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PHYSICAL PRINCIPLES OF VECTOR BALLISTOCARDIOGRAPHIC MEASUREMENT

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AIR RESEARCH AND DEVELOPMENT COMMAND

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Preface:

The purposes of this contract were stated:

(1) To improve the technique of recording and measuring the internal force patterns of the heart and great vessels.

(2) To establish relationships between ballistocardiographic data, and the normal and abnormal physiology of the cardiovascular system.

It developed first that the technique of getting an external view of the internal forces generated by the moving heart and blood, was barred in every mode of motion by extraneous forces. These reaction artefacts on the body from its necessary supports to earth, had been recognized in part and remedies devised for the head-foot BCG, prior to our contract. We have deepened and extended the understanding of the nature and coupling of these extraneous forces, in all six dimensions (3 translation, 3 rotation) of body-ballistic motion. We also have devised and constructed practical means of overcoming them. This has led to problems of designing structures with maximum rigidity and minimum weight, requiring the detailed application of aerodynamic and airframe principles, analysis and methods. It also led to devising and testing various solutions to the problem of six-dimensional seismic support; taking account also of the special biophysics of the human physique. The final solutions employed are given in this report. The steps intermediate thereto were given in progress reports previously submitted.

The study of relationships between BCG data and cardiovascular forces, was limited to the biophysical aspects of body dynamics. We did not reach the stage of investigating controlled hemodynamic alterations, as to their BCG manifestation. But we have investigated certain biological matters which underly the interpretation of all such physiological experiments. Such matters are the transmission of cardiovascular force information, from its diverse sources to the local transducer signals which define the BCG in practice. Without understanding this detailed coupling of the observation to the biological activity, we cannot hope to realize detailed biological meaning of BCG observations. These relationships are discussed in our first two chapters.

This writing is therefore as much a Technical Report on Physical Principles of BCG Measurement, as it is a final report on our contract undertaking. As such, it is phrased to reach the understanding of those whose use it may best serve, namely the physiologists, biophysicists, engineers and mathematicians associated with scientifically advancing our understanding of hemodynamics in humans.

The research reported herein is the result of extensive teamwork. Wolf W. vonWittern contributed much to the early conceptual basis and the study of low frequency suspensions. Sidney Roedel and James Harrison did the exploratory developmental work. After v.Wittern's departure to Karlsruhe, Dr. Paul A. Crafton helped with the engineering analysis and on physical models. The detailed work on structural stress and the generalized analysis in Chap. III, IV and V, were carried out by Mr. Joseph M. Gwinn III, associated with the G. L. Martin Company. Fabrication of the rugged but lightweight structure was by them, as was assistance in the analog computations. Thomas Englar and Robert Cramer explored the simpler LCR method of computing. To Mr. John G. Kummell the author owes much for detailing, supervision and development of successive structures, and to Mr. Stanley Przyborowsky for its execution. Joseph H. Condon contributed analytical insight to the differential pendulum, negative spring and other problems. Mr. Peter Hume devised the ingenious modification of Fourier Analysis presented; which was subsequently developed by Mr. G. N. Webb and Robert N. Glackin. Finally, the competent handling of this difficult typescript should be credited to Mrs. R. B. Garrett.

Physical Principles of Vector Ballistocardiographic Measurement

Introduction:

A report under this title should interest two types of readers: those dealing with ballistocardiography (BCG) for clinical or physiological information, and those concerned with medical methodology or with the physics of biological systems. Unfortunately these groups do not have a common language: the principles and concepts of dynamics which describe multi-dimensional systems in several modes of motion, are central to engineering and physical training, but are absent from medical or biological training. Although these principles are widely applicable in biology, and essential to understanding the methods and rationale of BCG measurements, a survey of basic mechanics would be out of place in this report. We choose therefore to aim at the understanding of engineers and physicists who may be called to work in this field; and of those biologists and clinicians whose special concern with ballistocardiography gives them some prior grasp[1] of the mechanics involved.

We will include a review of these mechanical principles, but for readers untrained in biology, will first show from the physical point of view, why ballistocardiography is needed, what it consists of, justification of the physical assumptions (model) used, and what has been done so far.

Chap. 1.

A. SETTING and PHILOSOPHY:

1. Perspectives of Ballistocardiography in relation to medicine, cardiovascular physiology, and biophysics.

Present knowledge of the cardiovascular system rests on a rather limited range of information; we lack a whole area of data, which prevents our forming clear concepts of how the heart and vascular system performs as a whole, or of defects in that performance. This situation may be described in non-medical terms.

The areas of information best developed, are those from electrocardiography (EKG) and from auscultation. The EKG gives information on what may be called the ignition, trigger or firing system. There are various ways in which firing can be faulty: parts of the musculature that cannot fire (infarcts), incorrect order of firing, irregularity, delayed timing, weak or overdeveloped firing. Insofar as the magnitude of the EKG reflects the thickness of the heart muscle, and its extent in the chest, the electrical information does tell something about the potential strength of the muscular pump. But in no way does the EKG foretell the mechanical performance, which only begins after firing: namely, the force, velocity pattern or volume of the stroke, nor the character of the systemic flow after ejection. Neither does the EKG give indication of impending failure, or of how far the mechanism is reaching into its reserve capacity just to satisfy the ordinary demands of the body. Though the pump's ignition may be operating, the EKG does not reveal that the conduits are clogged, or their walls strained to the limit; whether the gas lines (coronaries) are half closed, the motor idling or straining*

*Small changes in the recovery wave of the "ignition system" between firing indicates but does not measure strain.

under load or damage. The EKG measures cardiac status and performance very inadequately, and vascular not at all; and so must be supplemented by many other tests.

Failing "ignition" is, of course, important. While some of the cellular electro-physiology is understood, much of the coordinated mechanism of the EKG is still controversial. The ambiguity is being attacked by several new analytical approaches[2,3,4,5] so that misfiring sequences should be more accurately described.

Auscultation (stethoscopy) gives mechanical or performance information to a limited and equivocal degree, about certain restricted parts of the system. The intake and output performance of all four heart valves can be characterized as normal, opening imperfectly, or closing imperfectly. The anatomical character of such defects can be inferred, but not accurately. Many other sounds (murmurs) are thought related to degree of defect, but with low reliability. As regards detailed physical bases of these sounds, it can be said they are completely unknown as yet. That is, the hydromechanical mechanisms by which turbulent blood produces murmur or clicks in this kind of an environment, are entirely unexplained: more is known about automobile valves and knocks by auscultation, than those of the heart. Although the basic problems are now being attacked, this source of cardiovascular information has progressed but little in two centuries; and at best is restricted in its purview, in relation to both capacity and performance of the normal or the subclinically weakened cardiovascular <u>system</u> under load.

Blood <u>pressure</u> data taken by catheter provides most of our current information on the dynamical or performance characteristics of the heart and vascular system. Experiments with dogs (and recently with humans) at rest or exercised, normal and diseased, have given much information of how this system which supplies all the others, responds to stress or demand. However, taking pressures is by no means a simple, non-dangerous procedure, especially near the left heart; nor does the information so gained, really determine the mechanical action desired.

Somehow, the flow or motion of the blood must be observed, in order to characterize performance of a system primarily designed to provide blood flow. Completely knowing all the pressures in the system throughout the cardiac cycle, does not suffice to specify the flow. In principle, if also one knew all impedances, flow would be known; but we do not yet know the nature of these impedances, nor can we measure them. Indeed other than the pressure-flow relation there is no direct measure of impedance: just as in the case of electric current networks. Like active electrical networks and systems, these impedances change rapidly, to provide general and local control of the flow: so that the temporal behavior of the impedance pattern must be known also, to understand the rationale of cardiovascular performance. The problem of externally estimating cardiac and vascular status and performance, (at rest, under normal load, or sustained stress, or sickness) therefore reduces to finding and developing the interpretation of measurement of flow: the thing for which the cardiovascular system is designed. Flow is the main study of ballistocardiography, and ultimately of this research.

Direct measures of flow are just now coming into existence, but until recently all flowmeters required procedures which either changed the flow or measured it erroneously. Among these the best were the orifice[6] and bristle methods[7]. Preliminary models exist of flowmeters which can be sewn into animals, to measure flow correctly and without changing what they measure. But these (magnetic and supersonic) methods cannot be used to study normal humans or patients without surgery. A catheter method of promise[41] has appeared. This may serve well for research in the hands of experts, and in a few locations, but is unsafe for routine or group studies of cardiovascular conditions; and too stringent for many subjects. The catheter method itself can psychosomatically and by stimulation of the arterial inner wall, seriously alter the cardiovascular function. While this will not prevent eventually accumulating important and necessary data, it is a barrier for routine and general use, either as a scientific or a practical test.

In sum, because of these defects of observation by EKG, stethoscope and pulse pressure, we have now and in prospect no better methods of assessing the functional status of cardiac and vascular performance as to blood flow, than those of <u>plethysmography</u> and <u>ballistics</u>.

Plethysmography (measurement of blood flow from swelling) gives information more local than general; i.e. best when related to limbs. It may give valuable information on the state and function of the peripheral vascular system, if one can distinguish between <u>deep</u> and <u>superficial</u> flow (which have separate controls). The measurement of central flow by electrical "impedance plethysmography"[43] is still too unspecific to be successful.

The <u>ballistic</u> method of evaluating cardiovascular function to which this report refers, has moved ahead considerably in the last fifteen years. The rather superficial method of correlations (which in early stages of medical science is alone available), has sustained medical interest by a few outstanding successes and a general positive relationship to heart and artery disease, and to the athletic heart. But throughout medical science, such statistics eventually give place to the study of functional relations in both the physiological and mathematical sense.

So in ballistocardiography (BCG) (the study of motions of the body during the cardiac cycle), the correlation of BCG wave-forms with normality vs. disease, is now declining in importance. Emphasis has shifted to more scientific questions. To what degree do individual cardiac and vascular mechanisms, account for each detail of the observed body-motion? How do the observed displacement, velocity, acceleration of the body relate to those of the blood at rest and in specified stress? How do known alterations of the cardiac action, and of the (complex) impedance of the vascular network with age or drugs, alter the blood flow in detail? In particular, to what extent are these measurable externally, by measuring the vector components of the body motions? Clearly, to answer such questions is to acquire important practical information for medicine and for physical fitness, as well as basic understanding for physiology and biophysics.

Modern methods of recording these body motions reveal such rich and characteristic detail, that such crude categories as "normal vs. abnormal" become too complex to have scientific or medical meaning. But scientists close to BCG development, find physiological relationships emerging, and measurement problems yielding steadily and rationally; so that this approach to external and objective specification of cardiovascular functions, offers increasing encouragement.

In interpreting the BCG, it turns out that we must give much attention to individual differences, and to distinguish vascular from cardiac dynamics. The statistical approach common in medicine, of correlating recorded details with normality and disease, worked poorly enough with the EKG, but is worse with the BCG. Instead of one, there are four major cycles of activity to interpret, all varying with the individual. Without the functional (mechanistic) approach to supplement the usual normal-abnormal rubrics, the very richness of physiological information in the BCG defeats statistical methods. This has discouraged the correlational school of physicians. To compound the problem, vascular hemodynamics is still an infant science, and cardiac dynamics as well, lacking measurements of flow, is still but little understood. So in this mid-epoch, the demand for practical interpretation of the BCG finds scant satisfaction: much as was true with virus or allergies before their mechanisms became known.

Indeed the present situation is self-defeating, (1) in that external measurements of cardiovascular dynamics by this method are so poor technically, that only statistics instead of mechanisms are applicable. (2) Internal measurements on normal hemodynamics are not yet possible technically. (3) The low correlation of observations with individual cardiovascular status, caused by inhomogeneity of subjects, by artefact and by irrelevance of component information (arising from ignorance of mechanisms), has reduced the interest of the medical profession in this field. In this situation, we here attack the sector of reducing artefact and irrelevance in the record itself. The problem of cardiovascular dynamic mechanisms, is being attacked also by our laboratory.

To a considerable accuracy[8] the flow of blood in the body is actually measured by the body motion it produces. But the principles of measuring and interpreting the dynamic aspects of blood flow sensed through the body motion, are still being developed. At any instant the observations include multiple actions, and a vector summation of flow information in several directions at once. Were it not that the organization of this flow is quite different in the three major axes of the body, one might despair of separating the variables. Also, cardiovascular events are separated not only in direction but also in time, which encourages the hope that reliable and informative differential relations can be found, from adequate multidimensional data showing phase relations. The principles and methods of getting these data are a main objective of this report.

In interpreting body-ballistic measurements, one encounters strong individual differences among normals, such that the use of BCG wave amplitude ratios as in the EKG is inadequate as a criterion. The whole "normal vs. abnormal" view for the population as a whole, breaks down except in extreme or advanced cases. In this situation, an obvious remedy is to select sub-groups identified by other criteria, and to use dynamic as well as static descriptions: such groups are sex, age, relative size of heart, elastic status of vessels (physiological age), constitutional type. All these may be used as starting areas for systematic work on dynamic (stress-strain) analysis, of the successive complexes in the BCG cycle.

A second remedy is to separate the cardiac from the vascular factors in analysing cardiovascular function. This procedure is harder, involving as it does the selective use of short-acting drugs (both natural and synthetic) to create standard changes, and a clearer understanding of the typical ways in which the heart normally responds to standard changes in its hydromechanical load. It is encouraging to realize, however, that we now have an observing method which can ask and answer such questions on physiology and biophysics.

Thirdly, in cardiovascular mechanics in contrast to the present EKG, there are three kinds of information, whose intercorrelation may be used. The moment, momentum and force of the heart and blood (displacement, velocity, acceleration of the body) -- though related as derivatives and by frequency content -- give quite diverse information about cardiovascular properties and behavior. This focuses the analysis on the physiology of the individual, by introducing mechanical principles which hold for all individuals. The

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validity of this attack has a range, of course, which has yet to be determined.

In summarizing our perspective, we may say that contrasted with the information content of auscultation and the electrocardiogram, the ballistocardiogram intrinsically carries much more discriminable and significant data about the functional status and performance of the cardiovascular system. Physiologically and medically, the problems are to obtain reproducible data free of artefact and needless irrelevance; to establish bases of classifying and analysing these data; to relate them to other facts of physiology and pathology in a rational way. This whole program is progressing, but is still in an early stage of understanding.

2. Theory, methodology and the significance of BCG information.

The union of physics with biological science emphasizes that the way of observing things often decides the value of the observation. With the EKG the use of limb-leads for decades prevented the differential diagnosis of infarction. Reliance on the ear with the stethoscope, for generations prevented any detailed objective analysis of heart sounds. In the case of cardiovascular performance, restriction to blood-pressure data has misled and confused our understanding; the naive methods of recording the BCG now in clinical use, have obscured and discredited the information contained therein.

In a new field, one should design methods of observation which minimize irrelevancies and artefacts. This implies foreknowledge and criteria of what is relevant and true: an absurd position. But the simpler problem of electrocardiography (another field of vector dynamics) showed that advances in theory, have each time brought advances in method. The planar analysis of EKG limb-leads followed the theories of Waller and Einthoven. The spherical approximation and potential theory introduced by Wilson, at once stimulated the clinical study of the newer chest and dorsal leads. Burger's theory of lead-space, extended by Frank, led in turn to new methods which reduce the artefact arising from heart-position and body-configuration, and so let us view the heart-vector more directly.

Similarly in studying cardiovascular dynamics by the BCG, the theoretical analysis of body motions by Curtis and Nickerson[10] (perceived by Gordon in 1877) was further explored and verified experimentally for acceleration by v.Wittern[12] and for displacement by Burger[9]. Without expecting that treating the body as a rigid-mass in this way will ultimately provide basic understanding of the ballistic effects, this simple concept of the BCG* stands alongside Wilson's potential theory, in cutting away the ad hoc clinical methods which went before. The theoretical advances by Burger and v.Wittern generated new methods and revealed details which at once superseded the obscure criteria of normal vs. abnormal that clogged the clinical literature. So we may legitimately expect that this interplay of theory and method, which we extend in this report from the head-foot to the other vector components of the body motion, will lead to better ways of recording and further eliminate current obscurities. However, we are not entitled to expect that for three-dimensional body dynamics, the basic theory will be simple: any more than was Wilson's for the three-dimensional EKG. Luckily it happens that public interest in the dynamics of missles and planes, at least makes the technical vocabulary of our analysis familiar to many.

Certainly the theory of rigid (as opposed to elastic) bodies in vibration, is only a first approximation; so that as new aspects are revealed, the BCG recording methods which result from these rigid-body concepts likewise will become outmoded. (1) The modern methods and lines of progress, result from asking certain physical questions about blood-flow in the body; but the answers given by these methods are colored by the rigid-body theory of the measurement. (2) Because of the questions asked, information arising from other forces (tissue vibration, action of joints, etc.) may rightly be regarded as irrelevant, artefactual, and to be avoided. (3) Scientific methodology is concerned with separating variables, to make information from various aspects as independent (orthogonal) as possible. Consequently

*The name "Ballistocardiogram" given in 1940 to the record of whole-body motions under cardiovascular forces, has since proved almost a misnomer. Very little of the information recorded is attributable uniquely to the heart, in the sense that the EKG is.

this whole physical approach differs basically from the biological approach. The latter presumes that the observations relate to the biological events, in an intrinsic way, which is not really determined either by the observer's method or his conceptual approach. However, we use here the more physical strategy which emphasizes dependence of observations on method, to attack the problem of determining the cardiovascular dynamics from body motion.

Furthermore, this information is considered significant for our purposes, if it brings out various independent aspects of the displacement, velocity or acceleration of the blood flow, with a minimum of background or distortion from other factors. The ultimate test of this "significance" is indeed a cross-correlation, but not with other information of the same kind as in statistics, nor with disease symptoms or pathology; rather with direct internal observation of blood flow in relation to controlled variables of cardiac function. Consequently, BCG methodology itself - our concern at the moment - cannot even be developed without intimate knowledge of the physics of blood-flow, seen in the detailed anatomical context of the human body, in a way specifically related to the particular BCG methods proposed. However, as a problem in biophysics, at each stage we design not for the actual complexities of the body, but for our abstraction: a reduced or simplified model which includes only as much at this stage as we choose to. The real art is to include enough features in the abstraction, to make the results useful as well as interesting.

In summary, we may attribute the slow growth of understanding in ballistocardiography in the last fifteen years, to lack of an adequate theory, as with the EKG. The rigid-body theory of the body, plus conservation of momentum (as developed by Curtis, Burger and v.Wittern) admittedly does not conform to the bodily detail, but does separate the variables for clarity by introducing a model. When finally this must be related to details of blood flow, the model probably must be modified.

B. GENERAL AIMS OF THIS THEORETICAL AND EXPERIMENTAL PROGRAM.

The present program attempts (a) to clarify the physical problems arising in vector ballistocardiography, and (b) to spotlight the kinds of information available. With this information clearly in view, we attack (c) the practical problem of inventing devices and procedures to observe it correctly.

We concentrate on the physical problems of three-dimensional mechanics as the most important at this stage, for several reasons: (a) the rotational motion clearly affects the translational motion used to define cardiovascular force. This coupling must be reduced, before one can deal intelligently with the translational vector components of the cardiac hemodynamics. (b) In the anterior-posterior motion and in pitch-rotation, one should find unique views of the cardiac output (stroke volume); which in other directions of motion (e.g. head-foot) may be mixed with other information (e.g. arterial elasticity). Due to this mixing, the vector summation may not always be desirable. (c) The structural difference of the body along its three axes, raises new problems of passive body-mechanics. (d) The ultimate aim (stress-testing the cardiac performance) for hemodynamic reasons requires a sitting posture, which in turn requires solving similar problems of experimental design (e.g. a vertical suspension of "zero" frequency) mentioned in (b) above.

The physical information we seek is of three kinds. (a) An effective record of the vector components of body motion, separated in such a way (orthogonal and informationally homogeneous) that vector composition becomes permissible*. (b) A study of this vector motion in its various dynamical aspects

* Some BCG vector composition which is not permissible in this sense, has been done for several years.(20, 21, 22, 24, 15, 55).

(displacement of c.g., momentum, and force); these have separate significance both physically and physiologically. (c) A study of each of the six motional components (3 translation, 3 rotation) with relation to its distinct physiological meaning; eliminating those data which are least useful.

Besides clarifying and formulating the detailed physical observables to be recorded and the parameters involved, there is finally an equally demanding practical problem of creating a method to get this information. That is, even with improved understanding of what we want to know from ballistocardiography in the head-foot axis, methods must be worked out in practice, de novo, for the five other axes of motion. The first two years of this "hardware" part of the task, were spent in exploring various practical means that suggested themselves, for (1) observing translation, free of rotational component (2) fabricating body-suspensions which did in fact fulfill dynamical requirements for frequency and stiffness not previously achieved in applied mechanisms. Null-frequency suspensions in three rotational dimensions were being worked on (secretly), under the name of inertial guidance systems. Our problem however, though dynamically similar, is in six dimensions, requiring isolation from the surround in translation as well as in rotation. This resembles in essence designing an artificial oloud or flying carpet, strong enough for the subject to lie or sit upon.

A qualitative review of trial solutions rejected and accepted has been given in previous progress reports*. The quantitative results found practical will be presented in this final report. Accompanying these trial solutions, of course, is the mass of detailed design, calculations, drawings, mechanical

* Investigation of the Unloaded Internal Ballistocardiogram: Its Biophysical and Physiological Relation to Cardiac Performance. Contract #AF-18(600)-1107 9/22/56.

work and out-bugging which go with each proposition tested. These cannot be detailed here. However, we will present (a) the precise dynamical basis and values which emerged as acceptable for this problem, and their limitations in general and in particular. We will present (b) experimental procedures which can be used to put this theory in action for: (1) determination of parameters, (2) interaction between degrees of freedom, (3) spectral range of fidelity and (4) evidence of practical validity of this approach, from records of humans.

C. SPECIFIC AIMS AND PROCEDURES.

To design dynamical measurements on a biological system one must start with designing specific hardware and experimental procedures, which intrinsically determine what is observed. If one chooses to observe motions of the whole body, one must decide upon what set of concepts are to govern the design details. These concepts have already been subjected to discussion by the author[1] and others[9], and will be further criticized here. Furthermore, these basic concepts once adopted imply a physical model; which then invokes the corresponding physical theory and laws. Most of this theory is quite old, but has been redeveloped in convenient form, for the motion of missiles and stress-testing aircraft. It is the theory of two rigid bodies, coupled in vector rotation and translation in free space. The range of validity of this theory for our purpose will be discussed.

After arguing the conceptual basis, we will apply rigid body mechanics to the special interaxial couplings and body dimensional peculiarities of ballistocardiography, which differ somewhat for the various kinds of body supports used. The dynamical and physiological interpretation of the records in turn depend on these supports. A theoretical solution of the differential equations in certain planes (by analog computer) will be shown vs. frequency the boundary conditions being certain intrinsic frequencies seen in BCG records.

Our experience from head-foot recording is transferred to the five other modes of motion, to get order-of-magnitude values for the individual's body-parameters in these other axes.

We aim to show a practical way how to record the translational motions independently of the rotational, especially difficult as regards body roll. This way minimizes mechanical coupling between these basically independent aspects, without requiring routine use of complex computing equipment to separate the variables. This aim is specially important because the data of multidimensional or vector recording can easily get out of hand. Separating the rotation vector from the translation space-vector, enables us (for the first time) to display correctly the phase relations of the cardiovascular events in each plane of projection. While we have not yet carried this aim so far as to correct thoroughly for orthogonality, standardization and directional transmission of the body (as has recently been done for the EKG) this elimination of (rot-trans) interaction, does take a step nearer to sensing the "intrinsic" or essential geometry of cardiovascular dynamics. Since observation of the cardiovascular system through body-dynamics, is only one of several important aspects, it is important to reduce it to essentials. The "space-vector" presentation probably better uses the synthetic powers of the observer's mind, and exhibits certain details in a form more striking than does the older BCG record against time; but both enhance understanding.

In summary, for this aspect of the problem of cardiovascular dynamics, we aim

1. To establish a physical basis for data in several degrees of freedom, whose assumptions, range and degree of validity are clear.

2. To design rationally and to execute structures which produce such data, in a way which separates the variables.

3. To discuss the limitations and interpretation of these data, in each degree of freedom:

a) Graphically, in its aspects of frequency spectrum, the time axis, and phased vectors (Lissajous complexes) both in plane projection on the body axes and in space view (intrinsic axes).

b) Physically, as to the meaning of the displacement moments, momentum and force, derivable from motions of the body.

Our procedures have been:

a) To improve the conceptual foundation of ballistocardiography as a vector problem.

b) To formulate and carry out the six-dimensional analysis entailed by the conceptual basis chosen.

c) To devise and construct mechanical means of taking the measurements required.

d) To display in several forms, criticize and interpret these measurements as meaningful for the cardiovascular system.

D. HISTORY: Prior development of the theory and practice of the ballistic measurement of cardiovascular dynamics.

Understanding of this problem has progressed in two parallel streams or dynamic views: the force aspect (expositions of Starr and v.Wittern)[11][12] and the displacement aspect (Burger's exposition)[13][9]. Lying between is the momentum aspect (whose integral is the displacement, and whose derivative the force). This third aspect is best shown by the data of Nickerson[14]. His method, by virtue of a suspension having broad mechanical resonance, records mainly body-velocity, proportional to cardiovascular momentum[1], and so exhibits the best correlation with cardiac output. However, the records of this method, like those of Starr and Dock, are shifted in phase, distorted and confused by resonance of the body-support coupling, which unrecognizedly intermixed the other dynamic aspects. W. v.Wittern resolved this confusion about force recording, by taking the acceleration record from pendular suspensions like Burger's [similar to those of Gordon (1877) and Henderson (1905)]. As a result, we now have available bodies of data based on two complementary views of the blood-body dynamics: the displacement[16] and the force in the head-foot direction. These two views conflict in no way, but supplement each other's information, and contribute to each other's understanding.

1. Force ballistocardiography.

Records of head-foot body-acceleration, insofar as the body accelerates as a unit, measure the forces exerted thereon by the cardiovascular system. For frequencies up to 8 c/s or so, this unitary motion of the body in the head-foot direction has been corroborated roughly by several workers[12,1,17] using an indirect method (external shakers).

Firstly, the proof is rough, because it refers only to the impedance of the system as seen from the shaking position. Local recordings from the body itself resemble each other somewhat in displacement[12] up to 8 c/s, but poorly in acceleration. This results from local resonances (head, shoulders, etc.) and local forces (apex beat).

Secondly, the shaker-table evidence of body unity is indirect. Shaking the body from without is by no means comparable to shaking it from within as is done by the cardiovascular system. Shaking by dorsal contact (v.Wittern's method) actuates the coupling springs in a particular stiffness configuration and drives them in-phase. Thus the constituent masses of the body are made to vibrate in a particular amplitude distribution at each frequency. This is not the same amplitude distribution with which these submasses vibrate, when driven by the feet at various footboard pressures (Cunningham's method), or when driven by the heart and aorta via the skeleton. These differences of effective mass-distribution vs. frequency, are greatest in the vicinity of the natural frequency of the supine body (in response to a step displacement). As a result, the shape of the BCG record for frequency components above 4 c/s, depends appreciably on the use of foot-boards (Scarborough), ankle-supports (Smith)[18], shoulder braces, etc.[12]

In sum, we can readily measure acceleration of support; but to multiply this by a unitary body-mass to arrive at "force" on body, we assume an approximation which shows increasing error above about 8 c/s (Cunningham). Consequently we cannot regard wave details of the acceleration BCG in the 12-30 c/s band, as representing "forces" which are on the same scale, or in phase, with the slower force components. It is therefore better to scale such records frankly in acceleration units rather than force units[19]; and not refer in any quantitative sense to the "force BCG". Qualitatively, however, the "force" interpretation is legitimate and helpful for relating the observations to cardiovascular dynamics, in contrast to the "momentum" and "displacement" aspects which also operate on the same assumptions of body-unity.

This report, concerned with improving the dynamical foundation of these cardiovascular measurements, must omit the growing body of literature on physiological interpretation, as to details of the "force" record and the evidence for it.

We turn now to review the work done on recognition of the vector nature of these forces. Lateral components of the BCG arise from the oblique posture of the heart in ejection (and bilateral non-symmetry of vessels as well) which varies with individuals, body habitus, respiratory phase, and age. It is found that persons having quite disorderly head-foot records, may have quite regular lateral components, which increase systematically as the heart tilts with age.[20] In most cases the lateral (RL) record looks more oscillatory, especially in diastole. As with the head-foot "force" record, we must first scrutinize the method and its assumptions, before we accept such lateral records as valid or try to interpret them physiologically.

(1) In the lateral mode the body-unity assumption rests on much shakier grounds. Transverse rigidity of the body is far less than head-foot; in fact, most of what transverse rigidity the supine body has, comes from the support or bed rather than the skeleton. Because of the smaller mass (thoracic cage) engaged in lateral motion, the smaller lateral forces give R.L. accelerations large enough to compare with the head-foot. This scale-factor needed for the vector sum has not been recognized in the literature.

(2) The lateral record is more likely to contain "local BCG" aspects than the head-foot, because "averaging by support" transverse to the spine is poorer. The role of a platform in summing or averaging forces from the body is important in the head-foot direction but more so laterally. The HF force-pattern entering the platform varies with the subject's pressure on the footboard.[13] In this way, the contribution via spine + legs + pedal

compression becomes more or less strong, compared with the contribution via shear-of-dorsal-tissues. The pattern of HF motion varies accordingly, mainly in smaller details. This factor has not been studied systematically, although v.Wittern has attempted to accentuate the HF forces of thoracic origin by coupling a foot-strap to the bed at hip-level.

(3) As to method, the lateral motion is currently recorded[20] by constraining rotation (yaw) of the body-support, at the foot end. Thus, by moving the axis of rotation (yaw) from the body's c.g. to beyond the feet, the inertia seen by the driving force is increased, and the motion is reduced. But it is simplified also, in that there is no phase-reversal of the lateral motion at the center of rotation. However, this yaw artifact introduces another scale-factor in the lateral (RL) record; partly determinable, but varying by twenty-five percent between individuals.

In the lateral (RL) record it is plain that the motion observed depends strongly on where the body transmits force to the platform. The spine acts as a flexible shock-absorber, for lateral internal impacts originating between shoulders and hips. Since the latter points lie on opposite sides of the c.g. of support, this factor may even reverse the phase of the output (for higher frequencies), depending simply on where the dorsal contacts are, which contribute to the lateral summation. In other words, there is local relative motion between the body and the support, which only forms the average observed. This averaging is inevitable to some degree, since even in head-foot the right and left sides of the body do not displace equally to the apex beat and the aorta is not strictly axial. Consequently, lateral chocking of the body must in some way be standardized[21]. Clearly also, although the hips do contribute part of the HF drive to the platform, they do not assist the RL drive and so become mainly an inertial load in the lateral force.

As a result of these three factors (flexure, rotation, local effects) the standardization of a method for lateral recording becomes especially important, if one is to speak of "the" lateral component of "the" cardiovascular force cycle. Various workers have used several quite different methods. Braunstein[22] used a platform of considerable inertia mounted on stiff springs, which (like the Starr platform in the HF mode) follows the body in RL motion only below about 3 c/s, above which it gets out of phase (both laterally and in yaw), at frequencies low in the BCG spectrum. Scarborough and Talbot[21] used a lighter platform mounted transversely to a Starr bed (stiff to earth), with hip and shoulder boards to pick up the lateral body-motion. The thoracic platform of Dock[23,24] moves laterally against stiff metal springs, with respect to which (again like the Starr bed) the body oscillates on its own tissue-springs, both in translation and roll (independently). Both these oscillations appear in the record, often as dominating artefacts. v.Wittern[25] and Honig[20] used a quite light pendular platform pivoted at feet. Since this is constrained to move in a horizontal plane, the body rolls somewhat: which shows in the record. The mercury bed (Talbot and Deuchar)[26] is mechanically similar and produces the same artefact.

Indeed it can be said that no lateral BCG record has been published so far, which is acceptable physically. The errors can probably not be characterized in detail until good records have been taken for comparison, but there are at least two and sometimes three basic artefacts in current records, which prevent quantitative interpretation[1]. With the high-frequency supports of Braunstein, Scarborough and Talbot, and Dock, there is firstly a lateral resonance in translation, corresponding to the head-foot resonance at 4-5 c/s seen in the Dock and Starr head-foot BCG[2]. With the ultra-low frequency supports of v.Wittern, Honig, Talbot and Deuchar, Scarborough and Soper[27], the translational softness of the suspension prevents such resonance in lateral translation. Secondly, because the body oscillates in roll (depending on the above-axis impact of the heart and roll constraint, e.g. flatness of back) a resonance in rotation also emerges.[3] Thirdly, there is the coupling in recording the transverse motion of platform, between roll and lateral motion: that is, the "true roll-BCG" (as distinguished from roll resonance) exerts its tangential forces on the bed. These forces combine with the true lateral ones, to give a mixed or composite RL record, containing both translation and rotation (with the latter not proportionate to former) and two characteristic resonance frequencies.

Both these latter errors in the BCG have been demonstrated by Scarborough[28], by taking RL records from a subject first prone, then supine. If roll artefact is absent, the record appears inverted. If excessive, the records look alike; all gradations between occur.

It becomes clear that any body support which fails to follow the body freely in "roll" (i.e., is aperiodic in roll) provides the conditions for roll-resonance, and also for a mixed roll-lateral BCG. Any relative motion between body and support then creates (torsional) restoring forces about the body's long axis (β), when the body rolls under the transverse off-axis impacts of the heart. This error occurs with all BCG's currently in use: they are all maintained horizontal, i.e., stiff about this β axis. Secondly, even if the platform were mounted in compliant gimbals about this axis, one must still devise a way to sense the lateral BCG alone, sans roll BCG.

It results that a true "frontal-plane BCG" has not yet been published, either in components against time, or in vector form: although many authors (cited above) have thought to do so. Research programs even exist here and abroad specifically for "vector" BCG studies, whose equipment is intrinsically incapable of recording true lateral components (not to mention the vertical). The nature of this error is clarified by considering how radically the shape and timing of the HF record changed when its resonances were removed around 1955. A main activity of this contract has been to understand and remedy these defects of the frontal-plane BCG.

Anterior-posterior (AP) or vertical component of the "force" BCG.

It is possible to mount a platform on a set of springs which have high and equal stiffness in all three axes(29)(24). Here again the AP record is found to be distorted by translational resonance, as well as admixture of the resonance in roll. But although the RL (x) and HF (y) motions are relatively easy to make so compliant as to be aperiodic(26), a way to do this has not heretofore been found simultaneously for the vertical (z) motion.

The literature of mechanics actually provides no ready solutions for this practical problem; although suspensions of very low vertical frequency have been achieved by air springs for busses; and by coil springs and gravity for seismography(30) and for calibrating low-frequency accelerometers. Previous progress reports on this contract, have outlined our experience in using both these approaches. Later in this report we will describe a successful quantitative solution applicable for ballistocardiography by use of non-linear negative springs.

There exists in principle(31), a different solution to the problem of recording the xyz components of body motion, unmixed with "roll" information. This is to construct a suspension which provides a system of "torque rods" (as commonly used in automobiles) to prevent roll and pitch of the body. If the body is fastened down to such a table so stiffly that roll vibrations can occur only at very high frequency (15 to 20 c/s), with respect to the table, and the torque rods are coupled rigidly enough to prevent rotation of the table to the same degree, then the frequencies below this in the records of xyz and $\ddot{xyz}$, will have no rotational artefact. Such a table has been designed and constructed in this project. It results that to combine the practical requirements of extreme compliance in 3 translations, with extreme stiffness in 3 rotations, with errors in the .0001" range, demands an extraordinary amount of fine machine work. We have concluded that such a suspension is impractical to advocate for general use, and that tying down the subject to this degree inflicts excessive discomfort. This last has been shown to alter the BCG(32). Furthermore, the use of tight body constraint to earth in roll produces an underdamped resonance condition which is still well within the upper BCG range of frequencies.

In principle, again, one can construct a BCG to measure rotational components directly as Ernsthausen has done(33). Here the body rests on a tubular bar, whose rotation may be defined and measured about any single axis (vertical, horizontal or oblique) passing through it. If then one records the rotations and determines the three inertias, one can in principle compute the forces (as well as torques) producing them. At the time this work was published, attention had not been focussed on the emergence in practice of a resonance between body and support, in the case that the latter is stiffly coupled to earth. The subsequent demonstration of this factor by Burger(13) and by v.Wittern(12), proved at once the practical impossibility of correctly recording cardiovascular forces by this "torsional BCG" approach. The reasons in rotation are precisely the same, as account for the large errors in phase and amplitude of the Starr and Dock methods in translation. This torsional BCG is also dynamically impure or "mixed", in the sense of the following paragraph.

Theory of "force BCG".

Theoretical development in this field has proceeded along two lines. The first explored the consequences of considering the body a rigid mass for motion in the HF (y) axis. The analytical solution of this problem by the author[1], its graphical interpretation and practical limitations, were almost simultaneously confirmed by Burger, et al.[9]. The author's further exposition[1] of the "mixed" dynamical content of existing BCG "force" records, was also confirmed by Burger[9]. That is, the Starr (and Dock) records can legitimately be interpreted as "force" only below the body resonance frequency. At and around this resonance (4-5 c/s) the record becomes the "derivative of force" (jerk) and the finer details (> 6 c/s) the "second derivative of force".

Secondly, the basis of this whole analysis was challenged by Cunningham (Griswold and Cunningham) (17) as a result of their experiments. This analytical approach showed that the motion of the body when shaken via the feet, followed a law (in amplitude and phase) resembling above 10 c/s that of acoustic vibrations propagated in an elastic medium. Below 10 c/s a number of separate damped-resonators showed; suggesting that at quite low frequency the body is uncoupled from its unity into separate oscillators, and above this to an acoustic continuum. This serious challenge to any use at all of the unified body-mass concept, requires rebuttal. Since these newer data and concepts conflict with the experimental results of Wittern and of Talbot and Harrison, they must be explained by the difference in technique, and some reason suggested for a choice between the two views.

We may suggest that the disagreement results from how the body is driven: In their case, by an external sinusoidal force acting through a series of leg joints designed for compression only. Such a force must pass through tendons; which having the properties of collagen and elastin known to be viscoelastic[34], can be expected to show phase-lag increasing with frequency. However, when the actual ballistic drive is from inside the body, we suggest that the strong longitudinal tethering of arteries, and of the diaphragmatic foramina, respond better dynamically than do joints seen in pressure-release, transmitting via tendons in parallel.

Further work is needed, clearly, on the colligative properties of the body in small scale vibrations. However, this should be done with due regard to pre-stressing: which is the normal relation of joints, and with us has proved a basic consideration in transferring body motions to a support, for dynamic measurement.

Another analysis treating the body as a non-unitary structure, has been offered by Burger[35]. This differs less radically from our first-order approximation of body unity than Cunningham's. He evaluated v.Wittern's[12] suggestion that heart mass be regarded separately as a mechanical element. The calculations show that the mass of the heart made less difference, than the spring and damper couplings from heart to body. That is, if underdamped, a strongly selective response at the frequency of the heart as a passive oscillator, should result. Since no such transient occurs, Burger concluded that the heart's coupling must be overdamped; which seems reasonable dynamically in the closed and inflated chest, and corresponds with direct x-ray observation. Burger's argument applies only insofar as the heart's suspension participates in the pattern of forces acting on the body. However, this participation is limited to the events at isometric contraction and early ejection, which cover only a small interval of the BCG cycle; beyond this time the heart inertial reaction no longer enters the driving force. Subsequently such forces come from changes of blood momentum other than by the heart; so that Burger's analysis refers only to features of the record in an initial ten percent of the cycle. That is to say, in his basic equation,

$$m_h \ddot{x}_h + \beta_h (\dot{x}_h - \dot{x}_s) + D_h (x_h - x_s) = m_s \ddot{x}_c = F\ *$$

h = heart
s = rest of subject
c = center of gravity of rest of body

the driving force on the right can indeed be identified with heart force. But this equation expresses the total driving force F* only during this short period. Immediately thereafter, one should insert in the equation also, the inertia-elastic reaction [due to "wave-guide" impedance (Van der Tweel)[36]] of the aorta. So, if there occurs at all a heart-resonance as predicted by the values used in Burger's calculation, it should show up as differences in the HI segment of the BCG only. For this reason, the name BCG is not well-chosen.

The nature of the body-driving force was considered qualitatively by Honig[37]. He did not attempt to subdivide the body masses, nor to propose any dynamic model which is exact or simple enough to formulate mathematically. Honig focussed on the later events in systole and early diastole, rather than the motion of the heart itself, confined to early systole. His model emphasizes first the role of ejection in stretching arterial walls, whereby the cardiac work is progressively stored and then discharged as a wave of kinetic energy. While it is true that a relation exists between heart work and body motion, it is not the simple one Honig assumed, i.e., that energy of body motion is proportional to combined potential and kinetic energy of blood ejection. What is conserved between

* The notation of this paper is confusing. Contrary to this notation, the motion of the whole center of gravity ($\ddot{x}_c$)=0 at all times. The expression $m_s \ddot{x}_c$ refers to the driving force on the body or moving cardiovascular masses.

the two is momentum, rather than energy; so that the energy of body motion is a minute fraction of the kinetic energy of the blood. This fraction varies continuously, according to the mass of blood instantaneously in motion. The true relation involves the time derivative of the energy, in Lagrange's equation, of which Newton's law of conserved momentum is a special case. It results that Honig's conclusions as to the irrelevancy of the BCG to cardiac strength, are based on erroneous physics.

Secondly, emphasis is laid on reflection at vessel junctions, as generating counter forces which give BCG waves. This is based on assumptions of resonance used by physiologists to explain the pulse wave. This interpretation, while still under discussion, loses force on quantitative examination,(38) and as other explanations of the pulse-form come to light. The BCG record is indeed sensitive to limb position and limblessness, which suggests a reflection component under these special circumstances. However, the circulatory dynamics also changes, so that new changes of momentum enter even when arms or legs are bent.

Several physiologists, like Honig, are interested in establishing the relation between changes in cardiovascular mechanisms and changes in the BCG record. In the author's estimation this can only be done by further hemodynamic understanding, based on actual measurement of local blood flow velocities in chronic animals. The attempts to account for the force record on the basis of simple localized action, like cardiac ejection pattern (Starr), heart energetics and vascular resonance (Honig), or mass displacements of blood, do not take account of the summations of local dynamic action involved. The concept of a series of discrete "events" accounting for the separate waves (Starr, v.Wittern) is probably a good first approximation, but the theory of a hydrodynamic network (Noordergraaf) better admits the pertinent variables. Pending fuller development of this theory, the interpretation of the body-ballistic record, will rest on the deceptive ground of correlation. At present, such correlation in one dimension is possible when the heart is idling, as on a bed; but the other components of the force vector, and the action of the heart under normal and heavy loads, which is of greatest interest, cannot be studied without further advances in methodology such as those in this report.

One may summarize the progress in understanding the "force" aspects of the external ballistics of blood flow, as follows:

(1) By making the rude approximation that the body moves as a whole, in response to the sequence of composite forces, one can more clearly spotlight certain other large factors in the force picture. These are: (a) the extraneous forces arising from spring-coupling to earth in translations and rotations; and (b) the role of the support, both as to its inertia, its coupling stiffness and damping, and as to its averaging action in defining what is being measured. Better understanding and control of these factors, in turn helps one take better practical account of departures from the unitary-body assumption.

(2) Coupling the body to earth via stiff supports has been shown to produce several effects: (a) strong resonant oscillation of the body on its own (dorsal) tissues, which distorted and confused the record; (b) mechanical differentiation and integration of the record (on either side of the resonance frequency), which makes erroneous any interpretation of the record as one of "force"; (c) external reactions from the ground, which enter the body via various contacts, act on the body in ways dynamically different from internal forces of the cardiovascular system. Consequently it has proved[1] impossible practically or theoretically, to simply subtract their resonance-producing effect from the record, to get the "undistorted" or true record.

(d) Interaction forces between the axes of translation and rotation are created by grounding, so that one cannot simply combine the components of translation. When the head-foot coupling to ground was removed (Burger, v.Wittern, Talbot) it then became clear that interaction attributable to remaining coupling to ground (e.g., RL, AP, roll, yaw), still remained to distort the other components of motion.

(3) Once the stiffness of support to earth was so reduced that any resonant distortion from this cause was pushed downward out of the BCG spectrum, then the weight of the platform was shown[1] to remain as an important source of error. Due to its inertia, a platform even as light as 1/10 the body weight, gets out of phase with the body-motion at BCG frequencies as low as 10 c/s. Although this relative motion never produces much resonance (since light platforms are overdamped by the viscosity of body-tissues) important phase and amplitude losses occur[1]. In other words, these errors from poor coupling between body and support, formerly at 4 c/s with stiff supports, are now moved up to 10 c/s or so, but are still well within the spectrum. One may reduce this decoupling error appreciably by lightening the support[39] but its weight must be 7 lbs. or more (for an adult bed) to keep adequate flexural, lateral and torsional rigidity, as we will show. Further improvement was had by v.Wittern's procedure, of coupling the body better to its suspension. This has complications, if one forgets the averaging property of a bed. A shoulder-contact for instance, enhances the contribution of local vibrations from the loose shoulder-structures. v.Wittern[31] advocates a foot-pressure derived from a spring-rope (rather than a footboard) in effect; this restricts the high frequency BCG information (in HF direction), to the thoracic region.

In sum, by adopting temporarily this theory of the unitary body, we have been (1) led to practical changes in method which have brought out a new characteristic form for the head-foot BCG record; and (2) we have discovered similar errors to be overcome in the other components of BCG response. If then we pass to the theory of the non-unitary body (Burger, Cunningham), our attention focusses on stiffness of coupling between body parts, and the frequencies at which they "break loose" under shaking by the heart. This invalidates (a) the ascription of all resonance artefacts to the body-bed coupling alone and (b) the use of [platform acceleration x whole-body-mass] as a measure of force on body, by which one can legitimately call the record a "force" BCG. One must therefore speak warily of the body-acceleration BCG record in the y direction as measuring a y-"force" in all its details; and must examine this usage even more carefully in the roll (β), lateral (x), and anterior-posterior (z) modes. In passing ultimately to using the body in the seated position (as we shall propose), these questions of local-resonance and of mass-unity vs. frequency must all be raised again.

2. Displacement ballistocardiography.

The simplest physical picture of ballistocardiography, is that of the displacement of blood in an isolated system: the body. With no external force, the center of gravity cannot move, so that at any instant, the sum of the moments (displacement x mass) of the moving blood volumes must equal the change in moment of the rest of the body (displacement x mass). The time derivatives of this express the conservation of momentum, and the second derivative, the Newton's law of action. This view emphasizing displacement, is easily conceived, and was the basis of physical interpretation by Gordon[44], Henderson,[45] Burger and Noordergraaf. In contrast have been the force or impact interpretation developed by Starr[11], Curtis and Nickerson[14], v.Wittern[12], and Talbot[1]. Physically, the latter is better suited to visualizing the case when other external forces (from earth or suspensions) act on the body. The "moment" (mass x displacement) and its derivatives (the relative momenta and accelerations) aid more in understanding the internal dynamics. The forces acting internally on the body are very complex. Thus, at the J peak, there has developed a maximum flow velocity (not a head-on impact) around the aortic arch, yielding a maximum pressure on its outer circumference, a maximum tug on the tethering of the aorta, mainly to spine and secondarily on pleural mediastinum and pericardial membranes, (transmitting forces to sternum, diaphragm and ribs). To trace these forces requires better knowledge of structural connections and stiffness as well as blood acceleration patterns than we yet have. Starr has worked for years to identify the force pattern at the heart itself, in a situation overcomplicated by other forces.

However, the summation of blood displacements in detail, has proved feasible, though laborious. Noordergraaf, taking account of the measured pulsatile propagation, has constructed the contribution of head-foot moments from all the great arteries, and shown[8] the sum to equal the displacement BCG.

If this is done accurately, the time derivatives should equal the velocity and acceleration BCG's taken on a light bed: which indeed they do[46].

This displacement BCG (called "the BKG" in Europe) or "whole body plythysmogram" (Nyboer)[43] has great rational appeal, showing clearly how each of the vessels gives part of the record. However, the displacement record itself contains so little detail, that clinicians are hard put to associate it with local physiological or pathological changes. It has a range of forms associated with body habitus and other factors; but the differential dynamical aspects of heart action are too small to see on this record. Since these aspects are also faster, they are considerably enhanced in the derivative; so that the blood-velocity record really begins to exhibit alterations in cardiac physiology or function.

In our problem, we have felt that the dynamical changes in the cardiovascular system were of most interest, and so have centered the analysis on the accelerations of the body and how to observe them. The approach to the latter depends largely on the concepts used: next to be discussed.

Chap. II. Concepts: the quantitative basis of ballistocardiography.

Three physical principles have been invoked as the conceptual basis of studying the status and operation of the cardiovascular system thru body vibrations: (1) conservation of center of gravity, by which a body freely suspended is displaced equally and oppositely to the displacements of all cardiovascular material within,

$$\sum_i m_i x_i = 0 \text{ or } \sum m_{c.v.} x_{c.v.} \approx m_B x_B$$

(2) Conservation of momentum in a free system, by which the velocity of the body follows equally and oppositely the summed momentum of the blood and vascular masses,

$$\sum_i m_i v_i = 0$$

(3) Equality of action and reaction, by which a body freely suspended accelerates equally and oppositely to the summed momentum changes (forces) inside it,

$$\sum_i m_i a_i = 0 \text{ or } \sum \frac{d}{dt} mv = 0 \text{ since m and v vary.}$$

These three types of information, although related mathematically, are not dimensionally the same, and have separate and distinct physical, physiological and clinical significance. Which is most "important" is not yet known; indeed importance may depend mainly on usefulness or intelligibility. But technically, the potential information is proportional to the frequency spectrum, which is always greater in the acceleration record.

However, the technical requirements for recording of the three aspects of motion, though the same in kind are not in degree. As with heart sounds, the high frequency components of the motions seen clearly in velocity and force records, are much smaller in the BCG than are the components of displacement at heart frequency or lower; so that to sense them well enough for low noise in the second derivative, the free suspension mentioned above must be designed primarily for the force record.

Consequently we will confine our discussion and theory to the problem of observing $\ddot{x}_B\big]_{ijk}$ and $\ddot{\alpha}_B\big]_{ijk}$ as the component aspects of translation and rotation to be made available in the respective axes.

To apply analysis in force ballistocardiography, one must carefully define what masses and what accelerations are to be observed. This we will discuss under "assumptions".

Dynamical assumptions.

We will assume:

(1) Forces within the body are transmitted to the suspension. $F_B = F_S$

(2) The body moves as a unit, i.e. may be treated as rigid in all directions and about all axes, to a sufficient approximation.

(3) The acceleration of the suspension in all axes measures the corresponding accelerations of body, and so gives the force on the body, taking its mass as a whole, i.e. $F_B = m_B \ddot{x}_S$.

(4) The suspension also moves essentially as a unit. The justification for such assumptions, as regards the head-foot (HF or y) direction, has been described by the author(1) in some detail. However, the introduction of the frontal planar and 3-D vector components, demands a second look; as well as recent papers(39) (48) on theory of body ballistic suspensions.

We will examine these assumptions in order.

1. Transmission of body forces to suspension.

In a biophysical system, the coupling of information from the living material to the measuring device becomes of utmost importance. Several questions arise: (a) how quantitatively do forces from accelerating material in the cardiovascular system, reach the bed or suspension? (b) Over how wide a frequency range? (c) And to what degree is this transmission of force information alike for all subjects?

a. Outward transmission and coupling. From the softness of inner suspensory tissues and the relatively heavy body shell, one would expect most of the force to be shunted off into motion of internal parts at heart frequency, leaving little for the body. The fact remains, that the slower force components in HF do move the whole body with almost the exact acceleration calculated [46] [1] from blood motion, and with frequency components measurable to 30 or 40 c/s[49]. (The amount of attenuation and shift of the faster components must await further knowledge of the internal dynamics.) Mechanistically it is reasonable for this to be true in the HF direction, from the stiffness of aortic tethering to the spine and the strong mediastinum dorsad. The transmission laterad should be pporer and more variable; nevertheless lateral ballistic force records do show low-frequency detail calculable to first order. But dorsad, the transmission of force information out to the support should again be good, though poorer ventrad, Consequently it is worthwhile to have the suspension sense forces in all three directions, provided one can couple the forces well through the body in the lateral (RL) direction. This transverse internal coupling in the chest, being weak, varies with respiration and organic differences more than in the HF or AP directions.

Although we have little control of the transmission of forces inside the body, we can do much to assure transmission from the outside to the platform. The HF (y) and AP (anteroposterior) (z) transmission can be made good to probably 20 c/s[1] (of support following body) by prestressing the dorsal tissues, with the couplings now used. This also applies to pitch (X axis). Yaw (γ) and roll (β) coupling are related to the lateral fixation, and depend on chocking methods and strapping which will be discussed, under the various supports. The lateral (RL or x) coupling from body to support is the most critical; variations in it can radically change the higher frequency components of R.L. forces (accelerations) observed. With suitable methods, distinct 20 c/s components of lateral motion may reasonably be expected in the record.

It is fashionable now on theoretical grounds, to view dimly the possibility of sensing externally the distinguishable details of internal cardiovascular forces, momenta and displacement, just as of flying fifty years ago. But the few laboratories now working to improve methods and principles of observation, do not find this to be so. Every change toward better coupling of body and support has improved reliability, increased detail and intelligibility. It has become clear that we are moving toward reproducible and highly detailed BCG information. While this does not automatically give interpretation, it is prerequisite. The assumption that cardiovascular forces can "get out" into the support with good frequency range and in a systematic way, is justified by experience.

b. The frequency range for which provision must be made in a good vibration-measuring system, has two aspects; (1) The support must not evoke resonances (if undamped) - or phase distortion (if well damped) - which can be driven by any component of the emerging force. (2) Motional coupling exists between the modes of a solid structure excited off its elastic center, so that the highest driving frequency in any one direction governs the design. Since in the HF direction body-driven frequencies up to 40 c/s [49] in the force record have been demonstrated, our design criterion will be this rigidity in all fundamental modes of support vibration. However, the high degree of damping from the body, suggests this is excessive.

c. Transmission of force information to the support, differs between individuals. It depends on tissue coupling, which varies in elasticity and stiffness; on the area of body contacts; and on their location referred to the main internal generators of cardiovascular force, i.e. the dorsal thorax. Other parts of the body probably act as passive dampers (e.g. buttocks) and may reduce faster components. Such questions are important in a detailed way; but can all be studied later on a clinical basis. This matter of positive and negative transmission of body forces must eventually be studied, as part of the examination of the "integrating function of the support".

d. Definiteness of design-criteria would be improved by statistical analysis of individual variability in body conformation. For the present stage of exploring principles, we have designed on an average basis, with small safety-factors. The internal variability of individuals (size of heart, impedance of joints, bones, etc.) may affect the biophysical interpretation of data, but is not taken into account as yet in design of methods.

2. Our assumption that the body moves as a unit (i.e. a rigid body) is the basis of the analysis and (analog) computation in this report. This should be taken as a first-order approximation, sufficiently valid at frequencies up to about 8 c/s in the head-foot HF (y) direction, and much less in the RL (x) and AP (z) directions.

The strength and purpose of this assumption is that it lets us proceed to calculations of body suspension systems, as to the rigidity (stiffness or high frequency response) which will be <u>sufficient</u> to avoid resonance artifacts. That is, if the body oscillates (a) is an assembly of coupled masses or (b) as a propagative continuum (or both), then the structural stiffnesses in the support found sufficient to respond at high natural frequency to the body taken as a whole, will yield still higher frequency toward the body parts. This results from the lower masses of the parts and the higher vibrational modes of the suspension excited. Whether these stiffnesses are necessary will vary a great deal with the coupling between our support and the body. If this were uniform (body "poured in") the damping of the structure by the body would be so great, that no resonances could occur, and phase shift with frequency would be very gradual.

In essence, some definite assumption must be made about the rigidity of the body-mass, in order to calculate the inertia and damping terms of the support under design. The acoustic-vibration propagation theory [proposed by Cunningham[17]], while reasonable at first sight (but see below) for small vibrations in the longitudinal direction, cannot hold in the transverse, anterior-posterior, pitch, roll or yaw modes because of (a) the small dimensions of the body in relation to wave-length of such waves, and (b) the strong damping of propagated waves by tissues [Oestreicher[42]]. In general neither the all-rigid nor all-acoustic theories of force transmission probably holds through the BCG vibrational spectrum, but are superimposed along with more complex coupling of body parts at intermediate frequencies and in the several directions.

More specifically, in the head-foot direction, the recent work of Noordergraaf in predicting the observed acceleration BCG of the whole body to over 10 c/s, is sufficient validation of our assumption for this axis alone. But the full implications of this assumption for the other axes, should be noted. Although we may be justified in assuming a rigidity in the head-foot direction somewhat exceeding that demonstrated (Chap.I) the rigidities transverse to the spine (lateral and anterior-posterior) are clearly much less. The body below the hips, as well as the arms, contains no net component of blood motion away from the spine, and so is passive in this region. The masses in these portions of the body mass m_B

do not enter the equation $\sum m_B\ddot{x}_B = 0$ or $\sum m_B\ddot{z}_B = 0$ from which forces on the body are to be calculated. Similarly in roll, $\sum I_B\ddot{\beta}_B$ does not contain the roll-inertia of the hips, thighs, and legs; since these portions suffer no roll-torque from the cardiovascular drive. Finally, it must be admitted that the body-inertia in yaw and pitch, stands in similar relation to the confinement of driving torques to the thoracic cage, which comprises perhaps 1/3 of the average body mass.

In the face of these elementary facts of anatomy, we propose simply to ignore (in the x, z, α, β, γ axes) the inertial effect of the passive (extra thoracic) parts of the body-mass in the calculations. To a first approximation, they load the driving forces indirectly (via the support), and may be interpreted as increasing in a complex way the effective mass of the support in these axes. It will be the task of future work in this field of "external ballistocardiography", to devise means of overcoming this crude approximation as to body inertia.

[We have made a beginning in this direction by constructing an ultra-low-frequency 3D suspension for the thorax alone (Chap. I). This, when modified for freedom of rotations this report shows necessary, should be a clear advance toward a BCG that is more quantitative in the x, z, α, β and γ axes.]

We conclude that the strategy in this game, is first to solve the problem in terms of the very low frequency BCG components (< 6 c/s) comprising the dominant features of the whole-body BCG pattern. The process of removing artefacts of earth-coupling and rotational admixture by developing suitable whole-body suspensions, results in methods modifyable in detail, which let us better evaluate the bodily sub-vibrations and so to reduce errors of rendering them. We would emphasize the need of successive approximations, and of carrying along simpler concepts like the unitary-body assumptions, to their acceptable limit.

3. The acceleration of the suspension, measures that of the body.

This third assumption implies not only that the whole body has unitary acceleration vectors (rotation and translation) as above, but that the support is so connected to it, that it executes the same motions: at low frequencies, anyhow. [Part of our task is to calculate, and measure just how, at higher frequency, the inertial properties of the support make it depart from the body motion in the various fundamental modes of motion: $x, y, z, \alpha, \beta, \gamma$]. If only the body were actually rigid but with a soft viscoelastic envelope, this assumption about the support would present small difficulty. In practice, the motion of "the body" has no operational meaning apart from that measured on the support. If one attaches light accelerometers to various parts of the body surface to compare the motions or read "the" body motion, one encounters a biophysical tangle. Firstly, each accelerometer generates a spurious local natural frequency due to its own inertia and stiffness of surface attachment. Secondly the different local parts of the body surface are differently attached (and under differing prestress) to the spine and to other routes of momentum transmission. There is a "local surface BCG" for every square inch of the body, with large local differences over the thorax especially. The only way a body-volume BCG can be defined at all, is in terms of some "standard" system of body-contacts which summates and so smooths the local surface differences, and minimizes local "bounces" like those of the chest and sides near the heart (and shoulders, neck, feet, etc.) which are loosely or flexibly coupled to the spine and in a quite variable way.

Of logical necessity, in this way the support itself defines the "body" motion; so that "relative motion of body and support" ~~+/-~~ turns out to have no highly precise meaning. [Any engineer realizes this is also quite generally true, e.g. the "point of insertion of a beam", etc.]. We therefore assume here explicitly, that the body "has" an acceleration which is that of the support (other forces being absent) as seen at low frequency; and that any concept of the body motion can only be had in terms of some integrating support (or observing platform) rendered weightless.

It follows that the need for this kind of extrapolative definition of the "motion of the body" as a whole, at once affects our interpretation of the high-frequency details of blood-motion in terms of body-motion via the laws of momentum. In addition, the operational meaning of such concepts as "propagative transmission" of heart impacts, is rendered vague. Real accelerometers (linear) put on the body are subject to transient local tilts and jiggles, which can produce as large local artefactual differences in phase as those being verified. Only by large-area summation can reproducible high-frequency details be recorded usefully. So a clear measure of the degree of failure ot the "unitary body" theory, is hard to demonstrate -- though not impossible.

4. The suspension moves as a unit (is rigid). Clearly this fourth assumption is necessary, if accelerometers attached to the suspension are to define the BCG. In the head-foot (y) direction, any simple tubular frame[39] fulfils this criterion. But to record lateral (x) accelerations, such simple hammocks or even honeycomb sheets are not stiff enough. The former vibrates in lateral flexure, and both in vertical flexure. Trussing to kill the latter[37] still leaves compliance in torsion toward the strong forces in roll [rotation (β) about the body axis]. But because there exists xβ coupling (Chap. III) the suspension must also be sufficiently stiff (unitary) in torsion. When this requirement is filled, we can say the

suspension moves as a unit for the purposes of this problem. It is a major problem in mechanical design, to satisfy this requirement for torsional rigidity along with the lightness (1/10 the body weight) needed, with a mechanical assembly of body size.

Other concepts and assumptions needed for physical analysis of the BCG.

Simply to treat the motion of the body under cardiovascular forces, as a problem of rigid body or even coupled-system dynamics, overlooks some dominating principles concerning information. Unlike the dynamic assumptions stated above, these additional simplifications are dictated by the viewpoint of medical physics, rather than by physics.

1. An acceptable solution must result in separation of variables. This is the conditional of orthogonality, implied in the use of the tetrahedral lead system in electrocardiography. Applied to ballistocardiography, this requires not only that we seek natural axes of the system for translation, but also that rotational data be not mixed with translational. This turns out to imply the assumption of small angular velocities, so that centrifugal gyroscopic and coriolis forces are negligible (which is true).

In principle, if enough is known about the coupling between modes of motion, for a system of rigid bodies, one can readily compute the pure axial motions, in spite of interaction. For two reasons, this possibility is excluded. (a) Unlike the ECG, where simple resistive networks are now often used for this very purpose[51], the BCG 3-D computation involves complex reactive terms, which require a rather large analog computer of forbidding cost. (b) A BCG design which does not succeed in separating (uncoupling) the translational and rotational modes, introduces body artefacts so large as to obscure, even reverse the appearance of some components. If the system were indeed a pair of coupled rigid bodies (body and support), these spurious resonances could readily be rejected by filters. But in the rotational modes, with a real body, this assumption proves too simple, and the filter remedy will not work. There appears no choice but to uncouple these modes from the beginning, and avoid computers.

2. With systems as complex as those in biology, one must choose what information is most valuable, and not simply record every manifestation that can be measured. In the y axis (head-foot) alone, the displacement, velocity and acceleration BCG yield quite different biological information. When we add x and z data to y we must raise the whole question of the value of retaining all three derivatives in all axes: a total of nine traces (apart from the necessary ECG, respiration and phonogram). Clearly to record the three rotational components and their derivatives as well, would require strong biophysical justification on purely informational grounds, however equivalent these axes may be physically.

For this reason, in the analysis given here, we will concentrate on recording the translational motion, solving but not studying in detail the rotations or how to read them from supports. This simplification is dictated not only by the general intractibility of excess information, but also by the special relation of blood displacement to the physiology of the heart muscle and vascular structure. The body rotations depend on features of body architecture which contribute little added understanding of normal blood flow. If reasons should arise from pathology to study the body's rotations (e.g. pitch) these will acquire informational value. Thirdly, we mention a need of unity and adhering to a restricted field, to penetrate further in this research.

3. Errors. To distinguish between "true" and erroneous data, labeling the latter "artefact" or "noise" is mainly an ex-post-facto art. Nevertheless in biological measurements, there is generally some baseline activity unrelated to the problem at hand. The measurement problem in biology often consists of so devising the method that only pertinent information is seen, and other information rejected. For example in bloodflow measurement (magnetically) the ECG may come through to mix with the flow information, and must be rejected.

So with the BCG we find both extraneous and internal "noise", quite apart from misinformation mentioned above, where one mode of mechanical motion leaks through into another mode. A very practical requirement of BCG design, is to reduce couplings to extraneous systems. Though no mechanical system (in a gravitational field) can be studied completely free of coupling to earth, this simplification will be assumed in our analysis. Actually, this coupling proves a major problem and source of error, both in practical BCG supports, and in the analog computers used to solve the dynamic equations.

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Chap. III. DYNAMICS OF THE SUPPORT SYSTEM

In order to obtain a repeatable and meaningful record of acceleration, it is necessary to secure the subject to a rigid frame and measure the motions of the frame. It would be ideal if this frame would follow the body vibrations without distortion, at least up to 40 c/s, the bandwidth of interest for cardiovascular performance. However, the spring and damping of the body tissue between body and frame stand against this requirement. Consequently a knowledge of the phase shift and amplitude change as between body and frame is needed, to interpret the data taken from the frame.

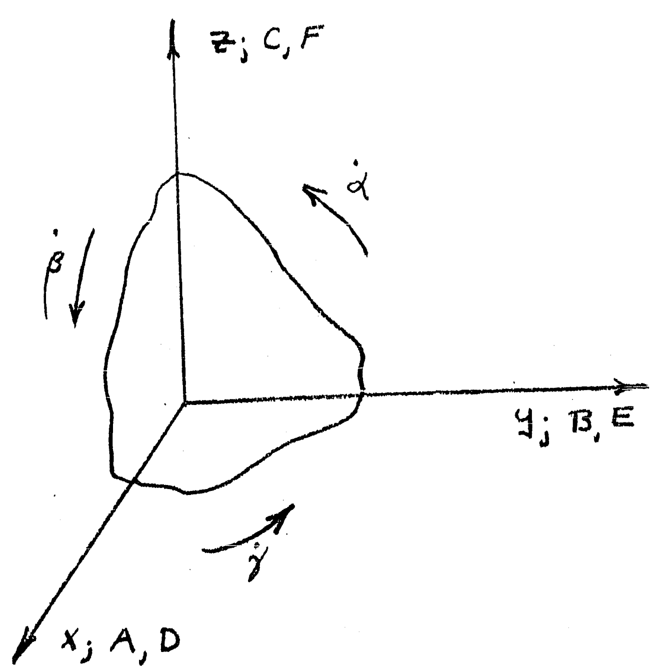
This knowledge is expressed to a first approximation in the equations of motion for the frame and body, regarded as a two-mass system in relative motion both of translation and rotation. These equations are derived here, experimental means of getting numerical values of the coefficients are suggested, and the nature of the solutions computed and shown. The complexity of the equations (12 simultaneous second order differential equations) require the analogue computer for solutions. Passive (LCR) analogs gave fair solutions, but the difficulties of varying parameters in rotational modes led us to use active (operational amplifier) computers (Reeves) for the larger networks. For this, the Martin Company gave generously of time and assistance in their computer laboratory.

Solving the equations of motion in this way simulates the actual problem in a simplified form; it stimulates and points up physical thinking and observation with real subjects. The computer program has given a clearer physical understanding if the genesis and interaction of forces producing translational and rotational motion. Its solutions to the equations of motion are needed to interpret the acceleration record for cardiovascular performance analysis, and to provide data for improving the design of the frame and suspension relations, by varying the parameters embedded in the coefficients.

Apart from the practical problem of improving BCG methods, there is the biophysical problem of enunciating principles by which the BCG record may be rationally understood. The computer program here serves to evaluate the range of validity for the theories of the BCG that are under discussion. Experience accumulated in this way should lead to better solutions with live subjects and real instrumentation.

A. EQUATIONS OF MOTION

The most general equations of motion for a two-mass system in 12 degrees of freedom with static coupling are excessively complicated, and were whittled down considerably before a solution was considered. To get an idea of the complication and the simplifications introduced, consider the equations of motion of a single mass in six degrees[50] of freedom: Figure 1



$$m\,(\ddot{x}\overbrace{-\dot{y}\dot{\gamma}+\dot{z}\dot{\beta}}^{\text{Coriolis}}) = X$$

$$m\,(\ddot{y}-\dot{z}\dot{\alpha}+\dot{x}\dot{\gamma}) = Y$$

$$m\,(\ddot{z}-\dot{x}\dot{\beta}+\dot{y}\dot{\alpha}) = Z$$

$$\dot{h}_x - h_y\dot{\gamma}+h_z\dot{\beta} = L$$

$$\dot{h}_y - h_z\dot{\alpha}+h_x\dot{\gamma} = M$$

$$\dot{h}_z \underbrace{- h_x\dot{\gamma}+h_y\dot{\alpha}}_{\text{gyroscopic}} = N$$

where

$$h_x \equiv A\dot{\alpha} - F\dot{\beta} - E\dot{\gamma}$$

$$h_y \equiv B\dot{\beta} - D\dot{\gamma} - F\dot{\alpha}$$

$$h_z \equiv C\dot{\gamma} - E\dot{\alpha} - D\dot{\beta}$$

and A, B C are moments of inertia defined by $A = \iiint (y^2+z^2)\,dm$ while D E F are products of inertia defined by $D = \iiint yz\,dm$

The forces on the left are inertia forces; those on the right are external forces (and torques) such as the drive. If now a second mass is coupled to this mass, the forces arising from the stiffness and damping of this coupling would appear on the right-hand side as shown here, although these forces also would involve the dependent variables (relative displacements and velocities). Before considering these coupling forces, in our case we may simplify considerably the expression for the inertia forces. This follows from two observations regarding our system:

(1) Displacements and velocities are assumed <u>small</u> enough so that their squares and products can be neglected; (see Appendix A). This allows us to set the Coriolis accelerations [consisting of products of a linear and an angular velocity] and the gyroscopic moments [consisting of the products of angular momentum and angular velocity] equal to zero.

(2) The body exhibits a plane of <u>symmetry</u>, the YZ saggital plane. Hence, products of inertia DEF are zero except in this plane of symmetry, i.e., $D \neq 0$, $E = F = 0$. So in our case, the above equations reduce drastically and separate the forces and torques:

$$
\begin{aligned}
X &= m\ddot{x} & L &= A\ddot{\alpha} \\
Y &= m\ddot{y} & M &= B\ddot{\beta} - D\ddot{\gamma} \\
Z &= m\ddot{z} & N &= C\ddot{\gamma} - D\ddot{\beta}
\end{aligned}
\qquad (B)
$$

However, we are still left with dynamic coupling of rotational reactions, introduced by the product-of-inertia D about the transverse body (X) axis. Because the summation in D ($\iiint yz\,dm$) does not contain a square, the $\pm$ contributions on either side of the body's xy plane tend to cancel one another. So the product-of-inertia D will be an order of magnitude or so less than the moments of inertia, B and C. Hence, the terms containing D can also be dropped, and the equations become simply those of a completely symmetrical body for small motion:

$$
\begin{aligned}
m\ddot{x} &= X & A\ddot{\alpha} &= L \\
m\ddot{y} &= Y & B\ddot{\beta} &= M \\
m\ddot{z} &= Z & C\ddot{\gamma} &= N
\end{aligned}
\qquad (C)
$$

We turn our attention next to the right-hand sides of these equations, which involve the motion of both frame and body, because the stiffness and damping forces depend upon the relative motion of the frame with respect to the body. The plane of body symmetry helps here because with no dynamic coupling, motions in the plane of symmetry do not excite motions out of that plane. Hence, the equations can be separated into symmetric and asymmetric as follows:

<u>In plane of body's symmetry</u>: saggital or YZ plane

$$m\ddot{y} = Y, \quad m\ddot{z} = Z, \quad A\ddot{\alpha} = L \qquad (D)$$

<u>Out of plane of body's symmetry</u>: transverse xz and frontal xy plane

$$m\ddot{x} = X, \quad B\ddot{\beta} = M, \quad C\ddot{\gamma} = N \qquad (E)$$

This means that torques in pitch such as L create only y, z, α motion; while torques in roll or yaw such as M or N create only X, β and γ motion.

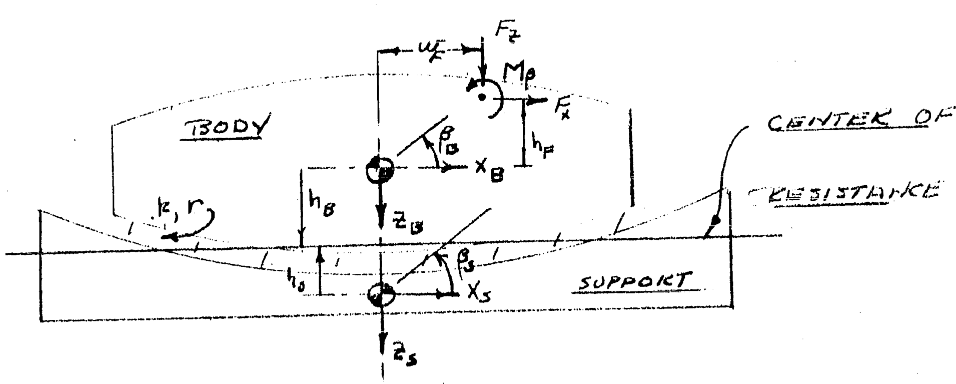
The asymmetric equation will be a little more involved because of static coupling between <u>forces</u> and <u>moments</u> in the transverse xz and frontal xy planes.

While the derivation of detailed expressions for Y, Z .... will be left to specific designs.

[the human bed, mercury bed and chair] - the equations of motion in one plane (the transverse xz) will be derived here to demonstrate certain general features for all equations, and later to develop experimental methods of coefficient determination.

<u>1. Assumptions</u> and geometry

Fig. 2



The above diagram is an abstract of the transverse plane of the body on a bed seen by an observer looking footward along the spine. The conventions of direction are standard in BCG usage[19]. The body and bed are treated here as rigid bodies coupled together through the damping and spring characteristics of the body tissue. This coupling is assured because the forces from the heart are not great enough in size to separate the body from the bed or to slide the body with respect to the bed.

While dependent non-linearly with tissue stress, they are linear for the small motions (.001") we are concerned with. That is, in the equations that follow, spring and damping forces k and r are taken as linearly proportional to displacement and velocity.

"Center-of-resistance" is defined by the axis of a static force whose application causes no rotation. By way of illustration, consider how the center-of-resistance is determined for the x-direction. Apply a force in the x-direction on the body; if the body rotates in addition to translating, change the point of application. Keep changing until a point is found at which a force produces translation, without rotation. The axis through this point in the x-direction is the center of x-resistance. In the same way, the centers of y- and z-resistance can be obtained.

The concept of center-of-resistance allows rotational moments due to translational displacements and velocities, to be distinguished from rotational moments due to angular displacements and velocities. The driving (heart and blood) forces and moments are noted by F_x, F_z, and M_β, which cause the motion we are trying to measure.

Since the xz plane is not one of symmetry, the equations of motion in this plane, as derived earlier, are

$$m\ddot{x} = X, \quad B\ddot{\beta} = M$$

for both support and body, yielding a total of 4 equations. Note that the equation for vertical motion ($m\ddot{z} = Z$) is not involved in this motion because it is in the (saggital) plane of symmetry which, in the figure, is a plane perpendicular to the paper through the Z-axis. That is, z forces being symmetrical cause no roll motion.

2. Derivation of Equations of Motion

Substituting the equivalent of X and M in terms of the known physical properties of the body and support, we have from Fig. 2 for the equations of motion* of the body in translation:

$$m_B\ddot{x}_B + r_x(\dot{x}_B + h_B\dot{\beta}_B - \dot{x}_s - h_s\dot{\beta}_s) + k_x(x_B + h_B\beta_B - x_s - h_s\beta_s) = F_x \tag{1}$$

and in rotation:

$$B_B\ddot{\beta}_B + r_\beta(\dot{\beta}_B-\dot{\beta}_s) + k_\beta(\beta_B-\beta_s)$$

$$+h_B[r_x(\dot{x}_B+h_B\dot{\beta}_B-\dot{x}_s-h_s\dot{\beta}_s)+k_x(x_B+h_B\beta_B-x_s-h_s\beta_s)] = M_\beta+F_z w_1-F_x h_F \tag{2}$$

and for the supports, in translation:

$$m_s\ddot{x}_s+r_x(\dot{x}_s+h_s\dot{\beta}_s-\dot{x}_B-h_B\dot{\beta}_B)+k_x(x_s+h_s\beta_s-x_B-h_B\beta_B)= 0 \tag{3}$$

and in rotation:

$$B_s\ddot{\beta}_s+r_\beta(\dot{\beta}_s-\dot{\beta}_B) + k_\beta(\beta_s-\beta_B)$$

$$-h_s[r_x(\dot{x}_s+h_s\dot{\beta}_s-\dot{x}_B-h_B\dot{\beta}_B)+k_x(x_s+h_s\beta_s-x_B-h_B\beta_B)] = 0 \tag{4}$$

All distances have signs. A distance is positive when measured in the positive direction of the coordinate. For example, the distance to the center-of-resistance h_B is negative because it is in the negative z-direction from the body c.g., the origin of the body coordinate axes.

The various forces or moments and their signs are derived by imagining a force or moment applied to produce a positive displacement. The reactions are summed up on the left-hand side, being opposite in direction and hence sign from the driving force.

* Rewritten from III (D) in terms of displacements and for each mass.

Consider the terms of equations (1). Application of a force F_x is instantaneously reacted by the inertia force $M_B\ddot{x}_B$. As time passes, velocity is developed, giving rise to the damping force

$$r_x(\dot{x}_B+h_B\dot{\beta}_B-\dot{x}_S-h_S\dot{\beta}_S).$$

The damping force depends only on the relative motion between body and support. Owing to the coupling of body and support, angular and linear velocities are built up for both support and body. The expression for damping accounts for these various motions in developing a relative velocity. Eventually, the velocity leads to displacements, bringing in a reaction from the stiffness of body tissue, given by $k_x(x_B+h_B\beta_B-x_S-h_S\beta_S)$. As with damping, the spring force depends on relative motion only, hence the similarity between the damping and stiffness terms.

Equation (2) expresses the equilibrium of moments acting on the body. The applied moment $-F_z w_F + h_F F_x + M_\beta$ (M being a pure couple) is reacted first by the inertia moment, then by damping moments, followed by spring moments. These latter two moments are due to pure relative rotation expressed by $r_\beta(\dot{\beta}_B-\dot{\beta}_S)$ for damping torque and $k_\beta(\beta_B-\beta_S)$ for stiffness. Additional rotational reactions are suffered if a relative linear motion exists, and is obtained by multiplying the force due to linear motion by the appropriate distance to the center-of-resistance, as shown above.

The signs on these last terms will be explained to illustrate our sign convention. Looking at equation (2) for body moment equilibrium, note that application of a positive moment $F_x h_F$ is reacted by a positive linear displacement of the body greater than that of the support. The linear displacement of the <u>body</u> at the contact with the support is $x_B+h_B\beta_B$.

The displacement of the support at the contact is $x_S + h_S\beta_S$, the positive sign being required because although h_S is negative, β being positive counterclockwise, gives a negative displacement at the contact point. Taking the difference, assuming the body displacement larger, we have for the relative linear displacement between body and support

$$[(x_B + h_B\beta_B)-(x_S + h_S\beta_S)]$$

multiplication of which by k_x gives the translational spring force. The damping force is obtained by replacing displacement by velocity, and multiplying by r_x. The total translational force reacting the applied rotational moment is

$$r_x[(x_B + h_B\dot{\beta}_B)-(\dot{x}_S + h_S\dot{\beta}_S)] + k_x\,[(x_B + h_B\beta_B)-(x_S + h_S\beta_S)]$$

This must be multiplied by h_B to obtain this reacting <u>moment</u>.

No term representing the coupling to ground has been included in the equations. It has been assumed that the very soft suspension, 1/3 c/s cutoff, will keep the influence of ground to an unobservable size.

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B. EXPERIMENTAL DETERMINATION OF COEFFICIENTS

Numerical values of the coefficients in the equations of motion are, of course, needed for applications involving these equations. Although estimates of certain coefficients are possible from theoretical considerations, much easier and more reliable are estimates based on experimental procedures. We shall describe here the procedures required to measure all coefficients excepting those associated with specific shapes of the support, leaving this information to those sections dealing with support design.

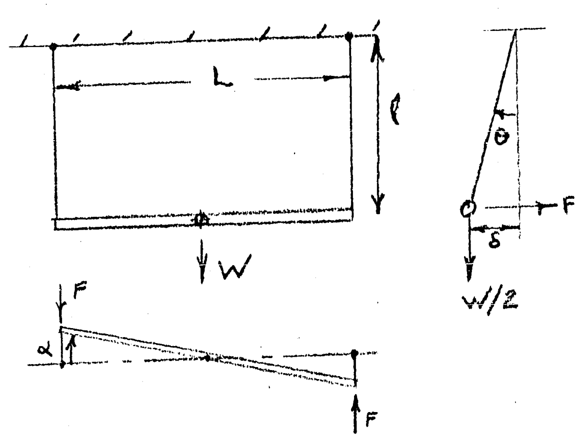
1. Moments of Inertia

Moments of inertia are conveniently determined by oscillating the item in some kind of pendulum, or spring-mass combination. The period of oscillation is related to the radius of gyration by a relation depending on the type of restoring force. We shall next derive these relations for the pendulums and spring-mass combinations used here. A detailed description of procedures will be taken up in the sections on support design.

In this analysis, since the subject and support are treated as rigid bodies, the inertias must be found at very low test frequencies: not higher than one cps or so.

a. The Bifilar Pendulum

To determine inertia about a vertical axis. (Z)



$$F = \frac{W}{2}\theta$$

$$\ell\theta = \frac{L}{2}\alpha$$

$$F = \frac{WL}{4\ell}\alpha$$

The bifilar pendulum is useful for determining moments of inertia when the swing must be about a vertical axis, such as a subject sitting in a chair. As seen in the figure, a bifilar pendulum is simply a two-strand pendulum with the restoring moment FL being provided by the angular displacement θ, of the strands. We shall derive the period of the bifilar with strands of equal length, because these are the most convenient to use. This requires that the c.g. be midway between the strands if all the energy of the oscillation is to be confined to motion about the vertical axis. Unequal c.g. spacing would lead to oscillation about a horizontal axis too. From the expression for F, we see immediately that the torsion spring rate is

$$\frac{FL}{\alpha} = k_\alpha = \frac{WL^2}{4\ell} = \text{torque/unit angle}$$

The frequency is computed from the usual relationship

$$f = \frac{1}{2\pi}\sqrt{\frac{k_\alpha}{I}} = \frac{1}{T}$$

$$= \frac{1}{2\pi}\sqrt{\frac{W L^2}{4\ell}\,\frac{g}{W\rho^2}}$$

where ρ is the radius of gyration in the expression $I_Y = m\rho^2$.

Solving for ρ the required quantity

$$\rho = \frac{TL}{4\pi}\sqrt{g/\ell} \qquad \text{in terms of T, L and } \ell. \tag{5}$$

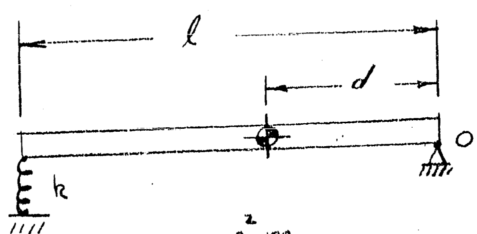
The test procedure is simple. Choose the strand spacing L and strand length ℓ, so as to get a period greater than one second but no greater than five seconds or friction will influence the results. (For a recumbent adult, if ℓ = 6 ft., L = 40"). Adjustments after tests of the length ℓ and spacing L may be necessary to get repeatable results.

Set the pendulum in motion, making sure that the amplitude is small enough to insure a linear oscillation. Keeping $\delta/\ell < .2$ during the timing will suffice. Also, make sure the oscillation is about an axis through the c.g., by making the initial rotation about this axis. Repeat until the resulting motion is the required one. Time the oscillation over five complete oscillations and obtain an average period T. Repeat several times and average again. The value of ρ is available from equation (5) knowing L, ℓ and T.

The inertia I_ρ about the long axis y of the body is determined similarly, with $L \cong$ 12" for a standing adult.

b. Wheelbarrow

Moments of inertia I_x about a horizontal axis x for long slender shapes such as a body on a bed, can readily be obtained by resting one end on a knife-edge O and the other end on a vertical spring of stiffness k, shown in the adjacent figure. The restoring moment about an axis through the knife-edge equals the force at the spring $k\ell\theta$ times the length ℓ, from which is obtained the torque/unit angle $k_\theta = k\ell^2$



$I = \rho_o^2 m$

From the relation for frequency

$$f = \frac{1}{2\pi}\sqrt{\frac{k_\theta}{I}}$$

follows the expression for radius of gyration ρ_o in terms of period T_o:

$$\frac{1}{T_o} = \frac{1}{2\pi}\sqrt{\frac{k\ell^2}{\rho_o^2 m}}$$

$$\rho_o = \frac{T_o}{2\pi}\sqrt{\frac{k\ell^2}{m}}$$

Applying the moment-of-inertia transfer equation, we obtain the moment of inertia about the horizontal axis through the center of gravity:

$$\rho = \sqrt{\rho_o^2 - d^2}$$

$$= \sqrt{\frac{T_o^2}{4\pi^2}\frac{k\ell^2}{m} - d^2}$$

$$I_d = m\rho^2$$

In the test, adjust k and ℓ to obtain a period T_o of between one and five seconds for reasons already discussed. (For a recumbent adult, k = 2.5 lb/in). Make sure the oscillation is in the vertical plane only. Leakage of energy to other degrees of freedom will reduce the accuracy of estimating ρ by altering T_o.

2. Determination of body spring and damping coefficients.

As far as we know, no one has ever made a serious attempt to obtain complete information on the spring and damping characteristics of the body tissue in this kind of motion. Some time ago, a measurement of head/foot characteristics was obtained here by restraining the bed in this direction by a very stiff spring and then exciting the head/foot motion of the body by a blow on the bed. Decay rates and frequencies of the lightly damped oscillation of the semi-fixed bed provided the basic data from which body damping and spring characteristics were calculated.

The schemes for obtaining these coefficients described below is based on the transient response of the bed referred to above. We do not know now that this technique will give consistent results; it is advanced here as theoretically feasible but not necessarily practical.

For illustrative purposes, determination of the stiffness and damping coefficients by transient response of the bed will be discussed for motion in one degree of freedom. We shall then give a detailed account of how this method will be applied to estimating coefficients controlling the motion of the bed in the sagittal plane.

For free motion in one degree of freedom of the bed

$$m\ddot{y} + r\dot{y} + ky = 0 \qquad (1)$$

has for a characteristic equation

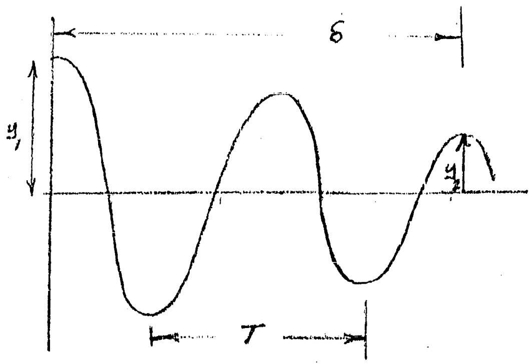
$$m\lambda^2 + r\lambda + k = 0$$ where λ may be regarded as a (complex) frequency.

whose roots λ_i give the exponents in the general solution to (1): $y = \sum A_i \varepsilon^{\lambda_i t}$

These roots are easily calculated as

$$\lambda_i = -\frac{r}{2m} \pm i\sqrt{\frac{k}{m} - \left(\frac{r}{2m}\right)^2} = -\gamma\omega \pm i\sqrt{\omega^2 - \gamma^2\omega^2} \qquad (2)$$

For a lightly damped oscillation, the record will look as in the sketch.



The measurements to be made are shown. The time δ is chosen large enough to make the ratio y_2/y_1, considerably less than one in order to be insensitive to error in measurements of y_1 and y_2. Damping, r, and spring, k, are related the test data by

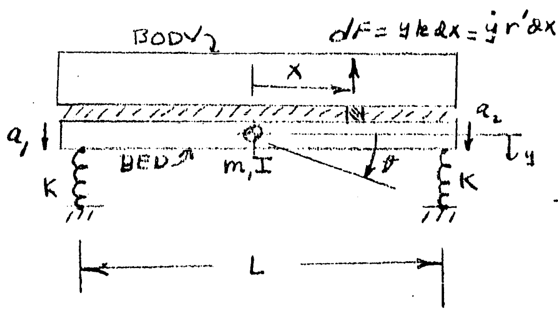
$$\frac{r}{2m} = \frac{\ln y_1/y_2}{\delta}$$

$$\sqrt{\frac{k}{m} - \left(\frac{r}{2m}\right)^2} = \frac{2\pi}{T} = \bar{\omega} \qquad (3)$$

Knowing the mass (m) of the bed, k and r can be extracted from these relations.

Sagittal coefficients

First we introduce stiff springs K from bed to ground, so that the natural frequency of the bed relative to ground is high compared to that of the bed to body. In this case, the amplitude of the body oscillation will be small compared to the amplitude of the bed, so that the bed displacement is relative to ground, which we can measure, is nearly equal to the displacement of the bed relative to the body. The equations of bed motion for this configuration are:



$$m\ddot{y} + \left[\int_0^L r'\,dx\right]\dot{y} + \left[\int_0^L k'\,dx + K\right] y = 0$$

$$I\ddot{\theta} + \left[\int_{-L/2}^{L/2} r' x^2\,dx\right]\dot{\theta} + \left[\int_{-L/2}^{L/2} k' x^2 + K L^2/2\right]\theta = 0 \qquad (4)$$

In this analysis, spring and damping from shear have been neglected because it is expected that they will be small for sagittal motion. Putting in effective values of k' and r', we have

$$m\ddot{y} + \bar{r}'L\,\dot{y} + (\bar{k}'L + K)y = 0 \tag{5}$$

for the equation of vertical motion. Assuming the pick-offs are accelerometers of output a_1 and a_2, this equation becomes in terms of the sum of the output -

$$m(\ddot{\overbrace{a_1 + a_2}}) + \bar{r}'L\,(\dot{\overbrace{a_1 + a_2}}) + (\bar{k}'L + K)(a_1 + a_2) = 0 \tag{6}$$

Observe that (6) is similar to (1). Hence, from (3)

$$\frac{\bar{r}'L}{2m} = \frac{\ln (y_1 / y_2)}{\delta}$$

$$\sqrt{\left(\frac{\bar{k}'L + K}{m}\right)^2 - \left(\frac{\bar{r}'L}{2m}\right)^2} = 2\pi/T \tag{7}$$

where the quantities y_1, y_2, δ and T are to be taken from a decay record of the sum of the accelerometer outputs. How the bed is excited does not matter in general, but if the frequency spectrum is scanned for maximum response to sinusoidal drive, the period T is easily determined as the frequency of maximum amplitude. Then by removing the drive, the decay over a time δ yields the ratio y_1/y_2.

What we have done to this point is to determine effective values of damping and stiffness.

We have now to determine the position $\bar{x}$ where these effective values act. This is determined from the moment equation by replacing the integrals by effective values and expressing the equation in terms of the difference of accelerometer readings - thus:

$$I\,\ddot{\overbrace{(a_1 - a_2)}} + \bar{r}' L \bar{x}^2 \dot{\overbrace{(a_1 - a_2)}} + \left(\frac{KL^2}{2} + \bar{k}' L \bar{x}^2\right)(a_1 - a_2) = 0 \qquad (8)$$

From the decay record of $a_1 - a_2$, one measures y_1 and y_2 and δ, giving $\bar{x}$:

$$\frac{\bar{r}' L \bar{x}^2}{2I} = \frac{\ln y_1/y_2}{\delta} \qquad (9)$$

No frequency (period) measurement is necessary since only one quantity is unknown. The damping relation was chosen for its higher sensitivity. However, a frequency $(\frac{2\pi}{T})$ measurement might be needed as an aid to getting good accuracy in the measurement of $\bar{k}'$ because it appears in combination with a large term - the K spring. It may even be necessary to take several readings in computing $\bar{k}'$ and then apply the methods of statistics to get a meaningful estimate of $\bar{k}'$.

C. SOLUTION OF THE EQUATIONS OF MOTION

As mentioned in Chapter I, the equations of motion of the body-support system have been solved(1)(9) for the simple case of head-foot (y) motion, which is relatively free of coupling with rotation. However, the lateral (x) motion of the body couples strongly with rotation in both yaw and roll, because the cardiovascular driving forces do not pass through the body's center of gravity.

In the case of yaw [motion about the γ or AP axis in the xy plane] the cardiovascular y-forces are so nearly along the body axis, and the inertia is so large about an AP axis through the navel (approx. cg), that the contribution of y-forces to rotation in yaw, is small. However, transverse (x) forces in the body's frontal plane are large because of the tilt of the heart. Consequently the body tends to rotate counter-clockwise in yaw with a motion:

$$I''_{\gamma_B} = X\ell + M_\gamma$$

[where ℓ is the instantaneous distance of transverse force X from the cg, and M_γ is the pure couple exerted by the blood in rounding the aortic arch]

However, a pendular body support rotates at quite low frequency in yaw (see bifilar calculation above) so that the coupling of x and γ is compliant enough to be negligible if the x motion is not constrained. Moreover in this xy plane the coupling between body and support may be made relatively stiff, so their relative motion about the γ axis is negligible.

Hence there is little need to examine the xyγ system of equations for interaction between body and support. Moreover information-wise, we do not need to solve for M_γ (in the above equation) because the cardiovascular "news" so obtained would be trivial. The value of x (transverse component of cardiovascular force) however, is so strongly involved in roll, that the xyγ system of force equations alone, will not define it.

In the case of roll (forces in xzβ plane, transverse to the body), we find a major difficulty in decoupling rotation from translation. This is because (a) the roll inertia I_β is much less than are I_α and I_γ [$\rho_\beta^2/\rho_\gamma^2 = (13/40)^2 = 1/10$ for the radii of gyration] and (b) the cardiovascular forces at beginning and end of ejection and of filling, act on the body quite ventrad (3"-6") to the body c.g.. As a result with this moment arm, a rightward force X tends to roll a light platform under the body toward the left, the while it translates the body (and platform) to the right. If now there is spring coupling with resonance $\tilde{\omega}_\beta$ in roll, a roll BCG not only adds to the lateral, but has a highly variable phase and amplitude relation.

In sum, the yaw BCG does not interfere with the lateral BCG as does the roll response, because (1) the body inertia I_γ is much larger; (2) the X force is on the same side of the body c.g. (i.e. headward) as is the best coupling to the support; (3) the support can be free to rotate with the body, avoiding excitation of springs k_γ; (4) the stiffness of coupling k_γ from body to support is naturally much higher than is k_β.

For these reasons we will confine our analysis of the interaction error due to rotatory coupling in measuring the BCG, to the motions seen in the transverse (xzβ) plane. We will show analytically, that for supports which are not free to roll with the body, the coupling from body to support in roll acts as a high-cut filter to the lateral BCG: (both as to resonance and cutoff). Further, for deep-chested individuals (high rotational moment of lateral forces) a double-resonance occurs. This creates a frequency band in mid-spectrum, where support and body act in phase-opposition, and so suppresses important details in the lateral BCG. These relations result when physiological values of body stiffness are inserted in the xzβ equations, as follows.

Particular BCG Equations of Motion for the xzβ plane and their solutions.

Equations 1 - 6 (Table 1) show the relations between the forces and motions, as well as the torques and rotations defined in Fig. 2. Implicit are the assumptions that through the frequency spectrum, there exist constant coefficients (parameters) of stiffness and damping (k's and r's) for each mode when the other motions are zero. Thus, with locked support, a pure couple will excite a roll frequency

$$\bar{\omega}_\beta = \sqrt{k_\beta/I_\beta - (r_\beta/2I_\beta)^2}$$

$$= \omega\sqrt{1-\zeta^2}$$

[where k_β is the equivalent shear stiffness seen by a pure rotation] and a damping factor r_β [which governs the β motion of the body alone]

Similarly $\bar{\omega}_x$ is the natural frequency of x vibration when rotation is locked; although physically k_x is a stiffness aspect of the identical tissues excited in a different way than for k_β.

In practice one cannot measure the force coefficients k and r, but only $\bar{\omega}$ and ζ the frequency and damping of the single (locked) modes of vibration. Experience shows that referred to the support, whole body frequencies are $\gg$ 3 c/s or $\bar{\omega} \gg 18$ with damping factor $\zeta = \frac{r}{2m\bar{\omega}} \leqslant .25$ where $mg = W \gg 50$ lb.

or $Q = \frac{1}{2\zeta} \gg 2$ in terms of resonance ratio

so that the damping term amounts to: $\frac{r}{2m} = \zeta\bar{\omega} = 5$

The complex frequency is: close to monotonic $\omega^2 = (\omega\zeta)^2$

The decrement $\ln(y_1/y_2) = \frac{\Upsilon r}{2m} = 5/3$

shows damping $y_2/y_1 = \mathcal{E}^{-5/3} = \frac{1}{5}$ which is quite strong.

In order to solve these **six** equations, one must calculate the k and r from observed values $\bar{\omega}$ and ζ, assuming suitable masses or inertias.

Table 1. Equations of Motion ($x\ z\ \beta$ plane)

rewriting with moment arms <u>positive</u>, and rearranging for computer connections

x Forces on body and support:

(B) $$m_B \ddot{x}_B + r_x \left[(\dot{x}_B - \dot{x}_S) + (h_B \dot{\beta}_B + h_S \dot{\beta}_S)\right]$$

$$+ k_x \left[(x_B - x_S) + (h_B \beta_B + h_S \beta_S)\right] = F_x \quad \text{(drive)}$$

(S) $$m_S \ddot{x}_S - r_x \left[(\dot{x}_B - \dot{x}_S) + (h_B \dot{\beta}_B + h_S \dot{\beta}_S)\right] - r'_x \dot{x}_S \quad \text{(ground)}$$

$$- k_x \left[(x_B - x_S) + (h_B \beta_B + h_S \beta_S)\right] - k'_x x_S \quad \text{(ground)} = 0$$

β Torques on body and support:

(B) $$I_B \ddot{\beta}_B + r_\beta (\dot{\beta}_B - \dot{\beta}_S) + h_B r_x \left[(\dot{x}_B - \dot{x}_S) + (h_B \dot{\beta}_B + h_S \dot{\beta}_S)\right]$$

$$+ k_\beta (\beta_B - \beta_S) + h_B k_x \left[(x_B - x_S) + (h_B \beta_B + h_S \beta_S)\right] = -F_z \omega_f - F_x h_f$$

(S) $$I_S \ddot{\beta}_S - r_\beta (\dot{\beta}_B - \dot{\beta}_S) - h_S r_x \left[(\dot{x}_B - \dot{x}_S) + (h_B \dot{\beta}_B + h_S \dot{\beta}_S)\right]$$

$$- k_\beta (\beta_B - \beta_S) - h_S k_x \left[(x_B - x_S) + (h_B \beta_B + h_S \beta_S)\right] = 0$$

z Forces on body and support:

(B) $$m_B \ddot{z}_B + r_z (\dot{z}_B - \dot{z}_S) + k_z (z_B - z_S) = F_z \quad \text{(drive)}$$

(S) $$m_S \ddot{z}_S - r_z (\dot{z}_B - \dot{z}_S) - k_z (z_B - z_S) = + r'_z \dot{z}_S + k'_z z_S \quad \text{(ground)}$$

where	m_B = .375 inch slugs	k_x = 375 lb/in	k_z = 1500 lb/in
	m_S = .025 inch slugs	r_x = 5 lb.sec./in	r_z = 11 lb.sec./in
	I_B^* = 14.5	k_β = 21,600 lb.in/rad	h_B = 2" h_S = 4"
	I_S^{**} = .833	r_β = 200 lb.in.sec./rad	h_S = 2" ω_f = 2"

* assumed 8"x20"

** assumed 1"x20"

C 1. Solution by analog computer (differential analyser).

Discussion:

We have seen that the forces and torques in the transverse ($xz\beta$) plane prove particularly interesting, because of (a) the small inertia (I_β) of the recumbent human body in roll, and (b) the application of cardiovascular force on the body, on the (anterior) side of the β axis (thru c.g.) away from the contact with support. Qualitatively, the resulting lateral (x) response of the body to such forces, is bucked out by the body's rolling motion about its c.g.. This unfavorable situation does not exist for any other rotational axis, and so becomes the most important dynamical relation to solve for.

In Table I are gathered the $xz\beta$ equations for body and support, as previously developed. These equations show the following features, which stand out clearly in the analog connections (Fig. 2B).

(a) In every equation there occurs the term in square brackets

$$[(x_B-x_S)+(h_B\beta_B+h_S\beta_S)]$$

This is a force in shear proportional to the relative motion of body and support, whether displacement or velocity. Note that the translational term depends on the difference of the motions, but the rotational term on the sum, because of the body and support are tangent.

(b) This composite force in shear operates on both body and support, to oppose the relative translation and rotation of both: the latter in proportion to the distances of the c.g.'s from the interface of shear motion, h_B and h_S. [Since this interface is curved, one must establish experimentally an equivalent straight line of "resistance" to which h is measured]. While h_S is constant, the dimension h_B varies considerably with a subject's "thickness"; so that the coupling between lateral and roll BCG will vary similarly.

(c) There also occur spring and damping forces in "pure" relative roll

$(\beta_B-\beta_S)$. For this to happen without relative motion in shear, i.e.

$$[(x_B-x_S)+(h_B\beta_B+h_S\beta_S)] = 0$$

there must be a relative vibration of both bodies in pure roll $(\beta_B-\beta_S)$, so as to produce alternating compression of tissues on both sides of the midline. This compressional vibration thus sees mainly the compressional tissue stiffness in the z direction but partly also stiffness in shear. The natural frequency of this vibration if excited separately, should be higher than that of tissue in shear, due partly to greater stiffness in compression and to lower inertia in roll.

(d) A theoretical dynamic system set up on an analog computer, actually needs light coupling to "earth" just like a real system. This is shown in Fig. 2B for both translation and rotation, using both "springs" k' and "dampers" r'. During operation the integrated amplifier drifts will move the output off scale, if equivalent springs K' to earth are not added. With such springs, the output will slowly oscillate till damped out by the added coefficient r'. So the analog clearly parallels real systems designed to have zero force to earth, such as the "ideal BCG". This is because such real systems also show drift with the least wind, so that practical suspensions must have some "ultra-low-frequency" springing rather than be completely "aperiodic" (e.g., mercury bed).

(e) The particular dynamical problem whose solution is shown in Fig. 2B involves locking the platform rotationally, without stiffening the translation. The analog then acts like current clinical BCG beds of ultra low frequency. One accomplishes this in the computer, by grounding at $\dot{\beta}$ and β. When these terms drop out of all the equations, the "relative" rotations become simply β_B: body roll on the support, as a recumbent subject on a table.

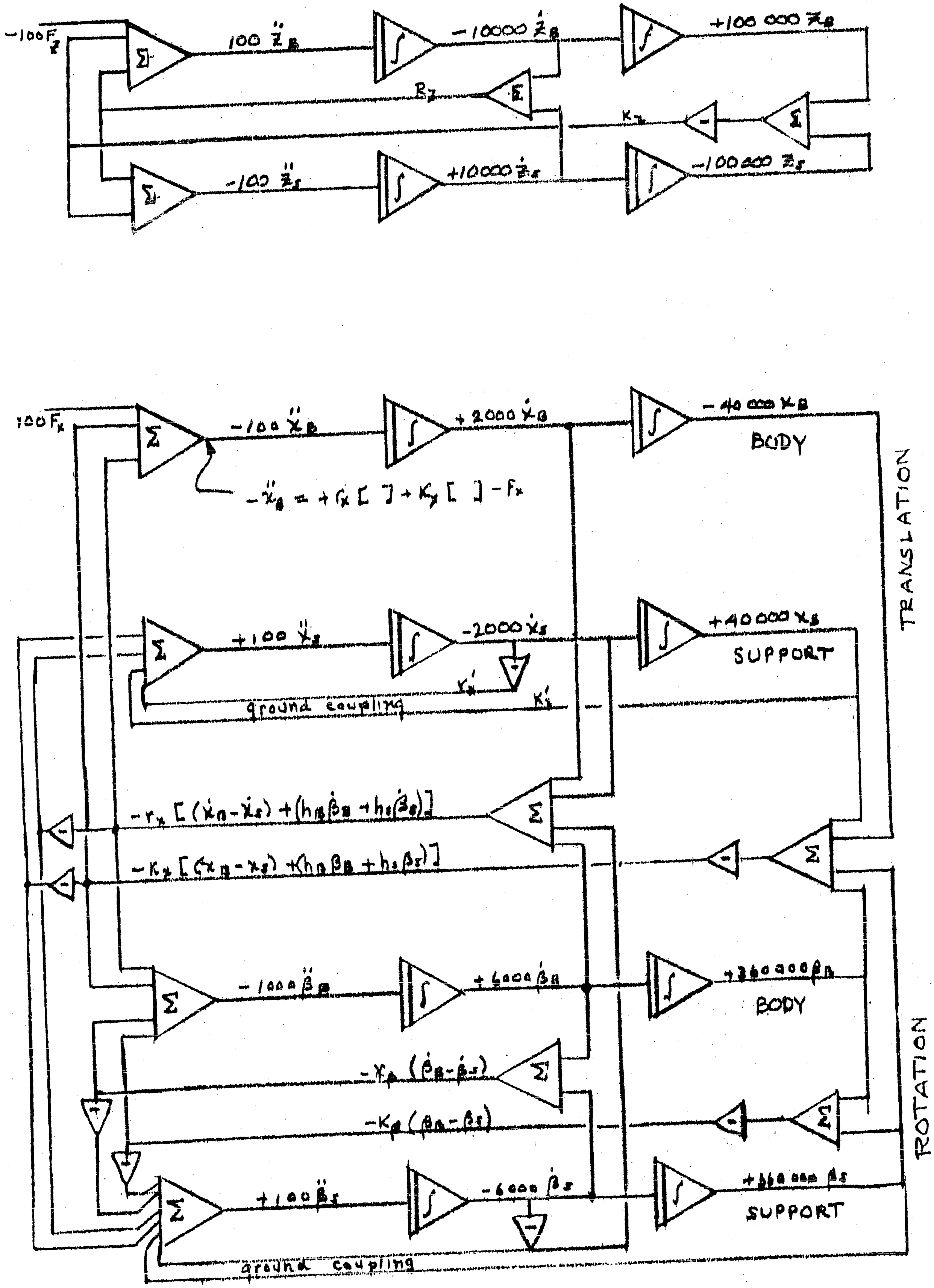


FIG. 2B COMPUTER ANALOG OF XZβ DYNAMICS

Results of Computer solution of motion in XZβ plane.

The lateral motion $\ddot{x}_s$ of the support when driven by a force on the body F_x, appears in figure 2C as a function of frequency. The roll $\ddot{\beta}_B$ of the body is shown in figure 2D. The dotted curves show the response when support is free to roll with the body; while the solid curves show the resonances generated when the support is not permitted to roll, as with current BCG practice.

The line T = 0 would be the response were the heart forces directed thru the roll axis. The lines T show the change due to the torque resulting from the anterior position of the heart. 2T is for normal heart position. When platform is free to roll (dashed lines), the lateral amplitude $\ddot{x}_s$ decreases somewhat at all frequencies as the heart force moves forward, due to energy going into roll. This enhanced roll motion appears in Fig. 2D (line 2T). However, the body would roll somewhat even if the heart force passes through the roll axis, due to inertia reactions of platform.

The freely rolling platform resonance shows translational resonance at the frequency $\bar{\omega}_{sx}$ of the support on its coupling to the body. This resonance corresponds exactly to the cutoff seen in the head-foot (y_s) frequency-response curves of ULF support(/); it is caused by finite mass of support m_s and its coupling k_x to the body, and so inevitable. For convenience this cutoff was set at 10 c/s, corresponding to a platform "heavy" in translation.

With support <u>not</u> free to roll with the body (a) a strongly resonant peak (Q = 2.5) appears at a frequency $\bar{\omega}_{B\beta}$ where body-roll-resonance is in phase with translation; (b) just beyond this is a sharp trough (attenuation to about 1/5 amplitude) where the roll goes out of phase with the lateral motion; and finally (c) an increase to the undistorted (dotted) amplitude, exceeded somewhat with zero torque; because the body's roll inertia adds to its translational inertia. The dual peaking from phase interference is characteristic of coupled resonant systems.

Our analog also describes (fig. 2D, solid lines) the roll-motion of the body itself on a platform that won't roll. Even without impressed torque from the heart (T = 0), the platform translation causes a strong body-roll. But with the normal anterior cardiac drive, the energy goes increasingly into roll at the higher BCG frequencies, and less into lateral motion of the platform.

In net effect, the roll constraint on the platform introduces a lateral BCG resonance followed by cutoff at and beyond the natural frequency of body-roll on the support. Since the dorsum is rounded the ribs and shoulders are compliant when chocked, the stiffness in roll is intrinsically low; so with the low inertia about the β axis, the roll-resonance is hard to raise above 6 c/s. Such a periodicity is indeed seen commonly in lateral BCG records, as well as the predicted absence of sharp wave-forms (containing higher frequencies). The analog computer, using the simple two-mass model of Fig. 2A, therefore has accounted for the major artefacts of the (lateral) RL-(ULF)BCG. It also predicts that these artefacts vanish when the support is free to roll.

The case of the (HF)BCG is not shown here, but again exhibits complex resonance in the RL record, due to coupling between the resonances $\bar{\omega}_{Bx}$ and $\omega_{B\beta}$; this time with higher Q and lower frequency ($\bar{\omega}_{Bx}$ = 6 c/s).

The frequency at which body is resonant in roll can be shifted upward (on the computer) to imitate tethering-straps fastened across the chest, and a form-fitting support. Since these will also raise $\bar{\omega}_{sx}$ (the lateral resonance) the BCG pass-band can be increased. If instead the support is free to roll, the resistive torques between body and platform $k[h\beta_B + h_S\beta_S]$ and $k[\beta_B - \beta_S]$ are not forced into action, and such tethering becomes unnecessary.

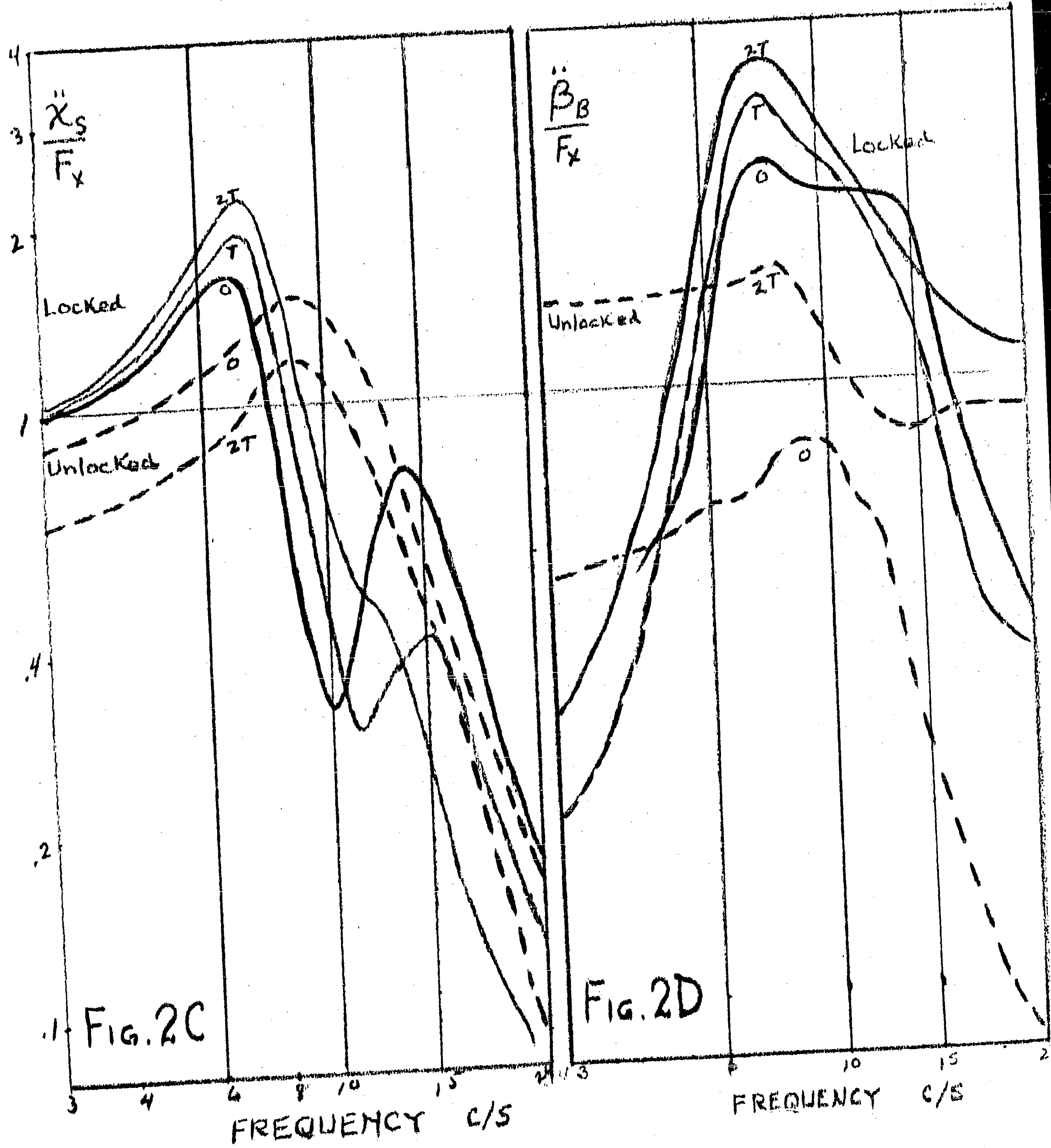
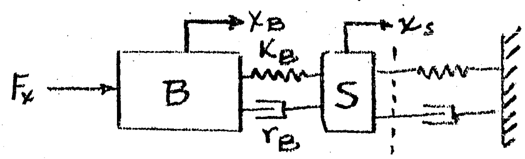


Fig. 2C Reeves computer results:
Lateral acceleration of support by lateral force in body [solid lines] (0): force through c.g. (platform not free to roll), showing resonances due to inertial reaction of support, tangential to body. (T) and (2T): lateral force more anterior (in position of heart), showing enhanced roll-resonance and cut-off resulting. [Dotted lines]: (0) and (2T) lateral heart force acting through c.g., and anterior to it (platform free to roll); showing simpler cutoff, due only to translational body-coupling to support (of 1/10 body mass).

Fig. 2D: Roll acceleration of body by lateral force in body [Solid lines]: Support not free to roll -- showing resonant body-roll so created. [Dotted lines]: Support free to roll -- curve (0) force thru c.g.; body roll ceases above frequency of platform decoupling. Curve (2T) force in heart position; body-roll (driven) persists without transmission to support.

C-2. Passive analog solution. [L C R circuit]

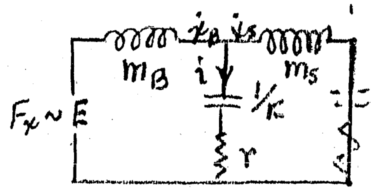
Just as in pure y or z motion of a two-mass system, one can set up a simple mechanical analog whose forces are:



where $v = \dot{x}_B - \dot{x}_S$

$$m_B x_B + r_B(x_B - x_S) + k_B(x_B - x_S) = F_x$$

equivalent electrically to:



where $i = \dot{x}_B - \dot{x}_S$

Similarly there is a passive circuit equivalent to the $x\beta$ equations of Table 1. This circuit can be realized by multiplying the rotational quantities by a distance h, which makes them linear and so able to combine with the linear equations. [In fact the linear networks of the active analog, receive rotational couplings by this same process.]

A peculiarity of the passive (LRC) model is that components required by certain electrical analogs (resulting from particular mechanical assumptions) are unobtainable, e.g. variable transformers. However, it was found[47] that a more specialized model than Fig. 2, with mechanical parameters lumped in a particular way would express the same mechanics. This is shown in Fig. 3A.

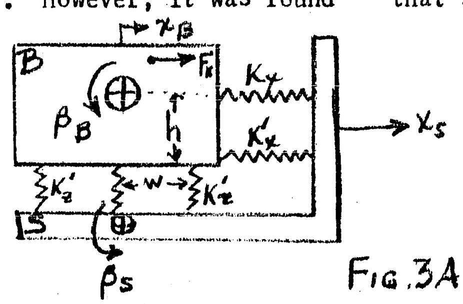


FIG. 3A.

For the body:

$$m_B\ddot{x}_B + (r_x + r_x')(\dot{x}_B - \dot{x}_S) + (k_x + k_x')(k_B - x_S) = F_x$$

$$I_B\ddot{\beta}_B + [r_x(\dot{x}_B - \dot{x}_S) + k_x(x_B - x_S)]\,h + [r_z'(\dot{z}_B - \dot{z}_S) + k_z'(z_B - z_S)]\,w = -F_x h_F + F_y w_f$$

where $K_{\beta x}$ = torque/angle

= force .h/dist/h

$= \dfrac{\text{force}}{\text{dist}} \cdot h^2$

$= k_x' h^2$

For the support, similarly:

$$m_S\ddot{x}_S - (r_x + r_x')(\dot{x}_B - \dot{x}_S) - (k_x + k_x')(x_B - x_S) = 0$$

$$I_S\ddot{\beta}_S + [r_z w(\dot{z}_B - \dot{z}_S) + r_x h(\dot{x}_B - \dot{x}_S)] + [k_z' w(z_B - z_S) + k_x h(x_B - x_S)] = 0$$

This particular mechanical hookup may be analyzed rather simply, electrically [at far less cost than an active (operational) analog computer] with passive (LCR) elements: Fig. 3B.

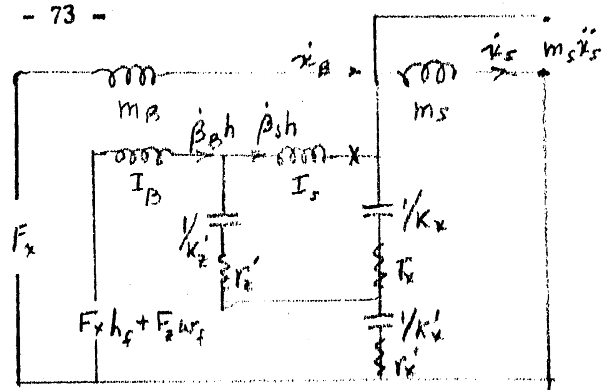


FIG. 3B.

Experimentally the values of the inductors* (the most costly elements) determine the impedances throughout the circuit. These are set at an arbitrary level. The frequency level (machine time) is chosen at 100 x real frequency, to put the performance spectrum in the range 5 - 5,000 c/s, where the inductors have least error from intrinsic R and C. This puts the real spectrum at the BCG range [.05-50 c/s]. The L and C = 1/K values are calculated (as for the active network) from the assumptions for real-time resonances and damping:

$\bar{\omega}_{B\beta} = 5$ c/s $= \sqrt{K_\beta / I_\beta}$ $\qquad \bar{\omega}_{Bx} = 10$ c/s $= \sqrt{K/m}$ $\qquad \bar{\omega}^2 = \frac{1}{LC} = \frac{K}{m}$

$Q_{B\beta} = 2 = L\frac{\bar{\omega}}{R_x'}$ $\qquad Q_{Bx} = 2 = \frac{L\omega}{R_x + R_x'}$

To simulate constraining the roll of support ($\beta_S = 0$), sever the circuit at (x) so that body roll β_B is excited via k_x' and k_z'. There results a lateral motion of the support $\ddot{x}_s$ (lateral BCG spectrum) shown in Fig. 3C. It is clear that the body-roll strongly distorts the lateral BCG, especially when the cardiovascular driving force creates (as usual) a clockwise moment. This moment opposes platform motion $\ddot{x}_s$ below $\bar{\omega}_{B\beta}$ and assists above this roll resonance. There results a pair of system-frequencies $f_1 f_2$ (displaced from the locked natural frequencies $\bar{\omega}_\beta \bar{\omega}_x$) which dominate the lateral BCG spectrum seen with the usual horizontal (non-roll) platform. This agrees with the "active analog" analysis above, confirming the passive analog.

*Hycor type EM-6.

Results of passive analog solution: response spectrum of the transverse BCG.

A. Effect of body coupling and platform mass.

Before investigating the effect of adding BCG motion in the roll mode, the purely translatory solutions were reexamined. This was done to tie in with previous theory[1], and to specify quantitatively the amount of coupling ("tethering") needed between body and support in any mode, in terms of the "natural frequency" of that coupling referred to a "locked" support. This is the method of determining coefficients examined theoretically in Chap. III.

In Fig. 3C the solid lines show the influence of relative mass of support with very stiff body-coupling ($\bar{\omega}_B = 10$ c/s). In dotted lines are shown heavy and light supports, with very weak body-coupling ($\bar{\omega}_B = 2$ c/s).

It is clear that the support could indeed be rather heavy, provided the coupling is extremely tight $[K_{10\ c/s} = 25 \times K_{2\ c/s}]$; but that if no particular strapping or stressing of the body is used, the mass or inertia of the support must be very much less than that of the body, to obtain satisfactory responses to the sharp (20 c/s) components of the BCG. This is particularly pertinent in the roll mode. In Table I the ratio of body inertia about the roll axis to that of the support is $I_B/I_S]_\beta = 17.5$ According to Fig. 3D with such a ratio no fastening in roll should be required, even on a flat-surfaced support.

B. Effect of body roll on lateral response.

When the "rotational" meshes of the circuit (Fig. 3B) are added, the important region 3-10 c/s acquires a dual resonance, with striking effects on the X_S/F_X spectrum.

The circled line in Fig. 3D shows the response to a lateral driving force F_{Bx} acting directly through the β axis (back alongside the dorsal

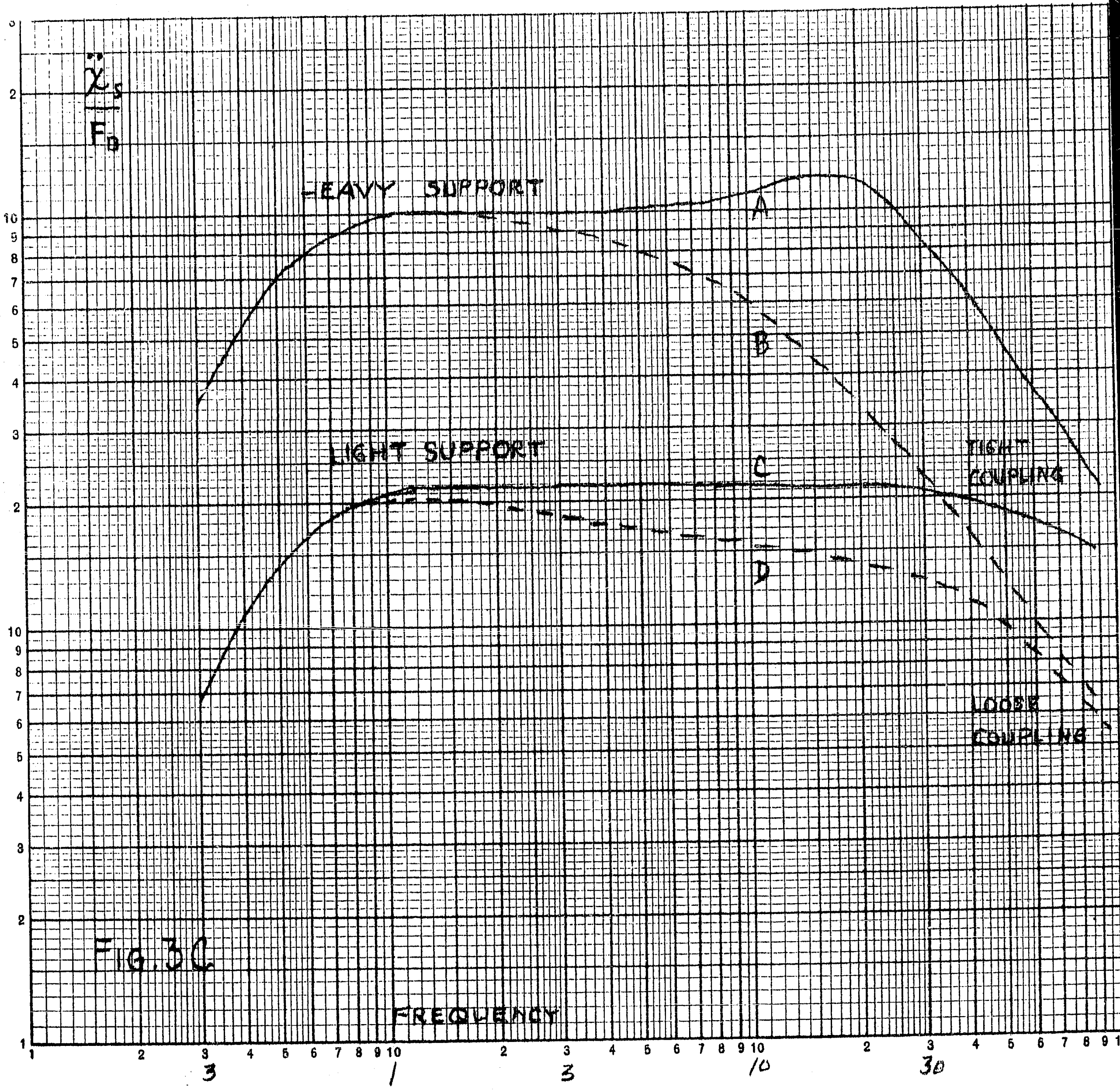


Fig. 3C: Computation on LCR analog.

Effect of body-coupling compliance compared with effect of heavy-platform on lateral BCG spectrum:

	relative mass (m_S/m_B)	lateral stiffness (lb/in)
A	1/4	75
B	1/20	45
C	1/4	3
D	1/20	3

Showing resonance with tight coupling and heavy support, and loose coupling sufficient with light support.

.... Following page:

Fig. 3D: LCR analog computations. Effect of non-rolling support on lateral BCG. [Solid lines] resonance and absorption due to enforced body roll (T, 2T, 3T increasing anterior heart position) [dotted lines] resonance artefact predicted for counter-clockwise torque (-2T) exerted by heart in dextrocardia.

Fig. 3B: Roll effects on lateral BCG without torque on body from blood flow:

A. Neither body nor support can roll, both move laterally.
B. Support cannot roll, body can roll, both move laterally.
C. Both body and support can roll, and move laterally.
(roll resonance absent; lateral response reduced)

aorta). The roll resonance resembles that in Fig. 2C for the same case, except that here $\bar{\omega}_\beta$ is set at 3.5 c/s: somewhat less tethering in roll. Now when the drive is moved anteriorly to the natural position of the heart (curve T in Fig. 3D) a torque T_β is produced by F_{Bx}. A similar peaking and sharp attenuation result in the active analog (Fig. 2C, curve T). As this torque is increased (2T and 3T) (as with deep chests), resonance in roll so increases as to obliterate the trough by phase opposition.

The dotted lines show the change in roll artefact which would occur, were the moments from ejection to become counterclockwise (seen footward). [This could happen with an upright aorta or dextrocardia.] In this case the S-shaped distortion of response begins at lower frequency, suppressing the important 3 c/s BCG component; while a broad resonance 4-10 c/s appears - a band-pass effect.

Finally, the passive computer was caused to show the motion of the system when the support was free to roll with the body-forces. The spectrum Fig 3E (C:dot-dash) is the lateral motion of support reacting to body roll, but now free of roll resonance. Mechanically, the springs responsible for this resonance are not excited by such a support. This result agrees with Fig. 2C. (dotted lines)

Critique:

These response-spectrum studies of the transverse BCG have been mainly exploratory, to gain control of the analytical method and models used. The resonance phenomena shown are produced by these models, and from the parameters (stiffness and damping) selected for the various modes of motion. The damping factors were purposely cut down to exaggerate the amplitude (Q) of the resonances, and to seek their physical basis.

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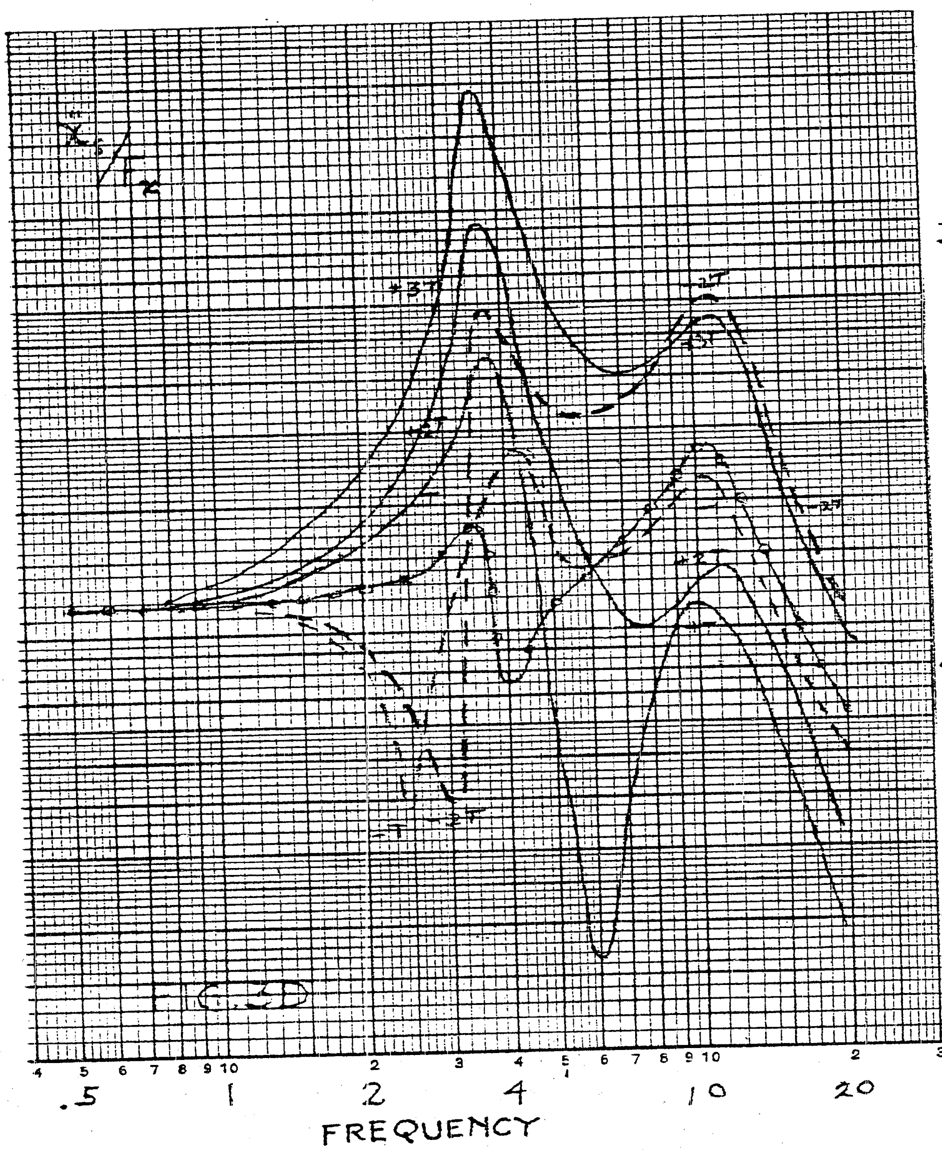

FIG. 3D

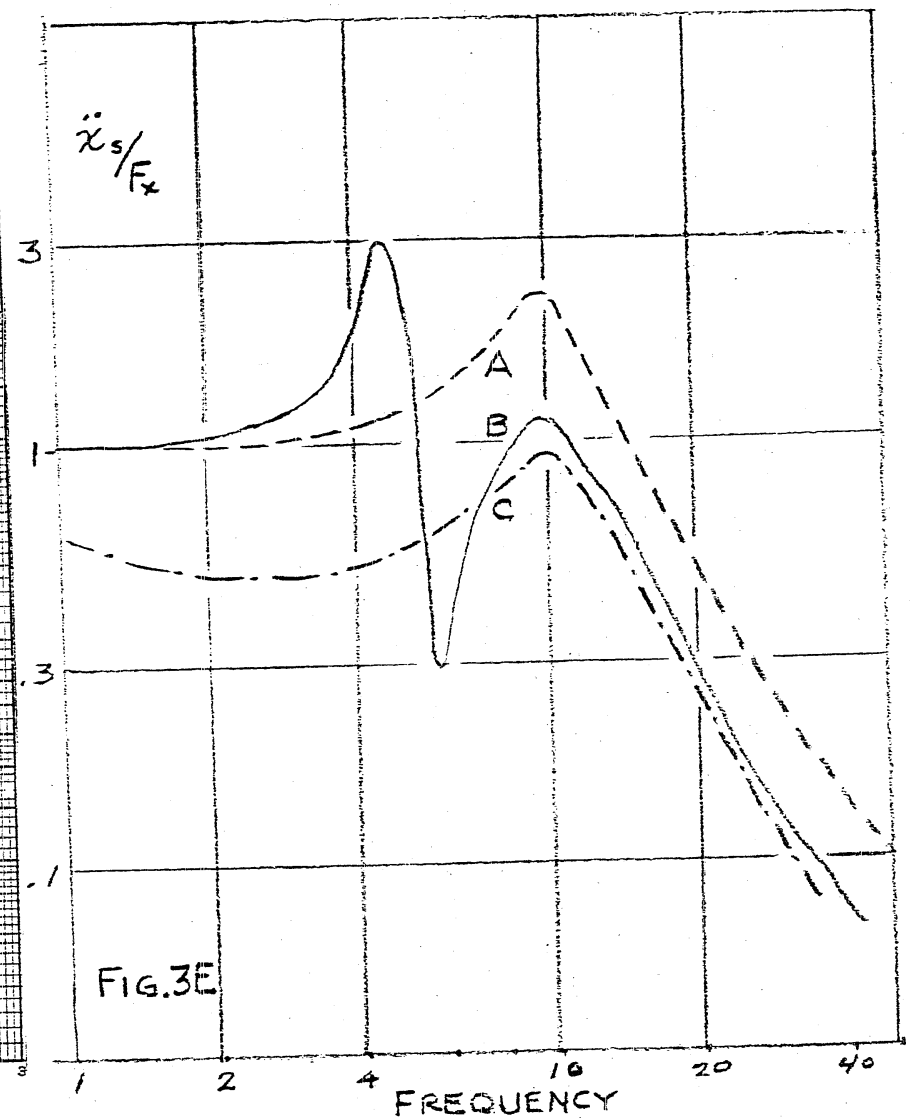

FIG. 3E

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The rotational resonances clearly depended on certain rotatory moments exerted by the cardiovascular force and the dynamic reactions to it: (a) about the axis of the body (h_f); (b) from the body-axis to the surface of the support (h_B); and (c) from this surface to the axis (c.g.) of the support (h_s). These important measures differ appreciably among individuals and for variously designed supports; and must be considered in interpreting the usual lateral BCG.

As in both BCG and ECG, one considers the heart-axis in interpreting the record; so in BCG, the effective thickness of chest (h_f+h_B) matters whenever a suspension is used where body-roll affects the lateral record. This complication in BCG interpretation can be avoided, if one gimbals the support to exclude the admixture of roll.

Our comparison of active computer and passive analog methods, in practice, shows that they serve different purposes. The LCR circuit is quite adequate to demonstrate the qualitative relations, and explore the frequency and damping aspects of the coupling between body and support. The differential analyzer lets one vary separate factors (such as the h's) more readily, and study the phase relations among all the motions and their derivatives. This was helpful in understanding in detail the sharp slopes of the response curves.

The analog computer has some disadvantages in practice. It is slower for getting simple response curves, because at each frequency several variables were recorded and later measured. With one-percent precision in the calculator one takes more careful settings. A large network (Fig. 2B) runs into difficulties (not found on the LCR) as to signal level, and as to circuit instability from cumulative phase-shift. This slowed down the problem to 1/10 real time vs. 100 x real time in the passive analog. The connections are much more numerous and complex, and require frequent checking. The main advantage is that the computer imposes less limitation on the model. Thus the LCR model required introducing compression springs (capacitors) to simulate the "pure" roll or stiffness; while the computer executed the corresponding mathematical operation $K(\beta_B - \beta_S)$ with a simple voltage divider (coefficient potentiometer). In general, computers can avoid specifying a particular physical mechanism, when this is unknown or unnecessary. More complex mechanics (such as interacting rotations in our case) requiring three-dimensional networks, may involve couplings which can not be realized by practically feasible LCR models.

Further study is needed of the dynamics of rotation-translation coupling. For example, the yaw BCG also participates in the lateral motion; and in a more complex way, since the body-mass is distributed less symmetrically about the yaw axis, than about the roll axis. It must be verified experimentally, that a support with low frequency in yaw, does not intermix the yaw BCG with the lateral BCG. In this case, due to the several moment-arms involved, the accompanying analysis requires an analog computer rather than an LCR model.

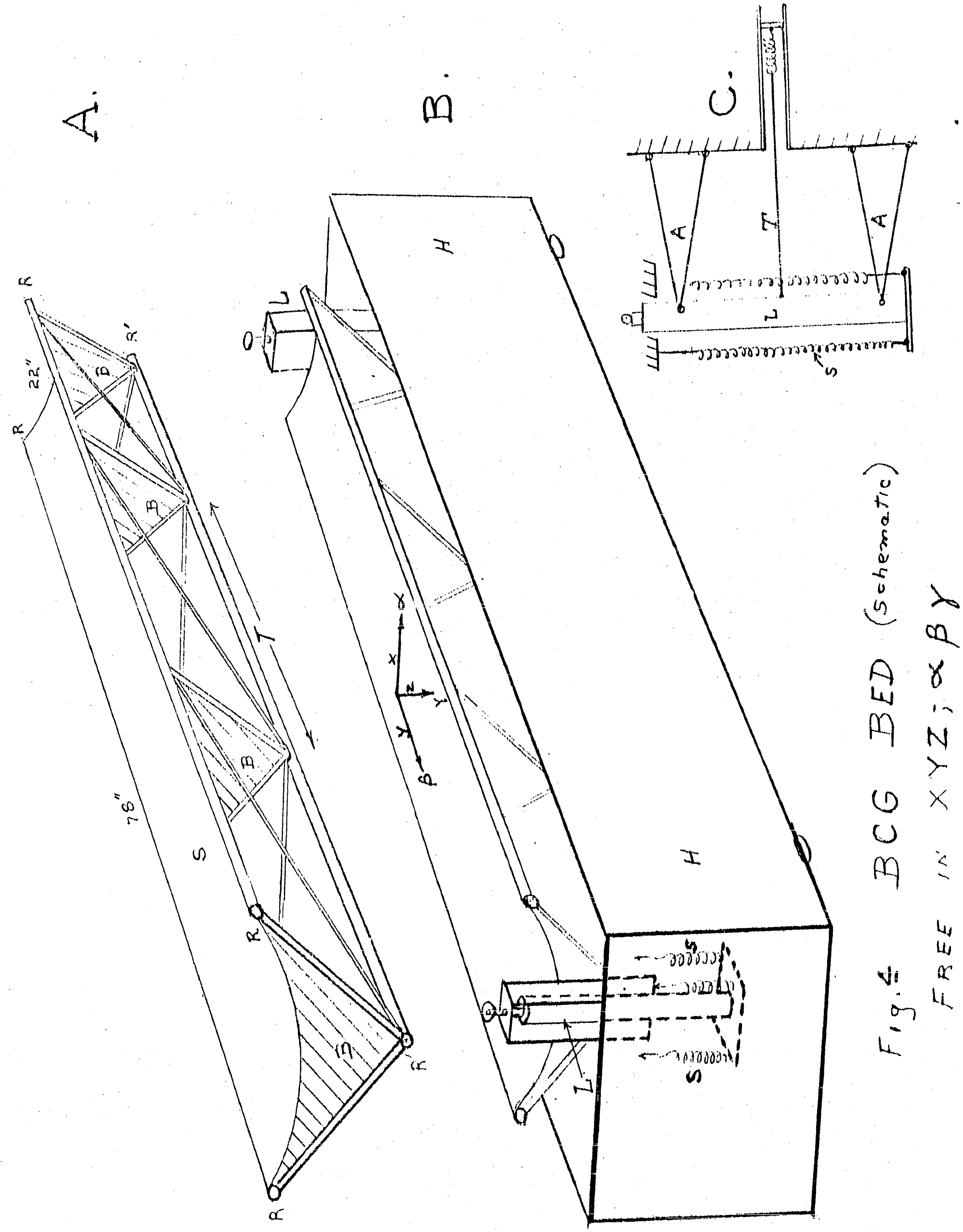


Fig. 4: A. Light-weight, torsion and vibration-free support of max. strength-weight ratio.

B. Housing for 3D ultra-low frequency suspension.

C. Relation of positive and negative springs of vertical suspension to u.l.f. leg (see fig. 6).

IV. DESIGN OF PRACTICAL SUPPORT SYSTEMS: RECUMBENT HUMAN SUBJECT DETAILED ANALYSIS OF STRUCTURE

Since a great deal of data has been obtained on a recumbent human subject, the bed was chosen as a support so that a comparison can be made between data obtained from our design and others. This comparison is expected to resolve much of the discussion concerning artefacts, since presumably our design has reduced artefacts of the suspension to an unobservable value. Even if it doesn't, the frequency response test of the support/patient combination will reveal any resonances present which might alter the ballisto record. With this information, such artefacts can be eliminated from the record.

Our objective here is to present the details of the analysis used to establish the design. Figure 4 shows the general arrangement of bed, suspension and housing. The bed consists of a sheet of aluminum (S) hung from two parallel side rails (R), held apart by a series of bulkheads (P). The bulkheads take very little of the patient's weight, most of it being diaphragmed to the rails. The rail loads are delivered to the truss panels T, from which the load is fed to earth at the ends via the two vertical legs L. Note the high torsional rigidity provided by the torque box consisting of the sheet and two side trusses.

Vertical and pitch frequencies are controlled by the vertical coil springs S (Fig. 4B) at each end. Alone, these springs produce a frequency of 10cps. Horizontal tension T on the leg in opposition to the A-frames (Fig.4C) produces a negative (toggle) spring effect, which reduces their stiffness sufficient to bring the vertical frequency down to 1/3 cps or so.

Roll frequency is controlled by additional vertical springs on either side of the leg (not shown) acting between bed and base of outer leg.

Lateral frequencies (including head-foot, side and yaw motion) are controlled by the legs at each end, each equivalent to a 20' pendular suspension. As will be described in detail later, the leg contains a positive and negative pendulum, whose combined stiffness yields a 1/3 cps natural frequency in linear motion.

The housing H is required to protect the bed from everyday abuse and to provide a loading platform from which the patient can transfer his weight to the bed with a minimum of concentrated and dynamic loading on the bed itself. The extreme lightness of bed construction required to get the low mass makes it **important** to mount the patient without damage to the bed.

This housing must establish rigid references for **setting** the vertical suspension (legs) plumb to high accuracy ($.01^{\circ}$). It must deflect little under loading, to minimize the technicians' concern with mechanical adjustments.

A. STRUCTURE OF BED ALONE

To achieve a bed of very low mass required detailed attention to stress and rigidity. First, however, design criteria must be established.

1. <u>Design criteria and loads.</u>

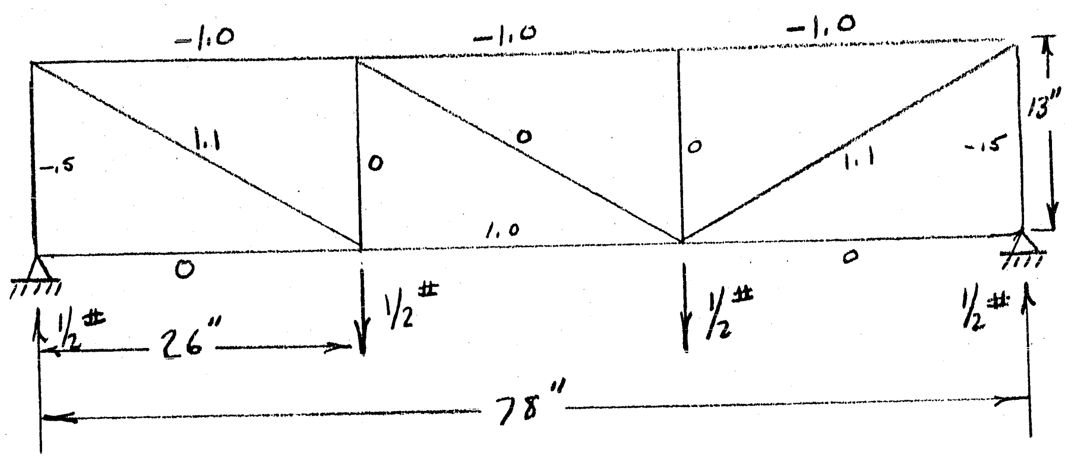
A design load of 300 lbs. was used for static deflection and stress calculations. Temporary pressures from this load of 30 lb/in^2 are possible from one hand. Test pressures are much lower for the recumbent patient, estimated at 1 lb/in^2.

To insure no artefacts in the record from the bed, the stiffness should be great enough to provide a minimum natural frequency of 40 cps. This is not an easily applied criterion because the mass entering into oscillation of this high a frequency is not that of the entire body, upon which the criterion is based, but something much less. Probably only 20 to 30 pounds mass - the tissue between the thorax and bed - are affected. To be definite. 25 lb. mass will be used in all stiffness calculations involving the 40 cps requirement.

This natural frequency requirement will be applied to the first force-free mode of bending and of torsion. The bending axes will be chosen as vertical and horizontal.

2. Vertical frequency.

Vertical and lateral stiffness come from the tubular frames. The bulkheads and sheet serve only to maintain the geometry and to deliver load to the frames, as far as vertical frequency is concerned. The sheet, of course, completes the torque box, so necessary to torsional rigidity. Shown below is one of the truss frames, loaded by unit force, 1/2 lb at each bay:



The stiffness is computed from the formula for deflection, Δ, at the center:

$$\Delta = \sum \frac{LF^2}{AE}$$

- L = length of truss member
- A = area of truss member
- F = load in truss member
- Δ = deflection
- E = Young's modulus

Horizontals are all 1"x.03" dural tubes. Center diagonals are 5/8"x.030" dural tubes. End diagonals are 3/8"x.030". From the above formula:

$$E\Delta = \frac{2\ (1.1)^2\ (29)}{.625\ (.03)\pi} + \frac{2\ (.5)^2(13)}{.625\ (.03)\pi} + \frac{4\ (1.0)^2(26)}{\pi\ (.03)}$$

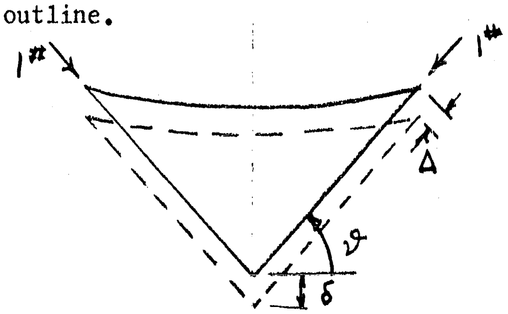
$$= 3300 \text{ lb/in}$$

Since this is due to a one pound load, stiffness in the plane of the frame is:

$$k_F = \frac{1}{\Delta} = \frac{E}{3300}$$

$$= 3000 \text{ lb/in}$$

The vertical stiffness of the bed depends on the stiffness of both frames and the geometry. The dotted outline in the sketch shows the deflection of an imaginary center bulkhead under unit frame loads, from the zero load, the solid outline.



The vertical load on the bed = 2 sin θ;

the vertical deflection of the bed under this load = $\delta = \Delta/\sin\theta$.

Hence, the stiffness of the bed,

$$k_B = \frac{2\sin^2\theta}{\Delta} = 2\,(9/13)^2\,3000$$

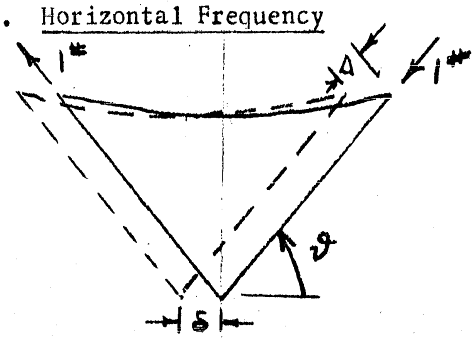
$$= 2900 \text{ lb/in in the vertical plane.}$$

The frequency of the bed referred to support is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2900}{25/386}}$$

$$= 34 \text{ c/s} \doteq 40 \text{ c/s}$$

3. Horizontal Frequency



Side force = 2 cos θ

Horizontal deflection

$= \delta = \Delta/\cos\theta$. Hence,

the horizontal stiffness is

$$k_B = \frac{2\cos^2\theta}{\Delta}$$

$$= 2\left(\frac{10}{12}\right)^2 3000$$

$$= 3500 \text{ lb/in}$$

from which is computed the frequency

$$f = \frac{1}{2\pi}\sqrt{\frac{3500}{25/386}}$$

$$= 37 \text{ c/s} \doteq 40 \text{ c/s}$$

4. Torsional frequency

The torsional rigidity is determined by the torque box formed by the two trusses and the sheet. For convenience, the shear rigidity of the trusses are assumed at least equal to the sheet. The shear rigidity of the curved sheet is assumed equal to that of a flat sheet for low stress levels. In this case, we can use a simple formula to compute torsional rigidity:

$$k_t = \frac{4G\,A^2 t}{UL} \qquad \left\{ \begin{array}{l} G = \text{shear modulus} \\ L = \text{length of torque tube} \\ A = \text{area enclosed by tube} \\ t = \text{sheet thickness} \\ U = \text{perimeter of tube section} \end{array} \right.$$

Substituting in numerical values -

$$k_t = \frac{4(4\text{x}10^6)\ (60)^2(.015)}{48\ (78)}$$

$$= .23\text{x}10^6$$

from which is computed the torsional frequency

$$f = \frac{1}{2\pi}\sqrt{\frac{k_t}{T}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{.23\text{x}10^6}{25\ (3)^2/386}}$$

$$= 100\ \text{c/s} > 40\ \text{c/s allowed}$$

4. Local Frequency

The frequency of the truss members must themselves be above the critical frequency. The diagonals have the lowest frequency, being the longest and thinnest of the members. The expression for frequency of these tubes of diameter D and length L is:

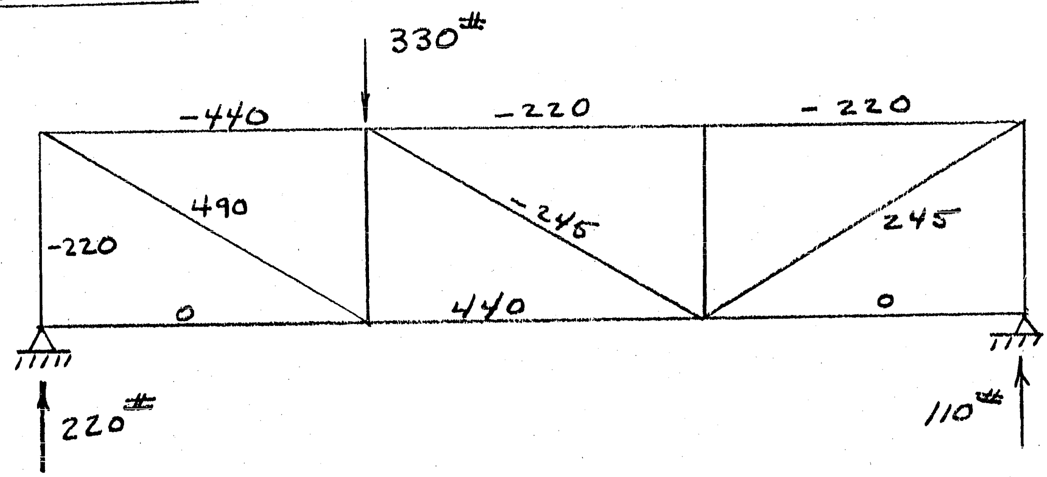
$$f = \frac{D}{L^2}\ (11\text{x}10^6)$$

For the diagonal of the truss under consideration here -

$$f = \frac{.375}{(29)^2} \ (11 \times 10^6)$$

$= 49$ c/s 40 c/s allowable

6. Truss stress



The 330 lb truss load arises from assuming a 300 lb subject placing his entire weight at that station.

Direct stress on diagonal members

$= 490/.375\ (.030)\pi = 13{,}800\ \text{lb/in}^2$

$< 50{,}000\ \text{lb/in}^2$ allowable

Direct stress on longitudinal members

$= 2(440)/1\ (.030)\pi = 9{,}400\ \text{lb/in}^2 < 50{,}000\ \text{lb/in}^2$

in the lower longitudinal member which carries load from both frames.

Diagonal Euler Load

$$P = \frac{2\pi^2 EI}{L^2} = 2\pi^2\ (10^2)\ \frac{\pi(.31)^3(.03)}{(29)^2}$$

$=$ 660 lb allowable

$>$ 245 lb actual

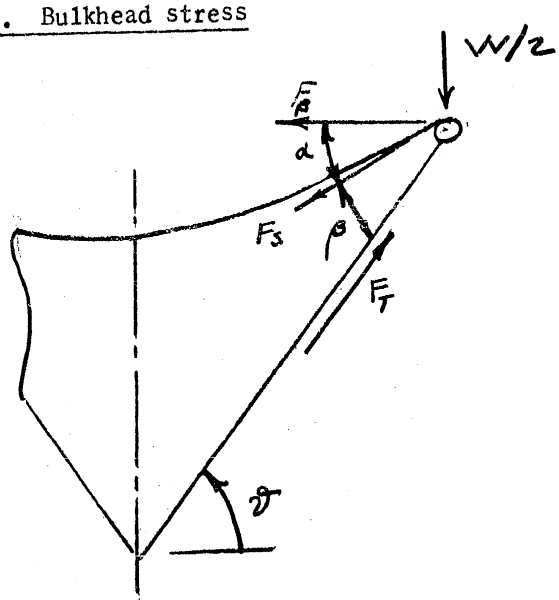
Horizontal Euler Load

$$P = \frac{2\pi^2 EI}{L^2} = \frac{2\pi^2(10^2)\ \pi(.5)^3\ (.03)}{(26)^2}$$

$=$ 3500 lb

$>$ 400 lb actual

7. Bulkhead stress



The function of the bulkhead is to maintain the geometry of the bed. If the sheet stresses F_S were parallel (β=0) to the truss reactance F_T, the loading in the plane of the bulkhead would be essentially zero, except for some direct loading on the top due to deflection of the sheet itself.

Loading due to Angle β

Static equilibrium requires

$$F_T \sin \theta = F_S \sin \alpha = W/2$$

since F_S is due to a load in the center directly above the bulkhead. This will be the critical loading for the bulkhead with W equal to the entire patient weight, taken here as 300 lb.

The net squeezing load F_β is the difference between the horizontal components of F_S and F_T -

$$F_\beta = F_S \cos \alpha - F_T \cos \theta$$

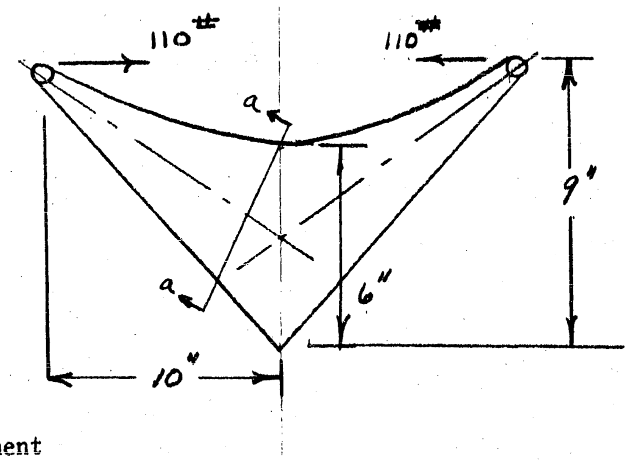
$$= \frac{W}{2} (\cos \alpha - \cos \theta)$$

For $\theta = 45^\circ$ and $\alpha = 30^\circ$ -

F_β = 110 lb.

β - stress at section a-a perpendicular to center line

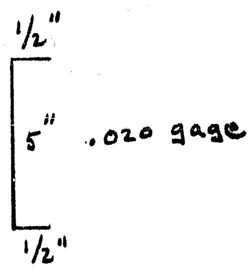
(This section probably has the highest β-stress)



Bending moment

M = 5(110)=550 in lb.

Moment of inertia



$$I = [\frac{5^3}{12} + 2(1/2)(2.5)^2](.020)$$

$$= .326 \text{ in}^4$$

Bending stress is, therefore:

$$S_b = \frac{MC}{I} = \frac{550(2.5)}{.326} = 4200 \text{ lb/in}^2$$

Direct stress

$$S = P/A = \frac{110}{6\,(.02)} = 920 \text{ lb/in}^2$$

Buckling of flange and web

Stresses are low so that no buckling is expected.

Total stress

Direct + bending $\cong$ 5000 lb/in^2 total which is well under allowable tensile stress, but is probably near the allowable buckling stress.

B. SUSPENSION: STIFFNESS

Considerable care was required in getting a suspension of the correct frequency and damping characteristics. Analysis establishing the design is contained in this section.

The damping is to be very small so that frequency criteria are based on undamped natural frequencies.

1. Vertical stiffness.

The design frequency of the vertical springs without negative compensation is 1 c/s, being a practical lower limit. To keep the springs below the deck requires a length no greater than 20". A practical solid height for a spring extended to 20" is 10".

Varying patient weights are accommodated by using several 25 lb. springs with one at each end being adjustable for leveling and positioning the bed. The 25 lb. is chosen specifically, of course, to meet the frequency conditions:

$$f = 1 = \frac{1}{2\pi}\sqrt{\frac{k}{25}(384)}$$

from which -

$$k = (2\pi)^2 \frac{25}{365} = 2.5 \text{ lb/in}$$

Extension of this spring to 10 inches requires 25 lb., showing that the spring does have the required frequency of 1 c/s.

Spring Design

From reference (40), we have the following relations for the coil spring:

The stress relation:

$$C \geq \sqrt{\frac{PKG}{\pi \tau_W Hk}}$$

The stiffness relation:

$$d = \sqrt{\frac{8 HC^3 k}{G}}$$

The surge frequency relation:

$$f_s = 14{,}000/HC^2$$

d = wire diameter
r = coil radius
H = solid height (10")
k = spring stiffness (2.5 lb/in)
P = spring max. load (25 lb)
C = 2 r/d index of curvature
K = function of C accounting for stress concentration due to curvature
= 1.2 for $C > 4.0$
G = shear modulus (11×10^6 lb/in^2)
τ_W = working shear stress (75,000 lb/in^2)
f_s = surge frequency

Substituting in the equation for C -

$$C = \sqrt{\frac{25\ (1.2)\ (11 \times 10^6)}{\pi\ (75{,}000)\ (10)\ (2.5)}}$$

$$= 7.5$$

The index C must be greater than this to satisfy the stress attainable.

Choosing C to be this minimum value, we have for the wire diameter -

$$d \geq \sqrt{\frac{8\ (10)\ (7.5)^3 2.5}{11 \times 10^6}}$$

$$\geq .0875''$$

The coil radius r is determined from the definition of the index C -

$$C = 7.5 = 2\ r/d$$

Solving for coil radius,

$$r = 7.5\ d/2$$

$$= 7.5\ (.0875)/2$$

$$= .328''$$

The surge frequency

$$f_s = 14{,}000/10\ (7.5)^2$$

$= 25$ c/s, a minimum value, so that C cannot be larger.

This is needed also to determine the ratio of maximum surge-force on the support, to maximum BCG force on the support.

From appendix C, this ratio is shown to be

$$R = \frac{k}{b\omega_s m} \qquad \text{where } \omega_s = 2\pi f_s$$

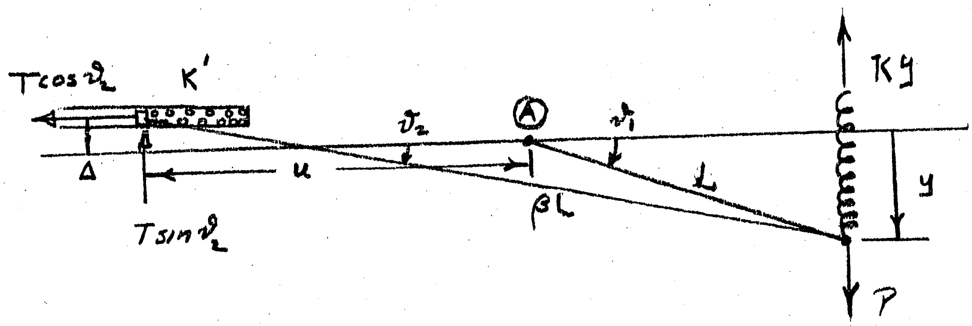
$$= \frac{2.5}{1(2\pi 25)(.5)}$$

$$= .032 \qquad (\text{negligible})$$

Design conclusion:

Use a .65" mean diameter coil with .0875" diameter wire heat-treated to 250,000 lb/in^2.

Negative spring for vertical motion



Above is a sketch showing how the positive spring K is compensated to achieve the very low frequency of 1/3 c/s. The toggle rod L is under compression by the wire βL attached to the spring K' under initial tension T_0 due to its initial deflection X'. This arrangement produces a destabilizing force P which subtracts from the supporting force K_y, to yield a net force

$$F = K_y - P \qquad [1]$$

Differentiating with respect to y yields the net spring rate

$$k = \frac{\delta F}{\delta y}\bigg|_{y = y_0} = K - \frac{\delta P}{\delta y}\bigg|_{y = y_0} \qquad [2]$$

The equilibrium position y_0 due to the misalignment Δ is given by the condition

$$F = 0 = K\, y_0 - P\bigg|_{y = y_0} \qquad [3]$$

a. Unstable Force ~ P

Taking moments about (A) gives the relation

$$P\, L \cos \theta_1 = T\, (u \sin \theta_2 - \Delta \cos \theta_2) \qquad [4]$$

By geometry, for small angles

$$\sin \theta_1 = y/L \qquad \sin \theta_2 = \frac{y + \Delta}{\beta L} \qquad [5]$$

$$\cos \theta_1 \cong 1 - \frac{1}{2}\left(\frac{y}{L}\right)^2 \qquad \cos \theta_2 \cong 1 - \frac{1}{2}\left(\frac{y + \Delta}{\beta L}\right)^2$$

with

$$u = (\beta-1)L + \frac{y^2}{2L} - \frac{(y+\Delta)^2}{2\beta L}$$

Because of the sliding contact at the spring K', the tension is computed as

$$T = K'[X' - y^2/_{2L} + (y+\Delta)^2/_{2\beta L}] \qquad [6]$$

Combining equations 4, 5 and 6, we obtain for the unstable force -

$$P = \frac{K'y}{\beta L^2}\left\{ X'(\beta-1)L + \frac{X'\beta L}{2}\left[\left(\frac{y}{L}\right)^2 - \left(\frac{y+\Delta}{\beta L}\right)^2\right] - \frac{\beta-1}{2}L^2\left[\left(\frac{y}{L}\right)^2 - \beta\left(\frac{y+\Delta}{\beta L}\right)^2\right]\right\}$$

$$- \frac{K'}{\beta L^2}\left\{ X'L - \frac{L^2}{2}\left[\left(\frac{y}{L}\right)^2 - \beta\left(\frac{y+\Delta}{\beta L}\right)^2\right]\right\} \qquad [7]$$

In order to extend the linearity of P with respect to vertical displacement y, we may choose the initial displacement x' of the K' spring so that for $\Delta = 0$, the coefficients of y^2 in the first curly bracket add to zero. This requires a value

$$\frac{X'}{L} = \frac{\beta-1}{\beta+1} \qquad \text{where } \beta = 2 \quad L = 8'' \qquad [8]$$

$$X' = L/3 = 2.7''$$

This assumption resembles setting the curvature (second derivative) of K to zero at the origin, except the above includes the displacement Δ which affects the curvature of K.

Simplifying by putting [8] in [7]:

$$P = \frac{K'y}{\beta}\,\frac{\beta-1}{\beta+1}\left(\beta-1 - \frac{y\Delta}{L^2}\right)$$

$$- \frac{K'\Delta}{\beta}\left\{\frac{\beta-1}{\beta+1} - \frac{1}{2}\left[\left(1-\frac{1}{\beta}\right)\left(\frac{y}{L}\right)^2\right]\right\} \qquad [9]$$

for the residual force about the zero position.

b. Spring rate.

The coefficient of y in [9] gives the first order spring rate of the unstable force. Combining this with the positive spring, gives the overall stiffness of the vertical spring system:

$$k = K - T_o(\beta-1)/\beta L \;=\; R - T_o/2L \quad \text{in this case} \quad [10]$$

c. Equilibrium position ~ y_o

Putting [9] into [3] with [10], we obtain the dependence of zero position on leveling:

$$y_o = \frac{T_o}{W}\,\frac{g\,\Delta}{\beta L\omega^2} \qquad [11]$$

For all practical purposes, k = 0. Hence, from [10] the tension in the negative spring wire is:

$$T_o = \frac{\beta L}{\beta-1}\,K$$

$$= \frac{2\,(8)\,10}{1} = 160 \text{ lb. max.} \quad [10 = \text{stiffness of 4 vertical springs}]$$

more accurately, $k = 1$ at $f = 1/3$ c/s for 150 lbs

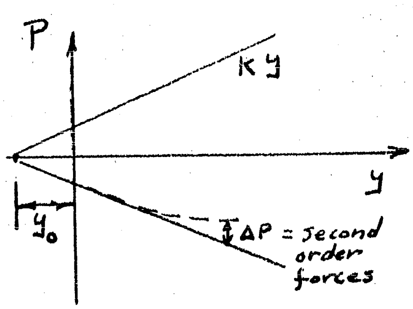
$$T_o = (K - k)\,36 = (10-1)\,16 = 144 \text{ lbs.}$$

Substituting m [11] with W = 175 lb/2, we find for sensitivity to leveling:

$$y_o/\Delta = 10 \quad \text{where} \begin{cases} \omega = 2 \text{ rad/sec} \\ \beta = 2 \\ L = 10'' \end{cases}$$

d. Second order forces

The presence of out-of-level Δ, as just shown, very seriously affects equilibrium position. The question here is how much out-of-level can the design tolerate without affecting the spring rate. The second order forces



($\sim y^2$) are small compared to either positive or negative forces (proportional to y), but not so small compared to their difference. If we could operate the leg in the displaced position, no difficulty would arise. However, the design requires that the equilibrium position be held at y=o by cranking the patient up and down with the positive spring. This subjects us to changes in spring rate due to second order forces.

The condition for judging how much Δ can be tolerated is when the ratio (R) of the forces involved are greater than some design value, say 10 percent. From [1], [10] and [11], the relation between y_0 and this ratio R, is

$$\frac{y_0}{L} = \sqrt{\frac{2\beta R}{\beta - 1}} \quad \text{where } R = \frac{\Delta P}{P} = \frac{\Delta K}{K} \qquad [12]$$

For a 10 percent restriction,

$$y_o = 10 \sqrt{\frac{2\ (2)(.1)}{(2-1)}}$$

$$= 6.3 \text{ in.}$$

so Δ = .63 in. out of level to change spring rate by 10 percent or frequency by 5 percent.

5. Results.

Although the equilibrium position is quite sensitive to level of the tension wire, a given position can be maintained by cranking the positive spring without fear of changing stability. Hence the tension wire need not be releveled for practical operation of this suspension.

2. Horizontal stiffness.

The legs, one at each end of the bed, provide horizontal restoration(Fig.6). As indicated earlier, these legs embody a differential (negative and positive) pendulum which is equivalent to a pendulum adjustable from 8 to 20 ft. long. The frequency of linear motion in a horizontal plane is, according to the well-known relation: $f = 1/2\pi \sqrt{g/\ell}$.

A pendulum restoring force is more convenient than springs because the frequency is independent of the subject weight, a widely varying quantity in this application.

General theory.

The essential features of the differential pendulum leg are illustrated in the accompanying sketch, where the displacements have been greatly exaggerated for clarity.

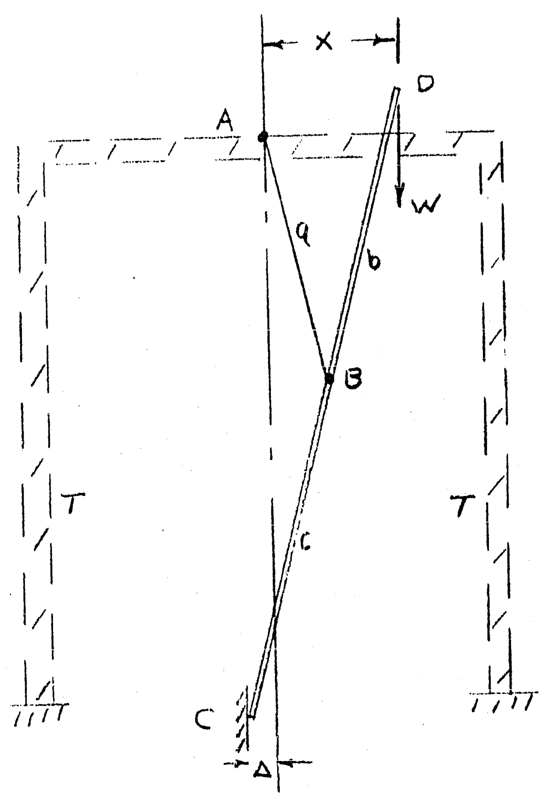


Fig. 5

The outer tube T has a bridge at A from which is suspended the positive pendulum of length a. At its extremity, the point B, is hung the inner tube (b+c), the negative pendulum, which is constrained to vertical motion only at C (by the outer tube). The bed weight rests on the upper end D of the inner tube, and is free to swing around in any horizontal direction (circular pendulum). Actually, the wire a runs down the inner tube (b+c) via a slide clamp at B and receives the load at C. As will be seen later, the frequency is controlled by sliding the clamp at B along the inner tube (b+c).

The point C is supposed to be on a plumb line through A, but any looseness in the fit at C or rocking of the outer leg T can cause a departure from plumb, indicated by Δ. In the scale shown Δ and x would be so small that a and (b+c) would appear on a single line.

A displacement x is restored by the force in the wire a, but is aided by the leverage of the inner tube against the side of the outer tube at C.

Frequency.

The frequency of horizontal motions x, y, γ (yaw) with this suspension depends mainly on the geometry (as with simple pendulums) and not on the load.

If we neglect the stiffness of the wire, it is shown (appendix D) the horizontal frequency is that of a simple pendulum, multiplied by a difference of two small quantities, easily adjusted to near zero:

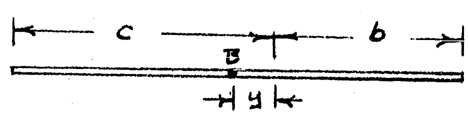
$$f = \frac{1}{2\pi}\sqrt{g/a}\sqrt{\frac{c^2-ab}{(2a)^2}}$$ when plumb (see below for "out of plumb")

That is, instead of a simple pendulum of frequency $\frac{1}{2\pi}\sqrt{g/a}$ we have a differential pendulum adjustable to zero frequency (infinite period when $c^2 = ab$).

Frequency control in this design, results from moving a slide at point B in the previous figure, a distance y along the inner tube (c+b). (See Fig. 6)

If B is the center position of (b+c),

we have b+y = c-y



So that

$$c-b = 2y = \xi$$

With y replacing ξ, the expression for frequency in Appendix D becomes

$$f = \frac{1}{4\pi a}\sqrt{g\,(2\xi+\eta)} = \frac{\sqrt{g}}{4\pi a}\sqrt{4y+\eta}$$ where $\eta = b-a = .75''$

$\ll a = 10''$

when $f = 0$: $y_o = -.187''$

when $f = 1/3$ c/s; $y = .96''$

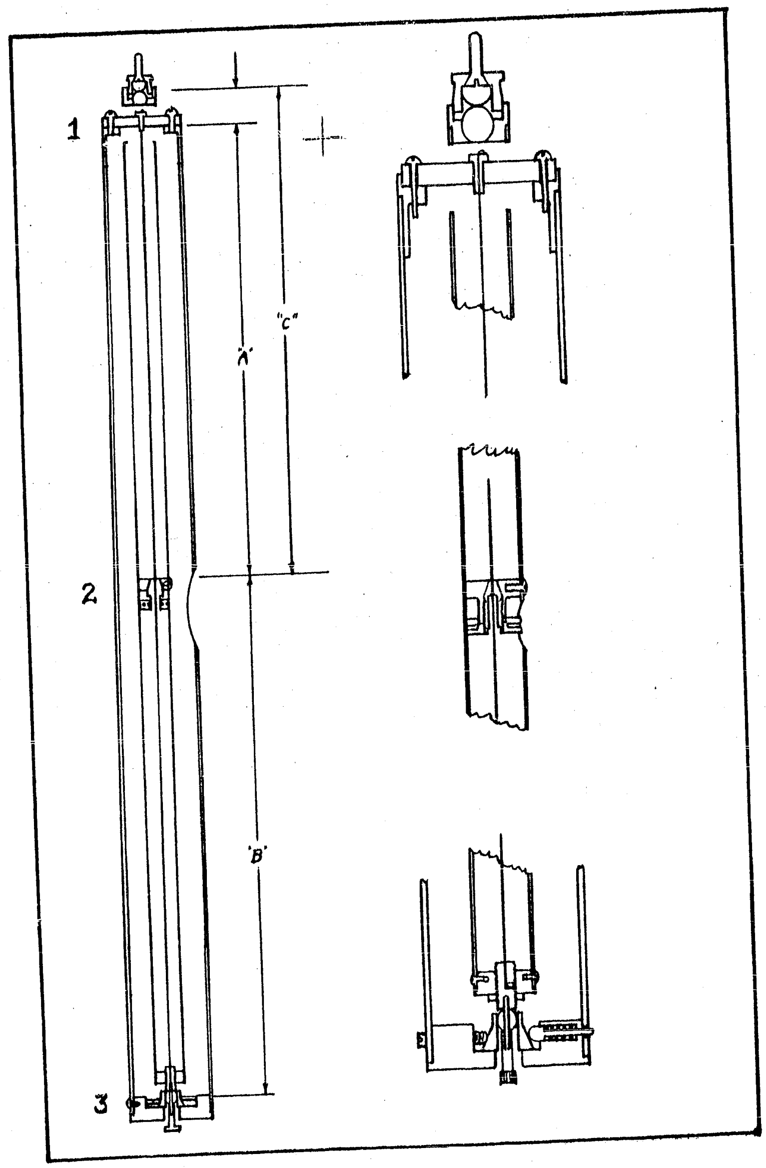


Fig. 6: 20 inch leg affording 1/3 c/s frequency in all horizontal directions. Load rolls on ball at top. Positive pendulum wire clamped inside negative pendulum tube.

Stiffness of wire

However, the suspension wire required to support a 300 lb. subject adds a spring restoring force which can add appreciably to the gravitational action of the pendulum, when the latter is adjusted to compensate this spring and gets to very low frequency. The complete expression (appendix D) for frequency becomes, including stiffness factor α:

$$f = \frac{1}{4\pi a}\sqrt{g\ (2\xi + \eta + \frac{5}{\alpha})}$$

where

$$\alpha = \sqrt{W/EI} \quad W = \text{load}$$

$$I = \frac{\pi r^4}{4}$$

i.e. frequency depends on load

For this leg, with .040 wire, the effect of wire stiffness comes out:

$$2\xi + \eta + 5/\alpha = \frac{(4\pi af)^2}{g} \qquad \alpha = \sqrt{150/3x10\ x12x10^{-8}} = 6$$

$$(4y+.75)\div.83 = 4.6 \qquad \text{when} \quad f = 1/3\ c/s$$

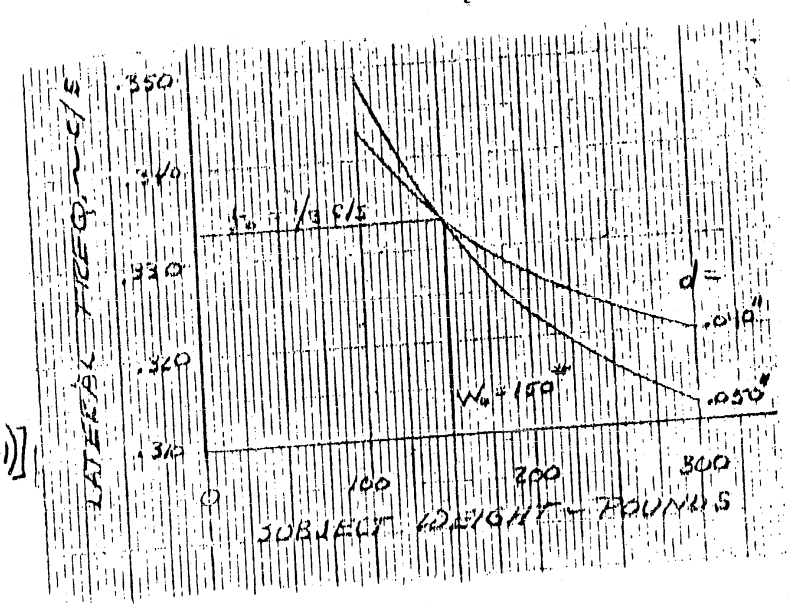
$y = .96"$ clamp point with no stiffness

$y = .75"$ or .2" shift due to stiffness

Thus a .040 wire causes a change in frequency of $\frac{.81}{4.6} = 10$ percent which varies with the square of wire diameter (this is observed).

The stress on .040" music wire is $150\# \div 12x10^4 in^2 = 125{,}000\ lb/in^2 < 200{,}000$ which gives a safety factor of 2. The effect of larger wire on frequency is shown in the graph. Evidently the wire could be increased without becoming spring-dominant.

$$f = f_0\left[1 + \frac{59}{12}\left(\frac{10d}{4\pi a}\right)^2\left(\sqrt{\frac{W_0}{W}} - 1\right)\right]$$



Out-of-plumb effects:

(a) On rest position:

From appendix D, we see that out-of-plumb (Δ) results in a static displacement of lateral position in amount

$$x_o = \frac{\Delta a}{\xi + \eta/2}$$

In terms of frequency (circular) [see above]

$$x_o = \frac{\Delta g}{2 a \omega^2}$$

from which we see how much the frequency affects this sensitivity to out-of-plumb. For typical working values

$$x_o = \frac{(384)\Delta}{20\ (4)}$$

$$= 4.8\Delta$$

Reasonable out-of-plumb (Δ = .01") results in x_o = .050", a considerable part of the allowable .125" travel. It is important that accurate plumb be maintained so as to avoid equilibrium positions of the inner tube near the stops.

(b) On frequency:

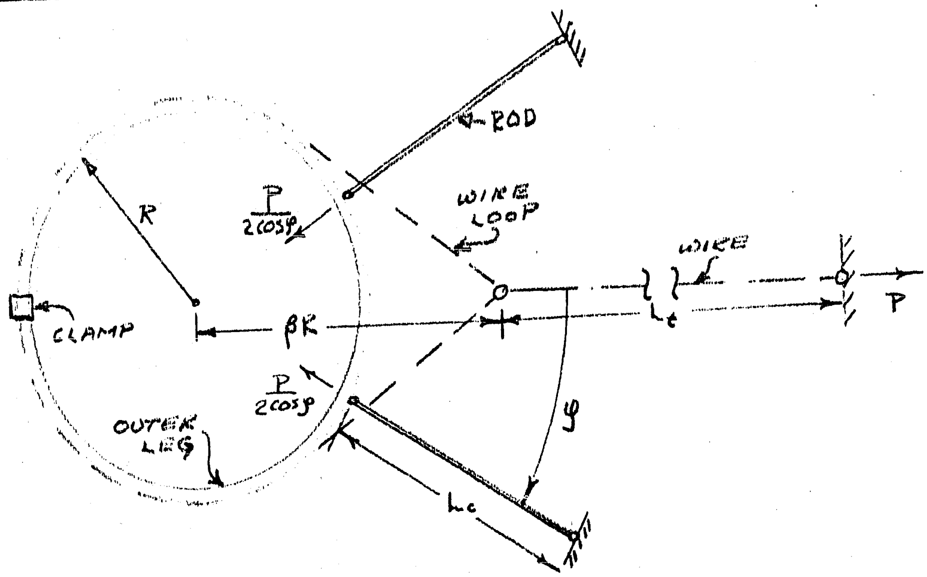
The frequency is negligibly sensitive to out-of-plumb increment

since $\frac{x}{a} = 10^{-4}$ and $\Delta \cong x_{o/4}$ so $\Delta/a \sim 10^{-4}$.

Stretch of the wire:

Reduces the frequency by the relation $\delta f = -f \frac{\delta a}{a}$; thus decreasing stability. Under full load of 150 lbs., .040 wire stretches = .080 inches. At 1/3 c/s, $\delta f = 1/3\ (\frac{.080}{10})$ = .003 c/s = 1 percent

Rotational stability of outer leg



The outer leg is constrained to vertical motion (normal to paper) along its axis by the rods. The tension P applied by a loop of wire around the outer leg provides the negative spring rate required to bring the natural frequency in vertical motion down to 1/3 c/s.

1) Stability.

The restoring moment about tube due to a small arbitrary rotation δ from initial plane of symmetry, is given by

$$M = PR\vartheta(1/\cos\varphi - \beta)$$

where it has been assumed that R/L_c and R/L_t are small compared to unity. The sign of this moment must be negative for stability. This is assured if

$$\beta > 1/\cos\varphi$$

If $\beta = \frac{1}{\cos\varphi}$ then the wire exerts no restoring force on twist of leg.

2) Alignment.

The question here is what mis-alignments can be tolerated without loss of stability. For this question, rotations θ are no longer small ($\sin\theta \neq \theta$).

We first observe that if the loop and wire do not have their line of action exactly midway between the rods, no first order changes in stability occur. Hence all mis-alignments considered here will be from the symmetrical configuration as shown.

Suppose in fixing the clamp, the tube and rods were not centered relative to the wire, by an amount θ_T. The restoring moment upon applying the tension load would be

$$M = PR\left[\frac{\sin(\theta + \theta_T)}{\cos\varphi} - \beta \sin\Delta\theta\right]$$

where $\Delta\theta$ is the change from the initial position θ_T.

The condition for stability follows from $\partial M/\partial\theta < 0$ -

$$\beta > \frac{1+\theta_T}{\cos\varphi} \quad , \quad \theta_T < 20^\circ$$

The added rotation $\Delta\theta$ to balance out the unstable moment from the mis-alignment θ_T corresponds to a zero restoring moment -

$$\tan\Delta\theta = \frac{\theta_T}{\beta\cos\varphi - 1} \quad , \quad \theta_T < 20^\circ$$

If very little stability is present [$\beta \cong \frac{1+\theta_T}{\cos\varphi}$], then $\Delta\theta$ is near 45° required, which will allow very little misalignment θ_T before collapsing the configuration.

Suppose, instead, the end of the tension wire were displaced an amount δ. This would be equivalent to an initial misalignment of the loop in amount $\delta\beta R/L_t$, which is second order because δ and $\beta R/L_t$ are both small compared to one.

3) Conclusions.

For the present configuration $\beta = 4$ and $\varphi = 30^{o}$.

Putting these values in the stability condition

$$\beta - 1/\cos\varphi = 4 - 1/.866 = 2.85 > 0$$

which indicates substantial stability.

It is concluded that if the wire loop is clamped to the outer leg, no special sensitivity to alignment is expected.

C. SUSPENSION: DAMPING

A small amount of damping is desirable to settle the system out following the many and varied external disturbances that will occur. Without some damping, the patient and support would be in a continual state of low frequency vibration.

1. Formulation of damping required.

The damping coefficients in each axis of the system are related to the respective mass and spring characteristics as follows:

for linear motion: $r_i = 2\ m\ \omega_i \zeta$ along any axis $\underline{i}$

for angular motion: $r_i = 2\ I_i \omega_i \zeta$ about any axis $\underline{i}$

where ω_i (with subscript) denotes the natural frequency associated with this axis.

(a) The inertial coefficients are determined as follows:

m = mass of bed plus patient

$= 175/384 = .45$ slugs" [(inch) slugs]

I = moment of inertia

For the transverse axes, considering the bed and patient as a long, uniform rod, the inertia would be

$I = m\,L^2/12$ where L is the length of the bed. Based on this

$I_\alpha = I_\gamma = mL^2/16 = .45\ (78)^2/16 = 170$ slug in^2.

[Division by 16 instead of 12 accounts for the un-uniformity of mass distribution along the bed, it being denser near the c.g. than at the ends].

For roll axis, the inertia is more or less a guess at the radius of gyration (ρ) in the basic formula,

$I = m\rho^2$

$I_\beta = .45\ (5)^2 = 11.3$ slug in^2.

(b) The (undamped) natural frequencies (ω_i) are found as follows:

For the linear motions our design criterion was a linear natural frequency

$$\omega_i = \omega_x = \omega_y = \omega_z = 2 \text{ rad/sec.}$$

The stiffness (k) giving this frequency was provided by the legs at each end of the bed.

For angular motions, the natural frequencies may be found as follows: The legs resist an angular displacement (α) of the bed about the pitch (x) axis by a torque $k_z L^2 \alpha/4$. Substituting the general relation for angular natural frequency about any axis θ (corresponding to $k_i = m\omega_i^2$):

$$\omega_\theta^2 = \frac{\text{torque}/\theta}{I}$$

Then about this axis

$$\omega_\alpha^2 = (K_z L^2/4) \div (mL^2/16)$$

$$\omega_\alpha = 2\omega_z \qquad \text{since } k_z = m\omega_z^2$$

Thus, $\omega_\alpha = \omega_\gamma = 4$ rad/sec.

That is, the natural frequency in pitch and yaw are twice that in translation.

The roll frequency ω_β is determined separately, by coil springs designed to give $\omega_\beta = 2$ rad/sec.

(c) Damping coefficient assumed.

The damping ratio ζ is chosen as .25 for each axis to keep the phase shift low at 1 c/s when the cut-off frequency is 1/3 c/s = 2 rad/sec. Substituting these values in the relations for the linear damping coefficients required:

$$r_x = r_y = r_z = 2\ (.45)\ (2)\ (.25)$$

$$= .45 \text{ lb/in/sec.}$$

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For the rotational damping coefficients required

$$r_\alpha = r_\gamma = 2\ (170)\ (4)\ (.25)$$

$$= 340 \text{ in/lb/rad/sec.}$$

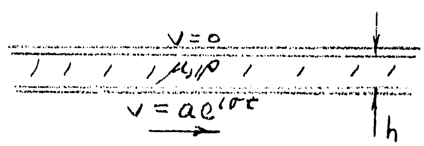
$$r_\beta = 2\ (11.3)\ (2)\ (.25)$$

$$= 11.3 \text{ in/lb/rad/sec.}$$

Individual damping elements must be sized and positioned to meet these conditions on damping.

2. Type of damping element used

The greatest energy absorption per unit volume is obtained from Couette flow, characterized by two close plates separated by a thin viscous fluid. The resistance of one plate relative to another is given by the relation, as per page 621 of Lamb.(52)



$$D = \mu A\ (1+i)\ \beta \coth\ [(1+i)\beta h]\mathbf{v}$$

where $\beta = \sqrt{\dfrac{\sigma}{2\,\nu}}$ and A equals the wetted area. In this application

$$(\beta h)_{max} = \sqrt{\frac{60}{2\ (30)}}\ (.01) = .01$$

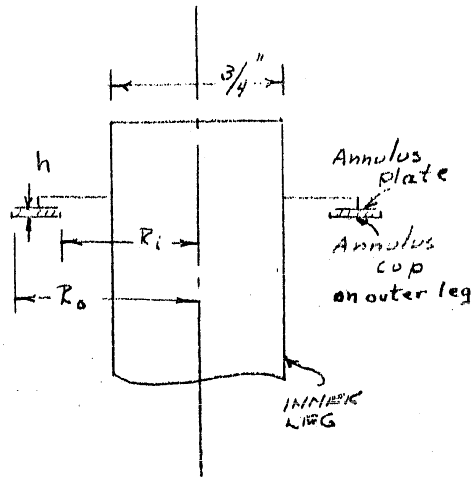
for which the expression for drag (D) reduces to its steady-state value

$$D = \frac{\mu A}{h}\ v$$

from which is obtained the resistance coefficient

$$r = \frac{\mu A}{h}$$

3. Translational damper design: lateral motion.



Lateral damping is provided by Couette flow between the annulus plate and cup separated by a viscous fluid of thickness h and viscosity 10,000 centipoises $= .0015\ in^2/sec$. For thicknesses of ten thousandths or so, and frequencies below 40 c/s, the damping coefficient is not dependent upon frequency and assumes its static value,

$$r = \mu A/h$$

where A is the wetted area and μ is the viscosity of the viscous layer. In terms of the configuration here,

$$r = \frac{\mu\pi}{h}(R_o^2 - R_i^2)\ \text{lb/in/sec.}$$

Specifically, for a damper at each end, we have for the outer radius

$$R_o = \sqrt{\frac{rh}{2\mu\pi} + R_i^2} = \sqrt{\frac{.45\ (.010)}{(.0015)2\pi} + (.625)^2}$$

to give the required damping. R_i was chosen as .625 to allow 1/8" travel of the upper end of the inner leg without striking the annulus cup which is fixed to the outer leg.

Interference due to leg rotation is determined to be

$$= \frac{1}{8}\ \frac{1}{20}\ \frac{1}{1} = .0063" < .010 \text{ allowed.}$$

4. Translational dampers: vertical motion.

The area required for vertical damping is the same as for lateral motion -

$$A = \frac{rh}{\mu} = \frac{.5\ (.010)}{.0015}$$

$$= 3.3 \text{ in}^2 \text{ or } 1.7 \text{ in}^2 \text{ at each end.}$$

5. Rotational damper design: yaw motion.

Since the yaw dampers are common to the lateral dampers and these are fixed at the ends of the bed, the yaw damping requirement cannot be met. Applying the formula of previous action with $y_A = \frac{78}{2} = 39''$, we find for the damping ratio in yaw -

$$r_\alpha = \frac{.0015\ (3.3)(39)^2}{.010}$$

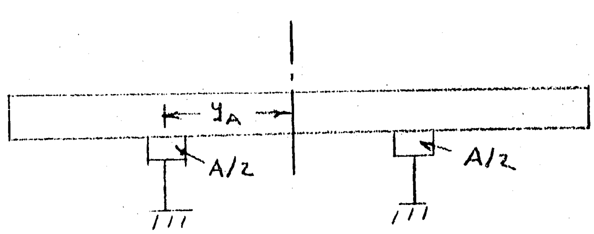
$$= 750$$

Since the required damping is 340, the damping ratio actually obtained is higher by the ratio 750/340 = 2.2. Hence, for this case, $\zeta = .55$. This brings the natural frequency down to

$$\omega_\alpha' = \omega_\alpha \sqrt{1 - .55^2}$$

$$= .83\ \omega_\alpha$$

6. Rotational damper design: pitch motion.



Pitch damping is obtained by distributing the area (A) required for vertical damping about the z axis. For one-half the area located a distance y_A on both sides of the z-axis, the pitch damping coefficient becomes

$$r_\alpha = 2\,\frac{(\mu A/2)}{h}\;y_A^{\,2}$$

Solving for the distance y_A, we have

$$y_A = \sqrt{\frac{r\alpha}{\mu A}}$$

$$= \sqrt{\frac{.010\ (340)}{.0015\ (3.3)}}$$

$= 26.3$ inches required to meet the pitch damping requirement.

The placement of these dampers presents a problem. They cannot be connected directly to the bed for this would influence roll and lateral motion. One possibility is to connect the dampers to a bar supported at each end by the outer legs.

A simpler solution is to connect the dampers directly to the outer legs which will, of course, increase the damping ratio. Substituting for $y_A = 78/2$ in the above formula,

$$r_\alpha = \mu A y_A^{\,2}/h$$

$$= \frac{(.0015)(3.3)(39)^2}{.010}$$

$$= 750$$

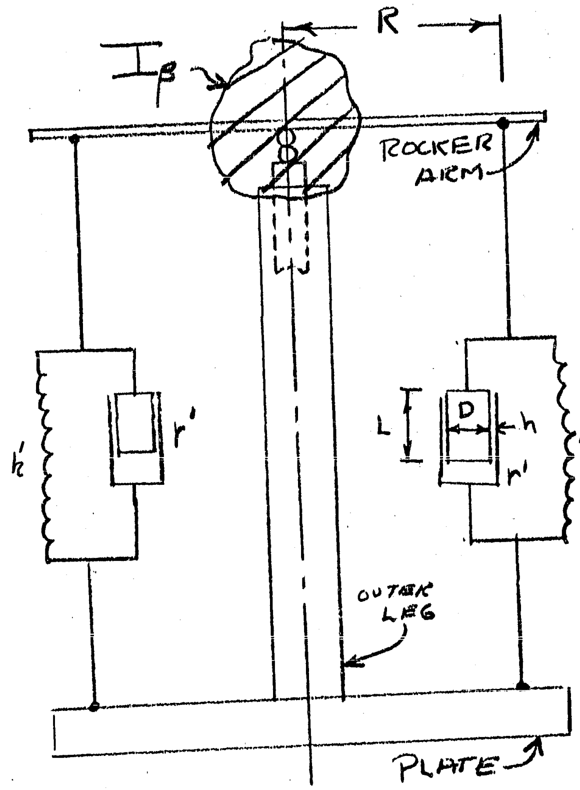
Since the required damping is 340, the damping ratio is increased by the ratio $750/340 = 2.2$. For this case, $\zeta = .25\ (2.2) = .55$. The natural frequency is reduced to

$$\omega_\alpha' = \omega_\alpha \sqrt{1 - .55^2}$$

$$= .83\ \omega_\alpha$$

This solution is attractive for it may well reduce the amplitude of the very low frequency oscillation due to breathing. This oscillation is troublesome because its large amplitude throws the pen off when the sensitivity is adjusted for the low amplitudes of the vibrations arising from cardiac forces.

7. Rotational damper design: roll motion



The subject [illegible]d support, represented by the inertia in roll, $I_\beta = 11.3$ slug in.^{2}, is restrained and damped as shown in the sketch (not to scale). Because this suspension acts about the roll axis, only one end of the bed need be so equipped. The rocker arm is attached to the bed and rolls with it. Frequency and damping in roll are controlled by the springs k', the oil cylinder r' and the rocker arm radius R. The springs k' and damper r' are tied to earth in roll only by the plate which is fixed to the outer leg which constrained to vertical motion by the A-frames.

The roll frequency (ω_β) and damping (ζ_β) are controlled by the choice of k', r' and R for the relations:

$$\omega_\beta^2 = 2\,R^2 k'/I_\beta$$
$$\zeta_\beta = 2\,R^2 r'/2I_\beta\omega_\beta \qquad [1]$$

a. Variation in patient size

The patient size as far as we are concerned is represented by his roll inertia $I_\beta = m\rho^2$ where m = mass of patient plus bed and ρ = its radius of gyration.

Approximating the patient by a circular cylinder of uniform density, we have

$I_\beta \sim m^2/L$ since $\rho^2 \sim r^2$, $m \sim r^2 L$ where L = length of cyl., r = radius of cyl.

The weight of our average patient plus bed is 175 lbs. The lightest and heaviest patient-bed combinations are approximately one-half and double this value. If the height (i.e. L) of the patient remained independent of weight, its roll inertia would vary as the square of mass, for which case, the inertia of the lightest and heaviest patient would be 1/4 and 4 times the average. However, patient height does change positively with radius (i.e. weight). Account of it here is taken by assuming the inertia of the lightest and heaviest patients 1/3 and 3 times the inertia of the average patient.

We turn our attention now to the problem of how to accommodate these variations in patient size. We must hold roll frequency and damping a constant, and to do this, equation [1] requires

$$\left.\begin{array}{l} k'R^2/I_\beta \\ r'R^2/I_\beta \end{array}\right\} = \text{constant}$$

Our choice is to hold the spring (k') and damper (r') constant and vary rocker arm radius (R) because it is the easiest way of meeting the requirement, now reduced to

$$R^2/I_\beta = \text{constant} \qquad [2]$$

b. Rocker arm variation

Although the roll inertia varies by a factor of 9 (1/3 to 3), the radius R need vary but by a factor of 3 to meet conditions [2]. Thus if the minimum rocker arm radius is 2", the maximum is 6"; a minimum of 2.5" requires a maximum of 7.5" and so on.

The channel on the end is 4" or 5" across, presenting a lower limit on the minimum rocker arm radius. Since this minimum radius is in doubt, calculations will be made for three values: R_{min} = 2.5", 2.75", 3.0".

c. Spring design

From equation [1],

$$k' = \omega^2_\beta I_\beta / 2R^2$$

The desired frequency is 1/3 cps = $2\pi/3$ rad/sec. Roll inertia has been calculated as 11.3 slug in.^{2} for the average patient/bed combination. Since the calculations to follow are for minimum radius, the inertia of the lightest patient = 11.3/3 must be used.

$\underline{R_{min} = 2.5"}$

$$k' = \left(\frac{2\pi}{3}\right)^2 \left(\frac{11.3}{3}\right)/2\ (2.5)^2$$

$$= 1.33\ \#/\text{in.}$$

$\underline{R_{min} = 2.75"}$

$$k' = 1.33\ (2.5/2.75)^2$$

$$= 1.10\ \#/\text{in.}$$

$\underline{R_{min} = 3.0"}$

$$R' = 1.33\ (2.5/3.0)^2$$

$$= .92\ \#/\text{in.}$$

We aim to pick a spring that can be set at any of these stiffnesses by a change in solid height. From the stiffness condition

$$Hk' = \frac{d^2G}{8C^3}$$

where

G = modulus of shear = $11\text{x}10^6$ for steel

C = index of curvature = 2 r/d

d = wire diameter

r = coil radius (mean)

For a .05 wire diameter and 10 index of curvature

$$Hk' = (.050)^2\ (11\text{x}10^6)/8\ (10)^3$$

$$= 3.45"$$

The coil radius

$$r = \frac{d}{2}\,c$$

$$= \frac{.05}{2}\,(10)$$

$$= .25$$

OD = 2r+d = .54"

OD = 2r-d = .46"

Solid heights for the three springs under consideration are:

$$H_{2.5} = 3.45/1.33$$
$$= 2.6"$$

$$H_{2.75} = 3.45/1.1$$
$$= 3.1"$$

$$H_{3.0} = 3.45/.92$$
$$= 3.75"$$

The roll spring is .54" OD, 46" ID, made of .050 diameter steel spring wire, and has a solid height of 3.75" stretched to 6" working height including a 1" allowance for clamping. This spring will be cut to length depending upon minimum rocker arm radius.

Surge frequency

$$f_s = 14{,}000/HC^2 = 14{,}000/100\ (3.75)^2$$
$$= 10 \text{ cps} - \text{(a bit low, but O.K.)}$$

<u>d. Damper design</u>

From equation [1]

$$r' = I_\beta \omega_\beta \zeta_\beta / R^2$$

Putting in required values, we have

<u>$R_{min} = 2.5"$</u>

$$r' = \frac{11.3}{3} \quad \frac{2\pi}{3}\ (.25)/(2.5)^2$$
$$= .32 \text{ lb/in/sec damping rate}$$

<u>$R_{min} = 2.75"$</u>

$$r' = .315\ (2.5/2.75)^2$$
$$= .26$$

<u>$R_{min} = 3.0"$</u>

$$r' = .315\ (2.5/3.0)^2$$
$$= .22$$

As with the spring, we seek a design which can be set at any of these damping coefficients by changing immersion of the oil piston. In terms of the design parameters, the immersion length

$$L = 4'h/\mu\pi D$$

where

h = clearance (.010 typical)

μ = viscosity of oil (.0015 max.)

D = diameter of piston

L = immersion of piston in cylinder

A small diameter is desirable to allow the smallest rocker arm radius.

For $D = 1/2"$ -

$$L_{2.5} = .32\ (.010)/.0015\pi(1/2)$$

$$= 1.5"$$

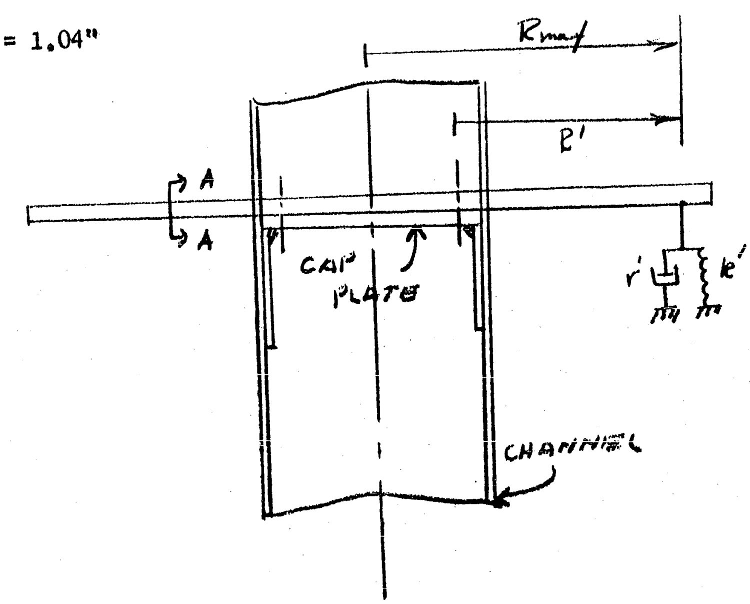
$$L_{2.75} = 1.5\ (.26/.32)$$

$$= 1.24"$$

$$L_{3.0} = 1.5\ (.22/.32)$$

$$= 1.04"$$

e. Rocker Arm Design



The stiffness of the rocker arm must be large compared to the k' spring if the k' spring is to control the natural frequency in roll. Assuming a 10:1 ratio on frequency requires a 100:1 ratio on stiffness. Imposing this condition on the rocker arm section (I).

$$100\,k' = 3\,El/r'^2$$

where R' = distance between spring attaching point and the nearest bolt to the plate cap. This can be expressed approcimately as

$$R' = R_{max} - R_{min} + 1''$$

The 1" term is to allow for rivet spacing and width of damping piston.

$\underline{R_{min} = 2.5}$

$$R_{max} = 3\,(2.5)''$$
$$= 7.5''$$
$$R' = 7.5-2.5+1$$
$$= 6''$$
$$I = 100\,(1.33)\,(6)^3/30\times10^6 \qquad \text{(Dural)}$$
$$= .00095\ in^4$$

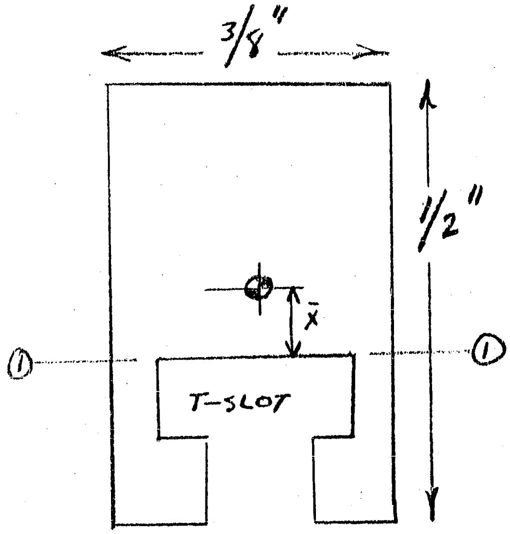
$\underline{R_{min} = 2.75''}$

$$R_{max} = 3\,(2.75) = 8.25''$$
$$R' = 8.25-2.75+1.0=6.5''$$
$$I = 100(1.10)(6.5)^3/30\times10^6$$
$$= .00101\ in^4$$

$R_{min} = 3.0''$

$$R_{max} = 3(3) = 9''$$
$$R' = 9-3+1=7''$$
$$I = 100\,(.92)(7)^3/30\times10^6$$
$$= .00104\ in^4$$

Although sections such as a round tube, a channel or I-beam are more efficient in bending, a solid section is chosen here for two reasons: (1) the tie-down at the cap plate must be rigid, (2) a T-slot is convenient for sliding adjustment of the rocker arm position.



SECTION A-A

The c.g. position $\bar{x}$ from the axis (1)-(1) is obtained from summation of first moment areas about that axis -

$$\bar{x} = \frac{AiXi}{Ai}$$

$$= \frac{1/4(1/8)^2+1/8(1/8)3/16}{3/8(1/2)-1/4(1/8)-1/8(1/8)}$$

$$= \frac{.00684}{.14}$$

$$= .049"$$

The inertia about the $\bar{x}$ axis is the summation of second moment areas about that axis -

$$I = AiXi^2 + I_o - \bar{x}^2 \; Ai$$

$$= 3/8(1/2)[(1/2)^2/12]-1/4(1/8)[(1/16)^2+(1/8)^2/12]-1/8(1/8)[(3/16)^2+(1/8)^2/12]$$

$$-(.049)^2(.14)$$

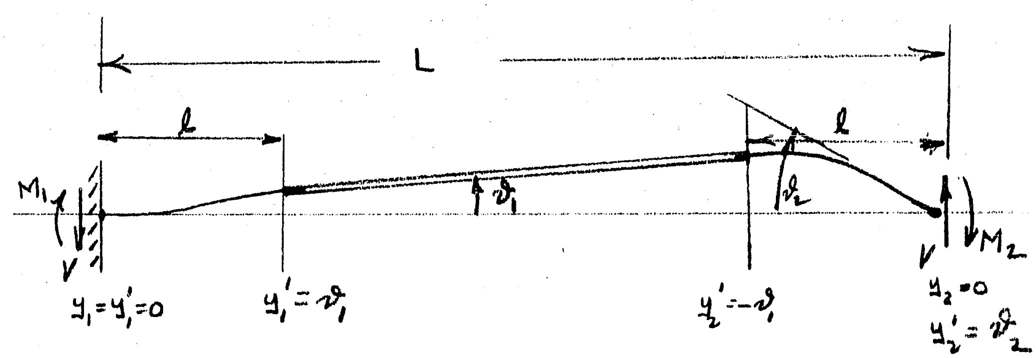
$$= .0039-.000162-.0000567-.000337$$

$$= .00334 \text{ in}^4 \quad .001 \quad \text{O.K.}$$

f. Effect of wire hinge on Spring Rate

Although the spring/damper combination is pinned at each end to allow adjustment of the rocker arm length according to patient weight, the pin joint is clamped by friction during test as far as small displacement is concerned. In this condition, rotation of the rocker arm will bend the wire hinge at the extremeties. Of interest here is the spring rate introduced by this energy storage.

Shown below is a sketch of the rod and wire hinge in its deflected position with the important quantities noted -



Integrating the basic equation for bending

$$EI y'' = M(x)$$

over each wire of length l and relating displacements and slopes at $x_1 = x_2 = l$, we obtain the very simple expression

$$M_2/\theta_2 = EI/l$$

for angular spring rate. The simplicity is due to the condition that the wire length l is small compared to overall length L. This angular spring rate is converted to linear spring rate by dividing by the rocker arm length squared -

$$\Delta K' = EI/R^2 l$$

This increment in spring rate must be small compared to the design spring rate controlling roll natural frequency, given by

$$k' = \omega_\beta^2 I_\beta / 2R^2$$

Forming this ratio, we obtain

$$\frac{\Delta k'}{k'} = \frac{2 E I}{\ell \omega_\beta^2 I_\beta}$$

Putting in numbers

$$\frac{\Delta k'}{k'} = \frac{2\ (30 \times 10^6)\ \pi\ (.03)^4/64}{(1.0)\ (2^2)\ (11.3/3)}$$

= 16 percent for the light patient

= 5 percent for the average patient

= 2 percent for the heavy patient

A .030" wire hinge would need to be 1" long, to be soft enough for this application

V. DESIGN OF PRACTICAL SUPPORT SYSTEMS: MERCURY BED.

1. Purpose of bed.

At first glance, it would seem that BCG vectors (in the frontal plane at least) could be obtained from a patient floating on a board or boat in liquid. Indeed our simplest ULF-BCG suspension[26] was a plywood board (stiffened longitudinally by 1" dural angles) floating on mercury. The HF records on this proved similar to pendular beds, and afforded good preliminary results. However, lateral BCG records on this, like all others which are stiff in roll, contain the artefacts and errors discussed in Chap. III. Consequently, valid frontal-plane BCG vectors are not to be had from a mercury (or water) bed.

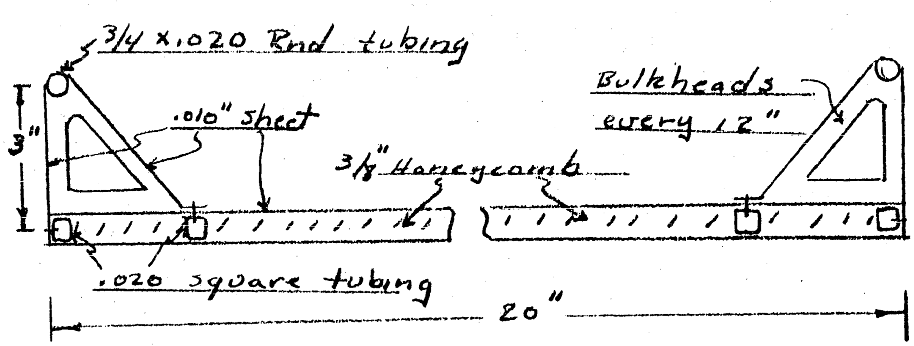
2. Scope of report.

Questions have been raised however, as to the physics of the mercury-bed: whether frictional drag, end-waves or surface tension affect its motion. Further, this bed is highly sensitive to ambient floor vibrations; indeed may be unusable in modern steel buildings with vibrating machinery, so sensitive are the BCG accelerometers.

We therefore have examined these problems analytically, with a view (a) to understanding the physical validity of this "aperiodic"[26] BCG suspension, and (b) to approaching rationally the control of vibrations transmitted to the bed via the mercury.

A. DESCRIPTION

Our first attempt to design the bed was a 2" board of honeycomb. However, the weight was high and the bending excessive. Sketched below is a cross-section of the design finally chosen and the one in service now.



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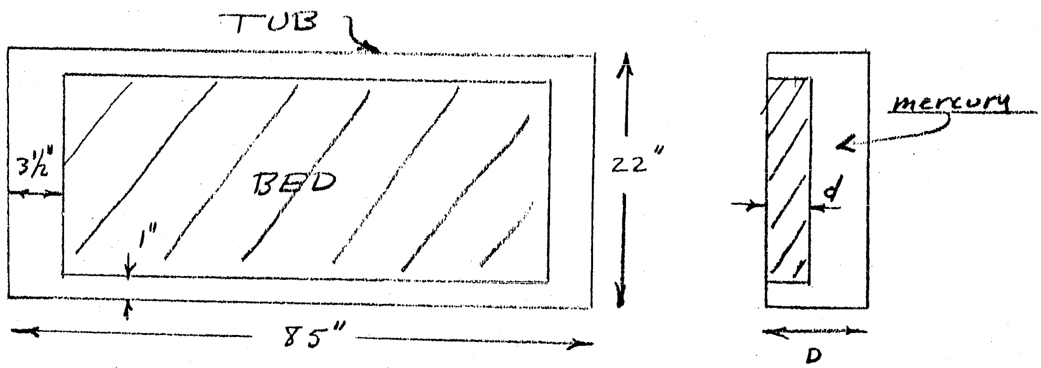
The length of the bed is 6-1/2 feet. It weighs 11 pounds. The side rails not only help to pick up lateral forces but also are the main longitudinal bending members. Lateral bending is of course taken by the honeycomb.

The bed rests on the mercury, and as shown in the sketch below is limited by the tub to a couple of inches of travel in the plane of the bed.



The mercury weighs 375 lbs., since mercury weighs .49 lb/in^3, the total

mercury volume = 375/.49 = 764 cu. in.

The patient and bed weigh a maximum of 315 lbs., which have a

displaced volume = 315/.49 = 642 in^3.

Referring to the sketch above, the height of the mercury (D) is the total volume divided by tub base area:

$$D = \frac{764 + 642}{75\ (22)}$$

$$= .85''$$

The height (d) of the displaced volume is simply that volume divided by its base area

$$d = \frac{642}{20(72)}$$

$$= .45''$$

Hence, the layer of mercury between bed and tub bottom equals

D-d = .40 inches clearance.

In this section we shall present the criteria chosen for design of the bed, and the analysis of stress and stiffness required for detail design.

1. Criteria.

a) All static stress and deflections are based on a 300 lb. concentrated load in the center of the bed. This is for simplicity and is not a real loading condition. Hence, no local stresses are considered.

b) A local loading condition of 100 lbs. concentrated on one rail is assumed to assure integrity of the rail under loading by a patient catching himself in a half-fall.

c) Static deflection shall not reduce the clearance between bed and tub bottom to less than 1/8".

d) Free-free bending and torsion frequency shall be at least 40 c/s with 25 lb. distributed mass of the patient plus mercury vertical mass. The bending frequency criterion is to be applied about both lateral and longitudinal axes.

e) All stresses shall be less than 40,000 $lb/in.^2$ for dural to assure no permanent yielding.

2. Section properties.

a) Bending - longitudinal: bending rigidity is due to the rail configuration as shown in the sketch. Honeycomb board outside the right-hand square tube is assumed ineffective. The effective area in bending of the round tube and of the square tubes is .075" which gives a section moment of inertia of .64 $in.^4$.

b) bending - lateral: bending rigidity is due to the honeycomb acting between rails. The inertia of the section is .0007 in^4 per inch of honeycomb in the longitudinal direction.

c) Torsion: the honeycomb board provides all the torsion resistance of the bed. For this section, the torsion constant K - defined by the relation

$\theta = TL/KG$ where θ = angle of twist

T = applied torque

K = torsion constant

G = shear modulus

L = twisted length

is equal to .056 in^4.

3. Static deflection

The critical condition for static deflection is the patient sitting in the middle of the bed. In this condition, a 300 lb patient would bend the bed a total of .18 inches of which about .1 inch would extend below the equilibrium position of the bottom of the bed. Since for this size patient the clearance is nearly .4 inches, the bed will not contact the bottom of the mercury tub.

In the test condition where the patient is reclining, the deflection is far less because the load is reacted out in compression across the honeycomb. Bending arises only to the extent that the patient's weight is not uniformly distributed in the H/F direction.

4. Bending frequency

The fundamental frequency of a free-free beam in bending is given by

$$f_1 = \frac{3.58}{L^2} \sqrt{\frac{EIg}{W/L}}$$

Assuming the bed to weigh 15 lb plus the 25 lbs for tissue and mercury, we have for a 78" bed, a frequency of 42 c/s in longitudinal bending, somewhat above the desired 40 c/s. The lateral bending frequency is much higher owing to the short length across the bed.

5. Torsion frequency

The fundamental frequency of a free -free rod in torsion is given by

$$f_1 = \frac{1}{2\pi} \sqrt{\frac{KGg}{W\rho^2 L}}$$

Again, assuming 40 lb mass affecting frequency, and assuming a radius of gyration (ρ) of 5", we obtain a frequency of 5 c/s, considerably lower than desired. However, the great symmetry about the torsion axis precludes to a great extent excitation of torsional oscillation, and hence, no attempt was made to raise the frequency closer to the desired 40 c/s.

6. Rail Stress

The stress on the 3/4" tube which serves as the rail runs about 30,000 lb/in^2 under a 100 lb load between bulkheads.

D. PERFORMANCE

It is supposed that the mercury bed has been designed stiff enough to transmit cardiac response without distortion. Even if the natural frequency of the bed is not as computed owing to the difficulty of determining the body mass entering into the motion at the higher frequency, the damping provided by body tissue in all probability keeps the distortion from bed resonances within acceptable bounds.

If the mercury is not the perfect isolator as supposed, and permits floor vibrations to drive the bed, as long as these vibrations are not amplified, their amplitude does not seriously interfere with obtaining useful cardiac records.

1. Mercury forces on bed

The tub containing the mercury, in general, follows the floor motions. Of interest here is the extent to which the bed floating on the mercury "sees" the motion of the tub.

a). Friction.

The wide, flat bottom of the bed suggests that lateral and head-foot tub motions might be transmitted to the bed by laminar friction. It is assumed that the mercury follows the tub motion at the depth sustained by the bottom of the floating bed. The ratio of the acceleration of the bed by drag forces to the tub acceleration, we derive to be

$$\text{Ratio} = \frac{A}{m_B}\sqrt{\mu\rho/\omega}$$

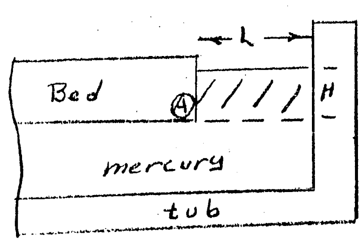
For a 150 lb bed at 40 c/s, this amounts to .0044. This is negligible even if the tub accelerations are as large as BCG accelerations. Hence friction is not an important means of force transfer thru the mercury.

b). Piston action.

Piston action refers to the transfer of force thru the mercury acting as an incompressible fluid.

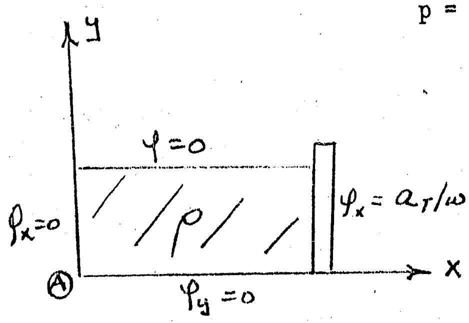
In the vertical direction, the mercury acts as a solid because of the symmetry and the fact that the accelerations are small compared to gravity. Hence, vertical accelerations of the tub are transmitted directly to the bed except at the sides and ends where the mercury is free to flow around the bed. Pitch and roll of the tub are also transmitted to bed by this same mechanism.

In the horizontal direction the free surface provides considerable relief from acceleration by the tub. Assume that the dashed line is a slip plane, so that only the mercury enclosed within the cross-hatched area is involved in the transfer of force from the tub to the bed. The flow in this area, is irrotational, whose velocity potential φ obeys Laplace equation. The



pressure is obtained from the unsteady Bernoulli equation

$$p = \rho \frac{\partial \varphi}{\partial t}$$



The solution to Laplace's equation fitting the boundary conditions shown in the sketch is the Fourier series in y:

$$\varphi = \sum_{1,3}^{\infty} \frac{8H}{\omega(n\pi)^2} \frac{\cos n\pi y/2H \cosh n\pi x/2H}{\cosh n\pi L/2H}$$

Substituting in the pressure relation, and computing for H/4 as the average pressure transmitted, we have for the ratio of bed acceleration to tub acceleration:

ratio: $\dfrac{8AH\rho \cos \pi/8}{\pi^2 m_B \cosh \pi L/2H}$

For a 150 lb. bed, this amounts to 10^{-4} for the lateral direction and for less for head-foot because of a much greater value of L. Again, this is negligible unless the tub accelerations are far greater than BCG accelerations.

c). Wave action.

Energy storage in gravity gives rise to standing surface waves of frequency

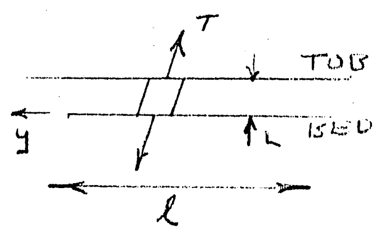
$$\omega = \sqrt{\frac{n\pi g}{L} \tanh \frac{n\pi h}{L}}$$

where h is the height of the free mercury surface above the tub bottom, L is the length of the free surface and n is any integer except zero. Substitution in this relation shows many standing wave frequencies common to the BCG spectrum. Noise from the tub will drive these standing waves on a random schedule resulting in amplification of tub noise. These frequencies will probably not be distinguishable in the usual recording of BCG data.

This is not to say, however, that the bed will receive accelerations greater than those of the tub. Since wave action affects the surface level of the bedside area exposed to lateral motion of the mercury, it would require a tremendous amplification by wave action to increase the bed acceleration above that of the tub. Hence, wave action will be presumed unimportant in the transfer of tub forces to the bed.

d). Surface tension.

If no slip occurs where the surface of the mercury contacts the bed and tub, the field of surface tension provides a means for the transfer of force to the mercury bed from the tub. A displacement y of the bed relative to the tub is resisted by a shear force dependent on distance L from the side, which we derive as:



$$F = \ell T y / L$$

where T surface tension and ℓ is the length under tension.

The primary force of surface tension acts on the head-foot direction because of the large values of ℓ and small value of L. For two sides, the force per unit of relative displacement

$$k = 2T\ell/L \qquad \text{where } \ell = 72''$$

$$= .43 \text{ lb/in.} \qquad L = 1''$$

$$T = .003 \text{ lb/in}$$

Isolation of a 150 lb bed occurs at frequencies above

$$f = \frac{1}{2\pi}\sqrt{\frac{.43\,(400)}{150}}$$

$$= .17 \text{ c/s}$$

Since this frequency is not on the spectrum of BCG, surface tension is not an important mode of force transfer from tub to bed.

Surface tension provides for force transfer in the lateral direction by changing the tension field in the vertical plane rather than in the horizontal plane (This results from meniscus and gravity factors). An estimate of the magnitude of this force will not be made as it requires treatment of a non-linear differential equation. This equation is soluble in terms of elliptic functions but is not available to us at this writing, plus the fact that the force will probably be of order .4 lb/in also. Again, isolation is at frequencies below BCG interest and hence of no consequence here.

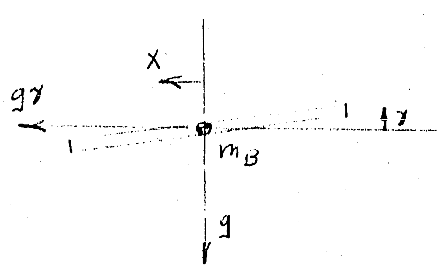
e). Summary

Piston action in the vertical direction as well as pitch and roll are the only important modes of force transfer across the mercury to the bed. Hence, isolation of the bed from tub motion by mercury is good except in these modes. If these motions of the mercury bed are not coupled to horizontal motion, then isolation is complete as far as BCG data is concerned.

In the next section we show these modes are indeed coupled to horizontal motion.

2. Bed coupling to tub drive.

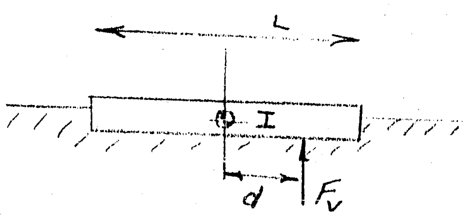
If the c.g. of the bed is not directly above the center of pressure of the mercury in vertical drive, the bed will be set in angular motion. This angular motion is coupled to horizontal motion, through the action of gravity. Any tilt of the bed away from level allows it to see the gravity vector in its own plane which for small angles of tilt can be treated as the horizontal plane. The acceleration on the bed is related to the angle of tilt γ by the relation:



$$\ddot{x} = g\gamma/m_B$$

Note that mass unbalance of the bed as described above, is not the only source of tilt. We have already shown that angular motion of the tub is transmitted directly to the bed. In this case, the angle of tilt γ is the same as the tub's.

The angle of tilt due to mass unbalance depends on the dynamic response of the bed on the mercury. The equation governing the response is:



$$I\ddot{\gamma} + k_T\gamma = dF_V$$

where

$k_T = \rho g \ell L^3/12$ is the stiffness of the mercury to rotation,

dF_V is applied moment from the tub,

ρ is the density of the mercury.

The natural frequency is somewhere between 25 and 30 c/s depending on the assumptions regarding moment of inertia. This frequency is essentially the same in pitch and roll, and is the principal source of tilt in the BCG frequency spectrum from vertical tub drive. Obviously these free oscillations can also be excited by angular vibration of the tub.

Since the excitation of the oscillations is random, it is not possible to be quantitative regarding the bed accelerations. However, knowing the source of the disturbance has directed us in the design for isolation of the bed from the tub and its support.

3. Isolation.

The presence of the 25 c/s resonance of the bed in pitch and roll means that true isolation of the bed is not possible. Hence, attempts at isolation really consist of keeping the transfer of floor noise to the tub frame to a minimum.

The natural frequency of the tub in bending about a lateral axis is also 25 c/s. Direct support from the floor in the center of the span raises this to something like a 100 c/s but since it also introduces more noise energy, the result is more bed noise than without this modification. Isolation material between floor and support does not improve the situation.

The drum-head frequency of the mercury and plywood bottom of the tub, due to bulkheads stands at 70 c/s. This is presumed high enough for purposes here.

In general the transmission of floor noise should be through a soft, moderately damped medium. This will attenuate the noise at all frequencies and at the same time damp the troublesome low frequency oscillations associated with the soft suspension. No matter what the stiffness employed, the damping should be around critical to obtain attenuation of the wide band noise input from the floor. [Solutions so far tried fall in the latter category. At a later date, the more difficult to achieve soft mount with moderate damping may be attempted.]

4. Conclusions.

a) Structurally, it did not prove possible to design a 72" platform having the required stiffnesses, within the desired weight limit of 7 lbs. The lightest possible material (3/8" honeycomb) having the strength and rigidity for the loads, spans and frequencies required, when moment was added for longitudinal strength, was 50 percent too heavy. This is because the mercury is so shallow, that sufficient moment of inertia in the cross-section could not be used, without giving the support low frequency in roll: so low is the center of buoyancy. The calculations indicate that a round-bottomed cockle in water might be acceptable, if it can have cross-struts on torque.

b) However, in both principle and practice, if we do allow flexure while loading and will accept a heavier bed with increased body-coupling (see Chap. III), a mercury bed is indeed a valid ulf-BCG support. We have then shown that friction, horizontal piston-action to the nearby tub-wall, and wave action all introduce negligible reaction forces to BCG motion within the action spectrum thereof. Moreover, surface tension is shown to introduce appreciable restoring force in shear only below 1/8 c/s, and does indeed confer a zero position when walls are nearby. Thus the bed is not completely aperiodic. In restricted tubs, mercury-pendulum action is also present at very low frequency, and seriously interferes with recording the displacement BCG without held-respiration.

For acceleration records the mercury suspension requires an extremely quiet location, so hard is it to decouple from the floor, in this frequency range. In particular, dynamic unbalance of the subject on the platform (either in roll or pitch, both being normally present) turns out to excite natural frequencies of oscillation (ca 25 c/s) in these rotary modes. These in turn, communicate (with a boat) to the RL and HF modes, and amplify common floor noises on the BCG acceleration transducers.

This sensitivity of the mercury bed to vertical vibration has been calculated. It is enhanced by structures used to elevate the bed to a convenient working height. Such structures are extremely hard to bring above 25 c/s in all possible modes.

So while the "mercury bed" (covered with oil against evaporation, of course) is a useful, physically correct, and practical device for exploratory work in favorable situations, it cannot be recommended for general use.

CHAP. VI. A METHOD FOR ROUTINE HARMONIC ANALYSIS OF BCG RECORDS.

Background and objectives

Harmonic analysis of BCG records is needed:

a) To establish the amplitude-frequency passband required in accelerometers and recorders. These are usually specified as needing two to ten times the passband of significant information present. This is important, because present evidence shows significant information to 35 c/s or more[49]. Present ECG recorders are linear to 45 c/s, adequate for ECG, but not for certain common BCG features like splitting of the J wave.

b) To establish the phase-frequency passband required. Especially good rendition of phase at high frequency is needed where details having stable form, move systematically on a main wave (with respiration) as in the BCG. It is also necessary to take special account of the phase distortion in commercial amplifiers and recorders, whose passband has been extended by tuned networks[53]. Their phase-shift spectrum may be unacceptable, though the amplitude spectrum is adequate.

c) To establish characteristic differences between classes of BCG records. Thus the Fourier spectrum of subjects over 50, in the head-foot seems to contain less high-frequency components than that of younger subjects. The lateral BCG seems to contain more high-frequency than the head-foot record. This also appears true of "abnormal" records.

d) The human body resting elastically on mechanical supports, may exhibit several resonant components that should be sorted out, identified and dealt with, in seeking the physical foundations of the BCG. Various tissues and organs, held up as culprit oscillators, should be confirmed as such by harmonic analysis of "local" records.

With the ECG there is no physiological reason to postulate component physical oscillators of any kind. Applying the Fourier analysis to get harmonic components to such records, has technical rather than biological interest. But in the genesis of the BCG, both physical oscillators and traveling waves of relatively fixed harmonic content are known to contribute. So the discovery of underlying hemodynamic and transient mechanical oscillations (commensurate or incommensurate with phase related to the cardiac cycle), may help in understanding underlying BCG mechanisms.

Frequency analysis not only is important for ballistocardiography, but its requirements are rather demanding. With many physical and biological phenomena, the amplitude spectrum tends to be monotonic. However, in audition (e.g. with heart sounds), the spectrum exhibits several quite discrete bands of activity. Similarly the BCG (acceleration) record, while dominantly composed of 3-8 c/s information, contains much information in higher frequency regions. These are the flexures, knees, jogs, splittings and isolated waves, which are so individual as to constitute a cardiovascular "signature" (unlike the ECG). Although such single details contribute little energy [in the technical sense of spectral $(\text{amplitude})^2$], and may be lost in spectral noise if one is not careful, they still bear significant information. To evaluate them, requires reading the amplitude of at least 30 harmonics of the fundamental (cardiac) period, as well as the corresponding phase spectrum. The phase is of no consequence in audition, because the ear analyses sound regardless of it. With the BCG, phase relations are essential, as explained above.

This task cannot be accomplished with standard Fourier analysers, because of the usual complexity of such analysis. Instruments which read the phase as well as the amplitude components of an unknown wave-form, require determining and recording both sin and cos components of each harmonic, and subsequent calculation of phase. This cannot be done on a clinical basis without impossible expense of help and time. A direct reading of amplitude and phase is required.

This objective, part of our program from the beginning, has been carried out in our laboratory by Mr. Peter Hume.* His design specification called for a rapid method of reading to the 50th harmonic, the amplitude and phase spectra of a BCG record, in order to apply it on a clinical research scale. While this can be done with high accuracy on a large IBM machine, we required a small laboratory equipment, to read only to fair accuracy (ca. 2 percent). The final design as developed and tested, will be described.

Principle of method.

The Fourier analysis consists in finding the vector amplitude $\tilde{A}_n$ and the phase φ_n of each of n harmonic coefficients, defined as:

$$\tilde{A}_n = \int_{-\infty}^{+\infty} F(t) \sin(n\,\omega t)dt \qquad \varphi_n = \cos^{-1}(\tilde{A}_n, \tilde{A}_1)$$

The operation required, is to multiply the unknown function F(t) by each of its harmonics and integrate. The integrals of all terms other than $\int A_n \sin^2 n\,\omega t = A_n$ drop out. In our design we chose to scan the desired waveform at 60 c/s on a curve follower, and multiply it by the nth harmonic of 60 c/s. [There are many "panoramic analysers" which do this, but being designed mainly for acoustical end-uses, only the amplitude spectrum is given, not phase].

Incorporation of stray 60 c/s harmonics, was avoided by using a 15 c/s intermediate frequency, as follows. For the 5th harmonic, 315 c/s was multiplied in, rather than 300 c/s. When the product of this (5th) "reference frequency" f_R with the 300 c/s component of unknown wave F(t) is integrated

* in connection with a Master's Thesis in Electrical Engineering at Johns Hopkins University, June, 1958.

[so dropping all other time-averaged component products] there remains only the 15 c/s beat or "intermediate frequency" (product of 300 and 315.) A tuned filter easily segregates this I.F.; this when phase modulated (in phase and in quadrature) with the 15 c/s fundamental yields two components which add vectorially to display directly on a scope the required A_5 with phase φ_5. One next switches the f_R to 375 c/s, and reads off A_6 and φ_6. These can be read as fast as they can be plotted: about 5 min. for 30 harmonics, for both amplitude and phase.

Details of method

Several new approaches were incorporated, as well as the standard methods.

a) Multiplication of the unknown by a small off-frequency, to obtain (after integration) a Fourier coefficient having a constant AC frequency, is a new analytical approach, devised by Mr. Hume. This enables the subsequent operations to be carried out electronically by AC circuitry, free of drifts.

b) The use of a constant beat or intermediate frequency also obviates setting a wide-band phase-shifter as is used in standard harmonic analysers.(54) This saves an operation at each reading, and simplifies equipment.

c) The reference harmonics (multipliers) are set up at approximately $f_n + 15$, in the form of 50 fixed LC beat-oscillators (BFO). As each is selected, it is pulled into exactly $f_n + 15$ by a frequency control loop via a reactance tube. This control accomplished by comparison of (R_n+15) to nx60 against a 15 c/s bistable oscillator driven precisely i. frequency and phase from the 60 c/s line.

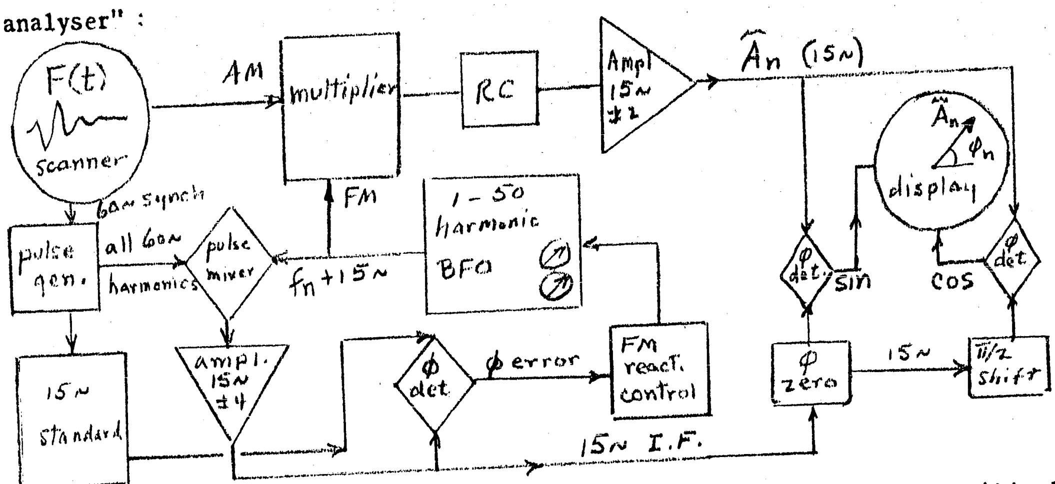
d) The reference harmonic oscillators are also all phased in with the fundamental, since the frequency feedback operates by sensing phase-error.

This novel feature avoids multiplying the unknown separately by the sin and the cos of the reference frequency, so saving another operation. This phase control is accomplished by a pulse transformer, triggered at the 60 c/s crossing. This generates the 50 required harmonics of 60 c/s in-phase and with nearly equal amplitude.

e) The readout at 15 c/s, regardless of harmonic, makes for uniformity of display, free of interference by line frequency.

f) An AM-FM transistor multiplier good to 3 kc was devised for this problem.

Since Fourier analysis is a standard tool of wide utility for physical systems, we will include the block diagram of this rapid "frequency-phase analyser" :



Functionally, the scanner delivers the desired function F(t) repetitively on a 60 c/s time-base to a multiplier. Here it amplitude-modulates a 500 kc carrier, frequency modulated by the desired harmonic. However, the multiplying harmonic is locked at 15 c/s above the desired component of F(t) both in frequency and phase. The product is both integrated and filtered by a low-pass RC to produce a 15 c/s wave A_n having the required amplitude and phase relation to the time of origin of F(t). This latter is guaranteed by the blocking oscillator (pulse generator) so triggered.

There remains only to split this vector into x and y components for cathode-ray display, referring their phase to a 15 c/s standard itself synchronized with the inception of $F(t)$.

For the precautions and details which confer the desired performance and accuracy, the reader is referred to the thesis mentioned above.

CHAP. VII. CONCLUSIONS

The contribution of this investigation lies in establishing better techniques as well as improved understanding of the relationship between BCG records and hemodynamic events. The approaches used were both analytical and experimental. They deal, however, with biophysical rather than the physiological aspects of the problem of assessing externally the internal hemodynamics.

We have confined our attention to developing a first-order theory and experimental method for analysing cardiovascular vector ballistics. This emphasis on the force or acceleration aspect, has excluded developing the theory and observation of the transient momentum (flow) and transient displacement (distributive) aspects of circulatory mechanics. [A method successful for acceleration ballistics also serves the latter purposes, since it must deal with finer detail and more artefacts than appear in the velocity and displacement records.]

In the past ten years a great deal of work on correlational ballistocardiography, has been devoted to analysing BCG wave-forms which told more about the passive vibratory properties of body and support, than about the subject's hemodynamics. Although such raw correlations are often a necessary first step in a biological problem, much effort has been wasted for lack of distinguishing between artefact and systematic information about functional relationships. We therefore began our analysis in Chap. II, with a clear statement on the pros and cons of a physical model able not only to provide the necessary basis for quantitatively dealing with the vector mechanics involved, but also incorporating enough of the biological realities to be meaningful.

The symmetry properties of a recumbent human, brought out in Chap. III, and the location of the cardiovascular acceleration patterns, focused interest on the separation of dynamical variables possible in the transverse ($xz\beta$) plane. The concept of line-of-resistance used in aero-vibration testing, afforded a clear way to formulate the relation between rotational and lateral translatory motion. Using this concept, there also results an experimental method of measuring the effective body-parameters in all dimensions, without making direct observations on the body. [Not only would the latter be impractical, but also operationally meaningless, in terms of what is agreed on as the meaning of the BCG observation.] These parameters (inertias and stiffness) must be considered for each subject, if one is to realize the full value of BCG detail.

For frontal-plane ballistocardiography, whose lateral component is recognized as significant both physiologically and pathologically, the importance of the roll component was developed in detail. The relations of this component to the location of heart and the dorsal configuration (h_f and h_B) were explored analytically, and the phasic interaction of roll and lateral BCG discussed. We showed the efficacy of uncoupling the roll artefact and the degree to which it could be done, with the physical model assumed.

We may add that since completing this contract, we have used the resulting construction, to verify our analysis, and to show that extraneous waves in the lateral BCG drop out, when the support is free to roll. The timing or phase-error in systole is also reduced.

In Chap. IV we attacked the problem of a physical structure which is sufficiently light and rigid in all deflections and torsions, to transmit without vibrational artefact the high stresses associated with gimballing the body axially. To prevent intermodal transfer of vibrations in the BCG spectrum, required a high ratio of stiffness to mass in all parts. The weight was brought down to the BCG requirement only by using minimum metallic sections and by diaphragming loads in a tandem torque-box design. Advanced air-wing procedures were needed to fulfil this requirement. It does not follow that all adequate 2D and 3D supports will require such highly engineered construction. This is an experimental model designed to explore body-vibrational variables. So subtle are the vibrational couplings in an extended structure, that we constructed a reference standard, against which to test simpler designs. The method of Cunningham[55], modified for freedom in roll, might well suffice.

The suspension of this structure to earth at a damped frequency of 1/3 c/s (ulf-BCG) in all translational and rotational modes, constituted a difficult problem in mechanical design. Both theory and experience indicate that free motion in roll and yaw at least, are essential to get a lateral BCG record without serious artefact. The 20" differential pendulum shown here, and its predecessor[27] (developed under this contract) have proved practical substitutes for the tall structures generally used to see properly the <u>horizontal</u> BCG. However, such a short leg becomes mandatory if one is to achieve a suspension which also gives <u>vertical</u> freedom from earth, to bring in the AP (z) forces. The negative spring developed for this purpose, is the only practical means so far devised for realizing this component of the BCG vector. We have also shown analytically and practically, a method of adjustable damping in all six directions, compatible with the vectorially free suspension. Proper damping is critical for ballistocardiography, because of phase-distortion at low frequencies.

In proposing correct physical principles for defining the observations to be used in the practice of vector ballistocardiography, we are simply following the lead of Wilson, Burger and Frank for the ECG. Since there may be simpler ways of achieving this, we scrutinized in Chap. V the theory of liquid suspensions for correct frontal-plane BCG recording. It results that in principle such suspensions are quantitatively free of constraint through the BCG spectrum; and may be used under favorable circumstances. However, connection to earth in the vertical direction, couples in horizontal forces via roll and pitch, which in practice introduce unwanted horizontal oscillations and ambient vibration.

The importance of frequency analysis of the BCG on a clinical scale, was explained in Chap. VI; and a means of doing it practically was developed. Clinical applications of this method have not been carried out within the period of this contract.

The emphasis of this report on BCG methodology and the principles underlying it, is by no means misplaced. Naive and erroneous methodology and physical interpretation, have had much to do with leaving us with a plethora of ambiguous and erratic BCG records; which in turn have discredited the possibility of finding reliable hemodynamic information in the BCG. It has resulted that investigators in this field are not encouraged to continue. It is true that the record is complex, but it carries a wealth of physiological information to interpret. The importance of reading hemodynamic information externally (in the light of its absence by other practical approaches), would seem to warrant full support of investigation in this field. Any impression that such information cannot be forthcoming, in the author's view results in part from wrong observing methods, and in part from inadequate understanding of what is going on physically in the body. Both these deficiencies are of a type that can be remedied. We hope that this report will contribute to that end.

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References:

1. Talbot, S. A., and Harrison, W. K. Jr.: Circulation 12:577,845,1022 (1955).

2. Schmitt, O. H.: Arch. Int. Med. 96:574 (1955).

3. Frank, E.: Ann. N.Y. Acad. Sci. 65:980 (1957).

4. McFee, R., and Johnston, F. D.: Circulation 9:868 (1954).

5. Ernsthausen, W.: Z.f. Kreislaufforschung, 45:893 (1956).

6. Shipley, R. E., Gregg, D. E., and Schroeder, E. F.: Am. J. Physiol. 138:718 (1943).

7. Brecher, G. A.: Venous Return, Grune and Stratton, N. Y., 1956.

8. Noordergraaf, A.: Physical Basis of Ballistocardiography (Thesis) Uitgeverij Excelsior, Hague, Netherlands (1956).

9. Burger, H. C., and Noordergraaf, A.: Am. H.J., 51:127,179; 52:653 (1956).

10. Nickerson, J. L., and Curtis, H. J.: Am. J. Physiol. 142:1 (1944).

11. Starr, I., Rawson, A. J., and Schroeder, H. A.: Am.J. Physiol. 123:195(1938).

12. v.Wittern, W. W.,: WADC Tech. Rep. 52-340, Nov. 1952.

13. Burger, H. C., Noordergraaf, A., and Verhagen, A. M. W.,: Am.H.J. 46:71 (1953).

14. Nickerson, J. L., Mathers, J. A.: Am. H.J. 47:1 (1954).

15. Brandt, J.L., Cuccese, A., Dock, W., and Schrager, A.: J. Clin. Inv. 30:971 (1951).

16. Elsbach, H.: (Thesis) Klinische Onderzoekingen met den Laag-Frequente BCG, Uitgeverij Excelsior, Hague (1954).

17. Wilson, G. P., Cunningham, D. M., and Griswold, H. E., Jr.: Circulation 15:280 (1957).

18. Smith, J. E., and Bryan, S.: Am. H.J., 45:40 (1953).

19. Scarborough, W. R., and Talbot, S. A.: Circulation 14:435 (1956).

20. Honig, C. R., and Tenney, S. M.: Am. H.J. 52:167 (1956).

21. Scarborough, W. R., Beser, J., Talbot, S. A., Mason, R. E., Singewald, M. L., and Baker, B. M., Jr.: Bull. J. Hopkins Hosp. 87:235 (1950).

22. Morris, G. W., and Braunstein, J. R.: Circulation 11:767 (1955).

23. Dock, W.: Am. J. M. Sc. 228:125 (1954).

24. Tannenbaum, O., Vessell, H., and Schack, J. A.: Am. H.J., 48:562 (1954).

25. v.Wittern, W. W.: Unpublished communication.

26. Talbot, S. A., Deuchar, D. C., Davis, F. W., Jr., and Scarborough, W. R.: Bull. J. Hopkins Hosp. 94:27 (1954).

27. Talbot, S. A., Soper, W., Condon, J. H., and Scarborough, W. R.: Am. J. Cardiol., Aug. 1958 (in press).

28. Scarborough, W. R.: Unpublished data.

29. Cunningham, D. M.: USPHS Atl. City Conf. on BCG (1954).

30. Blumberg, R. K.: Thesis, Dept. of Geology, Harvard Univ. (1948).

31. v.Wittern, W. W.: Personal communication.

32. Edson, J.(abstract):BCG Club, Atl. City, May, 1957.

33. Ernsthausen, W., v.Wittern, W. W., and Reismann, K.: USAF-AMC Rep. MCRED 696-116A (1948).

34. Lawton, R. W.: Circulation Research 3:403 (1955).

35. Burger, H. C., Noordergraaf, A., and Kamps, H.J.L.: Am.H.J. 53:907 (1957).

36. Van der Tweel, L. H.: Am. H. J. 53:4 (1957).

37. Honig, C. R., and Tenney, S. M.: Am. H.J. 54:98 (1957).

38. Womersley, J. R.: An Elastic Tube Theory of Pulse Transmission, WADC, TR 56-614, Jan. 1957.

39. Rappaport, M. B.: Am. H.J. 52:483 (1956).

40. Wahl, A. M.: Mechanical Springs, Penton, Cleveland (1949).

41. Fry, D. L., Mallos, A. J., and Casper, A. G. T.: Circ. Res. 4:627 (1956).

42. Oestreicher, H. L.,: J. Acoust. Soc. 23:707 (1951).

43. Nyboer, J., and Watson, T. R.: J. Lab. Clin. Med. 42:270 (1955).

44. Gordon, J. W.: J. Anat. Physiol. 11:533 (1877).

45. Henderson, Y.: Am. J. Physiol. 14:277 (1905).

46. Noordergraaf, A.: Fed. Proc. 17:118 (1958).

47. Suggested by Dr. Paul A. Crafton.

48. Honig, C. R., and Tenney, S. M.: Am. H. J. 54:678 (1957).

49. Talbot, S. A.: USPHS Conference on BCG, Atl. City (1954) (Unpublished).

50. Durand, F.: Aerodynamic Theory, Vol. 5. Springer, Berlin, (1935)

51. Dower, G. E., and Osborne, J. A.: Am. H. J. 55:523 (1958).

52. Lamb, H.: Hydrodynamics. Dover, 1945.

53. e.g. Sanborn Company 350 series recorders.

54. Instrument Test Equipment Company.

55. Cunningham, D. M., Donaldson, R., and Griswold, H. E., Jr.: Clin. Res. Proc. 5:59 (1957).

Appendix A (Chap. III-A)

Replacing the variables by

$$x = x_o \; \mathcal{E}^{j\omega t}$$

$$y = y_o \; \mathcal{E}^{j\omega t}$$

and so on, the acceleration amplitude in the x-direction becomes

$$\omega^2 \left[(x_o) + (y_o \gamma_o) + (z_o \beta_o) \right]$$

Previous measurements indicate linear displacements of order .010 inches. Observed angular motion does not exceed 0.1^o = .002 rad., and can safely be taken less than .01 rad. Substituting, we have for maximum x-acceleration

$$(.010)\omega^2 \left[1 + .02 \right]$$

We see that neglecting squares and products of velocities and displacements introduces an error no greater than two percent.

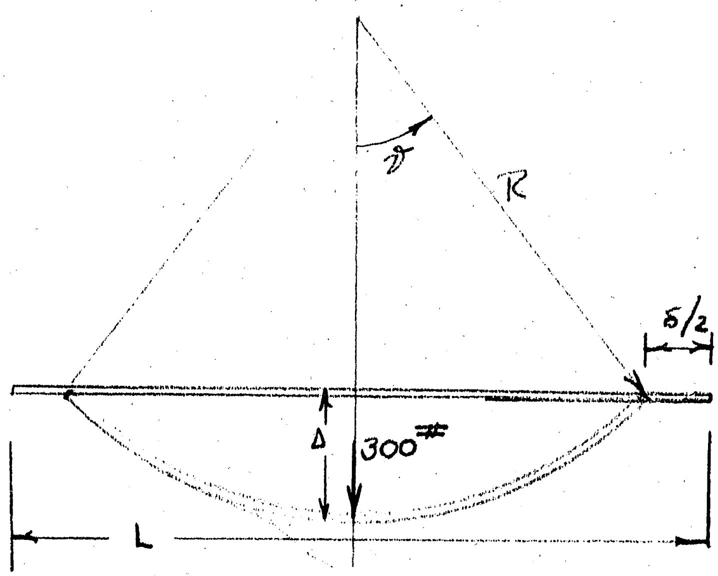
In the example just shown, it is the coriolis forces that are being dropped. This is the sideward push experienced by a person walking along a radius of a carousel.

Expanding the expression for angular acceleration about the x-axis, we have

$$\dot{h}_x + D(\dot{\gamma}^2 - \dot{\beta}^2) + \dot{\gamma}\dot{\beta}(C-D) + \dot{\gamma}(F\dot{\gamma} - E\dot{\beta})$$

The terms containing squares and products are gyroscopic moments, and can be dropped since they are small compared to $\dot{h}_x$, by the reasoning advanced for dropping the coriolis forces.

BED SHORTENING UNDER LOAD



The arc length equals the bed length in a purely bending deformation:

$$2\,\theta\,R = L \qquad [1]$$

The displacement is related to the geometry:

$$\Delta = R\,(1-\cos\theta) \cong R\theta^2/_2 \text{ for } \theta \ll 1. \qquad [2]$$

The shortening is given by the difference between the original length and the projection of the arc on the horizontal:

$$\delta = L - 2\,R\sin\theta \cong L - 2R\,(\theta - \theta^3/_{3!})$$

Since $\quad L - 2R\theta = 0$:

$$\delta = R\theta^3/_3 \qquad [3]$$

Eliminate R between [1] and [2]

$$\left.\frac{L}{2\theta} = \frac{2\Delta}{\theta^2}\right\}\ \theta = 4\,\Delta/L$$

Eliminate θ^2 between [1] and [2]

$$\left.\left(\frac{L}{2R}\right)^2 = \frac{2\Delta}{R}\right\}\quad R = L^2/8\Delta$$

Substituting in [3]

$$\delta = \frac{L^2}{8\Delta} \left(\frac{4\Delta}{L}\right)^3 \frac{1}{3}$$

$$= \frac{64}{24} \frac{\Delta^2}{L}$$

$$\boxed{\delta = 2.7\,\Delta^2/_L}$$

From the work on the frame

$$k_B = 2900 \text{ \#/in}$$

which allows for Δ due to a 300# load

$$= 300/k_B = 300/2900$$

$$\cong .1"$$

thus -

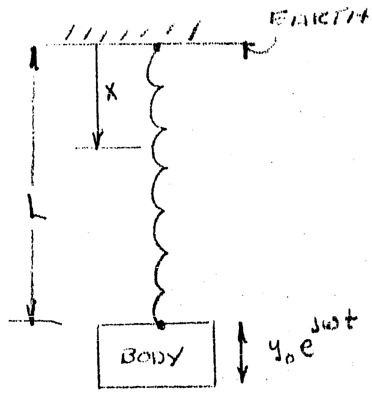
$$\delta = 2.7\,(.1)^2/78$$

$= .00035"$ **shortening of bed under load.**

Appendix C.

<u>Force on body due to surge of vertical supporting springs</u>

Ref.: Mechanical Springs
by Wahl.



The equation of motion of a particle at x on the spring is given by equation 224 of the reference as

$$\frac{\delta^2 y}{\delta t^2} + 2b\,\frac{\delta\, y}{\delta\, t} = a^2 \quad \frac{\delta^2 y}{\delta x^2}$$

when y is the displacement of particle from its rest position, b is the damping coefficient and a is the speed of propagation of a small disturbance ("sound speed") along the spring. We are interested in the steady forced response to the end being driven by $y_o e^{j\omega t}$. Assume a solution of the form

$$y = F(x)e^{j\omega t}$$

and try to satisfy the boundary conditions:

$$y = o \text{ at } x = 0$$

$$y = y_o e^{j\omega t} \text{ at } x = L$$

After substituting in the differential equation, and cancelling $e^{j\omega t}$, as it can never be zero, and rearranging, we obtain the eqn. for F:

$$F'' + \left(\frac{\omega^2 - 2\,bj\omega}{a^2}\right) \quad F=0$$

which has for a general solution

$$F = A \sin x \sqrt{\frac{\omega^2 - 2\,bj\omega}{a^2}} + B \cos x \sqrt{\frac{\omega^2 - 2\,bj\omega}{a^2}}$$

Multiplying by $e^{j\omega t}$ yields the solution

$$y = [A \sin x \sqrt{\quad} + B \cos x \sqrt{\quad}]e^{j\omega t}$$

For small values of the damping which can be expected for springs, the square root can be approximated as $\frac{\omega}{a}(1 - \frac{b}{\omega} j)$.

Hence, $$y = [A \sin x \frac{\omega}{a}(1 - \frac{b}{\omega} j) + B \cos x \frac{\omega}{a}(1 - \frac{b}{\omega} j)]\, e^{j\omega t}$$

At $x = 0$, $y = 0$: hence $B = 0$

AT $X = L$, $y = y_o\, e^{j\omega t}$; hence

$$A = \frac{y_o}{\sin \frac{\omega L}{a}(1 - \frac{b}{\omega} j)}$$

and the solution for y which satisfies the boundary condition is

$$y = y_o \frac{\sin [\frac{\omega x}{a}(1 - \frac{b}{\omega} j]}{\sin [\frac{\omega L}{a}(1 - \frac{b}{\omega} j]}\, e^{j\omega t}$$

For small damping, maximum amplitude occurs for $\frac{\omega_1 L}{a} = \pi$, which defines ω_1, the surge resonant frequency. Since the $\sin \pi (1 - \frac{b}{\omega_1} j) = \pi\, b/\omega_1$, the expression for y_{max} becomes (at surge resonance)

$$y_{max} = \frac{y_o\, \omega_1}{\pi\, b} \left(\sin \frac{\pi x}{L}\right) e^{j\omega_1 t} \qquad [1]$$

This itself is a maximum when the oscillatory functions multiply to unity, and for the final max. of y, we find

$$y_{max} = \frac{y_o \omega_1}{\pi b}$$

BODY FORCE

From equation 220 of the reference and the fact that

$$\frac{\delta y}{\delta s} = \frac{L}{2\pi rn} \frac{\delta y}{\delta x}$$

we obtain for the spring force at any point x

on the spring

$$P = Lk \frac{\delta y}{\delta x} \qquad [2]$$

where k is the usual spring constant. Since we are interested in the max. force, differentiate eqn. [1] to get

$$\frac{\delta y}{\delta x} = \frac{y_o \omega_1}{Lb} \cos \frac{\pi x}{L} \; e^{j\omega_1 t} \qquad [3]$$

The force on the body is computed by substituting x = L into [3], which yields for the force, as per eqn. [2]

$$(P_{Body})_{max} = \frac{y_o \omega_1 k}{b} \quad \text{at surge resonance}$$

where the oscillatory functions have been set equal to one.

RELATIVE FORCE

If the body is following the support as desired, the inertial force is:

$P_{Body} = m\ddot{y} = m\,\omega_1^{\,2}\, y_o$, where m equals the mass of body plus support.

$$\text{Relative force} = \frac{P_{spring}}{P_{body}} = R = \frac{y_o\ \omega_1\ k/b}{m\ \omega_1^{\,2}\ y_o}$$

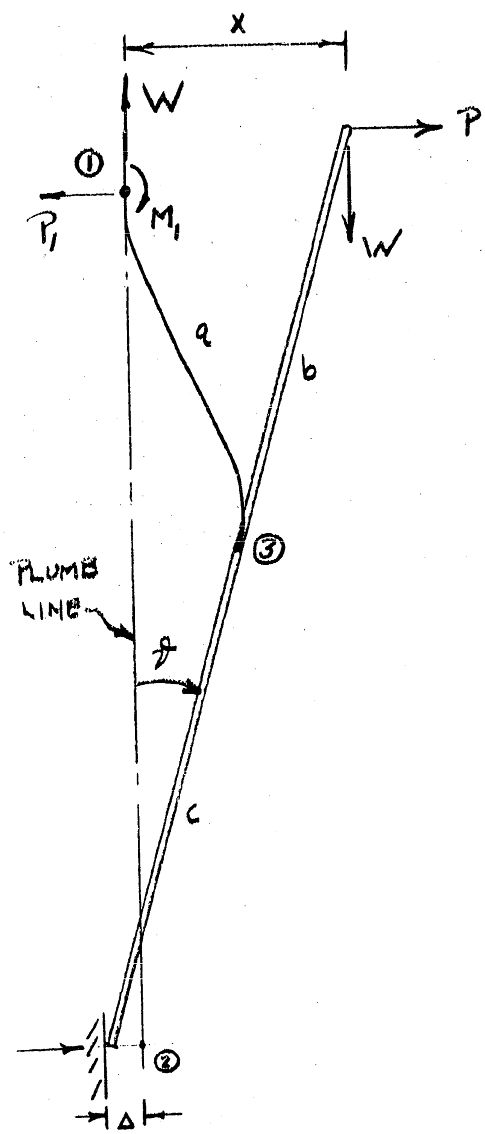
or,

$$R = \frac{k}{b\ \omega_1\ m} \qquad [4]$$

This applies also to the case where earth is driving the body through the spring.

NATURAL FREQUENCY AND EQUIL.

POSITION OF LEG



To the left is shown the differential pendulum which provides low frequency lateral restraint for the bed. One is mounted at each end of the bed and serves as the legs for reacting the force of gravity, thru differential vertical springs acting at (1) to provide low frequency vertical restraint. We are concerned here with the frequency, and equilibrium position when the restraint at (2) is a distance Δ out-of-plumb from the upper restraint at (1).

Descriptive

The bed rests on the upper end of the rod (b+c) which by virtue of the restraint at (2) is a positive pendulum. Opposing this is the wire (a) fixed at (1) and (3). A slide at (3) allows adjustment. The fixity at (1) and (3) injects Hookian resistance in addition to the pendulous restraint. (see Fig.6,pg.94)

Static Equilibrium

From a summation of horizontal forces -

$$P = P_1 - P_2 \qquad [1]$$

From a summation of moments about the point (2) -

$$M_1 + Wx + P\,(b+c) - P_1\,(a+c) = 0 \qquad [2]$$

Treating the rod as a free body which itself must be in moment equilibrium about point (3) -

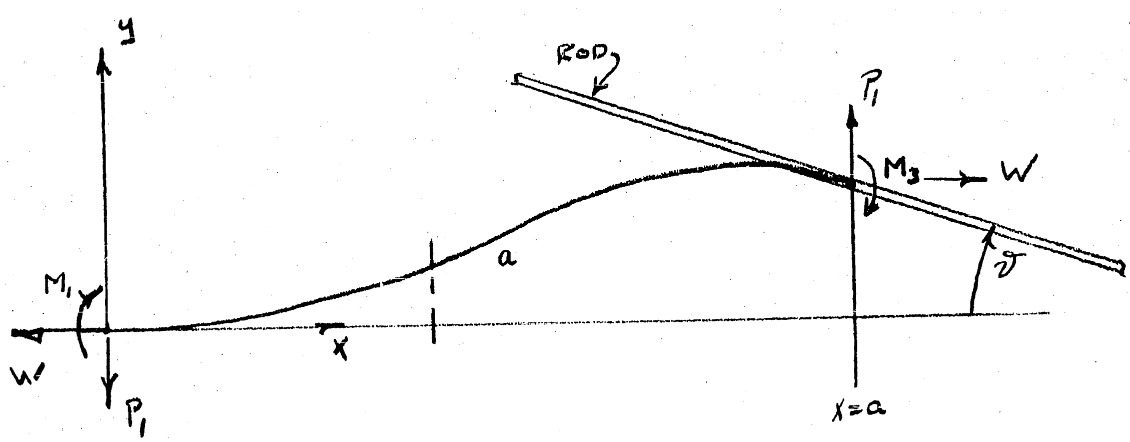
$$M_3 + P_2\,c - Pb - W\theta b = 0 \qquad [3]$$

Geometry gives us the relation -

$$\theta = \frac{x + \Delta}{b+c} \qquad [4]$$

Determination of M_1 and M_3

The moments M_1 and M_3 arise because of the fixity at these points, and introduce flexural resistance to the leg. Shown below is the wire of length a removed as a free body. It has been turned on its side for convenience:



The basic differential equation for bending of a thin beam is

$$EIy'' = M(x)$$

where

E = modulus of elasticity

I = section inertia moment

M = bending moment at x

y = displacement, assumed small enough for $y' \ll 1$.

The bending moment at x is positive if the curvature is concave up. Placing ourselves at a point x and looking to the left, we compute the moment to be -

$$EIy'' = Wx - P_1 y + M_1 \qquad [5]$$

The general solution of this equation is

$$y = A \cosh \alpha x + B \sinh \alpha x + \frac{P_1}{W} x - \frac{M_1}{W}$$

where A, B and M_1 are determined from the boundary conditions. At x = 0, the deflection (y) and the slope (y') are zero. This yields

$$\left. \begin{array}{l} B = M_1/W \\ A = P_1/\alpha W \end{array} \right\} \quad \alpha = \sqrt{\frac{W}{E\,I}}$$

At x = a, the slope must be the same as the rod because of the fixity i.e. $y' = -\theta$. Differentiating (6) and setting the result for x = a equal to $-\theta$, we obtain:

$$y' = -\theta = \frac{P_1}{W} + \alpha \left(\frac{-P_1}{W\alpha} \cosh \alpha a + \frac{M_1}{W} \sinh \alpha a\right)$$

from which

$$\frac{M_1}{W} = \frac{P_1}{\alpha W} (\operatorname{ctnh} \alpha a - \operatorname{csch} \alpha a) - \frac{\theta}{\alpha} \operatorname{csch} \alpha a \qquad [7]$$

The moment M_3 is determined from the differential equation as follows:

$$M_3 = EIy''\Big|_{x=a} = EI\alpha^2 \left(\frac{P_1}{\alpha W} \sinh \alpha a + \frac{M_1}{W} \cosh \alpha a\right)$$

Observing that $EI\alpha^2 = W$ and replacing M_1/W by its value from [7] -

$$\frac{M_3}{W} = \frac{-P_1}{\alpha W} (\operatorname{ctnh} \alpha a - \operatorname{csch} \alpha a) - \frac{\theta}{\alpha} \operatorname{ctnh} \alpha a \qquad [8]$$

The expressions for M_1 and M_3 can be greatly simplified by using the fact that for beams in the class of wire such as being used here, αa is large compared to unity. This allows with very little error, the following approximations:

$$\frac{M_1}{W} \simeq \frac{P_1}{\alpha W} \qquad [9]$$

$$\frac{M_3}{W} \simeq -\left(\frac{P_1}{\alpha W} + \frac{\theta}{\alpha}\right) \qquad [10]$$

Frequency

We are now ready to determine frequency. This is done by expressing the applied load P as a function of displacement (x) and design variables. The coefficient of x is, of course, the stiffness (k) which **is related to frequency.**

Solving for P_1 from (2) and (9) -

$$\frac{P_1}{\alpha W} + x + \frac{P}{W}(b+c) - \frac{P_1}{W}(a+c) = 0$$

from which

$$\frac{P_1}{W} = \frac{x + P/W\,(b+c)}{a + c - 1/\alpha}$$

$$\simeq \frac{\left[x + \frac{P}{W}(b+c)\right]\left[1 + \frac{1}{\alpha\,(a+c)}\right]}{(a+c)} \qquad [11]$$

because $1/\alpha\,(a+c) \ll 1$.

Solving for P_1 from (1), (3) and (10) -

$$\left(\frac{\theta}{\alpha} + \frac{P_1}{\alpha W}\right) + c\left(\frac{P_1}{W} - \frac{P}{W}\right) - \frac{P}{W}b - \theta b = 0$$

from which

$$\frac{P_1}{W} \simeq \frac{\left[\frac{P}{W}(c+b) + \theta\,(b-1/\alpha)\right](1-1/\alpha c)}{c} \qquad [12]$$

Equating $\frac{P_1}{W}$ from [11] and [12]

$$c\left[x + \frac{P}{W}(b+c)\right]\left[1 + \frac{1}{\alpha(a+c)}\right] = (a+c)\left[\frac{P}{W}(c+b) + \theta\,(b-1/\alpha)\right](1-1/\alpha c)$$

Solving for P/W -

$$\frac{P}{W} = \frac{xc\left[1+\frac{1}{\alpha(c+a)}\right]-\left(\frac{x+\Delta}{b+c}\right)(b)(a+c)(1-1/\alpha b+1/\alpha c)}{(c+b)\left[a-\frac{c}{\alpha(a+c)}\right]}$$

Collecting terms in x and Δ -

$$\frac{P}{W} = \frac{\left[c-\frac{b(a+c)}{b+c}+\frac{1}{\alpha}\left(\frac{c}{c+a}+\frac{b}{c}+1\right)\right]x}{a(c+b)\left[1-\frac{c/a}{\alpha(c+a)}\right]} \cong \frac{c^2-ab}{a} \qquad [13]$$

Near neutral equilibrium, which is of interest to us, the differences among the lengths a, b and c are small, but are decisive in determining frequency. For convenience, let us shift notation to these differences and a single length (a). Thus -

$$\left.\begin{array}{l} b-a=\eta \\ c-b=\xi \end{array}\right\} \quad \begin{array}{l} a \cong b \cong c \\ \xi \text{ and } \eta \ll a \end{array}$$

Equation [13] becomes in terms of this new notation plus taking advantage of the smallness of ξ and η

$$\frac{P}{W} = \frac{2\xi+\eta+5/\alpha}{(2a)^2}x - \frac{\Delta}{2a} \qquad [14]$$

The frequency is obtained from the coefficient of x as ω^2:

$$f = \frac{1}{2\pi a}\sqrt{g\left(\frac{\xi}{2}+\frac{\eta}{4}+\frac{5}{4\alpha}\right)} \cong \frac{1}{2\pi}\sqrt{g/a\ \frac{c^2-ab}{(2a)^2}} \qquad [15]$$

Equilibrium position $\sim x_o$

If the leg is out-of-plumb, the rest position of x will be other than zero, as determined by setting P = 0 in equation [14]:

$$\frac{2\xi+\eta+5/\alpha}{(2a)^2}x_o = \frac{\Delta}{2a}$$

or,

$$x_o = \frac{2a\Delta}{2\xi+\eta+5/\alpha}$$

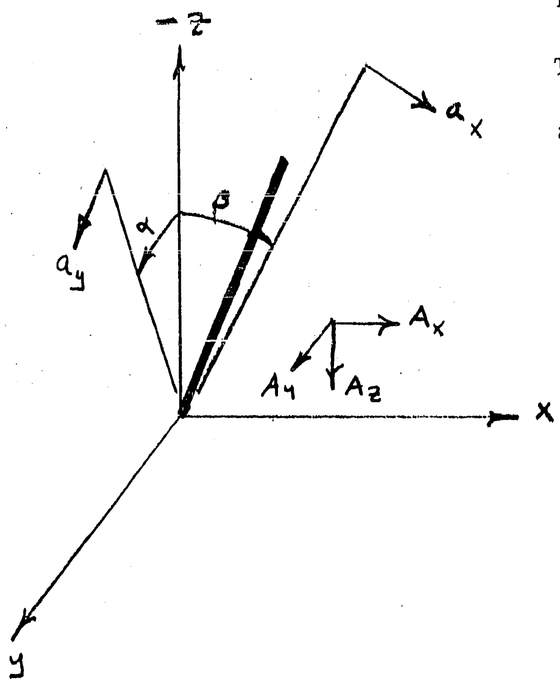
In terms of frequency -

$$x_o = \frac{\Delta g}{2a\omega^2}$$

where ω is circular frequency ($2\pi f$).

ACCELEROMETER TILT

One very desirable position for locating the accelerometers recording lateral accelerations is the top of the inner leg. However, since we expect to be recording very small accelerations it may be that the gravity component due to tilt is of the same order of magnitude. If this is so, another location will have to be chosen such as the bed, or to isolate these accelerometers from the gravity effect.



The accompanying sketch shows the geometry. The A's are the applied accelerations, the a's are the accelerometer readings.

Ideally

$$a_x = A_x$$

$$a_y = A_y$$

$$a_z = A_z$$

The vertical accelerometer is mounted on the outer leg, and so does not enter our considerations here.

In reality, the accelerometers pick up components of all accelerations, as per the following:

$$a_x \cong A_x + \beta A_z + \text{terms in } \alpha^2 \text{ and } \beta^2 + \text{----} .$$

$$a_y \cong A_y + \alpha A_z + \text{terms in } \alpha^2 \text{ and } \beta^2 + \text{-----} .$$

Each accelerometer is in error (e) to first order in their respective angular displacement, i.e. typically

$$e_y = \alpha A_z$$

If we suppose that all accel. outputs have been zeroed electrically prior to test, then the angular displacement α is due only to amplitude of body vibration.

It is known from our measurements that a max. (displacement of the bed) is .001" due to an accel. of .001 g. In our special case* the error is, then in a gravity field ($A_z \cong g$) -

$$e_y = \alpha g$$

$$= \frac{.001\ g}{20}$$

$$= .00005\ g\text{'s}.$$

This is quite small compared to the maximum -

$$\text{Ratio of accel.} = \frac{.00005}{.001} = 5 \text{ percent}$$

Accordingly, it is permissible to locate the lateral accelerometers on the inner leg.

* The dominating frequency is about 3 c/s with the BCG.

STAT

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