ON THE TURBULENCE IN THE PROTOPLANETARY CLOUD

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1. The problem of turbulence in the protoplanetary cloud is of importance for planetary cosmogony. Chaotic macroscopic motions probably existed in the cloud during its formation. Further evolution of the cloud depended to a great extent upon whether these original motions damped in a short time, or turbulence supported by some source of energy existed during planet formation. According to Kuiper's and Fessenkov's hypotheses massive protoplanets formed as a result of gravitational instability and turned into planets after the dissipation of light elements. Large-scale turbulent motions with mean velocities exceeding the thermal velocities of atoms and molecules would prevent, however, gravitational instability in the cloud, even if its mass was of the order of the mass of the sun. According to Edgeworth and to Gurevitch and Lebedinsky the planets grew gradually from small condensations formed in a flattened dust disk. But even small scale turbulent motions would prevent extreme flattening of the disk necessary for gravitational instability. The problem of turbulence is also connected with the problem of present distribution of angular momentum between the sun and planets, as large-scale turbulence produces redistribution of matter and of angular momentum in the cloud.

2. The hypothesis of the presence of large-scale turbulence in the protoplanetary cloud was introduced by von Weizsacker [1]. But Weizsäcker's arguments do not proove its existence. Reynolds number is very large for the cloud (about 10^{10}). But for a rotating medium Reynolds number cannot be considered as a criterion of turbulence. Weizsäcker regards turbulence as a result of convective instability. But he uses the criterion of convection for non-rotating medium, which is inapplicable in the case of the rotating cloud. The problem needs therefore further study.

3. In order to reveal the main features of motions in a flat protoplanetary cloud one can use the results of investigations of fluid motion between two rotating coaxial cylinders. Rayleigh [2], Taylor [3] and Synge [4] proved that such a motion of incompressible fluid is stable if the angular momentum increases outwards

$$\frac{d}{d\tau}\left(\omega\tau^2\right) > 0 \qquad (1)$$

This condition had to be satisfied for the protoplanetary cloud. If we neglect the pressure gradient in the cloud and its own gravitation as compared with the gravitation of the sun, the angular momentum will be proportional to $\sqrt{\tau}$. Then condition (1) becomes identical with the condition of stability of circular orbits well known in stellar dynamics. But condition (1) was obtained for an incompressible fluid and does not take into account the possibility of convection. On the other hand Weizsäcker using the criterion of convection left out of account the condition of stability of circular orbits. These two conditions were combined in the paper of the author and E.L.Rousool (5). The condition of convection for a flat rotating cloud (cylindrical rotation) was found:

$$\left(\omega\tau^2\right)\frac{d}{d\tau}\left(\omega\tau^2\right) < \frac{\tau^3}{2\rho^2}\frac{dp}{d\tau}\left[\left(\frac{d\rho}{d\tau}\right)_{ad} - \frac{d\rho}{d\tau}\right] =$$

$$= \frac{\tau^3}{2\gamma\rho T}\frac{dp}{d\tau}\left[\frac{dT}{d\tau} - (\gamma-1)\frac{T}{\rho}\frac{d\rho}{d\tau}\right] \qquad (2)$$

When considering small disturbances it is possible to approximate smooth functions ρ and T for small intervals of r by power functions

$$\rho \sim \tau^{-a_1} \quad , \quad T \sim \tau^{-a_2} \qquad (3)$$

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The condition of convection is then reduced to

$$\left[2 - (1 - \tfrac{1}{\gamma})(a_1 + a_2) \right](a_1 + a_2) > \frac{GM}{RT\tau} \qquad (4)$$

The protoplanetary cloud being largely an HI region one can take as maximum value of T in the right hand side of the inequality (4) the temperature of the black body in transparent cloud $T_0 \approx \frac{300}{\sqrt{r_{ae}}}$, where r_{ae} is the distance from the sun in a.u. Then

$$\frac{GM}{RT\tau} > \frac{360}{\sqrt{\tau_{ae}}} \qquad (5)$$

and the condition of convection (4) is not satisfied at any values of a_1 and a_2. Hence the undisturbed protoplanetary cloud is stable in respect to small disturbances and convection could not arise in it at any admissible values of temperature and of density gradients.

    4. The possibility of large-scale turbulence during a long time is open to serious objections from energetic considerations. Solar radiation entering the flat cloud is insufficient to support turbulence. Gravitational energy of the parts of the cloud approaching the sun suffices only for a short time. Weizsäcker's value of the mean turbulent velocity of about one tenth of the orbital velocity leads to the time of disintegration of the cloud of about 10^3 years, whilst for the planet formation 10^8 years are needed according to Weizsäcker himself. It seems probable that the ratio of the mean turbulent velocity to the orbital and the ratio of the mixing length to the distance from the sun are of the same order of magnitude. Chandrasekhar and ter Haar (6) have found $\ell = 0.62$ r from the law of planetary distances (but this argument does not seem convincing) and take the value of turbulent velocity to be slightly higher than one half of the

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orbital velocity. Karman's formula [7] for the mixing length
in a rotating medium leads to a still higher value, namely
$\ell = 2kr \approx$ 0.8 r. Under these conditions the time of di-
sintegration of the cloud is less than 10^2 years and the for-
mation of the planets is impossible. Large-scale turbulent mo-
tions, if such existed at the initial stage of the evolution,
had to damp rapidly. According to energetic considerations on-
ly motions of a scale by thousand times less than it follows
from Karman's formula could exist for a long time.

5. It is of interest to investigate the problem of the
transfer of matter and angular momentum during the existence
of turbulence in the cloud. According to Weizsäcker turbulent
friction diminished the angular momentum of the rapidly rota-
ting inner parts of the cloud, which therefore approached the
sun. The outer parts acquired the momentum and went away from
the sun. Weizsäcker uses shearing stresses depending on the
gradient of angular velocity

$$\tau'_{r\varphi} = \eta\, l\, \frac{d\omega}{dr} \qquad (6)$$

But this tensor of molecular viscosity stresses is valid,
strictly speaking, only for the case of small free paths and
is unfit for large-scale turbulent motions. Prandtl found ano-
ther expression for the stresses as a function of the gradient
of angular momentum:

$$\tau'_{r\varphi} = \eta\, \frac{1}{r}\, \frac{d}{dr}(\omega r^2) \qquad (7)$$

Karman [7] gives the same expression (7) without any comment
on Weizsäcker's using expression (6). In the solar system an-
gular velocity decreases with the distance from the sun, while
the angular momentum increases. Hence the direction of the
transfer of matter and angular momentum in the cloud according
to Prandtl's and Weizsäcker's formulae are opposite.

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Taylor [8] believes that the steady value of angular mo-
mentum in the central region of turbulent flow (the inner cy-
linder is rotating) found experimentally by him and Wattendorf,
[9] , contradicts Prandtl's formula, as the latter gives in
this case zero shearing stresses and would make impossible the
transport of angular momentum. However the equalization of an-
gular momentum in the main part of the flow agrees with Prandtl
expression. The accuracy of the experiment is not sufficient
to state that the derivative of angular momentum is exactly
zero . We can only say that the derivative is very small, but
this conclusion follows just from Prandtl's formula, if the
turbulent viscosity is great. The same takes place in the strai-
ght flow in tubes. The almost flat velocity profile far from
the walls of the tube and its sharp bending near the walls can
be explained, if we suppose that turbulent viscosity is high
far from the walls and decreases rapidly when approaching the
walls (as the first or the second power of the distance from
the walls, for example). A similar suggestion about turbulent
viscosity in a rotating flow permits to explain, by using
Prandtl's formula, the almost constant value of angular mo-
mentum far from the walls and its sharp fall near the walls.
Neither the relation (6) resulting from the molecular visco-
sity tensor, nor Taylor's suggestion of vorticity conservati-
on permit to explain this peculiarity of turbulent rotational
motion.

Probably Prandtl's formula is not quite accurate, because
of the semiempirical character of the turbulence theory. On
the ground of new interpretation of the mixing length J.Wasiu-
tynsky [10] has obtained the expression for stresses in a mo-
re general form. For the case of cylindrical rotation:

$$\tau'_{\tau\varphi} = \frac{\rho K_\tau^\tau}{\tau} \frac{d(\omega \tau^2)}{d\tau} - 2\rho K_\varphi^\varphi \omega \qquad (8)$$

At $K_\varphi^\varphi = 0$ (purely radial exchange) one obtains Prandtl's formula; at $K_\varphi^\varphi = K_\imath^\imath$ (isotropy) one obtains a formula of molecular viscous stresses with the exception that turbulent viscosity figurates instead of the molecular. He found the condition of non-decreasing turbulence for incompressible ideal fluid

$$\left[2 K_\varphi^\varphi \omega \imath - K_\imath^\imath \cdot \frac{d(\omega \imath^2)}{d\imath} \right] \frac{d\omega}{d\imath} \leq 0 \qquad (9)$$

It is not clear whether this generalization is only formal, or characterizes the turbulent motions more exactly. In this case it is not clear, which values of the ratio $K_\varphi^\varphi / K_\imath^\imath$ are more probable in the actual turbulent flow. For the solar system ($\omega \sim \imath^{-3/2}$) the turbulence decreases according to this formula, if $K_\varphi^\varphi < \frac{1}{4} K_\imath^\imath$. The sign of the stresses is then given by Prandtl's formula and the transfer of matter and of angular momentum is opposite to that found by Weizsäcker. It seems probable according to energetic considerations that this situation took place for large-scale turbulence. It might be believed that small-scale turbulence would be more isotropic. But small-scale turbulence would be incomparable with the theoretical value of mixing length found by Karman for a rotating system. It is not clear whether such turbulent motions are possible.

Being only an astronomer the author fails to estimate correctly the theories and criteria of turbulence and should like to know the opinions about these questions of specialists in this field.

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