6/17/98

Approved For Release 2002/07/12 : CIA-RDP78B04747A002700020042-5

"Maximum Utilization of Laser Energy in a Collimated Beam"

A coherent collimated beam of uniform intensity and maximum energy content is needed for many applications such as photographic printing. Unfortunately, one of the most useful light sources, the laser, does not possess a uniform spatial intensity, i.e., across the beam. In the lowest mode of operation the laser intensity obeys a Gaussian distribution (Fig. 1) which may be expressed in a normalized form as: $I(r) = \exp(-r^2)$ where r is the radial distance from the optical axis in cylindrical coordinates. This memo will consider four ways to form a collimated beam with uniform or near uniform intensity. They are presented in ascending order of laser energy utilized in the final beam.

First, and most wasteful of energy, is to use the laser as a conventional light source to illuminate a pin hole and collimate the central region of the resulting diffraction pattern. A great deal of energy is needlessly lost in illuminating the pin hole and in the unused portion of the diffraction pattern. No use is made of the laser's spatial coherence, i.e., coherence perpendicular to the direction of propagation. A pin hole must be used with conventional sources to achieve the spatial coherence that the laser already possesses.

The second method recognizes the spatial coherence of the laser; the central portion of the laser beam is selected by a stop of radius R_1 and collimated; a "pin hole" is not used. The fraction of the energy utilized is:

$$F = \frac{\int_0^{2\pi} d\theta \int_0^{R_1} \exp(-r^2)\, r\, dr}{\int_0^{2\pi} d\theta \int_0^{\infty} \exp(-r^2)\, r\, dr}$$

$$= 1 - \exp(-R_1^2)$$

$$= 1 - u$$

where $u = \exp(-R_1^2)$ is the ratio of the minimum or edge, to the maximum or central intensity. Energy for $r > R_1$ is lost.

STAT

Branch Research

Neither of the first two methods yield perfectly uniform beams; the uniformity achieved is at the loss of most of the initial laser energy. In a third method, apodization, a filter with a transmission $t(r)$ is placed in the collimated beam, formed as in the second method, to improve the uniformity and thereby allow a larger portion of the laser beam to be admitted by a larger stop while maintaining the same variation in intensity. There will be a maximum and minimum intensity in the apodized beam. For a specified maximum apodized intensity any filtering that decreases the intensity below this maximum represents wasted energy; thus in any region where the incident intensity is less than the apodized maximum the filter should be perfectly transmitting and in regions of intensity greater than the apodized maximum the filter should just reduce it to the apodized maximum. The incident beam is a monotonically decreasing function of r so that the incident beam will take on the maximum value of the apodized beam only once at $r = R_n$. Thus the optimum apodized beam intensity, $I_2(r)$, is constant for $r < R_n$ and equal to the incident beam, $I_1(r)$, for $R_n < r < R_1$:

$$I_2(r) = t(r) I_1(r)$$

$$= \begin{cases} \exp(-R_n^2) : & \text{for } r < R_n \\ \exp(-r^2) : & \text{for } R_n < r < R_1 \end{cases}$$

The transmission of the optimum apodizing filter is:

$$t(r) = \begin{cases} \exp(r^2 - R_n^2) : & \text{for } r < R_n \\ 1 & \text{for } R_n < r < R_1 \end{cases}$$

The ratio of minimum to maximum intensities is:

$$a = \frac{\exp(-R_1^2)}{\exp(-R_n^2)}$$

The fraction of the incident energy utilized in the apodized beam is:

$$P = \int_0^{R_1} 2\pi I_2(r) r \, dr$$

$$= (R_1^2 - R_n^2) \exp(-R_n^2) - \exp(-R_1^2)$$

$$= (R_1^2 - \ln a) \exp(-R_n^2).$$

The values of R_n and R_1 that maximize the energy utilized for a specified a may be found by setting $dP/dR_n = 0$. This gives $R_n = (-\ln a)^{1/2}$ and $R_1 = (a - \ln a)^{1/2}$. The fraction

of the energy utilized because $P = \exp(-n)$. Although apodization results in uniformity of illumination and improves the energy utilization much of the energy is still lost. The filter that produces the uniformity absorbs the peak intensity.

The fourth method uses two aspherical surfaces to redistribute the peak intensity into the region of less intensity. The center, $r \leqslant R_1$, of an incident collimated laser beam of intensity $I_1(r)$ (Fig. 1) is recollimated into a beam of uniform intensity, $I_2(r)$ for $r \leqslant R_2$. r_1 and r_2 are the radial distances in cylindrical coordinates at the two aspheric surfaces for one ray.

$$I_1(r) = \exp(-r^2) \; ; \text{ for } r \leqslant R_1$$

$$I_2(r) = I_2 = (\text{a constant}) \; ; \text{ for } r \leqslant R_2$$

Conservation of energy requires:

$$\int_0^{2\pi} d\theta \int_0^{r_1} I_1(r)\, r \; dr = I_2 \int_0^{2\pi} d\theta \int_0^{r_2} r \; dr$$

This yields $r_2 = C(r_1)$ where for convenience we define:

$$C(r_1) = R_2 \left[\frac{1 - \exp(-r_1^2)}{1 - \exp(-R_1^2)} \right]^{1/2} .$$

I_1 is a plane of equi-phase; if all the optical path lengths, $p(r_1)$, are equal I_2 will also be an equi-phase plane; i.e., I_2 will also be collimated. The difference between two optical paths is:

$$p(r_1) - p(0) = 0$$
$$= n' \left[X(r_1) - X(0) \right] + \left[X(0) - X(r_1) \cos\theta \right]$$

where n' is the ratio of the refractive index between the two aspheric surfaces to that of the surrounding medium; $X(r_1)$ is the "lens thickness" measured along the optical path and θ is the angle through which a ray is deflected. For a glass element in air the first bracket represents the difference in optical paths in the lens of refractive index n'; the second represents the path difference in air. This gives:

$$X(r_1) = X(0) \frac{(n'-1)}{(n' - \cos\theta)}$$

Branch Research

Geometric considerations yield the relationship for the radial positions of an entering and emerging ray as: $r_2 = r_1 + X(r_1) \sin\theta$. These equations may be combined to give:

$$\frac{\sin\theta}{n' - \cos\theta} = \frac{C - r_1}{X(0)(n'-1)}$$

Snell's law for a ray incident on the first surface is: $\sin i = n' \sin\phi$, where i and ϕ are the conventional angles of incidence and refraction. From geometric considerations we find: $i = \theta + \phi$ and $\tan i = -dz/dr_1$ where z is distance along the optical axis. These equations may be combined to yield for the first surface:

$$\tan i = -\left(\frac{\sin\theta}{1/n' - \cos\theta}\right)$$

Since:

$$\frac{\sin\theta}{n' - \cos\theta} = \frac{C - r_1}{X(0)(n'-1)}$$

it follows that the slope of the first surface is:

$$\tan i = \frac{(C - r_1)n'}{X(0)(n'-1)}$$

$z_1(r)$, the first aspheric surface is:

$$z_1(r) = -\frac{1}{X(0)}\left(\frac{n'}{n'-1}\right)\int_0^r \left[C(r) - r\right] dr$$

The integrand, $C(r_1) - r_1$, is the radial displacement experienced by a ray that enters the first surface at r_1.

If a ray passes through an object with two parallel surfaces it emerges from the second surface displaced but parallel to the entering ray, i.e. the ray has been recollimated. The ray entering the first surface at r_1 leaves the second surface at $r_2 = C(r_1)$; at these two points the surfaces will appear parallel to a ray if they possess the same slope. Thus for the second surface $z_2(r)$, $r_2 = C(r_1)$ is substituted in the previous expression for $z_1(r)$. All of the incident energy, $P = 1 - \exp(-B_1^2)$, is utilized in the final beam.

Figure 3 shows the fraction of laser energy utilized in the collimated beam as a function of n, the ratio of minimum to maximum intensities, for an

unmodified laser beam (Method 2), and for the apodized beam (Method 3). Figure 4 shows the radial dimensions R_1 and R_n as a function of n for these two methods. Figure 5 illustrates the relative intensities across the unmodified and apodized beam for several variations in intensity

In the fourth method a single element with two aspheric surfaces was considered. In practice it might be desirable to split this single element to form an equivalent air element between two glass elements (Fig. 6). The outer surfaces of these elements are planes parallel to the equi-phase planes of the collimated beam. The central air element formed by the two glass surfaces of refractive index n has a relative refractive index of $n' = 1/n$.

Figures 6, 7 and 8 illustrate the aspheric surfaces just described. The incident and emerging beams were chosen to be of the same diameter, i.e., $R_1 = R_2$. Figures 7 and 8 show the normalized (normalized quantities are capitalized) aspheric surfaces utilizing P = 63% (for $R_1 = R_2 = 1$) and P = 98% (for $R_1 = R_2 = 2$) of the energy respectively. The surfaces are shown as a function of sr where s has been introduced to permit a change of scale. The normalized surfaces are:

$$Z_i(sr) = \left[s(n-1)X(0)x_i - sr \right]$$

The subscript, i = 1 and 2 represent the first and second surfaces respectively. The normalized radial displacement of each ray is shown as:

$$D(sr) = s(x_2 - x_1) = C(sr) - sr \quad \text{for the ray entering at } sr$$

The difficulty in manufacturing an aspheric is determined in part by the maximum deviation of the aspheric surface from the best fitting spherical surface. The closest fitting spherical surfaces (S_i) given were determined graphically. The equation for a sphere of radius r_0 may be written as $r^2 + (r_0 - z)^2 = r_0^2$; for $|z| \ll r_0$, which is true for slight curvature, this becomes $z = r^2/2r_0$. r_0 was found to be the same for the first and second surface. For the normalized surface this becomes $Z = (sr)^2/2R_0$ where $R_0 = sr_0/(n-1)X(0)$ is the normalized radius. The normalized maximum deviation between this sphere and the aspheric surface is $\Delta_i = (n-1)X(0)\delta_i$ where δ_i is the actual deviation for a particular r_i, $X(0)$ and s. The maximum deviation occurs at the origin, edge and once inbetween.

error in the manufacture of one surface can be compensated for in the other surface so that the wave front will be perfectly flat and the only consequence will be a slight non-uniformity in intensity.

TABLE 1

Aspheric Elements

P	R_o	Δ_1	Δ_2
63%	8.5	.0080	.0064
98%	2.6	.12	.04

a. Normalized quantities (dimensionless)

P	d	ℓ	r_o	n = 1.5		n = 2	
				δ_1	δ_2	δ_1	δ_2
%	centimeters			microns			
63	1	20	85.	2.0	1.6	1.0	0.8
63	1	5	21.3	8.0	6.4	4.0	3.2
63	4	12	51.	53.	43.	26.	22.
98	1	20	26.	7.5	2.5	3.8	1.3
98	1	5	6.5	30.	10.	15.	5.0
98	4	12	15.6	200.	67.	100.	33.

b. Specific Examples

Figure 6 shows the two elements redistributing 98% of the incident light ($R_1 = R_2 = 2$) for n = 1.5, s = 1 and $X(0) = 8$. The incident and resultant intensities are shown.

 [redacted] has pointed out that these optics act as a telescope with variable magnification since the relative ray spacing is changed by different amounts at different radial distances while collimation of the rays is maintained. This may be observed directly by noting that the point $F(r_1)$ of Figure 2, which represents the conventional focal length, varies with radial position. Telescopes with variable magnification have been built [redacted] using spherical optics. It is possible to achieve at least some degree of [illegible] spherical optics. One

method being considered would utilize the inherent spherical abberation found in spherical optics.

At the plane reflector of a hemispherical laser resonator the planes of constant phase vary from a plane at the reflector to a spherical surface at a distance equal to the radius of the spherical reflector. An approach similar to that used in this memo will permit optics to redistribute and collimate the light directly from the laser without considering the incident beam to be initially collimated.

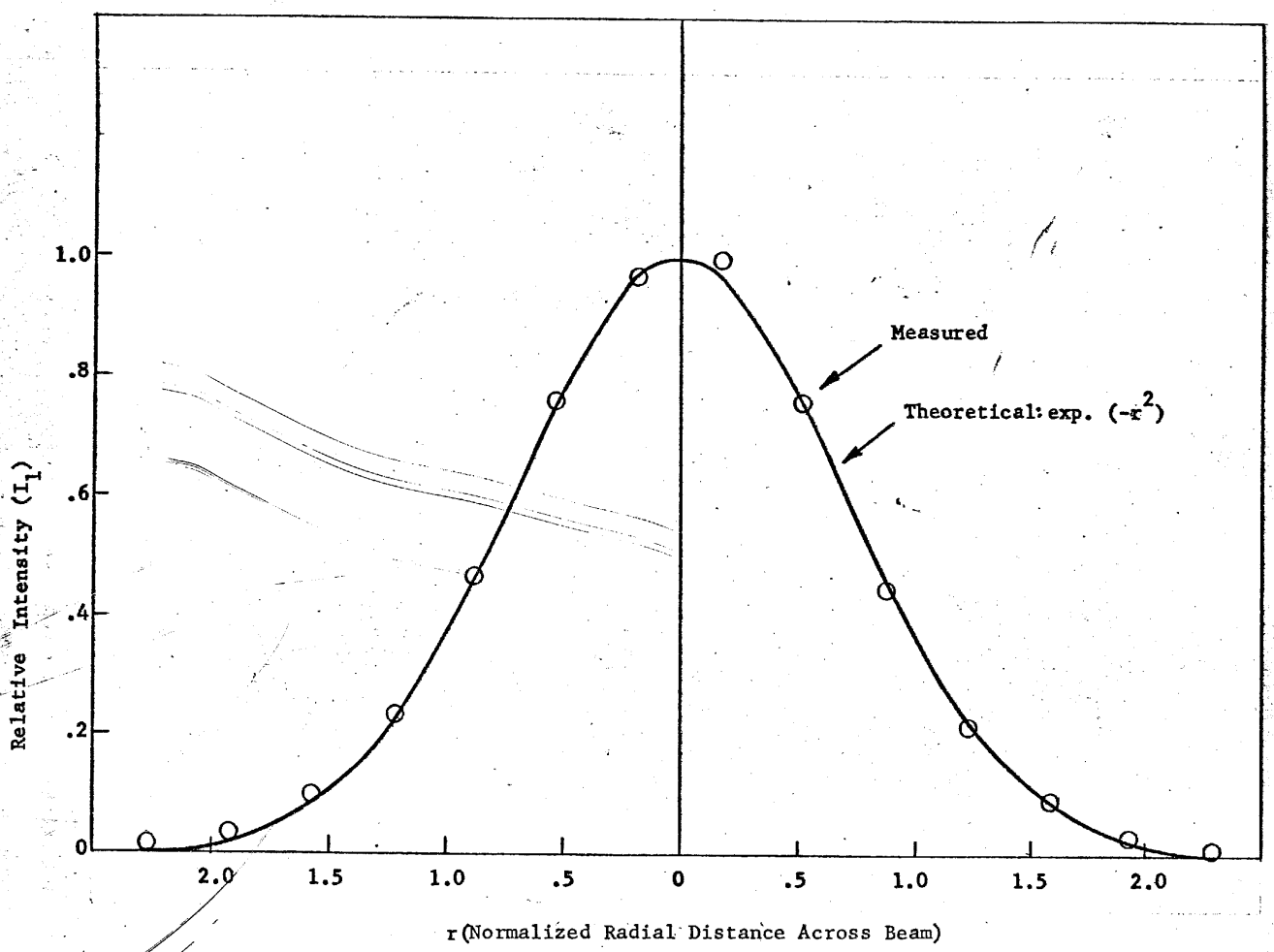
If both maximum uniformity and intensity are required apodization or redistribution of the light should be considered. If great uniformity is not required then the last two methods offer little advantage over the unmodified beam. The first method mentioned is not particularly useful but could describe a laser replacement for a conventional source in existing optical equipment. In the last three methods the laser, not a "pin hole", serves as the apparent source and thus the laser must be rigidly mounted into the optical system.

At present the methods outlined in this memo appear to offer the best possibility of achieving an intense monochromatic collimated beam with uniform intensity. However, this result may be attainable by modifications directly on the laser. Different reflector geometries, reflectors with non-uniform reflectance and non-linear operation of the lasing material might provide greater uniformity.

JAE/mg

6/3/63

Branch [redacted] Research



[redacted]

Figure 1. Normalized Laser Intensity [redacted] Model #112 with Hemispherical Reflectors; RF Current = 78 ma; Intensity Measured at the Plane Reflector)

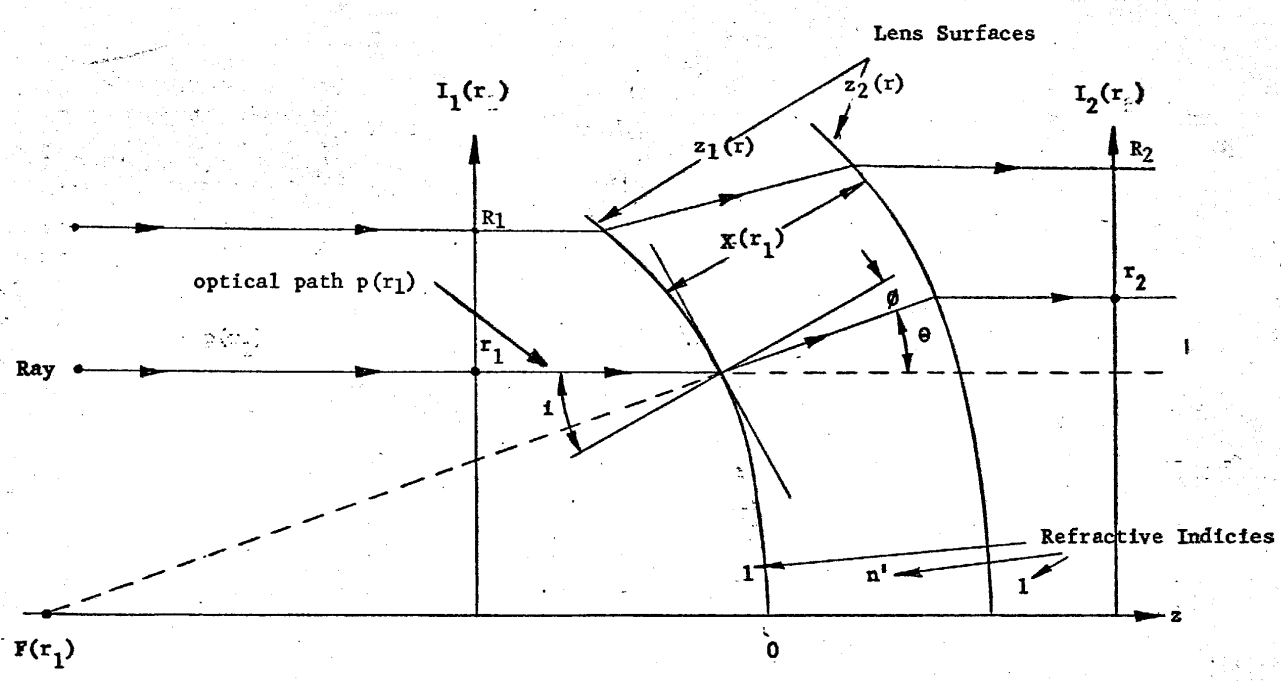


Figure 2. Redistribution of Light Intensity in a Collimated Beam

Branch Research

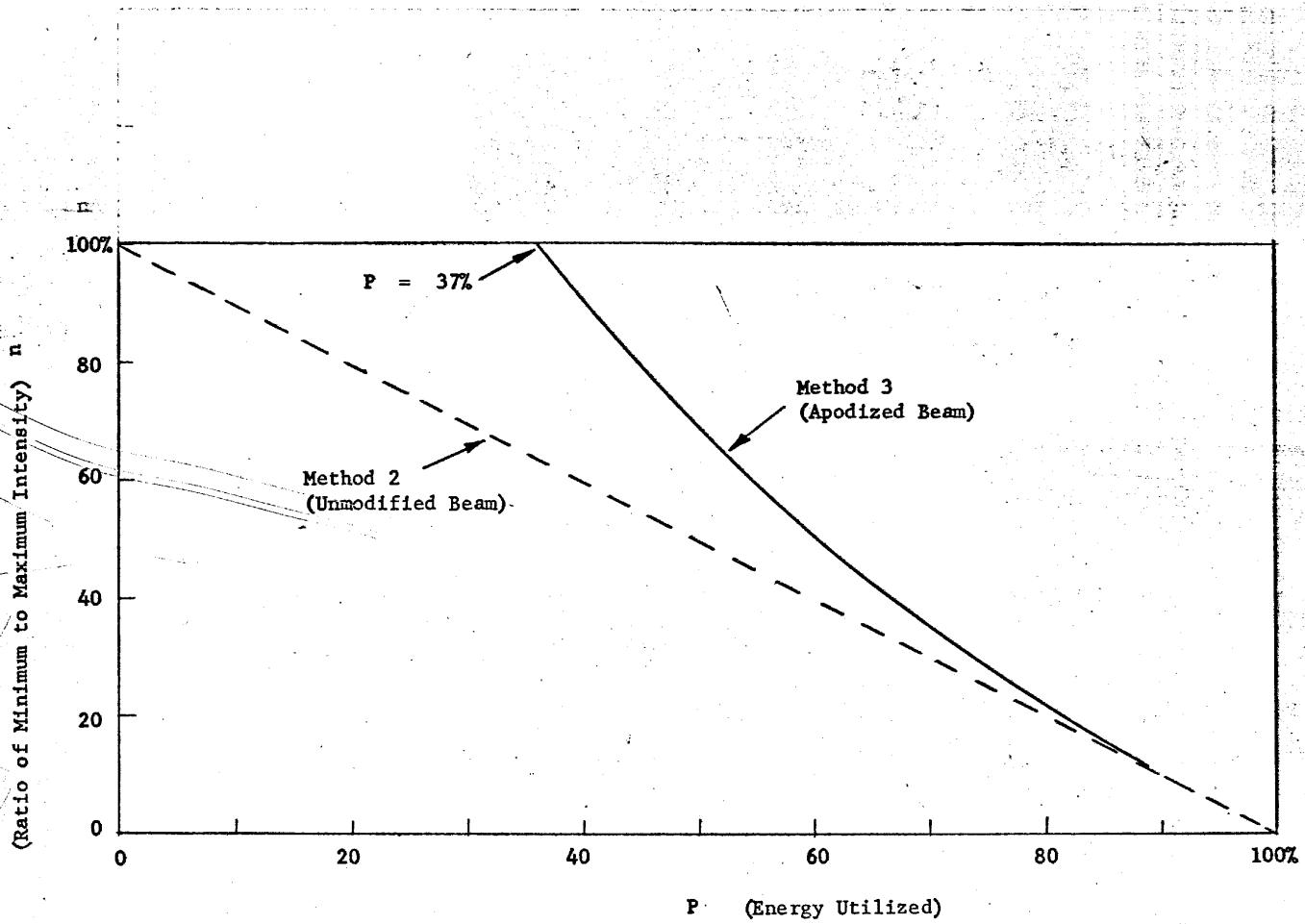


Figure 3. Laser Energy Utilized in a Collimated Beam for a Given Ratio of Intensities

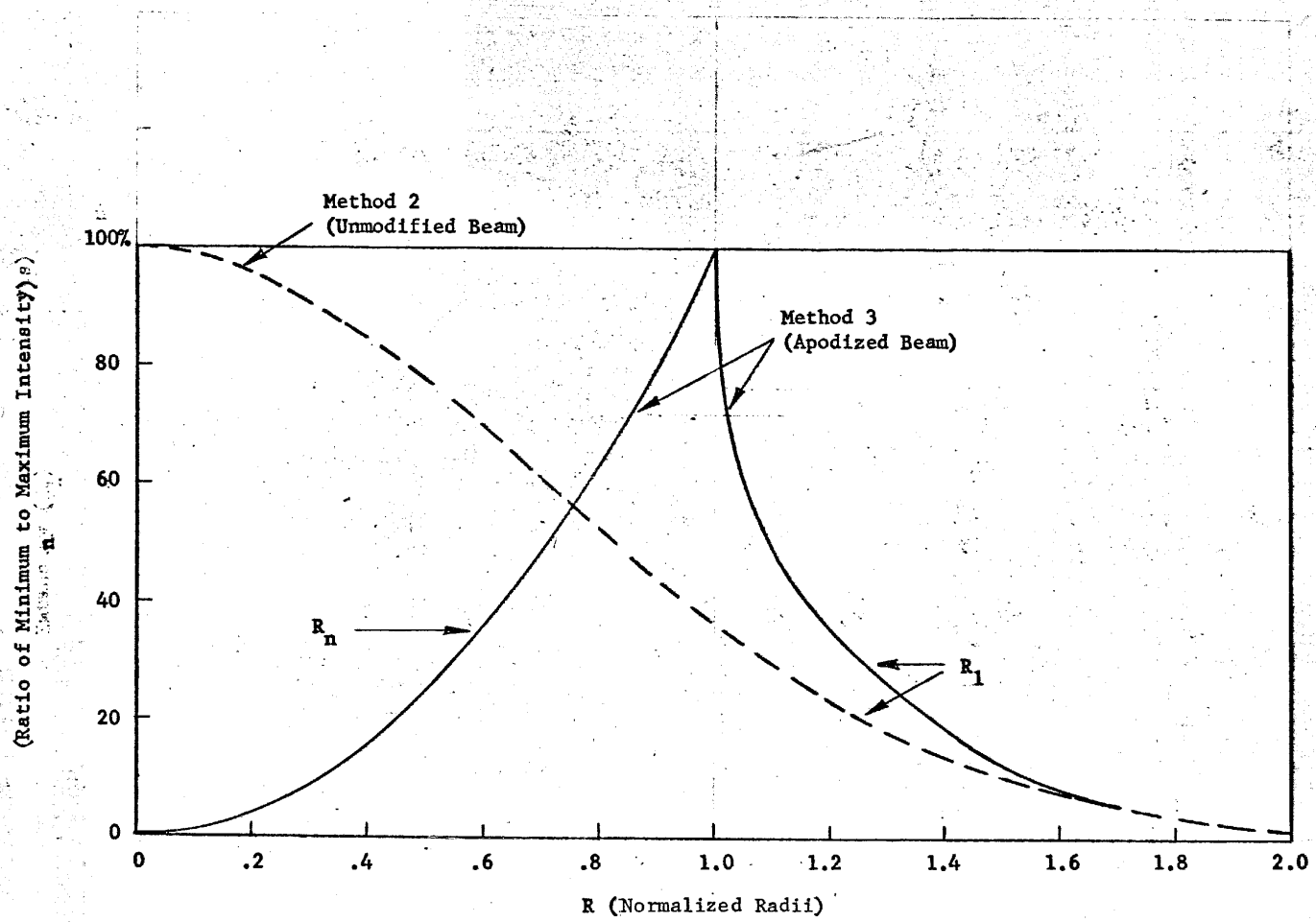
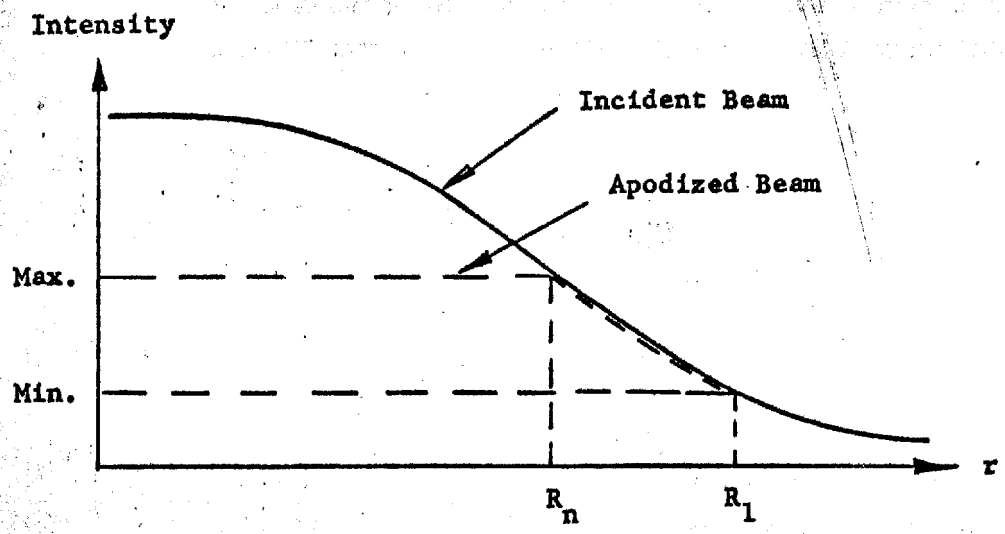
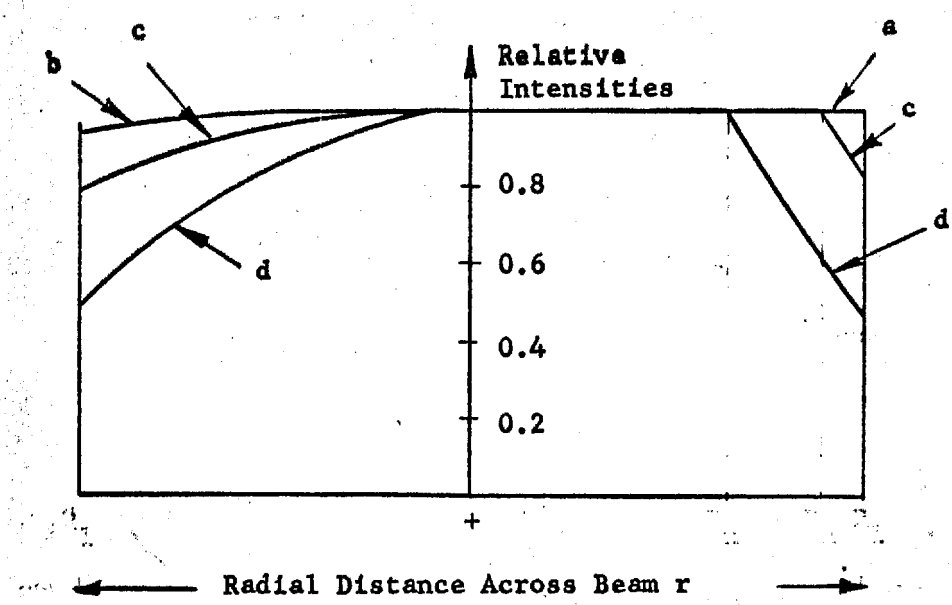


Figure 4. Ratio of Intensities as a Function of Beam Radii

Branch Research



Apodized Laser Beam



Curve	n	Method 2 (Unmodified Beam) P	Method 3 (Apodized Beam) P
a	100%	(0)%	37%
b	95	5	(39)
c	80	20	45
d	50	50	61

Figure 5. Unmodified and Apodized Beam

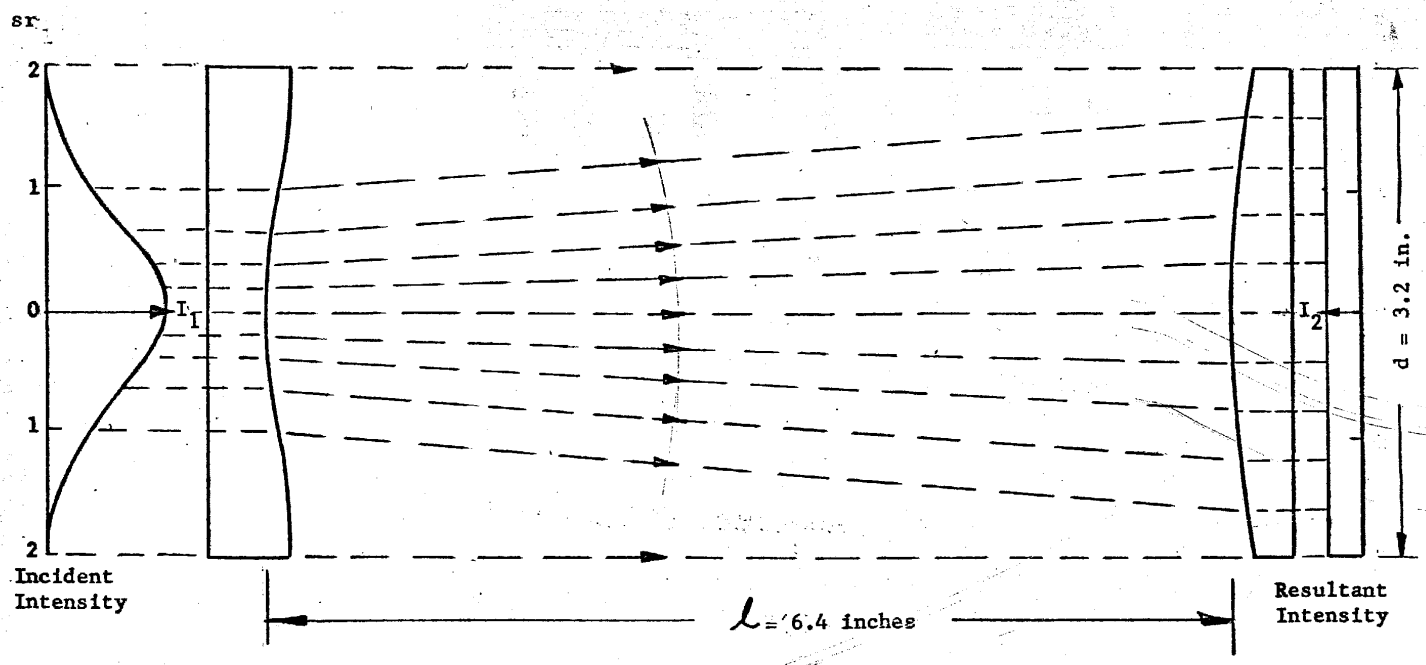


Figure 6. Optics Redistributing and Recollimating P = 98% of the Incident Laser Energy (For: $R_1 = R_2 = 2''$, $X(o) = 8''$, $n = 1.5$ and $s = 1.25$)

Branch Research

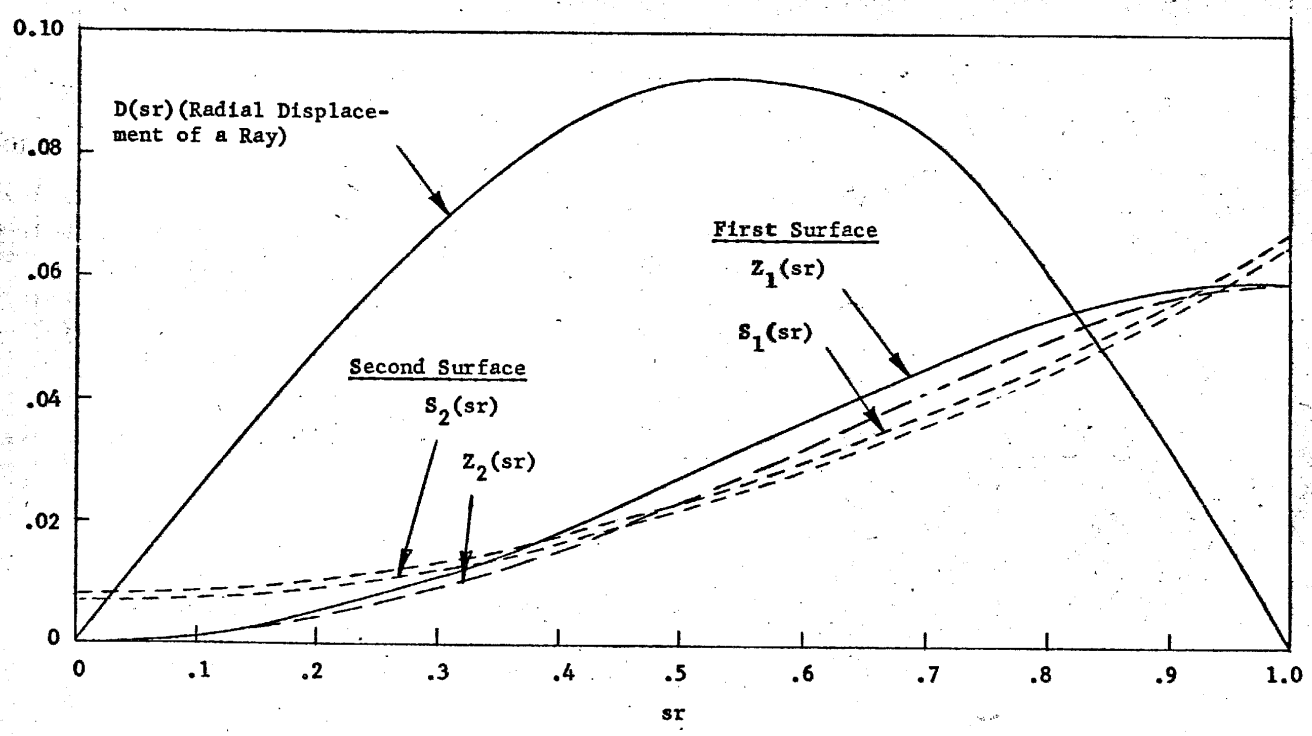


Figure 7. Normalized Aspheric Surfaces Utilizing P = 63% of the Incident Energy

STAT

Branch Research

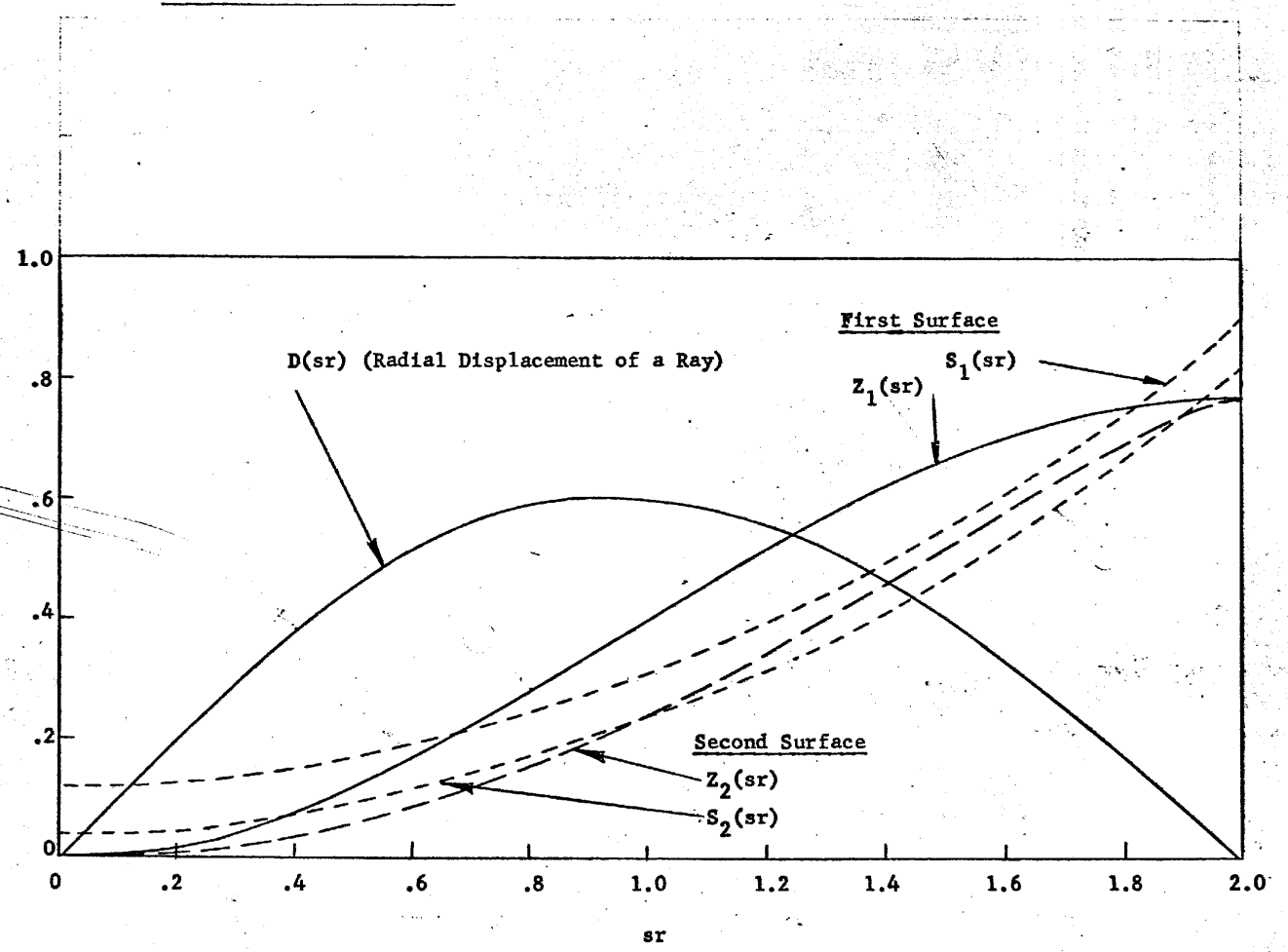


Figure 8. Normalized Aspheric Surfaces Utilizing P = 98% of the Incident Energy