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Title: CHERENKOV'S EFFECT AND THE COMPLEX DOPPLER EFFECT (USSR)

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Source: Doklady Akademii Nauk SSSR, Vol LXXIII, No 1, (pp 55-59)
Russian periodical, (every two weeks), Ju~~ne~~ly 1950

CONFIDENTIAL

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CHERENKOV'S EFFECT AND THE COMPLEX DOPPLER EFFECT

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There are many complex waveguides in which conditions are such that the phase velocity of wave propagation is less than the velocity of light in a vacuum -- these conditions, namely, being created by the special disposition of metallic partitions without the use of dielectrics. When a charged particle moves uniformly along such a system, ~~the~~ and its velocity ~~of which (i.e., particle)~~ exceeds the phase velocity of wave propagation, just as when a charged particle moves in a dielectric, then electromagnetic waves are radiated (Cherenkov's effect).

This radiation can be simply determined in the case of linear periodic structures which are a series of identical compartments interconnected with each other by apertures through which a charged particle moves.

We shall seek the transverse part of a vector potential $\vec{A}$ in the following form:

$$\vec{A} = \sum_{\lambda} q_{\lambda}(t) \cdot \vec{A}_{\lambda}(\vec{r})$$

where $\vec{A}_{\lambda}(\vec{r}) \cdot \exp(i\omega_{\lambda} t)$ represents the various waves which are able to be propagated in the periodic structure in the absence of a charge (NOTE: $\lambda \equiv \kappa, s$ is the set of continuous (κ) and discrete (s) parameters characterizing the wave); q_{λ} are certain functions of time. The spatial function $\vec{A}_{\lambda}(\vec{r})$ can be represented in the following form:

$$\vec{A}_{\lambda}(\vec{r}) = \vec{a}_{\lambda}(\vec{r}) \cdot exp(i\kappa x), \quad -\frac{\pi}{l} \leq \kappa \leq \frac{\pi}{l}$$

where $\vec{a}_{\lambda}$ is a periodic function of the x coordinate (along which the compartments are arranged), the function being normalized according to the following condition:

$$\int_{V_1} |\vec{a}_\lambda|^2 dv = 4\pi c^2$$

(V_1 is the volume of one compartment and ℓ is the period of the structure).

The function $q_\lambda(t)$ satisfies the equation of an oscillator of frequency ω_λ, which (i.e., oscillator) is under the action of a forcing force:

$$\ddot{q}_\lambda + \omega_\lambda^2 q_\lambda = f_\lambda(t) \tag{1}$$

where

$$f_\lambda = \frac{1}{CN} \int \vec{j}\, \vec{A}^* \cdot dv$$

($\vec{j}$ is the current density connected with the particle; N is the number of compartments; and V_n is their volume).

The energy of the electromagnetic field equals:

$$H(t) = \frac{N}{2} \sum_\lambda (\dot{q}_\lambda^2 + \omega_\lambda^2 q_\lambda^2)$$

If radiation occurs, then H is proportional to t for $t \to \infty$. Such an asymptotic behavior holds true only for resonance between (a) the eigenoscillations (that is, the characteristic or proper oscillations) q_λ of the oscillators and (b) the forcing force $f_\lambda(t)$ (NOTE: see 1, 2, and 3 in the Bibliography).

When a charge e moves uniformly with velocity v along the x-axis, then equation (1) above assumes the following form:

$$\ddot{q}_\lambda + \omega_\lambda^2 q_\lambda = \frac{ev}{c} \sum_{n=-\infty}^{\infty} b_\lambda^{(n)*} \cdot \exp\left[-i\left(\kappa + \frac{2\pi n}{\ell}\right)\right] \tag{2}$$

(where $b_\lambda^{(n)} = \frac{1}{\ell} \int_0^\ell \vec{e}\, \vec{a}_\lambda(x,0,0) \cdot e^{-\frac{2\pi i n x}{\ell}} \cdot dx$ and e is the unit vector of polarization); namely, along the x-axis the value of the current density $\vec{j}$ is:

$$j_x = ev \cdot \delta(x - vt), \quad j_y = j_z = 0.$$

The force is characterized by the spectrum of frequencies:

$$\Omega_{\kappa n} = \left(\kappa + \frac{2\pi n}{\ell}\right) v$$

The condition of resonance, which (i.e., resonance) is the condition governing radiation, has the following form:

$$\omega_\lambda = \omega_{\kappa s} = \Omega_{\kappa n} = \left(\kappa + \frac{2\pi n}{l}\right) v . \qquad (3)$$

In an unbounded dielectric we have:

$\omega = kc/\sqrt{\varepsilon}$ (k is the wave vector; ε is the dielectric constant),

$\Omega_{\kappa n} = kv \cdot \cos\theta$ (θ is the angle between the direction of wave propagation and the velocity of the particle),

$n = 0$, and

equation (3) leads to the well-known condition that governs the possibility of the Cherenkov effect:

$$\cos\theta = \frac{c}{v\sqrt{\varepsilon}} \leqslant 1 .$$

In the case of periodical structures the condition (3) can be effected, generally speaking, for various combinations of the quantities s, n, κ.

In an ordinary waveguide filled with a dielectric, we have

$$\omega_\lambda = \sqrt{\omega_0^2 + u^2\kappa^2}$$

where ω_0 is the limiting frequency and $u = c/\sqrt{\varepsilon}$. The condition governing the Cherenkov effect (3) leads to the relation $\omega_\lambda \equiv \sqrt{\omega_0^2 + u^2\kappa^2} = v\kappa$; hence it follows that the frequency that can be radiated equals:

$$\omega_\lambda = \omega_0 / \sqrt{1 - u^2/c^2} .$$

This relation determines the discrete spectrum of frequencies, since u is a certain function of the frequency $u = u(\omega)$.

The general formula for the intensity I of radiation has the following form:

$$I = \lim_{t\to\infty} \frac{H(t)}{t} = \frac{e^2 v^2 l}{4c^2} \cdot \left\{ \sum_{n,\lambda'_j} \frac{|b_\lambda^{(n)}|^2}{\left|\frac{d\omega}{d\kappa} - v\right|_{\lambda=\lambda'_j}} + \sum_{n,\lambda''_j} \frac{|b_\lambda^{(n)}|^2}{\left|\frac{d\omega}{d\kappa} + v\right|_{\lambda=\lambda''_j}} \right\} \qquad (4)$$

where λ'_j is the set of quantities (κ,s) satisfying the equation $\omega_\lambda - \Omega_{\kappa n} = 0$, and λ''_j is the set of quantities satisfying the equation $\omega_\lambda + \Omega_{\kappa n} = 0$.

In the case of a cylindrical waveguide this formula leads to the following result:

$$I = \frac{2e^2 v}{R^2} \sum_s \frac{1}{J_1^2(\mu_s)} , \qquad (5)$$

where R is the radius of the waveguide, J_1 is Bessel's function, mu μ_s is the roots of the Bessel function J_0; the summation is carried out over those values of s for which we have $u(\omega_{\kappa s}) \leq v$. Here the radiated frequencies $\omega_{\kappa s}$ are determined from the relation $\omega_{\kappa s} \equiv u\sqrt{\kappa^2 + \mu_s^2/R^2} = \kappa v$ (With increase in s the frequency increases, but the phase velocity $u(\omega_{\kappa s})$ tends toward c; therefore, starting from a certain s, the condition $u < c$ ceases to be fulfilled).

If the oscillator moves along the x-axis with a velocity v, ~~the~~ and its eigenfrequencies and moment ~~of which (i.e., oscillator)~~ equal ω_0' and d' respectively, then we have:

$$j_x = \omega_0 d \cdot \cos\omega t \cdot \delta(x - vt) , \quad j_y = j_z = 0 ,$$

where $d = d' \cdot \sqrt{1 - v^2/c^2}$ and $\omega = \omega_0' \cdot \sqrt{1 - v^2/c^2}$.

The condition governing radiation, that is the condition of resonance, has now the form:

$$\omega_\lambda = v\left(\kappa + \frac{2\pi n}{l}\right) \pm \omega_0 . \qquad (6)$$

This relation determines the Doppler effect in periodic structures. For given v, ω_0, s we obtain from (6) a series of discrete values of kappa κ to which correspond definite discrete frequencies of the radiation.

When the oscillator moves in an unbounded dielectric, condition (6) assumes the following form:

$$\omega_\lambda \equiv \omega = ck/\sqrt{\varepsilon} = kv \cdot \cos\theta \pm \omega_0 ;$$

hence the well-known formulas (see 4 in the Bibliography) for the complex Doppler effect immediately follow:

$$\omega = \omega_0 / \left[1 - \frac{v\sqrt{\varepsilon}}{c} \cdot \cos\theta\right], \quad \frac{v\sqrt{\varepsilon}}{c} \cdot \cos\theta < 1;$$
$$\omega = \omega_0 / \left[\frac{v\sqrt{\varepsilon}}{c} \cdot \cos\theta - 1\right], \quad \frac{v\sqrt{\varepsilon}}{c} \cdot \cos\theta > 1. \tag{7}$$

(NOTE: In (7) epsilon ε is a function of ω : $\varepsilon = \varepsilon(\omega)$).

The intensity I of radiation is determined by the following general formula:

$$I = \frac{\omega_0^2 d^2 l}{16 c^2} \left\{ \sum_{n, \lambda_j'} \frac{|b_\lambda^{(n)}|^2}{\left| \frac{d\omega}{dk}\lambda - v \right|_{\lambda = \lambda_j'}} + \sum_{n, \lambda_j''} \frac{|b_\lambda^{(n)}|^2}{\left| \frac{d\omega}{dk} + v \right|_{\lambda = \lambda_j''}} \right\}, \tag{8}$$

where λ_j' is the set of quantities (κ,s) determined from the equations $\omega_\lambda = v(\kappa + 2\pi n/l) \pm \omega_0$ and λ_j'' is the set of quantities (κ,s) defined by the equation $\omega_\lambda = -v(\kappa + \frac{2\pi n}{l}) \pm \omega_0$.

Submitted 18 March 1950

Submitted 2 May 1950,

by Acad S. I. Vavilov

BIBLIOGRAPHY

1. W. Heitler, Quantum Theory of Radiation, Moscow 1940.
2. V. Ginzburg, DAN SSSR, Vol 56, p 699 (1947).
3. A. Bohr, Kgl. Danske Vidensk Selskab, Mat-fys Medd., 24, 19 (1948).
4. I. Frank, IAN SSSR, ser fiz; Vol 6,2 (1942).

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