25X1A

4 January 1968

MEMORANDUM FOR: [redacted]

SUBJECT: Certain Probabilities in Search Problems when the Targets are Subject to Death

1. First Problem.

Let us assume that a sensor system surveys the target area at the rate of L looks per day. Let us assume that, because of the combination of sensor and target parameters, the probability that the sensor will detect the target or any one pass is P. Let us furthermore assume that the average lifetime of the target, expressed in days, is T.

For an on-going surveillance program-one that has reached the ready state condition of knowledge-the expected fraction of the targets which are correctly stated to be in existence is a useful general measure of the overall capability of the surveillance system.

Clearly, the probability that the existence of a target is correctly stated, given that it was most recently seen "i" looks ago equals the probability that the target is still alive after the period of time which "i" looks consume, given that it was alive "i" looks ago.

The problem arises in determining the probability on the right hand side of the above equation. This probability depends not only on the average life of the target, T, but also on the actual form of the various mortality functions. Two such more mortality functions will be examined. Case 1: On targets surviving exactly T days, the probability of death at any age than T is zero; the probability of death in the T day is 1. Case 2: The probability of death in any day, given that the target was alive at the beginning of the day, is independent of the age of the target. This is the so called mark off assumption. If, furthermore, the probability that a target dies between any two successive looks of the sensor is small, then we may use a poisson approximation that the probability of death between any two looks is $\frac{1}{LT}$.

The expected lifetime of the target in both Case 1 and 2 is T days.

The expected coverage of the surveillance can, of course, be obtained by multiplying the probability that the target was seen "i" looks ago between probability that the target is still alive if it was known to be alive "i" looks ago and summing over "i" the range of summation is from "i" to infinity. The results are:

Case 1.

$$\text{Expected Correctness} = 1 - \frac{1}{PLT} + \frac{(1-P)^{LT}}{PLT}$$

Case 2.

$$\text{Expected Correctness} = \frac{P \quad (LT - 1)}{PLT + 1 - P}$$

Numerically it appears that in the usual domain of the parameter, there is little difference in the two values. For example, if $L = 1$, $T = 100$, and $P = 1/2$, the expected correctness according to Case 1 is .9800+; according to Case 2 it is .9802-. For extreme cases,where the probability of a death of the target becomes significant, the two sets diverge. For instance, if we take $L = 1$, $T = 100$, as before, but reduce P to the value to $\frac{1}{100}$, we have a case in where the probability that the target dies before it is seen is significant. In this case the value given by the Case 1 formula is .366 and by the Case 2 formula .495.

2. Second Problem.

Let us assume the same target and sensor parameters as before, and assume that a new target comes into existence. Let us assume the probability that this new target will have been detected by the sensor on the "nth" look in the target area or earlier. Let us assume the mortality functions of Case 2 above. Then, probability target will have been detected on or before "nth" look

$$= \frac{P}{1-P} \quad \frac{[Z(1-P)] - [Z(1-P)]^{N+1}}{1 - [Z(1-P)]}$$

where,

$$Z = 1 - \frac{1}{LT}$$

It is interesting to note that there is some finite probability the target will have died before it has been detected or this probability of never detecting the target is

$$= 1 - \frac{1}{1 + \frac{(1 - Z)}{PZ}}$$

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