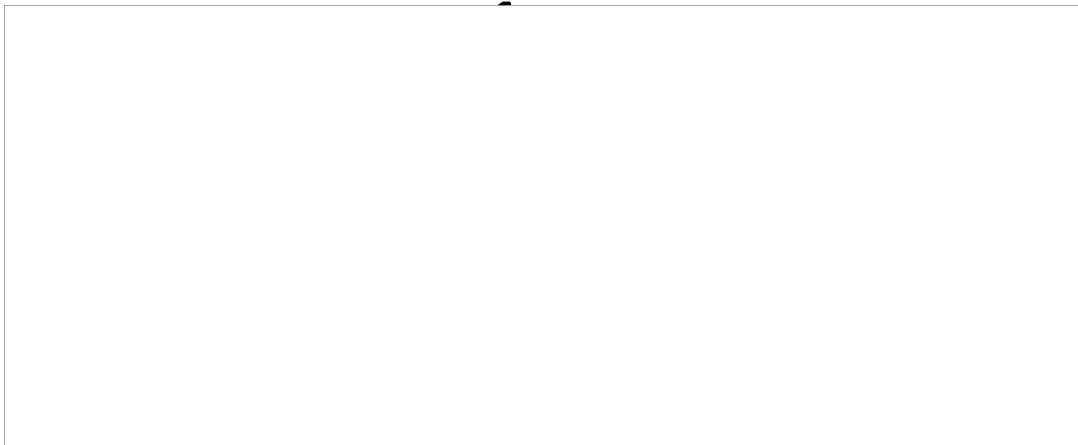50X1-HUM

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AEROPHOTOTOPOGRAPHY
(Aerofototopografiya)

by

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Chapters V - VIII

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CHAPTER V

DEVELOPING THE MAP CONTOURS

36. Use of Aerial Photographs for Plan Plotting

In aerophotosurveying, all features of the ground relief are recorded on the photograph. In order to make a map from this photograph, it is first necessary to interpret the print and identify the contours and then to transpose these contours onto a map. This chapter will deal with the various methods of transposing the relief features from a photographic print to a map.

If the photograph was taken at a strictly horizontal position and had no tilt, the image, in case of flat terrain, will resemble the actual map plan but at a certain scale and at arbitrary orientation. The aerial photograph is of much greater value than prints made by geodetic surveys since it captures much more relief detail which, in turn, makes it easier to correctly orient the contours and furnishes valuable data for many branches of the national economy (land management, hydrotechnology, planning of communication lines, geology, etc.). The relative position of contours is also much more accurately recorded on an aerial photograph.

If the aerial photograph was taken at a tilt, its image cannot be used directly to make a map. The distortion of the photograph (i.e., deviation from its plan), as shown by eq. (10), increases from the center toward the edges. In cases of small angles of tilt, the central section of the print may be considered as undistorted. For example, if $\alpha < 0.5°$, $f_k = 70$ mm, and $r = 65$ mm, the maximum distortion at the edges of the central section will not exceed $\delta\alpha = 0.5$ mm.

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However, if the angle of tilt during exposure is ±3° or even 5°, the resultant distortion will not interfere too much with using these photographs for map making. The distortion can be easily corrected by the transformation process which is a means of projecting the print onto a tilted surface, compensating for the original tilt from the horizontal. During the transformation process the image is also adjusted to the desired scale, which is either enlarged or diminished compared to the original.

A more serious drawback is the displacement of points on a print, caused by the nature of the relief. If the magnitude of these displacements does not exceed the specified accuracy limits of the map, which is common in areas of flat contours, then these prints are used directly for map making. If, however, relief distortion exceeds specified limits, then a special process of transformation is employed (transformation by sections) or the map is prepared by stereophotogrammetric mapping methods.

This means that aerial photographs, irrespective of their position in space (angle of tilt) can be used to determine the contours of a map without field surveying of the relief. Field surveys are used only to obtain supplementary information to establish details not captured by the aerial photographs.

However, the photograph gives the contours of the terrain but does not orient them with respect to the coordinate system of the area (map), and appears at a slightly different scale. It is therefore mandatory, even in the case of strictly vertical photographs, to have two control points (whose locations on the map are known). These are required for establishing the proper scale ratio between map and photograph and to orient the photograph with the coordinates of the map. However, photographs with tilt are used in the processing, which must be transformed. To transform the photograph, at least four control points of the photo must be available on the map. Some of these control points are determined by geodetic means and part by photogrammetric measurement of the prints.

Aerial photographs are not used exclusively for photogrammetric purposes. They

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are often of great help to the topographer in plane-table field surveying. There
are cases, especially under wartime conditions, when only a few sectional photo-
graphs or a partial strip of prints are available for the terrain to be mapped, which
do not cover the entire area. In such cases, for the sake of speed, the topographer
must make full use of the available prints by interpreting them and transposing these
data to a map. To do this, the topographer must be able to measure distortion and
to compensate for it.

Another valuable use for aerial photographs is in the revision and restoration
of obsolete maps. Most of the corrective work in this case can be performed in the
drafting room or laboratory by using the unchanged or basic relief of the map as
control points. Field surveying is necessary in this case only to establish the
detail which did not appear on the aerial photographs. In revising and restoring
obsolete maps measurement of the relief distortion of the aerial photographs is re-
quired.

37. **Photomap and Mosaic**

The easiest and fastest way of making a map from aerial photographs is by the
preparation of a photomap. This fact has led to extensive use of photomaps in map-
ping flat terrain.

The photomap is an area plan, assembled from aerial photographs which had been
corrected for scale and for angle of tilt (transformed).

Preparation of a photomap comprises the following steps:

1) Photogrammetric densification of the control network;

2) Transformation of aerial photographs;

3) Assembly of aerial photographs by control points.

To make a photomap, a grid of geodetic control points is needed, while photogram-
metric work requires the processing of the photographs at a station using complicated
instruments, such as transforming printers. In cases in which high map accuracy is

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not required, e.g. for general study of an area, preliminary exploration, small-area topographic work (relief survey, field interpretation) an uncontrolled mosaic is often used.

An uncontrolled mosaic represents an assembly (composite) of aerial photographs, joined by their common contours, without the use of special control points. Contact prints are commonly used in such mosaics. These prints are not corrected for tilt distortion and are not brought to a common scale.

38. Making an Uncontrolled Mosaic

The simplest method of assembling a mosaic from adjacent photomaps is through contour points located in the overlap section of the prints (Fig. 53). However, due to scale variations and tilt distortion, it is impossible to make all contour lines coincide, so that it usually is attempted to match points having the least distortion, e.g., point a and b of prints 1 and 2 (Fig. 54), or points c and d of prints 2 and 3, or points k and b of prints 1 and 23 in the adjacent flight strips located along the line ab, cd, kb of the overlapping prints. After placing the photographs in their overlap position, the accuracy of overlap is checked by making pin pricks through the top print onto the bottom print for selected contour points. For example, if the points c and d do not coincide, because of scale differential of the prints, then analogous points c_2, c_3 and d_2, d_3 are determined on line cd in such a way that points c_3 and d_3 would be located at equal distances from points c_2 and d_2, but in opposite directions.

The contour points a and b, c and d are selected relative to the terrain, usually having the same elevation, so that their relief distortion variance would be at a minimum.

After matching the prints in overlap, they are held flat with weights and cut with a scalpel along line ab. The line of cut is usually not straight, in order to intersect a given contour line as few times as possible and at less sharp angles.

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The cutting line is made in zones of best coincidence of contour lines and in zones of similar hatchures depicting the terrain.

Next the prints are cemented to a sheet of cardboard or heavy paper, leaving the edges unglued since they may be cut off in mounting the next photograph.

After cementing from 2 to 4 prints of one flight strip to the board, the adjacent prints of another flight strip are mounted, and the work is continued simultaneously on two to three flight strips. For example, print 23 (Fig. 54) is mounted by
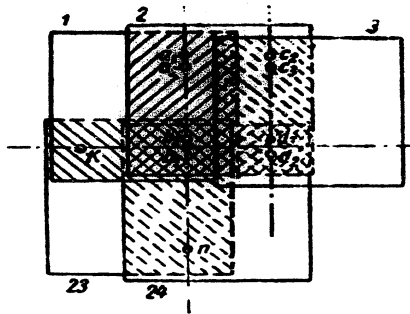
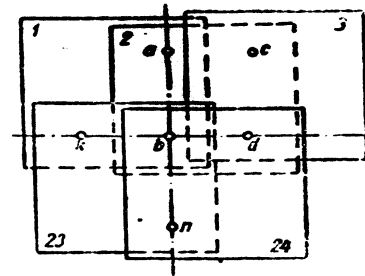Fig. 53 - Overlapping the Prints   Fig. 54 - Selecting the Direction
                                                        of Cut

the above method, while print 24 and the following prints are mounted simultaneously along both sides of the overlap, side lap and end lap, along the points b, n and b, d. If the contour points do not coincide, the print is mounted in such a way that the discrepancy is at a minimum. The prints are cut at this time along the lines of the end lap. A strip of thin celluloid (0.2 mm) is placed under the prints to be cut, in order to protect the print below it as well as the base mount. The side lap is trimmed off after all or a major portion of a flight strip has been mounted.

During mounting of the prints, there is a cumulation of errors due to inaccurate mounting as well as to print distortion. In order to minimize the errors of assembly, the mosaic is laid from the center of the area to be assembled, i.e., from

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the center of the middle flight strip.

To minimize the errors, due to rotation of the photographs and the bend of the flight strip, the mounting is often done along the initial radial (direction between the centers of the prints). The center points are pin pricked on the prints (contour points with a circle outline of $r = f_k : 50$ around the principal point, which can be easily located on adjacent photographs). These points are also pricked on the adjacent prints. The radials are then marked on all the prints from the center point of a given photograph to the images of the center points of adjacent photographs (Fig. 55).
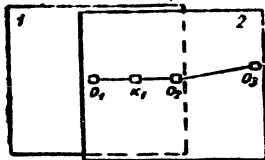
Fig. 55 - Initial Radial

The prints of a strip in such a case are no longer mounted according to the points a and b, but by superimposing the initial radials $o_1 o_2$ of the first photograph and $o_2 o_1$ of the second print. Having aligned these directions, the prints are shifted along them until their common contour point k_1 is aligned. This point lies half way along the initial radial or, otherwise, at the center of side lap. The rest of the flight strip is mounted, pasted, and trimmed by the above-described method.

39. Transforming Aerial Photographs

In transforming of prints, they are corrected for distortion due to angle of tilt during photography and are made to conform to a common scale. The solution of this problem lies in the process of conversion (transformation) during which the image of the tilted print is projected onto a horizontal surface, compensating for the initial tilt.

The principle of transformation can be explained as follows:

Taking into account the external and internal elements of orientation we place the photograph into such a position, with respect to the horizontal plane, as it

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occupied at the instant of exposure. Or, having illuminated the print from above, we can establish the relationship of its projected rays aS, nS, bS, cS, oS, dS (Fig. 56) which will pass through the points A, N, B, C, O, D, on the horizontal plane of the screen E, the image on which will be corrected to resemble the image of a true vertical photograph. By altering the height of screen E with respect to area T, i.e., the distance of the screen from the center of projection S, the scale
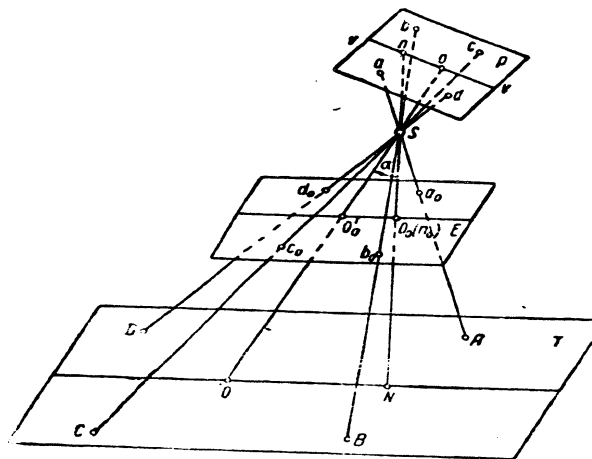

Fig. 56 - Principle of Transformation

of the image on the screen is changed, making it conform to the desired scale of the print. By placing a sheet of photographic paper on the screen and exposing it, a print corrected for tilt distortion and at the desired scale is obtained.

There are several ways to accomplish transformation of prints. These can be divided into four categories: optico-mechanical (transforming printers), optico-graphic (projectors and tracing instruments), mechanical-graphic (perspectograph) and graphic. The first two methods are conventionally used for production.

40. Graphic Method for Transforming Aerial Photographs

The graphic transformation of prints is performed with the aid of perspective

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grids whose construction was described in Chapter IV. After constructing the perspective grids on the plan, the perspective contours of the photograph are transposed onto the plan square by square.

When making a plan where $\alpha < 3°$ it becomes difficult to construct a grid of the type described in Chapter IV, since the line of the true horizon $h_i h_i$ in this case lies at a great distance. In such cases, the grid is plotted on the plan with the following arrangement of the perspective image (print).

Let us assume that the following contour points are available on the plan: a_o, b_o, c_o, d_o, n_o, k_o, which represent the points a, b, c, d, n, k on the photograph
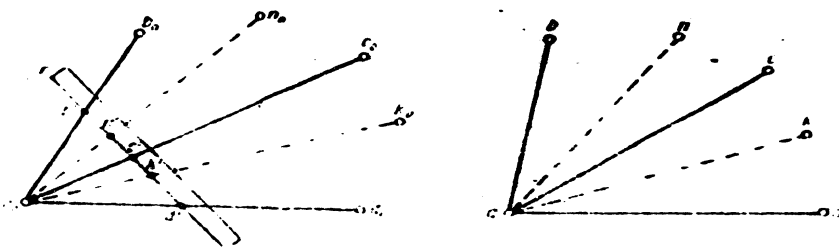

Fig. 57 - Constructing the Family of Radials

(Fig. 57). Let us draw a family of radials to all points on the map, denoting the apex by a_o. Next intersect this family by an arbitrary line rr and mark on it the points of intersection for b', n', c', k', d'. Then, draw a family of corresponding radials to the points b, n, c, k, d from point a. By transferring the line rr, with its points of intersection, from the map to the print and aligning its points b', c' and d' with the radials ab, ac, and ad, then all other points on rr (e.g., n', k') will coincide with their corresponding radials (an, ak).

The above-described principle of perspective representation can be used for transferring radials to definite points from the photograph to the map. In this case, the number of points on the map, corresponding to points of the photograph, must be not less than four.

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Let us assume that we have the points a,b,c,d on the photograph and the corresponding control points a_o, b_o, c_o, d_o on the map (Fig.58). Joining these points with straight lines, we select on them points 1,2,3...,4,5,6..., 7,8,9...,10,11,12. In selecting these points it is simpler to have them divide the lines into equal portions, which, however, is not necessary. Next draw radials from point A to points 1,
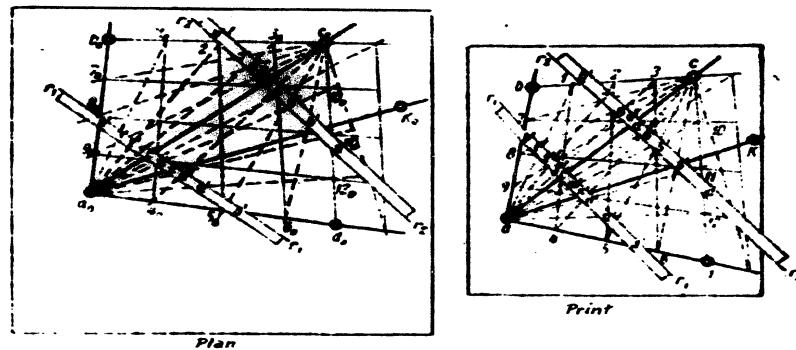

Plan   Print

Fig. 58 - Constructing the Perspective Grid

2,3,10,11,12. Placing a strip of paper across these radials in an arbitrary position, we mark on its edge r_1r_1 the points b', c', d', 1', 2', 3', 10', 11', 12', at the intersection with the radials. On transferring the strip of paper to the map, its points b', c', d' are matched with the rays a_ob_o, a_oc_o, and a_od_o, after which the points 1', 2', 3', 10', 11', 12' from r_1r_1 are pricked on the map. By drawing rays from point a_o, through the pricked points marked above, we obtain at the intersection of these rays with the sides b_oc_o and c_od_o the wanted points 1_o, 2_o, 3_o, 10_o, 11_o, and 12_o which are the projections of points 1,2,3,10,11,12 of the photograph.

Taking the point c as the vertex of the rays, radials from the point c to points 7,8,9,4,5,6 are drawn on the photograph. Then, these radials are transferred to the map with the aid of the strip of paper r_2r_2 which is now oriented by the ra-

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dials $c_o b_o$, $c_o a_o$, $c_o d_o$. The intersection of the transformed rays (from point c_o) with the sides $a_o b_o$ and $a_o d_o$ will give the wanted points 4_o, 5_o, 6_o, 7_o, 8_o, 9_o. On joining the resultant points on the map by straight lines correcponding to a single photograph, a projected grid is obtained, which makes the transformation possible.

The contours must be transferred to the grid with the aid of proportional dividers.

41. Optico-Mechanical Method of Transforming Aerial Photographs

As pointed out previously, the optico-mechanical method of transformation is accomplished with special instruments, known as transforming printers.

The principles of transformation, using a transforming printer of category I, are described in Section 39 (Fig.59).

In transforming printers of category I, the elements of interior orientation are used for correlating the projected rays of the photograph. To accomplish this, the principal point of the photograph, contained in the magazine, is aligned with the principal point o' of the camera K of the transforming printer. The distance So' between the nodal point of the lens S and the principal point o' of the camera of the transforming printer is adjusted to be equal to the focal length f_k of the aerial camera. The photograph is illuminated from the rear by a light source and is projected through the lens S onto the screen E, which can be rotated about the horizontal axis aa. The horizontal axis of rotation of the screen is attached by means of the stand 2 to the disk 3 which can be rotated with the screen about the vertical axis by a handwheel 5. This rotation makes it possible to give the screen any desired position of tilt in space. The carrier 4 can be moved along the base 1 of the instrument, by turning the crank 6, parallel to the principal ray o'S, which permits the required scale of the image to be set on the screen E. However, moving the screen away from the lens will disturb the optical conjugation of the planes P and E, i.e., the sharpness of the image will be lost. To maintain good definition at var-

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ious distances from the screen, the transforming printer should be equipped with a large variety of lenses of different focal lengths. Due to this fact, work with transforming printers of category I is inconvenient so that they are not used in practice.

Transforming printers of category II are now widely used. They distort the pencil of the projected rays, obtained during photography, but give the correct perspective of the plane of the photograph P and of the plane of the map E. The principle of operation of the transforming printer of category II is as follows:
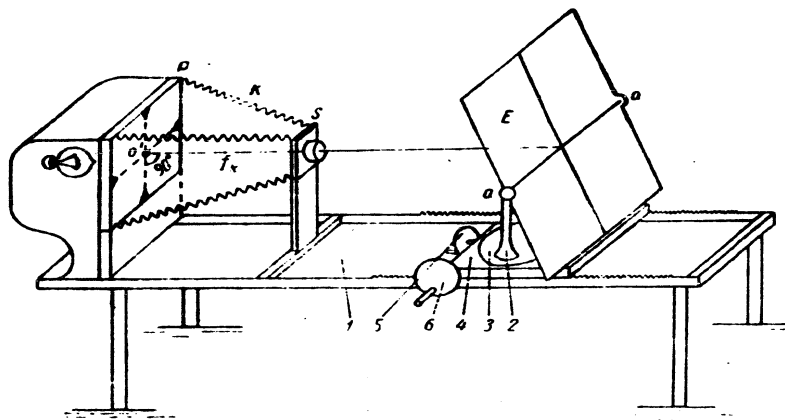

Fig. 59 - Transforming Printer of Category I

The basic principles of projective geometry, as described in Chapter IV, indicate that to construct a perspective image of a given object, it is necessary and sufficient to know the location of the principal vanishing point i, of the isocenter c, and of the picture base tt and that a parallel displacement of the line tt, in the plane of the picture, does not disturb the similarity of the perspective representation but merely changes the scale. Consequently, if the position of these main elements i, c, and tt remains constant, any change in the other elements will

not disturb the respective coincidence between the planes of the photograph and map.

Let P be the true position of the picture plane and E_o the true position of the plane of the screen, i.e., parallel to the plane of the area (Fig.60).

Now if the screen is moved from the position E_o to some other position E, and the lens from the position S to a position S', and if the condition is satisfied that the location of points i and c on the plane P remains constant for the new angle α', then the mutual perspective relation of the photograph P and the map of the plane E will remain undisturbed, i.e., an image corresponding to the horizontal photograph is obtained in the plane E, which solves the problem of transformation,
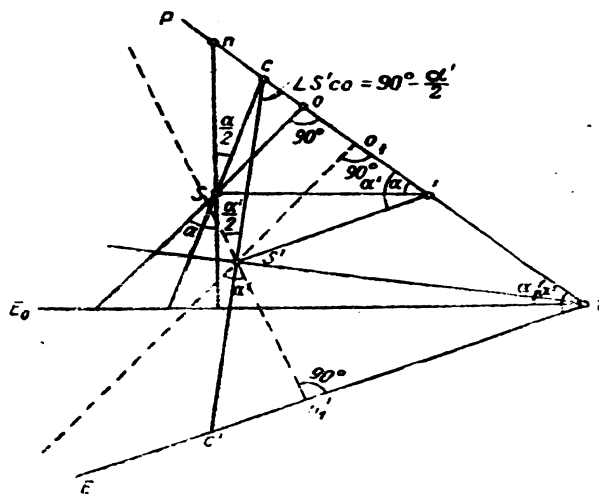

Fig. 60 - Transformation of Category II

From the condition that the position of points i and c at the angle α' is to remain constant, the new position of the lens S' is defined as the point of intersection of the ray iS', forming the angle α'c with the picture plane ic (since, from the definition of the principal vanishing point, the ray S'i must be parallel to the plane E of the plan) with the ray cS' forming the angle $90° - \frac{\alpha'}{2}$ with the line ci.

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From the isosceles triangles ciS and ciS' it follows that $iS' - ic = f_k : \sin \alpha$. Also note that triangles ciS and ciS' are isosceles, since the angles formed by the sides cS, for the first triangle, equals $90° - \frac{\alpha}{2}$, and the angles formed by the sides cS', for the second triangle, equal $90° - \frac{\alpha'}{2}$. This means that $iS' - ic = iS - = f_k : \sin \alpha$.

Therefore, in performing transformations of the second category the following geometric conditions must be satisfied.

1. The plane of the screen must be parallel to the plane passing through the horizon trace of the photograph and the new center of projection.

2. The point c which had been the isocenter with respect to the terrain must remain the isocenter with respect to the plane of the screen or, in other words, the new center of projection must lie in the plane of the principal line of the photograph and also on a circumference of the radius $f_k : \sin \alpha$, drawn from the principal vanishing point i.

In addition to the specified geometric rules, certain optical laws have to be observed in the design of transforming printers, to ensure satisfactory definition of the image on the screen under any conditions of its tilt and measured scale.

The two optical laws of transformation are:

1. Maintaining optical conjugation of the points on the photograph and screen, which are located on the principal optical axis of the lens. This law is usually expressed by the known optical formula: $\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{F}$, where d_1 and d_2 are distances from the negative to the lens and from the lens to the screen, while F is the focal length of the lens of the transforming printer.

2. Maintaining the optical conjugation of the tilted plane of the negative and screen. This law will be satisfied if the plane of the negative and screen intersect the principal plane of the lens of the transforming printer along a single straight line tt.

With respect to design, these conditions are satisfied independently, and taken

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into consideration in the construction of various transforming printers.

42. Short Description of Transforming Printers

a. The MGI Transforming Printer

The MGI transforming printer was designed in the shops of the Moscow Geodesic Institute (now MIIGA i K), under the supervision of Professors P.P.Sokolov and N.M.Aleksapol'skiy. The instrument is designed for transforming photographs 18 × 18 cm in size, at small angles of tilt (about 5°). A general view of the MGI printer is given in Fig.61.
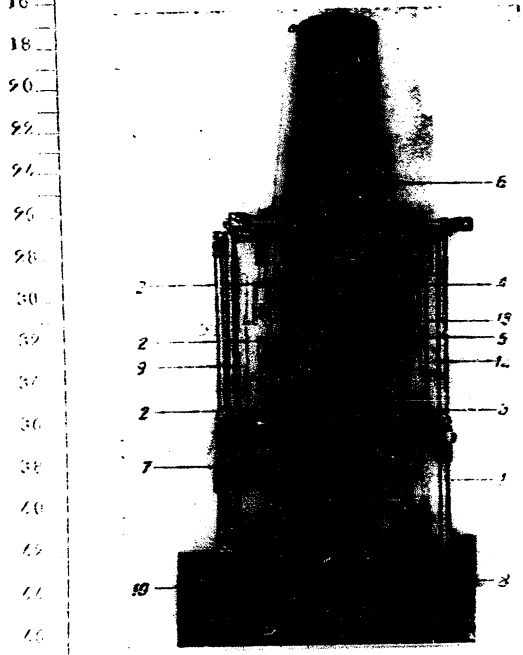
The parts of the apparatus are mounted on a special frame 1, which forms the base of the instrument. The screen 3, the lens 5, and the negative holder 4 are attached to the horizontal axis 2. The lens illuminator 6 is placed over the negative. The horizontal axis of rotation of the screen is rigidly mounted to the base of the instrument, and the lens and negative are suspended on brackets that can be shifted along the vertical guide wheels of the base plate. This shift produces a change in the distances from the lens to the screen and from the lens to the negative, which results in a change in the scale of the image on the screen. The scale is changed with the aid of the special device 7, termed a scale inverter, which ensures constant optical conjugation of the points of the negative and the screen with respect to the vertical axis of

Fig.61 -   The MGI Transforming
           Printer

the transforming printer. The scale inverter is actuated with the aid of the pedal-operated wheel 8.

The screen, the negative holder and the objective are all rotatable about their horizontal axes 2 for setting the proper angle between the negative and the screen (the rotation of the lens is necessary to maintain optical conjugation at tilts of the negative and of the screen). The rotation of the screen, negative, and lens about their horizontal axes is effected by means of the special devices 9 and 12, assuring satisfaction of the other geometric and optical conditions of transformation.

The device 9, termed a perspective inverter, automatically ensures optical conjugation of the planes of the negatives and of the screen at the tilts produced over the pedal-operated wheel 10.

The geometric conditions of transformation in the MGI transforming printer are automatically satisfied by using the parallelogram inverter 12. The parallelogram inverter keeps the screen at all times parallel to the line S'i (see Fig.60)

Fig.62 - Large Transforming Printer

while equality of the distance S'i to the quantity $f_k : \sin \alpha$ is achieved by setting the so-called adjustable focal length f_o on the scale 13. The tilting of the screen and negative in the MGI printer is effected only in one direction (about the axes 2); consequently, the principal line in the transforming printer always occupies an entirely definite position, perpendicular to the axes 2. Therefore, it becomes
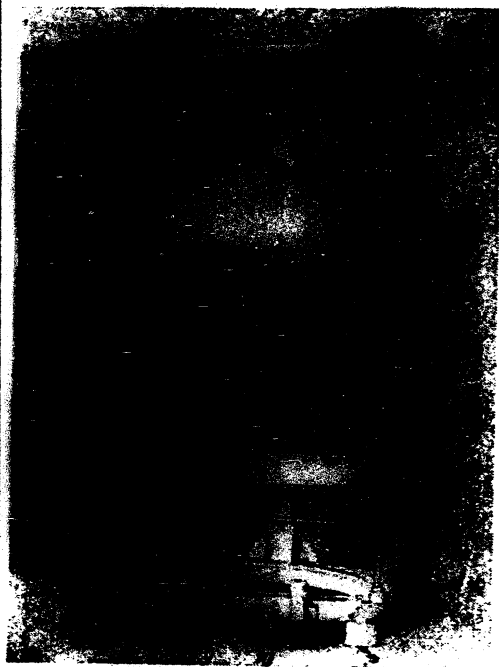
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necessary to rotate the negative in its plane in order to make the principal line of the aerial photograph (the negative) coincide with the principal line of the transforming printer. This rotation is effected by manual rotation of the negative holder in its plane.

The lens of the transforming printer has a variable diaphragm and is equipped with a red optical filter. As stated previously, the MGI transforming printer is designed for the transformation of aerial photographs with angles of tilt not exceeding 10° at f_k = 200 mm, or 5° at f_k = 100 mm. The scale ratio may be varied from 0.5 to 2.0.

b. **Large Transforming Printer (FTB)**

The FTB transforming printer (Fig.62) is a precision instrument that allows the transformation of aerial photographs as large as 30 × 30 cm at angles of tilt up to 36.° and scale ratios ranging from 0.7 to 5.0. The large printer has exactly the same principal motions as the MGI printer, but the control of the motions has a different design, due to its basically different realization of the conditions of correct transformation.

The Screen 3 of this transforming printer may be tilted about the horizontal axes 2, which are rigidly attached to the base 1 of the instrument. The lens carrier 5 and the negative 4 can be displaced vertically with the aid of the pedal-operated wheel 8. When this is done, the optical conjugation of the negative and screen points about the axes of the instrument is obtained with the aid of the rectangular scale inverters 7.

The lens system in the large transforming printer has no tilt mountings and its axis is always vertical. The condition of perspective optical conjugation is satisfied by means of the corresponding tilts of the negative and screen, using the perspective inverter 9. The screen itself is tilted by turning the pedal-operated wheel 10.

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The geometric conditions of correct transformation are not satisfied automatically in the large printer but by a combination of separate movements. When the screen is tilted, so-called longitudinal eccentring of the negative is introduced, which consists in a linear displacement of the negative along the principal line.

Fig. 63 - The Small Transforming Printer

The negative is displaced until the isocenter coincides with the bisector of the angle α' (Fig.60). The longitudinal eccentring of the negative is effected over the handle 14.

In addition to the longitudinal eccentring, the negative may also be given a transverse eccentricity – a linear displacement along the axes 2. The negative is

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rotated in its plane by the handle 11.

The illuminator of the large transforming printer consists of the elliptic mirror 6, with an electric bulb placed in its focus. To ensure greater uniformity of illumination on the negative, a frosted disk of Astrolone, celluloid, wax, or other transparent material is provided.

c. The Small Transforming Printer (FTM)

This printer (Fig.63) is designed for processing aerial photographs up to 30 × × 30 cm in size at angles of tilt not greater than 9° and at scale ratios ranging from 0.7 to 2.5. The screen 3 of the small printer, exactly as with the 2 preceding instruments, is attached to the base 1 of the instrument, while the negative and lens may be displaced vertically by means of the pedal-operated wheel 8. The screen may be tilted about the two mutually perpendicular axes 2 by manipulating the wheels 10'. The tilt of the screen is transmitted over the flexible shafts 9' and the cor-

Table 7

Type of Transforming Printer	Size of the Photograph cm	Angle of Tilt	Scale Ratio	Focal Length of the Aerial Camera	Focal Length of the Lens	Size of the Screen cm	Height of the Instrument m
MGI	18 × 18	15°	0.5 - 2.1	100 - 250	150	38 × 45	3.0
Large FTB	18 × 18 30 × 30	45	0.7 - 5.0	as desired	180	100 × 60 and 100 × 100	2.8
Small FTM	18 × 18 30 × 30	14.5	0.7 - 2.5	as desired	180	60 × 60	2.4

responding transmissions to the lens 5, which also can be rotated about two mutually perpendicular axes. The negative 4 has no tilting arrangements. The geometric conditions of transformation, exactly as in the large transforming printer are not automatically fulfilled. Since the screen is tilted about two axes, the negative also moves for its linear eccentring in two mutually perpendicular directions. Longitudinal eccentring is effected by the aid of the screw 14, while transverse ec-

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centring is performed manually. The negative does not rotate in its own plane. Optical conjugation is obtained over the scale inverter 7.

The illuminator 6. of the small transforming printer, as in the large type, consists of an elliptic mirror.

The principal characteristics of the transforming printers are given in Table 7.

43. Transformation of Photographs in the Transforming Printer

Aerial photographs may be transformed by the optico-mechanical method, either from known elements of exterior orientation or from control points.

When the elements of orientation of the aerial photographs are known, transformation can be performed by setting the transforming printer to the corresponding angles between the plane of the photograph and that of the screen, and to the required distance between screen and center of projection. In modern production practice, however, the elements of exterior orientation are usually known with insufficient accuracy. For this reason, the reconstruction of the perspective correspondence between the planes of the photograph and the screen (transformation) is solved by using control transformation points. Such points must be first identified and pricked on the photograph, and then located on the system of coordinates of the map and drawn on the plotting board placed on the screen of the printer. The plan position of the transformation points is defined from the photogrammetric density of the plan geodetic base.

The transformation of an aerial photograph is accomplished by the displacement of the images of its transformation points from their position on the board.

If we project the transformation points a, b, c, d of the photograph P (Fig. 56) through the lens S onto the screen E and match resultant projections with the plan points a_o, b_o, c_o, d_o, then the corresponding angle between the planes P and E will be defined, and the photograph P will be transformed. On the screen E we obtain an image of the photograph P, reduced to a horizontal photograph and to the scale of

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the map. The images of the transformation points of the photograph are matched with their positions on the screen by means of: a) rotation of the photograph (and of the map with the control points on the screen) in their plane, to orient the image with respect to the map; b) tilting the photograph; c) changing the height of the screen to reduce the image of the photograph so obtained to the required scale.
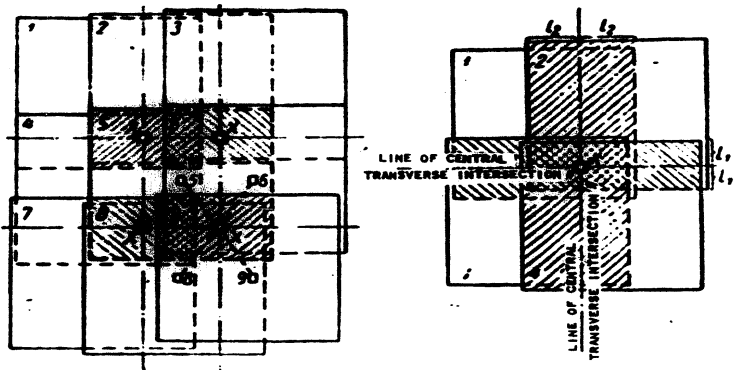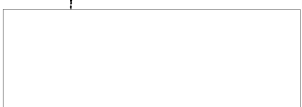
Fig.64 - Transformation Points

For transforming the photograph on a transforming printer of category I, it is sufficient to know the position of three non-colinear transformation points which are recognized on the photograph. In this case, the problem consists in determining the spatial position of the plane of the screen with respect to the reconstructed pencil of rays. The spatial position of the plane is determined by three points. At different intersections of the pencil of rays by the plane of the screen, differ- ent positions of the images of the transformation points of the photograph will be obtained; by matching them with the control points of the board, the singular posi- tion of the plane of the screen is obtained.

In the general form, however, when a pencil of projecting rays condensed dur- ing photography is destroyed, as it happens in transforming printers of category II,

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three transformation points are insufficient for transformation of the photograph.

It is proved in projective geometry that if, for two mutually projective planes (of the photograph and of the map), the mutual perspective relation between four non-coplanar corresponding points of these planes is reconstructed through a certain center of projection (Fig.56), then the perspective relationship is automatically restored for any other pairs of corresponding points of these planes, i.e., the mutual perspective relation of the planes themselves is now reconstructed. Therefore, for transforming a photograph, it is necessary and sufficient to have four transformation points which do not lie in a single straight line.

The transformation points x are usually selected near the working angles of the photograph (Fig.64). They are, consequently, located at the center of overlap of four adjacent aerial photographs from two adjacent flight strips, i.e., at the center of their end and side laps.

The existence of supplementary control points markedly increases the accuracy of transformation, especially in working with large and small transforming printers.

The center of the photograph is the most advantageous position for such an excess point. For this reason, a fifth excess transformation point at the center of the photograph is very desirable; this is usually obtained when the transformation points are determined photogrammetrically.

The images of the transformed points are matched with their positions on the screen of the transforming printer (on the plotting board) by the following manipulations:

MGI Transforming Printer	Change scale, tilt the screen, rotate the photograph in its plane;
Large Transforming Printer (FTB)	Change scale, tilt the screen, rotate the photograph in its plane, eccenter along principal vertical (longitudinal);
Small Transforming Printer (FTM)	Change scale, tilt the screen, about two of its axes, longitudinal and lateral eccentricity.

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44. Preparing the Photographs and the Base Plan for Transformation

Prior to transformation, the photographs are prepared as follows:

1. *Preparation of Negatives.* This preparation consists in pricking the transformation points and the field control points. The size of the punched holes is kept to 0.1 - 0.2 mm so that the prick marks are shown with sufficient clarity on the transforming printer.

2. *Compiling the Transformation "Basis".* Due to the fact that the rigid plotting board on which the control and transformation points of the map are mark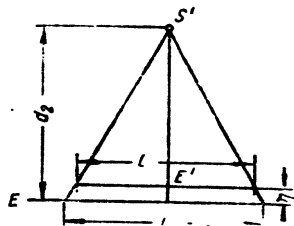ed, generally does not fit onto the screen of the printer, the points are copied from the map onto a strip of transparent paper by prick marks and from there onto the tracing paper. Usually, such base composite is prepared for an entire flight strip of the mosaic. For prominent relief of the control points of the area, " corrections for relief " are introduced. These points are marked in India ink by circles of approx. 2 mm diameter.

Fig. 65 - Introducing the Correction for Deformation of the Paper

3. *Determination of "Shrinkage for the Photographic Paper".* Since, after transformation, the photograph is developed, washed and dried, the photographic paper becomes deformed or " shrinks ".

This shrinkage is determined by placing a piece of cardboard of determined thickness under the base sheet. After transforming the photograph in this position, the cardboard is removed and the print is exposed onto a sheet of photographic paper, placed directly on the screen. If no cardboard is used, the length of the rays from the lens of the transforming printer to the screen increases, causing the image to increase to a size of L (Fig.65). Therefore, the problem consists in using a cardboard having a thickness η corresponding to the shrinkage indicated by the size increase of the image. If S' denotes the lens of the transforming printer, E

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the screen, and E' the cardboard liner, then:

$$\eta : d_2 = (L - \ell) : L$$
$$\eta : d_2 \left(1 - \frac{\ell}{L} \right) \tag{23}$$

where $\ell : L = K$ denotes the coefficient of reduction in size of the image when using a cardboard liner and which must be equal to the coefficient of shrinkage of the photographic paper; ad_2 is the distance from the lens to the screen.
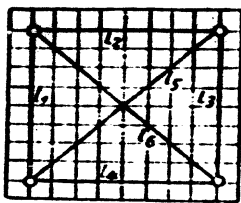

Fig. 66 - Measuring the Sample on a Grid

To determine the coefficient K of the shrinkage of the photographic paper, several contact prints are made of an exact grid of squares having sides of 5 mm each and, after drying, the segment ℓ_i (Fig.66) is measured for correspondence with the known original L_i.

Thus, the coefficient of drying can be determined from the relation $K = \Sigma \ell : \Sigma L$ so that eq.(23) will be:

$$\eta = d_2 \left(1 - \frac{\Sigma \ell}{\Sigma L} \right)$$

45. Technique of Transforming a Print on a Transforming Printer

The screen is set in a horizontal position, and the eccentricity values are set to readings of zero. The negative is inserted (with the emulsion side down) into the magazine of the transforming printer, with the center of the print aligned with the center of the negative holder. The cardboard spacer is placed on the screen over which the base with transformation points is placed. The negative and the base copy are rotated so that the print will lie diagonally along the axis of the screen and the illuminated images of the transformation points would lie in a spread from the center of the base.

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By varying the scale of the image, the transformation light spots 1 and 3, located along the axis of the screen, are then matched with the corresponding points on the base (Fig.67, points of the base in black). If after this, the points 2 and 4
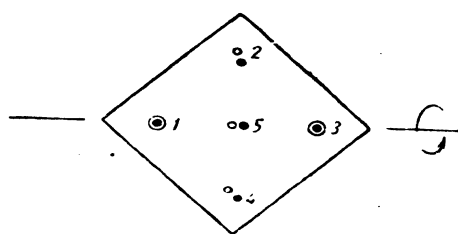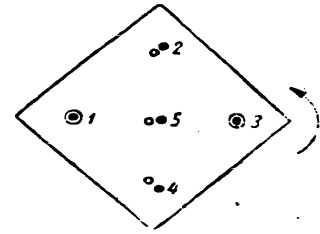

Fig.67 - Change in Tilt of the
Screen

Fig.68 - Rotation of the Photograph
in its Plane

fail to coincide, the image trace 5 - 4 should be enlarged, and the trace 5 - 2 reduced in size; this is achieved by tilting the screen toward the observer (point 4 is lowered and point 2 is raised). After tilting the screen, all image points and

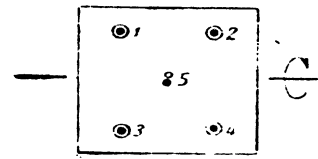Fig.69 - Noncoincidence of Points
caused by Relief
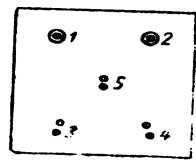
Fig.70 - Additional Tilt of Screen

the points on the base may still fail to coincide, as shown in Fig.68. In this case, obviously the part of the image around the point 1 should be enlarged, while the part of the image around point 3 should be reduced (the sides 1 - 2, 1 - 5, 1 - 4 to be enlarged, and the sides 2 - 3, 5 - 3, 4 - 3 to be reduced). This is achieved by rotating the print (together with the base) in its plane (in the small transforming

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printer by tilting the screen laterally). On such rotation, the part of the photo-graph near point 3 having the sides 2 - 3, 3 - 4 will be projected onto the part of the screen that had been moved upward and thus will be reduced in size (compressed).
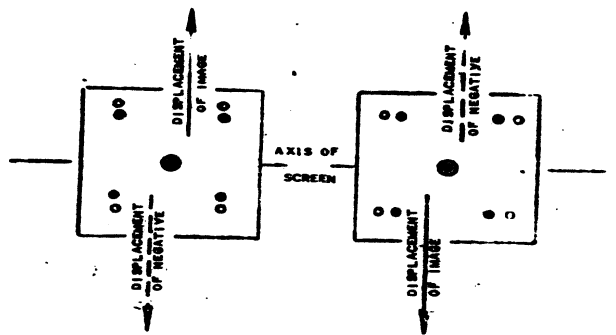

Fig.71 - Longitudinal Eccentring

The part of the print near the point 1 having the sides 2 - 1, 1 - 4 will be enlarged (expanded) and shifted to the lower portion of the screen.

It should be borne in mind that the change in the image produced by any of these manipulations is more pronounced in the lower part of the screen.
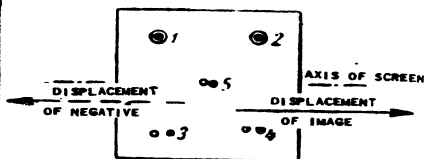
Fig.72 - Lateral Eccentring

Due to the fact that each manipulation some-what disturbs all previous settings, the image points are matched with those of the base by successive approximations, repeating the above operations and correcting the scale setting each time.

Occasionally, a case arises when the four orientation points coincide, but the central point will still be out of alignment along the principal line (Fig.69). This discrepancy is explained either by errors in the determination of the points for transformation or the influence of the dis-placement of the picture points due to relief. At the points 1, 2, 3, and 4, these

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errors may be inaccurately eliminated, thus leading to errors in setting the tilt, and to non-coincidence of the point 5. This non-coincidence may be somewhat improved by tilting the screen, since this will cause the points 3 and 4, located in the low-ered part, to be displaced faster, i.e., the sides 1 - 3 and 2 - 4 will be changed more than the sides 1 - 2 and 3 - 4. The screen is given a tilt (raising the part with the points 3 and 4) until the position shown in Fig.70 is reached. By changing the scale (enlarging it) a better matching of the points is accomplished.

In the transformation of photographs on the large and small transforming printer where the geometrical conditions of transformation are satisfied with a simultaneous eccentring of the photograph, the pattern of non-coincidence shown in Fig.71 or 72 will be obtained, after the above manipulations have been made.

At longitudinal eccentring of the negative along the principal line, the image is elongated along this line, if it is displaced to the lowered part of the screen and is compressed during displacement into the raised part of the screen. In this case, the displacement of points (Fig.71) is accomplished by longitudinal eccentring in the direction shown in the diagram (with the screen tilted toward the observer), followed by a change in scale. In the position shown in Fig.72, however, the dis-placement of the points is accomplished by application of transverse eccentring, as indicated in the diagram for the case of tilting the screen on the side of the ob-server. In lateral eccentring, a torsion of the image occurs since, during dis-placement of the image along the axis of the screen, points 3 and 4 located in the lowered part of the screen, will be displaced faster than points 1 and 2.

When transforming the photographs on the large transforming printer, no later-al eccentring is used or only on a small scale, in view of the lack of centring of the negative in the holder.

The above manipulations will cause the image points to be matched with the cor-responding points of the base by a series of successive approximations. Due to er-rors in the position of the transformation points and due to the displacement of the

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picture points caused by relief, it may be impossible to achieve complete coinci-
dence of the points. These misalignments must not exceed 0.3 mm. At larger errors,
the transformation points must be checked for accuracy.

After the photograph is transformed, the base and cardboard liner are removed
from the screen, the lens is covered with a red filter, a sheet of photographic
paper is placed on the screen and the photograph is exposed after removing the fil-
ter.

During exposure and developing, all photographs on the plotting board are made
uniform in tone, i.e., the images of the same contours are produced at equal photo-
graphic density.

46. Application of Corrections for Relief

As shown previously, the transformation of photographs is achieved by the method
of polyconic projection for the case of photographs with a tilted plane, by project-
ing them to a horizontal plane. This method of transformation compensates for the
distortions of the photograph caused by tilt
but not for distortions caused by ground re-
lief.

Assuming that photograph P (Fig.73) con-
tains an image a of the terrain point A, and
that $a_o a$ denotes the distortion of the image
point a due to ground relief. An orthogra-
phic projection of the point A is made onto
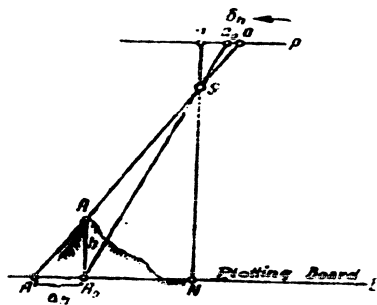the screen E at the point A_o. During trans-
formation, as indicated in Fig.73, the ray

Fig.73 - Correction for Relief

aS will not coincide with the ray SA_o. The point a will form the image A' on the
screen, which is displaced from the map location A_o of the point A by a distance of
Δ_h. This is a direct result of the distortion $a_o a$ of the point a caused by relief.

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If the relief is negligible, these distortions are insignificant and can be disregarded (up to 0.3 mm). However, at higher distortion it becomes absolutely necessary to correct for relief during transformation.

Corrections for relief distortion can be made at the locus of the transformation points on the photograph a (correction δ_h) or at the locus of the transformation points A_o on the base (correction Δ_h). The magnitude of the correction is determined by eq.(12), except that its sign will be opposite to that obtained from eq.(12).

Therefore, for correction we have:

$$\delta_h = -\frac{dh}{H} \; ; \; \Delta_h = +\frac{Dh}{H} \tag{25}$$

In practice it is inconvenient to make corrections for relief on the negative, since this will result in pin holes, so the corrections for relief are usually made at the locus of the points on the plotting board.

In the case that the correction of the point has a positive value of relief correction, the mark on the photograph is moved toward the nadir point, and the mark on the plotting board away from the nadir point. For the case of negative correction, the opposite procedure is used, and the correction on the photograph is made away from the nadir point and on the plotting board, toward it.

To keep the relief distortion of a transformed image to as low a value as possible for a given scale of transformation, the correction values h are calculated by eq.(25) from the average height of ground relief. This average height will be the plane of transformation. For example, if the elevations of the points on a photograph vary from 200 to 400 m, then the factors of compensation are calculated from the average plane of elevation, which is at 300 m, so that these compensations will fluctuate from - 100 to + 100 m. The survey elevation H will then also be calculated from the mean plane of elevation.

The correction at the locus of the transformation points, to compensate for re-

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lief distortion, does not eliminate the distortions of the photographic images but does permit the correct transformation of the print.

Corrections for relief distortion are made for purposes of transformation of prints and also for solving other photogrammetric problems, e.g., those involved in determining the true location of points on a photograph or points on a map, which have a projected relationship with the print.

47. Transformation of Aerial Photographs by Zones, for Areas of Bold Ground Relief

The correction of transformation points for relief distortion allows the transformation of prints for any type of terrain, but the distortions of the points on a transformed photograph with bold relief still exceed the limits of 0.3 - 0.4 mm, i.e., reduce the accuracy of the map.

The transformation of the photograph by zones or sections of various elevation planes permits elimination of this defect and reduction of the relief distortion to the desired degree of accuracy.

Let P (Fig.74) be the photograph of the area of bold relief T. Divide the area into various sections by elevation intervals equal to h', as shown in Fig.74. Then, the change in elevation from one section of the area to the next can be observed as the general height fluctuation for the mean plane of elevation of the area T. The deviation in the height of individual points from the mean plane of elevation, for a given section can be observed in relation to the boundary limits of this section, or as particular elevations h which cause relief distortion of points on the print at the scale of the particular section of the photograph. This scale is determined at the average plane of elevation. Therefore, in the photograph given in Fig.74, the scale of section 3 will be larger than that of section 2, while the scale of section 2 will be larger than that of section 1.

The transformation of the photograph by sections is accomplished by converting the sections (average planes of the sections) to the scale of the map.

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Let us take any section, say section 2, as the starting point. The transforma-
tion points are corrected for relief, with the corrections calculated from the aver-
age plane of elevation for section 2, which completes transformation of the print.
Thus, section 2 of the print will appear on the screen E" at the specified scale.
The distortion of individual points for this section will not exceed ± 0.3 to ± 0.4 mm.
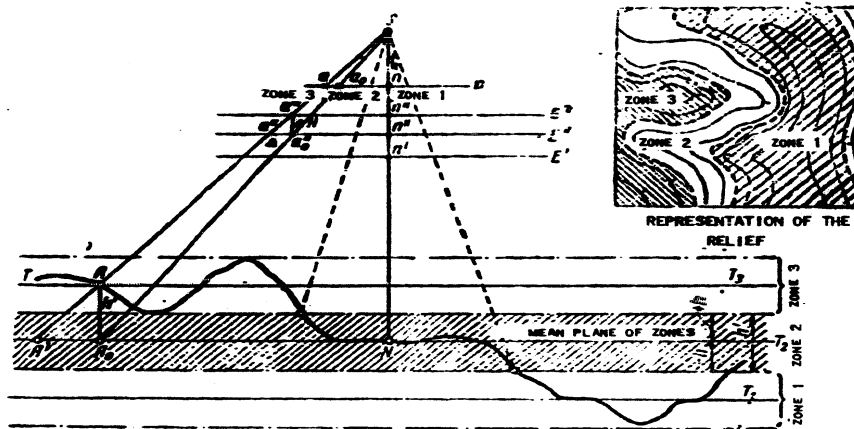

REPRESENTATION OF THE
RELIEF

Fig. 74 - Transformation by Zones

Point A on the mean plane of elevation for section 3 will appear at a", when
it should have appeared at point a_0''. Consequently, if the scale of the image is
decreased by moving the screen from position E" to E''', so that point a" would be
reduced in size by Δ, then this point will appear at the position a''', which is cor-
responds to the position a_0''. Then, the scale for the third section will appear on
the screen E''' (average plane T_3), equal to the specified scale at which section 2
was transformed. In this position, the distortion of the photograph prints for the
third section will not exceed the specified limits.

By increasing the scale of the image (moving the plane of the screen to loca-
tion E'), a scale equal to the specified scale is obtained for the first section of

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the photograph.

Actually, the transformation of the photograph by sections is done for the average elevation plane of the initial section. The adjustment to the average elevation plane of the other sections is done by changing the scale or, in other words, by changing the value of the sections by Δ.

In this case, Δ represents the difference in correction for relief of various sections, at an arbitrary elevation of a more distant point A. These values are determined by the elevation of point A, from the average plane of elevation of the various sections. In practice, all corrections for point A are marked on the plotting board and, after transformation of photograph along its points corrected for the first section is completed, only the scale is changed for the other sections, by means of displacing the image of point a by the correction value, corresponding to this zone.

For each position of the screen E" and E"', a photograph of the print is made, and is marked by its zone number.

Transformation by zones or sections is very complex and is used in production when there are three sections or less to a print.

When the relief of the terrain is very bold, no photomaps are made; Instead a graphic plan is prepared.

48. Assembling the Photomap

The assembly of a photomap consists of: placing the transformed prints on a mounting board in accordance with their transformation points; trimming off the overlapping edges of adjacent prints; and cementing the prints to the board.

Prior to assembly, the tonality of the photographs is checked for uniformity. Next, a hole of 0.5 mm is pricked through each transformation and control point by means of a punch. The punch comprises a needle mounted in a cylindrical bushing with tapered ends. The needle is kept in the extended position by spring pressure

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and is retracted into the bushing under pressure. After placing the needle on the center of the transformation point, pressure is applied to the bushing, which punches a hole in the print. The center of this hole is the location of the transformation point.

Before assembly, the transformation as well as the effects of drying of the prints are checked. To do this, each print is placed so that the marked transformation dots are at the center of the punched holes. These dots and holes must coincide. The error of coincidence for them must not exceed 0.4 to 0.5 mm. If this limit is exceeded, or if the tonality of the photograph is not uniform, the transformation must be repeated.

Assembly of the prints is started after the checking is completed. The first print is put in position on the base, followed by the remainder of the prints, so as to have the centers of the punched holes coincide with the corresponding points marked on the control network or so that the variations in alignment of the holes and the dots are kept to a minimum. (Misalignment of the corresponding dots and hole centers must be equal and in opposite directions from the center of the hole.) Next the print is fastened down with weights to prevent shifting. The second print is located on the base in the same way and is fastened down too.

Next, the line of cut along the end lap of both photographs is marked. This trim line is marked within 1 cm from the center of overlap. The direction of cut is selected under the following conditions: a) both prints should have the same tonality along the trim line, b) the trim line must intersect a minimum number of contour characteristics and not come close to relief boundary outlines (e.g., roads)or individual objects (houses), c) the trim line must not intersect contours or population centers at an acute angle.

Coincidence of relief lines along the cut is checked for proper fit. For this purpose, characteristic points (angles) of the contours of the upper photograph are pricked by pins to check whether the prick coincides with the corresponding contour

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line on the lower print. The cut is made where (taking the above conditions into account) the agreement of contours of adjacent photographs is best. The discrepancy between the contours and the cut lines should not exceed 0.7 - 0.8 mm. In no case should the photographs be moved away from the transformation points, in order to reduce the discrepancy of contours.

The above conditions being satisfied, a thin strip of celluloid is placed between the prints and the mount and both prints are cut along the trim line with a scalpel. The edges of the prints must be even, smooth, and must coincide with each other. The trimmed-off overlap strips are saved for correcting the photomap and are marked with the number of the photograph.

The edges of the trimmed prints are raised (the other part being held down by weights) and the mounting board is coated with amyl acetate cement (at the spots where the photographs will not be trimmed); the prints are cemented to the mount, carefully smoothed down, especially along the trim line, and are covered with weighted glass plates. The third print of the flight strip is mounted in the same way as the second, and the same procedure is used for all remaining photographs of the strip.

The second flight strip is mounted in the same way, with the prints first being trimmed along the line of end lap, while the side lap is trimmed after all photographs of adjacent flight strips have been mounted. During this operation, as in the case of cutting along the end lap, the terrain alignment is checked on adjacent photographs and the overlap is trimmed along the line within 1 cm of the center of side lap. After this trimming, the complete print is cemented down and smoothened. The trimmed edges of adjacent photographs must coincide exactly. There should be no overlap or space between the edges.

All other flight strips are mounted in the same way. To prevent glueing of the prints along the borders, the mount is covered with wax paper up to the frame line. After the complete photomap is mounted, the overlap along the edge of the borders is

cut off, using a straightedge. Prior to glueing down the corner prints of the photo-map, the frame corners are marked at the end points of the lines of the control net-work and are pierced with a punch. A thin strip of celluloid is placed under the print to avoid a second prick at the established position of the frame corner.

After completing the assembly of the photomap, the prints are left in position under pressure of the weight for two or three days, after which time the excess glue is wiped off with acetone.

In cases in which the prints had been transformed by sections, the laying of the prints on the mount is also done by zones according to the relief of the terrain shown in Fig.74. In this case, only the scaled part of each print is used, i.e., only the second zone is cut of the print for the second zone, and only the third section of the print for the third zone. The cutting between sections is performed after both prints of the adjoining zones have been aligned with the transformation points punched after correction for relief with respect to the plane of mean eleva-tion of the given section of the print.

In addition to the described steps of assembling an uncontrolled mosaic and photomap it occasionally becomes necessary to assembly accurate cartographic mater-ial, whenever the geodetic data for the prints are not available. In this case, it is impossible to assemble an accurate photomap, which means that unoriented photo-maps are prepared instead, which are sometimes called semicontrolled mosaics. The method of composing them does not differ from that used for photomaps. In such cases, the control points for transformation and scale determination are established by the photogrammetric method, after which the prints are transformed and mounted. Thus, unoriented photomaps are somewhat less accurate than controlled mosaics pro-duced from geodetic data but the method of camera work is the same.

49. Putting the Photomap into Final Shape

Finishing the photomap comprises the following steps:

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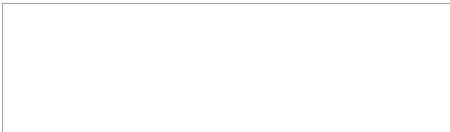
a) The corners and sides of the frame are marked in India ink by a square with 2-mm sides.

b) The trigonometric points and control points are marked in India ink with corresponding conventional symbols. The symbol must be centered at the side of the line and not of the punch hole.

c) The corners of the frame are marked by their rectangular and geographic coordinates.

d) The ends of the kilometer grid are entered and numbered.

e) The entries made outside the frame include: the name of the trapezoid entered over the northern edge of the map, at the middle; the numerical scale designation, below the southern edge; the date of preparation of the photomap and the signatures of its compilers at the right; the scheme of the frame with the diagonals of the photomap and an indication of their theoretical dimensions at the eastern border.

50. Proof Checking and Correcting the Photomap

The proof checking of a photomap consists in checking and determining: a) Accuracy of entry and outline of the frame of the photomap and ends of the kilometer grid, b) accuracy of coincidence of the photographs with the transformation points, c) divergence of the contours at all cuts between the photographs, d) divergence of the contours at the borders of adjoining mounts.

These corrections are entered on a special correction sheet.

The corrections are made, observing the following requirements:

1. The frame sides, the diagonals of the board, and the kilometer grid ends are checked by commonly accepted method of the use of a straightedge. The maximum tolerance for deviations on the sides of the frame and the ends of the kilometer lines is 0.1 mm, and 0.2 mm for the diagonals.

2. The accuracy of superposition of the photographs on the transformation

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points is determined from the deviations of the centers of the punched holes from the pricks of the corresponding points on the board. A diagram of the position of all points is plotted on the correction sheet (Fig. 75) on which the deviations of the centers of the punched holes from the pricks are entered in tenths of a milli- meter. These deviations must not exceed 0.4 mm.

3. The divergences along the contour lines at the lines of cut are determined by matching the cut-off edges of the prints, saved at the time of assembly of the photomap.

A diagram of cuts is reconstructed on the correction sheet (Fig. 76), on which are recorded the location and amount of misalignment. The misalignment is checked at 3-4 cm intervals along the line of cut. To determine the degree of misalignment,
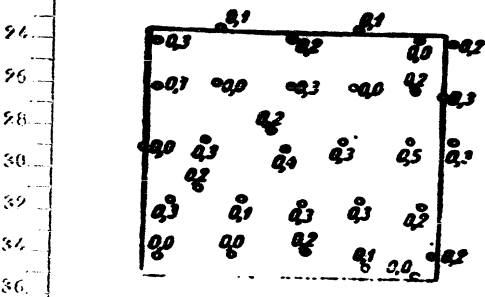
Fig. 75 - Correction from the Points   Fig. 76 - Correction from Cuts

the corresponding cut-off of one of the adjacent photographs is placed along the joint, and then at intervals of 3-4 cm the most distinct contour points located as close as possible to the line of cut are selected and pricked. The cut-off part is then removed, and the deviation of the prick from the corresponding contour point of the photomap is measured and recorded on the correction sheet.

The misalignment of contours must not exceed 0.7-0.8 mm, and even these maximum discrepancies are permissible only in exceptional cases.

4. The correction for match along the borders of adjacent photographs when the

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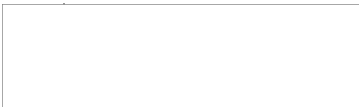
photomaps are cut along the line of the border, is done (as in the corrections along the cuts) by laying the cutoffs of one photograph on the other and superimposing them with the lines of a kilometer grid. By pricking corresponding contours as close as possible to the border, the discrepancy between them is measured.

If a flap is left in cutting off the photomap (usually 8 mm on the scale of cartographic work), the contour from one photomap is transferred to the other with the aid of compasses, after determining the distance of the contours from the frame and the length of the perpendicular to the border. The distance between the contour so transferred and the one on the photomap gives the wanted deviation.

The deviation of the contours along the border must not exceed 1.0 mm.

The data of these corrections are entered on a special correction form (Fig. 77), on whose margin the places and results of measurements, according to the kilometer grid, are noted.

Fig. 77 - Correction Along
the Frame

If discrepancies exceeding the established tolerances are found anywhere during the correction, or if the number of discrepancies should reach the maximum tolerance, the corresponding places on the photomap must be re-laid.

51. Map Phototriangulation

To expedite the transformation and assembly of aerial photographs in making a photomap, the map location of four control points on the prints must be known. This can be done by a geodetic survey of the area or by the photogrammetric method, with the aid of plan aerial triangulation and photopolygonometry (see Sect. 112). This permits to greatly shorten the difficult work of field surveying. In the last case, only two points have to be field-surveyed, and these separated by several prints.

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The most that are ever needed are four points per photograph, as for transformation, and these can be determined by instruments. The purpose of plan aerial triangulation is to determine additional control points, and thus to define the position of contour points by laboratory instruments.

Aerial phototriangulation is a method of determining the position of points on a photograph by means of constructing a network of triangles whose dimensions are measured on the aerial photographs.

To construct such a network of triangles, the angles of these triangles must be measured (not less than two angles per triangle) on the aerial photographs. In the analysis of angular distortion of prints, it was established that the angles inscribed by lines on a photograph are distorted due to the tilt of the photograph and the tilt of the terrain, and are not equal to the angle inscribed on the terrain.

The value of the angles of a tilted photograph and terrain are described by a radial drawn from the isocenter of a flat plane. The relief of the area creates distortion only in the directions not passing through the nadir point.

It is on these remarkable properties of the isocenters and nadir points that the method of phototriangulation is based.

If the photograph was horizontal during exposure, its isocenter and nadir point will coincide with its principal point. In the case of a horizontal or plan survey, however, which is the only one that will be considered in the following passages, when the angles of tilt of the photographs are small ($\alpha < 3°$), the isocenter and nadir point will be close to the principal point, and if they are replaced by the principal point, or even by any desired contour point close to it - termed "central point" - no practically perceptible distortions will result (as proved in Sect.34).

Consequently, the angles can be measured on an aerial photograph, or radials drawn on it, for the construction of a phototriangulation grid only from these base points (principal point, nadir point, isocenter) or from the central points of the photograph.

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In processing the photographs of a mountainous region, when the distortion of directions due to relief is very considerable, the nets should be constructed from the nadir point; since the true nadir points are unknown, they are determined as the so-called arbitrary (most probable) nadir points. (see Chapter XII).

The construction of phototriangulation nets may be carried out by various methods, either on a single flight strip or for several flight strips together. The most widely used, simplest, and at the same time fairly accurate form of phototriangulation net is what is called a rhomboid chain, which is usually constructed within the limits of a single flight strip.

A phototriangulation rhomboid chain is constructed in the following manner.

Given is the flight strip of air photographs 1,2,3...(Fig.78), with a 60% end lap, on which every three adjacent photographs mutually overlap. In this case, the the central point (or nadir point) of each photograph will be represented on both adjacent photographs. For example, point 2 of the second photograph is mapped as point 2' on the first and third photographs; similarly, the contour points a and b, termed tie points and chosen in the zone of triple overlap, are mapped on all photographs. Consequently, for each photograph we may measure (or graphically construct) the angles ψ and $\mathcal{E}$, formed by the radials from its central point to the central points, mapping them on adjacent photographs, and the tie points, as shown in Fig.78.

Let us assume the coordinates of the central points of the first and second photographs to be known. Then, knowing the angles ψ_1 and ψ_2, we may, by direct intersection, determine the coordinates of point a. In exactly the same way, the angles $\mathcal{E}_1$ and $\mathcal{E}_2$ can be used for determining the coordinates of point b. Knowing the coordinates of the points a and b, and the central point of the second photograph, the position of the central point of the third photograph can be determined twice by resection from the angles ψ_3 and ψ_4 and $\mathcal{E}_3$ and $\mathcal{E}_4$.

The figure formed by these four triangles (1a2, 1b2, 2a3, 2b3) constitutes a

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rhomboid; therefore such a scheme of a phototriangulation net is termed a rhomboid chain.

For the photographs 2, 3, and 4, a new rhomboid can be constructed by the aid of the point c and d, after which the distance b_3, and so on can be determined for
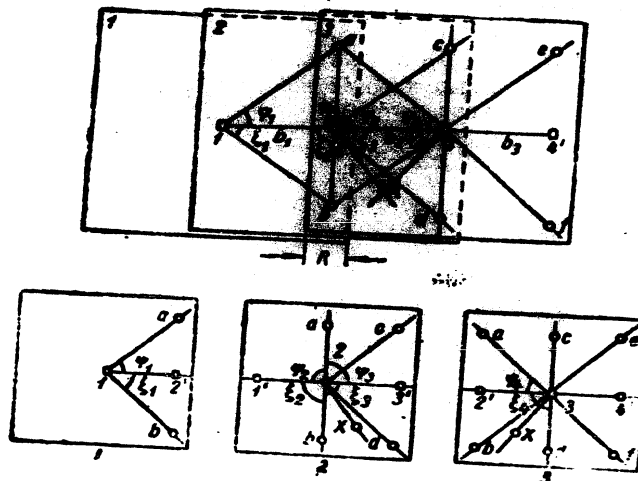

Fig.78 - Rhomboid Phototriangulation Chain

any number of photographs of the flight strip. By constructing such a rhomboid system, the radials from the central points can be laid to any desired points of the photograph, e.g., to point x (Fig.78) and determine their new position by intersection. In this way, the horizontal position of four points for each photograph can be determined, as required for transformation and mosaic assembly.

From this scheme of constructing a rhomboid phototriangulation net, and from the condition that the tie points belong to three adjacent photographs, i.e. the condition that there be a zone of triple overlap, it follows that a 60% end lap between the aerial photographs on the flight strip is required for constructing a rhomboid phototriangulation net.

The construction of such a grid may be made either analytically, for which

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purpose the angles ψ and ε are measured with special instruments, or graphically, by copying the radials from the photograph onto tracing paper, which is termed a central radial tracing. A central radial tracing is shown in the lower part of Fig.78. As a result of the great amount of work involved when using the analytical method, the net is usually constructed graphically.

To construct the nets, either the negatives of the aerial photographs or contact prints made from them may be used. The former have the advantage of being less subject to deformation, while the photographic image is better viewed on the latter. When negatives are used, the points of the net are pricked on them and copied onto the central radial tracings. When using contact prints, the central radial tracings may be copied from the points pricked on the photograph, or the radials may be drawn directly on the back of the photographs, by which the net is constructed. In productive operations, negatives are usually employed.

The graphic construction of rhomboid phototriangulation nets consists of the following processes:

    a. Selection and pricking of the points on the photographs;

    b. preparation of the central radial tracings;

    c. construction of the net;

    d. reduction and connection of the net.

52. Selection and Pricking of the Points

The pricked contour points must be distinct, sharp, and easily recognizable on on the photographs. Such points, e.g., may be plowed fields or other agricultural tracts, the intersections of roads or ditches, the corners of low structures, small shrubs, or haystacks (pricked in the center). It is forbidden to prick: blurred, unclear, indistinct, or rounded outlines; outlines intersecting each other at an angle of less than 30° (or more than 150°); corners of shadows or the shady sides of objects; high buildings or trees; shrubs and haystacks more than 0.3 mm in di-

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ameter on the photograph, incised water channels; variable outlines (boats and rafts on the water, automobiles and trains on roads, etc.) Points on steep slopes are also unsuitable.

The pricking of points, on the accuracy of which the accuracy of the phototriangulation depends to a great extent, must be done with extreme care and precision. The center of the prick must exactly coincide with the point to be pricked, which must be represented in the form of a geometric point. The prick must be small, with a diameter of not more than 0.1 mm.

a. The Pricking of Central Points

The central points must be pricked at the best contour point located closest to the principal point of the photograph (and not further from it than the radius determined by the expression $r = f_k : 50$). A special transparent sheet is prepared for selecting the contour of the central point: The coordinate induces of the photographs are pricked on the celluloid, and the opposite points are connected by straight lines intersecting in the principal point of the picture; then, a small circle with this point as its center and with a radius of $f_k:50$, is cut out of the celluloid. The contour point taken as the central point must be easily identified on both adjacent photographs of the flight strip, on which the point is also pricked. On the photographs (negatives) the prick is surrounded with a square in India ink.

In constructing the nets for subsequent stereoscopic processing of the photographs, rather than pricking the central points, the nadir points are pricked, regardless of whether they coincide with any marked contour points.

If the central point (or the nadir point) does not coincide with any contour points, e.g., if it coincides with a lake and cannot be recognized and pricked on the adjacent photographs of the flight strip, then the initial radials (the radials to the central points of the adjacent photographs) on these photographs are drawn

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"artificially" (cf. Sect.53).

b. <u>Pricking the Ground Points of Control</u>

Ground points of control serve to orient the phototriangulation nets on the plotting board and to reduce them to the assigned scale. The location and density of the points of ground control are dictated by the required accuracy of triangulation, based on the laws of cumulation of errors in the nets. Ordinarily the points of ground control are given in each of a number of photographs (from 6, 8, and up to 20 photographs, depending on the scale of the flight strip and the scale of the map to be prepared) in the zone of side lap between the flight strips and, wherever possible, in a zone of triple overlap of the photographs in one flight strip. After determining the points of ground control on the ground, they are identified and pricked on the contact prints, putting a sketch and description of the contour point so selected on the back of the photograph.

The pricking of the points of ground patrol in the process of phototriangulation consists in pricking the fixed contour point on all negatives of both the given and the adjacent flight strips, on which the identified contour is shown. In the identification and pricking of points in the negatives, the field prick on the negative, and also the sketch and description on the back of the photograph are used as a guide. The prick points on the negatives are marked with triangles in India ink.

c. <u>Pricking of the Tie Points</u>

It follows from the principle of constructing a phototriangulation net and from Fig.78, that the tie points a and b should be selected on contour lines lying on the center line of the triple overlap of the photographs of one flight strip and as far as possible from the center lines of the photographs, but not closer than 1 cm from the edge of the photograph (Fig.79). On the end photographs 1 and 3, these points are located at approximately equal distances l from the edges of the

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photographs.

The tie points are pricked on each individual flight strip independently, and are not repricked on the adjacent flight strips. On the negatives they are encircled in India ink.

d. Pricking of the Transformation Points

The transformation points are also marked on the contour points of the photograph, located at the center of the quadruple (end and side) lap of four adjacent
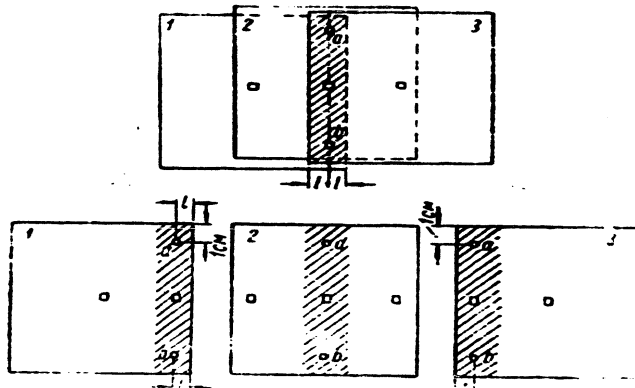

Fig.79 - Tie Points

photographs of two adjacent flight strips (Fig.64).

If four such photographs are placed side by side, as shown in Fig.80, then the transformation points must be selected at equal distances l_1 from the end edges of the photographs and at equal distances l_2 from the side edges of the photographs.

The rectification points are pricked on both flight strips, when viewing them simultaneously. They are encircled in India ink on the negative, with the subscript "x".

The total number and position of the points pricked on the photograph is shown in Fig.81. In addition to the points given in this general scheme of their loca-

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tions, the ground points of control are also pricked on a few photographs.

53. Plotting the Initial Radials

In cases in which the central points or the nadir points are not recognized on the adjacent photographs, the initial radials are plotted "artificially".
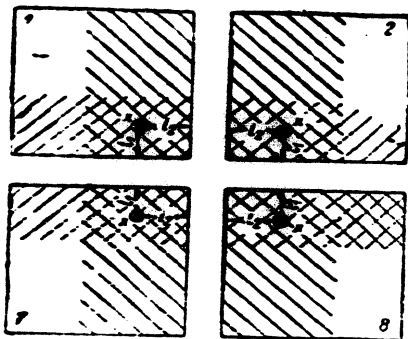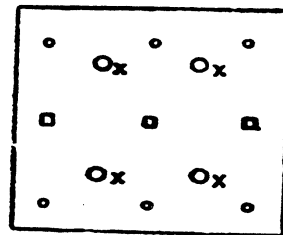
Fig.80 - Selection of Transformation Points

Fig.81 - Location of the Points of the Phototriangulation Net on the Photograph

Artificial drawing of the initial radials is based on the fact that a straight line on the ground is also represented by a straight line on the photograph; consequently, if the terrain is flat, contour points lying on a straight line on one
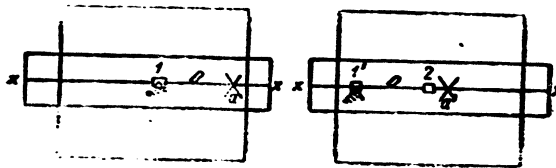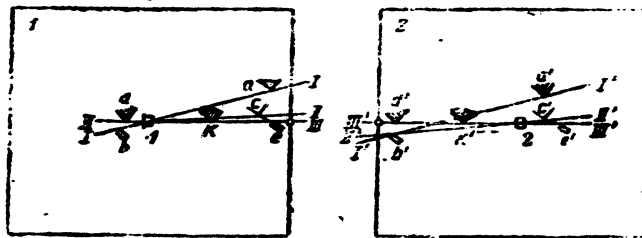
Fig.82 - Plotting the Initial Radials

photograph must also lie on a straight line on the other. In broken terrain, this postulation is valid only for an initial radial passing through the nadir points of both photographs. If such straight lines, containing corresponding contour points

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pass simultaneously through the nadir point (or the central point) on each of the photographs, then these straight lines will form a straight line interconnecting the

Fig.83 - Plotting the Initial Radials by Successive Approximation

nadir points (or central points) i.e., they will form the initial radial (Fig.82).

a. Plotting the Radials with a Slotted Celluloid Templet

First case. One of the nadir points (or center points) is identified on the adjacent photograph.

Let the central point I be identified on the second photograph, while the central point 2 of the second photograph does not coincide with the contour and cannot be pricked on the first photograph. Then, taking a strip of clear celluloid film with a straight line xx etched on it by a fine needle (Fig.82), this line is then superimposed on the pricked central points 1' and 2' of the second photograph. All the contour points of the photograph coinciding with the line xx lie on the initial radial 1' - 2, After selecting the contour point a' most remote from the point 1' and coinciding with the line xx, this point is then identified on the second photograph. Placing a second strip of celluloid, with a straight line etched on it, on the points 1 and a of the first photograph, the required initial radial is obtained, which is marked on the edge of the photograph by a prick through the celluloid.

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Second case. The nadir points (or central points) are not identified on adjacent photographs.

In this case, the initial radial is determined by the method of successive approximations. The line xx of the celluloid templet is placed on the first photograph in position I (Fig.83) and is matched with the prick of the central point I;
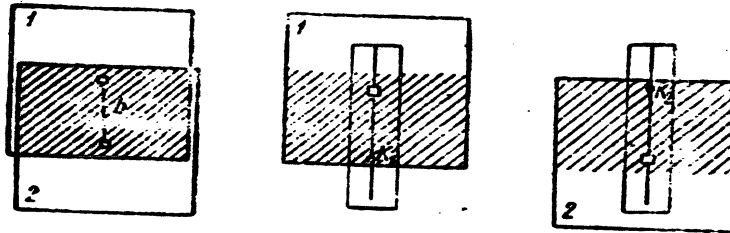

Fig.84 - Tracing the Initial Radials on Rotating the Photographs through 90°

the contour point a is selected in the region of the central point 2, on the assumed line of the initial radial. In this position of the templet line, we select on it the contour point b in the region of the central point 1 or, preferably, in the exterior part of the initial radial, which increases the accuracy of the determinations.

Setting the second line xx in the position I' on the corresponding points a', b' of the second photograph, we note that the dash does not pass through the central point 2. We then rotate the line around point b' into position II' to its coincidence with the prick of the central point 2, and mark the new contour point c' on the templet. Then, the line xx on the first photograph is rotated about the central point 1 into the position II, to coincide with the contour point c. The line now coincides with point b, on which a new contour point d is then selected. The line xx is rotated on the second photograph about the central point 2 into the position III', to coincide with the point d, on which, in the region of the center 2,

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a new contour point e' is selected. Next, the line xx on the first photograph is rotated about the central point 1 into the position III, to match the contour point e. This rotation is, in practice, so small that the point d does not leave the templet line. This results in a position in which the line xx on the first and second photographs passes through a series of corresponding contour points d, k, e, (d', k', e') and through the pricks of the central points 1 and 2. These lines are consequently the same and, since they pass through central points (or nadir points), they are initial radials. The initial radial so obtained is noted on the edge of each photograph by a prick through the celluloid.

b. Stereoscopic Plotting of the Initial Radials

The stereoscopic plotting of the initial radials gives more accurate results, since the stereoscopic method of observation is considerably more exact than the monocular*.

For stereoscopic plotting of the initial radial, the adjacent photographs 1 and 2 of one flight strip are rotated through 90° with respect to the normal position and are placed in the position shown in Fig.84. In this position they are viewed under the stereoscope and shifted until the images of the two aerial photographs merge into one distinct stereoscopic image. In this position of the photographs, the earth's surface will be perceived as a plane.

If the photographs are then superimposed with fine lines traced on glass or Plexiglass plates (Lobanov rulers), these dashes will likewise be perceived stereoscopically with respect to the contours of the photographs. Thus, if the line on the left photograph passes through the contours a_1, b_1, c_1 (Fig.85), while the line on the right photograph does not pass through these contours and is rotated with respect to them through a certain angle γ, then the line will be stereoscopically perceived not as lying in the apparent plane of the ground, but inclined to it in

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* The principles and methods of stereoscopic vision are discussed in Chapter VIII.

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space. In other words, if the points a, b, and c of the left and right photographs
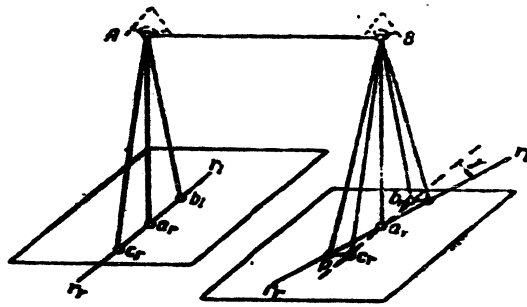lie on parallel lines, they will not have the parallax distortion p and will appear
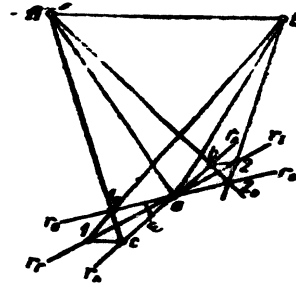

Fig.85 - Stereoscopic Plotting of   Fig.86 - Parallactic Shift of the
           the Initial Radials   Templet Line

to be located at the same height. The points of the right line $r_r r_r$, as a result
of rotation with respect to the left line, cause a parallax distortion p from the
points a, b, c, and from the points of the left line $r_l r_l$ superimposed on them.
This makes them appear as lying at a height differing from that of the points a, b,
and c. Figure 86 shows how this perception of the noncoincidence of the templet
lines with the corresponding contours is evolved. The parallax distortion $cl = p$
of the dash r_r at the points c leads to an intersection of the visual rays Ac and
Bl at the points c and l of the dashes, in the point l_0, lying above the plane of
the apparent locality and its point c in the intersection of the rays Ac and Bc.
The parallax distortion of the dash r_r at point b leads to an intersection of the
visual rays Ab and B2 at the point 2_0 lying below the plane of the apparent local-
ity. At the point a, where there is no parallactic shift (parallax difference),
the floating line $r_0 r_0$ intersects the apparent ground plane.

If the templet line passes on both photographs through corresponding contour
points and, consequently, does not produce parallax distortion, the line, when
viewed stereoscopically, will appear to coincide with the apparent plane of the

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locality at all points. Consequently if, by rotating the line, a position such that it merges with the apparent plane of the terrain is reached, it will pass through corresponding contours on both pho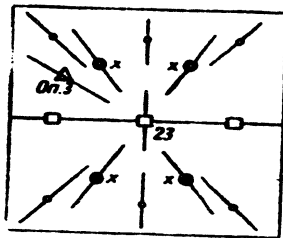tographs. If in so doing, the line on both left and right photograph can be made to pass through the pricks of the nadir points (or the central points), then the position of the lines will give the required initial radial, which is then pricked by a needle through the special openings K on the ruler (Fig.84).

Fig.87 - The Radial Tracing

The same principle is used in the stereoscopic plotting of initial radials with more complicated instruments, where the templet lines may be fixed and the photographs, placed in special holders, rotated instead, and where, instead of the lines, special floating marks are used.

54. Laying the Rhombic Net

To obtain in graphic form the radials from the central point to all other points pricked on the photograph, all points are carefully copied on tracing paper, encircled with a pencil line, and marked with subscripts as on the photograph. Then, from the central point, the radials are traced to all other points by a calibrated ruler, with India ink or ordinary ink, as shown in Fig.87.

The radials must be fine (0.1 mm) and must pass exactly through the centers of the pricks of the points. At the central point, the photograph reference number is entered.

In accordance with the general scheme of constructing a phototriangulation net (Fig.78), each succeeding central point (e.g., the center of photograph 3) is determined with respect to the preceding ones (centers 1 and 2) by resection along three radials (at the points a, 2, and b) on a single straight radial (2 - 3) - the

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initial radial. For this reason, the procedure of graphic construction of a plane

phototriangulation rhombic net is as follows.

On a certain commercial base (tracing paper, Goznak paper, Astrolon), on which

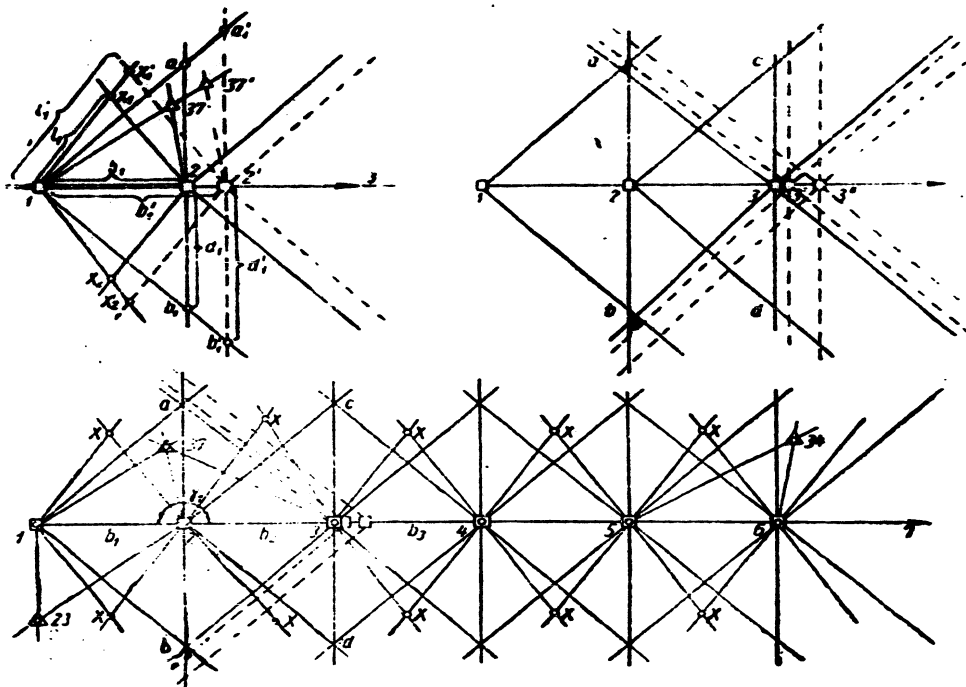the construction is pricked through to the points of the nets, the first radial

Fig.88 - Construction of the Rhombic Net

tracing of central radials is placed. This is superimposed by a second tracing, in

such a way that the corresponding initial radials 1 - 2 of the two tracings will

coincide (Fig.88).

The distance between the central points of these tracings, which is termed the

first base b_1 of the net, determines the scale of the entire net. If, after match-

ing the first two tracings along the initial direction, the second tracing is

shifted along the initial radial without distrubing the coincidence so obtained,

then the base will be shortened (or lengthened); at the same time, all the triangles formed by the resections at the remaining points of the net will be proportionally reduced or enlarged.

In this way the selection of the value for the first base determines the construction scale of the net.

After laying the first two tracings, the positions of points a and b are obtained at the intersection of the corresponding radials, which positions are necessary for determining the position of the third tracing with respect to the first two, and the initial radial 2 - 3 on the central point of the third tracing.

The third tracing is matched with the initial radial 3 - 2 with the corresponding radial 2 - 3 (position 3", Fig.88) which has already been obtained in the net. In so doing, the radial 3 - a and 3 - b need not necessarily pass through the points a and b previously obtained. Since these directions must pass through the tie points a and b, the tracing 3 is shifted along the superimposed initial radial 2 - 3, moving along the rays 3 - a and 3 - b, toward the points a and b, until they coincide. If the ray 3 - a has been made to pass through the point a (always being careful to prevent the initial radials from becoming separated, and being sure that they remain superimposed), then, as a result of the existence of errors in the radials, ray 3 - b may perhaps not pass through the point b, thus forming a small triangle of error (position 3 in Fig.88). The altitude of the triangle of error, measured from the base of its side along the lateral radial 2 - a, must not exceed 0.3 mm. If it does, the existence of a gross error in the radials must be assumed, which will have to be corrected by verification, first of the initial radials and then of the radials to points a and b.

If the altitude of the triangle of error is within the permissible range, the third tracing is shifted along the initial radial, with the coincidence of the initial radial being maintained under all circumstances, to the position 3, until the ray 3 - b passes through the center of the triangle of error obtained in the po-

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sition 3'. In this case, the triangle of error at point b will be halved, but the same triangle of error arises at point a. These connected triangles of error at points a and b must be equal in size, but opposite in direction, i.e., with respect to the lateral rays 2 - a and 2 - b, the vertexes of the triangles must be turned in opposite directions (position 3). The altitudes of the triangles after the connection must not exceed 0.15 mm. The initial radials here must under all circumstances coincide.

The closure of the triangles by rotation of the tracing involving a shift in the initial radials, is categorically forbidden, since this would break up the net.

By establishing the position of the third tracing, the positions of the points c and d and of the initial radial 3 - 4 are obtained, which are required for determining the position of the following or fourth tracing. The fourth and all succeeding tracings are laid in the same way as the third one.

This results in a phototriangulation net of arbitrary scale, which is termed a "free" net, i.e., one which has not been reduced to the scale of the plotting board and is not oriented on it.

The points of the net obtained at the intersection of radials and also the central points, are pricked with a needle into the base, on which a sheet of tracing paper has been placed. At the points where permissible triangles of error have been obtained, the points are pricked in the center of such triangles. All points on the base are encircled with a bow compass (its diameter in the image, on reduction should be 1.5 mm); the central points in addition are surrounded by a square, with a subscript showing the reference number of the photograph, while the ground points of control are surrounded by a triangle bearing a number or place-name as subscript. The name of the trapezoid and the number of the flight strip are inscribed on the base.

It is recommended that the laying of free phototriangulation nets be performed on a mounting board with transillumination. Otherwise, especially under field con-

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ditions where the plan position of points in difficultly accessible areas is to be
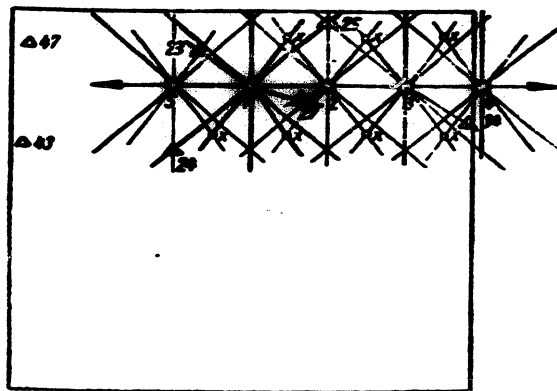determined, the nets of plan phototriangulation are constructed directly on the

plotting board to the assigned
scale.

The construction of nets at
the scale of the base (the plot-
ting board) is done as described
above, while the position and
length of the first base (i.e., the
first two tracings) must be found
from ground points of control de-
termined on the terrain itself
(Fig.89).

Fig.89 - Construction of a Net at the Scale
of the Base

The position of the first
tracing is determined by the Bolo-
tov method, i.e., the tracing 1 is placed on the plotting board in such a way that
the central rays to the ground control points 23, 24, 25, and 27 pass through the
pricks of these ground control points on the plotting board. The position of the
first tracing already characterizes the orientation of the net on the board, since
the initial radials to the adjacent photographs thus are given an accurately de-
fined position.

The second tracing is matched with the corresponding initial radial from the
first tracing and is then moved along it until the radials to points 25 and 27 pass
through the pricks of these points on the plotting board. Since, in this case,
the resections of the ground control points in the net coincide with the position
of the points of the field control on the board, it follows that the base b_1 , as
also the entire net, now has a scale equal to that specified. The technique of the
further construction of the net does not differ from that described above.

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Passing now to the construction of a net on one side (right), and then also on the other side (the left) to the field control points, for example to point 34 on tracings 3 and 4, we may obtain a position where the intersection of the field control point in the net does not coincide with its position on the board. The mean allowance of the error of closure m so obtained is defined by

$$m = \pm 0.35 \; Rb \; \frac{m_\varepsilon}{\rho} \; \sqrt{2 \; n^3 - 3n^2 + 5.7 \; n} \qquad (26)$$

where n is the number of bases from initial to final; m_ε is the error in the radial expressed by the value 4-5'; R is the scale ratio, equal to the ratio between the scale of the survey and the scale of the map; and b is the value of the base on the photograph, in millimeters.

The error of closure is eliminated by gradually shifting the tracings in the direction of the true position of the ground control point, but in so doing we attempt not to disturb (within the limits of graphic accuracy) the coincidence of the initial directions and the intersection of the rays of the tie points.

After connecting the region of the net on the right side from the first base, the construction of the net left of the first base is continued until closure at the minor field control points. After completing the construction and closure of the net, we prick its points on the plotting board.

In those cases where the extension of the nets is performed on an assigned scale (as described above for a construction directly on the board), the laying must be started from the center, extending the net to both edges. This will result in the smallest deviation from the specified scale and also furnish proper orientation of the net. For cases of free net construction, i.e., nets at an arbitrary scale and with arbitrary orientation followed by reduction on the plotting board, the laying of the net is usually started from the edge (left); the accuracy of the net will be the same in either case.

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55. Reduction of Horizontal Phototriangulation Nets

The process of reducing free nets to scale and orienting the board or inserting the net between field control points is termed reduction of phototriangulation nets.

Reduction may be performed either by an optico-graphical method, using special projectors known as reducing printers, or by the graphical method.

The principal method used is that of optico-graphical reduction.

Figure 90 shows a schematic diagram of a projector reducing printer, where 2 is the holder carrying the phototriangulation net (on wax paper or Astrolon) and 3 is the lens projecting the image of the net, illuminated by the lamps, onto the screen 1. The planes 2 and 1 must be parallel; in that case, the image on the screen will be similar to the net.
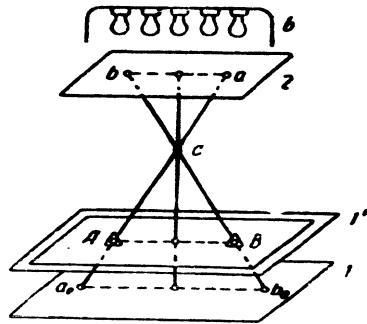
If the position of the screen is shifted from position 1 to position 1', then, as indicated by the sketch, the image a_0b_0 is reduced to the size AB. The problem of changing the scale of the net is thus solved simply and simultaneously for all points of the net. To preserve the sharpness of the image, the lens 3 must be advanced slightly.

Fig.90 - Schematic Diagram of a
Reducing Printer

If a plotting board on which the points A and B (field control points) have been plotted is placed on the screen, then the line AB may be placed along the line ab of the image by rotating the board on the screen. By changing the scale of the image still more, the projection of the points AB can be matched with the points A and B of the board. This solves the problem; the image of the net is reduced to the scale of the board and is oriented on it by the two control points A and B.

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Thus, to reduce the net it is essential to have not less than two control points. To lay the net, the photographs are so selected that the minor field control points are located along the edges of the net (for some intermediate nets or flight strips, points in common with adjacent nets are sometimes used as minor control points).

After matching the minor control points of the image and of the board, all points of the image of the net are transferred to the board with a sharp hard pencil, i.e., all the rectification and central points are so transferred.

The nets can be reduced on any projection instrument (reducer, rectifier, projector) which allows the scale of the image to be changed while maintaining its sharpness and assuring similarity of the image to the projected net; this latter condition requires that the planes of the negative and screen are parallel. However, the transforming printers and the projectors now in actual production have holders that are too small to accomodate a net, and this requires a preliminary photographic reduction of the net, while still preserving similarity. This inconvenience is eliminated in a special instrument, the Popov reducing printer.

Fig.91 - Popov Reducing Printer

The Popov printer (Fig.91) consists of a projector whose schematic diagram is shown in Fig.90. However, the screen 1 of this instrument, in contrast to the diagram (Fig.90), is fixed, and the scale is changed by shifting the holders 2 with

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respect to the screen, and by shifting the lens 3 to maintain sharpness. These shifts (change of scale while preserving sharpness) are made automatically, using the special device 4, termed a rhombic scale inverter which is actuated by the hand-wheel 5.

The holder consists of a frame with a cut glass plate 60 × 60 cm, on which the net is placed, covered by a cover glass, and illuminated by the electric lamps of
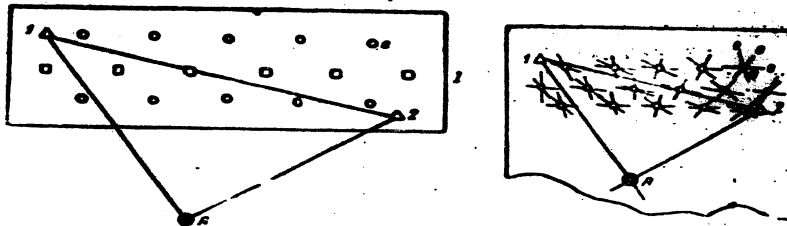
Fig.92 - Graphic Reduction

the illuminator 6. The scale factor of the instrument can be varied from 0.6 to 1.5. The size of the screen is 100 × 100 cm and the height of the instrument, 2.55 m.

56. Graphic Reduction

Graphic reduction is used in field work when no reducing printer is available. The principle of graphic reduction consists in the intersection of points in the net with the control points, i.e., with a certain assigned base.

Let the problem posed be the reduction of the free phototriangulation net I on the plotting board by the control points 1 and 2 (Fig.92). The points of the net, subject to reduction, are pricked into the tracing. The control points 1 and 2 are connected on it by a straight line which will serve as a base. In addition, to obtain optimum intersection of the points near line 1-2, the arbitrary auxiliary point A is selected on the tracing and is connected by straight lines with both control points. For long nets several auxiliary points are selected.

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The tracing is then placed on the board in such a way that the points 1 of the tracing and the board coincide, while the radial 1 - 2 of the tracing passes through point 2 of the board. After orienting the tracing in this way, all of its points and the auxiliary point A are pricked onto the board and the radial drawn to them from point 1.

The tracing is then advanced along line 1-2 so that the control points 2 of the tracing and the board coincide and the radial 2 - 1 of the tracing passes through 1 of the board, after which all points of the net and the auxiliary point A are again pricked on the board. The new pricks are then connected with point 2. The intersections of corresponding radials furnishes the wanted position of the points of the net at the scale of line 1-2, on the plotting board. The same intersection is obtained on the board for the position of the auxiliary point A.

The points of the net lying near the line 1-2 intersect poorly with points 1-2. To obtain a reliable intersection, point A of the tracing is matched with the position of this point on the board, and the tracing is so oriented that the points 1 and 2 of the board lie on lines A-1 and A-2 of the tracing. After pricking all points of the net in this position on the board, the new pricks are connected with point A. The radials from point A to the points of the net yield reliable inter- sections for such points.

CHAPTER VI

TOPOGRAPHIC INTERPRETATION

57. Basic Principles of Topographic Interpretation of Aerial Photographs

One of the basic processes in making a topographic map from the information gathered through aerial surveys is photographic interpretation; in other words, the recognition of the characteristics of the area as recorded on the aerial photograph and which are vital for making a map of the area. Depending on the purpose for which the map is intended - for topographic, ground surface, geological, agricultural, or any other purpose - the corresponding interpretation will be called topographic, surface, etc. In this study, intended for topographers, only the basic principles of topographic interpretation are discussed.

As is known, the various elements of an area, such as: roads, forests, cultivated areas, populated areas, etc., must be indicated on a topographic map. All these topographic details must be placed within their respective positions and boundary limits on the map, and must be given identifying marks and designations. For example, in drawing a road into a topographic map, it is absolutely necessary to describe the character of the road by an identifying symbol which would indicate whether the road is an expressway, an improved dirt road, a side road or what other type, and also to indicate the populated areas which it joins. In drawing in the outlines of a forest it is necessary to indicate the type of trees and to identify clearings and burnt-over areas. All the other elements are also described in similar breakdowns. The number of separate elements as well as the degree of detail in

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which each element is described depends on the scale of the map being prepared and on the natural features of the area. For example, a foot trail does not have much significance in the Central European region of the Soviet Union but is very important in the relatively inaccessible regions of the North and Far East. The physical elements, entered on the topographic map, are defined by means of a specially developed key chart; by the naming of populated areas, by boundaries and landmarks, by waterways, etc.

Many different objects are recorded on the aerophototopographic survey chart, objects which have to be drawn into the topographic map. This great detail recorded on a photograph raises the problem of defining where certain parts of the landscape appear on the print and for determining their characteristic features. If this problem could be solved completely in studying aerial photographs, the field survey would be limited to compiling the names of towns etc., and all of the interpretation would be done with instruments. However, the small scale of aerial photographs and the necessity to secure data on the terrain which cannot be recognized or recorded by an aerial photograph, require additional field surveying to obtain such data. Therefore, topographic surveyors provided with aerial photographs, go over the photographed terrain, marking all elemental boundaries required for map preparation on the prints. He also marks on the prints objects which were not recorded by the photograph and inserts the names of places. In practice, the field survey is usually combined with a photointerpretation. For example, aerial photographs at a scale 1:50,000 to 1:70,000 cannot be interpreted to show, nor is it possible to identify on them, lines of communication. It is difficult to determine the nature of a forest, etc. Even with prints at a larger scale it is impossible to establish the type or the functional use of a photographed building. Is the building of stone or of wood? Considerable additional data has to be secured.

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58. Identifying Features, which can be Used in Photointerpretation

All identifying features, used in laboratory interpretation of aerial photographs, are classified into two groups, the first group comprising direct data and the second group indirect data. The first group, as used for the photointerpretation, contains the data that positively identify the object in question, its shape, size, height, and color. The indirect group refers to data that establish the relative position of an object with respect to the surrounding objects, the interpreted classification of the landscape as a whole, and to supplemental data obtained from existing statistics and descriptions.

The shape of an object is one of the most important facts in identification. The object is presented on prints without any distortions of its shape when the optical axis of the camera is exactly vertical and the object is on a horizontal plane. For example, buildings appear on prints as rectangles, roads appear as developed ribbons which represent improved highways if they are rather straight, and railroads if they are quite straight. Rivers and streams appear as winding lines, etc. If the optical axis is slanted, the shape of the image of an object on the photograph differs from the actual shape. This distortion is very small on prints used for map making. The height of an object, however, has a great influence on the change in shape of an image. This is so because the higher elevations of an object will be distorted on the image (relief distortion), as in the case of an aerial photograph of a town, the high buildings may not only make the adjoining streets appear narrower than they are, but may in places hide the street completely. Therefore, it is advisable to judge the shape of a high building from an aerial photograph only on the basis of stereoscopic examination (Sect.69), when the interpreter is able to see the depth of field of an object, with the aid of a stereo pair. It is often possible to draw correct conclusions as to the shape of an object from only one print, but in this case the shadow thrown by the object, as recorded on the print, is of great assistance.

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For a comprehensive interpretation, the size of an object is a desirable cri-
terion. Knowing the applicable scale for the print (the scale being governed by the
focal length of the camera and the altitude of flight during photography) and the
approximate dimensions of the actual object, it is easy to determine the actual size
of the object as shown by the print on a reduced scale. This permits a more accu-
rate description of the object to be interpreted. Thus, e.g., rectangles on the
prints represent individual buildings, gardens, agricultural areas, and some other
elements of the landscape whose dimensions can be calculated by their representa-
tive size. At the same time, on large-scale prints (1:5000 to 1:10,000), individual
buildings often appear larger than large agricultural areas on small-scale prints
(1:50,000 to 1:70,000).

One of the important factors of interpretation is the height of objects, which
is determined by their shadows or through stereoscopic examination of the prints.
To determine the height of an object by its shadow, it is important to know not only
the direction of sunlight at the time the photograph was taken (which can be deter-
mined by the direction of the shadow) but also the height of the sun above the hori-
zon. It is definite that any variation in the height of the sun will cause a vari-
ation in the length of the shadow of an object. This makes it impractical to judge
height by shadow alone. Furthermore, in many cases, high objects do not throw a
shadow. The surest way to judge height of objects is by stereoscopic examination
of a pair.

Color of an object is a very important factor in interpretation or, to be more
specific, the ability of the object's surface to reflect ranges of the color spec-
trum. An object absorbs some rays of the sun and reflects others. The reflected
rays have a certain range within the spectrum, giving an idea about its color, by
its image. Any light reaching the light-sensitive emulsion of a film first passes
through a thick layer of atmosphere and then through the camera lenses. The re-
sultant effect on the film is well known (Sect. 11). This light-sensitive surface

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reacts differently to the different rays of the light spectrum. Thus the variable
quantity of the reflected rays and their spectral composition create a variable den-
sity on the composite image. This makes it possible to judge the color-reflecting
ability of the corresponding object. By using different types of film, which have
different coefficients of color sensitivity, and by using optical filters of differ-
ent color, the images of variable objects on the prints are not reproduced in stand-
ard tones of shading. For example, when using panchromatic film, water surfaces
appear as dark tones (appearing in light tones on the negative), whereas infrachro-
matic film will cause these same water surfaces to appear in light tones on the
print. This variance of tone, in photographing the same objects, is advantageous
for interpretation, since it permits better distinction between objects, which
would be reproduced in the same tone on one film but not on another.

The ability of terrain features to reflect colored light, the spectral sensi-
tivity of film, and the use of optical filters were used as basis for special refer-
ence charts published, which are composites of samples for photointerpretation. By
comparing the aerial photographs, with respect to their geometric (shape and size)
and optical (tone qualities) characteristics, with developed samples, it is possi-
ble to interpret with greater accuracy.

Factual data of interpretation give a basis for indirect data. For example,
a road ending at a river and continuing on the other side of the river indicates
the existence of a bridge or a ford, numerous paths converging at one point in a
populated area indicate the presence of a well at that point, etc. In addition,
supplemental practical data are of importance as is also a geographic study of the
entire terrain. For example, a topographic surveyor working in northern regions
invariably will study its geographic characteristics to be prepared for the type of
features he might have to face during interpretation. Data obtained from rosters
and lists in larger towns of the region and lists of names greatly facilitate the
process of interpretation.

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59. The Sequence of Steps in Photointerpretation

Prior to interpreting an aerial photograph, the topographer makes a study of the region from literature data and determines the basic characteristics of the region he will be confronted with during interpretation. If the scale for the map composite is to be larger than 1:100,000, the topographer will make a detailed stereoscopic examination of all photographs and mark them with a route for future field surveying. This route must include all populated areas and all sections of the prints where interpretation by instruments will be uncertain, such as boundary areas of elements and also their composition. Certain areas for which photointerpretation may be accurate enough, still absolutely require a field survey.

Field surveying consists in recording objects on the photographs, which are not distinguishable on these photographs and which are needed for making the topographic map (communication lines, wells, bridges, etc.). The surveyor also draws in the established boundaries of elements which includes the collected list of pertinent names. The elements which were not distinguishable are drawn on the photograph, fixing their location by distances measured from the distinguishable objects.

All identified objects are drawn into the photograph in the field, using designated key charts. Every day, after returning from the field, the topographer makes the day's entry on the photographs permanent by going over them with India ink. If the interpretation is being made from contact prints or from enlargements, each photograph is marked at the start of the work with the boundaries of the work areas, which progress approximately along the center line up and down and across the composite. The interpretation is done along these working areas of a print. Simultaneously with plotting the interpreted objects on the photograph, names are inserted, the course of rivers is indicated, and all other specified data are recorded.

When making a map at a scale 1:10,000 of an area which is difficult to penetrate, it is senseless to attempt a field survey by the above method. In such cases, after a comprehensive study of the prints obtained, the topographer goes into the

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field with only the prints which are most representative of the general area. After the field survey, these prints can be studied as standard examples and used as reference indicators for an interpretation of the rest of the prints. In this case, geographic observations recorded in report form are of great assistance.

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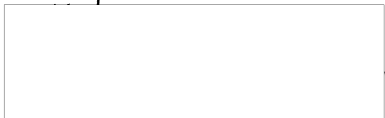
CHAPTER VII

TOPOGRAPHIC-GEODETIC WORK IN VERTICAL-COMBINED AEROPHOTOSURVEYING

60. Tying in Aerial Photographs on the Plan

Tying in of aerial photographs consists in defining the plan position of several contour points of the terrain which can be recognized on the aerial photograph (identification points).

The number of plan identification points and their distribution on the photographs depend on the scale of the map being prepared, the scale of flight, the photogrammetric method of filling in or densifying the control net that is being used, and also on the physical, geographic, and economic characteristics of the terrain. To compute the number of identification points, the formula for cumulative errors is used which, for a plan phototriangulation series, takes the form:

$$m = \pm 0.35 \; Rb \; \frac{m_\varepsilon}{\rho} \sqrt{n^3 + 11.3 \, n + 6.5 \, \frac{1}{n} + 33} \tag{27}$$

where m denotes the mean-square error in the position of median points of the phototriangulation series at the scale of the map; R is the reduction factor (ratio of the scale of flight to the scale of the map); b is the size of the base at the scale of the photograph; m_ε is the mean-square error in one direction; and n is the number of bases between identification points. The allowable error in the position of points on the plan is given in the respective instructions so that, in computing

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the density of the geodetic base, it can be assumed that its magnitude is known. Then, knowing the distance b between the principal points of the photographs, the scale of the map being compiled, the scale of flight, and the mean-square error in one direction m_e, the equation

$$n^3 + 11.3\, n + 6.5\, \frac{1}{n} + 33 = \frac{9\, m^2 \rho^2}{R^2\, b^2\, m_e{}^2} \qquad (27a)$$

will yield the permissible number of photographs between plan control points, and consequently, the distance L between bench marks. This distance is obtained from the equation

$$L = B\,(n-1) \qquad (27b)$$

where B is the length of the photographic base on the terrain.

61. Drawing up a Plan for Tying in Identification Points

Bearing in mind all of the above conditions, a plan for positioning geodetic tie-in contour points (or identification points) can be drawn up.

The planning is carried out by rough mosaic mounting of the aerial photographs on which all known points of the geodetic grid are entered (approximately or exactly, depending on identification) along with the proposed plan identification points.

The plan identification points are usually so selected that they fall in the center of the side lap between flight strips; then each identification point can be used for two flight strips. The plan identification point must not be located along the center line of the photographs since, in that position, it will be very difficult to incorporate into the phototriangulation grid.

In planning, the topographic features of the terrain must be taken into consideration to draw up a project easily reduced to practice. When marking off a point, not only the ease and reliability with which it can be identified but also the

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possibility of determining its geodetic coordinates must be considered. Therefore, in drawing up a plan, stereoscopic methods for viewing aerial photographs are a prerequisite.

The photographic representation of contour points must be clear and sharp.

The following may be selected as identification points: crossings of roads or paths, intersections or corners of tilled fields, foundations of buildings or structures not covered by shade, unshaded bases of isolated trees or bushes, etc.

Selection of identification points on poorly defined contours or contours that are circular in form should be avoided since they cannot be accurately identified.

• PROVISIONAL MARK
OF CENTRAL PRINT

▲ PLAN IDENTIFICATION
POINT
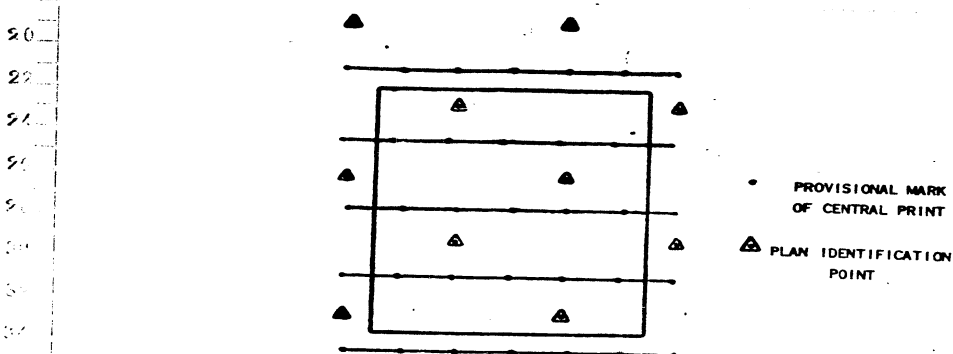
Fig. 93 - Standard Scheme of Locating Plan Identification Points
on a Trapezoid at a Scale 1:25,000

For example, identification points in ravines, among trees and bushes, in the deltas of rivers or brooks, or at the crowns of trees are unsuitable.

. From the viewpoint of geometric location, the identification points should be positioned so that they can be intersected by no less than four central points of the photographs, i.e., they should be in the center of the zone of side lap of adjacent flight strips, and should preferably be in zones of triple end lap. The density of identification points will depend, as indicated above, on the scale of the map and other conditions. Thus, e.g., for a map at a scale 1:100,000, the intervals

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between plan identification points along the line of flight may be as large as 60-100 km, while for maps at a scale 1:25,000 or 10,000 (Fig.93) it has been established that "plan identification points should be located at intervals of 4.5 to 9 km. Aerial photographs with insufficient end lap and toning should be tied in with additional identification points" (Bibl. 17 and 18).

After a plan is drawn up, all points selected for tying in and all bench marks are plotted on the reproduction of the rough mosaic and designated by conventional symbols.

The aerial photographs and the reproduction of the rough mosaic, with the selected points marked on it, are turned over to the personnel charged with tying in the aerial photographs.

62. Performing Topographic-Geodetic Work for Tying in Aerial Photographs

The task of tying-in aerial photographs to points of the geodetic bases consists of the following operations:

1) Identifying the contour points (identification points), marking them on the aerial photographs with pin pricks, and sketching the identification points (contours);

2) Fixing the identification points on the terrain;

3) Topographic-geodetic surveying to determine the plan position of the identification points.

Before tying-in is started, a thorough reconnaissance of the terrain should be made to confirm the choice of geodetic methods and to check the disposition of planned identification points.

a. Identifying, Pin Pricking, and Sketching Points

The process of identification consists in collating and matching identification points with the corresponding points on the terrain, of marking them on the aerial photographs with pin pricks, and of sketching their position. These are all simple

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but very important operations.

The process of identification requires great care and full attention, since the possibility of oversights in identification must be avoided at all cost. An incorrectly identified point, even if its geodetic coordinates are correct, is completely useless.

Identification on the terrain should be carried out with an error not exceeding the graphic accuracy of identification on the photograph. It may be taken for
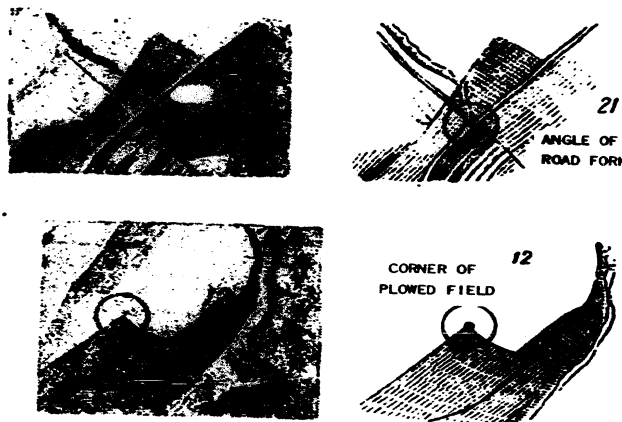

Fig. 94 - Marking of an Identification Point on a Photograph
and its Contour Sketch

granted that a pin prick can be made on a photograph with an accuracy of 0.1 mm. Therefore, errors in identifying contour points on the terrain should not exceed 0.1 mm relative to the scale of the photograph. When identifying a point on the terrain, the topographer should carefully compare all contours on the photograph near the selected identification point with the terrain to make certain that the point he is about to pin-prick on the photograph is the point he is sighting. Such a comparison is mandatory, since one photograph may contain several identical contours, so that the wanted point can be positively identified only by comparing the

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image of the contours next to the point with the terrain. After the contour of ter-
rain selected as an identification point has been positively identified by comparing
the terrain with the photograph, the point is pricked on the photograph. Some kind
of firm backing such as celluloid should be placed under the aerial photograph when
pin-pricking points to ensure proper coincidence. A thin, sharp needle should be
used so that the pin pricks are round, small in diameter, and visible against the
light.

The pin prick should be encircled on the reverse side of the photograph and
signed. Here also a sketch is entered of the identified point at a scale larger
than that of the aerial photograph, for
easy reading. When sketching a contour,
the image of the contours on the photo-
graph should be used as guide and the ter-
rain as a control, to achieve maximum co-
incidence with the sketch of the aerial
photograph. When hatching the sketch of
the contour, the relative degree of dark-
ness of the photograph should be retained.

PLAN |—— 1.5M ——|
PROVISIONAL MARK
⊙ — "IDENTIFIED"
— — DITCH
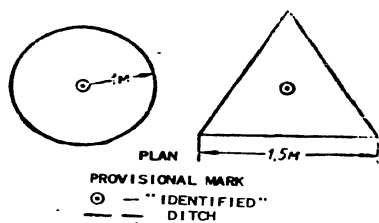
Fig. 95 - Marking an Identification
Point on the Terrain

Terrain contours which do not show up on the photograph should not be shown on the
contour sketch. Brief explanatory descriptions should be made on the contour
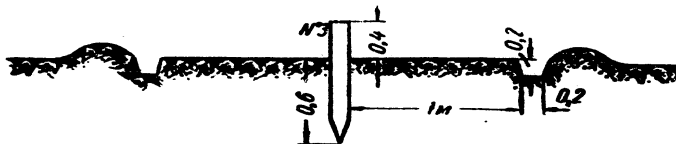
Fig. 96 -   Digging for Reference Marks

sketch, e.g. "corner of a field", "bush", "cross roads", "corner of a house", etc.
(Fig. 94).

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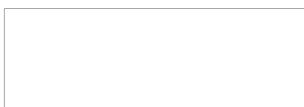
The contour sketch should be drawn with an ordinary black pencil, clearly and accurately, indicating the date of identification and the name of the sketcher. Careful and accurate sketches of identification points are helpful as a control of identification and in transferring identification points from one photograph to another.

b. Marking Identification Points on the Landscape

Identification points are marked on the landscape with a stake about 1 m long and 0.1 m thick which is driven into the ground to a depth of 0.6-0.7 m. A spot is smoothed off on the upper part of the stake, and the name or number of the identification point is written on it.

Identification points are also marked by ditches dug in the form of an equilateral triangle with 1.5 m sides or a circle with a radius no smaller than 1 m (Fig. 95).

The ditch should be one shovel-width wide and deep (0.2-0.3 m); see Fig. 96. The earth removed from the ditch should be piled outside the ditch so that it does not interfere with the setting up of the instrument.

A surveyor's stake may be placed on the identification point to facilitate sightings from other points.

If these reference marks must be preserved for a long time, they should be reinforced in a more permanent manner.

c. Topographic-Geodetic Measurements

Various methods of topographic-geodetic measurement are used depending on terrain conditions.

The analytic grid (small triangulation grid) and the graphic (geometric grid) are most widely used for locating identification points when mapping open or semi-concealed terrain, and the transit traverse is primarily run through the concealed terrain. Combined use of both these and other methods of tying-in are often met in

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practice. Constructing a control survey net by graphic methods is permissible only for maps at a scale 1:100,000.

Since the majority of plan identification points are isolated points, it is most convenient to locate them by direct or reverse intersection with triangulation points (Fig.97.)

However, identification points cannot always be obtained directly by triangulation or intersection; in such cases some point near the identification point must be selected and the identification point must be tied in with this point by making additional geodetic measurements.
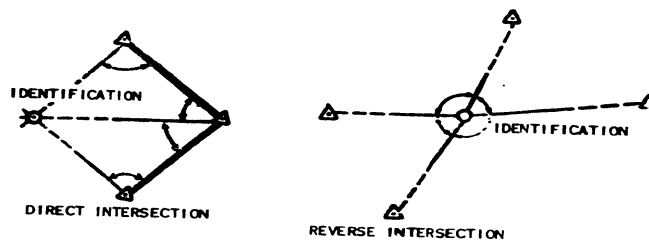
Fig. 97 - Locating Identification Points by Intersections

Thus, e.g., the polar method of tying-in can be used, with direct measurement of the line to the identification point, the so-called tying-in by points (Fig. 98). This method may also be used when some point of the geodetic grid cannot be identified on the aerial photograph. In such cases, the known contour adjacent to the point which cannot be identified is also tied in by a point.

When direct measurement of the distance to the identification point is impossible, the distance can be determined geodetically by constructing a triangle and measuring the elements necessary to determine the plan position of the identification point.

As noted above, for concealed terrain, transit or tachymetric traverses are run. Tachymetric traverses for tying-in identification points are run for small

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scales of photography (1: 50, 000; 1: 100, 000) or for cases where it is difficult to measure lines of the strip, e.g., when the land is concealed, intersected, or swampy.
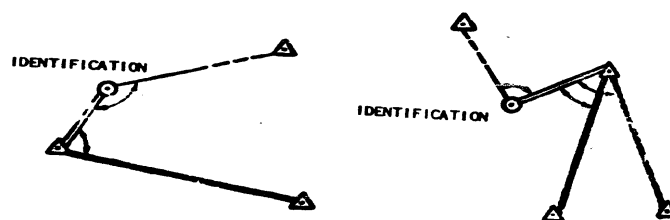
Transit traverses are run:

Fig. 98 - Polar Method of Tying-In

a) Between two geodetic points, in the form of separate elongated traverses;

b) In the form of a system of traverses based on geodetic points and forming one or several junction points;
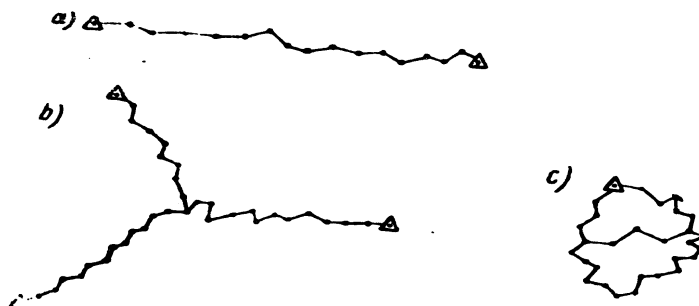
c) In the form of closed polygons based on one geodetic point (Fig. 99).

Fig. 99 - Diagram of Transit Traverses

"Hanging" traverses should not be permitted, the traverse should be tied into a geodetic base point or into the extension of an earlier traverse.

Graphic (plane-table) tying-in of identification points is carried out by expanding the geometric grid.

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In analytic, and even more so in graphic tying-in, there may be opportunities for locating additional identification points. Advantage should be taken of such opportunities if this can be done simultaneously with carrying out the basic task, the tying-in of planned identification points.

In addition to the most common methods, indicated above, various forms of triangular circuits, complete and incomplete central systems, and combined intersections may be used, their choice depending on the specific conditions encountered.

For the analytic tying-in of aerial photographs, the basic measuring instrument is a theodolite of 30" accuracy; for graphic tying-in, a plane-table with a telescopic alidade is generally used.

The technical methods of making angular and linear measurements for tying-in of aerial photographs are the same as for ordinary geodetic work, while the few specific requirements and technical tolerances (configuration of grids, length of traverses, allowable errors of closure, etc.), depending on the scale of the map and the method of tying-in, are contained in corresponding instruction manuals, which should be consulted in carrying out the work.

On completion of the work of tying-in aerial photographs, the following materials should be submitted.

For analytic tying in:

    1) The aerial photographs (contact prints) with identification points and contours marked on them;

    2) A reproduction of the rough mosaic with identification points and geodetic control points marked on it;

    3) A diagram showing the tying-in of identification points.

    4) A survey record.

For graphic tying-in (in addition to the aerial photographs and the reproduction of the rough mosaic):

    1) The plotting board with points located on it;

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2) Data on the margins of the plotting board of the plane-table;

3) Topographic records.

63. Surveying Relief on Photomaps, Aerial Mosaics, and Photographs

Compared to plane-table survey, in which the topographer enters on a blank sheet of paper contours of his own choice and the relief, the survey of relief on a photomap, aerial mosaic, or photograph differs as follows.

Primarily, the contours of the earth's surface are depicted on the photograph, meaning that the survey of relief is based on known contours; secondly, the photographs, in addition to the contours of the locality (estate boundaries, roads, populated points, etc.), contains contours related to the relief of the terrain, e.g., edges of ravines, lines of water courses, cliffs, water holes, etc., i.e., elements which form the skeleton of the relief.

The base lines of the relief or its skeleton, i.e., the lines of water courses, ridges, summits, etc., plotted by the topographer during the plane-table survey in the form of broken lines, are of considerable value in relief surveying. In a given case, these lines need not be plotted, since they are already given in much greater detail on the photograph than on the plane-table survey.

The basic methods of plane-table surveying (Bibl. 19), in their application to plotting horizontals and sketching relief on photomaps, aerial mosaics and photos remain unchanged in principle. However, there are certain specific differences which have a noticeable effect on the nature of the work.

It is known that a photomap is a mosaic of transformed prints glued to a stiff mounting; the photographs are usually printed on glossy paper. Such a photomap is inconvenient for work under field conditions, so that the photomap is usually replaced by a reproduction on mat paper, cemented to a rigid support (aluminum or plywood).

Before beginning the field work, the accuracy of the photomap reproduction must

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be checked. The borders and the kilometer grid of the coordinates can be checked
with a precision ruler or a beam compass by comparing the dimensions with their
theoretical magnitudes. The accuracy with which geodetic points have been plotted
must also be checked. Deviations in dimensions of the borders should not exceed
0.3 mm, while errors in the position of control points with respect to the border of
the trapezoid should not exceed 0.2 mm.

The photographic reproduction of the photomap should be of a uniform grey or
light-brown shade and of normal sharpness and density, so that lines drawn in by

Scale 1:20000
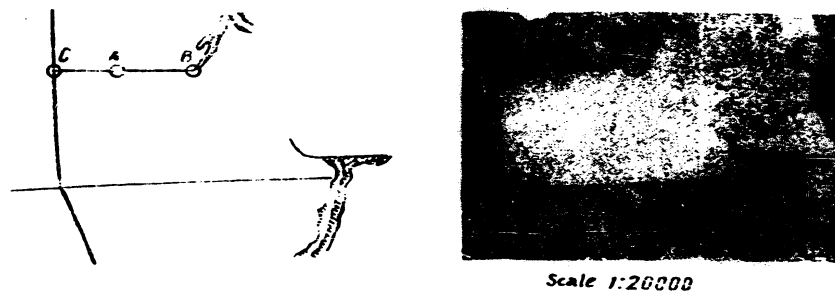
Fig. 100 - Determining the Plan Position of Halting Points by
Measuring the Distance along the Contour with a
Leveling Rod

pencil will be readily discernible on all parts of the photomap. At this time, the
elevation control points, elevation reference marks, and the descriptions of their
position should be checked.

In addition to the photomap, a set of aerial photographs with 60% end lap of
the given strip of terrain is required for stereoscopic study of the relief.

64. An Elevation Control System for Surveying Relief on Photomaps

If a photomap of scale 1:25,000 to 1:10,000 has 20 to 30 elevation control
points which are uniformly distributed over the plotting board, there is no need to

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expand the elevation control net, since this number of control points will permit accurate determination of the elevation of any intermediate point. If the density
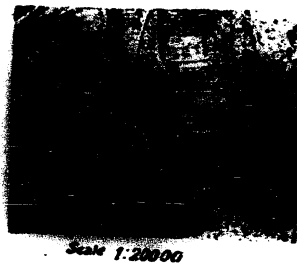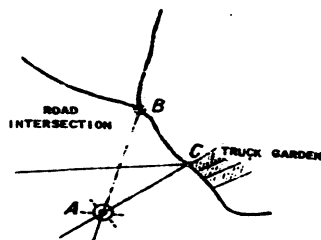
Fig. 101 - Determining the Plan Position of Halting Points
by Measurement from Two Contour Points

and distribution of elevation control points is inadequate, the elevation control network will have to be densified.
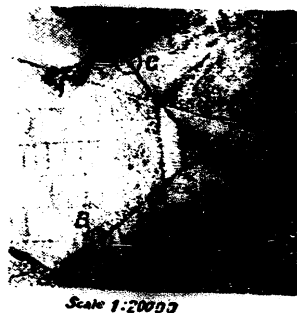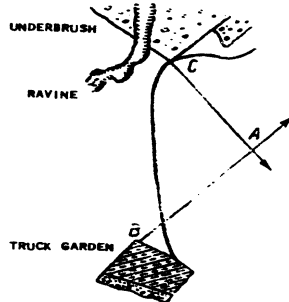

Fig. 102 - Determining the Plan Position of Halting Points
by Intersection from Two Points

Densification of the elevation network is carried out either by expanding the geometric grid, or by extending the basic plane-table elevation traverses. The most convenient method of constructing an elevation grid is selected on the basis of

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the number and distribution of elevation control points on the plotting board, and also on the basis of terrain features (density of forests, irregularity of relief, and other characteristics). Actual inspection of the terrain along with a stereoscopic analysis of the photograph will help in selecting the most convenient method of constructing the elevation grid.

The most common method of constructing the elevation control network is by using the basic plane-table elevation traverses, whose points can generally be identified on the photomap. It is one of the characteristics of vertical-combined surveying that the distance between points can be determined directly from the photomap as the distance between identified points, rather than measuring them with a range finder. If it is impossible to identify points of the traverse in the plan position, these can be fixed by the distance measured with a ruler from a contour point of the traverse, while orienting the photomap by a compass or by the contours on the photomap, checking the orientation with other more distant contour points or geodetic points.

In addition to the method described above, many other combinations of methods may be used to determine the plan position of points. Several of the most typical methods are given below.

1. By measuring the distance along a straight contour from its turning point with a leveling rod, using another angle along the line or another contour for a control (Fig. 100).

2. By measuring the distance from two well-identified contour points of the photomap with a leveling rod. The point will be found at the intersection of these radials, laid off to scale with compasses (Fig. 101). The photomap is oriented by compass or by contour points, using distant objects as controls.

3. By intersection from two or three well-identified contour points of the photomap, which points are oriented in advance (Fig. 102).

4. By reverse intersection from geodetic control or identifying contours, with preliminary orientation of the photomap.

In addition to these methods, characteristic for vertical-combined surveying, plane-table survey methods (a geometric grid, based on the plane-table traverse, and a graphic solution of reverse intersection) are also used. The latter is chiefly used in parts of the photomap with an insufficient number of identifiable points.

The basic plane-table elevation traverse is run between elevation control points (control points, reference marks, triangulation points), and also between points of tied-in basic plane-table elevation traverses, by using a telescopic alidade or transit with a vertical circle that reads to 1' or 30" accuracy, or a telescopic alidade equipped with a cap-altimeter designed by engineer G. Yu. Stodolkevich.

The transfer of elevations with a telescopic alidade is generally carried out by taking readings in two directions to obtain direct and reverse elevation, while
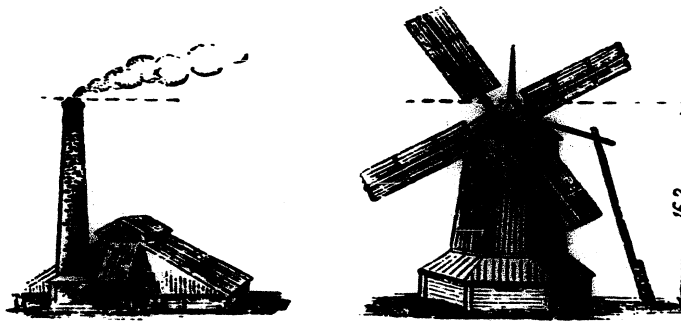
Fig. 103 - Sighting on Landmarks for Orientation

angles are measured in the two positions by the vertical circle.

While running the basic elevation traverses, observations are made on points of the trigonometric grid and on other local landmarks which have significant elevation. The distance to these objects is taken from the photomap.

The elevations transferred from points of the traverse by numerous sightings on local landmarks, used for orientation, may be used later in sketching the relief. In observing such landmarks, the horizontal stadia wire of the telescope tube of the

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alidade is sighted on prominent characteristic points or lines of the object, such as the top of a chimney or tower, the base of the spire of a windmill, etc. The sighted place is then entered in the record, with a sketch (Fig. 103).

Points of the basic traverse, used later for plotting the relief and for tying-in of survey traverses, should be marked with a wooden stake and capped. For the scales of topographic maps compiled by vertical-combined surveying (mainly 1: 10,000 and 1: 25,000), published instructions are available that give the respective toler-ances for lengths of traverses and lines, errors of closure, and other technical rules to be used as guides. In the given case, greater lengths are permitted for plane-table elevation traverses than for plane-table surveying, since in running them off, the plan position of points is determined from the photomap.

This fact is important, since it makes it possible to have a less dense geode-tic base on the plotting board which, in turn, affects the organization of the work as a whole.

After computing the average increments in elevation between points of the trav-erses and drawing up a diagram of observations, the heights of points of the basic plane-table elevation traverse are equalized by the vanishing point method or by the polygonal method. Then other adjacent traverses and connecting points are tied in.

Elevation points are plotted on a tracing of elevations (Fig. 104) which con-tains also their numbers and elevation.

Also plotted on the elevation tracing are geodetic control points, reference marks for geometric leveling, survey base points, survey traverse points, and iso-lated points for which observations have been entered on, record (local landmarks, characteristic points, etc.). Magnetic declinations are also recorded on the trac-ing.

Running of elevation traverses is done under various conditions, depending on the existence and location of elevation control points and on the physical, geo-graphic, and economic character of the region surveyed. All these affect the trav-

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TRACING OF ELEVATIONS
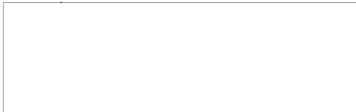N-37-86-V-a
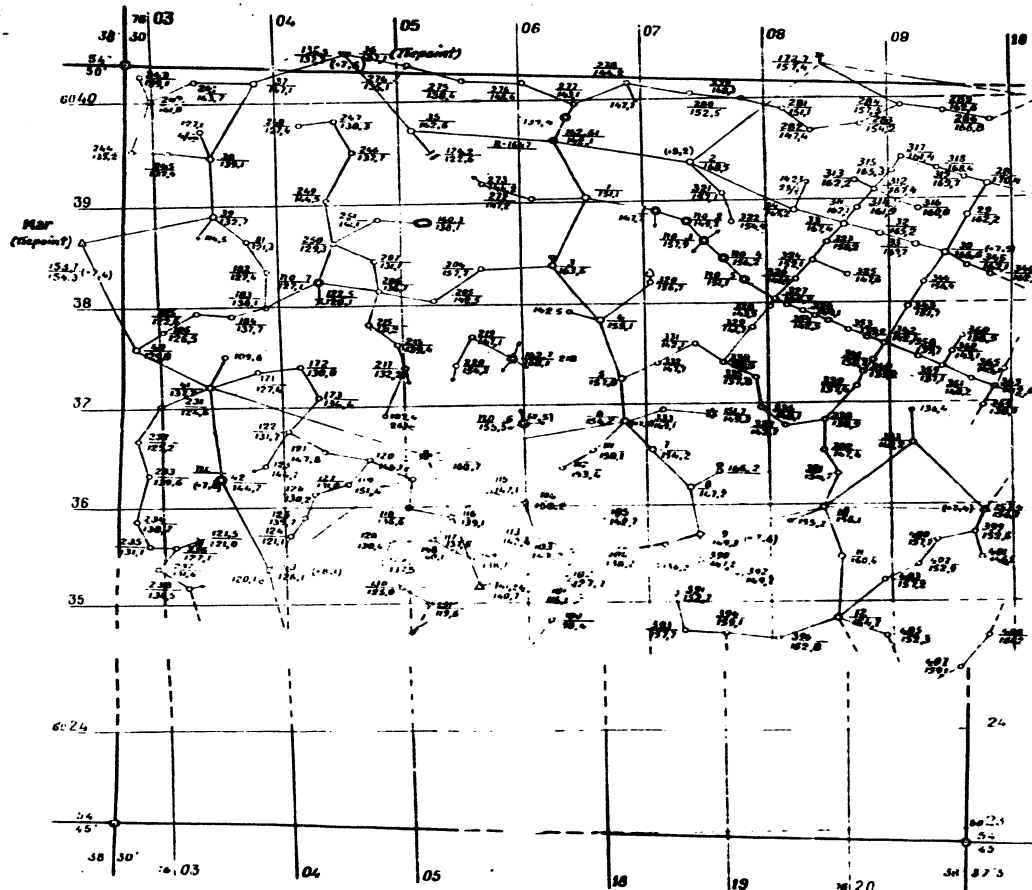
Chief of Party   Scale 1: 25,000   Drawn by Topographer 1st Class
IVANOV between 5 May and
15 September 1940

Magnetic declination + 7° 15';   Correction after comparing with compass - 1° 3';

Corrected declination + 5° 45'

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erse and the nature of work performed. For example, depending on the availability of points and the nature of the terrain, it may be expedient to extend the elevation grid in parts, but not over the entire plotting board at once. In open and treeless terrain, the work is different from what it would be in wooded terrain. If there are large bodies of water on the terrain, the water surface can be used as a reflector for transferring elevations, etc.

All these and other individual variations in the terrain should be considered and put to good use in running basic plane-table elevation traverses.

65. Transitional Survey Points

Full use should be made of points of the basic elevation traverses as transitional survey points in plotting relief. Besides these points, the elevation and position of other transitional points, necessary for the survey, should be defined. The density and distribution of these points will depend on the skill of the topographer and on the nature of the relief and density of trees in the area. In open places, the number of transitional points will be smaller than in wooded sectors, while an area with broken or sharp relief will require more transitional points than a level area.

Transitional survey points may be located on the photomap in various ways.

In the plan position, these are located by:

a) Identifying a contour point. Any contour, its angle, or the intersection of contours identified on the photomap, can be used as transitional survey points;

b) Identifying lines on which the point is located (landmarks, roads, etc.), and measuring the distance from one, two, or more identified contours (see Fig. 100);

c) Measuring the distances from two or more identified points located near by; the point will then be located at the intersection of the radials, read from the staff and laid off to scale on the photomap with compasses (see Fig. 101);

d) Intersecting with two or more identified contours at a determined point (see Fig. 102);

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e) Reverse-intersecting from geodetic control or identified contours with preliminary orientation of the photomap;

f) Any of the methods used in plane-table surveying, or a combination of them. A frequently used method of locating points is by reverse intersection with orientation of the plotting board by means of sketched-in bearings. For example, assume that a traverse is run along a boundary which is poor in contours (swamp, tundra, meadow, etc.). From the last positively identified contour point, a bearing is taken to a transition point not identified on the photomap. On this transition point, the halting point of the plotting table is oriented by a line drawn to this point from the preceding point, and the point itself is reverse-intersected with surrounding geodetic or contour points.

In orienting the photomap, besides using control points, use prominent distant contour points and identify them carefully.

With respect to elevation, transitional survey points are obtained:

a) Primarily, by running survey elevation traverses through a point (through the leveling staff). Vertical angles are measured twice in each direction, on two different points of the staff. Survey elevation traverses are run between elevation control points (triangulation points, bench marks, points on basic elevation traverses) and tied in. Traverses consisting of no more than two points, not counting the starting point, do not have to be tied in.

In some cases, it may be preferable to mark the terrain position of transitional points for further use by driving a stake and capping it, blazing a tree, or other method;

b) as isolated points - by sightings on two or three points, whose distances are taken from the photomap.

Elevations of the tied-in transitional points should be entered in the record and also marked on the elevation tracing. Numbering of all points on the elevation tracing, in the record, and in the catalog should be the same, to avoid error. In

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addition, it is advisable to number all basic and survey transitional points with serial numbers, starting with "one". This method of numbering precludes the possibility of giving the same number to two points.

66. Surveying Relief

To depict relief with horizontals, elevation points, so-called stake or rod points, are required; these can be obtained from transition points.

One of the outstanding characteristics of surveying relief on the photomap is the fact that points and contour identified on this map can be used for determining
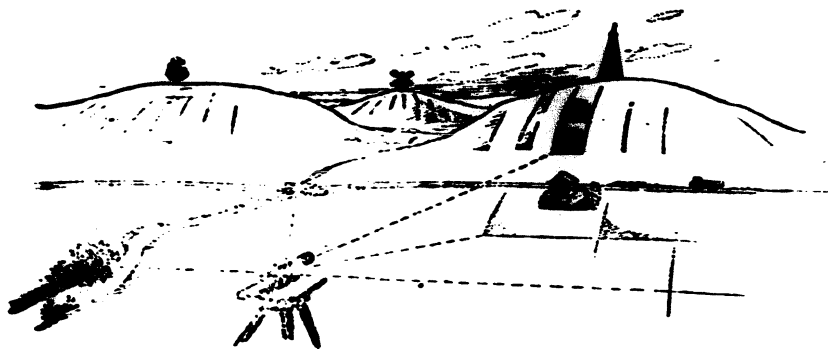
Fig. 105 - Sighting on Contours of the Terrain

the elevation of points without using the leveling staff. To do this, the vertical angle is measured by sighting the horizontal stadia wires of the telescope of the alidade on the base (ground) of the appropriate contour (identification point), while the corresponding distance is taken from the photomap (Fig. 105). This, of course, does not mean that leveling staff sightings are neven taken, since the relief always contains sections without identifying contour points, whose elevation and location must be determined by rod readings. Contour points should be used as much as possible to obtain elevation reference marks, corresponding to their number on the photomap.

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It is known that, in ordinary plane-table surveying, sightings are taken at the height of the instrument to simplify computing the height of the stakes; when taking bearings on contours, the stadia hair is sighted on the ground. This means that, in the latter case, the height of the instrument must be taken into consideration.

The technical methods of measuring angles remain the same as for plane-table survey, i.e., vertical angles on the stakes are measured with the vertical circle in one position, having the value of the null setting close to or equal to zero. Sightings on stakes are not entered in the record, except for observations along the shores of bodies of water (unless previously sighted from control points), which are made with the circle in two positions.

Elevations of typical terrain features are recorded both on the photomap and on the elevation tracing. Stake points are usually selected on typical terrain features, i.e., at discontinuities along water divides and water-course lines (peaks, bases, ridges, bottoms of hollows, ravines, anticlines, points where the slope of a hill changes, and shores of bodies of water, etc.).

Stations, from which the terrain is studied and heights of the stakes taken, should be so selected that the relief can be seen clearly in all directions both horizontally and in depth, i.e., in such a way that the configuration of the terrain presents itself most clearly to the eye.

It is natural that questions as to the number of transition points and stakes and their location cannot be answered in a sufficiently general way to cover all situations met in practical work. Primarily, these will depend on the scale of the map and secondly, on the number of elevation readings taken.

However, correct positioning of elevation stakes is as important as the number of stakes; they should characterize the configuration of the relief, as far as its general character and individual details are concerned. Correct positioning of the elevation stakes, will considerably facilitate correct interpolation of horizontals and plotting of the relief.

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Plotting the forms of relief on the photomap is easier than in plane-table sur-veying since the topographer has before him the base (skeleton) lines of relief, i.e., lines of water courses, edges of ravines, gullies. In plane-table surveying, the base lines of relief are also plotted but in a more schematic, less objective manner than is accomplished by photography.

Having the base lines of relief available greatly simplifies and facilitates the task of positioning transition points and elevation stakes. For example, on a
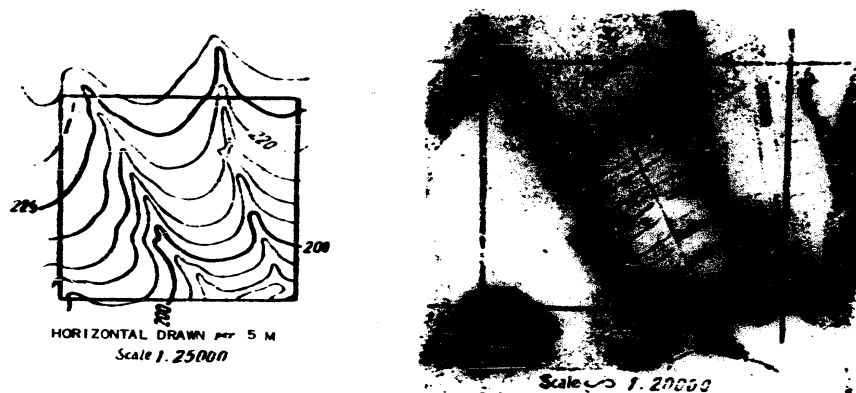
Fig. 106 - Representation of the Horizontal Relief of
Terrain, as Shown on an Aerial Photograph

photomap or aerial photograph, edges of ravines and gullies are shown in detail, difficult to attain in plane-table surveying without a large number of stake points.

Figure 106 shows part of the contour lines from a topographic map at a scale 1:25,000 and the corresponding aerial photograph. Ravines and water courses in hollows are clearly visible on the photograph.

Experience has shown that the characteristics of a photomap permit a reduction in the number of elevation stakes by at least half, as compared to plane-table sur-veying, which considerably speeds up the work.

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Experience has also shown, that in surveying relief in general, and on a photo-map in particular, several general principles should be observed:

1. Plotting of relief should be done for an entire massif rather than for small sectors, since, in the analysis of the section photographed, the aggregate of the contour lines continuously extended over a stretch of terrain helps in the interpretation of the general nature of the relief and of individual parts.

Before starting to plot the relief, the terrain should be studied closely in the field. This can be done simultaneously with running basic elevation traverses.
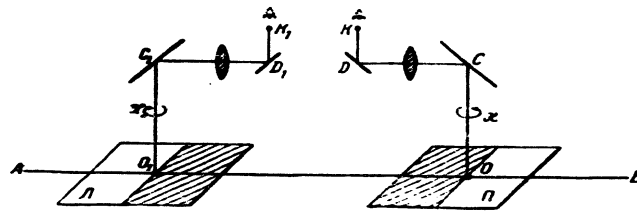
Fig. 107 - Schematic Diagram of a Stereoscope

Moreover, by using a set of contact prints and a simple collapsible mirror stereo-scope (stereoscopes L-3 or D-5; see Sec. 70), the entire area can be viewed, especially its concealed parts (woods, undergrowth, etc.). For example, if there are large forests on the surveyed section, it is preferable to plot first the open areas around the woods, which makes it possible to analyze the general nature of the relief and makes it easier to plot the relief in the forest. Swampy spots which extend for a considerable distance and are impassable in early summer, may be explored in late summer or after the first frosts of autumn. Proper study of local conditions helps in efficient planning of the sequence of operations.

2. Although the number of transition points, as noted above, depends on the scale of the map, on the ground relief, density of trees, and other factors, an increase in number of transition points at the expense of a decrease in stake points

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speeds up the plotting of relief and improves its quality.

It is poor practice to fix on distant stakes and extend the contours to places where the terrain cannot be seen clearly. In such cases, it is preferable to take an extra transition point, so as to see the surveyed relief more clearly.

In open places with complex microrelief, the relief can be plotted from stake points. To do this, the instrument is set over a stake, the photoplan is oriented by contours, and the relief is sketched by eye, using the elevation of the halting point (stake) as a guide.

3. As a rule, contours are drawn on the photomap in the field; photographic plotting of contours in the drafting room according to stake readings taken in the field is permissible only if a stereoscope is used and the plotted relief is later checked in the field. The smaller the scale of the map, the more use has to be made of the stereoscope.

In covered areas, where difficulties are encountered in sighting stakes, use of the stereoscope considerably facilitates the sketching of relief. The relief is plotted on the aerial photograph under the stereoscope, and the contours are later transferred to the photomap. Noting the position of horizontals on the aerial photograph with respect to well-defined nearby contours, permits drawing the contours directly on the photomap, using the corresponding contours of the photograph and the photomap for orientation.

The aerial photographs are set up in the stereoscope as shown in Fig. 107, which is a schematic diagram of the positions of the two aerial photographs forming the stereo pair, for viewing with the stereoscope. The l denotes the left photograph and the letter r, the right photograph; $O_1C_1D_1K_1$ and OCDK are the paths of the central rays traveling from the left and right photographs to the eyes of the observer. The eyes of the observer are at K and K_1. Mirrors are located at points C_1, D_1, C, and D. The aerial photographs are set up so that their mutual end lap is toward the inside and so that the left photograph is seen by the left eye and the right photo-

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graph, by the right eye; the photographs should be rotated in their planes until their initial radials lie on the straight line AB; they then should be moved along this line until a stereo image of the terrain can be seen without strain.

While viewing stereo models, the so-called pseudoscopic effect may take place, in which it will seem that slopes of the terrain seen through the stereoscope do not correspond to reality but will seem turned in the opposite direction. For example, it may seem that rivers are flowing uphill, and that the height of the stakes does not correspond to their location. This effect must be corrected by turning the photographs in their own plane.

4. Since the differentiation of relief proceeds from bottom to top, along the lines of water courses, each upper contour line will, as a rule, reflect to some extent the curvature of those below. When sketching the relief from bottom to top, i.e., from transition points located lower and closer to water courses, we must remember that from below and to the side, relief is seen more clearly than from above, where the landscape seems flatter and declivities seem rounded. On the other hand, when plotting from the top downward, the relief can be seen for greater distances than from below. These factors should be kept in mind when locating transition points.

5. In surveying wooded areas where several ravines and hollows are poorly visible on the photomap, and in other cases in which the base lines of the relief are not clearly visible, these lines should first be roughed in on the photomap, using the stereoscope.

6. The character and form of contour plottings should correspond to the actual forms of the relief of the earth's surface. In most cases, the contours are sketched in flowing, rounded lines, corresponding to the unvaried, smooth forms of the relief of a plain. However, sharp, jagged forms, characteristic of mountainous regions, may be encountered, in which case the contours should show up in the sketch as pointed, angular lines.

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7. Contours of additional intersection (half-contour lines) should be used gen-
erously to depict such terrain features as rims of terraces and cones of washouts,
and to emphasize important relief elements that do not stand out in scale: sink
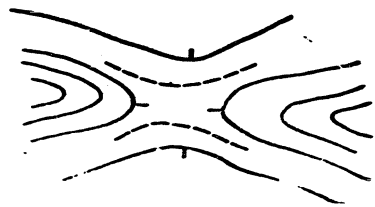

Fig. 108 - Saddle   Fig. 109 - Bend of a Slope

holes, landslides, and other designated elements. If these special relief features
can be shown by means of the basic contours, there is no need for using additional
contours, since they may crowd the map and make it difficult to study the relief and
contours.
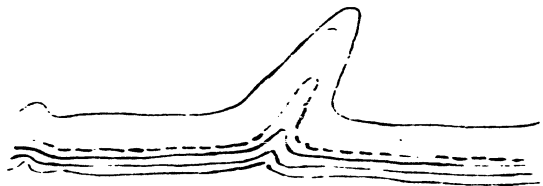
Fig. 110 -   Curve of a Declivity and Hollow

Practical experience has established several rules for drawing additional con-
tours which may be summarized as follows (Bibl. 20):

a) In sketching summits and saddles additional contours must be run on both
sides of the summits or saddles, so-called key contours (Fig. 108):

b) The additional contour characterizing the curve of a slope should not be

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filled in until the slope becomes uniform (Fig. 109);

c) The additional contour characterizing the bottom of a valley should be run along both slopes and should intersect the line of the water course; an unsymmetrical break in the contour can only appear if the opposing slopes have different formations;

d) The additional contour characterizing a declivity should not be placed at the entrance to a hollow, but should be run through an opposite declivity to show the relief of the bottom of the hollow (Fig. 110).

8. A basic knowledge of the laws of geomorphology is very important in sketching relief. When interpolating contour lines, not only the elevation readings of the stakes but also the geomorphological character of the relief described must be taken into consideration. Sometimes it is necessary, depending on the scale, to exaggerate or generalize contours to emphasize characteristic or important relief features. When using a very large scale, the depicition of relief loses the character of a sketching, since the contours are obtained from exact measurements and the scale of the map shows up all relief features. The smaller the scale of survey, the more important will plotting become.

Knowledge of the principles of geomorphology helps in developing the skill of recognizing the main typical relief feature. Changing the form of the contour, even within the limits of graphic accuracy, i.e., within 0.2 mm or slightly more, relief features can be emphasized when viewing contours as a unit rather than as separate items.

This can be demonstrated on an example:

Figure 111 shows part of a topograpnic map at a scale 1:25,000, whose contours are at 5 m intervals and show three depressions. The curvilinear rounded contours do not show the abruptness of the gullied, eroded landscape and the actually very pronounced rim of the hollow does not stand out, despite the fact that the contours were geometrically correctly plotted from elevation readings of stakes.

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Figure 112 shows the same depressions with their form slightly changed and the contours displaced by 0.2-0.5 mm along the rims of the hollows and the slopes between. Along the rims (by the rim is meant the line of transition of the slope to the wall of ravine, hollow, or valley, where a sharp change is noticeable), the contours were brought closer together, the transition from the slope to the walls of the hollow was made more angular, and the curvilinear form of the contours showing the slopes between the hollows was made straighter. After correction, the relief looks quite
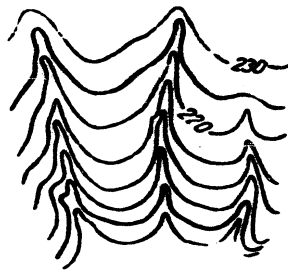
Fig. 111 - "Smoothed-Out"
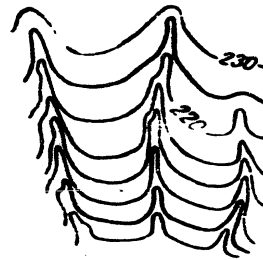
Fig. 112 - A Truer Rendition of the same Relief

different; the rims of the hollow can be seen. The relief does not look "smoothed out," since the straightened contours, in combination with the lines of the rims of the hollows give the impression of hollows that cut sharply into the slopes i.e., they reflect the morphological nature of the landscape more accurately.

The above example shows that geomorphological features, when depicted on a map, can always be confirmed topographically.

67. Surveying Relief on Aerial Photographs and Aerial Mosaic

All of the above statements as to sketching relief on photomaps also apply to aerial photographs, since identification of relief on photographs basically proceeds in the same order and by the same methods as on a photomap. The difference lies in

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the fact that, instead of working on an entire sheet such as a photomap, the work
is done on individual photographs, each of which has a different, nonstandard scale
which is inconvenient to use (e.g., 1:15,650 or 1:24,700). These facts somewhat com-
plicate the general work process, but plotting relief on aerial photographs (on con-
tact prints or enlargements) has the advantage over plotting on photomaps in that it
can be done as soon after the flight as the photographs can be developed. There are
cases in which such a procedure, which reduces the time spent on field work, is nec-
essary.

If there are geodetic control points on the section photographed, densification
of the elevation control network is possible by expanding the geometric grid. Points
determined in this manner are identified on the aerial photographs and used to sur-
vey the relief. If the coordinates of the plan control net are not given, the ele-
vation control net is densified as on the photomap, by running basic elevation trav-
erses. In the latter case, the traverse should preferably be run through the middle
of the flight strip.

Two sets of photographs are needed in surveying relief, one printed on mat
paper for surveying relief and interpretation, the other printed on glossy paper for
plan tying-in of the aerial photographs.

In surveying relief, rather than the entire photograph only the working part is
used, delineated by lines passing through the center between end and side lap of the
photograph and the flight strips. Fig. 113 shows a group of six photographs from two
adjacent flight strips with an end lap of 60% and a side lap of 40%.

The lines passing through the centers of overlap are shown as broken lines,
while the hatched area A, B, C, D of photograph No. 337, is the part which will have
the relief drawn in.

The boundaries of the working area are either marked off on the rough mosaic by
using a plot on which the center lines of the overlaps are traced, or else these
lines are sketched directly on the photographs. In practice, the boundaries of the

working area are outlined by determining the points A, B, C, D from nearby contours and interconnecting these points by straight lines. The edges of the working area are marked with India ink.

The outline of the trapezoid to also transferred to the photographs from the rough mosaic.

It may happen that a border line intersects a complex object (a ravine, a popu- lated point, etc.); in such cases the border line should be interrupted and made to
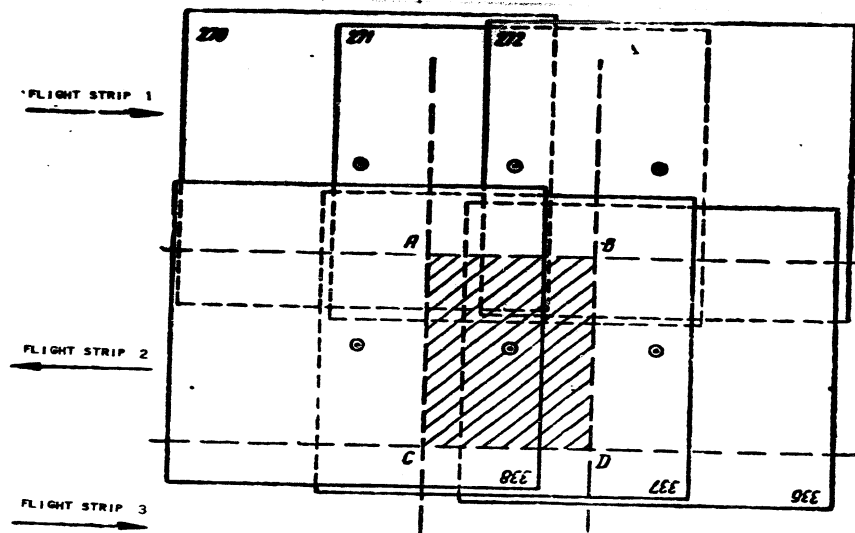

Fig. 113 - Working Area of the Photograph

go around the object, so that the entire object can be shown in one photograph.

Since the scales of untransformed photographs vary, the scale should be deter- mined individually for each photograph or even for each part of a photograph.

The scale of an aerial photograph can be determined from the equation:

$$1 : m = d : D$$

where m denotes the numerical scale denominator, d the distance between contour

points measured on the photograph, and D the corresponding distance measured on the terrain.

If the scale is computed for the entire photograph, it should be calculated twice along mutually perpendicular (approximately) lines passing through the central part of the photograph. The contour points of the ends of these lines should be symmetrical (approximately) with respect to the center of the photograph (Fig. 114).
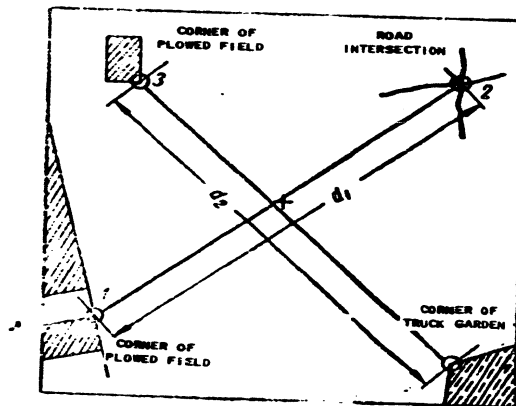
Fig. 114 - Selecting Points for Computing the Scale of a Photograph
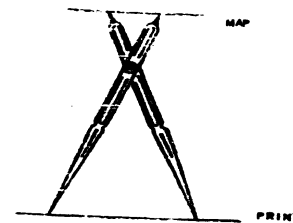
Fig. 115 - Proportional Dividers

Distances on photographs can be measured in the drafting room by using an accurate ruler or sliding calipers; in the field, they can be measured with beam compasses. Distances on the terrain are determined from the coordinates of points, from measurements on the plotting board if the points are graphically interconnected, or from measurements with surveying rods.

On dividing the distances measured on the photograph by distances measured on the terrain (both distances must be expressed in the same units), the numerical scale of the aerial photograph is obtained. Since the measurements are made twice in two directions, we obtain two values, the average of which will be the mean scale of the aerial photograph. In most cases, the scales obtained will not be suitable for practical use (e.g., 216 m to 1 cm).

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For laying off or determining distances from aerial photographs, proportional dividers (Fig. 115) can be used, which are easily adjusted so that one end opening will correspond to the scale of the map and the other to the scale of the aerial photograph.

However, use of proportional dividers is inconvenient since two operations are involved: taking distances from a transverse scale with one end, and laying them off with the other end on the photograph, necessitating constant reversal of the dividers, a procedure that slows down the work and distracts the attention. Therefore, it is preferable to use specially designed interpolating scales, computed for varying pitch of scale change. Such scales may be conveniently arranged in the shape of a wedge, two sides of which form the legs of a right triangle, on which distances and their scales are laid off.

Figure 116 shows a wedge scale with three distances laid off on it:

    I - At the scale 1:20,000   - 230 m
    II - At the scale 1:19,000   - 300 m
    III - At the scale 1:21,000   - 454 m

Such a scale can be used not only for determining and laying off distances, but also for determining the scale of an aerial photograph. The scale should be made of cardboard, celluloid, aluminum, or other material of low deformation and long service life.

Work on aerial mosaics does not differ greatly from work on aerial photographs, with the basic difference consisting in the fact that the aerial mosaic represents an assembly of aerial photographs. In working with aerial mosaics, measuring radials across cuts of the mosaic should be avoided. If the mosaic is laid separately for each flight strip, the working areas determined from the zones of end overlap are marked off, as is done in the case of aerial photographs.

When working with an aerial mosaic, it should be mounted on a plotting board; for work with aerial photographs, it is preferable to use a small plotting board or, if a large board must be used, to substitute a light plywood board of smaller dimen-

...sions for the regular board.

Field assembly of photomaps, aerial photographs, and aerial mosaics is done in four colors: contours, in black; relief, in red or brown; water-filled ditches, in green; and swamps or marshes, in blue. Particular attention should be paid to making
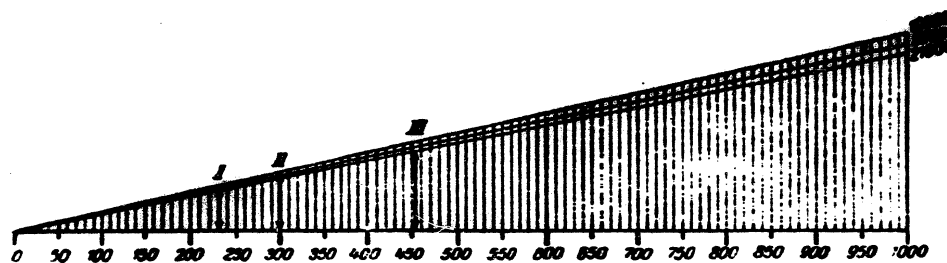

Fig. 116 - Wedge Scale

the contours and relief sharp and clear. Rules and methods for field assembly are given in instruction booklets.

After completion of the work, the following data are submitted: for surveying with a photomap - the photomap, field notebooks, data on boundaries, elevation trac-ings, and technical notes; for surveying with aerial photographs - aerial photo-graphs, field notebooks, diagrams of working areas sketched, and technical notes.

68. Revising Obsolete Maps

Working methods and measurement techniques in revising obsolete maps are basi-cally the same as for making a survey. Organization of the work may be different, depending on the number of corrections and the nature of the terrain. Revision of obsolete maps involves corrections for contours and relief. Since the relief of an area changes very little in time, compared with the contours, revision and correc-tion of obsolete maps primarily concerns the contours. Correction of relief may be done incidentally, particularly in cases where it was incomplete or incorrect in the previous survey.

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Revision of old maps by the aerophotosurveying consists basically of the following processes: the aerophotosurvey itself, collection of cartographic data and information on changes, interpretation and comparison of the aerial photographs with the map, transfer of the changes from the aerial photographic materials to the map, and compilation of the map.

Aerophotosurveying. For revision of obsolete maps, the survey is carried out in the same way and with the same overlap of the photographs as for ordinary aerial surveying. The scale of the photographic survey is established to correspond with the scale of the map being revised. In selecting the scale, it should be remembered that an aerophotosurveying scale close to 1:25,000 is most convenient for interpretation. If the scale is further reduced, some details will no longer show up on the photographs. When aerial geodetic enterprises revise a topographic map of scale 1:50,000, they generally use an aerophotosurveying scale close to 1:25,000.

Collection of Cartographic Data and Information on Changes. The most convenient procedure is chosen on the basis of the available information and cartographic materials. Therefore, when the map of a given region is to be revised, all available cartographic-geodetic material is collected, carefully studied, and evaluated. Evaluation consists in comparing old coordinates, elevations, and topographic plans with new ones. As a rule, selective comparison will help in reaching conclusions as to the best method of using old cartographic-geodetic data when revising the map of a given region.

Information needed for revising maps may be found in both central and regional institutions. These offices, should have exhaustive information on populated centers, their names, industrial enterprises, farms, roads and their conditions, etc. Comparison of this information with the map and aerial photographs will establish the accuracy of the information.

Interpreting and Comparing Aerial Photographs with a Map. In interpreting aerial photographs and comparing them with a map, a stereoscope is used. In revis-

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ing, brown or black copies of the original map, on which the points requiring revision are marked, are usually preferred. From the comparison, a plan for studying the map is laid out, i.e., the points to be interpreted in the drafting room and the points requiring field interpretation are noted. When using data from aerophotosurveying to revise a map, field work should be kept to a minimum.

Important changes or new features (populated points, railroads, highways, industrial enterprises, and farms) should be studied in the field, while changes in small or unimportant objects (small settlements not located on railways or highways, dirt roads, field roads, forest contours, etc.) can be plotted in the drafting room, without field study.

Transferring Changes from Aerial Photographic Material to the Map. In transferring changes from the aerophotosurveying material to the map, the following procedures may be required: 1) transfer of the changes from the photomap and 2) transfer of the changes from the aerial photograph to the map.

1. In regions where all of the interpretation is done in the field, photomaps may be used and corrections made from them. In making the photomap, positively identified contours from the old map can be used as control points. Photomaps are prepared for the entire trapezoid of the original to be corrected, or for part of it. In the latter case, the part of the photomap is mounted on the corresponding part of the original. Contours are drawn on the photomap from the interpreted aerial photographs, while the horizontals are transferred from the original.

If necessary, the relief can be corrected directly from the photomap in the field, or plotted in the drafting room from aerial photographs, using a stereoscope or a topographic stereometer. The elevation points needed for sketching the relief can be taken from the old map.

2. Transfer of changes from aerial photographs to the map may be effected in one of the following ways:

    a) by using proportional dividers, or compasses in combination with a wedge scale;

b) by graphic-mechanical means, using a pantograph;

c) by optico-mechanical means, using a transforming printer, projector, or multiplex.

Either one of these methods may be used, depending on individual conditions. If the work is done in the field, transferring must be done with an ordinary compass or with a proportional dividers. In the drafting room, this can be done by one of the means indicated under b) or c).

When using the transforming printer, it is convenient to print the transformed negative on special transparent film with a dull backing. If this film is superimposed on the original, the impression will be that of a photographic image on the film as well as an image of contours and relief on the original. By matching the transformed aerial photograph with the general contours of the map, the changed contours can be transferred from the aerial photograph to the original by tracing or other means.

Finishing the Maps. After the original is revised by one means or another, a blueprint is made and finished with the appropriate conventional symbols.

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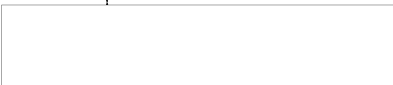
CHAPTER VIII

STEREOPHOTOGRAMMETRIC RELIEF PLOTTING WITH THE STEREOSCOPE

69. Monocular and Stereoscopic Vision

All photogrammetric plans are based on the identification of coordinates of photograph points; the accuracy of this identification determines the accuracy of the map. The accuracy with which these coordinates are defined, in turn, depends on the accuracy of the measurements made, which is controlled by the conditions under which the eye of the observer functions.

The human eye (Fig.117) has the shape of a nearly perfect sphere with a radius of about 12 mm; its outer surface consists of three coats. The outermost coat of the eye is the sclera 1 whose anterior portion is transparent and is called the cornea 2. Underneath the sclera is the choroid 3 which terminates in the opaque iris 4. The front of the iris has a central perforation 5 which is able to change its diameter and is called the pupil of the eye. It serves as a diaphragm, controlling the amount of light that enters the eye. The third internal coat is the retina 6 which is composed of numerous photosensitive elements (rods and cones) of a diameter of 3-6 μ, which transmit a stimulus through the nervous system to the

Fig.117 - Arrangement of the Eyeball

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brain of the observer*. The spot 7 where the optic nerve enters the retina is call-
ed the blind spot, since it has no rods or cones and thus cannot react to an optical
stimulus. In the center of the retina, facing the pupil, there is the macula lu-
tea 8, about 2 mm in diameter, which is more sensitive to light than the rest of
the retina. The center of the yellow spot is concave (0.4 mm diameter) and consists
of a few cones. In the front center of the eye, beyond the pupil, is the crystal-
line lens 9 which has the form of a double-convex lens. The space between the lens
and the cornea is filled by the aqueous humor, while the space between the lens and
the retina is filled by the vitreous humor. The index of refraction of both humors
is about the same.

Thus the eye can be considered an optical system which, by means of the crys-
talline lens, throws a real inverted image of an object on the surface of the reti-
na. Since different objects lie at variable distances from the eye, the crystal-
line lens, to obtain a sharp image on the retina, changes its curvature with the aid
of the ocular muscle. This property of the eye is called accommodation. However,
if the diameter of the circle of diffusion does not exceed the diameter of the pho-
tosensitive element of the retina (3-6 μ), then the eye does not change its accom-
modation. Therefore, under normal illumination, when the pupil diameter is equal
to 4-5 mm, and the focal length of the eye is 15 mm, optical infinity of the eye
begins at 10-12 m; and increasing this distance does not change the accommodation.
If the distance between the eye and an object increases then the image thrown on the
retina will decrease in size; however to compensate for this, the accommodation
remains near normal, corresponding to infinite distance.

Using these facts as basis, the optimum distance of vision is taken as 250 mm,
at which distance the image on the retina is rather sharp, while accommodation is
performed without particular effort. At distances of less than 250 mm from the eye,

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* μ = micron: a unit of length equal to 0.001 mm.

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the curvature of the crystalline lens has to undergo abrupt changes, thus inducing extreme eye fatigue. Both monocular (with one eye) and binocular (with two eyes) observation is used in making photogrammetric measurements. Monocular observation allows approximate judgement as to the spatial position of objects, but as a rule is used only to estimate their plane position. The accuracy of monocular observation depends to a considerable extent on the resolving power of the eye, i.e., on the smallest angle at which the eye of the observer is still able to perceive two points separately. The resolving power of the eye depends on the size of the photosensitive element, on the type of objects being observed, and on the conditions of observation. Its average value is taken as 40". Poor illumination of the observed object, eye fatigue, color of the object, and other factors may markedly reduce the resolving power of the eye and thereby lower the accuracy of measurement.

Binocular and, in particular, stereoscopic observation is used much more extensively. This means binocular observation which allows perception, with sufficient accuracy, of the depth of the observed objects. In stereoscopic observation, the visual axes of both eyes (the lines joining the center of the crystalline lens with the central depression of the macula lutea) intersect at the observed point. The angle of intersection γ of these axes (Fig.116), subtended by the eye base b (the distance between the centers of the observer's eyes) at the observed object, is termed the angle of convergence or angular parallax. A change in distance between the observer and the object results in a change in angular parallax, and likewise in the accommodation of the eye, so that, as indicated above, these two quantities are to a certain extent interrelated.

The relationship between the angular parallax and the distance is determined by solving the right triangles S_1AA_o and S_2AA_o, from which it follows that

$$\tan \gamma' = \frac{S_1A_o}{AA_o} \; ; \quad \tan \gamma'' = \frac{S_2A_o}{AA_o} \; ; \quad S_1A_o + S_2A_o = S_1S_2 = b, \; \gamma' + \gamma'' = \gamma$$

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Since the angles γ' and γ'' are small, we may put $\tan \gamma' + \tan \gamma'' \approx \tan \gamma$, so that

$$\tan \gamma = \frac{b}{AA_o} = \frac{b}{Z} \tag{28}$$

where Z denotes the distance or, in other words, the length of the perpendicular
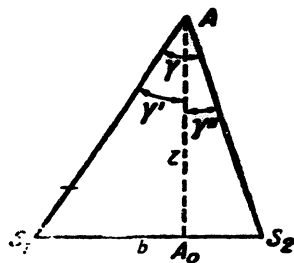
Fig.118 - Angular Parallax
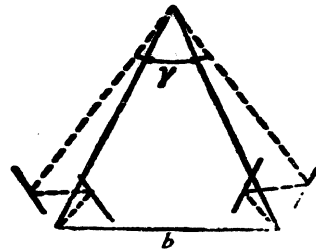
Fig.119 - Enlargement of the Eye Base by Using a Stereoscope

dropped from the observed point to the vertical plane containing the eye base. The change ΔZ leads to the change $\Delta \gamma$, defined by the expression

$$\tan (\gamma + \Delta \gamma) = \frac{b}{Z + \Delta Z} = \frac{\tan \gamma + \tan \Delta \gamma}{1 - \tan \gamma \tan \Delta \gamma}$$

Whence:

$$Z + \Delta Z = \frac{b - b \tan \gamma \tan \Delta \gamma}{\tan \gamma + \tan \Delta \gamma} \quad \text{or} \quad Z = \frac{b}{\tan \gamma}$$

Therefore,

$$\Delta Z = \frac{b - b \tan \gamma \tan \Delta \gamma}{\tan \gamma + \tan \Delta \gamma} - \frac{b}{\tan \gamma} = -\frac{b \tan \Delta \gamma (1 + \tan^2 \gamma)}{\tan \gamma (\tan \gamma + \tan \Delta \gamma)}$$

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At small angular parallax and minor changes in these angles, the term $\tan^2\gamma$ in the denominator may be neglected, as well as the term $\tan \Delta\gamma$ in the numerator. If we also note that $\tan \gamma = b:Z$, we may write

$$\Delta Z \approx - \frac{Z^2}{b} \tan \Delta\gamma \qquad (29)$$

A change in angular parallax is perceived by the observer with an accuracy that assures a reliable determination of a change in distance. The smallest change in angular parallax that can be perceived by an observer is termed the resolving power of stereoscopic vision and is on the average taken to be 20".

The resolving power of stereoscopic vision is affected by the brightness of objects, their form and dimensions, the contrast between objects, and by eye fatigue so that the conditions of observation greatly affect the accuracy of determination of distance differences. To increase the accuracy of depth perception of objects, auxiliary optical systems are used. In this case, the accuracy of perception increases with enlargement of the eye base, and with the use of an observation system with an angular magnification greater than unity. Thus, e.g., the use of a binocular with 6 × magnification results in an increase in the resolving power of stereoscopic vision to 3 - 4" and, consequently, to reduce the error in determination of the distance difference. The eye base is lengthened, for example, by using a stereoscope (Fig.119), consisting of four mirrors arranged in two parallel pairs. The observation of objects through such a stereoscope increases the angular parallax and, consequently, the accuracy of determination of the distance difference.

In practice, the visual observation of objects by instruments is used in stereoscopic range finders, in which the eye base of the observer is lengthed and a special magnifying viewer is provided. Observations with a stereoscopic range finder allow highly accurate distance determination for objects located in enemy territory.

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70. Stereoscopic Observations of Perspective Images

Stereoscopic vision produces a polyconic projection of the observed object on the retina, and creates a three-dimensional image of it. If, therefore, precisely the same images are produced on both retinas, the observer will see the object in space. This effect can be produced if perspective images of the object are placed before the observer's eyes in such a way that each eye has its own corresponding perspective image. In aerial photography, the aerial photographs actually are such images if the left aerial photograph is placed before the left eye of the observer and the right image before the right eye. In that case, the area of terrain photographed on two adjacent photographs will be perceived stereoscopically by the observer, i.e., the observer will see the spatial arrangement of the given area. Actually, if the left eye of the observer views the left photograph, containing the images a_1c_1 of any points of the terrain A, C (Fig.120), while the retina of the right eye receives the image of the corresponding points a_2c_2 of the right aerial photograph, then the observer will see the points A and C in space just as though they were before him in reality, i.e., point A will appear farther away from the observer than point C. Such a perception of an object is termed the direct stereoscopic effect.
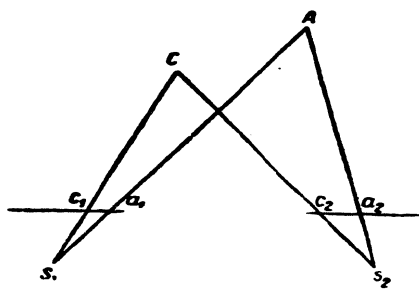
Fig.120 - Direct Stereoscopic
Effect
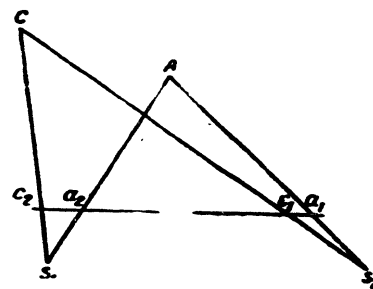
Fig.121 - Reverse Stereoscopic
Effect

If, however, the photographs are interchanged (Fig.121), i.e., the right-hand

photograph is viewed by the left eye and the left-hand photograph, by the right eye, then the observer will see a three-dimensional picture that is the reverse of the preceding, i.e., point C will appear farther away from the observer than point A; this case of observation is termed the reverse stereoscopic effect. It is not hard to see that, in the direct stereoscopic effect, the overlapping parts of contact prints will be arranged side by side while, in the reverse stereoscopic effect, they will be arranged outside the photographs, which sides are not in contact with each other.

If the aerial photographs are rotated through 90° in their own plane, then, while observing any corresponding points of two photographs, the visual axes of the eyes will be parallel to each other, so that the observed object will appear at an infinite distance, in which case all points of the object will seem to be located in a single plane. This is commonly interpreted to mean that the observation takes place under a null stereoscopic effect. Obtaining a reverse or null stereoscopic effect is possible only through use of perspective images, since the stereoscopic effect is always direct when real objects are observed with the naked eye.

Perception of the spatial position of photographed objects with the naked eye
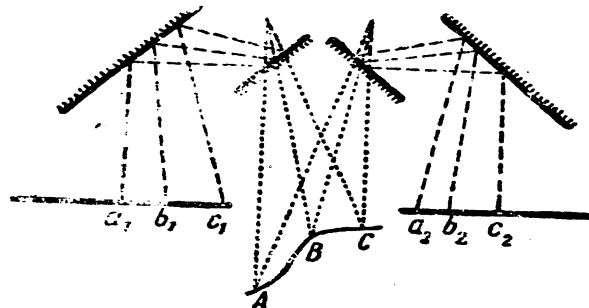
Fig. 122 - Construction of the Stereoscopic Model

is made difficult by the fact that each eye is able to observe only a single photograph intended for this eye. Such perception is considerably facilitated by the use

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of stereoscopes consisting of four mirrors arranged in parallel groups of two. By placing aerial photographs (or any other perspective images) under the external circles of the instrument and directing the visual axis of the eyes to the internal mirrors, the observer readily obtains a three-dimensional representation of the photographed objects, since the left eye cannot view the right-hand photograph in the stereoscope, nor can the right eye view the left-hand photograph. Construction of a three-dimensional image in the stereoscope is shown in Fig.122.

Current models of widely used stereoscopes are: the folding type (LZ), "Tsiklop", D-5, and the Bashtan.

Fig.123 - The Folding Stereoscope

The portable stereoscope LZ (Fig.123) consists of four mirrors 1, 1', 2, and 2', between which the lenses 3 and 3' are placed, allowing aerial photographs to be used with a double magnification. The mirrors and lenses are mounted to the holder 4, which is attached to the four-leg stand 5, by which the stereoscope is set up on a table, above aerial photographs. In packing, the legs of the instruments are folded toward the holder, which keeps the dimensions of the packing box small. Since the eye bases of different observers are not uniform, varying in length within a range of 58 to 72 mm, the inner mirrors of the stereoscope may be moved toward the outside. Despite the magnification produced by the lenses, it is

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expedient in many cases to remove them from the instrument to avoid additional errors in the stereoscopic observation, owing to the lens errors (distortion). Then, the observation will have to be conducted without magnification.

The "Tsiklop" stereoscope designed by F.V.Drobyshev (Fig.124) consists of a single pair of mirrors arranged in front of one eye of the observer, which feature has given the instrument its name. There is no optical system in front of the other eye of the observer, and he views the second photograph directly. Both mirrors of

Fig.124 - The Tsiklop Stereo-
scope

the stereoscope are mounted in a single mounting 1, connected by the stand 2 with the base 3. During manufacture of the mounting, a fixed angle of 15° is set between the mirrors, in contrast to other stereoscopes, in which these mirrors are always parallel. Due to this angle, a vertical ray is viewed after reflection from the outer mirror, at a vertical angle of 30°, so that the holder for the photograph, placed on the base 3, deviates from the perpendicular to this ray and will be inclined by an angle of 30°. This arrangement permits separating the prints from each other.

For packing, the mounting of the mirrors is removed from the stand, which in turn is removed from the base; the packing box required for the stereoscope is therefore small, which allows it to be housed in the field tent and used in field work for the selection of control points or for sketching the relief.

The stereoscope D-5 designed by F.V.Drobyshev (Fig.125) is also of the four-mirror type, which has retractable lenses. The lenses have a magnification of 2 × but a field of view of only 60 mm. When the lenses are retracted, the field of

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view covers the entire stereo pair but without magnification.

The stereoscope consists of a holder to which all the four mirrors and the lenses are mounted. The holder is attached to the base with a swivel bracket. The
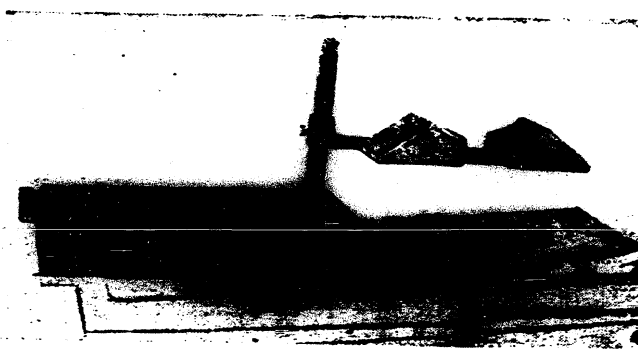
Fig.125 - Stereoscope D-5

holder, the bracket and the base are dismountable, so that the size of the carrying case becomes small. This makes the stereoscope adaptable for field use. The holder is easily adjusted and locked on the bracket, and the interior mirrors can be adjusted to the interpupillary distance.

The Bashtan stereoscope (Fig.126) is intended for stereoscopic viewing of two pictures of different scales, e.g., an aerial photograph and the corresponding area of a photomap. The instrument consists of the base 1, the carrier 2, and the outer mirror 3, which forms an an angle of 45° with the plane of the carrier. The stand 4 with the inner mirror 5, two ocular apertures 6, and the lens 7 are connected with the track 8, along which, when the screw is turned, the carrier 2 with the photograph is moved. This displacement is parallel to the eye base of the observer, the direction of which is determined by the ocular aperture 6. In operation, the observer views the photomap with one eye through the lens 7, and the photograph on the carrier 2 with the other eye through the two mirrors. The ratio of the lengths of

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the viewing rays should correspond to the ratio of the scales of the contact print to the photomap, i.e., the distance to the photograph must be as many times greater than the distance to the photomap as the scale of the contact print is larger than the scale of the photomap.
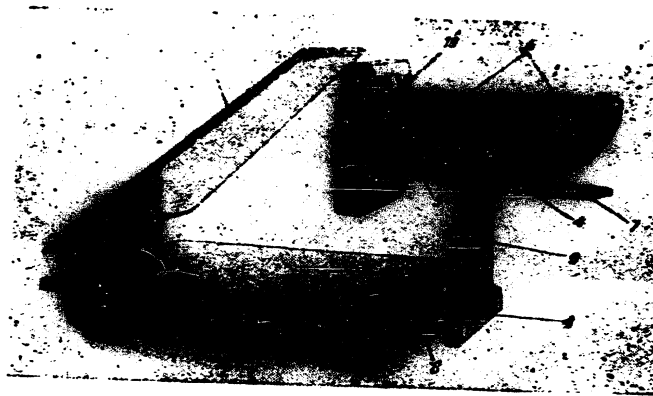
Fig.126 - The Bashtan Stereoscope

To obtain equality of similar relations, the contact print, together with the mirror 3, is moved with respect to the stand 4. However, the difference in accommodation of the eyes of the observer in this case will not allow both pictures to be seen with sufficient distinctness if the distance to the photograph and the distance to the photomap differ by over 30% of this distance. To eliminate this difference in accommodation, the lens 7 is provided, which forms an image of the photomap at a distance equal to the distance to the contact print. Any shift in the carrier 2 along the track must thus be accompanied by a displacement of the lens with the aid of the screw 9. In addition to the lens 7, a lens 10, placed between the two mirrors, is provided to compensate the difference in accommodation.

In working with the Bashtan stereoscope, the observer simultaneously views the contact print and the corresponding area of the photomap. This area may be obtained from assembly of a preceding or subsequent photograph, or an adjacent

photograph from another flight strip. The stereoscopic perception in various parts will, therefore, differ. The part constituting a subsequent photograph may give a direct stereoscopic effect; that of a preceding exposure, a reverse stereoscopic effect; an adjacent photograph from another flight strip, a null stereoscopic effect; while the part consisting of the same photograph will give no stereoscopic effect at all. However, this difference in stereo perception does not interfere with using the Bashtan stereoscope for stereoscopic transfer of a sketched relief from the photograph to the photomap.

Stereospectacles are used to some extent for the stereoscopic study of aerial photographs. These consist of two pairs of mirrors or prisms in a mounting. In use, the photographs are placed on a table and viewed by the observer through these stereoscopic spectacles. A disadvantage of such an instrument is that it requires the head of the observer to remain motionless, since even the slightest turns of the head will change or cancel the stereoscopic effect.

71. Sketching of Relief on Stereoscopes, Based on Bench Marks

The stereoscopes described in the preceding Section allow the relief to be sketched from aerial photographs of bench marks. For this purpose any stereoscope can be used, which is removed from the packing box and placed on the table. The material for sketching the relief will be contact prints if the photographed terrain is hilly or mountainous, or a mosaic and contact prints for level terrain. This condition is necessary since, on level terrain, a small area photographed on a single aerial photograph does not provide an opportunity for correctly detecting and representing the character of the relief, and random, partial fluctuations of the relief may lead to untrue concepts. Conversely, in mountainous or hilly terrain, the relief is sufficiently distinct, but a mosaic prepared from contact prints would be very inaccurate.

Within the limits of each contact print the elevations of a number of bench

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marks must be known. The number and disposition of these bench marks determines the
scale of the map to be prepared and the character of the relief. These points are
selected on characteristic sites: summits, tops of a slope, terraces, drainage sys-
tems of rivers, channels cut by water, saddles, divides, foots of mountains, crests
of ravines, etc., and their elevations are determined by field geodetic operations
or by photogrammetric densification of a field vertical control net. The bench
marks are selected on the basis of a preliminary stereoscopic study of the aerial
photographs of each region, with the participation of a geographer, who determines
the patterns of relief formation and the conditions under which it may be correctly
represented by contour lines.

In working with contact prints of mountainous or hilly regions, these are
placed under the stereoscope in such a way that the lines joining the principal
points of both photographs are approximately parallel to the eye base of the observ-
er, and that the distance between the photographs is such that optimum three-
dimensional perception is obtained. After arranging the photographs under the ste-
reoscope so that their overlapping parts are in contact with each other, they are
moved apart until stereoscopic perception is obtained. A more careful orientation
is made on the base of the visual stereoscopic model of the locality and of the
existing bench marks. Thus, the presence of water bodies, rivers, swamps, etc. can
be of substantial assistance. Such water bodies should be arranged on the stereo-
scopic model according to their known patterns of arrangement on the ground. To
obtain correct correspondence between the model being viewed and the actual land-
scape, one of the aerial photographs is rotated in its plane.

After such orientation, the photographs are fastened by disks to the table, on
one of which (usually the right one) the contour lines are drawn.

Here the existing bench marks and visible stereoscopic relief of the terrain
are the guiding factors. In studying the relief of the terrain under the stereo-
scope, it is necessary to pay attention to the fact that the visible relief almost

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always appears more prominent than it is in reality. This exaggeration is directly
proportional to the ratio between the distance from the observer's eye to the photo-
graph, and to the focal distance of the camera. When an aerial camera with a focal
length of 70 mm is used, at a distance of 250 mm from the eye to the photograph in
the stereoscope (distance of optimum vision), the relief seen in the stereoscope is
3.5 times as great on the ground. Mountain peaks will thus appear to tower more
over the surrounding territory, and ravines to be more sharply incised. Such an
exaggeration of relief allows minor variations in it to be more rapidly and easily
detected, and the observer quickly becomes accustomed to this lack of correspond-
ence with the actual scene.

As already pointed out, the relief is sketched on the basis of a grid of bench
marks. In viewing the stereoscopically placed photographs, the observer, by visual
interpolation between the bench marks, determines the position of the contour lines
of the section selected, using the available geographic description of the area and
the conditions of relief formation as guide. The standards established during the
surveying of each region are of great significance in sketching the relief, since
they give the basic type of relief and the methods of depicting it by contour lines.

The relief is sketched with a soft pencil on a contact positive print, and it
is only after the relief has been mapped over the whole area of the photograph that
the contour lines are inked in and the principal contours marked. On a steep slope,
not all of the contour lines need be sketched on the photographs, in view of the
small distance between them, but it is advisable to avoid such cases. Only indi-
vidual elements (cliff, boulders) are indicated by conventional symbols, but all the
remaining forms of relief are expressed by contour lines.

The relief of level regions is sketched on uncontrolled mosaics. In this case,
the contact print is placed under the left eye of the observer, while the portion
of the mosaic corresponding to it is placed under the right eye. In sketching the
relief, the entire mosaic, covering the area of a trapezoid of the given scale, is

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stereoscopically viewed. For this purpose, all contact prints composing the mosaic are successively placed under the stereoscope. The contact print and the mosaic are oriented, after which the observer sketches the outline of the relief on the mosaic. This skeleton relief is tied in with the bench marks and with the hydrographic grid and outlines the principal divides, valleys, and drainage systems of rivers. Only after the skeleton relief has been traced on all areas of the mosaic is it possible to proceed to its more detailed delineation, since otherwise the principal forms of the relief may be incorrectly mapped as a result of minor changes within the limits of a single photograph. In level terrain, the work should be started by drawing the contour lines delineating the drainage system, which must first be entered in blue ink on the mosaic.

72. Measuring Elevations from Photographs

Frequently, the given number of geodetic points of elevation is insufficient for correct interpretation of the relief of a photographed area, specifically in cases in which it is difficult to establish elevation points by geodetic means. This necessitates determination of the elevation of points by photogrammetric means based on measuring the horizontal parallax of the photographed points.
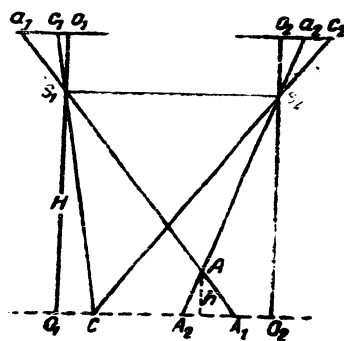
Fig. 127 - Variation in Horizontal Parallaxes

Let the camera axis, at the time of photographing a pair of overlapping photographs, be pointed in a vertical position, and let both centers of projection lie in one horizontal plane. Point A (Fig. 127), which has an elevation of h relative to the horizontal plane, passes through the point C and is located at points a_1 and a_2 on the aerial photograph. The image of point C on the prints will be c_1 and c_2. If, as the origin of coordinates on the

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photograph the principal point is taken and if the axis lying on a vertical plane is denoted by xx, representing the eye-base axis, then the abscissas of points a_1, a_2, c_1, and c_2 will be x_{a_1}, x_{a_2}, x_{c_1}, and x_{c_2}. Continuing the projection of the rays S_2A and S_1A until they intersect the horizontal plane at points A_1 and A_2 will yield:

$$O_1A_1 - O_2A_2 - O_1C + O_2C = A_1A_2 \qquad (30)$$

where the distance S_1S_2 between the centers of projection of the prints is called the photographic base since all projected rays intersect the ends of this radial. The angle that the projected ray forms with the horizontal plane at the surface is called the angle of parallax. The deviation of the abscissas of identical points on two photographs is known as the horizontal parallax and is denoted by p. From the above:

$$P_a = x_{a_1} - x_{a_2}; \quad P_c = x_{c_1} - x_{c_2} \qquad (31)$$

If both sides of eq.(30) are multiplied by the factor $\dfrac{f_k}{H}$, or brought to the scale of the print, then:

$$\frac{C_1A_1f_k}{H} = o_1a_1 = x_{a_1}; \quad \frac{O_2A_2f_k}{H} = o_2a_2 = x_{a_2}$$

$$\frac{O_1Cf_k}{H} = o_1c_1 = x_{c_1}; \quad \frac{O_2Cf_k}{H} = o_2c_2 = x_{c_2}$$

or

$$x_{a_1} - x_{a_2} - (x_{c_1} - x_{c_2}) = (A_1A_2)\ \frac{f_k}{H} = P_a - P_c = \Delta_P \qquad (32)$$

where Δp is the horizontal parallax difference of two points.

At the same time, the solution of the similar triangles S_1S_2A and $A_1\hat{A}A_2$ yields:

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$$A_1 A_2 = \frac{Bh}{H - h}$$

Thus,

$$\Delta p = \frac{bh}{H - h} \tag{33}$$

since

$$b = B \frac{f_k}{H} \tag{34}$$

Consequently, the horizontal parallax difference of two points depends on the height of one of these points above the horizontal plane which contains the second point, and is determined by the variance in the difference of values of the abscissas of identical points on two prints.

If point A on the surface is located in the same plane as point C then, if $h = 0$, the horizontal parallax difference Δp will also be zero. Due to this and on the strength of eq.(32), the variance in the abscissas (horizontal parallax) of identical points on two photographs will be equal. Hence,

$$x_{a_1} - x_{a_2} = x_{c_1} - x_{c_2}$$

Since the initial point C was located in the initial plane, it follows from Fig.127 that

$$O_1 C - O_2 C = B$$

or

$$x_{c_1} - x_{c_2} = b = P_c \tag{35}$$

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i.e., the horizontal parallax of the point located in the initial plane, will equal the photographic base within the boundaries of the photograph.

On the basis of eq.(33), it is easy to develop

$$h = \frac{H \, \Delta p}{b + \Delta p} \qquad (36)$$

Therefore, if the abscissas of identical points are measured on the prints and the value of the horizontal parallax difference is calculated, then it is possible to calculate the actual elevations of the corresponding points on the map.

73. Determination of the Elevation of Points

As pointed out previously, the photogrammetric determination of the elevation of points consists in measuring the absolute parallax differences on the photograph and in calculating the elevations from them. The steroscope is equipped with additional measuring devices for measuring the absolute-parallax differences. When such devices are used, the instruments are known as measuring or topographic stereoscopes. The measuring element of such stereoscopes consists of marks of various types, which are displaced with respect to the aerial photograph for determining the absolute-parallax difference.
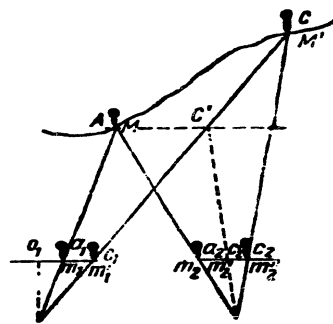
Fig.128 - Measurement of Elevations with the Aid of Wandering Marks

If, when viewing two aerial photographs in the stereoscope, a given mark in form of a dot, line, cross, etc. is placed on each of them, the two marks, when viewed with both eyes, will appear to be merged into one spatial mark, located arbitrarily with respect to the stereoscopic model of the terrain. Now let (Fig.128) the left-hand

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mark m_1 coincide with the point a_1 of the left-hand photograph, and the right-hand mark m_2 with the point a_2 of the right-hand photograph. The observer will then see the wandering mark M as coinciding with the point A of the model. A simultaneous displacement of both marks in any direction (e.g., by the value $m_1'm_1 = m_2'm_2$) will allow matching the wandering mark with the point C', located at the same distance from the observer as the point A. The displacement of only a single mark in a direction parallel to the visual space of the observer, however, will lead to an apparent displacement of the wandering mark in depth.

Now, if the left-hand mark is left in the position m_1' while the right-hand mark is moved from the position m_2' to the position m_2'', then the floating mark m' will be farther away from the observer than it previously appeared to be. Thus, a simultaneous displacement of both marks or the separate displacement of one mark makes it possible to superimpose the floating mark on any point of the model being viewed.

Such a solution of the problem involves a displacement of the marks with respect to fixed photographs, but the result will be the same if the aerial photographs are moved and the marks remain fixed. In this case (Fig.129), the wandering marks will always appear to be at a constant distance and in a single place, while the model of
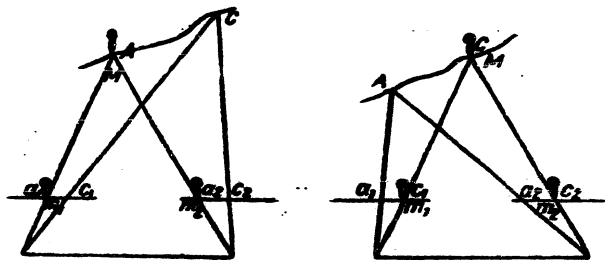
Fig.129 - Displacement of a Wandering Mark

the terrain is displaced in all directions with respect to the mark. Ordinarily, in topographic stereoscopes, some of these displacements are made with the marks and

some with the aerial photographs.

The marks employed in topographic stereoscopes are classified as point, line, and area marks (Fig.130). With point marks, the wandering mark appears in the form of a dot (point, intersection of short lines, end of a short line, etc.) which is successively displaced from various points of the model. Each mark must, therefore, be susceptible to displacement from any point of the overlapping parts of aerial photographs.
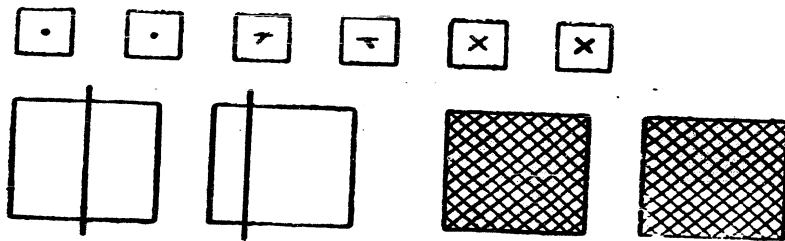
Fig.130 - Forms of Marks

To accomplish this, both marks (or both aerial photographs) are displaced together along two mutually perpendicular axes, one of which is parallel to the eye base of the observer. In addition, one of the marks (or one of the aerial photographs) has an independent displacement with respect to the other along an axis parallel to the eye base. If the marks consist of lines perpendicular to the eye base of the observer and if the entire aerial photograph is perceived, the necessary motions will be confined to the displacement of both marks jointly along an axis parallel to the eye base, and to the separate displacement of one of them along that same axis. Finally, with plane marks, when the entire overlap area of both photographs is filled with numerous marks that form a plane, a simultaneous displacement of both marks is no longer required, and only the separate motion of one of them along the line of the eye base is necessary. It is true that in this case it is

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very time-consuming to fill the entire area with points, and that the joint displacements of both marks still takes place, although within very narrow ranges.

The marks may be produced mechanically or optically. In the former case, they consist of thin metal or textile filaments or hairs, stretched over the aerial photographs, or of glass plates engraved with the marks, and placed over the photographs. In the latter case, the marks are located outside the aerial photographs, but are projected onto the plane of the photographs by an optical system. The first method is more convenient and does not require a special optical system, but it is less advantageous from the operational point of view since it prevents free work of the observer on the photographs.

Thus, whether he has one kind of marks or another, arranged in the field of vision of the viewing system, the observer may superimpose the wandering mark on any desired point of the model and read off the displacement on the corresponding scales. The relationship between the linear displacement of one mark and the apparent displacement in depth of the wandering mark is readily established from Fig.128.

Let the left mark coincide with the point a_1 of the left photograph and the right mark with the point a_2 of the right photograph. Then, the floating mark will appear to coincide with the point A of the model. If both marks are displaced by the same quantity $a_1 c_1 = a_2 c_2'$, then the wandering mark is shifted to the position C', located at the same distance from the observer as the point A. In order to displace the space mark from the point C of the model, the right mark must be displaced, independently of the left mark, by the quantity $c_2' c_2$. On the basis of Fig.128, we may write:

$$a_1 c_1 = o_1 c_1 - o_1 a_1 = x_{c_1} - x_{a_1};$$

$$a_2 c_2 = o_2 c_2 - o_2 a_2 = x_{c_2} - x_{a_2};$$

$$a_2 c_2 - a_1 c_1 = c_2' c_2 = x_{c_2} - x_{a_2} - x_{c_1} + x_{a_1} =$$

$$= x_{a_1} - x_{a_2} - (x_{c_1} - x_{c_2}) \qquad (37)$$

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A comparison of eqs.(37) and (32) indicates that the independent displacement, by $c_2'c_2$, of the right-hand mark is the absolute-parallax difference, which can be determined in this way. Consequently, to determine the X-parallax difference between the points A and C, the wandering mark must be successively displaced from the points A and C of the model, and the value of the independent displacement of the right (or the left) mark must then be measured. The difference of elevation between these points can then be calculated from eq.(33).

The variations in parallax difference obtained by this method must be measured very accurately as indicated by eq.(33). Assume that photographs, 180 × 180 mm, in size, with a 60% end lap were obtained in aerial surveying. Then the photographic base at the scale of the photograph, (distance between principal points) will be 72 mm. The flight altitude was 3600 m, and is used as the initial altitude, while the relative elevation to be measured was 1m. In this case, the horizontal-parallax difference will be

$$\Delta p = \frac{72 \text{ mm} \times 1 \text{ m}}{3600 \text{ m} - 1 \text{ m}} = 0.02 \text{ mm}$$

Therefore, in measuring the parallax of up to 1 m, this must be done with an accuracy of 0.02 mm. This indicates the importance of accuracy in measuring parallax. It also is necessary to measure accurately the abscissas of identical points with special measuring instruments. This is particularly true when the points are located on contours that are not too clear.

Despite the convenience in using topographic stereoscopes to determine relative elevation from prints, the method has not received wider use in the USSR. The reason for this is the fact that the horizontal-parallax difference, measured on the photograph, depends not only on the elevation difference of the points but also on the elements of interior orientation of the photographs, thus making a calculation of the elevation difference from eq.(36) possible only for cases of ideal photo-

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graphs, when the optical axis of both prints are strictly vertical and the projection
centers are located in the same horizontal plane. In all other cases, the correc-
tions for the difference between the actual conditions of exposure and the assigned
conditions must be applied to the measured horizontal-parallax differences. The
corrections for horizontal parallax are correlated with the elements of exterior
orientation and also with the current coordinates of the control points.

74. Elements of Exterior Orientation

The elements of exterior orientation of prints determine their position with
respect to a given system of space coordinates and are characterized by six quanti-

ties. The system of space coordinates is
usually described by the vertical planes
(Fig.131) XZ and YZ and a horizontal plane
XY. The outlines of these planes are the
directions of the axes of the system of
space coordinates. The first three elements
of exterior orientation represent the linear
coordinates of the center of projection and
are denoted by X_S, Y_S, Z_S. The position of
the optical axis of the camera is defined by
the angles α_x and ω. The angle α_x is the
projection of the angle of tilt α of the
optical axis to the coordinate plane XZ,

Fig.131 - Elements of Exterior
Orientation

while the angle ω represents the angle formed by the optical axis with its projec-
tion onto the plane XZ and is, therefore, measured in the tilted plane. Finally,
the sixth element is the angle of rotation χ of the photograph in its own plane,
and is measured on the photograph between the path of the plane XZ and the xx axis
of the photograph. Consequently, there are six elements of exterior orientation for
each photograph, of which three are linear and three are angular. Therefore, the

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spatial position of two photographs is determined by twelve elements of exterior orientation.

75. The Coordinates of Photograph Points

The position of any of the image points on a photograph is determined in accordance with a previously selected rectangular system of coordinates.

Since the elementary object of processing in a stereophotogrammetric survey consists of a pair of photographs, it follows that the axes of coordinates are selected simultaneously for two photographs. The principal point of the photograph is often selected as the origin of such a system of coordinates, in view of the fact that its position can easily be obtained from the images of the coordinate marks. The direction of the axis of the system of coordinates is given by a line connecting the principal points of the two photographs, while lines perpendicular to the xx axis and passing through one of the principal points are taken as the yy axis. In this way, each pair of photographs has one axis of abscissas, xx and two axes of ordinates, yy and y'y'.

In processing a second pair of photographs, the direction of the xx axis will change, since it will be given by a line connecting the principal points of the second and third photographs (instead of the first and second) so that the direction of the ordinate axis yy will also change. For this reason, there may be two different directions of the coordinate axes on the second photograph, depending on whether it is paired with the preceding or the following photograph.

In accordance with the system of coordinates selected, the position of each point of the photograph may be expressed in a linear or an angular form. In the former case (Fig.132), the wanted data are the quantities x, x', y, and y', of which the coordinates x and y define the position of a certain point a_1 of the left photograph with respect to its principal points, while the coordinates x' and y', in turn, determine the position of the corresponding point a_2 of the right photo-

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graph with respect to the origin of the coordinates at its own principal point.

If the position of a point on the photograph is expressed by angular coordinates, then the quantity sought will be the angles β_x and β_y (Fig.133), which are
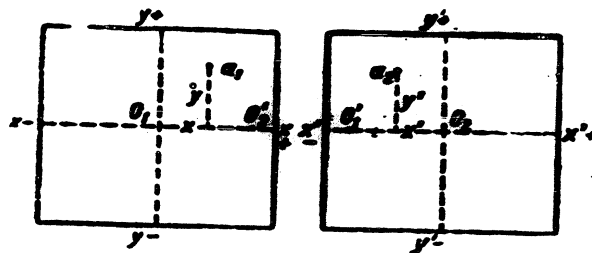
Fig.132 - Linear Coordinates of the Photograph Points

the projections of the angle β formed respectively by the projecting and principal rays on two planes. One of these planes is the plane containing the projection

Fig.133 - Angular Coordinates of the Points of an Aerial Photograph

center and the abscissa axis of the photograph, while the other plane is an inclined plane produced by the projecting ray Sa, whose locus on the plane of the photograph is perpendicular to the abscissa axis.

By solving the right triangles Soa_0 and Sa_0a, the relation between the linear and angular coordinates of the points of the photograph is readily established, bearing in mind the fact that the distance between the projection center and the principal point of the photograph is the focal length. Solving the triangle Soa_0, formed after projecting the point a of the photograph onto the abscissa axis, will yield

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$$\tan \beta_x = \frac{x}{f_k} ; \quad Sa_o = f_k : \cos \beta_x \tag{38}$$

Since the angle at point a_o in the triangle $Sa_o a$ is a right angle, it follows that

$$\tan \beta_y = \frac{y}{Sa_o}$$

or, after substituting the value of Sa_o,

$$\tan \beta_y = \frac{y}{f_k} \cos \beta_x \tag{39}$$

By analogy, the following equations are obtained for the second photograph:

$$\tan \beta_x' = \frac{x'}{f_k} ; \quad \tan \beta_y' = \frac{y'}{f_k} \cos \beta_x'$$

Knowing the linear coordinates of the points of the photographs, this makes it easy to obtain their angular coordinates, and vice versa.

76. Horizontal-Parallax Difference

The deviation of the elements of exterior orientation from the specified values of ideal exposure conditions results in a change in the value of the abscissa for identical points on the photographs and thus in a change in the horizontal-parallax difference. Consequently, the horizontal-parallax difference is defined by the equation:

$$\Delta p = x_{a_1} - x_{a_2} - x_{c_1} + x_{c_2} \tag{32}$$

Then, as a result of the change in the abscissa, we have:

$$\Delta' p = x_{a_1}' - x_{a_2}' - x_{c_1}' + x_{c_2}' \tag{40}$$

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Taking into account that

$$\Delta'p = \Delta p + \delta p; \quad x'_{a_1} = x_{a_1} + \Delta x_{a_1}; \quad x'_{a_2} = x_{a_2} + \Delta x_{a_2};$$

$$x'_{c_1} = x_{c_1} + \Delta x_{c_1}; \quad x'_{c_2} = x_{c_2} + \Delta x_{c_2}$$

where δp is the change in the horizontal parallax, while Δx_{a_1}, Δx_{a_2}, Δx_{c_1}, and Δx_{c_2} denote the change in the abscissa, it can be stated that:

$$\delta p = \Delta'p - \Delta p = \Delta x_{a_1} - \Delta x_{a_2} - \Delta x_{c_1} + \Delta x_{c_2} \tag{41}$$

The values of Δx_{a_1}, Δx_{a_2}, Δx_{c_1}, and Δx_{c_2} may be defined from the correlation of the elements of exterior orientation with the change in abscissas.

Assume that the exposure was done at a strictly vertical camera axis, for both cameras, but at an altitude which was different from the assigned altitude. Then the distance X (Fig.134) along the abscissa axis, and between any two points on the earth's surface, will be represented at the assigned altitude H_o by the distance x_1 on the photograph. At the actual flight altitude H it will be x_2, i.e.,

$$x_1 = X \frac{f_k}{H_o} \quad \text{and} \quad x_2 = X \frac{f_k}{H}$$

Therefore, the change in the abscissa on the photograph, due to a change in height, will be:

$$x_2 - x_1 = \Delta x_1 = X \frac{f_k}{H} - X \frac{f_k}{H_o} = \frac{X f_k}{H_o H} (H_o - H)$$

or

$$x_2 - x_1 = \frac{x_1}{H} (H_o - H) \tag{42}$$

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On changes in the angle of tilt α_x of the optical axis, the relation between the coordinates x_2 of the tilted photograph and x_1 of the horizontal photograph is
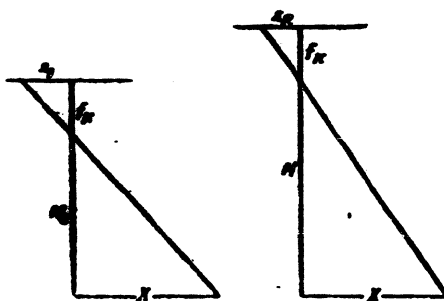
Fig.134 - Influence of Different
Flight Altitudes on
the Abscissa
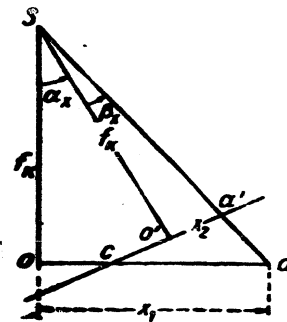
Fig.135 - Influence of Longitud-
inal Angle of Tilt on
the Change in Abscissa

found by solving the triangles Soa and So'a', formed by the direction of the projected ray Sa and the optical axis So or So'.

Then (Fig.135),

$$x_1 = f_k \tan(\alpha_x + \beta_x), \quad x_2 = f_k \tan \beta_x$$

or

$$x_2 - x_1 = \Delta x_2 = f_k \left[\tan \beta_x - \frac{\tan \alpha_x + \tan \beta_x}{1 - \tan \alpha_x \tan \beta_x} \right] =$$

$$= - f_k \frac{\tan \alpha_x + \tan \alpha_x \tan^2 \beta_x}{1 - \tan \alpha_x \tan \beta_x} \tag{43}$$

At low angles of tilt α_x ($\alpha_x \leqslant 30$), the second term of the denominator will be small and can be neglected in order to simplify the solution. Then,

$$\Delta x_2 \approx - f_k \tan \alpha_x - f_k \tan \alpha_x \tan^2 \beta_x$$

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Knowing that,

$$\tan \beta_x = \frac{x_2}{f_k}$$

it follows that

$$\Delta x_2 \approx - f_k \tan \alpha_x - \frac{x_2^2}{f_k} \tan \alpha_x \tag{44}$$

On varying the angle ω, i.e., the lateral angle of tilt, the photograph is displaced from its horizontal position (Fig.136) into a tilted position, while the projected ray Sa maintains its former direction. The projection of the point a onto the yy axis will be a_0 for a true vertical or horizontal photograph, while the projection of the point a', again onto the yy axis, will be a_0' for a tilted photograph. In accordance with the notations, introduced above, the lateral angle of tilt oSo' will be ω, and the angle
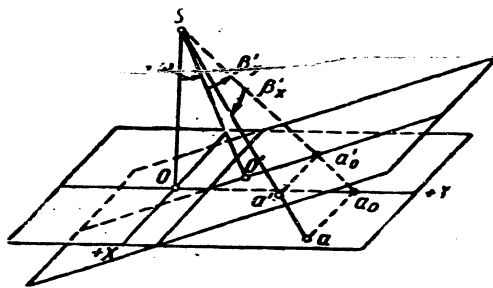
Fig.136 - Effect of the Lateral Angle of Tilt on the Change in Abscissa

between the direction of the projected ray and that of the principal ray of the tilted photograph will be β, while its projections onto the coordinate planes a_0S_0 and aa_0S will be β_y' and β_x'. The difference between the angles β_y' and β_x' and the angular coordinate β_y and β_x consists in the fact that the angle β_y' is measured in the plane containing the optical axis and the radial of the yy axis, whereas the angle β_y is measured in the plane containing the projected ray and its projection onto the plane Sox. Similarly, the angle β_x is measured in the plane Sox, while the angle β_x' in the plane containing the direction of the projected ray and its projec-

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tion onto the plane Soy.

The lines $a'a_o'$ and aa_o will be parallel to each other since both are perpendicular to the plane $Soa_o'a_o$, so that the triangles $a'a_o'$ and Saa_o are similar. From the similarity of these triangles, it follows that

$$a'a_o' = aa_o \frac{Sa_o'}{Sa_o}$$

Likewise,

$$a'a_o' = x_2; \quad aa_o = x_1; \quad Sa_o' = \frac{f_k}{\cos \beta_y'}; \quad Sa_o = \frac{f_k}{\cos (\beta_y' + \omega)}$$

Thus,

$$x_2 = x_1 \frac{\cos (\beta_y' + \omega)}{\cos \beta_y'} = x_1 \frac{\cos \omega \cos \beta_y' - \sin \omega \sin \beta_y'}{\cos \beta_y'} =$$

$$= x_1 \cos \omega - x_1 \tan \beta_y' \sin \omega$$

Noting that the solution of the right triangle $So'a_o'$ yields

$$\tan \beta_y' = \frac{y}{f_k}$$

and considering, from the smallness of the angle ω ($\omega \leqslant 30$), that $\cos \omega = 1$, it follows that

$$x_2 - x_1 = \Delta x_3 \approx - \frac{x_1 y}{f_k} \sin \omega \qquad (45)$$

Finally, if the photograph is rotated in its own plane through the angle χ (Fig.137), with the xx axis of the photograph preserving its former direction, the abscissa of some point a can be expressed as

$$x_2 = r \cos (\varphi + \chi)$$

while its value, before the rotation, was

$$x_1 = r \cos \varphi$$

In these equations, φ denotes a polar angle, and r the radius vector to a point. Then,

$$x_2 - x_1 = \Delta x_4 = r \cos (\varphi + \chi) - r \cos \varphi =$$

$$= r (- \cos \varphi + \cos \varphi \cos \chi - \sin \varphi \sin \chi)$$

or, noting that

$$1 - \cos \chi = 2 \sin^2 \frac{\chi}{2}; \quad \sin \varphi = \frac{y}{r};$$

$$\cos \varphi = \frac{x}{r}$$

Fig.137 – Effect of the Angle of Rotation of the Photograph on the Change in Abscissa

we have

$$\Delta x_4 = - y \sin \chi - 2 x \sin^2 \frac{\chi}{2} \qquad (46)$$

At a low angle of rotation of the photograph in its own plane

$$x_2 - x_1 = \Delta x_4 \approx - y \sin \chi \qquad (47)$$

Thus, the combined effect of all above elements of exterior orientation on the abscissas of photographed points is expressed by the equation

$$\Delta x = \Delta x_1 + \Delta x_2 + \Delta x_3 + \Delta x_4 = - \frac{x}{H} (H - h_o) -$$

$$- f_k \tan \alpha_x - \frac{x^2}{f_k} \tan \alpha_x \frac{xy}{f_k} \sin \omega - y \sin \chi \qquad (48)$$

This equation defines the variation of the abscissas of any points on the left and right photographs. Therefore,

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$$\Delta x_{a_1} = - \frac{x_{a_1}}{H} (H_1 - H_o) - f_k \tan \alpha_{x_1} - \frac{x_{a_1}^2}{f_k} \tan \alpha_{x_1} -$$

$$- \frac{x_{a_1} y_{a_1}}{f_k} \sin \omega_1 - y_{a_1} \sin \chi_1$$

$$\Delta x_{a_2} = - \frac{x_{a_2}}{H} (H_2 - H_o) - f_k \tan \alpha_{x_2} - \frac{x_{a_2}^2}{f_k} \tan \alpha_{x_2} -$$

$$- \frac{x_{a_2} y_{a_2}}{f_k} \sin \omega_2 - y_{a_2} \sin \chi_2$$

$$\Delta x_{c_1} = - \frac{x_{c_1}}{H} (H_1 - H_o) - f_k \tan \alpha_{x_1} - \frac{x_{c_1}^2}{f_k} \tan \alpha_{x_1} -$$

$$- \frac{x_{c_1} y_{c_1}}{f_k} \sin \omega_1 - y_{c_1} \sin \chi_1$$

$$\Delta x_{c_2} = - \frac{x_{c_2}}{H} (H_2 - H_o) - f_k \tan \alpha_{x_2} - \frac{x_{c_2}^2}{f_k} \tan \alpha_{x_2} -$$

$$- \frac{x_{c_2} y_{c_2}}{f_k} \sin \omega_2 - y_{c_2} \sin \chi_2$$

$$\left. \right\} \quad (49)$$

where the subscript "1" refers to the left photograph of the stereo pair, while the subscript "2" refers to the right photograph. Substituting these values in eq.(41), the change in horizontal parallax difference, in relation to the change in the elements of exterior orientation and the current coordinates of the points on the photograph can be calculated. For simplifying this expression, we note that, in accordance with eqs.(32) and (35),

$$x_{c_2} = x_{c_1} - b; \quad x_{a_2} = x_{a_1} - b - \Delta p$$

Moreover, assuming that the point c_1 coincides with the principal point of the left photograph, that at low angles of tilt the difference between the ordinates

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of corresponding points will be very small, and that the deviation of the elements
of exterior orientation from the ideal ($\alpha_{x_1} = \alpha_{x_2} = \omega_1 = \omega_2 = \chi_1 = \chi_2 = H_1 - H_o = H_2 - H_o = 0$) is also small, we may write

$$x_{c_1} = y_{c_1} = y_{c_2} = 0; \quad y_{a_2} \approx y_{a_1}; \quad \tan \alpha_{x_1} = \alpha_{x_1} \sin 1'$$

$$\tan \alpha_{x_2} = \alpha_{x_2} \sin 1'; \quad \sin \omega_1 = \omega_1 \sin 1'; \quad \sin \omega_2 = \omega_2 \sin 1'$$

$$\sin \chi_1 = \chi_1 \sin 1'; \quad \sin \chi_2 = \chi_2 \sin 1'$$

Then,

$$\delta p = \Delta x_{a_1} - \Delta x_{a_2} - \Delta x_{c_1} + \Delta x_{c_2} = - \frac{x_{a_1}}{H} (H_1 - H_o) -$$

$$- \frac{x_{a_1}^2}{f_k} \alpha_{x_1} \sin 1' - \frac{x_{a_1} y_{a_1}}{f_k} \omega_1 \sin 1' - y_{a_1} \chi_1 \sin 1' + \frac{x_{a_1}}{H} (H_2 - H_o) -$$

$$- \frac{b}{H} (H_2 - H_o) - \frac{\Delta p}{H} (H_2 - H_o) + \frac{x_{a_1}^2}{f_k} \alpha_{x_2} \sin 1' -$$

$$- \frac{2b x_{a_1}}{f_k} \alpha_{x_2} \sin 1' - \frac{2 \Delta p x_{a_1}}{f_k} \alpha_{x_2} \sin 1' +$$

$$+ \frac{2b \Delta p}{f_k} \alpha_{x_2} \sin 1' + \frac{b^2}{f_k} \alpha_{x_2} \sin 1' + \frac{\Delta p^2}{f_k} \alpha_{x_2} \sin 1' +$$

$$+ \frac{x_{a_1} y_{a_1}}{f_k} \omega_2 \sin 1' - \frac{b y_{a_1}}{f_k} \omega_2 \sin 1' - \frac{\Delta p y_{a_1}}{f_k} \omega_2 \sin 1' +$$

$$+ y_{a_1} \chi_2 \sin 1' + \frac{b}{H} (H_2 - H_o) - \frac{b^2}{f_k} \alpha_{x_2} \sin 1' = - \frac{x_{a_1}}{H} (H_1 - H_2) -$$

$$- \frac{x_{a_1}^2}{f_k} (\alpha_{x_1} - \alpha_{x_2}) \sin 1' - \frac{2b x_{a_1}}{f_k} \alpha_{x_2} \sin 1' - \frac{x_{a_1} y_{a_1}}{f_k} (\omega_1 - \omega_2) \sin 1' -$$

$$- \frac{b y_{a_1}}{f_k} \omega_2 \sin 1' - y_{a_1} (\chi_1 - \chi_2) \sin 1' - \frac{\Delta p}{H} (H_2 - H_o) -$$

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$$- \frac{2\Delta px_{a_1}}{f_k} \alpha_{x_2} \sin 1' + \frac{2b\Delta p}{f_k} \alpha_{x_2} \sin 1' -$$

$$- \frac{\Delta py_{a_1}}{f_k} \omega_2 \sin 1' + \frac{\Delta p^2}{f_k} \alpha_{x_2} \sin 1'$$

Then, assembling the above quantities in accordance with the coordinates of current points, it is not difficult to construct the final expression

$$\delta p = - \frac{x_{a_1}}{f_k} \left(\delta H + \frac{2b\alpha_{x_2}}{\rho} \right) - \frac{x_{a_1}^2}{f_k \rho} \left(\alpha_{x_1} - \alpha_{x_2} \right) -$$

$$- \frac{x_{a_1} y_{a_1}}{f_k \rho} \left(\omega_1 - \omega_2 \right) - \frac{y_{a_1}}{\rho} \left(\chi_1 - \chi_2 + \frac{b}{f_k} \omega_2 \right) -$$

$$- \frac{\Delta p}{H} (H_2 - H_o) - \frac{2\Delta px_{a_1}}{f_k \rho} \alpha_{x_2} + \frac{2b\Delta p}{f_k \rho} \alpha_{x_2} - \frac{\Delta py_{a_1}}{f_k \rho} \omega_2 + \frac{\Delta p^2}{f_k \rho} \alpha_{x_2} \qquad (50)$$

where

$$\delta H = \frac{f_k}{H} (H_1 - H_2); \quad \rho = \frac{1}{\sin 1'}$$

Equation (50) permits calculating the change in the horizontal parallax difference as a function of the change in the elements of exterior orientation.

Let

$$f_k = 70 \text{ mm}; \quad H = 3500 \text{ m}; \quad b = 72 \text{ mm}; \quad x_{a_1} = 70 \text{ mm}; \quad y_{a_1} = 70 \text{ mm};$$

$$H_1 - H_2 = + 20 \text{ m}; \quad \alpha_{x_1} = + 2°; \quad \alpha_{x_2} = - 1°; \quad \omega_1 = - 1°; \quad \omega_2 = + 2°;$$

$$\chi_1 = - 3°; \quad \chi_2 = - 1°; \quad \Delta p = 0$$

Then,

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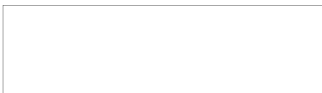
$$\delta H = + 0.40 \text{ mm}; \quad \frac{2b\alpha_{x_2}}{\rho} = - 2.53 \text{ mm}$$

$$\alpha_{x_1} - \alpha_{x_2} = + 3°; \quad \omega_1 - \omega_2 = - 3°; \quad \chi_1 - \chi_2 + \frac{b}{f_k} \omega_2 = 0°$$

i.e., the first term will be equal to +2.13 mm, the second -3.60 mm, the third +3.60 mm, and the fourth will be zero. Therefore, for perfectly flat terrain where the horizontal parallax difference should be zero, it actually equals -2.13 mm, and the elevation difference, calculated from eq.(36), will be 109.6 m. Thus, if the effect of the elements of exterior orientation on the change in the horizontal parallax difference is disregarded, the elevation differences calculated from eq.(36) will show very large errors, interfering with the preparation of maps with satisfactory accuracy. This fact is responsible for the limited usefulness of measuring or topographic stereoscopes.

77. Plotting the Relief by Sections

An analysis of eq.(50) which expresses the change in horizontal parallax difference in relation to the change in the elements of exterior orientation, indicates that the first and fourth terms will be directly proportional to the change in the abscissas and ordinates of the observed point, while the second and third terms are tied in with the current coordinates by a more complex relation. Therefore, if it is assumed that the second and third terms are equal to zero, the change in the horizontal parallax difference with any change in the abscissa or ordinates of a current point, can be graphically represented by a straight line. For this purpose, segments equal to the corresponding abscissas or ordinates, are laid off on the straight line representing the xx axis or yy axis of the photograph (Fig.138). Along the perpendicular, erected on this straight line at the point with the abscissa x = 100 mm, the values for the changes (δp) in the horizontal parallax dif-

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ferences, calculated from eq.(50) are laid off for the value $\chi_a = y_a = 100$ mm.
Then, connecting the point of origin of the straight lines with the end of the per-
pendicular, a graph of the corrections for
all intermediate points is obtained, since,
for these, the corrections will be repre-
sented by the length of the perpendiculars
from the ground line to the drawn line.

Construction is possible when the geo-
detic elevations of two points are known,
since the change in the horizontal parallax
difference (if $\alpha_{x_1} = \alpha_{x_2}$ and $\omega_1 = \omega_2$) is
directly proportional to the linear coordi-
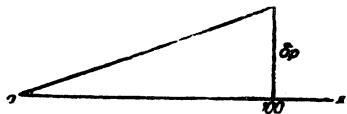
**Fig.138 - Graph of Correction
for Linear Interpo-
lations**

nates of these points. In this case, (Fig.139), measuring the horizontal parallax
difference for two points a and c having known geodetic elevations on the topo-
graphic stereoscope, and comparing these elevations with those calculated from
eq.(36), the difference due to effect of the elements of exterior orientation is
obtained. By connecting the two points with a
straight line and measuring the horizontal-paral-
lax difference of any point (e.g., c) on this
line, with respect to one of the points of origin,
linear interpolation (as mentioned above) will
yield the change in this difference so that its
correct value can be calculated from
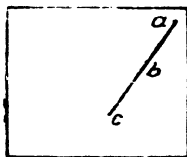
**Fig.139 - Linear Interpo-
lation on a
Straight Line**

$$\Delta p = \Delta'p = \delta p \qquad (51)$$

where $\Delta'p$ is the measured horizontal parallax difference and δp is its difference
obtained from the graph.

Such a construction can also be performed in the case where the influence of

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the second term (i.e., $\alpha_{x_1} = \alpha_{x_2}$) while the third term of eq.(50) is not equal to zero. However, in contrast to previous statements, linear interpolation may be performed in this case if the original geodetic points are located either along the ordinate axis ($x_1 = x_2$), or along the abscissa axis ($y_1 = y_2$). For these points, the change in parallax will be directly proportional to only one coordinate (y or x).

Therefore, an over-all analysis of eq.(50) which relates the change in the horizontal parallax difference to the elements of exterior orientation, permits the following conclusions: If two points, having geodetic elevations are located along the yy axis, the change in horizontal-parallax difference of these points will be directly proportional to the difference in their ordinates, which makes it possible to determine these changes for any intermediate point by means of linear interpolation. In all other cases, the change in the horizontal parallax difference can be determined by linear interpolation only with a certain amount of error, which will be the smaller the smaller the difference in abscissas of the selected points and the smaller the difference in the angles of tilt.

For a practical solution of this problem, the entire area of the photograph is divided into sections (zones), within whose limits the change in horizontal parallax difference will be considered to obey a linear interpolation law. This zone is usually provided with several (usually four) geodetic elevation marks. Picking two of these points for the starting points (a and b), their horizontal-parallax differences are measured and compared with those previously calculated from eq.(33). Simultaneously, parallax differences are measured at other points as well, including some located on the straight line ab. The differences between measured and calculated values are used for calculating the change in the horizontal-parallax differential. The resultant value is interpolated for intermediate points. By subtracting the change in horizontal-parallax differential, obtained by interpolation, from the measured value, the corrected horizontal parallax difference is obtained which is used for calculating the elevation difference.

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The number of sections that the field of the photograph is divided into depends on the required accuracy of plotting the relief and the quality of the calculation data. For making maps at a scale of 1:100,000, the print is divided into two or three sections, when aerial photographs are used. Simultaneously, when determining the values of elevation for a series of intermediate points of each section, the relief is plotted together with the elevation marks located within the boundaries of the section.

78. Densification of the Control Network by the Straight-Line Method

The geometric principle of the straight-line method, proposed by G.V.Romanovskiy, encompasses the fundamental law of projective geometry that a straight line in space is mapped by a straight line on the picture plane. This is due to the fact that, through the center of projection and the spatial straight line, a plane may be drawn that always intersects the plane of the aerial photograph along a straight line. However, the location of the image of three points of a locality along a straight line on the aerial photograph does not mean that the corresponding straight lines of the terrain are collinear.

Fig.140 — Selection of Points on the Aerial Photograph for Making a Straight-Line Method

Now let the three points a, c, and d of the aerial photograph (Fig.140) be collinear. Then, through this line and the center of projection S_1 a plane can be produced in which the corresponding points A, C, and D of the terrain are located, regardless of whether or not they lie on a single spatial straight line or at the

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locus indicated in Fig.140. At the same time, the images of these three points on the second photograph will usually be non-collinear, except for isolated cases. In order that the images of three points of the terrain shall be, at the same time, collinear on two adjacent photographs, it is a necessary condition that, through these three points and the two centers of projection, two planes are drawn intersecting both picture planes in straight lines. This condition will be satisfied if the three points of the locality lie on a single straight line in space or if these two planes coincide, i.e., if the three ground points and the two centers of projection are coplanar.

The second of these cases corresponds to the arrangement of points on aerial photographs in straight lines roughly parallel (for the case of a plan aerial photograph) to the direction of the base line. Therefore, to exclude the second case from consideration, the points on the photographs must be selected along directions roughly perpendicular to the directions of the base line. In that case, the collinear location of the three points a, c, and d on the left and right aerial photographs indicates that the corresponding ground points A, C, and D lie on a single spatial straight line. On the other hand, if on one photograph three points a_1, c_1, and d_1 are collinear, then the deviation of the point d_2 of the second aerial photograph from the straight line joining the point a_2 and c_2

Fig.141 - Determining the Elevation Difference of a Point Located on a Straight Line in Space

indicates that the three ground points A, C, and D do not lie on a single straight line in space. In this case, the deviation of the point d_2 from the line a_2c_2 will be a result of the relative elevation of the point D above the line AC. Thus, a study of the character of the location of three points on two aerial photographs

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permits determination of the mutual location of the three ground points.

In densification of a basic vertical control net by the straight-line method,

Fig.142 - Determining the Excess
of a Point by Measuring
the Photograph
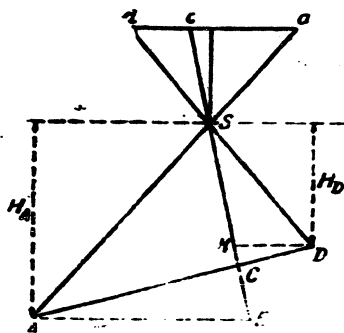
the elevations of points A and C of the ground are determined geodetically, and the elevation of any other point, e.g., point D, on the same straight line, is then found photogrammetrically. In Fig.141, let all three points A, C, and D be located on a single straight line, and let the elevations of points A and C be known. Then the elevation of point D is found from the similarity of the triangles ACC_o and ADD_o, whence,

$$A_D = A_A + \frac{D_1 + D_2}{D_1} (A_C - A_A) \qquad (52)$$

To determine the elevation of point D from eq.(52), the distances D_1 and D_2 between known and determined points must be known. These distances can be replaced by radials measured between the images of these points on the aerial photograph.

Let (Fig.142) an aerial photograph occupy a strictly horizontal position, and let the three ground points A, C, and D be located on some inclined straight line. Produce from the points A and C horizontal straight lines to their intersection with the projecting ray SC at the points E and K. Then, the triangles acS and AES, dcS and DKS, and ACE and CDK will be similar. The similarity of these triangles gives

$$AE = \frac{H_A}{f_k} \ ac; \quad DK = \frac{H_D}{f_k} \ dc; \quad \frac{AC}{CD} = \frac{AE}{KD}$$

where H_A and H_D are the height of the camera station above the plane containing A

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and D. Noting that $AC = D_1$ and $CD = D_2$, and denoting ac by d_1 and cd by d_2, we may write

$$\frac{D_2}{D_1} = \frac{H_D d_2}{H_A d_1} = \frac{(H_A - h)d_2}{H_A d_1}$$

or

$$\frac{D_2 + D_1}{D_1} = \frac{d_1 + d_2}{d_1} - \frac{h}{H_A}\frac{d_2}{d_1} \tag{53}$$

If we assume that $(d_1 + d_2) : d_1 = Q$, then

$$\frac{D_1 + D_2}{D_1} = Q - (Q - 1)\frac{A_D - A_A}{H_A} \tag{54}$$

or, from eq.(52),

$$A_D - A_A = h = \left[Q - (Q - Q)\frac{h}{H_A} \right]\left(A_C - A_A \right)$$

Finally,

$$h = \frac{Q(A_C - A_A)}{1 + (Q - 1)\dfrac{A_C - A_A}{H_A}} \tag{55}$$

Equation (55) clearly indicates that, to determine the elevation difference of the point D over the point A, the elevation difference between the points C and A located on the same straight line must be known, and the distances d_1 and d_2 between the images of these points on the aerial photograph must be measured.

If the point D of the ground does not lie on the straight line in space joining the points A and C, then the determination of its elevation is performed in two stages. At first the elevation of some point D' (a fictive point) located on the

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straight line AC is found, and then the elevation of the actual point C with respect
to the fictive point. The solution of this problem is illustrated in Fig.143. On
the left photograph, let the images a_1, c_1, and d_1 of three ground points be col-
linear, while, on the ground, let the point D be higher than the line AC. Then the
images a_2, c_2, and d_2 of these same three points on the right photograph will not be
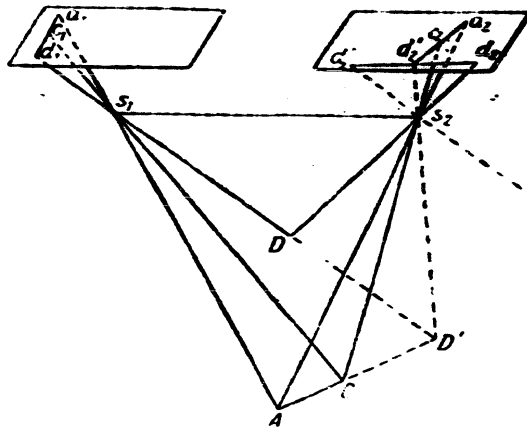
Fig.143 - Determining the Excess of the Actual Point over the
Fictive Point

collinear, i.e., point d_2 will deviate from the line a_2c_2 by a certain quantity
d_2d_2'. Then, prolong the line AC to its intersection with the projecting ray S_1D
at the point D'; the resultant point D' will be the fictive point whose image on
the right photograph will be at the point d_2', lying on the line a_2c_2. If, through
the right projection center S_2, the ray S_2d_2 is drawn parallel to the ray S_1d_1,
then its intersection with the plane of the right photograph at the point d_2'' will
lie on the line d_2d_2', since the rays S_2d_2, S_2d_2', and S_2d_2'' will be coplanar. If

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then the plane $S_1 S_2 D$ is drawn, the rays $S_2 d_2$ and $S_1 d_1$ will lie on this plane (and, consequently, also the ray $S_2 d_2''$ which is parallel to it). The same plane will then contain the point D' and, obviously, also the ray $S_2 D'$.

Assuming the planes of both photographs and the photographic base $S_1 S_2$ to be horizontal, it is logical that the triangles $d_2'' d_2 S_2$ and $S_1 S_2 D$, and also $d_2'' d_2' S_2$ and $S_1 S_2 D'$, will be similar. For this reason, the sides of the triangles have the same ratio to each other as the focal length of the camera has to the flight altitude H_D and H_D' above the planes drawn through the points D and D', respectively. Thus,

$$d_2'' d_2 = \frac{S_1 S_2 f_k}{H_D}; \quad d_2'' d_2' = \frac{S_1 S_2 f_k}{H_D'}$$

or

$$d_2'' d_2 - d_2'' d_2' = d_2 d_2' = \frac{S_1 S_2 f_k}{H_D H_D'} \left(H_D' - H_D \right) = \frac{B f_k}{H_D H_D'} \Delta h$$

where Δh denotes the elevation of point D over D'. The quantity $d_2 d_2'$ represents the horizontal parallax difference, determining the distance of point d_2 from the line drawn between points a_2 and c_2. For this reason, the elevation reference mark of the ground point D will be expressed by the relation

$$A_D = A_A + \frac{Q (A_C - A_A)}{1 + (Q - 1) \dfrac{A_C - A_A}{H_A}} + \frac{\Delta p H_D H_D'}{B f_k} \qquad (56)$$

Thus the straight-line method makes it possible to determine the elevation mark of any point D whose image is located one one of the photographs, on a line joining the images of two other points whose elevation is known. For this purpose, the difference in horizontal parallaxes Δp and the radials d_1 and d_2 on the photograph must be measured from the known point to the unknown point.

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There are two variants for the densification of the elevation control network by the straight-line method. In the first case, densification takes place within the limits of a single pair of photographs in the zone of their end lap. The
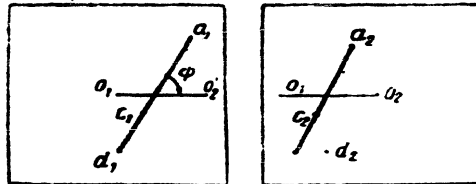
Fig. 144 - Selection of the Straight Line in the Zone of End Lap

straight line is selected to lie in a direction roughly perpendicular to the base line (Fig.144), forming an angle of not less than 30° with it. Two of the points of this line must have geodetic elevations, preferably at the end of the marked line (in that case Q will be less than unity, which increases the accuracy of the deter- minations).

The photographs are placed in the measuring stereoscope and adjusted (oriented) so that the marked line is perpendicular to the xx axis of the instrument. The horizontal parallax is measured at all points and will be equal at points A and C. The horizontal parallax difference Δp of point D with respect to the two other points is used for calculating the elevation of point D. If the straight line forms an angle ψ, different from 90°, with the base line, then the measured horizontal parallax difference $\Delta p'$ will be inaccurate; to obtain its accurate value, it must be divided by cos (90° - ψ) or by sin ψ. Then, the equation for calculating the elevation of the points will take the form

$$A_D = A_A + \frac{Q\,(A_C - A_A)}{1 + (Q - 1)\,\dfrac{A_C - A_A}{H_A}} + \frac{\Delta p'\,H_D H_D'}{Bf_k\,\sin\psi} \qquad (57)$$

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In the second case, the densification of the basic elevation control network is performed within the limits of a few stereo pairs, using the photographs from two adjacent flight strips. The straight line must be located in the zone of side lap (Fig.145) roughly parallel to the direction of the flight strip, and must have geo- detic elevation points at its ends. These two points are joined by a straight line,
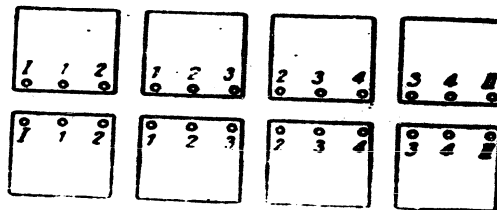
Fig.145 - Condensation of Straight Line in Zone of Side Lap

on which points of densification are selected so that not less than three points are located in the side lap of the first photographs of both flight strips. In this case, the first of the points selected must be simultaneously depicted on the second photograph of both flight strips, and the second point must be in the zone of triple overlap, i.e., appear on the sec- ond and third photographs. Thus, two points will be marked on the second photographs, supplemented by a third point located simul- taneously on the third and fourth photographs. Similar selection of points is continued until the end of the straight line so marked.
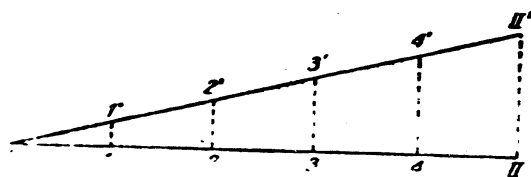
Fig.146 - Tying-in of Points by the Straight-Line Method

To measure the horizontal parallax difference, the two first photographs of both flight strips are adjusted in the measuring stereoscope until the straight line is perpendicular to the xx axis of the instrument. The datum points in the densifi-

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cation of the elevation control network obtained by the straight-line method comprise a datum point having a known geodetic elevation, and the point 1 whose elevation is arbitrarily assigned. From the known points of elevation, the straight-line method is used to determine the first unknown and then the next, etc. until the end of the marked line is reached. Arbitrary elevation values are used. The progressive method is used until the last photograph is reached, so that the elevation of points 3 and 4 can be used for determining the elevation of point II having a geodetic elevation. The difference between the arbitrary and the geodetic values of elevations of point II will be the error in the arbitrarily selected elevation of point 1. To determine the correct value of the elevation of that point, the difference at point II must be divided by the number of bases into which the straight line had been divided, and the quotient must be subtracted from the elevation taken for point 1. Now if the reference mark for point 1 (Fig.146) was in error by the quantity 1-1', then this error increases to the quantity II-II' at point II, while all intermediate points are in the positions 2', 3', and 4'. To obtain the correct marks of all points selected, the discrepancy II-II' must be divided proportionally to the distance to the points selected and the resultant quantity must be applied, with reversed signs, as corrections to the elevations so determined.

The errors of elevation densification by the straight-line method increases with increasing number of photographs between the points of known geodetic elevation, because of the unavoidable errors in photogrammetric constructions. An additional increase in error is caused by relief of the photographed terrain, so that the straight-line method is useful only on level or slightly undulating terrain.

79. Parallactic Rulers

The densification of the elevation control network by the straight-line method may be conveniently carried out with the parallactic sine ruler developed by F.V. Drobyshev and shown in Fig.147. The two glass plates are provided with lines, representing the measuring marks of a stereoscope. One of the sides of each plate

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has a beveled edge forming an angle of 5°44' with the traced lines. The beveled edge of the left plate is graduated in millimeters, and an index line is marked on the beveled edge of the right plate. 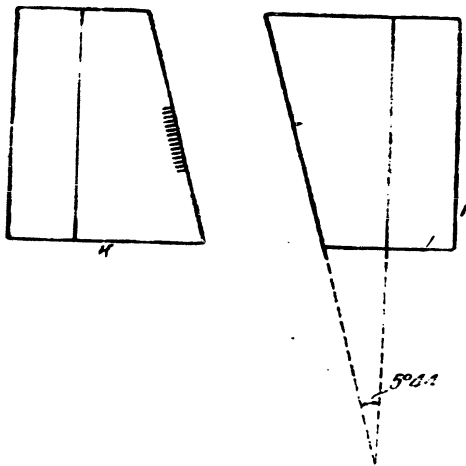In measuring the horizontal parallax difference during densification of the elevation network by the straight-line method, the first plate is placed on the left photograph so that its line coincides with the selected line. In the same way, the plotted line of the second plate is matched with the straight line a_2c_2 on the right photograph, after which both plates with the photographs are brought together until their beveled edges coincide. If a simple stereoscope is placed over the photographs, the observer will see a stereoscopic model of the terrain and a spatial line cutting the model at points A and C. In this position, the initial reading is taken on the index of the beveled edge of the plate.

Fig. 147 - Parallactic Sine Ruler

On displacement of one of the plates along the beveled edge of the other, the distance between the plotted lines, which remain parallel to one another, will change, which, stereoscopically, corresponds to a displacement of the spatial line in depth. As a result of this displacement the floating mark can be superimposed on the point D of the model, after which the new position of the plate is read off on the scale of the beveled edge. The displacement of the plate along the beveled edge will be equal to the change in distance between the rulers, divided by the sine of the angle formed by the edge and the line, a statement based on Fig. 148. Since this angle is equal to 5°44', its sine will be 1 : 10, i.e., the resultant displacement

of the plate will be ten times as great as the change in distance between the gradu-
ation lines, which is equal to the horizontal parallax difference between points D
and A. Consequently, to obtain the desired horizontal parallax difference, the
measured displacement of the index of the plate must be divided by ten and used for calculating the elevation difference.
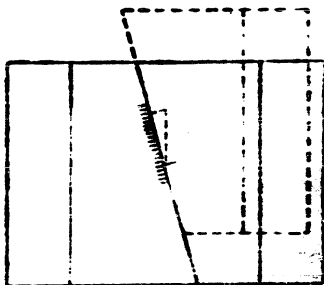
In determining elevation by the straight line method with the aid of a measuring ste-reoscope or parallactic rulers, the photo-graphs need not necessarily be so adjusted that the straight line is strictly perpendic-ular to the xx axis of the instrument or coincides with the index line. In this case, taking one of the points of the straight line with a known elevation as the datum point, the horizontal parallax difference of the two remaining points is measured by the

Fig. 148 - Measuring the Horizon-
tal Parallax Difference
with a Parallactic
Ruler

above-described method. Then, by simple calculations, the same horizontal parallax
difference of the third point with respect to the two other points which would have
been measured if the orientation of the photographs had been strict, is now deter-
mined. This procedure considerably facilitates orientation of the photographs.

The calculation of the horizontal parallax differences is performed in a record
on the form presented in the Table given below.

In Column 1 of this record, the number of the aerial photograph with which the
measurements were made is entered; in Column 2, the number of the orientation, which
is always done twice (as a cross check); in Column 3, the number of the point on the
straight line selected; and in Column 4, the readings on the scale of the parallax
screw. Taking the reading at point 1 for the datum and subtracting it from all

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Record for Measuring the Horizontal Parallax by the Straight-Line Method

Pair No.	Orientation No.	Point No.	Reading mm	$\Delta p'$	l	Q	$\delta p'$	$\Delta p'$	Δp
1	2	3	4	5	6	7	8	9	10
1748 1795	I	1	67.41	0	0.0	-	-	0.00	0.00
		2	67.95	+0.54	39.6	1000	-0.54	0.00	0.00
		3	67.87	+0.46	76.4	1.939	-1.04	-0.58	-0.60
	II	1	68.35	0	0.0	-	-	0.00	
		2	68.14	-0.21	39.2	1.000	+0.21	0.00	
		3	67.33	-1.02	75.8	1.934	+0.40	-0.62	

other readings, will give the measured horizontal parallax differences which are entered in Column 5. In Column 6, the distances measured on the photographs along the straight line from the initial point to the two other points are entered, and in Column 7, the ratio of these distances to the distance to a second point of known elevation. The product of these ratios by the measured horizontal parallax difference of point 2 (a point of known elevation), used with reversed sign, gives the quantity $\delta p'$, which is entered in Column 8. The sum of the values of Columns 5 and 8, entered in Column 9, represents the horizontal parallax difference which would have been measured at strict orientation of the photographs. The same calculations are made for both orientations, and the average of these values is entered in Column 10.

The calculations of the elevations of points by the straight-line method is done in a special record of the following form.

In this record, the number of the aerial photograph is entered in Column 1; the number of the point of the straight line in Column 2; the flight altitude above the plane passing through the datum point, in Column 3; the coefficient calculated from the equation given in Column 12 is entered in Column 4 and serves for the transition from the measured horizontal parallax differences to the elevation differences; in

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Record for Calculating Point Elevations by the Straight-Line Method

Photo-graph No.	Point No.	H m	k	Δp m	Δh m	Q	Qh m	δ' m	h m	A m	Remarks
1	2	3	4	5	6	7	8	9	10	11	12
1748	1			0.00	0.0					194.5	$k = \dfrac{H_D H_D'}{Bf_k \sin \psi}$
1795	2	3500	17.48	0.00	0.0	1.000	-16.2	-	-16.2	178.3	
	3			-0.60	-10.4	1.936	-31.4	-	-41.8	152.7	$\delta' = Q\,(A_2 - A_1) \times$ $\times (Q - 1)\,\dfrac{A_2 - A_1}{H}$

Column 5, the parallax values of Δp are transcribed from the record. The product of the data in Columns 4 and 5 is next entered in Column 6. From the same record, the values of Q are entered in Column 7. The known elevations of points 1 and 2 are then entered in Column 11; the elevation of point 2 with respect to point 1 is next calculated and entered in Column 10. The resultant elevation difference (in this particular example -16.2 m) is multiplied by the value of Q and the result entered in Column 8. Since the calculated quantity represents merely the numerator of the second term of eq.(56), the correction δ' is applied to it at differences in elevation of more than 50 m, to allow for the deviation of the denominator from unity. The correction is calculated from the equation in Column 12, while the result of the calculations is entered in Column 9. The sums of the quantities entered in Columns 6, 8, and 9 give the values shown in Column 10; by adding these to the reference marks of the datum points, the geodetic elevations (Column 11) of the points to be determined are obtained.

In the case of transverse lines passing across a number of stereo pairs, the arrangement of the record becomes slightly more complicated, but the principle of calculating the elevation of the points remains the same.