☐ UNCLASSIFIED ☐ INTERNAL USE ONLY ☐ CONFIDENTIAL ☐ SECRET

ROUTING AND RECORD SHEET

SUBJECT: (Optional)

FROM:	EXTENSION	NO.
Ch/BMSB		
		DATE

TO: (Officer designation, room number, and building)	DATE RECEIVED	DATE FORWARDED	OFFICER'S INITIALS	COMMENTS (Number each comment to show from whom to whom. Draw a line across column after each comment.)
1. Dr. Ken Kress				
2. 255 So. Bldg.				FYI & retention
3.				
4.				
5.				
6.				
7.				
8.				
9.				
10.				
11.				
12.				
13.				
14.				
15.				

FORM 3-62 610 USE PREVIOUS EDITIONS ☐ SECRET ☐ CONFIDENTIAL ☐ INTERNAL USE ONLY ☐ UNCLASSIFIED

Physiological correlates of
Unpublished manuscript.)

nt

L PROBLEM. Pamphlet
Murphy. Reprinted from
numbers of this JOURNAL.

estion of the survival of
nded that Dr. Murphy's
ginally appeared in the

ival Evidence," presents
ed form.
nfronting the Survival
lem of finding evidence
some other hypothesis.
Survival," discusses the
nized wholes cannot be
ts) for psychical research.
clairvoyant processes is
to relate to the future and
l evidence are indicated.

Quantum Theory and Parapsychology

J. H. M. WHITEMAN

INTRODUCTION

In recent years a number of attempts have been made to sketch explanations of certain psi phenomena along lines which the authors consider to be compatible with modern physics. There is held to be a special need today for such attempts, since "many scientists reject psi because of its apparent nonphysical nature" (28, p. 84), or since rejection of the "naturalistic" world-view by prominent writers on psychical research "is responsible for a good deal of the current prejudice on the part of natural scientists against parapsychological research" (12, p. 226). A "physical" theory of psi, it seems to be argued, might convert many opponents; but presumably the authors would not put forward such explanation unless they believed it to be the right kind of explanation to aim for.

Nearly always the basic assumption seems to be that modern physics *is* naturalistic, i.e., one-level deterministic (9, p. 17), and thus the need for compatibility with modern physics demands the production of a naturalistic theory for psi. An opposing view, which I maintain is in its broad lines the view of the great majority of quantum physicists and also of philosophers of science oriented to theoretical physics, is that modern physics, far from being naturalistic, has overthrown the naturalistic world-view. This "opposing view," which became fairly general nearly fifty years ago and has become much more compelling still in recent years, unfortunately receives little publicity outside the specialist journals. Consequently it has received so far almost no attention in parapsychology. But presuming that it is as widely accepted by authorities as I am maintaining, it is clear that the task for parapsychology, in countering attacks on its alleged "nonphysical" character, is not to try to devise naturalistic explanations, but to show how the critics might see that parapsychology is compatible with modern physics, if only they knew enough about both.

It is chiefly the problem of precognition that has called forth attempts at explanation along substantially naturalistic lines. Professor C. T. K. Chari, in a timely and extensively documented paper (10), has set out to rebut these insofar as they propose to modify the formalism of quantum theory or its interpretation. With his general conclusion, that "all attempts to crack the riddles of psychical research by relying on quantum mechanics are, for the present, premature and hazardous" (10, p. 203), I am in agreement, and would even express the conclusion far more strongly if the attempts referred to have the purpose of reducing parapsychology to physics. Nevertheless, after the elimination of "partially baked ideas" (in I. J. Good's sense [17]) and other formalistic proposals against which more positive objections can be laid, there still remain certain fundamental questions. How far is the experimental basis of "main-stream" quantum theory (2, p. 1) still virtually unchallengeable? What is the nature of the "epistemological problem" raised by it? Why is quantum theory considered to have overthrown the "classical ontology"? And what is the relevance, if any, of the epistemological problem and the overthrow of the classical ontology to parapsychology?

It is clear, I think, that this last question must be strictly attended to if parapsychology is to be placed on a sound scientific basis; and the others are of course involved in it. I shall try to answer each of the questions in as concise a way as possible, and then to show how, compatibly with the answers, we may build the beginnings of an axiom system for parapsychological phenomena.

"Main-Stream" Quantum Theory

Distinguishing, for the present, "elementary" quantum mechanics from quantum field theory, one can say that "main-stream" quantum mechanics is a mathematical language devised basically to formalize certain remarkable experimental results with which the classical "language" appears to be quite incompatible. "I know from my own experience," Max Born wrote, "and I could call on Heisenberg for confirmation, that the laws of quantum mechanics were found by a slow and tedious process of interpreting experimental results" (8, p. 86). And Bastin, more recently, has said, "One can scarcely deny that the quantum theoretical formalism took the form it did precisely to accommodate those puzzling aspects of the quantal state of matter which [the] realist position seeks to find an excuse to ignore" (2, p. 8).

There are four questions which need to be put concerning main-stream quantum mechanics. Is the mathematics always rigorous and consistent within a certain postulational framework? Does it always work out in satisfactory agreement with experimental results? Are the remarkable experimental results just referred to, from which the character of the formalism is inferred, beyond dispute? And is the inference justified?

The answers to these subsidiary questions, I suggest, are as follows: The mathematics is rigorous and consistent if developed

203), I am in agreement,
far more strongly if the
reducing parapsychology
ation of "partially baked
her formalistic proposals
be laid, there still remain
s the experimental basis
). 1) still virtually un-
pistemological problem"
lered to have overthrown
relevance, if any, of the
of the classical ontology

must be strictly attended
und scientific basis; and
shall try to answer each
ssible, and then to show
ay build the beginnings
phenomena.

THEORY

ary" quantum mechan-
ay that "main-stream"
guage devised basically
ntal results with which
incompatible. "I know
ce, "and I could call on
of quantum mechanics
of interpreting experi-
nore recently, has said,
a theoretical formalism
nodate those puzzling
ch [the] realist position
).
e put concerning main-
ics always rigorous and
nework? Does it always
perimental results? Are
eferred to, from which
eyond dispute? And is

ons, I suggest, are as
consistent if developed

in a certain way (not usual in textbooks of physics [33, pp. 319 f.]), up to a certain point, where additions and adaptations are made, often prompted by classical ideas and open to various mathematical or phenomenological objections—a small qualification, since every scientific theory has a dubious fringe at its growing points. The theory is in quite extraordinary agreement with experimental results, again up to a point, where the adjustments made begin to look very artificial, the phenomenology becomes very confused, and the experimental results are not predicted with entire accuracy or completeness (but quantum field theory, within its range of application, goes a long way toward rectifying these deficiencies). The experimental results which the consistent core of the theory is designed to formalize have been tested repeatedly in the most thorough way, and it does not seem remotely possible that they could ever be overthrown. As regards the question whether some other kind of formalism might serve equally well, I think we can confidently answer that the features of the formalism which seem most surprising, because quite foreign to classical physics, could not be dispensed with because they correspond to features of the experiments to be explained; but the formalism might possibly be expanded or replaced by a nearly equivalent one if those features are preserved.

It is part of main-stream quantum theory, and essential for understanding the objections to the classical ontology, that one should realize what the basic experimental results are and consequently why the formalism has to be of the kind it is. All this is exemplified most clearly in various *thought-experiments*, of which two kinds call for consideration here.

These are diffraction or interference experiments in which a beam of monochromatic light is directed into the apparatus and certain effects are eventually detected on a sensitized screen. If the beam is of strong or average intensity, bright lines or bands are seen in certain positions; but if the intensity is greatly diminished, the continuous effect breaks up into single scintillations or absorptions detected by a photomultiplier. This fact strongly suggests that the beam "really" consists of particles, though the sense in which the term "really" is used remains obscure. It is to be noted, however, that interference and diffraction effects involve the canceling out of the intensity in certain regions, and this seems quite inexplicable unless the intensity is structured as a wave (for neither mass nor energy can cancel themselves out). The question as to what it is consistent to believe "really" happens in the apparatus before the screen is reached thus becomes acute.

In the "grating experiment" (25, p. 18) the beam is directed at a diffraction grating from which one half can be removed, leaving half as many lines, spaced as before. Simple geometrical theory shows that such removal should greatly worsen the resolving power of the grating (line-breadth), assuming that the wave meets the whole of the grating in position on each occasion; and this theoretical conclusion is accurately verified experimentally for normal intensities. If, now, the intensity is very greatly reduced, and the wave then breaks up into small wave-packets, the resolving power will similarly depend on the size of the packets and not at all on how far the grating extends on either side beyond the packets as they impinge on it. Thus we have a clear test whether the wave which is producing scintillations can or cannot logically be considered to have broken up into separate packets. The verdict of experiment is that, however weak the intensity of the incoming beam, it behaves as an unbroken wave, not as an ensemble of separate packets.

In the variant of the two-slit interference experiment carried out by Jánossy and Náray in 1957 (21, pp. 2 f.; 33, p. 316) the beam was split by a semi-transparent mirror into two beams which were caused to interfere. The intensity was reduced until less than one quantum of energy was in the apparatus at any time. Single absorptions were nevertheless still detected, and their statistical distribution was conspicuously the same as for intensities ten thousand times greater, being always in agreement with what geometrical theory predicts for a complete wave not broken up into wave-packets.

Although in these two cases the arguments proceed along quite different lines, the same conclusion seems to follow inescapably. As Messiah puts it: "On [the] way up to the detecting apparatus, everything happens as if light were propagated as a wave [and definitely not as an ensemble of either wave-packets or particles]; the corpuscular aspect manifests itself only at the instant of detection" (25, p. 20).

Thus the formalism needs to contain two distinct kinds of elements: (*a*) The mathematical expression for a sinusoidal wave that does not break up into packets and is not detectable *in loco* (for if a new detector were inserted to explore the effect at any place, this would make a new experiment and bring about a new corpuscular effect); in short, the wave is physically *unobservable* as such. (*b*) A mathematical operation for deriving from this wave, on the occasion of an observation, a statistical distribution which individual absorptions must fall into, although individually their place and timing cannot be predicted. The theory of linear operators in a Hilbert Space fulfills precisely these conditions, and with appropriate choice of wave-function and identification of operators it provides the basis for main-stream quantum theory and all its immensely varied and accurate confirmations.

ɔle geometrical theory
:en the resolving power
ıt the wave meets the
:casion; and this theo-
erimentally for normal
eatly reduced, and the
:s, the resolving power
:kets and not at all on
beyond the packets as
test whether the wave
ıot logically be consid-
The verdict of experi-
the incoming beam, it
ıble of separate packets.
experiment carried out
; 33, p. 316) the beam
two beams which were
:ed until less than one
t any time. Single ab-
, and their statistical
as for intensities ten
agreement with what
ave not broken up into

ts proceed along quite
to follow inescapably.
ıe detecting apparatus,
gated as a wave [and
ɛ-packets or particles];
· at the instant of de-

two distinct kinds of
for a sinusoidal wave
ɔt detectable *in loco* (for
the effect at any place,
ıg about a new corpus-
y *unobservable* as such.
rom this wave, on the
ɔution which individual
dually their place and
f linear operators in a
ions, and with appro-
ication of operators it
um theory and all its
ns.

Von Neumann, to whom the rigorous Hilbert Space formalization is principally due (31), wrote at a time when it was not known that absorptions can continue even when there is less than one quantum in the field, and also long before the development of high-energy physics. In trying to present a complete mathematical theory of the "measurement process" he was led to introduce, over and above the basic analysis just given, several assumptions, techniques, or interpretations which have since been ruled out, disproved, or widely and seriously questioned. It is necessary to give some attention to these here, since the redundant assumptions of von Neumann rest on presuppositions of the classical ontology and are still often adopted (by those unfamiliar with the situation in the philosophy of science) as the basis for attempted solution of the "epistemological problem" arising.

One of the inadmissible assumptions is that the probability density which is ultimately derived ($|\psi|^2$) refers, even before any observation has been made, to an "ensemble" of "systems" to be regarded as present in space at the time in question. It is thus conceived of as a classical probability density in the space of the apparatus before anything is detected, and is declared to yield the "probability of finding a particle" in the region specified and at the time in question, the particles being (contrary to the experiments just mentioned) already in existence as such. Now such a probability can only be verified by repeating the experiment many times under identical conditions and in each case exploring the region in question with detecting apparatus. But this of course destroys the conditions of the experiment. Verification by such means is impossible; the kind of verification actually obtainable in quantum mechanics applies only at the *final* stage (where absorptions occur) and not in the intervening field. Thus the postulate is "definitively *ad hoc*" (33, p. 120) and scientifically inadmissible. Reference may also be made to Messiah's further objections to the "ensemble theory" (25, pp. 158-159) and to Feyerabend's remarks (15, p. 98).

What is called "von Neumann's Theorem," that quantum mechanics is "complete" in the sense that there can be no "hidden variables," is by common consent today considered to fail because of the over-stringent postulates imposed (3, 6, 14).

The Projection Postulate, or theory of "reduction of the wave-function," is the assumption that "measurement" abruptly reduces the state of the "system" being measured to an eigenstate corresponding to the eigenvalue which appears as the measure. This has come in for sustained criticism on various grounds, chiefly because the concept of "measurement" is vague (the measure being the outcome of an experiment extended in space and time indivisibly), and because the postulate is clearly false if, as in absorptions, the photon presumably indicating the measure is destroyed (24, 30).

The chief confusions in the use of the term "measurement" seem to arise from the lack of distinction between a *first interaction*, as between a field and two slits, and a *second interaction*, as at a sensitive screen (34, p. 73). To suppose that the measure at the screen tells us that a certain "system" at the slits had that measure, again runs counter to the basic analysis detailed above. If it is not clearly understood, as Bohr never tired of insisting (7, 29), that a particular measure cannot be attached to a wave or "system" until *after* the final interaction, continual pseudo-problems and paradoxes arise. (Similarly, in parapsychology we could distinguish a "first interaction" between an agent and some physically unobservable and probabilistic connecting means, and a "second interaction" between the connecting means and the observation by a particular percipient, as specified by the total conditions on some particular indivisible occasion.)

I omit discussion of von Neumann's "theory of mixtures" as not being immediately relevant to parapsychology.

Still another serious point of criticism concerns von Neumann's attempts to formalize the "measurement process" as a single encounter between a "system" and the "apparatus," presumed also to have its wave-function. The proposed attachment of a wave-function to the macroscopic apparatus has led to absurdities, as in the example of "Schrödinger's cat" (16, p. 14). It is perhaps sufficient simply to repeat von Weizsäcker's remark that "the problem of measurement certainly is not sufficiently described in von Neumann's way, because there is not the confrontation between one object which is the quantum object and another object which is either the measuring device or the human being" (2, p. 71). (Similarly, in parapsychology there could be no "confrontation" between a general psychical connecting means and a measuring device or percipient, since this would at once produce something specific.)

As regards the meaning of the term "Copenhagen Interpretation," the difficulty is that many writers interpret this "interpretation" as including some or all of the questionable von Neumann assumptions, while others—Messiah, for instance (25, pp. 48, 152, 158 f.)—keep close to the basic analysis above, and thus to the original views of Bohr in regard to the "wholeness" or "indivisibility" of an experiment on the occasion of an observation. Even Heisenberg seems to accept von Neumann's ideas on "mixtures" (20, p. 125), while arguing for the "Copenhagen Interpretation." It is misleading to say, however, as Ballentine does (1, p. 361) and is quoted by Chari, that "Heisenberg combines the 'subjective interpretation of probability' with the Aristotelian notion of

is destroyed (24, 30).
"measurement" seem
n a *first interaction*, as
d interaction, as at a
at the measure at the
slits had that measure,
iled above. If it is not
nsisting (7, 29), that a
vave or "system" until
roblems and paradoxes
ld distinguish a "first
ysically unobservable
"second interaction"
rvation by a particular
ns on some particular

ry of mixtures" as not
gy.
cerns von Neumann's
ocess" as a single en-
tus," presumed also to
ment of a wave-func-
absurdities, as in the
t is perhaps sufficient
that "the problem of
scribed in von Neu-
ontation between one
other object which is
n being" (2, p. 71).
e no "confrontation"
ans and a measuring
e produce something

penhagen Interpreta-
erpret this "interpre-
onable von Neumann
ance (25, pp. 48, 152,
ove, and thus to the
oleness" or "indivisi-
an observation. Even
ideas on "mixtures"
agen Interpretation."
e does (1, p. 361) and
bines the 'subjective
ristotelian notion of

potentia" (10, p. 195). Heisenberg's position, in accord with the theory of mixtures, is simply that uncertainty may be due to inadequate preparation as well as to the "objective" limitations of the uncertainty relations (15, p. 92; 20, p. 53; 23, p. 22).

Ballentine's paper argues for von Neumann's "ensemble theory" (without considering the evidence and arguments ruling this out) and against what he calls the "Copenhagen Interpretation." For this purpose he produces Bohr's very argument of "wholeness" on the occasion of observation, and supposes that in consequence the reference can never be to an "individual system," and so must be to an "ensemble." I agree with Peierls (26) and Feyerabend (15, p. 310) that the confusions have gone so far that the term "Copenhagen Interpretation" would be better dropped, provided it is realized that there *is* a core of main-stream quantum theory which is scarcely open to dispute.

The Epistemological Problem

What are here called the "redundant assumptions of von Neumann" are the basis for his "theory of measurement," about which Jauch, Wigner, and Yanase say: "As has been pointed out many times before, von Neumann's theory, if followed to its ultimate consequences, leads to an epistemological dilemma" (22, p. 145). In their view the alternatives are (*a*) to refrain from making any definite statement about "physical reality" and to concern oneself only with experimental results and the formalism simply as a summary of these (this is the *positivist* position), or (*b*) to assert that the wave-function is indeed a description of "physical reality," but may change discontinuously and erratically as a result of the act of measurement. Neither of these points of view, they say, seems satisfactory.

A third course is to abandon the redundant assumptions of von Neumann and the ideas on "classical ontology" in which they originate. Even after this has been done, however, an epistemological problem remains, centering around the concepts of "reality" and "completeness." It arises from two interrelated difficulties. Firstly, there is the fact that we have no way of predicting the place where an absorption is to take place, or the time when an absorption or emission will occur. Secondly, there is the peculiar ontological status of the wave-function inasmuch as it is unobservable *in loco* and has no one specific form out of many possible, yet it is posited as a part-cause (along with the experimental conditions in their wholeness) of the specific measure found on the occasion in question. There is thus a double hiatus of ignorance or experimental incapacity at the observational level, and, correspondingly, of predicting incapacity at the causal level represented by the formalism. Attempts to solve *this* epistemological problem usually amount to attempts to fill in the hiatus in the formalism with further hypotheses, hoping that the experimental hiatus can then be considered to have been sufficiently accounted for.

The existence of this double hiatus reveals a striking similarity between the epistemological problem which arises in parapsychology and that which arises in quantum theory. In each case there is the impossibility of accounting for a very wide field of evidence in terms of the "classical ontology"; there is the question of the peculiar ontological status or "reality" of the physically unobservable causes which are postulated as an explanation; and there is the question whether it is right or not to demand, as explanation, a complete system of laws which are acceptably "physical" in character. In each case there is a crisis, a problem of "reality," and a problem of "completeness" at the physical level.

I shall deal first with the problems in quantum theory, noting, however, that both in quantum theory and in parapsychology any solutions offered must rest on some fairly definite world-view, and that the same kind of world-view should serve for both cases.

All the solutions that have been offered appear to fall into one or the other of two categories, according to the kind of world-view assumed. In order to bring hidden assumptions to light it will first be necessary, I think, to state in explicit terms the more familiar world-view. This seems to be still commonly assumed by most scientists other than quantum theorists, although, as I shall maintain, it is definitely ruled out by many powerful considerations, apart from its incompatibility with main-stream quantum theory.

It has been given various names. As a basis of nineteenth-century physics it is called the "classical ontology." In Whitehead's terminology, and viewed as a basis for conceptions of space and time, it is the theory of "simple location" (32, p. 72; 33, p. 289). Where space alone is concerned, it has been called the "container theory," following Einstein (13, p. xiv). And as a general philosophical standpoint it was described above as "one-level naturalism."

In this "classical" nineteenth-century world-view everything in nature is characterized by a precise position and a precise time in a unique space and a unique time. It is admitted that experimental measures of space and time only approximate to the (presumed) precise theoretical measures; but this fact, it is claimed, needs no other explanation beyond saying that the divergences result from unpredictable deficiencies of the apparatus or personal differences on the various occasions of measurement. The origin and action

rrespondingly, of pre-
ited by the formalism.
em usually amount to
lism with further hy-
tus can then be con-
for.
ls a striking similarity
arises in parapsychol-
ry. In each case there
wide field of evidence
is the question of the
e physically unobserv-
lanation; and there is
iand, as explanation, a
ptably "physical" in
blem of "reality," and
il level.
antum theory, noting,
In parapsychology any
finite world-view, and
ve for both cases.
ppear to fall into one
he kind of world-view
ons to light it will first
ms the more familiar
nly assumed by most
ough, as I shall main-
werful considerations,
eam quantum theory.
of nineteenth-century
gy." In Whitehead's
ceptions of space and
32, p. 72; 33, p. 289).
called the "container
d as a general philo-
ve as "one-level na-
ld-view everything in
and a precise time in
tted that experimental
ite to the (presumed)
is claimed, needs no
vergences result from
r personal differences
The origin and action

of these personal differences are supposed to be brought within the scope of this theory of "simple location" in the following way:

In the strictest form of this world-view it is supposed that a complete system of laws exists for objects or fields characterized in this way, including objects or fields corresponding exactly with everything of the personal life of individuals, so that, given any situation, the outcome at any future time is, in principle, exactly predictable. Such a system of laws is of course not known, but it is supposed that it nevertheless "exists" in such a way as to be effective for exactly limiting the outcome at any future time. The view that such an exact and complete correspondence exists between everything of personal life and the fully determined placing and motion of simply-located objects and fields is known as psychophysical parallelism. This is the metaphysical system which Behaviorism seems to presuppose.

A "weakened" one-level naturalism would result if the rigors of this definition were slightly relaxed in order to admit free will, human intention, memory, and so on, in such a way that while the operation of these factors is wholly confined to physical objects and fields (for nothing else is supposed to exist, on this theory), the laws which they obey are considered to be not suitably put in the same category as "physical" laws.

The problems arising in main-stream quantum theory for those who believe in a one-level naturalism seem to be (*a*) that the experimental hiatus negates the assumption of a complete system of laws from which prediction is actually possible; this negation is also echoed in the formalism; (*b*) that the wave-function is, in principle, physically unobservable and moreover expresses only a range of possibilities; and (*c*) that there are many independent arguments, which some philosophers of science and mathematicians consider conclusive, against one-level naturalism.

In respect of these three difficulties, the policy of those who believe, consciously or unconsciously, in one-level naturalism may be (*a*) to claim that the experimental hiatus will be removed by new discoveries, or is in principle removable; the wave-function will then merely have to be changed or supplemented; and (*b*) to ignore the philosophical and mathematical arguments as meaningless within the language of the assumed naturalism.

Thus de Broglie's proposal, also touched on by Bohm (4, p. 80) in 1957, that all fields consist of simply-located particles and that these are guided by the ψ-field (denominated a "pilot wave") to positions in accord with the known probability distribution, seems to be aimed at preserving the Newtonian concept of *particle trajectories* along with that of simply-located fields. It fails in many ways: for instance, because the particles are "definitively *ad hoc*"

(33, p. 120), because the difficulty is only transformed into the worse difficulty of accounting for the right steering of the particles by a general wave into particular positions, and because it is incompatible with the occurrence of absorptions when less than one quantum is in the apparatus at any time.

It is not possible here to review in any adequate way the philosophical and mathematical arguments against one-level naturalism. They are developed at length in my book, *Philosophy of Space and Time* (33). Three points may perhaps be noted: (*a*) Practically nothing of the "naturalist's" program can be actually put into effect, even in principle. The conditions at any given time cannot be measured with absolute accuracy, nor can an infinity of measures for them be formulated; the time itself cannot be measured with absolute accuracy; and the laws, even if they were known, could not be applied to an infinity of initial observations. In any case, the initial measurements would need to be made in other experiments at previous times, and thus would not in fact determine the experimental conditions as a whole for the particular experiment in question. The fantasy that this kind of prediction is possible results from a wrong analysis of what is actually done in applications of quantum theory. (*b*) There are continual "category mistakes." The representations of objects and fields are mathematical constructs, and the laws are conceptual relations between conceptual events, but these are supposed to operate on non-conceptual objects. In other words, the laws are *a priori* exact, but their field of application is empirical and inexact. (*c*) The theory presents only an impersonal and unknowable substratum of nature, as if given by a God-like allocation of space and time labels regardless of operational possibilities.

Let us now consider the fact that the wave-function, insofar as it is a plain description of the phenomenological situation, is a conceptual structure corresponding with a range of possibilities (each with a certain mathematical probability) and not with any one specific measure; and let us put with this the fact that a specific measure is yielded when, corresponding to a specific occasion in its wholeness, a certain *operation* is made on the wave-function. These facts seem to be little more than an explicit demonstration, in the field of physics covered by quantum theory, of a fundamental distinction between what is in potential and what is actualized according to the occasion.

As soon as we accept such a world-view (or such feature in our world-view) and abandon ideas of simple location, the difficulties posed in the epistemological problem begin to evaporate. It is not to be expected that we should at once attain to a complete and detailed justification of the foundations of quantum theory. It

transformed into the
tecring of the particles
ns, and because it is
ptions when less than
ne.
lequate way the philo-
st one-level naturalism.
Philosophy of Space and
noted: (*a*) Practically
be actually put into
any given time cannot
an infinity of measures
nnot be measured with
ey were known, could
ations. In any case, the
e in other experiments
determine the experi-
ticular experiment in
ction is possible results
lone in applications of
tegory mistakes." The
thematical constructs,
een conceptual events,
conceptual objects. In
heir field of application
nts only an impersonal
given by a God-like
s of operational possi-

ve-function, insofar as
logical situation, is a
range of possibilities
) and not with any one
the fact that a specific
a specific occasion in
on the wave-function.
xplicit demonstration,
eory, of a fundamental
nd what is actualized

or such feature in our
cation, the difficulties
to evaporate. It is not
ain to a complete and
f quantum theory. It

should be sufficient, for the time being, that the conceptual difficulties previously encountered are largely overcome. Some further remarks on the "potentiality" or "hierarchical" view are needed, however, since although I believe something of the kind is accepted by nearly all quantum theorists, the essentials are easily misunderstood or associated with unacceptable presuppositions such as those of von Neumann.

What is in *potential* is to be thought of as normally *subconscious, general, conceptual-formative, time and space comprehending*; but it enters into specific manifestations (actualization) by a limitation of the potentiality according to the total conditions on the occasion of manifesting. A useful illustration is to be found, I think, in the Pythagorean formula for the length of the hypotenuse of a right-angled triangle. Since this is true wherever Euclidean geometry is true, it could be said to permeate local space, though the exactly deduced result is made approximate by the operation of additional potentialities. But nature does not secure the results by making the sort of construction with squares in the space of the physically observed phenomenon, or by any other of the constructions of that kind used in proofs. On the contrary, the result comes about by the fact that (local) concepts of space are organized conceptually according to the Euclidean Isometry Group, and (local) space is inconceivable except in those terms.

Thus potentiality acts by an internal logic, inherent in the phenomenon itself, but not at the same "level" because not in the same category of existence. Another illustration of this is in the expression of emotion and thought in the features of the face. Here it is even more evident that the potentiality (i.e., logically prior causes) and the manifestation are in different categories, and that the potentiality is not strictly in the same space as its manifestation. The actualization takes place, moreover, not by a mechanistic fitting together of geometrical constructions simply located in the one space, but by a direct rule of correspondence.

We come now to the crucial question: Does a thoroughgoing recognition of the distinction between potentiality and actualization, besides being *prima facie* implied by the nature of the wave-function and by the philosophical and mathematical arguments mentioned, relieve the difficulties involved in the epistemological problem in quantum theory? If it does, there would not seem to be any ground for refusing to apply it also to parapsychology.

First and perhaps most urgently, there is the question how absorption can take place even when there is less than one quantum in the apparatus. A partial answer is at once provided by saying that the quantum is *not* to be located in the apparatus, any more than the Euclidean Isometry Group is to be located in the lengths being measured. We cannot go further without tackling the more positive question whether it is now possible to conceive how absorption can take place in distinct quanta at particular positions and times, although the wave which we use to represent the potentiality is extended in space and time; in other words, is there really a sudden "collapse" of the wave-front into a point-absorption?

In the view which we are now considering, the wave-function represents a control-mechanism for absorption, and not anything which is simply located in actuality. It is constructed by "backward extrapolation" (34, pp. 73 f.) from the observational results, and since we have no way of predicting the place and time of individual absorptions, our construction must be of a general kind, like an average. But while it is impossible (for the reasons given above) to carry out such prediction in full and precise detail, other potentialities, individually formalizable perhaps, but not capable of being handled mathematically in their infinity, must have an effect on the manner of actualization. Every slight irregularity in the production of fields and construction of the apparatus must have some effect.

One may occasionally narrow down the range of choices by fresh discoveries; the wave-function thus becomes less general, but still stands for a potentiality (34, p. 76). This is not therefore a "collapse" on actualization, but only a more refined extrapolation.

In the further study of actualization it is necessary to distinguish between *overall* potentialities and *small-bias* potentialities. When, for example, a large number of throws with a six-faced die are classified, certain overall potentialities are seen to dominate until, when the representation has become continuous, the effect of the small-bias potentialities has almost disappeared. In a single throw, on the other hand, it is the small-bias potentialities which dominate, and the effect of the overall potentialities cannot be distinguished. The mathematical probability 1/6 for each face (called "objective") is only a convenient measure of the overall potentiality in the experiment as a whole.

It is clear also that a special kind of potentiality is operative on the instant of actualization in order to *integrate* or *consummate* all the other potentialities, and that this results in the manifesting of a "choice" (of the particular molecule to be affected, in the case of the two-slit or grating experiment). This, we can say, is the chief reason why the experiment must be treated as a whole when we attempt to draw conclusions from what is observed. (A similar integration or consummation of all potentialities clearly takes place in psychological responses.)

It may be remarked that quantum theory has no way of formulating the integration and choice otherwise than as the bare un-

:hout tackling the more
sible to conceive how
a at particular positions
: use to represent the
in other words, is there
nto a point-absorption?
-ing, the wave-function
ition, and not anything
constructed by "back-
.e observational results,
the place and time of
ist be of a general kind,
(for the reasons given
ind precise detail, other
rhaps, but not capable
infinity, must have an
ry slight irregularity in
of the apparatus must

inge of choices by fresh
es less general, but still
ot therefore a "collapse"
extrapolation.
necessary to distinguish
is potentialities. When,
ith a six-faced die are
seen to dominate until,
nuous, the effect of the
ired. In a single throw,
ialities which dominate,
annot be distinguished.
ace (called "objective")
rall potentiality in the

entiality is operative on
grate or *consummate* all
s in the manifesting of
be affected, in the case
we can say, is the chief
d as a whole when we
s observed. (A similar
lities clearly takes place

' has no way of formu-
: than as the bare un-

predictable result of a mathematical operation; for it has no way of formulating all the small-bias potentialities. There are two possible views in regard to the bringing about of single absorptions by such integration, but further discussion here would take us too far afield.

Lastly, it may be asked why *one quantum only* is absorbed. Quantum theory assumes this in order to "save the appearances," and no deeper explanation seems forthcoming. A clue, however, may be provided by the fact that quantum-mechanical *energy* is measured by *frequency* of pure sinusoidal constituents in the wave-function. Hence interaction would appear to occur not by the transfer of localized packets of energy working into one system from another, but by a process analogous to *resonance* in "stationary" (i.e., stabilized) states of the atomic systems reacting to a "free field." In quantum field theory, interaction between fields has in fact to be conceived of in such a non-localized way.

Much space could be devoted to examining the views of Heisenberg on *potentia* (18, pp. 13, 27 f.; 19, p. 149; 20, p. 53), of Bohm on "potentiality" (5, p. 241), of Popper on "propensity" (27), and of Dobbs in interpreting Popper's "propensity" as a "potentiality" (11, pp. 323-327), and to considering how far elements of the simply-locational view, belief in the redundant assumptions of von Neumann, and varying usages of such terms as "subjective" and "objective" have confused the issue. I think it is more helpful, however, simply to bear in mind that the exposition of potentiality presented here is a further articulation of Bohr's insights (rejecting a few incompatible elements) and, as such, is already latent in main-stream quantum theory.

The Relevance of Quantum Theory to Parapsychology

The analysis of the epistemological problem in quantum theory leads inevitably to an extension of the findings (notably the distinction between potentiality and actuality) beyond the limits of physical experimentation to experience in general.

Let us return first to a consideration of the small-bias potentialities which cannot be formalized in their completeness. These are evidently of two kinds. One kind will be operative when an experiment, after having been set up, proceeds without human influence and the results are recorded photographically. Another kind will be operative when human influence is unavoidable, as for instance if the experimenter is breathing into the apparatus or walks about in the laboratory during the experiment. If the experimenter sees, hears, or feels the result, still other potentialities must be operative to bring about his sensing what he actually does sense.

The state of affairs then appears to be that potentialities of the apparatus and of the experimenter must intermingle, so to speak, and mutually adjust themselves in the act of integration which is observation. This is also an actualization (since at a particular place and time) derived from what is potentially extended over space and time, and also from what is dispositional in the observing individual.

The admission of a sphere of potentiality where human consciousness is concerned is nothing more than a theory of the unconscious made explicit, with paradoxes removed, and confirmed as a viable theory by the existence of its analogue in quantum theory. It needs to be further remarked, however, that individuals are not in that case sealed off from one another, for their ability to observe "the same" physical objects shows that there is a *common* potentiality ("intersubjectivity," or "common unconscious") involved in all human observation. We must therefore admit a double aspect of human potentialities: first, as originating in one individual and specially referring to him or his physical body (the "subjective" aspect), and second, as being involved in any integration with potentialities of other objects or individuals in the wholeness of an occasion of actualization for him (the "intersubjective" or "objective" aspect).

Continuing further over the range of human experience, we come to the consideration of states of thought, mental imagery, dreams, and ESP (including precognition). Consistently with the above analysis, we must admit "spheres of actualization" and corresponding potentialities in each case, not necessarily all different. But even though the spheres may be different, the same human potentialities may be actualized in them in different ways. Thus fear may manifest itself in one way in the physical state and in another way in the dream state because the additional potentialities required for actualization are different in the two cases.

When all kinds of awareness (each with its sphere of actualization and potentiality substructure) are admitted as subjects for scientific study, the world-view that results is of the kind called hierarchical. Natural science results from the arbitrary limitation of attention to the effectively *impersonal* physical sphere, with human observation of the events in question replaced by recorded artifacts. Discarding this arbitrary limitation, we have now to examine how the hierarchical world-view may help us toward an understanding of phenomena in parapsychology (as well as those in more ordinary psychology). Since I am attempting here only an introductory sketch aimed principally at showing the relevance of quantum

Declassified and Approved For Release 2014/01/09 : CIA-RDP79-00999A000200010009-6

what he actually does

ıt potentialities of the
ermingle, so to speak,
f integration which is
ce at a particular place
extended over space
nal in the observing

v where human con-
in a theory of the un-
ved, and confirmed as
ie in quantum theory.
at individuals are not
heir ability to observe
is a *common* potenti-
nscious") involved in
dmit a double aspect
in one individual and
ody (the "subjective"
any integration with
n the wholeness of an
ersubjective" or "ob-

n experience, we come
ntal imagery, dreams,
ently with the above
lization" and corres-
sarily all different. But
same human potenti-
ways. Thus fear may
te and in another way
potentialities required
es.
sphere of actualization
s subjects for scientific
nd called hierarchical.
iitation of attention to
th human observation
d artifacts. Discarding
xamine how the hier-
an understanding of
ose in more ordinary
only an introductory
elevance of quantum

theory, I propose merely to develop the hierarchical view a little further, in places again appealing to quantum theory for illustration so as to show how room is definitely found for telepathy, clairvoyance, and precognition, which are notoriously inexplicable on the simply-locational world-view.

In order to be as definite as possible in a compact space, I shall now set down the chief points argued for above along with further suggestions in the form of axioms, with definitions of "technical terms" shown in italics, and accompanying comment where it seems necessary:

Axiom 1. There is a *potentiality-sphere* for events in the physical world, whether or not they are observed by any individual. An individual can participate in this, and it can be called his *physical-perception* sphere. Actualization of physical phenomena for him results from integration of potentialities of that kind with his subjective potentialities.

Axiom 2. Every individual has a potentiality-sphere of thought and images, to be called his *thought-image sphere.*

Axiom 3. The various *levels* of potentiality for an individual include a level of values and higher intentions (the *ideal sphere*) influencing the thought-image sphere, and this in turn influences the sphere of physical perception and action there. Actualizations at any one level act as potentialities at a *lower* level, so that the lower the level, the lesser the generality and the greater the specificity.

Axiom 4. Objects and individuals, singly or in consociation, have distinguishable potentialities, which continually *develop*. Such development is not in physical time, but in what may be called a *conceptual* or *quasi time.*

Comment. (*a*) Compare the two-slit experiment, where "at the slits" only the wave-function is determined, as an "overall" potentiality; but it is determined precisely in conceptual space and time. When a screen is placed in front, the manifestation becomes specific. Such working-out of potentiality proceeds, by its inherent logic, as far, and in as much detail, as is possible until further actualizations occur to specify the development more precisely (34, pp. 76 f.).

(*b*) Conceptual time becomes actually observable when, for instance, a melody and all its structure in notes of various time-values is perceived by "stopping of time." The relative durations retain their cognitive character in spite of the fact that the comprehending of the whole melody, when correlated with physical events, is comprised within a few moments of physical time (33, p. 293).

AXIOM 5. After actualization of an event in any sphere has taken place, there exists a new and fulfilled potentiality in regard to that total event, with a *past-coloring*, in the thought-image sphere of each individual who has experienced it; this is his individual *memory* of the event. If no individual has experienced the event, there is still a general fulfilled potentiality established in the physical potentiality-sphere; this may be called a *cosmical* memory.

COMMENT. Without such a "memory," how could the "wholeness" in quantum theory comprise events at different times?

AXIOM 6. Before actualization of an event has taken place, particular stages in its working-out constitute potentialities which may be called *provisional*, and have a *future-coloring*. Actualization of any of these in an individual's thought-image sphere may occur in integration with his subjective potentialities. If this is relatively free from subjective distortions and is not the result of (subjective) reasoning from theories, regularities, etc., it is called a *precognition*.

AXIOM 7. The kinds of potentiality affecting an individual in any actualization are those of (*a*) his individual disposition, needs, and memory at all levels; (*b*) events at any level: fulfilled, presently actualized, or provisional; and (*c*) other individuals: their disposition, needs, memory, or events in their lives.

COMMENT. Again, "wholeness" in integration is not possible if the potentialities of individuals present are to be absolutely cut off from each other.

AXIOM 8. The development of potentialities and their integration as necessary in any actualization take place by various kinds of *resonance*, i.e., special relevance or resemblance. Resonating potentialities have various degrees of intensity and stability, and various kinds and degrees of *openness*.

COMMENT. (*a*) Resonance may be conceived of as a mutual adaptation of developing potentialities to each other. It would occur by some almost instantaneous or automatic logical process, prior to actualizations in physical space and time, as the molecule to absorb a quantum from a field is "chosen" almost instantaneously. Each potentiality may be conceived of as structured out of categories of archetypal ideas (the Euclidean Isometry Group is such a structure; by it, equal triangles can be "congruent," wherever placed); the adaptation would then be essentially by conceptual *sameness* of some kind causing a link or transfer of characteristics.

(*b*) Relative lack of openness for an individual results when he is *fixated* on other potentialities. The chief fixations are emotionality, fixed ideas, absorption in physical phenomena, and absorption in discursive thinking (accompanied by emotion and words and faintly

ɛnt in any sphere has taken
tentiality in regard to that
thought-image sphere of
it; this is his individual
ıas experienced the event,
established in the physical
ı *cosmical* memory.

," how could the "whole-
ts at different times?

vent has taken place, par-
e potentialities which may
-*coloring*. Actualization of
mage sphere may occur in
lities. If this is relatively
t the result of (subjective)
, it is called a *precognition*.

ecting an individual in any
ıal disposition, needs, and
level: fulfilled, presently
er individuals: their dis-
ıeir lives.

egration is not possible if
re to be absolutely cut off

ntialities and their inte-
ake place by various kinds
resemblance. Resonating
tensity and stability, and

eived of as a mutual adap-
ch other. It would occur
ic logical process, prior to
as the molecule to absorb
ost instantaneously. Each
tured out of categories of
Group is such a structure;
," wherever placed); the
nceptual *sameness* of some
ceristics.

ividual results when he is
fixations are emotionality,
ımena, and absorption in
ion and words and faintly

merged in a general impression of the physical world and body). The last two of these characterize the *normal physical state*.

(*c*) There is an analogue of fixation in physics, better described as *limitation* or *exclusion*; namely, the exclusion of all resonance between objective physical and individual potentialities, directionally from the individual to the object (save indirectly, by muscular action). Such exclusion may be considered normal, but cannot be absolute, since voluntary mental action can initiate electromagnetic action and thus observable physical changes. It is not known precisely in what ways individual potentialities can combine with impersonally physical ones so as to effect physical changes.

In terms of these axioms, telepathy may be explained simply as resonance between the thought-image spheres of individuals. Clairvoyance is resonance between the potentiality-sphere of a physical object or event and the thought-image sphere of an individual. Precognition is resonance between a provisional potentiality in the physical sphere and the thought-image sphere of an individual. In each case one individual (the percipient) must be in a thought-image state, and his potentialities there for resonance of the respective kind must have a certain degree of openness and stability.

Summary

Endeavoring now to sum up the relevance of quantum theory to parapsychology, I think we can safely say that this does *not* lie in the inducement which its formalism offers for extension by means of mathematical theories of imaginary time or mass, joint probability distributions, homology, etc., while phenomenological analysis of the basic experimental situations in quantum theory is not attempted. Indeed, phenomena such as telepathy, clairvoyance, and precognition, which can only be observed by an individual human being and cannot be recorded photographically or by any other direct physical means, are reasonably held to be not "physical" and thus not within the province of quantum theory.

What I have tried to show is that there is an intimate epistemological relevance of quantum theory, rightly understood, to the problem of providing a scientifically acceptable conceptual framework within which parapsychological phenomena make sense as part of nature and human life in their entirety.

In short, the hierarchical world-view, although supported by powerful arguments from philosophy, is likely to prove so puzzling for people brought up in the traditions of "one-level naturalism" that they might ordinarily be excused for saying: "This is hazily unscientific word-play, unconvincing as an account of telepathy, clairvoyance, and precognition." The clear content of the words is missed because of a natural conservatism which tries to adhere to quite another system of ideas.

But now we have seen, in quantum theory, a supremely scientific theory which in its foundations is not a "saving of appearances" but a direct phenomenological analysis of experimental data; and this theory presents a precise mathematical formalism of the very points considered most hazy and difficult to grasp; namely, conceptual structures having the character of "potentiality," a "non-actual" (but nevertheless "real") category of existence and thus a "degree of reality" transcending the "simply-locational" in time and space, and the production of an actual phenomenon from that potentiality according to the whole conditions on the occasion in question.

Hence in quantum theory we have an explicit nature-given illustration of how these conceptions which are being offered are not only perfectly feasible and precise, but have been tested out and found to yield results scientifically confirmed, where older world-views have conspicuously failed.

References

1. Ballentine, L. E. The statistical interpretation of quantum mechanics. *Reviews of Modern Physics*, 42, 1970, 358-381.
2. Bastin, T. The function of the Colloquium. In T. Bastin (Ed.), *Quantum Theory and Beyond.* Cambridge: Cambridge University Press, 1971. Pp. 1-12.
3. Bell, J. S. On the problem of hidden variables in quantum mechanics. *Reviews of Modern Physics*, 38, 1966, 447-452.
4. Bohm, D. *Causality and Chance in Modern Physics*. London: Routledge and Kegan Paul, 1957.
5. Bohm, D. On creativity. *Leonardo, I.* London: Pergamon Press, 1968.
6. Bohm, D. The role of hidden variables. In T. Bastin (Ed.), *Quantum Theory and Beyond.* Cambridge: Cambridge University Press, 1971. Pp. 95-116.
7. Bohr, N. *Atomic Theory and the Description of Nature*. Cambridge: Cambridge University Press, 1932.
8. Born, M. *Natural Philosophy of Cause and Chance*. London: Oxford University Press, 1948.
9. Broad, C. D. The local historical background of contemporary Cambridge philosophy. In C. A. Mace (Ed.), *British Philosophy in the Mid-Century*. London: Allen and Unwin, 1957. Pp. 13-61.

ı account of telepathy,
· content of the words
ı which tries to adhere

·, a supremely scientific
saving of appearances"
experimental data; and
formalism of the very
:o grasp; namely, con-
'potentiality," a "non-
of existence and thus a
›ly-locational" in time
phenomenon from that
›ns on the occasion in

explicit nature-given
ı are being offered are
: have been tested out
›nfirmed, where older

·pretation of quantum
42, 1970, 358-381.
ım. In T. Bastin (Ed.),
lge: Cambridge Uni-

variables in quantum
38, 1966, 447-452.
dern Physics. London:

ndon: Pergamon Press,

s. In T. Bastin (Ed.),
lge: Cambridge Uni-

ption of Nature. Cam-
:32.
and Chance. London:

ound of contemporary
:d.), *British Philosophy*
'nwin, 1957. Pp. 13-61.

10. Chari, C. T. K. Precognition, probability, and quantum mechanics. *Journal* A.S.P.R., 66, 1972, 193-207.
11. Dobbs, H. A. C. Time and ESP. *Proceedings* S.P.R., 54, 1965, 249-361.
12. Dobbs, H. A. C. The feasibility of a physical theory of ESP. In J. R. Smythies (Ed.), *Science and ESP*. London: Routledge and Kegan Paul, 1967. Pp. 225-254.
13. Einstein, A. Foreword to M. Jammer, *Concepts of Space*. New York: Harper & Row, 1960. Pp. xii-xvi.
14. Feyerabend, P. K. Eine Bemerkung zum Neumannschen Beweis. *Zeitschrift für Physik*, 145, 1956, 421-423.
15. Feyerabend, P. K. On a recent critique of complementarity. *Philosophy of Science*, 35, 1968, 309-331; 36, 1969, 82-105.
16. Frisch, O. R. The experimentalist's point of view. In T. Bastin (Ed.), *Quantum Theory and Beyond*. Cambridge: Cambridge University Press, 1971. Pp. 13-21.
17. Good, I. J. *The Scientist Speculates*. New York: Basic Books, 1962.
18. Heisenberg, W. The development of the interpretation of the quantum theory. In W. Pauli (Ed.), *Niels Bohr and the Development of Physics*. London: Pergamon Press, 1955. Pp. 12-29.
19. Heisenberg, W. Die Plancksche Entdeckung und die philosophischen Grundfragen der Atomlehre. *Mitteilungen aus der Max-Planck-Gesellschaft*, 3, 1958, 140-161.
20. Heisenberg, W. *Physics and Philosophy*. London: Allen and Unwin, 1959.
21. Jánossy, L., and Náray, Z. Investigations into interference phenomena at extremely low light intensities, by means of a large Michelson interferometer. *Nuovo Cimento Supplemento*, 9, 1958, 588-598.
22. Jauch, J. M., Wigner, E. P., and Yanase, M. M. Some comments concerning measurements in quantum mechanics. *Nuovo Cimento*, 48B, 1967, 144-151.
23. London, F., and Bauer, E. *La théorie de l'observation en méchanique quantique*. Paris: Hermann, 1939.
24. Margenau, H. Measurements and quantum states. *Philosophy of Science*, 30, 1963, 1-16; 138-157.
25. Messiah, A. *Quantum Mechanics, I*. Amsterdam: North-Holland Publishing Co., 1961.
26. Peierls, R. E. Review of M. Jammer's *The Conceptual Development of Quantum Mechanics*. *Scientific American*, 216, 1967, 137. Quoted in (1), p. 358, and (10), p. 196.
27. Popper, K. R. The propensity interpretation of the calculus of probability and the quantum theory. In S. Körner (Ed.), *Observation and Interpretation*. London: Butterworths, 1957. Pp. 65-70.

Declassified and Approved For Release 2014/01/09 : CIA-RDP79-00999A000200010009-6

28. RUDERFER, M. Correspondence. *Journal* A.S.P.R., 62, 1968, 84-86.
29. SCHILPP, P. A. (Ed.). *Albert Einstein, Philosopher-Scientist.* Evanston and Chicago: Northwestern University Press, 1948.
30. SNEED, J. D. Von Neumann's argument for the Projection Postulate. *Philosophy of Science*, 33, 1966, 22-39.
31. VON NEUMANN, J. *Mathematical Foundations of Quantum Mechanics.* Princeton: Princeton University Press, 1955.
32. WHITEHEAD, A. N. *Science and the Modern World.* Cambridge: Cambridge University Press, 1926.
33. WHITEMAN, J. H. M. *Philosophy of Space and Time.* London: Allen and Unwin, 1967.
34. WHITEMAN, J. H. M. The phenomenology of observation and explanation in quantum theory. In T. Bastin (Ed.), *Quantum Theory and Beyond.* Cambridge: Cambridge University Press, 1971. Pp. 71-84.

Department of Applied Mathematics
University of Cape Town
Rondebosch, C. P., South Africa