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Title: A NEW ELECTRICAL APPARATUS FOR HARMONIC ANALYSIS AND SYNTHESIS

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Source: Izvestiya Akademii Nauk SSSR, Otdeleniye Tekhnicheskikh Nauk,
No 3, 1946

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A NEW ELECTRICAL APPARATUS FOR HARMONIC ANALYSIS AND SYNTHESIS

Izv. Akad Nauk, SSSR, Ot Tekh Nauk, No 3, 1946
P. 389-400

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1. INTRODUCTION

The problems of harmonic analysis are not new. Nevertheless, however, they have lost none of their importance and a fast, reliable and, at the same time, sufficiently accurate method for finding the coefficients of a Fourier series has still not been found, inspite of many intensive searches.

Methods that involve actual computations and numerical analysis require a great amount of work and time for testing and therefore prove to be but slightly convenient in those cases where large-scale investigations are necessary; that is, where a large number of curves are resolved into Fourier series with many harmonics. Existing types of mechanical harmonic analyzers that possess more or less widespread application (Mader, Coradi, Henrici, etc) -- although they facilitate considerably the work of harmonic analysis -- suffer from a number of serious defects, whose presence often prevents their use.

In working with these apparatuses one must draw a graphical representation of the curve to be analyzed, as many times as a harmonic must be found. This work itself requires sufficient experience and errors depend upon the personal qualities of the person operating the apparatus.

In order to make the results more accurate, it is necessary to repeat several times the drawing of the curve and to calculate the average value of the coefficients found.

Such a method of operation does not even give high accuracy. This is explained by the fact that a graphico-mechanical method is employed here, and the accuracy of any graphical calculation is always limited by the accuracy of construction of the graphs.

Moreover, none of these apparatuses permit one to conduct synthesis; that is, to obtain the 'summed' curve corresponding to given coefficients of a harmonic series. Such a problem, however, is often encountered by researchers and require as much work as does analysis for its solution.

In this article the author proposes a new apparatus based on principles other than those of previous analyzers. This apparatus permits one to conduct

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analysis directly from the values of the curve's ordinates given in tabular form. In operations with it, high qualifications are not required and the accuracy is independent of the person operating the apparatus and of predetermined technical conditions (the apparatus can, in principle, be made so that it always gives results with a given degree of accuracy). The speed of operation on it exceeds considerably that on other analyzers; thus it is hoped that this apparatus can find application especially in those cases where large-scale computations are required. The laboratory of electro-modeling and electrometry of the Power Institute, Academy of Sciences USSR, is executing the first practical design and construction of such an apparatus which permits one to find in the course of one-half hour 18 harmonics of a curve being analyzed and with an accuracy up to 1% of maximum. In this apparatus one can find with the same speed 36 ordinates (every 10°) of a total curve, if the curve is given analytically in a Fourier series whose number of harmonics does not exceed 18.

A. L. Goflin worked out the construction part of the apparatus.

2. SCHEME OF OPERATION OF THE APPARATUS DURING HARMONIC ANALYSIS

The apparatus is intended to find the coefficients of the Fourier series into which a given function f(x) must be expanded; that is, to find the coefficients A_k and B_k (where k is 0, 1, 2, 3...) of the following series:

$$f(x) = A_0 + A_1 \sin x + A_2 \sin 2x + \ldots + B_1 \cos x + B_2 \cos 2x + \ldots \qquad (a)$$

The main detail of the apparatus is the multiplier, which is represented schematically in Figure 1 [Note: All figures are appended in the annex].

The primary coil of transformer T (terminals MN) is connected to an alternating-current circuit. The voltage of the secondary coil is supplied to the potentiometer.

By moving the slide P on the potentiometer coil one can supply any part of the voltage U_2 to the voltage divider CD.

Potentiometer AB is graduated, which permits one to establish on CD a given part of U_2 directly according to a potentiometer scale.

The number of multipliers equals the number of ordinates into which a curve graduated into a number of portions is divided.

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As an example, an apparatus is described below which calculates with 36 ordinates and 18 harmonics. An apparatus can, in principle, be constructed for any number of ordinates and harmonics.

The apparatus described requires 36 multipliers.

Let us assume that the multiplier represented in Figure 1 corresponds to the n-th ordinate. In this case the voltage divider CD must have outputs selected in such a way that: 1) a part of the voltage applied to CD, which (part) equals

$$U_{1n} = U_{CD}/\sin(10n)^{\circ}/ \qquad (b)$$

acts between point 0 and 1 (Figure 1); 2) a part equal to

$$U_{2n} = U_{CD}/\sin(2\cdot 10n)^{\circ}/ \qquad (b')$$

acts between points 0 and 2;....m) a part equal to

$$U_{mn} = U_{CD}/\sin(m\cdot 10n)^{\circ}/ \qquad (c)$$

acts between points 0 and m.

As is seen from the formulas, only a fractional part of the voltage U_{CD} must act between point 0 and any other output, since the absolute value of the sine of any argument is always either zero or less than unity.

This fact makes it possible to realize a divider in the form of an ordinary resistance with taps without employing more complex means.

The number 10, which enters everywhere as a factor in the argument, possesses the following derivation: the whole period of the curve analyzed, which is assumed to equal 360°, is broken up in this case in 36 ordinates of equal parts which (parts) are spaced from one another, in degrees, by the amount $360/36 = 10^{\circ}$; consequently, the number of degrees from the origin to the n-th ordinate will be 10n.

The number of taps of the divider must equal the number of terms of the series — in this case 18.

If, besides a sine expansion, we also assume a cosine expansion, then the divider must possess additional taps besides the indicated taps. The voltages between these taps and the null point must equal respectively:

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$$U_{1n}' = U_{CD}/\cos(10n)^\circ/$$
$$U_{2n}' = U_{CD}/\cos(2\cdot 10n)^\circ/ \qquad (d)$$
$$\ldots\ldots\ldots$$
$$U_{mn}' = U_{CD}/\cos(m\cdot 10n)^\circ/$$

Thus in order to expand a certain function into a Fourier series containing 18 harmonics and having its terms made up of both sines and cosines, each divider must be provided with 36 taps. However the actual number of divider taps is considerably less, since many values of $/\sin(m\cdot 10n)^\circ/$ and $/\cos(m\cdot 10n)^\circ/$ coincide. For this reason the number of taps does not exceed 10.

Actually, since m and n are whole numbers, the product mn also can be only whole positive numbers; therefore the arguments of all sines and cosines will always be multiples of 10.

It is easy to see that the absolute values of the sines and cosines will not be repeated only when mn varies from 0 to 9. When mn equals 10, we have:

$$/\sin(10\cdot 10)^\circ/ = /\sin 100^\circ/ = /\sin 80^\circ/ = /\sin(8\cdot 10)^\circ/,$$
$$/\sin 110^\circ/ = /\sin 70^\circ/, \qquad (e)$$
$$/\sin 120^\circ/ = /\sin 60^\circ/, \text{ etc.}$$

Thus, 10 is the maximum number of taps on the divider dividing the voltage into quantities proportional to the absolute values of the sines of angles, which (values) are multiples of 10. [Note: sic; angles are multiples of 10.]

The resistance between the null point and the respective taps must be proportional to the following values:

$$R_{01} = /\sin 10^\circ/ = 0.174 \qquad R_{06} = /\sin 60^\circ/ = 0.866$$
$$R_{02} = /\sin 20^\circ/ = 0.342 \qquad R_{07} = /\sin 70^\circ/ = 0.940$$
$$R_{03} = /\sin 30^\circ/ = 0.500 \qquad R_{08} = /\sin 80^\circ/ = 0.985 \qquad (f)$$
$$R_{04} = /\sin 40^\circ/ = 0.643 \qquad R_{09} = /\sin 90^\circ/ = 1.000$$
$$R_{05} = /\sin 50^\circ/ = 0.766$$

A completely similar dependence can be obtained for the cosines.

In order to divide the voltage U_{CD} into parts proportional to $/\cos(10mn)^\circ/$, it is necessary to have between the null point of the divider and the respective taps resistances that are proportional to:

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$$R'_{01} = /\cos 10^\circ/ = 0.985 \qquad R'_{06} = /\cos 60^\circ/ = 0.500$$
$$R'_{02} = /\cos 20^\circ/ = 0.940 \qquad R'_{07} = /\cos 70^\circ/ = 0.342$$
$$R'_{03} = /\cos 30^\circ/ = 0.866 \qquad R'_{08} = /\cos 80^\circ/ = 0.174 \qquad (g)$$
$$R'_{04} = /\cos 40^\circ/ = 0.766 \qquad R'_{09} = /\cos 90^\circ/ = 0.000$$
$$R'_{05} = /\cos 50^\circ/ = 0.643$$

From a comparison of this table (g) with the preceding one (f), it is evident that:

$$R_{01} = R'_{08}, \quad R_{02} = R'_{07}, \quad R_{03} = R'_{06}, \ldots\ldots R_{08} = R'_{01};$$

that is, in order to form division by cosines one can use the same taps as those used for division by sines.

Let us consider now how to determine the coefficients of a Fourier series. Let us assume that we need to determine the coefficient for the K-th harmonic of the sine.

As is known from the theory of Fourier series:

$$A_k = \frac{2}{l}\int_0^{l} f(x)\cdot \sin\frac{2\pi}{l}x\cdot dx \qquad (1)$$

where l is the period of the function f(x).

The relation (1) can be approximately described in the form:

$$A_k \approx \frac{2}{l}\sum_{i=0}^{i=n} f(x_i)\cdot \sin k\frac{2\pi}{l}x_i\cdot \Delta x_i \qquad (2)$$

or in computation with degrees, instead of radians:

$$A_k = \frac{2}{l}\sum_{i=0}^{i=n} f(x_i)\cdot \sin k\frac{360^\circ}{l}x_i\cdot \Delta x_i \qquad (2a)$$

In our case the total period is divided up into 36 equal parts; therefore

$$\Delta x = \frac{l}{36}, \quad x = i\Delta x \quad (i = \text{the no. of the ordinate});$$

thus: $l = 36\cdot\Delta x$, and formula (2a) transforms into:

Declassified in Part - Sanitized Copy Approved for Release 2012/05/30 : CIA-RDP82-00039R000100080006-7

Declassified in Part - Sanitized Copy Approved for Release 2012/05/30 : CIA-RDP82-00039R000100080006-7

$$A_k = \frac{2}{36\Delta x}\sum_{i=1}^{i=36} f(x)\cdot \sin\left(k\frac{360}{36\Delta x} i\,\Delta x\right)\cdot \Delta x \qquad (2b)$$

Since Δx is constant, it can be carried from behind the summation sign; thus:

$$A_k = \frac{1}{18}\sum_{i=1}^{i=36} f(i\Delta x)\cdot \sin(k10i)\;; \qquad (3)$$

similarly

$$B_k = \frac{1}{18}\sum_{i=1}^{i=36} f(i\Delta x)\cdot \cos(k10i)\,. \qquad (4)$$

Let the curve represented in Figure 2 be expressed as a Fourier series. With this purpose in mind, let us break up the total period of the curve into 36 equal parts, as shown in the figure, and let us measure all the ordinates from y_1 to y_{36} corresponding to points of division of the period from 1 to 36.

Let the maximum ordinate of the curve equal Y_m.

Let a voltage be applied on the secondary coil of the transformer (Figure 1) equal to U_m. From the relation:

$$U_m = c_1 Y_m \qquad (5)$$

we determine the constant of proportionality:

$$c_1 = U_m / Y_m \qquad (5a)$$

(it is assumed here that identical voltages U_m act on the secondary coils of the transformer T of all multipliers, and that the primary coils are joined in parallel and connected to the network circuit).

Let us now set up with the aid of potentiometers AB, in the degree scale, the following voltages on the dividers:

1) on the divider of multiplier No 1: $U_{CD_1} = c_1 y_1$

2) on the divider of multiplier No 2: $U_{CD_2} = c_1 y_2$

....

m) on the divider of multiplier No m: $U_{CD_m} = c_1 y_m$

....

36) on the divider of multiplier No 36 $U_{CD_{36}} = c_1 y_{36}$

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Since the ordinates y can possess also negative values, a switching or reversing device is provided to set up negative values, which (mechanism) varies the voltage of the current in the potentiometer of the multiplier. Figure 3 represents schematically the switching, or reversing, device.

If y has a positive value, switches must be set on plus; for negative y it is on minus.

After all the multipliers are set for the above-mentioned positions (plus or minus), the following voltages will act between the terminals 0 and k of the dividers of the multipliers:

$$
\begin{aligned}
&\text{Multiplier No 1:} \quad U_{k1} = c_1 y_1 / \sin(1\cdot 10k)^\circ / \\
&\text{Multiplier No 2:} \quad U_{k2} = c_1 y_2 / \sin(2\cdot 10k)^\circ / \\
&\cdots\cdots\cdots\cdots\cdots\cdots \\
&\text{Multiplier No i:} \quad U_{ki} = c_1 y_i / \sin(i\cdot 10k)^\circ / \\
&\text{Multiplier No 36:} \quad U_{k36} = c_1 y_{36} / \sin(36\cdot 10k)^\circ / \, .
\end{aligned}
\qquad (6)
$$

If now the multipliers are joined in series with the sign of the sines taken into account (below we shall tell how the sign of the sines is considered), then we can omit the straight lines indicating absolute values in the expressions (6) and the total voltage acting between the origin 0 of the first multiplier and the end (terminal) of the divider of the succeeding multiplier will equal:

$$U_R = c_1 \cdot \sum_{i=1}^{i=36} y_i \cdot \sin(i\cdot 10k)^\circ \qquad (7)$$

Figure 4 shows schematically the hookup of multipliers for a determination of the coefficient of the k-th harmonic of the sine. To each coefficient A and B of the Fourier series will correspond its own hookup scheme.

In order to obtain the voltage in accordance with relation (7) one must connect in series the outputs of all the dividers for just a given number of a harmonic (taking the sign of the sine into account). The magnitude and sign of y_i do not influence at all the nature of the hookup. This fact permits one to make all connections beforehand on the controllers of the apparatus; in order to obtain the proper hookup it is only necessary to turn the controller-type switch to an assigned position.

Declassified in Part - Sanitized Copy Approved for Release 2012/05/30 : CIA-RDP82-00039R000100080006-7

Declassified in Part - Sanitized Copy Approved for Release 2012/05/30 : CIA-RDP82-00039R000100080006-7

In Figure 4, multiplier No 3 is connected in the reverse direction, which is done here since $\sin(3\cdot 10k)^{\circ}$ has a negative value. The sign of the sines and cosines is taken into consideration by this simple method.

Let us divide expression (3) by the magnitude of the resulting voltage U_R (relation 7) obtained between terminals P and Q (Figure 4):

$$\frac{A_k}{U_R} = \frac{1}{18c_1} \quad or \quad A_k = \frac{1}{18c_1} \cdot U_R \qquad (8)$$

Thus in order to determine the coefficient A_k it is necessary to measure the resulting voltage U_R and to multiply it by the coefficient $1/18c_1$.

The coefficients B_k are found in a similar way.

Obviously the hookup of multipliers for each coefficient must be its own. For this purpose a controller-type switch is used.

The determination of the coefficient A_0 assumes a somewhat special place. According to the theory of Fourier theories:

$$A_0 = \frac{1}{l}\int_0^{l} f(x)\,dx \quad \text{or approximately}$$

$$A_0 = \frac{1}{l}\sum_1^{n} f(x_i)\,\Delta x \qquad (8a)$$

In our case $f(x) = f(i\Delta x) = y_i$ and $l = 36\cdot\Delta x$ and Δx = constant; hence:

$$A_0 \approx \frac{1}{36}\sum_1^{36} y_i \quad \text{and since} \quad y_i = \frac{1}{c_i} U_{CD_i} \quad \text{we have}$$

$$A_0 \approx \frac{1}{36c_1}\sum_1^{36} U_{CD_i} \qquad (9)$$

That is, to determine A_0 it is necessary to connect the potentiometers in series and, after measuring the total voltage

$$U_R = \sum_1^{36} U_{CD_i} \qquad (9a)$$

to multiply it by the factor $1/36c_1$.

There is no necessity to use the divider in this case.

Declassified in Part - Sanitized Copy Approved for Release 2012/05/30 : CIA-RDP82-00039R000100080006-7

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3. HARMONIC SYNTHESIS

The basic element of the apparatus for harmonic synthesis is the same multiplier whose circuit scheme is shown in Figure 1.

The difference of this apparatus from the preceding one is that the individual multipliers in harmonic synthesis must correspond to the individual harmonics. Since the number of terms of the Fourier series in this case equals 36 (that is, 18 terms of the form $A_k \cdot \sin kx$ and 18 like $B_k \cdot \cos kx$, without counting A_o), then the total number of multipliers remain the same, equal to 36. The taps on the divider of a multiplier must correspond to definite ordinates.

In order to clarify the action of the apparatus let us assume that the multiplier represented in Figure 1 corresponds to the i-th harmonic of a sine. In this case the multiplier's divider must have taps that divide the applied voltage U_{CD} in the following manner:

$$\begin{aligned} &1)\ \text{voltage between point 0 and 1:} && U_{i1} = U_{CD}/\sin(1\cdot 10i)^{\circ}/ \\ &2)\ \text{voltage between point 0 and 2:} && U_{i2} = U_{CD}/\sin(2\cdot 10i)^{\circ}/ \\ &\quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots && \\ &m)\ \text{voltage between point 0 and m:} && U_{im} = U_{CD}/\sin(m\cdot 10i)^{\circ}/ \\ &18)\ \text{voltage between point 0 and 18:} && U_{i18} = U_{CD}/\sin(18\cdot 10i)^{\circ}/ \end{aligned} \qquad (9b)$$

It is evident from this table that the taps (outlets or outputs) on the divider of the i-th multiplier for synthesis must be the same as on the divider of a multiplier for analysis for the i-th ordinate.

The number of taps, or outlets, must equal 18 (for 18 sine harmonics); however, it is possible in this case to show that many outlets (outputs) will coincide among themselves and that the maximum number of non-coincident output equals 10.

Since the total number of terms in the Fourier series equals 36, the apparatus must have 36 multipliers; that is, the same number as for analysis.

An identical number of multipliers and identical outputs on the dividers make it possible to use one and the same apparatus for both analysis and synthesis. However, the circuit scheme of series-connection of multipliers for

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synthesis will differ from the circuit used in analysis. This fact compells us to have, in synthesis, a separate controller that has to be in the off position during operation with the controller for analysis, and vice versa.

The dividers for cosine harmonics do not differ at all from the dividers of sines, if we exclude the numbering of the outlet taps.

In harmonic synthesis the coefficients for the harmonics in the Fourier series must be given:

$$A_1 \quad A_2 \ldots A_{18} \quad ; \quad B_1 \quad B_2 \ldots B_{18} \quad . \qquad (9c)$$

Let the coefficient or factor, Am have the greatest value. Knowing the magnitude of the voltage on the terminals of the secondary coils of the multipliers' transformers (namely, Um) we can determine the constant of proportionality from the relation:

$$Um = c_2 Am, \text{ or } c_2 = Um/Am; \qquad (10)$$

then we set up with the aid of the multipliers the following voltages on the dividers:

1) on divider No 1: $U_{CD_1} = c_2 A_1$
2) on divider No 2: $U_{CD_2} = c_2 A_2$

....

18) on divider No 18: $U_{CD_{18}} = c_2 A_{18}$

19) on divider No 19: $U_{CD_{19}} = c_2 B_1$
20) on divider No 20: $U_{CD_{20}} = c_2 B_2$

....

36) on divider No 36: $U_{CD_{36}} = c_2 B_{18}$

Now let us assume that we need to obtain the value of the resulting curve at a point corresponding to the k-th ordinate. To do this let us connect in series all the k-th outputs (outlet taps) of the multipliers' dividers, taking into consideration the signs of the cosines and sines (namely, $\cos(k\cdot 10i)^\circ$ and $\sin(k\cdot 10i)^\circ$) according to a circuit scheme like that represented in Figure 4. The total voltage between the terminal clamps P and Q will be:

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Declassified in Part - Sanitized Copy Approved for Release 2012/05/30 : CIA-RDP82-00039R000100080006-7

$$U_{R_k} = \sum_{i=1}^{i=18} U_{CD_i} \cdot \sin(k \cdot 10 i)^\circ + \sum_{i=19}^{i=36} U_{CD_i} \cdot \cos[k \cdot 10(i-18)]^\circ \qquad (11)$$

or, by replacing U_{CD_i}, we obtain:

$$U_{R_k} = c_2 \left\{ \sum_{i=1}^{i=18} A_i \cdot \sin(k \cdot 10 i)^\circ + \sum_{i=19}^{i=36} B_{(i-18)} \cos[k \cdot 10(i-18)]^\circ \right\}. \qquad (11a)$$

The ordinate of curve f(x) corresponding to the abscissa $(360/36) \cdot k = (10k)^\circ$ is expressed, as we know, by the following dependence:

$$Y_k = \sum_{i=1}^{i=18} A_i \cdot \sin(k \cdot 10 i)^\circ + \sum_{i=19}^{i=36} B_{(i-18)} \cos[k \cdot 10(i-18)]^\circ. \qquad (12)$$

Comparing relations (11) and (12), we find:

$$Y_k = (1/c_2) \cdot U_{R_k} \qquad (13)$$

Connecting in series the various outputs of the multipliers, we can obtain by immediate measurements the voltages U_{R_k}, which are proportional to all the ordinates of the resulting curve.

4. THE APPLICATION OF THE CONTROLLERS

The apparatus for harmonic analysis and synthesis which is constructed on the above-discussed principle can perform even without the controllers which are used to obtain the coefficients of the various harmonics or in the synthesis-ordinate case.

To do this, the same number of individual dividers as the number of harmonics desired to be obtained must be connected alternately (by turns) to each secondary coil of a transformer. In this case, from each divider one must effect in toto one output which would yield a part of the voltage supplied (voltage) to the divider which (part) is proportional to the sine or cosine of the corresponding argument.

Declassified in Part - Sanitized Copy Approved for Release 2012/05/30 : CIA-RDP82-00039R000100080006-7

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The outputs of one number of a harmonic are connected, in this case, in a fixed manner according to the circuit schemes corresponding to various positions of the controllers. Here the number of dividers employed is increased considerably; however, for a small number of harmonics such a way of employing the apparatus can prove preferable.

5. MEASUREMENTS

In order to measure resulting voltages that give the value of the coefficients in a Fourier series (or the ordinates of a curve in synthesis), it is best of all to employ the compensation method, which reduces considerably errors of measurements.

The measuring potentiometer can be so graduated that the values of the coefficients in the series or the magnitudes of the ordinates are read off directly from a scale.

A magneto-electric galvanometer which is connected in series with a vibrating rectifier can serve as the zero indicator.

Since the apparatus and measuring device are fed from one and the same network and the results of measurements are obtained in relative quantities, the measured values do not depend upon the oscillations of the voltage from the feed circuit.

The general form of the apparatus, which was made in the Laboratory of Electrical Modeling in the Power Institute (Academy of Sciences USSR), is shown in Figure 5.

The apparatus is connected, by a flexible cord and a four-pole Y-connection to the measuring device (Figure 6), which is connected in its turn to a 110-120 volt alternating-current source.

Work with the apparatus is conducted thus: the outlet taps (outputs) of 36 potentiometers are worked out on the upper 'composing' panel of the apparatus, by proper selections; each potentiometer corresponds to a definite ordinate (during analysis) or to a definite coefficient (during synthesis). With the aid of flexible cords, the potentiometers are connected to the voltage dividers. A voltage proportional to the ordinate (or to the coefficient in the

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case of synthesis) is established at each potentiometer with an accuracy up to 1% of maximum.

The arrangement of the apparatus is set up by merely joining the connecting cords to the proper corresponding sockets, which is simply done. Thus is the entire curve to be analyzed 'composed', or set up. After selection, or 'composition', the switch of the apparatus k is set in the proper hole and turned 90° to one or the other side. Switching on of the controller is effected by the turned switch.

To each coefficient (in analysis) and to each ordinate (in synthesis) there corresponds a definite hole for the switch and a definite position of the switch in this hole. After turning the switch, the magnitude of the coefficient or ordinate is read off immediately from the scale of the measuring device.

6. TEST RESULTS

Several tentative analyses and syntheses of curves whose expansion and form were already known were carried out in order to test the apparatus. The values of the coefficients in the Fourier series and also of the ordinates of the whole curve are shown for one of the tests in Tables 1 and 2.

The following function is taken in these tables:

$$f(x) = \sin x + 0.8\sin 3x + 0.5\sin 5x + 0.2\sin 7x \qquad (14)$$

The values of the ordinates were calculated from this function at 10° intervals; and then the curve was selected, or composed, by picking of the proper wires on the panel of the analyzer in accordance with the table just obtained.

Table 1 shows the values of the coefficients that were obtained on the analyzer, in order to compare the series with the original values of the coefficients as computed by us; as is evident, the error nowhere exceeds 0.5% of maximum.

Table 2 shows the values of the ordinates of the same curve for 10° intervals as a) computed from the series and as b) obtained from the apparatus (that is, a test of the apparatus' operation during synthesis). Here too the error does not exceed 0.5% in magnitude. -- Submitted 12 Sep 1945, Power Institute imeni G. M. Krzhizhanovskiy, Acad Sci USSR. [Figures and Tables are appended in the annex.]

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Declassified in Part - Sanitized Copy Approved for Release 2012/05/30 : CIA-RDP82-00039R000100080006-7

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Table 1. The Coefficients in the Fourier Series (Analysis)

Coefficient	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}
Original	0	1	0	0.8	0	0.5	0	0.2	0	0	0	0
Measured	0	1.005	0.002	0.804	0.003	0.499	0.003	0.199	0.00	0.001	0.003	0.000
Error,%	0	0.5	0.2	0.4	0.3	0.1	0.3	0.01	0.0	0.1	0.3	0.00

(NOTE: The values of all B_k : $k = 1,2,3 \ldots 18$ and A_k for $k = 12, 13, \ldots 18$ were obtained equal to 0.)

Table 2. The Ordinates of the 'Summed' Curve (Synthesis) for the First Fourth of the Period

No of the Ordinate	1	2	3	4	5	6	7	8	9
x°	10°	20°	30°	40°	50°	60°	70°	80°	90°
y calculated	1.144	1.658	1.450	0.970	0.662	0.606	0.606	0.569	0.500
y measured	1.150	1.670	1.443	0.965	0.659	0.608	0.608	0.569	0.505

Annex-A

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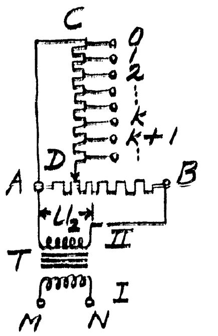
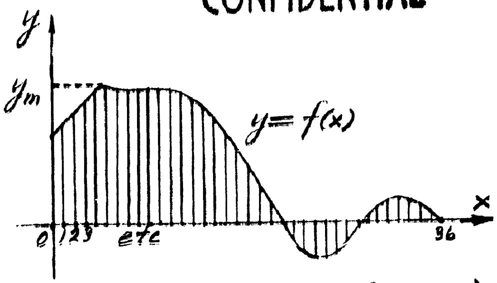


Figure 1.



(36 intervals, 10° apart)

Figure 2.

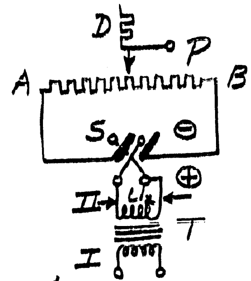
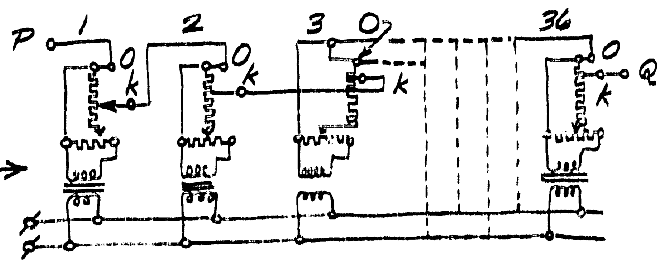


Figure 3.

Figure 4.



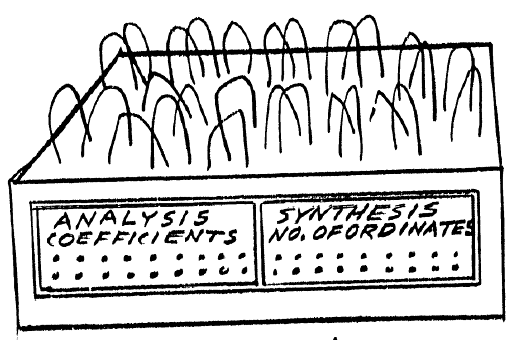


Figure 5

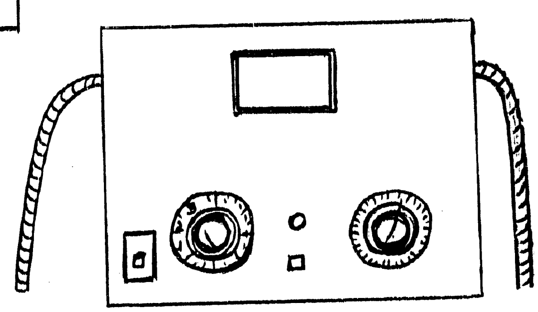


Figure 6

Annex-B

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