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The Elementary Mathematics Program
in Russia

American awareness that recent Soviet achievements in space technology bespeak a high order of mathematical competence has aroused interest in the teaching of mathematics at all levels in Soviet schools. The subject is a complicated one, as it is indeed in our own country, and much more detailed study must be made before anything like a fully adequate picture can be obtained. As a contribution along these lines the present article offers a translation of the official syllabus for the first four grades of elementary school.

As usual with documents of this sort the language is replete with technical jargon and is frequently terse to the point of ambiguity. In order that the material may be as meaningful as possible to the reader I have provided fairly extensive notes wherever these seemed desirable. The notes are based largely on my examination of a number of elementary textbooks and works for the training of Soviet teachers. The main contents of this article, that is the three sections entitled "Explanatory Remarks," "Methodological Instructions," and "Programs," are translated from the official Russian school programs for 1957-58.[1]

1 Programmy nachalnoi shkoly na 1957-58 uchebny god (Moscow, 1957), pp. 70-81. For further information on the official syllabi and their relationship to the Soviet school system see Izaak Wirszup, "Current School Mathematics Curricula in the Soviet Union and other Communist Countries," The Mathematics Teacher, LII, 5(May, 1959), p. 334 ff.

EXPLANATORY REMARKS

The teaching of arithmetic in Grades I-IV has as its aim to teach children correctly, knowingly, confidently, and rationally to perform operations with whole numbers and to apply acquired knowledge and experience to solving arithmetic problems and performing simple calculations. The teaching of arithmetic must help carry out the tasks of the communist upbringing of children.

The study of arithmetic in school should be so designed that number and measurement serve as a tool of cognition of surrounding reality.

During the instructional period of Grades I-IV the pupils should acquire:

A firm knowledge of whole numbers and operations with them and solid habits of oral and written computation with whole numbers, both abstract and denominate:

A firm knowledge of metric measures and measures of time, and the ability to use them in measuring;

An elementary knowledge of simple fractions;

A knowledge of a few elements of descriptive geometry and the ability to apply this knowledge in practice;

The ability to solve easy arithmetic problems.

Whole numbers and operations with them, studied in a definite order, comprise the basic content of the program of Grades I-IV.

First of all the children study numeration and the arithmetic operations (addition and subtraction) within the limit of 10, and then numeration and the four arithmetic operations within the limit of 20. Next comes the section "Numeration and the four operations within the limit of 100," the study of which begins in Grade I and is concluded in Grade II, after which begins the study of numeration and operations within the limit of 1000. From this section only numeration and the four operations in round hundreds are studied in Grade II.

In Grade III oral computations with round tens within the limit of 1000 are studied, and written methods of calculation within this limit are introduced.

After the study of the operations within the limit of 1000, numeration and the four arithmetic operations involving large numbers within the limit of a million are studied.

Pupils learn the reading and writing of numbers and familiarize themselves with the designation of the orders and their distribution into periods within the limit of six-digit numbers.[2]

In grade IV the knowledge which has been acquired by the pupils is expanded and systematized: the knowledge of numeration is expanded to include the periods of millions and billions; more difficult cases of multiplication (multiplication of numbers with zeroes at the end) and division (division with a remainder, etc.) are consolidated; the relationship between data and results of arithmetic operations is learned;[3] this last is used for checking operations, for solving ex-~~ercises with [illegible], and for solving problems.~~[4]

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[2] "Orders" are the successive positions occupied by the digits of a number. For example, a digit in the one's place is called a digit of the first order; a digit in the ten's place is called a digit of the second order; and so on. "Periods" refers to the successive groups of three digits, which in our national system (but not in the Russian) are set off by commas.

[3] As an example of what is meant by "the relationship between data and results" reference may be made to a textbook used in the fifth and sixth grades. (I. N. Shebchenko, Arifmetika. Uchebnik dlya 5 i 6 klassov semilyetei i srednei shkoly, [Moscow, 1958], pp. 49-50) Under this heading the author illustrated the relationship for addition by a discussion which concludes with the following : "In

ercises with x, and for solving problems.[4]

The course in primary arithmetic concludes with the study of simple fractions in Grade IV.

As a result of their study of whole and fractional numbers, pupils who complete Grade IV should:

Have a firm grasp of the terminology of each arithmetic operation;

Have a good command of the technique of written computation;

Know the formulations in which the relationship between the component parts of operations is expressed, and the rules for checking operations;

Understand the significance of each arithmetic operation (without memorization of specific operations) and know the basic circumstances in which each operation is applied to the solution of problems;

Be able to make use of the basic properties of operations in oral and written computations without the formulation of these properties, with the exception of the commutative property[5] of addition and

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order to find an unknown addend it is necessary to subtract the known addend from the sum of the two addends." This is further expressed with the notation that if $a+b=c$, then $a=c-a$ and $b=c-a$.

4 The word translated as "exercise" refers to a pure number problem, such as 18-13, whereas "problem" refers to what we frequently designate as "word problem." The expression "exercises with x" refers to such exercises as the following, taken from a fourth-grade Russian textbook: Find the value of x in $x-625=1{,}200$. (A. S. Polyak & G. B. Polyak, Arifmetika. Uchebnik dlya 4-go klassa nachalnoi shkoly. [Moscow, 1956].)

5 The principle that $a+b=b+a$, $ab=ba$.

multiplication, which pupils should be able to name and formulate;

Know the formation of the fractions (1/2, 1/4, 1/8, 1/5, 1/10) and how to add and subtract fractions having the same denominator and denominators which are multiples of each other.

Throughout every year of the primary arithmetic course great attention is devoted to the study of metric measurement, and also measurement of time.

So that the students will obtain a concrete idea of all the measures and learn to use them, in every grade, beginning with the first, the children themselves make up examples of various measures and practice measuring and weighing, developing judgment of eye, and determining approximate weight of solids by muscular sensation of weight.

In Grade IV the operations with compound denominate numbers[6] should be limited to the simplest cases of these operations with small, two-digit numbers to whatever extent is required for preparing children to study decimal fractions and for practical use in life. In particular, operations with numbers which express measures of time should be made easier.

A substantial part of the program of primary arithmetic is made up of geometric material, the study of which gives some knowledge and experience to the children and develops spatial concepts in them. The study of this material begins in Grade I and continues throughout the whole course of primary school, becoming gradually more complicated and closely interwoven with arithmetic. In Grades I and II, in the study of numeration and operations with numbers, squares, rectangles,

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6 Compound denominate numbers refer to magnitudes, expressed in units of more than one denomination, as 2 meters 15 centimeters.

triangles, circles, cubes, and rectangular solids are employed as didactic material. The children obtain visual images of these figures and bodies, become familiar with their names; draw and trace them on graph paper, and measure the length and width of rectangular figures.

Many exercises are carried out by children in the measurement of segments of a straight line with the aid of a centimeter ruler and of distances with the aid of a meter or a tape measure.

In Grade III children obtain practice in more difficult measurement of segments of a straight line and of demensions of small objects, expressing the results of the measurements in decimeters, centimeters, and millemeters. Measurements are carried out not only in the classroom, but also out in the open, where children learn to stake out and measure straight lines and gain practice in the development of visual estimation.

In Grade IV the pupils become acquainted with square meters and with the calculation of areas having rectangular form.

For the study of these subjects practical exercises are provided in measuring the area of the classroom floor, the illuminated area of the class, the area of the school gardens and of the schoolyard, and so on.

In the fall and spring pupils become acquainted visually with the are and the hectare on location, and also with the construction of a right angle, a square, and a rectangle.

After learning square measure and calculating area the pupils study cubic measure and learn to calculate the volume of bodies having right angular form (boxes, rooms). This must be limited to the solution of problems in which the dimensions of the sides are given and it is necessary to find the area or the volume.

In the calculation of area and volume a notation is used which is based upon the method of measurement of these magnitudes and is easier for students in the primary grades. (An example of this notation is 6 sq. ~~in.~~ m. x 4 = 24 sq. ~~in~~ m).

About half of the time devoted to arithmetic in classwork and homework should be used for teaching children the solution of arithmetic problems.

The ability to solve arithmetic problems is one of the basic aspects of the general-educational significance of the arithmetic course. The solution of problems fosters the development of the students' thinking and speaking, their discrimination, and their ability to determine the relationship among magnitudes and to make correct inferences; the solution of problems helps to prepare students for life.

The solution of problems helps the students to understand the concrete meaning of arithmetic operations, elucidates the diverse circumstances in which they are applied, provides elementary practice in the application of analysis and synthesis.

Beginning with the first steps in teaching and extending throughout the whole course of arithmetic, the solution of problems proceeds parallel with the teaching of operations.

In teaching the solution of problems it is necessary to adhere to a precise sequence in going from easy problems to more difficult ones, from the simple to the complex.

In Grades I, II, and III the pupils become acquainted with the basic aspects of simple problems in each of the four arithmetic operations.

In Grade I simple problems are solved in finding the sum and the difference, in increasing and decreasing given numbers, in finding the product (when a given number is repeated as an addend several times), and in partitive division.[7]

In Grade II problems are solved in comparison by subtraction,[8] in finding one of the addends from the sum of two numbers and one of the numbers, in finding the minuend from a given subtrahend and difference, in measurement division, in increasing and decreasing given numbers by "so many" times,[9] in finding the fractional parts of a number, and in comparison by division.[10]

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7 Partitive division refers to finding the size of the unit-group when given the whole group and the number of groups. For example, how many people are there in each group if 100 people are divided into 5 groups? It contrasts with measurement division, which consists of finding the number of groups when given the whole group and the size of the unit-group. For example, how many groups are there if 100 people are divided into groups of 20 people?

8 Literally, "difference comparison." Under this heading a Grade II text depicts two rectangular bars, one 7 centimeters long and the other 4 centimeters with the notation that one is "3 centimeters longer." (A. S. Pchyolko & G. B. Polyak, Arifmetika. Uchebnik dlya 2-go klassa nachalnoi shkoly. Moscow, 1956, p. 22).

9 The Grade II text mentioned above deals with this type of problem in pages 52 ff. Under the heading "Increasing a number several times" it depicts 6 hammers in contrast with 3 hammers, and states that the former are two times as many as the latter. It stresses the contrast between this and the process of increasing a number by some unit-number, as in the case of 5 cherries versus 3 cherries; the former are 2 more than the latter. The section "Decreasing a number several times" includes a problem "8 mushrooms divided by 2 equals 4 mushrooms." This also is contrasted with the process of decreasing a number by some unit-number: "8 mushrooms less 2 mushrooms equals 6 mushrooms."

10 Literally, "multiple comparison." Under this heading the work cited in Note 8 depicts (p. 74) two rectangular bars and notes that one is "3 times as long as" the other. [It may be well to note that the phrase b 3 raza dlinnyee does not mean, as it might seem to at first glance, "3 times longer (than)," but rather "3 times as long (as)."] Further along (pp. 77-78), it is stated that "In order to find how many times as large or as small one number is as compared to another, we must divide the larger number by the smaller." Comparison by division and comparison by subtraction are contrasted with each other in the following type of exercise: "5 centimeters is how many times as small as 35 centimeters? 14 rubles is how much less than 60 rubles?"

In Grade III, to supplement what was indicated above, simple problems are solved in finding the subtrahend from a given minuend and difference.

In addition to the simple arithmetic problems, composite problems of gradually increasing complexity are solved in primary school, beginning with Grade I.

In Grade II along with others are solved problems in direct and inverse reduction to a unit.[11] Among the composite problems are distinguished ordinary arithmetic problems, which are solved in close connection with the study of the arithemtic operations, and the so-called typical problems;[12] the latter are introduced only in Grades III and IV.

For examinations it is necessary to select problems which do not go beyond the scheduled requirements.

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11 As an example of "direct reduction to a unit" we can cite, from from p. 46 of the Grade II work mentioned above, the problem "4 pens cost 20 kopeks. How much were 3 such pens?" The steps in the solution of this are given as:

1) 20 k ÷ 4 = 5 k
2) 5 k x 3 = 15 k

As an example of "inverse reduction to a unit" the same work (p. 68) gives the problem "3 dresses were sewn from 12 meters of material. How many such dresses could be sewn from 20 meters of material?" Here the steps are:

1) ~~20~~ 12 m ÷ 3 = 4 m
2) 20 m ÷ 4 m = 5

12 These are problems which are "typical" or "characteristic" of "special methods" of solution, such as proportional division, sum and ratio, rule of three, and others. In one collection of problems and exercises for fourth grade, "typical problems" for each of the special methods, are listed by number at the beginning of the work but are scattered throughout ~~the 1946 and 1950 editions~~ the book under various topics. (See the 1946 and 1950 editions of N. N. Nikitin, G. B. Polyak, and L. N. Volodina, Sbornik arifmeticheskikh zadach i uprazhneny dlya chetvyortogo klassa nachalnoi shkoly).

Pupils should be able to do the following:

At the end of the first year of instruction: correctly write down the solution of problems and the results of operations, and explain them orally;

At the end of the second year of instruction: pose a question orally and name the operation for its solution, write down the operation correctly with a description of the components; coherently recount the successive steps in the solution of a problem after solving it (without prompting from the teacher);

At the end of the third year of instruction: briefly write down the conditions of a problem, independently formulate a plan for its solution, and write down the solution of the problem together with a written formulation of questions;

At the end of the fourth year of instruction: independently write down the conditions of a problem, coherently explain the steps in its solution and the choice of operations, precisely formulate and write down questions, and check the solution of simple problems.

In the program much attention is devoted to developing habits of <u>oral computation</u>.

In the course of the first two years of instruction the pupils use only oral methods of computation. Beginning with Grade III, written computations comprise the basic form of computation. However, work in familiarizing students with various methods of oral computation and in developing habits of fluent oral computation should continue up to the end of the course in arithmetic. In this connection special attention should be devoted to developing fluency in a computation within the limit of 100, and also with large numbers, operations with which reduce themselves to computation within the limit of 100; for example, 120 x 3 = 12 tens x 3; 480 ÷ 6 = 48 tens ÷ 6; 25,000 + 36,000 = 25 thous. + 36 thous.

It is necessary first to strive for mastery by the students of general methods of oral computation, and afterwards to acquaint them with special (short) methods of oral computation, devoting special attention to those methods which lead to easy and rapid computation; rounding, commutation of addends and of multiplicand and multiplier, successive multiplication and division ~~by x2, x4~~ in simple cases (e.g. multiplication and division by 2, 4, 8),[13] and short multiplication by 5, 25, 50.[14] The practice in oral computation within the capabilities of the children should involve not only abstract numbers but also compound denominate numbers, not only exercises but also problems to be worked out. Much attention should be devoted to the solution of oral problems.

Along with oral and written computations it is necessary to teach children computation on the abacus, which, as is known, finds wide application in living practice.

METHODOLOGICAL INSTRUCTIONS

Arithmetic teaches quantitative relationships of the real world. For the teaching of arithmetic to contribute to the correct reflection of these relationships in the consciousness of children, it must be closely tied in with life, with reality.

A pupil can choose correctly the operation for the solution of a problem only if he knows what connection exists among the magnitudes which are mentioned in the problem. Hence in solving problems it is

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13 e.g. 75 x 4 = 70 x 4 + 5 x 4 = 280 + 20 = 300, and
75 x 4 = 75 x 2 x 2 = 150 x 2 = 300

14 e.g. 63 x 5 = 63 x 10 ÷ 2

necessary to rely heavily on the living experience of children, to tie in closely the teaching of arithmetic with life.

The contents of problems should as much as possible reflect the laboring, productive reality of the people and should have a perceptive character.

Apart from the problems in arithmetic books, it is necessary in each class to solve problems based on numerical data taken directly from surrounding life, from local production familiar to the students.

It is necessary step by step to train children in the independent creation of problems analagous to those solved in class.

Students can gain practical experience in the solution of problems by carrying out simple calculations connected with some activity or other (excursions, school holidays, working on the student experimental plot of land, etc.). It is necessary to teach students to use reference works and tables given in the arithmetic book.

The study of arithmetic should help the child to achieve ~~fram~~ firm habits in computation and mensuration. The school should create firm computational habits by means of an adequate amount of practice in the solution of numerical exercises and problems, and also mensuration habits by means of practical work in measuring, weighing, etc. in class and on location. This work should be preceeded by detailed explanations by the teacher so as to secure a well-grounded mastery of arithmetic by the students. Practice in the solution of problems and numerical exercises should be carried out not only in class but also at home. Independent written work by the students should in large measure serve the aim of creating firm habits.

It is necessary to advance to the development of abstract mathematical concepts in primary school by starting with visual instruction. Accordingly in the teaching of arithmetic wide use should be made of visual aids: arithmetic box,[15] class abacus, models of metric measurements, geometric figures, measuring and drawing instruments (ruler, compasses, and drawing-triangle and drawing-square[16]), simple instruments for land-measurement (surveying-square[17], tape-measure or measuring chain), graphic illustrations.

The direct visual-tactile perception by the students contributes to the successful teaching of geometric material. The students should not only use ready-made graphic figures, given by the teacher, but also themselves make and reproduce geometric forms: model, cut, mount, draw, glue, and secure geometric figures by paper-folding.

It is important also to make use of self-made aids, as for example: for computation, a self-made abacus; for geometry, geometric figures; for measurement, models of measures.

The students themselves should be drawn into the preparation of these aids. The work of preparing visual aids helps children to achieve better mastery of those ideas which are illustrated with the help of the aids, and teaches students work habits and skills: cutting, mounting, sawing, planing, etc., using scissors, knife, saw, ruler, circle).

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15 This is described by V. G. Chichigin in his Metodika prepodavaniya arifmetiki (Moscow, 1952), p. 15 as a box filled with wooden cubes, rectangular solids, and plane figures, and widely used in elementary school in counting by ones and by groups, in the teaching of numeration, and in measuring volume.

16 The root ugol "angle" in the word ugolnik has led to the erroneous translation of the latter as "protractor." Actually, as indicated by the illustrations in the previously cited work by Nikitin et al (p. 81), the word refers either to a drawing-square or a drawing-triangle.

17 As the word ekker, a loan-word from the French équerre "square,"

A great diversity of activity in arithmetic is provided by mathematical games and ~~entertaining~~ recreational problems, which can be used in the lessons of Grades I and II and in extracurricular activity in Grades III and IV.

In the program for each class only new material has been pointed out. Along with the mastery of new material it is necessary to have systematic review of what has gone before. The teaching of each section of the course in arithmetic, and also of the material of each quarter and each school year ends with review.

PROGRAM[18]

Grade I[19]

Counting up to 10; familiarity with numbers up to 10. Addition and subtraction within the limits of 10, (66 hours). (Here and elsewhere in each program the hours cited at each paragraph include

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designates a special sort of square for outdoor work, I translate it as "surveying square." The instrument is depicted and described in the work cited in the previous note (pp. 93-94) and also in V. T. Snigiryev and Ya. F. Chekmaryev, Metodika arifmetiki. Posobiye dlya pedagogicheskikh uchilishch (Moscow, 1948), pp. 118-19. It consists of two small boards (35 x 5 x 2.5 centimeters) fastened crosswise to each other in the shape of a Greek cross and mounted horizontally atop a stake about four feet in height. The instrument is used for staking out right angles by sighting along headless nails at the ends of the boards to the tops of sticks held vertically some distance away.

18 The material in this section has already been published by Prof. Wirszup in the article cited in note 1. It has seemed useful to include my own translation nonetheless as it differs in a few points and is provided with explanatory notes.

19 For a comparison of the contents of Russian and American first-grade arithmetic texts see my article, "Beginnings of Mathematical Education in Russia," The Arithmetic Teacher, VI, 1 (Feb., 1959), 6-11.

the time in solving exercises and problems and in teaching metric measurement.)

Oral and written numeration up to 20. Addition and subtraction up to 20. Addition table. Increasing and decreasing a number by various units. Multiplication up to 20. Partitive division within the limits of 20. (100 hours).

Oral and written numeration up to 100. Addition and subtraction of round tens up to 100. Multiplication and division of round tens by a one-digit number within the limits of 100. (2[illegible] hours).

Measures and exercises in measurement. Meter, centimeter. Kilogram. Liter. Week, number of days in a week.

Familiarity with square, rectangle, triangle, and circle (their recognition and discrimination).

Problems. Solution of problems in one operation: by finding a sum or a difference, by finding the product (in the case of the repetition of a given number as an addend several times), by division into equal parts. Solution of problems in two operations.

Review of the material covered. (10 hours).

Grade II

Review of the material covered in Grade I. (12 hours).

Addition and subtraction up to 100. Comparison of numbers by subtraction. (40 hours).

Multiplication and division up to 100: familiarity with measurement division; multiplication table and division with the aid of a table.[20] (72 hours).

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20 The author of the 5th-6th grade textbook cited in Note 3 states (p. 37) that Russian children are taught the use of multiplica-

Increasing a number "so many" times; decreasing a number "so many" times; finding a fraction of a number; comparison of numbers by division within the limit of 100 without the aid of tables. (25 hours).

Oral and written numeration up to 1000. The four arithmetic operations on round hundreds up to 1000 with the use of oral methods of computation. (16 hours).

Measures and exercises in measurement. Measures of length: kilometer, meter, centimeter. Measures of weight: kilogram, gram.

Measures of time: year, month, day, hour, minute. (6 hours).

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tion and division tables "to speed up calculation," but it turns out that this involves more than the ordinary tables taught in our elementary schools. The student is informed that if he has to multiply, for example, 48 by 76, he should look either under 48 or 76 in a handbook conatining appropriate tables. In one such handbook (P. N. Gorkin, Tablitsy umnozheniya i delyeniya [Moscow, 1955]) the 48 table for multiplication is given as follows:

	0	1	2	3	4	5	6	7	8	9	
0	[illegible]	4	9	14	19	24	28	33	38	43	0
1	48	52	57	62	67	72	76	81	86	91	1
2	96	100	105	110	115	120	124	129	134	139	2
3	144	148	153	158	163	168	172	177	182	187	3
4	192	196	201	206	211	216	220	225	230	235	4
5	240	244	249	254	259	264	268	273	278	283	5
6	288	292	297	302	307	312	316	321	326	331	6
7	336	340	345	350	355	360	364	369	374	379	7
8	384	388	393	398	403	408	412	417	422	427	8
9	432	436	441	446	451	456	460	465	470	475	9
	0	8	6	4	2	0	8	6	4	2	

Here the bold-face numbers on the left and right represent the multiplier's digit in the tens place, and the bold-face numbers at the top represent the multiplier's digit in the one's place. To multiply 48 by 76, the student finds 364 at the intersection of the 7 row and the 6 column and annexes the number 8 at the bottom of this column, obtaining the answer 3648.

The straight line. Straight line segment and its measurement.

Problems. Solution of simple problems: by comparison by subtraction, by measurement division, by increasing and decreasing a number "so many " times, by finding a fraction of a number, by comparison by division.

Solution of composite problems in 2 to 3 operations.

Review of material covered. (12 hours).

Grade III

Review of what was covered in Grade II. (12 hours).

The four operations on round tens and hundreds up to 1000 with the use of oral methods of computation.

Written computations within 1000: addition and subtraction of three digit numbers; multiplication of two-digit and three-digit numbers by a one-digit number; division with remainder up to 100, with the use of tables; division of a three-digit number by a one-digit number. (44 hours).

Oral and written numeration of large numbers up to a million. Addition and subtraction, multiplication and division of many-digit numbers with one-, two-, and three- digit numbers.

Addition and subtraction on the abacus.

Naming the components of arithmetic operations. Checking operations. Parentheses (simple cases). (98 hours).

Measures and exercises in measurement. Table of measures of length: kilometer, meter, decimeter, centimeter, millimeter. Table of measures of weight: ton, centner, kilogram, gram.

Table of measures of time: century, year, month, day, hour, minute, second. (5 hours).

Geometric material. Measurement of segments. Simple measurements on location: staking out and measuring straight lines. Exercises in the development of visual estimation.

Rectangle and square; their sides and angles. Drawing a right angle, a square, and a rectangle with the help of a ruler and drawing-square and drawing-triangle. (8 hours).

Oral computations: fluent computation within 100 and in round numbers within 1000. The use in oral computation of the rounding method and the commutative property of addition and multiplication.

Problems. Solution of simple arithmetic problems and composite problems in 2 to 5 operations in close connection with the study of the arithmetic operations.

Solution of problems involving the simple rule of three,[21] proportional division,[22] finding the unknown from two differences,[23]

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[21] The "rule of three" is extensively discussed in the teacher-training textbook by V. G. Chichigin, Metodika prepodavaniya arifmetiki (Moscow, 1952), 233 ff. An example of the "simple" rule of three is the problem: "2 kg of sugar cost 12 rubles. How much do 3 kmg of sugar cost?" The "compound" rule of three is illustrated by the problem: "In 3 hours 5 pumps raised 1,800 buckets of water. How much water would 4 such pumps raise in 4 hours?" Chichigin suggests that the solution to such problems be taught by two methods, first by "reduction to a unit" and then with the help of proportion. For some historical notes on the "rule of three," a term which long ago disappeared from most textbooks of arithmetic, see the standard works by D. E. Smith, History of Mathematics, and Vera Sanford, A Short History of Mathematics.

[22] Under this heading a collection of problems and exercises for Grades V and VI includes the following: "Divide 765 into parts proportional to the numbers 1/5, 1/4, and 0.3," "Divide 144 into 3 parts, x, y, and z, so that x ÷ y = 3 ÷ 4 and y ÷ z = 4 ÷ 5." (S. A. Ponomavyov and N. I. Syrnev, Sbornik zadach i uprazhneny po arifmetike dlya 5-6 klassov semilyetnei i srednei shkoly Moskow, 1958], pp. 187-188). Shebchenko (op. cit., p. 190) gives the following rule for such problems: "In order to divide a number into parts proportional to given numbers, divide it by the sum of these numbers and successively multiply the resultant quotient by each of these numbers."

[23] "Problems in finding the unknown from two differences" is probably

opposite motion.[24] (10 hours). (Here are indicated only the hours for the solution of typical problems.)

Review of material covered. (12 hours).

Grade IV

Review of the material covered in Grade III. (12 hours).

Numeration of large numbers including millions and billions. Orders and periods. Addition and subtraction of many-digit numbers; the commutative property of addition; relationship between the components of addition and subtraction; checking addition and subtraction.

Addition and subtraction on the abacus.

Multiplication and division of many-digit numbers; the commutative property of multiplication; relationship between the components of multiplication and division; checking multiplication and division; order of performing arithmetic operations (review). (44 hours).

Compound denominate numbers. Simple and compound denominate numbers. Reduction ascending and descending[25] of denominate numbers

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another name for "problems in finding a number by the differences of two magnitudes." Under the latter heading the collection of 4th grade problems by Nikitin et al includes the following problem (p. 38): "An airman flew 9 hours one day and 12 hours the next day. The first day he flew 825 km less than the second. How many kilometers did the airman fly each day if his speed was the same?"

24 Under the heading "problems involving motion" the work cited in the previous note gives a number of problems involving motion in opposite directions, as for example (p. 41): "Two ships left simultaneously from docks 180 km apart and sailed toward each other without stopping. The first ship went 21 km per hour and the second 24 km per hour. (a) After how many hours did the ships meet each other? (b) How many kilometers did each ship go?"

25 Reduction ascending: 1,236 centimeters = 12 meters 36 centimeters.
Reduction descending: 12 meters 36 centimeters = 1,236 centimeters.

in the metric system of measurement. The four arithmetic operations on compound denominate numbers with the metric measures. Problems involving all operations with compound denominate numbers. (26 hours).

Geometric material. Familiarity with area. Units of measurement of area. Measuring and computing the area of a rectangle and a square. Table of quadratic measures. The are and the hectare. Solving problems in computation of area. Construction on location of a right angle, a square, and a rectangle. (14 hours).

Cubic measure. Familiarity with the cube: faces, edges, and vertices of a cube. The cube as a unit of measurement of volume. Measuring and computing the volume of right-angular bodies (boxes, chests, rooms). Table of cubic measures. Solving problems involving computation of volume. (14 hours).

Measures of time. Table ofmeasures of time (review); reducing measures to higher and larger order. The four operations on compound denominate numbers with measures of time (simple cases).

Problems involving computation of time within the limits of a day, year, and century (the last in whole years). (26 hours).

Simple fractions: 1/2, 1/4, 1/8, 1/5, 1/10. Partition. Numerator and denominator of fractions. Reduction of fractions. Addition and subtraction of fractions whose denominators are alike or multiples of each other. Solution of problems in finding various parts of a number. (18 hours).

Oral computations. Fluent computation within the limits of 100 and with round numbers within 1000. Use in simple cases of successive multiplication and division (by 2, 4, 8, etc.). Short multiplication by 5, 50, 25.

Problems. Solution of composite arithmetic problems in 2 to 6 operations in connection with studying the arithmetic operations.

Problems in computing the arithmetic mean. Problems solved by the method of ratios. Problems in finding two numbers from the sum and ratio.[26] (15 hours). (Here are indicated only the hours for the solution of typical problems.)

Review of material covered. (29 hours).

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[26] This designation covers also the finding of more than two numbers by the method indicated, as in the following example: "A tourist travelled 2,200 kilometers, going twice as far by ~~boat. How many kilometers did the tourist travel by boat, by~~ boat as by horse, and four times as far by railway as by boat. How many kilometers did the tourist travel by boat, by horse, and by railway?" The indicated solution utilizes the "method of parts" whereby the student finds the sum of the parts (1 + 2 + 8), the value of one part (2,200 ÷ 11), and the value of the remaining parts (200 x 2 and 200 x 8). (V. T. Snigiryov and Ya. F. Chekmaryev, <u>op. cit.</u>, pp. 75-76).