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Excerpts From "Mechanics of Aerosols"

N. A. Fuks

Mekhanika Aerozoley, Moscow, 1955, Chap I, pp 7-25; Chap II, pp 59-67; Chap IV, pp 107-123; 146-152; Chap V, pp 175-189, 215-219; Chap VI, pp 220-258.

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N. A. Fuks

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EXCERPTS FROM "MECHANICS OF AEROSOLS"

[Pages 7-25]

CHAPTER 1

CLASSIFICATION OF AEROSOLS. SIZE AND SHAPE OF PARTICLES IN AEROSOLS

1. Classification of Aerosols

Aerosols, or aerodispersion systems, are dispersed systems with a solid or liquid dispersed phase in a gas medium. Up to the present time there has been no single universal classification of aerosols and no single system of designation of various types of aerosols; technical literature in this respect is completely arbitrary. We suggest that a rational classification of aerodispersion systems must be based on differences between dispersion and condensation aerosols on one hand, and between systems with solid and liquid dispersed phases, on the other. In addition, the designation of individual types of aerosols must, as far as possible, coincide with the common, non-technical names given to aerosols (highway dust, natural fog, oven steam, etc).

Dispersion aerosols are formed through the dispersion (pulverization, atomization of solid and liquid bodies and transition of pulverized bodies into a suspension stated under the action of air currents, vibrations, etc. Condensation aerosols are formed through the volumetric condensation of oversaturated vapors and as the result of gas reactions leading to the formation of nonairborne products, such as soot. The differences between these two classes of aerodispersion systems, in addition to the method of origin, are that dispersion aerosols in most cases are considerably coarser than condensation aerosols, have a greater degree of polydispersion and, in the case of a solid dispersion phase, usually consist of individual, or slightly agglomerated particles of very irregular shape ("fragments"). In condensation aerosols the solid particles very quickly form loose aggregates consisting of very large numbers of primary particles which have regular crystalline or angular shapes.

The differences between aerosols with liquid and solid dispersion phases are seen in the fact that in the former the particles have a regular spherical shape, and in coagulation new spherical individual particles are formed. Solid particles may have extremely varied shapes which, upon coagulation form more or less porous aggregates, also with extremely varied shapes, the apparent density of which may be several times less than the density of the substance of which they consist.

Based on the foregoing, the designation of various types of aerosols in this book are the following.

Regardless of the degree of dispersion, both condensation and dispersion aerosols with liquid particles will be called fogs; in the Russian language they are referred to be the same word (natural, i.e., condensation fog, fog formed by the atomization of falling water, etc). In the given case the difference between condensation and dispersion systems is not very great.

Dispersed aerosols of solid particles, regardless of the degree of dispersion, will be called dusts. The existing opinion that only coarse-dispersion systems may be called dusts is incorrect: dusts with a high degree of dispersion may be formed by artificial separation, or by natural separation taking place in the atmosphere.

Finally, condensation aerosols with a solid dispersion phase will be called smokes. This may not include systems of condensation origin, containing both solid and liquid particles, the most important example of which are smokes formed as a result of incomplete combustion of fuels, smokes of hygroscopic substances (such as ammonium chloride), the particles of which may be solid, semi-liquid or liquid, depending upon the humidity of the medium, smokes of easily supercooled organic substances in which liquid particles gradually change into crystalline particles, etc. It is noted that the demarcation between smokes and condensation fogs, and even strict adherence to the proposed terminology sometimes is fairly difficult. However, this demarcation is still easier than that used by English language authors who combine both types of aerosols under the same term (smoke).

Furthermore, in practice, aerosols frequently are encountered which contain particles of both dispersion and condensation origin. Thus, furnace smoke always contains more or less quantities of salts mechanically carried away from the furnace grate; the so-called "atmospheric nuclei of condensation" consist partially of a fine spray of sea water, and partially of droplets of sulfuric acid formed by oxidation of sulfuric anhydride of furnace gases. The air of industrial centers contains large amounts of aggregates of soot, salts, products of the dry distillation of carbon and atmospheric moisture in proportions of from tens of microns to tens of millimeters in length. These aerosols may not be included in any of the existing classifications and the term proposed for them is "smeg" (smoke + fog).

All the above mentioned types of aerosols may have extremely varied dispersion, which has a great influence on almost all the properties of dispersed systems. Because of this it is appropriate to divide aerosols into high dispersion and coarse dispersion systems (see Figure 1).

In the theory of aerosols the word "cloud" is used by several foreign [1] and Soviet [2] authors to indicate all condensation aerosols with particles of 10^{-5} cm diameter. In the Russian language this word has an entirely different meaning: a cloud means a free aerodispersion system of any type (highway dust, dust cloud, a puff of gun smoke, etc) having definite size and shape. The word "cloud" will be used in this sense in this book, also.

2. The Size of Particles in Aerosols

In the problem of the lower limit of the size of particles in an aerosol it must be taken into account that the size of very fine particles (approximately 10^{-7} cm) may be determined by at least two methods:

(1) measurement of the movement of particles in an electric dield (see paragraph 27) with the use of an electrometer, and

(2) measurement of the coefficient of diffusion of particles, usually through the use of electrometric methods. Thus, in both methods only charged particles may be measured.

Experiments have shown that there are two types of charged particles in gases, called small (gaseous or light) ions, and large (heavy or medium) ions. The movement of the former is in the order of units, and of the latter in the order of 10^{-3} to 10^{-4} cm^2 per^{-1} sec^{-1}. At the present time it has been established that gaseous ions are molecular aggregates, formed from the charged central molecules (more properly, ions) and clouds of neutral gas molecules connected to them by electrostatic and molecular attraction.

Heavy ions, in distinction from light ions, are formed only in gases containing solid or liquid particles in suspension, i.e., comprise the charged portion high-dispersion aerosols. "Medium" ions, with mobility of 10^{-3} to 10^{-1} cm^2 per^{-1} sec^{-1} have been discovered, and particles with 0.2 cm^2 per^{-1} sec^{-1} mobility are contained in the combustion products of illumination gas and in sodium flames [3,4]. Mobility of the latter degree also is found in gaseous ions of the vapors of several organic substances [4], such as amyl alcohol. Thus, gaseous ions cannot be distinguished from charged particles in aerosols on the basis of mobility, but the different behavior of ions and particles with respect to coagulation may be put to use. In coagulation (recombination), the gaseous ions which form the neutral molecular complex immediately decompose, i.e., the ions disintegrate. In the coagulation of aerosol particles, which is not connected with the presence of charges, much larger particles are formed. Actually, the mobility of the above mentioned ions which are formed in flames undergoes a several hundred-fold decrease in approximately one second [3,4]. Because the probability of the presence of multiple charges on particles rapidly decreased with a reduction in the size of the particles (see equation (27.3)) it may be assumed that these ions exhibit a single, elementary charge. In this case a mobility of 0.2 cm^2 per^{-1} sec^{-1} corresponds to a radius of $1.5 \cdot 10^{-7}$ cm (see Table 3). In view of the fact that the charged particle content of an aerosol also decreases rapidly with a decrease in the size of particles it is quite possible that many very fine particles are present which cannot be detected or measured. (Particles of the order 10^{-7} cm may be detected with condensation nucleus meters, but their size remains undetermined by this method.) Curiously, this minimum size of aerosol particles has been obtained in electron microscope investigations because this size conforms to the crucial limits of magnification of the electron microscope. However, the abundance of particles with a radius of $r = 1.5 \cdot 10^{-7}$ cm (for example, in silver iodide smoke [5]) forces the conclusion that they contain still smaller particles. It is noted that only considerably larger particles of aerosols may be detected by ultramicroscopic methods.

This is the status of the experimental side of the problem. Theoretically it is quite possible that a substance with a very stable crystalline lattice may give an aerosol with particles 2 or 3 molecules in diameter. It is true, however, that because of very high speed of diffusion of such particles they precipitate extremely quickly on much larger particles (see paragraph 49), on walls, etc.

Turning to the problem of the upper limit of the size of particles in aerodispersion systems, it is noted that in systems with a non-mobile medium particles with a diameter of only several hundred microns precipitate so fast that detection of their suspension state is difficult. On the other hand, in rapidly rising or turbulent air currents, such as those found in cumulo-nimbus clouds from which rain is falling, the lifting of loose material by the air, fluidization of catalysts, during sand- and snow-storms, etc, particles several millimeters in diameter may be found in the suspended state. Because of the current importance of the above mentioned problems, these particles also must be taken into consideration in the mechanics of aerosols.

Thus, the study of aerosols encompasses systems with a very wide range of dispersion, from 10^{-7} to 10^{-1} cm. It is not surprising that the transition from the lower to the upper limit is accompanied not only by quantitative changes of almost all physical properties of aerosols, but also by changes in the laws which express these changes. This is especially apparent in the example of the law of resistance of a gaseous medium to the motion of particles. For very fine particles ($r < 10^{-6}$ cm), resistance is proportional to the speed and the square of the radius of the particle. Between 10^{-6} and 10^{-4} cm there is a gradual transition to the law of Stock: resistance remains proportional to speed, but the square function of the radius takes on a linear character. With a greater increase in radius there is an additional deviation from the law of Stock: at not too low speeds the proportionality of resistance to speed is disrupted, and at fairly high speeds and large diameter of particles the resistance is at the first approximation proportional to the square of the radius and the square of the speed (see equations 10.2, 10.3, and 10.4).

Changes in the character of laws which regulate several more important properties of aerosols are shown in Figure 1: here, in all cases, the transitional region is mostly between 0.5 to 1 · 10^{-5} to 10^{-4} cm. It is not surprising that the properties are combined in Figure 1 in groups 1 and 2, because these properties are connected with the ratio of the radius of the particles to the length of the free path of gas molecules composing air at an atmospheric pressure of approximately 10^{-5} cm, or the average length of a wave of visible light ($\lambda = 0.55 \cdot 10^{-4}$ cm). This coincidence has a circumstantial character for the remaining properties. The facts cited above enable the establishment of a natural classification of aerosols according to dispersion.

The group of high-dispersion aerosols with a radius of particles primarily less than 0.5 to 1 · 10^{-5} cm is characterized by the fact that the resistance to movement, the speed of evaporation and condensation of particles is proportional to r^2, the diffusion of light by particles is proportional to r^6, and the coagulation constant is a function of r. The particles are invisible through ordinary microscopes and may be detected by ultramicroscopy only under very favorable conditions. The vapor pressure of the dispersed phase of these aerosols noticeably exceeds the normal vapor pressure of the substance. Consequently, a rapid "eating up" of the smaller particles by the larger ones may take place in the latter. Finally, Brownian movement of the particles has a prevailing influence on precipitation due to gravity.

In coarse dispersion aerosols with a radius of particles greater than 10^{-4} cm both the size and shape of particles may be determined with the aid of a microscope: the coagulation constant does not depent on r; precipitation has considerable prevalence over Brownian movement, and all the laws mentioned above alternate with each other, as shown in Figure 1.

Finally, systems with a radius of particles of 0.5 - 1 · 10^{-5} to 10^{-4} cm are best isolated in a special group of aerosols of average dispersion and with transitional properties. It must be noted that this group plays a very great role in research on aerosols because this size of particles is very favorable for ultramicroscopy, which is one of the basic methods of studying aerosols. In addition, in the formation of condensation aerosols from substances with poor flexibility of the vapor phase this is the type of system which usually is obtained in practice.

3. Distribution of the Size of Particles in Aerosols

Most natural and artificially obtained aerosols have a fairly significant degree of polydispersion. (Relatively monodispersion aerosols, obtained from the Lamer [6] generator and in the formation of flower pollen and the spores of several plants: the particles of clover pollen lie within the limits 24.8 to 26.9 μ [7].) In view of the great degree of dependence of the physical properties of aerosols on their dispersion, as mentioned above, the "average" size of particles in most cases is inadequate for the classification of aerodispersion systems; it is necessary to take into account the distribution of the size of particles. Before the invention of the electron microscope the ordinary microscope was used for this purpose, which permits measurement only of the particles of coarse aerosols ($r > 3$ to $5 \cdot 10^{-5}$ cm) or the corresponding fraction of high dispersion aerosols. Only the general number of particles of the finer fractions of the latter were determined without measurement of their size, or else these fractions usually were disregarded. At the present time the distribution of the size of particles of aerosols may be determined in systems with a particle diameter of larger than 1 or $2 \cdot 10^{-7}$ cm.

The distribution of particle sizes may be expressed in several ways. The number of particles df, the radii of which are between (r, r + dr) may be expressed as:

$$df = f(r) \cdot dr \qquad (3.1)$$

under the conditions

$$\int_0^\infty f(r)\,dr = 1. \qquad (3.2)$$

The curve expressing the function $f(r)$ is called the distribution density curve of the differential curve of the distribution of the size of particles (Figure 2) or, more precisely, the curve of the computed distribution, in distinction from the curve of weight distribution which expresses the weight dg of particles with a radius $(r, r+dr)$:

$$dg = g(r)\,dr. \qquad (3.3)$$

in which

$$\int_0^\infty g(r)\,dr = 1. \qquad (3.4)$$

It must be noted that the area bound by the differential curve of distribution, the abscissa axis and two verticals to the points r_1 and r_2 expresses the weight (computed and gravimetric) of particles with a radius between r_1 and r_2 .

In terms of the function $g(r)$ the equation

$$g(r) = \beta m_r f(r),$$

may be constructed, in which m_r is the mass of particles with radius r ; β is the proportionality factor, which is easily determined by the integration:

$$\int_0^\infty g(r)\,dr = 1 = \beta \int_0^\infty m_r f(r)\,dr = \beta \overline{m},$$

where $\overline{m}$ is the average (arithmetical) mass of the particles of the aerosol.

The functions $f(r)$ and $g(r)$ may be combined in a simple equation

$$g(r) = \frac{m_r}{\overline{m}} f(r). \qquad (3.5)$$

Thus $\overline{m} = \gamma \overline{v}$, where γ is the density and $\overline{v}$ is the average volume of particles, and at constant γ , i.e., in a case in which the aerosol has uniform composition and contains no aggregates, the gravimetric distribution $g(r)$ is identical with the volumetric distribution $v(r) = \frac{v_r}{\overline{v}} f(r)$. In aerosols with heterogeneous composition (for example, In technical practice the function $G_a(r)$ is used very frequently in studying various industrial dusts. In determining its integral curve of gravimetric distribution it is called the "characteristic curve" of a given dust, or the "curve of residues," because it is determined by the weight of the dust which settles on a given screen or by the given speed of air movement in an air separator. The curve $G_b(r)$ is called the "curve of apertures." It is noted that:

$$F_a(r) + F_b(r) = G_a(r) + G_b(r) = 1 \qquad (3.8)$$

By way of clarification of the given example of distribution of the size of droplets in a stratus cloud [9] determined by microscopic measurement, it is noted that in distinction from a system with a liquid medium, the direct, continuous curve of distribution of which may be obtained through sedimentometric methods, in aerosols the weight of particles in an experiment is determined, the radii of which lie between end values, i.e., in addition to continuous curves, interrupted curves, or "histograms" also are determined.

The results of measurement of 100 droplets in the case under consideration are shown in Table 1.

TABLE 1

DISTRIBUTION OF SIZE OF DROPLETS IN A STRATUS CLOUD

Radius intervals of droplets, μ	2.5--4	4--5.5	5.5--7	7--8.5	8.5--10	10--11.5
Number of droplets	4	6	15	24	24	12

Radius intervals of droplets, μ	11.5--13	13--14.5	14.5--16	16--17.5	17.5--19
Number of droplets	4	4	4	1	2

The histogram shown in Figure 2 was constructed from these data. It may be used directly for the computation of various average sizes (see paragraph 4) but in most cases it is desirable to convert the histogram into a continuous curve, smoothing out any irregularities in this process which are caused by an insufficient number of measurements (for example, a physically improbable rise in the right region of the histogram).

This smoothing out of the curve is substantiated in experimental results by the circumstance that the number of measured particles is necessarily limited, and therefore statistical fluctuation is fairly noticeable. The direct drawing of a smoothed-out curve of this histogram is inconvenient. It is better to begin with the construction of experimental points (marked with "x's" in Figure 3) for the smooth integral curve $F_b(r)$. Thus it is relatively easy to correct occasional errors in measurement and fluctuations in the number of particles in each fraction. Graphic differentiation of the integral curve results in the differential curve $f(r)$ (see Figure 2).

In proceeding with the curve of weight distribution one could start with construction of a "gravimetric histogram" directly from experimental data, but in this case the fluctuations mentioned above are extremely marked. Therefore, it is better to begin with smoothing out the $f(r)$ curve of experimental data. Laying out the axis r for sufficiently narrow intervals Δr , the average mass of particles $m_r = \frac{4}{3}\pi\gamma r^3$ corresponding to the latter is calculated, which gives the curve $f(r)$ and enables determination of the mass $m_r f(r)\Delta r$ of the dispersed phase in each interval and the average mass $\bar{m}$, from which the desired function of gravimetric distribution may be derived through formula (3.5). The curve $g(r)$ in Figure 2 is constructed in this manner. From this, in turn, is derived the integral curve $G_b(r)$ (see Figure 3).

In grid or sedimentation analysis of dusts the function $G(r)$ is determined directly in the experiment. The reverse process of determining the calculated distribution from this function follows the same method. The construction of quadratic curves and other size distributions also is similar.

The selection of any method of expression for determination of the size of particles in an aerosol depends on the properties which are to be emphasized. For example, the computed distribution of the sizes of particles $f(r)$ (see equations (49.28) through (49.32)) must be used for computing the speed of thermal coagulation of aerosols. The speed of evaporation of coarse-dispersion aerosols at any given moment is determined by a linear distribution of the size of particles $rf(r)$, because the speed of evaporation of particles is proportional to their radius. The optical density of coarse-dispersion fogs is determined by a quadratic distribution of particle size $r^2f(r)$, because the reflection and diffusion of light by large droplets is proportional to the square of their radius. The degree of completeness of precipitation of aerosols due to the effects of gravity or inertia also is determined by this distribution.

The examples described above involved differential curves of distribution. Integral curves are applied primarily in the calculation of the degree of completeness of separation of the dispersed phases of an aerosol from the gaseous medium in the various aerosol generators and in expression of the distribution of particle sizes through empirical equations (see equation (3.9)). It is noted also that the gravimetric percentage fraction remaining on a grid screen of determined aperture size and passing through a grid of another size usually is used for the classification of industrial powdered substances, i.e., the size of the function $G(r)$ is given in two determined values of r.

It is necessary to mention a type of distribution curve frequently found in literature (Figure 4) in which the ordinate axes intersect at some distance from the initial coordinate. Curves of this type, which give a definitely false representation of the true character of the distribution of particle size, are obtained because of the terminal size of the intervals at which the size of the particles are graphed according to measured values. Actually, the curves in Figure 5 give the actual distribution of sizes. At sufficiently narrow intervals $(\Delta r = 0.1\mu)$, for example when particles are measured with an electron microscope, a histogram of the type shown in Figure 5a is obtained. "Smoothing out" this curve results in a curve which is very close to the curve of actual distribution. When particles are measured with an ordinary microscope, however, at best the smallest interval attainable is $\Delta r = 0.2\mu$, which gives the histogram shown in Figure 5b, and a curve similar to A in Figure 4. Finally, at intervals of $\Delta r = 0.5\mu$ obtained in ordinary hygienic procedures, the resulting histogram is of the type shown in Figure 5c, and the curve is of the type B, in Figure 4.

Usually, this masking of the true distribution occurs when a considerable portion of the distribution curve lies within that region of particle size where the method used for determination of the size is not favorable because the particles are too small; or in another case, when because of a very high degree of polydispersion the interval between sizes of particles is extended over several orders of greatness. Assuming that the radius of particles of several aerosols lies between 0.1 and 200μ and that the entire length of the abscissa is equal to 100 mm, then the part of the distribution curve corresponding to the limits 0.1 to 1μ ~~contains~~ an interval of only 0.5 mm, and it is impossible to construct a correct distribution on this graph.

It would seem that under these conditions a logarithmic scale could be used for the abscissa axis. Actually, in this condition of inequality the position of very fine particles is displaced because each order of greatness on the graph is not afforded an equal space. However, another difficulty arises at this point. If, taking a logarithmic scale for the radii, a distribution curve is constructed by simply plotting the function f(r) on the ordinate axis, then the area bound by this curve, the abscissa axis and verticals to the points r_1 and r_2 is equal to

$$\int_{r_1}^{r_2} f(r)\,d\lg r = \int_{r_1}^{r_2} \frac{1}{r} f(r)\,dr,$$

i.e., this area already is not proportional to the weight of particles with radius from r_1 to r_2, and the curve loses its descriptiveness. In order to retain the significance of this area $rf(r)$ must be plotted on the ordinate axis in addition to $f(r)$, but in aerosols with a high degree of polydispersion the curve $rf(r)$ practically coincides with the abscissa axis in the region of a small r value. Thus one is again confronted with the necessity of graphic expression of the distribution of sizes by means of a curve. In recent times there has been a tendency in aerosol research to reject the above described methods of graphic representation of the distribution of particle size and to substitute a system of coordinates in which the distribution is expressed by straight lines. This question is closely related to another problem which is discussed below.

The use of distribution curves for characterizing industrial aerosols, and in the solution of various theoretical and applied aerosol problems is very inconvenient. These curves are appropriately represented by formulas with a minimum number of coefficients, the size of which would characterize a given distribution. In the latter case it is desirable that the formula be applicable to the greatest possible number of aerodispersion systems, i.e., that only the size of the coefficient be changed in the transition from one system to another. With a sufficiently large number of coefficients all distributions encountered in practice could be represented by a single, unified formula. However, the selection of coefficients requires a great deal of work for each case, and in addition, it would be difficult to describe any given physical situation by these coefficients. Because of this, multiple-coefficient formulas have not received practical application. As a rule, only formulas with two coefficients are used, because this number of coefficients is in itself a minimum: one coefficient characterizes the average size of particles and the other characterizes the degree of polydispersion of the aerosol.

There still is no theoretical derivation of a dispersion formula for very complex and insufficiently studied processes of generation of both condensation and dispersion aerosols (with one exception; see below) but there are many empirical formulas which are used mostly for aerosols produced by the mechanical dispersion of solid and liquid bodies. The most well known of these are the following:

(1) Roller formula [9], of the type described in the foregoing:

$$G_b(r) = a r^{1/2} \exp(-s/r) \qquad (3.9)$$

(In this formula, at $r \rightarrow \infty$, $G_b(r) \rightarrow \infty$. Therefore, in this case the integral curve of distribution must be interrupted at that distribution of r_1 where $G_b(r_1) = 1$.)

which also is used for many industrial pulverized materials with extremely varied degrees of dispersion.

(2) Razin-Rammler formula [10]

$$G_a(r) = \exp(-a r^s) \qquad (3.10)$$

which is used for comparatively coarse-dispersion dusts and fogs produced by mechanical methods.

A considerably improved formula for these fogs has been suggested by Nukiyama and Tanasava [11]

$$f(r) = a r^2 \exp(-b r^s) \qquad (3.11)$$

where a and b are independent, but determined functions s, and the size of droplets is medium. These functions have been calculated by the present authors and are presented in table form.

Formula (3.9) may be represented in the form

$$\lg[G_b(r) / r^{1/2}] = \lg a - 0.434\, s/r . \qquad (3.12)$$

If $1/r$ is plotted along the abscissa and $\lg(G_b/r^{1/2})$ is plotted on the ordinate, then as far as is applicable, the experimental points must be plotted in a straight line, on which a and s may be easily determined.

Similarly to formula (3.10) we obtain

$$\lg G_a(r) = -0{,}434\, ar^s. \qquad (3.13)$$

In this case a value must be chosen for s at which the experimental points lie in a straight line for the selection of the coordinates r^s and $\lg G$. The experimental data are treated as in formula (3.11), which gives the formula

$$\lg [f(r)/r^2] = \lg a - 0{,}434\, br^s. \qquad (3.14)$$

Reduction of the formula to a stage in which the particle sizes are distributed in a straight line greatly eases the task of selection of coefficients in these formulas, and of smoothing out the curve of experimental data.

In a very few aerosols, such as that formed by plant spores [12] the curve of distribution is symmetrical, very similar to the Gauss curve, and corresponds to "normal" distribution:

$$f(r) = \frac{1}{\beta\sqrt{2\pi}} \exp[-(r-\bar{r})^2/2\beta^2], \qquad (3.15)$$

where $\bar{r}$ is the average radius of particles; $\beta^2 = \overline{(r-\bar{r})^2}$ is the average quadratic deviation (dispersion) of the size of the radius from $\bar{r}$.

As an auxiliary alternative the following formula is introduced.

$$\xi = (r-\bar{r})/\beta\sqrt{2}. \qquad (3.16)$$

The weight of particles with radius $\leq r_1$ is equal to

$$\int_0^{r_1} f(r)\,dr = \frac{1}{\beta\sqrt{2\pi}} \int_0^{r_1} e^{-(r-\bar{r})^2/2\beta^2}\,dr = \frac{1}{\sqrt{\pi}} \int_{-\frac{\bar{r}}{\beta\sqrt{2}}}^{\frac{r_1-\bar{r}}{\beta\sqrt{2}}} e^{-\xi^2}\,d\xi. \qquad (3.17)$$

Because according to the function $f(r)$ it is different from 0 only at $r \geqslant 0$, the lower limit in the integral may be taken as $-\infty$.

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The weight of particles with radius $\nless r_1$ is equal to

$$\int_0^{r_1} f(r)\,dr = \frac{1}{\beta\sqrt{2\pi}} \int_0^{r_1} e^{-(r-\bar{r})^2/2\beta^2}\,dr = \frac{1}{\sqrt{\pi}} \int_{-\frac{\bar{r}}{\beta\sqrt{2}}}^{\frac{r_1-\bar{r}}{\beta\sqrt{2}}} e^{-\xi^2}\,d\xi. \qquad (3.17)$$

Because according to the function $f(r)$ it is different from 0 only at $r \geqslant 0$, the lower limit in the integral may be taken as $-\infty$.

$$\int_0^{r_1} f(r)\,dr = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\frac{r_1-\bar{r}}{\beta\sqrt{2}}} e^{-\xi^2}\,d\xi = \frac{1}{2}\left[1 + Erf\left(\frac{r_1-\bar{r}}{\beta\sqrt{2}}\right)\right] = \frac{1}{2}(1 + Erf\xi_1), \quad (3.18)$$

where

$$Erf\xi_1 = \frac{2}{\sqrt{\pi}} \int_0^{\xi_1} e^{-\xi^2} d\xi \quad \text{(Kramp's function)} \quad (3.19)$$

---

but ξ_1 is equal to the value of ξ, corresponding to r_1.

The variable ξ is plotted along the ordinate on an arbitrary scale (Figure 6), and the corresponding values $0.5[1 + Erf(\xi)]$ are located on it, i.e., the weight of particles with the normal distribution under consideration, for which $r < \bar{r} + \beta\sqrt{2\xi}$. As before, r is plotted on the abscissa. If the integral curve $F_b(r)$ is constructed in this "probable" system of coordinates, expressing the weight of particles with radius less than r, then in the case of a normal distribution of particle sizes similar to equation (3.16) a straight line must result, intersecting the abscissa at the point $r = \bar{r}$. The tangent of the angle of intersection of this straight line with the abscissa is equal to $1/\beta\sqrt{2}$.

The data of Table 1 for a water fog do not give a probability grid of a straight line graph, as may be seen in Figure 6 in which the distribution of the size of droplets is indicated by crosses, in view of the asymmetry of the differential curve of distribution of this fog (see Figure 2). It must be mentioned that in the overwhelming majority of condensation and dispersion aerosols the distribution curves are asymmetrical with a greater deviation toward small values of r. Apparently this is connected with the already mentioned inequality of very small particles in the selection of the linear size of particles as the abscissa value in distribution curves. The distribution curves are more symmetrical if the logarithm of the radius is taken for the abscissa, and they frequently approach the shape of the Gauss curve. In this case the distribution may be expressed by the formula (logarithmic-normal distribution)

$$f(r)\,dr = \frac{1}{\lg \beta_g \sqrt{2\pi}} \exp\left[-\frac{(\lg r - \lg r_g)^2}{2(\lg \beta_g)^2}\right] d \lg r. \qquad (3.20)$$

Here $\lg r_g = \overline{\lg r}$, and consequently r_g is the average geometric radius of particles $(\lg \beta_g)^2 = \overline{(\lg r - \lg r_g)^2}$, i.e., is the average quadratic deviation of the logarithm of the radius. In foreign literature β_g usually is called the "standard geometric deviation." As is seen in Figure 7, the fog chosen by the present authors as an example (stratus cloud) gives a straight-line curve, 1, in a probability-logarithmic grid. (In passing straight lines through experimental points in probability networks, it must be remembered that points far removed from the axis have little statistical value because they correspond to a small number of particles.) L. Levin established the logarithmic-normal distribution of the size of particles in natural clouds on the basis of a great deal of material compiled by El'brus [13]. Recently, this type of distribution has been found in other aerosols of dispersion and condensation origin; in mineral [14] and uranium [15] dusts formed during mechanical crushing operations, in fogs formed by disc pulverization [16], in aerosols of NH_4Cl and H_2SO_4, formed by mixture of gaseous components [17], etc. In addition to the other distributions mentioned above, logarithmic-normal distribution undoubtedly also has theoretical significance [18]. In particular, as is shown by A. Kolmogorov [19], proceeding from simple hypotheses on the character of the process of crushing solid particles, it may be demonstrated that the distribution of the size of particles follows an asymptotic path in proportion to the degree the refinement of particles follows an asymptotic path in proportion to the degree the refinement of particles approaches the logarithmic-normal system [20]. It would be very interesting to clarify the conditions under which this distribution is obtained in condensation processes.

It is noted further, that in the case of logarithmic-normal calculated distribution of sizes (3.20) gravimetric and other derived distributions also become logarithmic-normal at the same value of β_g, i.e., all derived distributions are expressed by parallel straight lines [21].

Actually, in a distribution corresponding to the ν-th degree of radius the following expression is obtained (a is the normalizing factor, containing only constant values):

$$a \exp(2{,}302\,\nu \lg r)\frac{1}{\lg\beta_g\sqrt{2\pi}}\exp[-(\lg r-\lg r_g)^2/2\lg^2\beta_g] =$$

$$= \frac{a}{\lg\beta_g\sqrt{2\pi}}\exp\left[-\frac{(\lg r-\lg r_g)^2-2{,}302\,\nu\lg r\;2\lg^2\beta_g}{2\lg^2\beta_g}\right]=$$

$$= \frac{1}{\lg\beta_g\sqrt{2\pi}}\exp\left\{-\frac{[\lg r-(\lg r_g+2{,}302\,\nu\lg^2\beta_g)]^2}{2\lg^2\beta_g}\right\}.$$

The gravimetric distribution of the size of particles in a fog according to the calculated distribution, represented by straight line 1 in Figure 7, is expressed by the parallel straight line 2.

It is characteristic of all distribution formulas discussed in this paragraph that points which lie close to one or both regions of the curve more or less deviate from a straight line. However, this does not have great practical significance in the characteristics of aerosols because in integral distribution curves the marginal points correspond to a small weight of particles. Several authors [9] are inclined to aver that the formulas proposed by them are correct for all ranges, and they explain deviations by experimental errors, premature precipitation of particles, etc. The present author disagrees with this contention: as already stated, these formulas (with the exception of (3.20)) cannot be used for the expression of any theoretical idea. They are more or less lucky equations; purely empirical approximations of the actual distribution. Furthermore, because of the fact that each of these formulas has been successfully applied to one or another group of aerosols it follows that none of them has general significance. Nevertheless, the practical value of these formulas is beyond doubt because they are used in the determination of all properties of aerosols depending on their dispersion, in the comparison of aerosols using only two parameters, etc. The work involved in the selection of the fitting formula and in the determination of the size of the coefficients usually is considerably less than the work required by the measurement of a sufficiently large number of particles. Because of this, for the further development of knowledge of aerosols, and especially of the theory of the formation of aerosols, it is very desirable that the indicated processing of measurement data be conducted on the largest possible number of aerodispersion systems.

Knowledge of the distribution of the size of particles is extremely important in aerosol research because almost all properties of aerosols depend to a very great degree on their dispersion. Therefore, experimental aerosol research must be done on isodispersion systems, but because the production of the latter still has not been completely solved research must be conducted with polydispersion aerosols, In this case, more or less promising conclusions may be drawn from the results of these experiments, noting the distribution of the size of particles.

4. "Average" Size of Particles in Aerosols

Completely characterization of aerosols requires knowledge of the distribution of the size of particles. However, in practice the term "average" frequently is limited to those cases in which, for some reason or other, there has been no investigation of the distribution of the size of particles, but some other property of the aerosol, which depends upon its dispersion, has been measured, such as the coefficient of diffusion, expansion of x-ray lines in an x-ray photograph of precipitated particles, the diameter of the diffraction corona, etc. Very often the dispersion of aerosols is measured by the calculated-gravimetric method, i.e., the gravimetric concentration (mass of the dispersion phase per unit volume) and the calculated concentration (number of particles per unit volume) of the aerosol are measured; this includes the average mass of particles, and if their density is known, the average size of particles. (In Soviet literature the term "particular concentration" usually is used to designate the number of particles per unit aerosol volume. Because of the ambiguity ("particular" as opposite of "complete") of the term the present author will follow the example of G. Romashov [22] and employ the term "calculated" concentration.)

It must be borne in mind that the "average" particle sizes determined by various methods may be clearly distinguished from each other. Just as in the case of the calculated, gravimetric, etc, terms for the distribution of the size of particles, there are various average radius values:

(1) average arithmetic radius

$$r_1 = \bar{r} = \int_0^\infty r f(r)\,dr \approx \sum_\nu r_\nu N_\nu / N, \qquad (4.1)$$

where N_ν is the number of particles per ν size interval; r_ν is the middle of this interval; N is the total number of particles.

(2) average quadratic radius (surface average)

$$r_2 = \sqrt{\overline{r^2}} = \left[\int_0^\infty r^2 f(r)\,dr\right]^{1/2} \approx \left[\sum_\nu r_\nu^2 N_\nu / N\right]^{1/2}; \qquad (4.2)$$

(3) average cubic radius (average per volume of average weight)

$$r_3 = \sqrt[3]{\overline{r^3}} = \left[\int_0^\infty r^3 f(r)\,dr\right]^{1/3} \approx \left[\sum_\nu r_\nu^3 N_\nu / N\right]^{1/3} \qquad (4.3)$$

etc.

In these averages, derived from the calculated distribution of sizes, the large and small particles are equal. Suspension averages derived from the gravimetric distribution g(r) have greater practical value, such as

$$r_1' = \int_0^\infty r g(r)\,dr \approx \sum_\nu r_\nu g_\nu / G, \qquad (4.4)$$

where g_ν is the weight of particles per ν size interval; G is the total weight of particles.

The following also frequently are used:

(4) average geometric radius r_g, determined by the formula

$$\lg r_g = \overline{\lg r} = \int_0^\infty \lg r \cdot f(r)\,dr \approx \sum_\nu N_\nu \lg r_\nu / N; \qquad (4.5)$$

(5) calculated median radius r_m, determined from the condition $F(r_m) = F_b(r_m) = 0.5$. This implies that half of the particles have a radius greater than r_m and half less than r_m;

(6) gravimetric median radius $r_{m'}$, determined from the analogous condition $G_a(r_{m'}) = G_b(r_{m'}) = 0.5.$, i.e., the mass of particles with radius greater than $r_{m'}$ comprises one-half of the total mass of the aerosol.

It is noted that at normal distribution $r_m = \overline{r}$, but at the logarithmic normal distribution $r_m = r_g$.

By way of illustration various average particle sizes in the fog described above are calculated in the following. In this case it is advantageous to start directly with the experimental data (see Table 1) without smoothing out the raw data, and the calculation is conducted according to the formula

$$r_1 = \sum_\nu r_\nu N_\nu / N \qquad (4.6)$$

etc.

Thus we have: $r_1 = 8.9\mu$; $r_2 = 9.4\mu$; $r_3 = 9.9\mu$. From the curves for F_b and G_b in Figure 3 we have $r_m = 8.6\mu$ and $r_{m'} = 11.1\mu$; and from curves 1 and 2 in Figure 7 we obtain the values $r_m = 8.6\mu$ and $r_{m'} = 11.5\mu$.

Various average values are obtained in the experimental determination of the average size of particles, depending upon the method of measurement used. Thus, r_3 is determined by the calculated-gravimetric method, r_1 is determined by the "corona" method (diameter of diffraction rings), etc.

The rational selection of an average value for the characterization of an aerosol, as well as the selection of the distribution curve, is determined by the properties of the aerosol which are to be described. Therefore, the average quadratic radius r_2 must be chosen for expression of the optical density characteristics of coarse aerosols and the speed of their precipitation under the influence of a heavily charged field or under the influence of inertia, r_1 is used to characterize the speed of evaporation of aerosols, etc. In some special instances more complex average values must be established. Thus, the specific surface area of an aerosol, i.e., the surface per unit mass or volume of the dispersion phase, may be characterized by a particle in which the specific surface area is equal to the aerosol weight. The radius of this particle r_s is determined by the equation

$$\int_0^\infty 4\pi r^2 f(r)\,dr \Big/ \int_0^\infty \frac{4}{3}\pi r^3 f(r)\,dr = 4\pi r_2^2 \Big/ \frac{4}{3}\pi r_3^3 = 4\pi r_s^2 \Big/ \frac{4}{3}\pi r_s^3 \qquad (4.7)$$

or

$$r_s = r_3^3 / r_2^2.$$

In the fog under consideration $r_s = 11.0\mu$. This value is obtained in investigation of the problem of the absorption of light per unit volume of a substance in a coarse aerosol.

It is seen that in the given case the various average values do not differ greatly. The greater the degree of monodispersion, the smaller is the difference between average values. The concept of an average radius for aerosols with particle sizes which are extended over several orders of greatness loses all physical significance.

The problem of the average radius and distribution of sizes in aerosols with irregularly shaped particles will be discussed later.

[Pages 59-67]

CHAPTER II

14. The Movement of Aerosols within a Closed Space

In the case of aerosols contained in a limited space, the motion of particles involves both movement along with the suspension medium due to convection currents, artificial stirring, etc, and movement in relation to the medium. At present we are concerned only with the latter, and this problem will be studied from the point of view of the precipitation of particles due to gravity. In the precipitation of particles of an aerosol which fills a space bounded by walls, the speed of precipitation is V , and the motion of the suspension medium in the opposite direction has an average speed of ϕV , where ϕ usually is a very small volumetric size of the dispersion phase. Whereas in the immediate vicinity of the particles the medium is absorbed by the latter, at a greater distance from the particles (i.e., at a distance greater than the particle radius) the speed in the opposite direction is greater than ϕV . Thus the speed of precipitation of particles in the case at hand, in distinction from the movement of a free cloud, is less than the speed of isolated particles in an unbounded space by the factor $1 + k\phi$ where $k > 1$.

According to Cunningham [46] only one circumstance need be taken into consideration: in the derivation of the Stock formula one of the limiting conditions is that the speed of the medium is equal to zero at an infinite distance from the particle. In the precipitation of a system of particles in a closed space the speed of the medium must be taken as equal to zero at a distance ρ from the center of a particle (disregarding the opposing motion of the medium), where $2\rho \approx n^{-1/3}$ is the average distance between precipitating particles. Thus every particle is exposed to the same resistance which it would encounter in the center of a closed vessel with radius ρ . According to the calculations of Cunningham this resistance in Stock approximations is equal to $6\pi rV\eta(1 + 1.25\frac{r}{\rho})$. According to Oseyen the correction is increasingly smaller at greater values of the Reinholds number $V\rho\gamma/\eta$. All other authors arrive at a correction factor $1 + k\phi$ through fairly complicated, though not precise methods, in which the values of k are equal to 5.5 [85]; 7.0 [83]; and 4.5 [86]. Obviously, a precise solution of this problem is very difficult.

The difference between the correction factors of type $1 + k\phi$ (I) and $1 + k\frac{r}{\rho} \approx 1 + k\phi^{1/3}$ (II) is quite large, because at ordinary values of ϕ factor I is for practical purposes equal to unity, whereas factor II may be several percent greater than unity. The only research on the small values of ϕ , in which we are interested at this point has been done by Cermak [85], in which he studied the speed of precipitation of suspensions of a high degree of monodispersion, consisting of erythrocytes of various animals in water, with radii of 2.4, 3.0, 3.7, and 4.4μ . It was shown that at $\phi <$ 0.04 to 0.08 the experimental results agreed with the correction factor $1 + k\phi$, in which the value of k is between 4.8 and 6.9 for various erythrocytes. Unfortunately the speed of precipitation of isolated particles was not measured in this work, but was determined by extrapolation.

Because of the rather scanty data available it may be stated only that in the precipitation of aerosols in a closed space the resistance of the medium at small values of ϕ apparently is equal to $6\pi rV\eta(1 + k\phi)$, in which k is approximately 5 or 6. Recently, during the course of research on the fluidization of dusts (see equations 58.1 and 58.2) the problem of the speed of precipitation of concentrated dispersed systems acquired great importance. In the measurement of this speed fluidization was accomplished by the transformation of systems of uniform spheres or bodies with other shapes into a suspended state in a rising fluid stream. In this process the concentration of spheres is automatically determined because their speed of precipitation is equal to the speed of the fluid stream. The results of these experiments may be expressed by the formula

$$V_s' = V_s(1 - \phi)^{\alpha} , \qquad (14.1)$$

where V_s' is the speed of precipitation of the system of particles, and V_s represents isolated particles. Louis and Bauerman [87] and Richardson and Zaki [88] obtained the same value, 4.65, for the coefficient α in the case of the system of spheres. The work of Richardson and Zaki also included an approximated theoretical calculation of the speed of precipitation of the system of spheres. Proceeding from two models of the distribution of spheres within a space, these authors obtained two curves (V_s', ϕ), one of which was approximately 40% higher and the other 20% lower than the experimental curve.

In conclusion, mention is made of a phenomenon which is familiar to all who work with aerosols: in precipitating concentrated aerosols the upper limit of the dispersion phase usually is horizontal and flat. This phenomenon may be observed both in the laboratory and in natural fogs. The cause of this is that when the saturation of an aerosol exceeds that of a medium bordering on the aerosol, hydrostatic forces counteract the disturbance of the horizontal upper limit of the aerosol by the action of convection currents. In the given instance the aerosol behaves like a fluid. It is known that stabilization of the upper border is observed only when the dispersed phase moves as a whole with the medium, i.e., at a sufficiently high concentration of the aerosol (see preceding paragraph). Because of this, the surface of aerols charged with chlorine, carbon dioxide, etc, is very stable [89].

15. Vertical and Horizontal Electric Field Methods and their Application

The movement of aerosol particles in an electric field is observable primarily when movement within the field is accompanied by the attraction of gravity. The force acting on a particle in an electrical field is equal to qE , where q is the charge of the particle and E is the potential of the field. The speed of particles, in a case in which formula (8.2) applies, is equal to

$$V_E = qEB = qE\left(1 + A\frac{l}{r}\right) / 6\pi r\eta. \qquad (15.1)$$

The movement of aerosol particles in a vertical electrical field, i.e., when an electrical field is superimposed on the gravitational field of the earth, is very interesting because of its practical application. The vertical electrical field method developed by Millikan [90] and Ehrenhaft [91] is one of the most productive methods for study of aerodispersion systems and it plays a very great role in the development of our knowledge in this field. The method consists of aerosol particles in a test tube, which forms two horizontal condenser plates and side walls of an insulating material, and is provided with ports for observation, illumination and overcharging of the particles. Observation is through a horizontal microscope with an ocular grid. The potential of the field is $E = \Pi/h$, where Π is the potential difference, and h is the distance between the condenser plates. The intensity and direction of the electric field may be measured at will. The speed of precipitation without an electrical field V_S is determined, then the movement of the same particles is determined under the simultaneous action of electrical and gravitational fields $V_S + V_E$ or $V_S - V_E$, depending upon the direction of the electrical field. This is where the value V_E is involved. In addition, the charge of the field E_B sometimes is determined, which exactly counterbalances the action of gravitational force on the particles

$$E_B = mg/q = \frac{4}{3}\pi r^3\gamma g/q. \qquad (15.2)$$

In some designs the test tube is provided with facilities for changing the pressure within the tube within wide limits above or below atmospheric pressure. The techniques of the vertical field method are well covered in the literature [53, 92, 93], and enable solution of the following series of tasks.

A. Determination of the Size of Elementary Charge and Calculation of the Law of Resistance of a Gaseous Medium to the Movement of Small Particles

The method first used in the folution of these important tasks was the method of vertical electrical field [53]. In terms of V_s the equation may be expressed as

$$V_\varepsilon = mgB = mg\left(1 + A\frac{l}{r}\right) / 6\pi r\eta \qquad (15.3)$$

or

$$V_s = \frac{2r^2 g\gamma}{9\eta}\left(1 + A\frac{l}{r}\right). \qquad (15.4)$$

Multiplication of the square root extracted from (15.4) with (15.1) the following equation is obtained:

$$q = \left[\frac{18\,\pi V_E}{E}\left(\frac{\eta^3 V_s}{2\gamma g}\right)^{1/2}\right]\left(1 + A\frac{l}{r}\right)^{-\frac{3}{2}} = q_{St}\left(1 + A\frac{l}{r}\right)^{-\frac{3}{2}}. \qquad (15.5)$$

Assuming that the formula of Stock is applicable, the expression contained in brackets, i.e., the size of the charge of particles determined experimentally, is designated by q_{St} . In the case of liquid (oil) droplets all dimensional values which are factors of this expression, such as the viscosity of air, density of particles, field tension, and the velocities V_E and V_s , may be determined experimentally. When a particle is given an overcharge by x-rays or gamma rays the change in its charge Δq is equal to a small positive or negative whole number of elementary charges ε :

$$\Delta q = (\nu_1 - \nu_2)\,\varepsilon, \qquad (15.6)$$

where $\nu_1 - \nu_2$ usually equals ±1 , or more rarely, ±2 , etc. Similarly, $\Delta q_{St} = (\nu_1 - \nu_2)_{St}$. Thus ε_{St} may be easily determined by finding the common denominator of several values of Δq_{St} .

Because from equation (15.5) it follows, that:

$$\varepsilon = \varepsilon_{St} / \left(1 + A\frac{l}{r}\right)^{3/2}, \qquad (15.7)$$

then

$$\varepsilon_{St}^{2/3} = \left(1 + A\frac{l}{r}\right)\varepsilon^{2/3} = \varepsilon^{2/3} + A\varepsilon^{2/3}\cdot\frac{l}{r}. \qquad (15.8)$$

If the values of $\varepsilon_{St}^{2/3}$, which have been found for various droplets, are plotted for the function l/r , then in the case in which the Cunningham formula applies these points also must lie on a straight line (15.8). The point of intersection of this straight line with the ordinate gives the value of $\varepsilon^{2/3}$, and the tangent of the angle of inclination with the abscissa axis gives the value of $A\varepsilon^{2/3}$. In practice the true radius r is not determined directly from the experiment, but

$$r_{st} = \left(\frac{9V_s\eta}{2\gamma g}\right)^{1/2}, \qquad (15.9)$$

i.e., the size of the radius, calculated, assuming the accuracy of the formula of Stock. However, at small values of the correction element Al/r, r_{St} differs very slightly from r, and therefore if r is substituted for r_{St} in (15.8) the substitution has little effect on the results, in fact the experimental points lie quite within a straight line. This method resulted in the first accurate determination of the size of the elementary electrical charge and demonstrated the applicability of the formula of Cunningham for small values of l/r. In obtaining accurate results in the determination of ε the influence of Brownian movement must be eliminated as much as possible, i.e., comparatively large droplets must be used ($r = 2$ to 5μ).

For the same reason small droplets are not used in calculation of the law of resistance of the medium for large values of l/r, and measurements are taken at low pressures, i.e., at large values of l. In these experiments [39] the pressure in the test tube was brought down to 0.5 mm Hg, and the resistance of the medium was measured for values up to $l/r = 134$. As a result of these measurements the empirical formula (8.5) was established. The best method for studying the law of resistance for small particles consists of measurement of the speed of movement of a single particle at various levels of pressure [94, 95, 28].

It is noted that the first experiments with the vertical electric field method were conducted not with individual droplets, but with a water vapor fog obtained by condensation of vapor on gaseous ions. The water droplets in this case have the same elementary charge. V_s and V_E were determined for the movement of the upper limit of a cloud, and because of this the value of ε was approximately 30% lower than the true value [96]. This is not surprising, because the speed of motion of the upper boundary always is determined by the slowest particles (the smallest particles for V_s measurements and the largest particles for V_E measurements). Because of this, depressed values for the particle charge are obtained, similarly to equation (15.1).

All the discussion above pertains to individual, spherical particles. In the transition to non-spherical particles and to aggregates the following must be taken into consideration: in the case of aggregated particles γ indicates their apparent density. Therefore, if the true density is substituted for γ in (15.5), values somewhat lower than true may be obtained for q, and thus also for ε. This also gave rise to the hypothesis of the "subelectron" [97]. Furthermore, if the particle is not spherical, then r_e^2/x is substituted for r^2 in (15.4) (see 12.15), and $r_e x$ is substituted for r (see 12.18) in (15.1). As a result, the expression for q (see formula (15.5)) is a factor of $x^{2/3}$ in the denominator. Because for non-spherical particles $x > 1$ (see (12.18)), if this circumstance is not taken into consideration depressed values again are obtained for the elementary electrical charge.

Knowing the size of ε and the law of the resistance of the medium, the following tasks may be solved by the vertical field method.

B. Measurement of the Charge and Motion of Particles

Through repeated measurement of the speed of motion of a particle in a vertical electric field, directed both up and down, the arithmetical average of both speeds $V_E - V_s$ and $V_E + V_s$ is taken as equal to V_E

and the proportional number of elementary charges on a particle ν . If the latter is subjected to a several-fold overcharge, ν and the size of the charge on a particle $q = \nu \varepsilon$ is obtained. Knowing V_E and q , the motion of a particle B is determined according to formula (15.1).

The particle motion B also may be found by its speed of fall when pole V_s is absent (see formula 15.3), and by the strength of a counterbalancing pole E_B , which characterizes the mass of the particle (15.2). The shape and density of the particle is excluded from both of these methods.

A very substantial effect is due to the fact that the motion of particles in an electric field, determined by the first method, and the motion in the gravitational field, determined by the second method, may differ considerably because of the orientating action of the electrical field tends to divert particles which are aligned along other axes into alignment with the electric field (see paragraph 43). For some reason or other this circumstance has not been taken into consideration by any of the researchers working in this field, and explains many instances of disagreement between theory and experiment in investigation of the motion of aerosol particles in a vertical electric field."

C. Determining the Size of Particles

When determining the size of aerosol particles by their speed of precipitation under the attraction of gravity (i.e., according to formula 15.4) two difficulties are encountered:

(1) For particles in an aggregated state the apparent density, which is used in (15.4), is not known beforehand, as already has been mentioned.

(2) In the case of very small particles ($r \angle 10^{-4}$ cm) Brownian movement causes very large fluctuations in the measurement results. These

difficulties may be eliminated to a considerable extent through the use of the following variant method [98]. When V_E and q are found, as described above, they are used for the determination of r, using (15.1) or, in the case of non-spherical particles, $r_e x$ is determined. This enables elimination of particle density. The influence of Brownian movement in this case is lessened because the speed V_E (in distinction to V_s) may be made sufficiently large through increasing the tension of field E.

The size of particles with unknown density also may be found by measuring the speed of their precipitation under various pressures [99]. Since the length of the free path of gaseous molecules is inversely proportional to the pressure of the gas, equation (15.4) may be written in the form

$$V_s = \frac{2\gamma r^2 g}{9\eta}\left(1 + \frac{A'}{pr}\right). \tag{15.10}$$

Here $A' = Apl = Ap_0 l_0$, where l_0 is the length of the free path of a molecule at p_0 atmospheric pressure. Thus if V_{s1} and V_{s2} are the speeds of precipitation at pressures p_1 and p_2, then

$$\frac{V_{s1}}{1 + (A'/p_1 r)} = \frac{V_{s2}}{1 + (A'/p_2 r)}, \tag{15.11}$$

from which we obtain

$$r = \frac{V_{s2}p_2 - V_{s1}p_1}{V_{s1} - V_{s2}} \cdot \frac{A'}{p_1 p_2}. \tag{15.12}$$

Because similarly to this equation r is determined in relation to the difference between fairly close values, this method cannot give precise results. In addition, the theory of the motion of small particles in a gaseous medium in the transition zone (see paragraph 8) was developed only for particles of spherical shape, and in the case of other shapes it is unknown, and a coefficient must be introduced which allows for the different shapes, in the form of the correction factor $1 + Al/r$.

D. Determination of Apparent Density and the Dynamic Coefficient of the Shapes of Particles

When the particle radius is found by the method described above and the particle mass is determined by the strength of the counterbalancing field, γ may be determined, i.e., in the case of particles in an aggregate state, the apparent density. Using this method Whiteloy-Gray [31] discovered the value of particle density in several smokes, which is shown in Table 2. With this method correct results may be obtained only for spherical particles. For particles with other shapes the coefficient x must be introduced in (15.1) which, contrary to the above methods, gives an increased value for the equivalent radius r_e, and depressed values for the apparent density.

In the case of non-spherical individual particles, the density of which is known beforehand, their equivalent radius r_e may be determined by their charge and by the strength of the counterbalancing field using (15.2). Furthermore, the value x may be found through the speed V_E in an electric field, and the same value may be determined through the speed V_s under the influence of gravity. As already mentioned, various values may be obtained for x by this method because of the orientation of particles in an electric field.

Determination of particle density also is possible using an original modification of the vertical field method, proposed by Plachek [24], which used a non-uniform electric field with divergent lines of force. As the theory indicates, the force F acting on a non-charged particle in this field is equal to

$$F_E = x_E v \,\mathrm{grad}\, E^2, \qquad (15.13)$$

where ν is the volume of the particle, and x_E is a coefficient which is determined by the shape and dielectric permeability ε_k of the particle in the relationship $\frac{3}{8\pi}\frac{\varepsilon_k - 1}{\varepsilon_k + 2}$ for non-conducting, and $\frac{3}{8\pi}$ for conducting spheres. It is very important in the latter case that

x_E have the same value both for contiguous individual particles and for aggregates, if only the foremost particles of which they are composed have electrical contact. If, in the site in which the particles are located E and grad E are oriented vertically, then the force acting on the particles is

$$F_E = 2x_E vEdE/dz. \qquad (15.14)$$

If the gradient is directed upward, then under the conditions $F_E = mg$ or

$$EdE/dz = \frac{mg}{2x_E v} = \frac{\gamma g}{2x_E} \qquad . (15.15)$$

a particle will be counterbalanced in the air. As may be seen from (15.15), all non-charged spherical particles having uniform density and constant dielectric, are counterbalanced at the same value EdE/dz independent of their size. This conclusion has been proven in experiments with oil droplets and with individual mercury droplets. The upper face of the condenser was shaped in a form necessary for the production of a non-uniform field. In this case EdE/dz is proportional to the square of the tension on the condenser faces.

In these experiments the following important fact was observed: comparison of the value EdE/dz corresponding to the counterbalancing of oil and mercury droplets showed that the coefficien x_E has almost the identical value for these and other drops. Thus, oil droplets are practically polarized in a constant electric field and act as a conductor of electricity, apparently because of the ionizing impurities which they contain. Because mineral oils belong to the class of very poor electrical conductors, the same probably is true of particles of the majority of other substances. Because of this, for the sake of simplification aerosol particles will be considered as behaving like conductors in an electrical field.

In accordance with (15.15) the value of EdE/dz may be determined precisely through observation of individual droplets at various points of the electric field formed by the condenser. Through similar observation of aggregate particles of metallic aerosols the apparent density of these particles was determined by the method described above. In the given case the particles were counterbalanced at various values of $E dE/dz$, and thus had different apparent densities. By this method it was established that the particles of mercury fogs obtained by mechanical dispersion at low air pressure had normal density, but the density of particles of fogs obtained by dispersion at high pressure or by volatilization was 5- to 10-fold below normal (see (4.7)).

The vertical field method also was applied in research on the kinetics of evaporation of droplets, photo-effect on droplets, Brownian movement, movement in a temperature gradient field, and many other aerosol problems.

The motion of aerosol particles in the earth's gravitational field with the superposition of a horizontal electrical field also may be used for the determination of ε and for several other tasks mentioned above. In this case the motion of the particles describes an inclined straight line, and is described by the same equations (15.2) and (15.3), the only difference being that V_s in this case indicates vertical and V_E indicates the horizontal component of the speed of the particle. Equation (15.5) and all the remaining deductions remain in force. Both speed components are measured by photography under intermittent lighting [100] because this method enables working with much larger particles (up to $r = 10\mu$) than can be used in the vertical field method, and is an advantage in the precise determination of ε. On the other hand, multiple overcharging of particles in this experiment is excluded. Because this method has been used only for determining more precisely the size of an elementary charge, the approximate value of which was considered as known, the number of elementary charges on particles may be determined directly from the experiment (for variant methods see Chapter IV, paragraph 25).

[Pages 107-123]

CHAPTER IV. CURVILINEAR MOTION OF AEROSOL PARTICLES

25. General Theory of the Curvilinear Motion of Aerosol Particles. Precipitation of Aerosols in an Applied Horizontal Electric Field.

The theory of the curvilinear motion of aerosol particles is relatively simple with respect to the proportionality of resistance of the medium and the speed of particles, i.e., at small numerical values of Re. Assuming that the non-inertia character of the resistance of the medium, which was practically observed above with respect to the straight-line movement of particles, also holds true for curvilinear trajectory, we arrive at the following equation for the motion of particles in vector form:

$$m\frac{d\mathbf{V}}{dt} = -6\pi\eta r(\mathbf{V}-\mathbf{U}) + \mathbf{F}, \tag{25.1}$$

where **V** and **U** are the vectors of the speed of the particle and of the medium and **F** is the vector of external force.

In coordinate form (25.1) is written as

$$m\frac{dV_x}{dt} = -6\pi\eta r(V_x - U_x) + F_x; \qquad m\frac{dV_y}{dt} = -6\pi\eta r(V_y - U_y) + F_y, \tag{25.2}$$

where V_x is the component of the speed of a particle along the x-axis, etc.

As equation (25.2) indicates, the component speed of a particle along any axis complies with the same equation that applies to rectilinear motion because motion along the various axes are independent of each other. This circumstance greatly eases analysis of the curvilinear motion of particles.

The situation is different with a large numerical value for Re. In this case, considering that

$$\mathrm{Re} = \frac{2\gamma_g r\cdot|(\mathbf{V}-\mathbf{U})|}{\eta}, \tag{25.3}$$

we obtain the following vectorial equation for the movement of particles:

$$m\frac{dV_x}{dt}=\psi\left(\frac{2\gamma_g r\,|(V-U)|}{\eta}\right)\gamma_g\frac{\pi r^2}{2}(V_x-U_x)\,|(V-U)|+F_x \text{ etc} \tag{25.5}$$

where $|(V - U)|$ is the length of the vector $V - U$.

Plotting equation (25.4) along the coordinate axis, we obtain

$$m\frac{dV}{dt}=-\psi\left(\frac{2\gamma_g r\cdot|(V-U)|}{\eta}\right)\gamma_g\frac{\pi r^2}{2}(V-U)\cdot|(V-U)|+F, \tag{25.4}$$

The first member of the right hald of this equation depends both on the component relative speed along the x-axis, and on the absolute value of this speed. Thus the speeds along the various axes are not independent. The system of equation (25.5) is not generally solved, and analytic investigation of the curvilinear motion of particles at large values of Re is possible only in certain cases. Because of this we shall limit ourselves to the investigation of curvilinear motion at small values of Re .

The task of studying the curvilinear motion of aerosol particles may be divided into two groups: motion in media at rest and in moving media. In the first only particles moving in a horizontal direction under the action of an external force will be considered, such as an electrical field. This is one of the few examples of experimental research on the curvilinear motion of aerosol particles. Taking the above into consideration, we obtain the following type of equation for the settling motion of particles:

$$V_z=mgB=\tau g, \tag{25.6}$$

$$m\frac{dV_x}{dt}=F-6\pi\eta rV_x, \tag{25.7}$$

where x is the horizontal and z is the vertical direction below the axis.

If the force F is measured sinusoidally with time, $F=F_0\sin\omega t$, then the horizontal speed of a particle is expressed by formulas (19.6) and (19.11), and the horizontal displacement is indicated by the formula

$$x = -\frac{FB\cos(\omega t - \varphi)}{\omega\sqrt{1+\omega^2\tau^2}}, \tag{25.8}$$

which gives

$$\operatorname{tg}\varphi = \tau\omega = V_z\omega/g. \tag{25.9}$$

Furthermore, the vertical displacement of a particle during time t is equal to

$$z = \tau g t. \tag{25.10}$$

Thus, the trajectory of a particle is expressed by the sinusoid

$$x = -\frac{F_0 B\cos\left(\frac{\omega z}{\tau g} - \varphi\right)}{\omega\sqrt{1+\omega^2\tau^2}}. \tag{25.11}$$

Abbot [170] tested these conclusions experimentally by photographing water droplets of 30 to 40μ radius falling through a horizontal electric field of 60 hertz. The trajectory of the particles described precise sinusoidal curves. Through the use of special instruments the moments of time corresponding to the value $F = 0$ which enabled determination of the displacement of oscillation phase φ. The value φ found in this way agreed sufficiently with formula (25.9).

The small variations (1 or 2°) apparently are explained by deviations of the horizontal and vertical components of the motion of droplets in the formula of Stock.

This method doubtless may be used for the simultaneous determination of the size and charge of aerosol particles. However, in this respect it proved to be more advantageous to work with the constant of the strength, and the variable of the direction of the field, instead of working with the sinusoidal curve. In this case zigzag lines, composed of straight sections, are obtained. The vertical displacement of a particle per single period of oscillation is determined by the distance between zigzags, from which (25.10) τ and thence the particle radius are determined. The horizontal speed of a particle is equal to $V_x = BEq$ (E is the field tension, and q is the charge of the particle). Thus the tangent of the angle of inclination of trajectory sections from the horizontal are equal to $V_z/V_x = gm/Eq$ and, knowing the particle radius, the particle charge may be determined. This "oscillation" method, developed by N. Fuka and I. Petryanov [171], is very useful for studying the distribution of charges on particles of a fog with radius greater than 0.5μ. In the study of smokes it is essential that it be taken into consideration that the apparent density of particles may be considerably lower than the true value. The oscillation method was first proposed by Wells and Gerke [172]. However, these authors did not make measurements on the precipitation of particles but, assuming that particles had one elementary charge, measured the amplitude of displacement of particles as they were drawn slowly through a test tube, and computed the particle radius. However, the particles for which the oscillation method was used ($r > 0.5\mu$) carried, on the average, many more than a single elementary charge (see Table 11); thus the method of Wells and Gerke may be used only in certain cases, for example in aerosols produced by the "germination" method with amicroscopic nuclei [173].

A variant method developed by N. Rozenblume [174] is very useful for the study of charges in coarse dispersion aerosols under production conditions; the particles are allowed to fall through a horizontal sinusoidal field produced by a technically alternating current. In this method the particles form luminous horizontal lines, the length of which is equal to twice the amplitude of displacement. By means of an ocular grid and a timing device the speed of precipitation and the length of the above mentioned lines are determined simultaneously. The computation is described under paragraph 19.

26. Precipitation of Aerosols from a Laminar Flow Under the Action of Gravity

The most important division of the field of the mechanics of aerosols is that of the motion of particles in a stream of gas. This chapter explains the motion of particles in a laminar stream. The much more complex problem of motion in a tubular ribbon stream is discussed in Chapter VI.

The precipitation of aerosol particles will be considered first, in which the articles are in laminar flow in a horizontal tube of such small diameter that vertical convection currents are either lacking or are so limited that their speed is very small compared to the speed of precipitation of the particles. Thus, the calculations included below apply to aerosols in laboratory apparatuses, closed pipes, etc, but not to industrial precipitation chambers. These examples are limited to flat tubes in which the width is much greater than the height, and to round tubes.

The vertical motion of particles with respect to the medium is expressed by the formula (see (17.4))

$$V = V_s(1 - e^{-t/\tau}). \tag{26.1}$$

The member $e^{-t/\tau}$ may be ignored if the average time of the presence of a particle in the tube is high compared to the time of relaxation τ. In this case the vertical speed of the particles may be considered constant and equal to V_s, and the horizontal speed of the particles as equal to the speed of the medium at the same point. As is seen from the values of τ and V_s for particles of various sizes in Table 13, these conditions are fulfilled in ordinary use with aerosols.

The speed of passage through the tube is constant for all sections of the tube; thus, at a distance of the order $0.1\ R \cdot Re_f$ from the mouth of the tube (where R is the radius or half the height of the tube, and Re_f is Reinhold's number, related to the speed within the tube) there is a stationary distribution of speed values, expressed in the case of a flat tube by the formula

$$U_x = \left(\frac{3z}{h} - \frac{3z^2}{2h^2}\right)\overline{U}, \tag{26.2}$$

where h is half the height of the tube; $\bar{U}$ is the average speed; z is the distance from the bottom of the tube.

Thus, the line of flow in a transitional section of the tube is inclined from the horizontal.

The resultant speed of laminar flow at any point within a flat tube may be expressed by the function of flow ψ:

$$U_x = \partial\psi/\partial z; \quad U_z = -\partial\psi/\partial x. \tag{26.3}$$

The resultant speed of particles is equal, according to the above, to

$$\frac{dx}{dt} = U_x = \frac{\partial\psi}{\partial z}; \quad \frac{dz}{dt} = U_z - V_s = -\frac{\partial\psi}{\partial x} - V_s. \tag{26.4}$$

Excluding dt from these equations, we obtain the following differential equation for the trajectory of particles

$$-\frac{dx}{\partial\psi/\partial z} = \frac{dz}{(\partial\psi/\partial x) + V_s} \tag{26.5}$$

or, in another form

$$-V_s\,dx = \frac{\partial\psi}{\partial x}dx + \frac{\partial\psi}{\partial z}dz = d\psi. \tag{26.6}$$

Integrating for all lengths of the tube L, we obtain

$$V_s L = \psi_0 - \psi_L, \tag{26.7}$$

where ψ_0 and ψ_L are the values of the flow function at various points from the entrance and from the exit of the tube. The flow function expresses the volume of gas which flows through the tube per unit time between the bottom of the tube and a given line (surface) of flow per one cm width of the tube. Taking $\psi_L = 0$, we may find the limiting trajectory, which separates the trajectory of particles which precipitate and do not precipitate in the tube. For the limiting trajectory at the tube entrance $\psi = \psi_0 = V_s L$; consequently, if the total flow of gas in the tube is equal to Ψ, then the weight of the aerosol precipitating in the tube, or the "effectiveness of precipitation" Э, is equal to

$$\text{Э} = \psi_0/\Psi = V_s L/\Psi. \tag{26.8}$$

(This deduction was provided by the author G. Nathanson.)

Because

$$\Psi = 2h\bar{U}, \quad \text{Э} = \frac{V_s L}{2h\bar{U}}. \tag{26.9}$$

This conclusion is derived from the important deduction that the effectiveness of precipitation does not depend upon the distribution of speeds within the tube. The length of tube necessary for complete precipitation is equal to

$$L_{\text{кр}} = \frac{2h\overline{U}}{V_s}. \qquad (26.10)$$

The computation for round tubes is considerably more complicated. The distribution of speeds in this case is expressed by the formula

$$U = 2\overline{U}\left(1 - \frac{\rho^2}{R^2}\right), \qquad (26.11)$$

where ρ is the distance from the axis of the tube. For the sake of simplicity the distribution is taken as fixed at the entrance of the tube, and the length of tube L_{kp} necessary for complete precipitation of the aerosol, is computed. For this the trajectory of particles entering the tube at the uppermost point of the diameter of the tube and moving in a vertival plane through the axis of the tube must be found. The distribution of speeds in this plane is given by the expression $U = 2\overline{U}\left(1 - \frac{z^2}{R^2}\right)$, where z is the vertical distance from the axis.

As above, the differential equation of the particle trajectory is found,

$$\frac{dx}{2\overline{U}\left(1 - \frac{z^2}{R^2}\right)} = -\frac{dz}{V_s}. \qquad (26.12)$$

Integrating for z from -R to R, we have

$$L_{\text{кр}} = \frac{8R\overline{U}}{3V_s}. \qquad (26.13)$$

This formula was proposed by A. Chistov [176] for computing the precipitation of aerosols in industrial plants, however, as already has been stated, in this case the effects of convection currents must be taken into account. G. Nathanson derived the following formula for the effectiveness of precipitation in a round tube

$$Э = \frac{2}{\pi}\left(2\mu\sqrt{1-\mu^{2/3}} + \arcsin\mu^{1/3} - \mu^{1/3}\sqrt{1-\mu^{2/3}}\right), \qquad (26.14)$$

where $\mu = \frac{3V_s L}{8R\overline{U}}$.

At $\mu = 1$, i.e., $L = 8\,R\overline{U}/3V_s$, formula (26.14) gives Э = 1 in conformance with formula (26.13).

27. Precipitation of Aerosols from a Laminar Flow Under the Influence of an Electric Field

In considering the precipitation of charged particles from a laminar gas stream in a flat condenser the cases considered will be limited to that of very small particles, ignoring their inertia, i.e., as in the preceding paragraphs their motion will be taken as quasi-stationary (see equations (17.19) through (17.22)). The motion of particles in a direction perpendicular to the plane of the condenser is expressed by the equation

$$V_z = \frac{\Pi q B}{h}, \qquad (27.1)$$

where Π is the tension on the condenser plates; h is the distance between the latter; q is the particle charge; B is the mobility of the particle.

In the study of gaseous ions the speed of an ion in a field with a tension of $1\ B \cdot cm^{-1}$ is called the mobility of the ion. This designation also is carried over to heavy ions, i.e., very small, charged aerosol particles. Thus, "electrical mobility" also is connected with the "mechanical mobility" B in the formula

$$u = qB/300 \qquad (27.2)$$

(the coefficient 300 is ratio between the electrostatic unit of charge to the volt). As stated in paragraph 2 research on highly dispersed aerosols is conducted, for the most part, through electrometric measurement of their mobility u, and the latter is the basic characteristic of heavy ions. The relationship between the radius of particles and their mobility u in the presence of a single elementary charge $\varepsilon = 4.802 \cdot 10^{-10}$ el-st unit on the latter is shown in Table 3 and in Figure 10.

Here we must touch briefly on the size of the charges on aerosol particles. These charges may have various origins: in the production of dust aerosols the particles receive triboelectric charges; in the production of liquid aerosols the droplets become charged because of fluctuations in the concentration of ions in the liquid; in aerosols formed at high temperatures the charges are caused by thermionic emissions; finally, the precipitation of gaseous ions and electrons on aerosol particles, which is a general source of charges, is extremely important.

Regardless of the initial distribution of charges on aerosol particles which, due to the continuing precipitation of ions are continuously formed in a gaseous medium, the distribution gradually approaches the determined stationary state. As indicated by theoretical and experimental research in the aerosol laboratory of the Institute of Physical Chemistry imeni L. Ya. Karpov [177, 178], the stationary distribution of charges at a symmetrical bipolar ionization of the gaseous medium is expressed by the first approximation of the formula of Bol'tsman: the weight of particles with ν elementary charges (ν is a positive or negative integer) is equal to

$$f(\nu)=\frac{1}{\Sigma}e^{-\frac{(\nu\varepsilon)^2}{2rkT}}, \qquad (27.3)$$

where

$$\Sigma=\sum_{-\infty}^{+\infty}e^{-\frac{(\nu\varepsilon)^2}{2rkT}}.$$

Thus the stationary distribution of charges does not depend upon the degree of ionization of a gas, but upon the time necessary for the establishment of this distribution; the shorter the time, the greater the ionization.

The radii of the majority of amicroscopic particles in the atmosphere, called "condensation nuclei" and "heavy ions," have a size of the order 1 to $5 \cdot 10^{-6}$ cm. Because the distribution of charges in atmospheric aerosols is close to that of stationary aerosols, according to formula (27.3) we obtain the following (Table 11) distribution of charges, in which n_0 is the percentage of non-charged particles; n_1 is the number of particles with a single elementary charge, etc.

TABLE 11

DISTRIBUTION OF CHARGES OF PARTICLES OF HIGHLY DISPERSED ATMOSPHERIC AEROSOLS

$r \cdot 10^6$ см	n_0	n_1	n_2	n_3
1	90	10	—	—
3	55	43	2	—
5	43	48	8,6	0,4

Thus the percent of particles with several elementary particles is not large, and with the aid of Table 3, the size of these particles may be determined according to their mobility u without greatly increasing the errors. A classification of particles of atmospheric aerosols according to their mobility [179] which is used in atmospheric physics is included below, indicating their size (Table 12).

TABLE 12

CLASSIFICATION OF PARTICLES OF ATMOSPHERIC AEROSOLS ACCORDING TO THEIR MOBILITY u

Particles	u	$r \cdot 10^6$ см
Supercharged ions	$<2,5 \cdot 10^{-4}$	>5
Heavy (Lamzheven ions)	$2,5—10 \cdot 10^{-4}$	2,5—5
Large "average" ions	$10^{-3}—10^{-2}$	0,7—2,5
Small "average" ions	$>10^{-2}$	$<0,7$

If the tension is expressed in volts, formula (27.1) becomes

$$V_z = \frac{dz}{dt} = \Pi u / h. \qquad (27.4)$$

The motion of particles in a direction parallel to the condenser plates is expressed by the equation

$$\frac{dx}{dt} = U(z), \qquad (27.5)$$

where U is the speed of flow of the gas. From these equations it follows that a particle entering the condenser at the plate having a charge of the same sign reaches the other plate at a distance x_0 (Figure 24), equal to

$$x_0 = \frac{h}{\Pi u} \int_0^h U(z)\,dz = \frac{\overline{U} h^2}{\Pi u}. \qquad (27.6)$$

Similar to (26.10) this formula holds true for any distribution of speeds of flow in the condenser.

Thus, the distance which a particle traverses along the condenser is inversely proportional to its mobility u . A deservedly great amount of attention has been focused on this fact, although at that time the method for determination of the mobility of particles in highly dispersed aerosols proposed by Chenmen [180] was not yet available. The aerosol is produced through the slot-shaped nozzle B in the right-angle tube (Figure 25), through which pure air is blown. The aerosol stream has a direction parallel to the walls of the tube, and its speed is small compared to the speed of the air in the tube. Thus, the stream in the tube must be laminar. The tube walls A_1 and A_2 are condenser walls. Particles which are given similar charges by the wall A_1 move as discussed above and precipitate on the wall A_2 , traversing a distance x_o along the condenser (see (27.6)). The speed of the aerosol exactly in the center line between the walls is equal to the speed of air in this plane (1.5 $\overline{U}$). In this case the distance x_o will be twice as small as in the previous case.

Two methods may be used for the determination of x_o : microscopic study of the precipitation of particles, or measurement of the amount of electricity contained in the precipitated particles. In the first case the wall A_2 is covered with a copper grid which is coated with a thin layer of collodion, on which the particles precipitate. After the experiment is concluded the precipitate is studied under an electronic microscope.

The distribution of mobility u of aerosol particles (similarly to the distribution of their sizes) is expressed, as a rule, by a continuous curve (which, however, may have several maximum values). The number of particles with mobility lying between the limits u and u + du is equal to

$$dN = f(u)\,du. \qquad (27.7)$$

Because of this the spatial distribution of particles in a precipitate obtained as described above also may have the same characteristics: the number of particles in the band limited by the coordinates x and $x + dx$ is expressed by the formula

$$dN = \varphi(x)\,dx. \qquad (27.8)$$

The expressions (27.7) and (27.8) are equated. Because it follows from (27.6) that du is proportional to dx/x^2, then $f(u)$ is proportional to $x^2\varphi(x)$. Thus, determining $\varphi(x)$ experimentally, the distribution of mobility of particles $f(u)$ may be found. Measuring the size of particles on the microphotographs obtained, and calculating with the aid of Table 3 the mobility u which particles would have at one elementary charge, the actual number of charges per particle may be determined. A study of this type, which as yet has not been undertaken, would enable final solution of the problem of the number of elementary charges on very small particles.

In the second method a narrow electrode d (Figure 25) is inserted in the condenser very close to the wall A_2, but insulated from the latter. The electrode is connected with an electrometer and may be moved along the condenser. The charge obtained on the electrode, then, is equal to the number of particles precipitating on it and is increased by the average charge $\bar{q}$ of the particles, i.e., according to (27.8) it is equal to $\bar{q}\varphi(x)\Delta x$, where Δx is the width of the electrode. Thus, this method gives not the distribution of mobilities $f(u)$, but the function $\bar{q}(u)\,f(u)$, where $\bar{q}(u)$ is the average charge of particles with mobility u. Only in cases in which the size $q(u)$ may be calculated theoretically, or when the very small size of particles enables the assumption that they have only a single elementary charge, does this method enable determination of the distribution $f(u)$.

The "differential" method of determination of the mobility of aerosol particles described herein has a great advantage over widely used "integral" method, in that it yields directly the percentage of particles with a degree of mobility within given narrow limits. On the other hand, the current which flows through the electrode in this method is so small that its measurement is fairly difficult.

The general (integral) method for determination of the mobility of particles consists of the following: an aerosol passes through the condenser with a constant speed; one of the condenser plates is connected with an ammeter, and the current I , flowing through this plate at various degrees of tension II in the condenser is measured, i.e., the (II, I) curve (characteristic) is determined. The distribution $\bar{q}(u)\ f(u)$ [181, 182] is determined on the basis of this characteristic. Theoretically speaking, condensers of any shape may be used, but the cylindrical shape, which has many advantages, usually is used.

In the laminar flow of a gas through this type condenser (Figure 26) the speed of flow, and consequently the speed of particles in a direction parallel to the axis of the condenser $V_x(\rho)$, is a function of the distance ρ from the axis. The field tension in the cylindrical condenser is equal to

$$E = \Pi / \rho \ln\left(\frac{R_2}{R_1}\right), \tag{27.9}$$

where R_2 is the radius of the external plate, and R_1 is the radius of the internal plate.

The radial speed of particles within the condenser, then, is

$$V_\rho = \Pi u / \rho \ln\left(\frac{R_2}{R_1}\right). \tag{27.10}$$

Taking the outer plate as positively charge, a positively charge particle moving radially toward the inner condenser plate, which is connected with the ammeter, traverses in the time dt the distance

$$d\rho = -\Pi u dt / \rho \ln\left(\frac{R_2}{R_1}\right).$$

At the same time, the particle traverses along the condenser axis the distance

$$dx = U(\rho)\, dt.$$

Eliminating dt from these equations, we have

$$dx = -U(\rho)\rho \ln\left(\frac{R_2}{R_1}\right) d\rho / \Pi u. \qquad (27.11)$$

If a particle entered the condenser at a distance ρ_1 from the axis, it reaches the inner plate at a point with the coordinate

$$x = -\frac{\ln(R_2/R_1)}{\Pi u}\int_{\rho_1}^{R_1} U(\rho)\rho\, d\rho = \frac{\ln(R_2/R_1)}{\Pi u}\int_{R_1}^{\rho_1} U(\rho)\rho d\rho. \qquad (27.12)$$

Assuming that all the particles have a uniform mobility u , if the length of the condenser is L , those particles will reach the inner condenser plate, for which $x \leqslant L$. For these particles $\rho_1 \leqslant \rho_0$, where ρ_0 is determined by the equation

$$\frac{\ln(R_2/R_1)}{\Pi u}\int_{R_1}^{\rho_0} U(\rho)\rho\, d\rho = L. \qquad (27.13)$$

Thus at a given tension II , those particles which entered at a distance from the axis of less than ρ_0 will precipitate in the condenser. If n positively charged particles with charge q are contained per cm^3 , the number of particles passing through and precipitating on the inner plate per unit time for a ring-shaped section limited by a circumference with radii R_1 and ρ_0 (Figure 26) per unit time, is

$$N = \int_{R_1}^{\rho_0} nU(\rho)\, 2\pi\rho\, d\rho \qquad (27.14)$$

The current flowing through the inner plate, is

$$I = qN = 2\pi qn \int_{R_1}^{\rho_0} U(\rho)\rho\, d\rho \qquad (27.15)$$

or, according to equation (27.13)

$$I = 2\pi qnuL\Pi / \ln\left(\frac{R_2}{R_1}\right). \qquad (27.16)$$

Thus, the current is proportional to the tension on the condenser plates.

When the tension reaches

$$\Pi_s = \frac{\ln(R_2/R_1)}{uL}\int_{R_1}^{R_2} U(\rho)\rho\, d\rho, \qquad (27.17)$$

i.e., ρ_0 reaches R_2 , all positive particles entering the condenser precipitate on it, and if the tension is increased even further the level of the current I_s remains constant (saturation current, Figure 27,a).

Whereas

$$\int_{R_1}^{R_2} 2\pi\rho\, U(\rho)\, d\rho = \Phi, \qquad (27.18)$$

where Φ is the volumetric speed of an aerosol in the condenser, then from equation (27.17) it follows that

$$u = \frac{\ln(R_2/R_1)\Phi}{2\pi L\Pi_s}. \qquad (27.19)$$

Thus the mobility of particles may be determined by the point of discontinuity of the characteristic. As in the precious examples all the formulas introduced above remain valid for any distribution of speed of flow in the condenser if the lines of flow are rectilinear and parallel. It is immediately clear that the saturation current is

$$I_s = qn\Phi \qquad (27.20)$$

from which the total of charges of positive particles contained in one cm^3 of aerosol may be determined. All the foregoing also applies to negatively charged particles if the condenser plate connected to the electrometer has a positive charge.

If an aerosol contains two groups of particles with mobility u_1 and u_2, charges q_1 and q_2, and concentrations n_1 and n_2, then the equations (27.16) and (27.17) apply to each group. In Figure 27 the characteristics for each group of particles are shown by broken lines, and the summary characteristic, determined by experiment, is shown by a solid line. The mobility of particles of both groups is indicated by the points of discontinuity in the summary characteristic.

The angle between the branches of the characteristic is more or less rounded, depending upon many causes. When only two groups are present this circumstance has no particular significance, but if there is a large number of groups a smooth curve may be obtained as well as the broken line. On the other hand, occasional errors in measurement of smooth curves, which must be obtained in the direct spectrum of the mobility of particles, easily, may lead to errors in conclusions as to the presence of discontinuities in the characteristics, i.e., concerning the existence of discrete groups of mobilities. It is beyond doubt that in many studies of highly dispersed aerosols of various origins in which discrete mobility groups were found the researchers actually were the victims of this type of self deception.

In the case of a smooth characteristic the distribution of mobilities is calculated in the following manner. First, it is assumed that all particles have a single elementary charge, and that at a tension Π_L all particles with mobility greater than u_L precipitate in the condenser. The relationship between Π_L and u_L is given by the equation

$$u_L = \frac{\ln (R_2/R_1)\Phi}{2\pi L \Pi_L}. \tag{27.21}$$

The current supported by these particles, according to (27.20), is

$$I_1 = \Phi \varepsilon n \int_{u_L}^{\infty} f(u)\,du. \tag{27.22}$$

Particles with mobility less than u_L, according to (27.16) produce a current equal to

$$I_2 = \frac{2\pi\varepsilon L\Pi_L n}{\ln (R_2/R_1)} \int_0^{u_L} u f(u)\,du. \tag{27.23}$$

Thus, the total current is equal to

$$I = I_1 + I_2 = \Phi\varepsilon n \int_{u_L}^{\infty} f(u)\,du + \frac{2\pi\varepsilon L\Pi_L n}{\ln (R_2/R_1)} \int_0^{u_L} u f(u)\,du. \tag{27.24}$$

Differentiating (27.24) for Π_L, we have

$$\frac{dI}{d\Pi_L} = -\Phi\varepsilon n f(u_L)\frac{du_L}{d\Pi_L} + \frac{2\pi\varepsilon L\Pi_L n u_L f(u_L)}{\ln (R_2/R_1)}\frac{du_L}{d\Pi_L} + \frac{2\pi\varepsilon L n}{\ln (R_2/R_1)} \int_0^{u_L} u f(u)\,du. \tag{27.25}$$

In accordance with (27.21) the first two members of the right side of this equation may be combined, giving

$$\frac{dI}{d\Pi_L} = \frac{2\pi\varepsilon L n}{\ln (R_2/R_1)} \int_0^{u_L} u f(u)\,du. \tag{27.26}$$

From which it follows that

$$I - \Pi_L \frac{dI}{d\Pi_L} = h = \Phi\varepsilon n \int_{u_L}^{\infty} f(u)\,du, \tag{27.27}$$

where h is the length of a segment intersecting the tangent to the characteristic at the point u_L along the ordinate axis (Figure 28). Thus, this segment is proportional to the number of particles, the mobility of which is between u_L and ∞, in which u_L is determined by the corresponding tension Π_L according to the formula (27.21). By this method the spectrum of mobilities of particles is determined or, in the case that the charges on the particles are not uniform, the function $\bar{q}(\mu)\, f(u)$ is determined. This method has the disadvantage that small errors in the determination of I lead to significant errors in the size of segments on the ordinate axis, and consequently, also in the value of the distribution function.

In research on the spectrum of mobility of particles in the atmosphere, because of the usual variations in the general content of aerosol particles in the atmosphere with the passage of time, the method described above may lead to incorrect results. In this case it is advantageous to use two condensers [181]: the air passes through the first condenser with tension Π_1, the current of which (not measured, I_1) is expressed by formula (27.24), and then passes through the second condenser, in which the tension is such that practically all particles are precipitated. The current level I_2 in the second condenser, which is measured by an electrometer, is equal to $\Phi \varepsilon n - I_1$. If the tension in the first condenser is equal to zero, then the current passing through the second condenser is $\Phi \varepsilon n = I_o$, and is proportional to the volume of charged particles contained in the air. The measurements are performed as follows: after determining the level of the current I_2, corresponding to a given tension Π, I_o is measured and the value of the relationship $(I_o - I_2)/I_o = I_1/I_o$, after which Π is increased somewhat, the corresponding current I_2 is determined and I_o is measured again, etc. This is the "normalization" of I_1 to a constant volume of charged particle content of the air. Although the proposition that the distribution of the mobility of particles in the atmosphere does not depend upon their concentration lies at the basis of this method, it is accurate only in special cases, and the two condenser method gives much more desirable results than the single condenser method. In both methods it is desirable to eliminate gaseous ions from the air before hand, so that the air may pass through the secondary condenser at a low tension, sufficient for the removal of all gaseous ions with their high mobility of the order one. At the same time the number of particles charged by this condenser is not large.

It is emphasized that all the foregoing is true only for the laminar flow of a gas through the condenser. The phenomena are considerably different for turbulent flow, and the current which flows through the condenser at a given tension is markedly less than given by formula (27.24) [181] (see equations (46.7) through (46.10)).

As already mentioned, electrometric determination of the mobility of particles is the basic method of investigation of highly dispersed aerosols, and particularly of the highly dispersed portion of atmospheric aerosols, because in this case the particles are too small for ultramicroscopic study. On the other hand, the precision of the electrometric method decreases with an increase in the size of particles. Because of this, in conformance with the equations introduced above, either the condenser current must be increased, or the speed of flow of the aerosol must be decreased. However, at very high field strength electrical dissipation, local discharges and other complications arise, and at slow speeds of flow convection currents in the condenser create serious difficulties. The upper limit of applicability of the electrometric method may be taken at $r = 10^{-5}$ cm, which is somewhat greater than the lower limit of applicability of ultramicroscopy with aerosols. Thus these two methods supplement each other and enable investigation of aerosols of practically any degree of dispersion.

It is noted that in electrometric investigation of highly dispersed, non-charged aerosols they first must be charged with the aid of radioactive preparations [183], etc. The following observations [184] are important in the practical application of the latter: if the air intake tube is made of rubber a considerable loss in the number of charged condensation nuclei is noted in the tube because non-charged nuclei are found in very small numbers. This effect is increased if the tube is bent, and the effect disappears if the tube is coated with a conducting liquid. Apparently opposite charges are formed on the concave and convex walls of the tube, and the interior or the tube acts like an electric field.

Cochet [185] studied the precipitation of non-charged droplets in air at rest in a charged, horizontal cylinder of infinite length (for example, in electric conduits), i.e., with simultaneous action of gravity and the force of electrical induction. Ignoring the inertia of particles and assuming that the air resistance is proportional to the first degree of the speed of the particles, Cochet found the following expression for the coefficient of precipitation of droplets (see equations (31.15) and (31.16))

$$\vartheta = \left(\frac{9\lambda E_0^2}{8\gamma g R}\right)^{1/3}, \qquad (27.28)$$

where $\lambda = (\varepsilon_k - 1)/(\varepsilon_k + 2)$; ε_k is the dielectric permeability of the particles; R is the radius of the cylinder; E_o is the field tension at the cylinder surface, combining the charge per unit length of the cylinder with the formula $E_o = 2q/R$.

Formula (27.28) is applicable only when $E_0 > \sqrt{4\pi\gamma g R/3\lambda}$; in the opposite case ϑ may be determined only by mathematical methods. The quantity ϑ is not dependent upon the size of the particles because both the gravitational effect and the force of induction are proportional to the volume of particles.

Experimental determination of ϑ by means of photographing the trajectory of particles which precipitate on a charged cylinder have shown that at $R = 0.6$ cm, $E_o = 47$, or 88 el. st. un., and $r < 25\mu$ the correlation between theory and experience is very close, i.e., the inertia of particles actually may be ignored. At $r = 40\mu$ and $E_o =$ $= 88$ el. st. un. the experimental value of ϑ is 14% greater than that computed according to formula (27.28) because of the inertial effect.

The precipitation of particles in an electric field is widely applied in industry for cleansing gases of dusts, fog, etc. In the electric filters used for this purpose the negatively charged inner plate of the condenser has a very small radius (on the order of 1 mm), which creates a coronal discharge and consequently a high concentration of negative gaseous ions and electrons, which transmit the charge to the particles. Thus, two processes take place simultaneously in an electric filter: the charging and precipitation of particles. (The field of an electric filter also caused the "parallel" coagulation of particles (see paragraph 52).) Furthermore, because of the strong ionic wind accompanying the coronal discharge, the flow of a gas in an electric filter has an irregular character, and the precipitation of particles must be computed as described in paragraph 44.

Recently, electric filters have appeared in which the two processes described above are separated spatially; in the first part of the filter the particles usually are only charged, and in the second only precipitation takes place [186]. In this design it was found possible to use positive coronal electrodes, which result in the formation of air oxidation products much less harmful to the organism (nitrogen oxides, etc) than those produced by the negative electrode. Because of this the latter method of precipitation of dust has great importance in air conditioning.

[Pages 146-152]

33. Slit Apparatuses

In aerosol research, especially in hygiene work, slit apparatuses are extensively used for the precipitation of particles [215]. In these apparatuses the aerosol passes through a flat slit or through a round opening with a speed on the order of 100 to 200 $m \cdot sec^{-1}$, behind which is a glass plate at right angles to the direction of flow. The aerosol stream leaving the slit impinges on the glass plate and spills over it. Due to inertia the particles leave the line of flow and precipitate on the plate. The precipitate, in the form of a straight line or round spot, is studied under the microscope. To facilitate adhesion of solid particles to the plate, the latter is coated with glycerine, a mixture of rosin and castor oil, polyisobutene, etc, or else the apparatus chamber is rendered highly humid through the use of wet paper. Because of adiabatic expansion of the gas leaving the slit moisture precipitates on the particles, which enables their adhesion to the plate. If the glass plate is replaced with a slowly rotating Petri dish covered with a layer nutrient agar, a convenient instrument is obtained for determining the number of aerosol particles per cubic centimeter of air which contain any type of microorganism [216].

In the impinger precipitation occurs with water at the bottom of the bottleneck. The aerosol is passed through a tube with an elongated tip, which is several millimeters from the bottom of the bottleneck. When the aerosol is passed through the apparatus the gas displaces water, the bottom of the bottleneck is moistened, and the particles which impinge on the wetted glass surface mix with the water. The particles which do not precipitate upon impinging on the bottom of the bottleneck apparently are caught by passage of gas bubbles through a water layer. In this case the particles are studied in the suspension which thus is obtained.

The various types of slit walls include flat (Figure 42,a) such as the impactor of May [217], or round, such as the apparatus of Owens [218] (Figure 42,b), or of Bausch and Lomb (Figure 42,c). In the case of a round aperture the walls are conical such as in the conimeter of Cochet (Figure 42,d). As will be seen below, the profile of the slit in several instances has significant influence on the functioning of the apparatus. The ratio of the distance between the walls and the plate to the width of the slit ranges from 1.0 to 2.0 in various designs and, according to many authors, has no noticeable effect on the efficiency of precipitation.

Davis [219] has made the only attempt at strict calculation of the movement of particles in slit apparatuses. In this work he assumes that the length of the slit is infinitely great compared to its width 2h and that the flow everywhere is potential, but inside the slit and on the plate at a sufficiently great distance from the slit the flow is rectilinear and parallel to the walls, with speed constant for all sections of flow (i.e., the boundary layer is lacking). On the basis of the latter the field current was determined, and in particular, the following relationship was deduced between the equations $\beta = d/2h$ and $\alpha = h/\Delta$, where d is the distance from the slit to the plate, and Δ is the thickness of the stream at an infinitely great distance from the slit (Figure 43):

$$2\beta = \frac{(\alpha^2+1)\ln(\alpha+1)}{\pi\alpha^2(\alpha-1)} + \frac{1}{\alpha}. \qquad (33.1)$$

Furthermore, the trajectory of particles and the effectiveness of precipitation in the function $Stk = l_1/2h$ for $\alpha = 1.0$, 1.6, 2.5, and 5.0, were calculated by the approximation method described in paragraph 31. The curves for $\alpha = 1.0$ ($\beta = \infty$) and $\alpha = 1.6$ ($\beta = 0.8$) are shown in Figure 44, including data obtained in other theoretical and experimental investigations described below. The values of $\sqrt{Stk} = \sqrt{l_1/2h}$, were plotted on the abscissa, which are proportional to the radii of the particles. It may be seen from the curves of Davis that there is a critical value of the Stock number (Stk_{kr}), below which precipitation in the slit apparatus is equal to zero. For $\alpha = 1$, $Stk_{kr} = 0.16$; for $\alpha = 1.6$, $Stk_{kr} = 0.12$. These values of Stk_{kr} are very close to those found experimentally (see below). However, in computing the movement of particles along the central axis of the line of flow (passing through the axis of the slit), Davis obtained a value of $Stk_{kr} = 0.32$, which coincides with the analytical results of L. Levin [220] (see (34.13) through (34.16)) for a value of $1/\pi$, but differs greatly with the experimental value. Similar discrepancies are obtained for critical flow at different values of α; also. An explanation of the cause of this discrepancy is rather difficult.

Attempting calculation of the effectiveness of precipitation of particles in a slit apparatus by the following primitive method, we have: assuming that $\Delta = h$, that the speed of flow U_0 is constant for all sections of the stream throughout its length, and that the line of flow in truncated sections is that of other circles with centers at the points AA (see Figure 43), and assuming that the tangential speed of particles and that of the medium coincide, then the speed of a particle normal to the line of flow is $U_\rho = U_0^2 \tau / \rho$, where ρ is the radius of the curved line of flow, and in time dt mixes in relation to the medium at a distance $d\rho = V_\rho \, dt$ in the direction ρ or at distance $dz = V_\rho \sin\varphi \, dt$ in the direction of the plate. Assuming that the degree of complete mixture of the particles with respect to the medium is small during the time of movement along the curve, i.e., is constant, we obtain the following equation for the component of this mixture in the direction toward the plate

$$\Delta z = \int \frac{U_0^2 \tau \sin\varphi}{\rho} dt = U_0 \tau \int_0^{\pi/2} \sin\varphi d\varphi = U_0 \tau = l_1. \qquad (33.2)$$

(In the work of Wilcox [221] the expression $\frac{\pi}{2} l_1$ for Δz is incorrect.)

Thus, the following simple expression is obtained for the effectiveness of precipitation

$$\vartheta = l_1 / h = 2\,\mathrm{Stk}. \qquad (33.3)$$

Although at large values of ϑ formula (33.3) more or less coincides with the experimental results it has one major disadvantage: according to this formula the Stock critical number does not exist, which contradicts the more strict derivation mentioned above, and contradicts factual data (see below). The cause of this error is not difficult to see: at small values of $\ni$ and Stk precipitation occurs only from the lines of flow closest to the axis of the slit and passing close to the point of stagnation, where the speed of flow is greatly decreased and the inertial mixing of particles from the line of flow is insignificant. In a simplified field current the speed of flow along these lines and the inertial mixing is great. For lines of flow far from the axis of the slit a simplified field current approaches the true value and our primitive calculation is more or less correct.

Turning to the experimental data on the effectiveness of slit apparatuses, it is necessary to bear in mind that for comparison with theory only the results of experiments on the precipitation of fogs may be used because experiments with solid particles result in many complications which will be discussed later. Apparently the most precise experiments were those performed by Ranz and Wong [222] on fairly monodispersion glycerine fogs with the radius of particles varying in different experiments from 0.17 to 0.69μ. The width of the flat slits was 0.2 to 0.7 mm, of round apertures 1.0 to 1.9 mm, and the speed of flow through the slit 10 to 180 m·sec^{-1}. The value of the function d/2h ranged within the limits 1 to 3, and had no marked influence on the results. The results of experiments with flat slits are shown in Figure 44 (experimental points and the curve which they describe). The value $\sim$0.11 is obtained for Stk_{kr}. A fairly accurate curve is obtained for a round aperture if the abscissa of all points of the curve for a flat slit are decreased 1.5-fold, which results in greater effectiveness than a flat slit with the same width.

The experiments of May [217] were conducted with a cascade impactor, in which the aerosol passes successively through four slit apparatuses with increasingly narrow slits, greater speeds of flow, and increasingly fine fraction. Figure 44 contains the results obtained at the third cascade, with one mm slit width, and a speed of flow of 50 m·sec^{-1}. It may be seen from the curve of effectiveness of precipitation included in May's article for each cascade in the function of the size of droplets that breaking off, for example, at Stk = 0.06 (see Figure 44), Stk_{kr} must be close to this value. However, May himself makes no reference to this fact, and he arbitrarily passes his curve (Э, Stk) through the first coordinate.

In the majority of work on slit apparatuses only a minimal size size of particles completely precipitated in the apparatus at a given speed of flow was determined (i.e., the minimal value of Stk for Э = 1). The results of expensive experiments of Wilcox [221] with apparatuses of various design and width of slit ranging from 0.25 to 6 mm at speeds of flow from 2.2 to 174 m·sec^{-1} are widely scattered, as may be seen in Figure 44, which renders the discovery of any dependent relationship between Stk values corresponding to complete precipitation, and the functioning of the apparatus quite difficult.

Furthermore, from Figure 44 it is seen that the coincidence between theoretical and experimental curves of the effectiveness of slit apparatuses, and between the data of various researchers is quite poor. At the present time it is difficult to establish the basic cause of these discrepancies. The characteristic difference between theoretical and experimental curves is that the latter bend markedly to the right at large value of Э, which means that the particles which are moving in the lines of flow farthest removed from the slit axis precipitate extremely poorly and that the theoretical calculations introduced above are not applicable to these particles. In all probability this is explained by turbulence which inevitably arises at the free surface of the stream and at its exit from the slit, and which causes scouring of this surface.

In work with solid aerosol particles the effectiveness of slit apparatuses may be considerably decreased due to blowing away of precipitated particles from the glass plate. Under the conditions which obtain in working with these apparatuses large particles are blown away easier than small particles, which occurs not only at the lower, but also at the upper limit of size of particles collected by the apparatus. Thus, according to the observations of Ts. Pik and L. Shurchilov [223] the maximum radius of particles of carborundum dust contained in the apparatus of Owens is equal to 1.8μ without wetting, and 3.7μ with wetting. According to the experiments of Jordan [224], in the apparatus indicated in Figure 42,a, quartz particles with $r = 2\mu$ and 1μ begin to blow away markedly from a dry glass plate at an air speed in the slit equal to 60 and 150 $m \cdot sec^{-1}$. The phenomenon of blowing off of particles in apparatuses with round apertures leads to a curious effect: a diffuse ring of 5 to 10 mm diameter, consisting of particles blown by the stream from the center of the precipitate often may be observed around a thick central precipitate with diameter of the order of 1 mm which is formed directly opposite the aperture when the speed of flow is sufficiently decreased [225].

The blowing away of particles in the Owens apparatus (see Figure 42,b) is especially great when the speed of the air stream is very high (300 $m \cdot sec^{-1}$); blowing away of particles also is marked in particles with $r = 0.5\mu$. The blowing away of particles is considerably less for the Bausch and Lomb apparatus (see Figure 42,c) at an air speed of approximately 150 $m \cdot sec^{-1}$ [218].

The best method for control of the blowing away of particles, apparently, is that of smearing the plate with a sufficiently viscous and adhesive substance. By this method the blowing away of particles of coal dust with radius up to 4μ was practically eliminated at a speed of flow in the slit of 200 to 300 $m \cdot sec^{-1}$. Without the coating particles with $r = 4\mu$ were completely blown away under these conditions. Moistening of the air gives considerably poorer results than coating of the plate, not only for coal, but also for hydrophillic quartz dust [218].

Another secondary phenomenon natural to slit apparatuses is the breaking up of hard and brittle substances such as quartz and orthoclase into particles of 5 to 10μ diameter [226]. It is interesting that soft, plastic particles, such as plant spores, are excluded from this entirely.

In working with dusts containing aggregated particles the breaking up of aggregates by the air current may be observed before the particles impinge on the plate. The fact that aggregates actually break up before coming in contact with the plate was demonstrated by Davies in the following manner [218]. Coal dust was passed through a 0.4 mm slit without a plate, and the calculated concentration of dust (by means of a thermoprecipitator), and the distribution of sizes of particles before and after the slit were determined. The dust was not changed at a speed of 50 $m \cdot sec^{-1}$, but at 170 $m \cdot sec^{-1}$ the number of small particles was considerably increased and the large particles were eliminated. When the dust was contained in an apparatus with a well smeared plate the number of particles with 1μ radius was unchanged, the number of particles of more than 1μ was decreased somewhat, and the number of particles of less than 1μ was increased several fold. The effect was somewhat decreased in moist air because the moisture makes the aggregates more stable. The effect depends greatly upon the profile of the slit: the effect is much greater in slits with a rounded profile than in conical slits. The cause of the breaking up of aggregates, according to the opinion of Davies, is the great relative speed of motion of particles and medium in entering the slit because of the inertia of the particles. Thus, in slits described in Figure 42,b the relative speed of particles with radius 10μ reaches 150 m·sec^{-1}. It is clear that with a conical profile, i.e., with a uniform increase in the flow, the relative speed must be less than in the case of a rounded profile. An important factor in the breaking up of particle aggregates is the nonuniformity of flow, and the presence of very great gradients of speed of flow at the exit from the slit (see (58.3)).

According to Davies, in slits with rounded profile the particles also precipitate on the walls of the slit itself. This phenomenon is enhanced by an increase in the linear speed of air in the slit, and because of this it is especially pronounced in the apparatus of Owens.

Several industrial apparatuses for the removal of dust from gases [227] are based on this principle of the impinging of an aerosol stream on a surface: the gas is passed through a large number of apertures of 2 to 5 mm diameter in metal plates and impinges on another plate placed parallel to, and at a distance of 2 to 3 mm from the latter. Unfortunately, there are no data on the effectiveness of these apparatuses with respect to particles of determined size, and comparison with theory is impossible.

[Pages 175-189]

CHAPTER V. BROWNIAN MOVEMENT AND DIFFUSION IN AEROSOLS

38. Diffusion Precipitation of Aerosols in an Immobile Medium

The precipitation of aerosols, caused by Brownian movement, on "walls," i.e., on surfaces in contact with aerosols of solid or liquid bodies, has great practical significance. According to the material described under paragraph 56 it may be assumed that the particles which touch the wall stick to it, and in the absence of strong vibration of air currents, do not return to a suspended state. The speed of precipitation of particles on walls may be determined here by computing the probability of the attainment of known limits composed of the determined shape of particles. This probability is computed with the aid of the function W^*, which is explained in paragraph 37. However, in most cases it is more convenient to treat this situation as the diffusion of a large number of uniform particles to an "absorbing" wall. Because the particles which touch the wall immediately leave the gaseous space, the concentration of particles at the wall may be taken as equal to zero. In all cases discussed below, except those which are indicated otherwise, it is assumed that the particles do not have a regular motion.

I. Taking the most simple case of a flat wall in contact with an unlimited volume of aerosol which has throughout a uniform starting concentration n_0 (the case of a vertical wall). The x-axis is placed perpendicular to the wall, and the starting coordinate is placed on the latter. The concentration of particles $n(x, t)$, it is known, must satisfy the equation

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial x^2} \qquad (38.1)$$

and also the starting condition

$$n(x, 0) = n_0 \text{ for } x > 0 \qquad (38.2)$$

and the limiting condition

$$n(0, t) = 0 \text{ for } t > 0. \qquad (38.3)$$

This problem is solved through the expression [274]

$$n(x, t) = \frac{2n_0}{\sqrt{4\pi Dt}}\int_0^x e^{-\xi^2/4Dt}\,d\xi = \frac{2n_0}{\sqrt{\pi}}\int_0^{x/\sqrt{4Dt}} e^{-\xi^2}\,d\xi = n_0 Erf\left(\frac{x}{\sqrt{4Dt}}\right), \qquad (38.4)$$

where Erf is the integral of probability (function of Cramp). The distribution of concentration of particles, expressed by formula (38.4), is shown in Figure 50.

From formula (38.4) it follows that the concentration gradient at the wall is $\partial n / \partial x_{x=0} = n_0 / \sqrt{\pi Dt}$, and the number of particles precipitating per cm^2 during the time $(t, t+dt)$ is equal to

$$I\,dt = D\frac{\partial n}{\partial x_{x=0}}\,dt = n_0\sqrt{\frac{D}{\pi t}}\,dt. \qquad (38.5)$$

From this, the number of particles which precipitate during time t is given by the expression

$$N(t)=\int\limits_0^t I\,dt=2n_0\sqrt{\frac{Dt}{\pi}}. \qquad (38.6)$$

In aerosol work the following simplified method is often used for the determination of the number of particles precipitating on a wall due to diffusion. Assuming that at any moment half of the particles are moving toward, and half of the particles are moving away from the wall, then during time t half of all particles contained in a layer next to the wall with thickness $\overline{\Delta x}$ will precipitate on the wall, where $\overline{\Delta x}$ is the average absolute displacement of particles during time t , expressed by formula (37.35). It is immediately apparent that this results in a number twice as small as that obtained with the correct formula (38.6). Thus, when using this method it must be assumed that all the particles contained in the layer mentioned, precipitate on the wall. The cause of this discrepancy becomes apparent by consideration of the number of particles which leave the aerosol during time t through a wall which is permeable for the particles, for example, through openings in a container wall.

In this case the condition $n(x, 0) = 0$ where $x < 0$, must be added to the precious starting condition $n(x, 0) = n_o$, where $x > 0$, noting that at the starting moment there are no particles at the wall surface. The limiting condition (38.3) is eliminated because the particles are not separated from the gaseous space. This problem is solved with the aid of the formula [272]

$$n(x,t)=\frac{n_0}{2}\left[1+Erf\left(\frac{x}{\sqrt{4Dt}}\right)\right], \qquad (38.7)$$

i.e., at a given moment the concentration of particles at the layer next to the wall remains constant, equal to $n_o/2$ (Figure 51), and the concentration gradient and the speed of diffusion at the wall is twice as small as with the absorbing wall. From the point of view of the

the Brownian movement of individual particles this difference is explained by the fact that the particles which pass through the permeable wall have a peculiar probability of return into the gaseous space which they left, while return from an absorbing wall is impossible; because of this, the removal of particles from an aerosol in the latter case is twice as fast.

The third passibility of a "reflecting" wall will not be considered here because this does not occur with aerosols., at least not at those degrees of dispersion at which Brownian movement and diffusion in general are of any significance.

II. Considering the probability of particles located at distance h from a flat wall and exhibiting regular motion in the direction of the wall at the speed V_0 reaching the wall during an interval of time (t, t+dt) , this possibility, which was designated $W^{**}\,dt$ under paragraph 37, is found with the aid of equation (37.22), using the corresponding starting and limiting conditions. However, it is simpler to solve an equivalent problem: at a moment $t = 0$, given a large number N of uniform particles at a distance h from an absorbing wall and which have a regular speed V_0 in the direction of the wall, what part of these particles will reach the wall during the time (t, t+dt) ? In this case equation (37.17) must be solved, with the first limiting condition (38.3) and the starting condition

$$n(x, 0) = 0 \text{ for } x \neq h; \quad \int_{h-\Delta}^{h+\Delta} n(x, 0)\,dx = N, \tag{38.8}$$

in which the integral is taken for an infinitely small interval 2Δ, containing the point h . This problem is solved through the use of the expression [275]

$$n(x, t) = \frac{N}{\sqrt{4\pi Dt}} \exp\left[-\frac{2V_0(x-h) + V_0^2 t}{4D}\right]\left\{\exp\left[-\frac{(x-h)^2}{4Dt}\right] - \exp\left[-\frac{(x+h)^2}{4Dt}\right]\right\}. \tag{38.9}$$

It is easily verified that this actually satisfies both equation (37.17) and the conditions mentioned. The probability of a particle's touching the wall during the time interval (t, t+dt) is found by

$$W^{\bullet\bullet}(x,t)\,dt = \frac{D}{N}\frac{\partial n}{\partial x_{x=0}}\,dt = \frac{h}{\sqrt{4\pi Dt^3}}\exp\left[-\frac{(h-V_0t)^2}{4Dt}\right]dt. \qquad (38.10)$$

From (38.10) it follows that the average time for particles reaching the wall is [see NOTE]:

$$\bar{t} = \int_0^\infty tW^{\bullet\bullet}\,dt = \frac{h}{\sqrt{4\pi D}}\int_0^\infty t^{-1/2}e^{-(h-V_0t)^2/4Dt}\,dt = h/V_0, \qquad (38.11)$$

i.e., the same as in the absence of Brownian movement. The average of the reciprocal of this time is equal to [see NOTE]:

$$\overline{\left(\frac{1}{t}\right)} = \int_0^\infty \frac{1}{t}W^{\bullet\bullet}\,dt = \frac{h}{\sqrt{4\pi D}}\int_0^\infty t^{-3/2}e^{-(h-V_0t)^2/4Dt}\,dt = \frac{2D}{h^2} + \frac{V_0}{h}. \qquad (38.12)$$

[NOTE] By substituting t = ht'/V these integrals become

$$\int_0^\infty t^{-1/2}e^{-\frac{t^2+1}{qt}}\,dt \text{ and } \int_0^\infty t^{-3/2}e^{-\frac{t^2+1}{qt}}\,dt,$$

the values of which may be found in handbooks.

From (38.11) and (38.12) the following important formula is obtained

$$D = \frac{h^2}{2}\left[\overline{\left(\frac{1}{t}\right)} - \frac{1}{\bar{t}}\right]. \qquad (38.13)$$

In the experiments described under paragraph 36 the time for obtaining an image of particles which intersect a horizontal ocular line with another line at a moment t = 0 , is measured. It is apparent that formula (38.12) is applicable to these experiments.

At V_o a simpler formula

$$W^{\bullet\bullet}\,dt = \frac{h}{\sqrt{4\pi Dt^3}}\,e^{-h^2/4Dt}\,dt, \qquad (38.14)$$

in addition to (38.10), in which case $\overline{\left(\frac{1}{t}\right)}$ is

$$\overline{\left(\frac{1}{t}\right)} = \frac{2D}{h^2}. \qquad (38.15)$$

This formula is used in the investigation of horizontal Brownian movement (see (36.1) and (36.2)).

Taking formula (38.10) in unmeasured form, with the average time for reaching the wall $\bar{t} = h/V_o$ per unit time, i.e., assuming $t' = t/\bar{t}$ and indicating the unmeasured value $\sqrt{V_0h/4D}$ by μ , (38.10) may be written as

$$W^{\bullet\bullet}\,dt' = \frac{\mu}{\sqrt{\pi t'^3}}\,e^{-\mu^2(1-t')^2/t'}\,dt'. \qquad (38.16)$$

Figure 52 contains curves which have been calculated according to this equation for $\mu = 1$, 5, and 30, which at $h = 0.4$ cm correspond to radii of 0.1, 0.3, and 1μ. The graph clearly shows the great change in the relative roles of precipitation and Brownian movement at the indicated interval of dispersion.

III. In the precipitation of an aerosol of limitless extent and uniform concentration n_0 on a horizontal wall, i.e., taking into account the precipitation of particles under the influence of gravity, expression (38.10) may be used, integrating for h from 0 to ∞. Here, we obtain the following formula for the number of particles precipitating per cm^2 of horizontal wall space during the time $(t, t+dt)$

$$I(t)\,dt = n_0\left\{\sqrt{\frac{D}{\pi t}}\,e^{-V_0^2 t/4D} + \frac{V_0}{2}\left(1 + Erf\sqrt{\frac{V_0^2}{4D}}\right)\right\}dt. \qquad (38.17)$$

At $t \gg \frac{4D}{V_0^2}$ this formula becomes $I(t) = n_0 V_0$, i.e., Brownian movement now has no influence on the speed of precipitation, and the latter is determined only as the speed of settling. At $t \ll \frac{4D}{V_0^2}$ (38.17) becomes $I(t) = n_0\left(\sqrt{\frac{D}{\pi t}} + \frac{V_0}{2}\right)$.

Thus in this case precipitation is composed of diffusion precipitation in the absence of settling (see (38.5)) and half of the sedimentation precipitation in the absence of diffusion. From this example it is clearly apparent that in the calculation of the speed of precipitation of aerosols on walls under the simultaneous influence of Brownian movement and external, or inertial forces, the simple addition of these effects may lead to serious errors. Unfortunately, because of great mathematical difficulties such additions sometimes must be limited.

It must be noted that the above formulas are correct only for a stationary medium, which is possible practically only for small volumes of gas. Even in smoke chambers, to say nothing of the open atmosphere, convection currents very greatly alter the entire picture.

IV. Ultramicroscopic computation of particles presents the following task, which is of interest to industry: what is the probability of a particle which is located anywhere between two parallel vertical walls precipitating on the walls because of diffusion during the time t . If the beginning coordinate is placed at one of the walls, this problem may be solved with the aid of equation (38.16) under the starting conditions

$$n(x,0)=n_0 \text{ at } 0<x<h;\ n(x,0)=0 \text{ at } x<0,\ x>h, \tag{38.18}$$

where h is the distance between walls, and under the limiting conditions

$$n(0,t)=0;\quad n(h,t)=0 \text{ at } t>0. \tag{38.19}$$

This problem is solved by the endless series [276]

$$n(x,t)=\frac{4n_0}{\pi}\sum_{\nu=1}^{\infty}\frac{1}{2\nu-1}\sin\left[(2\nu-1)\frac{\pi x}{h}\right]\exp\left[-\frac{(2\nu-1)^2\pi^2Dt}{h^2}\right]. \tag{38.20}$$

Thus, during time (t, t+dt) the number of particles precipitating is

$$Idt=2D\frac{\partial n}{\partial x}_{x=0}dt=\frac{8Dn_0}{h}\sum_{\nu=1}^{\infty}\exp\left[-\frac{(2\nu-1)^2\pi^2Dt}{h^2}\right]dt \tag{38.21}$$

and during time t ,

$$N(t)=\int_0^t Idt=\frac{8n_0h}{\pi^2}\sum_{\nu=1}^{\infty}\frac{1}{(2\nu-1)^2}\left\{1-\exp\left[-\frac{(2\nu-1)^2\pi^2Dt}{h^2}\right]\right\}=$$

$$=n_0h\left\{1-\frac{8}{\pi^2}\sum_{\nu=1}^{\infty}\frac{1}{(2\nu-1)^2}\exp\left[-\frac{(2\nu-1)^2\pi^2Dt}{h^2}\right]\right\} \tag{38.22}$$

particles precipitate. (Here the formula $\sum_{\nu=1}^{\infty}\frac{1}{(2\nu-1)^2}=\frac{\pi^2}{8}$ is used.) Analyzing this expression for the original number of particles contained between the walls per cm^2 of their surface, i.e., for n_0h , the following probability is obtained

$$W^*(t)=1-\frac{8}{\pi^2}\sum_{\nu=1}^{\infty}\frac{1}{(2\nu-1)^2}\exp\left[-\frac{(2\nu-1)^2\pi^2Dt}{h^2}\right]. \tag{38.23}$$

The average time that a particle remains between the walls is obtained according to (38.21) by the expression

$$\bar{t}=\frac{1}{n_0h}\int_0^{\infty}tIdt=\frac{8D}{h^2}\int_0^{\infty}t\sum_{\nu=1}^{\infty}\exp\left[-\frac{(2\nu-1)^2\pi^2Dt}{h^2}\right]dt=\frac{h^2}{12D}. \tag{38.24}$$

(Here the formula $\sum_{\nu=1}^{\infty}\frac{1}{(2\nu-1)^4}=\frac{\pi^4}{96}$ is used.)

In ultramicroscopic test tubes with a mechanically limited, calculated volume, the number of particles contained in a layer 0.1 mm thick between two parallel glass plates is calculated. (It must be noted that at present this method has become somewhat obsolete. The "continuous" method, developed by B. Deryagin and G. Vlasenko [277] is a more perfected method.) The opinion has been expressed that in this method of calculation some particles may settle on the walls because of Brownian movement before the calculation period. It is proposed that the time between the closing of the aerosol chamber and the calculation of particles be on the order of 0.1 sec. Then, according to formula (38.23) the values obtained at $W^* = 0.11$; for $r = 0.3\mu$ $W^* = 0.05$, i.e., the error mentioned will be comparatively small, for particles with 0.1μ radius, which corresponds approximately to the lower limit of applicability of ultramicroscopy to aerosols. The corresponding values of $\bar{t}$ are 3.8 and 16.7 sec. It is apparent, however, that investigations requiring longer observations, such as the measurement of Brownian movement, are impossible with such narrow test tubes. Because of this, cardioid, etc, condensers may not be used in aerosol work.

L. Radushkevich [278] investigated the speed of precipitation of stearic acid particles on the walls of a flat-parallel test tube with an aperture 0.1 mm wide, by ultramicroscopic calculation of the concentration of particles in the test tube. The average size of particles was determined by the calculated-gravimetric method. In comparatively iso-dispersed, prepared smokes, more or less satisfactory agreement was obtained between experimental results with the aid of formula (38.23), but in coagulated, more poly-dispersed smokes the speed of precipitation was greater than the theoretically computed value. L. Radushkevich correctly explained this discrepancy by the fact that the average cubic radius $\sqrt[3]{\overline{r^3}}$ of particles, determined by the calculated-gravimetric method, is considerably greater in the case of polydispersion smokes than the average value r', determined by the equation $\left(1+A\frac{l}{r'}\right)/r' = \overline{\left(1+A\frac{l}{r}\right)/r}$, which determines the speed of diffusion of polydispersion aerosols.

The change in the number of particles contained in a layer of particles with the progression of time, i.e., the function $1 - W^*$, is shown in Figure 53. In this graph time is expressed in unmeasured units Dt/h^2.

The case of an aerosol layer enclosed between parallel horizontal walls [279] also has attracted much attention. If the unmeasured variables $x' = x/h$, $t' = t/t_o$ (where $t_o = h/V_s$) are introduced and $\sqrt{V_s h/4D}$ is indicated by μ as above, then the following expression is obtained for the distribution of the concentration of an aerosol

$$n = n_0 \sum_{\nu=1}^{\infty} \frac{2}{\pi\nu} \frac{1-(-1)^{\nu} e^{-2\mu^2}}{1+\frac{4\mu^4}{\pi^2\nu^2}} \sin \pi\nu x' \exp\left\{\frac{2x' - \left(1+\frac{\pi^2\nu^2}{4\mu^2}\right)t'}{\mu^2}\right\}. \qquad (38.25)$$

Figure 54 shows the distribution at values $\mu = 0$ (no settling), 1.2, and 5. It may be seen from the graphs that at small values of μ i.e., in highly dispersed aerosols, a distinct upper limit cannot be obtained, even under ideal conditions of absence of convection currents and a completely isodispersed system.

V. If an aerosol with starting concentration n_o is contained in a spherical container with radius R, then after time t the average concentration will have dropped by the value [276]

$$\bar{n} = n_0 \frac{6}{\pi^2} \sum_{\nu=1}^{\infty} \frac{1}{\nu^2} e^{-D\pi^2\nu^2 t/R^2}. \qquad (38.26)$$

The number of particles precipitating on the walls during this time, then, is equal to $\frac{4}{3}\pi R^3 (n_0 - \bar{n})$. Figure 55 shows the relationship between $\bar{n}/n_o$ and an unmeasured value of Dt/R^2.

VI. In a similar aerosol problem, in which an aerosol is contained in a cylindrical vessel of infinite length and radius R, the

the following expression for the average concentration of particles at a moment t is obtained [276]:

$$\bar{n} = 4n_0 \sum_{\nu=1}^{\infty} \frac{1}{\beta_\nu^2} e^{-D\beta_\nu^2 t/R^2}. \qquad (38.27)$$

Here $\beta_1^2, \beta_2^2 \ldots$ are the quadratic zeros of the function of Bessel of the first type of zero order $I_0(x)$, having the following values: $\beta_1^2 = 5{,}784$; $\beta_2^2 = 30.47$; $\beta_3^2 = 74.89$; $\beta_4^2 = 132.8$; $\beta_5^2 = 222.9$.

Figure 56 shows the relationship between $\bar{n}/n_0$ and Dt/R^2. If $\bar{n}$ is calculated by the simplified method described above, assuming that during time t all particles which were located at a distance less than $2\sqrt{\frac{Dt}{\pi}}$, from the wall at the moment $t = 0$ will have precipitated on the wall, the dotted-line curve is obtained, which is suitable approximately up to the value $Dt/R^2 = 0.1$.

VII. The problem of diffusion to the surface of a sphere located in an infinitely large aerosol volume with starting concentration n_0 is very important in the physics of aerosols. The starting condition of this problem is $n(\rho, 0) = n_0$ at $\rho > R$; and the limiting condition is $n(R, t) = 0$ at $t > 0$. Because of the independence of the size of the coefficient of diffusion from the direction, in polar coordinates equation (37.18) becomes

$$\frac{\partial (n\rho)}{\partial t} = D\frac{\partial^2 (n\rho)}{\partial \rho^2}. \qquad (38.28)$$

Thus, function $\rho n(\rho, t)$ satisfies equation (38.28) and the conditions

$$\rho n(\rho, 0) = \rho n_0 \text{ at } \rho > R, \qquad (38.29)$$

$$n(R, t) = 0 \text{ at } t > 0. \qquad (38.30)$$

Solution of these problems gives the expression [280]

$$n = n_0\left[1 - \frac{R}{\rho} + \frac{2R}{\rho\sqrt{\pi}} \int_0^{\frac{\rho-R}{2\sqrt{Dt}}} e^{-\xi^2} d\xi\right]. \qquad (38.31)$$

From this it follows that after time $(t, t+dt)$ the number of particles which will have precipitated on the sphere is equal to

$$\Phi dt = 4\pi R^2 D \left(\frac{\partial n}{\partial \rho}\right)_{\rho=R} dt = 4\pi D R n_0 \left(1 + \frac{R}{\sqrt{\pi D t}}\right) dt \qquad (38.32)$$

and after time t:

$$N = \int_0^t \Phi dt = 4\pi R D n_0 \left(t + \frac{2R\sqrt{t}}{\sqrt{\pi D}}\right) \qquad (38.33)$$

At $Dt/R^2 \ll 1$ precipitation on the sphere takes place from a thin aerosol layer adjacent to the surface of the sphere at the same speed as precipitation on a flat wall (see formula (38.6)). At $Dt/R^2 \gg 1$ the distribution of the concentration of particles around the sphere remains practically constant, and is expressed by the equation

$$n = n_0\left(1 - \frac{R}{\rho}\right). \qquad (38.34)$$

Thus, the concentration gradient at the sphere surface is equal to

$$\frac{\partial n}{\partial \rho}_{\rho=R} = -\frac{n_0}{R}, \qquad (38.35)$$

but the speed of precipitation (i.e., the number of particles precipitating per second) on the sphere is constant and equal to

$$\Phi = 4\pi D R n_0. \qquad (38.36)$$

VIII. The solution of problems of the diffusion precipitation of aerosols on the outer surface of a cylinder of infinite length [281] is more complicated. Discussion of this problem here will be limited to the inclusion of a table of the values $\Phi'/\pi R^2 n_0$ (Φ' is the number of particles precipitating on a cylinder per unit length during time t) in the function Dt/R^2.

TABLE 15

DIFFUSION PRECIPITATION OF AN AEROSOL ON A ROUND CYLINDER OF INFINITE LENGTH

Dt/R^2	0,001	0,005	0,01	0,02	0,05	0,1	0,2	0,4	0,6	0,8	1,0	1,5	2,0	2,5	3,0	3,5
$\Phi'/\pi R^2 n_0$	0,072	0,164	0,235	0,337	0,550	0,805	1,190	1,632	2,279	2,721	3,125	4,04	4,87	5,67	6,38	7,10

A correction must be made for the "natural size" of particles (see (31.18)) must be made in the derivations introduced under paragraph 38. In this case this is quite simple: an "absorbing" surface is constructed parallel to the surface under consideration, at a distance r from the latter, equal to the radius of the particles. Because when a particle comes in contact with the actual surface its center comes in contact with the absorbing surface, these particles may be considered as points, and all the conclusions introduced above may be applied to the resulting system.

39. Diffusion of an Aerosol in Laminar Flow

The much more difficult problem of the diffusion of aerosols in laminar flow has been solved precisely for only very few cases:

I. The problem of the diffusion (or heat transfer) from liquids in laminar flow to the wall of a round tube in which they are contained has been solved at various times by different researchers who were not familiar with the work of their predecessors [282 to 285]. The conclusion that the speed of convection transmission of a substance is considerably greater than diffusion transmission was taken as the basis of these investigations. In other words, the x-axis was taken as parallel to the line of flow, making the member $D\partial^2 n/\partial x^2$ in the basis equation (37.17) very small in relation to $\partial(V_x n)/\partial x \approx U\partial n/\partial x$ (U is the speed of flow) and may be ignored.

Taking n_0 as the starting concentration of particles in the aerosol, $\bar{n}$ as the average concentration of the aerosol flowing through a tube with radius R and length x , and $\bar{U}$ as the average speed of flow in the tube, the following expression is obtained for $\bar{n}/n_0$:

$$\bar{n}/n_0 = 0{,}82\,e^{-3{,}66\mu} + 0{,}097\,e^{-22{,}2\mu} + 0{,}0135\,e^{-53\mu}, \qquad (39.1)$$

in which μ is an unmeasured value equal to

$$\mu = Dx/R^2\,\bar{U}. \qquad (39.2)$$

Here, we took coefficient values practically coinciding with those found by Nusselt [283] and Gormley and Kennedy [285]. Townsend [284] found the value 0.78 for the first coefficient. In the derivation of formula (39.1) it is assumed that the concentration decreases continuously along the axis of the tube and from the axis to the periphery, which agrees with the system of diffusion to a wall. Actually, at small values of μ the system cannot be applied in which $n = n_0$

for all cross sections with the exception of the layer adjacent to the wall, and formula (39.1) is not applicable, and at $\mu = 0$, $\bar{n}/n_0 = 0.93$ instead of 1. For this case a simplified method of calculation may be used, assuming that during time t all particles located at moment $t = 0$, i.e., at the entrance of the tube, at a distance

$$\delta = 2\sqrt{\frac{Dt}{\pi}}, \qquad (39.3)$$

from the wall will have precipitated on the wall. The distribution of speeds near the wall under laminar flow (see formula (26.11)) may be approximated by the expression

$$U = 2\bar{U}\left(\frac{R^2-\rho^2}{R^2}\right) = 2\bar{U}\left(\frac{R+\rho}{R}\right)\left(\frac{R-\rho}{R}\right) \approx 4\bar{U}\frac{\delta}{R}, \qquad (39.4)$$

where δ is the distance from the wall. The average speed in a layer of thickness δ adjacent to the wall is $2\bar{U}\delta/R$, and during time t a particle in this layer travels an average distance

$$x = 2\bar{U}\delta t/R. \qquad (39.5)$$

along the axis of the tube.

Eliminating t from (39.3) and (39.5), we have

$$\delta = \left(\frac{2DxR}{\pi\bar{U}}\right)^{1/3}. \qquad (39.6)$$

Thus, the number of particles passing per second through a cross section of the tube located at a distance x from the entrance, is equal to

$$\Phi = \int_0^{R-\delta} n_0 2\bar{U}\left(\frac{R^2-\rho^2}{R^2}\right)2\pi\rho\,d\rho \approx (R^2-4\delta^2)\pi n_0\bar{U} = \frac{R^2-4\delta^2}{R^2}\Phi_0, \qquad (39.7)$$

where Φ_0 is the corresponding number at the tube entrance. According to the relationship $\bar{n}/n_0 = \Phi/\Phi_0$,

$$\frac{\bar{n}}{n_0} = 1 - \frac{4\delta^2}{R^2} = 1 - 2{,}96\mu^{2/3}. \qquad (39.8)$$

A much more strict derivation (Leveque [286], and V. Levich [287]) leads to the formula

$$\frac{\bar{n}}{n_0} = 1 - 2{,}57\mu^{2/3}. \qquad (39.9)$$

Figure 57 contains curves which conform to formulas (39.1) (curve 1) and (39.9) (curve 2). These curves are applied when $\mu > 0.03$ and $\mu < 0.03$. In practice, the solid-line curve of Figure 57 may be used.

In the derivation of the formulas described above it is assumed that the poise flow expressed by equation (26.11) is established at the very beginning of the tube. In actuality this occurs at a distance approximately 0.1 R Re_f from the tube entrance, and the effect of the entrance section on the accuracy of the indicated formulas must be explained. In this, the portion α of an aerosol which precipitates on the walls of the entrance section of the tube is determined. Becuase due to diffusion a smaller part of the aerosol precipitates on the walls at a high speed of flow, and because the speed of flow decreases continuously from the entrance of the tube to the end of the entrance section, the upper limit is obtained for α, assuming that the poise flow is established at the beginning of the entrance section, i.e., $\alpha = 2.57\mu^{2/3}$. Taking $x = 0.1\ R\ Re_f$, we obtain $\alpha = 2.57(0.2\ D/\nu)^{2/3}$, where ν is the kinematic viscosity of the medium. For air $\nu = 0.15$ and $\alpha \approx 3\ D^{2/3}$. From Table 13 it is apparent that even for particles with $r = 10^{-6}$ cm, $\alpha < 0.01$. Thus, formula (39.9) is needed in material corrections only for aerosols with particle size of the magnitude 10^{-7} cm.

L. Radushkevich [288] applied formula (39.1) in the determination of the average size of particles of ammonia chloride smoke. In this experiment he passed the smoke through a system of vertical capillaries 20 cm long with a radius of 0.012 cm at an average speed of 1.84 $cm \cdot sec^{-1}$, and determined the concentration of the smoke before and after the passage by means of ultramicroscopic computation. In this experiment μ was of the order 0.5. The particle radius calculated from the $\bar{n}/n_0$ values found by this method were 20 to 30% greater than those obtained by the calculated-gravimetric method. This discrepancy probably is explained by the fact that the ammonium chloride smoke particles are more or less brittle aggregates, and their actual size is larger than that determined by the calculated-gravimetric method.

II. An analogous problem concerning a flat-parallel canal with distance h between plane surfaces [289] leads to the formula ($\mu = Dx/h^2\bar{U}$):

$$\bar{n}/n_0 = 1,066e^{-7,94\mu} + 0,0065e^{-68,8\mu}. \qquad (39.10)$$

(A more accurate formula [578] is:

$$\bar{n}/n_0 = 0,910\ e^{-7,54\mu} + 0,053\ e^{-86\mu} + \ldots$$)

Nolan and Guerrini [289] passed atmospheric air through a system of vertical flat-parallel canals and determined the percent of atmospheric condensation nuclei precipitating on the walls of the canals by the Aitken method. From this the two researchers obtained a value of 2.85 • 10^{-6} cm for the radius of condensation nuclei, using formula (39.10). They repeated this experiment with the canals in a horizontal position and, assuming that the difference between the results obtained in the two experiments corresponded to the number of nuclei precipitating due to sedimentation, they calculated the average speed of precipitation, from which they determined the mass and density of the condensation nuclei. The incompatible value which was obtained for the size of nuclei (1.7) indicates that such a simplified method of calculation is not applicable.

III. The problem of diffusion to the surface of a sphere from a stream has been studied theoretically and experimentally in the case of condensation of a vapor on the surface of droplets (or their vapor). The speed of diffusion to the sphere, expressed in the case of a stable medium by formula (38.36), is increased in the stream by the factor $1 + 0.27\ Re^{1/3}\ Sc^{1/3}$ [290], where $Sc = \nu/D$, which is the Schmidt number. However, in aerosols D is several orders smaller than in vapors; in vapors Sc is in the order of units, and in aerosols it is much larger ($\sim 10^5$ at $r = 10^{-5}$ cm; $\sim 10^3$ at $r = 10^3$ cm). Because of this experimental data on the diffusion of vapors may not be carried over to aerosols. The Sc number may serve as a means of comparison of the speed of convection and diffusion transferral of a substance (at constant Re) in the type of problem under consideration here. At Sc ≈ 1 the thickness of the diffusion layer and of the hydrodynamic friction layer around the streamlined body are approximately of the same order of greatness. At Sc >> 1 diffusion transmission is equal to convection transmission only at very small distances from the body surface, i.e., the diffusion layers are very thin in comparison to the friction layers. (The problem of the character of diffusion to the surface of streamlined bodies as a function of the order of greatness of Sc is discussed at great length by D. Frank-Kamenetskiy [291] and V. Levich [292.) Because of this diffusion has different mathematical expressions in these two cases. Because the value of Sc is very great for diffusion in solutions, also, we may apply the results of V. Levich [293] and obtain the following expression for the speed of diffusion to a sphere under Stock's flow:

$$\Phi = 7{,}9 n_0 D^{2/3} U^{1/3} R^{4/3}. \qquad (39.11)$$

As described above, in the derivation of this formula it is assumed that the thickness of the diffusion layer is very small compared to the radius of the sphere. Because at low values of U the thickness of the diffusion layer increases, formula (39.11) may not be applied at very low rates of flow.

IV. In conclusion, we may consider the case of free diffusion of an aerosol issuing from a naroow opening ("point source"), or from a flat slit ("linear source") placed perpendicular to the direction of flow of a medium in rectilinear motion at a uniform speed U . Under these conditions the precipitation of particles is ignored. The x-axis is set parallel to the flow, the starting coordinate is placed at the source and, in the case of a linear source, it is substituted by the y-axis.

First of all the problem of an instantaneous source is solved [294], because for solution in the case of a point source, substituted at the starting coordinate, there are several converted formulas (37.31) for a system of coordinates which are in motion in addition to the flow

$$n(x, y, z, t) = \frac{N}{\sqrt{(4\pi Dt)^3}} e^{-(x^2+y^2+z^2)/4Dt}, \tag{39.12}$$

where N is the number of particles issuing from the source at the moment $t = 0$. In a stable system of coordinates (39.12) this becomes

$$n(x, y, z, t) = \frac{N}{\sqrt{(4\pi Dt)^3}} e^{-[(x-Ut)^2+y^2+z^2]/4Dt}. \tag{39.13}$$

Similarly, in the case of an infinitely long linear source we have

$$n(x, y, z, t) = \frac{N'}{4\pi Dt} e^{-[(x-Ut)^2+z^2]/4Dt}. \tag{39.14}$$

where N' is the number of particles issuing per cm of length of the source.

V. In the case of continuous action of the source a stationary distribution of concentration of the aerosol in space must be postulated. Precise calculation of the latter is difficult. However, knowing what has been mentioned above about the comparative size of the convection and diffusion transmission of particles, i.e., eliminating the member $D\frac{\partial^2 n}{\partial x^2}$ from the equation (27.19) and introducing in the latter the term $\frac{\partial n}{\partial t} = 0$, the equation may be transformed, in the case of a linear source, to

$$\frac{\partial n(z, x)}{\partial x} = \frac{D}{U} \frac{\partial^2 n(z, x)}{\partial z^2} \tag{39.15}$$

at the limiting condition

$$n(z, 0) = 0 \text{ at } z \neq 0 \tag{39.16}$$

and at the normalization condition

$$U \int_{-\infty}^{\infty} n(z, x)\, dz = \Phi', \tag{39.17}$$

indicating that any plane perpendicular to the x-axis intersects that quantity of particles (equal to Φ' per cm of source length) per second, which emanate from the source. Solution of equation (39.15), which is mathematically equivalent to equation (37.16), gives a formula which may be obtained from (37.29) by substitution of t for x, $x - x_0$ for z D for D/U , and through multiplication by Φ'/U:

$$n = \frac{\Phi'}{\sqrt{4\pi DUx}} e^{-\frac{Uz^2}{4Dx}} \text{ (length of source)} \tag{39.18}$$

and for a constant point source the following formula is obtained

$$n = \frac{\Phi}{4\pi Dx} e^{-\frac{U(y^2+z^2)}{4Dx}}. \tag{39.19}$$

[Pages 215-219]

43. Brownian Rotation. Orientation of Aerosol Particles in an Electric Field.

Brownian rotation is determined by the equation [253]

$$\overline{\theta^2} = 2kTB_\omega t, \tag{43.1}$$

where $\overline{\theta^2}$ is the average square of the angle of rotation of particles around a given axis during time t, B_ω is the "rotational mobility" of particles around this axis, i.e., the angular velocity of rotation under the influence of a rotational moment of an equivalent unit. In other words

$$d\theta / dt = B_\omega P_\theta, \tag{43.2}$$

where P_θ is the moment of external forces relative to this axis.

For spherical particles

$$B_{\omega_0} = 1/8\pi\eta r^3. \tag{43.3}$$

However, in the physics of aerosols the rotation of apherical particles is of no interest because it does not appear and has no influence on the properties of aerosols. The rotation of non-spherical particles is apparent as flickering under side lighting, and they may be distinguished by this method from spherical particles. Furthermore, particles which are elongated in one or two directions are oriented by the action of hydrodynamic, electric, etc, forces, and thus the degree of orientation is determined by the relationship between the intensity of the orienting force and the intensity of Brownian rotation. Finally, Brownian rotation is very important in the coagulation of elongated particles.

In the case the rotation of an elongated ellipsoid with a small pole a and ratio β between the axes, the rotational mobility around the small axis is equal to [349]

$$B_\omega = \frac{3\left[\frac{2\beta^2 - 1}{\sqrt{\beta^2 - 1}} \ln\left(\beta + \sqrt{\beta^2 - 1}\right) - \beta\right]}{16\pi\eta a^3 (\beta^4 - 1)}. \tag{43.4}$$

The values of the ratio between the rotational mobility of a sphere with radius $r(B_{\omega_0})$ and an ellipsoid with small pole $r(B_{\omega})$ are shown in Table 20.

TABLE 20

ROTATIONAL MOBILITY OF ELONGATED ELLIPSOIDS ROTATING AROUND THE SMALL AXIS

β	2	3	4	5	6
B_{ω_0}/B_{ω}	3,0	7,0	13,5	23,1	36,4

In particles having the shape of flat discs with radius r, the rotational mobility around the diameter of the disc is

$$B_{\omega} = 3/32\eta r^3. \qquad (43.5)$$

The following formula [350] may be derived from (43.1) for the complete angle of rotation θ along the length of the axis of an elongated particle during time t :

$$\overline{\sin^2\theta} = \frac{2}{3}(1 - e^{-6kTB_{\omega}t}). \qquad (43.6)$$

At $t \to \infty$ $\overline{\sin^2\theta} = \frac{2}{3}$, which corresponds to the probability of any direction of the axis.

The orientation of elongated particles in an electrical field, caused by the polarization of particles is very important in the physics of aerosols. If non-charged particles having ellipsoidal rotation are placed in a uniform electrical field of strength E, so that the polar axis of the ellipsoid forms an angle θ with the direction of the field, the change in energy of the field caused by the polarized particles is equal to [351]

$$\Omega = -\frac{vE^2}{2}\left[\frac{\cos^2\theta}{\frac{1}{\varepsilon_k - 1} + \varkappa_1} + \frac{\sin^2\theta}{\frac{1}{\varepsilon_k - 1} + \varkappa_2}\right], \qquad (43.7)$$

where v is the volume, and ε_k is the dielectric permeability of the particle. The coefficients of form $\varkappa_1$ and $\varkappa_2$ are expressed in terms of the ratio between the large and small axes of ellipsoid β as follows:

For elongated ellipsoids:

$$\varkappa_1 = \frac{1}{\beta^2 - 1}\left[\frac{\beta}{\sqrt{\beta^2 - 1}}\ln(\beta + \sqrt{\beta^2 - 1}) - 1\right], \qquad (43.8)$$

$$\varkappa_2 = \frac{\beta}{2(\beta^2 - 1)}\left[\beta - \frac{1}{\sqrt{\beta^2 - 1}}\ln(\beta + \sqrt{\beta^2 - 1})\right]; \qquad (43.9)$$

for oblate ellipsoids:

$$\varkappa_1 = \frac{\beta^2}{\beta^2-1}\left[1 - \frac{1}{\sqrt{\beta^2-1}}\arcsin\frac{\sqrt{\beta^2-1}}{\beta}\right], \qquad (43.10)$$

$$\varkappa_2 = \frac{1}{2(\beta^2-1)}\left[\frac{\beta^2}{\sqrt{\beta^2-1}}\arcsin\frac{\sqrt{\beta^2-1}}{\beta} - 1\right]; \qquad (43.11)$$

and for conducting particles (43.7) the formula becomes

$$\Omega = -\frac{vE^2}{2}\left[\frac{\cos^2\theta}{\varkappa_1} + \frac{\sin^2\theta}{\varkappa_2}\right]. \qquad (43.12)$$

Because for practical purposes aerosol particles may be considered conductors (see (15.13) through (15.15)), in the following only formula (43.12) will be used.

Table 21 contains the values of coefficients $\varkappa_1$ and $\varkappa_2$ for a series of values of β.

TABLE 21

VALUES OF THE COEFFICIENTS OF THE SHAPE OF PARTICLES ORIENTED IN AN ELECTRICAL FIELD

Elongated Ellipsoids.

β	1	1,1	1,5	2	3	5	10	∞
$\varkappa_1$	0,333	0,310	0,233	0,174	0,109	0,056	0,020	0
$\varkappa_2$	0,333	0,345	0,383	0,413	0,446	0,472	0,490	0,5

Oblate Ellipsoids

$\varkappa_1$	0,333	0,347	0,446	0,527	0,635	0,751	0,860	1
$\varkappa_2$	0,333	0,320	0,277	0,236	0,182	0,125	0,070	0

Because the particles tend to place themselves in a manner so that the energy of the field will be minimal, from these data it follows that the value corresponding to the resistant attitude of elongated ellipsoids is $\theta = 0$ (the polar, i.e., longitudinal axis is parallel to the field), and the value for oblate ellipsoids is $\theta = \frac{\pi}{2}$ (the polar, i.e., short axis is oriented perpendicularly to the field). Thus the orientation of particles in laminar flow (see Figure 12 and (11.1)) is analogous to that in an electrical field. It must be remembered that the orientation of particles which are in motion with respect to the medium is opposite at large values of Re : the longitudinal axis is perpendicular to the direction of motion.

In the absence of Brownian movement an elongated particle which enters an electrical field will move along a resistant path, approaching it asymptotically. The orientation of particles in the presence of Brownian movement will be taken into consideration only for the case of elongated ellipsoids. The probability that the angle between the polar axis of a particle and the direction of the field will fall within the interval (θ, θ+dθ) is equal, according to the principle of Bol'tsman, to

$$W(\theta)\,d\theta = be^{-\Omega/kT}\sin\theta\,d\theta = b'e^{\lambda^2\cos^2\theta}\sin\theta\,d\theta. \qquad (43.13)$$

Here

$$\lambda^2 = \frac{E^2v\left(\frac{1}{x_1}-\frac{1}{x_2}\right)}{2kT} = \frac{2\pi E^2\left(\frac{1}{x_1}-\frac{1}{x_2}\right)a^3\beta}{3kT} \qquad (43.14)$$

is the ratio of electrical energy to the energy of Brownian rotation of a particle, a is the small pole of the ellipsoid, and b' is a constant coefficient, the value of which is determined by the normalization condition

$$1 = \int_0^{\frac{\pi}{2}} W(\theta)\,d\theta = \frac{b'}{\lambda}\int_0^{\lambda} e^{x^2}\,dx. \qquad (43.15)$$

From this the following expression is obtained for the average value of cos θ :

$$\overline{\cos\theta} = \int_0^{\frac{\pi}{2}} b'e^{\lambda^2\cos^2\theta}\sin\theta\cos\theta\,d\theta = \frac{e^{\lambda^2}-1}{2\lambda\int_0^{\lambda} e^{x^2}\,dx}. \qquad (43.16)$$

Table 22 contains the values of $\overline{\cos\theta}$ for several values of λ^2 .

TABLE 22

ORIENTATION OF ELLIPSOIDAL PARTICLES IN AN ELECTRICAL FIELD

λ^2	0,01	0,1	1	2	4	6	9	16
$\overline{\cos\theta}$	0,500	0,508	0,587	0,676	0,817	0,864	0,946	0,993

It is noted that $\overline{\cos\theta} = 0.5$ indicates an absence of orientation, and $\overline{\cos\theta} = 1$ indicates complete orientation. It is seen that practically complete orientation is attained at $\lambda^2 \approx 10$.

For ellipsoidal particles with an axis ratio of 3:1 and small pole value $a = 0.1\,\mu$, a fairly strong field of the order 1,000 v·cm^{-1} is required for complete orientation, but 30 v·cm^{-1} is sufficient at a value of $a = 1\,\mu$.

The phenomenon of orientation of particles in an electrical field is explained by the discrepancy between the measurements of the mobility of smoke particles under the influence of gravity and under the influence of an electrical field, which was mentioned in paragraph 19. It is obvious that in the latter case the mobility of particles which are oriented in the direction of their movement must be greater than in the former case, i.e., in the absence of orientation. Furthermore, particles which are oriented in an electrical field are dipolar, which leads to the so-called "directional" coagulation of aerosols in an electrical field (see paragraph 52). Changes in the intensity of light diffused by aerosols in an electrical field has been observed in the case of ammonium chloride smoke [352, 353]. Because the direction of observation, the illuminating rays and the field all are at right angles to each other, the observer seed the light reflected from the longitudinal sides of oriented particles, and the intensity of the reflected light increases as it approaches the plane of the field. In the case of ammonium chloride crystals the longitudinal axis coincides with the crystallographic axis, and because of this the phenomenon of double refraction also is observed in an oriented smoke. However, the orientation of aerosols in an electrical field still has not been investigated very thoroughly.

In view of the fact that the time of relaxation in the polarization of particles is insignificantly small in comparison with the time of relaxation of rotation under the influence of an external force, all the foregoing remains valid even for the orientation of particles in an alternating electrical field.

Similar orientation is observed in smokes of ferromagnetic substances in a magnetic field [354].

Droplets also may become oriented by strong electrical fields, which cause deformation of the latter due to induction. Assuming that deformed droplets have the shape of elongated ellipsoids of rotation (this assumption is emphasized by experiments) it may be shown [355] that the shape of the ellipsoid (axis ratio c/a) depends upon the value of rE^2/σ, where r is the radius of the non-deformed droplet, σ is the surface tension of the liquid, and depends upon the dielectric permeability of the liquid. Because of the complexity of the derived formulas only the values of c/a in the function rE^2 for water and dioctylphthalate are given in Table 23.

TABLE 23

DEFORMATION OF DROPLETS IN AN ELECTRICAL FIELD

rE^2 *	0	50	100	150	200
c/a (water	1	1.07	1.17	1.35	-
c/a (dioctylphthalate)	1	1.06	1.13	1.23	1.34

* E is expressed in $v \cdot cm^{-1}$.

At the same time the droplets are deformed, they also are oriented by the electrical field, as described above.

[Pages 220-258]

CHAPTER VI. CONVECTION AND TURBULENT DIFFUSION IN AEROSOLS

44. Precipitation of Aerosols Under Conditions of Convection and Mixing

In the foregoing, various cases of the movement of aerosol particles in an immobile medium and in laminar flow at given constant distributions of speed have been discussed. However, complete immobility of the medium, and lack of agitation of laminar flow occurs rather rarely, even under the limits of sensitivity of the author's observations, because of the presence of convection. Tests have shown [356] that in a gas which is in contact with a vertical wall, the temperature of which differs from the temperature of the gas by ΔT, a vertical current arises near the wall, the maximum speed of which, at a height z above the wall base, is

$$U = 0{,}55\sqrt{gz\alpha\Delta T}; \tag{44.1}$$

where α is the coefficient of expansion of the gas, equal to $1/T$. Thus, in a chamber one meter high at $\Delta T = 0.01^{\circ}$ the speed of convection flow is approximately 1 $cm \cdot sec^{-1}$, i.e., the speed of precipitation of large particles with radius approximately 10μ. (Convection is weaker in liquids than in gases because of a smaller coefficient of expansion.) Because of this convection currents are practically unavoidable in smoke chambers; they may be eliminated only by the use of small massive metallic vessels, and careful absorption of heat rays from light entering the vessel. For the same reason, true laminar flow of a gas is possible only in narrow tubes and canals; in wide tubes vertical convection currents inevitably arise. Although convection may be eliminated by creating a vertical temperature gradient in a vessel containing an aerosol, but this method is used comparatively rarely.

If one wall of a vessel maintains a constant temperature, higher than that of the other walls (such as in the heating of glass walls of smoke chambers by light used for illumination), the resultant convection often takes the form of a regulated circulation. The distribution of speeds in this case is obtained by calculation, from which the trajectory of particles also may be determined. The same calculation also is possible in a theoretical heat transmission case of free convection near heated bodies with straight sides. However, in practical work with aerosols convection usually has an irregular character, or else it may not be calculated, which eliminates the possibility of computing the trajectory of individual particles. A similar case is that of forced irregular convection caused by artificial mixing of an aerosol.

In these cases, however, statistical treatment of the movement of the medium and the particles suspended in it is impossible. Because of this, considerable idealization of the phenomenon of convection is necessary; it must be assumed that the convection transferral of the medium takes place through "convection diffusion" and that the intensity of convection is determined by the size of the "coefficient of convection diffusion." The motion of particles suspended in the medium is composed of the movement of the medium itself, i.e., regular flow and convective diffusion of the medium, and of the previously described movement of particles in relation to the medium.

In considering the precipitation of an aerosol in a chamber in the absence, and in the presence of convection, it is assumed that at a given concentration of particles the coagulation of the aerosol may be ignored. In the absence of convection, and with not very small particles (i.e., in the absence of diffusion erosion) in a monodispersion aerosol the upper limit of the latter precipitates with a constant speed V_s , and below the limit of concentration of the aerosol, which is constant for its entire volume, it does not change with time. In the case of a low degree of polydispersion the upper limit gradually dissolves, but remains clearly discernible for a considerable length of time. This is observed in actuality in the precipitation of more or less monodispersed fogs with a high weight concentration which facilitates "hydrostatic stabilization" of the upper limit (see paragraph 14).

Indicating the concentration of particles with radius $(r, r+dr)$ by the term $n(r)\,dr$, the number of particles precipitating per cm^2 of the bottom of the chamber per second is $V_s(r)\,n(r)\,dr$, and the number of particles precipitating during the time t is equal to

$$dN = V_s(r)\,n(r)\,t\,dr \qquad (44.2)$$

This formula is used only at $t \leq H/V_s(r)$, where H is the height of the chamber, because during time $H/V_s(r)$ all particles of the indicated size will have precipitated. The complete number of of particles precipitating during time t is obtained by integration of expression (44.2) for r .

With very intense convection (more accurately, if the average speed of convection currents considerably exceeds V_s) the concentration of the aerosols is practically constant for the entire volume of the chamber with the exclusion of the layers adjacent to the walls, but the concentration continuously decreases with the passage of time. The vertical component of the speed of convection and the coefficient of convective diffusion approaches zero toward the bottom of the chamber. At a rather small distance δ from the bottom the intensity of convective and molecular diffusion are equal, and in a layer next to the wall with thickness δ the latter prevails. Considering precipitation on the bottom of the chamber, it is assumed that the process has a quasistationary character, i.e., that during the time of passage of particles through the wall layer the concentration of the aerosol at a distance n_∞ from the bottom remains constant. In experiments on the precipitation of aerosols in chambers this condition usually is fulfilled.

The number of particles which pass down through a horizontal area 1cm^2 per sec is

$$I = V_s n + D_E \frac{dn}{dz} \qquad (44.3)$$

Here D_E is the effective coefficient of diffusion, including both the mechanism of diffusion and the dependent function of z . At a distance from the bottom greatly exceeding δ , $n = n_\infty$, and

$$I = V_s n_\infty. \qquad (44.4)$$

In accordance with the postulated quasistationary process of precipitation this formula expresses the number of particles which precipitate per sec per cm^2 of the bottom. Expressing equation (44.3) in the form

$$V_s (n_\infty - n) = D_E \frac{dn}{dz} \qquad (44.5)$$

and solving the latter for the limiting conditions $n = n_\infty$ at $z = \infty$ and $n = 0$ at $z = 0$, we obtain the following expression for the concentration of the aerosol at distance z from the bottom of the chamber

$$n = n_\infty \left[1 - \exp\left(-V_s \int_0^z \frac{dz}{D_E}\right)\right]. \qquad (44.6)$$

Thus, the complete flow of particles to the bottom of the chamber is composed of the sedimentation portion

$$I_1 = V_s n = V_s n_\infty \left[1 - \exp\left(-V_s \int_0^z \frac{dz}{D_E}\right)\right] \qquad (44.7)$$

and the diffusion portion

$$I_2 = D_E \frac{dn}{dz} = V_s n_\infty \exp\left(-V_s \int_0^z \frac{dz}{D_E}\right). \qquad (44.8)$$

It is easily seen that the latter becomes noticeable only at a very small distance from the bottom. Thus diffusion changes the distribution of concentration of particles close to the bottom, but does not influence the speed of precipitation.

In conformance with the foregoing, the speed of precipitation of particles with radius $(r, r+dr)$ on the bottom of the chamber is equal to $V_s(r)\,n(r)\,dr$, just as in an immobile medium, and $n(r)$ varies with t according to the equation

$$-H\,dn(r) = V_s(r)\,n(r)\,dt. \qquad (44.9)$$

From this it follows that

$$n(r) = n_0(r) \exp\left(-\frac{V_s(r)\,t}{H}\right), \qquad (44.10)$$

where n_0 is the starting concentration.

Thus the concentration of particles of every size decreases exponentially with time, but at various speeds. Because of this, with the passage of time the average dispersion of an aerosol in a chamber increases continuously. The total concentration of particles at moment t is equal to

$$n = \int_0^\infty n_0(r) \exp\left(-\frac{V_s(r)\,t}{H}\right) dr. \qquad (44.11)$$

The following expression is derived for the number of particles of a monodispersion aerosol which precipitate during time t per cm^2 of chamber bottom:

$$N = \int_0^t V_s n\, dt = V_s n_0 \int_0^\infty \exp\left(-\frac{V_s t}{H}\right) dt = n_0 H \left[1 - \exp\left(-\frac{V_s t}{H}\right)\right]. \qquad (44.12)$$

The investigation of an intermediate case with comparable speed of precipitation and convection is much more complicated and cannot be taken up here. It may be mentioned, however, that in the precipitation of aerosols in smoke chambers convection is very strong even in the absence of artificial stirring, and the character of the precipitation usually approaches the second of the types mentioned above. Thus, in the experiments of Gillepsie and Langstroth [357] the concentration of NH_4Cl smoke with particle radius 0.3 to 2.0μ was constant for all points of a chamber of 12 m^3 volume during the entire life of the smoke. In the experiments of Y. Vigdorchik [358] quartz dust particles with $r = 12.5\mu$ could be observed in a chamber 1.2 m high three hours after the dust was placed in the chamber. The same particles should have settled within 12 minutes in still air.

From formula (44.10) it follows, that

$$\ln n(r) = \ln n_0(r) - \frac{V_s(r)\,t}{H}, \qquad (44.13)$$

i.e., in the case of a monodispersion aerosol the change in the logarithm of the concentration with time is expressed as a straight line, the angle of inclination of which with the abscissa increases with an increase in the size of the particles. Unfortunately, almost all available experimental data on the kinetics of precipitation of aerosols in chambers refer to fairly polydispersed systems. Because with the passage of time the average size of particles in an aerosol, and consequently the angle of inclination of the curves (ln n, t) decrease, in polydispersion aerosols these curves bulge toward the abscissa, with the curvature increasing in proportion to the degree of polydispersion. This type of curve often is encountered [358].

From formula (44.12) it follows that in the case of convection precipitation of particles

$$-\frac{H}{V_s}\ln\left(1 - \frac{N}{n_0 H}\right) = t. \qquad (44.14)$$

This formula was introduced by Davies [218] for highly aggregated coal dust. Figure 65 contains curves for various radius intervals of particles in individual dust fractions. A linear relationship (44.14) may be observed for the fine fractions. The deflection of the curves for the coarse fraction are attributed by Davies to polydispersion with the fraction.

Considerably less research has been done on the precipitation of aerosols on the side (vertical) walls of chambers. For any mechanism of precipitation the speed of the latter is proportional to the concentration n of the aerosol, from which it follows that $dn/dt = -\beta n$. Thus, the kinetics of precipitation on vertical walls is expressed similarly to (44.13) by the equation

$$\ln(n_0/n) = \beta t, \qquad (44.15)$$

and the problem leads to the derivation of a theoretical expression for coefficient β.

Without artificial mixing, inertial precipitation of an aerosol on the flat walls of a chamber must be very small. Actually, under these conditions large particles $(r \approx 7\mu)$ do not precipitate at all on vertical walls [248]. Here we are concerned purely with diffusion precipitation, in which particles are led up to the wall by convective diffusion and precipitate due to molecular diffusion in the thin layer next to the wall (see equations (46.11) through (46.16)). Because the law of the change of the coefficient of convective diffusion under approach to the wall is not known the problem must be greatly simplified, and it must be assumed that only molecular diffusion takes place in a wall layer of width δ, and that outside this layer convective diffusion is so intense that the concentration of the aerosol outside the layer is constant. In this case the speed of precipitation of the aerosol on a vertical wall is expressed by the formula

$$I = \frac{Dn}{\delta} \text{ particles/cm}^2\text{.sec} \qquad (44.16)$$

where D is the coefficient of molecular diffusion of particles. Taking v as the volume, and S as the surface of the chamber side walls, then

$$-vdn = SIdt. \tag{44:17}$$

Thus, we obtain the following formula for the coefficient β in formula (44.15)

$$\beta = \frac{SI}{vn} = \frac{SD}{v\delta}. \tag{44:18}$$

It must be remembered that δ depends not only on the intensity of convection, but also on β. Actually, the size of δ, is determined by the condition that at distance δ from the wall the coefficients of molecular and convective diffusion are equal. The greater the coefficient of molecular diffusion, the greater is δ. Because of this there can be no simple proportionality between the speed of precipitation on the walls and D, and the speed of precipitation must be proportional to D^{α}, $0 < \alpha < 1$.

From the above the following conclusion may be drawn: the greater the degree of dispersion of an aerosol, the greater is the number of particles precipitating on the side walls and the smaller the number of particles precipitating on the bottom.

K. Shifrin and his associates [359] investigated (through measurement of the transparency of a smoke) the kinetics of precipitation of a smoke with $r \approx 0.5\mu$ on the walls of a cylindrical chamber 9m high and 4m in diameter, and found that equation (44.15) is applicable in this case, and that the concentration decreased two-fold during 2 to 3 hours. Because the starting concentration of the smoke was of the order 10^4 per cm^3, the decrease in concentration due to coagulation of the aerosol could be ignored. At the indicated degree of dispersion of the aerosol and chamber height precipitation on the bottom of the chamber played no noticeable role in the results. From the curves proposed by Shifrin it was possible to calculate that the ratio of the number I of particles precipitating per cm^2 of wall surface per second, to the concentration of the latter was 5 to 7 · 10^{-3} cm·sec^{-1}. From this, the value $\delta \approx 0.5\mu$ was obtained for the thickness of the wall layer.

In the experiments of Gillepsie and Langstroth [357], which were conducted with a chamber of 12 m^3 capacity and NH_4Cl smoke with particle radius in the interval 0.3 to 2.0 , the number of particles precipitating on the bottom, side walls and top of the chamber was determined directly through the use of plates of proper shape which were installed in the chamber. It was found that precipitation on the top was negligible, and precipitation on the side walls during the first 100 minutes was approximately one-third less than on the bottom, after which precipitation on the side walls practically ceased. From the graphs prepared by these researchers it may be roughly seen that the side of I/n in these experiments was on the order of 10^{-4}, and that the thickness of the wall layer $\approx 20\mu$, i.e., approximately two orders larger than the results of Shifrin. Because the speed of decrease in concentration of an aerosol observed by Shifrin in such a large chamber and in the absence of coagulation was in clear contradiction to the data of numerous works dedicated to the study of the coagulation of aerosols, and because direct data on the precipitation of aerosols on walls is much more promising than those obtained by calculation of the decrease in concentration, apparently the values for I/n and δ obtained by Gillepsie and Langstroth must be given preference until new data are obtained on this problem.

The charge of particles has a great influence on the speed of precipitation of aerosol particles on walls. In recent experiments of Gillepsie [360] the total number of SiO_2 particles with $r = 0.4\mu$ precipitating during the first few minutes of the life of the aerosol on the walls and bottom of a chamber of 0.2 m^3 volume increased 2- to 3-fold when the average number of elementary charges on the particles was increased from 7 to 14 (this increased the maximum charge from approximately 50 to 100). Proceeding on the assumption that at the speed of precipitation of the aerosol on the bottom of the chamber the charges could hardly have any marked influence, the increase of precipitation on the side walls in these experiments actually was much larger. It is curious that this influence of the charges gradually decreased, and totally disappeared after several score minutes. Because the charge of the aerosol was bipolar and fairly symmetrical, and electrostatic diffusion (see paragraph 24) did not play a marked role here, it is beyond doubt that the indicated effect was caused by induction attraction of the charged particles to the walls.

The very great influence of this effect may be calculated, assuming that the particles which reach the wall layer move toward the wall under the influence of the force of induction. The speed of this movement to the border of the wall layer according to formula (41.1) is equal to $q^2B/4\delta^2$, from which it is found that the number of particles precipitating per cm^2 of wall surface per second is $I_q = q^2Bn_o/4\delta^2$. The number of particles precipitating due to molecular diffusion is found by formula (44.16). We may establish the relationship

$$\frac{I_q}{I}=\frac{q^2Bn_0}{4\delta^2}\,\frac{\delta}{Dn_0}=\frac{q^2}{4\delta kT}. \qquad (44.19)$$

Taking $q = \varepsilon\nu = 4.8 \cdot 10^{-10}$ (ν is the number of elementary charges), $kT = 4 \cdot 10^{-14}$, $\delta = 20 \cdot 10^{-4}$ cm, we have

$$I_q/I=0{,}7\cdot 10^{-3}\nu^2. \qquad (44.20)$$

For particles with maximal charges $\nu = 100$, and $I_q/I = 7$. These particles precipitate much more rapidly than non-charged particles because the induction effect is insignificant for particles with small charges. Because of this, after the particles with large charges have precipitated further precipitation proceeds "normally."

It has been established in many studies that under artificial mixing of an aerosol the gravimetric concentration in a chamber decreases more rapidly than without mixing. This is partly explained by increased coagulation caused by the mixing (see paragraph 55), and consequently, by an increase in the size of particles. However, the main cause appears to be without doubt the inertial precipitation of particles on the chamber walls. This was directly indicated by Gillespie and Langstroth [357] who measured the speed of precipitation on the walls of the chamber and derived the coagulation constant method for NH_4Cl smoke at various speeds of mixing, which is described in paragraph 55. It was found that I/n increased linearly with the average speed of air flow in a chamber of one m^3 capacity, and at a speed of 50 $m \cdot min^{-1}$, I/n was approximately 5-fold larger than in the absence of mixing; artificial mixing had an insignificant effect on the size of the constant of coagulation. According to an experiment of Ye. Vigdorchik [358] when air in a chamber 1.2 X 1.2 X 1.2 m was mixed by a propeller at an average speed of flow of 4 $m \cdot sec^{-1}$ the concentration of quartz dust decreased 3 to 4-fold faster than in the absence of mixing. The cause of this phenomenon is a decrease in the thickness of the laminar wall layer δ, caused by the mixing, and inertial precipitation of particles. Actually, assuming that the chamber used in the experiments of Ye. Vigdorchik was cylindrical, it may be calculated that at the indicated air speed the centrifugal force at the wall, $mV^2/R = m400^2/60$, is almost three times greater than the force of gravity mg.

Actually, the picture of inertial precipitation with artificial mixing is much more complicated: inertial precipitation takes place in individual places, where vortexes with axes parallel to the walls formed by the mixing touch the latter. The inertial force generated in vortexes with small diameter may be considerably greater than the calculated figure. The character of the mixing also is very important. If the mixing is done with wide paddles which almost reach the walls of the chamber [361] the rate of decrease of the aerosol concentration (NH_4Cl smoke with $\bar{r} \approx 0.5\mu$) increases 3- to 4-fold even at an average air speed of 50 $cm \cdot sec^{-1}$. This type of mixing undoubtedly enables the formation of intense vortexes with axes parallel and close to the walls.

Calculation of the speed of inertial precipitation on the walls under artificial mixing is very difficult, but the number of particles precipitating on one cm^2 wall surface per second in any case must be proportional to $n\tau$, and the decrease in aerosol concentration with time must satisfy the equation

$$v\frac{dn(r)}{dt} = -Sbn(r)\tau(r),$$

where b is a coefficient depending upon the intensity and character of the mixing. From this it follows that

$$n(r) = n_0(r)\exp\left[-\frac{Sb}{v}\tau(r)t\right].$$

Since $V_s = g$, the total change in aerosol concentration with time (ignoring diffusion precipitation) is expressed by the equation

$$n(r) = n_0(r)\exp\left[-\left(\frac{Sb}{v} + \frac{g}{h}\right)\tau(r)t\right], \tag{44.21}$$

which has the same form as in the absence of inertial precipitation, but with the substitution of the coefficient g/h for $\frac{Sb}{v} + \frac{g}{h}$. In addition to equation (44.11) we now have

$$n = \int_0^\infty n_0(r)\exp\left[-\left(\frac{Sb}{v} + \frac{g}{h}\right)\tau(r)t\right]dr. \tag{44.22}$$

As already has been mentioned, the change in ln n with time in the case of a polydispersed aerosol may be expressed by an upwards convex curve. However, if noticeable coagulation of the aerosol takes place simultaneously with precipitation the increase in particle size caused by the former somewhat compensates for the decrease in average particle size caused by precipitation, which leads to rectification of the curve (ln n, t). Practically rectilinear curves sometimes are obtained [361].

The change in gravimetric concentration of a polydispersed aerosol with time is expressed by the equation

$$c = \int_0^\infty n_0(r)\, m(r) \exp\left[-\left(\frac{Sb}{v} + \frac{g}{h}\right)\tau(r)\, t\right] dr, \qquad (44.23)$$

where m(r) is the mass of a particle with radius r . In the case of an isodispersed aerosol $d\ln c/dt = d\ln n/dt$, but in polydispersion aerosols $|d\ln c/dt| > |d\ln n/dt|$, i.e., the gravimetric concentration decreases faster than the calculated concentration.

Actually we may write

$$\frac{d\ln c}{dt} = \frac{1}{c}\frac{dc}{dt} = \frac{1}{n\bar{m}}\frac{\bar{\bar{m}}dn}{dt}, \qquad (44.24)$$

where $\bar{m}$ is the average mass of particles contained in the aerosol, and $\bar{\bar{m}}$ is the average mass of precipitating particles. Since $\bar{\bar{m}} > \bar{m}$, then $|d\ln c/dt| > |d\ln n/dt|$, where the difference $|d\ln c/dt| - |d\ln n/dt|$ increases with the degree of polydispersion of the aerosol, and also with the absolute value of dn/dt , i.e., with the speed of artificial mixing. These conclusions have been verified by experiments [361]

In conclusion, reference is made to a phenomenon caused by natural convection, which is of interest in the treatment of rooms with aerosols. If a closed vessel with holes in opposite vertical walls is placed in the aerosol chamber many quite large ($r \sim 7\mu$) particles penetrate these holes because of horizontal convection ("draft") [248]. If there is only one hole there is no penetration. An analogous situation is found in cracks in wooden walls: an aerosol may penetrate quite deeply into holes which pass all the way through, but cannot penetrate blind holes.

45. Movement of Aerosol Particles in a Turbulent Stream

Theoretical and experimental research on the conduct of aerosols in turbulent flow naturally is more difficult than in the case of laminar flow. Regardless of the great successes which have been achieved in recent times in the study of turbulence, there are very few data available on the movement of suspended particles in a turbulent stream. In particular, the very important problem of the degree of blowing away of particles by turbulent pulsations has not been clarified.

According to contemporary views turbulent flow may be represented as the result of the superposition of a continuous spectrum of pulsations of varying scales on the basic (average) speed of flow. The first pulsation which arises through the breaking away of a vortex from the wall has a scale equal to the diameter of the tube through which the liquid flows; the speeds corresponding to this pulsation depend on the direction: in particular, the speeds of pulsations in a direction parallel to the walls are greater than in a perpendicular direction.

The energy of large-scale pulsations gradually turns into smaller and smaller scale pulsations, where for scales which are small in comparison with the diameter of the tube the pulsations are isotropic. According to the theory of A. Kolmogorov [362] the total energy of such pulsations of a scale $\leqslant \lambda$ is proportionate to $\lambda^{2/3}$. The rule remains valid because the transition of the energy to smaller-scale pulsations is not accompanied by a notable dissipation of energy (conversion into heat), i.e., for scales which are large in comparison with certain critical values of λ_o (the inner scale of turbulence). In the range $\lambda \angle \lambda_o$ the decrease in energy in the transition to smaller-scale pulsations is much faster.

Only the spectrum of pulsations at an immobile point may be studied experimentally, i.e., the value of function $F(\nu)$, indicating which part of the turbulent energy falls within a pulsation with velocity $\geqslant \nu \sec^{-1}$, recorded by a stationary observer (this is called the Eiler speed of pulsation). In the transition to a system of cocordinates moving with a stream with neutralized velocity U, we obtain the "scale spectrum" of pulsations, i.e., the $\Omega(\lambda)$ functions, indicating the fraction of the turbulent energy which is allotted per length of pulsation of scale $\leqslant \lambda$. However, the velocity spectrum of pulsations in a moving system is unknown (this speed is called the Lagrange speed).

Using the data of Simmons and Saltor [363], in which the spectrum of pulsations is measured in a 120-cm aerodynamic tube with a grid with 75-mm apertures, the degree of turbulence, i.e., the ratio u/U, where u is the total average quadratic speed of pulsation is equal to 0.03 and hardly depends upon Re_f. Thus, turbulent energy constitutes approximately 0.0009 of all the energy of the stream. The present author has constructed Table 24 from results obtained at $U = 7.5$ m·sec^{-1}, in which u_λ indicates the speed corresponding to pulsations of the scale $\leq \lambda$.

TABLE 24

SPECTRUM OF TURBULENCE IN AN AERODYNAMIC TUBE

AT $U = 7.5$ m·sec^{-1} AND $Re_f = 600{,}000$

λ, cm	$\Omega(\lambda)$	$u_\lambda/U \cdot 10^2$	u_λ, cm·sec^{-1}	$u_\lambda/\lambda^{1/3}$	λ/u_λ
37	0.74	2.6	19.5	5.9	2.0
19	0.58	2.3	17.3	6.5	1.1
7.5	0.36	1.8	13.5	6.9	0.55
3.7	0.19	1.3	9.8	6.3	0.37
3.0	0.13	1.1	8.3	5.8	0.36
2.0	0.07	0.8	6.0	4.8	0.33
1.5	0.05	0.67	5.0	4.4	0.30
1.0	0.019	0.41	3.1	3.1	0.32
0.75	0.008	0.26	1.9	1.7	0.40
0.50	0.0025	0.14	1.1	0.9	0.45

As is seen from the data of Table 24 the ratio $u_\lambda/\lambda^{1/3}$, which according to the principle of A. Kolmogorov must remain constant, begins to decrease sharply at approximately $\lambda = 2$ cm. A similar result is obtained at $U = 10.5$ m·sec^{-1}. Thus the inner scale of pulsation λ_o in an aerodynamic tube at Re of the order 10^6 has a value on the order of one cm. Because considerable error is possible in the experimental study of turbulence spectra this derivation should be proved with other experimental data. The following values of λ_o may be obtained from the formulas and experimental data of A. Obukhov and A. Yaglom [364]: 1.1 cm at $U = 12.2$ m·sec^{-1}, 0.7 cm at $U = 24.4$ m·sec^{-1}, and 0.6 cm at $U = 30.5$ m·sec^{-1}, i.e., the same order of values of λ_o. According to Obukhov [365] $\lambda_o = 0.5$ cm in atmosphere at a height of

1.5 m, and according to Taylor [366] λ_o = 2 cm at a height of 2 m, and 13 cm at a height of 30 m. According to Obukhov and Yaglom the lower internal scale $\Omega(\lambda)$ is proportional to λ^2, and according to Heisenberg [367] is proportional to λ^6, i.e., it decreases very rapidly with a decrease in the scale.

As already has been mentioned, the distribution of turbulent energy in the function of Lagrange speed or pulsation periods cannot be found directly from experimental data. The former are necessary for solution of the problem of the degree of increase in particles by turbulent pulsations. This problem may be solved very roughly in the following way. Assuming that pulsations with the scale λ are developed in motion in a stream with speed U of vortexial cords of diameter λ, the axes of which are perpendicular to the direction of the stream. The average speed of pulsation u may be taken as the speed of circulation at a distance $\lambda/4$ from the axis of the cord. Then, the Lagrange period of the corresponding pulsation t_L is equal to $0.5\pi\lambda/u_\lambda$, and the Eiler period $t_E = 2\lambda/U$. The relationship t_E/t_L is congruent to u/U, i.e., in the first approximation is equal to the degree of turbulence. The complete (conventionally 99%) blowing away of particles by pulsations, in conformance with Figure 20, takes place at $\tau/t_L \angle 0.02$, i.e., $\tau \angle 0.01$ according to Table 13, or at $r \angle 30\mu$ for particles with density 1.

Using the value for τ calculated under paragraph 18 for super-Stock particles at a typical average speed of pulsation of 30 cm/sec, or τ = 0.1 sec at r = 0.1 mm and τ = 6.3 sec at r = 1 mm and density 1, we find that in the first case the degree of blowing away of particles is 70%, and 2 % in the second case. Thus particles with size on the order of 1 mm practically are not involved in the pulsations of the medium.

However, these calculations are very rough, even though they apparently give the correct order of greatness of particles which are blown away and which are not blown away by the pulsations. Accurate solution of the problem of the degree of blowing away of particles is possible either through study of the turbulence spectrum with an instrument which is moving at the speed of the stream, or through ultramicroscopic observation in a stream which contains both very small and coarse particles.

The above problem is very important also in the solution of another basic task in the mechanics of aerosols, the vertical distribution of particles in a horizontal turbulent stream, which was first investigated by W. Schmidt [369]. If in the turbulent flow of an aerosol in a horizontal canal the precipitate is blown away and enters a state of suspension (this occurs, for example, in the pneumatic mixing of materials in a dust form), then a determined stationary distribution of particles according to height must be established in the canal. The number of particles passing down through a horizontal area one cm^2 per second under the influence of gravity is equal to $V_s n$. The number of particles passing through this area in the opposite direction because of turbulent diffusion is equal to $D_t\, dn/dz$ where D_t is the coefficient of turbulent diffusion of particles. Equating these expressions, we have

$$\frac{dn}{n} = -\frac{V_s}{D_t}\,dz, \tag{45.1}$$

from which we obtain

$$\ln\frac{n}{n_0} = -V_s\int_0^z \frac{dz}{D_t}, \tag{45.2}$$

where n_0 is the concentration at the bottom of the canal, and z is the distance from the bottom. Because D_t rapidly varies with z close to the botton, but varies slowly at greater distances from the bottom, D_t may be taken as constant in the central region of the canal, and (45.2) takes the form

$$\ln\frac{n}{n_0} = -\frac{V_s z}{D_t}. \tag{45.3}$$

It is emphasized that the distribution expressed by these formulas occur only in a stationary state, i.e., if the particles which have precipitated on the bottom of the canal are returned to the suspended state by the action of the stream. In the opposite case, it may be readily assumed that a large part of the aerosol succeeds in precipitating until this distribution is attained.

The theory of vertical distribution of particles in suspension in a turbulent stream which has been described herein was criticized by several hydrologists [370], the main objection being the usual assumption that the coefficients of turbulent diffusion of particles and the medium are equal. It may be seen from the foregoing, that for aerosol particles with $r \lesssim 30\mu$ this postulate apparently is justified. Furthermore, in the general expression for the coefficients of diffusion $D \approx l^2 t$, where l is the length of the "path," and t is the time consumed in traversing this path, in the case of turbulent diffusion l is the scale and t is the period of pulsation. Because of this, at incomplete blowing away of particles the relationship between the coefficients of turbulence of the diffusion of particles and the medium D'_t/D_t is equal to the relationship between the squares of the amplitude of pulsation of particles and medium, or the "degree of blowing away" of particles, calculated as described above. Formula (45.2) apparently remains applicable in the case of incomplete blowing away when the coefficient D'_t used in the formula has been derived in this manner.

Mention also must be made of the work of B. Brounshteyn and O. Todes [371]. Starting with the Bol'tsman distribution of concentration in a field with tension $n = n_0 \exp(-mgz/kT)$ and taking an analogy from the gas kinetics formula $\frac{3}{2}kT = mv^2/2$, where v^2 is the average square of the speed of pulsation of a particle and m is its mass and, consequently, $n = n_0 \exp(-3gz/v^2)$, these researchers used the formula [372] $v = [3\gamma_g/(\gamma_g + 2\gamma)]u$, where u is the speed of pulsation of the medium, for the derivation of a value for y which is correct for the movement of a particle in an ideal liquid. In the opinion of the present author the formal application of gas kinetics formulas to the movement of particles suspended in a turbulent medium is not justified. According to Brounshteyn and Todes the vertical distribution of particles does not depend upon the size of the particles which, however, contradicts experience. Furthermore, according to this method of calculation a speed of pulsation of the air of the order 1 km·sec^{-1} would be required by the condition in which the concentration of the aerosol would decrease no more than two-fold at a height of 10 cm!

More or less detailed experimental research on the vertical distribution of particles in a turbulent stream has been done only in water suspensions. In the experiments of Vanoni [373] D_t has been calculated for the profile of the speed of flow (see following paragraph). In the work of Kalinske [374] D_t was determined experimentally for the dispersion of a water solution jet in a stream. The vertical distribution of sand particles with radius on the order of several score microns (up to 70μ) which was found in these experiments more or less satisfactorily agreed with formula (45.2).

The only measurements on aerosols were conducted by M. Kalinushkin [375], who investigated the distribution of saw dust and other dust particles in a 25-cm horizontal round tube at $U = 10$ to 17 m·sec^{-1}. It is noted that in round tubes turbulent flow usually is accompanied by rotation of the gas around the axis of the tubr, with the result that the vertical distribution is transformed into radial distribution, in which the concentration of particles is maximum at the periphery of the tube and decreases in the direction of the tube's axis. To obtain normal vertical distribution in a round tube rotation must be eliminated by means of a screen. The data of M. Kalinushkin lie well within a straight line graph, corresponding to formula (45.3).

The data of Sherwood and Woertz [376] also may be introduced here, in which D_t was measured in an air stream in a canal with right-angle cross section, 5.3 cm in height and with a high ratio of width to height. At $Re_f = 10,000$, excluding the thin layers at the upper and lower wall surfaces, D_t was found to be practically constant, which is expressed by the empirical formula

$$D_t = 0{,}044\,\nu\,Re_f^{0{,}75} \tag{45.4}$$

where ν is the kinematic viscosity of the gas. For particles with $r = 5\mu$ and density 1, at $U = 13$ m/sec ($Re_f = 40,000$) the value of V_s/D_t was found to be equal to 0.016, and the ratio of concentration at the lower and upper walls in this case was 1.1, i.e., we have an almost uniform distribution for all sections of the channel.

We have described the action of a turbulent stream on particles suspended in it. The opposite problem, of the action of the dispersed phase on its carrier turbulent stream also is of great interest. From experiments it is known that the critical value of the Re_f number in clay suspensions is larger than in clear water [377, 378]. In the experiments of Vanoni [373] with sand suspensions in an open channel a marked decrease in the coefficient of turbulent viscosity was observed and, consequently, hydraulic resistance in comparison with clear water; this was also accompanied by a distinct drop in the degree of turbulence of the stream, and in the coefficient of turbulent diffusion.

The cause of these phenomena is the following: as we have seen, every particle of the dispersion phase participating in the pulsations of the medium is in continuous motion in respect to the layer of the medium adjacent to it due to the attraction of gravity. This is accompanied by the dispersion of mechanical energy (conversion into heat) which may be removed only from the energy of pulsation. This must result in a decrease in the degree of turbulence. Because the amount of energy dissipated by the particles per unit time is proportionate to the product of the weight of a particle and its speed of precipitation, i.e., the mass of a particle to the degree 5/3 ($m^{5/3}$), at a constant gravimetric concentration $c = nm$ the decrease in the degree of turbulence must increase with the height of the particles. The observation introduced under paragraph 29, that the resistance of a cyclone decreases with an increase in the concentration of dust, explains the decrease in turbulence under the influence of suspended dust. This already has been proposed by many researchers [379]. (In an article published in 1951 [380] the present author described a decrease in turbulence by the dissipation of energy caused by the relative motion between the particles and the medium caused by incomplete blowing away of particles by pulsations. As is apparent from the above this effect must be very small.)

The theory of this phenomenon was developed by G. Barenblatt [381]. The size of this effect is determined by the value of the nondimensional expression, which in the case of an aerosol has the form

$$K = -\frac{g\,dc/dz}{\gamma_g (dU/dz)^2} (>0), \qquad (45.5)$$

where z is the distance from the bottom and U is the average speed of flow.

At $K \ll 1$ the dispersion phase has no influence on the degree of turbulence nor on the profile of the speed of flow: only in this case is formula (45.2) applicable for the vertical distribution of concentration. At K equal to unity the mean-square speed of pulsations u is expressed by the formula $u = u_0(1 - K)^{1/4}$, where u_0 is the value of u for a pure gas. In this case formula (45.2) is not applicable and calculation of the distribution of concentration is complicated.

46. Precipitation of an Aerosol in Turbulent Flow

In turning to the problem of the precipitation of an aerosol in turbulent flow it is noted that almost all which has been said under paragraph 44 on gravitational precipitation in the presence of convection and artificial mixing is valid also in respect to turbulent flow. Because the mean-square pulsation speed in a direction perpendicular to the line of flow is approximately equal to 0.03 to 0.1 $\bar{U}$, where $\bar{U}$ is the average speed of flow [363. 382. 376. 383], the speed of precipitation of particles with radius less than 10μ in a horizontal tube is considerably less, even at a value of $\bar{U}$ of several meters per second, than the vertical component of the average pulsation speed, and these particles are more or less evenly distributed throughout all sections of the tube. The number of particles precipitating per second per unit length of a tube with radius R is $2RnV_s = 2Rgn\tau$, where n is the concentration of particles in the stream. During the passage of an aerosol layer one cm thick through a path dx, i.e., during time $dx/\bar{U}$, $2Rgn\tau\, dx/\bar{U}$ particles precipitate from the layer. Since the volume of this layer is equal to πR^2, we have the equation

$$-\frac{dn}{dx}=\frac{2Rgn\tau}{\pi R^2\bar{U}}=\frac{2n\tau g}{\pi\bar{U}R}. \qquad (46.1)$$

From this, analogously to (44.21), we derive the formula

$$n=n_0\exp\left[-\frac{2\tau g x}{\pi\bar{U}R}\right], \qquad (46.2)$$

which indicates the variation of the concentration of particles of a given size in the function of an aerosol passing through the path x. The speed of precipitation of the aerosol, which is proportionate to n, decreases exponentially in the direction of flow.

These considerations also are applicable to the theory of the cyclone. Assuming that turbulent mixing of the aerosol also takes place in a cyclone and, in conformance with formula (29.1), that the speed of flow at the external wall of the cyclone is equal to $U_0/2$, the radial speed of particles is equal to $U_0^2\tau/4R_2$, and the number of particles precipitation per cm^2 of wall surface per second is

$$I = \frac{nU_0^2\tau}{4R_2}. \tag{46.3}$$

Indicating the height of the spiral path in the cyclone by L/s (L is the height of the cyclone, and s is the number of turns), IL/s particles will precipitate per cm length of this spiral per sec. The number of particles which enter the cylcone per second, ascend the spiral and do not precipitate along its length is $HhnU_0$, where H and h are the height and width of the entrance opening of the cyclone. Therefore, we have

$$-\frac{1}{n}\frac{dn}{dx} = \frac{nU_0^2\tau L/4R_2 s}{nHhU_0} = \frac{U_0\tau L}{4R_2 sHh}, \tag{46.4}$$

where dx is the differential of the length of the apiral. Thus, the entire length of the latter $\approx 2\pi R_2 s$. and, integrating equation (46.4), we have for the relationship between the number of particles entering and leaving the cyclone, the formula

$$\ln\frac{n}{n_0} = -\frac{U_0\tau L\cdot 2\pi R_2 s}{4R_2 sHh} = \frac{\pi U_0\tau L}{2Hh} \approx \frac{\pi U_0\tau s}{2h} \tag{46.5}$$

(taking the height of the spiral path L/s as equal to the height of the entrance opening H). Thus, the efficiency of the cyclone is

$$\mathit{Э} = \frac{n_0 - n}{n_0} = 1 - \exp\left(-\frac{\pi U_0\tau s}{2h}\right), \tag{46.6}$$

i.e., considerably less than that calculated without taking account of turbulent mixing (formula (29.14)).

Although the lack of accurate experimental data on the efficiency of precipitation in cyclones as a function of the size of particles prevents the conduct of a quantitative test of formula (46.6), it is beyond doubt that it is closer to reality than formula (29.14). In conformance with the latter formula, particles with determined, large overall dimensions must precipitate completely. An actual 100% precipitation, even of large particles, is not observed in cyclones, but the efficiency of precipitation does increase continuously with an increase in particle size [384], which may be seen from formula (46.6).

Turbulence has a similar effect on the precipitation of highly dispersed aerosols in a condenser (paragraph 27): at a transition from laminar to turbulent flow the strength of the current at a given tension is considerably less than the strength of flow theoretically calculated for a laminar system [181].

Before discussing the diffusion precipitation of an aerosol from a turbulent stream several important facts will first be cited.

In the experimental investigation of the distribution of average speed of turbulent flow in tubes and in open channel it has been established that under these conditions a logarithmic profile of speeds, $U = a \ln z + b$, is observed, where U is the average speed at distance z from the wall. Proceeding on the ides of the "length of the path of mixture" (a value which plays the same role in the theory of turbulent viscosity and diffusion as is played by the length of the free path of a molecule in the theory of molecular viscosity and diffusion), Prandtl' [385] derived the formula

$$U / U^* = \frac{1}{\varkappa} \ln \frac{zU^*}{\nu} + C \qquad (z > \delta_L, \text{ see below}) \qquad (46.7)$$

in which $\varkappa$ is the constant of Karman, which combines the length of the path of mixture l with the distance from the wall according to the equation

$$l = \varkappa z; \qquad (46.8)$$

C is a constant which cannot be determined by theoretical means, and U^* is a value with a dimension of speed called the "dynamic speed," or the "speed of friction," and is linked through the equation

$$U^* = \sqrt{\tau / \gamma_g} \qquad (46.9)$$

with the impulses τ transmitted per second by the turbulent pulsations through a parallel wall of area one cm^2. Close to the wall τ may be considered constant (independent of z), and consequently equal to the force of friction between the flowing gas and the wall per cm^2 of the latter. The values $\varkappa = 0.4$, $C = 5.5$ were found through experiments, and so formula (46.7) may be written in the form

$$U / U^* = 2{,}5 \ln \frac{zU^*}{\nu} + 5{,}5 = 5{,}75 \lg \frac{zU^*}{\nu} + 5{,}5 \quad (z > \delta_L). \qquad (46.10)$$

It is noted also that U^* has the same order of magnitude for the mean square speed of turbulent pulsations u'.

This formula is not applicable at very small distances from the wall because at $z \longrightarrow 0$ it leads to $U \longrightarrow -\infty$, since at the wall surface the speed of flow is equal to zero. Because of this a thin laminar layer of thickness δ_L must be assumed at the wall surface, in which the gas moves in accordance with the ordinary laws of the flow of a viscous liquid, and the impulse is transmitted through molecular viscosity. Thus, $\tau = \eta\, dU/dz$ at $z < \delta_L$, from which it follows that in a laminar layer

$$U/U^* = \frac{\tau z}{\eta U^*} = \frac{U^{*2}\gamma_g z}{U^*\eta} = \frac{U^* z}{\nu} \qquad (z < \delta_L). \qquad (46.11)$$

Experience shows that the profile of speeds at $zU^*/\nu < 5$ is expressed by formula (46.11), at $zU^*/\nu > 20$ by formula (46.10), and in the intermediate zone is expressed by a continuous transitional curve [385]. Because of this the thickness of the laminar layer is of the order of magnitude

$$\delta_L \approx 10\nu/U^*. \qquad (46.12)$$

Using the idea of "turbulent viscosity," the equation

$$\tau = \eta_t\, dU/dz \qquad (z > \delta_L). \qquad (46.13)$$

may be written.

Inserting in the latter the expression from (46.7) for dU/dz and the expression from (46.9) for τ, we have $\eta_t = \gamma_g U^* z\varkappa$. From which the following formula for the "kinematic turbulent viscosity" $\nu_t = \eta_t/\gamma_g$ is obtained:

$$\nu_t = \varkappa U^* z. \qquad (46.14)$$

Because the mechanism of turbulent transmission of an impulse and mass are identical, the coefficients of turbulent diffusion D_t and the viscosity ν_t must have approximately the same value. Actually, according to Prandtl' $D_t/\nu_t = 1.4$ to 2.0, and according to the experiments of Sherwood and Woertz [376] $D_t/\nu_t = 1.6$. Using these values the coefficient of turbulent diffusion may be expressed by the formula

$$D_t = 1{,}5 - 2{,}0 \cdot U^* z \varkappa \approx 0{,}6 - 0{,}8\, U^* z \qquad (z > \delta_L). \qquad (46.15)$$

However, this formula is applicable only at $z > \delta_L$, i.e., outside of laminar flow. There are two points of view concerning the mechanism of diffusion inside this layer: according to Prandtl' and Taylor [387] turbulent pulsations are lacking inside a laminar layer, i.e., the transmission of substance occurs solely through molecular diffusion (the same as the transmission of momentum), and according to Landau and Levich [388] turbulent pulsations penetrate the laminar flow, dying out only at the wall surface, and in which D_t is proportional not to the first, but to the fourth degree of z, or

$$D_t \approx U^* z^4 / \delta_L^3. \tag{46.16}$$

At the wall surface there is a "diffusion substratum" of thickness δ_D, in which molecular diffusion predominates over turbulent diffusion. Here $\delta_L \gg \delta_D$. Thus, according to Prandtl' and Taylor the border of the diffusion layer, which in appearance coincides with the border of the laminar layer, is determined by the fact that on the former the coefficients of turbulent and molecular viscosity are equal, and according to Landau and Levich the border is determined by the fact that at this layer the coefficients of turbulent and molecular diffusion coincide. In the cases in which $\nu_t/\nu \approx D_t/D$, these borders actually do coincide, and the hypothesis of Prandtl' and Taylor appears applicable. Assuming that $\nu_t/D_t \approx 1$, it is necessary that the Schmidt number $Sc = \nu/D$ also be of the order of unity, which actually occurs, for example, in the diffusion of gases. However, in aerosols $Sc \gg 1$, i.e., $\nu_t/\nu \ll D_t/D$. Because in the zone adjacent to the wall the intensity of turbulent pulsations and the length of the mixing path and, consequently, ν_t and D_t in all cases decrease as the wall is approached. However, ν and D are constant values, and from the foregoing it follows that $D_t/D = 1$ at a considerably shorter distance from the wall than does ν_t/ν, i.e., δ_D for an aerosol must be considerably less than δ_L. In addition, under a given system of flow, i.e., at given δ_L the value of δ_D cannot be constant, but must vary sympathetically with D.

In particular, according to Landau and Levich, from the determination of δ_D it follows that $D = D_{t\,(z=\delta_D)} \approx U^* \delta_D^4 / \delta_L^3$, from which

$$\delta_D \approx D^{1/4} U^{*\,-1/4} \delta_L^{3/4} \approx \frac{0{,}57\delta_L}{\mathrm{Sc}^{1/4}}. \qquad (46.17)$$

In calculating the speed of diffusion precipitation of an aerosol on the walls of a tube changes in the concentration of the aerosol in the direction of flow are ignored, i.e., it is assumed that the concentration does not depend upon time, upon the x coordinates takin in the direction of flow, nor even upon the distance from the wall z . In view of the low speed of diffusion precipitation and the comparatively high speeds of flow under a turbulent system this simplification is completely justified. In this case the same number of particles I pass through every parallel wall of one cm^2 area per second, under which condition because of the great size of the coefficient of turbulent diffusion at points far from the walls the concentration of the aerosol may be taken as constant (n_0) throughout, with the exclusion of the wall layer. Indicating the effective coefficient of diffusion by D_E , which includes both the molecular and turbulent transmission of substance, the equation may be written in the general form

$$I = D_E \frac{dn}{dz} = \text{const.} \qquad (46.18)$$

In calculating I it is necessary to start from a definite hypothesis on the size of D_E in the function of the distance from the wall. It is easiest to make this computation on the basis of the Prandtl'-Taylor hypothesis, i.e., assuming that at $z < \delta_L$ only molecular diffusion occurs, but at $z > \delta_L$ only turbulent diffusion occurs. In the laminar layer

$$I = D \frac{dn}{dz} \quad (z < \delta_L), \qquad (46.19)$$

from which $n = Iz/D + C_1$ or, since at $z = 0$ $n = 0$

$$n = Iz / D \quad (z < \delta_L). \qquad (46.20)$$

In the turbulent zone D_E is expressed by the formula (46.15) and, indicating the coefficient 0.6 to 0.8 by α , we find

$$I = \alpha U^* z \frac{dn}{dz} \quad (z > \delta_L), \qquad (46.21)$$

whence

$$n = \frac{I}{\alpha U^*} \ln z + C_2. \qquad (46.22)$$

Indicating the distance from the wall by h, at which n has the constant value n_0, we obtain

$$n_0 = \frac{I}{\alpha U^*} \ln h + C_2 \qquad (46.23)$$

or, eliminating C_2 through (46.22) and (46.23),

$$n = n_0 + \frac{I}{\alpha U^*} \ln \frac{z}{h}. \qquad (46.24)$$

At $z = \delta_L$ the expressions (46.20) and (46.22) must coincide. Substituting z for δ_L in the latter and equating, we obtain

$$I = \frac{n_0 D}{\delta_L + \frac{D}{\alpha U^*} \ln \frac{\delta_L}{U^*}} \qquad (46.25)$$

or, substituting U^* with $10\nu/\delta_L$ according to (46.12)

$$I = \frac{n_0 D}{\delta_L + \frac{D\delta_L}{10\alpha\nu} \ln \frac{\delta_L}{n}} = \frac{n_0 D}{\delta_L \left(1 + \frac{1}{10\alpha \,\mathrm{Sc}} \ln \frac{\delta_L}{h}\right)}. \qquad (46.26)$$

Since in aerosols $\mathrm{Sc} \gg 1$, the second members in the parentheses must be ignored, and we finally obtain

$$I \approx \frac{n_0 D}{\delta_L}. \qquad (46.27)$$

Thus, according to Prandtl'-Taylor, the speed of diffusion precipitation is proportional to the first degree of the coefficient of thermal diffusion of an aerosol in which, as is apparent from formula (46.27), the concentration of an aerosol must be considered practically constant right up to the border of the laminar layer, which has been done in (44.14) and (44.15). An analogous computation based on the hypothesis of Landau and Levich is considerably more complicated, and only the final result is included here, in the form of the formula

$$I = \frac{\beta n_0 D}{\delta_D}, \qquad (46.28)$$

where β is the numerical coefficient of the order of unity, the value of which may not be determined theoretically. Since, according to (46.17) δ_D is proportional to $D^{1/4}$, I is proportional to $D^{3/4}$. This dependent relationship is explained by the fact that when the coefficient of thermal diffusion is decreased turbulent pulsations carry particles closer to the wall and thus partially compensate for the decrease in D.

Although the description of the process of diffusion precipitation of an aerosol from a turbulent stream in accordance with the hypothesis of Landau and Levich undoubtedly gives a better approximation of reality than according to the Prandtl'-Taylor theory, the mechanis, of this process in the final analysis may be explained only through experiment. It would be advantageous, in the latter, to use isodispersion fogs of sulfuric acid produced by the La Mer [6] generator, which would greatly facilitate determination of the amount of precipitate on the walls. Taking fogs with various sized particles, the dependence of I on D could be found, from which the law of variation of D_t with changes in the distance from the wall could be established.

It is suggested that formulas (46.27) and (46.28) are suitable for practical use. In this respect experimental data [389] obtained in smooth tubes at $Re_f < 10^5$ may be used, according to which

$$U^* \approx \frac{0{,}16\, U_M}{Re_f^{1/8}} \approx \frac{0{,}2\bar{U}}{Re_f^{1/8}}, \qquad (46.29)$$

where U_M is the speed of flow at the axis of the tube, and $\bar{U}$ is the average speed. From this we have

$$\delta_L = 10\nu / U^* \approx 50\nu\, Re_f^{1/8} / \bar{U} = 100\, R / Re_f^{7/8} \qquad (46.30)$$

(R is the radius of the tube), and

$$I = \frac{Dn_0}{\delta_L} \approx \frac{Dn_0\, Re_f^{7/8}}{100\, R} \text{ (from Prandtl'-Taylor)} \qquad (46.31)$$

Further, in accordance with (46.17)

$$\delta_D \approx 57\, RD^{1/4} / Re_f^{7/8}\, \nu^{1/4} \qquad (46.32)$$

and

$$I = \frac{\beta n_0 D}{\delta_D} = \frac{\beta D^{3/4} n_0\, Re_f^{7/8}\, \nu^{1/4}}{57\, R} \text{ (from Landau-Levich)} \qquad (46.33)$$

No experimental data are available on the non-evoked sedimentation of the precipitation of an aerosol on the walls of a tube under turbulent flow. Alexander and Coldren [390] atomized water by means of a jet through the axis of a short horizontal tube and measured the concentration of fog at various points of the tube, from which they calculated the speed of precipitation of the fog on the walls. However, in these experiments the drop in concentration from the axis of the tube to the periphery obviously has nothing in common with diffusion precipitation, and is related to the known profile of concentration in a free stream.

Turning to the inertial precipitation of an aerosol from a turbulent stream, it is possible to think that because of complete blowing away of particles by the turbulent pulsations inertial precipitation must be absent in this case.. However, the calculations in and following Table 24 are related to the central particles in a turbulent stream. Furthermore, observation has shown that the speed of pulsations increases in proportion to proximity to the wall, and begins to decrease only at a very small distance from the wall [382, 391, 392]. On the other hand, the length of the mixing path and, consequently, the diameter of the vortexes, continuously decrease as the wall is approached, but pulsations perpendicular to the wall observed by ultramicroscopic methods are of a very small scale, even when the distance from the wall is on the order of several microns [382]. Thus it is possible that near the walls a sufficiently large inertial force acts upon the particles to enable precipitation.

Unfortunately there are no experimental data on this problem. A copious precipitate of dust frequently is found on the walls of vertical tubes, which possibly is caused by inertial precipitation from a turbulent stream. It must be remembered that at a very high rate of flow solid particles which have precipitated may be blown away by the stream, and therefore the lack of a precipitate does not necessarily indicate the absence of precipitation. The solution of this problem would require experiments with fogs, or the walls must be smeared with a viscous liquid.

On the other hand, precipitation on walls caused by "constant" local vortexes which are caused by flow around various obstacles, etc, is commonly known. Figure 66 shows a photograph of a dust precipitate formed during the course of one year on the wall of a ventilated room in the vicinity of a telephone wire, caused by a thin stream of air issuing from a vertical slit located to the left of the wire. The dark, narrow line of dust is in front of the wire, and the thicker band is behind the wire. In addition, there is an inertial precipitate on the frontal surface of the wire, which was formed in the absence of vortexes. The vortexes formed by air flowing around an obstacle are shown diagrammatically in Figure 67. A very similar picture was observed in the theoretical investigations of N. Zhukovskiy [393] in the formation of snow banks near obstacles (fences), however, in this case the main effect was due to the sedimentation of arrested snow instead of inertial precipitation from vortexes.

Vortexial precipitation also is observed under known conditions by the flow of freely distributed particles around an obstacle, in which a precipitate is formed not only on the windward, or upstream side, but also on the leeward side of the body. According to the experiments of Yeomans [248] at a speed of flow of 8 $m \cdot sec^{-1}$ 2.5-fold more droplets of fog with $r = 5.6\mu$ precipitated on the leeward side of a glass disc 7 cm in diameter placed perpendicularly to the stream, than on the windward side. According to Landdahl [236] more particles precipitated on the leeward side of glass plates 2.5 cm wide at a speed of 0.5 $m \cdot sec^{-1}$ than at a speed of 1.5 $m \cdot sec^{-1}$. In the experiments of Asset and Pury [394] droplets of a isodispersed fog with $r = 6.5\mu$ did not precipitate on the leeward side of glass cylinders of 7.5 cm diameter nor on the hair of the uncovered forearms of persons, at a speed of flow of 2.3 $m \cdot sec^{-1}$. Precipitation on the hairs of the covered forearms of persons was approximately 4-fold less on the leeward side than on the windward. Apparently definite hydrodynamic conditions are necessary for precipitation to take place on a leeward surface: on one hand, the speed of flow in the vortexes must be adequate for inertial precipitation of particles, and on the other hand the vortexes must not be too rapid in the direction leading away from the obstacle around which the stream is flowing.

This possibly is related to the formation of a dense dust precipitate on the walls of diffusors at the exit; because this is the place where the vortexes tear away from the walls.

47. Distribution of Aerosols in the Atmosphere

The problem of the movement of aerosols in the atmosphere (smoke issuing from factory chimneys, camouflage, and insecticide fogs produced by special generators, etc) has great practical importance from a technical and hygienic point of view, such as in the problem of the prevention of pollution of inhabited localities from industrial aerosols. This problem may be of great importance in connection with atomic bomb devices, the explosions of which create enormous clouds of radioactive aerosols which extend through the atmosphere, the concentration of which constitutes a danger to human beings at distances of several hundred miles from the explosion site. However, despite the fairly large number of experimental and theoretical investigations which have been conducted, this problem still has not been sufficiently studied, mainly because of the extremely great complexity and variability of atmospheric streams, which give rise to enormous fluctuations in variables which have not been encompassed by mathematical analyses. Because of this the experimental proving of various theories is extremely difficult.

The movement of aerosols in the atmosphere is composed of the movement of the air itself, and the relative movement of particles and air, which for not very large particles boils down to their precipitation under the influence of gravity. At the outset we shall ignore this precipitation, i.e., we shall consider the distribution of highly dispersed aerosols in the atmosphere. In this case the laws of the distribution of aerosols and of gaseous poluents, are the same. These have been discussed in detail in articles by Sheleykhovskiy [395], Andreyev [396] and Sutton [397, 398], and therefore we shall focus attention on the principal facet of the problem, without special attention to the practical details. The problem to be considered first is the mechanism of turbulent dissipation in the atmosphere, because this problem is not covered very fully in the literature, and occasionally it is incorrectly described.

The general character of the dissipation of an aerosol in the atmosphere is as follows: issuing from any point a cloud or a continuous aerosol stream moves along with the wind and simultaneously is dissipated by the action of atmospheric turbulence. Molecular diffusion has practically no role in this process with the exception of a very thin air layer at the surface of a body in contact with the aerosol.

Atmospheric turbulence has several specific traits which must be mentioned briefly here. In turbulent flow in tubes, channels, rivers, etc, at each point of the stream there is an average speed of flow U which has a practically constant value and direction and which may be easily determined experimentally, upon which are superimposed irregular turbulent pulsations. The changes in U, if they take place at all (such as daily or seasonal changes in the speed of flow of rivers), have a period several orders greater than the period of the largest pulsations. Furthermore, U considerably surpasses the mean-square speed of pulsations. Thus, in this case the border between the neutral flow and pulsations is very sharp.

In the atmosphere the wind velocity, measured by wind vanes, etc, continuously changes both in direction and intensity, in which the size of the variations monotonously increases with the time during which it is observed. Here we have a full spectrum of pulsations, beginning with very small pulsations with periods measured in hundredths of a second and ending in daily and annual variations in the speed of the wind. Because of this no border may be established between the average and pulsations, the speeds of which cannot be established in this case, and the average wind velocity measured by various meteorological instruments for one-minute periods may be considered the pulsation speed in observations extending over a year.

Considering here the mechanism of diffusion in a turbulent stream, we shall begin with the simple case of flow in a tube or channel. Taking an aerosol stream issuing continuously through a long, narrow slit at any point O (Figure 68) perpendicular to the stream, line O - O' in Figure 68 indicates the stream of neutral flow, and the curves OA and OB indicate the contour of an aerosol stream fixed at a given moment, determined by the condition that the concentration of the aerosol along these curves is equal, for example, to 10% of the concentration at the axis of the stream. The mean square of the deviation of particles from the fixed aixs O - O', registered by a stationary observer (Eiler diffusion), is expressed by the general equation

$$\overline{x^2} = 2D_t t, \tag{47.1}$$

(for simplicity D_t is taken as the usual coefficient of turbulent diffusion, constant for the entire field of flow). Thence, for the distribution of concentration of an aerosol in flow (curve II) in the case of a continuously moving source the following equation, which was introduced above under (39.19), is obtained

$$n = \frac{\Phi'}{\sqrt{4\pi D_t U x}} e^{-\frac{U z^2}{4 D_t x}}, \tag{47.2}$$

and from the foregoing it follows that this formula gives the average concentration per unit time, calculated according to general laws.

The situation is different in the case of mutual diffusion of particles in a turbulent stream, i.e., movement of the particles in respect to an observer moving at the same rate as one of the particles (Lagrange diffusion). Because Eiler diffusion occurs in all pulsations, only pulsations of the scale λ participate in Lagrange diffusion, and λ is of the same, or smaller order of greatness than that which exists between particles x. These pulsations continuously increase the distance between particles, i.e., they lead to expansion of the stream. Large-scale pulsations with $\lambda \gg x$ cannot, obviously, change the distance between particles, i.e., the width of the stream, but decrease its width and break up the stream into individual puffs. Momentary distributions of the concentration in the stream (curve I) obviously are determined by Lagrange diffusion. By way of illustration a photograph of smoke issuing from a factory chimney under various degrees of atmospheric turbulence [399] is included below (Figure 69).

The scale of pulsations which are able to change this condition increases with an increase in the width of the stream, i.e., an increase in x. In other words, the Lagrange coefficient of diffusion D_L increases continuously in proportion to the movement and widening of the aerosol cloud, which also is a peculiarity of turbulent diffusion. It is emphasized again, that this circumstance is of importance in the distribution of momentary concentrations, but not in time-average concentrations. By empirical procedures Richardson [400] found the following relationship for D_L:

$$D_L \approx 0{,}2x^{4/3}. \tag{47.3}$$

As A. Obukhov [364] indicates, formula (47.3) may be theoretically derived for the range of pulsation scales to which the theory of Kolmogorov may be applied.

It is not difficult to see that at $x = 0$, actually $D_L = 0$, because particles distributed in a dense line cannot be easily broken up by the action of turbulent pulsations. On the other hand, in several cases, such as a wide river, turbulent pulsations at two points fairly widely separated may become independent of each other. In this case formula (47.3) is inapplicable, and the Lagrange coefficient of diffusion is equal to the sum of both (Eiler) coefficients of diffusion at these points

$$D_L = D_{l_1} + D_{l_2} \qquad (47.4)$$

in the same way that this occurs in molecular diffusion (see equation (49.8) through (49.11)). Generally speaking $D_L \leqslant D_{l_1} + D_{l_2}$, always, and thus the neutral stream MON (see Figure 68) always is wider than the momentary profile AOB. (The article of M. Yudin [401], in which the present considerations on turbulent dissipation in relation to a fixed axis and to the axis of a smoke stream is given a much more strict formulation, came to the attention of the author only after work on the present book had begun.)

Turning to diffusion in the atmosphere, mention must be made first of the substantial difference between vertical and horizontal pulsations. Although wind velocity usually has a small vertical component this may be ignored in the first approximation and the average vertical speed of the wind may be taken as zero; this eliminated consideration at this point of pulsations and average velocity. The coefficient of vertical turbulent diffusion (Eiler) D_z, determined by the kinetics of the vertical volume in the atmosphere and which plays a very great role in meteorology, has been the subject of numerous theoretical and experimental investigations, and its magnitude, which depends upon the altitude above the earth, wind velocity, and other meteorological conditions, the character of the locality, etc, in the first approximation may be taken as unknown.

The situation is complex with respect to horizontal diffusion; as we have already seen it is not possible to demarcate the average, and pulsation speeds; therefore, in the given case, the wind velocity U_{obs} must be taken in time-fixed observations t_{obs} and the average velocity for time t_{obs} determined, and the difference between momentary values of wind velocity and U_{obs} must be attributed to pulsations. Similarly the movement of the aerosol is taken to consist of a regular movement with speed U_{obs}, and of dissipation of its pulsations. It is completely clear that with an increase in t_{obs} both the mean square speed and the scale of pulsations will speed up because all changes in wind velocity for all large intervals of time are determined from the latter; consequently, the Eiler coefficients of diffusion D_{tx} also will increase. Therefore, the larger the t_{obs} the greater is the widening of the "time-neutralized aerosol jet," and the lower is the average concentration of the aerosol at points of the stream which are located at a determined distance from its axis, i.e., the maximum concentration of the stream. Thus, according to the experiments of V. Ryazanov [402] the maximum concentration of SO_2 in the immediate vicinity of a point source (factory chimney) at $t_{obs} = 5$ min, one day, one month, one year corresponds to 4.5, 0.94, 0.75, 0.3 $mg \cdot m^3$, respectively. Unfortunately, the law for changes in the coefficient of horizontal diffusion with the time of observation is unknown, without which theoretical calculation of the concentrations in an aerosol issuing from a point source (or linearly with a length equal to the distance from the source) is impossible.

With the exception of formula (47.3) nothing is known of the value of the Lagrange coefficients of diffusion in the atmosphere, either, which are necessary for computation of momentary concentrations in a stream. In addition, formula (47.3) is only a very rough approximation because it does not allow for the degree of turbulence of the wind.

The first attempts at the establishment of a theory of dissipation of gaseous mixtures in the atmosphere [294] used general diffusion equations in which the coefficient of turbulent diffusion and the wind velocity were taken as constant and independent of the distance from the ground z . This case coincides with the problem described at the end of paragraph 39, and the concentration of the mixture at a determined point in space is expressed as a function of the character of the source by formulas (39.13), (39.14), (39.18), or (39.19). However, the results of this primitive theory were at considerable variance with experience: at an increasing distance from the source the actual decrease in the concentration was much greater than that indicated by the formula. In the opinion of Sutton [403, 398], which was very popular for a long time, the cause of this variance was the increase in the effective coefficient of diffusion at greater distances from the source. Because in experiments the average concentration of the mixture always is measured at known intervals of time, the theory of Sutton may be correct only in respect to the horizontal dissipation of a stream from a point source, and not for the vertical dissipation of a stream issuing from an infinitely long (or, in practice, long in relation to the distance from the latter) linear source.

The differential equation of diffusion in a stream issuing from a linear source passing through the starting coordinate and perpendicular to plane xz has the form (see equation (39.15))

$$U\frac{\partial n}{\partial x}=\frac{\partial}{\partial z}\left(D_{tz}\frac{\partial n}{\partial z}\right), \qquad (47.5)$$

in which U and D_{tz} are functions of the height z (wind directed along axis x). (In this and in the following paragraph n may indicate the calculated, the gravimetric concentration of an aerosol and the concentration of a gaseous mixture.) In this we ignore, as usually is done in problems of diffusion in a stream the diffusion in the direction of the x-axis. The regional conditions of this problem are the following: $n=0$ at $x=\infty$ or $z=\infty$, $n=\infty$ at $x=z=0$, $\int_{-\infty}^{\infty} Undz=\Phi'$, where Φ' is the output per one cm of the source. The latter condition arises from the stationary nature of the process and the law of conservation of matter. The form of function $D_{tz}(s)$ is determined by the method described under equations (46.11) through (46.16) for a known wind profile, i.e., function U(z); in this the coefficients of turbulent diffusion and viscosity are taken to be equal.

The case of normal (dry adiabatic) temperature gradient in the atmosphere has been subjected to the greatest amount of experimental research. It has been established that in this case the wind profile is logarithmic, but it differs somewhat from the profile indicated in equations (46.7) through (46.10) which occur in "smooth" tubes, because under ordinary meteorological conditions the surface of the earth is "coarse" and flow over the latter is expressed by the formula

$$U/U^{*} = \frac{1}{\varkappa} \ln \frac{z-d}{z_0}, \qquad (47.6)$$

where z_0 and d are coefficients which depend upon the degree of coarseness of the earth's surface (height and density of vegetation, etc) and on the experimentally determined wind velocity [404].

Unfortunately, insurmountable difficulties are encountered in the solution of equation (47.5) in the case of a logarithmic wind profile. These difficulties are avoided by approximation of the expressions (46.7) and (47.6) with the aid of the graduated functions

$$U = U_1 \left(\frac{z}{z_1}\right)^q, \qquad (47.7)$$

or

$$U/U^{*} = m \left(\frac{z-d}{z_0}\right)^q \qquad (47.8)$$

and through the selection of coefficients q and m so that in a determined field the values z of the graduated function are as close as possible to a logarithmic configuration [404].

Proceeding from his point of view cited above and from several postulates of the statistical theory of turbulence, Sutton [403] arrived at the conclusion by semi-empirical methods, that the mean square relative motion of particles during time t is expressed by the formula

$$\overline{(\Delta z_1 - \Delta z_2)^2} = \frac{1}{2} C^2 (Ut)^{m} = \frac{1}{2} C^2 x^{m}, \qquad (47.9)$$

in which C is a generalized coefficient of turbulent diffusion and s is connected with q in equations of the wind profile (47.7) or (47.8), as follows:

$$s = 2/(q+1). \tag{47.10}$$

Taking the value 1/7 for q , which according to experimental data corresponds to a normal gradient and smooth surface, Sutton obtained the value s = 1.75 . From formula (47.9) it follows that coefficient C is connected with the Lagrange coefficient of turbulent diffusion D_L, for which the formula $\overline{(\Delta z_1 - \Delta z_2)^2} = 2D_L t$, is valid in the relationship

$$D_L = \frac{C^2}{4} U^s t^{s-1} = \frac{C^2}{4} x^{s-1} U. \tag{47.11}$$

In solving the basic differential equation (47.5) Sutton ignored the difference between the Lagrange and Eiler coefficients of turbulent diffusion, and also variations in the wind velocity and variations in the coefficients of diffusion with height, taking for U a wind velocity at a determined height (2 m) . Thus, in addition to (39.18) and (39.19) he obtains

$$n = \frac{\Phi'}{\sqrt{\pi C_z^2 U^2 x^s}} \exp\left(-\frac{z^2}{C_z^2 x^s}\right) \tag{47.12}$$

for a continuously acting linear source, and

$$n = \frac{\Phi}{\pi C_y C_z U x^s} \exp\left[-\frac{1}{x^s}\left(\frac{y^2}{C_y^2} + \frac{z^2}{C_z^2}\right)\right] \tag{47.13}$$

for a point source. Here, y is the horizontal distance from the axis of the stream C_y and C_z are values of C under vertical and horizontal diffusion, for which Sutton [397] derived an expression as a function of velocity and gustiness (i.e., degree of turbulence) of the wind. These formulas will not be used in the present book (indicating only that C is proportional to U in the ratio $\frac{s}{2} - 1$) because Sutton [405] uses other values for C_y and C_z in addition to the latter in the examples he cites, in the function of the height of source H , which have been obtained experimentally under determined conditions (on an open plain used for pasture, at an adiabatic gradient and wind velocity 5 m·sec^{-1} at a height of 2 m) (Table 25).

TABLE 25

SUTTON'S COEFFICIENTS OF DIFFUSION AT NORMAL GRADIENT (in a system of units of g·m·sec)

H, м	0	10	25	50	75	100	200
C_y	0,21	0,21	0,12	0,10	0,09	0,07	0,05
C_z	0,12	0,12					

Thus, in order to agree with the experimental values of Sutton a value which decreases with altitude must be used for the C coefficients, whereas in actuality the intensity of turbulent diffusion increases with altitude up to z, on the order of several hundred meters [406]. From this it follows that formulas (47.12) and (47.13) are, in substance, empirical, and which, as well be seen below, concur with experience at an appropriate selection of the values of the coefficients C_y and C_z.

These formulas have been derived for the case of dissipation in an unlimited volume. The important problem of local conditions at the earth surface in the solution of differential equations of turbulent diffusion also must be considered. For substances with a non-absorbent surface (for example, inert gases) this problem is easily solved: the local condition at a non-absorbent surface consists of the fact that the gradient of concentration in a direction normal to a wall is zero, i.e.,

$$\partial n / \partial z = 0 \quad \text{at} \quad z = 0. \tag{47.14}$$

Strictly speaking, for aerosols, the particles of which do not rebound from the wall but adhere to it, it may be postulated that n = 0 at z = 0. However, in this case, in the solution of differential equations of diffusion changes in the effective coefficient of diffusion in the wall layer with the distance from the wall should be taken into account (see equations (46.20) through (46.28)), which should lead to great mathematical difficulties. Therefore, proceeding on the assumption that the speed of diffusion of an aerosol in the wall layer is very small, i.e., the compressed portion of particles which are carried to this layer by turbulent pulsations do not precipitate on the wall but are carried away again, in this case it may be assumed that the wall is "reflecting for turbulent diffusion," and the conditions of (47.14) may be retained.

If the aerosol source is at a height H above the soil, then in formulas (47.12) and (47.13) it follows that z may be substituted for z - H . The formulas obtained in this way are applicable for those values of x at which the cloud touches the earth. In order to obtain a solution which satisfies the conditions of (47.14) the following method must be used: a fictitious source is taken at an altitude symmetrical with the source in relation to the earth surface, i.e., a point or a line with coordinates z = -H , x = 0 , and the output of this source is the same as that of the given data. Under these conditions, in addition to (47.12) and (47.13) the formulas are obtained

$$n = \frac{\Phi'}{\sqrt{\pi C_z^2 U^2 x^s}} \left\{ \exp\left[-\frac{(z-H)^2}{C_z^2 x^s}\right] + \exp\left[-\frac{(z+H)^2}{C_z^2 x^s}\right] \right\} \qquad (47.15)$$

for a linear source, and

$$n = \frac{\Phi}{\pi C_y C_z U x^s} \exp\left(-\frac{y^2}{C_y^2 x^s}\right) \left\{ \exp\left[-\frac{(z-H)^2}{C_z^2 x^s}\right] + \exp\left[-\frac{(z+H)^2}{C_z^2 x^s}\right] \right\} \qquad (47.16)$$

for a point source.

In an experimental test of these formulas Sutton [405] placed the aerosol source close to the earth surface, at a height (H = 0) . At various distances x from the source, in the direction of the wind, the maximum concentrations in the cloud at given x were measured, i.e., at the earth surface level (z = 0) in the case of a linear source, and along the axis of the cloud (z = y = 0) in the case of a point source. In the latter case, as has been mentioned above, the maximum concentration decreases with time t , during the course of which samples of air are taken for analysis. However, under the conditions of Sutton's experiments (normal gradient), at t > 3 minutes this decrease was hardly noticeable. According to formulas (47.15) and (47.16) the maximum concentration is equal to

$$n_{\max} = 2\Phi' \Big/ \sqrt{\pi C_z^2 U^2 x^s} \qquad (47.17)$$

in the case of a linear, and

$$n_{\max} = 2\Phi / \pi C_y C_z U x^s \qquad (47.18)$$

in the case of a point source.

At a gradient close to normal, which occurs in these experiments, s is equal to 1.75, as has been mentioned above, i.e., in the case of a linear source n_{max} is proportional to $x^{-0.875}$, and in the case of a point source is proportional to $-x^{-1.75}$. It was actually found that n_{max} is proportional to $x^{-0.9}$ in the first case, and to $x^{-1.76}$ in the second case.

Furthermore, from formula (47.15), assuming the dependent relationship of C on U, it follows that n_{max} $C_z^{-1}U^{-1}$ $U^{-0.875}$. Thus at wind ratios of 1:2:3 the corresponding value of n_{max} must be taken at $1:2^{-0.875}:3^{-0.875}$ = 1:1/1.84:1/2.61. The actual relationship was found to be 1:1/1.84:1/2.77.

The absolute value of n_{max} at $x = 100$ m, $U = 5\ \mathrm{m \cdot sec^{-1}}$, $\Phi = 1\ \mathrm{g \cdot sec^{-1}}$, was found to be $2.0\ \mathrm{mg \cdot m^3}$ in the case of a point source ($C_y = 0.21$, $C_z = 0.12$), and in which $n_{max} = 1.6\ \mathrm{mg \cdot m^3}$ must be used. In the case of a linear source, at $\Phi' = 1\ \mathrm{g \cdot sec^{-1} \cdot m^{-1}}$ the theoretical value is $n_{max} = 33\ \mathrm{mg \cdot m^{-3}}$, and experimental value $31\ \mathrm{mg \cdot m^{-3}}$.

Assuming that at the "border" of a cloud the concentration is equal to 0.1 of the maximum concentration, taking $H = 0$ in formula (47.15), we find that the "height" of the cloud z_o issuing from a linear source is determined by the equation

$$0{,}1 = \exp\left(-\frac{z_0^2}{C_z^2 x^2}\right), \qquad (47.19)$$

i.e., at $x = 100$ m, $z_o = 10$ m. The value found experimentally was $z_o = 10$ m.

Similarly, the theoretical value of the "width" of a cloud issuing from a point source at the earth surface was found to be 34 m at a distance of 100 m, and the experimental value obtained was 35 m.

According to the experiments of Ye. Teverovskiy [407] formula (47.17) with coefficient $s = 2$, is applicable over a fairly wide interval of the value of the temperature gradient. Ye. Teverovskiy found an average value of 0.027 for C_z in the case of areas with even, abundant vegetation, and 0.086 for broken terrain with tall vegetation. At this value of C_z was found to be practically independent of the wind velocity. From this it is apparent that the data of Table 25 refer only to areas in which the experiments of Sutton were carried out.

A more satisfactory theory of the dissipation of a cloud issuing from a linear source at the earth surface at a normal gradient is given by Calder [408]. If formula (47.6) is taken for the wind profile and the coefficients of turbulent diffusion and viscosity in a vertical direction are taken to be equal, then the expression

$$D_{tz} = \varkappa U^{*} (z - d). \qquad (47.20)$$

is obtained for D_{tz} according to formulas (46.11) through (46.16).

Since d usually has a value on the order of several centimeters it may be taken that $d = 0$. Table 26 which has been constructed on the basis of the experiments of Deacon [409], is introduced by way of illustration. In Table 26 h' indicates the height of the grass upon which the measurements were taken; U_1 and U_2 indicate wind velocity at a height of 1 m and 2 m, for which the parameters d, z_o and U^* were computed: $\tau_o = \gamma_g U^{*2}$ indicates the force of friction of the wind per cm^2 of earth surface (see (46.7) and ff). It is noted that direct measurement of τ_o gave results close to those calculated by formula (47.6) [410], which indicates that this is a promising method of calculation.

From Table 26 the increase in turbulent diffusion with the force of the wind and the degree of coarseness of the earth surface is clearly apparent.

TABLE 26

WIND PROFILE AND TURBULENCE OVER GRASS COVERED AREAS AT NORMAL GRADIENT

h, см	U_1, см·sec^{-1}	U_2/U_1	d, см	z_0, см	U^*, см·sec^{-1}	τ_0, dyne·см$^{-2}$	D_{tz}, см^2·sec^{-1} at $z=100$ см
30—70	100	1,45	15	15,9	23,9	0,68	810
	200	1,35	16	8,8	35,5	1,51	1190
	300	1,32	21	5,6	45,4	2,47	1430
	450	1,28	32	3,0	57,8	4,00	1570
1,5	100—800	1,112	0	0,20	6,4—51	0,05—3,2	260—2100
3,0	100—800	1,140	0	0,71	8—65	0,08—5,0	320—2600
4,5	200	1,191	0	2,65	22,0	0,58	880
	450	1,170	0	1,74	44,5	2,38	1780

Taking U = const in the integration of equation (47.5), then at D_{tz} proportional to z, the following solution is easily obtained:

$$n=\frac{\Phi'}{\varkappa U^* x}\exp\left(-\frac{Uz}{\varkappa U^* x}\right), \qquad (47.21)$$

whence

$$n_{\max}=\frac{\Phi'}{\varkappa U^* x}. \qquad (47.22)$$

Formulas (47.21) and (47.22) were tested [408] under conditions closely approximating those of the experiments of Sutton. The coefficient z_0 was equal to 3 cm, wind velocity at a height of 2 m was 5 m·sec^{-1}, $d = 0$, and $U^* = 50$ cm·sec^{-1}. At $\Phi' = 1$ g·m^{-1} the theoretical "altitude" of the cloud at a distance of 100 m was 10.4 m, and the experimental value was 10.0 m. The maximum concentration was ($z = 0$), theoretical 44 mg·m^{-3}, and experimental concentration was 36 mg·m^{-3}. Furthermore, it was found that $n_{\max}$ was proportional to U^{-1} and x^{-1}, in conformance with formula (47.22).

The great advantage of formula (47.21) over the formula of Sutton consists of the fact that it has been derived fairly strictly: the only inaccuracy in the derivation of the former is the postulate that U is constant in the integration of equation (47.5). In addition, formula (47.21) contains only experimentally determined values, and not coefficients, the values of which may be selected by comparison with experimental data.

In another work Calder [404] approximated the logarithmic profile with the aid of a graduated function (47.8), from which he found D_{tz} in function z by the method indicated and solved equation (47.5), still not postulating $U = \text{const}$, i.e., completely strict derivation. The formula obtained in this way is not included herein because of its complexity, but it gives a better comparison with experimental data than formula (47.21).

Deacon [409] gives an analogous derivation for non-adiabatic gradients, starting from the profile

$$U/U^{*} = \frac{1}{\varkappa(1-\beta)}\left[\left(\frac{z}{z_0}\right)^{1-\beta} - 1\right], \qquad (47.23)$$

in which $\beta > 1$ in the case of a superadiabatic gradient, and $\beta < 1$ in the case of inversion. Both cases give a satisfactory conformance with experimental results. For the profile $U = U_1(z/z_1)^q$ Frost [411] derived a formula for $n(x, z)$. It is noted that the calculation of D_{tz} for the wind profile is possible only at a height at which the value of τ may be taken as constant and equal to the force of friction of the wind at the earth surface τ_0, i.e., up to the order of several tens of meters. Thus the formulas for concentration in a cloud, derived for a normal gradient, are true only at a distance on the order of several hundred meters.

In view of the above mentioned peculiarities of horizontal diffusion in the atmosphere the case of a point source at the surface offers several fairly large difficulties. Through semiempirical methods Calder [408] derived the following formula for a normal gradient:

$$n = \frac{\Phi U}{2\varkappa^2\beta U^{*2}x^2}\exp\left[-\frac{U}{\varkappa U^{*}x}\left(\frac{y}{\beta} + z\right)\right], \qquad (47.24)$$

whence

$$n_{\max} = \frac{\Phi U}{2\varkappa^2\beta U^{*2}x^2}, \qquad (47.25)$$

where Φ is the output of the source, and β is the ratio of the mean squared speed of horizontal and vertical pulsations, the value of which may be determined easily by experiment. A test of this formula under the above described meteorological conditions (in connection with formula (47.21)) at $\beta = 2$ gave the following results. The altitude of the cloud at $x = 100$ m: theoretical 10.4 m, and experimental 10.0 m (i.e., the same as for a linear source); the width of the cloud: theoretical 41 m, experimental 35 m; n_{max} at $\Phi = 1\ g \cdot sec^{-1}$: theoretical 2.5 $mg \cdot m^{-3}$ and experimental 2.0 $mg \cdot m^{-3}$. The law of decrease of n_{max} with x: theoretical n_{max} proportional to x^{-1}, and experimental $x^{-0.8}$. Dependence of n_{max} on U: theoretical and experimental n_{max} proportional to U^{-1}. It is necessary again to note that the formula for a point source must be included in the nature of a substantial factor of time, during which the concentration measurements are made at various points, which in the experiments of Sutton and Calder were 3 to 5 minutes. The fact that the formulas of both authors give more or less similar results is explained mainly by the "decay" of the Sutton coefficients C_y and C_z as a result of measurement.

A recently published work of Crozier and Seely [573] describes a study of the distribution of concentration in an aerosol issuing from a point source on the earth surface, along the horizontal axis, prependicular to the direction of the wind. The test took place in an airplane flying through the aerosol stream at various altitudes. Thus a practically momentary determination of the distribution of concentration was attained. It was shown that even as far as several tens of kilometers from the source the concentration of the aerosol was approximately proportional to $\exp(By^n)$, where y is the distance from the momentary axis of the stream, B is a constant, and $n \approx 1.7$, which is fairly close to the Sutton index of 2.0 (see (47.13)).

In the opinion of the present author, further progress in this field will require:

1. perfection, by theoretical and experimental means, of present known on the magnitude of vertical Eilerian diffusion as a function of

altitude under various meteorological conditions and degrees of coarseness of the earth surface. Great advances have been made in this direction during recent times by Soviet geophysicists [412];

2. conduct of similar research on the Eilerian horizontal diffusion in the function of time of observation t_{obs} (see (47.4) and ff).

Apparently nothing has been done along this line, and thus a theory of dissipation from a point source still cannot be established.

3. insertion of experimentally determined functions $U(z)$, $D_{tz}(z)$ and $D_{ty}(z, t)$ in equation (47.5) or in an appropriate equation for a point source and precise solution of these equation with the aid of electronic computing machine for series of characteristic conditions of weather and locality.

The dissipation of smokes, gases, and aerosols discharged into the atmosphere by factory smoke stacks is of special interest for communal hygiene. In this case the concentration of polluents in the air at the earth surface is important; the following expression is obtained through formulas (47.15) and (47.16) for these concentrations:

$$n_{z=0}=\frac{2\Phi'}{\sqrt{\pi C_z^2U^2x^n}}\exp\left(-\frac{H^2}{C_z^2x^n}\right) \qquad (47.26)$$

for a linear source with altitude H , and

$$n_{z=0}=\frac{2\Phi'}{\pi C_yC_zUx^n}\exp\left(-\frac{y^2}{C_y^2x^n}-\frac{H^2}{C_z^2x^n}\right) \qquad (47.27)$$

for a point source.

In this case the values listed in Table 25 must be used for C_y and C_z .

Bosanquet and Pearson [413] obtained other expressions for concentration at the earth surface. In these derivations they took as their starting point the following formula for a linear source at the earth surface:

$$n=\frac{\Phi'}{b_zUx}e^{-z/b_zx}, \qquad (47.28)$$

where b_z is an empirical coefficient.

This formula is derived with the postulation that D_{tz} is proportional to z, and that $U = \text{const}$, analogously to formula (47.21), and is converted into the former if it is assumed that $b_z = \varkappa U^*/U$. Furthermore, Bosanquet and Pearson propose a "theory of reciprocity": an earth-surface concentration of a cloud issuing from a linear source of altitude H at D_{tz} proportional to z, is equal to concentration at altitude H of a cloud issuing from a linear source at the earth surface. From this formula

$$n_{z=0} = \frac{\Phi'}{b_z U x} e^{-H/b_z x}, \qquad (47.29)$$

is obtained.

From a point source these authors give the following semiempirical formula:

$$n_{z=0} = \frac{\Phi}{\sqrt{2\pi} b_y b_z U x^2} \exp\left(-\frac{H}{b_z x} - \frac{y^2}{2b_y^2 x^2}\right), \qquad (47.30)$$

where b_y is a coefficient analogous to b_z, and characterizes the horizontal diffusion.

In the experimental determination of the degree of pollution of the atmosphere resulting from the discharge from factory chimneys, there are very great fluctuations in the results, neutralized data are very inaccurate, and therefore it is fairly difficult to judge which formulas (Sutton, or Bosanquet and Pearson) conform the best to reality. As a result of numerous experiments, several authors [414, 415] arrive at the conclusion that with proper selection of coefficients these formulas give approximately the same degree of conformity with reality.

In the application of these formulas it must be taken into account also, that gases issue from a smoke stack with a high vertical speed and their temperature is much higher than that of the surrounding medium. The calculation of the rising of a smoke stream above the smoke stack because of these factors is given in the monograph cited earlier in this section, and will not be included here.

From the formulas introduced above it follows that the concentration of gases discharged from factory chimneys increases with increasing distance from the chimney, after which the concentration begins to drop. Location of the maximum concentration requires the determination of the maximum function $N(x)_{y=z=0}$ by the usual method. Thus from formula (47.27), we obtain

$$x_{\max} = H^{2/s} / C_z^{2/s}, \tag{47.31}$$

$$n_{\max} = 2\Phi' C_z / \pi C_y U H^2 e, \tag{47.32}$$

and from formula (47.30) we have

$$x_{\max} = H / 2b_z, \tag{47.33}$$

$$n_{\max} = 4b_z \Phi / \sqrt{2\pi} b_y U H^2 e^2. \tag{47.34}$$

Because 2/s is close to 1, we obtain the following conclusion from both formulas, which has practical importance: the distance from the chimney to the point of maximum pollution of the air is proportional to the height of the chimney, and the maximum concentration of pollution is inversely proportional to the square of the height of the chimney and the wind velocity. For further details the reader is referred to the monographs of P. Andreyev [396] and G. Sheleykhovskiy [395].

In conclusion, mention must be made of the external appearance of an aerosol stream issuing from a smoke stack [416]. At very small values of D_{tz} which occur at inversion, a very narrow vertical stream is obtained which widens very gradually. The stream is sharply defined at a very great distance from the chimney and is bounded by two straight lines intersecting in an angle of only a few degrees. However, in a horizontal plane the stream is sharply bent, which clearly indicates the difference between vertical and horizontal diffusion.

At a normal adiabatic gradient the angle of solution of the stream is much greater. Upon careful observation it may be detected that in vortexes with horizontal axes the rotation is such that they almost touch the underlying layer of air. Actually, in this case turbulence is caused mainly by local convection currents which arise because of uneven heating of the earth's surface. By way of illustration photographs are included (Figure 69) of smoke streams at a wind velocity of 10 $m \cdot sec^{-1}$ at the level of the top of the chimney, and at various values of the Eiler coefficient of vertical diffusion expressed in $cm^2 \cdot sec^{-1}$.

48. Precipitation of Aerosols from the Atmosphere

Turning to the problem of the movement in the atmopshere of coarsely dispersed aerosols with a marked rate of precipitation, we note the following: the movement of concentrated clouds issuing from any source is at first determined by the density and size of the clouds moving as a whole with the gas in which they are contained (see paragraph 13). In particular, aerosols of thermal origin have a much higher temperature than the surrounding air, and because of this they have a considerable initial vertical velocity. For aerosols issuing from vertical tubes, the speed acquired by the aerosol in the tube is added to the latter. At the time when the temperature of the cloud levels out with the temperature of the surrounding air due to turbulent mixing the density of the cloud and of the air are practically equal and this type of movement ceases. Here the influence of gravity appears in the precipitation of particles with relation to the medium. As mentioned above, for isodispersion clouds a uniform precipitation of the cloud of speed v_s is superimposed on its dissipation, the former corresponding to the speed of precipitation of particles in stable air. A polydispersion cloud may be considered as consisting of several isodispersion clouds precipitating at various speeds.

Unfortunately, the calculation of the speed of precipitation of aerosol particles (number of particles precipitation per second per cm^2) on the earth surface from a moving cloud or aerosol stream, and which has great practical significance, is very difficult. In the case of relatively fine particles and small distances x from the source ($xV_s/U \ll H$) the distribution for gaseous mixtures described under the preceding section may be assumed without great error, from which the concentration of the aerosol at the earth surface $n_{z=0}$ may be found. The speed of precipitation at any point of the earth surface is expressed by the formula

$$I = n_{z=0} V_s. \tag{48.1}$$

If V_s is equal to the speed of vertical widening of the aerosol stream (for example, in the cases mentioned following Table 26, at $V_s \approx 0.1U \approx 40$ cm·sec^{-1}, i.e., at $r \approx 0.1$ mm) this simplification is not applicable. This case was described by Johnston and his associates [417]. Because during time t the aerosol particles drop a distance V_st with respect to non-precipitated particles, Johnston substituted z for $z + V_st = z + V_sx/U$ in the expressions for gaseous streams, for calculating the distribution of concentrations in an aerosol stream. Thus, it is proposed that the distribution of concentration in precipitating and nonprecipitating streams is the same. However, this is possible only in the case of constant wind velocity (independent of z) and constant coefficient D_{tz}. In actuality, dissipation in an aerosol stream will differ more greatly from dissipation in a nonprecipitating stream in proportion to the precipitation of the former.

After the above mentioned substitutions in the formulas of type (47.15) and (47.16) Johnston multiplied the second member in the two-group parentheses by the "coefficient of reflection" of aerosols by the earth α. At $\alpha = 1$ the formulas obtained correspond to a smoke stream reflected from the earth, at $\alpha = 0$ the stream precipitates as though the earth was completely saturated with the latter. Johnston erroneously assumed that $\alpha = 0$ corresponds to a stream absorbed by the earth surface; the difference between these two cases is discussed under paragraph 38, I. Starting for the expression for the concentration of an aerosol at the earth surface which contains α , and formula (48.1), an equation for the material balance in a precipitating stream is obtained. From this equation is obtained a formula for the calculation of α according to which at $x = 0$, $\alpha = 1$, and α decreases with an increase in the distance from the source, passes through 0 and at $x \longrightarrow \infty$ approaches -1 . This result clearly indicates that coefficient α has no physical concept and that the formulas proposed by Johnston are in substance, empirical. This work has been given in detail because along with the completely unfounded formulas of Bosanquet [413] these two works are the only references to this problem in the literature.

A strict solution of this problem requires (in the case of a linear source) starting from the differential equation (47.5), with a supplementary member, necessary for calculation of precipitation:

$$U\frac{\partial n}{\partial x} - V_s\frac{\partial n}{\partial z} = \frac{\partial}{\partial z}\left(D_{tz}\frac{\partial n}{\partial z}\right), \qquad (48.2)$$

and solution of this equation, giving theoretical or experimental expressions for U and D_{tz} in function z with limiting condition (48.1). Nevertheless, an approximate solution of this problem which is sufficiently accurate for practical purposes is possible by much simpler means.

Davies [418] investigated the case of the precipitation of a cloud consisting of very large droplets with altitude H , emphasizing not the site at which the cloud precipitates on the earth, but the distribution of concentration in the precipitate, i.e., the Lagrange diffusion in the cloud. Starting from the theory of Sutton, Davies obtained the following distribution formula

$$n(x, y) = \frac{Q\cos\theta}{\pi C^2 L^s}\exp\left(-\frac{y^2\cos^2\theta + x^2}{C^2 L^s}\right), \qquad (48.3)$$

where x and y are the coordinates at the earth surface, with the origin in the center of the cloud; Q is the mass of the cloud;

θ is the angle of inclination of the trajectory of the precipitating cloud with the vertical ($\theta = \arctan (U/V_g)$); L is the length of the trajectory ($L = H/\cos \theta$); and C is the neutralized Sutton coefficient.

According to Davies, experiments with water vapor clouds ($r \approx 0.5$ mm) produced at an altitude of 300 to 1,500 m at a normal gradient agreed with formula (48.3). In these tests C was determined by special experiments.

In conclusion, mention must be made again of the vertical distribution of aerosol particles in the atmosphere. Numerous observations have shown that the concentration and average size of particles decrease in proportion to the distance from the earth surface. There is continuous rotation of aerosol particles in the atmosphere: along with precipitation there is the reverse process of the formation and dissipation of particles in the atmosphere. It is not very accurate to speak here of any average stationary condition, but the formula previously introduced under (45.2) is repeated, even for a determined distribution of particles according to altitude:

$$\ln (n / n_0) = - V_s \int_0^z \frac{dz}{D_{tz}}, \qquad (48.4)$$

which connects the distribution of particles of a given size (characterized by speed V_g) with altitude and the turbulence of the atmosphere.

FIGURE CAPTIONS

[Pages 7-25]

Figure 1. Dispersion and basic properties of aerosols. [Within figure, top to bottom]: Radius of particles, cm; Resistance of the medium; Rate of evaporation; Rate of cooling; Coagulation constant; Light diffusion; Range of microscopic visibility; Range of ultramicorscopic visibility; Predominance of diffusion or precipitation; Diffusion; Precipitation; Vapor pressure; (water droplets).

Figure 2. Differential curves of distribution of particle size. [Within figure]: length of particle.

Figure 3. Integral curve of distribution of particle size. [Within figure]: length of particle.

Figure 4. False curves of distribution.

Figure 5. Origin of false curves of distribution. [Within figure]; length of particle; length of particle; length of particle.

Figure 6. Distribution curves in a probability grid. [Within figure]: Radius less than r ; length of particles.

Figure 7. Distribution curves in a probability-logarithmic grid. [Within figure]: Radius less than r ; length of particles.

[Pages 59-67] [No figures]

[Pages 107-123]

Figure 24. Precipitation of particles in a condenser.

Figure 25. Differential method of determining mobility.

Figure 26. Usual method of determining mobility.

Figure 27. Characteristics of aerosols with one and two mobility values.

Figure 28. Characteristic of an aerosol with continuous determination of mobility.

[Pages 146-152]

Figure 42. Profiles of various slit apparatuses.

Figure 43. Field of flow in a slit apparatus.

Figure 44. Efficiency of slit apparatuses. [Within figure]: Davies; Wilcox; Ranz and Wong; May.

[Pages 175-189]

Figure 50. Distribution of concentration of an aerosol near a flat wall.

Figure 51. Distribution of concentration of an aerosol near an opening.

Figure 52. Precipitation of Brownian particles.

Figure 53. Distribution of concentration of an aerosol in a vertical flat-parallel slit.

Figure 54. Distribution of concentration of an aerosol in a horizontal flat-parallel slit.

Figure 55. Change in the concentration of an aerosol in a spherical vessel.

Figure 56. Change in concentration of an aerosol in a cylindrical vessel.

Figure 57. Drop in concentration of an aerosol when passed through a cylindrical capillary.

[Pages 215-219] [No figures]

[Pages 220-258]

Figure 65. Precipitation of an aerosol on the bottom of a chamber.

Figure 66. Vortexial precipitation of particles behind an obstruction.

Figure 67. Air vortexes before and behind an obstruction.

Figure 68. Aerosol stream in turbulent atmosphere.

Figure 69. Smoke streams at various degrees of atmospheric turbulence.