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WAVE FLOW OF THIN LAYERS OF A VISCOUS LIQUID

II. The Flow in Contact with a Gas Stream and Heat Transfer

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(The problem of the action of a gas stream on a liquid is studied. It is shown that, accepting the disruption of a gas stream in passing over the wave surface of a liquid, the increased drop in pressure of a gas stream flowing in a tube with wet walls can be explained and quantitatively determined. The expression is given which corresponds to ampirical results for the critical velocity at which "choking" occurs in a vertical tube along whose walls the liquid flows against the gas. An explanation is given for the increased heat transfer in thin layers of a liquid during wave flow).

Introduction

In the action of a gas current on the surface of a thin layer of a liquid flowing along a wall, there are uncovered a series of characteristic phenomena.

If the test is conducted in a tube in which the liquid flows along the inner wall, then for equal initial gas velocities the drop in pressure in a gas stream along the tube appears to be noticeably greater for wet walls than for dry walls. In tubes of small diameter a compound drop in pressure is found [1].

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If gas flow and liquid flow are opposed to each other then, with an increasing gas velocity, a point is reached at which the liquid is carried off by the gas and the counter flow cannot exist. When this effect is observed in the tube and when the critical velocity is attained, the disruption of the proper flow is accompanied by an accumulation of liquid in the tube. This condition is called "choking."

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P. Semenov [2] pointed out that during liquid flow even with a gas at rest there occurs a drop in pressure along the tube.

To explain all these effects on the basis of the notion of laminar flow did not appear possible, since the action of a gas stream on the flat surface of a fluid is not sufficiently great to produce the disruptions observed in the character of the flow of a liquid and a gas. If the surface of the liquid has a wave character, the action between the gas stream and liquid can be significantly greater. There thus appears a possibility of quantitatively explaining the described effects.

The Influence of a Gas Stream

Let us examine the pressure exerted by a gas stream on a wave surface. We shall select the distribution of the coordinates as in a sufficient distance from the surface (of the order of a wave length), the velocity of a gas current parallel to axis x has a constant value and the pressure equals U_0 . We will limit ourselves to the examination of the case in which the surface of the liquid flow has a simple sinusoidal form. We shall first assume it to be fixed. Then the equation of the wave surface will be:

$$y = a_0\phi = a_0 \cdot \alpha \cdot \sin Nx \qquad (1)$$

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The function describing the velocity of gas flow, closely complying with the given limiting conditions will be:

$$\psi = U_0(y - a_0\alpha \cdot \sin Nx \cdot e^{-Ny}) \qquad (2)$$

This equation gives, for a small value of $a_0\alpha/\lambda$ curves corresponding to the surface profile of the waves. When $y = \infty$, it will give the constant velocity U_0 . The rectangular components of the gas velocity will be:

$$U_x = \frac{\partial\psi}{\partial y} = U_0 \cdot (1 + a_0 \alpha N \cdot \sin Nx \cdot e^{-Ny})$$
$$U_y = -\frac{\partial\psi}{\partial x} = U_0 \cdot (a_0 \alpha \cdot N \cdot \cos Nx \cdot e^{-Ny}) \qquad (3) \quad p20$$

The excess pressure at the surface will be

$$p - p_0 = -\frac{1}{2}\rho_2 \cdot (U_x^2 + U_y^2)\Big|_{y=0} = \qquad (4) \quad p20$$
$$= -\frac{1}{2}\rho_2 U_0^2 \cdot [1 + 2a_0\alpha \cdot N \cdot \sin Nx + a_0^2\alpha^2 \cdot N^2]$$

where ρ_2 is gas density. If we consider that the waves move with phase velocity k, and are limited by the term depending on ~~x~~ α, then we obtain

$$p - p_0 = -a_0\alpha \cdot N\rho_2 \cdot [U_0 - k]^2 \cdot \sin Nx \qquad (5)$$

In this expression, it is necessary to consider which signs are taken before U_0 and k when liquid flow and gas current are in the same or opposite directions.

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The pressure gradient in the fluid stream will be:

$$\frac{\partial p}{\partial x} = -a_0\alpha \cdot N^2 \rho_2 \cdot (U_0 - k)^2 \cdot \cos Nx \qquad (6)$$

If equation I (25) takes into consideration both the pressure gradient of capillary forces and the obtained gradient (6) and the first approximate solution is obtained, then the action of a gas stream on a liquid surface will apparently be reflected in the solution, so that ϕ will be given by expression (1) in which the number N will always be greater than n of the previous solution I (26), obtained in the absence of a gas stream. Consequently the action of a gas stream will result in the de-

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crease of wave length on the surface. The physical cause of this effect of a gas stream on wave length comes from the known nature, shown by Helmholtz, a dynamic reaction occurring between the borders of two streams of different densities. Calculations show that, within the limit of those velocities yielding the effects described in the introduction, the difference between n and N caused by the gas current is insignificant. It cannot exert an essential influence on the general picture of the liquid flow. There will not be an increase in the pressure-drop in the gas along the length of the tube, since it can happen only when non-reversible processes occur at the passing of the gas around the wave surface. Thus such a form of reaction between gas and liquid streams cannot explain the increased pressure-drop observed during liquid flow along tube walls.

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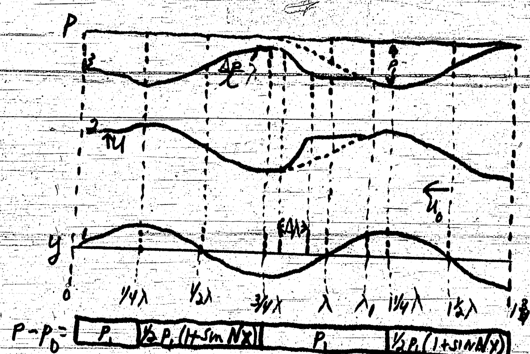
It is known that increased pressure-drop occurs in rough tubes. This effect is explained as due to the pressure losses occurring in passing around the protrusions of the rough wall surface. The effect taking place when a gas stream passes around the separate waves forming the liquid surface is naturally assumed to be similar to that which occurs on the protrusions of a rough tube. For this purpose the waves on the liquid surface must be assumed to have a profile which can disturb their smooth flow and create the conditions necessary for the disruption of the current on the downward side of the waves.

This effect can be represented similarly to the scheme in Fig 1.

Fig 1



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Curve 1 represents the wave surface. No 2 gives the velocity at a certain, but not too great distance from the wave surface. If a disruption did not occur, then they would follow the curve along the dotted line. At disruption, the velocity at point λ on the downward part of the wave retains its high value instead of decreasing smoothly. Now, the decrease in velocity will occur irreversibly due to the dissipation of kinetic energy in the interval $\Delta\lambda$, after which, when the current again encounters the ascending part of the wave, the velocity will have its normal value and the whole process will repeat itself. Curve 3 depicts the pressure produced by a gas at a certain short distance from the wave surface. The dotted line represents the case without a disruption in velocity, and the continuous line represents the case with a disruption in velocity. In the latter case after disruption at λ , the pressure at the surface will not regain its initial value smoothly, but for a rather long interval will retain a value close to the minimum. After this, when the pressure rises in the interval $\Delta\lambda$, it will not assume its exact initial value but will remain smaller due to the residual drop in pressure $(\Delta p/l)\cdot\lambda$, which is equal to the pressure drop on the wave length in a tube of radius r . This quantity, as will be shown later, is equal to

$$\frac{\Delta p}{l}\lambda = \gamma\cdot\frac{2a_0\alpha}{r}\cdot p_1 \qquad (7)$$

In order to describe the process quantitatively, we will assume that, in those parts of the gas current where the velocity at the walls is increasing, its variation and the corresponding variation in the pressure occur similarly in the reverse process and follow expression (5). This assumption can be justified in the case of small values for the amplitude in comparison with a wave length long enough for such a cycle to be created. Then from (5) we find that the maximum variation in the pressure p_1 at the surface will be equal to

$$p_1 = -2a_0 \cdot N\rho_2 (U_0 - k)^2 \qquad (8)$$

The determination of the place as being of disruption λ, at the wave peak in the examined problem presents significant difficulties. As is known, it depends on the hydrodynamic condition of the gas stream and on the wave pattern. At the point of disruption, there occurs an abrupt change in pressure on the liquid surface and this can bring about a local change in wave contour. Thus, the data available for determining conditions of disruption at the walls are hardly applicable in our case. But if we assume that it occurs close to $1\frac{1}{4}\lambda$, then it will not influence the quantitative result if it is placed on the peak of the wave $\lambda_1 = \frac{1}{4}\lambda$. The length $\Delta\lambda$, in which the equalization of the pressure occurs, can be considered small in comparison with the wave length λ. The quantity $(\Delta p / l) \cdot \lambda$ of the drop in comparison with P_1 as can be seen from (7), can be disregarded. Then we obtain the distribution of the pressure $p - p_0$ along the wave, corresponding to the quantities written in the lower line in Fig 1. Having made these allowances, let us calculate the tangential pressure τ_m, produced by the gas stream on the surface of the liquid. The force along the X-axis acting on the infinitismal element of the wave surface will be equal to $p \cdot dy$ and consequently the average pressure along the length of the stream will be

$$\tau_m = \frac{1}{\lambda} \int_0^{\lambda} p \cdot dy \qquad (9)$$

Substituting the value of y from (1) and integrating with the values of p as given below Fig 1, we get

$$\tau_m = \frac{a_0 \alpha \cdot N p_1}{2\pi} = -\frac{1}{\pi}(a_0 \alpha N)^2 \cdot \rho_2 \cdot (U_0 - k)^2 \qquad (10)$$

In the reverse direction of the stream, the expression will be the same, but the sign before τ_m will change. It is obvious that this sign will coincide with the sign of the velocity U_0. For the assumed distribution of pressure, the obtained quantity τ_m will be a maximum. To account for the influence of the assumptions made, we will introduce the coefficient γ which should be less than unity. Then we have for the actual tangential pressure

$$\tau = \gamma \cdot \tau_m \, , \quad \gamma \leq 1 \qquad (11)$$

To the energy E_j in I (35), transmitted to the liquid by the force $j\rho$ per unit length of stream, it is now necessary to add the energy transmitted during wave motion with velocity k against the force τ. Thus the average overall energy, transmitted to the fluid stream per unit length, will be

$$E_J = j\rho \cdot Q \pm \gamma \tau_m k \qquad (12)$$

Introducing the new designation J and the quantities Q and z, we get

$$E_J = J\rho \cdot Q = \rho Q (j \pm \gamma z a_0^{-1} \cdot \rho^{-1} \cdot \tau_m) \qquad (13)$$

or, introducing the value of τ_m from (9);

$$J = j \pm \gamma \cdot \frac{1}{\pi} \cdot z a_0 \alpha^2 \cdot N^2 \cdot \frac{\rho_2}{\rho} \cdot (U_0 - k)^2 \qquad (14)$$

In these expressions the plus sign refers to currents in the same direction, while the minus sign refers to the counterflow of liquid and gas.

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A physical interpretation of the obtained expression can be made thus: as a result of the action of gas stream the flow of the liquid layer occurs as if there acted on it in the direction X, a volumetric force having the acceleration J. This is a basic quantitative factor for the action of a gas stream on the wave flow of a liquid. We will calculate it by changing j to J in the basic equation I (19).

The pressure gradient in the liquid, created by the gas stream on the amplitude of the waves is determined by the form of curve 3 in Fig 1. It consists of two parts: the first corresponds more to the abrupt variation in pressure in the element $\Delta\lambda$ and the second corresponds to a smooth variation in the half waves.

On the first-harmonic term $\sin Nx$ of the function ϕ entering into the solution of equation I (19), only pressure changes having the same period may exhicit a substantial effect. It can be assumed from this that the pressure variations in the short element $\Delta\lambda$ will not have any noticeable effect on the term with the first-harmonic. Consequently, in the first approximation, the computation may be limited only to the variation in gradient occurring smoothly in the extent of the halfwaves; and over the wave length, its first-harmonic will equal

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$$\frac{\partial p}{\partial x} = -\frac{1}{2} a_0 \alpha N^2 \rho_2 \cdot (U_0 - k)^2 \cdot \cos Nx \tag{15}$$

Let us introduce this value into equation I (19) and also the following designation:

$$2d = \frac{1}{2} \frac{\rho_2}{\sigma} \cdot (U_0 - k)^2 \tag{16}$$

Using expression I (IV), we get the equation for determining the first-harmonic of function ϕ:

$$a_0 \cdot \delta \cdot (\ddot{\phi} + n^2\phi + 2d \cdot N^2 \alpha \cdot \cos Nx) + 3\left(J - \nu \cdot \frac{2V_0}{a_0^2}\right)\phi + \left(J - 3\frac{\nu V_0}{a_0^2}\right) = 0 \tag{17}$$

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For a steady-state periodic solution the constant term and the coefficient of ϕ must be set equal to zero, as before. If this condition is maintained, we find that N is determined from the following equation:

$$N^2 - 2d \cdot N - n^2 = 0 \tag{18}$$

whence we obtain
$$N = d + \sqrt{d^2 + n^2} \tag{19}$$

or if d is small compared to n :

$$N = n\left[1 + \frac{d}{n} + \frac{1}{2} \cdot \left(\frac{d}{n}\right)^2 + \dots\right] \tag{20}$$

Thus, when the ratio d/n is small, the wave length will be changed only a little by the gas stream. The effects described in the first paragraph usually occur under these conditions.

With a small value for d/n, the approximation of the second order I (IV) obtained by us will also be correct here. In this case, when V_0 and consequently n in I (IV) are equal to zero, there arise the usual standing waves $\phi = \alpha \sin 2d \cdot x$. They are found when the flow of the fluid is retarded. The condition of equating to zero the coefficient of ϕ in equation (16) gives

$$j \pm z a^{-1} \rho^{-1} \tau = z \frac{\nu}{a_0^3} \cdot Q \tag{21}$$ p.24

To maintain the energy balance, it is necessary that the energy $\overline{E_J}$ (13) equal the dissipated energy $\overline{E_\mu}$ I (34). Comparing them, we get

$$j \pm z a^{-1} \rho^{-1} \tau = 3 \frac{\nu}{a_0^3} \cdot Q \cdot F \tag{22}$$

Comparing this equation with the previous we get

$$z = 3F \tag{23}$$

This corresponds to the previous relation I (42) for determining F. Thus function F remains as before. It can be assumed from this that the values I (VI) obtained for a^2 and z and F are retained. Substituting them and (10) in expression (22), we get

$$j \pm \gamma\, 0.16\, a_0 N^2 \cdot \frac{\rho_2}{\rho} (U_0 - k)^2 = 2.4 \frac{\nu}{a_0^3} \cdot Q \qquad \text{(I)}$$

This expression, together with (19) and (16) and I (IV), make it possible to compute N and a_0, for any given Q and U_0 and for other physical constants of the currents. The problem requires a lengthy calculation and is simplified only when n/d is small or large.

The Pressure-Drop in the Tube

If the liquid flows along the inner walls of a tube of radius r and if we equate the tangential force along the perimeter to the pressure-drop along the tube, we then get:

$$\frac{\Delta p}{l} = \frac{2\pi}{r} = \frac{2\gamma}{\pi r} (a_0 a N)^2 \cdot \rho_2 \cdot (U_0 - k)^2 \qquad \text{(II)}$$

From this expression, by substituting the value of p from (8), we get the previously obtained expression (7).

Determining the pressure-drop according to expression (II) is connected with calculating N and a_0. This problem is noticeably simplified if n/d is small (that is, $n \cong N$). It is also simplified when n is small in comparison with $2d$ (that is $N \cong 2d$). But in this case, it is usually difficult to satisfy those assumptions made in creating these expressions.

There is little experimental data on the measurement of pressure drop. P. Semenov [2] studied this problem very minutely, but his conditions relate to high-speed gas streams in the direction of liquid flow and opposed to gravity; they make the application of expression I complex and less reliable.

If in expression (II) $U_0 = 0$ (i.e., the gas is considered stationary), then the pressure-drop in the tube will remain and will be dependent on the waves moving with phase velocity k. In the given case N can be made equal to n. Then, using expression I (43) and I (VIII) we get

$$\frac{\Delta p}{l} = \frac{4\gamma}{r} \cdot \left(\frac{jQ^2}{\nu}\right)^{4/3} \cdot \frac{\rho_2}{\sigma} \qquad \text{(III)}$$

This expression gives, when $\gamma = 1$, the upper limit of the pressure-drop. P. Semenov found it at the ends of the tube in the absence of gas current (quantitative data were not cited in the work. Proving this simple relationship might be interesting.

Flow of a Liquid With and Against a Gas Stream

As was already shown, flow conditions of a liquid during the action of a gas stream take a simple form when d/n is small. Calculations show that this condition is close to the one which exists when the characteristic effects of the reaction between the currents, as described in the beginning of this article, occur. Let us assume that d/n is small; then, disregarding the quantity d, we substitute in expression (I) the value of n^2 from I (IV) and the values from I (VI); we get

$$a_0^3 \pm 0.27\gamma \frac{\rho_2}{\sigma j}(U_0 - k)^2 Q^2 a_0 - 2.4\frac{\nu}{j}Q = 0 \qquad (24)$$

The upper sign refers to the positive direction of the velocity U_0, and the lower refers to the negative direction. This equation, for given U_0, connects Q and a_0. Let us introduce the expressions:

$$s = 0.09\gamma \frac{\rho_2}{\sigma j}(U_0 - k)^2; \quad q = 1.2\frac{\nu}{j} \qquad (25)$$

then (24) appears as:

$$a_0^3 \pm 3sQ^2a_0 - 2qQ = 0 \qquad (26)$$

Let us investigate the roots of this equation. For positive values of Q, there are always solutions in which a_0 will have a positive and real value. In uniflow a_0 will decrease with U_0 and because of this the flow cannot lose its stability. In counterflow a_0 will increase with U_0 and when it becomes greater than the corresponding laminar flow, the stability of flow with further increase in a_0 is not evident. When Q is negative, a_0 can have a real value in uniflow (i.e., the liquid is carried away by the gas against a force $g\rho$. Equation (26) will then appear as

$$a_0^3 - 3sQ^2a_0 + 2qQ = 0 \qquad (27)$$

This equation has a positive and real root for a_0 when

$$(sQ^2)^3 \geqslant (Qq)^2 \qquad (28)$$

whence it follows that

$$Q^4 \geqslant q^2 \cdot s^{-3} \qquad (29)$$

Thus for any given q and s, there exists a minimum value of the discharge rate Q carried by the gas stream. The quantity for minimum feed Q_m, at which a reverse current is possible, will from expression (28) be

$$Q_m = q^{1/2} \cdot s^{-\frac{3}{4}} \qquad (30)$$

~~Accordingly, the thickness of the layer at minimum feed at which~~ the liquid can begin to be carried away by the gas, from equation (28) will be equal to

$$a_m = q^{\frac{1}{2}} \cdot s^{-\frac{1}{4}} \qquad (31)$$

When a feed higher than the minimu, the equation will give two possible values for α_0 .

Experimental research or a suitable analysis can show which of the possible flows appears most stable.

"Choking"

Fradkov [1] conducted experiments with the flow of thin layers of liquid air in tubes 10 cm long with a radius of 0.23 cm, with counterflow of air, at 1 at. abs. In his tests the supply came through an opening in the wall of the tube of ~4cm from the upper edge. Thus the fluid, flowing into the tube, could flow either up or down. When, with a determined velocity of the gas and, consequently, the quantity S in expression (26), the supply Q reaches the quantity Q_m, then, in compliance with obtained conditions, there will appear a possibility for the fluid instead of flowing down, to be carried up by the gas. This effect was called "choking." The corresponding velocity U_k , at which it begins, is called the critical velocity and was determined experimentally by Fradkov.

Substituting the values of S and q in expression (30) of (25), we get for U_k :

$$U_k = \gamma^{-\frac{1}{2}} \cdot 3.5 \left(\nu j^{\frac{1}{2}} \cdot Q_m^{-2} \right)^{\frac{1}{3}} \cdot \left(\frac{\sigma}{\rho_2} \right)^{\frac{1}{2}} + k \qquad \text{(IV)}$$

Here all quantities are known except γ , the correction coefficient for τ_m (11), which takes into consideration the uncertainty in conditions of disruption. At first we shall set $\gamma = 1$. Let us assume that phase velocity k can be disregarded in comparison with U_k .

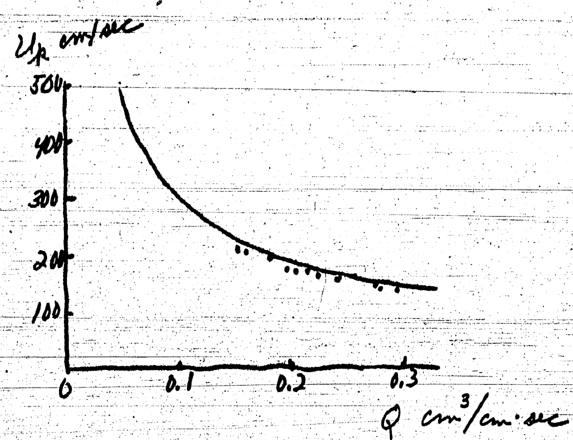

Fig 2

In Fig 2 the drawn curve represents the theoretical values of U_K, calculated according to expression (IV). Here the values of the physical constants presented in the table of the first article, air density at 1 abs. and 82°K, $\rho_2 = 4.4 \cdot 10^{-3}$, were used.

Fradkov's experimental data are also shown in Fig 2.

As can be seen theoretical and experimental results fully agree, although there are no arbitmary quantities in expression (IV). This coincidence is significantly better than expected from the assumptions and approximation made during calculation. Because of this the validity of expression (IV) has to be tested on a series of liquids to show whether or not this coincidence is accidental. In any case, it shows that expression (IV) apparently gives the correct relation of the critical velocity to the physical constants of the currents.

The Influence of Tube Dimensions

In determining the quantity τ , the distribution of velocity U_0 along the tube was disregarded. The distribution is due to viscosity effects in the gas stream. The assumptions made are all the more substantiated when the pressure drop determined by expression (II) is greater than that which occurs in a tube without a liquid layer. In the opposite case, distribution of the velocities at the wall of the tube will be determined not so much by the resistance caused by passing over the waves as by the virtual viscosity of the actual turbulent flow of the gas. Because of this, U_0 at the wall will be less than the average velocity of the gas flow in the tube. Using our present knowledge of the roughness ζ of tube walls and, in the case of a liquid layer, expressing it as the ratio of the radius of the tube to the amplitude of the waves, we get

$$\zeta = \frac{r}{2a_0 k} \tag{32}$$

The rougher the wall, the smaller is this quantity. From Nikuradze's [3] tests the effect of roughness on the pressure drop in a tube is known to depend on the Re . For example, when $\zeta \leqslant 15$ and $Re \geqslant 10^4$ the pressure drop is determined practically by the roughness alone. In the cited experiments of Fradkov, the tube was of sufficiently small cross section and ζ was small enough to satisfy the assumptions.

Due to the indicated allowance the velocity U_k given in expression (IV), for the same discharge rate Q and with an increase in tube diameter and consequently with the increase of ζ will be less than the corresponding average velocity of the gas in the tube. This is found in test [1]. In all expressions into which τ enters, if they are to be applied to tubes with a significant cross section, a correction coefficient must be introduced which will depend on ζ and Re . Practically this coefficient can be incorporated with γ .

Heat Transfer

Heat transfer in a liquid stream during wave flow will be distinguished from heat transfer during laminar flow. If you assume that in both cases turbulent heat transfer is absent, then the transfer of heat will be determined by the thickness of the layer. If you designate κ as the heat conductivity of the liquid, then in laminar flow the coefficient of heat transfer will be

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$$a_l = \kappa/m \tag{33}$$

In wave flow, it will have a mean value since the thickness of the layer varies along the length. This can be made equal to

$$\alpha_v = \kappa/a \tag{34}$$

Substituting the value of a from I (6) and using the value of the integral in I (39) and the values in I (VI), we get

$$\alpha_v = \kappa\beta^{-1} \cdot \overline{(1+\phi)^{-1}} = 1.21\ \kappa/m \tag{35}$$

i.e., it is 21 percent greater than in laminar flow. Experimental data on the determination of the coefficient of heat transfer in a flowing layer are contradictory. There is usually found a higher value than that given by expression (33). In the work of Hebbard and Badger. [4] this coefficient is given as 1.25 ± 0.05 which corresponds with out values. The coefficient of heat transfer is always affected by any disturbance of laminar flow, and, in wave flow this can easily occur. Because of this, the quantity (33) can be regarded as the minimum, practicable at low Re and in the absence of a gas stream.

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Conclusion

The study of the described effects, because of their complexity, must be done by approximate methods and the validity of some of the assumptions and simplifications are difficult to evaluate except theoretically. Because of this, further and more detailed experimental verification is necessary. Existing experiments were not conducted in a manner which facilitates the development of theoretical investigations. For example, there are as yet no determinations for phase velocity, wave length and amplitude. Such experimental research is desirable for further study of this type of flow.

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