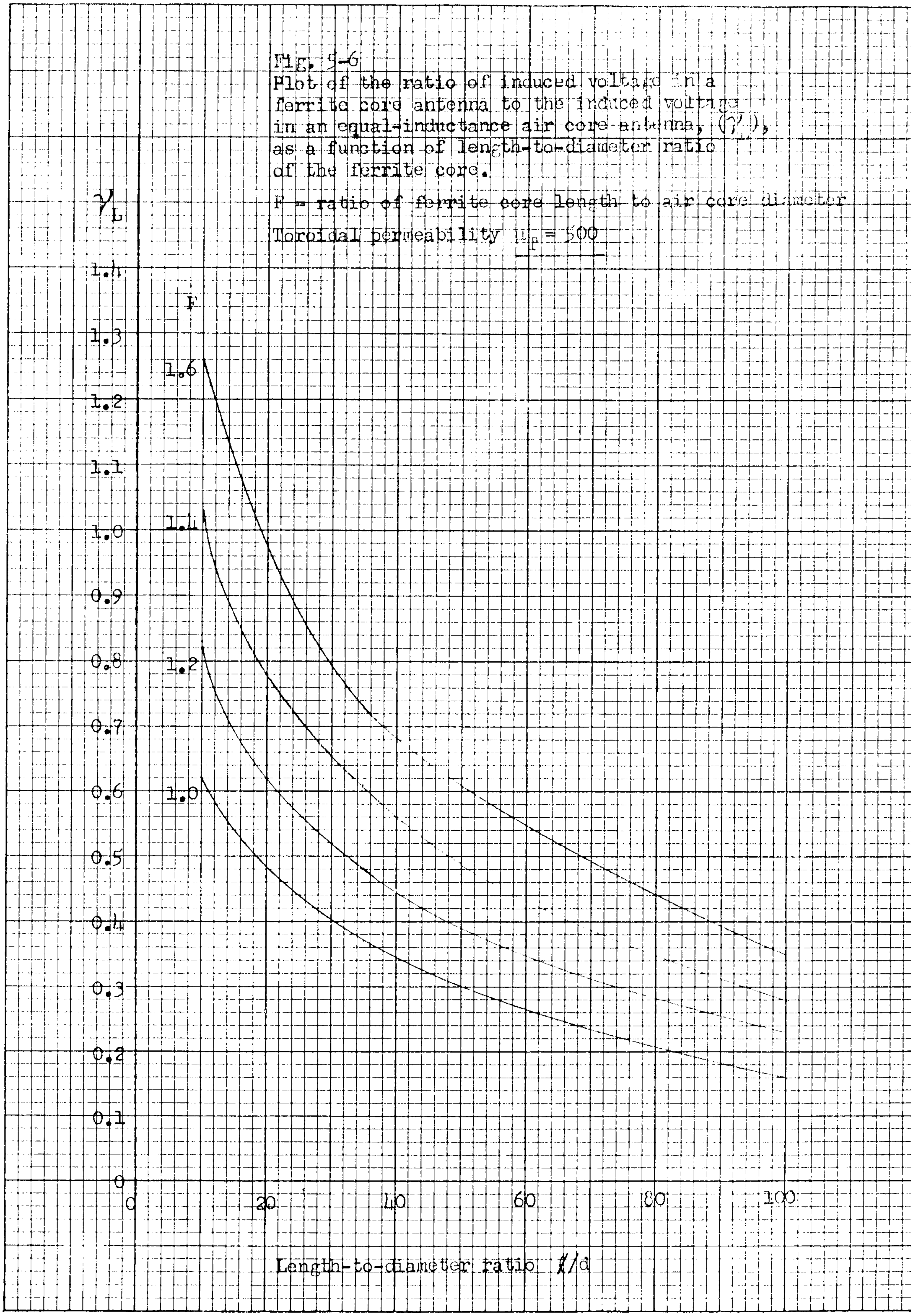


Fig. 9-6
Plot of the ratio of induced voltage in a ferrite core antenna to the induced voltage in an equal-inductance air core antenna, (γ'_L), as a function of length-to-diameter ratio of the ferrite core.

F = ratio of ferrite core length to air core diameter

Toroidal permeability $\mu_p = 500$

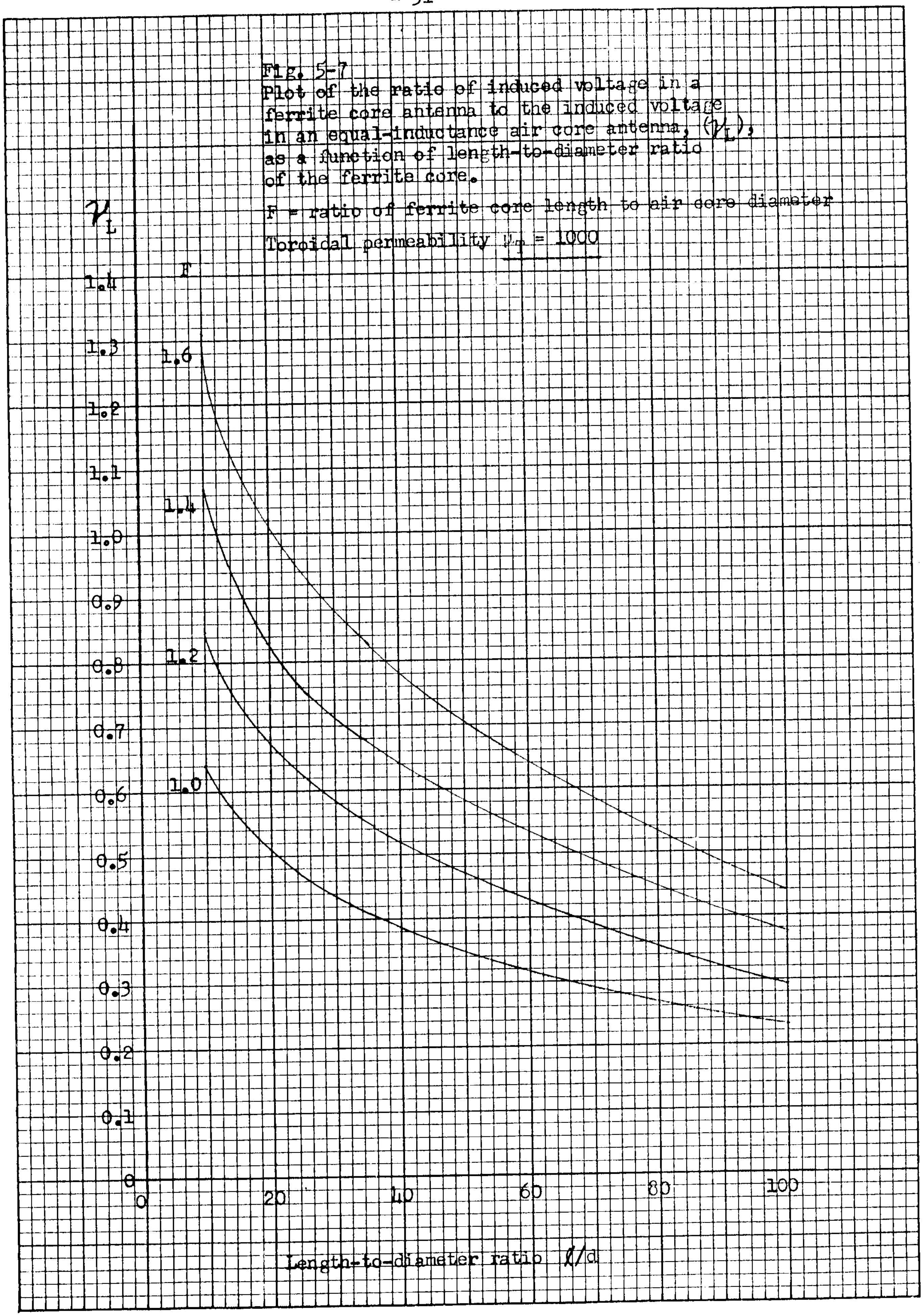


Fig. 5-7
Plot of the ratio of induced voltage in a ferrite core antenna to the induced voltage in an equal-inductance air core antenna, (γ_L), as a function of length-to-diameter ratio of the ferrite core.

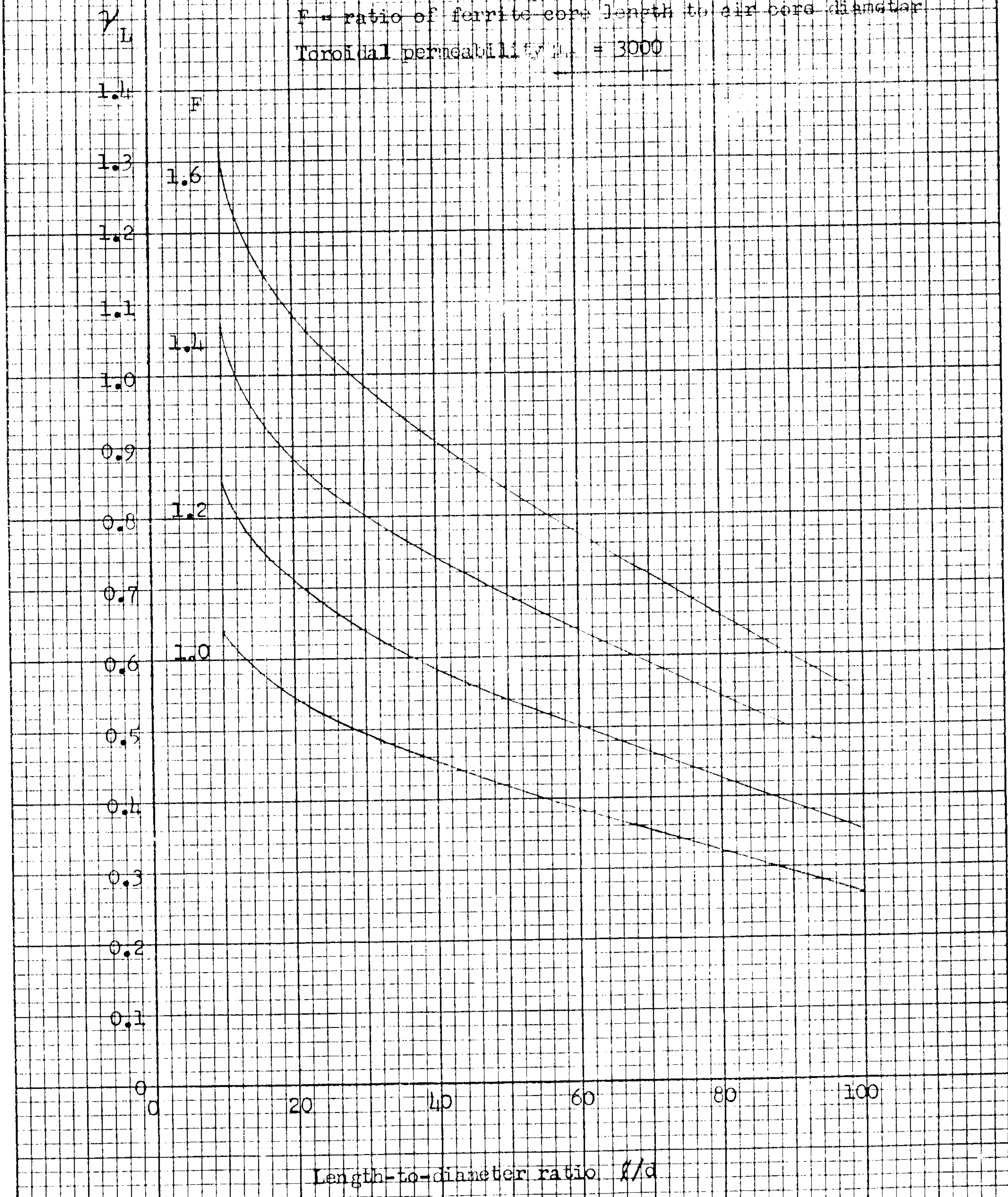
F = ratio of ferrite core length to air core diameter

Toroidal permeability μ_T = 1000

Fig. 5-8
Plot of the ratio of induced voltage in a ferrite core antenna to the induced voltage in an equal-inductance air core antenna, (ν_L), as a function of length-to-diameter ratio of the ferrite core.

F = ratio of ferrite core length to air core diameter

Toroidal permeability μ = 3000



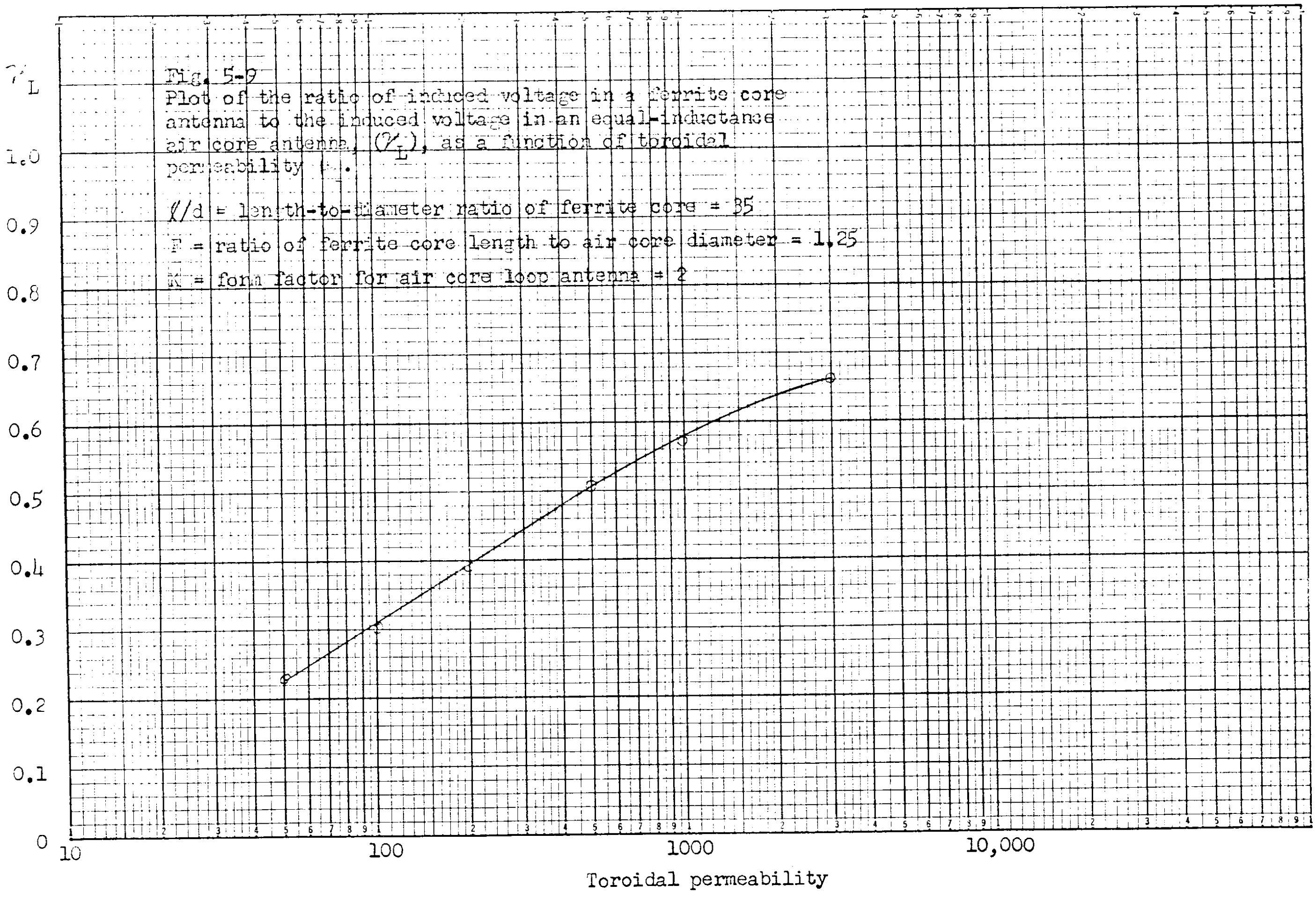


Fig. 5-9
Plot of the ratio of induced voltage in a ferrite core antenna to the induced voltage in an equal-inductance air core antenna, (γ_L), as a function of toroidal permeability (μ).

ℓ/d = length-to-diameter ratio of ferrite core = 35

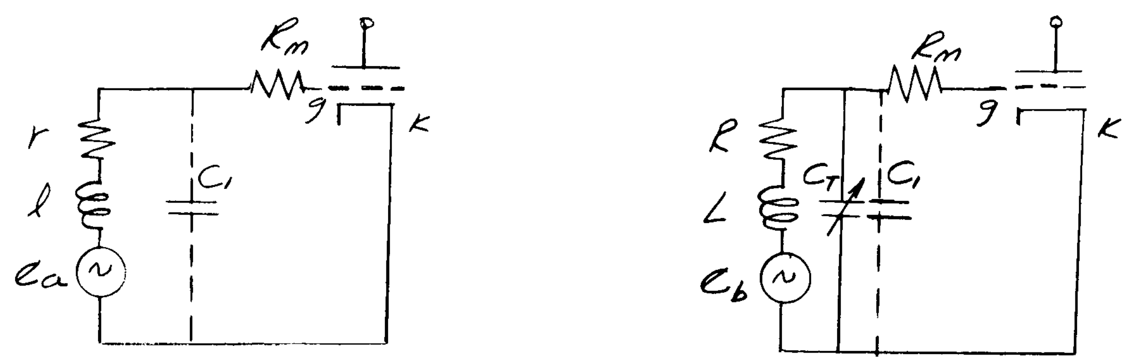
F = ratio of ferrite core length to air core diameter = 1.25

K = form factor for air core loop antenna = 2

An important observation can be made from the equations of this section. Reference to Equations (5-1) and (5-2) shows that the induced signal voltages for both ferrite and air core loops are proportional to number of turns. The thermal noise developed in each is proportional to the square root of its resistance. The resistance in turn is proportional to the inductance ($R = \omega L/\bar{Q}$) for a tuned antenna. Equations (5-3) and (5-5) show that the inductances are proportional to number of turns squared. Thus, it follows that for both ferrite and air core tuned antennas, the signal-to-thermal-noise ratio is independent of number of turns and, hence, independent of impedance level. Therefore, signal sensitivity can be maximized by maximizing the number of turns without sacrificing signal-to-thermal-noise ratio.

VI. COMPARISON OF TUNED AND UNTUNED ANTENNAS ON A SENSITIVITY BASIS AND ON A SIGNAL-TO-THERMAL-NOISE BASIS FOR (a) Broadband Antenna; (b) Narrow-Band Antenna

In a loop antenna which has a maximum dimension very much smaller than the wavelength being received, an equivalent circuit representation is simply an RL impedance in series with an induced emf. The resistive part of the internal impedance, of course, is a source of thermal noise. Consider now two basic antenna circuits by referring to the circuits (a) and (b) in Figure 6-1.



(a) Untuned: $\left[\omega_0^2 < \frac{1}{l C_1}\right]$ $Q \triangleq \frac{\omega l}{r}$

(b) Tuned: $\left[\omega_0^2 = \frac{1}{L(C_1 + C_T)}\right]$ $\bar{Q} \triangleq \frac{\omega L}{R}$

FIGURE 6-1

EQUIVALENT CIRCUITS FOR TUNED AND UNTUNED LOOP ANTENNA CIRCUITS

For the untuned antenna of Figure 6-1a, the signal voltage at the grid is given by

$$e_{g_a} = e_a \tag{6-1}$$

and the signal-to-noise ratio at the grid is

$$(S/N)_a = \frac{e_a}{\sqrt{4\ KT\ \triangle f(r+R_n)}} \tag{6-2}$$

For the tuned antenna of Figure 6-1b

$$e_{g_b} = \overline{Q}\ e_b \tag{6-3}$$

and

$$(S/N)_b = \frac{\overline{Q}\ e_b}{\sqrt{4\ KT\ \triangle f(\overline{Q}^2\ R + R_n)}} \tag{6-4}$$

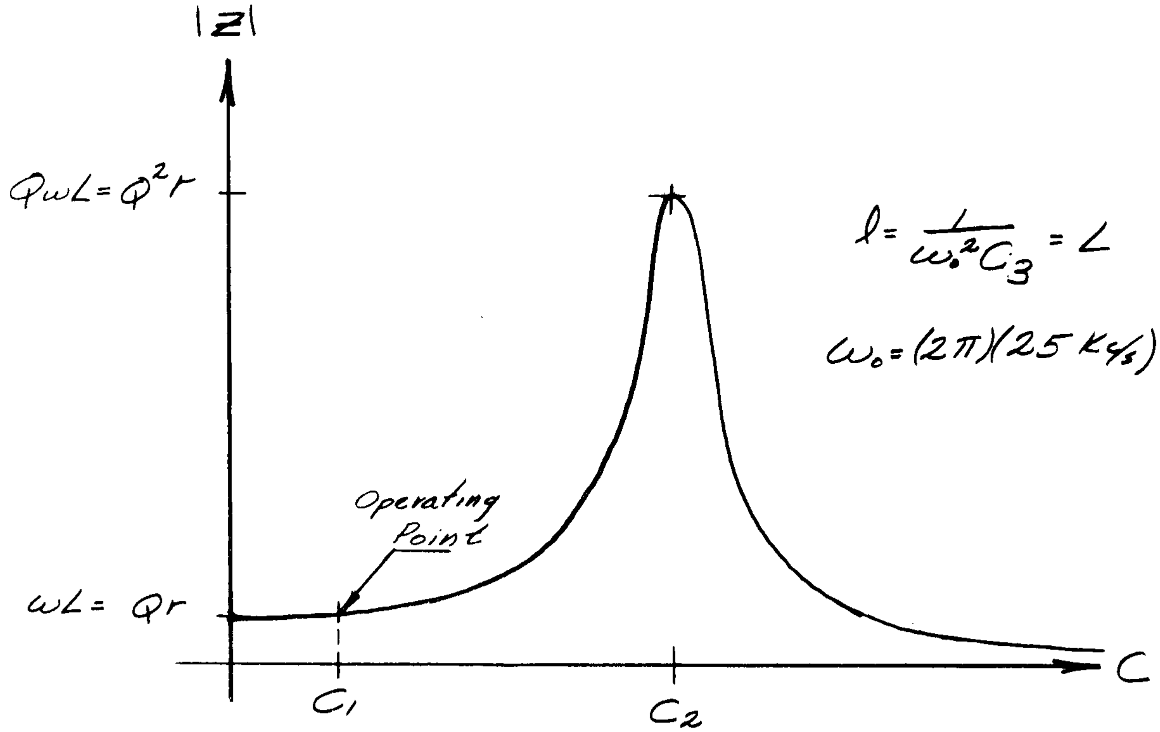
Thus by way of comparison, we have

$$\frac{e_{g_a}}{e_{g_b}} = \frac{1}{\overline{Q}}\left(\frac{e_a}{e_b}\right) \tag{6-5}$$

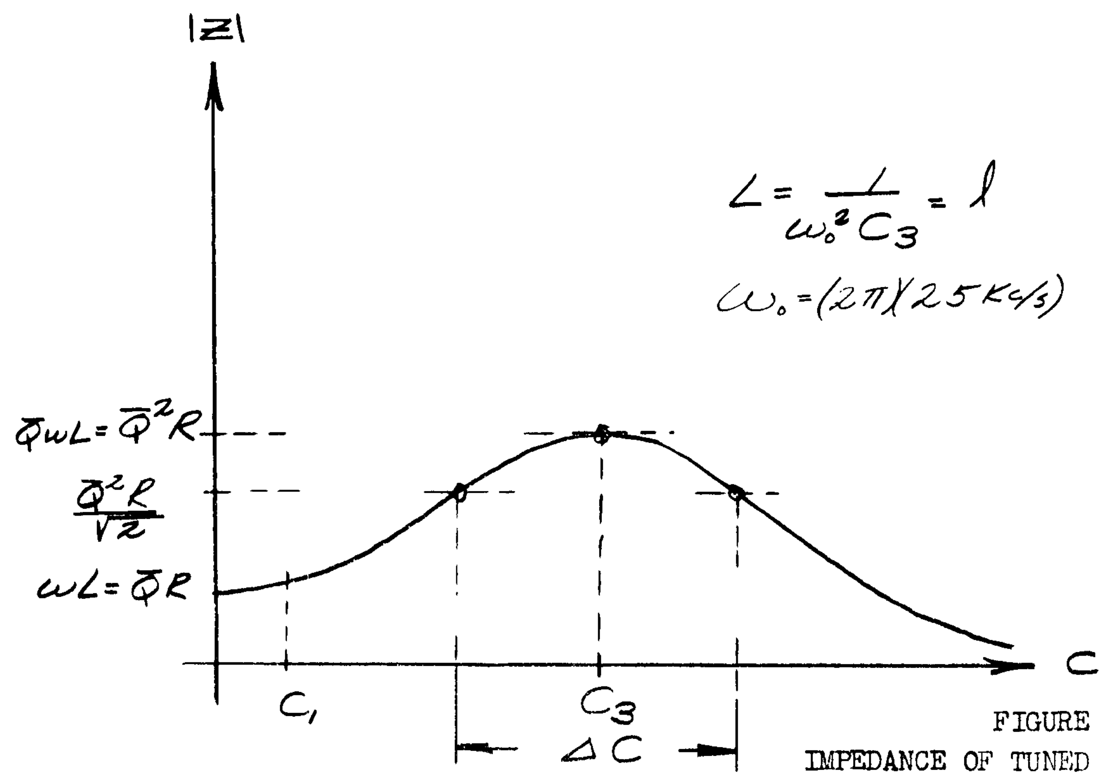
and

$$\frac{(S/N)_a}{(S/N)_b} = \frac{e_a}{e_b}\frac{\sqrt{\overline{Q}^2\ R + R_n}}{\sqrt{\overline{Q}^2\ (r + R_n)}} \tag{6-6}$$

In practice, the maximum value of inductance for either the tuned or untuned antenna is determined by minimum capacity restrictions. For example, the minimum capacity of the tuned antenna must be appreciably greater than the stray capacity introduced when a body is brought near the antenna. Clearly, if such a minimum capacity is not provided, the proximity of external objects will have undesired detuning effects on the antenna (see Figure 6-2b). Similarly, for the case of the untuned antenna (see Figure 6-2a), the maximum inductance is limited to a value such that the capacity which causes it to resonate is appreciably greater than the total capacity appearing across the inductance (including stray capacity of nearby objects). Figure 6-2a shows that the untuned antenna operates as a high Q circuit with a minimum capacitance C_1 across its terminals. Although not desired, resonance will occur if the shunting



(a) Untuned Antenna



(b) Untuned Antenna

FIGURE 6-2
IMPEDANCE OF TUNED AND UNTUNED ANTENNAS

capacity is permitted to increase to the value C_2. Thus, the maximum inductance permitted in the untuned antenna is such that a capacitance C_2 or greater is required for resonance. To prevent de-tuning effects

$$C_2 \gg C_1 + C_x \tag{6-7}$$

where C_x = external stray capacity.

On the other hand, Figure 6-2b shows that the simple resonant (tuned) antenna operates as a low Q circuit in order to pass the desired band of frequencies with the resonance capacity C_3 across its terminals. If $\Delta C/2$ is the change in C_3 required to reduce the impedance magnitude to 0.707 of its resonant value, then

$$\Delta C = \frac{2\, C_3}{1 + \bar{Q}} \tag{6-8}$$

De-tuning effects are prevented by making

$$\Delta C/2 \gg C_x \tag{6-9}$$

or

$$C_3 \gg (1 + \bar{Q})\, C_x \tag{6-10}$$

For $\bar{Q}$ = 12.5 and typical values of $C_1 \approx 60$ μμf and C_x = 10 μμf, the minimum values of C_2 and C_3 are comparable although C_2 can usually be made somewhat smaller than C_3.

As a result of the preceding capacitance considerations, it may be said that slightly higher values of inductance may be used in the untuned antenna. Assuming, however, that

$$C_2 = C_3 = C_T \tag{6-11}$$

and

$$\ell = L = 1/\omega^2 C_T \tag{6-12}$$

will simplify the following work without introducing appreciable error. Hence, comparing similar antennas (equal inductances), the induced voltages are equal, i.e.,

$$e_a = e_b = e \tag{6-13}$$

The passband of the tuned antenna must be at least as great as the passband of the receiver. Therefore, let

$$\overline{Q} = \frac{f_o}{\Delta f}, \quad \text{where } f_o = \text{center frequency} \qquad (6\text{-}14a)$$

$$\Delta f = \text{passband of receiver}$$

$$\overline{Q} = \frac{\omega L}{R} \qquad (6\text{-}14b)$$

For the untuned antenna, there is no restriction on the Q of the coil; hence, it will be made as large as possible. Thus, for a broadband system $\overline{Q} \ll Q$; whereas, in a narrow-band system, $\overline{Q}$ approaches the maximum obtainable Q, i.e., for a narrow-band system $\overline{Q} \rightarrow Q_{max}$.

On the basis of the preceding remarks, we will re-examine Equations (6-5) and (6-6) to obtain a useful comparison of (a) induced voltages and (b) signal-to-noise ratios for the untuned and tuned antennas.

a. Induced Voltages (Eq. 6-5)

For the equal inductance case, the ratio of signal voltages at the grid for the two antennas becomes

$$\frac{e_{g_a}}{e_{g_b}} = \frac{1}{\overline{Q}} \qquad \text{for } \ell = L \qquad (6\text{-}15)$$

As noted before, the inductance of the untuned antenna can be made somewhat greater than that of the tuned antenna. A good range of values may be

$$\frac{1}{\overline{Q}} < \frac{e_{g_a}}{e_{g_b}} < \frac{2}{\overline{Q}} \qquad (6\text{-}16)$$

b. Signal-to-noise ratios (Eq. 6-6)

Two cases are considered, viz., the signal-to-noise ratio in a broadband antenna circuit and that in a narrow-band circuit.

BROADBAND CASE $\bar{Q} << Q$:

Corresponding to $C_T = 100\ \mu\mu f$ = minimum allowable capacity, we obtain the maximum inductance to be

$$L_{MAX} = 400 \text{ mh}$$

If $f_o = 25$ kc and $\Delta f = 2$ kc, then

$$\bar{Q} = \frac{f_o}{\Delta f} = 12.5$$

$$R = \frac{\omega L}{\bar{Q}} = 5 \text{ k ohms}$$

$$r = \frac{\omega L}{Q} = \frac{62.8 \text{ k ohms}}{Q}$$

- If $Q = 250$, then

$$r = 250 \text{ ohms}$$

Values of R_n for triodes are given by

$$R_n \approx \frac{2.5}{g_m}$$

Thus, a reasonable range of values is

$$100 < R_n < 400$$

It follows then that if

$$\bar{Q}^2 R >> R_n \qquad (6\text{-}17a)$$

and

$$r \approx R_n \qquad (6\text{-}17b)$$

then

$$\frac{(S/N)_a}{(S/N)_b} \approx \sqrt{\frac{R}{2r}} > 1 \quad ; \qquad Q >> \bar{Q} \qquad (6\text{-}18)$$

For the values selected

$$\frac{(S/N)_a}{(S/N)_b} \approx 3.16$$

NARROW-BAND CASE $\bar{Q} \doteq Q$

If $R \doteq r \approx R_n$, then

$$\frac{(S/N)_a}{(S/N)_b} \approx \sqrt{\frac{r}{r+R_n}} \approx 1/2; \quad \bar{Q} \longrightarrow Q \tag{6-19}$$

VII. E-FIELD PICKUP

As was pointed out in Section II, static noise in the region of 25 kc is primarily an E-field phenomenon. Thus, insensitivity to E-fields is an important characteristic of an antenna which is to operate in a region where local electrostatic disturbances are appreciable.

A loop antenna is designed to couple the H-field while an open antenna is designed to respond to the E-field. A loop antenna does, however, have a certain amount of sensitivity to the E-field. The degree of this sensitivity is dependent upon several factors including geometry, number of turns, and isolation from ground. The last point refers to the fact that, if one side of an antenna is connected to ground through the receiver and associated power line, the "aerial effect" or E-field sensitivity is increased(3).

It is possible to reduce the E-field sensitivity of a loop antenna by proper shielding. A complete treatment of this topic is beyond the scope of this report, and discussion will be limited to a few comments on the shielding of air core and ferrite core loops.

One common method of shielding air core loops is shown in the following diagram.

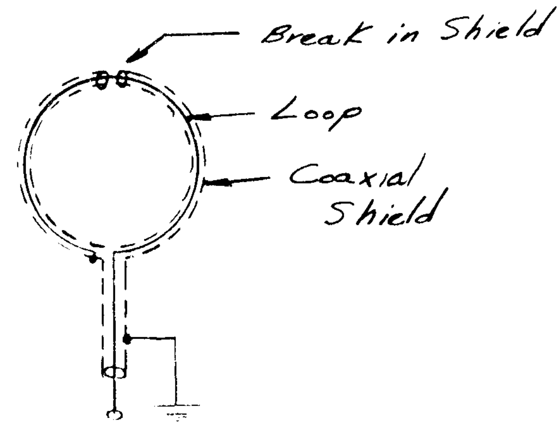


FIGURE 7-1

A METHOD OF SHIELDING AN AIR CORE ANTENNA

In the diagram, the loop consists of a single turn for simplicity; whereas, actually it may be a multi-turn loop. The break in the shield is to prevent the shield from acting as a shorted turn.

For the ferrite antenna, one possible method of shielding is shown in the following diagram. Again, the shield must have an airgap - this time a lengthwise slot is employed.

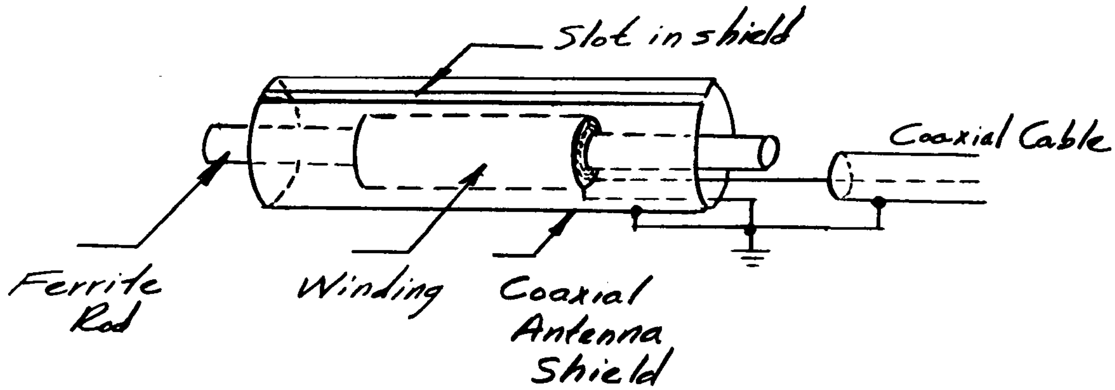


FIGURE 7-2

A METHOD OF SHIELDING A FERRITE ANTENNA

In order to achieve high sensitivity, the antenna may be wound with a large number of turns (Section V). For maximum turn antennas, it is desirable to keep the capacity between the windings and the shield as low as possible so that the number of turns need not be reduced. For this case, it is worthwhile to point out that for an air core loop with many turns the diameter of the wire bundle becomes appreciable relative to the loop diameter, and construction of a shield spaced far enough from the windings to keep capacity down to a tolerable value could present practical difficulties; e.g., the resultant package would be large and bulky.

The line geometry of the ferrite could be of a real advantage in this situation. A cylindrical slotted shield as shown in the diagram could be constructed relatively easily, and the resultant physical package would be more compact than that of the air core loop.

VIII. EXPERIMENTAL METHODS AND RESULTS

Introduction

Whenever possible in the study, theoretical results were checked experimentally in the laboratory. Also, in some cases, experimental data were required for the analytical study. The test equipment and methods of test used will be described briefly. Also included will be a summary of results of some of the attempts to experimentally verify various important equations and results derived analytically.

Test Equipment and Methods

It was necessary to have a uniform magnetic field of known value available for comparing the performance of various antennas and for other test purposes. To accomplish this, a large test loop was constructed. The loop consisted of ten turns of wire forming a circle nine feet in diameter. This coil or loop was supported by a wooden framework which was assembled with plastic bolts to insure that the magnetic field would not be distorted by any metal. The test loop was excited by an audio power amplifier which, in turn, was driven from a General Radio Type 700-A oscillator. The output frequency of the signal generator was monitored by a Hewlett-Packard Electronic Counter, Model 522-B, which indicated the frequency to the nearest cycle. The magnetic field at the center of the test loop is given by

$$H = \frac{NI}{2R} \qquad (8-1)$$

where N = 10 turns, R is the radius = 4.5 feet = 1.37 meter, and I is the rms value of the current flowing in the loop. In order to measure the current in the test loop, a 1000 ohm precision resistor was placed in series with the loop and the voltage across the resistor was recorded. A block diagram of the complete test setup follows.

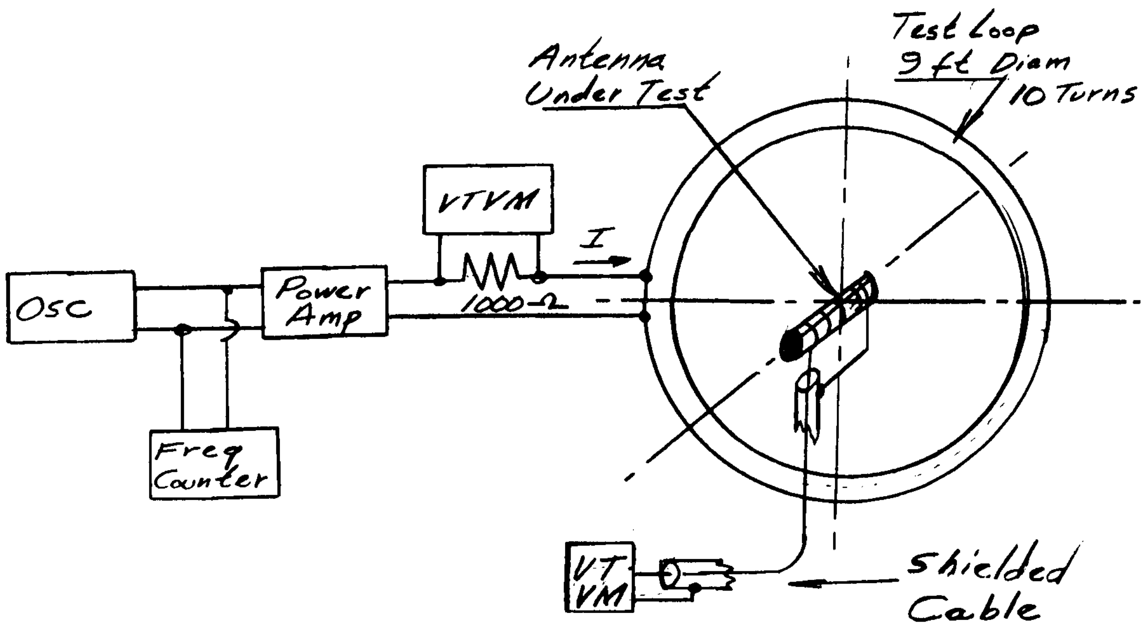


FIGURE 8-1

EXPERIMENTAL SETUP FOR ANTENNA TESTING

As the diagram indicates, the antenna to be tested is placed in the center of the test loop and a shielded cable is brought out to a vacuum tube voltmeter (a Hewlett-Packard Type 400-C was used). The antenna may be tuned or untuned as desired. The magnetic field is very nearly constant for a small region at the center of the test loop. Figures 8-2 and 8-3 show voltage induced in a small test coil as a function of distance from the center of the loop in the plane of the loop and perpendicular to the plane of the loop. Equation (3-1) shows that the induced voltage is proportional to magnetic field intensity.

Using the test setup described above, it is possible to determine the Q and the induced voltage of a tuned antenna. The resonant frequency f_o of the antenna is found by sweeping the oscillator until the voltmeter across the antenna reads maximum. Then the 0.707 point frequencies are noted and $\bar{Q}$ (loaded Q) is found from

$$\bar{Q} = f_o / \Delta f \qquad (8\text{-}2)$$

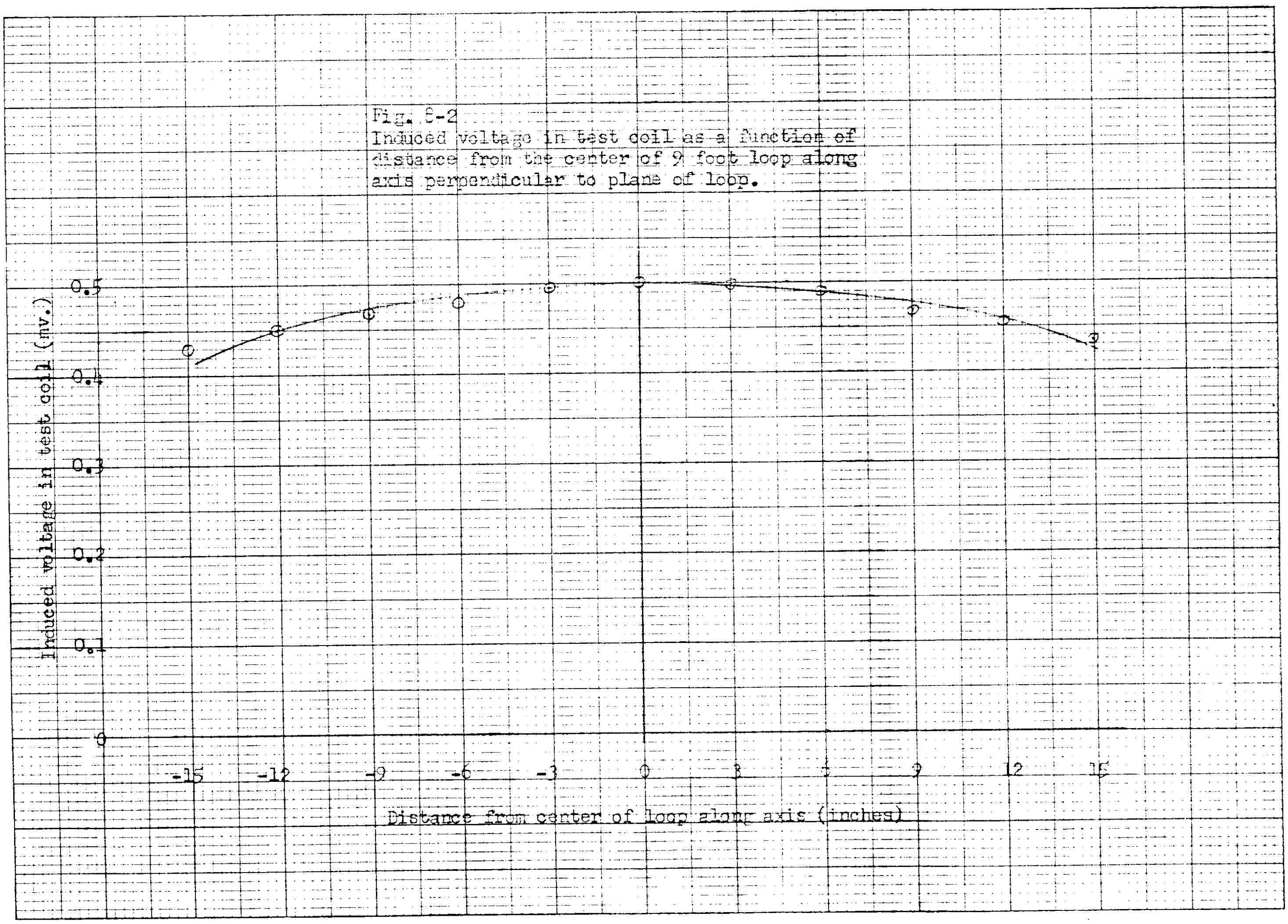
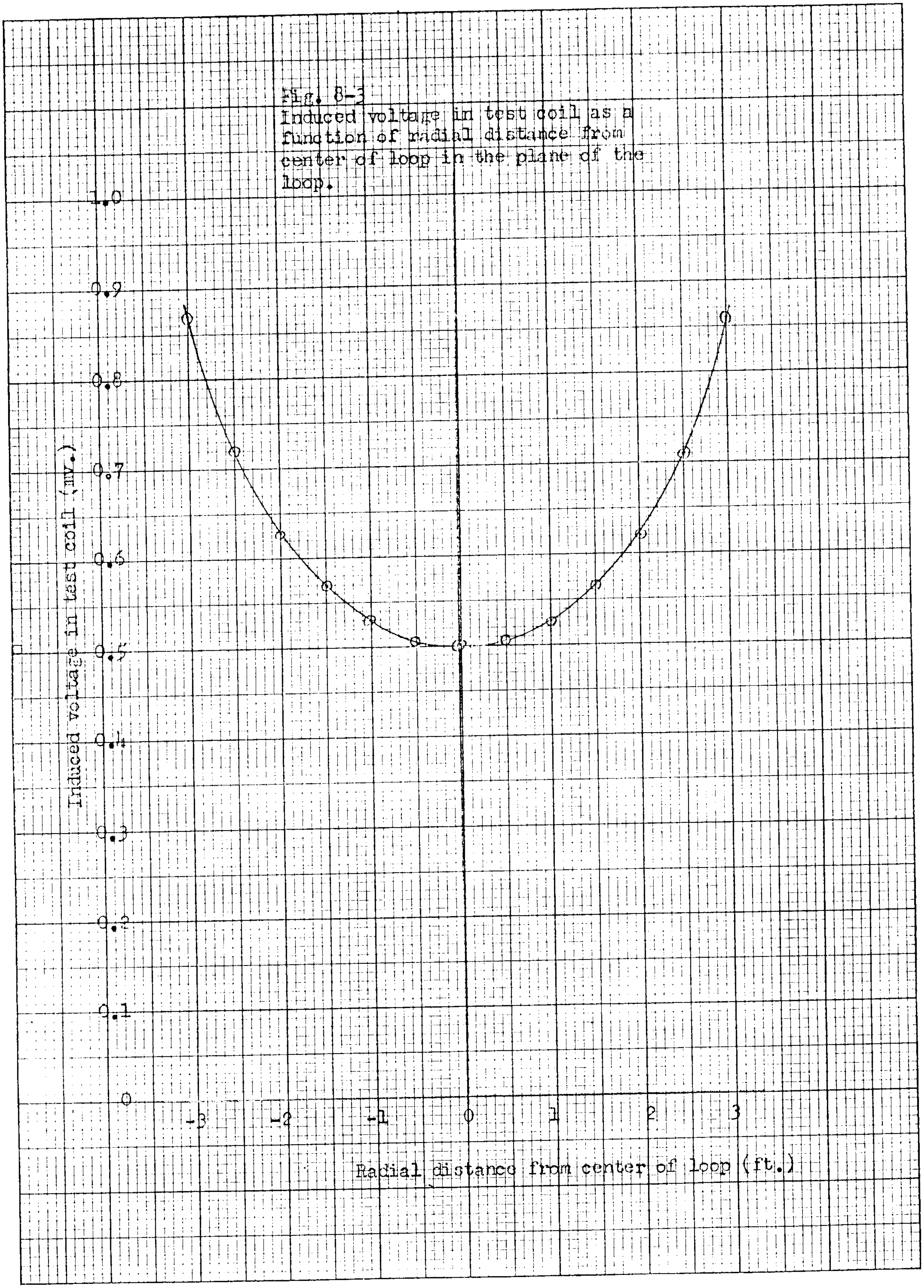


Fig. 8-2
Induced voltage in test coil as a function of distance from the center of 9 foot loop along axis perpendicular to plane of loop.

Fig. 8-3
Induced voltage in test coil as a function of radial distance from center of loop in the plane of the loop.



The induced voltage is given by the output voltage divided by:

$$e_i = e_o/Q \tag{8-3}$$

A General Radio Type 650-A Impedance Bridge was frequently used for measurements of L, R, and C. The instrument was equipped with a Type 650-P1 Oscillator-amplifier.

It was frequently desired to measure the apparent inductance, true inductance, and distributed capacity of a coil or antenna. This was accomplished with a Boonton Type 160-A Q-Meter. The instruction manual for this meter explains the procedure. Since the instrument had an oscillator frequency range of 50 kc - 75 mc, it was necessary to use an external oscillator in order to accomplish measurements in the 25 kc region. The oscillator, power amplifier, and frequency counter previously mentioned were used for this purpose.

The ferrite rod which was used for most of the tests was a low-loss* rod approximately sixteen inches long by seven-sixteenths inch in diameter. The rod had a measured** toroidal permeability of approximately 150 and an effective permeability of approximately 90 (for a full length winding). In order that various different coil windings could be used on the same ferrite rod, the coils were wound on thin cardboard coil forms which fit tightly over the rod but could be slid on and off the rod.

An air core loop antenna was also constructed for purposes of comparison with the various ferrite antennas and for making measurements to check the validity of the analytical equations of performance. This air core antenna consisted of 500 turns of #22 Heavy Double Celanese Bare Magnet Wire. A sketch of the antenna is shown below. When tuned to resonance at 25 kc, this air core loop exhibited an unloaded Q of 34.

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* Unloaded Q's of as much as 250 were observed.

** These measurements are described later in this section.

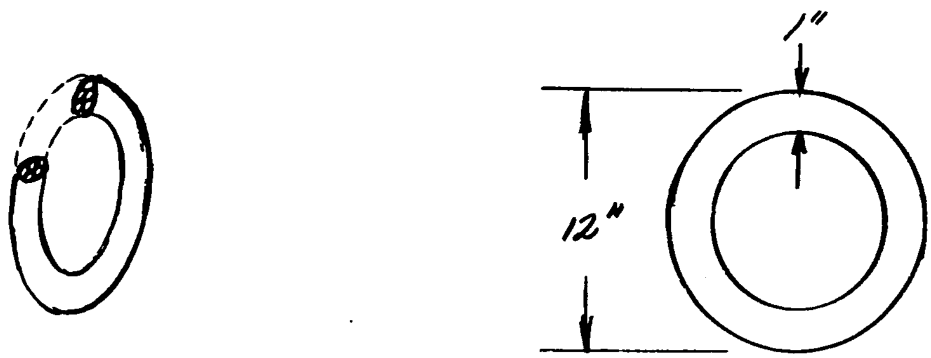


FIGURE 8-4

EXPERIMENTAL AIR CORE LOOP ANTENNA

Experimental and Analytical Results

1. Inductance of Air Core Loop Antenna

The inductance of the air core loop antenna described in the preceding paragraph was measured with the G-R Impedance Bridge and with the Q-meter. Both instruments gave the same value:

$$L = 141 \text{ mh}$$

The inductance of this antenna was calculated from the following formula which is repeated here

$$L \approx n^2 \mu_o R K$$

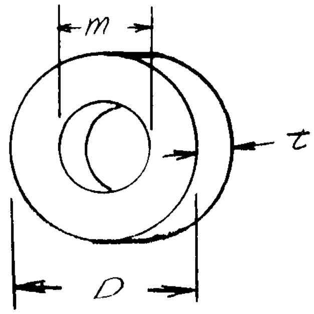
where K = form factor $= \ln \frac{8R}{a} - 2$

In this formula, R is the mean radius of the air core loop (6") and a is the radius of the loop cross-section (1/2"). This calculation yields the value:

$$L \approx 123 \text{ mh}$$

2. Toroidal Permeability of Ferrite Rod

To measure the toroidal permeability of the ferrite rod, a small toroid was cut from the rod. The dimensions (measured with a micrometer) are shown in the following sketch:



m = 3/16" drill = 0.1875 in.

t = 0.128 in.

D = 0.442 in.

FIGURE 8-5

DETAIL OF FERRITE TOROID

The mean path length S and the cross-sectional Area A were calculated:

$$S = 0.0251 \text{ m}$$

$$A = 10.48 \times 10^{-6} \text{ m}^2$$

The toroid was close-wound with 51 turns of AWG 32 wire. The inductance as measured on the GR Impedance Bridge was:

$$L = 200 \times 10^{-6} \text{ h}$$

The toroidal permeability was calculated from the equation[1]

$$\mu_T = \frac{SL}{n^2 A \mu_o}$$

which yielded the value

$$\mu_T = 147$$

From the curves in Bozorth[5] (pp 846-848), the effective permeability can be determined. For $\mu_T = 150$ and length-to-diameter ratio of 35, the value is

$$\mu_{rod} \approx 120$$

which for a full length winding on a ferrite rod would correspond to:

$$\mu_f = \frac{\mu_{rod}}{\sqrt{2}} \approx 85$$

3. Effective Permeability*

The equation for the induced voltage in a ferrite core antenna is given by Equation 3-4 and is repeated here:

$$e_i = \mu_o \mu_f \omega A_f n_f H$$

By placing an antenna in a known H-field and measuring the induced voltage, the value of μ_f can be determined. This was done for an antenna with a full length winding, and the following results were obtained:

$$\mu_f = 90$$

$$\mu_{rod} = \mu_f \sqrt{2} = 127$$

Another value for μ_f for the rod was obtained from Figure 3-5.

$$\mu_f = 92$$

The values of μ_f and μ_{coil} for this graph were obtained in the following manner: Four coils of different lengths were constructed such that, with ferrite core in place, each had an inductance of 13.2 mh. The air core inductance of each coil was measured, and μ_f was determined as the ratio of the coil inductance with ferrite core to the coil inductance with air core. The values for μ_{coil} were determined from the Equation (3-5) for the inductance of a ferrite antenna coil length a:

$$L_f = \frac{\mu_{coil} \mu_o A_f n_f^2}{a}$$

4. Ratio of Ferrite-to-Air Core Induced Voltages

Equations (5-1) and (5-2) can be used to predict the induced voltages in the ferrite and air core antennas. For an H-field of 3.28×10^{-3} amp-turns/m and a frequency of 16,650 cps, these equations yield

$$e_f = 7.42 \text{ mv}$$

$$e_a = 15.7 \text{ mv}$$

$$e_f/e_a = 0.473$$

---

* The same rod was used for all permeability measurements. This is also the rod from which the previously mentioned toroid was ultimately cut.

If the inductances of the two antennas are equal:

$$\nu_L = e_f/e_a = 0.473$$

An experimental check on these values was made by constructing a ferrite antenna (full length winding) with an inductance equal to that of the air core loop (141mh). The induced voltages were measured for these antennas at the frequency and H-field intensity specified above. The values obtained were:

$$e_f = 7.44 \text{ mv}$$

$$e_a = 14.1 \text{ mv}$$

$$\nu_L = e_f/e_a = 0.528$$

Another check for this value of ν_L can be obtained from the graph, Figure 5-9. From this graph for a value of μ_T of 150, the value of ν_L is 0.36. This curve was plotted for the case of form factor K = 2 and, therefore, must be corrected for the actual case where K = 2.57 for the air core loop constructed in the laboratory. As is pointed out in Section V, the correction factor is $\sqrt{K/2} = \sqrt{2.57/2}$. Thus, the corrected value for ν_L is

$$\nu_L = 0.41$$

5. Power Measurements

Figure 8-6 shows the results of a number of power measurements performed on four antennas. For these measurements, the load resistor was varied and readings of bandwidth and power delivered to the load were taken. The test H-field was the same for all four antennas. These curves show the power delivering capabilities of the antennas; also, they are significant in another respect. It will now be shown that the same factors which govern the power output of a ferrite antenna also govern its signal-to-thermal-noise ratio. The equivalent circuit for a ferrite antenna is

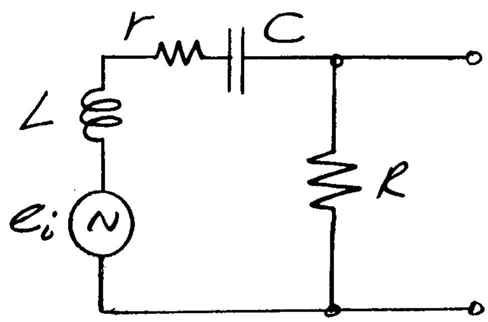


FIGURE 8-7

EQUIVALENT CIRCUIT FOR A FERRITE ANTENNA

In this diagram, r represents core loss while R is the load resistor. Note that

$$\text{Unloaded } Q = Q = \omega L/r$$

$$\text{Loaded } Q = \overline{Q} = \omega L/(r+R)$$

It can be shown that the power delivered to the load is given by:

$$P = \frac{e_1^2}{\omega L}\left(1 - \frac{\overline{Q}}{Q}\right)\overline{Q} \qquad (8\text{-}4)$$

Make the following substitutions:

$$e_i = \mu_f \mu_o \omega A_f n_f H$$

$$L = \frac{\mu_{coil} \mu_\omega A_f n_f^2}{a}$$

which then reduces to:

$$P = \left[\frac{\mu_f^2 A_f a}{\mu_{coil}}\right] \mu_o \omega H^2 \overline{Q} \qquad \overline{Q} \qquad Q \qquad (8\text{-}5)$$

For the induced voltages-to-thermal-noise ratio:

$$e_f = \mu_f \mu_o \omega A_f n_f H$$

The thermal-noise voltage is:

$$e_{Tr} = \sqrt{4 K T \triangle f R} = \sqrt{4 K T \triangle f \omega L/\overline{Q}}$$

$$L = \frac{\mu_{coil} \mu_o A_f n_f^2}{a}$$

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These equations reduce to:

$$(e_f/e_{TN})^2 = \left[\frac{\mu_f^{\,2}\, A_f\, a}{\mu_{coil}}\right] \frac{\mu_o\, \omega\, H^2\, \overline{Q}}{K\, T \Delta f} \qquad (8\text{-}6)$$

Note that in the equation for P and for $(e_f/e_{TN})^2$, the only quantities not predetermined are $\dfrac{\mu_f^{\,2}\, A_f\, a}{\mu_{coil}}$ for both cases. It is shown in Section III that the optimum coil length is a full length winding ($a = \ell$). Under these conditions, Equations (8-5) and (8-6) reduce to:

$$P = \frac{\mu_{rod}}{2}\, A_f\, a \quad \mu_o\, \omega\, H^2\, \overline{Q} \qquad a = \ell_f \qquad (8\text{-}7)$$

$$(e_f/e_{TN})^2 = \frac{\mu_{rod}}{2}\, A_f\, a \quad \frac{\mu_o\, \omega\, H^2\, \overline{Q}}{K\, T \Delta f} \qquad a = \ell_f \qquad (8\text{-}8)$$

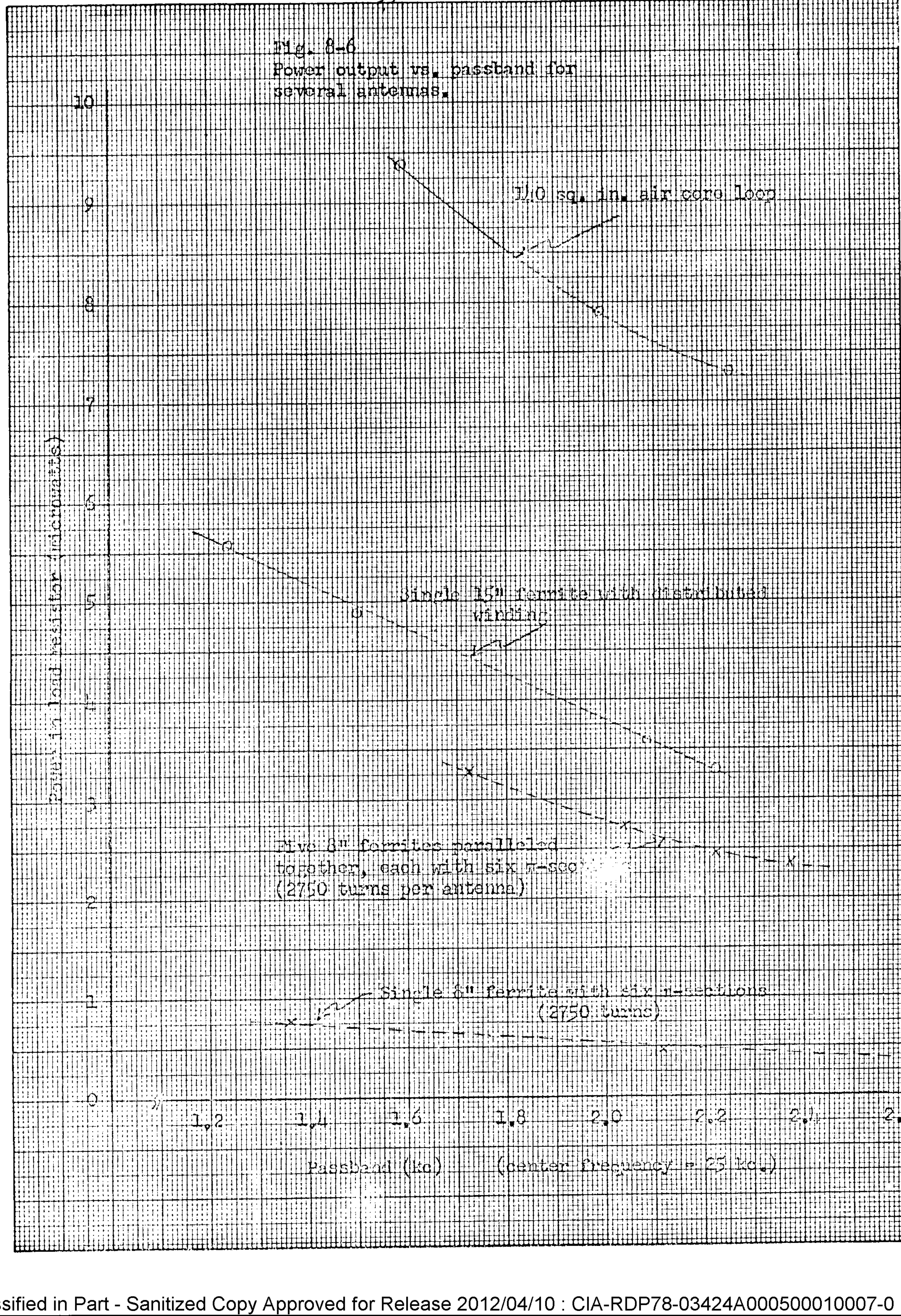

Fig. 8-6
Power output vs. passband for several antennas.

APPENDIX

VOLTAGE INDUCED IN FERRITE-ROD ANTENNA BY A PLANE (UNIFORM) E.M. FIELD AS A FUNCTION OF COIL LENGTH

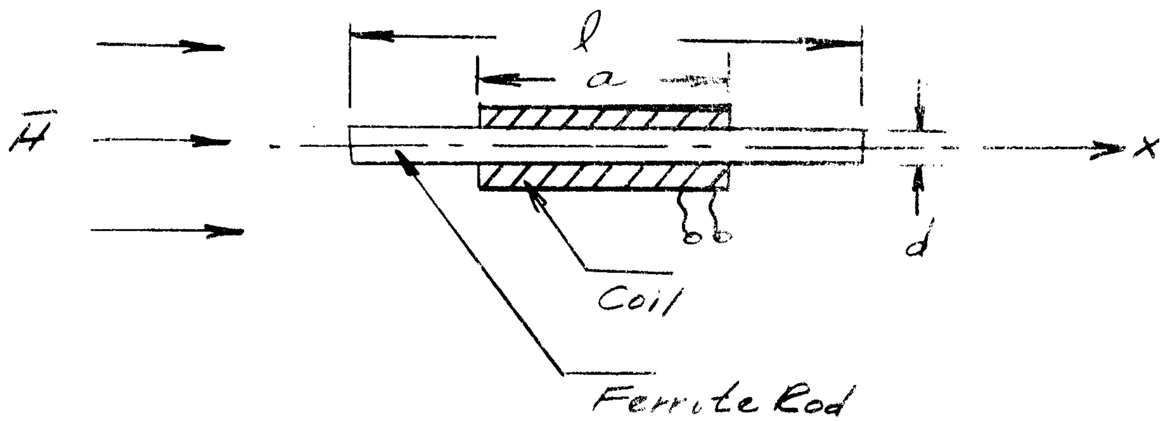

FIGURE A-1

DETAIL OF FERRITE ROD

Let k = number turns per unit of coil length

$n = ka$ = total number turns

$\phi_{rod}(x)$ = flux passing through rod cross-sectional area, A_{rod}, at any plane x along ferrite rod.

The voltage induced in the coil is given by

$$e = \int_{-a/2}^{+a/2} k \frac{d}{dt} \left[\phi_{rod}(x)\right] dx \qquad \text{(A-1)}$$

Assuming k and A_{rod} are constants and that all time variations are sinusoidal, then (A-1) can be written as

$$e = 2\,\omega\,k\,A_{rod} \int_{0}^{+a/2} B_{rod}(x)\; dx \qquad \text{(A-2)}$$

where $B_{rod}(x)$ = average flux density over a rod cross-section located a distance x from rod length center

ω = circular frequency

In the absence of the ferrite antenna, the flux density is assumed to be uniform and is given by

$$B_S = \mu_o H_S \quad \text{(where subscript S refers to signal)} \qquad \text{(A-3)}$$

When the ferrite antenna is introduced, the flux density changes - the external field is no longer uniform in space. The average flux density established inside the ferrite antenna can be written in terms of original flux density as follows

$$B_{rod}(x) = m_f(x)\,\mu_o\,H_S \qquad \text{(A-4)}$$

$m_f(x)$ = step-up flux density ratio as a function of position along rod length (Eq. (A-4) is a defining equation for $m_f(x)$).

Thus,

$$e = 2\omega\, k\, A_{rod}\, H_S\, \mu_o \int_0^{a/2} m_f(x)\, dx \qquad \text{(A-5)}$$

The step-up flux density ratio can be obtained empirically by measuring the voltage induced in a small search coil at various positions along the axis of the ferrite rod. Reference (4), page 191, shows that the step-up flux density variations are given by:

$$m_f(x) = \mu_{rod}\left(1 - 3.6\,\frac{x^2}{\ell^2}\right) \qquad \text{(A-6)}$$

Substituting equation (A-6) into (A-5) yields

$$e = \omega\, A_{rod}\, \mu_o\, \mu_{rod} \left[Ka\right]\left[1 - 0.3\,\frac{a^2}{\ell^2}\right] H_S \qquad \text{(A-7a)}$$

but Ka = total number of turns on coil = n_f

$$\therefore \quad e = \omega\, A_{rod}\, \mu_o\, \mu_{rod}\, n_f \left[1 - 0.3\,(a/\ell)^2\right] H_S \qquad \text{(A-7b)}$$

Equation (A-7b) illustrates the dependency of induced voltage upon coil length and, therefore, provides a useful criterion for the determining of the most desirable coil length. Values of μ_{rod} for various ℓ/d ratios and toroidal initial permeabilities are available in the literature[5]. For convenience, μ_{rod} vs ℓ/d for various values of $\mu_{toroidal}$ have been plotted in Figure 5-2 in this report.

The factor μ_f describing an effective relative permeability with respect to an axial external field is obtained from Equation (A-7b) as

$$\mu_f = \mu_{rod} \left[1 - 0.3 \, (a/\ell)^2 \right] \tag{A-8}$$

This effective relative permeability describes the increase in induced voltage due to a uniform, axial magnetic field when a ferrite rod core is introduced into an air core coil. Experimental data gathered in the course of this investigation agree with Equation (A-8) with a good degree of accuracy (see Figure 3-5 in this report).

REFERENCES

1. "Reference Data for Radio Engineers", 3rd Edition, Federal Telephone and Radio Corp.

2. Attwood, S.S., "Electric and Magnetic Fields", John Wiley and Sons (1949)

3. Cornelius, P. "The Aerial Effect in Receiving Sets with Loop Aerial", Philips Technical Review, 7, No. 3 (Mar. 1942)

4. Blok, H. and Rietveld, J.J., "Inductive Aerials in Modern Broadcast Receivers", Philips Technical Review, 16, No. 7 (Jan. 1955) pg 181-212.

5. Bozorth, R.M., "Ferromagnetism", D. Van Nostrand Co., Inc.

6. Grimmett, C.A., "Ferrite Cored Antenna for Broadcast Band Receivers", G.E., TIS 53E310.

7. Goldman, S., "Frequency Analysis, Modulation and Noise", McGraw-Hill (1948).

NOMENCLATURE

μ_{coil} = relative permeability describing the increase in self-inductance of a ferrite-cored antenna coil over the self-inductance of the same antenna coil with an air core.

μ_T = relative initial permeability of ferrite material in a toroidal shape.

μ_{rod} = ratio of the average flux density across the center cross section of a ferrite rod to the flux density in the absence of the ferrite rod. (Magnetic field before introduction of ferrite rod is uniform and oriented so as to be parallel to the ferrite rod axis when the rod is placed in the field.)

μ_f = relative permeability describing the increase in the voltage induced in an antenna coil (in a uniform axial magnetic field) with a ferrite core over that induced with an air core.

L = ratio of voltage induced in a ferrite core antenna to voltage induced in an equal-inductance air core antenna.

F = ratio of length of ferrite rod to diameter of an air core loop.

K = form factor used in calculating inductance of air core loop.

D = diameter of air core loop.

ℓ = length of ferrite rod.

d = diameter of ferrite rod.

a = length of coil or winding on ferrite rod.

ℓ/d = length-to-diameter ratio for ferrite rod.

subscript 'a' refers to air core loop

subscript 'f' refers to ferrite core loop