TWO APPROACHES TO GRAVITY REFERENCED ORIENTATION

Approved For Release 2001/08/02 : CIA-RDP80T00703A000800040001-0

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TWO APPROACHES TO GRAVITY REFERENCED ORIENTATION

1. First Approach

Given camera parameters and overlapping exposures of an object the shape and configuration of an object may be determined by the use of conjugate images in suitable reciprocal collinearity equations. If only the shape of the object is required all exposures may be referred to any orthogonal coordinate system inherent to the object. However, if the object's orientation is a function of the direction of gravity then an arbitrary coordinate system with an unknown relation to the direction of gravity is unacceptable.

In the absence of any gravity referenced control data the search for the gravity direction in the arbitrary coordinate system must be found in those motions under the influence of gravity. The motions of the camera associated with pitch, roll, and yaw are a coupling consequence of the drag resistance of the camera through a space under continuous restoring force of gravity. Translations from camera station to camera station are mechanically imposed while the changes in orientation occurring during translation have the direction of gravity as the axis of equilibrium. These axes of equilibrium are illustrated in Figure 1. If x_v, y_v, and z_v are the axes of a camera vehicle Z_g is the direction of gravity and Y_g is the horizontal direction of motion. At rest x_v, y_v, z_v coincides with X_g, Y_g, and Z_g. Pitch (π) is the vertical angle y_v axis defines with the horizontal X_g, Y_g plane. Roll (ρ) is the rotation of x_v, z_v plane about the y_v axis from the X_g, Y_g plane. Thus both pitch and roll refer to the X_g, Y_g plane but π is a vertical angle that always contains Z_g and ρ is not a vertical angle except when $\pi = 0$. ρ lies in the x_v, Z_v plane when $\pi \neq 0$. Yaw (ζ) is the rotation about Z_g of the y_v axis or the horizontal angle the y_v axis makes with Y_g axis.

Assuming the camera to be rigidly attached to its transport vehicle the camera axes may be treated as vehicle axes with constant angles relating

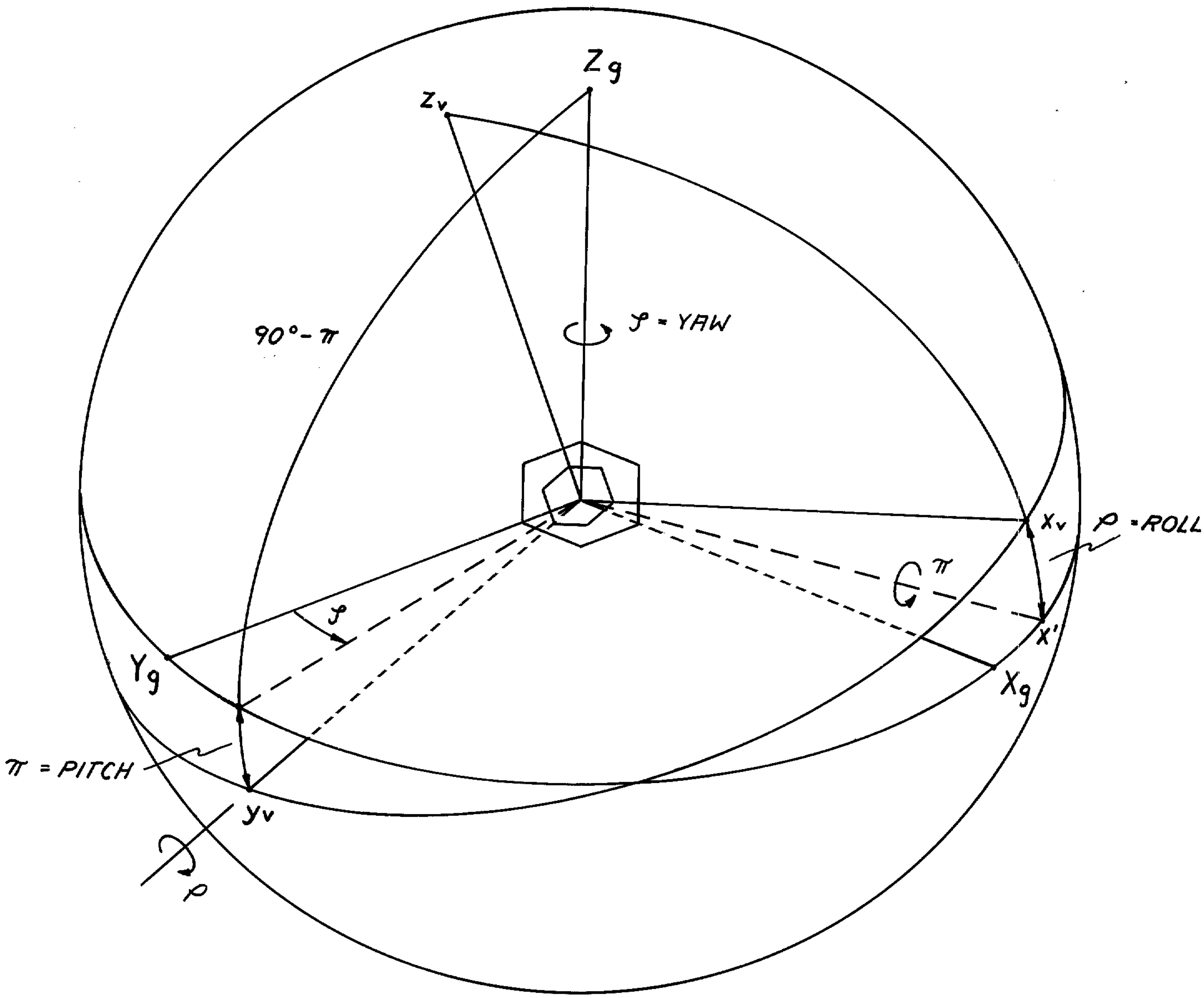

Figure 1 PITCH, ROLL AND YAW

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the vehicle axis to the camera axes. Let π_o, ρ_o, and ζ_o be the constant pitch, roll, and yaw of the camera with the vehicle at equilibrium whence changes in vehicle orientation do not alter the camera relation to the vehicle.

It is believed that a given vehicle had undergone normal pitch and yaw rotations with minimal or nominal zero roll rotations. This means the x_v axis lies in the X_g, Y_g plane and defines a level line which with horizontal change associated with yaw defines a horizontal plane whose perpendicular is the direction of gravity. This geometry is shown in Figure 2 with two different orientations. It is evident if yaw is a constant x_v does not change its horizontal direction without which the direction of Z_g cannot be determined. Figure 3 illustrates the proposed solution based on the assumption of significant pitch and yaw and small or neglectable roll.

Assume $\rho = 0$ and X'_g or the axis of pitch is a level line. X'_g in the presence of yaw about Z_g defines a plane whose perpendicular is Z_g. X_a, Y_a, Z_a is the arbitrary static object space coordinate system to which the camera undergoing yaw and pitch rotations refer. The camera orientations with respect to the arbitrary datum is established. It is desired to refer the camera orientation to the direction of gravity and X'_g, Y'_g system normal to Z_g. X'_g and Y'_g lie in a horizontal plane but because of yaw are different for each exposure. However, the camera axes defines constant angles $x_o X'$, $y_o X'$, and $z_o X'$ with the yawing X'_g (x_v) axis. It is necessary to determine both these constant angles and the instantaneous direction of X'_g in order to determine the direction Z_g. Given the orientation matrix of a camera axes referred to the arbitrary datum.

	x	y	z
X_a	$\cos \alpha_x$	$\cos \alpha_y$	$\cos \alpha_z$
Y_a	$\cos \beta_x$	$\cos \beta_y$	$\cos \beta_z$
Z_a	$\cos \beta_x$	$\cos \beta_y$	$\cos \beta_z$

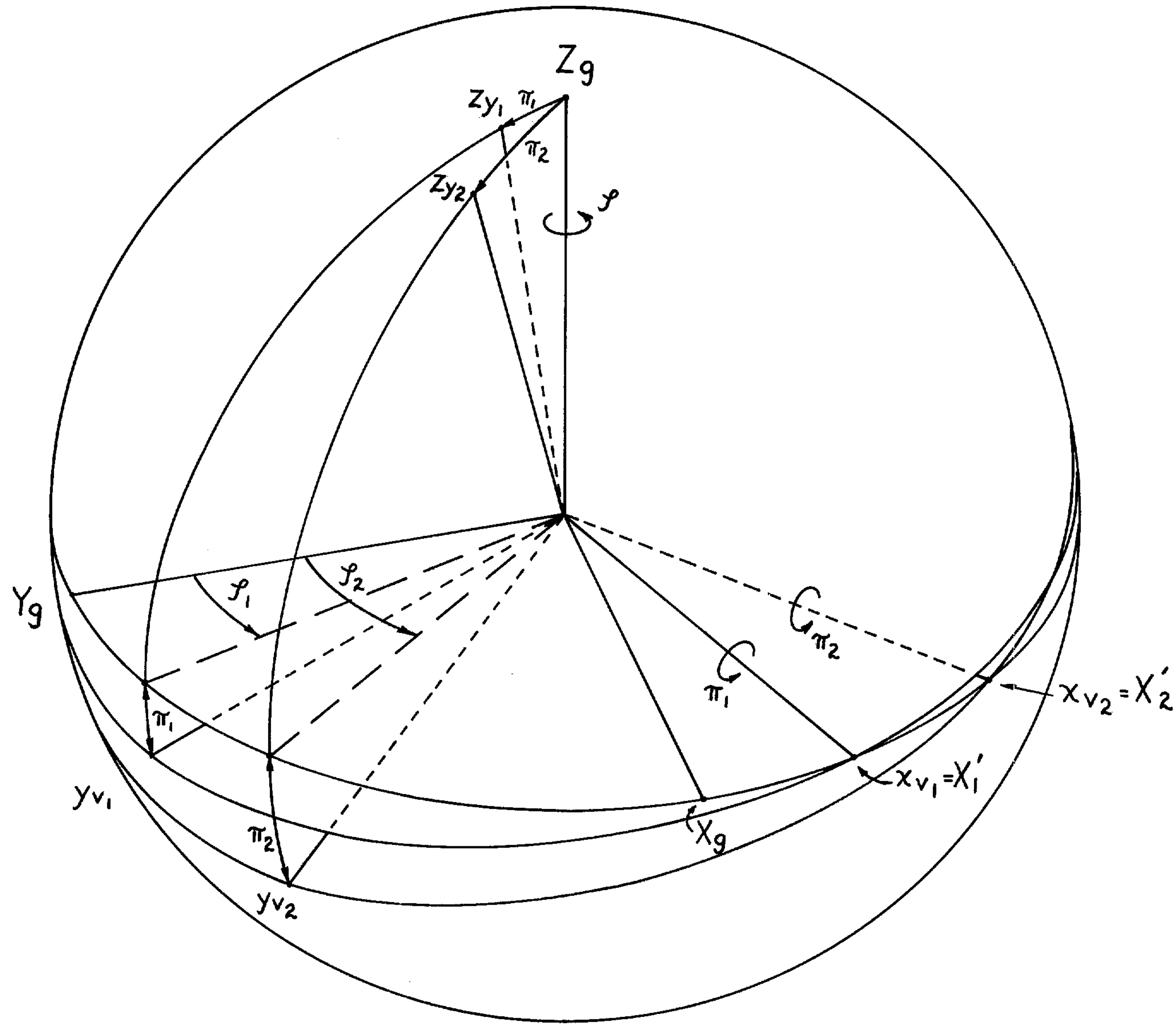

Figure 2 CHANGE IN π AND ƒ WITH ρ =0 FOR TWO EXPOSURES.

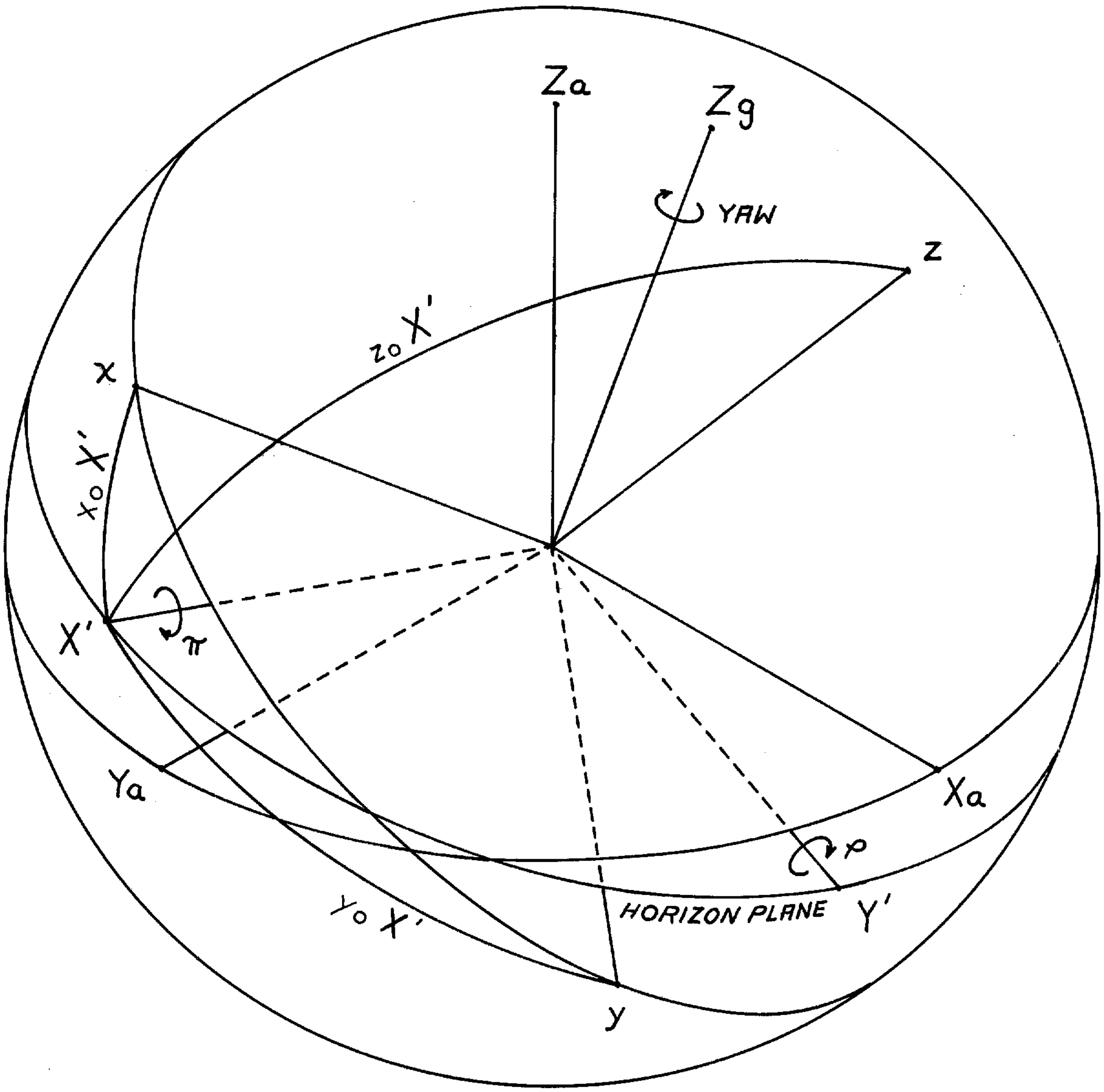

Figure 3 GEOMETRY OF SOLUTION FOR DIRECTION OF Zg IN Xa,Ya,Za COORDINATE SYSTEM.

As usual the columns are the direction cosines of the camera x, y, z axes with respect to the X_a, Y_a, Z_a axes and the rows are direction cosines of the X_a, Y_a, Z_a axes with respect to the camera x, y, z axes. If one knew the direction cosines of the instantaneous X'_g axis referred to the arbitrary datum one could write for the constant camera-vehicle angles. the following equations:

$$\cos \alpha_{X'} \cos \alpha_x + \cos \beta_{X'} \cos \beta_x + \cos \gamma_{X'} \cos \gamma_x = \cos xoX'$$
$$\cos \alpha_{X'} \cos \alpha_y + \cos \beta_{X'} \cos \beta_y + \cos \gamma_{X'} \cos \gamma_y = \cos yoX'$$
$$\cos \alpha_{X'} \cos \alpha_z + \cos \beta_{X'} \cos \beta_z + \cos \gamma_{X'} \cos \gamma_z = \cos zoX'$$

Since the right hand terms are constant

$$\cos xoX' \cos \alpha_x + \cos yoX' \cos \alpha_y + \cos zoX' \cos \alpha_z = \cos \alpha_{X'}$$
$$\cos xoX' \cos \beta_x + \cos yoX' \cos \beta_y + \cos zoX' \cos \beta_z = \cos \beta_{X'}$$
$$\cos xoX' \cos \gamma_x + \cos yoX' \cos \gamma_y + \cos zoX' \cos \gamma_z = \cos \gamma_{X'}$$

Now since Z_g is perpendicular to X'_g

$$\cos \alpha_{X'} \cos \alpha_{Z_g} + \cos \beta_{X'} \cos \beta_{Z_g} + \cos \gamma_{X'} \cos \gamma_{Z_g} = 0.$$

While $\alpha_{X'}$, $\beta_{X'}$, and $\gamma_{X'}$ vary for each exposure they are expressible in terms of the known arbitrary orientation matrix and the constant angles of the vehicle level X' (x_v) axis with respect to the camera x, y, z axes.

Substituting

$$(\cos \alpha_{Z_g} \cos xoX') \cos \alpha_x + (\cos \alpha_{Z_g} \cos yoX') \cos \alpha_y + (\cos \alpha_{Z_g} \cos zoX') \cos \alpha_z$$
$$+(\cos \beta_{Z_g} \cos xoX') \cos \beta_x + (\cos \beta_{Z_g} \cos yoX') \cos \beta_y + (\cos \beta_{Z_g} \cos zoX') \cos \beta_z$$
$$+(\cos \gamma_{Z_g} \cos xoX') \cos \gamma_x + (\cos \gamma_{Z_g} \cos yoX') \cos \gamma_y + (\cos \gamma_{Z_g} \cos zoX') \cos \gamma_z$$
$$= 0.$$

Division by $\cos \gamma_{Z_g} \cos zoX'$ gives

$$(\tan \eta_{Z_g} \tan \eta_{X'}) \cos \alpha_x + (\tan \eta_{Z_g} \tan \xi_{X'}) \cos \alpha_y + (\tan \eta_{Z_g}) \cos \alpha_z$$
$$+(\tan \xi_{Z_g} \tan \eta_{X'}) \cos \beta_x + (\tan \xi_{Z_g} \tan \xi_{X'}) \cos \beta_y + (\tan \xi_{Z_g}) \cos \beta_z$$
$$+(\tan \eta_{X'}) \cos \gamma_x + (\tan \xi_{X'}) \cos \gamma_y = - \cos \gamma_z$$

which contains 8 unknowns in parentheses. Assume an 8 x 8 is solved to obtain preliminary values of $\tan \eta_{Z_g}$, $\tan \xi_{Z_g}$, $\tan \eta_{X'}$, and $\tan \xi_{X'}$

Then

$$\cos \alpha_{Z_g} = \frac{\tan \eta_{Z_g}}{(1 + \tan^2\eta_{Z_g} + \tan^2\xi_{Z_g})^{1/2}}$$

$$\cos \beta_{Z_g} = \frac{\tan \xi_{Z_g}}{(1 + \tan^2\eta_{Z_g} + \tan^2\xi_{Z_g})^{1/2}}$$

$$\cos \gamma_{Z_g} = \frac{1}{(1 + \tan^2\eta_{Z_g} + \tan^2\xi_{Z_g})^{1/2}}$$

$$\cos xoX' = \frac{\tan \eta_{X'}}{(1 + \tan^2\eta_{X'} + \tan^2\xi_{X'})^{1/2}}$$

$$\cos yoX' = \frac{\tan \xi_{X'}}{(1 + \tan^2\eta_{X'} + \tan^2\xi_{X'})^{1/2}}$$

$$\cos zoX' = \frac{1}{(1 + \tan^2\eta_{X'} + \tan^2\xi_{X'})^{1/2}}$$

Inasmuch as $\rho = 0$ any 8 x 8 formed from sequential exposures in flight will yield poor results because of the small changes in ξ_o. To get good results 40 to 50 exposures must be normalized to an 8 x 8 where by Least Square values may be obtained and residuals generated. Since the right hand term of the original equation is zero or cos 90° the residuals obtained are the cosines of angles less than or greater than 90° or $\cos(90° \pm \rho) = + \sin \rho$. Thus the residuals are a consequence of the assumption $\rho = 0$. A large number of condition equations will include the necessary spread in yaw - especially if the values are selected from different flight directions.

Assuming those values of $\sin \rho$ having 3 times the mean deviation from the average $\sin \rho$ are rejected from the solution an iterative form of the equations is normalized and solved. The iterative form is a 4 x 4

$$A\ \Delta\eta_{Z_g} + B\ \Delta\xi_{Z_g} + C\ \Delta\eta_{X'} + D\ \Delta\xi_{X'} = \underline{\sin \rho}$$

where

$$A = (\tan \eta'_{X'} \cos \alpha_x + \tan \xi'_{X'} \cos \alpha_y + \cos \alpha_z) \sec^2\eta'_{Z_g}$$

$$B = (\tan \eta'_{X'} \cos \beta_x + \tan \xi'_{X'} \cos \beta_y + \cos \beta_z) \sec^2\xi'_{Z_g}$$

$$C = (\tan \eta'_{Z_g} \cos \alpha_x + \tan \xi'_{Z_g} \cos \beta_x + \cos \gamma_x) \sec^2\eta'_{X'}$$

$$D = (\tan \eta'_{Z_g} \cos \alpha_y + \tan \xi'_{Z_g} \cos \beta_y + \cos \gamma y) \sec^2\xi'_{X'}$$

The primed η' and ξ' denote the best values obtained from the normalized 8 x 8 which are still considered preliminary. Solving normalized equations for $\Delta\eta_Z$, $\Delta\xi_Z$, $\Delta\eta_{X'}$, and $\Delta\xi_{X'}$

$$\eta_{Z_g} = \eta'_{Z_g} + \Sigma\Delta\eta_{Z_g}$$

$$\xi_{Z_g} = \xi'_{Z_g} + \Sigma\Delta\xi_{Z_g}$$

$$\eta_{X'} = \eta'_{X'} + \Sigma\Delta\eta_{X'}$$

$$\xi_{X'} = \xi'_{X'} + \Sigma\Delta\xi_{X'}$$

Naturally values tan η'_{Z_g}, tan ξ'_{Z_g}, tan $\eta'_{X'}$, and tan $\eta'_{X'}$ are revised with each iteration until the $\Sigma \sin^2 \rho$ = a minimum.

An important point not mentioned is the fact that the cameras cannot be interchanged nor can the camera x, y axes be different from exposure to exposure. Each camera defines different constant angles with the vehicle and any one camera x, y axes defines constant angles if the camera x, y axes are not rotated from frame to frame.

Suppose for some reason the camera x, y axes were rotated 90°, 180°, or 270° as is indicated in Figure 4.

In each case for convenience in relative orientation the camera axes has been rotated from the original orientation x_o, y_o to x, y for which the matrix is given. It is only necessary to restore the matrix to the original axes when the rotation is 90°, 180°, or 270° to make the sign and/or column change indicated below:

90° Rotation

$$\begin{vmatrix} a_{x_o} & a_{y_o} & a_{z_o} \\ b_{x_o} & b_{y_o} & b_{z_o} \\ c_{x_o} & c_{y_o} & c_{z_o} \end{vmatrix} = \begin{vmatrix} -a_y & a_x & a_z \\ -b_y & b_x & b_z \\ -c_y & c_x & c_z \end{vmatrix}$$

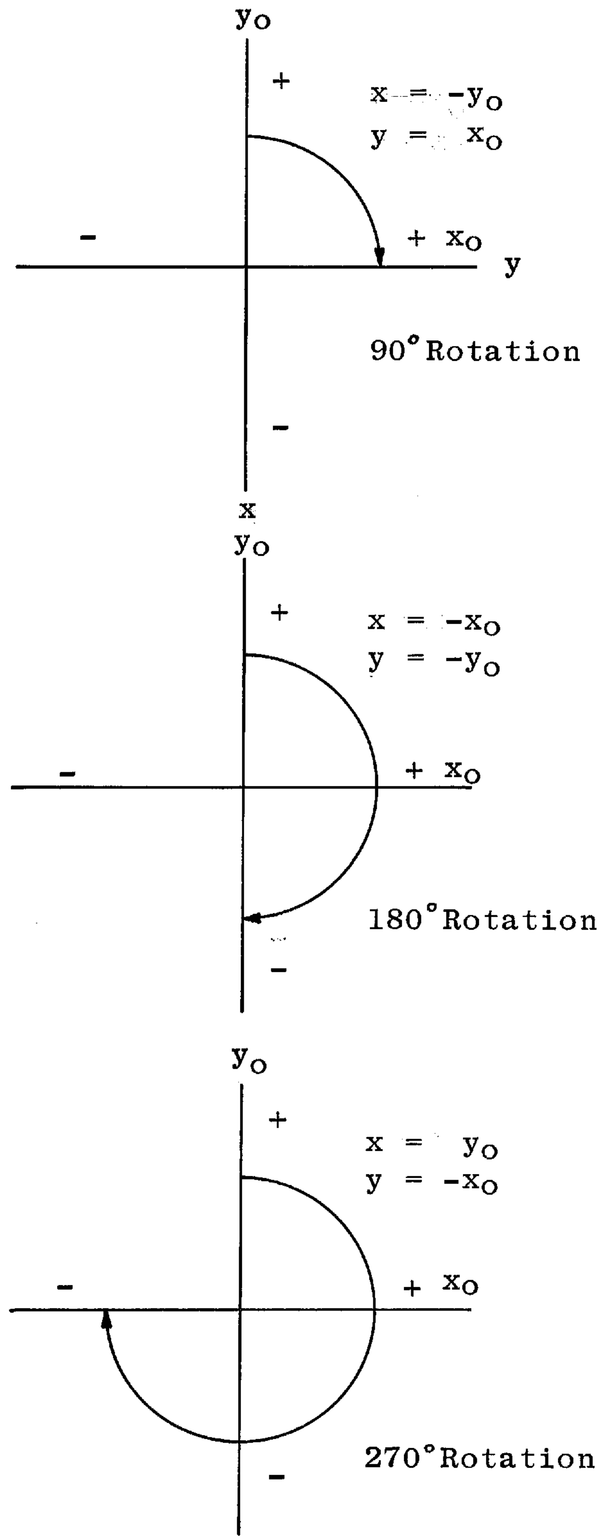

Figure 4 ORTHOGONAL ROTATIONS

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180° Rotation

$$\begin{vmatrix} a_{x_o} & a_{y_o} & a_{z_o} \\ b_{x_o} & b_{y_o} & b_{z_o} \\ c_{x_o} & c_{y_o} & c_{z_o} \end{vmatrix} = \begin{vmatrix} -a_x & -a_y & a_z \\ -b_x & -b_y & b_z \\ -c_x & -c_y & c_z \end{vmatrix}$$

270° Rotation

$$\begin{vmatrix} a_{x_o} & a_{y_o} & a_{z_o} \\ b_{x_o} & b_{y_o} & b_{z_o} \\ c_{x_o} & c_{y_o} & c_{z_o} \end{vmatrix} = \begin{vmatrix} a_y & -a_x & a_z \\ b_y & -b_x & b_z \\ c_y & -c_x & c_z \end{vmatrix}$$

A test solution of an 8 x 8 for each of two cameras yielded both encouraging and discouraging results. The standard coordinates tan η_Z and tan ξ_Z were roughly the same in magnitude and sign for both cameras. This would not be possible unless the solution was based on correct theorems. The values tan η'_X and tan $\xi_{X'}$ showed large variations as might be expected from 8 closely grouped exposures and/or containing an unknown camera x, y rotation. The signs of tan $\eta_{X'}$ and tan $\xi_{X'}$ were opposite for the two cameras as was expected.

The consistent values of η_Z and ξ_Z and the consistent signs of the $\eta_{X'}$ and $\xi_{X'}$ values suggest the solution is valid and the assumptions are valid. The large spread in the values of $\eta_{X'}$ and $\xi_{X'}$ obtained by two camera solutions indicates the x, y axes must be rotated back to the original camera orientation and a large over determination is necessary to insure a spread in yaw and to average out the variation of ρ from zero.

In view of the sensitivities of the 8 x 8 and the linearized 4 x 4 to both unknown x, y rotations and departures from the assumption of $\rho = 0$ an unduly large effort is necessary to pre-process each orientation matrix employed. To surmount this difficulty a solution is proposed not dependent on $\rho = 0$ and independent of x, y rotations.

2. Second Approach

It may be noted that the z columns of the orientation matrix are unaffected by an x, y rotation. The following approach is suggested to provide a solution that is not dependent on $\rho = 0$ and is independent of an x, y rotation. The 8 x 8 and linearized 4 x 4 indicate ρ is not quite as small as thought and π is not quite as large as thought. If this is true, each axis (x, y, z) of each camera with η exposures generates a right cone with a sinusoidal surface and whose axis is the direction of gravity. An equation is written for each exposure of each camera:

x axis

$$\left(\frac{\cos \alpha_G}{\cos x_g}\right) \cos \alpha_{x_1} + \left(\frac{\cos \beta_G}{\cos x_g}\right) \cos \beta_{x_1} + \left(\frac{\cos \gamma_G}{\cos x_g}\right) \cos \gamma_{x_1} = 1$$

$$\left(\frac{\cos \alpha_G}{\cos x_g}\right) \cos \alpha_{x_2} + \left(\frac{\cos \beta_G}{\cos x_g}\right) \cos \beta_{x_2} + \left(\frac{\cos \gamma_G}{\cos x_g}\right) \cos \gamma_{x_2} = 1$$

....

$$\left(\frac{\cos \alpha_G}{\cos x_g}\right) \cos \alpha_{x_n} + \left(\frac{\cos \beta_G}{\cos x_g}\right) \cos \beta_{x_n} + \left(\frac{\cos \gamma_G}{\cos x_g}\right) \cos \gamma_{x_n} = 1$$

y axis

$$\left(\frac{\cos \alpha_G}{\cos y_g}\right) \cos \alpha_{y_1} + \left(\frac{\cos \beta_G}{\cos y_g}\right) \cos \beta_{y_1} + \left(\frac{\cos \gamma_G}{\cos y_g}\right) \cos \gamma_{y_1} = 1$$

$$\left(\frac{\cos \alpha_G}{\cos y_g}\right) \cos \alpha_{y_2} + \left(\frac{\cos \beta_G}{\cos y_g}\right) \cos \beta_{y_2} + \left(\frac{\cos \gamma_G}{\cos y_g}\right) \cos \gamma_{y_2} = 1$$

....

$$\left(\frac{\cos \alpha_G}{\cos y_g}\right)\cos \alpha_{y_n} + \left(\frac{\cos \beta_G}{\cos y_g}\right) \cos \beta_{y_n} + \left(\frac{\cos \gamma_G}{\cos y_g}\right)\cos \gamma_{y_n} = 1$$

z axis

$$\left(\frac{\cos \alpha_G}{\cos z_g}\right) \cos \alpha_{z_1} + \left(\frac{\cos \beta_G}{\cos z_g}\right) \cos \beta_{z_1} + \left(\frac{\cos \gamma_G}{\cos z_g}\right) \cos \gamma_{z_1} = 1$$

$$\left(\frac{\cos \alpha_G}{\cos z_g}\right) \cos \beta_{z_n} + \left(\frac{\cos \beta_G}{\cos z_g}\right) \cos \beta_{z_2} + \left(\frac{\cos \gamma_G}{\cos z_g}\right) \cos \gamma_{z_2} = 1$$

....

$$\left(\frac{\cos \alpha_G}{\cos z_g}\right) \cos \alpha_{z_n} + \left(\frac{\cos \beta_G}{\cos z_g}\right) \cos \beta_{z} + \left(\frac{\cos \gamma_G}{\cos z_g}\right) \cos \gamma_{z_n} = 1$$

The geometry of the above three solutions is illustrated in Figure 5. x_g, y_g and z_g are the cone angles each axis generates with the direction and gravity. α_G, β_G, and γ_G are the direction angles of the direction of gravity in the arbitrary coordinate system.

Only the third array is independent of an x, y rotation. However, if there were no x, y rotation the first and second array would yield the same direction of gravity except for the cone angle generated. The approximate orientation of the system at rest is

$$x_g = 114° \ (65°)$$

$$y_g = 0°$$

$$z_g = 24°$$

This means that the x solution in the absence of an x, y rotation would yield results comparable to the z solution whereas the y solution being very approximately a level line (direction of motion) would yield a poor solution. The y cone degenerates to a sinusoidal plane. Since the departures from cosine 90° are equal to or greater than several degrees, the cosine of 90° + 2° has twice the numerical magnitude as the cosine

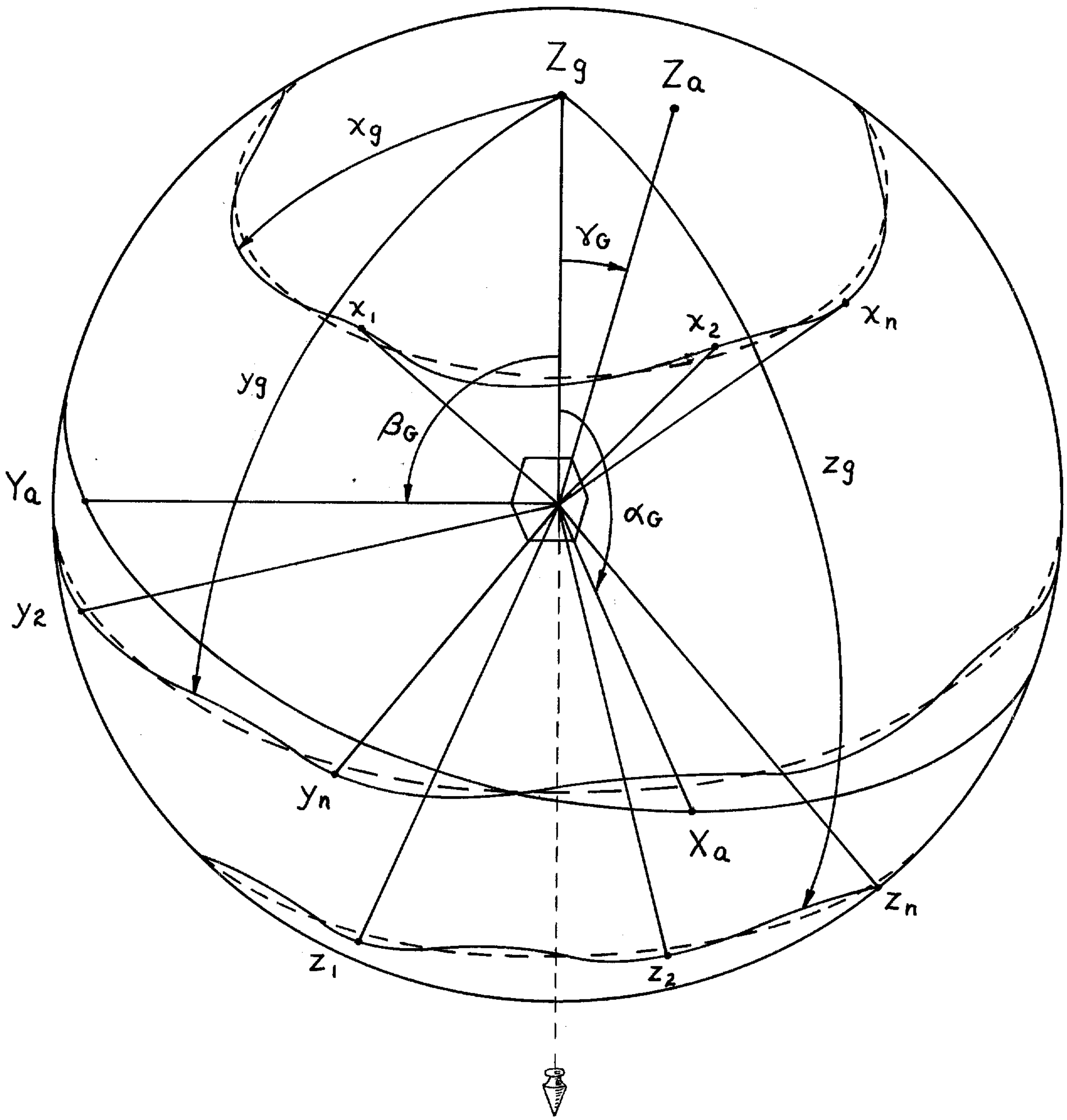

Figure 5 SINUSOIDAL CONES GENERATED BY x,y and z AXES ABOUT Z_G

of 90° + 1° for example. $\cos y_g$ being a denominator can affect the numerator $\cos \alpha_G$, $\cos \beta_G$, $\cos \gamma_G$ by several 100% with an error of several degrees. Assume the values in parentheses have been solved from a normalized 3 x 3

$$\cos x_g = \frac{1}{\left[\left(\frac{\cos \alpha_G}{\cos x_g}\right)^2 + \left(\frac{\cos \beta_G}{\cos x_g}\right)^2 + \left(\frac{\cos \gamma_G}{\cos x_g}\right)^2\right]^{1/2}}$$

$$\cos y_g = \frac{1}{\left[\left(\frac{\cos \alpha_G}{\cos y_g}\right) + \left(\frac{\cos \beta_G}{\cos y_g}\right)^2 + \left(\frac{\cos \gamma_G}{\cos y_g}\right)^2\right]^{1/2}}$$

$$\cos z_g = \frac{1}{\left[\left(\frac{\cos \alpha_G}{\cos z_g}\right) + \left(\frac{\cos \beta_G}{\cos z_g}\right)^2 + \left(\frac{\cos \gamma_G}{\cos z_g}\right)^2\right]^{1/2}}$$

whence

$$\cos \alpha_G = \left(\frac{\cos \alpha_G}{\cos x_g}\right) \cos x_g = \left(\frac{\cos \alpha_G}{\cos y_g}\right) \cos y_g = \left(\frac{\cos \alpha_G}{\cos z_g}\right) \cos z_g$$

$$\cos \beta_G = \left(\frac{\cos \beta_G}{\cos x_g}\right) \cos x_g = \left(\frac{\cos \beta_G}{\cos y_g}\right) \cos y_g = \left(\frac{\cos \beta_G}{\cos z_g}\right) \cos z_g$$

$$\cos \gamma_G = \left(\frac{\cos \gamma_G}{\cos x_g}\right) \cos x_g = \left(\frac{\cos \gamma_G}{\cos y_g}\right) \cos y_g = \left(\frac{\cos \gamma_G}{\cos z_g}\right) \cos z_g$$

The particular values of x_g, y_g, and z_g for each exposure are obtained by substitution of the equated values of $\cos \alpha_G$, $\cos \beta_G$, $\cos \gamma_G$ back into the original condition equations:

$$\cos \alpha_G \cos \alpha_{x_n} + \cos \beta_G \cos \beta_{x_n} + \cos \gamma_G \cos \gamma_{x_n} = \cos x_{n_g}$$

$$\cos \alpha_G \cos \alpha_{y_n} + \cos \beta_G \cos \beta_{y_n} + \cos \gamma_G \cos \gamma_{y_n} = \cos y_{n_g}$$

$$\cos \alpha_G \cos \alpha_{z_n} + \cos \beta_G \cos \beta_{z_n} + \cos \gamma_G \cos \gamma_{z_n} = \cos z_{n_g}$$

whence

(pitch) $\Delta\pi_n = y_{n_g} - y_{g_o}$

(tilt) $\Delta t_n = z_{n_g} - z_{g_o}$

(roll) $\Delta\rho_n = \rho_n - \rho_o$

where

$$\tan \rho_o = \left(\frac{\sin x_{g_o}}{\cos z_{g_o}}\right)$$

$$\tan \rho_n = \left(\frac{\sin x_{n_g}}{\cos z_{n_g}}\right)$$

These quantities are only of academic interest.

To test the above equations, the x, y, and z columns of eight camera orientation matrices were normalized and solved by the method of Least Squares. The results are given below:

	x	y	z
$\cos \alpha_G$	.0250543	-.1388537	.1172539
$\cos \beta_G$	.2918100	.9568414	.2801572
$\cos \gamma_G$	.9561481	-.2552922	.9527663

As anticipated, the direction cosines of x and z are consistent to the first order where as those of y are absurd. By treating $\cos y_g$

as zero the solution for cos α_G, cos β_G, and cos γ_G is as follow:

$$\cos \alpha_G = .0962929$$

$$\cos \beta_G = .4908134$$

$$\cos \gamma_G = .8503334$$

which means treating y as sweeping out a sinusoidal plane in place of a conic removes the absurd aspects of the values even though the results are not as logical as those obtained with z.

Were it not for the problem of x, y rotation a solution embracing a conic could be employed with the rows as well as the columns.

X row

$$\left(\frac{\cos x_g}{\cos \alpha_G}\right)\cos_x + \left(\frac{\cos y_g}{\cos \alpha_G}\right)\cos_y + \left(\frac{\cos z_g}{\cos \alpha_G}\right)\cos \alpha_z = 1$$

or

Y row

$$\left(\frac{\cos x_g}{\cos \beta_G}\right)\cos_x + \left(\frac{\cos y_g}{\cos \beta_G}\right)\cos_y + \left(\frac{\cos z_g}{\cos \beta_G}\right)\cos_z = 1$$

or

Z row

$$\left(\frac{\cos x_g}{\cos \gamma_G}\right)\cos_x + \left(\frac{\cos y_g}{\cos \gamma_G}\right)\cos_y + \left(\frac{\cos z_g}{\cos \gamma_G}\right)\cos_z = 1$$

Solution of normalized 3 x 3 in each of the above forms yielded reasonable results for the third row but for the reason of weak geometry, absurd results in the first and second row.

The evidence points toward the most reliable solution being obtained with the third column. To this end 67 starboard and 71 port third column matrices were employed in a Least Square determination of cos α_G, cos β_G, cos γ_G, and cos z_g. The results are given below:

	Starboard (67)			Port (71)		
	Cosine	Angle		Cosine	Angle	
z_g.	.9072857	24°	52'	.9085228	24°	43'
α_G	.1948335	78°	46'	.1669294	80°	23'
β_G	.2858874	73°	23'	.3628651	68°	43'
γ_G	.9382475	20°	14'	.9167680	23°	33'

Inasmuch as the interlocking angle between the two cameras is some value greater than 48° and inasmuch as the bisector of this angle approximates the vertical, solved for values of z_g starboard 24° 52' and z_g port of 24° 43' establishes confidence in the above direction angles of the vertical. The difference in the starboard and port gravity angles may be attributed entirely to the noise of the sinusoidal assumptions.

3. Precision of the Gravity Direction Angles

In order to demonstrate the precision of the determination a numerical example of 8 starboard 3rd column values normalized to a 3 x 3 is given:

$$\left(\frac{\cos\ \alpha_G}{\cos\ z_g}\right) \cos\ _z + \left(\frac{\cos\ \beta_G}{\cos\ z_g}\right) \cos\ _z + \left(\frac{\cos\ \gamma_G}{\cos\ z_g}\right) \cos\ _z = 1$$

a	b	c	
.2978860	.5787331	.7591650	= 1
-.2073995	.4288371	.8792521	= 1
-.2193539	.4242990	.8785523	= 1
.1588703	.6334470	.7573012	= 1
-.0026143	-.0984186	.9951417	= 1
.1853262	.5969892	.7805498	= 1
-.2711262	.3807350	.8840427	= 1
.4051708	.0442197	.9131710	= 1

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Normal Equations

$$\left(\frac{\cos \alpha_G}{\cos z_g}\right)[aa] + \left(\frac{\cos \beta_G}{\cos z_g}\right)[ab] + \left(\frac{\cos \gamma_G}{\cos z_g}\right)[ac] = [a]$$

$$\left(\frac{\cos \alpha_G}{\cos z_g}\right)[ab] + \left(\frac{\cos \beta_G}{\cos z_g}\right)[bb] + \left(\frac{\cos \gamma_G}{\cos z_g}\right)[bc] = [b]$$

$$\left(\frac{\cos \alpha_G}{\cos z_g}\right)[ac] + \left(\frac{\cos \beta_G}{\cos z_g}\right)[bc] + \left(\frac{\cos \gamma_G}{\cos z_g}\right)[cc] = [c]$$

The brackets denote sums.

.4771323	.1166048	.2437452	=	0.3467600
.1166048	1.6131149	2.4138944	=	2.9888415
.2437452	2.4138944	5.9097528	=	6.8471758

Solving by Crammer's Rule

$$\left(\frac{\cos \alpha_G}{\cos z_g}\right) = .1267239 \qquad \left(\frac{\cos \gamma_G}{\cos z_g}\right) = 1.0297208$$

$$\left(\frac{\cos \beta_G}{\cos z_g}\right) = .3027854$$

$$\cos z_g = \frac{1}{\left[\left(\frac{\cos \alpha_G}{\cos z_g}\right)^2 + \left(\frac{\cos \beta_G}{\cos z_g}\right)^2 + \left(\frac{\cos \gamma_G}{\cos z_g}\right)^2\right]^{1/2}}$$

$$= 0.9252666$$

$$+ \; 22^\circ \; 17'.5$$

$\cos \alpha_G = .1172539$		$\alpha_G = 83^\circ \; 17'$
$\cos \beta_G = .2801572$		$\beta_G = 73^\circ \; 44'$
$\cos \gamma_G = .9527663$		$\gamma_G = 17^\circ \; 41'$

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To determine the residuals the values of $\cos \alpha_G$, $\cos \beta_G$, and $\cos \gamma_G$ are substituted in the original equations:

$$\cos z_g - (\cos \alpha_{z_n} \cos \alpha_G + \cos \beta_{z_n} \cos \beta_G + \cos \gamma_{z_n} \cos \gamma_G)$$

$$= \cos z_g - \cos z_{g_n}$$

where z_g is the Least Square value and z_{g_n} is the value obtained from any condition equation n

now $\cos z_{g_n} = \cos z_g \cos \Delta_{z_{g_n}} + \sin z_g \sin \Delta_{z_{g_n}}$

To the first order $\cos \Delta_{z_{g_n}} = 1$ therefore $\cos z_g - \cos z_{g_n} = - \sin z_g . \Delta_{z_{g_n}}$

whence $\dfrac{\cos z_g - \cos z_{g_n}}{\sin z_g - \sin 1'} = \Delta_{z'_{g_n}}$ (in minutes of arc)

With this formulation the following residuals were obtained:

$\Delta_{z_{g_1}} = 44'.0$

$\Delta_{z_{g_2}} = -75.0$

$\Delta_{z_{g_3}} = -44.8$

$\Delta_{z_{g_4}} = 69.3$

$\Delta_{z_{g_5}} = 45.4$

$\Delta_{z_{g_6}} = -67.0$

$\Delta_{z_{g_7}} = 73.5$

$\Delta_{z_{g_8}} = -42.3$

The values are tilt oscillations on the surface of the cone generated by the starboard optical axis and are equal in magnitude to what might be expected. Since the $\Sigma\Delta_{z_g} = 3'.5$ the value of z_g obtained is the most probable value exhibited by the 8 starboard exposures.

Thus

$$\Sigma\Delta_{z_g} = 3.5$$

$$\sigma = \left(\frac{\Sigma\Delta_{z_g}}{n}\right) = 57'.7 \text{ (ignoring signs)}$$

and the mean error of e_{z_g}

$$e_{z_g} = \sqrt{\frac{\Sigma\Delta_{z^2_g}}{n-u}} = 75'.0$$

where n = the number of equations and u = the number of direction angles.

In order to determine the mean error of α_G, β_G, and γ_G it is necessary to compute the relative weights of the gravity direction angles from a modified form of the normalized 3 x 3.

$$\cos \alpha_G\,[aa] + \cos \beta_G\,[ab] + \cos \gamma_G\,[ac] = 1,\ 0,\ 0$$

$$\cos \alpha_G\,[ab] + \cos \beta_G\,[bb] + \cos \gamma_G\,[bc] = 0,\ 1,\ 0$$

$$\cos \alpha_G\,[ac] + \cos \beta_G\,[bc] + \cos \gamma_G\,[cc] = 0,\ 0,\ 1$$

Using the first right hand column for $P\alpha_G$, the second for $P\beta_g$ and the third for $P\gamma_g$ the following equations and numerical values are obtained

$$P\alpha_G = \frac{\Delta}{[bb]\,[cc] - [bc]^2} = \frac{1.7293830}{3.7062241} = .4666159$$

$$P\ \beta_G = \frac{\Delta}{[cc]\cdot[aa] - [ac]^2} = \frac{1.7293830}{2.7603222} = .6265149$$

$$P\ \gamma_G = \frac{\Delta}{[aa]\cdot[bb] - [ab]^2} = \frac{1.7293830}{.7560725} = 2.2873243$$

where

$$\Delta = [aa]\left\{[bb]\cdot[cc] - [bc]^2\right\} - [ab]\left\{[ab]\cdot[cc] - [bc]\cdot[ac]\right\} + [ac]\left\{[ab]\cdot[bc] - [bb]\cdot[ac]\right\}$$

An alternate method of computing weights was employed as a check. To determine the weight of variable α_g, β_G and γ_G are expressed in terms of α_g in the β_G and γ_G normal equations. These values are substituted in the normal equation α_G. The resulting coefficient of α_G is the weight of α_G. To determine the weight of variable β_G, α_G and γ_G are expressed in terms of β_G in the α_G and γ_G normal equations. These values substituted in the normal equation in β_G form the weight of β_G in the resulting coefficient. Similarly the coefficient of γ_G gives the weight of γ_G when α_g and β_g expressed in terms of γ_G are substituted in the normal equation in γ_G.

$P\ \alpha_g$

$$[bb]\ \cos\ _G + [bc]\ \cos\ _G = -[ab]\ \cos\ \alpha_G$$

$$[bc]\ \cos\ _G + [cc]\ \cos\ _G = -[ac]\ \cos\ \alpha_G$$

$$\cos\ \beta_G = \left\{\frac{[ac]\cdot[bc] - [ab]\cdot[ac]}{[bb]\cdot[cc] - [bc]^2}\right\} \cos\ \alpha_G$$

$$\cos\ \gamma_G = \left\{\frac{[bc]\cdot[ab] - [bb]\cdot[ac]}{[bb]\cdot[cc] - [bc]^2}\right\} \cos\ \alpha_G$$

Substituting in the α_G normal equation

$$P\ \alpha_G = [ab] + [ab]\left\{\frac{[ac]\cdot[bc] - [ab]\cdot[cc]}{[bb]\cdot[cc] - [bc]^2}\right\} + [ac]\left\{\frac{[bc]\cdot[ab] - [bb]\cdot[ac]}{[bb]\cdot[cc] - [bc]^2}\right\}$$

$$P\ \beta_G$$

$$[aa]\ \cos \alpha_G + [ac]\ \cos\ _G = -[ab]\ \cos \beta_G$$

$$[ac]\ \cos \alpha_G + [cc]\ \cos\ _G = -[bc]\ \cos \beta_G$$

$$\cos \alpha_G = \left\{ \frac{[bc]\cdot[ac] - [ab]\cdot[cc]}{[aa]\cdot[cc] - [ac]^2} \right\} \cos \beta_G$$

$$\cos \gamma_G = \left\{ \frac{[ac]\cdot[ab] - [bc]\cdot[aa]}{[aa]\cdot[cc] - [ac]^2} \right\} \cos \beta_G$$

$$P\ \beta_G = [ab] \left\{ \frac{[bc]\cdot[ac] - [ab]\cdot[cc]}{[aa]\cdot[cc] - [ac]^2} \right\} + [bb] + [bc] \left\{ \frac{[ac]\cdot[ab] - [bc]\cdot[aa]}{[aa]\cdot[cc] - [ac]^2} \right\}$$

$$P\ \gamma_G$$

$$[aa]\ \cos \alpha_G + [ab]\ \cos \beta_G = -[ac]\ \cos \gamma_G$$

$$[ab]\ \cos \alpha_G + [bb]\ \cos \beta_G = -[bc]\ \cos \gamma_G$$

$$\cos \alpha_G = \left\{ \frac{[bc].[ab] - [ac].[bb]}{[aa].[bb] - [ab]^2} \right\} \cos \gamma_G$$

$$\cos \beta_G = \left\{ \frac{[ab].[ac] - [aa].[bc]}{[aa].[bb] - [ab]^2} \right\} \cos \gamma_G$$

$$P\ \gamma_G = [ac] \left\{ \frac{[bc].[ab] - [ac].[bb]}{[aa].[bb] - [ab]^2} \right\} + [bc] \left\{ \frac{[ab].[ac] - [aa].[bc]}{[aa].[bb] - [ab]^2} \right\} + [cc]$$

Exactly the same numerical values were obtained $P\ \alpha_G = .4666159$

$P\ \beta_G = .6265150$

$P\ \gamma_G = 2.2873241$

The mean error of each direction angle is computed with

$$e\,\alpha_G = \frac{e_{z_g}}{\sqrt{P\ \alpha_G}} = \frac{75.0}{.683} = 109'.8 = 1°\ 49.8$$

$$e\,\beta_G = \frac{e_{z_g}}{\sqrt{P\ \beta_G}} = \frac{75.0}{.792} = 94'.8 = 1°\ 34.8$$

$$e\,\gamma_G = \frac{e_{z_g}}{\sqrt{P\ \gamma_G}} = \frac{75.0}{1.512} = 49'.6$$

While these errors by ordinary standards are large those based 67 on starboard exposure and 71 port exposures are much smaller. Even these error values are adequate for the purpose of referring the object space coordinate system to the direction of gravity.

4. Further Adjustment

Inasmuch as cos α_G, cos β_G, and cos γ_G are dependent functions it would seem that improved values would be obtained if the condition equation were linearized in terms of two independent Eulerian angles. Let

$$\cos \alpha_G = \cos \theta_G \sin \gamma_G$$

$$\cos \beta_G = \sin \theta_G \sin \gamma_G$$

$$\cos \gamma_G = \cos \gamma_G$$

whence

$$(\cos \theta_G \sin \gamma_G) \cos \alpha_z + (\sin \theta_G \sin \gamma_G) \cos \beta_z + \cos \gamma_G \cos \gamma_z = \cos z_g$$

If

$$\cos z_g - (\cos \alpha_G \cos \alpha_z + \cos \beta_G \cos \beta_z + \cos \gamma_G \cos \gamma_z) = -\sin z_g \, \Delta_{z_g}$$

or

$$A \cdot \Delta\theta_G + B \cdot \Delta\gamma_G = \Delta_{z_g}$$

where

$$A = (-\cos \beta_G \cos \alpha_z + \cos \alpha_G \cos \beta_z) / \sin z_g$$

$$B = [(\cos \alpha_G \cos \alpha_z + \cos \beta_G \cos \beta_z) \cot \gamma_G - (\cos \gamma_G \cos \gamma_z) \tan \gamma_G)] / \sin z_g$$

n condition equations of the above form may be normalized to a 2 x 2 in $\Delta\theta_G$ and $\Delta\gamma_G$. A test solution demonstrated that the best values had already been obtained inasmuch as on the first iteration $\Delta\theta_G$ and $\Delta\gamma_G$ were significantly zero. Had the condition equation been linearized, it would have the following form:

$$\sin \alpha_G \cos \alpha_z \, \Delta\alpha_G + \sin \beta_G \cos \beta_z \, \Delta\beta_G + \sin \gamma_G \cos \gamma_z \, \Delta\gamma_G = \sin Z_g \, \Delta_{z_g}$$

Assume there is a particular value of α_G, β_G, and γ_G that satisfies the equation:

$$\cos \alpha_{G_n} \cos \alpha_{z_n} + \cos \beta_{G_n} \cos \beta_{z_n} + \cos \gamma_{G_n} \cos \gamma_{z_n} - \cos Z_g = 0$$

Since $\cos z_g - \cos z_{g_n} = -\sin z_g \, \Delta_{z_{g_n}}$

$$\cos \alpha_{G_n} = \cos \alpha_G + \Delta \cos \alpha_{G_n}$$

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$$\cos \beta_{G_n} = \cos \beta_G + \Delta \cos \beta_{G_n}$$

$$\cos \gamma_{G_n} = \cos \gamma_G + \Delta \cos \gamma_{G_n}$$

and

$$\Delta \cos \alpha_{G_n} = \cos \alpha_G - \cos \alpha_{G_n} = -\sin \alpha_G \; \Delta\alpha_G$$

$$\Delta \cos \beta_{G_n} = \cos \beta_G - \cos \beta_{G_n} = -\sin \beta_G \; \Delta\beta_G$$

$$\Delta \cos \gamma_{G_n} = \cos \gamma_G - \cos \gamma_{G_n} = -\sin \gamma_G \; \Delta\gamma_G$$

Substituting we obtain to the first order the form obtained by linearization:

$$\Delta \cos \alpha_{G_n} \cos \alpha_z + \Delta \cos \beta_{G_n} \cos \beta_z + \Delta \cos \gamma_{G_n} \cos \gamma_z$$

$$= \cos z_g - (\cos \alpha_G \cos \alpha_z + \cos \beta_G \cos \beta_z + \cos \gamma_G \cos \gamma_z) = -\sin z_g \; \Delta_{z_g}$$

$$\text{or } + \frac{\sin \alpha_G}{\sin z_g} \cos \alpha_z \; (\Delta\alpha_G) + \frac{\sin \beta_G}{\sin z_g} \cos \beta_z \; (\Delta\beta_G) + \frac{\sin \gamma_G}{\sin z_g} \cos \gamma_z \; (\Delta\gamma_G) = + \Delta_{z_g}$$

New normal equations in $\Delta\alpha_G$, $\Delta\beta_G$, and $\Delta\gamma_G$ are easily formed since all coefficient changes are constant. It is necessary to accept the values of α_G, β_G, and γ_G as the best possible if the correction determined from the revised normal equations are significantly zero.

$$\frac{\sin^2\alpha_G}{\sin^2 z_g} [aa] \; \Delta\alpha_G + \frac{\sin \alpha_G \sin \beta_G}{\sin^2 z_g} [ab] \; \Delta\beta_G + \frac{\sin \alpha_G \sin \gamma_G}{\sin^2 z_g} [ac] \; \Delta\gamma_G = [a.\Delta_{z_g}] \; \frac{\sin \alpha_G}{\sin z_g}$$

$$\frac{\sin\alpha_G \sin\beta_G}{\sin^2 z_g}[ab]\,\Delta\alpha_G + \frac{\sin^2\beta_G}{\sin^2 z_g}[bb]\,\Delta\beta_G + \frac{\sin\beta_G \sin\gamma_G}{\sin^2 z_g}[bc]\,\Delta\gamma_G = [b.\Delta z_g]\,\frac{\sin\beta_G}{\sin z_g}$$

$$\frac{\sin\alpha_G \sin\gamma_G}{\sin^2 z_g}[ac]\,\Delta\alpha_G + \frac{\sin\beta_G \sin\gamma_G}{\sin^2 z_g}[bc]\,\Delta\beta_G + \frac{\sin^2\gamma_G}{\sin^2 z_g}[cc]\,\Delta\gamma_G = [c.\Delta z_g]\,\frac{\sin\gamma_G}{\sin z_g}$$

3.2707100	.7726223	.5110397	=	.0414955
.7726223	10.3315155	4.8919810	=	.1753744
.5110397	4.8919810	3.7896887	=	.0940528

Solution of the above normal equations gives in minutes of arc

$\Delta\alpha_G = 0'.009$

$\Delta\beta_G = 0'.013$

$\Delta\gamma_G = 0'.007$

These quantities show no improvement is justified.

5. Application of the Gravity Direction Cosines to a Gravity Referenced Datum

There are a number of possibilities. The simplest notion is to allow the intersection of the $Z_G Z Z_a$ plane with the $X_G\, Y_G$ plane to be the Y_G axis and the intersection of the $X_a\, Y_a$ plane with the $X_G\, Y_G$ plane be the X_G axis. This concept is illustrated in Figure 6.

Given

$\cos\alpha_G$, $\cos\beta_G$, $\cos\gamma_G$

$$\tan\theta_G = \frac{\cos\alpha_G}{\cos\beta_G}$$

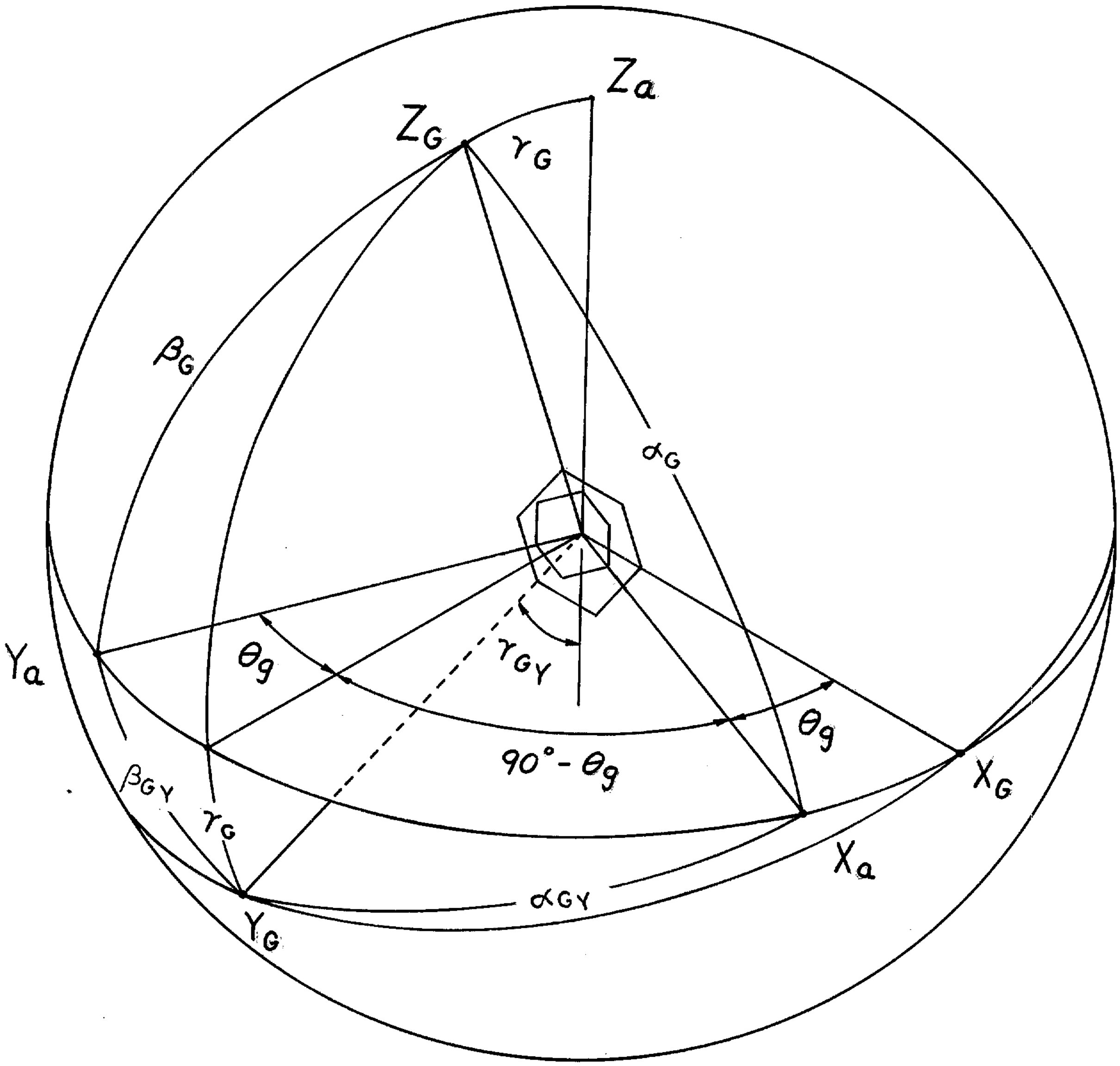

Figure 6 SIMPLEST CONCEPT OF GRAVITY REFERENCE PLANE

whence

$$\cos \alpha_{G_X} = \cos \theta_G$$

$$\cos \beta_{G_X} = -\sin \theta_G$$

$$\cos \gamma_{G_X} = 0$$

$$\cos \alpha_{G_Y} = \sin \theta_G \cos \gamma_G$$

$$\cos \beta_{G_Y} = \cos \theta_G \cos \gamma_G$$

$$\cos \gamma_{G_Y} = \sin \gamma_G$$

Such a datum orientation is not likely to be entirely satisfactory inasmuch as the arbitrary datum selected is probably inherent to the object such as the X axis defining the axis of the object. In such a case, it may be desirable to let X_a or Y_a be the preferred axis projected to the X_G Y_G plane from Z_G. This selection is illustrated in Figure 7. Since $\underline{/Z_G}\, X_G = 90°$

$$\cos \alpha_{G_X} = \sin \alpha_G$$

from the low of cosines

$$0 = \cos \alpha_G \cos \beta_G + \sin \alpha_G \sin \beta_G \cos s_{XY}$$

$$0 = \cos \beta_G \cos \gamma_G + \sin \beta_G \sin \gamma_G \cos s_{YZ}$$

$$0 = \cos \gamma_G \cos \alpha_G + \sin \gamma_G \sin \alpha_G \cos s_{ZX}$$

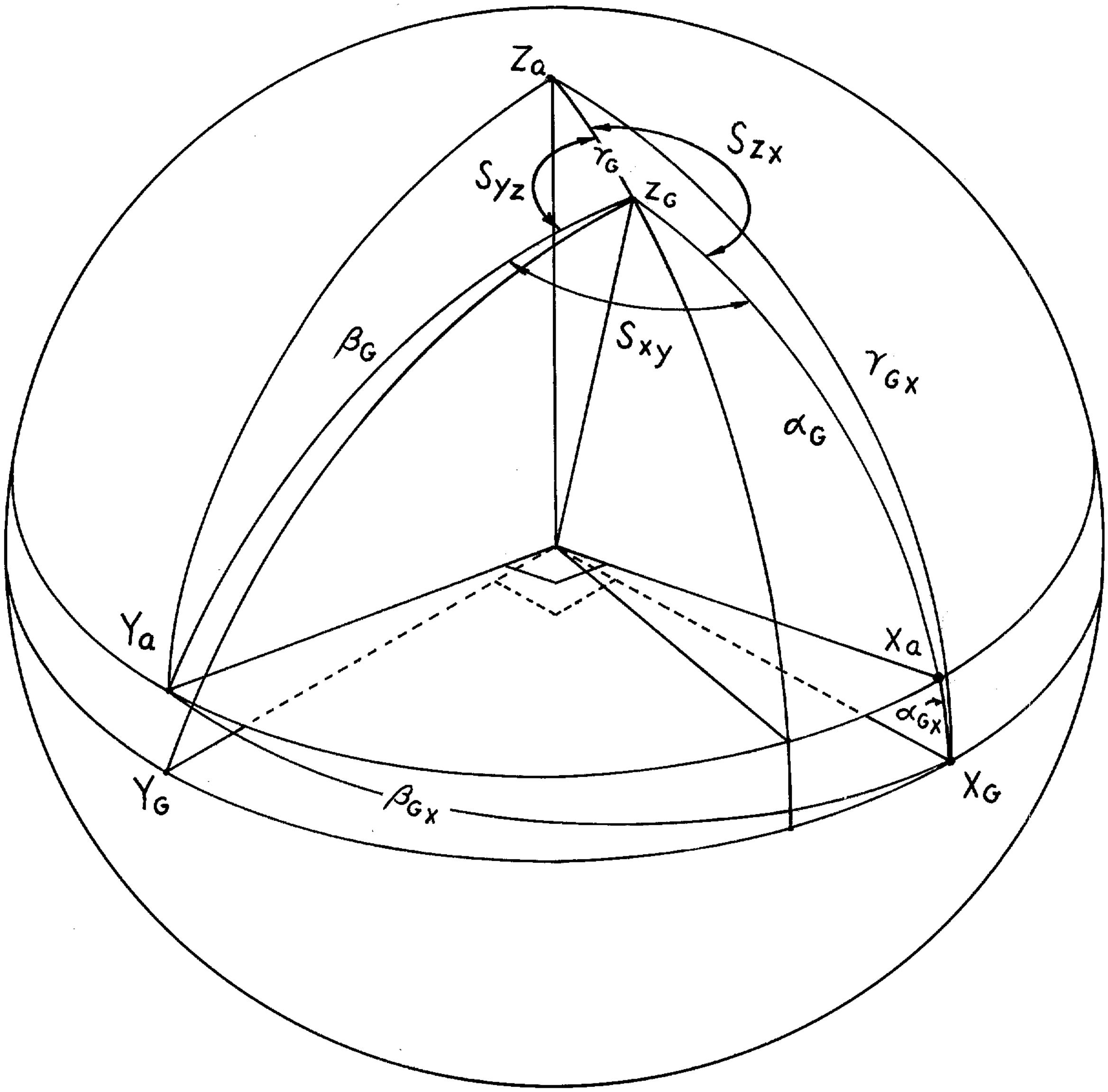

Figure 7 PROJECTION OF ARBITRARY AXIS X_a TO X_G IN PLANE $Z_G X_a$

whence

$$\cos s_{XY} = -\frac{\cos \alpha_G \cos \beta_G}{\sin \alpha_G \sin \beta_G}$$

$$\cos s_{YZ} = -\frac{\cos \beta_G \cos \gamma_G}{\sin \quad _G \sin \quad _G}$$

$$\cos s_{ZX} = \frac{\cos \gamma_G \cos \alpha_G}{\sin \quad _G \sin \quad _G}$$

Therefore

$$\cos \beta_{G_X} = -\sin \beta_G \frac{\cos \alpha_G \cos \beta_G}{\sin \alpha_G \sin \beta_G} = -\frac{\cos \alpha_G \cos \beta_G}{\sin \alpha_G}$$

$$\cos \gamma_{G_X} = -\sin \gamma_G \frac{\cos \gamma_G \cos \alpha_G}{\sin \gamma_G \sin \alpha_G} = -\frac{\cos \alpha_G \cos \gamma_G}{\sin \alpha_G}$$

The direction of Y_G is deduced from

$$\cos \alpha_{G_Y} \cos \alpha_G + \cos \beta_{G_Y} \cos \beta_G + \cos \gamma_{G_Y} \cos \gamma_G = 0$$

$$\cos \alpha_{G_Y} \cos \alpha_{G_X} + \cos \beta_{G_Y} \cos \beta_{G_X} + \cos \gamma_{G_Y} \cos \gamma_{G_X} = 0$$

$$\cos \alpha_{G_Y} = \cos \beta_G \left(-\cos \gamma_G \frac{\cos \alpha_G}{\sin \alpha_G}\right) - \left(-\cos \beta_G \frac{\cos \alpha_G}{\sin \alpha_G}\right) \cos \gamma_G = 0$$

$$\cos \beta_{G_Y} = \cos \gamma_G (\sin \alpha_G) - \cos \alpha_G \left(-\cos \gamma_G \frac{\cos \alpha_G}{\sin \alpha_G}\right)$$

$$= \cos \gamma_G \frac{(\sin^2\alpha_G + \cos^2\alpha_G)}{\sin \alpha_G}$$

$$= \frac{\cos \gamma_G}{\sin \alpha_G}$$

$$\cos \gamma_{G_Y} = \cos \alpha_G \left(-\cos \beta_G \frac{\cos \alpha_G}{\sin \alpha_G}\right) - \cos \beta_G \sin \alpha_G$$

$$= - \frac{\cos \beta_G}{\sin \alpha_G}$$

Summarizing for the projection of either X_a or Y_a

X_a Projected

$$\cos \alpha_{G_X} = \sin \alpha_G$$

$$\cos \beta_{G_X} = -\cos \beta_G \frac{\cos \alpha_G}{\sin \alpha_G}$$

$$\cos \gamma_{G_X} = - \cos \gamma_G \frac{\cos \alpha_G}{\sin \alpha_G}$$

$$\cos \alpha_{G_Y} = 0$$

$$\cos \beta_{G_Y} = \frac{\cos \gamma_G}{\sin \alpha_G}$$

$$\cos \gamma_{G_Y} = \frac{\cos \beta_G}{\sin \alpha_G}$$

Y_a Projected

$$\cos \alpha_{G_X} = - \frac{\cos \gamma_G}{\sin \quad_G}$$

$$\cos \beta_{G_X} = 0$$

$$\cos \gamma_{G_X} = \frac{\cos \alpha_G}{\sin \beta_G}$$

$$\cos \alpha_{G_Y} = -\cos \alpha_G \frac{\cos \beta_G}{\sin \quad_G}$$

$$\cos \beta_{G_Y} = + \sin \beta_G$$

$$\cos \gamma_{G_Y} = - \cos \gamma_G \frac{\cos \beta_G}{\sin \beta_G}$$

In the most general sense, any preferred line $\bar{o}$ may be projected to the X_G Y_G plane. Assume it is desired to project any selected line to the X_G Y_G plane and let the projected line be the X_G or Y_G axis. The direction cosine of Z_G and o are known whence

$$\cos \alpha_{G_X} = \frac{\cos \beta_G \cos \gamma_o - \cos \beta_o \cos \gamma_G}{\sin G_o}$$

$$\cos \beta_{G_X} = \frac{\cos \gamma_G \cos \alpha_o - \cos \gamma_o \cos \alpha_G}{\sin G_o}$$

$$\cos \gamma_{G_X} = \frac{\cos \alpha_G \cos \beta_o - \cos \alpha_o \cos \beta_G}{\sin G_o}$$

where

$$\cos G_o = \cos \alpha_G \cos \alpha_o + \cos \beta_G \cos \beta_o + \cos \gamma \gamma_G \cos \gamma_o$$

Then

$$\cos \alpha_{G_Y} = \cos\beta_G \cos \gamma_{G_X} - \cos \beta_{G_X} \cos \gamma_G$$

$$\cos \beta_{G_Y} = \cos \gamma_G \cos \alpha_{G_X} - \cos \gamma_{G_X} \cos \alpha_G$$

$$\cos \gamma_{G_Y} = \cos \alpha_G \cos \beta_{G_X} - \cos \alpha_{G_X} \cos \beta_G$$

The projection of any line is illustrated in Figure 8. If it is desired to let Y_G be the perpendicular to the Z_o plane, only the subscripts are changed.

In any case, whatever the decision the matrices and coordinates are rotated to a gravity reference:

given

$\cos \alpha_{G_X}$ $\cos \alpha_{G_Y}$ $\cos \alpha_G$

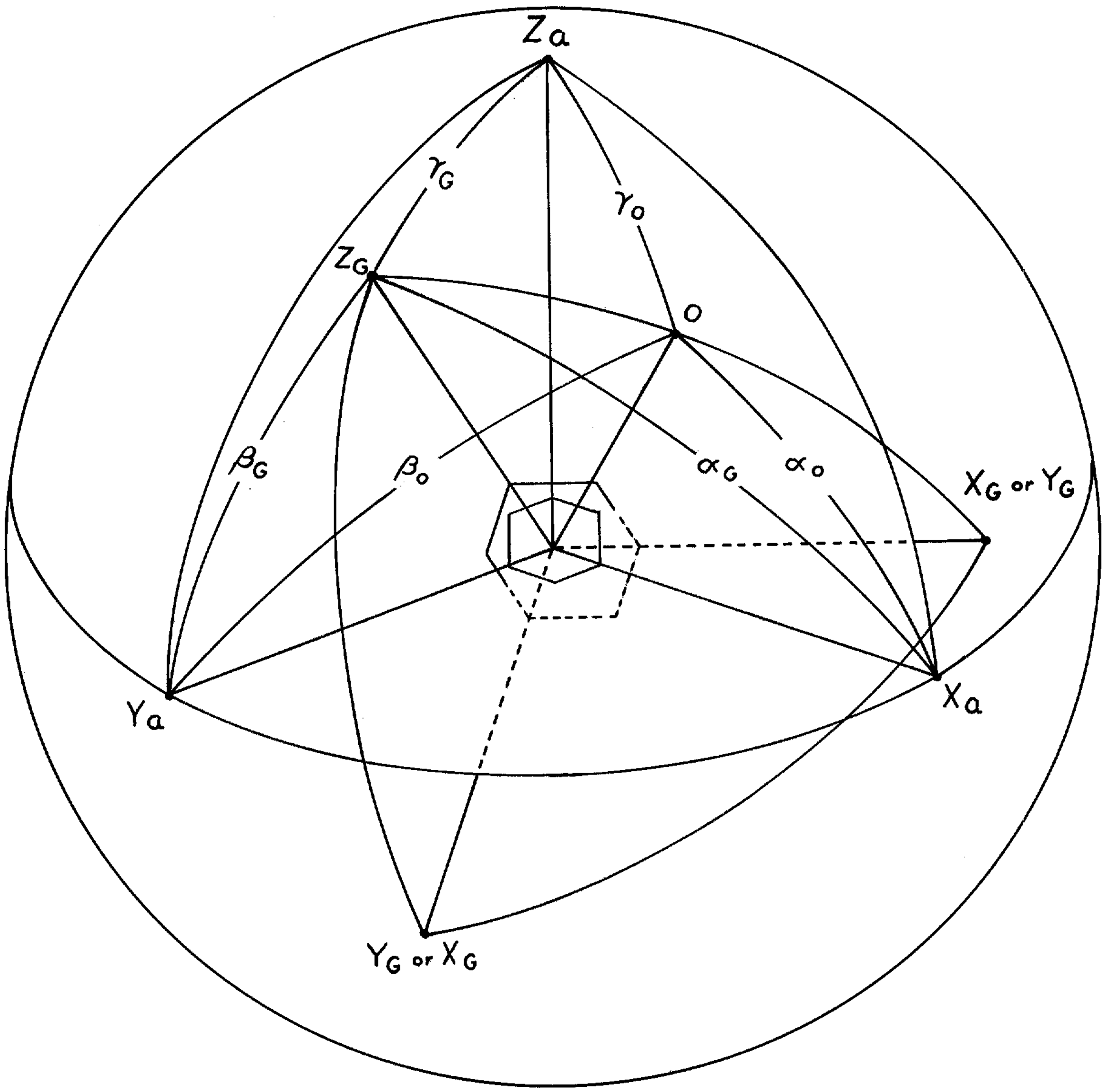

Figure 8 PROJECTION OF ANY LINE 0 TO THE X_G Y_G PLANE

$$\begin{array}{lll} \cos \beta_{G_X} & \cos \beta_{G_Y} & \cos \beta_G \\ \cos \gamma_{G_X} & \cos \gamma_{G_Y} & \cos \gamma_G \end{array}$$

and an orientation matrix

$$\begin{array}{lll} \cos \alpha_x & \cos \alpha_y & \cos \alpha_z \\ \cos \beta_x & \cos \beta_y & \cos \beta_z \\ \cos \gamma_x & \cos \gamma_y & \cos \gamma_z \end{array}$$

The gravity reference matrix is reduced as follows:

$$\cos \alpha_{x_g} = \cos \alpha_x \cos \alpha_{G_X} + \cos \beta_x \cos \beta_{G_X} + \cos \gamma_x \cos \gamma_{G_X}$$

$$\cos \alpha_{y_g} = \cos \alpha_y \cos \alpha_{G_X} + \cos \beta_y \cos \beta_{G_X} + \cos \gamma_y \cos \gamma_{G_X}$$

$$\cos \alpha = \cos \alpha_z \cos \alpha_{G_X} + \cos \beta_z \cos \beta_{G_X} + \cos \gamma_z \cos \gamma_{G_X}$$

$$\cos \beta_{x_g} = \cos \alpha_x \cos \alpha_{G_Y} + \cos \beta_x \cos \beta_{G_Y} + \cos \gamma_x \cos \gamma_{G_Y}$$

$$\cos \beta_{y_g} = \cos \alpha_y \cos \alpha_{G_Y} + \cos \beta_y \cos \beta_{G_Y} + \cos \gamma_y \cos \gamma_{G_Y}$$

$$\cos \beta_{z_g} = \cos \alpha_z \cos \alpha_{G_Y} + \cos \beta_z \cos \beta_{G_Y} + \cos \gamma_z \cos \gamma_{G_Y}$$

$$\cos \gamma_{x_g} = \cos \alpha_x \cos \alpha_G + \cos \beta_x \cos \beta_G + \cos \gamma_x \cos \gamma_G$$

$$\cos \gamma_{y_g} = \cos \alpha_y \cos \alpha_G + \cos \beta_y \cos \beta_G + \cos \gamma_y \cos \gamma_G$$

$$\cos \gamma_{z_g} = \cos \alpha_z \cos \alpha_G + \cos \beta_z \cos \beta_G + \cos \gamma_z \cos \gamma_G$$

More important perhaps is the rotation of the space coordinates to the gravity referenced datum

$$X_a \cos \alpha_{G_X} + Y_a \cos \beta_{G_X} + Z_a \cos \gamma_{G_X} = X_G$$

$$X_a \cos \alpha_{G_Y} + Y_a \cos \beta_{G_Y} + Z_a \cos \gamma_{G_Y} = Y_G$$

$$X_a \cos \alpha_G + Y_a \cos \beta_G + Z_a \cos \gamma_G = Z_G$$