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CHAPTERS XI - XIX

FROM A BOOK ON

DYNAMIC METEOROLOGY

V. A. BELINSKIY

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CHAPTERS

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CHAPTER XI

VARIATIONS IN GEOSTROPHIC WIND WITH HEIGHT

Section 1. Basic Equations

The geostrophic wind equations show that, due to altitude variations in atmospheric density and in the pressure gradient force, the velocity and direction of the geostrophic wind are functions of altitude. This conclusion remains true even when the horizontal component of wind velocity is constant.

In the following paragraphs the regularities, to which the variations of geostrophic wind with height are subject will be studied.

Assuming that $l = 2\omega \sin\varphi$, we write the geostrophic wind equation as follows:

$$u = -\frac{1}{l\rho}\frac{\partial p}{\partial y}; \quad v = \frac{1}{l\rho}\frac{\partial p}{\partial x}; \quad -\frac{1}{\rho}\frac{\partial p}{\partial z} - g = 0. \qquad (1)$$

Eliminating the density with the aid of the equation of state we obtain:

$$\frac{u}{T} = -\frac{R}{l}\frac{\partial \ln p}{\partial y}; \quad \frac{v}{T} = \frac{R}{l}\frac{\partial \ln p}{\partial x}; \quad \frac{g}{T} = -R\frac{\partial \ln p}{\partial z} \qquad (2)$$

Differentiating the first two equations (2) by z, and the last one by y and x, respectively, and eliminating the compound derivatives, we find:

$$\frac{\partial}{\partial z}\left(\frac{u}{T}\right) = \frac{1}{l}\frac{\partial}{\partial y}\left(\frac{g}{T}\right); \quad \frac{\partial}{\partial z}\left(\frac{v}{T}\right) = -\frac{1}{l}\frac{\partial}{\partial x}\left(\frac{g}{T}\right). \qquad (3)$$

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Completing the differentiation and reducing by T, we obtain the precise formulas, which determine the variations of the velocity components of the geostrophic wind with changes in altitude.

$$\frac{\partial u}{\partial z} = \frac{u}{T}\frac{\partial T}{\partial z} - \frac{g}{fT}\frac{\partial T}{\partial y},$$
$$\frac{\partial v}{\partial z} = \frac{v}{T}\frac{\partial T}{\partial z} + \frac{g}{fT}\frac{\partial T}{\partial x}. \qquad (4)$$

Let us ascertain the condition, under which the geostrophic wind does not change with height, that is, under what condition $\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = 0$. . Eliminating from (4) u, v, g, as per (1), we obtain:

$$\frac{\partial u}{\partial z} = -\frac{R}{fp}\left(\frac{\partial p}{\partial y}\cdot\frac{\partial T}{\partial z} - \frac{\partial p}{\partial z}\cdot\frac{\partial T}{\partial y}\right),$$
$$\frac{\partial v}{\partial z} = \frac{R}{fp}\left(\frac{\partial p}{\partial x}\cdot\frac{\partial T}{\partial z} - \frac{\partial v}{\partial z}\cdot\frac{\partial T}{\partial x}\right). \qquad (5)$$

It follows from (5) that, in the absence of change in the geostrophic wind with height the following condition must be satisfied:

$$\frac{\partial p}{\partial x} : \frac{\partial p}{\partial y} : \frac{\partial p}{\partial z} = \frac{\partial T}{\partial x} : \frac{\partial T}{\partial y} : \frac{\partial T}{\partial z}, \qquad (6)$$

This means that the isobaric surfaces must, at the same time, be isothermic surfaces, and therefore, also isosteric surfaces, i. e. the air must be barotropic. If this condition does not exist, the velocity of the geostrophic wind will vary with height. Thus, the variation of the geostrophic wind with height is a result of the baro-

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clinicity of the atmosphere.

Now, let us integrate (3) within the limits from Z_0 to Z.

$$\left.\begin{aligned} u &= \frac{T}{T_0} u_0 - \frac{gT}{l} \int_{z_0}^{z} \frac{1}{T^2} \frac{\partial T}{\partial y} dz, \\ v &= \frac{T}{T_0} v_0 + \frac{gT}{l} \int_{z_0}^{z} \frac{1}{T^2} \frac{\partial T}{\partial x} dz. \end{aligned}\right\} \qquad (7)$$

Equations (7) show that the geostrophic wind at a given atmospheric level can be considered as consisting of two components: (a) a wind conditioned upon the presence of a pressure gradient force at the lower level, and equal to the geostrophic wind at the same level, changed in the ratio $\frac{T}{T_0}$, and (b) a wind conditioned upon the horizontal temperature gradient in the contemplated layer. The latter is usually called the "thermic" wind. We will designate it by $\Delta c_T (\Delta u_T, \Delta v_T)$. We will call the first component, by analogy, the "baric" wind, which we will designate $c_B (u_B, v_B)$. It is obvious that

$$c = c_B + \Delta c_T; \quad u = u_B + \Delta u_T; \quad v = v_B + \Delta v_T \qquad (8)$$

Since we consider the wind as geostrophic, its variations must reflect the variations, with altitude, of the pressure gradient force. Substituting in (7) for u, v, values in accordance with condition (1), we arrive at the following correlationships, which determine the variation in the components of the pressure gradient force, with a change in altitude:

$$\left.\begin{aligned} -\frac{\partial p}{\partial x} &= -\frac{p}{p_0}\left(\frac{\partial p}{\partial x}\right)_0 - gp\int_{z_0}^{z} \frac{1}{T^2} \frac{\partial T}{\partial x} dz, \\ -\frac{\partial p}{\partial y} &= -\frac{p}{p_0}\left(\frac{\partial p}{\partial y}\right)_0 - gp\int_{z_0}^{z} \frac{1}{T^2} \frac{\partial T}{\partial y} dz. \end{aligned}\right\} \qquad (9)$$

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Equations (9) show that the pressure gradient force at a given upper level can also be considered as consisting of two components: (a) a pressure gradient force conditioned upon the pressure distribution at the lower level, and (b) a pressure gradient force conditioned upon the temperature distribution in the given atmospheric layer.

By analogy with the "thermic" and "baric" winds, we will call the last item the "thermic" component of the pressure gradient force, and will designate it by $\Delta G_T (\Delta G_{Tx}, \Delta G_{Ty})$, and the first item -- the "baric" component, which we will designate $G_B (G_{Bx}, G_{By})$. Obviously:

$$G = G_B + \Delta G_T; \quad G_x = G_{Bx} + \Delta G_{Tx}; \quad G_y = G_{By} + \Delta G_{Ty} \qquad (10)$$

where $G(G_x, G_y)$ is the pressure gradient force at the upper level.

Supposing first, that in the atmospheric layer, within which the variations of the geostrophic wind with height are being considered, the horizontal temperature gradient is absent,

$$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial y} = 0,$$

that is, we will first contemplate the variation of the "baric" wind with altitude.

As per (7) and (8), the velocity of the "baric" wind is determined by:

$$u_B = \frac{T}{T_0} u_0; \quad v_B = \frac{T}{T_0} v_0, \qquad (11)$$

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from where:

$$c_B = \frac{T}{T_0} c_0, \quad \tan\theta_B = \tan\theta, \tag{12}$$

that is:

$$c_B = \frac{T}{T_0} c. \tag{13}$$

This means, that the direction of the "baric" wind (determined by the angle θ_B, which is formed by the wind with the x-axis) does not change with altitude, coinciding all the time with the direction of the geostrophic wind at the lower level, but the only thing that changes is the absolute value of velocity.

Assuming that $\Delta c_B = c_B - c_0$, $\Delta T = T - T_0 = -\gamma \Delta z$ where $\gamma = -\frac{\partial T}{\partial z}$ - the vertical temperature gradient, we obtain:

$$\frac{\Delta c_B}{\Delta z} = -\gamma \cdot \frac{c_0}{T_0}. \tag{14}$$

Thus, with the reduction of temperature with altitude ($\gamma > 0$), the velocity of the "baric" wind is diminished ($\Delta c_B < 0$); in the isothermic layer ($\gamma = 0$) it remains unchanged ($\Delta c_B = 0$); and in the inversion layer ($\gamma < 0$) it is increased. However, the absolute value of the variation of the "baric" wind with altitude, is very small. Specifically, the variation in the velocity of the "baric" wind, within an atmospheric layer of 1 kilometer, even at the tremendous velocities of 50 meters per second, does not exceed 1 - 2 meters per second. This is within the precision limits of the existing methods of gauging wind velocity in the free atmosphere. Therefore, the variation in the velocity of the "baric" wind, can be practically completely overlooked, and the "baric" wind considered as having, at

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all altitudes, a velocity and direction equal to the same of the geostrophic wind at the lower level $c_B = c_0$, and therefore:

$$c = c_0 + \Delta c_T \tag{15}$$

Let us consider now the variation, with altitude, of the baric component of the pressure gradient force. As per (9) and (10), the baric component is determined by:

$$G_{Bx} = \frac{p}{p_0} G_{0x}, \quad G_{By} = \frac{p}{p_0} G_{0y}, \tag{16}$$

from where:

$$G_B = \frac{p}{p_0} G_0, \quad \tan \psi_B = \tan \psi_0, \tag{17}$$

where ψ_B and ψ_0 are angles formed with the x - axis by vectors G_B and G_0, respectively. Therefore,

$$G_B = \frac{p}{p_0} G_0. \tag{18}$$

This means that the baric component of the gradient does not change its direction with altitude, but changes only its absolute value.

Assuming that $\Delta G_B = G_B - G_0$, and $\Delta p = p - p_0$, we obtain from (17):

$$\frac{\Delta G_B}{G_B} = \frac{\Delta p}{p} \tag{19}$$

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or, utilizing the basic static equation $\Delta p = -g\rho_m \Delta z$, and the equation of condition $p = \rho R T$, and assuming, by approximation, $\frac{\rho_m}{\rho_0} \cong 1$, we find that:

$$\frac{\Delta \mathfrak{G}_B}{\Delta z} = -\frac{g}{RT}\mathfrak{G}_0. \tag{20}$$

Thus, the value of the baric component of the gradient, is always decreased with altitude.

The variation in the barometric component of the pressure gradient force may attain a considerable magnitude, therefore, it cannot be overlooked. Furthermore, in the absence of a horizontal temperature gradient, the isobar, with increase in altitude, will gradually diverge even though the inclination of the isobaric surfaces will vary very little. This follows from the correlationship

$$\tan\beta = \frac{l}{g}c = \frac{l}{g}\frac{T}{T_0}c = \frac{T}{T_0}\tan\beta_0. \tag{21}$$

Section 2. "Thermic" Wind and Temperature Gradient.

Under the preceding title, we have shown, that in the absence of a horizontal temperature gradient, the magnitude of the geostrophic wind varies very little with height and its direction varies not at all.

Let us now clarify in greater detail the effect of the horizontal temperature gradient upon the variation of the geostrophic wind

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with height. Assuming that the pressure gradient force at the lower level is absent, i. e. that

$$\left(\frac{\partial p}{\partial x}\right)_0 = \left(\frac{\partial p}{\partial y}\right)_0 = 0$$

and, therefore,

$$u_0 = v_0 = 0.$$

Then, the variation in the geostrophic wind and the pressure gradient force, will be determined by the horizontal temperature gradient only. By analogy with the "thermic" wind, we will call the horizontal temperature gradient the "thermic" gradient. Designating the thermic wind vector by Δc_T, and its components by Δu_T, Δv_T, we have:

$$\Delta u_T = -\frac{gT}{l}\int_{z_0}^{z}\frac{1}{T^2}\frac{\partial T}{\partial y}\,dz,$$

$$\Delta v_T = \frac{gT}{l}\int_{z_0}^{z}\frac{1}{T^2}\frac{\partial T}{\partial x}\,dz. \qquad (1)$$

Due to erratic variations of temperature with altitude, the integrals in the right-hand part of equation (1) cannot be computed by their aggregate values, and we will limit ourselves to their average values

$$\int_{z_0}^{z}\frac{1}{T^2}\frac{\partial T}{\partial y}\,dz = \frac{1}{T_m^2}\left(\frac{\partial T}{\partial y}\right)_m \Delta z,$$

$$\int_{z_0}^{z}\frac{1}{T^2}\frac{\partial T}{\partial x}\,dz = \frac{1}{T_m^2}\left(\frac{\partial T}{\partial x}\right)_m \Delta z, \qquad (2)$$

where T_m, in the first approximation, as the arithmetical mean temperature of the layer; $-\left(\frac{\partial T}{\partial x}\right)_m$, $-\left(\frac{\partial T}{\partial y}\right)_m$ as the arithmetical mean of the "thermic" gradient components; Δz is the thickness of the atmospheric layer. Designating the vector of the mean "thermic"

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gradient of the layer by Γ, and its components by:

$$\Gamma_x = -\left(\frac{\partial T}{\partial x}\right)_m \text{ and } \Gamma_y = -\left(\frac{\partial T}{\partial y}\right)_m :$$

Substituting these values into equation (1) and reducing by the approximately equal in value T_m and T, we obtain (leaving out index m):

$$\Delta u_T = \frac{g\Delta z}{lT} \cdot \Gamma_y ,$$

$$\Delta v_T = -\frac{g\Delta z}{lT} \cdot \Gamma_x . \qquad (3)$$

The absolute value of the "thermic" wind velocity will be:

$$\Delta c_T = \frac{g\Delta z}{lT} \Gamma , \qquad (4)$$

and its direction will be determined as follows (Study Figure 97 in original text), where the x - axis is along the isotherm, and the y - axis in the direction of the diminution of temperature.

Figure 97. The "thermic" wind is in the direction of the isotherm, with the cold area remaining on the left in the northern hemisphere (a), and on the right in the southern hemisphere(b).

Then $\Gamma_x = 0$, and, consequently, $\Delta v_T = 0$. In the northern hemisphere, when $\varphi > 0$ and $\Gamma > 0$, $\Delta u > 0$. Thus, we arrive at the

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TABLE 41

VARIATIONS OF THERMIC WIND WITH ALTITUDE

Γ \ φ	10°	20°	30°	40°	50°	60°	70°	80°	90°
1 degree per 100 kilometers	14.4	7.3	5.0	3.9	3.3	2.9	2.7	2.6	2.5
2 degrees per 100 kilometers	28.8	14.6	10.0	7.8	6.6	5.8	5.4	5.2	5.10
3 degrees per 100 kilometers	43.2	21.9	15.0	11.7	9.9	8.7	8.1	7.8	7.5
5 degrees per 100 kilometers	72.2	36.7	25.0	19.5	16.3	14.5	13.3	12.7	12.5

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differentials, when $\Gamma < 1$ degree per $\frac{100}{\Delta}$ kilometers, can the value of the "thermic" wind be overlooked (in the formula), and this for high latitudes only. In the low latitudes, and, in the presence of higher temperature gradients, even in the high latitudes, the value of the "thermic" wind is not to be overlooked, since in this case, its value is of the same order of magnitude, as the velocity of the geostrophic wind.

As per expression (9) in Section 1 and expression (2) in this section, the thermal component of the pressure gradient force is determined by equations

$$\Delta G_{T_k} = \frac{g\Delta z}{RT_m} \cdot \frac{p}{T_m} \Gamma_k ,$$
$$\Delta G_{T_y} = \frac{g\Delta z}{RT_m} \cdot \frac{p}{T_m} \Gamma_y . \qquad (6)$$

Equations (6) show, that the thermal components of the pressure gradient force ΔG_T is directed along the temperature gradient Γ, and its absolute value is in direct ratio to the absolute value of the temperature gradient

$$\Delta G_T = \frac{g\Delta z}{RT_m} \cdot \frac{p}{T_m} \Gamma . \qquad (7)$$

Expressing ΔG_T in millibars per 100 kilometers, Γ in degrees per 100 kilometers, Δz in kilometers, and assuming that g = 10 meters per second, T = 273 degrees Centigrade, R = 287 M^2 Sec^{-2} $degrees^{-1}$, we obtain the following values for the variation of the "thermic" component of the pressure gradient force within the 1 - kilometer atmospheric layer, as depicted below in Table 42, and computed from the equation

$$\frac{\Delta G_T}{\Delta z} = 0.00046 \cdot p \cdot \Gamma ,$$

[See page 13 for Table 42]

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TABLE 42

VARIATIONS IN THE THERMAL COMPONENT OF THE PRESSURE GRADIENT FORCE, WITH ALTITUDE

Γ \ p millibars	1000	900	700	500	300	200	100 millibars
1 degree per 100 kilometers	0.46	0.41	0.32	0.23	0.14	0.09	0.05
2 degrees per 100 kilometers	0.9	0.8	0.6	0.5	0.3	0.2	0.1
3 degrees per 100 kilometers	1.4	1.2	0.9	0.7	0.4	0.3	0.1
5 degrees per 100 kilometers	2.3	2.0	1.6	1.1	0.7	0.5	0.2

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With increase in altitude, the thermal component of the pressure gradient force is ever more increasing, in conformance with which there is a steeper inclination of the isobaric surface, as determined by equation

$$\tan\beta = \frac{\Gamma}{T}\Delta z. \qquad (8)$$

It follows from (8) that the isobaric surfaces, with reference to the horizon, will always have a considerable smaller angle of inclination, than the corresponding isothermic surfaces. These angles of inclination satisfy the equation

$$\frac{\tan\beta}{\tan\beta_T} = \frac{\Delta T}{T} \qquad (9)$$

Where β_T is the angle of inclination of the isothermic surface, and ΔT is the temperature differential at the boundaries of the contemplated atmospheric layer.

Section 3. Variation in Geostrophic Wind with Height in Relation to the Mutual Disposition of the Pressure Gradient Force and the Thermal Gradient.

In the presence of the pressure gradient force and the thermal gradient, the variations of the geostrophic wind with height may be diverse, with relation to the mutual disposition of the above gradients, or in other words, with relation to the mutual disposition of the isobars and isotherms.

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As pointed out above, the velocity vector of the geostrophic wind at an altitude, can be considered as a product of two vectors: vector C_0, i. e. the geostrophic wind below, and vector Δc_T, i. e. the "thermic" wind, which is predicated on the mean thermal gradient in the contemplated atmospheric layer

$$c = c_0 + \Delta c_T . \tag{1}$$

Vector C_0 is determined in magnitude and direction by the pressure gradient force vector G_0,

$$c_0 = \frac{1}{l\rho} G_0 , \tag{2}$$

and vector Δc_T is determined in magnitude and direction by the thermal gradient vector Γ, it being known that

$$\Delta c_T = \frac{g \Delta z}{l T} \cdot \Gamma . \tag{3}$$

Finally vector C is determined by the pressure gradient force vector G

$$G = \frac{p}{p_0} G_0 + \Delta G_T \tag{4}$$

Thus, the pressure gradient force vector G can be computed as the vector sum of the pressure gradient force G_0, which is multiplied by $\frac{p}{p_0}$ times, and the thermal component of the pressure gradient force ΔG_T, in accordance with (7) of Section 2 which equals

$$\Delta G_T = \frac{g \Delta z}{R T_m} \cdot \frac{p}{T_m} \Gamma . \tag{5}$$

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The greater the thickness of the contemplated atmospheric layer, the less the effect of the first items in equations (1) and (4), and the more the effect of the second items. In conformance with this, the direction of vector C, with altitude, approximates more and more the direction Δc, and the direction of vector $\mathcal{G}$ approximates the direction of vector Γ. Thus, with increase in altitude, the direction of the geostrophic wind approximates the direction of the isotherm, and the direction of the pressure gradient force approximates the direction of the thermal gradient.

Figures 98 and 99 are a diagrammatic presentation of the variation, with altitude, of the velocity of the geostrophic wind and the pressure gradient force. They show clearly that the altitudinal changes in the velocities of the wind will vary, with relation to the disposition of the pressure gradient force and the temperature gradient.

Figure 98. The geostrophic wind, with higher altitudes, more and more approximates the isotherm

Figure 99. Variations in the pressure gradient force, with altitude.

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Above, we discussed the altitudinal variation of the "thermic" wind vector Δc_T. This variation determines the altitudinal change in the geostrophic wind. For practical purposes, the wind is considered in meteorology not as a vector, but as an aggregate of the two scalar magnitudes, the velocity modulus of the motion of the air and the angle determining the direction of the air current. We will derive the corresponding formulas, which express the altitudinal variations in the modulus of velocity and the direction of the wind, with relation to the angle α between the pressure and the thermal gradients.

Let us direct the x - axis along the isobar, and the y - axis along the pressure gradient force at the lower level (Figure 100). The "thermic" wind Δc_T will be directed along the isotherm, forming the angle α. with the isobar. The wind velocity c will be the vector sum $c_0 + \Delta c_T$. Let us designate the angle formed by velocity c with the x - axis (the isobar at the lower level) as θ. It is obvious that

$$u = c \cos\theta,$$
$$v = c \sin\theta \qquad (6)$$

and

$$\Delta u_T = \Delta c_T \cos\alpha,$$
$$\Delta v_T = \Delta c_T \sin\alpha. \qquad (7)$$

Figure 100. Altitudinal variation in the velocity and direction of the geostrophic wind, with relation to the angle between the pressure and temperature gradients.

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Differentiating equation (6), we derive:

$$\Delta u = \Delta c \cos\theta - c \sin\theta \Delta\theta,$$
$$\Delta v = \Delta c \sin\theta + c \cos\theta \Delta\theta. \qquad (8)$$

Since we are disregarding the change in the wind under the effect of the pressure component of the gradient,

$$\Delta u = \Delta u_T,$$
$$\Delta v = \Delta v_T.$$

therefore,

$$\Delta c \cos\theta - c \sin\theta \Delta\theta = \Delta c_T \cos\alpha,$$
$$\Delta c \sin\theta + c \cos\theta \Delta\theta = \Delta c_T \sin\alpha, \qquad (9)$$

whence, after solving (9) for Δc and $\Delta\theta$,

$$\Delta c = \Delta c_T \cdot \cos(\alpha - \theta),$$
$$c\Delta\theta = \Delta c_T \cdot \sin(\alpha - \theta) \qquad (10)$$

or, taking into consideration (3)

$$\frac{\Delta c}{\Delta z} = \frac{g}{\bar{T}} \cdot \Gamma \cos(\alpha - \theta),$$
$$\frac{\Delta\theta}{\Delta z} = \frac{g}{\bar{T}} \frac{1}{c} \Gamma \sin(\alpha - \theta). \qquad (11)$$

Equations (11) show that the altitudinal change in the geostrophic wind depends substantially upon the angle $(\alpha - \theta)$, formed between the isotherms and the isobars. When $(\alpha - \theta) = 0$, i. e. when the isobars coincide with the isotherms, then $\frac{\Delta\theta}{\Delta z} = 0$, and there is no rotation of the wind with higher altitude. $\frac{\Delta\theta}{\Delta z} = 0$ also in the case when $(\alpha - \theta) = 180$ degrees. If $(\alpha - \theta) = \pm 90$ degrees, $\frac{\Delta c}{\Delta z} = 0$, i. e. the wind changes direction only, without a change in the magnitude of its velocity, it being the case, that when $(\alpha - \theta) > 0$,

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there is left-hand rotation of the wind, and when $(\alpha-\theta)<0$, there is right-hand rotation. Generally, the smaller the angle $(\alpha-\theta)$, the more rapid the change in the magnitude of the wind velocity, and the slower the change in its direction. When the thickness of the contemplated atmospheric layer is not too great, the angle $(\alpha-\theta)$ can be considered as the angle that is formed by the thermal gradient with the pressure gradient force at the lower level. When the thickness of the contemplated atmospheric layer is considerable, the angle $(\alpha-\theta)$ is an angle that is formed by the pressure gradient force with the thermal gradient at the center level of the atmospheric layer. By the same token, the magnitude of C, in this case, is to be its magnitude at the center level of the contemplated atmospheric layer.

Table 43 characterizes the altitudinal variations in the velocity modulus of the geostrophic wind for various values of the angle $(\alpha-\theta)$ and Γ. The Table is computed for T = 273 degrees Centigrade; φ = 60 degrees; Δz = 1 kilometer, by the formula $\frac{\Delta c}{\Delta z} = 2.9 \cdot \Gamma \cos(\alpha-\theta)$.

[See page 20 for Table 43]

The variations in the direction of the wind within a 1-kilometer atmospheric layer, for various values of $(\alpha-\theta)$ and C, are given in Table 44.

[See page 21 for Table 44]

The computations were made on the assumption that φ = 60 degrees, T = 273 degrees Centigrade, Γ = 5 degrees per 100 kilometers, by the formula

$$\frac{\Delta\theta}{\Delta z} = 83 \cdot \Gamma \frac{\sin(\alpha-\theta)}{c}.$$

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TABLE 43

VARIATION OF VELOCITY MODULUS OF GEOSTROPHIC WIND

r \ d - θ	0	10	20	30	40	50	60	70	80	90
1 degree per 100 kilometers	2.9	2.9	2.7	2.5	2.2	1.9	1.4	1.0	0.5	0.0
2 degrees per 100 kilometers	5.8	5.7	5.4	5.0	4.4	3.7	2.9	2.0	1.0	0.0
3 degrees per 100 kilometers	8.7	8.6	8.1	7.5	6.6	5.6	4.3	3.0	1.5	0.0
5 degrees per 100 kilometers	14.5	14.3	13.6	12.6	11.1	9.3	7.2	5.0	2.5	0.0

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TABLE 44

CHANGE IN DIRECTION OF GEOSTROPHIC WIND

$\alpha-\theta$ / Γ	0	10	20	30	40	50	60	70	80	90
1 M/sec	0	14	29	41	59	63	72	78	82	83
3 m/sec	0	5	10	14	18	21	24	26	27	28
5 m/sec	0	3	6	8	11	13	14	15	16	17
10 m/sec	0	1	3	4	5	6	7	8	8	8
20 m/sec	0	1	1	2	3	3	4	4	4	4
30 m/sec	0	0	1	1	2	2	2	3	3	3

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The above Table shows that in the case of high wind velocities (i. e. in the presence of great pressure gradient forces), the change of direction in the wind, with higher altitudes, is small. Inversely, in the case of mild winds, the change in direction, with higher altitude, is sharp. For example, within the 1 - kilometer air layer, a wind with a velocity of 20 meters per second, will turn only 4 degrees, while a 3 meter per second wind, under the same conditions, will turn 28 degrees. Low-velocity winds, irrespective of their direction at lower level, will rapidly approximate the isotherm at higher altitude. In the presence of great velocities, the change in direction is slower. However, at great altitudes, in a homogeneous air mass, the air still has a tendency to move in the direction of the isotherms, since accretion of velocity in this case is great, but the direction coincides with the isotherm. Since the actual wind at great atmospheric altitudes frequently is not much distinct from the geostrophic wind, the predominance of the general westerly current, which is observed in the troposphere of the temperate latitudes, may be explained by the diminution in temperature from the equator to the pole through out the troposphere. In the stratosphere, the western component of the wind diminishes with altitude, in conformance with the inversion of the thermal gradient.

The above considerations make it possible to reverse the order of computation: by the altitudinal change in the velocity and direction of the wind, to determine the horizontal temperature distribution.

For example -- acording to pilot balloon observations, the

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velocity of the wind at the lower level equals C_0 (Figure 101), and at the upper level, it equals C. By determining vectorially $\Delta c = c - c_0$, we derive all the required values for the determination of the thermal gradient Γ in the contemplated atmospheric layer. Indeed, the thermal gradient Γ is perpendicular to Δc and directed to the left of it (in the northern hemisphere), and its magnitude is determined by the equation:

$$\Gamma = \frac{fT}{g} \cdot \frac{\Delta c}{\Delta z} . \qquad (12)$$

These considerations can be utilized in the extrapolation of synoptic charts. Given a network of pilot ballon observation points, it is possible to get adequately complete data on the pressure and temperature distribution beyond the cut-off of the synoptic chart.

However, in the practical application of these deductions, it must be remembered, that all the above said relates to the geostrophic wind, and therefore, it holds true only for homogeneous atmospheric layers, located outside the ground turbulent zone, i. e. for atmospheric altitudes in excess of 500 - 1000 meters.

Figure 101. Determination of the horizontal temperature gradient by the altitudinal changes in the geostrophic wind [Can be studied from original text].

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Section 4. Variations of Temperature and Pressure in Time with Relation to the Altitudinal Change of Direction of the Geostrophic Wind.

The variation in the temperature of the air at a given point can be expressed by:

$$\frac{\partial T}{\partial t} = \frac{dT}{dt} - u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} - w\frac{\partial T}{\partial z} . \qquad (1)$$

Assuming that the air masses are shifting without a change in temperature, i. e. $\frac{dT}{dt} = 0$. Then, due to the geostrophic wind being horizontal:

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} = c\Gamma\cos(c,\Gamma) = -c\Gamma\sin(\alpha-\theta).$$

Converting to finite increments, we derive

$$\frac{\Delta T}{\Delta t} = -c\Gamma\sin(\alpha-\theta), \qquad (2)$$

where $\frac{\Delta T}{\Delta t}$ is the temperature variation for a unit of time, and $(\alpha-\theta)$ is the angle between the thermal and the pressure gradients (or between the isobar and the isotherm). The angle is counted off from the pressure gradient to the thermal gradient.

From (2) it can be seen that the change in temperature with time, which is taking place at a given point, on account of the inflow of air with a different temperature, is in direct ratio to the thermal gradient and wind velocity, and also to the sine of the angle formed by the wind direction and the isotherm. The direction of the wind is determined by the mutual disposition of the gradients: if the thermal

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gradient is to the left of the pressure gradient ($\alpha-\theta>0$), there is cooling ($\frac{\Delta T}{\Delta t}<0$), if it is to the right of it ($\alpha-\theta<0$), there is warming ($\frac{\Delta T}{\Delta t}>0$).

Combining of expressions (2) and (11) from Section 3, permits the tying in of the change in temperature which is caused by advection, with the altitudinal change in direction of the geostrophic wind. Indeed, when we eliminate $\sin(\alpha-\theta)$, we derive:

$$\frac{\Delta T}{\Delta t} = -\frac{l}{g} T c^2 \frac{\Delta \theta}{\Delta z}. \qquad (3)$$

Equation (3) shows that in the northern hemisphere, during the left-hand rotation of the wind with height ($\frac{\Delta \theta}{\Delta z}>0$), the temperature falls with time ($\frac{\Delta T}{\Delta t}<0$), and, inversely, during the right-hand rotation of the wind ($\frac{\Delta \theta}{\Delta z}<0$), the temperature increases ($\frac{\Delta T}{\Delta t}>0$). The velocity of the temperature change with time is the greater, the greater the velocity of the wind, and the sharper the change in the direction of the wind with height.

It can also be seen from expression (3) that even with a slow rotation, but high velocities, of the wind, considerable changes in temperature can occur. With slow velocities, but fast rotation of the wind, very little advective change in temperature takes place. The inflow of air with a different temperature, and by virtue of that, a different density, is usually accompanied by a change in pressure, it being the case that warming is accompanied by a drop in pressure, and cooling by an increase in same. Therefore, the rotation of the wind with height can be tied in with a change in pressure with time.

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Assuming at a certain height h, the pressure P_h being constant, we will, from the barometric equation

$$p_h = p_0 \cdot e^{-\frac{gh}{RT_h}} \tag{4}$$

which is true in the case of the geostrophic wind, derive by differentiating by t:

$$\frac{\Delta p_0}{\Delta t} = -\frac{gh}{RT} \cdot \frac{p_0}{T} \cdot \frac{\Delta T}{\Delta t} \tag{5}$$

or, by substituting for $\frac{\Delta T}{\Delta t}$, as per equation (3):

$$\frac{\Delta p_0}{\Delta t} = \frac{l}{RT} \cdot p_0 c^2 h \cdot \frac{\Delta \theta}{\Delta z} \tag{6}$$

Equation (6) shows that, in the case of a right rotation of the wind with height ($\frac{\Delta \theta}{\Delta z} < 0$), the pressure, due to advection of masses with a different temperature, drops ($\frac{\Delta p_0}{\Delta t} < 0$), and in the case of a left rotation of the wind with height ($\frac{\Delta \theta_0}{\Delta z} > 0$), the pressure increases ($\frac{\Delta p_0}{\Delta t} > 0$). The square of velocity, contained in equation (6), points to the tremendous significance of great wind velocities in the effecting of a change in pressure.

Section 5. Variation of Wind with Height in Occluded Cyclones and Anti-Cyclones.

Results derived in the preceding paragraphs, may be applied, in first approximation, also to the case of curvilinear isobars, if only their curvature is not great.

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Let us visualize an occluded symmetrical cyclone with circular isobars (Figure 102). Let us assume for simplification that the temperature gradient is everywhere the same, and has a planetary direction (i.e. from south to north), and that, consequently, the isotherms run latitudinally, from west to east. The direction of the wind at the lower level, with relation to the points of the compass, will be different in the different parts of the cyclone, while the direction of the "thermic" wind will be the same everywhere (from west to east); its magnitude will also be the same everywhere. Then the nature of the variation of the wind with height will be different in different parts of the cyclone, as can be seen from the diagram.

Figure 102. Variation of geostrophic wind with height in different parts of the periphery of the cyclone.

Together with that, but in a different manner, there will occur variations of temperature and pressure in time.

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Figure 102 shows diagramatically the variation of geostrophic wind with height, in different parts of the periphery of the occluded cyclone, where the curvature of the isobars is so infinitesimal, that the curvature radius attains a length of 500 kilometers. The velocity of the wind at the lower level is accepted as equal to 6 meters per second, which conforms to a pressure gradient force of 1 millibar per 100 kilometers. The temperature gradient equals 1 degree Centigrade per 100 kilometers.

The "thermic" wind developing in the 1 - kilometer atmospheric layer under the effect of this gradient, is accepted at all heights to be equal to 3 meters per second.

At a point located to the south of the center of the cyclone the velocity of the wind is increased and the pressure gradient force is also increased, that is the isobars become more crowded with altitude. There is no change in temperature and pressure.

At a point to the north of the center of the cyclone, the velocity of the wind decreases with height. At a certain height it crosses its zero value, then again increases, having reversed its direction. The pressure gradient force diminishes with height, passes through its zero value, then begins to increase again, having reversed its direction. There is no change in temperature and pressure.

In the eastern half, the wind rotates, with height, to the right with the more intensive rotation taking place in the northern part of this half. Conformantly in the eastern half the temperature increases and the pressure drops.

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Inversely, in the western half the wind with height, rotates to the left, also with higher intensity in its northern part, with temperature falling and pressure increasing. Thus the pressure minimum is displaced gradually from west to east in the direction of diminishing pressure. The shifting of the pressure minimum takes place along the isotherm in such a way that the low pressure area remains on the left. The air masses at great heights are displaced in the same direction, generating a current which, according to S. I. Troitskiy, is called the leading current.

Figure 103 shows a pressure area at a height of 1.2 and 5 kilometers, obtained by superimposing the rectangular pressure area, conditioned by the thermal gradient, upon the circular pressure area of the lower level. The distances between the circular isobars are assumed as 110, 130, and 260 kilometers respectively, while the distances between the straight-line isobars are 220, 120 and 70 kilometers, respectively.

With height, the configuration of the isobars is gradually changed in such a way that they gradually become disconnected, approximating the isotherms. The center of low pressure is displaced with height, into the area of the low temperatures in such a manner that the cyclone axis is not vertical, but inclined into the cold area at a small angle with the horizon.

The change in configuration of the isobars, with height, will be the greater, the greater the thermal gradient, and the smaller the pressure gradient force at the lower level.

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What occurs in an anti-cyclone, is as if it were a mirrored reflection of what takes place in a cyclone. Namely, with the same disposition of isobars and isotherms, at a point to the south of the center of the anticyclone, the velocity of the wind diminishes with height until at a certain height it crosses its zero value, then again increases, having reversed its direction. The pressure gradient force with height, is reduced until at some certain height it crosses its zero value, then again increases, having reversed its direction. There is no change in temperature or pressure.

Figure 103a. Changes in pressure field with height, over cyclone.

Figure 103 b. Changes in pressure field with height, over cyclone.

At a point, to the north of the center of the anti-cyclone, the

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wind velocity and the pressure gradient force increase, with height, without changing direction so that the isobars become more crowded with height. There is no change in temperature and pressure.

In the eastern half of the anti-cyclone the rotation of the wind with height is to the left, and is accompanied by cooling and an increase in pressure. Inversely in the western half there takes place a right rotation of the wind; warming and a drop in pressure. Thus, the anti-cyclone too travels along the isotherm from west to east, leaving the area of low temperatures on the left. The axis of the anticyclone is also inclined at a very small angle to the horizon, but already into the area of high temperatures.

The results of our observations are basically true for any arbitrary location of the thermal gradient with relation to the points of the compass. In this case, it is sufficient to qualify not the eastern and western part of a cyclone or anticyclone, but their forward and rear parts.

Figure 103 c. Change in the pressure field, with height, over a cyclone.

Thus, the forward part of a cyclone and the rear part of an

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anti-cyclone are characterized by a right rotation of the wind and warming, while the rear part of an anti-cyclone and the forward part of a cyclone -- by a left rotation of the wind and cooling. Since at great heights the wind approaches the isotherms, there becomes established in the migrating cyclones and anticyclones, at great height, a general steering current, in the direction of which various pressure cycles move. The above results remain true also for the non-frontal parts of non-occluded cyclones. Only in a front area, conditions may differ substantially from the above described, on account of the discontinuity of pressure gradients and the thermal field.

Let us, finally, consider the change in geostrophic wind with height, in stable pressure areas. A pressure area is stable when the barometric pressure in it does not change, and the area itself does not migrate. The condition of stability, therefore, can be expressed by equation

$$\frac{\Delta p_0}{\Delta t} = \frac{l}{RT} p_0 c^2 \cdot h \frac{\Delta \theta}{\Delta z} = 0, \qquad (1)$$

from which, in accordance with (11) of Section 3,

$$\frac{\Delta \theta}{\Delta z} = \frac{g}{lT} \cdot \frac{1}{c} \cdot \Gamma \sin(\alpha - \theta) = 0 \qquad (2)$$

which is only possible in two cases:

(1) when $\Gamma = 0$, (3)

(2) when $\sin(\alpha - \theta) = 0$, (4)

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i. e. when the pressure gradient and the thermal gradient are parallel.

Thus, the stability of a pressure area, which is characterised by the invariability in the direction of the wind, is realized in 3 cases:

(1) When temperatures are leveled off in a horizontal direction, as is the case in an old occluded cyclone.

(2) When the low pressure area coincides with the cold area, while the high pressure area coincides with the warm area, as is the case in the so-called high cyclones and anticyclones, when the wind velocity increases with height, without change of direction.

(3) When, with the low pressure area coinciding with the warm area, and the high pressure area coinciding with the cold area, the wind diminishes with height, and after crossing its zero value, reverses direction.

The axes of stable cyclones and anti-cyclones are almost vertical.

It may occur that conditions of stability (3) and (4) hold true only for a certain part of the pressure area, while at the same time, the remainder of the above area is unstable.

All the above said merely relates to the change in pressure and temperature which is dependent on the advection of the air of a different temperature. Other processes may substantially distort the above described phenomena, particularly in the case when the advection of heat is not very intensive. This explains the rather small

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coefficients of
correlation between the temperature variations, computed theoretically, and the ones actually observed.

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CHAPTER XII

THE DYNAMICS OF PRESSURE VARIATIONS

Section 1. The Connection Between the Change in the Temperature of an Air Column and the Change in Pressure.

One of the most important problems in modern dynamic and synoptic meteorology, is the ascertaining of the causes of local pressure changes.

Directly linked with this is

The central problem of modern meteorology, the genesis and development of cyclones and anticyclones.

As of the present, there is no satisfactory and universally accepted theory on the genesis and development of pressure centers. The main, but not the only, difficulty is the ascertaining of the causes for the accumulation of tremendous volumes of air in the anticyclones, and for their being thrown out in the cyclones.

From the time of the first investigations into the problem of atmospheric pressure, the attempt to learn the physical causes that bring about local changes in terrestrial atmospheric pressure, was not too successful.

To date, some meteorologists still cling to the opinion that the effect of cold and warm air, respectively, upon surface pressure, is different, although it is quite obvious that a change of temperature can have an effect only upon the height of the air column, but not upon its mass.

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Consequently, the change in pressure at the terrestrial surface is tied in with the spatial distribution of pressure, temperature, and wind velocity.

The most rational method for the solution of this problem is by way of integration of the equations of hydro-dynamics, together with the equation of the inflow of energy. This was the line of thought taken by I. A. Kibel' (in Chapter XX we will discuss the results which he obtained).

In this chapter we will discuss, in all possible detail, the results obtained on the basis of the so-called "equation of tendency", which makes it possible to approach the study of local changes in pressure. But, before proceeding with the derivation and analysis of the equation of tendency, we will look into some problems tied in with the erratic application of the barometric formula to the clarification of the causes of a change in pressure.

We will discuss the correlationships existing between the variations in pressure at the bottom of the air column and the variations in its temperature as deducted from the barometric formula.

In Chapter V we pointed out that with an atmosphere in a state of repose, the pressure is measured by the weight of a single air column, extending from the point of observation to the upper limit of the atmosphere. Therefore, the pressure will change, when the mass of air, located over the contemplated level, will change. This is the controlling condition for the analysis of the formulas, which are frequently misinterpreted.

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Let us mentally divide the contemplated air layer into n layers of equal thickness h. Designating the pressure at the boundaries of these layers by P_0, P_1,.....P_n, and the mean temperatures of the layers by T_1, T_2,.... T_n, and, by the use of the barometric formula, we derive:

$$\ln p_0 = \ln p_1 + \frac{gh}{RT_1}; \ln p_1 = \ln p_2 + \frac{gh}{RT_2}; \ldots; \ln p_{n-1} = \ln p_n + \frac{gh}{RT_n}.$$

In adding up these equations, by their respective terms, and in reducing the congruent terms, we derive

$$\ln p_0 = \ln p_n + \frac{gh}{R} \sum_{i=1}^{n} \frac{1}{T_i}. \qquad (1)$$

By differentiating, we obtain

$$\frac{\partial p_0}{\partial t} = \frac{p_0}{p_n} \frac{\partial p_n}{\partial t} - \frac{gh}{R} \cdot p_0 \sum_{i=1}^{n} \frac{1}{T_i^2} \frac{\partial T_i}{\partial t}. \qquad (2)$$

The local derivatives, which can be interpreted as changes in temperature and pressure during 12 or 24 hours, can be determined with the aid of aerological sounding.

Let us now analyze equation (2), remembering that it holds true only for the case, when the entire contemplated layer is in a state of static equilibrium. Let us first assume that the temperature of the intermediary layers remained unchanged. Then

$$\frac{\partial p_0}{\partial t} = \frac{p_0}{p_n} \cdot \frac{\partial p_n}{\partial t}. \qquad (3)$$

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Thus the change in surface pressure will be $\frac{p_o}{p_n}$ times greater than the corresponding change in pressure above. From this, the deduction was made that the changes in surface pressure are determined by the changes in the pressure above. This belief was even further substantiated, when aerological ascents established that diurnal changes in pressure at great heights are on the same order as the changes in pressure near the terrestrial surface. However, equation (3) cannot mean that the changes in pressure $\frac{\partial p_n}{\partial t}$ at a height where the pressure equals P_n, induces a change in pressure $\frac{p_o}{p_n} \cdot \frac{\partial p_n}{\partial t}$ below. This is incorrect because of the fact that the change in pressure by δp_n shows only that over the level where the pressure is increased, the mass in the unit section column increased by $\frac{\delta p_n}{g}$. And if there is no advection in the air of the lower layers, the overall air mass will increase by $\frac{\delta p_n}{g}$, and not by $\frac{p_o}{p} \cdot \frac{\delta p_n}{g}$.

This point being of great importance, we will elaborate on it from a different viewpoint. Let us assume that, at the earth's surface, the pressure changed by δp_o, which for purposes of being specific, we will consider as positive. This increase in pressure means that the mass of the air column has increased by $\frac{\delta p_o}{g}$. Let us imagine that the column of air is restricted from the sides by absolutely impermeable walls, to a height where the pressure equals p_n (Figure 104).

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Figure 104. Change in pressure at the earth's surface depends on air mass inflow.

Under these conditions, the increase in the mass of the air column by $\frac{\delta p_0}{g}$ is induced only by advection at and above level H, so that the column of air beneath the H level is compressed, and a part of the air, which was initially above the H level, will descend underneath this level. Therefore, the change in pressure δp_n, at this level H, equals the weight of the air added to the air which was at and above level H, minus the weight of the air which descended underneath the level H because of compression. The increase in mass by $\frac{\delta p_n}{g}$, consequently, is only a part of the total mass which was added to the mass that was above level H. Thus, the pressure below should increase more than by δp_n, since the change in this pressure depends on the general increase in the mass of the air column. Consequently, the above equation (3); in reality, expresses not the pressure change δp_n at level H, which caused a $\frac{p_0}{p_n}$ times greater change in surface pressure, but the entire change in pressure δp_0, which is tied in with the inflow of air masses above the level H, and which is reduced $\frac{p_n}{p_0}$ times on account of the compressibility of the air.

Let us assume now, that there is no change in pressure above,

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and that only a change in temperature in one of the [mentally] segregated air layers, took place. Then

$$\frac{\partial p_0}{\partial t} = -\frac{g h}{R} \frac{p_0}{T_\kappa^2} \frac{\partial T_\kappa}{\partial t}. \qquad (4)$$

This equation shows that with one and the same change in temperature, taking place at different heights, the change in the surface pressure will be greater in the case when the temperature change occurs at a greater height, since temperature decreases with height. However, one and the same temperature change at a greater height acts upon a smaller mass (if the [intermediate] layers are considered to be of the same thickness), since density decreases with height. Therefore, the change in the general mass of the column will be the smaller, the higher the layer in which a change of temperature occurs. In order to explain this paradox, let us analyze the change in pressure at the lower boundary of the layer in which temperature changes occurred.

Since, as per assumption, the pressure at the upper boundary of the layer remained unchanged, the change in pressure at the lower limit of the layer will be expressed by equation.

$$\frac{\partial p_{\kappa-1}}{\partial t} = -\frac{g h}{R} \cdot \frac{p_{\kappa-1}}{T_\kappa^2} \frac{\partial T_\kappa}{\partial t}. \qquad (5)$$

In accordance with (5), the change in pressure at the bottom limit of the layer will be the smaller, the higher up the layer is located, i.e., the smaller the mass of this layer. Consequently, for the layer located at a greater height, the change in pressure

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will be smaller, regardless of the fact, that the change in temperature is the same. But, according to (3), the change in the surface pressure is tied in with change (5) by the formula.

$$\frac{\partial p_0}{\partial t} = \frac{p_0}{p_{K-1}} \cdot \frac{\partial p_{K-1}}{\partial t}. \qquad (6)$$

Incorporating (6) and (5), we derive (4). This return to the initial equation (4) shows that the change in the weight of the air column, induced by the temperature change in the selected layer, is in proportion to the pressure at the bottom level of this layer. Thus, the effect of the assigned temperature change in a layer of assigned thickness, upon the weight of the entire air column, is the smaller, the higher the location of this layer in the atmosphere, since in the latter case a smaller mass of air exerts its effect upon it. But, when a state of equilibrium is maintained in the lower layers (due to the inflow of air masses), the mass of the air column must change in such a way, as will cause the surface pressure to change in accordance with equation (4).

Section 2. The Effect of Advection in the Air at Great Heights Upon the Change in Surface Pressure.

The real correlationships between the changes in pressure and temperatures in the atmosphere, are considerably more complex than was assumed in Section 1. The truth is, that the changes in the conditions of the lower atmospheric layers are not precisely such, as will counterbalance the change in the weight of the layer above. The air that was located in the contemplated layer, does

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not remain motionless, but is displaced vertically, since the lower air layers become compressed. Figure 105 shows diagrammatically the displacement that occurred.

Figure 105. Effect of advection in the air masses at great heights upon the change in surface pressure.

We measure only the local changes in pressure occurring at a fixed point in space, as, for instance, in the layer ABCD, prior to, and in the layer A'B'C'D', after the advection in air masses of various temperatures. These are the changes, which were discussed in Section 1. In reality, a change in an air mass during advection is affected by those "individual" changes that occur in an individual physical air particle, which, prior to advection, was located in layer ABCD, and, after advection, in layer EFGH. Therefore, the barometric formula cannot make apparent the dynamics of the change in surface pressure, stipulated upon advection occurring at great heights. For this, it is necessary to turn to the individual changes in pressure $\delta\pi$ at a surface, which, at all times, consists of the same air particles, and which moves up and down like a piston. Let us imagine an air column of unitary cross section,

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restricted from the sides by rigid walls, but open on top. This means the change in pressure occurring in the contemplated air column, is only possible at the expense of the inflow and outflow of air masses through the top open end, i.e. at the expense of advection of the air masses occurring at great heights at a time when there is no inflow of air masses at the bottom layers. Let us segregate in this column an air layer with a thickness dz. Let us contemplate the changes in the condition occurring in the air column due to advection taking place at great heights. We will designate the changes in values, occurring with time, by differential operator δ ; the individual changes, by index i, and the local changes, by index e.

Let us examine the individual displacement of particles, and also the individual and local changes in temperature $\delta_i T$, $\delta_e T$ and pressure $\delta_i p$, $\delta_e p$. When these changes are determined, it becomes possible, by comparison with observations, to decide upon the degree of accuracy of the assumption about advection at great heights.

The individual change in pressure $\delta_i p$ at all levels is, obviously, equal to the total advection of the air masses $\delta\pi$ at great heights. Obviously, the local change in surface pressure δp_o , also equals the total advection of the above air masses. Therefore:

$$\delta_i p = \delta p_o = \delta\pi. \qquad (1)$$

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The local change in pressure at any point, is equal to the sum of the individual change in pressure and the effect of vertical air displacement:

$$\delta_e p = \delta\pi + g\rho\delta z. \qquad (2)$$

The same is true for temperature:

$$\delta_e T = \delta_i T + \gamma\delta z, \qquad (3)$$

Where δz is the change in the height of the surface made up of those physical particles, for which the individual changes in condition are measured.

We will assume that the change in the condition of the contemplated air column, is polytropic. From the polytropy equation, we derive:

$$\frac{\delta_i p}{p} = \kappa\frac{\delta_i \rho}{\rho} \quad \text{or} \quad \delta_i \rho = \frac{\rho}{p}\,\frac{\delta\pi}{\kappa}. \qquad (4)$$

Designating the thickness of the elementary layer by dz, we derive, geometrically that:

$$d(\delta z) = \delta(dz) \qquad (5)$$

(Since $d(\delta z)$ is the displacement differential of the upper and lower limit of the layer, and $\delta(dz)$ is the change in the thickness of the layer). From the equation of conservation of matter, it

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follows that

$$\delta(\rho dz) = 0$$

or

$$\frac{\delta_i \rho}{\rho} + \frac{\delta(dz)}{dz} = 0. \qquad (6)$$

Incorporating (4), (5), and (6), we derive:

$$\frac{d(\delta z)}{dz} = -\frac{1}{\kappa}\frac{\delta\pi}{p},$$

or, by integration, and taking into consideration that $(\delta z)_{z=0} = 0$ we obtain:

$$\delta z = -\frac{\delta\pi}{\kappa}\int_0^z \frac{dz}{\rho}. \qquad (7)$$

Equation 7 shows the magnitude of the displacement of the surface, which consists of one and the same particles and lies at level z. Let us assume that the distribution of temperature with height, is linear. Then, in place of p, under the sign of the integral, we can substitute $p_0\left(\frac{T}{T_0}\right)^{\frac{g}{R\gamma}}$. Upon integration, we derive:

$$\delta z = \frac{\delta\pi}{\kappa}\frac{R}{R\gamma - g}\cdot\frac{T_0}{p_0}\left[\left(1 - \frac{\gamma z}{T_0}\right)^{\frac{R\gamma - g}{R\gamma}} - 1\right] \qquad (8)$$

or, separating into a series and limiting ourselves to terms of the first and second degree, we obtain:

$$\delta z = -\frac{\delta\pi}{\kappa}\cdot\frac{z}{p_0}\left(1 + \frac{1}{2}\frac{gz}{RT_0}\right). \qquad (9)$$

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Equations (8) and (9) show that the individual particle descends ($\delta z < 0$), if, in the upper layers of the atmosphere, some mass of air ($\delta\pi > 0$) is added, and rises ($\delta z > 0$), if, at great heights, there is an outflow of air ($\delta\pi < 0$). These displacements of the individual particle are the greater, the greater its height and the smaller the exponent of the polytropy. Thus, for example, with adiabatic transfer of the change in pressure (k=1.4), the vertical displacements are 1.4 times smaller than in the case of isothermic transfer (k=1.0). This is due to the fact that, with adiabatic compression, the entire air column warms and expands, as a result of which the vertical displacement of individual air particles becomes smaller, as compared with what happens in the case of isothermic compression, when no such effect occurs.

Substituting (7), (8), and (9) into equation (2), for the local change in pressure, we derive:

$$\delta_e p = \delta\pi\left[1 - \frac{g\rho}{\kappa}\int_0^z \frac{dz}{p}\right],$$

$$\delta_e p = \delta\pi\left[1 + \frac{1}{\kappa}\frac{g}{R_y - g}\cdot\frac{\rho R T_0}{p_0}\left\{\left(1 - \frac{\gamma z}{T_0}\right)^{\frac{R_y - g}{R_y}} - 1\right\}\right], \quad (10)$$

$$\delta_e p = \delta\pi\left[1 - \frac{1}{\kappa}\frac{g\rho z}{p_0}\left(1 + \frac{1}{2}\frac{gz}{RT_0}\right)\right].$$

Equation (10) shows that local change in pressure, at the terrestrial surface, assumes its maximum value, which is equal to the entire advection value, and decreases with height. This being the case, local changes in pressure, occurring at the same height, will differ, with relation to the character of the transfer of

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this change in pressure. In the case of adiabatic transfer, the local changes in pressure will be greater, than in the case of isothermic transfer.

Finally, let us compute the individual and local temperature changes. From the equation of polytropy, we derive:

$$\delta_i T = \frac{\kappa-1}{\kappa} \cdot \frac{T}{p} \delta\pi. \qquad (11)$$

Substituting (11) into (3), and taking into consideration (7), (8), and (9), we obtain:

$$\delta_e T = \frac{\kappa-1}{\kappa} \cdot \frac{T}{p} \cdot \delta\pi \left[1 - \frac{\gamma}{\kappa-1} \cdot \frac{p}{T} \int_0^z \frac{dz}{p}\right],$$

$$\delta_e T = \frac{\kappa-1}{\kappa} \cdot \frac{T}{p} \cdot \delta\pi \left[1 + \frac{1}{\kappa-1} \frac{R_\gamma}{R_\gamma - g} \left\{1 - \left(\frac{T}{T_0}\right)^{\frac{g-R_\gamma}{R_\gamma}}\right\}\right],$$

$$\delta_e T = \frac{\kappa-1}{\kappa} \cdot \frac{T}{p} \cdot \delta\pi \left[1 - \frac{\gamma}{\kappa-1} \cdot \frac{p}{T} \cdot \frac{z}{p_0} \left(1 + \frac{1}{2} \frac{gz}{RT_0}\right)\right]. \qquad (12)$$

With the aid of the derived formulas, the vertical displacement of a particle, as well as the changes in pressure and temperature, at a given height, which are predicated upon advection at great heights, can be calculated, when the initial pressure and temperature distribution is known. In practical computations, equations (9), (10), and (11) are used with more facility when taken in a particular form, with the assumption that the change in the condition of the air column occurs adiabatically, and by interpolating the mean barometric temperature T_m:

$$\delta z = -\frac{\delta\pi}{k} \cdot \frac{RT_m}{g} \left(\frac{1}{p} - \frac{1}{p_0}\right),$$

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$$\delta_e p = \delta\pi\left[1 - \frac{1}{k}\cdot\frac{T_m}{T}\left(1 - \frac{p}{p_0}\right)\right], \qquad (13)$$

$$\delta_e T = \frac{R}{c_p}\cdot\frac{T}{p}\cdot\delta\pi\left[1 - \frac{\gamma c_v}{g}\frac{T_m}{T}\left(1 - \frac{p}{p_0}\right)\right].$$

Section 3. The Equation of Tendency.

A local change in pressure for a 3-hour period, is conventionally called, in meteorology, the barometric tendency.

The equation of tendency is derived by incorporating the third of the equations of motion in the atmosphere (in the form of the equation of state) with the equation of continuity.

It may seem that the equation of continuity, in a non-stationary status, would be the means for the determination of the change of pressure with time in a spatial distribution of velocity and density. However, for the practical application of this mathematical solution, it is necessary to know the distribution of the meteorological elements with more precision than what is deducible from observations. Due to the importance of this problem, we will analyze it in more detail.

Pressure p at some level h is, with a high degree of precision, the weight of a mass of air, situated above this level, per unit of area.

$$p = \int_h^\infty g\rho\, dz. \qquad (1)$$

Consequently, local change in pressure can be expressed through a change in mass:

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$$\left(\frac{\partial p}{\partial t}\right)_h = \int_h^\infty g \frac{\partial \rho}{\partial t} dz. \qquad (2)$$

Substituting into (2), instead of $\frac{\partial \rho}{\partial t}$, its value from the equation of continuity, we derive:

$$\left(\frac{\partial p}{\partial t}\right)_h = -\int_h^\infty g\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)dz - \int_h^\infty g\left(\rho\frac{\partial w}{\partial z} + w\frac{\partial \rho}{\partial z}\right)dz -$$

$$-\int_h^\infty g\left(u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y}\right)dz. \qquad (3)$$

The calculation of these integrals with the infinite upper limit may cause a misunderstanding on account of the vertical velocity in infinity being indeterminate, and also on account of the change in the force of gravity with height.

Therefore, it is physically more justified to consider not the entire thickness of the atmosphere, but only the thickness of a certain layer, delimited by levels h_0 and h_1. Then, disregarding the change g with height, we derive:

$$\frac{\partial p_0}{\partial t} = \frac{\partial p_1}{\partial t} - g\int_{h_0}^{h_1}\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)dz + g(\rho_0 w_0 - \rho_1 w_1) -$$

$$-g\int_{h_0}^{h_1}\left(u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y}\right)dz. \qquad (4)$$

This equation, known as the equation of tendency, shows that the change in pressure at a motionless fixed point at level h_0, is determined by 5 factors:

(1) by the change in pressure at the upper level predicated on processes occurring above this level, and equal to

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$$\Delta p_1 = \frac{\partial p_1}{\partial t}; \tag{5}$$

(2) by the horizontal divergence of velocity in the layer $h_0 - h_1$,

$$\Delta p_2 = -g \int_{h_0}^{h_1} \rho \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) dz, \tag{6}$$

it being the case that a positive divergence corresponds to a drop in pressure at level h_0, and negative divergence (or convergence), to an increase in pressure;

(3) by the vertical transfer of air masses through the bottom limit h_0 of the layer

$$\Delta p_3 = g \rho_0 w_0, \tag{7}$$

it being the case that an ascending motion $(w_0 > 0)$ induces an increase in pressure at level h_0, and a descending motion $(w_0 < 0)$ induces a decrease in same;

(4) by the vertical transfer of air through the upper limit h_1 of the layer

$$\Delta p_4 = -g \rho_1 w_1, \tag{8}$$

with the ascending current $(w_1 > 0)$ inducing a drop in pressure at level h_0, and the descending current $(w_1 < 0)$ inducing an increase in same;

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(5) by the horizontal transfer, or the advection of the mass in layer $h_o - h_1$

$$\Delta p_5 = -g\int_{h_o}^{h_1}\left(u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y}\right)dz, \qquad (9)$$

with the substitution of a cold mass by a warm mass resulting in a drop of pressure, and vice versa.

We will now evaluate the magnitude of each of the terms in equation (4). We will consider 3 hours as a unit of time; then, $\Delta p_1 \ldots \Delta p_5$ will represent barometric tendencies, predicated on the corresponding one of the above indicated factors. If there are no changes in the air mass of the layer $h_o - h_1$, the change in pressure above will induce an identical change in pressure below, i.e., in this case, the change in pressure is transferred in the same manner, as in non-compressible liquid.

For further analysis, we will interpolate the mean values of the derivatives of the components of velocity:

$$\frac{\partial u}{\partial x} = \frac{\int_{h_o}^{h_1}\rho\frac{\partial u}{\partial x}dz}{\int_{h_o}^{h_1}\rho dz} = \frac{g\int_{h_o}^{h_1}\rho\frac{\partial u}{\partial x}dz}{p}; \quad \frac{\partial v}{\partial y} = \frac{g\int_{h_o}^{h_1}\rho\frac{\partial v}{\partial y}dz}{p} \qquad (10)$$

where

$$p = g\int_{h_o}^{h_1}\rho\, dz = p_o - p_1 \qquad (11)$$

(in order to economize writing space, we avoid new designations for mean velocities). From (6), we derive:

$$\Delta p_2 = -p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right). \qquad (12)$$

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Correlationship (12) shows that the change in pressure predicated on the divergence of velocity, [redacted] is in direct ratio to the mean velocity divergence, and in direct ratio to the difference in pressures at the layer boundaries, i.e., to the mass of the layer. Let us further visualize the plane velocity divergence as a sum of the divergence of velocity magnitude and the divergence of velocity direction.

Assuming that $u = c\cos\alpha$, $v = c\sin\alpha$, where c is the velocity magnitude, and α is the angle formed by a velocity with a positive direction of the x - axis, we obtain:

$$\frac{\partial u}{\partial x} = \frac{\partial c}{\partial x}\cos\alpha - c\sin\alpha\frac{\partial \alpha}{\partial x},$$

$$\frac{\partial v}{\partial y} = \frac{\partial c}{\partial y}\sin\alpha + c\cos\alpha\frac{\partial \alpha}{\partial y}.$$

Let us now guide the x - axis along the direction of the current s, and the y - axis along a normal n to the latter. Angle α will then become zero, and we derive:

$$\frac{\partial u}{\partial x} = \frac{\partial c}{\partial s}; \frac{\partial v}{\partial y} = c\frac{\partial \alpha}{\partial n}.$$

Consequently,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial c}{\partial s} + c\frac{\partial \alpha}{\partial n}. \qquad (13)$$

So that the velocity divergence consists of the divergence of the velocity magnitude and the divergence of the velocity direction, i.e., of the deflection of velocity in a direction normal to

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the current. Then,

$$\Delta p_2 = -p\left(\frac{\partial c}{\partial s} + c\frac{\partial \alpha}{\partial n}\right) = \Delta p_2' + \Delta p_2'', \qquad (14)$$

where

$$\Delta p_2' = -p\frac{\partial c}{\partial s}; \quad \Delta p_2'' = -pc\frac{\partial \alpha}{\partial n}. \qquad (15)$$

Assuming that p = 100 millibars, Δs = 100 kilometers, we obtain:

$$\Delta p_2' = -10^2 \cdot \frac{\Delta c}{10^5} \cdot 60 \cdot 60 \cdot 3 = -10\Delta c \text{ or } \Delta c = -0.1\Delta p_2'.$$

Thus, for a change in pressure of 1 millibar in 3 hours, to occur, it is sufficient that a mean change in the velocity magnitude at the lowest 1 - kilometer layer, equal to 10 centimeters / seconds per 100 kilometers, occurs.

By the same token, assuming that p = 1000 millibars, c = 10 meters/seconds, Δn = 100 kilometers, we derive:

$$\Delta p_2'' = -10^2 \cdot 10\frac{\Delta \alpha}{10^5} \cdot 60 \cdot 60 \cdot 3 \cong -100\Delta\alpha \text{ or } \Delta\alpha = 0.01\Delta p_2''.$$

Consequently, for a change in pressure of 1 millibar/3 hours to occur, it is sufficient that a mean deflection in the direction of the wind in the lowest 1-kilometer layer, equal to 0.01 of a radian, or 0.5 degrees/100 kilometers, occurs.

These computations show that even an insignificant divergence of velocity induces considerable local changes in pressure.

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We further evaluate the effect of vertical currents upon the magnitude of the barometric tendency.

Assuming that g = 10^3 centimeters/seconds^2, ρ_0 = 10^{-3} grams/centimeters^3, we obtain $\Delta p_3 = 10^3 \times 10^{-3} \cdot w \cdot 60 \cdot 60 \cdot 3 \cdot 10^{-3} = 10 \cdot w$, or $w = 0.1 \Delta p_3$ centimeters/seconds.

Thus, for a change of pressure by 1 millibar/3 hours to occur, a vertical velocity of 1 millimeter/second is sufficient.

Finally, we evaluate the effect of advection of the mass. Computing the mean /average?/ value of the integral in equation (9), we obtain:

$$\Delta p_5 = -\left(u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y}\right) = -c \frac{\partial p}{\partial n} \cos\left(c, \frac{\partial p}{\partial n}\right), \quad (16)$$

which is based on principles:

$$u = \frac{\int_{h_0}^{h_1} u \frac{\partial \rho}{\partial x} dz}{\int_{h_0}^{h_1} \frac{\partial \rho}{\partial x} dz} = \frac{g \int_{h_0}^{h_1} u \frac{\partial \rho}{\partial x} dz}{\frac{\partial p}{\partial x}};$$

$$v = \frac{g \int_{h_0}^{h_1} v \frac{\partial \rho}{\partial y} dz}{\frac{\partial p}{\partial y}}; \quad p = -g \int_{h_0}^{h_1} \rho \, dz = p_0 - p_1.$$

Assuming that

$$\cos\left(c, \frac{\partial p}{\partial n}\right) = 1, \quad c = 10 \text{ meters/second}, \quad \Delta n = 100 \text{ kilometers},$$

we find that:

$$\Delta p_5 = -\frac{10 \cdot \Delta p \cdot 60 \cdot 60 \cdot 3}{10^5} \cong -2 \cdot \Delta p,$$

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i.e., for a change in pressure by 1 millibar/3 hours to occur, a wind velocity of 5 meters/second parallel to a pressure gradient force of 1 millibar/100 kilometers in the lower 1-kilometer layer, is sufficient.

The above computations show that it is practically impossible to utilize the equation of tendency for quantitative computations of a change in pressure, since it would require such precision in measuring the meteorological elements as is not attainable in practical experience. Therefore, deductions from the analysis of the equation of tendency are limited to the qualitative range.

The difficulties connected with the application of the equation of tendency, are obviously predicated upon the fact that the integrals entering into formula (4), extend over a layer of very great thickness, where the determination of the mean velocity values is very difficult. In this layer of great thickness, conditions may change with height in such an arbitrary manner, that the effect of some sub-layers is erased as a result of the effect of other sub-layers. Therefore, in the final analysis the changes in pressure being observed turn out to be much smaller than could have been expected. Consequently, a local change in pressure may be considered as a rather small difference between rather large magnitudes. This specifically is the reason for the difficulties in its pre-computation.

In the practice of the Soviet weather service, barometric topography maps are used widely. It is, therefore, feasible to cite the equation for local change in pressure based on the

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barometric topography maps. We take a partial derivative by z from an individual derivative $\frac{dp}{dt}$:

$$\frac{\partial}{\partial z}\left(\frac{dp}{dt}\right)=\frac{\partial}{\partial z}\left(\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial y}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}\right)= \tag{17}$$

$$=\frac{\partial^2 p}{\partial z\partial t}+u\frac{\partial^2 p}{\partial z\partial x}+v\frac{\partial^2 p}{\partial z\partial y}+w\frac{\partial^2 p}{\partial z^2}+\frac{\partial u}{\partial z}\cdot\frac{\partial p}{\partial x}+\frac{\partial v}{\partial z}\cdot\frac{\partial p}{\partial y}+\frac{\partial w}{\partial z}\cdot\frac{\partial p}{\partial z}=$$

$$=\frac{d}{dt}\left(\frac{\partial p}{\partial z}\right)+\frac{\partial u}{\partial z}\cdot\frac{\partial p}{\partial x}+\frac{\partial v}{\partial z}\cdot\frac{\partial p}{\partial y}+\frac{\partial w}{\partial z}\cdot\frac{\partial p}{\partial z}.$$

Eliminating from (17) $\frac{\partial w}{\partial z}$ with the aid of the equation of continuity, which can be used in the form of:

$$\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{d\rho}{dt}-\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right). \tag{18}$$

We then obtain:

$$\frac{\partial}{\partial z}\left(\frac{dp}{dt}\right)=\frac{d}{dt}\left(\frac{\partial p}{\partial z}\right)-\frac{1}{\rho}\frac{\partial p}{\partial z}\frac{d\rho}{dt}-\frac{\partial p}{\partial z}\frac{\partial u}{\partial x}+\frac{\partial u}{\partial z}\frac{\partial p}{\partial x}-\frac{\partial p}{\partial z}\cdot\frac{\partial v}{\partial y}+ \tag{19}$$

$$+\frac{\partial v}{\partial z}\frac{\partial p}{\partial y}.$$

The first two terms of the right-hand part can be converted in this manner:

$$\frac{d}{dt}\left(\frac{\partial p}{\partial z}\right)-\frac{1}{\rho}\frac{\partial p}{\partial z}\cdot\frac{d\rho}{dt}=\rho\frac{d}{dt}\left[\frac{1}{\rho}\frac{\partial p}{\partial z}\right]=-\rho\frac{dg}{dt}=0, \tag{20}$$

if the individual change in the acceleration of gravity with time is disregarded. In the remaining terms of the right-hand part we will dissociate from parentheses

$$\frac{\partial}{\partial z}\left(\frac{dp}{dt}\right)=-\frac{\partial p}{\partial z}\left[\frac{\partial u}{\partial x}-\frac{\partial u}{\partial z}\frac{\frac{\partial p}{\partial x}}{\frac{\partial p}{\partial z}}+\frac{\partial v}{\partial y}-\frac{\partial v}{\partial z}\frac{\frac{\partial p}{\partial y}}{\frac{\partial p}{\partial z}}\right]. \tag{21}$$

Remembering that the ratios $\frac{\partial p}{\partial x}:\frac{\partial p}{\partial z}$ and $\frac{\partial p}{\partial y}:\frac{\partial p}{\partial z}$ are equal to the tangents of the angles β_x, β_y which are formed

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by the isobaric surface with the coordinate axes x and y, equation (21) can be converted to:

$$\frac{\partial}{\partial z}\left(\frac{dp}{dt}\right)=-\frac{\partial p}{\partial z}\left[\frac{\partial u}{\partial x}+\frac{\partial u}{\partial z}\left(\frac{dz}{dx}\right)_p+\frac{\partial v}{\partial y}+\frac{\partial v}{\partial z}\left(\frac{dz}{dy}\right)_p\right], \quad (22)$$

Noting further that:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial z}\left(\frac{dz}{dx}\right)_p=\left(\frac{\partial u}{\partial x}\right)_p; \quad \frac{\partial v}{\partial y}+\frac{\partial v}{\partial z}\left(\frac{dz}{dy}\right)_p=\left(\frac{\partial v}{\partial y}\right)_p, \quad (23)$$

where index p shows that the values of functions u and v are taken not at points lying in plane z = const., but at points lying in the isobaric plane p = const.

Equation (22) then becomes:

$$\frac{\partial}{\partial z}\left(\frac{dp}{dt}\right)=-\frac{\partial p}{\partial z}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)_p. \quad (24)$$

Integrating equation (24) within the limits from h_1 to h_2 we obtain:

$$\frac{dp_2}{dt}-\frac{dp_1}{dt}=-\int_{p_1}^{p_2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)_p dp. \quad (25)$$

Solving this equation for the local derivative $\frac{\partial p_1}{\partial t}$ and substituting for the integral its mean value we finally obtain:

$$\frac{\partial p_1}{\partial t}=\frac{\partial p_2}{\partial t}-\left(u_1\frac{\partial p_1}{\partial x}+v_1\frac{\partial p_1}{\partial y}\right)+\left(u_2\frac{\partial p_2}{\partial x}+v_2\frac{\partial p_2}{\partial y}\right)+ \quad (26)$$

$$+g(\rho_1 w_1-\rho_2 w_2)-p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)p,$$

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where as before $p = p_1 - p_2$, while u and v are determined by equations

$$\frac{\partial u}{\partial x} = \frac{\int_{p_0}^{p_1} \frac{\partial u}{\partial x} dp}{p}, \quad \frac{\partial v}{\partial y} = \frac{\int_{p_0}^{p_1} \frac{\partial v}{\partial y} dp}{p}. \tag{27}$$

Thus, a local change in pressure at the lower level h_1 is determined by the following factors:

(1) by the local change in pressure $\Delta p_1 = \frac{\partial p_2}{\partial t}$ at the upper level h_2, it being the case that this change in pressure is transferred down without any additional change if it is not accompanied by an inflow or outflow of air masses, vertical or horizontal;

(2) by advection of the pressure at lower level h_1

$$\Delta p_2 = -\left(u_1 \frac{\partial p_2}{\partial x} + v_1 \frac{\partial p_1}{\partial y}\right) = -c_1 \frac{\partial p_2}{\partial u} \cos\left(c_1 \frac{\partial p}{\partial n}\right), \tag{28}$$

it being the case that pressure will increase, when the angle between the wind velocity c and the pressure gradient force $\left(-\frac{\partial p}{\partial n}\right)$ is acute, and will drop when the latter angle is obtuse; in the case of a right angle, the pressure does not change;

by advection of the pressure at the upper level

$$\Delta p_3 = \left(u_2 \frac{\partial p_2}{\partial x} + v_2 \frac{\partial p_2}{\partial y}\right) = c_2 \frac{\partial p_2}{\partial n} \cos\left(c_2 \frac{\partial p_2}{\partial n}\right), \tag{29}$$

it being the case, that the pressure below will increase when the angle between the wind velocity c and the pressure gradient force $\left(-\frac{\partial p}{\partial n}\right)$ is obtuse and will drop when same angle is acute; when the angle is 90 degrees, i.e. when the wind blows along the isobar,

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there is no change in pressure;

(4) by the vertical transfer of an air mass through a horizontal area which is lying at level h_1

$$\Delta p_4 = g\rho_1 w_1,$$

it being the case that an ascending air current $(w_1 > 0)$ is accompanied by an increase in pressure at level h_1, and a descending air current - by a pressure drop;

(5) by the vertical transfer of an air mass through a horizontal area at level h_2

$$\Delta p_5 = -g\rho_2 w_2$$

it being the case that an ascending air current at level h_2 is accompanied by a drop in pressure at level h_1, and a descending current - by an increase in pressure;

(6) by a horizontal divergence of velocity at some intermediate isobaric surface located between the isobaric surfaces p_1 = const. and p_2 = const.

(30)

$$\Delta p_6 = \int_{p_1}^{p_2} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)_p dp = -p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)_p = -p\left(\frac{\partial c}{\partial s} + c\frac{\partial \alpha}{\partial n}\right)_p,$$

it being the case that a divergence of velocity induces a drop in pressure while convergence (negative divergence) of the latter induces an increase in pressure.

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CHAPTER XIII

SURFACES OF SEPARATION

Section 1. Surfaces of Discontinuity. Discontinuities of the Zero and the First Order.

A thoughtful analysis of synoptic charts and vertical sections of the atmosphere brings us to the conclusion that alongside of areas filled with homogeneous air masses, characterized by slow and gradual change in the meteorological elements, there occur frequently in the atmosphere narrow transition zones which divide homogeneous air masses, and are characterized by a rapid change in most of the meteorological elements.

Usually these transition zones are so narrow that on small scale maps they are presented diagrammatically as lines along which the meteorological elements change abruptly, i.e. they sustain a discontinuity. Such a diagrammatic method, widely used in synoptic practice, is very convenient for a theoretical study of the characteristics of transition zones.

So that, instead of transition zones with a rapid change in meteorological elements, we will contemplate the discontinuity surfaces of these elements dividing two air masses. The results obtained by the use of this concept the more precisely conform with reality, the more abrupt the change in the meteorological elements in the transition zone, which was presented diagrammatically as a surface of discontinuity.

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Figure 106 represents a scalar plane temperature field. At the entry to, as well as at the exit from the layer of rapid temperature change, the isotherms are strongly refracted; they are, however, not discontinued. If the temperatures at two infinitely close to each other points in this layer, are measured, the temperature values obtained will differ infinitesimally. If, however, there is a discontinuity surface in the temperature field, the isotherms, in approaching this surface at some section, will unite with it, then will leave it from the other side at a point non-coincidental with the point of entry. The difference in the temperatures at two infinitely close to each other points, lying to different sides of a discontinuity surface, will have a finite value.

Figure 106. Scalar plane temperature field: (a) discontinuity of temperature, (b) layer of rapid change in temperature.

Figure 107 represents a vectorial plane velocity field. In passing through the layer of the rapid change in velocity, the lines of the stream are curved, while, in passing through the surface of discontinuity, they break.

As will be shown below, surfaces of discontinuity, so significant in meteorology, are at the same time surfaces of separation for two air masses. Examples of such surfaces are fronts,

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inversions, the tropopause. All these surfaces have the characteristic of becoming displaced together with the air masses that are separated by them, neither overtaking them nor lagging behind them. In any case, particles of one air mass will always lie to one side of the discontinuity surface, while the particles to the other side of the discontinuity surface, will always belong to the other air mass. It is natural to assume that the discontinuity surface for a long time consists of the same particles of air which are being displaced in space, together with the discontinuity surface.

Figure 107. Vectorial plane velocity field: (a) layer of rapid change in velocity; (b) discontinuity of velocity.

Alongside of such surfaces of discontinuity, which separate the air masses, there may be propagated in the atmosphere surfaces of discontinuity with entirely different characteristics. Examples of such surfaces are the discontinuity surfaces generated by the flight of an artillery shell, a blast, or sound waves. These discontinuity surfaces are characterized by the fact that their displacement velocity in space is not tied in with the velocity of the air particles, but exceed the latter many times over. It is obvious that these surfaces at every new moment of time will consist of new air particles.

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From a mathematical viewpoint, discontinuities are of this or of that order according to whether discontinuity is sustained by the meteorological element itself, or its derivatives.

Let Φ be some meteorological element. If at some surface the function Φ proper sustains a discontinuity this surface is called a discontinuity surface of the zero order. If, however, in passing through a surface, Φ is changing continuously but a discontinuity is sustained by the first derivative of Φ, through some argument, we have a discontinuity surface of the first order. Similarly a discontinuity surface of the second order, etc., can be defined. In the atmosphere we usually deal with the elements proper or their gradients. We will, therefore, limit ourselves to the consideration of discontinuities of the zero and the first orders.

In the following sections we will show that front surfaces are discontinuity surfaces of the first order for pressure, and discontinuity surfaces of the zero order for temperature, density, specific volume, wind velocity.

The tropopause which separates the air masses of the troposphere from those of the stratosphere, is a discontinuity surface of the first order for temperature, density, specific volume, and wind, and is not a discontinuity surface of the zero, or the first order for pressure.

Section 2. Dynamic and Kinematic Conditions.

It is usually assumed that, at the meteorologically

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significant discontinuity surfaces, pressure and the normal component of velocity do not sustain a discontinuity. This being of cardinal importance, we shall clarify it in more detail.

For simplification we consider a uniform flow of an ideal compressible liquid in a cylindrical pipe of unitary cross-section. Let us guide the x-axis along the axis of the pipe. Let the velocity of each particle of the liquid be parallel to the x-axis. Since the motion, by condition is uniform all the values such as density ρ , pressure p, velocity u will be functions of x and t only.

In a general case we must consider the discontinuity surface as being displaced in the pipe at a velocity c_F with relation to a stationary system of coordinates and at a velocity c_R , with relation to the liquid, it being necessary to assume, generally speaking, that c_R has different values to the different sides of the discontinuity surface.

If there were no discontinuity in the liquid, the uniform flow could have been expressed by two equations - the equation of Euler

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + X \qquad (1)$$

and the equation of discontinuity

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x} = 0. \qquad (2)$$

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In the presence of a discontinuity surface, the equations, expressing the motion of the liquid, will contain in addition to u, ρ, p, which, generally speaking, have different values to the different sides of the discontinuity surface ($u_1 \rho_1 p_1$ - to the right, $u_2 \rho_2 p_2$ - to the left of it), also the velocity of the discontinuity surface c_R or c_F.

Figure 108. Derivation of an analogue to the equation of discontinuity.

With the aid of Figure 108, a tabulation describing the displacement of the discontinuity surface can be easily made:

Moment of time	t	$t+\Delta t$
Coordinates of the discontinuity surface	x	$x+c_F\, dt$
Coordinates of the liquid particles forming discontinuity surface S	x	$x+u_2 \Delta t$
Coordinates of the liquid particles which formed the discontinuity surface S'	$x+c_F \Delta t-u_1 \Delta t$	$x+c_F \Delta t$
The mass of liquid contained in volume, which is delimited by surfaces consisting of liquid particles	$\rho_1(c_F-u_1)\Delta t$	$\rho_2(c_F-u_2)\Delta t$
The amount of motion of this mass of liquid	$\rho_1(c_F-u_1)\Delta t.u_2$	$\rho_2(c_F-u_2)\Delta t.u_1$

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Since the mass contained between the surfaces which consists of the same particles, remains continuous, we can write:

$$\rho_1 (c_F - u_1) = \rho_2 (c_F - u_2) \tag{3}$$

or, by interpolating the propagation velocity (c_2) of the discontinuity surface,

$$\rho_1 c_{R1} = \rho_2 c_{R2}. \tag{4}$$

Equation (4), tying in the propagation velocity of the discontinuity surface with the density of the liquid is, as can be seen, an analogue to the equation of discontinuity.

In order to derive in the presence of a discontinuity surface the equation of the motion of the liquid, which is analogous to the equation of Euler, we shall utilize the law of conservation of the quantity of motion. The change in the quantity of motion is equal (taking into consideration (3)) to

$$\rho_2 (c_F - u_2) \Delta t \cdot u_1 - \rho_1 (c_F - u_1) \Delta t \cdot u_2 = \rho_2 (c_F - u_2)(u_1 - u_2) \Delta t.$$

The effective forces are the pressures applied to sections S and $\tilde{S}$, perpendicular to the pipe axis, and external forces.

By virtue of the smallness of the contemplated volume, the impulse of the external forces can be disregarded, since the sum of the external forces will be in this case a magnitude of the second order of smallness. Therefore, the impulse of all the forces can be taken as equal to $(P_1 - P_2)\Delta t$.

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$$\rho_2 (c_F - u_2) \Delta t \cdot u_1 - \rho_1 (c_F - u_1) \Delta t \cdot u_2 = \rho_2 (c_F - u_2)(u_1 - u_2) \Delta t.$$

The effective forces are the pressures applied to sections S and $\bar{S}$, perpendicular to the pipe axis, and external forces.

By virtue of the smallness of the contemplated volume, the impulse of the external forces can be disregarded, since the sum of the external forces will be in this case a magnitude of the second order of smallness. Therefore, the impulse of all the forces can be taken as equal to $(P_1 - P_2)\Delta t$.

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Therefore, by the theorem on the quantity of motion, we derive:

$$\rho_2 (c_F - u_2)(u_2 - u_1) = p_1 - p_2 . \tag{5}$$

This is the analogue to the equation of Euler. Substituting

$$u_1 = c_F - c_{R1} = c_F - \frac{\rho_2}{\rho_1} c_{R2} ,$$

$$u_2 = c_F - c_{R2} \tag{6}$$

into (5), we derive:

$$c_{R2} = \sqrt{\frac{\rho_1}{\rho_2} \frac{p_1 - p_2}{\rho_1 - \rho_2}}$$

$$c_{R1} = \sqrt{\frac{\rho_2}{\rho_1} \frac{p_2 - p_1}{\rho_2 - \rho_1}} , \tag{7}$$

Let us evaluate the order of magnitude for the propagation velocity of the discontinuity surface c_R, i.e. its displacement velocity with relation to the particles of the liquid.

For simplification, we assume that the liquid is barotropic, that is, that:

$$p = f(\rho).$$

Then, by the theorem of finite increments, we obtain:

$$\frac{p_1 - p_2}{\rho_1 - \rho_2} = \frac{f(\rho_1) - f(\rho_2)}{\rho_1 - \rho_2} = f'(\tilde{\rho}),$$

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where $\tilde{\rho}$ is some average value of ρ, contained between ρ_1 and ρ_2.

In the adiabatic process

$$p = p_0 \left(\frac{\rho}{\rho_0}\right)^{\kappa},$$

where $\kappa = \frac{c_p}{c_v}$. Then:

$$f'(\tilde{\rho}) = \kappa \frac{\tilde{p}}{\tilde{\rho}}$$

$$c_{R2} \approx \sqrt{\frac{\kappa p}{\rho}}. \qquad (8)$$

The expression in the right hand part, as is known, determines the velocity of sound. Under normal atmospheric conditions, its magnitude is 332 meters per second.

Thus, the velocity of propagation of the discontinuity surface is a magnitude of the order of velocity of sound. In meteorology, such velocities are not significant. Therefore, in declining to consider discontinuity surfaces, which are propagating at velocities of sound, we must assume:

$$c_{R2} = c_{R1} = 0,$$

i.e. we must consider only such discontinuity surfaces which are displaced together with the liquid and consequently consist at all times of the same particles. Thus, the discontinuity surfaces which are contemplated in meteorology are at the same time the surfaces of division, it being the case that they at all times separate

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the same air masses. These surfaces are propagated at velocities considerably below the velocity of sound.

From the fact that for surfaces of separation $c_R = 0$ it follows that

$$p_1 = p_2 = p. \tag{9}$$

Such is the dynamic condition which is almost fulfilled for surfaces of separation.

If $c_R = 0$, then in accordance with (6),

$$c_F = u_1; \quad c_F = u_2.$$

consequently,

$$u_1 = u_2 \quad or \quad v_{n^1} = v_{n^2}, \tag{10}$$

i.e. the normal component of velocity, in passing through the surface of separation, remains continuous.

Since velocity v, in passing through the surface of separation, sustains a discontinuity, while its normal component remains continuous, it is clear that the discontinuity is sustained by the tangent component v_t of velocity.

In conclusion we will ascertain the rationality of equality (10) in the case of a layer in which a rapid change of meteorological elements occurs.

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Let us delimit the transition zone by two surfaces S_1 and S_2 (Figure 109). Such boundaries are, of course, only conditional, in reality, we cannot point to them. However, for our argumentation, this conditionality has no essential significance. It is obvious, that if

$$v_{n1} = v_{n2}$$

the transition zone, with the rapid change in meteorological elements is being displaced without changing its width.

If $v_{n2} > v_{n1} > 0$, the boundary S_1 becomes displaced more rapidly than the boundary S_2, which means that the transition zone becomes narrower and, consequently, the discontinuity is accelerated.

If $v_{n1} < v_{n2}$, the widening of the transition zone, and the deceleration of the discontinuity occur.

Figure 109. The acceleration and deceleration of discontinuity in the transition zone.

Section 3. The Incline of the Surface of Separation.

The most important synoptic application of the theory of discontinuity surfaces, is tied with the question relating to the incline of the surface of separation. We will first consider the

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simplest case of a stationary surface of separation. Let the two air masses, a warm one M_1, and a cold one M_2, be contiguous. Assuming for simplification that the surface separating them is a plane surface, we can prove that the angle of incline of the stationary separation surface to the horizon, cannot assume any arbitrary values, since it is tied in by a definite correlationship with the discontinuity in velocity and density.

We will first assume that both air masses are at a standstill with relation to the earth. It is obvious that the system consisting of these two masses M_1 and M_2, will be in stable equilibrium only [redacted] when the surface of separation is a horizontal plane, with the cold mass lying under the warm mass.

The surface of separation of two masses at a standstill cannot be vertical. Although each one of the masses separately is in a state of equilibrium the entire system consisting of both masses with a vertical surface of separation cannot be in a state of equilibrium. Even with the same pressure below there will be a difference in pressures at the separation surface of the two contiguous masses due to the different change of pressure with height, which will result in the overflow of the warm air onto the cold air and the underflow of the cold air under the warm air. Therefore, in the final analysis the cold mass M_2 will find itself underneath the warm mass M_1.

If, at least, one mass is in motion the separation surface will not remain horizontal but will be inclined at some very small angle to the horizon, it being the case that under certain conditions

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this angle will not change with time.

Let us now develop the formulas which determine this angle of incline of the surface of separation.

As proved above the pressure in passing through the surface of separation must remain continuous. Therefore, the equation of the surface of separation can be taken as

$$p_1 - p_2 = 0 \tag{1}$$

or in its differential form:

$$\left(\frac{\partial p_1}{\partial x} - \frac{\partial p_2}{\partial x}\right)dx + \left(\frac{\partial p_1}{\partial y} - \frac{\partial p_2}{\partial y}\right)dy + \left(\frac{\partial p_1}{\partial z} - \frac{\partial p_2}{\partial z}\right)dz = 0, \tag{2}$$

whence by designating α_x, α_y the angles formed by the separation surface with the axes x, y and by θ, - the angle formed with the x-axis which is tangent to the front, i.e. to the line at which the surface of separation crosses the surface of the earth we derive:

$$\left.\begin{aligned} \tan\alpha_x &= \frac{dz}{dx} = -\frac{\frac{\partial p_1}{dx} - \frac{\partial p_2}{\partial x}}{\frac{\partial p_1}{\partial z} - \frac{\partial p_2}{\partial z}}, \\ \tan\alpha_y &= \frac{dz}{dy} = -\frac{\frac{\partial p_1}{\partial y} - \frac{\partial p_2}{\partial y}}{\frac{\partial p_1}{\partial z} - \frac{\partial p_2}{\partial z}} \\ \tan\theta &= \frac{dy}{dx} = -\frac{\frac{\partial p_1}{\partial x} - \frac{\partial p_2}{\partial x}}{\frac{\partial p_1}{dy} - \frac{\partial p_2}{dy}} \end{aligned}\right\} \tag{3}$$

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Orientating the coordinate axes so that the y-axis is directed along a tangent to the front we have $tan\,\theta = \infty$, [redacted] whence [redacted]

$$\frac{\partial p_1}{\partial y} - \frac{\partial p_2}{\partial y} = 0 \qquad (4)$$

and consequently $tan\,\alpha_y = 0$. To determine the angle of incline α of the surface of separation to the horizon we derive

$$tan\,\alpha = -\frac{\frac{\partial p_1}{\partial x} - \frac{\partial p_2}{\partial x}}{\frac{\partial p_1}{\partial z} - \frac{\partial p_2}{\partial z}} . \qquad (5)$$

Thus the incline of the surface of separation is determined exclusively by the distribution of pressure at the front, it being the case that the angle of incline is the greater, the greater the discontinuity of the normal to the front pressure gradient component, and the lesser the discontinuity of the vertical pressure gradient.

Let us note that equation (5) is completely general and rigid. Below we will derive a series of formulas not completely rigid but convenient for application.

Interpolating, in the place of the vertical component of the pressure gradient, density, we have as per the basic equation of statics:

$$\frac{\partial p_1}{\partial z} = -g\rho_1 ; \quad \frac{\partial p_2}{\partial z} = -g\rho_2 .$$

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In the place of the horizontal component of the pressure gradient, we will interpolate the angle of incline β of the isobaric surface. Since:

$$\frac{\partial p_1}{\partial z} = -\frac{\partial p_1}{\partial z}\left(\frac{dz}{dy}\right)_{p_1} = g\rho_1 \tan\beta_1,$$

$$\frac{\partial p_2}{\partial y} = -\frac{\partial p_2}{\partial z}\left(\frac{dz}{dy}\right)_{p_2} = g\rho_2 \tan\beta_2,$$

then:

$$\tan\alpha = \frac{\rho_2 \tan\beta_2 - \rho_1 \tan\beta_1}{\rho_2 - \rho_1} \qquad (6)$$

Usually $\tan\alpha \gg \tan\beta$ consequently the surface of separation rises at a considerably steeper angle than the isobaric surfaces. If the motion of the air is geostrophic it is convenient to express the incline of the surface of separation through the temperature and the wind velocity at that surface. We again orientate the coordinate axes in such a way that the y-axis is parallel to the front. Equations of the geostrophic wind for the warm and cold masses will be as follows:

$$-2\omega \sin\varphi v_1 = -\frac{1}{\rho_1}\frac{\partial p_1}{\partial x}, \quad -2\omega \sin\varphi v_2 = -\frac{1}{\rho_2}\frac{\partial p_2}{\partial x},$$

$$2\omega \sin\varphi u_1 = -\frac{1}{\rho_1}\frac{\partial p_1}{\partial y}, \quad 2\omega \sin\varphi u_2 = -\frac{1}{\rho_2}\frac{\partial p_2}{\partial y},$$

$$\frac{1}{\rho_1}\frac{\partial p_1}{\partial z} - g = 0, \qquad -\frac{1}{\rho_2}\frac{\partial p_2}{\partial z} - g = 0.$$

Substituting these equations into formula (5) we derive:

$$\tan\alpha = \frac{2\omega \sin\varphi}{g}\,\frac{v_1\rho_2 - v_2\rho_1}{\rho_2 - \rho_1}. \qquad (7)$$

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Equation (7) is not convenient for application since density is not measured in meteorology. Eliminating density with the aid of the equation of state $\rho = \frac{p}{RT}$, and utilizing the dynamic condition $p_1 = p_2$ at the discontinuity surface we derive the so-called Margules formula:

$$\tan \alpha = \frac{2\omega \sin \varphi}{g} \frac{T_1 v_2 - T_2 v_1}{T_1 - T_2} \qquad (8)$$

The air humidity may be taken into account by interpolating into equation (8) the actual temperature value.

Let us derive the formula for the approximate determination of the angle of incline of the surface of separation by interpolating the discontinuities in temperature and wind velocity.

$$\Delta T = T_1 - T_2; \quad \Delta v = v_1 - v_2 .$$

Then

$$\tan \alpha = \frac{2\omega \sin \varphi}{g} \left(v_2 - T_2 \frac{\Delta v}{\Delta T} \right) \qquad (9)$$

or

$$\tan \alpha = \frac{2\omega \sin \varphi}{g} \left(v_1 - T_1 \frac{\Delta v}{\Delta T} \right). \qquad (10)$$

Thus the incline of the surface of separation is tied in with the incline of the isobaric surface at any one of the contemplated air masses and also with the discontinuity in velocity and wind

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[sic. should read: "discontinuity in temperature and wind velocity]. Since the angle of incline of the isobaric surface, as we shall see below, is considerably smaller than the angle of incline of the surface of separation, the first terms within the parentheses of equations (9) and (10) may be disregarded. Then:

$$\tan\alpha \cong -\frac{2\omega \sin\varphi}{g} \cdot T \cdot \frac{\Delta v}{\Delta T} \qquad (11)$$

where T is the mean temperature.

Assuming that $\omega \cong 7.3 \cdot 10^{-5}$, $T = 275$, $g \cong 10\ m/sec^2$ we derive the following formula for the computation of the angle of incline α:

$$\tan\alpha = 4 \cdot 10^{-3} \sin\varphi \frac{\Delta v}{\Delta T}, \qquad (12)$$

where Δv is expressed in meters per second, and ΔT in degrees centigrade.

For a rapid determination of the angle of incline a nomogram computed by formula (11) can be used. The magnitude of the angle of incline of the surface of separation is several tens of minutes, rarely attaining one degree, while the isobaric surface is inclined at an angle of several tens of seconds, rarely attaining 2-3 minutes in value. Therefore, under usual conditions the incline of the isobaric surface is approximately 30 times smaller in magnitude than the incline of the surface of separation. This justifies the assumption previously made while deriving equation (11).

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Section 4. A Surface of Separation In a Pressure Field.

We have proved above that when the y-axis is parallel to the front, then:

$$\frac{\partial p_1}{\partial y} = \frac{\partial p_2}{\partial y} . \qquad (1)$$

This means that the pressure gradient component, tangent to the surface of separation in passing through the front changes continuously.

As to the pressure gradient component normal to the front, it sustains a discontinuity, it being the case that

$$\frac{\partial p_1}{\partial x} \gtrless \frac{\partial p_2}{\partial x}$$

with relation as to whether angle α is acute or obtuse, i.e. as to whether the x-axis is directed away from the less dense (warm) mass or toward the more dense (cold) mass, or the other way around.

If angle α is acute, i.e. the x-axis is directed from the warm into the cold air mass, then

$$\frac{\partial p_1}{\partial x} < \frac{\partial p_2}{\partial x} \qquad (2)$$

and when angle α is obtuse:

$$\frac{\partial p_1}{\partial x} > \frac{\partial p_2}{\partial x} \qquad (3)$$

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This means that the isobars at the front become broken in such a way that the cusp always faces the high pressure side, and the isobar at the front has an infinitely great cyclonic curvature. In the vicinity of fronts the isobars are of a type shown in Figures 110 and 111.

Figure 110. Disposition of a separation surface in a pressure field with front being displaced. The upper row is an elevation view, the bottom is a plan view.

Figure 111. Disposition of a separation surface in a pressure field, with front stationary. The upper row is an elevation view, the bottom - is a plan view.

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From equation (5) in Section 3, it also follows that isobaric surfaces break at the surfaces of separation forming a trough of infinitely great curvature.

It should be mentioned that the incline of isobaric surfaces is so small that a break in them will be noticeable only when a vertical scale many times greater than the horizontal scale is used.

The types of isobars and isobaric surfaces which may be disposed in the vicinity of a front are depicted in Table 45.

Table 45.

Types of isobars and isobaric surfaces in the vicinity of a front.

Type	$\frac{\partial p}{\partial y}$	$\frac{\partial p}{\partial \psi}$	α and β
A_1	$\frac{\partial p_1}{\partial y}=\frac{\partial p_2}{\partial y}$	$\frac{\partial p_2}{\partial \psi}>\frac{\partial p_1}{\partial \psi}>0$	$0<\beta_1<\beta_2<\alpha<\frac{\pi}{2}$
A_2		$\frac{\partial p_1}{\partial \psi}>\frac{\partial p_2}{\partial \psi}>0$	$0<\beta_2<\beta_1<\frac{\pi}{2}<\alpha<\pi$
B_1		$\frac{\partial p_1}{\partial \psi}<\frac{\partial p_2}{\partial \psi}<0$	$0<\alpha<\frac{\pi}{2}<\beta_1<\beta_2<\pi$
B_2		$\frac{\partial p_2}{\partial \psi}<\frac{\partial p_1}{\partial \psi}<0$	$\frac{\pi}{2}<\alpha<\beta_2<\beta_1<\pi$
C_1		$\frac{\partial p_1}{\partial \psi}<0<\frac{\partial p_2}{\partial \psi}$	$0<\beta_2<\alpha<\frac{\pi}{2}<\beta_1<\pi$
C_2		$\frac{\partial p_2}{\partial \psi}>0>\frac{\partial p_2}{\partial \psi}$	$0<\beta_1<\frac{\pi}{2}<\alpha<\beta_2<\pi$
D_1	$\frac{\partial p_1}{\partial y}=\frac{\partial p_2}{\partial y}=0$	$\frac{\partial p_2}{\partial \psi}>\frac{\partial p_1}{\partial \psi}>0$	$0<\beta_1<\beta_2<\alpha<\frac{\pi}{2}$
D_2		$\frac{\partial p_1}{\partial \psi}>\frac{\partial p_2}{\partial \psi}>0$	$0<\beta_2<\beta_1<\frac{\pi}{2}<\alpha<\pi$
E_1		$\frac{\partial p_1}{\partial \psi}<\frac{\partial p_2}{\partial \psi}<0$	$0<\alpha<\frac{\pi}{2}<\beta_1<\beta_2<\pi$
E_2		$\frac{\partial p_2}{\partial \psi}<\frac{\partial p_1}{\partial \psi}<0$	$\frac{\pi}{2}<\alpha<\beta_2<\beta_1<\pi$
F_1		$\frac{\partial p_1}{\partial \psi}<0<\frac{\partial p_2}{\partial \psi}$	$0<\beta_2<\alpha<\frac{\pi}{2}<\beta_1<\pi$
F_2		$\frac{\partial p_2}{\partial \psi}>0>\frac{\partial p_2}{\partial \psi}$	$0<\beta_1<\frac{\pi}{2}<\alpha<\beta_2<\pi$

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Let us consider now the equation of a front derived from the condition of continuity in the change of pressure in passing through a surface of separation:

$$p_1(x, y, t) - p_2(x, y, t) = 0.$$

Accepting a line normal to the front as the x-axis for the individual particle which is being displaced together with the front we have:

$$\frac{d(p_1 - p_2)}{dt} = \frac{\partial(p_1 - p_2)}{\partial t} + u\frac{\partial(p_1 - p_2)}{\partial x} + v\frac{\partial(p_1 - p_2)}{\partial y} = 0.$$

But

$$\frac{\partial p_1}{\partial y} = \frac{\partial p_2}{\partial y}.$$

Consequently,

$$u = -\frac{\frac{\partial(p_1 - p_2)}{\partial t}}{\frac{\partial(p_1 - p_2)}{\partial x}}. \qquad (4)$$

But in Section 2 it was clarified that the normal component of the velocity of an individual particle contiguous to the front must equal the velocity of front displacement c_f, therefore also

$$c_f = -\frac{\frac{\partial p_1}{\partial t} - \frac{\partial p_2}{\partial t}}{\frac{\partial p_1}{\partial x} - \frac{\partial p_2}{\partial x}}. \qquad (5)$$

Above it was shown that $\frac{\partial p_1}{\partial x} - \frac{\partial p_2}{\partial x} \neq 0$. Therefore, if the front is being displaced at a velocity $c_f \neq 0$, then, the numerator in formula (5) is different from zero. Let us select the

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coordinate axes so that c_f is positive and let us ascertain the variations in barometric tendency during the passing of the front. In the case of a warm front we have:

$$\frac{\partial p_1}{\partial t} - \frac{\partial p_2}{\partial t} = -c_f\left(\frac{\partial p_1}{\partial x} - \frac{\partial p_2}{\partial x}\right), \qquad (6)$$

in the case of a cold front:

$$\frac{\partial p_2}{\partial t} - \frac{\partial p_1}{\partial t} = c_f\left(\frac{\partial p_1}{\partial x} - \frac{\partial p_2}{\partial x}\right). \qquad (7)$$

The approach of a type A front (see Figure 112) will be accompanied by a drop in pressure and after the passing of the front the drop in pressure will be retarded or completely stopped.

The approach of a type B front will be accompanied by an increase in pressure and after the passing of the front the increase in pressure will be accelerated.

Finally in the case of the passing of a front of the C type the considerable drop in pressure will be replaced by a noticeable increase in same.

The nature of the change in pressure during the passing of fronts of the above described types is presented in Figure 112.

Figure 112. Characteristics of the barometric tendency which is predicated on the transfer of a pressure field during the passing of a front.

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Such would be the character of the change in pressure during the passing of a front if it were predicated exclusively upon the transfer of a pressure field together with the front. However, in addition to being transferred together with the front, the pressure field changes at each of its points also under the effect of other causes. Therefore, the nature of the curves representing the barometric tendency changes essentially. However, in all cases of passing of a front the typical broken line with its apex downward is preserved in the barogram.

Figure 113. Types of barometric tendencies during the passing of fronts: a, b, c - for warm fronts; d, e, f - for cold fronts.

All the above said pertains only to a discontinuity surface of the first order for pressure, i.e. to the frontal surfaces of the troposphere. It cannot be applied in the case of the tropopause. It is easily proved that the tropopause is a discontinuity surface of pressure of an order higher than the first. Aerological investigations established the fact that at the tropopause the temperature changes continuously with only the temperature gradient sustaining a discontinuity. From the equation of state $\rho = \frac{p}{RT}$ it follows that density, too, does not sustain a discontinuity at the tropopause. Consequently, $\rho_1 = \rho_2$ and $\frac{\partial p_1}{\partial z} = \frac{\partial p_2}{\partial z}$. Since $\tan\alpha$ does not identically equal infinity, the numerator of the formula (5) in Section 3 must become zero. Consequently:

$$\frac{\partial p_1}{\partial y} = \frac{\partial p_2}{\partial y}.$$

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Thus the isobars do not break at the [redacted] line of crossing of the tropopause with a horizontal surface but have as compared with contiguous points a maximum finite cyclonic curvature. Analogously at the tropopause:

$$\frac{\partial p_1}{\partial t} = \frac{\partial p_2}{\partial t}.$$

Only the secondary pressure derivatives will sustain a discontinuity at the tropopause, i.e. the tropopause itself is a discontinuity surface of the second order, with relation to pressure.

Section 5. Surfaces of Separation in a Geostrophic Wind Field.

We will now ascertain the distribution of geostrophic wind in the vicinity of a front. As per equation (1) in Section 4, we have $\rho_1 u_1 = \rho_2 u_2$ or $\Delta u = u_1 - u_2 \approx 0$, that is, the velocity component normal to the front does not sustain a discontinuity. From equation (11) in Section 3 it follows that:

$$\Delta v = v_1 - v_2 = -\frac{g}{2\omega \sin\varphi} \frac{T_1 - T_2}{T} \cdot \tan\alpha \qquad (1)$$

i.e. the tangent component of the wind sustains a discontinuity if the temperature (or density) sustains a discontinuity. Consequently, the frontal surface for the wind is a discontinuity surface of the zero order.

The magnitudes $g, \omega, T_1 - T_2, T$, located in the right-hand part of equation (1) are essentially positive by virtue of which the sign of Δv depends only upon the signs of $\sin\varphi$ and $\tan\alpha$.

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Selecting the x-axis so that $\tan\alpha > 0$ for the warm front we derive the following correlationships:

Northern hemisphere	$\tan\alpha > 0$	$v_1 - v_2 < 0$
$\varphi > 0$	$\tan\alpha < 0$	$v_1 - v_2 > 0$
Southern hemisphere	$\tan\alpha > 0$	$v_1 - v_2 > 0$
$\varphi < 0$	$\tan\alpha < 0$	$v_1 - v_2 < 0$

Figure 110 shows a diagram of the values of the geostrophic wind at a front. We see that the discontinuity of the wind has cyclonic characteristics both for the warm and the cold fronts. We thus obtain a result which is in conformity with the behavior of the isobars in the vicinity of a front. It must be kept in mind, however, that the rule derived for the isobars was derived with complete rigidity. It is, therefore, true for all fronts without exception while the rule for the wind was derived only after a series of simplifying assumptions, and is, therefore, less rigid.

In individual cases it may occur that the horizontal accelerations are so significant that they will change the cyclonic rotation of the wind at the front into an anticyclonic rotation. However, such cases are rare and of short duration.

Thus, any front may be detected by the cyclonic rotation of the wind, irrespective of the circumstance as to what type of air mass lies to the right of the front, and what type of air mass lies to the left of it. In determining the relative displacement of air masses at a front the following rule may be followed:

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"If the observer facing the front is being displaced together with one air mass, the second air mass lying beyond the front will be displaced, with relation to the observer, to the left." This postulate is true in the case of any air mass and any front.

Let us finally note that during the passing of a front the direction of the wind at a given point changes in such a way that "the wind turns to the right". (Figure 114).

Figure 114. During the passing of any front the wind with time turns to the right.

However, even though a front is accompanied by a wind rotation, it does not follow that any rotation line of the wind is a front. A line of rotation of the wind will be a front only in that case when there is present a locality of discontinuity of density or, what is the same thing, a discontinuity of temperature.

Section 6. Change in Gradient Wind with Height, in the Area of a Front.

Equation (1) in Section 5 may be applied to the analysis of wind distribution along a vertical line. This formula shows that in its ascent through a frontal surface the gradient wind changes with a sudden shift, it being the case that the vector of wind acceleration in passing from a cold mass into a warm mass is in

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direct ratio to the incline of the front $\tan \alpha$ and to the jump in temperature at the front and is directed parallel with the line of the front in such a manner that the wedge of cold air remains to the left and the warm air remains to the right. This rule follows from the analysis of the problem on the rotation of the gradient wind, with height in a homogeneous air mass (Chapter XI).

In the case of a warm front $\tan \alpha > 0$ (Figure 115, A), and, consequently $\Delta v = v_1 - v_2 < 0$, i.e., before a warm front the gradient wind sustains a discontinuity in such a way that the rotation of the wind with height in the area of a front is to the right (Figure 115, A). In the case of a cold front, $\tan \alpha < 0$, and

$$\Delta v = v_1 - v_2 > 0$$

Therefore, behind a cold front the rotation of the wind with height at the frontal surface is to the left (Figure 115, C).

Figure 115. Change in wind with height in the vicinity of a front: (a) warm front, (b) warm air mass; (c) cold front.

W — — — — Tropopause — — — — — — — — — — E

Figure 116. Distribution of wind at the tropopause.

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If the front is stationary, the gradient wind with height becomes decelerated with a sudden shift, retaining the direction of the cold current, or becomes accelerated with a sudden shift, or, finally, reverses its direction with a sudden shift.

Equation (1) in Section 5 shows that sharply accentuated fronts with a significant discontinuity of temperature and a steep incline are accompanied by a considerable change in the gradient wind with height. And, inversely, fronts mildly accentuated in a temperature field, and with small angles of incline are accompanied by a mild shift in wind with height.

Thus, radiosonde observations can be used for the determination of the location of fronts in the free atmosphere. But, together with that, as already mentioned above a sudden shift in wind will indicate the presence of a front only when a discontinuity in density (or temperature) occurs. Therefore, radiosonde observations can only be used as a supplement to temperature sounding.

We shall also note that when the wind with height rotates to the right the height of the frontal surface is increased in the direction of the wind (Figure 115, A). When the wind with height rotates to the left the height of the frontal surface decreases in the direction of the wind (Figure 115, C).

We have seen above that the tropopause is a discontinuity surface of the first order with relation to density and the pressure gradient. Therefore, the tropopause is a discontinuity surface of the first order, also with relation to wind velocity, i.e., the velocity of the wind at the tropopause will change continuously.

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As is known, the tropopause under normal conditions is inclined to the Pole. Figure 116 shows the possible distribution of the wind with height in the vicinity of the tropopause.

Section 7.

Disposition of a Surface of Separation in a Non-Stationary Wind Field.

Let us assume that at an initial moment a certain surface of separation is inclined to the horizon more steeply than a stationary surface of separation, the inclination of which is determined from the formula (8) in Section 3, i.e., in Figure 117.

$$\tan\alpha > \tan\alpha_0 .$$

Figure 117. Evolution of accelerations in the vicinity of a non-stationary surface of separation, with $\alpha > \alpha_0$.

Let us assume in addition that no perturbations are acting upon the currents. An elementary but cumbersome analysis shows that in this case a discontinuity of the acceleration components, normal to the front, occurs, it being the case that

$$\dot{u}_1 - \dot{u}_2 = \frac{\Delta T}{T} \cdot g \cdot (\tan\alpha - \tan\alpha_0) > 0$$

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Together with this near the frontal surface there also occurs a discontinuity of the vertical components of acceleration

$$\dot{w}_1 - \dot{w}_2 = \frac{\Delta T}{T} \cdot g(\tan\alpha - \tan\alpha_0) > 0.$$

This means that the air masses which were previously being displaced parallel to the surface of separation sustain non-uniform accelerations in a direction of a normal to the front, it being the case that the algebraic accelerations in the warm mass will be greater than the same in the cold mass. Together with this, the vertical accelerations in the warm mass will also be greater than in the cold mass.

The following three cases may occur:

1) $\dot{u}_1 > \dot{u}_2 > 0$; $\dot{w}_1 > \dot{w}_2 > 0$; $\dot{v}_1 < \dot{v}_2 < 0$;

2) $\dot{u}_1 > 0 > \dot{u}_2$; $\dot{w}_1 > 0 > \dot{w}_2$; $\dot{v}_1 < 0 < \dot{v}_2$;

3) $0 > \dot{u}_1 > \dot{u}_2$; $0 > \dot{w}_1 > \dot{w}_2$; $0 < \dot{v}_1 < \dot{v}_2$.

In the first case both masses are being displaced in the direction of the cold air with the velocity of the warm air accelerating more rapidly than the velocity of the cold air.

This motion is accompanied by the appearance and accretion of vertical velocities, with the vertical velocities for the warm air greater than the same for the cold air. Therefore, in both the warm and cold air there will develop vertical velocities, with the velocity of the warm air greater than that of the cold air. It follows from this that in case No. 1, we are dealing with a warm front. The oncoming warm air displaces the receding cold air with

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ascending currents developing in both the cold and the warm air (the ascending warm air overtakes the ascending cold air).

The emerged normal components of velocity induce tangential accelerations which are predicated on the rotation of the earth. These supplementary accelerations in the first case have the same direction, with the absolute value of the warm mass acceleration greater than that of the cold mass and with the warm air remaining to the right with relation to the cold air.

In case No. 2, the air masses being considered are being displaced toward each other, i.e., both masses are moving toward the front. It being understood that the front proper will not undergo displacement only in the case when the accelerations of the air masses in their absolute values are equal to each other. If the accelerations in their absolute values are not the same, the front will undergo displacement in the direction of the cold mass (when $|\dot{u}_1| > |\dot{u}_2|$), and in the direction of the warm mass (when $|\dot{u}_1| < |\dot{u}_2|$). In conformity with this the front may be stationary (when $|\dot{u}_1| = |\dot{u}_2|$), warm (when $|\dot{u}_1| > |\dot{u}_2|$) or cold (when $|\dot{u}_1| < |\dot{u}_2|$).

In this case regardless of the nature of the front an ascending motion will develop in the warm air and a descending one in the cold air.

Accelerations which are tangent to the front are of opposite directions and they contribute to the evolution of cyclonic motion at the front.

Finally in case No. 3, both air masses will be displaced in the direction of the warm mass with the cold mass sustaining a

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greater (in absolute value) acceleration than the warm mass. This motion will be accompanied by descending currents developing in both air masses, it being the case that the cold mass will be descending more rapidly than the warm mass.

The accelerations which are tangential to the front will again be in the same direction, with the acceleration of the warm mass in absolute value being lower than the acceleration of the cold mass.

The accelerations analyzed above result in that (1) either the warm air will overtake the receding cold air and rise at a greater vertical velocity along the wedge of the simultaneously rising cold air. Together with this the existing discontinuity in velocity at the front will increase, which circumstance entails a greater angle of inclination of the stationary surface of separation. (2) Or the warm air will flow into the wedge of the descending cold air encountered by it. In this case the present discontinuity in velocity will increase, which circumstance also entails a greater value in the angle of inclination of the stationary surface of separation. (3) Or the cold air will flow under the receding warm air, with the warm air in its receding, lagging in its vertical motion, behind the more rapidly descending cold air. The discontinuity in the tangential velocities will also become greater, which circumstance will entail for purposes of equilibrium a steeper angle of inclination of the surface of separation.

In all cases as a result of accelerations the angle of inclination of the surface of separation will be diminished while the

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intensified discontinuity in the tangential component of wind velocity is increased so that an excessively low-sloping surface of separation (Figure 118) is already not in conformity with conditions of equilibrium. In such a case accelerations having an inverted sign begin to develop at the surface of separation, which results in that either (1) both masses will be displaced in the direction of the warm mass, descending at the same time, with the warm mass moving more rapidly and overtaking the descending cold mass, or (2) both masses will flow in different directions, with the warm mass descending along the wedge of the ascending cold mass, or (3) both the warm and cold mass will move in the direction of the cold mass, ascending simultaneously, with the cold mass receding more rapidly, overtaking the warm mass along a vertical line.

Together with this in all the above cases the tangential components of the generated accelerations have anticyclonic characteristics, as a result of which, with time, the discontinuity in velocity at the front is gradually diminished, and may even change its sign. The angle of inclination of the surface of separation will increase while at the same time for a stationary surface the requisite angle of inclination will already be smaller. Thus, we returned to the initial postulate from which we began our analysis.

A more detailed investigation shows that a surface of separation oscillates somewhat about the position of equilibrium, which is not coincident with the position of a stationary surface but deflected from the latter in the same direction as is the initial position of a surface of separation.

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Figure 118. Evolution of accelerations in the vicinity of a non-stationary surface of separation, with $\alpha < \alpha_0$.

Section 8. Classification of Fronts.

The above described characteristics of fronts relating to their position in a pressure field, wind field, and temperature field, are applicable to both shifting fronts and stationary (motionless) fronts. By these characteristics the presence or absence of fronts in a synoptic chart or in a vertical section may be judged.

However, with relation to the movement of fronts and to the relative position of the warm and cold air masses and their stability it is possible to establish a series of additional characteristics for any given type of front.

Thus, it becomes possible to classify fronts by their characteristics. Such a classification relegates a contemplated front to its proper type.

Fronts may be classified by their geographic index. There are four basic types of air masses: arctic, polar, tropical and equatorial air.

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The Polar air is frequently separated from the Arctic air by an Arctic front. The Tropical air is separated from the Polar air by a Polar front.

The tropical air as it migrates toward the equator is gradually being transformed into equatorial air. Therefore, there is no front formation between equatorial and tropical air. However, when the equatorial air of the summer hemisphere encounters the equatorial air of the winter hemisphere, there is the formation of an Intra-Tropical front.

Thus, by geographical classification there are three primary permanent fronts: the Arctic, the Polar, and the Intra-Tropical fronts.

In addition to these primary fronts, there may from time to time within the boundaries of one main air mass form secondary fronts, provided that temperature and velocity distributions promote frontogenesis. Arctic, tropical and equatorial air masses are usually characterized by great horizontal homogeneity in temperature, as a result of which no secondary fronts are evolved within the boundaries of these air masses. In the polar air, however, secondary fronts are evolved very frequently between the "stale" and the "fresh" polar air, and also between the continental Polar air and the maritime Polar air.

Daily synoptic experience shows that most of the fronts found in synoptic charts are tied in with the primary fronts. A synoptic chart showing numerous secondary fronts usually indicates flaws in the analysis.

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Besides the general frontal characteristics, such as discontinuity in temperature, discontinuity in the tangential component of wind velocity and others which were discussed in the preceding text and which are applicable to all fronts, there are no other particular characteristics which would determine the geographical classification of the fronts. Nevertheless, the usefulness of such a classification is obvious. Since the main fronts predominate over the secondary ones, by the frequency of their emergence, by the vastness of the area occupied and by the duration of their existence, the clarification of weather data that is tied in with the existence of fronts for the purpose of a synoptic analysis is concerned, first of all, with the primary fronts. Only in the case when a particular weather feature under analysis cannot be explained by the existence of the main front, is attention to be drawn to the secondary fronts. In so doing it is recommended to pay particular attention to the clarification of the processes promoting the formation, as well as the disappearance, of these secondary fronts. The most essential classification of fronts is their classification with relation to the characteristic of the movement of the front and the migration of the air masses. Within this classification there are 4 types of fronts:

(1) A warm front, i.e. a front moving in such a way that the warm air displaces the cold air;

(2) A cold front, i.e. a front moving in such a way that the cold air displaces the warm air;

(3) A stationary front, i.e. a front that is motionless;

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in this case there is no displacement of one air mass by another;

(4) An occluded front, i.e. a front generated as a result of the warm air being forced out after a more rapidly moving cold front reaches a slowly moving warm front. An occluded front is generated when between the cold air receding before a warm front and the cold air advancing behind the cold front there exists a discontinuity in temperature.

It can be shown that the movement of a front is determined by the formula

$$c_f = u_1 - w_1 \cot\alpha = u_2 - w_2 \cot\alpha. \qquad (1)$$

When the vertical velocities w are very small it can be assumed with approximation that the velocity of the movement of the front is equal to the wind velocity component which is normal to the front. It is, of course, understood that in this case we are considering not the velocity of the surface wind but the velocity of the geostrophic wind which is observed above the friction level. Consequently a fast moving front must cross the isobars at an almost right angle, with the isobars themselves running adequately close to each other.

Stationary fronts must be parallel to the isobars. Strictly stationary fronts occur very rarely in the atmosphere. Very slowly moving fronts are encountered more frequently. Such a slow moving front, called a quasi-stationary front, greatly complicates the problem of weather forecasting, since its subsequent movement is

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determined by accelerations which are extremely difficult of evaluation.

Warm and cold fronts on synoptic maps are usually easily distinguished one from the other, even when the temperature differential by surface observations is not sharply expressed. The systems of ~~cloud formations~~ and hydro-meteoric phenomena for these two types of fronts are different. The difference between a warm and a cold front is also manifested in the field of a barometric tendency and in a pressure field.

An occluded front has a more complex structure due to the superimposition of a warm front upon a cold one. If the air behind a cold front is colder than the air ahead of it, the occlusion has the characteristics of a cold front. If temperature conditions are the reverse of the above the occlusion resembles a warm front. An occluded front usually leads to the evolution of an unstable cyclonic wave.

Finally fronts can also be classified by the characteristics of the vertical velocity distribution and the conditions of stability of the air masses contiguous to the front.

Observations show that frontal cloud systems are usually formed in warm air, which is obviously tied in with the fact that ascending velocities in a warm mass are greater than those in a cold mass. The latter circumstance is predicated on the fact that the cold air, confined from below by the terrestrial surface, and from above by the frontal surface, finds itself within more rigid kinematic boundaries than the warm air.

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Therefore, the classification of fronts or surfaces of separation is done with relation to the behavior of the warm air mass.

There are surfaces of up-slope sliding or anafronts when $w_1 > 0$, and surfaces of downslope sliding or katafronts, when $w_1 < 0$. Differentiating also warm $(c_f > 0)$, cold $(c_f < 0)$, and stationary $(c_f = 0)$ fronts and remembering correlationship (1), we obtain a considerable number of theoretically possible types of fronts.

We saw above that if a real surface of separation up-slopes more steeply than a stationary surface there will occur an underflow of cold air under the warm air and the displacement of the latter. Such a surface is called a surface of passive up-slope sliding. It is natural to connect this surface with a cold front since in this case the cold air actively spreads under the warm air lifting it upwards.

In the case of a warm front the warm air actively flows over the cold air, forces it away and displaces it, rising along the surface of separation. Therefore, a warm front is a surface of active up-slope sliding.

We also saw above that if the slope of a surface of separation is not as steep as the slope corresponding to a stationary surface there begins a flowoff of cold air from under the warm air and the displacement of the cold air by the descending warm air, while the receding cold air is rising. Such a surface of separation is called a surface of passive downslope sliding. It is obvious

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that a surface of passive downslope sliding may be connected with a warm front.

Finally if a surface of downslope sliding is tied in with a cold front the warm air is displaced in the same direction as the cold air overtaking it and descending along the surface of separation. Such a surface of separation is called a surface of active downslope sliding.

The above named four principal types of surfaces of sliding differ essentially from one another by various characteristics, first of all, by the angle of slope, then, also by the system of cloud formations and precipitation which are tied in with the front.

It follows from the above that a surface of passive up-slope sliding should have a maximum angle of slope, while a surface of passive downslope sliding - a minimum angle of slope. We arrive at this conclusion not only by the analysis of the accelerations which are generated at the surface of separation as a result of its deflection from a position which is conformant to stationary conditions. The effect of friction against the terrestrial surface which was up to now disregarded leads to the same result. Indeed in the case of a cold front the friction of the air against the earth will retard the displacement of the oncoming cold air, contiguous with the earth's surface, and the surface of separation will therefore have a greater angle of slope than in the absence of friction. At times under the effect of friction the angle of slope of the cold front in the lower atmospheric layers may be even in excess of 90 degrees, as a result of which the invasion of cold air,

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in this case first begins at a height and reaches the earth's surface only afterwards. In such a case the wedge of cold air resembles rather a shaft. In the case of a warm front, friction causes a decrease in the angle of slope of the surface of separation, it being the case that behind a warm front passing at a height extends a thin film of cold air several tens or hundreds of meters in height.

From what was said about the angles of slope of surfaces of sliding the conclusion can be drawn that a surface of passive up-slope sliding has the maximum angle of slope. Aerological sounding shows that the slope of the latter attains values on the order of 1/25 - 1/80. Then follow surfaces of active up-slope sliding with a slope of 1/50 - 1/200, - and surfaces of active downslope sliding. Surfaces of passive downslope sliding have a minimum slope, within the values of 1/400 - 1/1000.

A warm front is in a predominant number of cases an anafront, i.e. a surface of up-slope sliding. A warm front will be a katafront, i.e. a surface of downslope sliding, in rare cases only.

There are two types of cold fronts. A cold front of the first type up to a great height is an anafront. A cold front of the second type is an anafront in its lower part only and beginning with a height of 1-2 kilometers it already becomes a katafront. Anticyclonic compression inversions which as a rule do not cross the earth's surface are also katafronts. Katafronts as a rule do not extend to the terrestrial surface, it being the case that their lower parts are diffused before reaching the earth's surface, as a result of the divergence of the currents, or they become anafronts due to

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the increase in slope angle.

All possible movements of air at surfaces of sliding are analyzed in Table 46. The analysis shows that at surfaces of up-slope and down-slope sliding ascending as well as descending movements of air are possible.

Table 46. Possible movements of air at surfaces of sliding.

Surface		Up-slope sliding ($w_1 > 0$)	Downslope sliding ($w_1 < 0$)
Warm front	u_1	$u_1 > c_f > 0$	1) $0 < u_1 < c_f$, if $\lvert w_1 \rvert < c_f \tan\alpha$ 2) $u_1 < 0$, if $\lvert w_1 \rvert > c_f \tan\alpha$
$c_f > 0$	u_2, w_2	1) $u_2 > c_f > 0$, $w_2 > 0$ 2) $c_f > u_2 > 0$, $w_2 < 0$, if $\lvert w_2 \rvert < c_f \tan\alpha$ 3) $u_2 < 0$, $w_2 < 0$, if $\lvert w_2 \rvert > c_f \tan\alpha$	
	α	$\frac{1}{50} > \tan\alpha > \frac{1}{200}$ Active	$\frac{1}{400} > \tan\alpha > \frac{1}{1000}$ Passive
Cold front	u_1	$u_1 > 0 > c_f$, if $w_1 > \lvert c_f \rvert \tan\alpha$ $u_2 < 0$, if $w_1 < \lvert c_f \rvert \tan\alpha$	
$c_f < 0$	u_2, w_2	1) $u_2 > 0$, $w_2 > 0$, $\lvert w_2 \rvert > \lvert c_f \rvert \tan\alpha$ 2) $u_2 < 0$, $w_2 > 0$, $\lvert w_2 \rvert < \lvert c_f \rvert \tan\alpha$ 3) $u_2 < 0$, $w_2 < 0$	
	α	$\frac{1}{25} > \tan\alpha > \frac{1}{80}$ Passive	Active
Quasi-stationary front	u_1	$u_1 = w_1 \cot\alpha > 0$	$u_1 = w_1 \cot\alpha < 0$
$c_f = 0$	u_2, w_2	1) $u_2 > 0$, $w_2 > 0$, $u_2 = w_2 \cot\alpha$ 2) $u_2 < 0$, $w_2 < 0$, $u_2 = w_2 \cot\alpha$	$u_2 > 0$, $w_2 > 0$ $u_2 < 0$, $w_2 < 0$

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Section 9. Frontogenesis and Frontolysis.

Frontogenesis is a process which leads to the evolution of a new or the accentuation of an already existing discontinuity. Frontolysis is a process which leads to the destruction or abatement of a discontinuity.

The main characteristic of surfaces of discontinuity in the free atmosphere, is a discontinuity in temperature. We will speak of the potential or of the equivalent potential (in the case of saturation) temperature.

However, not every discontinuity in temperature bespeaks the presence of a surface dividing two different air masses. A discontinuity in temperature may sometimes be induced by local causes the effect of which is only of short duration. Therefore, merely a discontinuity in temperature does not yet lead to the formation of a surface of separation which is of significance for the atmospheric processes.

Already Helmholz in his analysis of the general circulation in the atmosphere, pointed out that a surface of separation is generated when two air masses of different origin and temperature meet.

In order to segregate the principal front generating factor let us analyze the equation of thermal inflow $\frac{dq}{dt} = c_p \frac{T}{\theta} \frac{d\theta}{dt}$ and solve it with relation to the local derivative $\frac{\partial \theta}{\partial t}$:

$$\frac{\partial \theta}{\partial t} = -u \frac{\partial \theta}{\partial x} - v \frac{\partial \theta}{\partial y} + \frac{\theta}{T} \cdot \frac{1}{c_p} \cdot \frac{dq}{dt}$$

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(we limit ourselves to the consideration of the case of two-dimensional motion).

Let us assume, for simplification, that the isotherms of the potential (or equivalent potential) temperature are straight lines. Then directing the x-axis parallel with the isotherms we derive

$$\frac{\partial \theta}{\partial t} = -v \frac{\partial \theta}{\partial y} + \frac{\theta}{T} c_p \frac{dq}{dt}.$$

The variation in the temperature differential of two neighboring points is determined by equation:

$$\frac{\partial(\theta_1 - \theta_2)}{\partial t} = -\left(v_1 \frac{\partial \theta_1}{\partial y} - v_2 \frac{\partial \theta_2}{\partial y}\right) + \frac{\theta}{T} c_p \left(\frac{dq_1}{dt} - \frac{dq_2}{dt}\right). \quad (1)$$

Equation (1) shows that the variation in the temperature differential of two neighboring points, is determined by two factors: advection and the inflow of heat. Assuming there is no advection or its value is so insignificant that it can be completely disregarded, then, the variation in the temperature differential of the above points will be determined only by the difference of the heat inflow into the surrounding air masses. Let us consider all 3 forms of heat transfer: radiation, turbulent conduction, and latent heat transfer [advection]. In the free atmosphere conditions of radiation at two neighboring points cannot be essentially different. Therefore, in the free atmosphere there is no possibility for the evolution or disappearance of a front caused by radiation. On the contrary, in the presence of sharp contrasts in the character of the underlying surface, radiation conditions at the earth's surface may be essentially different (for example, at a seacoast).

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However, such contrasts are of short duration and in the diurnal cycle they change their sign. In addition they are limited to the very lowest layer of the atmosphere only several dekameters in thickness. The discontinuity in temperature tied in with these contrasts is not accompanied by a discontinuity in wind velocity, pressure gradient force and other frontal characteristics.

Since the discontinuity in temperature caused by the difference in conditions of radiation is locally limited and of no general significance, the thermal inflow caused by radiation cannot lead to frontogenesis or frontolysis.

As to thermal turbulence conduction, its effect is expressed as is known in the equalization of temperature and, therefore, it cannot lead to frontogenesis. Frontolysis, however, essentially depends on thermal turbulence conduction.

The temperature differential caused by evaporation or condensation in the free atmosphere cannot be considerable. Only at the earth's surface there may occur conditions favorable to the evolution of an essential temperature differential. This differential, however, is not typical and is only of limited local significance.

Thus, frontogenesis can be tied in only with heat advection in the atmosphere.

For simplification let us assume that at an initial moment the temperature is a linear function of the ordinate y, with:

$$\frac{\partial \theta_1}{\partial y} = \frac{\partial \theta_2}{\partial y} = \frac{\partial \theta}{\partial y} > 0.$$

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If the y-axis is directed toward the side of increasing temperatures, then:

$$\frac{\partial(\theta_1-\theta_2)}{\partial t} = -\frac{\partial\theta}{\partial y}(v_1-v_2)$$

i.e., in a linear temperature field the change with time in the temperature differential of two neighboring points is tied in with a discontinuity in the velocity component normal to the isotherm, with the change being in direct ratio to the discontinuity of this velocity component.

Thus, frontogenesis will occur only in such a field of flow in which the isotherms converge. Under reversed conditions frontolysis will take place.

Therefore, frontogenesis and frontolysis will develop most actively in a deformation field, when two air masses are in motion toward each other, while, at the same time spreading out sideways.

In the simplest case of a flat simple field of deformation the lines of flow are equilateral hyperbolae (Figure 119).

Figure 119. Frontogenesis in a field of deformation.

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To such a field of flow in a pressure field corresponds a col (saddle). The point in the center of the hyperbolic field is called the neutral or hyperbolic point. The wind velocity at this point is zero. The axis in moving along which two air masses approach each other, is called the axis of compression, and the other axis - the axis of elongation.

From Figure 119 it can be seen that the most favorable conditions for the development of frontogenesis will occur in the case when the isotherms are parallel to the axis of elongation. They will then without fail converge while remaining parallel to their previous direction. If the isotherms are parallel to the axis of compression, they will without fail diverge while remaining parallel to their previous disposition.

Let us evaluate for these simplified cases the order of velocity of frontogenesis and frontolysis. Assuming that $v_1 = -v_2 = 10$ m/sec. we derive the following hourly changes of the temperature differential for two particles of air belonging to various currents:

$\left\lvert \frac{\partial \theta}{\partial y} \right\rvert_{mean}$	1°/1000 km.	1°/500 km.	1°/10 km.
$\frac{\partial(\theta_1 - \theta_2)}{\partial t}$	0.7°	1.4°	7°

This small tabulation shows the velocity of frontogenesis and frontolysis is particularly great in those cases where there is already present a frontal zone. Consequently a front may be considerably accentuated or diffused within a very short time of, let us say, a few hours.

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If the isotherms form with the axis of elongation angle α, which is smaller than 45 degrees, the isotherms under the effect of the field of deformation will converge, i.e. there will develop frontogenesis, although at a slower rate than in the case when angle α equals zero, it being the case that the isotherms will so turn that angle $\alpha \rightarrow 0$.

If the isotherm forms with the axis of elongation angle α which is greater than 45 degrees (Figure 120), the isotherms will first diverge, i.e. frontolysis will take place and the isotherms turn so that angle $\alpha \longrightarrow 45$ degrees. In further turning the isotherms will again converge, i.e. there will again begin the development of frontogenesis. Thus, only the isotherms which are perpendicular to the axis of elongation will at all times diverge. All other isotherms which are lying in a field of deformation will finally converge.

Figure 120. Frontolysis in a field of deformation.

As already pointed out above, a field of deformation in a

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pressure field is matched by a saddle which is evolved during a cross-like disposition of two cyclones and two anticyclones. Therefore, frontogenesis mainly takes place in areas where such a pressure field occurs most frequently.

It is obvious that in the atmosphere a field of deformation is not two-dimensional. In addition to the two horizontal axes of deformation there must exist in the atmosphere in a general case a third axis of deformation lying in a vertical plane inclined to the horizon. Let us analyze the field of deformation in a vertical plane.

If the horizontal axis is the axis of compression, then the inclined axis will be the axis of elongation (Figure 121). Repeating the argumentations from above we satisfy ourselves that in this case the isentropic surfaces will converge, which corresponds to frontogenesis. However, if the horizontal axis is the axis of elongation (Figure 122), the isentropic surfaces will diverge, which corresponds to frontolysis.

Kinematic conditions for the formation of inversions of subsidence differ from front disintegration conditions in [redacted] that in the case of frontolysis the axis of compression is almost horizontal and in the case of inversion of subsidence it is almost vertical.

Figure 121. Frontogenesis in a vertical plane.

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Figure 122. Frontolysis in a vertical plane.

Therefore, in the case of an inversion of subsidence, the isentropic surfaces in converging form an almost horizontal inversion layer.

The above pertains to the simplest diagrams which, however, illustrate adequately well the principal front generating part played by the relative disposition of a temperature field and a field of current. The depicted diagrams may be approximated to reality if the effect of the simple transfer is eliminated, and the fact pointed out that all the above said relates to a system of coordinates/ which is being displaced together with the transfer current.

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CHAPTER XIV

THE KINEMATICS OF A PRESSURE FIELD

Section 1. Characteristic Curves and Important Points in a Pressure Field

In this chapter we will analyze the very important practical question pertaining to the regularities and variations in a pressure field, the clarification of which does not require the application of physical and dynamic concepts, but is based entirely on the given instantaneous distributions of pressure and pressure variations.

Atmospheric pressure is the only meteorological element in the representativeness of which there is never a doubt, and the measuring of which by meteorological stations is done with great precision. Reflecting in itself the state (weight) of the entire air column at the point of observation, the pressure of the air is to a lesser degree than other meteorological elements, subject to incidental variations. On the other hand, pressure and its horizontal distribution have a determinant value in the distribution and the variations of a series of meteorological elements. With the particular characteristics of a pressure field and their displacement, the features that characterize the weather and its changes are tied in.

Thus, for example, the velocity and direction of the wind depend, to a determining degree, on the pressure gradient force; in the atmospheric layers above the friction level, where the effect of friction

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against the terrestrial surface becomes infinitesimally small, the pressure field pretty nearly coincides with the field of wind. The cyclonic and anticyclonic curvature of the isobars and the divergence of the isallobaric gradient lead to the evolution, in the atmosphere, of vertical motions. Frontogenesis is predicated on the appearance and accentuation in the discontinuity of the pressure gradient force, etc.

Extensive experience accumulated by synopticists during the recent decades points to the fact that the principal peculiarities of a synoptic map find their expression in the pressure field of the map.

It is, therefore, natural that the study of the pressure field, its characteristics and evolution, occupies one of the central places in meteorology.

The purpose of this chapter is to derive the basic correlationships between a pressure field, on the one hand, and subsequent pressure variations, on the other hand, while always keeping in view their weather forecasting value.

A pressure field on a synoptic map, drawn for an arbitrary moment of time t, may in the simplest manner, be represented by a group of isobars. In the immediately following moment of time, the isobars, generally speaking, will become displaced and deformed, occupying a new position. The displacement of the isobars may be determined quantitatively if their momentary distribution and the momentary change in pressure are known.

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The equation for a group of isobars may be put down as:

$$p(x,y,t) = a,$$

where a is the parameter characterizing a given isobar. The isobars are curves characterizing the distribution of pressure; they can, therefore, be called characteristic curves. All points lying on any isobar, are conventional points in a pressure field. But alongside of an infinite number of conventional points lying on any one isobar, the pressure field, generally speaking, contains a finite number of particular points, which are isolated from the rest of the points in the pressure field and are characterized by the fact that they do not belong to any one isobar. These so-called isolated particular points in a pressure field, may be centers of either high pressure, or low pressure. In addition, the pressure field may contain a finite number of particular points, at which the isobar will cross itself. These are the so-called neutral or hyperbolic points.

In analyzing the changes taking place in a pressure field, it becomes necessary to interpolate characteristic curves of a higher order. Let

$$\frac{\partial p}{\partial t} = T(x,y,t)$$

represent a change in pressure (or the barometric tendency). The isolines of equal barometric tendencies

$$\frac{\partial p}{\partial t} = T_0, T_1, T_2, \ldots$$

(the so-called isallobars), are also characteristic lines in a pressure field.

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In addition, a pressure field may contain other more complex characteristic curves, which are distinct by a continuous value in certain derivative functions of pressure. Such characteristic curves may, for example, be the axes of troughs and ridges, which are the geometrical loci for the maximum curvature points of the isobars, the maximum curvature isolines of the pressure configuration, the fronts, i. e., lines, which are characterized by a discontinuity in the pressure field gradients, and the like.

In the following section, we will derive the general formulas, which determine the velocities and accelerations of the characteristic curves.

Section 2. The Motion of Lines

Let us visualize, in a plane (x,y) a family of curves

$$f(x,y,t)=a \qquad (1)$$

which are dependent on two parameters t and a. The first paramter is the time t, and the second parameter is the given curve of the family being analyzed.

With the running of time t, the curve, corresponding to a definite value of parameter a, will be displaced and deformed. To determine the new position of the characteristic curve in the plane (x, y), it is sufficient to substitute into equation (1) the corresponding value for t. Let us note that we are not interested in the position/

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into which a definite physical particle of our curve becomes displaced, since there is no physical law regulating the confomity of points in two curves answering to the same value of parameter a. [redacted] Indeed [redacted] the characteristic curves we are studying are by no means physical. They do not consist of the same physical particles, and are merely geometrical abstractions, connecting geometrical points which are characterized by equal values of pressure, or a definite value in tendencies, and the like. But even in the case when the lines under study are physical ones (for example, vortex lines), they will not always consist of the same particles. Only in the case of a barotropic liquid, it is possible to establish a single-value conformity between the physical particles for two positions of a vortex line.

Nevertheless, in order to determine the displacement of characteristic curves, we must so follow up the elements of a curve on a synoptic map, that it should be possible to identify them from one map to the other.

We must then, individualize the points of a curve in such a way that we obtain a concept of the motion of the individual elements of the contemplated curve. To attain this, we utilize the following method. We interpolate a second family of curves

$$\varphi(x, y, t) = b \qquad (2)$$

where b is the constant magnitude for the given curve. Let us call this new family of curves the "directing" family (Figure 123)

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Figure 123. Characteristic curves (family a) and directing curves (family b).

Thus, in the plane (x, y) there are, generally speaking, two families of curves subject to displacement and deformation. Assigning definite values to parameters a, b, t, we derive two curves: a characteristic curve of family (1) and a directing curve of family (2). These curves will, generally speaking, cross themselves at one or several points.

We now follow up the motion of one of the cross points. The point in curve a selected for this, we will call the "individualized" point. As distinct from all other points in the curve, this individualized point has the additional characteristic of remaining, during the entire time of moition of curve a, also in the definite curve b of the directing family (2).

In order to determine the coordinates of the point of crossing of the curves a and b, we must solve the system of equations

$$\left.\begin{aligned} f(x,y,t) &= a, \\ \varphi(x,y,t) &= b. \end{aligned}\right\} \qquad (3)$$

This system (3) has a solution only in the case when functions f and φ satisfy the following conditions : (1) these functions must be continuous in the neighborhood (x_0, y_0, t_0); (2) they must have

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continuous individual derivatives $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial \varphi}{\partial x}, \frac{\partial \varphi}{\partial y}$ in the neighborhood of the same system of values of (x_0, y_0, t_0); (3) the functional determinant

$$D = \begin{vmatrix} \frac{\partial f}{\partial x}, & \frac{\partial f}{\partial y} \\ \frac{\partial \varphi}{\partial x}, & \frac{\partial \varphi}{\partial y} \end{vmatrix} \qquad (4)$$

must be other than zero for the values (x_0, y_0, t_0).

If all the above conditions are satisfied, there exists only a singular system of functions

$$\left.\begin{aligned} x &= x(a, b, t), \\ y &= y(a, b, t), \end{aligned}\right\} \qquad (5)$$

which are identical to the system of equations (3), and are reducing themselves into x_0, y_0, when $t = t_0$. This result has a definite kinematic meaning.

Let us consider equations (5) as the "laws of motion" of the individualized point. Then, the derivatives $\frac{dx}{dt}, \frac{dy}{dt}, \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}$ can be considered, respectively, as the components of the "velocity" and "acceleration" of the "individualized" point of the curve. The terminology used is to be taken in a conditional sense only, since it must not be forgotten that the "individualized" point of the characteristic curve is not a physical particle and that the analogy used is necessary for the corresponding mathematical argumentation only; this analogy may, in some cases, be disturbed, particularly, since individualization may be attained by an infinite number of methods.

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In order to compute the components of velocity and acceleration of the "individualized" point, which components we will designate respectively as c_x, c_y, A_x, A_y, we utilize the rule for the differentiation of implicit functions.

Differentiating equation (3) by t, we obtain

$$\left.\begin{aligned} \frac{df}{dt} &= \frac{\partial f}{\partial t} + c_x \frac{\partial f}{\partial x} + c_y \frac{\partial f}{\partial y} = 0, \\ \frac{d\varphi}{dt} &= \frac{\partial \varphi}{\partial t} + c_x \frac{\partial \varphi}{\partial x} + c_y \frac{\partial \varphi}{\partial y} = 0, \end{aligned}\right\} \tag{6}$$

whence

$$\left.\begin{aligned} c_x &= \frac{1}{D}\left(\frac{\partial f}{\partial y}\cdot\frac{\partial \varphi}{\partial t} - \frac{\partial \varphi}{\partial y}\cdot\frac{\partial f}{\partial t}\right), \\ c_y &= \frac{1}{D}\left(\frac{\partial \varphi}{\partial x}\cdot\frac{\partial f}{\partial t} - \frac{\partial f}{\partial x}\cdot\frac{\partial \varphi}{\partial t}\right). \end{aligned}\right\} \tag{7}$$

Differentiating (6) once more by t,

$$\frac{d^2 f}{dt^2} = \frac{\partial^2 f}{\partial t^2} + 2c_x\frac{\partial^2 f}{\partial x \partial t} + 2c_y\frac{\partial^2 f}{\partial y \partial t} + 2c_x c_y \frac{\partial^2 f}{\partial x \partial y} + c_x^2 \frac{\partial^2 f}{\partial x^2} + c_y^2 \frac{\partial^2 f}{\partial y^2} + A_x \frac{\partial f}{\partial x} + A_y \frac{\partial f}{\partial y} = 0, \tag{8}$$

$$\frac{d^2 \varphi}{dt^2} = \frac{\partial^2 \varphi}{\partial t^2} + 2c_x\frac{\partial^2 \varphi}{\partial x \partial t} + 2c_y\frac{\partial^2 \varphi}{\partial y \partial t} + 2c_x c_y \frac{\partial^2 \varphi}{\partial x \partial y} + c_x^2 \frac{\partial^2 \varphi}{\partial x^2} + c_y^2 \frac{\partial^2 \varphi}{\partial y^2} + A_x \frac{\partial \varphi}{\partial x} + A_y \frac{\partial \varphi}{\partial y} = 0,$$

whence

$$A_x = \frac{1}{D}\left[\left(\frac{\partial^2 \varphi}{\partial t^2} + 2c_x\frac{\partial^2 \varphi}{\partial x \partial t} + 2c_y\frac{\partial^2 \varphi}{\partial y \partial t} + 2c_x c_y \frac{\partial^2 \varphi}{\partial x \partial y} + c_x^2 \frac{\partial^2 \varphi}{\partial x^2} + c_y^2 \frac{\partial^2 \varphi}{\partial y^2}\right)\frac{\partial f}{\partial y} - \left(\frac{\partial^2 f}{\partial t^2} + 2c_x\frac{\partial^2 f}{\partial x \partial t} + 2c_y\frac{\partial^2 f}{\partial y \partial t} + 2c_x c_y \frac{\partial^2 f}{\partial x \partial y} + c_x^2 \frac{\partial^2 f}{\partial x^2} + c_y^2 \frac{\partial^2 f}{\partial y^2}\right)\frac{\partial \varphi}{\partial y}\right], \tag{9}$$

$$A_y = \frac{1}{D}\left[\left(\frac{\partial^2 f}{\partial t^2} + 2c_x\frac{\partial^2 f}{\partial x \partial t} + 2c_y\frac{\partial^2 f}{\partial y \partial t} + 2c_x c_y \frac{\partial^2 f}{\partial x \partial y} + c_x^2 \frac{\partial^2 f}{\partial x^2} + c_y^2 \frac{\partial^2 f}{\partial y^2}\right)\frac{\partial \varphi}{\partial x} - \left(\frac{\partial^2 \varphi}{\partial t^2} + 2c_x\frac{\partial^2 \varphi}{\partial x \partial t} + 2c_y\frac{\partial^2 \varphi}{\partial y \partial t} + 2c_x c_y \frac{\partial^2 \varphi}{\partial x \partial y} + c_x^2 \frac{\partial^2 \varphi}{\partial x^2} + c_y^2 \frac{\partial^2 \varphi}{\partial y^2}\right)\frac{\partial f}{\partial x}\right].$$

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For the existence of the above computed continuous derivatives of the first and second orders of coordinates of the "individualized" point with relation to time, which we are considering as the "velocity" and "acceleration" components of the individualized particle, it is necessary that the functions φ and f, in addition to the above enumerated conditions, satisfy two new conditions:

(4) The existence of continuous individual derivatives in terms of time $\frac{\partial f}{\partial t}$ and $\frac{\partial \varphi}{\partial t}$ is a condition sine qua non; (5) the existence of continuous individual derivatives of the second order, from functions f and φ, by all independent variables in the neighborhood of (x_0, y_0, t_0), is also indispensable.

And so, we derived formulas which determine the components of momentary velocity and momentary acceleration of the individualized points in characteristic curves. These formulas appear rather complex, but in a number of cases they can be considerably simplified. A particularly simple aspect can be given to those formulas, when the given family of curves is only one family (1), which characterizes the distribution of any one meteorological element or its derivatives, and the second family, which we called the "directing", may be selected arbitrarily, in conformity with the conditions of the problem.

In this case, the "directing" family has a purely geometrical meaning. A case is also possible, however, when the "directing" family is tied in with some physical peculiarities of the contemplated field. For instance, when the characteristic curve is the axis of a trough, the directing curve may be the axis of a ridge of high pressure. In

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that case, the thus individualized point will have a definite physical meaning, it will be the knot, or in other words, the neutral or hyperbolic point.

We will consider the first case only, when the selection of the directing family is arbitrary.

Supposing that the directing family is an aggregate of stationary straight lines parallel to the x - axis; then we have

$$f(x,y,t)=a;\quad \varphi=y=b, \tag{10}$$

and all individual derivatives of function φ, except $\frac{\partial\varphi}{\partial y}=1$, will become zero:

$$\frac{\partial\varphi}{\partial x}=\frac{\partial^2\varphi}{\partial x^2}=\frac{\partial^2\varphi}{\partial x\partial y}=\frac{\partial^2\varphi}{\partial y^2}=\frac{\partial\varphi}{\partial t}=\frac{\partial^2\varphi}{\partial t^2}=0,$$

and therefore, $D=\frac{\partial f}{\partial x}$. Equations (7) will give

$$c_x=-\frac{\frac{\partial f}{\partial t}}{\frac{\partial f}{\partial x}};\quad c_y=0, \tag{11}$$

and from equation (9) we derive:

$$A_x=-\frac{\frac{\partial^2 f}{\partial t^2}+2c_x\frac{\partial^2 f}{\partial x\partial t}+c_x^2\frac{\partial^2 f}{\partial x^2}}{\frac{\partial f}{\partial x}};\quad A_y=0. \tag{12}$$

From the above it is clear that we do not impose any limitations upon function f, with the exception of required continuity and differentiability, and the condition $D \neq 0$. Therefore, formulas (11), (12), (14), (15) are applicable to a very wide class of functions.

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We will now apply the formulas derived for velocity and acceleration to the study of the displacement of the most important characteristic curves in the pressure field.

Section 3. The Velocity and the Acceleration of Isobars.

Let us assume that a characteristic curve, the motion of which is being analyzed, is an isobar

$$p(x, y, t) = a. \tag{1}$$

Then, directing the x - axis along the isobaric ascendant (Figure 124) we will derive at a given point:

$$c_i = -\frac{\frac{\partial p}{\partial t}}{\frac{\partial p}{\partial x}}, \tag{2}$$

$$A_i = -\frac{\frac{\partial^2 p}{\partial t^2} + 2c_i \frac{\partial^2 p}{\partial x \partial t} + c_i^2 \frac{\partial^2 p}{\partial x^2}}{\frac{\partial p}{\partial x}}. \tag{3}$$

Figure 124. Computation of velocity of isobars.

Equations (2) and (3) determine with adequate precision the values of the velocity and acceleration of an isobar at an arbitrary selected point, i.e., they establish the momentary kinematic state of a pressure field.

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But formulas (2' and (3) can also be utilized for the extrapolating of the pressure distribution for a small interval of time t. The displacement S_t of the selected isobar in the direction of a normal will be determined by the following approximated extrapolation formula.

$$S_i = c_i t + A_i \frac{t^2}{2}, \qquad (4)$$

where S_i is the displacement of the isobar along the initial normal, and t is the time of forecast.

Let us write down formula (2) in a different way. Let us note that in our selection of coordinate axes $\frac{\partial p}{\partial \psi}$ is the isobaric ascendant. Operating with unitary isobars, we can write down:

$$\frac{\partial p}{\partial \psi} = \left|\nabla p\right| = \frac{1}{h}$$

where h is the distance between two neighboring single isobars. If we will, in addition, assume that

$$\frac{\partial p}{\partial t} = T$$

where T is the barometric tendency, then formula (2) will become

$$c_i = -Th \qquad (5)$$

Thus, the velocity of the displacement of an isobar along a line normal to itself is equal to the negative magnitude of the barometric tendency multiplied by the distance between unitary isobars. From equation (5) it follows that, in the area of positive

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tendencies ($T>0$), the isobars are displaced along the pressure gradient force ($c_i<0$), and in the area of negative barometric tendencies ($T<0$), along the ascendant of pressure ($c_i>0$). If acceleration, which is determined by equation (3), is small, then for a not too long forecasting period, the interpolation formula (4) becomes

$$S_i = c_i t.$$

Here, t represents the number of 3 - hour intervals, for which the barometric tendency is usually computed. With the aid of formula (4), the change in the pressure gradient force in the neighborhood of a shifting isobar, can be computed. Indeed, let two neighboring isobars p_1 and p_2 shift uniformly at velocities respectively equal to c_1 and c_2. Then the change in distance between the isobars, for a unit of time, will be $(c_2 - c_1)$, and for the forecasting period t, the distance h will be

$$h = h_0 + (c_2 - c_1)t. \qquad (6)$$

If the velocities of the isobars are accelerated, then

$$h = h_0 + (c_2 - c_1)t + (A_2 - A_1)\frac{t^2}{2}, \qquad (7)$$

and the pressure gradient force will be $\frac{1}{h}$.

Since the velocity of the wind is usually in direct ratio to the pressure gradient force, equations (6) and (7) make it possible

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to compute the anticipated wind velocity.

With time, however, the density of the isobars, and consequently, the velocity of the wind, and also the orientation of the isobars, and consequently, the direction of the wind, change. Limiting ourselves to a case of uniform displacement of the isobars, we derive the velocity of rotation

$$\frac{\partial c_i}{\partial y} = -\left(I_y \cdot h + T \cdot \frac{\partial h}{\partial y}\right), \qquad (8)$$

where $I_y = \frac{\partial T}{\partial y} = \frac{\partial^2 p}{\partial y \partial t}$ is the isallobaric ascendant component along the isobar. Thus the momentary velocity of rotation for uniformly shifting isobars is predicated on: (1) the density of the isobars h, (2) the divergence of the isobars $\frac{\partial h}{\partial y}$, (3) the barometric tendency, and (4) the relative disposition of the isobars and isallobars. If the isobars are parallel to each other, $\frac{\partial h}{\partial y} = 0$, whence

$$\frac{\partial c_i}{\partial y} = -I_y h$$

As can be seen from equation (8), the algebraic sign of the term $\frac{\partial c_i}{\partial y}$ is opposite to the sign of the term I_y. Therefore, in an area where the isallobaric ascendant component is positive ($I_y > 0$), the isobars turn to the left, and vice versa. Thus, in an area of isallobaric minimum, the curvature of the isobars becomes cyclonic, and in an area of isallobaric maximum, it becomes anticyclonic.

When the isobars are parallel with the isallobars, $I_y = 0$, and, consequently,

$$\frac{\partial c_i}{\partial y} = -T \frac{\partial h}{\partial y} \qquad (9)$$

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In the case of divergence of the isobars, they are deflected in the direction of falling pressure, while in the case of convergence, the isobars are deflected in the direction of rising pressure.

The above analysis shows, how important it is to note the relative disposition of isallobars and isobars, in order to determine the changes occurring in the distribution of pressure.

Let us now turn to the analysis of equation (3), which determines the acceleration of the isobars. With the coordinate axes selected, as above, the terms in formula (3) are as follows: $\frac{\partial p}{\partial x}$ is the pressure ascendant, which already occurred in formula (2); $\frac{\partial^2 p}{\partial t^2} = \frac{\partial T}{\partial t}$ is the barogram curvature, or in other words, the local change in barometric tendency; $\frac{\partial^2 p}{\partial x \partial t} = \frac{\partial}{\partial x}\left(\frac{\partial p}{\partial t}\right) = I_x$ is the isallobaric ascendant component, normal to the isobar; $\frac{\partial^2 p}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial p}{\partial x}\right)$ is the change in pressure ascendant along a normal to the isobar, or in other words, the curvature of the pressure configuration.

All values entering into equation (3), with the exception of $\frac{\partial^2 p}{\partial t^2}$, can be taken from one synoptic map. Only for the value of $\frac{\partial^2 p}{\partial t^2}$, it is necessary to refer either to a barogram, or to two consecutive 3-hour interval synoptic maps, which contain the values of the barometric tendency. It must be noted that the barogram curvature essentially affects the acceleration value of the isobar. Expressing, as before, the pressure gradient force by the distance between two unitary isobars, we derive:

$$\frac{\partial p}{\partial x} = \frac{1}{h}; \quad \frac{\partial^2 p}{\partial x^2} = -\frac{1}{h^2}\frac{\partial h}{\partial x}.$$

Assuming, in addition, that $\frac{\partial^2 p}{\partial x \partial t} = I_x$, we re-write equation (3) as follows

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$$A_i = -\frac{\partial^2 p}{\partial t^2} \cdot h - 2c_i I_\varphi h + \frac{c_i^2}{h} \frac{\partial h}{\partial \varphi}. \qquad (10)$$

Thus, the acceleration of the isobar can be considered as composed of 3 parts:

$$A_i = A' + A'' + A''', \qquad (11)$$

where

$$A' = -h \frac{\partial^2 p}{\partial t^2}, \qquad (12)$$

$$A'' = -2c_i I_\varphi h, \qquad (13)$$

$$A''' = \frac{c_i^2}{h} \cdot \frac{\partial h}{\partial \varphi}. \qquad (14)$$

The most difficult problem is the determination of the first acceleration component A', since it is expressed by a value which is not directly measured, and cannot be computed with the aid of one synoptic map. This value $\frac{\partial^2 p}{\partial p^2}$ is the curvature of the barogram. Typical barograms are shown in Figure 125.

Figure 125. Various types of barograms.

Figure 125 shows that, if a barogram is a straight line, i.e., when the change of pressure is uniform, and consequently has a constant tendency, or is expressed by a curve, which has a point of

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recurvature then

$$\frac{\partial^2 p}{\partial t^2} = 0 \quad \text{and} \quad A' = 0. \tag{15}$$

If the barogram has a cyclonic curvature (Figure 125, a, d, g,), which is possible, either with an accelerated rise in pressure (a), or with a retarded fall in pressure (d), or finally, with a transition from falling to rising pressure (g), then:

$$\frac{\partial^2 p}{\partial t^2} > 0 \quad \text{and} \quad A' < 0. \tag{16}$$

Finally, when the curvature of the barogram is anticyclonic (figure 125, b, e, h), which is possible, either with a retarded rise in pressure (b), or with an accelerated drop in pressure (e), or finally, with a transition from rising to falling pressure (h), then:

$$\frac{\partial^2 p}{\partial t^2} < 0 \quad \text{and} \quad A' > 0. \tag{17}$$

However, the barogram makes it relatively simple to determine only the algebraic sign of the derivative $\frac{\partial^2 p}{\partial t^2}$ (by the curvature characteristic). But the value of this derivative cannot be determined by the barogram. There are 3 different methods for the determination of the value of $\frac{\partial^2 p}{\partial t^2}$:

(1) This value may be obtained as the difference between two consecutive 3-hour interval barometric tendencies, if 3 hours is accepted, as it is in practice, as the unit of time

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$$\frac{\partial^2 p}{\partial t^2} = T_1 - T_0 , \qquad (18)$$

it being the case that these tendencies are to be taken for neighboring moments in time, separated by a 3-hour interval.

(2) Since a barometric tendency, in its turn, is the difference between two consecutive pressures,

$$\frac{\partial p}{\partial t} = T = p_1 - p_0 ,$$

the value of $\frac{\partial^2 p}{\partial t^2}$ can be derived from three consecutive pressure observations, by the formula

$$\frac{\partial^2 p}{\partial t^2} = (p_2 - p_1) - (p_1 - p_0) = p_2 - 2p_1 + p_0 . \qquad (19)$$

For greater precision in the determination of $\frac{\partial^2 p}{\partial t^2}$ it is necessary to use the barometer readings, and not the barograph recordings, since the barometer readings are more reliable: they are less subject to all sorts of accidental effects, which distort the magnitude of the barometric tendency as recorded by the barograph.

The difficulty in determing the derivative $\frac{\partial^2 p}{\partial t^2}$ lies also in that, in order to find it, it is necessary to have the values of the barometric tendencies for two consecutive moments, separated by a time interval not to exceed three hours. Incidentally, with ring devices, in popular use by the Soviet Weather Service, the value of $\frac{\partial^2 p}{\partial t^2}$ can be determined without much difficulty. With the aid of these "ring devices", the pressure differentials, which must be known for the computation of $\frac{\partial^2 p}{\partial t^2}$ value, can be obtained with adequate precision.

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(3) Let us, finally, show that the $\frac{\partial^2 p}{\partial t^2}$ value can be derived, if the velocity of the isallobars is determined preliminarily with adequate precision.

Indeed, by applying equation (11) from Section 2 to the isallobar, we find

$$c_T = -\frac{\frac{\partial T}{\partial t}}{\frac{\partial T}{\partial x}} = -\frac{\partial^2 p}{\partial t^2} \cdot H, \qquad (20)$$

where c_t is the isallobar velocity in the direction of the pressure ascendant, and H is the distance between the adjacent unitary isallobars.

Equation (20) is entirely analogous to equation (2) for isobar velocity. The velocity of the isallobars is positive, when the barogram curvature is anticyclonic (Figure 125, b, e, h), and negative when the barogram curvature is cyclonic (Figure 125, a, d, g). If the barometric tendency is uniform $\frac{\partial T}{\partial t} = 0$, the isallobar is stationary $(c_T = 0)$, with the exception of the center of the isallobaric area, where $\frac{\partial T}{\partial x} = 0$, which is a particular point, for which c_t becomes an indeterminate value of the $\frac{0}{0}$ type.

When two consecutive synoptic maps, separated by a 3-hour interval, are available, the isallobar velocity c_t can be determined from these maps. When the distance H between two adjacent unitary isallobars is known we derive:

$$\frac{\partial^2 p}{\partial t^2} = -\frac{c_T}{H}, \qquad (21)$$

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and the appropriate acceleration component will be determined by

$$A' = c_T \cdot \frac{h}{H} \cdot \tag{22}$$

So the first acceleration component A' may be expressed by any of the formulas below:

$$A' = \begin{cases} -(T_1 - T_0)h, \\ -(p_2 - 2p_1 + p_0)h, \\ c_T \cdot \frac{h}{H}. \end{cases} \tag{23}$$

The formulas (23) show that the acceleration of the isobars, all other conditions being equal, will be the greater, the less crowded the isobars are (h is greater), and the more crowded the isallobars (H is smaller), and also, the greater the velocity of the isallobars (c_t is greater).

Thus, the acceleration component A' will be at its maximum in the area of rapidly shifting isallobars, which coincides with the isallobar densification area and with the isobar rarification area. A' becomes zero, or approximates zero only when the change in pressure is uniform or almost uniform, which on the synoptic map, corresponds to zero or very small isallobaric gradients, i.e., stationary isallobaric areas. Such a disposition is observed in the centers of isallobaric systems, and also in the warm sector of a cyclone.

Let us also note that A' and c_t have identical algebraic signs. The motion of the isobars is accelerated, when the isallobars are shifting along the isallobaric ascendant, and conversely, the motion

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of the isobars is retarded, when the isallobar velocity is directed along the isallobaric gradient.

Let us now analyze the second acceleration component A" which is determined by correlationship (13).

Assuming $I_x = \frac{1}{H_x}$, where H_x is the distance between adjacent unitary isallobars, which is measured along a normal to the isobar, we derive:

$$A_i'' = -2c_i I_k h = -2c_i \frac{h}{H_k} = 2T \frac{h^2}{H_k}. \quad (24)$$

All values, entering into formula(24) can be taken off one synoptic map.

The sign of the acceleration component A" is determined by the sign of the product of barometric tendency by isallobaric ascendant component, in accordance with the diagram shown in Figure 126.

Figure 126. The sign diagram for the acceleration component.

This diagram shows that the isobar acceleration is positive, if in the area of positive barometric tendencies, the baric and isallobaric ascendants form between themselves an obtuse angle.

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The acceleration component A" becomes zero in the three following cases:

(1) When the barometric tendency equals zero. This occurs in the neighborhood of the axis of symmetry for the troughs and ridges. In these areas the velocity of the isobars also becomes zero, and the displacement of the isobars is determined exclusively by the value of [redacted] A_i'.

(2) When the isallobaric gradient I = 0, i.e., in the centers of the isallobaric areas.

(3) When the isallobars cross each other at straight angles to the isobars, i.e., when cos (I,x) = 0.

Finally, the third acceleration component [redacted] A_i''' can also be computed with the aid of data taken from one synoptic map:

$$A_i''' = \frac{c_i^2}{h} \cdot \frac{\partial h}{\partial x} = T^2 h \frac{\partial h}{\partial x}. \qquad (25)$$

To compute $\frac{\partial h}{\partial x}$, we designate by h_1 the distance between isobars $(P_0 - 1)$ and P_0, and by h_2 the distance between isobars P_0 and $P_0 + 1$ (Figure 127).

Figure 127. Computation of $\frac{\partial h}{\partial x}$.

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For the mean isobar P_0, we can assume $h = \frac{1}{2}(h_1 - h_2)$, then

$$\frac{\partial h}{\partial K} = \frac{h_2 - h_1}{\frac{1}{2}(h_1 + h_2} \qquad (26)$$

substituting (26) into (25), we derive after reduction:

$$A_i''' = T^2(h_2 - h_1). \qquad (27)$$

Consequently, the acceleration will be positive, if, in the direction of the pressure ascendant, the isobars are rarefied, and vice versa. Acceleration A''' is reduced to zero, when:

(1) In the areas, where the isobars are disposed equidistantly, as is the case in the forward and rear parts of moving cyclones and anticyclones, and also in the warm sector of a cyclone.

(2) In areas where the barometric tendency is equal to zero.

There are frequently areas on a synoptic map, with relation to which the displacement of isobars can be computed without taking into account the acceleration, or some of the terms in the acceleration formula. Therefore, in order to avoid complex and clumsy computations, it is necessary to locate such areas on the map, and make the computations with relation to them.

Section 4. Velocity and Acceleration for Troughs and Ridges.

In the preceding section we showed that the displacement of the isobars depends to an essential degree on the barometric tendency

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curvature $\frac{\partial^2 p}{\partial t^2}$, i.e., on the local change in the tendency, since this value enters into the isobar acceleration formula. At the points, where the curvature of the barogram attains considerable values, its determination is technically very difficult. The curvature of the barogram is particularly great when passing through the observation point of ridges, troughs, and baric centers. Deferring the analysis of the movement of baric centers to the next section, we will discuss in this section the movement of troughs and ridges only. It should be again underscored that, in principle, the location of troughs or ridges can be pre-computed by the pre-computation of the movement of isobars, and only the practical difficulties appearing in the determination of $\frac{\partial^2 p}{\partial t^2}$, compel the search for a different method of pre-computation of velocity and acceleration for troughs and ridges. For brevity, we will use the term "baric axis", since the formulas for the troughs and ridges are identical. The problem posed is naturally divided into two parts:

(1) The pre-computation of the baric axis displacement.

(2) The pre-computation of the change in pressure along the baric axis.

We will occupy ourselves with the first part of the problem only. The baric axis is the geometrical locus for the points of maximum curvature of the isobars. Let us interpolate the equations of the baric axis.

If the equation of the curve is given in its explicit form

$$y = y(x,t)$$

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where t is its parameter, then its curvature σ is determined by formula

$$\sigma = \frac{y''}{\left[1+y'^2\right]^{3/2}} \qquad (1)$$

If the equation of the curve is given in its implicit form

$$f(x,y,t) = \text{const.},$$

then, since

$$y' = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}; \quad y'' = -\frac{\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2 - 2\frac{\partial^2 f}{\partial x \partial y}\cdot\frac{\partial f}{\partial x}\cdot\frac{\partial f}{\partial y} + \frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2}{\left(\frac{\partial f}{\partial y}\right)^3}$$

its curvature will be determined by:

$$\sigma(x,y,t) = -\frac{\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2 - 2\frac{\partial^2 f}{\partial x \partial y}\cdot\frac{\partial f}{\partial x}\cdot\frac{\partial f}{\partial y} + \frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2}{\left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{3/2}} \qquad (2)$$

While the curvature of the isobar

$$p(x,y,t) = \text{const.}$$

will, consequently, be:

$$\sigma(x,y,t) = -\frac{\frac{\partial^2 p}{\partial x^2}\left(\frac{\partial p}{\partial y}\right)^2 - 2\frac{\partial^2 p}{\partial x \partial y}\cdot\frac{\partial p}{\partial x}\cdot\frac{\partial p}{\partial y} + \frac{\partial^2 p}{\partial y^2}\left(\frac{\partial p}{\partial x}\right)^2}{\left[\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2\right]^{3/2}} \qquad (3)$$

We are searching for points, at which the curvature of the isobar will be at its maximum; these points, of course, also lie on the isobars. Therefore, the coordinates of the required points must also satisfy the corresponding equation of the isobars. For definitions we will consider x as an independent variable, and y as the function

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of x, which satisfies the equation of the isobar

$$P = \text{const.} \tag{4}$$

The requisite condition, in order that the value $\sigma(x,y)$ is the maximum, will be expressed by the equalities:

$$\left.\begin{aligned} \frac{\partial\sigma}{\partial x} + \frac{\partial\sigma}{\partial y}\cdot\frac{\partial y}{\partial x} = 0, \\ \frac{\partial p}{\partial x} + \frac{\partial p}{\partial y}\cdot\frac{\partial y}{\partial x} = 0. \end{aligned}\right\} \tag{5}$$

and this is possible only when:

$$\begin{vmatrix} \frac{\partial\sigma}{\partial x}, & \frac{\partial\sigma}{\partial y} \\ \frac{\partial p}{\partial x} & \frac{\partial p}{\partial y} \end{vmatrix} = 0. \tag{6}$$

This correlationship must be satisfied by the points, for which the curvature of the isobars is at its maximum, and which lie on these isobars. Correlationship (6), then is the equation of the baric axis.

Designating the left-hand part of (6) by F, we derive the equation of the baric axis in the form of

$$F = \begin{vmatrix} \frac{\partial\sigma}{\partial x}, & \frac{\partial\sigma}{\partial y} \\ \frac{\partial p}{\partial x}, & \frac{\partial p}{\partial y} \end{vmatrix} = 0. \tag{7}$$

While the velocity of the baric axis, according to (11) in Section 2 will be:

$$c_L = -\frac{\frac{\partial F}{\partial t}}{\frac{\partial F}{\partial x}}, \tag{8}$$

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where:
$$\left.\begin{aligned}\frac{\partial F}{\partial t} &= \begin{vmatrix} \frac{\partial^2\sigma}{\partial x\,\partial t}, & \frac{\partial\sigma}{\partial y} \\ \frac{\partial^2 p}{\partial x\,\partial t}, & \frac{\partial p}{\partial y}\end{vmatrix} + \begin{vmatrix} \frac{\partial\sigma}{\partial x}, & \frac{\partial^2\sigma}{\partial y\,\partial t} \\ \frac{\partial p}{\partial x}, & \frac{\partial^2 p}{\partial y\,\partial t}\end{vmatrix}, \\ \frac{\partial F}{\partial x} &= \begin{vmatrix} \frac{\partial^2\sigma}{\partial x^2}, & \frac{\partial\sigma}{\partial y} \\ \frac{\partial^2 p}{\partial x^2}, & \frac{\partial p}{\partial y}\end{vmatrix} + \begin{vmatrix} \frac{\partial\sigma}{\partial x}, & \frac{\partial^2\sigma}{\partial x\,\partial y} \\ \frac{\partial p}{\partial x}, & \frac{\partial^2 p}{\partial x\,\partial y}\end{vmatrix}.\end{aligned}\right\} \qquad (9)$$

Equation (8) can be somewhat simplified by a special selection of the coordinate axes. Namely, we direct the x-axis along a tangent to the isobar so that at the initial moment $\frac{\partial p}{\partial x} = 0$; it is obvious that, in this case, the x-axis will not necessarily be normal to the baric axis (Figure 128).

Figure 128. The baric axis and the characteristic line.

Then, the momentary velocity of the baric axis, will be expressed as:
$$c_L = -\frac{\frac{\partial^2\sigma}{\partial x\,\partial t}\cdot\frac{\partial p}{\partial y} - \frac{\partial^2 p}{\partial y\,\partial t}\cdot\frac{\partial\sigma}{\partial y}}{\frac{\partial^2\sigma}{\partial x^2}\cdot\frac{\partial p}{\partial y} - \frac{\partial^2 p}{\partial x^2}\cdot\frac{\partial\sigma}{\partial y}}, \qquad (10)$$

with σ being determined by correlationship (3).

Differentiating (3), we derive a precise, but not practically applicable, formula, which determines the momentary velocity of the trough in a direction tangent to the isobar:
$$c_L = \frac{1}{A}\left\{3\frac{\partial^2 p}{\partial x\,\partial t}\left[\frac{\partial^3 p}{\partial x^2\,\partial t}\frac{\partial p}{\partial y} - 2\left(\frac{\partial^2 p}{\partial x\,\partial y}\right)^2 + \frac{\partial^2 p}{\partial x^2}\left(\frac{\partial^2 p}{\partial x^2} - \frac{\partial^2 p}{\partial y^2}\right)\right] - \right.$$
$$- \frac{\partial p}{\partial y}\left[\frac{\partial^4 p}{\partial x^3\,\partial t}\frac{\partial p}{\partial y} + \frac{\partial^3 p}{\partial x^3}\cdot\frac{\partial^2 p}{\partial y\,\partial t} - 3\frac{\partial^3 p}{\partial x^2\,\partial t}\cdot\frac{\partial^2 p}{\partial x\,\partial y} - \right. \qquad (11)$$
$$\left.\left. - 3\frac{\partial^2 p}{\partial x^2}\frac{\partial^3 p}{\partial x\,\partial y\,\partial t}\right]\right\}$$

where
$$A = \frac{\partial p}{\partial y}\left(\frac{\partial^4 p}{\partial y^4} - 2\frac{\partial^2 p}{\partial x^2}\cdot\frac{\partial^3 p}{\partial x^2\,\partial y}\right) + 3\left(\frac{\partial^2 p}{\partial x^2}\right)^2\cdot\left(\frac{\partial p}{\partial y} - \frac{\partial p}{\partial x}\right).$$

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line, for which the condition $\frac{\partial p}{\partial x} = 0$, will hold at all times. But such a line will coincide with the baric axis at the initial moment only, and with continued motion, will gradually recede from the latter.

Figure 128 shows the location L of the baric axis at the initial moment of time, and its location L' after time interval t. L' gives the location of a characteristic line, which is determined by the condition $\frac{\partial p}{\partial x} = 0$, and which coincided at the initial moment with the trough. If the baric axis is sharply accentuated, the line $\frac{\partial p}{\partial x} = 0$ will be very close to it. Considerable difference in location occurs only when the baric axis is not sharply accentuated. But this case is of no practical interest.

Section 5. Displacement of Baric Centers. Other formulas for the Extrapolation of a Pressure Field

The pressure at a baric center has its relative minimum and maximum values. Generally speaking, two baric axes can be drawn through the baric center: one through points corresponding to the maximum curvature of the isobars, and the other one, through points corresponding to the minimum curvature of the isobars. The problem of the movement of the baric center, then, reduces itself to the problem of the movement of the point at which the two baric axes cross each other.

We will limit ourselves to the analysis of the case when the baric centers, as points, will remain for a certain time interval, stationary in the pressure field, i.e., points at which at all times

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exist the correlationships:

$$\frac{\partial p(x,y,t)}{\partial x}=0, \quad \frac{\partial p(x,y,t)}{\partial y}=0, \tag{1}$$

although the pressure values at these points change with the course of time.

It is known that a function of two variables, which satisfies condition (1), has a minimum, if

$$\frac{\partial^2 p}{\partial x^2}>0; \quad \frac{\partial^2 p}{\partial y^2}>0; \quad \Delta=\left(\frac{\partial^2 p}{\partial x\,\partial y}\right)^2-\frac{\partial^2 p}{\partial x^2}\cdot\frac{\partial^2 p}{\partial y^2}<0, \tag{2}$$

and a maximum, if

$$\frac{\partial^2 p}{\partial x^2}<0; \quad \frac{\partial^2 p}{\partial y^2}<0; \quad \Delta=\left(\frac{\partial^2 p}{\partial x\,\partial y}\right)^2-\frac{\partial^2 p}{\partial x^2}\cdot\frac{\partial^2 p}{\partial y^2}<0. \tag{3}$$

Let us assume that function p(x,y,t) has continuous derivatives of the first and second order, by all arguments. Then, using the rule of differentiation of implicit functions, we derive from (1) the following expressions for the velocity components of the baric center:

$$\left.\begin{aligned} c_x&=\frac{dx}{dt}=\frac{1}{\Delta}\left(\frac{\partial^2 p}{\partial x\,\partial t}\cdot\frac{\partial^2 p}{\partial y^2}-\frac{\partial^2 p}{\partial y\,\partial t}\cdot\frac{\partial^2 p}{\partial x\,\partial y}\right),\\ c_y&=\frac{dy}{dt}=\frac{1}{\Delta}\left(\frac{\partial^2 p}{\partial y\,\partial t}\cdot\frac{\partial^2 p}{\partial x^2}-\frac{\partial^2 p}{\partial x\,\partial t}\cdot\frac{\partial^2 p}{\partial x\,\partial y}\right).\end{aligned}\right\} \tag{4}$$

If the baric axes, passing through the baric center, do not rotate, $\frac{\partial^2 p}{\partial x\,\partial y}=0$, and formulas (4) are simplified:

$$c_x=-\frac{\frac{\partial^2 p}{\partial x\,\partial t}}{\frac{\partial^2 p}{\partial x^2}}, \quad c_y=-\frac{\frac{\partial^2 p}{\partial y\,\partial t}}{\frac{\partial^2 p}{\partial y^2}} \tag{5}$$

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For this simplest case the acceleration components of the baric center all have a relatively simple form:

$$\left.\begin{aligned} A_x &= -\frac{\frac{\partial^3 p}{\partial x \partial t^2}\cdot\frac{\partial^2 p}{\partial x^2} - 2\frac{\partial^2 p}{\partial x \partial t}\cdot\frac{\partial^3 p}{\partial x^2 \partial t}}{\left(\frac{\partial^2 p}{\partial x^2}\right)^2}, \\ A_y &= -\frac{\frac{\partial^3 p}{\partial y \partial t^2}\cdot\frac{\partial^2 p}{\partial y^2} - 2\frac{\partial^2 p}{\partial y \partial t}\cdot\frac{\partial^2 p}{\partial y \partial t}}{\left(\frac{\partial^2 p}{\partial y^2}\right)^2} \end{aligned}\right\} \qquad (6)$$

Analogous formulas can be deduced for the displacement of a front, for a change in pressure on the axis of a baric system and at a baric center, for frontogenesis and frontolysis, also for the generation and diffusion of baric centers.

For example, with the aid of general formulas it is simple to derive the velocity and acceleration of a front, the equation for which can be taken in the following form:

$$p_1(x, y, t) - p_2(x, y, t) = 0. \qquad (7)$$

If axis y is directed along a front, and the x-axis perpendicular to it then, as was shown in the preceding chapter,

$$\frac{\partial p_1(x, y, t)}{\partial y} - \frac{\partial p_2(x, y, t)}{\partial y} = 0 \qquad (8)$$

and

$$\frac{\partial p_1(x, y, t)}{\partial x} - \frac{\partial p_2(x, y, t)}{\partial x} \neq 0. \qquad (9)$$

Therefore, the velocity of a front C_f is represented by:

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$$c_f = -\frac{\frac{\partial p_1}{\partial t} - \frac{\partial p_2}{\partial t}}{\frac{\partial p_1}{\partial x} - \frac{\partial p_2}{\partial x}}, \qquad (10)$$

and its acceleration by

$$A_f = -\frac{\left(\frac{\partial^2 p_1}{\partial t^2} - \frac{\partial^2 p_2}{\partial t^2}\right) + 2c_f\left(\frac{\partial^2 p_1}{\partial x \partial t} - \frac{\partial^2 p_2}{\partial x \partial t}\right) + c_f\left(\frac{\partial^2 p_1}{\partial x^2} - \frac{\partial^2 p_2}{\partial x^2}\right)}{\frac{\partial p_1}{\partial x} - \frac{\partial p_2}{\partial x}} \qquad (11)$$

In the case of an occluded cyclone, the cold front overtakes the warm front, forcing out the warm air mass. Therefore, the process of occlusion may be evaluated quantitatively by the diminution of the distance, along the x-axis, between the warm and the cold fronts:

$$S_{xf} - S_{wf} = (c_{xf} - c_{wf})t + (A_{xf} - A_{wf})\frac{t^2}{2}, \qquad (12)$$

where the magnitudes relating to the cold front are marked by the index K, and the magnitudes relating to the warm front are marked by the index W.

For a more detailed study of the kinematics of a pressure field, we refer our reader to the manual Methods for the Extrapolation of a Pressure Field (Hydro-meteorological Publishing House, 1940), and first of all, to the essential articles by A. F. Dyub'yuk and E. S. Kuznetsov.

The extrapolation formulas, deduced in this chapter, and also analogous formulas, are purely kinematic. Therefore, nothing in the nature of causative connections between various changes in a pressure field, can be derived from them. A practicing synopticist, of course,

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constantly resorts to extrapolation, when making deductions with relation to future changes in a pressure field on the basis of changes that have been observed. The extrapolation formulas that were cited, and similar ones, make it possible to use the extrapolation method on an improved theoretical basis. They make it possible for forecasters, in evaluating the anticipated changes in a pressure field, to take into consideration the structure of such a field.

However, the essence of extrapolation lies specifically in the fact that we endeavor, on the basis of infinitesimally small variations, occurring at a given moment, to pre-compute the value of the function being studied for the more or less remote future. The question arises, whether the infinitesimally small [variations at the given moment] is more within our reach than the finite [change] in the remote future. The answer is no. Firstly, because the devices we use are not that perfect, and secondly, and this is the main reason, because the statistical regularity, which we ascribed to the function being extrapolated (continuity and differentiability), is retained only while we contemplate the function in large aspect. With the transition to small details, this statistical regularity disappears. Therefore, formal extrapolation will never give us the opportunity to penetrate into the future.

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CHAPTER XV

TURBULENT EXCHANGE, THE SPECTRAL STRUCTURE OF TURBULENCE

Section 1. The Basic Formulas of Turbulent Exchange

Already the first experiments in the study of turbulence showed that as soon as the Reynolds number exceeds the critical value, the current loses its laminar character. The trajectories of separate particles stop being rectilinear, they become tangled and intermixed. Only the mean velocity of the current retains a definite magnitude and direction, while each air particle attains its own additional velocity.

In studying this motion by the Euler method, turbulence is manifested by pulsation in time of the magnitudes of velocity and other magnitudes. But in studying the motion by the Lagrange method, the turbulence manifests itself in chaotic motion of the particles along complex trajectories, it being the case that the particles constantly exchange places. Such a molar motion resembles to a considerable degree the chaotic motion of the molecules.

The most characteristic feature of turbulent processes occurring in the atmosphere, is the turbulent exchange of air masses and the intermixing connected with it. In molar transfer, the "element of turbulence", or the individual vortex does not simply penetrate into the surrounding mass of air, but intermixes with it. Therefore, with time the element of turbulence is as if dissolved in the surrounding

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air. This constitutes the principal difference between a vortex in the atmosphere, as a real viscous liquid, and a vortex in an ideal liquid. A vortex evolved in an ideal liquid shifts without intermixing with the surrounding liquid, and exists eternally. Therefore, the concept of "intermixing" is extraneous to an ideal liquid, while for the real atmosphere, it is the basic and most characteristic property of motion.

And in the same manner as the chaotic motion of the molecules leads to the development of relaxation processes, such as viscosity, heat conductivity, diffusion, the chaotic motion of the elements of turbulence (moles and vortices) leads to the development of analogous relaxation processes. These processes, in contradistinction to molecular processes, are called "virtual or turbulent viscosity", "virtual or turbulent heat conductivity", "virtual or turbulent diffusion".

In the atmosphere, the effect of molar turbulent exchange, or intermixing, is much greater than the effect of molecular exchange; therefore, the latter can be completely disregarded.

The mathematical theory of turbulent exchange has been developed for the last 30 years. We will analyze the thoery of turbulent exchange on the basis of a strict and consecutive thesis published in 1931 by the Soviet scientist L. V. Keller.

For simplification, we will contemplate an air current over a horizontal surface conditioned by a pressure gradient force. We will

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assume that the motion of the air is also horizontal, but only in its mean value. We will direct the x-axis along the mean current. Then, the mean velocity components along the axes y and z will equal zero.

As to the acceleration components, they are all different from zero. We will limit ourselves to the consideration of motion in a plane.

Let us consider some characteristic of air which we will designate as substance. This designation is conditional, since under it we will consider not only substance as such (dust, water vapors, and the like), but also quantitative characteristics of the air.

We assume that substance possesses the following three distinct features, by which it is set apart:

(1) Substance is not destroyed in transfer with an elementary particle of air, i.e., it remains constant until such time as the contemplated particle is intermixed with the surrounding air.

(2) The quantity of substance is preserved in the intermixing of two air masses.

(3) Substance is "passive". That means that its admixture to the air does not essentially affect the development of turbulence.

Substance is a quantitative characteristic of air, in contradistinction to its qualitative characteristics. This difference is based on the manner in which these two characteristics of the air behave in the process of intermixing. If we intermix two masses of

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air M' and M" with characteristics Q' and Q", the substance of the mixture will be

$$MQ = M'Q' + M''Q'', \tag{1}$$

and the qualitative characteristic of the mixture will be:

$$Q = \frac{M'Q' + M''Q''}{M' + M''} \tag{2}$$

As an example of quantitative characteristics, or substances, can be cited: the quantity of motion, enthalpy, electric volumetric charge, kinetic energy.

Some qualitative characteristics are: density, velocity, temperature.

It must be emphasized particularly that in the exchange of air masses, it is not temperature and velocity that intermix, but respectively, enthalpy and quantity of motion. Only in the case of a non-compressible liquid, can we discuss conditionally the exchange or intermixing of temperature and velocity.

Let us designate the specific quantity of substance, as related to a unit of mass, by S, and the density of the substance i.e., its quantity as related to a unit of volume, by σ . It is obvious that $\sigma = \rho \cdot S$. We will cite all the arguments with relation to the density of the substance, and write down the result for the specific quantity of the substance by analogy.

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Let us consider an adequately large horizontal area f, which is being displaced together with the mean air current (Figure 129).

Figure 129. Determination of the current of the characteristic stipulated by turbulence.

Each element of turbulence, moving, with its mean value together with the horizontal area, will cross the latter in ascending or descending, at its vertical velocity. Together with such a vertical exchange of air masses, along a vertical line, the substance will also be transferred.

The quantity of substance, being carried in a unit of time across the elementary horizontal area, equals $\sigma \cdot w \cdot df$, and the total quantity of substance Q, which in its mean value, will pass in a unit of time across a unitary horizontal area, will be:

$$Q = \frac{1}{f} \int_{(f)} \sigma \cdot w \cdot df. \qquad (3)$$

We will assume that the substance of a particle is a function of the [space] coordinates and time: $\sigma(x, y, z, t)$. But for the averaged value, the fluctuations in the distribution of the substance over the area f are smoothed out, so that $\sigma = \sigma(z, t)$.

The vertical velocity of the particle is also a function of

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space coordinates and time: $w(x,y,z,t)$.

It is important to emphasize that the horizontal area f, with relation to which the averaged value of the product $\sigma \cdot w$, is computed that is, the vertical current of the substance, must be very large in size. At any rate, it must be large to the extent that the aggregate of various values of the magnitudes σ and w, is sufficiently great to allow the deduction from it of an aggregate of magnitudes according to some additional characteristic, and still remain with a new aggregate that is adequately significant for purposes of determination.

We make such a selection by two characteristics: by the initial height Z_0, at which the substance was located prior to its transfer onto the area f, and by the initial velocity value W_0. It is obvious that at the initial height Z_0, velocities may be arbitrary. Therefore, to each value of W, there may be a great number of various corresponding values of $\sigma(x_0, y_0, z_0, t_0)$. Then, integral (3) can be resolved into a great number of integrals, each one of which extends over a horizontal area with corresponding values for Z_0 and W_0. Extending the integral over such an area, we derive, after dividing it by the magnitude of the area, some averaged value $\sigma(z_0, t_0)$, retaining for the previous designation for purposes of brevity. Thus, we derive some averaged field of substance σ, it being the case that the averaging of the value now is done not with relation to time, as was the case previously, but with relation to area.

Let us further assume that the distance $(Z - Z_0)$ passed by the substance carrying particle, and the time interval $(t - t_0)$ are ade-

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quately small. We will expand the area-wise averaged value of the substance into a Taylor series, and limit ourselves to the first terms:

$$\sigma(z,t) = \sigma(z_0,t_0) + \frac{\partial\sigma}{\partial z}(z-z_0) + \frac{\partial\sigma}{\partial t}(t-t_0), \quad (4)$$

whence

$$\sigma(z_0,t_0) = \sigma(z,t) - \frac{\partial\sigma}{\partial z}(z-z_0) - \frac{\partial\sigma}{\partial t}(t-t_0). \quad (5)$$

Multiplying (5) by w, and integrating by area f, we obtain

$$Q = \frac{1}{f}\int_{(f)} \sigma(z_0,t_0)\cdot w\cdot df$$

or, taking into account equation (5),

$$Q = \frac{1}{f}\int_{(f)} \left[\sigma(z,t) - \frac{\partial\sigma}{\partial z}(z-z_0) - \frac{\partial\sigma}{\partial t}(t-t_0)\right] w\cdot df. \quad (6)$$

In the integral equation (6), the following magnitudes are constants: σ, $\frac{\partial\sigma}{\partial z}$, $\frac{\partial\sigma}{\partial t}$ as averaged magnitudes with relation to the entire area; t_0 as the parameter, which does not depend on the location of the element df in the horizontal area f; and t, since the integration is done by the area, for a fixed moment of time.

The variables, for each element of df, are: the vertical velocity W, at which the air particle is carried through the horizontal area, and the distance $(Z - Z_0)$ from the initial level, from which the air particle came.

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Carrying out the constants beyond the sign of the integral, we obtain:

$$Q=\left[\sigma(z,t)-\frac{\partial\sigma}{\partial t}(t-t_0)\right]\frac{1}{f}\cdot\int_f w\,df-\frac{\partial\sigma}{\partial z}\cdot\frac{1}{f}\int_f w\cdot(z-z_0)\,df.$$

Assuming that the volume of descending air carried across the area f is equal to the volume of ascending air carried across the area f, we can consider the first integral as equal to zero. Then, for the current of the substance we obtain:

$$Q=-\frac{\partial\sigma}{\partial z}\cdot\frac{1}{f}\int_{(f)} w(z-z_0)\,df. \qquad (7)$$

Let us assume:

$$K=\frac{1}{f}\int_{(f)} w(z-z_0)\,df. \qquad (8)$$

Then, the current of the substance, stipulated by a turbulent exchange, will be expressed by the formula:

$$Q=-K\cdot\frac{\partial\sigma}{\partial z}. \qquad (9)$$

The magnitude K we will call the coefficient of turbulence. As can be seen from equation (8), K is the averaged value of the product of W and $(Z - Z_0)$:

$$K=\overline{w(z-z_0)}. \qquad (10)$$

We remember that Z and Z_0 are the coordinates of the same

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particle at moments of time t and t_0, and that $w = \frac{dz}{dt}$; the averaging was done for one moment of time with relation to an adequately large horizontal area. ~~It is usually considered that such an averaging may be substituted by an averaging with relation to an adequately large horizontal area.~~ It is usually considered that such an averaging may be substituted by an averaging with relation to an adequately large time interval, for some fixed point Z_0.

We further have:

$$K = \overline{w(z-z_0)} = \overline{(z-z_0)\frac{d}{dt}(z-z_0)} = \frac{d}{dt}\overline{\left(\frac{z-z_0}{2}\right)^2} \qquad (11)$$

If K is a constant, independent of time, the differential equation (11) can easily be integrated:

$$2K(t-t_0) = \overline{(z-z_0)^2} . \qquad (12)$$

In the right-hand member of equation (12) is the mean square of the departure of the particle from its initial level $\sum^2_{z-z_0}$. With K constant, this mean square is in direct ratio to time t - t_0, so that the magnitude

$$K = \frac{\overline{(z-z_0)^2}}{2(t-t_0)} = \frac{\sum^2_{z-z_0}}{2(t-t_0)} \qquad (13)$$

is one half of the mean square of departure, with relation to a unit of time.

The coefficient of turbulence K can also be expressed differently:

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$$K = \overline{w(z - z_0)} = r \Sigma_w \Sigma_{z-z_0}, \qquad (14)$$

where r is the correlation factor between the vertical velocity W and departure $(Z - Z_0)$; Σ_w and Σ_{z-z_0} are the mean quadratics, respectively for W and $(Z - Z_0)$.

Let us designate the product $l' = r \cdot \Sigma_{z-z_0}$ which is the dimension of length, as the "length of the path of intermixing". It is obvious that

$$K = l' \cdot \Sigma_w . \qquad (15)$$

It is probable that the conditional length which we just introduced, is really tied in with the mean distance of transfer for particles in a turbulent exchange. But the very concept of mean distance of transfer, as will be seen further below is rather conditional. Therefore, the introduction of the "mean path of intermixing", is caused principally by the impossibility of measuring the actual path of intermixing.

Repeating the argumentations used for the deduction of equation (9), for a specific quantity of substance S, we obtain:

$$Q = -A \frac{\partial s}{\partial z} \qquad (16)$$

where

$$A = \rho \overline{w(z - z_0)} \qquad (17)$$

is the so-called coefficient of turbulent exchange, which was first

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derived by Shmidt, in a somewhat different form. If the variations in a somewhat different form. If the variations in density, which are infinitesimally small as compared to the variations in W and $(Z-Z_0)$, are disregarded, then the correlationship between the coefficient of turbulence (K) and the coefficient of turbulent exchange (A) will be:

$$A=\rho K \tag{18}$$

It follows from the above that the magnitude of the coefficients of exchange and turbulence is entirely independent of what the substance is or of its distribution, and is determined exclusively by the degree of turbulence in the atmosphere. Therefore, the current of characteristic, determined by equation (9) or (16), is also entirely independent of what this characteristic is, but is determined merely by its gradient and the degree of turbulence in the atmosphere.

This means that, after having somehow determined the coefficient of exchange we can compute the current of any substance. To do so, it is only necessary to know the mean value of the vertical gradient of the substance.

It is specifically this fact that makes the formula of turbulent exchange particularly fruitful. Incidentally, the importance of the result obtained is not to be overestimated. In the deriving of formulas (9) and (16) we imposed a series of limitations, some of which are very exacting. Let us remember the most important of them:

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(1) Formulas (9) and (16) are applicable only to substances, which have the characteristic of non-destructibility; therefore, their application for example, to the transfer of a quantity of motion is not altogether regular, since a quantity of motion is gradually destroyed under the effect of internal friction; it also changes under the effect of other forces such as the pressure gradient force.

(2) Formulas (9) and (16) were derived as a result of averaging with relation to a very large area, which may be substituted by an averaging with relation to a very large time interval. Therefore, they are applicable to processes that change very slowly.

Numerous computations of the values of A and K, made on the basis of observations of the transfer of various substances in the atmosphere, have shown that A (and, consequently, also K) varies very intensively with height and with time. Therefore, the assumption that A = const., and K = const. is very rough, and at times, distorts the very essence of the phenomenon.

The transfer of a substance along a vertical line leads to the fact that the area-wise averaged value of the substance varies with time in a turbulent air current. Let us derive the formulas which determine this variation.

Let us imagine an elementary parallelepiped with a unitary base and a height Δz, which is being displaced, without rotation, together with the mean air current. Due to turbulent exchange, the substance will be flowing into the parallelepiped through the bottom

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base, and flowing out of it through the upper base. The currents of the characteristic through the lower and upper bases, will be respectively, as follows: $Q(z) = -K(z)\left(\frac{\partial \sigma}{\partial z}\right)_z$ and $Q(z+\Delta z) = -K(z+\Delta z)\left(\frac{\partial \sigma}{\partial z}\right)_{z+\Delta z}$.

The difference between the quantities of the inflown and outflown substance, per unit of time, equals

$$\left(K\frac{\partial^2 \sigma}{\partial z^2} + \frac{\partial K}{\partial z}\frac{\partial \sigma}{\partial z}\right)\Delta z = \frac{\partial}{\partial z}\left(K\frac{\partial \sigma}{\partial z}\right)\Delta z,$$

with the terms of the second order of smallness having been discarded. This difference is equal to the variation, per unit of time, of the substance, contained in the shifting elementary volume $\frac{d\sigma}{dt} \cdot \Delta z \cdot 1$. Equating these magnitudes and reducing by Δz, we derive:

$$\frac{d\sigma}{dt} = \frac{\partial}{\partial z}\left(K\frac{\partial \sigma}{\partial z}\right). \tag{19}$$

If K is independent of height,

$$\frac{d\sigma}{dt} = K\frac{d^2\sigma}{dz^2} \tag{20}$$

Repeating these argumentations for the specific quantity of substance S, we obtain:

$$\frac{ds}{dt} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(A\frac{\partial s}{\partial z}\right), \tag{21}$$

and for the case, when A does not depend on Z,

$$\frac{ds}{dt} = \frac{A}{\rho}\frac{\partial^2 s}{\partial z^2}. \tag{22}$$

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If variation ρ is disregarded, equation (21) for the specific quantity of substance coincides with equation (19) for the density of the substance.

Equations (20) and (22) ~~are~~ called equations of the vortex diffusion, or Fick equations, since they are completely analogous with the equations derived by Fick for molecular diffusion.

Let us note that the individual derivative $\frac{d}{dt}$, in the left-hand part of the derived equations, can be substituted by a local derivative $\frac{\partial}{\partial t}$, since we are considering the value of the substance, averaged with relation to area, which as per condition, varies only with height, with no horizontal changes. This condition in the atmosphere may be considered most of the time as fulfilled.

In concluding this section, we may say that in the case of a horizontal motion the coefficient of exchange condenses in itself the essence of atmospheric turbulence. The magnitude of the coefficient of exchange incorporates in itself both, the mean intensity as well as the mean dimension of additional motions in the atmosphere. It is remarkable that the magnitude of A may be determined from observations of almost every one of the meteorological elements.

Section 2. Transfer of a quantity of Motion.

In the preceding section we derived the formulas of exchange with relation to an arbitrary substance. We shall now apply these formulas to existing substances, and first of all to a quantity of motion.

Strictly speaking, a quantity of motion is not a passively

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transferable substance. It also does not satisfy the basic characteristic of a substance - non-destructibility. In reality, a quantity of motion is destroyed by forces of internal friction, and is also destroyed or regenerated with a transfer of a quantity of motion into layers with different values for the pressure gradient force.

We can, however, assume that in a not very thick layer, in which the process of transfer of a quantity of motion takes place, the pressure gradient force does not vary with height, and the variation in the quantity of motion under the effect of friction is so small that, in the first approximation, this variation may be overlooked. Let us first of all consider the analogy between the turbulent and molecular transfer of a quantity of motion. The molecular transfer of a quantity of motion brings us to the Newton law of internal friction:

$$\tau = \mu \frac{du}{dz}$$

where $\mu = \frac{1}{3}\rho l c$, the coefficient of internal friction, ρ - density, l - the length of the mean passage path of the molecule, c - the mean velocity of the molecules.

With turbulent exchange, additional stresses develop in a liquid, stipulated by the transfer of a quantity of motion and of the direction of the mean velocity gradient of the current. The additional stress, stipulated by the transfer of a quantity of motion in a turbulent current, in the case of planar motion will be $-\rho \overline{u'w'}$.

In this case the molecularly stipulated stress does not disappear, and the additional turbulent stress is superposed upon it so that

$$\tau = \mu \frac{du}{dz} - \rho \overline{u'w'} \qquad (1)$$

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Since the additional turbulent stress is considerably greater than the molecular stress, the latter may always be disregarded. The additional stress can only increase the total stress, but not diminish it, - therefore:

$$\tau = -\rho\overline{u'w'} = -r\Sigma_{u'}\Sigma_{w'} > 0, \qquad (2)$$

and the correlation factor between u' and w' must be a negative magnitude.

For a precise calculation of the additional stress, it is necessary to find all the individual values for u' and w', and then determine the mean value of their products $\overline{u'w'}$. But magnitudes u' and w' cannot be measured directly. Therefore, the determination of the additional stress reduces itself to finding the functional relationship between u' and w', on the one hand, and the mean velocity of the current, which it is possible to measure, on the other hand.

Prandtl' suggested a series of fruitful, although non-rigid hypotheses on the character of such connections, having developed in 1925 the theory of transfer of a quantity of motion.

Since each particle of the liquid, alongside the horizontal velocity component, has a vertical component w', the trajectories of the particles, in the case of planar motion, are sloping lines. Along these trajectories, according to Prandtl', at a velocity w', is transferred from layer to layer, some additional quantity of motion

$\rho u'$.

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Let us assume that a particle captures from some level Z_1 a quantity of motion, corresponding to the mean velocity of current U_1 at the same level, and carries it to another level Z_2, where this particle is intermixed with the surrounding liquid, acquiring velocity U_2 . A segment of the verticle projection of the trajectory of the particle, contained between the indicated levels, is called, according to Prandtl', the "path of intermixing", which we will designate by l'.

Thus, at levels Z_1 and Z_2, the individual air particle possesses the same quantity of motion as other particles moving at the mean velocity of the current and, consequently , its motion does not differ from the mean motion of the other particles. Only in transferring from level Z_1 to level Z_2, the particle manifests its individuality, which is acquired at level Z_1 and lost at level Z_2.

If the path of intermixing is small in comparison with the dimensions of the current it can be assumed.

$$\frac{u_2 - u_1}{l'} \approx \frac{du}{dz}. \qquad (3)$$

Prandtl' assumes

$$u_2 - u_1 = u'$$

i.e., he identifies the difference of mean velocities at various levels with a pulsating velocity observed in a motionles air. Consequently, in accordance with Prandtl' :

$$u' = l' \frac{du}{dz}. \qquad (4)$$

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If U' is substituted in equation (2), the additional stress will be expressed by formula

$$\tau = -\overline{\rho w' u'} = -\overline{\rho w' l'} \frac{\partial u}{\partial z}. \qquad (5)$$

On the other hand, the current of a quantity of motion is expressed by:

$$Q = -\overline{\rho w' u'} = -A \frac{d\bar{u}}{dz}. \qquad (6)$$

Therefore, for the coefficient of exchange A, we derive:

$$A = \rho \overline{w' l'} \qquad (7)$$

It is obvious that the structure of the coefficient of exchange is identical with the structure of the coefficient of viscosity μ. But the numeral factor 1/3 is not present in the coefficient, since we introduced l' for one direction only - perpendicular to the current, while the mean length of the path of the free passage of the molecules is introduced when molecular exchange, occuring in three dimensions, is considered. In place of the mean velocity of the molecules, into the coefficient of exchange enters the vertical component of pulsation velocity, which is perpendicular to the mean velocity of the current. In place of the mean path of the free passage of the molecules, into formula (7) enters the length of the path of intermixture, i.e., the segment of the vertical projection of the trajectory of the particle, which the latter covers prior to complete intermixture with the surrounding particles.

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But into formula (7), which determines the coefficient of exchange, there still enters the additional momentary pulsation velocity W', which can be expressed through mean velocity in the following manner. It is natural to assume that W' must be tied in with the rotation of a particle of the liquid, or what is the same, with the velocity vortex, which upon being averaged may be expressed by $\frac{du}{dz}$.

Prandtl' accepts also here the principle of proportion:

$$w' \approx l'' \frac{du}{dz}. \tag{8}$$

Combining (8) and (5), we finally derive:

$$\left|\overline{u'w'}\right| = l^2\left(\frac{du}{dz}\right)^2, \tag{9}$$

where $l^2 = l'l''$. The coefficient of exchange will now be expressed by formula

$$A = \rho l^2 \left|\frac{du}{dz}\right|. \tag{10}$$

The Prandtl' thesis is not rigid. As we have seen in his process of deduction, he makes a series of rather non-obvious assumptions, some of them even in contradiction to experimental data.

(1) Assuming that $U_2 - U_1 \equiv U'$, Prandtl' identifies the difference in the mean velocities at points lying at various distances from the hard wall, with pulsating velocity at the given point. The pulsation of velocity is thus considered as a relay of mean velocities trans-

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ferable from various points of the current. Such an assumption is unfounded and is contradictory to the results of experiments in aerodynamic tubes, which have shown that, while the mean velocities of the current further away from the walls differ very little from each other, the pulsation of velocity may attain 20 percent of the averaged velocity .

(2) Prandtl' assumes that the process of intermixing is discontinuous. At first, velocity is simply transferred, without any change; then after the shifting of the particles to a distance equal to the path of intermixing the particle is suddenly intermixed with its surrounding particles, and acquires a velocity equal to the mean velocity of the current at this level. It is not clear why the process of intermixing should be discontinuous, why the particle should retain the velocity transferable by it without experiencing the effect of viscosity. We are still more perplexed, since according to Prandtl', it is not the momentary velocity of the particle, at the beginning of its path, that is being transferred, but its averaged velocity.

(3) Prandtl' assumes that the vertical pulsation velocity W' serves only for the transfer of a horizontal quantity of motion, while, in reality it is being transferred itself, with the order of magnitudes U' and W', as shown by measurements, being the same.

(4) Prandtl' introduces the length of the path of intermixing into a differential expression, relating to a finite point of the path, while the length of the path of intermixing is known a priori to be an integral magnitude, relating to the initial and final points of the path.

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In spite of the serious inaccuracies and vaguenesses in the Prandtl' thesis, it is widely used in hydro- and aero-dynamics. This can be explained by the fact that the magnitudes l' and l" in the Prandtl' formulas can be considered not as the paths of intermixing, but as magnitudes, which are proportional to the paths of intermixing. Through such generalization, the geometrical obviousness of the thesis is diminished, yet formally it becomes stricter, it being the case that it becomes possible to impart several other derivations to the Prandtl' formulas. We will cite the simplest of these derivations based on the method of dimensional equation. We are searching for a functional relationship of the type

$$\left|\overline{u'w'}\right| = f\left(\frac{du}{dz}\right). \tag{11}$$

The left-hand part of (11) has a dimensionality L^2T^{-2}, and in the right-hand part is a function of a magnitude, the dimensionality of which is T^{-1}. It is obvious, that into the right-hand part, it is necessary to interpolate some length l.

We will search for the function f in the form of a mononomial

$$\left|\overline{u'w'}\right| = l^m\left(\frac{du}{dz}\right)^n, \tag{12}$$

where m and n are the, so far, undetermined exponents. Equating the exponents for L and T in the right- and left-hand parts of equality (12), we obtain m = 2; n = 2, whence

$$\overline{u'w'} = l^2\left(\frac{du}{dz}\right)^2 \tag{13}$$

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This, then is the Prandtl' formula, but coefficient l is now indeterminate. It follows from this that the interpolation of the path of intermixing is formally not obligatory, yet it is tempting by its physical (it is even better to say, geometrical) descriptiveness.

By measuring the tangents of stress and velocity at various points of the turbulent current with the aid of formula

$$\tau = \rho l^2 \left(\frac{d\bar{u}}{dz}\right)^2 ,$$

it is possible to arrive at the magnitude of the path of intermixing, as per Prandtl':

$$l = \sqrt{\frac{\tau}{\rho}} \left(\frac{du}{dz}\right)^{-1} . \qquad (14)$$

Measurements taken in cylindrical tubes showed that magnitude l varies with distance to the wall, within the limits of from zero to 14 degrees radius of the tube. It is conventionally considered that l is a linear function of the distance to the wall $l = \kappa z$, it being the case that $\kappa \cong 0.4$.

The above results are particularly important for the study of air currents in the surface atmospheric layer.

Yet, the application of these results to the free atmosphere is not always possible. Thus, for instance, in the free atmosphere, mean velocity frequently does not change with height, i.e., U = const. In this case, in accordance with (10), the coefficient of exchange becomes zero, i.e., turbulent exchange in the free atmosphere, in this case, must

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not occur. [However,] observations of temperature and humidity changes in the free atmosphere show that, with $\frac{du}{dz} = 0$, turbulent exchange does not cease by far.

The concept of the path of intermixing has the advantage of geometrical obviousness. The concept of the coefficient of exchange is not that obvious, since it depends not only on the purely geometrical magnitude l, but also on velocity and density. However, only the understanding of the coefficient of exchange makes the direct computations in the numerous problems of atmsopheric turbulence, or better said, in the problems of turbulent exchange, possible.

Incidentally, the above relates only to conditions occurring in the free atmosphere, far away from the terrestrial surface. For the surface atmospheric layer, the introduction into meteorology of the concept of the path of intermixing, as a measure of turbulence, is of tremendous importance, due to the simple geometrical dependence of the path of intermixing on the height and roughness of the underlying surface.

Section 3. The Transfer of Vortices

Almost simultaneously with Prandtl', Taylor developed the theory of the transfer of vortices. Taylor considers that the chaotic character of the turbulent motion in a liquid is tied in with irregularities in the distribution of static pressure. Therefore, during the displacement of the finite mass of liquid from one layer into another, the pulsation of pressure, and pressure gradient forces generated thereby, must not be overlooked. It is wrong to assume that, in the transfer of

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a mass, its quantity of motion is reserved.

In limiting himself to the consideration of planar motion, Taylor assumes that, in the transfer of a particle of liquid, it is not the quantity of motion of the transferable mass of liquid that is preserved, but only the intensity of the vortex.

Taylor's argumentations are adequately convincing. Indeed, the intensity of the vortex in a turbulent current is preserved longer than the quantity of motion of the transfer mass, even though the intensity of the vortex also varies, of course, due to the presence of viscosity.

Let us take the equation of the motion of an ideal non-compressible liquid, occurring in a plane XOZ, in the form

$$\frac{\partial u}{dt} = -\frac{\partial p}{\partial x} - \frac{1}{2}\frac{\partial}{\partial x}(u^2+w^2) - w\eta \qquad (1)$$

where

$$\eta = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}.$$

Averaging equation (1) and assuming that the averaged values $\bar{u}^2$, u'^2, w'^2, along the x-axis, do not vary, we derive:

$$\frac{\partial \bar{u}}{\partial t} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} - \overline{w'\eta'}. \qquad (2)$$

Comparing equation (2) with the previously derived averaged Reynolds equation (see Section 12, Chapter VIII), which, in our case, has the form of

$$\frac{\partial \bar{u}}{\partial t} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} - \frac{d}{dz}(\overline{u'w'}), \qquad (3)$$

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We see that the variation in the quantity of motion can be expressed by $-\overline{w'\eta'}$, or by $-\frac{d}{dz}(\overline{u'w'})$; consequently,

$$-\overline{w'\eta'} = -\frac{d}{dz}(\overline{u'w'}). \qquad (4)$$

Since, in accordance with Prandtl',

$$\left|\overline{u'w'}\right| = l^2\left(\frac{d\bar{u}}{dz}\right)^2,$$

then

$$\overline{w'\eta'} = \frac{d}{dz}\left\{l^2\left(\frac{d\bar{u}}{dz}\right)^2\right\} \qquad (5)$$

On the other hand, having in mind that the averaged value of the vortex is $\frac{d\bar{u}}{dz}$, and applying the Prandtl' hypothesis to the pulsation of velocity $w' = l\frac{d\bar{u}}{dz}$ and the pulsation of the vortex

$$\eta' = l\frac{d\bar{\eta}}{dz} = l\frac{d^2\bar{u}}{dz^2},$$

we derive:

$$\overline{w'\eta'} = l\cdot\frac{d\bar{u}}{dz}\cdot l\cdot\frac{d\bar{\eta}}{dz} = l^2\cdot\frac{du}{dz}\cdot\frac{d^2u}{dz^2}. \qquad (6)$$

So that, for the transfer of vortices, we derive two separate results. Starting with equation (3), which is derived on the basis of the theory of transfer of quantity of motion, we derive (assuming that $\frac{\partial p}{\partial x} = 0$)

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$$\frac{\partial \bar{u}}{\partial t} = \overline{w'\eta'} = \frac{d}{dz}\left\{ l^2 \left(\frac{d\bar{u}}{dz} \right)^2 \right\} \qquad (7)$$

And, starting with equation (2), derived on the basis of the theory of transfer of vortices, we obtain:

$$\frac{\partial \bar{u}}{\partial t} = \overline{w'\eta'} = l^2 \cdot \frac{d\bar{u}}{dz} \cdot \frac{d^2\bar{u}}{dz^2} \qquad (8)$$

Therefore, by introducing the coefficient of exchange $A = \rho l^2 \frac{d\bar{u}}{dz}$, or, better yet, the coefficient of turbulence $K = \frac{A}{\rho} = l^2 \frac{d\bar{u}}{dz}$, we derive:

$$\frac{\partial \bar{u}}{\partial t} = \frac{d}{dz}\left(K \frac{d\bar{u}}{dz} \right), \qquad (9)$$

$$\frac{\partial \bar{u}}{\partial t} = K \frac{\bar{d^2 u}}{dz^2} \qquad (10)$$

it being the case that, for simplicity, we dropped the sign of the modulus in the expressions, which determine A and K, since in the contemplated case, velocity is always accelerated upward so that $\frac{d\bar{u}}{dz} > 0$.

Comparing equation (9) with equation (10), after rewriting it in an expanded form:

$$\frac{\partial \bar{u}}{\partial t} = K \frac{\overline{d^2 u}}{dz^2} + \frac{\partial K}{\partial z} \cdot \frac{\partial \bar{u}}{\partial z}. \qquad (11)$$

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The right-hand part of (11), according to argumentations cited in Section 2, is the difference between the quantity of motion that entered into a unitary volume, and the quantity of motion that left the unitary volume. The first term in the right-hand part $K\frac{d^2u}{dz^2}$, according to (10) is the real increment in the quantity of motion. Therefore, the second term $\frac{\partial K}{\partial z}\frac{du}{dz}$ in the right-hand part (11) is the quantity of motion, which is destroyed as a result of the pulsation of pressure.

Equations (9) and (10) coincide in one case only: when $\frac{\partial K}{\partial z}=0$, and, consequently:

$$K=l^2\frac{d\bar{u}}{dz}=const.$$

Since K generally depends on Z, the theory of transfer of the quantity of motion and the theory of transfer of the vortices, should, generally speaking, lead to different results. Since both these theories are not rigid, the final choice between the two may be made only by comparison with experimental data. However, experiments conducted, as of this date, do not yet furnish any basis for giving preference to any one of these two theories.

Section 4. The Transfer of Other Atmospheric Characteristics

We derived the general expression for a current of substance in the following form:

$$Q_s=-A_s\frac{\partial s}{\partial z}. \qquad (1)$$

This expression, strictly speaking, is only true for substances, possessing the characteristics of non-destructibility in transfer, con-

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servability in intermixing, and passivity. Such characteristics are possessed by material admixtures suspended in the atmosphere, such as water vapor, smoke, fine dust, etc. But, usually, this equation is also applied to such substances as quantity of motion, heat, and also the intensity of vortices, which, strictly speaking, do not possess all three characteristics, particularly the characteristic of passivity.

As we have already seen, the coefficient of turbulent exchange A, does not depend on the transferable substance, but is only some statistical characteristic of atmospheric turbulence. But the character of turbulence varies from point to point within wide limits. Turbulence depends on temperature stratification, wind distribution by height, and in the lowest layer, also on the distance to the terrestrial surface, and the roughness of the underlying suface.

During a transfer of a quantity of motion, the current of the quantity of motion is identified with the displacement tension, and the coefficient of turbulent viscosity ν must coincide with the coefficient of turbulent exchange A.

During turbulent heat transfer, the potential heat content (enthalpy), with relation to a unit of mass, is considered to be the specific quantity of the transferable substance, i.e.,

$$s = c_p \theta \qquad (2)$$

where θ is the potential temperature. Consequently, the turbulent heat current will be determined by equation

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$$Q_\theta = -c_p A_\theta \frac{\partial \theta}{\partial z} . \qquad (3)$$

V. Shmidt postulates the truth of the general equation of turbulent exchange also for the case of the transfer of heat. Then, it should be:

$$A_\theta = A_s = A . \qquad (4)$$

However, the assumption of equality for the coefficients of turbulent exchange for a quantity of motion, heat, and admixtures, is not founded on proof. A quantity of motion and heat, no doubt, possess the characteristic of conservability, in the intermixing of two air masses. But a quantity of motion does not possess the characteristic of non-destructibility. Masses of air, in their passing from one layer to another, may acquire a forward motion under the effect of other forces, for example, local pressure gradient forces. Under the effect of these forces, a quantity of motion can be recreated or destroyed. Heat is not a passively transferable substance, since the transfer of heat leads to variations in temperature stratification, and this factor plays a substantial part in the development of turbulence. Therefore, there is no basis to the assertion that A_θ, A_s and A are equal to each other. It may only be affirmed that the order of these magnitudes cannot be essentially different, but their numerical values, generally speaking, are not equal.

The coefficient of turbulent viscosity A, and also the coefficients of turbulent exchange A_s and A_θ have the same dimensionality $[L^2 T^{-1}]$ and, in the absence of turbulence, are simultaneously

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reduced to zero. Therefore, the ratio

$$\frac{A_s}{A} = \alpha_s \qquad (5)$$

is a dimensionless parameter which is a function of dimensionless characteristics of a turbulent current. If a liquid is non-compressible (or only mildly compressible), the only characteristic of a turbulent current (in the absence of a free surface) is the Reynolds number. Therefore, parameter α_s can only be a function of [redacted] Re.

As was shown above, the numbers R_e in the atmosphere are very great. The above developed semi-empirical theory of turbulence is specifically applicable to the area of very great values of the number Re. Then, all functions of the Reynolds number are approximately equal to the ultimate values, with $Re \longrightarrow \infty$. In this case, α_s = const., $A_s = \alpha_s A$, and the basic equation of turbulent exchange, as per Obukhov, assumes the form

$$Q_s = -\alpha_s A \frac{\partial s}{\partial z} \qquad , \qquad (6)$$

or, if A substituted by its value, equal to $\rho l^2 \left|\frac{d\bar{u}}{dz}\right|$,

$$Q_s = -\alpha_s \rho l^2 \left|\frac{d\bar{u}}{dz}\right| \cdot \frac{\partial s}{\partial z} \qquad (7)$$

where α_s is a dimensionless parameter which depends on the nature of the transferable substance. For a quantity of motion, $\alpha_s = 1$. For other substances, α_s is of the order of one, although, generally

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speaking, is not equal to one, and has different values for different substances.

The formula for a turbulent heat current will be written down, if (3) is taken into account:

$$Q_\theta = -a_\theta \rho c_p l^2 \left|\frac{d\bar{u}}{dz}\right| \frac{\partial \theta}{\partial z}. \qquad (8)$$

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Section 5. The Effect of Thermal Stratification on the Development of Turbulence. The Richardson Number.

Until now we studied the development of turbulence in a homogeneous non-compressible liquid and took the dimensionless Reynolds number $Re = ul/\nu$. However, the Reynolds number is inadequate for the study of atmospheric turbulence.

In establishing his criterion of turbulence, Reynolds proceeded from the premise that the current was physically homogeneous and that only the dimensions of the boundaries and velocity varied. But in the atmosphere the development of turbulence depends most essentially on the vertical temperature gradient. The presence of substantial temperature gradients is conducive to the development of turbulence while small gradients and inversions serve to retard the development of turbulence. Therefore the experiments conducted in aerodynamic tubes where the temperature gradient is zero are comparable with one another but not with conditions as they exist in the atmosphere.

Aerodynamic laboratories recently began to use special tubes of a rectangular cross section in which the upper and lower sides can be heated or cooled. This creates various temperature gradients in the air current. The first experiments conducted in such tubes have already shown that in the substitution of an isothermic current by a current of stable stratification the critical velocity, in passing from laminar motion to turbulent motion, changes from 20 centimeters per second to 100 centimeters per second. With stable stratification, the two types of flow were

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observed simultaneously; turbulent flow in the bottom part of the tube and laminar flow in the upper part of the tube.

Thus the Reynolds number in itself is inadequate for the turbulence characteristic of a current in the presence of temperature non-homogeneity.

Prandtl' has shown that the criterion of turbulence with non-homogeneous density stratification in the current is a new dimensionless parameter which subsequently was named the Richardson number Ri.

We shall find the expression for the Richardson number by proceeding from considerations of dimensionality. We first ascertain the factors by which the development of turbulence is determined. Such factors are: (1) the vertical velocity gradient of the current $\frac{\partial u}{\partial z}$; (2) the density of the liquid ρ; (3) the vertical density gradient $\frac{\partial \rho}{\partial z}$; and (4) force of gravity g.

By using these magnitudes we compose an expression:

$$Ri = \rho^{\alpha}\left(\frac{\partial \rho}{\partial z}\right)^{\beta} \cdot \left(\frac{\partial u}{\partial z}\right)^{\gamma} g^{\delta}$$

and we require that this expression be dimensionless:

$$M^0 \cdot L^0 \cdot T^0 = \left(\frac{M}{L^3}\right)^{\alpha} \cdot \left(\frac{M}{L^4}\right)^{\beta} \cdot \left(\frac{1}{T}\right)^{\gamma} \cdot \left(\frac{L}{T^2}\right)^{\delta}.$$

Comparing the exponents, we see that they must satisfy the following conditions:

$$\begin{aligned} \alpha + \beta &= 0, \\ -3\alpha - 4\beta + \delta &= 0, \\ -\gamma - 2\delta &= 0. \end{aligned}$$

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Expressing all the unknown exponents through δ:

$$\alpha = -\delta;\ \beta = \delta;\ \gamma = -2\delta,$$

We obtain:

$$Ri = \left[g \frac{\frac{1}{\rho}\frac{\partial \rho}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2} \right]^{\delta}; \qquad (1)$$

δ may be any number. It is simplest to accept δ as equal to 1. Then the Richardson number will assume the form:

$$Ri = g \frac{\frac{1}{\rho}\frac{\partial \rho}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2} \qquad (2)$$

The development of turbulence in the atmosphere is affected by: (1) vertical velocity gradient of the current; (2) vertical potential temperature gradient; (3) the potential temperature proper θ ; and (4) force of gravity.

Arguing as before, we derive the turbulence criterion for the atmosphere in the same form as established by Richardson in 1920, proceeding from considerations of energetics:

$$Ri = \frac{g}{\theta} \frac{\frac{\partial \theta}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2} \qquad (3)$$

or

$$Ri = \frac{g}{T} \cdot \frac{\left(\frac{\partial T}{\partial z} + \gamma_a\right)}{\left(\frac{\partial u}{\partial z}\right)^2} \qquad (4)$$

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The Ri number also determines the degree of atmospheric turbulence. There are three distinct cases:

(1) $Ri > 0$; then $\frac{\partial \theta}{\partial z} > 0$. The stratification of the atmosphere is stable. The development of turbulence is retarded. The greater the value of Ri, the more stable the stratification of the atmosphere. Consequently, with values of Ri greater than some critical value, atmospheric turbulence is entirely diffused. The critical value Ri_{crit} of the Richardson number was evaluated more than once by various authors who arrived at different conclusions. Thus, according to Richardson, $Ri_{crit} = 1$; according to Prandtl', $Ri_{crit} = \frac{1}{2}$; according to Taylor, $Ri_{crit} = \frac{1}{4}$; according to Tolmiye~~n~~, $Ri_{crit} = \frac{1}{24}$. Reliable values for Ri_{crit} may be obtained only as a result of processing specially organized and thoroughly selected observations. [redacted] For this, as can be seen from the formula for Ri_{crit}, [redacted] simultaneous determinations are required for the distribution with height of both temperature and wind. In accordance with Sverdrup, the processing of these observations leads to $Ri_{crit} = \frac{1}{11}$.

(2) $Ri < 0, \frac{\partial \theta}{\partial z} < 0$. Consequently, the stratification of the atmosphere is unstable. The turbulence energy is increasing at the expense of a diminution of the energy of non-stability.

(3) $Ri = 0, \frac{\partial \theta}{\partial z} = 0$. The stratification of the atmosphere is indifferent. In this case, turbulence is developed in the same manner as in a homogeneous liquid, i.e., the above derived equations of the semiempirical theory of turbulence are applicable.

A. M. Obukhov generalized this theory for the case of a

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thermally non-homogeneous atmosphere. Proceeding from considerations of congruency, Obukhov assumes that all dimensionless characteristics of a current of liquid of variable density are functions of the Richardson number. In addition Obukhov assumes that all the coefficients of turbulent exchange for various substances are in proportion to each other in the same manner as in the case of a homogeneous liquid. Then, in accordance with Obukhov, to describe the various processes of turbulent exchange in a non-homogeneous atmosphere, it is sufficient to know only one "universal" function of the Ri number which enters as a correction factor into the equations of turbulent exchange.

Let A_0 be the coefficient of turbulent exchange in an adiabatic stratification of the atmosphere when Ri = 0. Then with the same distribution of wind velocity, the coefficient of turbulent exchange in the case of a non-adiabatic atmosphere will be:

$$A = \varphi(Ri) \cdot A_0 . \qquad (5)$$

As to the type of function φ (Ri), Obukhov reasons as follows. By determination of function φ (Ri), when Ri = 0, $\varphi(0) = 1$. With the Ri number increasing, the stratification of the atmosphere becomes more stable and turbulence intensity subsides. Consequently, φ (Ri) is a monotonic decreasing function. At some critical value Ri_{crit} turbulence is completely diffused so that

$$\varphi(Ri) = 0 \quad \text{when} \quad Ri > Ri_{crit}$$

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Interpolating the universal function into the equations of turbulent exchange for the case of a transfer of a quantity of motion and the case of a transfer of heat, we derive:

$$\tau = -\varphi(Ri)\cdot\rho l^2\left|\frac{d\bar{u}}{dz}\right|\frac{d\bar{u}}{dz}, \tag{6}$$

$$Q_\theta = -a\varphi(Ri)\cdot c_p\cdot\rho l^2\left|\frac{d\bar{u}}{dz}\right|\cdot\frac{d\theta}{dz}. \tag{7}$$

Section 6. Roughness

Tests in aerodynamic tubes have shown that the critical value of the Reynolds number Re is a function of temperature distribution or, in other words, a function of the Richardson number Ri.

However, observations show that atmospheric currents are always turbulent even when there is considerable stability in the atmosphere. Therefore, the deciding factors in the development of turbulence in this case are those irregularities of the terrestrial surface, which may be combined under the statistical concept of roughness.

The preceding sections discussed currents that can be reproduced in the laboratory - in tubes with smooth walls. Irregularities in the walls essentially distort the results, leading to the development of turbulence at those velocities at which

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the current would remain laminar if the walls of the tubes were smooth.

In aerodynamics and hydrodynamics two types of roughness are distinguished: (1) angular roughness which is characterized by sharp angular protuberance of extremely irregular shape; (2) undulatory roughness which is characterized by more gradual and regular wave-like protuberances.

The roughness of the natural surface of the land mass is of various forms and cannot be classified under the above mentioned two types of roughness.

Thick grass is an extreme example of such roughness. While an air current penetrates freely into the upper part of the vegetation cover, the thickly crowded stems create an impenetrable impedance to the penetration of the wind down to the surface of the earth.

In forests, on the contrary, the wind penetrates freely between the trunks of the trees while between the tree tops there is almost no movement of air.

The roughness of the sea surface is undulatory; a sort of mobile undulatory roughness.

Not being able to investigate the effect on the air current of each individual impedance, we are forced to approach the concept of roughness statistically, considering that the number of individual protuberances is great and their shape is much varied.

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We characterize the entire surface as rough to a certain degree, evaluating its summary effect with the aid of a certain number which we call the coefficient of roughness or the measure of roughness.

The principal result of roughness is the fact that directly at the wall the mean path of intermixing has a finite value different from zero.

This circumstance, according to Prandtl', is taken into account by the following artificial method. We shift the origin of the coordinates under the contemplated surface by just as much as it is necessary to bring about the occurrence of the observed mean path of intermixing already at the surface level. Thus,

$$l = x(z + z_0)$$

where z is the distance from the point to the rough surface, the constant $x = 0.4$, and z_0 is some length which is dependent on the linear dimensions of the roughness. From the tests conducted in tubes with smooth walls but with sand grains pasted onto the latter, it was established that $z_0 = \frac{1}{30} h_0$, where h_0 is the mean diameter of the grains. However, the coefficient 1/30 has no constant value even for the case of tubes.

For natural surfaces the roughness of which differs essentially from the artificial roughness of the tube walls pasted with sand, such as the surface of a grass cover, z_0 may be considered as having a value of 1/7 - 1/8 from h_0, where h_0 is the mean height of the grass cover.

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The effect of roughness and height upon the path of intermixing under atmospheric conditions is pronounced up to little heights only, while in the free atmosphere this effect is absent. Therefore the path of intermixing cannot increase indefinitely with the increase in height. This regularity occurs only in the surface layer of the atmosphere which is characterized by the absence of change in the direction of the wind. We will study the distribution of wind velocity in this atmospheric layer in Section 1, Chapter XVI.

Thus, the development of turbulence in the atmosphere is promoted by the non-homogeneity of the temperature field and by the extremely diverse boundary conditions on the earth's surface, i.e., the roughness of the latter.

With non-homogeneity of the temperature field is tied in the development of "thermal" or "static" turbulence stipulated by the non-homogeneity in the heating of various parcels of the underlying surface. Thermal turbulence was previously considered as the "emersion" in the atmosphere of isolated, more or less superheated, masses of air in the form of "bubbles"; the place vacated by them filled in with air which flows under these "bubbles" from all sides.

Another theory for the explanation of thermal turbulence was offered recently. Thermal turbulence is tied in with the development of limited closed air circulations in the atmosphere. In this case a cumulus cloud will develop in the ascending branch of this closed circulation, while in the descending branch the clouds

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will be diffused and the descending branch itself will correspond to the breaks between the clouds.

"Dynamic" turbulence is stipulated mainly by the formation of vorticities at the terrestrial surface obstructions. Since the sizes and shapes of these obstructions are extremely diverse the eddies, which "clog up" the atmosphere, are very diverse in form and in magnitude.

However, the nature of these eddies and also their size and form remain unclarified.

Section 7. Horizontal Turbulent Exchange of Masses.

We have seen above that the current of a characteristic is proportional to the gradient of this characteristic and to the coefficient of turbulent exchange. If the turbulent exchange of masses is the same in all directions, the current of the characteristic will flow almost exclusively in a vertical direction since all the meteorological elements vary in a vertical direction much more rapidly than in a horizontal. The coefficient of exchange in a vertical direction is of the order 10^1 — 10^2 gram centimeter^{-1}/second^{-1}.

However, the coefficient of turbulent exchange in a horizontal direction can be considerably greater than the same in a vertical direction. Therefore, regardless of the small values of the horizontal gradients, the horizontal flow of a characteristic may play an important part in the analysis of atmospheric processes of a large scale.

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Due to the strong gravitational stability of the atmosphere, which expresses itself in the increase of the potential temperature with height, the turbulent exchange will occur along the isentropic surfaces, that is, surfaces of equal entropy or surfaces of equal potential temperature.

In order to study the turbulent exchange taking place along an isentropic surface, which we will call for brevity, "lateral intermixing", meteorologists suggested a method which became known as "isentropic analysis".

To trace the trajectory of an air particle, the distribution of specific humidity is plotted on the charts of isentropic surfaces. The values of potential temperature and specific humidity, which are conservative characteristics, can be considered as the Lagrange coordinates of an air particle. For the complete determination of an air particle it is necessary to interpolate a third Lagrange coordinate, i.e., a third conservative characteristic. As such, the so-called "potential vortex" is interpolated.

Let us take the equation of motion in the following simplified form:

$$\left.\begin{aligned} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - 2\omega \sin\varphi v &= -\frac{1}{\rho}\frac{\partial p}{\partial x}, \\ \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + 2\omega \sin\varphi u &= -\frac{1}{\rho}\frac{\partial p}{\partial y}, \\ g &= -\frac{1}{\rho}\frac{\partial p}{\partial z} \end{aligned}\right\} \quad (1)$$

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Let us segregate an adequately thin layer of air in the atmosphere and assume for simplification that this layer is barotropic

$$\rho = \rho(p)$$

Noticing that

$$\left.\begin{aligned} \frac{\partial}{\partial y}\left(\frac{1}{\rho}\right) &= -\frac{1}{\rho^2}\frac{d\rho}{dp}\cdot\frac{\partial p}{\partial y}, \\ \frac{\partial}{\partial x}\left(\frac{1}{\rho}\right) &= -\frac{1}{\rho^2}\frac{d\rho}{dp}\cdot\frac{\partial p}{\partial x} \end{aligned}\right\} \qquad (2)$$

we will differentiate the first of equations (1) by y, and the second by x, and subtract the second result from the first result. Then, taking into account the correlationships (2), we derive:

$$\frac{d\zeta}{dt} + \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right) 2\omega\sin\varphi + \left(2\omega\sin\varphi + \zeta\right)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0, \qquad (3)$$

where

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad \text{and} \quad \frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}.$$

Since

$$\frac{\partial}{\partial t}(2\omega\sin\varphi) = 0$$

then

$$\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right) 2\omega\sin\varphi = \frac{d}{dt}(2\omega\sin\varphi),$$

and correlationship (3) assumes the form of

$$\frac{d}{dt}(\zeta + 2\omega\sin\varphi) + (\zeta + 2\omega\sin\varphi)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0. \qquad (4)$$

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Let us take the equation of continuity in the form

$$\frac{d\rho}{dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \frac{\partial \rho w}{\partial z} = 0. \qquad (5)$$

Multiplying equation (5) by gdz and integrating within the limits from z_0 to z_1 we derive:

$$\int_{z_0}^{z_1} g\frac{d\rho}{dt}\,dz + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\int_{z_0}^{z_1} g\rho\,dz + \int_{z_0}^{z_1} g\frac{\partial \rho w}{\partial z}\,dz = 0, \qquad (6)$$

it being the case that z_0 and z_1 are considered as functions of x, y, t. Consequently,

$$\left.\begin{aligned} &\frac{d}{dt}\int_{z_0}^{z_1} g\rho\,dz = \int_{z_0}^{z_1} g\frac{d\rho}{dt}\,dz + g\left(\rho_1\frac{dz_1}{dt} - \rho_0\frac{dz_0}{dt}\right), \\ &\int_{z_0}^{z_1} g\rho\,dz = p_1 - p_0, \\ &\int_{z_0}^{z_1} g\frac{\partial \rho w}{\partial z}\,dz = g\left[(\rho w)_1 - (\rho w)_0\right] = g\left(\rho_1\frac{dz_1}{dt} - \rho_0\frac{dz_0}{dt}\right), \end{aligned}\right\} \qquad (7)$$

Incorporating equations (6) and (7), we obtain:

$$\frac{d}{dt}(p_0 - p_1) + (p_0 - p_1)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \qquad (8)$$

Finally, eliminating from equations (4) and (8) the horizontal divergence of velocity $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$, we derive:

$$\frac{\zeta + 2w\sin\varphi}{p_0 - p_1} = \text{const.} \qquad (9)$$

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Expression (9) remains constant although the relative vortex ζ varies with the latitude of the locality and the mass of the air layer. Thus, if the air is reduced to a standard latitude φ and to a standard pressure differential $p_o - p_1$, the resulting vortex can be considered as a conservative characteristic of the air particle. This specific characteristic is called the potential vortex.

Studying the displacement of the lines of equal specific humidity on isentropic charts and considering that this displacement is tied in with lateral intermixing or with horizontal turbulent exchange, which is almost the same, the order of magnitude of the coefficient of horizontal exchange can be evaluated. It turns out to be equal to $4 \times 10^5 - 5 \times 10^7$ gram centimeter $^{-1}$ second $^{-1}$.

Thus the coefficient of horizontal turbulent exchange is at least 10^3 times greater than the coefficient of vertical turbulent exchange. Yet the vertical wind velocity and temperature gradients are approximately 10^2 times greater than the respective horizontal gradients. Consequently, the horizontal turbulent exchange is of no less important value than the vertical turbulent exchange. Let us note, however, that the order of magnitude of the coefficient of horizontal exchange cannot be considered as finally established.

Section 8. The Spectral Structure of Turbulence.

Let us now analyze in more detail the internal structure of

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the turbulent current. The pulsating motion, which is superposed on the averaged current, can be considered as a result of superposing motions of turbulence which are tied in with vorticity formations (elements of turbulence) of various scales.

A scale of a motion is the order of magnitude of the distances in which the velocity of the motion changes substantially. It was established experimentally that the appearance of pulsations of a various scale is tied in with the Reynolds number, namely with the increase in the Re number, the scale of pulsations decreases, and with very great Re numbers pulsations of the scales varying from very large to infinitesimally small take place in the turbulent current.

The part played by the pulsations of various scales in the turbulent current varies in importance. The basic part is played by pulsations of very large scales of the order of magnitude of the typical lengths L of the regions in which the turbulence occurs. These large scale pulsations have the maximum "amplitudes" -- of the order of variation of the averaged velocity Δu for the distance L. The "frequencies" of the large scale pulsations are on the order of u/L. Small scale pulsations have greater "frequencies" and lesser "amplitudes".

Thus, we may speak of turbulence of a varying scale, of the spectral structure of turbulence. Conformantly, we may speak of the spectral distribution of the kinetic energy of the turbulent current. The basic part of the kinetic energy is tied in with large scale turbulence while a comparatively small part of the kinetic

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energy is tied in with small scale turbulence.

The spectral representation of turbulence makes it possible to understand the peculiar variations in pulsation velocity along the current at a given moment of time. The variations in pulsation velocity over great distances are stipulated by large scale turbulence. Therefore they are comparable with a magnitude of the order Δu. Variations in pulsation velocity over small distances are stipulated by small scale turbulence. They are, therefore, small as compared to Δu although great by comparison with the variations of the averaged velocity over small distances.

By the same token, in observations conducted at different times at a fixed geometrical point in space, considerable variations in velocity, of the order of Δu which are comparable with the typical time $T = L/u$ for the basic current, are stipulated by large scale turbulence. On the contrary, insignificant variations in velocity over small intervals of time $\tau < T$ are tied in with small scale turbulence.

We saw above that the Re number which determines the characteristics of the current depends substantially on the typical length l. To evaluate the characteristics of the current which are tied in with the spectral structure of turbulence, a qualitative concept of the Reynolds numbers (Re_λ), characteristic for turbulence of various scales, may be introduced. Designating the order of magnitude of the turbulence scale by λ, and the order of magnitude of the pulsation velocity by v_λ, we determine the corresponding Re number by the formula

$$Re_\lambda = \frac{v_\lambda \lambda}{\nu} \qquad (1)$$

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The value of the Re number is greater, the greater the scale of the motion. But with Re numbers of high value the Re_λ number for large scale turbulence also has a high value. However, high value Re numbers correspond to low viscosity. Therefore we have the following result.

For large scale turbulence, which is of basic significance for turbulent currents, viscosity does not play an essential part. Therefore it can be disregarded and, considering the liquid as ideal, its motion can be studied by utilizing the Euler equations. It is obvious that with large scale turbulence no noticeable dissipation of kinetic energy takes place. On the contrary, the smaller the scale of turbulence, the smaller is the Re_λ number and, consequently, the greater the effect of friction.

These considerations lead to the following conclusion about the process of "transformation" and dissipation of kinetic energy in a turbulent current.

The kinetic energy of large scale turbulence, which is comparable with the scale of the entire current, is evolved at the expense of the non-stability of the basic averaged motion. With the disintegration of the large scale vortices, part of the kinetic energy of large scale turbulence is gradually transformed into kinetic energy of small scale turbulence. In such transformations practically no kinetic energy is dissipated. Thus, in the turbulent current there is a continuous flow of energy from large scale turbulence to small scale turbulence. And only in the micro-scale elements of turbulence, of the order of $\lambda_0 \ll \lambda$, kinetic energy

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is dissipated, i.e., transformed directly into heat energy.

Thus all magnitudes related to a turbulence of the scale $\lambda \gg \lambda_0$ cannot be dependent on ν, i.e., on viscosity. More specifically, these magnitudes cannot vary with variations of ν if all other conditions which determine the motion do not change.

Consequently, the number of parameters which determine the characteristic of the turbulent current decreases. This circumstance makes it possible to utilize considerations of congruency for the derivation of a series of important results.

As an example, let us evaluate the order of magnitude of energy dissipation in a turbulent motion. Let us designate the amount of energy which is dissipated in a unit of time in a volumetric unit of a liquid by ϵ :

$$[\epsilon] = \frac{erg}{cm^3 sec} = \frac{g}{cm\ sec^3}$$

Although dissipation, in the final analysis, is stipulated by the viscosity of the liquid, the value of ϵ can be expressed only through the values of ρ, l, Δu, which are characteristic for large scale turbulence. The only combination of these values which has the dimensionality of dissipation ϵ is $\frac{\rho(\Delta u)^3}{l}$. Therefore:

$$\epsilon \sim \frac{\rho(\Delta u)^3}{l} \qquad (2)$$

However expression (2) for the kinetic energy dissipation is not a convenient one for comparing with the expression for energy

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dissipation as derived above (Section 8, Chapter VIII).

As was indicated above, it is accepted in the semiempirical theory of turbulence that the motion of a turbulent liquid becomes subject to the equations of the motion of a viscous liquid if the kinematic coefficient of viscosity ν is substituted by the "kinematic coefficient of turbulent viscosity" ν_{turb}. Let us now evaluate the magnitude of ν_{turb} with the aid of magnitudes Δu, l, which are characteristic of large scale turbulence.

Out of these magnitudes only one combination that has the dimensionality of kinematic viscosity: $\Delta u \times l$, can be composed. Therefore

$$\nu_{turb} \sim \Delta u \cdot l \tag{3}$$

The ratio of the coefficient of turbulent viscosity to the coefficient of molecular viscosity is the Re number, whence

$$\nu_{turb} : \nu = \frac{\Delta u \cdot l}{\nu} = Re. \tag{4}$$

Consequently, ν_{turb} increases with the increase in Re. Substituting into expression (1) ν_{turb} with the aid of expression (2), we derive:

$$\epsilon \sim \nu_{turb} \cdot \rho \left(\frac{\Delta u}{l}\right)^2. \tag{5}$$

This expression corresponds to the conventional expression for the dissipation of energy (Sections 8 and 14, Chapter VIII), namely:

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The kinematic coefficient of viscosity ν determines the dissipation of energy by the derivatives of the true velocity along the coordinates.

The kinematic coefficient of turbulent viscosity ν_{turb} determines the dissipation of energy by the derivatives of the mean velocity of the current along the coordinates.

Section 9. "The Law of Two Thirds", by Kolmogorov - Obukhov.

The hypothesis of homogeneous and isotropic turbulence, which was introduced into the statistical theory of turbulence, makes it possible to clarify some important characteristics of turbulent motion. This hypothesis is effectively corroborated by measurements in aerodynamic tubes with artificially created turbulence which is induced by sending a current through a screen.

However, in the majority of practically important cases, particularly in investigating natural turbulent currents, this hypothesis furnishes only a very rough approximation to reality. Indeed, assuming the completely incidental character of turbulent pulsations, it is natural to expect a tendency toward isotropy in the turbulent current. However, natural currents are located at all times in the field of gravitational force. The field of gravitational force in its nature is not isotropic, therefore the presence of the gravitational force hinders the establishment of isotropy in a turbulent current. In addition isotropy is disturbed by the presence of solid boundaries nearby.

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But the part played by volumetric forces diminishes together with the decrease in the scale of the phenomenon, which follows from the general theory of congruence. Therefore, when the phenomenon is of a sufficiently small scale, the volumetric forces can be disregarded.

Proceeding from the above considerations, A. N. Kolmogorov assumes that in an adequately small region of an arbitrary current with an adequately great Re number which does not lie near the boundaries, the hypothesis of "local isotropy" is substantiated with fair approximation.

By a "small region" Kolmogorov understands a region of which the dimensions are small as compared to the characteristic dimensions of current L, and dimensions of time are small as compared to the characteristic time: $T = L/u$.

Thus Kolmogorov considers such scales l of a phenomenon, the order of which is sufficiently small as compared to the scale of the basic motion so that the effect of external forces hindering the establishment of isotropy of the current $l \ll L$ can be disregarded. At the same time the order of the scales of the phenomenon is adequately great as compared to the dimensions of the smallest vortices, the energy of which is dissipated into heat energy: $l \gg \lambda_0$.

Turbulence on such a scale possesses the following characteristics:

(1) It is isotropic; (2) the advent of the kinetic energy of the turbulence of the contemplated scale is realized by way of

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transformation into it of the kinetic energy of large scale turbulence; (3) the expenditure of kinetic energy is realized by way of its transformation into energy of a smaller scale turbulence and not by way of its dissipation into heat energy.

This simple diagram of the phenomenon made it possible for Kolmogorov to establish a very important structural rule of the turbulent current which received the name of the "rule of the two thirds".

Let us first ascertain which parameters may determine the characteristics of a turbulent motion in a segregated region of scale l, which is such that

$$\lambda_0 \ll l \ll L.$$

The general dimensions of the current L, Δu, and the velocity of the mean motion u, at the given values of ρ and ϵ, have no effect on the local characteristics of turbulence, by virtue of the selected scale l. For the same reason viscosity ν is not a determining parameter for a turbulence on a scale l.

The determining parameters are ρ, l, and ϵ -- the dissipation of energy in a unit of time related to a unit of volume. Although dissipation of energy is tied in with micro-scale turbulence, the magnitude ϵ also determines the characteristics of a turbulent motion on a larger scale. This is explained by the fact that the magnitude ϵ, as was shown above, represents a "current of energy" directed from large-scale to small-scale turbulence.

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Let us now consider the magnitude

$$R_u = \overline{[v_l(M') - v_l(M'')]^2}$$

i.e., the mean square of the difference between velocity components directed along a straight line connecting points M' and M". In the case of local isotropy, the function R_u depends only on the distance r between points M' and M", on the value of dissipation ϵ, and density ρ.

Composing the dimensionalities of the magnitudes

$$[R_{ll}] = L^2T^{-2};\ [\epsilon] = ML^{-1}T^{-3};\ [\rho] = LT^{-3};\ [r] = L.$$

The only possible combination of these magnitudes produces the following expression:

$$R_u = C\left(\frac{\epsilon}{\rho}\right)^{2/3} \cdot r^{2/3} \qquad (1)$$

The above formula (1) expresses the "rule of the two thirds" by Kolmogorov - Obukhov.

An analogous formula is derived for

$$R_{nn} = \overline{[v_n(M') - v_n(M'')]^2} \qquad (2)$$

i.e., for the mean square of the difference between the velocity components directed perpendicular to the straight line connecting points M' and M".

This rule affirms that the mean square of the difference of

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some velocity component v_j at two points is in direct ratio to the distance between these points raised to the power of 2/3.

The rule of the two thirds, which was established by Kolmogorov on the basis of considerations derived from the theory of dimensionality, was also arrived at by A. M. Obukhov on the basis of the analysis of the energy balance of a turbulent current.

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CHAPTERS

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CHAPTER XVI

VARIATION OF WIND WITH HEIGHT IN A TURBULENT ATMOSPHERE

Section 1. Variation in the Velocity of Wind and the Coefficient of Turbulent Exchange in the Lowest Atmospheric Layer.

The variation of wind with height which is one of the most important problems of meteorology has a considerable history behind it. Not mentioning the numerous empirical theses on the subject, let us point out that the first theoretical research pertaining to it relates to the eighties of the last century.

Distribution of wind with height, in the atmospheric layer up to 500-1000 meters in height, is subject to the effect of turbulent exchange and friction against the terrestrial surface. However, with relation to the entire atmosphere, the distribution of wind with height is also affected by the Coriolis force and by the variation with height of the pressure gradient force. The effect of the latter factor on the changes in geostrophic wind was discussed in the preceding text (see Chapter XI), and the part played by the Coriolis force will be clarified in Section 3 of this Chapter.

In this section we will limit ourselves to the study of the distribution of wind with height or, shorter yet, the study of the contour of the wind in the lowest layer of the atmosphere only several dekameters thick, characterized by the absence of change in the direction of wind with height, which absence testifies to the

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vanishing effect of the earth's rotation upon the distribution of the wind in this layer. For purposes of brevity we will call this layer the "surface" layer.

In the surface layer most of the meteorological elements (wind velocity, temperature, humidity) undergo maximum changes with height, as a result of which the effect of turbulent exchange in this layer is particularly great.

Observations show that at times the distribution of wind velocity in the surface layer corresponds to the contour of the flow velocity in pipes with rough walls, but this correspondence is true only in the case of a thermally homogeneous current. In the presence of thermal stratification, the contour of the wind in the atmosphere differs considerably from the contour of the flow velocity in pipes.

In the surface layer the forces of inertia, the Coriolis force, and the horizontal pressure gradient force can be disregarded since these forces are small compared to the friction forces. We will consider the motion of the air in the surface layer as motion along a plane and the tangential stress as a constant in magnitude as well as direction, consequently,

$$\tau = \rho\left(l\frac{du}{dz}\right)^2 = const., \qquad (1)$$

whence

$$A\frac{du}{dz} = const. \qquad (2)$$

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and

$$l \frac{du}{dz} = u^* = const., \tag{3}$$

where

$$A = \rho l^2 \frac{du}{dz} \tag{4}$$

is the coefficient of turbulent exchange and u* is a constant, the so-called friction velocity.

Assuming further that in the surface layer the path of displacement with height varies in accordance with the linear rule:

$$l = \kappa(z + z_0), \tag{5}$$

where $\kappa = 0.4$, and z_0 is the measure of roughness. Eliminating from (3) and (5) the path of displacement l, we have:

$$\frac{du}{dz} = \frac{u^*}{\kappa(z + z_0)}, \tag{6}$$

and integrating (6) we derive:

$$u = \frac{u^*}{\kappa} \ln \frac{z + z_0}{z_0} \tag{7}$$

(the boundary condition we take in the form $u = 0$, with $z = 0$).

The logarithmic law of wind velocity distribution in the surface layer was established empirically several times. The corresponding empirical formula is:

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$$u = a + b \ln(z + c). \qquad (8)$$

However, only the interpolation of the path of intermixing makes it possible to penetrate adequately into the mechanism of the phenomenon and discover the dependence of the coefficients in the empirical formula (8) on the measure of roughness. Taking formula (8) in the aspect

$$u = a + b \ln c + b \ln \frac{z+c}{c}$$

and assuming that

$$c = z_0; \quad b = \frac{u^*}{\varkappa}; \quad a = -\frac{u^*}{\varkappa} \ln z_0$$

we arrive at formula (7).

We now determine for the surface layer the coefficient of turbulent exchange (with conditions same as above). Taking into account (4), (5), and (6), we find immediately that

$$A = \rho l^2 \frac{du}{dz} = \rho l u^* = \rho u^* \varkappa (z + z_0) \qquad (9)$$

Thus, with the logarithmic rule for the distribution of wind in the surface layer the coefficient of turbulent exchange increases linearly with height.

Since the true mean wind velocity u is in proportion to u*, the coefficient of turbulent exchange is in direct ratio to wind velocity and, therefore, the following correlationships can be put down for A:

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$$\frac{A}{\rho} = K = a_0 + a_1 z, \tag{10}$$

$$\frac{A}{\rho} = K = au \tag{11}$$

This derived result is very important for the correct understanding of processes taking place in the surface layer.

In the preceding text we more than once mentioned the possible analogy between the chaotic turbulent and chaotic molecular motions. However, this analogy is a limited one. Indeed, we have seen above that the coefficient of molecular viscosity is a constant, which depends neither on the density nor on the velocity of the current, and is determined only by the nature of the gas. But the coefficient of turbulent exchange which is an analogue of the coefficient of viscosity, as we have just seen, depends on:
(1) density ρ, (2) the distance to the underlying surface z,
(3) the mean velocity u, and (4) the measure of roughness z_0.

The equations for the processes tied in with turbulent exchange are, therefore, more complex than the equations for the processes tied in with molecular exchange. Only in particular cases can these equations be simplified, by assuming A = const. For the surface layer however, such an assumption is inadmissible.

The mean value of the coefficient of exchange under mean wind value and mean condition of roughness at a height of 2 meters equals 1-2 grams centimeters $^{-1}$ seconds $^{-1}$, while the mean value for the molecular viscosity of the air equals 1-2 x 10 $^{-4}$ grams centimeters $^{-1}$ seconds $^{-1}$. Consequently, the coefficient of exchange at this height is approximately 10,000 times the coefficient of viscosity.

The magnitude of the coefficient of exchange A and the measure of roughness z_0 can be computed on the basis of measuring the wind velocity at two given levels in the surface layer. Writing down formula (7) for some definite level z_1,:

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$$u_1 = \frac{u^*}{\chi} \ln \frac{z_1 + z_0}{z_0} \qquad (12)$$

and eliminating from (7) and (12) the auxiliary magnitude u*, we obtain:

$$u = u_1 \frac{\ln \frac{z + z_0}{z_0}}{\ln \frac{z_1 + z_0}{z_0}} \qquad (13)$$

Substituting into (9) in place of $\frac{du}{dz}$, its value,

$$\frac{du}{dz} = \frac{u_1}{z + z_0} \cdot \frac{1}{\ln \frac{z_1 + z_0}{z_0}}$$

we derive the following expression for the coefficient of turbulent exchange:

$$A = \rho \chi^2 (z + z_0) u_1 \frac{1}{\ln \frac{z_1 + z_0}{z_0}} \qquad (14)$$

These results are true for a homogeneous current in a non-compressible liquid; in the conditions of the surface atmospheric layer, they are true only in the absence of thermal stratification. In this case the character of the motion is determined by viscosity and roughness only. The only characteristic of the current is the Reynolds number. But since in the surface layer the Reynolds number varies little, it loses its significance as a characteristic for atmospheric turbulence. Additionally, in the surface layer as a rule there always takes place thermal stratification which plays a very important, frequently a paramount part in determining the character of the motion. Therefore, the character of motion in the surface layer is determined not only by the Reynolds number Re, but also by the Richardson number Ri:

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$$Ri = \frac{g \cdot \frac{d \ln \theta}{dz}}{\left(\frac{du}{dz}\right)^2} . \qquad (15)$$

The Richardson number Ri, in the conditions of the surface layer, may vary within rather wide limits assuming alternately positive and negative values. With adiabatic stratification of the atmosphere Ri = 0 and the character of the motion is determined only by Re. In this case the above derived formulas hold true. With stable stratification Ri > 0, with unstable stratification, Ri < 0.

It should be noted that due to the small thickness of the surface layer, the adiabatic stratification can hardly be distinguished from the isothermic stratification. Therefore, the following types of temperature distribution with height are distinguished in the surface layer: isothermia (considering it as adiabatic stratification), inversion (which, of course, corresponds to stable stratification), and temperature drop which is sometimes called convection and corresponds to super-adiabatic gradients.

Attention was also called several times to the fact that the logarithmic rule for the velocity distribution of the wind in the surface layer is only a particular case of the more general law of exponents (the law of the exponent $1/p$). It was also pointed out that the logarithmic rule holds true only for an adiabatic atmosphere and that in the presence of inversions this rule must be substituted by the law of exponents of the variety:

$$u = u_1 \left(\frac{z}{z_1}\right)^{\frac{1}{p}} \qquad (16)$$

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where p is some dimensionless parameter. Observations have shown that formula (16) holds true in the case of deep inversions only.

The Soviet scientist D. Y. Laykhtman suggested a more general expression for the distribution of wind velocity and the coefficient of turbulent exchange in the surface layer. Let us assume, following the argumentation by Laykhtman, that the length of the path of intermixing is expressed by:

$$l = \kappa \left(\frac{z}{z_0}\right)^n \cdot z_0 \qquad (17)$$

where n is an, as yet, undetermined exponent which is dependent on the character of atmospheric stratification, i.e. on the Ri number. With n = 1, we have the particular case which was discussed above. Substituting (17) into (3), we derive:

$$du = \frac{u^*}{\kappa} \cdot \frac{1}{z_0} \left(\frac{z}{z_0}\right)^{-n} dz,$$

and integrating, we obtain:

$$u = \frac{u^*}{\kappa} \cdot \frac{z_0^{n-1}}{1-n} \left[z^{1-n} - z_0^{1-n}\right]; \qquad (18)$$

since

$$u_1 = \frac{u^*}{\kappa} \cdot \frac{z_0^{n-1}}{1-n} \left[z_1^{1-n} - z_0^{1-n}\right],$$

we derive finally:

$$u = u_1 \frac{z^{1-n} - z_0^{1-n}}{z_1^{1-n} - z_0^{1-n}} \qquad (19)$$

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Let us ascertain the magnitude of exponent n. In the surface layer, $\frac{du}{dz}>0$. But

$$\frac{du}{dz}=(1-n)u_1\frac{z^{-n}}{z_1^{1-n}-z_0^{1-n}}$$

Consequently, $1-n>0$, whence $n<1$.

It is also known from observations that the vertical velocity gradient diminishes with distance from the rough surface, i.e.

$$\frac{d^2u}{dz^2}=-n(1-n)\frac{u_1z^{-n-1}}{z_1^{1-n}-z_0^{1-n}}<0$$

consequently, $n>0$, and n is a positive proper fraction

$$0<n<1. \tag{20}$$

In order to make expression (17) comparable with the law of exponent $1/p$ as expressed by equation (16), we interpolate in the place of n a new parameter p, tied in with n by the correlationship:

$$1-n=\frac{1}{p}. \tag{21}$$

It is obvious that p varies within the following limits:

$$1\leqslant p\leqslant\infty. \tag{22}$$

Hence, we derive the following Laykhtman formulas:

$$l=\kappa\left(\frac{z}{z_0}\right)^{1-\frac{1}{p}}, \tag{23}$$

$$u=u_1\frac{z^{\frac{1}{p}}-z_0^{\frac{1}{p}}}{z_1^{\frac{1}{p}}-z_0^{\frac{1}{p}}}, \tag{24}$$

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$$\frac{A}{\rho} = K = l^2 \frac{du}{dz} = \frac{\varkappa}{\rho} \frac{u_1 z^{1-\frac{1}{p}}}{z^{\frac{1}{p}} - z_0^{\frac{1}{p}}}. \qquad (25)$$

Formula (24) is transformed into the logarithmic rule of the wind when $p \to \infty$, i.e. when $n \to 0$, and into the linear rule, when $p \to 1$, i.e. when $n \to 0$.

The accuracy of the Laykhtman formulas was checked by the Central Geophysical Observatory on numerous observation substances in a layer up to 6 meters high. The observation data are in conformity with the Laykhtman theory.

The ratio between the wind velocities at two levels of the surface layer is tied only with one parameter p, i.e., in the final analysis with the Richardson number.

Numerous observations have shown that the coefficient of turbulent exchange in the surface layer increases rather rapidly with height; at some level, its growth is practically stopped and at times even a decrease is observed. Such behavior of the coefficient of turbulent exchange is qualitatively quite explicable. In the lowest atmospheric layer the proximity of a solid boundary hinders the development of exchange, causing the coefficient of turbulent exchange near the terrestrial surface to be very small. With distance from the solid boundary the exchange becomes livelier and the coefficient of exchange increases with height. This increase, however, cannot continue indefinitely. Since the sources of vortices, "clogging" the atmosphere, are principally the irregularities of the terrestrial surface, it would seem that with greater distance from the latter the coefficient of turbulent exchange having attained its maximum would begin to decrease. But

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this is frequently not the case because the irregularities of the terrestrial surface are not the only sources of vortices that clog the atmosphere. Vortices are also evolved far away from the terrestrial surface due to the presence in the atmosphere of discontinuity surfaces. Therefore, the coefficient of exchange which is the statistical characteristic of turbulence does not with height diminish to zero, but assumes some almost constant value.

Consequently it can be assumed in most cases that the coefficient of turbulent exchange varies linearly with height up to some height h, after which it remains constant. This height h which generally speaking does not coincide with the upper limit of the surface layer is not constant. It is tied in with the stratification of the atmosphere as well as with the mean velocity of the current. With unstable stratification and great velocities of the current, the height h increases while with stable stratification and low velocities of the current it diminishes.

Section 2. Distribution of the Coefficient of Turbulent Exchange in the Boundary Layer.

A. M. Obukhov made a theoretical study on the character of the change of the coefficient of exchange with height.

The boundary layer in the atmosphere plays a double part. Firstly, the friction stresses are transferred into the atmosphere through the boundary layer, this being a result of the effect of the underlying surface upon the migrating air masses. Secondly, through the boundary layer the heat exchange between the active

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layer of soil and the free atmosphere takes place.

This double part played by the boundary layer is utilized by A. M. Obukhov for the theoretical determination of the change in the coefficient of exchange in the boundary layer with height. We will cite the argumentation by A. M. Obukhov in this section.

Let us designate by τ_0 the friction stress at the soil and by q_0 the flow of heat through a unitary area of the horizontal surface in a unit of time. Let us assume that in the boundary layer there was established some stationary cycle, it being the case that all external forces are vanishingly small as compared to the forces of turbulent friction. Let us assume in addition that the internal sources of heat(/latent/heat of condensation, radiation) are absent. With these assumptions the current of the quantity of motion and the thermal current do not change with height:

$$\tau(z) = \tau_0 = const., \tag{1}$$

$$q(z) = q_0 = const. \tag{2}$$

Seemingly, within the limits of the boundary layer, with small values of z, these assumptions are satisfied with adequate precision.

Let us put down for the surface layer the equations of turbulent exchange for a quantity of motion and heat:

$$\tau = -\varphi(Ri)\rho l^2 \left|\frac{du}{dz}\right| \cdot \frac{du}{dz} \approx \tau_0 = const., \tag{3}$$

$$q = -a\varphi(Ri)c_p \cdot \rho l^2 \left|\frac{du}{dz}\right| \cdot \frac{d\theta}{dz} \approx q_0 = const. \tag{4}$$

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In the place of τ and q, Obukhov interpolates the "velocity of friction" u* and the "velocity of heat transfer" q^*, determining them by equalities

$$u^* = \sqrt{\frac{\tau}{\rho}} \tag{5}$$

$$q^* = \frac{q}{c_p \rho T}. \tag{6}$$

Disregarding the small variations in ρ and T within the limits of the surface layer, Obukhov assumes:

$$u^* = u_0^* = const., \tag{7}$$

$$q^* = q_0^* = const. \tag{8}$$

Eliminating φ(Ri) from (3) and (4) and dividing (4) by (3) we obtain

$$\frac{1}{T}\frac{d\theta}{dz} : \frac{du}{dz} = \frac{q^*}{\alpha u^{*2}} = const. \tag{9}$$

Integrating (9) by z and disregarding the small variations of T, we find:

$$\theta(z) = \frac{q^* T}{\alpha u^{*2}} \cdot u(z) + const. \tag{10}$$

Since $\frac{q^* T}{\alpha u^{*2}} \approx$ const. the temperature distribution in the surface layer turns out to be congruent with wind distribution.

Let us turn to the deduction of the correlationship which determines the Richardson number. Interpolating into formula (5) the magnitudes τ from formula (3) which formula determines the transfer of the quantity of motion we obtain:

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$$u^* = \sqrt{\varphi(Ri)} \cdot l \cdot \frac{du}{dz}. \qquad (11)$$

Substituting into formula (6) the expression (4) for the velocity of heat transfer we obtain:

$$q^* = \frac{q}{c_p \rho T} = \frac{a\varphi(Ri)}{T} l^2 \left|\frac{du}{dz}\right| \cdot \frac{d\theta}{dz}.$$

Dividing term for term the above expression by the cube of the right-hand and left-hand part of (11), we obtain:

$$\frac{q^*}{u^{*3}} = \frac{a}{\sqrt{\varphi(Ri)}} \cdot \frac{1}{lT} \frac{\frac{d\theta}{dz}}{\left(\frac{du}{dz}\right)^2}. \qquad (12)$$

Multiplying both parts by g and remembering that

$$Ri = \frac{g}{T} \frac{\frac{d\theta}{dz}}{\left(\frac{du}{dz}\right)^2},$$

we obtain an equation which determines Ri:

$$\frac{Ri}{\sqrt{\varphi(Ri)}} = \frac{1}{a} \cdot g \cdot \frac{lq^*}{u^{*3}} \qquad (13)$$

where $l = xz$.

Above (Section 5, Chapter XV), it was shown that φ (Ri) is a monotonic decreasing function. Therefore, the left-hand part of (13) is a monotonic increasing function of Ri. Therefore, equation (13) has only one solution for Ri.

It is obvious that on the earth's surface, i.e. when $l = 0$, $Ri = 0$. Let us evaluate the magnitudes of Ri near the earth's surface. To accomplish this we will find the value of the derivative $\left(\frac{\partial Ri}{\partial z}\right)_{z=0}$; we transpose equation (13) so that:

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$$Ri = \frac{\kappa}{\alpha} \cdot g \cdot \frac{zq^*}{u^{*3}} \sqrt{\varphi(Ri)} . \qquad (14)$$

Differentiating equation (14) by z, we obtain:

$$\frac{\partial Ri}{\partial z} = \frac{\kappa}{\alpha} \cdot g \frac{q^*}{u^{*3}} \sqrt{\varphi(Ri)} + \frac{\kappa}{\alpha} \cdot g \frac{q^*}{u^{*3}} \cdot z \frac{\varphi'(Ri)}{2\sqrt{\varphi(Ri)}}, \qquad (15)$$

whence

$$\left(\frac{\partial Ri}{\partial z}\right)_{z=0} = \frac{\kappa}{\alpha} \cdot g \cdot \frac{q^*}{u^{*3}} \qquad (16)$$

Since $\varphi(0) = 1$. Consequently, near the earth's surface $Ri \approx 0$. on this basis Obukhov assumes that within the boundary layer with an arbitrary distribution of temperature, there exists a sub-layer, in which the effect of thermal stratification is infinitesimally small, and the character of turbulence is determined by dynamic causes only. Obukhov calls this sub-layer the sub-layer of "dynamic turbulence".

Obukhov further evaluates the height of the sub-layer of "dynamic turbulence", designating as height some scale characteristic L of this sub-layer, and determining it in the following manner:

$$L = \frac{1}{\alpha \left(\frac{\partial Ri}{\partial z}\right)_{z=0}} = \frac{1}{\kappa} \frac{u^{*3}}{g \cdot q^*} \qquad (17)$$

or, returning to τ and q,

$$L = \frac{1}{\kappa} \frac{c_p T \cdot \tau^{3/2}}{g \sqrt{\rho} \cdot q} \qquad (18)$$

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This scale characteristic does not depend on the form of the universal function φ (Ri), but is determined only by the current of the quantity of motion t and the thermal current q .

Interpolating, in the place of τ the wind velocity u_0 at some certain height ($\approx$ 30 meters), which is dependent on the roughness of the surface, Obukhov computed by the empirical formula $\tau = 0.0025 \rho u_0^2$, the following Table of values for the height of the sub-layer of dynamic turbulence (Table 47):

Table 47. Height of the sub-layer of dynamic turbulence (in meters).

q Calories / cm^2 min		0.01	0.02	0.05	0.1	0.15	0.2	0.3
u_0=20 u* meters/ second	u* cm/ sec							
0.5	2.5	0.182	0.091	0.036	0.018	0.120	0.008	0.006
1	5	1.45	0.73	0.29	0.14	0.10	0.07	0.05
2	10	11.63	5.81	2.34	1.16	0.78	0.58	0.39
3	15	39.2	19.6	7.9	3.9	2.6	1.9	1.3
4	20	93.0	46.5	18.7	9.2	6.2	4.6	3.1
5	25		90.8	36.5	18.2	12.1	9.1	6.0
6	30			63.1	31.4	21.0	15.7	10.5
10	50					97.0	72.7	48.4

As can be seen from the Table, in the region of the most probable values of u_0 and q , the height of the sub-layer of dynamic

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turbulence varies within the limits of 8 to 50 meters.

Obukhov further investigated the distribution by height of the Richardson number and the coefficient of turbulence. For convenience, the following designations were introduced:

$$\frac{Ri}{Ri_{ult}} = \eta; \quad \frac{z}{L_1} = \zeta; \quad \varphi(Ri) = \varphi_1(\eta); \quad \beta = \frac{1}{\alpha (Ri)_{ult}} \tag{19}$$

where ζ is a dimensionless height, β is a constant number of the first order.

Since

$$\varphi(0) = 1,$$
$$\varphi'(Ri) \leqslant 0,$$
$$\varphi(Ri) = 0 \quad (\text{when } Ri \geqslant Ri_{ult})$$

the function $\varphi_1(\eta)$ satisfies the conditions

$$\varphi_1(0) = 1,$$
$$\varphi_1'(\eta) \leqslant 0,$$
$$\varphi(\eta) = 0 \ (\text{when } \eta \geqslant 1).$$

For definiteness, we consider the case of stable temperature distribution when Ri $>$ o and the thermal flow is directed from the atmosphere to the soil. In this case q and q^* are considered positive:

$$Ri = \frac{k}{\alpha} z \cdot \frac{g q^*}{u^{*3}} \sqrt{\varphi(Ri)} = \frac{z}{\alpha L} \sqrt{\varphi(Ri)}. \tag{20}$$

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Then equation (13) can be written down in the following dimensionless form:

$$\frac{\eta}{\sqrt{\varphi_1(\eta)}} = \frac{z}{L}\frac{1}{\alpha(Ri)_{\kappa\rho}} = \beta\zeta = \xi. \qquad (21)$$

Then a new function is interpolated

$$\eta = \psi(\xi)$$

- the result of solving equation (21) for η.

Returning now to the previous variables Obukhov derives the following expression which determines the distribution of Ri by height:

$$Ri(z) = Ri_{u/t} \cdot \psi\left(\beta\frac{z}{L}\right). \qquad (22)$$

On the basis of the established characteristics of the function φ_1 (See Section 5, Chapter XV), Obukhov shows that function ψ has the following characteristics:

$$\left.\begin{aligned} &\psi(0)=0;\ \psi(\xi)\leqslant 1;\ \psi'(\xi)\geqslant 0,\\ &\varphi(\xi)\longrightarrow 1 \quad when\ \xi\longrightarrow+\infty,\\ &\frac{\psi(\xi)}{\xi}\longrightarrow 1 \quad when\ \xi\longrightarrow 0. \end{aligned}\right\} \qquad (23)$$

Thus, Obukhov arrives theoretically at a very interesting result to the effect that in the boundary layer of a stable atmosphere with $z > L$, the Ri number asymptotically approximates the $Ri_{u/t}$.

Finally, Obukhov computes theoretically the change in the coefficient of turbulence K with height.

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Since

$$\tau = K\rho \frac{du}{dz}, \tag{24}$$

then

$$K = \frac{\tau}{\rho \frac{du}{dz}} = \frac{u^{*2}}{\frac{du}{dz}}. \tag{25}$$

Interpolating the height L of the sub-layer of dynamic turbulence, Obukhov obtains:

$$K = u^* \sqrt{\varphi(Ri)}\, kz = L k u^* \sqrt{\varphi(Ri)}\, \frac{z}{L} \tag{26}$$

or, in terms of the variables η and ζ:

$$K = kLu^* \sqrt{\varphi(\eta)} \cdot \zeta. \tag{27}$$

But $\frac{\eta}{\sqrt{\varphi(\eta)}} = \beta\zeta$, consequently $\sqrt{\varphi(\eta)} \cdot \zeta = \frac{\eta}{\beta}$, and

$$K(z) = \frac{k \cdot u^* L}{\beta} \cdot \psi\left(\beta \frac{z}{L}\right). \tag{28}$$

The behavior of function ψ, basically speaking, is known; therefore, it is simple to investigate the variations of $K(z)$ with relatively small (as compared with L) and relatively great values of z.

Since $\frac{\psi(\xi)}{\xi} \to 1$ when $\xi \to 0$, then, with $z \ll L$,

$$\psi\left(\frac{\beta z}{L}\right) \approx \frac{\beta z}{L},$$

therefore

$$K(z) = k u^* \cdot z. \tag{29}$$

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Further: $\psi(\xi) \longrightarrow 1$, when $\xi \longrightarrow +\infty$; consequently, with $z \gg L$

$$K(z) = \frac{\kappa u^* L}{\beta}.$$

Thus, Obukhov arrived theoretically at the following result: with low values of z, the coefficient of turbulence K increases linearly with height, and with high values of z, the coefficient of turbulence asymptotically approximates its ultimate value

$$K_\infty = \frac{\kappa u^* L}{\beta} \tag{30}$$

If the scale L is eliminated from (30) with the aid of (17), K_∞ will be expressed thus:

$$K_\infty = \frac{1}{\beta} \frac{u^{*4}}{g q^*} \tag{31}$$

Obukhov computed the following Table of ultimate values of A_∞ gram/centimeter^{-1} second^{-1} (Table 48).

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Table 48. Ultimate value of the coefficient of turbulent exchange A_∞ gram centimeter^{-1} second^{-1} for a stable atmosphere.

q Calories / cm^2 min		0.01	0.02	0.05	0.1	0.15	0.2	0.3
u_o=20 u* meters/second	u* cm/sec							
0.5	2.5	0.0224	0.0112	0.00443	0.00221	0.00158	0.00108	0.000738
1	5	0.3567	0.1722	0.0738	0.0369	0.0246	0.0123	0.0123
2	10	5.7195	2.8536	1.1439	0.5658	0.3813	0.2829	0.1845
3	15	28.905	14.514	5.781	2.829	1.968	1.476	0.98
4	20	91.512	45.756	18.327	9.102	6.150	4.551	3.08
5	25		111.684	44.895	22.386	14.883	11.193	7.38
6	30			93.111	46.371	30.873	23.124	15.38
10	50					238.743	178.719	119.19

As seen from the above Table, the values of the parameters of turbulence theoretically derived have the same order of values of these parameters, established from observations.

Obukhov cites the following example to show how well the theoretical values of the turbulence parameters coincide with their observational values:

When u_o= 5 meters/second, u* = 0.25 meter/second, and q = 0.1 calorie/square centimeter, K_∞ = 1.82 square meters/second, A_∞ = 22.4 grams/centimeter^{-1} second^{-1}, L = 18.2 meters, $\frac{\partial \theta}{\partial z}$ = 0.31 degrees centigrade/100 meters, $\frac{\partial T}{\partial z}$ = -0.69 degrees centigrade/100 meters.

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This example shows that the value of the temperature gradient computed by the theoretically derived value of K_∞, also conforms well to observational magnitudes. Let us take note of the fact that in his computations Obukhov assumed a value for Ri ultimate that was obtained by Sverdrup from observations, namely: (Ri) ultimate = $\frac{1}{11}$.

Section 3. Variation of Wind with Height and Turbulent Exchange in the Planetary Boundary Layer.

One of the most known manifestations of the effect of turbulence in the atmosphere is undoubtedly the variation of the velocity and direction of the wind with height in the lowest atmospheric layer. In a homogeneous air mass the increase in the velocity of the wind with height is manifested with perfect clarity, also its rotation to the right in the northern hemisphere, and to the left in the southern hemisphere.

Such a change in the velocity of the wind begins at a height of 10-20 meters, and extends in our latitudes to a height of about 1 kilometer. Below the level of 10-20 meters the direction of the wind remains constant and the velocity magnitude varies logarithmically or in accordance with the law of exponents.

The change in the direction of the wind with height in the higher atmospheric layers is obviously stipulated by the combined effect of the deflecting force of the earth's rotation and turbulence.

The layer in which the wind rotates with height, the

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rotation stipulated by the combined effect of turbulence and the deflecting force of the earth's rotation, is called the planetary boundary layer.

The most substantial characteristic of this layer is the angle β, which is formed by the direction of the wind v with the direction of the pressure gradient force (Figure 130). This angle is known as the "angle of deflection". The angle θ, formed by the direction of the wind with the isobar, constituting the complement of the angle of deflection, is called the "angle of inclination" of the wind.

Figure 130. Angle of deflection β and angle of inclination θ of the wind.

Within the limits of the planetary boundary layer the angle of inclination of the wind is more or less substantially different from zero and the angle of deflection is always an acute angle. The magnitude of the angle of deflection is a function of the height z.

As shown by numerous observations the angle of inclination of the wind depends not only on the height, but changes also with time (annual and diurnal cycles), latitude and orography of a locality. Thus, for example at the earth's surface the angle of inclination of the wind is usually smaller in the summer than in the winter, smaller in the higher latitudes than in the lower ones,

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smaller over the surface of the sea than it is over land.

We will now analyze theoretically the change in the velocity and direction of the wind with height under the effect of turbulence and the deflecting force of the earth's rotation. Assuming [redacted] the motion to be horizontal and the field of velocities along a horizontal to be homogeneous, we will proceed from the averaged equations of motion of the atmosphere which can be written down as follows:

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\omega_z v + \frac{\partial}{\partial z}\left(\frac{A}{\rho}\frac{\partial u}{\partial z}\right),$$
$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - 2\omega_z u + \frac{\partial}{\partial z}\left(\frac{A}{\rho}\frac{\partial v}{\partial z}\right). \qquad (1)$$

Let us also make the following simplifying assumptions:

(1) The motion of the air is homogeneous and rectilinear, therefore

$$\frac{du}{dt} = \frac{dv}{dt} = 0.$$

(2) The pressure gradient does not change with height. Therefore, directing the x-axis along the isobar, and the y-axis along the gradient, we have:

$$\frac{\partial p}{\partial x} = 0; \quad \frac{\partial p}{\partial y} = \text{const}.$$

(3) Density does not change with height, i.e. ρ = const.

(4) The coefficient of turbulent exchange does not change with height, i.e. A = const.

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Then, the equations of motion will assume the following form:

$$\left.\begin{aligned} 2\rho\omega_z \cdot v + A\frac{d^2u}{dz^2} &= 0, \\ -2\rho\omega_z \cdot u + A\frac{d^2v}{dz^2} &= \frac{\partial p}{\partial y}. \end{aligned}\right\} \qquad (2)$$

Now we formulate the boundary conditions:

(1) At the earth's surface there takes place adhesion of air so that

$$u = v = 0 \quad \text{when} \quad z = 0;$$

(2) At great heights the wind becomes a geostrophic wind (u_g), consequently, with $z \to \infty$ $u \to u_g$, $v \to 0$.

The problem of integrating the system of equations (2) with two unknown functions u, v, can be reduced to the problem of integrating one equation but with a complex unknown function $w = u + iv$. To accomplish this we multiply the second one of equations (2) by $i = \sqrt{-1}$, and add the result to the first equation.

$$A\frac{d^2w}{dz^2} - 2\rho\omega_z \cdot i \cdot w = i\frac{\partial p}{\partial y}. \qquad (3)$$

Thus, we obtain an ordinary linear differential equation of the second order with constant coefficients, which can be re-written as follows:

$$-\frac{1}{a^2}\frac{d^2w}{dz^2} + w = u_g, \qquad (4)$$

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where

$$a^2 = \frac{2\rho\omega_z \cdot i}{A}; \quad u_g = -\frac{1}{2\rho\omega_z} \cdot \frac{\partial p}{\partial y}. \tag{5}$$

The general integral w of the non-homogeneous equation (4) equals the sum of any particular integral w_1 of this equation and the general integral w_2 of a homogeneous equation

$$\frac{d^2 w_2}{dz^2} = a^2 w_2. \tag{6}$$

As the particular integral of equation (5) we can take

$$w_1 = u_g = -\frac{1}{2\rho\omega_z}\frac{\partial p}{\partial y} = const. \tag{7}$$

i.e. the velocity of the geostrophic wind. The general integral of equation (6) will be

$$w_2 = C_1 e^{az} + C_2 e^{-az} \tag{8}$$

where C_1 and C_2 are arbitrary constants of integration.

Adding up (7) and (8), we derive the general integral of a non-homogeneous equation

$$w = C_1 e^{az} + C_2 e^{-az} + u_g. \tag{9}$$

The arbitrary constants of integration are determined from the boundary conditions. Since with unlimited increase in z, the wind velocity remains limited,

$$C_1 = 0. \tag{10}$$

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Satisfying the first boundary condition $u = v = w = 0$ with z = 0, we derive:

$$C_2 = -u_g. \tag{11}$$

Consequently, the final solution of equation (4) is like this:

$$w = u_g\left[1 - e^{-\alpha z}\right]. \tag{12}$$

We now separate the imaginary and the essential parts of solution (12).

In accordance with (5),

$$\alpha = \sqrt{\frac{2\rho\omega_z i}{A}} = \sqrt{\frac{\rho\omega_z}{A}}\cdot\sqrt{2i} = \alpha(1+i), \tag{13}$$

where

$$\alpha = \sqrt{\frac{\rho\omega_z}{A}}$$

since

$$(1+i)^2 = 1 + 2i - 1 = 2i,$$

consequently

$$w = u_g\left[1 - e^{-\alpha z}\cdot e^{-i\alpha z}\right] \tag{14}$$

Applying the Euler formula

$$e^{-i\alpha z} = \cos\alpha z - i\sin\alpha z$$

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we find

$$w = u + iv = u_g\left[1 - e^{-\alpha z}(\cos \alpha z - i \sin \alpha z)\right] \tag{15}$$

and finally

$$\left.\begin{aligned} u &= u_g\left[1 - e^{-\alpha z}\cos \alpha z\right], \\ v &= u_g \cdot e^{-\alpha z} \cdot \sin \alpha z. \end{aligned}\right\} \tag{16}$$

Thus, we derived the components of wind velocity as functions of height z. We now find the absolute value of velocity $c = \sqrt{u^2 + v^2}$ and the angle of inclination θ :

$$c = u_g\sqrt{1 - 2e^{-\alpha z}\cos \alpha z + e^{-2\alpha z}}, \tag{17}$$

$$\tan \theta = \frac{v}{u} = \frac{e^{-\alpha z} \cdot \sin \alpha z}{1 - e^{-\alpha z} \cdot \cos \alpha z} \tag{18}$$

Formulas (17) and (18) show that with the increase in height z, the velocity value increases and the angle of inclination diminishes so that the wind approximates the geostrophic wind in magnitude as well as direction. This process is the more rapid, the greater the value of $\alpha = \sqrt{\frac{\rho w \sin \varphi}{A}}$, i.e. the lower the turbulent exchange and the higher the latitude of a locality.

From (18) it follows that at some heights the wind assumes the direction of the geostrophic wind so that $\theta = 0$ with $\alpha z = \pi, 2\pi, \ldots, n\pi$ The minimum height

$$D = \frac{\pi}{\alpha} = \pi\sqrt{\frac{A}{\rho w \sin \varphi}}, \tag{19}$$

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at which the wind assumes the direction of the geostrophic wind, is called the level of friction. Although with subsequent increase in height the wind is somewhat deflected from the direction of the geostrophic wind, these deflections are so small that practically they cannot be detected, as can be seen from Table 50. Therefore, the level of friction which is determined by equation (19) can be accepted as the upper limit of the planetary boundary layer. Table 49 gives the thickness of the planetary boundary layer for various values of the coefficient of turbulent exchange and latitude with $\rho = 1.25 \times 10^{-3}$ gram centimeter^{-3}.

Table 49. Level of friction with coefficient of turbulent exchange constant.

A \ φ	10°	30°	50°	60°	90°
10 gram cm^{-1} sec^{-1}	790	470	380	350	330
50	1770	1040	840	790	740
100	2500	1470	1190	1120	1040
150	3060	1800	1460	1370	1280

As can be seen from formula (19) and Table 49, the level of friction is the higher, the greater the coefficient of turbulent exchange and the lower the latitude of a location.

The velocity value at the level of friction which is expressed by formula

$$c_D = u_g\sqrt{1+2e^{-2\pi}+e^{-2\pi}} = u_g(1+e^{-\pi}) > u_g, \quad (20)$$

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is greater than the velocity of the geostrophic wind since $e^{-\pi}$ is an essentially positive magnitude. Thus, the velocity of the wind attains in absolute magnitude the velocity value of the geostrophic wind somewhat below the level of friction.

When $z \to \infty$ $e^{-z} \to 0$, and the wind becomes geostrophic.

When $z = 0$ formula (18) results in an undeterminate form of the type $\frac{0}{0}$. Evaluating it in accordance with the L'Hospital's rule we derive:

$$\lim_{z \to 0} \tan\theta = \lim_{z \to 0} \frac{\frac{d}{dz}(e^{-\alpha z} \sin \alpha z)}{\frac{d}{dz}(1 - e^{-\alpha z} \cos \alpha z)} = 1.$$

Thus, at the earth's surface the angle of deflection under all conditions equals 45 degrees.

Assigning definite values to α it becomes possible to compute, by (17) and (18) the corresponding values for θ and $\frac{c}{u_g}$. Assuming that $\varphi = 45^\circ$, $\rho = 10^{-3}$ gram/cm^3, A = 25.2 gram cm^{-1} sec^{-1} and, consequently $\alpha = 0.004775$ m^{-1}, we derive the following Table of values for θ and $\frac{c}{u_g}$ for various heights (Table 50).

z =	10	20	40	100	200	400	800	1200 m
$\frac{c}{u_g}$ =	0.075	0.145	0.286	0.584	0.893	1.068	1.005	0.999
θ =	43	42	39	31	20	5	-1	0

Table 50. Variation in the velocity and direction of the wind with height in the planetary boundary layer.

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In constructing by table 50 a diagram of the wind velocities as function of the height z, we derive a spiral which is known in geophysics under the name of the Eckman spiral (in honor of Eckman who was the first one to solve the problem of the change in direction of ocean currents under the effect of turbulence and the Coriolis force). The Eckman spiral (Figure 131) is an equi-angular logarithmic spiral with an angle of 45 degrees. It passes through the origin of coordinates and approaches asymptotically the point $c = u_g$.

Figure 131. The Eckman spiral.

Now even with the aid of simplifying assumptions we have shown that under the combined effect of the Coriolis force and turbulence, the velocity of the wind increases with height, simultaneously turning to the right (in the northern hemisphere). At some height the wind becomes equal to the geostrophic wind, first in magnitude and then continuing to increase at some greater height becomes equal to the geostrophic wind in direction. At the earth's surface the angle of deflection is always equal to 45 degrees.

Accumulated empirical data shows that the factual mean distribution of the wind by height at an adequate distance from the earth's surface, concurs well with the theoretical distribution,

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but in the lowest dekameters there is no such concurrence. Thus, at the earth's surface the angle of inclination of the wind is considerably less than 45 degrees and in addition it does not remain constant, but varies within rather wide limits, decreasing over the sea during the summer and during the day and increasing over land during the winter and during the night. The factual magnitude of the wind velocity at the anemoscope level is considerably above its theoretical value.

The subsequent efforts of the scientists were directed toward the further development of the theory relating to the change in the wind with height, the beginning of which was laid by the work of Eckman and Okkerblom.

Gessel'berg and Sverdrup took into account the change with height in the pressure gradient force, considering this change as taking place in accordance with the linear rule, but assumed, as before, that ρ and A are constants. However, they completely excluded from consideration the layer below the anemoscope level, assuming justly that conditions of turbulent exchange predominating there are different from those in the layers above.

With the indicated assumptions, Gessel'berg and Sverdrup laid out a model of wind distribution with height, which concurred well with results of observations. Figure 132 shows a rectified (in accordance with Gessel'berg and Sverdrup) Eckman spiral for the atmospheric layer above the level of the anemometer. For comparison purposes the same Figure shows a spiral constructed as a result of developing the data obtained by 99 observations in Lindenberg. Figure 133 shows the distribution by height of the

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absolute magnitude of velocity, obtained both theoretically and empirically. Both curves coincide rather closely.

Figure 132. The Eckman spiral for the layer above the anemometer. The solid line is the mean of all observations, the dotted line - the mean of all computations.

Figure 133. The absolute magnitude of wind velocity as a function of height above the anemometer. The solid line is the observation curve, the dotted line - the computation curve.

The research by Gessel'berg and Sverdrup substantially improved the Eckman-Okkerblom theory. However, the solution they arrived at is by far not an exhaustive one. The change in the angle of inclination with relation to the time of the year and orography still remains an open question.

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A considerable contribution to the solution of the contemplated problem was made by the labors of Soviet scientists, which we are going to analyze in the Section immediately following.

Section 4. Variation of Wind with Height in the Presence of a Variable Coefficient of Turbulent Exchange.

The state of turbulence at any point is determined by one number - the coefficient of exchange. Having assumed in the preceding paragraph that the coefficient of exchange was the same for all heights, we have by the virtue of this accepted the state of turbulence as the same for all heights.

Yet, the state of turbulence and, consequently, the coefficient of exchange in the atmosphere are subject to considerable variations. The principal factors affecting the value of the coefficient of exchange are: the velocity of the geostrophic wind (or the pressure gradient that determines it), the roughness of the underlying surface, the stability of atmospheric stratification (vertical temperature gradient), the height above the earth's surface. Since the coefficient of exchange is not measured directly, but is computed on the basis of measurements of other meteorological elements, it follows that a correctly assigned character of variation of the coefficient of exchange is very important to such computations. The assumption of constancy of the coefficient of exchange by height, in some cases, does not allow the correct evaluation from observations of even the order of magnitude of the coefficient of exchange.

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On the other hand the assumption that the coefficient of exchange in the entire planetary boundary layer increases according to the linear law leads to incorrect results, namely, to excessively large values of the coefficient of exchange.

In their latest works, Soviet scientists use one of these two computation patterns, outlining the variation of the coefficient of exchange with height.

Pattern A: the coefficient of turbulent exchange increases with height asymptotically approaching the constant value.

$$\nu = \nu_\infty (1 + \epsilon - e^{-mz}) \qquad (1)$$

where ν_∞, ϵ is molecular viscosity. This pattern was suggested by B. I. Izvekov.

Pattern B: the coefficient of exchange first increases with height according to the linear law up to a certain height h, after which it remains constant, i.e.

$$\left.\begin{aligned} \nu &= cz \quad \text{when } z \leqslant h, \\ \nu &= ch \quad \text{when } z > h. \end{aligned}\right\} \qquad (2)$$

Pattern A is deficient in the sense that its use leads into more complex computations. Pattern B is more convenient for computations, although it leads to the solution of a two-layer problem.

In this Section we will analyze the problem of the variation of wind with height, accepting pattern B as the coefficient of exchange. This pattern was first suggested by M. E. Shvets and

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M. I. Yudin who derived an adequately simple and precise solution of the contemplated problem.

Yudin and Shvets, assuming the motion to be homogeneous, proceed from the following equations:

$$\left.\begin{aligned} \frac{\partial}{\partial z}\left(\nu\frac{\partial \rho u}{\partial z}\right)+2\omega_z\cdot\rho v=\frac{\partial p}{\partial x},\\ \frac{\partial}{\partial z}\left(\nu\frac{\partial \rho v}{\partial z}\right)-2\omega_z\rho u=\frac{\partial p}{\partial y}. \end{aligned}\right\} \qquad (3)$$

Multiplying the second equation by i, subtracting result from first, and interpolating the complex velocity

$$w=u-iv, \qquad (4)$$

the two equations (3) are reduced to one:

$$\frac{\partial}{\partial z}\left(\nu\frac{\partial \rho w}{\partial z}\right)+2i\omega_z\rho w=\frac{\partial p}{\partial x}-i\frac{\partial p}{\partial y}. \qquad (5)$$

With the aid of a new unknown function - quantity of motion

$$M=M_x-iM_y=\rho w, \qquad (6)$$

Equation (5) is rewritten in the following form:

$$\frac{\partial}{\partial z}\left(\nu\frac{\partial M}{\partial z}\right)+2i\omega_z M=\frac{\partial p}{\partial x}-i\frac{\partial p}{\partial y}. \qquad (7)$$

It is assumed that the pressure gradient does not change with height.

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For the layer with the linear variation of the coefficient of turbulence, which is lying below level h, equation (7) will assume the form:

$$c \cdot z \cdot \frac{\partial^2 M}{\partial z^2} + c \frac{\partial M}{\partial z} + 2i\omega_z M = \frac{\partial p}{\partial x} - i \frac{\partial p}{\partial y}. \qquad (8)$$

This is one of the forms of the well known Bessel's equation, which is integrated by the use of special so-called cylindrical functions which have been extensively studied:

$$M = D \cdot I_0(2\kappa\sqrt{i} \cdot z) + E N_0(2\kappa\sqrt{i}\, z) + G, \qquad (9)$$

where $D = D_1 + iD_2$ and $E = E_1 + iE_2$ - are arbitrary complex integration constants, which are determined from the boundary conditions; I_0 is a Bessel function of the zero order; N_0 is a Neumann function of the zero order; $\kappa^2 = \frac{2\omega_z}{c}$, $G = -\frac{1}{2\omega_z}\left(\frac{\partial p}{\partial y} - i\frac{\partial p}{\partial x}\right)$ is the geostrophic wind.

For the layer above level h we derive the Okkerblom solution already analyzed before:

$$M = Ae^{-n\sqrt{-i}\, z} + B \cdot e^{n\sqrt{-i}\, z} + G, \qquad (10)$$

where A and B are arbitrary integration constants, which are determined from the boundary conditions and $n^2 = \frac{2\omega_z}{hc}$.

Yudin and Shvets assume the following boundary conditions:

(1) At level z_0 the air adheres, therefore:

$$w = 0 \text{ when } z = z_0. \qquad (11)$$

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(2) At infinity the force of friction is finite:

$$c \cdot h\left(\frac{\partial M}{\partial z}\right) < \infty \quad \text{when } z = \infty. \tag{12}$$

(3) At level $z \longrightarrow 0$ the Prandtl' correlationship

$$M = \frac{1}{\kappa}\sqrt{\tau_0 \rho_0}\ln\frac{z}{z_0} \quad \text{when } z \longrightarrow 0 \tag{13}$$

and the Kibel' correlationship

$$\frac{\partial M}{\partial z} \longrightarrow \frac{\tau_0}{cz} \quad \text{when } z \longrightarrow 0. \tag{14}$$

take place.

In addition at level $z = h$ the solution derived for the lower layer must continuously turn into the solution derived for the upper layer. This means that at level $z = h$ function M and its derivative $\frac{\partial M}{\partial z}$ do not sustain a discontinuity:

$$M_{h+0} = M_{h-0}; \left(\frac{\partial M}{\partial z}\right)_{h+0} = \left(\frac{\partial M}{\partial z}\right)_{h-0}. \tag{15}$$

The formulated boundary conditions give 10 correlationships for the determination of 11 magnitudes: c, h, τ_0 and 8 arbitrary integration constants. Accepting one of these magnitudes as a parameter, all the other magnitudes can be expressed in terms of said parameter.

Introducing one more new variable

$$\kappa = 2\kappa\sqrt{z},$$

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we notice that the following designations occur for functions I_0 and N_0:

$$I_0(\varkappa\sqrt{i}) = ber\,\varkappa + i\,bei\,\varkappa, \tag{16}$$

$$N_0(\varkappa\sqrt{i}) = ner\,\varkappa + i\,nei\,\varkappa. \tag{17}$$

The Bessel and Neumann functions are well known. So that, when $x \longrightarrow 0$

$$\left.\begin{aligned} &ber\,\varkappa \longrightarrow 1,\\ &bei\,\varkappa \longrightarrow 0,\\ &ner\,\varkappa \longrightarrow -\frac{2}{\pi}\ln\frac{2}{\gamma\varkappa},\\ &nei\,\varkappa \longrightarrow \frac{1}{2}, \end{aligned}\right\} \tag{18}$$

where γ is the so-called Euler constant which is equal to 1.7811.

Then

$$|M| \longrightarrow \frac{2}{\pi}\ln\sqrt{z}\,|E|. \tag{19}$$

Substituting (19) into (13), we obtain

$$\frac{\varkappa}{\pi}|E| = \sqrt{\tau_0\rho_0}, \tag{20}$$

and from the Kibel' ratio (14), it follows:

$$|E| = \frac{\pi\tau_0}{c}. \tag{21}$$

Eliminating τ_0 from (20) and (21), we find:

$$|E| = \frac{\pi}{\varkappa^2}c\rho_0 \tag{22}$$

For further computations Yudin and Shvets assume:

$$\frac{\pi}{\varkappa^2} = 20 \quad (\varkappa = 0.395).$$

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They then interpolate a new parameter α - the angle of inclination of the wind, i.e. the angle formed by the direction of the wind with the isobar:

$$\tan\alpha = \frac{v(0)}{u(0)} = -\frac{E_2}{E_1}. \tag{23}$$

Then equation (22) can be transformed into:

$$|E| = \sqrt{E_1^2 + E_2^2} = \sqrt{E_1^2(1+\tan^2\alpha)} = 20c\rho_0,$$

from where

$$\left.\begin{aligned} E_1 &= 20c\rho_0\cos\alpha, \\ E_2 &= 20c\rho_0\sin\alpha. \end{aligned}\right\} \tag{24}$$

Satisfying the boundary condition (13), we derive:

$$DI_0(x_0\sqrt{i}) + EN_0(x_0\sqrt{i}) + G = 0. \tag{25}$$

Since x_0 is very small the authors consider it possible to utilize the approximation values of cylindrical functions for small argument values. Then, taking into account (18) we derive:

$$D + E\left(-\frac{2}{\pi}\ln\frac{2}{\gamma x_0} + \frac{1}{2}i\right) + G = 0. \tag{26}$$

In order to satisfy condition (12), it is necessary that B = 0. Finally satisfying conditions (15) we have:

$$Ae^{-nh\sqrt{-1}} = DI_0(x_h\sqrt{i}) + EN_0(x_h\sqrt{i}), \tag{27}$$

$$-An\sqrt{-i}\,e^{-nh\sqrt{-1}} = \frac{x\sqrt{i}}{\sqrt{h}}\left[DI_0'(x_h\sqrt{i}) + EN_0'(x_h\sqrt{i})\right]. \tag{28}$$

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Since

$$\frac{\kappa}{\sqrt{h}} = n,$$

then, combining (27) and (28) we derive:

$$D[I_0(\kappa_h\sqrt{i}) + iI_0'(\kappa_h\sqrt{i})] + E[N_0(\kappa_h\sqrt{i}) + iN_0'(\kappa_h\sqrt{i})] = 0. \qquad (29)$$

Further mathematical research is conducted by the authors as follows. Assuming for brevity that

$$I_0(\kappa_h\sqrt{i}) + iI_0'(\kappa_h\sqrt{i}) = j_1 + ij_2,$$

$$N_0(\kappa_h\sqrt{i}) + iN_0'(\kappa_h\sqrt{i}) = n_1 + in_2$$

and eliminating D with the aid of (26) we derive:

$$E = (n_1 + in_2) = \left[G + E\left(-\frac{2}{\pi}\ln\frac{2}{\gamma\kappa_0} + \frac{1}{2}i\right)\right](j_1 + ij_2) = 0, \qquad (30)$$

and separating the empirical part from the imaginary we find:

$$\left.\begin{aligned} &E_1 n_1 - E_2 n_2 + j_1\left[E_1\frac{2}{\pi}\ln\frac{2}{\gamma\kappa_0} + \frac{1}{2}E_0\right] - \\ &\quad - j_2\left[E_2\frac{2}{\pi}\ln\frac{2}{\gamma\kappa_0} - \frac{1}{2}E_1\right] = Gj_1, \\ &E_1 n_2 + E_2 n_1 + j_1\left[E_2\frac{2}{\pi}\ln\frac{2}{\gamma\kappa_0} - \frac{1}{2}E_0\right] + \\ &\quad + j_2\left(E_1\frac{2}{\pi}\ln\frac{2}{\gamma\kappa_0} + \frac{1}{2}E_2\right) = Gj_2. \end{aligned}\right\} \qquad (31)$$

Dividing both parts of the equations by 20ρ and taking into account correlationships (24) we derive:

$$\left.\begin{aligned} &c\left(n_1\cos\alpha + n_2\sin\alpha - \frac{1}{2}j_1\sin\alpha + \frac{1}{2}j_2\cos\alpha\right) + \\ &\quad + \frac{2}{\pi}c\ln\frac{2}{\gamma\kappa_0}\left(j_1\cos\alpha + j_2\sin\alpha\right) = \frac{G}{20\rho_0}\cdot j_1, \\ &c\left(n_2\cos\alpha - n_1\sin\alpha - \frac{1}{2}j_1\cos\alpha - \frac{1}{2}j_2\sin\alpha\right) + \\ &\quad + \frac{2}{\pi}c\ln\frac{2}{\gamma\kappa_0}\left(j_2\cos\alpha - j_1\sin\alpha\right) = \frac{G}{20\rho_0}\cdot j_2. \end{aligned}\right\} \qquad (32)$$

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Thus equations (32) [the original text is referring to equations (33) which is an error] contain the following parameters:

(1) The velocity of the gradient wind at the earth's surface

$$\frac{G}{\rho_0} = U.$$

(2) The angle of inclination α when $z = 0$.

(3) Magnitudes which are determined by the rule for the variation of the coefficient of turbulence c.

$$\nu_h = \sqrt{\frac{8\omega_z}{c}} \cdot h.$$

(4) Magnitudes $\nu_0 = \sqrt{\frac{8\omega_z}{c}} z_0$, which characterize the roughness of the terrestrial surface.

Equations (32) allow of the solution of two practically important problems:

(A) At the given values of the geostrophic wind U, angle of inclination α and roughness of the terrestrial surface z_0, - it is necessary to determine the rule for the variation of the coefficient of turbulence with height, i.e. parameters c and h.

(B) At the given values of the geostrophic wind U, the roughness of the surface z_0, and one of the parameters determining the variation of the coefficient of turbulence, - it is necessary to determine the other parameter and the angle of inclination

In order to illustrate the solution derived and for the

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execution of computations the authors developed a series of auxiliary tables.

After the rule for the variation of the coefficient of turbulence with height was determined, the component velocities at any height may be determined. For the layer $z \leqslant h$,

$$M = -\left[E\left(-\frac{2}{\pi}\ln\frac{2}{\gamma\kappa_0} + \frac{1}{2}i\right) + G\right] I_0(\kappa\sqrt{i}) + EN_0(\kappa\sqrt{i}) + G. \quad (33)$$

Separating the empirical part from the imaginary part we find:

$$\begin{aligned} u = &-20c\frac{\rho_0}{\rho}\left(-\frac{2}{\pi}\ln\frac{2}{\gamma\kappa_0}\cos\alpha + \frac{1}{2}\sin\alpha\right) ber\,\kappa + \\ &+ 20c\frac{\rho_0}{\rho}\left(\frac{1}{2}\cos\alpha - \frac{2}{\pi}\ln\frac{2}{\gamma\kappa_0}\sin\alpha\right) bei\,\kappa + \\ &+ \frac{\rho_0}{\rho}U(1 - ber\,\kappa) + 20c\frac{\rho_0}{\rho}\cos\kappa\, ner\,\kappa + 20c\frac{\rho_0}{\rho}\sin\alpha\cdot nei\,\kappa, \end{aligned} \quad (34)$$

$$\begin{aligned} v = &\ 20c\frac{\rho_0}{\rho}\left(-\frac{2}{\pi}\ln\frac{2}{\gamma\kappa_0} + \frac{1}{2}\sin\alpha\right) bei\,\kappa + \frac{\rho_0}{\rho}U\, bei\,\kappa - \\ &- 20c\frac{\rho_0}{\rho}\cos\alpha\, nei\,\kappa + 20c\frac{\rho_0}{\rho}\sin\alpha\cdot ner\,\kappa. \end{aligned}$$

For the layer where $z > h$,

$$M = Ae^{-n\sqrt{i}z} + G, \quad (35)$$

it being the case that the value of arbitrary constant is determined from condition (27).

Shvets and Yudin computed several examples of the distribution of wind with height, assuming the following values for the parameters:

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	I	II	III	IV	V	VI
U =	3.88	5.75	12.8	20	6.62	10.52 m/sec
z_o =	0.1	0.1	0.1	0.1	0.1	190 M
ν =	0.077	3.44	3.82	42.3	20.9	1.77 m^2/sec

The analysis of Tables compiled by the authors led them to formulate the following deductions resulting from theoretical research and agreeing well with observations:

(1) The angle of inclination of the wind is increased with the increase in the roughness of the underlying surface.

(2) Turbulent viscosity is increased with an increase in the pressure gradient.

(3) The velocity of the wind at the height of the anemometer is about one half of the geostrophic wind velocity.

(4) With increased height the wind in magnitude and direction approximates the geostrophic wind, it being the case that the velocity magnitude attains the geostrophic value before the direction does.

(5) With greater turbulent viscosity the angles of inclination are smaller.

(6) In the case of winds of great force the height at which the wind attains the geostrophic values is greater than in the case of mild winds.

(7) The ratio between the velocity of the wind at the anemometer height and the velocity of the geostrophic wind, is tied in

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with this velocity, roughness of the underlying surface and the angle of inclination. With a decrease in any of these 3 values, the above ratio is increased.

(8) At some heights the value of the velocity of the wind exceeds the value of the velocity of the geostrophic wind at the earth's surface.

Section 5. Diurnal Variation in the Velocity of the Wind.

At the earth's surface under conditions prevailing over a plain the velocity of the wind has a clearly pronounced diurnal variation unless this variation is disrupted by perturbations, which are stipulated by changes in weather. The wind is at its maximum velocity soon after midday. At night there is a wide minimum. The angle of deflection of the wind from the isobar is at its minimum during the day, and at its maximum during the night. The diurnal variation in the velocity of the wind is tied in with the season of the year, the underlying surface, and the air mass. Over the surface of the sea this diurnal variation is almost completely absent.

Above a certain level the velocity of the wind has an inverted diurnal variation: the maximum is observed at night and the minimum during the day. The height of rotation of the diurnal variation of the velocity of the wind varies within wide limits, dependent on the season of the year and the force of the wind. During the summer when turbulence is greater, the height of rotation attains 300 meters, while during the winter it is only 30-50 meters.

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A computation shows that the diurnal variation in the velocity of the wind cannot be explained by the diurnal variation in pressure since wind velocities induced by the diurnal variation in pressure do not exceed 10 centimeters per second, while actually observed diurnal wind velocity ranges are tens of times greater than the above indicated value.

On the other hand, the enumerated peculiarities of the diurnal variation in the wind indicate clearly that the diurnal variation in the velocity of the wind is tied in with the diurnal variation in turbulence.

The first attempt to explain the diurnal variation in the velocity of the wind with the aid of the equations of hydromechanics was made by B. I. Izvekov. He considered the kinematic coefficient of turbulent viscosity as non-varying with height, but as a periodical function of time.

$$\nu = \nu_0 + \epsilon \sin \omega t,$$

where ν_0 is the mean diurnal value of the kinematic coefficient of turbulent viscosity, ϵ is the diurnal variation amplitude of this coefficient and ω is the angular velocity of the earth.

However, having assumed the coefficient of turbulent viscosity as non-varying with height, Izvekov arrived at rather unsatisfactory results. The velocity of the wind, as per Izvekov, does not approximate with height the velocity of the gradient wind. The condition of the air adhering to the earth's surface also remains unsatisfied. Finally, the rotation of the diurnal variation of wind velocity at the upper levels of the boundary layer is also absent.

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M. E. Shvets solved the problem of the diurnal variation of wind velocity by assuming that ν increases linearly up to some height h, after which it remains constant. In addition, Shvets considers ν a periodical function of time, with the period equal to 24 hours.

Taking the equation of motion in a turbulent atmosphere in the form of

$$\left.\begin{aligned} -\rho\frac{\partial u}{\partial t}+2\omega_z\rho v+\frac{\partial}{\partial z}\nu\frac{\partial\rho u}{\partial z}&=\frac{\partial p}{\partial x},\\ -\rho\frac{\partial v}{\partial t}-2\omega_z\rho u+\frac{\partial}{\partial z}\nu\frac{\partial\rho v}{\partial z}&=\frac{\partial p}{\partial y}\end{aligned}\right\} \qquad (1)$$

and interpolating the complex velocity $w=u-iv$, Shvets reduces the problem to the integration of one equation in partial derivatives:

$$-\rho\frac{\partial w}{\partial t}+\frac{\partial}{\partial z}\nu\frac{\partial\rho w}{\partial z}+2i\omega_z\rho w=\frac{\partial p}{\partial x}-i\frac{\partial p}{\partial y}. \qquad (2)$$

Shvets considers that in equation (2) the term $-\rho\frac{\partial w}{\partial t}$ can be disregarded since the oscillation range of wind velocity does not usually exceed 10-20 percent of the mean velocity value of the wind. Therefore, in the place of equation (2) the equation following below is considered:

$$2i\omega_z\rho w+\frac{\partial}{\partial z}\nu\frac{\partial\rho w}{\partial z}=\frac{\partial p}{\partial x}-i\frac{\partial p}{\partial y}, \qquad (3)$$

it being the case that magnitudes ν, w, p are tied in parametrically with time t so that

$$\left.\begin{aligned} \nu&=c(t)\cdot z \quad \text{when } z\leqslant h(t),\\ \nu&=c(t)h(t) \quad \text{when } z>h(t).\end{aligned}\right\} \qquad (4)$$

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Shvets formulates the boundary conditions as follows:

(1) At the level of roughness of the underlying surface the velocity of wind becomes zero:

$$w = 0 \quad when \quad z = z_0 \tag{5}$$

(2) The force of friction at infinity is finite:

$$\nu \frac{\partial \rho w}{\partial z} < \infty \quad when \quad z = \infty \tag{6}$$

(3) At level z = h the condition of conjugateness of two solutions is fulfilled, i.e. functions ρw and $\frac{\partial \rho w}{\partial z}$ are continuous.

(4) When $z \longrightarrow 0$ the ultimate correlationship of Prandtl' takes place:

$$\left|\rho w\right|_{z \to 0} \longrightarrow \frac{1}{\kappa}\sqrt{\tau_0 \rho_0}\ln\frac{z}{z_0}; \quad \tau_0 = \lim_{z \to 0} \nu \left|\frac{\partial \rho w}{\partial z}\right|. \tag{7}$$

A solution of equation (3), which satisfies the given boundary conditions was obtained before (see Section 4):

$$\rho w = -\left[E\left(-\frac{2}{\pi}\ln\frac{2}{\gamma \kappa_0} + \frac{1}{2}i\right) + G\right] \cdot I_0(\kappa\sqrt{i}) + E N_0(\kappa\sqrt{i}) + G \tag{8}$$

when $z \leqslant h$,

where I_0, N_0 are, respectively, the Bessel and Neumann functions,

$$\kappa_0 = \sqrt{\frac{8\omega_x \cdot z_0}{c}}; \quad \kappa = \sqrt{\frac{8\omega_x z}{c}}; \quad G = -\frac{1}{2\omega_x}\left(\frac{\partial p}{\partial y} - i\frac{\partial p}{\partial x}\right); \tag{9}$$

E - the arbitrary integration constant, γ = 1.7811 - the Euler constant

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$$\rho w = Ae^{-\sqrt{\frac{-2\omega_z i}{ch}}} + G \qquad \text{when } z > h, \qquad (10)$$

where A is the arbitrary integration constant.

It was also shown there that after determining the arbitrary constants of integration there remain two equations tying in three magnitudes: $c(t), h(t), \alpha(t)$, where α is the angle between the direction of the wind and the direction of the isobar at the earth's surface.

These equations can be written like this:

$$\left.\begin{aligned} \frac{n_1 j_2 - n_2 j_1}{j_1^2 + j_2^2} + \frac{1}{2} &= -\frac{G}{20 c \rho_0} \sin\alpha; \\ \frac{n_1 j_1 + n_2 j_2}{j_1^2 + j_2^2} - \frac{2}{\pi} \ln\frac{2}{\gamma k_0} &= -\frac{G}{20 c \rho_0} \cos\alpha, \end{aligned}\right\} \qquad (11)$$

where

$$\left.\begin{aligned} j_1 + i j_2 &= I_0\left(\sqrt{\frac{8\omega_z h}{c}}\, i\right) + i I_0'\left(\sqrt{\frac{8\omega_z h}{c}}\cdot i\right), \\ n_1 + i n_2 &= N_0\left(\sqrt{\frac{8\omega_z h}{c}}\, i\right) + i N_0'\left(\sqrt{\frac{8\omega_z h}{c}}\, i\right). \end{aligned}\right\} \qquad (12)$$

Eliminating from equation (11) angle α we derive one equation tying in $c(t)$ and $h(t)$:

$$\left(\frac{n_1 j_2 - n_2 j_1}{j_1^2 + j_2^2} + \frac{1}{2}\right)^2 + \left(\frac{n_1 j_1 + n_2 j_2}{j_1^2 + j_2^2} - \frac{2}{\pi}\ln\frac{2}{\gamma k_0}\right)^2 = \left(\frac{G}{20 c \rho_0}\right)^2. \qquad (13)$$

Accepting as the coefficient of turbulent viscosity ν at a height above the level z = h, the condition

$$\nu = c(t)h(t) = \nu_0(1 + \epsilon \sin \omega t), \qquad (14)$$

Shvets determines c(t) and h(t) from equations (13) and (14), and then by the Tables he constructed he derives the distribution of the wind with height.

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Irrespective of the schematic characteristics of rule (14), the variations of the coefficient of turbulent viscosity with time, as derived theoretically by Shvets, will agree with observational values.

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CHAPTER XVII

THE ENERGY OF ATMOSPHERIC MOTIONS

Section 1. Transformation of Energy in the Atmosphere.

For the atmospheric processes under study in dynamic meteorology, the following forms of energy are of essential significance:

(1) Radiation energy (R);

(2) Kinetic energy of averaged large scale currents of the order of general circulation or currents in cyclones and anticyclones (E);

(3) Kinetic energy of the turbulent motions which is tied in with the presence in the averaged currents of additional velocities (E');

(4) Potential energy of the air masses (Π);

(5) Internal energy of air (U).

We are not concerned here with electric and magnetic energy, which have considerable significance for some individual phenomena in the atmosphere, but for the processes under study in dynamic meteorology, are not essential.

The potential and internal energy of the air are in direct ratio to temperature T; therefore, as will be shown below there is a close and direct relationship between them to the effect that the increase in the internal energy of the atmosphere is always accompanied by an increase in its potential energy and vice versa.

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A continuous transformation of energy from one form into another one is continuously taking place in the atmosphere. The basic mechanisms for such transformation are absorption and radiation, mechanical work, and the dissipation of mechanical energy into heat.

The following transformations of energy are possible in the atmosphere:

1) $R \rightarrow (U+\Pi)$,
2) $(U+\Pi) \rightarrow R$,
3) $(U+\Pi) \rightarrow E$,
4) $(U+\Pi) \rightarrow E'$,
5) $E \rightarrow (U+\Pi)$,
6) $E' \rightarrow (U+\Pi)$,
7) $E \rightarrow E'$

Radiation energy cannot be converted into kinetic energy directly. It has no significance for the atmosphere until such time as the atmosphere will absorb it, i.e. until such time as it will be transformed into internal energy. It being the case that in atmospheric conditions (in a gravitational field) an increase in internal energy is always accompanied by an increase in potential energy, therefore, $R \rightarrow (U+\Pi)$. Internal energy can again be transformed into radiation energy by way of atmospheric radiation, it being the case that a decrease in internal energy is always accompanied by a corresponding decrease in potential energy $(U+\Pi) \rightarrow R$.

The internal and potential energies, accumulated as a result of absorption of solar radiation, may subsequently be transformed into kinetic energy which sometimes assumes the form of kinetic energy of large scale motions $(U+\Pi) \rightarrow E$, or the form of kinetic

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energy of turbulent motions $(U+\Pi) \rightarrow E'$. The transformation of potential and internal energies into kinetic energy is one of the most important aspects of modern dynamic meteorology, since the general circulation of the atmosphere and the evolution of cyclones and anticyclones are specifically tied in with such transformation. In its turn kinetic energy may be transformed into potential and internal energies either by way of direct transition of kinetic energy into potential $E \rightarrow (U+\Pi)$, or by way of dissipation of mechanical energy into thermal energy. In the latter case, the gradual process of degradation of kinetic energy by way of the breaking up of vortices takes place. The kinetic energy of large currents is converted into kinetic energy of progressively smaller vortices $E \rightarrow E'$ until such time as it is dissipated into heat energy under the effect of viscosity $E' \rightarrow (U+\Pi)$.

The process of transformation of kinetic energy of large currents into turbulent kinetic energy is an irreversible process.

A process which would lead to the straightening of the lines of flow and to the increase in total velocity of a current at the expense of a diminution in the velocities of the additional motions is not possible in nature.

Section 2. Equation of the Energy Balance of an Individual Air Particle.

Let us develop the equation of the energy balance of an individual air particle having a mass equal to unity. We will assume that friction is completely absent, and the air is in motion as the ideal liquid. In this case the equations can be written down this way:

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$$\left.\begin{aligned} \frac{du}{dt} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\omega \sin\varphi v, \\ \frac{dv}{dt} &= -\frac{1}{\rho}\frac{\partial p}{\partial y} - 2\omega \sin\varphi u, \\ \frac{dw}{dt} &= -\frac{1}{\rho}\frac{\partial p}{\partial z} - g. \end{aligned}\right\} \qquad (1)$$

Multiplying equation (1) respectively by u, v, w and adding the results we derive the equation of the mechanical energy balance or the equation of live forces:

$$\frac{d}{dt}\left(\frac{c^2}{2} + gz\right) = -\frac{1}{\rho}\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial s}\cdot c, \qquad (2)$$

where c is the velocity of the wind, $\frac{\partial p}{\partial s}$ is the component of the pressure ascendent in the direction of velocity.

We note that the deflecting force of the earth's rotation did not enter the equation of live forces, which was to be expected, since the amount of work performed by the force of inertia is identically equal to zero, due to the direction of velocity and the force of inertia being perpendicular to each other.

When the motion is horizontal $\frac{d}{dt}(gz) = 0$, an equation (2) is simplified:

$$\frac{d}{dt}\left(\frac{c^2}{2}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial s}\cdot c. \qquad (3)$$

It follows from equation (3) that if pressure drops in the direction of the motion $\left(\frac{\partial p}{\partial s} < 0\right)$, kinetic energy is increased and vice versa. Consequently, the counter-gradient throwing out of air is accompanied by a loss of kinetic energy.

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Combining the equation of live forces (2) with the equation of heat inflow for the case of an ideal gas

$$\frac{dq}{dt} = c_v \frac{dT}{dt} + p \frac{d}{dt}\left(\frac{1}{\rho}\right), \tag{4}$$

we derive the equation of energy balance for an ideal gas which is in motion without friction:

$$\frac{dq}{dt} = \frac{d}{dt}\left(\frac{c^2}{2} + gz + c_v T\right) + p \frac{d}{dt}\left(\frac{1}{\rho}\right) + \frac{1}{\rho}\frac{\partial p}{\partial s} \cdot c. \tag{5}$$

Equation (5) shows that the energy imparted from without to a unit of mass of an ideal gas moving without friction is expended for the conversion of the sum of the kinetic, potential and internal energy of the gas for the work of expansion, and for the overcoming of the pressure gradient effect.

Noticing that

$$c\frac{\partial p}{\partial s} = \frac{dp}{dt} - \frac{\partial p}{\partial t},$$

we write the equation of energy balance in the following form:

$$\frac{dq}{dt} = \frac{d}{dt}\left(\frac{c^2}{2} + gz + c_v T + \frac{p}{\rho}\right) - \frac{1}{\rho}\frac{\partial p}{\partial t}. \tag{6}$$

If the motion is a settled one, $\frac{\partial p}{\partial t} = 0$, and the change in the atmosphere is adiabatic, $\frac{dq}{dt} = 0$, equation (6) is integrated for time:

$$\frac{c^2}{2} + gz + c_v T + \frac{p}{\rho} = const. \tag{7}$$

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But

$$c_v T + \frac{p}{\rho} = c_v T + RT = c_p T.$$

Consequently, the equation of energy balance in this case assumes the form of

$$\frac{c^2}{2} + gz + c_p T = const. \qquad (8)$$

This equation mutually ties in the parameters of the state of the same air particle at different moments of time. But since we consider the motion as a settled one, the lines of flow and the trajectories of the particles in this case coincide. Therefore, equation (8) also ties in the parameters of the state of various particles lying on the same line of flow. However, the values of the constant for various lines of flow will, generally speaking, vary.

Equation (7) is a generalization of Bernoulli's equation for a non-compressible ideal liquid.

$$\frac{c^2}{2} + gz + \frac{p}{\rho} = const. \qquad (9)$$

Bernoulli's equation and its generalization has numerous applications, particularly in instrumental meteorology. It explains, in particular, why in the case of great velocities, some impedance in the current may perceptibly distort the value of barometric pressure. This equation serves as the theoretical basis for the design of instruments measuring the velocities of the current by registering and reading the pressures that develop in the current. Bernoulli's equation shows the extent of the error that is made

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in measuring pressure and temperature with the aid of instruments installed in high speed aircraft.

Section 3. Equations of the Energy Balance of an Arbitrary Air Mass.

Proceeding from the law of the preservation of energy we can write down the equation of the energy balance of an arbitrary mass of liquid as follows:

$$\frac{dE}{dt} + \frac{dU}{dt} = Q + \int_{\tau} \rho(F, v)\, d\tau + \int_{\Sigma} (p_n, v)\, d\sigma \qquad (1)$$

(See Section 7, Chapter VIII), where $\frac{dE}{dt}$ is the variation in a unit of time of the kinetic energy of the contemplated mass of liquid; $\frac{dU}{dt}$ is the variation in a unit of time of the internal energy of the same liquid; Q is the heat inflow from without; $\int_{\tau} \rho(F, v)\, d\tau$ is the work of all volume forces in a unit of time; $\int_{\Sigma} (p_n, v)\, d\sigma$ is the work of all surface forces in a unit of time.

Let us transpose the equation of energy balance to a form suitable to the study of the atmosphere. Utilizing the results obtained in Section 13 of Chapter VIII, we can consider the variation in the entire kinetic energy of the current $\frac{dE}{dt}$ as the sum of variations of the kinetic energy of the mean motion $\frac{d\bar{E}}{dt}$ and the kinetic energy of the pulsations $\frac{dE'}{dt}$:

$$\frac{dE}{dt} = \frac{d\bar{E}}{dt} + \frac{dE'}{dt}, \qquad (2)$$

it being the case that the variation in the kinetic energy of the pulsations of the contemplated air mass can be expressed as:

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$$\frac{dE'}{dt} = \int_\tau T\,d\tau - \int_\tau D\,d\tau, \qquad (3)$$

where T stands for the transformation of the kinetic energy of the mean motion and the kinetic energy of the pulsations as related to a unit of volume, and D is the dissipation of mechanical energy as related to a unit of mass:

$$T = \mu\left\{2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2\right\}, \qquad (4)$$

$$D = \mu\left\{2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2\right\}. \qquad (5)$$

Substituting into the energy balance equation (1) in place of $\frac{dE}{dt}$, its expression as derived from (2) and (3):

$$\frac{dE}{dt} = \frac{d\bar{E}}{dt} + \int_\tau T\,d\tau - \int_\tau D\,d\tau, \qquad (6)$$

we reduce the energy balance equation to the following form:

$$\frac{d\bar{E}}{dt} + \frac{dU}{dt} = Q + \int_\tau \rho(F, v)\,d\tau + \int_\Sigma (p_n, v)\,d\sigma - \int_\tau T\,d\tau + \int D\,d\tau. \qquad (7)$$

Let us now consider the expression for the work of mass forces. We will remember that in studying atmospheric motions in dynamic meteorology we are dealing with two mass forces only: the deflecting force of the rotation of the earth and the force of gravity. Since the deflecting force of the earth's rotation is always directed perpendicular to velocity, its work is identically equal to zero. Therefore, the deflecting force of the earth's

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rotation does not enter the energy balance equation.

Let us now interpolate into the energy balance equation in the place of the only mass force (the force of gravity), the geopotential Φ :

$$F = -\operatorname{grad}\Phi. \tag{8}$$

Since the geopotential Φ is the potential energy as related to a unit of mass, the entire potential energy Π of the contemplated mass of liquid will be expressed by formula

$$\Pi = \int_\tau \rho\Phi\, d\tau. \tag{9}$$

Taking into account that $\rho\, d\tau = const.$, we derive:

$$\frac{d\Pi}{dt} = \frac{d}{dt}\int_\tau \rho\Phi\, d\tau = \int_\tau \rho\frac{d\Phi}{dt}\, d\tau = \int_\tau \rho\frac{\partial\Phi}{\partial t}\, d\tau + \int_\tau \rho(v, \operatorname{grad}\Phi)\, d\tau. \tag{10}$$

But the geopotential of a point does not depend on time $\frac{\partial\Phi}{\partial t} = 0$, consequently:

$$\frac{d\Pi}{dt} = \int_\tau \rho(v, \operatorname{grad}\Phi)\, d\tau. \tag{11}$$

Transposing the integral which expresses the work of the mass force we derive:

$$\int_\tau \rho(F, v)\, d\tau = -\int_\tau \rho(v, \operatorname{grad}\Phi)\, d\tau = -\frac{d\Pi}{dt}. \tag{12}$$

Consequently, the energy balance equation assumes the form of

$$Q = \frac{d}{dt}(\bar{E} + E' + \Pi + U) - \int_\Sigma (p_n, v)\, d\sigma \tag{13}$$

or

$$Q = \frac{d}{dt}(\bar{E} + U + \Pi) - \int_\Sigma (p_n, v)\, d\sigma + \int_\tau T\, d\tau - \int_\tau D\, d\tau.$$

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Equation (13) shows that the inflow of heat from without is expended in order to bring about the variation in the sum total of kinetic (energy of the averaged motion and pulsations), potential, and internal energies, and also to bring about the work in a unit of time, of forces of pressure and viscosity acting upon the surface which confines the contemplated volume of liquid.

Let us interpolate into the energy balance equation the enthalpy ϑ.

The enthalpy of a unit of mass is determined by formula $\vartheta = u + p\alpha$, where u is the internal energy of a unit of mass, α is the specific volume, and the enthalpy of the entire mass of liquid is expressed by the formula

$$\vartheta = \int_{\tau} \rho(u + p\alpha)\, d\tau = \int_{\tau} \rho u\, d\tau + \int_{\tau} \rho(p\alpha)\, d\tau. \qquad (14)$$

Differentiating this equality by time and taking into account that $\rho\, d\tau = const.$ we derive:

$$\frac{d\vartheta}{dt} = \frac{dU}{dt} + \int_{\tau} \rho \frac{d(p\alpha)}{dt} d\tau. \qquad (15)$$

Eliminating with the aid of (15) internal energy from equation (13), we obtain the energy balance equation in the following form:

$$Q = \frac{d}{dt}\left[\bar{E} + E' + \Pi + \vartheta\right] - \int_{\tau} \rho \frac{d(p\alpha)}{dt} d\tau - \int_{\Sigma} (p_n, v)\, d\sigma. \qquad (16)$$

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Transposing the surface integral $\int_{\Sigma}(p_n, v)\,d\sigma$ which expresses the work in a unit of time of the forces of pressure and viscosity acting upon the surface Σ confining the volume τ, we derive:

$$\int_{\Sigma}(p_n, v)\,d\sigma = -\int_{\Sigma} p(n, v)\,d\sigma + \int_{\Sigma}(p_n + p\cdot n)\cdot v\,d\sigma.$$

where n is the unit vector of a normal to surface $d\sigma$; the first right-hand integral expresses the work of pressure forces, the second - the work of viscosity forces.

Utilizing the Gauss' theorem and taking into account the Euler operator, we derive:

$$\int_{\Sigma} p(n, v)\,d\sigma = \int_{\Sigma} p v_n\,d\sigma = \int_{\tau} div\,(pv)\,d\tau = \int_{\tau}\left(\frac{\partial pu}{\partial x} + \frac{\partial pv}{\partial y} + \frac{\partial pw}{\partial z}\right)d\tau =$$

$$= \int_{\tau}\left(\frac{\partial p}{\partial x}u + \frac{\partial p}{\partial y}v + \frac{\partial p}{\partial z}w\right)d\tau + \int_{\tau} p\,div\,v\,d\tau =$$

$$= \int_{\tau}\left(\frac{dp}{dt} - \frac{\partial p}{\partial t}\right)d\tau + \int_{\tau} p\,div\,v\,d\tau. \qquad (17)$$

In addition:

$$\int_{\tau}\rho\frac{d(p\alpha)}{dt}\,d\tau = \int_{\tau}\rho p\frac{d\alpha}{dt}\,d\tau + \int_{\tau}\frac{dp}{dt}\,d\tau = \int_{\tau} p\,div\,v\,d\tau + \int_{\tau}\frac{dp}{dt}\,d\tau,$$

therefore:

$$-\int_{\tau}\rho\frac{d(p\alpha)}{dt}\,d\tau - \int_{\Sigma} p(n, v)\,d\sigma = \int_{\tau}\frac{\partial p}{\partial t}\,d\tau.$$

Consequently, the balance of energy equation assumes the form:

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Transposing the surface integral $\int_{\Sigma}(p_n, v)\,d\sigma$ which expresses the work in a unit of time of the forces of pressure and viscosity acting upon the surface Σ confining the volume τ, we derive:

$$\int_{\Sigma}(p_n, v)\,d\sigma = -\int_{\Sigma} p(n, v)\,d\sigma + \int_{\Sigma}(p_n + p\cdot n)\cdot v\,d\sigma.$$

where n is the unit vector of a normal to surface $d\sigma$; the first right-hand integral expresses the work of pressure forces, the second - the work of viscosity forces.

Utilizing the Gauss' theorem and taking into account the Euler operator, we derive:

$$\int_{\Sigma} p(n, v)\,d\sigma = \int_{\Sigma} p v_n\,d\sigma = \int_{\tau} div\,(pv)\,d\tau = \int_{\tau}\left(\frac{\partial pu}{\partial x} + \frac{\partial pv}{\partial y} + \frac{\partial pw}{\partial z}\right)d\tau =$$

$$= \int_{\tau}\left(\frac{\partial p}{\partial x}u + \frac{\partial p}{\partial y}v + \frac{\partial p}{\partial z}w\right)d\tau + \int_{\tau} p\ div\ v\,d\tau =$$

$$= \int_{\tau}\left(\frac{dp}{dt} - \frac{\partial p}{\partial t}\right)d\tau + \int_{\tau} p\ div\ v\,d\tau. \qquad (17)$$

In addition:

$$\int_{\tau}\rho\frac{d(p\alpha)}{dt}\,d\tau = \int_{\tau}\rho p\frac{d\alpha}{dt}\,d\tau + \int_{\tau}\frac{dp}{dt}\,d\tau = \int_{\tau} p\ div\ v\,d\tau + \int_{\tau}\frac{dp}{dt}\,d\tau,$$

therefore:

$$-\int_{\tau}\rho\frac{d(p\alpha)}{dt}\,d\tau - \int_{\Sigma} p(n, v)\,d\sigma = \int_{\tau}\frac{\partial p}{\partial t}\,d\tau.$$

Consequently, the balance of energy equation assumes the form:

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$$Q = \frac{d}{dt}\left[\bar{E} + E' + \Pi + \Im\right] - \int_{\tau} \frac{\partial p}{\partial t}\, d\tau - \int_{\Sigma} v(p_n + pn)\, d\sigma. \qquad (18)$$

This balance of energy equation shows that the inflow of heat from without to a volume of liquid is expended in order to effect a change in the sum total of kinetic energy (the averaged motion and turbulence), potential energy and enthalpy, also a local change in pressure and for the work of the viscosity forces at the boundaries of the volume.

Section 4. The Energy of a Vertical Column of Air.

Let us consider a vertical air column, the cross section of which is unity, and which extends from level $z = 0$ to the level $z = h$. The internal energy of this air column is determined by formula

$$U = c_v \int_0^h T\rho\, dz. \qquad (1)$$

Taking into account the basic equation of statics in the atmosphere

$$dp = -g\rho\, dz,$$

we derive:

$$U = \frac{c_v}{g} \int_{p_h}^{p_0} T\, dp. \qquad (2)$$

Formula (2) is sometimes more suitable for the computation of internal energy of an air column than formula (1).

The potential energy of the contemplated air column is

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$$\Pi = \int_0^h g z \rho \, dz, \tag{3}$$

from where by interpolating pressure with the aid of the basic equation of statics we find that

$$\Pi = \int_{p_h}^{p_0} z \, dp = -h p_h + \int_0^h p \, dz,$$

but

$$p = R \rho T;$$

Consequently,

$$\Pi = -h p_h + R \int_0^h T \rho \, dz. \tag{4}$$

The enthalpy of the same air column we find from formula

$$\vartheta = c_p \int_0^h T \rho \, dz = \frac{c_p}{g} \int_{p_h}^{p_0} T \, dp. \tag{5}$$

Combining equalities (1) and (4) we derive:

$$\Pi = -h \cdot p_h + (\chi - 1) \cdot U, \tag{6}$$

where

$$\chi = \frac{c_p}{c_v} = 1.4,$$

Consequently,

$$U + \Pi = -h p_h + \chi U = -h \cdot p_h + \vartheta. \tag{7}$$

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If the air column extends into infinity, i.e. when $h \to \infty$ the product $h \cdot p_h \longrightarrow 0$, since pressure p diminishes with height in accordance with the law of exponents while the height itself increases linearly. Therefore, for an infinitely high air column the following correlationships are true:

$$U + \Pi = \text{э}, \tag{8}$$

$$\Pi : U = \varkappa - 1 = 0.4. \tag{9}$$

Consequently, any change in the potential energy of the infinitely high air column is accompanied by a proportional change in the internal energy of this column of air, and vice versa. Therefore, in studying the processes of transformation of energy in the atmosphere, the sum of potential and internal energy is usually considered. This sum may be substituted by enthalpy. Let us again emphasize that such a correlationship is true in the case of an infinitely high column of air only, while for an arbitrary finite volume this correlationship is not satisfied as can be seen from equation (7) which was constructed for an air column of a finite height. Equation (7) shows that for a finite volume of air Π is not only not equal to $(\varkappa - 1)U$, but always less than this value.

Section 5. Energy Liberated in the Vertical Redistribution of Air Masses.

Let us consider some schematic examples of transformation of potential and internal energy into kinetic energy. We assume that the atmosphere is in unstable equilibrium, which can be

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induced either by vertical stratification or by an unstable horizontal distribution of air masses. In order to bring the atmosphere out from a state of unstable equilibrium, the application of some outside impulse is required. If the equilibrium is disturbed the potential and internal energies are transformed into kinetic energy. While the sum total of potential and kinetic energies diminishes, converging toward a minimum, kinetic energy is increased converging toward a maximum. The state that corresponds to the maximum value of kinetic energy, we will call conditionally the "final" state. It is clear that this "final" state is not preserved since after it has been attained the reverse transformation of kinetic energy into potential and internal energies sets in.

Let us consider a column of air confined from below by the terrestrial surface, from the sides - by rigid vertical walls and open on top. The redistribution of masses will take place in the lower part of the column only up to a certain height h, with the upper part of the column moving up and down like a rigid piston weighing $p_h \mathcal{B}$, where p_h is the pressure at the upper boundary of the masses undergoing redistribution, and $\mathcal{B}$ is the area of the base of the column.

Example 1. In a column of air (Figure 134), the pressure of which at the surface of the earth equals p_0, two air masses (designated by indexes 1 and 2) lie one over the other.

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Figure 134. For the computation of kinetic energy liberated in the process of disturbing an unstable equilibrium of air masses in a vertical direction.

At the upper limit of the first air mass pressure p_h prevails. The second air mass is potentially warmer than the first one. As a result of this the redistribution of masses sets in and in the final state the first mass lies over the second one (1' and 2'). It being the case that the height of both layers is correspondingly changed from h_1 to h_1' and from h_2 to h_2'. The pressure at the boundaries of the masses changes from p_i to p_i'. The level with the pressure p_h also changes.

In the process of redistribution of the masses the sum of the potential and internal energies of a unitary column of air is partially transformed into kinetic energy and in addition is partially expended in the work involved in the expansion of the air column which is obviously equal to $p_h(h'-h)$. Consequently,

$$(\Pi+U)-(\Pi+U)'=E'+p_h(h'-h), \tag{1}$$

where the prime indicates the values of the respective magnitudes in the final state. But in accordance with expression (7) in Section 4,

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$$\Pi + U = \vartheta - hp_h,$$

$$(\Pi + U)' = \vartheta' - h'p_h.$$

Consequently,

$$E' = \vartheta - \vartheta'. \qquad (2)$$

Thus, in order to determine the kinetic energy liberated in the process of the vertical redistribution of masses it is sufficient to work out the difference in the enthalpy values of the contemplated air column.

Let us assume that in the initial state the first and the second masses are each in indifferent equilibrium, i.e. each one of the masses has everywhere the same potential temperature, and further that the redistribution of the masses is adiabatic in its nature, i.e. each of the masses retains a constant potential temperature during the process of redistribution. Let there be, at boundary i, in the initial state, a jump in temperature from T_{i2} to T_{i1}. Considering $p_0, T_{02}, h_2, h_1, T_{i1}$ as given magnitudes, we compute the enthalpy of the unitary air column in the initial and final states. According to (5) in Section 4,

$$\vartheta = \frac{c_p}{g}\int_{p_h}^{p} T\,dp.$$

Since the equilibrium is an indifferent one, we, by the use of the equation of the adiabat

$$T = T_0\left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}},$$

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derive:

$$\int_{p_h}^{p} T\,dp = T_0 p_0^{-\frac{\varkappa-1}{\varkappa}} \int_{p_h}^{p_0} p^{\frac{\varkappa-1}{\varkappa}}\,dp = \frac{\varkappa}{2\varkappa-1}(T_0 p_0 - Tp). \qquad (3)$$

Thus, the enthalpy of a unitary column of air in the initial state will be expressed as follows:

$$\vartheta = \frac{c_p}{g}\frac{\varkappa}{2\varkappa-1}(p_0 T_{02} - p_i T_{i2} + p_i T_{i1} - p_h T_{h1}),$$

it being the case that

$$T_{h1} = T_{i1} - \gamma_a h_1;\ T_{i2} = T_{02} - \gamma_a h_2;\ p_i = p_0\left(\frac{T_{i2}}{T_{02}}\right)^{\frac{\varkappa}{\varkappa-1}}. \qquad (4)$$

The enthalpy of the unitary air column in the final state, i.e. after the redistribution of the masses, will equal:

$$\vartheta' = \frac{c_p}{g}\frac{\varkappa}{2\varkappa-1}(p_0 T_{01}' - p_i' T_{i1}' + p_i' T_{i2}' - p_h T_{h2}'), \qquad (5)$$

with pressures p_0 and p_h at the lower and upper boundaries of the contemplated air column remaining neutral, and $p_i' = p_h + p_0 - p_i$. Since the redistribution of the masses takes place adiabatically, the distribution of temperature in each layer is determined by the equation of the dry adiabat, so that

$$T_{01}' = T_{h1}\left(\frac{p_0}{p_h}\right)^{\frac{\varkappa-1}{\varkappa}};\quad T_{i1}' = T_{h1}\left(\frac{p_i'}{p_h}\right)^{\frac{\varkappa-1}{\varkappa}},$$
$$T_{i2}' = T_{02}\left(\frac{p_i}{p_0}\right)^{\frac{\varkappa-1}{\varkappa}};\quad T_{h2}' = T_{02}\left(\frac{p_h}{p_0}\right)^{\frac{\varkappa-1}{\varkappa}}. \qquad (6)$$

The thickness of the layers after the redistribution of the masses is determined from correlationships

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$$\left.\begin{aligned} T_{i1}' &= T_{01}' - \gamma_a h_1', \\ T_{h2}' &= T_{02}' - \gamma_a h_2'. \end{aligned}\right\} \qquad (7)$$

The mass of the unitary air column is, obviously, equal to $\frac{p_0 - p_h}{g}$. Therefore, the square of mean velocity will be:

$$c^2 = \frac{2g}{p_0 - p_h}(\vartheta - \vartheta'). \qquad (8)$$

Computations made in accordance with the above formulas show that in the process of vertical redistribution of air masses, a tremendous amount of kinetic energy is liberated. Therefore, great velocities may develop in the process of such redistribution. Consequently, the kinetic energy of cyclones may be partially connected with such redistribution of the air masses.

Example 2. Upon the surface of the earth are resting two air masses lying side by side (Figure 135)

Figure 135. For the computation of kinetic energy liberated with the disturbance of the unstable equilibrium of masses lying alongside each other.

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They have equal bases B/2 and equal heights h and are restricted from the sides by vertical walls, and from the top by a layer of air in stable equilibrium which moves up and down as a rigid piston. Let the first mass be potentially colder than the second.

In the initial state a system of two air masses may be in equilibrium if there is a rigid partition between them.

If the vertical partition is removed, then, due to the presence of a horizontal pressure gradient, the cold air begins to shift, underflowing the warm air. Thus, the cold air will be disposed below the warm air with the cold air compressed and the warm air expanded. With this, the upper boundary will rise or descend. If the upper boundary rises, work is performed by the pressure of the air, which is disposed under it; if the upper boundary descends work is performed by external pressure.

Assuming that at the initial moment both air masses are in indifferent equilibrium. After adiabatic redistribution, the masses again remain in indifferent equilibrium.

The enthalpy in the initial state equals:

$$\vartheta = \frac{c_p}{g} \frac{\varkappa}{2\varkappa - 1} \frac{B}{2} \left[p_{01} T_{01} - p_{h1} T_{h1} + p_{02} T_{02} - p_h T_{h2} \right]. \qquad (9)$$

Let's consider p_h, h, T_{h1}, T_{h2} as given magnitudes. The other magnitudes are determined from the correlationships following from the state of indifferent equilibrium and the adiabatic character of the process:

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$$T_{o1} = T_{h1} + \gamma_a h; \quad T_{o2} = T_{h2} + \gamma_a h,$$

$$p_{o1} = p_h \left(\frac{T_{o1}}{T_{h1}}\right)^{\frac{k}{k-1}}; \quad p_{o2} = p_h \left(\frac{T_{o2}}{T_{h2}}\right)^{\frac{k}{k-1}}. \tag{10}$$

The enthalpy in the final state will equal:

$$\vartheta' = \frac{c_p}{g} \cdot \frac{k}{2k-1} B\left[p_o' T_{o1}' - p_i' T_{i1}' + p_i' T_{i2}' - p_h' T_{h2}'\right]. \tag{11}$$

In the final state, the temperature at the surface of separation of two air masses sustains a discontinuity from a lower value of T_{i1}' to a higher value of T_{i2}'. The upper layer of the second mass remains at the same pressure p_h; therefore, $T_{h2}' = T_{h2}$ The surface of separation is determined by the fact that the pressure prevailing at it is as expressed by formula

$$p_i' = p_h + \frac{1}{2}(p_{o2} - p_h),$$

since the second mass spreads out over an area twice as large.

Due to the fact that the redistribution of the masses is adiabatic and the equilibrium during the initial and final states is indifferent in its nature, we have:

$$T_{i2}' = T_{h2}\left(\frac{p_i'}{p_h}\right)^{\frac{k-1}{k}}, \quad T_{i1}' = T_{h1}\left(\frac{p_i'}{p_h}\right)^{\frac{k-1}{k}}, \quad T_{o1}' = T_{h1}\left(\frac{p_o'}{p_h}\right)^{\frac{k-1}{k}}, \tag{12}$$

where $p_o' = \frac{p_{o1} + p_{o2}}{2}$.

Computations show that with a temperature differential of 10 degrees centigrade, the following velocities C develop at each horizontal level with various height of layer h :

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h = 2000 3000 6000 meters

c = 13 16 23 meters/second $^{-1}$

The above evaluations of the velocities developing during the transformation of potential and internal energies into kinetic energy, were derived as a result of too crude a schematization, which is very far from the actual processes taking place in the atmosphere. However, the significance of these evaluations consists in that they underscore sharply the important part played by the unstable distribution of atmospheric air masses, as condensers of tremendous reserves of potential energy, which, with the redistribution of the masses, is being transformed into kinetic energy.

A thorough analysis shows that the principal source of kinetic energy in the atmosphere is the unstable distribution of the air masses. The previously predominant opinion that the source of kinetic energy was the pressure field has proved to be erroneous.

Schematic computations show that the horizontal distribution of pressure is perfectly inadequate for the explanation of the kinetic energy generated in a cyclone. The horizontal distribution of pressure may produce only about 5 percent of the total reserve of the kinetic energy of a cyclone. The basic source of kinetic energy of atmospheric motions is the potential energy of a state, which energy is increased with a rise in temperature stipulated by the inflow of heat. The pressure gradients express the difference in the potential energy of two adjacent air columns. Therefore, the pressure field is only the mechanism for the conversion of potential energy into kinetic energy, since, as a

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(the averaged values are not marked and the coefficient of turbulent viscosity is substituted by the coefficient of exchange A).

It is easy to evaluate the transformation of the kinetic energy of an averaged atmospheric motion only for the simplest case when $u = u(z)$; $v = v(z)$; $w = 0$. In this case, the formula for T is simplified and assumes the form of

$$T = A\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right]. \qquad (2)$$

Let us suppose further that the coefficient of turbulence, density and the pressure gradient do not change with height and that velocity does not change with time. Such a supposition corresponds to the stationary cycle of mean velocities when the loss of kinetic energy, stipulated by transformation, is continuously replenished. In Section 3 of Chapter XVI we have shown that the components of the wind velocity with the assumptions as made, are expressed as:

$$u = u_g(1 - e^{-\alpha z}\cos \alpha z),$$

$$v = u_g \cdot e^{-\alpha z}\sin \alpha z,$$

where

$$u_g = \frac{1}{2\rho w \sin \varphi}\frac{\partial p}{\partial y}$$

is the velocity of the geostrophic wind, and

$$\alpha = \sqrt{\frac{\rho w \sin \varphi}{A}}$$

consequently,

$$T = 2\rho w \sin \varphi u_g^2 e^{-2\alpha z}. \qquad (3)$$

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Since above the level of friction $D = \pi/\alpha$, the velocity of the wind with height under the effect of turbulence changes very little, transformation of the kinetic energy of the averaged motion into the energy of turbulence above this level occurs only on a very small scale. Therefore, to evaluate the order of magnitude for the transformation of kinetic energy in the atmosphere, it is sufficient to integrate expression (3) from the terrestrial surface to the level of friction D:

$$T^* = \int_0^D T\,dz = u_g^2 (1 - e^{-2\pi}) \sqrt{A\rho\omega \sin\varphi} \qquad (4)$$

Let us compare the magnitude of the transformation of kinetic energy with the total reserve of kinetic energy in a unitary air column extending from the surface of the earth to the level of friction D . Since

$$\mathcal{E}^* = \frac{\rho}{2} \int_0^D (u^2 + v^2)\,dz = \frac{\rho}{2} D \cdot u_g^2 \left\{ 1 - \frac{(1 + e^{-\pi})^2}{2\pi} \right\}, \qquad (5)$$

$$\frac{T^*}{\mathcal{E}^*} = \frac{2\omega \sin\varphi}{\pi} \frac{1 - e^{-2\pi}}{1 - \frac{(1 + e^{-\pi})^2}{2\pi}}. \qquad (6)$$

Computations show that at latitude $\approx 45^\circ$, about 15 percent of the kinetic energy reserve is transformed in one hour. Consequently, in 7 hours the entire kinetic energy reserve would have been transformed into turbulent energy if it would not be replenished at the expense of solar radiation being transformed into internal and potential energies.

More precise evaluations show that the transformation of kinetic energy in a column of air, having a cross section of one

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square meter and a height of 10 kilometers from the surface of the earth, equals 5×10^7 erg meter^2/second, it being the case that in the lower part of the column (up to a height of 1 kilometer) transformation equals 3×10^7 erg m^2/sec. If the mean wind velocity throughout the entire air column is taken as 10 meters/second, the entire amount of kinetic energy in the column will be 5×10^{12} erg. Consequently, in 10^5 seconds = 1.16 diurnal periods, the entire kinetic energy of the mean motion is transformed into turbulent energy. An elementary computation shows that about 2 percent of inflowing solar radiation is adequate to replenish the loss of kinetic energy.

Section 7. Equation of the Energy Balance of Atmospheric Turbulence.

As was shown above (Section 13 of Chapter VIII), the change in the turbulent energy of a unit of mass of a non-compressible liquid $\frac{dE'}{dt}$ is determined by the transformation of the kinetic energy of the averaged motion T and the dissipation D of mechanical energy into work:

$$\frac{dE'}{dt} = T - D. \qquad (1)$$

But the transformation of the kinetic energy of an averaged motion is not the only source for the replenishment of turbulent energy in the atmosphere. In Section 5 we have shown that the change in the internal energy of an air column is tied in closely with the change in the potential energy of this column. If the atmosphere is in stable stratification, an air particle (individual vortex) on reaching an above lying layer, will find itself colder than the surrounding air, which condition forces it to descend.

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By the same token an individual vortex in descending in a stably stratified atmosphere will find itself warmer than the surrounding air, and will tend to rise to its previous level. Since, in a turbulent current, there is continuous ascending or descending of individual vortices, the work performed in the process is at the expense of a decrease in the kinetic energy of turbulence. Consequently, the change in the kinetic energy of atmospheric turbulence is determined not only by the transformation and the dissipation of energy, but also by the work which is being performed in overcoming the stable vertical stratification of the atmosphere. It turns out that the work being performed is considerably in excess of the dissipation of energy into heat.

A. M. Obukhov takes the turbulent energy balance equation in the following form:

$$\frac{dE'}{dt} = T - \frac{d\Pi}{dt} - D, \qquad (2)$$

where E' is the kinetic energy of turbulence, Π is the potential energy of the current, $\frac{d\Pi}{dt}$ is the work expended to overcome the stable stratification of the atmosphere.

In order to analyze the turbulent energy balance equation, A. M. Obukhov interpolates two turbulence characteristics: v' which is the pulsation velocity, and l, which is the scale of turbulence, which corresponds to the path of intermixing, thereby reducing the turbulence energy balance equation to the form:

$$\frac{dE'}{dt} = K\left(\frac{\partial v}{\partial z}\right)^2 - K_T \frac{g}{T}\frac{\partial \theta}{\partial z} - D, \qquad (3)$$

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where

$$K = v'l; \quad K_T = \alpha v' l,$$

$$D = \beta \frac{v'^3}{l}; \quad E' = \frac{v'^2}{2}$$

(See Sections 5 and 8 in Chapter XV).

Coefficients α and β in the first approximation are considered as constants, which will be evaluated below.

Obukhov contemplates the established turbulence cycle, for which the energy balance equation has the form

$$\frac{\partial E'}{\partial t} = v'l\left(\frac{\partial v}{\partial z}\right)^2 - \alpha v' l \frac{g}{T}\frac{\partial \theta}{\partial z} - \beta \frac{v'^3}{l} = 0$$

or

$$v'\left[l^2\left(\frac{\partial v}{\partial z}\right)^2 - \alpha l^2 \frac{g}{T}\frac{\partial \theta}{\partial z} - \beta v'^2\right] = 0. \tag{4}$$

Equation (4) can be broken down into 2 equations:

$$v' = 0, \tag{5}$$

$$\left(\frac{\partial v}{\partial z}\right)^2 - \alpha \frac{g}{T}\frac{\partial \theta}{\partial z} = \beta \frac{v'^2}{l}. \tag{6}$$

Equation (5) characterizes the established laminar cycle and equation (6) - the established turbulence cycle. Equation (6) can have a substantial solution only on condition that

$$\left(\frac{\partial v}{\partial z}\right)^2 - \alpha \frac{g}{T}\frac{\partial \theta}{\partial z} \geqslant 0. \tag{7}$$

Interpolating into the analysis the Richardson number

$$Ri = \frac{g}{T}\frac{\frac{\partial \theta}{\partial z}}{\left(\frac{\partial v}{\partial z}\right)^2},$$

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we discover that with an established turbulence cycle:

$$Ri < (Ri)_{ult} = \frac{1}{\alpha} = \frac{K}{K_T}. \qquad (8)$$

With $Ri > (Ri)_{ult}$, the only substantial solution of the energy balance equation is $v' = 0$, which corresponds to the established laminar motion.

To evaluate the magnitude of β, we transpose equation (6) to the form

$$\frac{\beta v'^2}{l^2 \left(\frac{\partial v}{\partial z}\right)^2} = 1 - \frac{Ri}{(Ri)_{ult}},$$

from where

$$v' = \frac{1}{\beta} \sqrt{1 - \frac{Ri}{(Ri)_{ult}}} \left|\frac{\partial v}{\partial z}\right| \cdot l. \qquad (9)$$

In order that formula (9) coincides with the basic correlationship of the semi-empirical theory $v' = l \frac{\partial v}{\partial z}$, with an adiabatic stratification of the atmosphere when $Ri = 0$, it is necessary to accept $\beta = 1$.

Besides, the magnitude for the dissipation of energy can be disregarded. Therefore, the equation of energy balance can be written down as:

$$\frac{dE'}{dt} = T\left(1 - \frac{Ri}{(Ri)_{ult}}\right). \qquad (10)$$

In this case, the established turbulence cycle is possible only at entirely definite values of the $Ri = (Ri)_{ult}$. If $Ri < (Ri)_{ult}$

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I.e. if the transformation of energy is in excess of the work expended for the overcoming of the stable stratification, the turbulence energy will increase. In the opposite case, the turbulence energy will diminish. Considerable vertical gradients of mean velocity with all other conditions being equal, are characteristic of a considerable value of transformation of energy. Therefore, turbulence energy will increase even with inversions and the inversions themselves will disintegrate, if, as it frequently occurs in the atmosphere, the inversions are tied in with an abrupt discontinuation in the velocity of the wind.

The problem of the numerical value of $(Ri)_{ult}$ has not been finally solved as yet.

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CHAPTER XVIII

GENERAL CIRCULATION OF THE ATMOSPHERE

Section 1. General Considerations.

The atmosphere is always in a state of complex and continuous motion which is varying with time. Analyzing synoptic maps constructed for a limited territory, we see the unusual diversity of atmospheric motions. All these various motions, whatever their cause, enter into the general circulation of the atmosphere. The motion of the air in cyclones and anticyclones, orographic winds, breezes, and similar phenomena are various forms in which the general circulation of the atmosphere is manifested. The motions in the atmosphere are so diverse that the probability of the recurrence of the same complex of motion of the entire atmosphere can definitely be considered to be equal to zero.

It is therefore clear that to describe the general circulation of the atmosphere in all details is impossible, and that out of the general circulation only the most characteristic and important features which may be preserved from year to year can be segregated. But the theoretical analysis of even these most characteristic features of the general circulation encounters insurmountable difficulties.

In the present state of our knowledge we lack even one somewhat satisfactory theoretical diagram of the general circulation.

In order to derive a satisfactory theoretical diagram of the general circulation, we must first of all arrive at a

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mathematical formulation of the laws controlling the general circulation of the atmosphere. The general laws of physics and hydro-dynamics remain perfectly true, but they must be expressed in concrete mathematical form. In this attempt most of the difficulties are encountered in the composition of the equation of the inflow of heat. These difficulties are tied in, first of all, with the circumstance that, in the atmosphere, heat exchange is effected through radiation, turbulent heat conductivity and water changing its phase.

Under the conditions of general circulation of the atmosphere all these three forms of heat exchange are equally important and are, therefore, to be taken into account. Particularly complex in their characteristics, as was seen above, are radiation and turbulent heat exchange. In deriving equations of motion, the accounting for turbulent viscosity presents particular difficulties. Therefore, no rigid mathematical formulation of the problem of general circulation is, as yet, possible. Nevertheless, the investigation of the problem of the general circulation of the atmosphere continues, and it proceeds in two directions: (1) Some meteorologists, proceeding principally from the analysis of mean climatological charts of pressure and wind, attempt the construction of empirical, purely qualitative theories of the general circulation of the atmosphere (Bjerknes, Bergeron, Prandtl'). (2) Other meteorologists offer semi-empirical theories of general circulation, proceeding from a given temperature distribution. Here it is necessary to take notice of the most significant progress attained by Soviet theoreticians (Kochin, Blinov). However,

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the characteristic of the factual general circulation of the atmosphere is of equal importance to all meteorologists.

Section 2. Review of the General Circulation of the Atmosphere.

In order to bring out the most typical features of the general circulation of the atmosphere, mean climatic charts, most frequently January and July charts, are used. (See Figures 136 and 137)

Figure 136. Mean distribution of pressure for January.

Figure 137. Mean distribution of pressure for July.

In comparing the two charts for the mean distribution of pressure in January and in July, it, first of all, becomes obvious that low pressure is predominant over the continents in the summer, while high pressure is predominant over the oceans. The reverse is true for the winter months. There is a lowered pressure in the region of the equator. In the southern hemisphere there is

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observed throughout the year a sharply pronounced subtropical belt of high pressure having its central area about Parallel 30 south, it being the case that over the oceans this high-pressure belt is manifested more sharply than over the continents. The maximum mean pressure in this zone is 764-768 millimeters. In the northern hemisphere, the high pressure belt is observed only in the winter, it being the case that it is strongly distorted by the effect of the Asiatic land mass, over which maximum pressure prevails during the winter. These subtropical regions of high pressure girdle the earth, not in a continuous manner, however, but divided into 5 closed regions forming the subtropical oceanic anticyclones, which are preserved without any particular changes throughout the year. Two such oceanic subtropical anticyclones are disposed in the northern hemisphere: the Azorean (North Atlantic) and the Honolulu (North Pacific) anticyclones. In the southern hemisphere there are 3 such anticyclones: the South Atlantic (the St. Helena Island), the South Pacific, and the St. Mauritius Island (in the South Indian Ocean).

At the equatorial periphery of these subtropical anticyclones with exceptional steadiness throughout the year predominant winds of a moderate force, the trade winds. In the northern hemisphere their direction is to the northeast, in the southern hemisphere - to the southeast. In approaching the thermal equator the trade winds become mild east winds and here is disposed the doldrum zone. The width of this zone varies, and frequently it disappears completely, in which case the trade winds of the two hemispheres meet with the resulting formation of so-called tropical

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fronts. Since the air of the trade wind currents is very damp, and since in its migration toward the equator, it encounters a warmer underlying surface, the air masses carried by the trade winds when passing near the equator, possess great reserves of moist-labile energy. That is why the equatorial region of the doldrums is known for its frequent thundershowers, cloudbursts and squalls.

The tropical front is also tied in with abundant precipitation. Constant as the trade winds are, they still shift somewhat in conformity with the peculiar features of the area. The Atlantic trade winds show more constancy than the Pacific trade winds, which in January and February in the area of the Caroline Islands, are completely discontinued. However, more substantial changes in the trade winds are observed during the transition from one season to the other. The entire system of trade winds becomes displaced, first to the north, then to the south, following the sun. The central area of the doldrum zone, remaining at all times in the northern hemisphere, occupies in September its northernmost position, reaching to 10 degrees of latitude north. It is characteristic that the central area of the high pressure belt does not become displaced in this process. The seasonal changes in the trade-winds are stipulated not by the displacement of the high-pressure zone, but by a change in its intensity.

As already mentioned above, the evolution during the summer of an area of high pressure over the continent cannot take place on account of the intensive heating of the land surface.

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Due to sharp diurnal changes there is an accumulation over the landmass of considerable moist-labile energy which is resolved in short-duration cloudbursts, involving considerably larger areas over land than over the ocean.

The mightiest belt of low pressure occurs in July over the southern part of the Asiatic continent. The most powerful cyclonic formation - the Asiatic depression is evolved here, and it generates a peculiar phenomenon in the general circulation - the famous Asiatic or Indian southwesterly monsoon. The Indian monsoon is distinguished by exceptional constancy during the season, and it embraces a vast area. In this respect, the Indian monsoon may be compared with the trade winds.

The central strip of subtropical anticyclones is the zone of calm. These are the so-called "horse latitudes" of the subtropical seas.

At the polar periphery of the subtropical anticyclones predominate westerly winds. The zone of the westerly winds extend approximately to the latitude of 65-70 degrees.

Thus, the dimensions of the zone of the westerly winds are no smaller than those of the zone of the trade winds. The westerly winds, however, lack the constancy of the trade winds and frequently winds of a different direction, tied in with the passing of cyclones and anticyclones, are observed in that zone.

In the southern hemisphere the low-pressure zone of the extra-tropical latitudes is not broken into individual closed

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depressions, which is due to the homogeneity of the underlying surface.

Until recently it was assumed that at the poles high pressure constantly predominates. However, observations by Soviet scientists conducted lately in the Arctic have shown that shifting cyclones may penetrate into very high latitudes. With all this, synoptic maps of mean pressure distribution show high pressure areas in the Arctic as well as in the Antarctic. At the periphery of these high pressure areas predominate easterly winds, which when encountering in the high latitudes the westerly winds from the temperate latitude, form Arctic fronts, and in penetrating into lower latitudes, polar fronts.

There is as yet no adequate data on the factual distribution of wind in the free atmosphere. However, a fairly adequate concept of the mean wind field in the free atmosphere can be obtained by the height-calibrated pressure field, although data on the distribution of pressure in the atmosphere based on aerological ascents is even scarcer than wind observation results. But the mean pressure field for various levels of the free atmosphere can be computed by the given mean pressure field at sea level and by the mean values of the temperature gradients. The mean temperature distribution with height is available for a series of points in the northern hemisphere for the summer as well as for the winter and, therefore, the computation of the pressure field at various levels in the free atmosphere of the northern hemisphere is not particularly difficult. However, for the southern hemisphere where no adequate network of

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temperature sounding stations is available, such a computation cannot be made.

Due to the presence of a planetary horizontal temperature gradient, the mean direction of which in the troposphere of the northern hemisphere is from south to north, the isobars with height are more and more approximating the latitudinal direction.

Already at the 2-kilometer level (Figures 138-142) the pressure maximum at the North Pole has disappeared. At all heights, from 2 to 8 kilometers the pole is the center of low pressure. The Azorian and Honolulu pressure maximums and also the South Asia depression are still present at the 2-kilometer level, but are considerably weakened. A mild anticyclone appeared over North Africa. At the 4-kilometer level the Honolulu high-pressure area disappeared completely, the Azorian maximum and the South Asia depression are considerably weakened, while the North Africa maximum was accelerated. In addition a mild maximum appeared to the north of the Himalayas.

Figure 138. Mean distribution of pressure in the northern hemisphere at the height of 2 kilometers for January

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Figure 139. Mean distribution of pressure in the northern hemisphere at the height of 4 kilometers for January

Figure 140. Mean distribution of pressure in the northern hemisphere at the height of 2 kilometers for July

Figure 141. Mean distribution of pressure in the northern hemisphere at the height of 4 kilometers for July

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Figure 142. Mean distribution of pressure in the northern hemisphere at the height of 8 kilometers for July

At the 8-kilometer level the North Africa maximum became intensified, the Azorian maximum shifted westward, with its center over Mexico. The maximum that appeared at the height of one kilometer to the north of the Himalayas, now extends from the Persian Gulf to Eastern China.

In the winter the flattening of the pressure field with height proceeds more rapidly than in the summer. Already at the 4-kilometer level the isobars run almost latitudinally with no closed isobars anywhere. Thus, the pressure field at this level is a gigantic depression, embracing the entire northern hemisphere and having its center near the North Pole.

Since the wind in the free atmosphere approximates the geostrophic wind the above synoptic maps for the mean distribution of pressure at various heights are also characteristic of the wind field at these heights.

It follows from the above that at heights from 2 to 8

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kilometers, let us say, up to Parallel 30 north, westerly winds will predominate. At greater heights the region of westerly winds extends almost to Parallel 10 north. In the winter at a height above 4 kilometers the circulation of the air around the subtropical anticyclones is completely discontinued. At this level the westerly wind predominates in all latitudes with such conditions preserved up to heights of 12-16 kilometers.

During the summer at a height of 8 kilometers easterly winds predominate over the tropical areas of Africa, India, and North America.

No final deduction as to the winds predominating over the Pacific and Atlantic oceans can be made by the above maps on account of the small values of the pressure gradients.

This, then, is the concept of the distribution of wind in the upper layers of the troposphere. Near the equator, up to a height of 10000 meters is a region where easterly winds predominate. As the scene shifts toward the north, the wind at the heights becomes consecutively southeasterly, then southerly, then southwesterly, and in the tropics and further north - westerly. In the southern hemisphere an analogous distribution of the wind along the latitude is to be expected, namely, at the equator, the wind is easterly, then, as the scene shifts toward the south, the wind becomes northeasterly, northerly, northwesterly and westerly.

As we have seen in the region of the trade winds, the wind in the upper atmospheric layers has a direction directly opposite to the latter and is known as the anti-trade wind. Of course, the

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anti-trade wind is not just the simple returning phase of the trade wind and, therefore, the trade wind and the anti-trade wind do not form a mutually simple closed circulation in a vertical plane. Observations show that the vertical depth of the trade winds is small. It is frequently not over several hundred meters, and, at any rate, not over 1.5 kilometers. The upper limit of the trade wind is the trade wind inversion. Then follows a transition layer with low velocities of non-stable direction. In this layer, the direction of the motion coincides at times with the direction of the trade winds, but most frequently it has a westerly component.

The anti-trade wind appears only at a considerable height, ranging between 2-7 kilometers.

Observations over the North Atlantic have shown that the transition from trade wind to anti-trade wind is accompanied by left rotation of the wind with height.

The upper boundary of the trade wind, as well as the lower boundary of the anti-trade wind, rises from the Pole to the equator. The height of these boundaries changes also with longitude. In some places above the anti-trade winds, upper trade winds blowing from the east are observed.

Further north there are winds which at all heights blow predominantly toward the west. In the winter these winds have a component directed toward the Pole and in the summer - a component directed toward the equator.

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From time to time cyclones and anticyclones disturb these mean aspects of the wind procedures, causing winds of any direction to occur at all levels.

In the high latitudes at all levels on the average the prevailing winds blow predominantly towards the west.

It is also essential to have at least a rough idea about the vertical movements of the air as forming a part of the general circulation of the atmosphere. As yet, meteorology does not know of any methods for the direct measuring of vertical air velocities. It is, therefore, natural that no such observational data exists, and emphasis must be placed on indirect analysis, which indicates the predominant signs of the vertical velocities. In this category, in particular, belongs the analysis of precipitation distribution. In the zone of subtropical maximums precipitation is low. It is, therefore, natural to assume that the predominant air movement there is a descending one. The mean vertical velocity of such a descending motion rarely exceeds 0.5 centimeter/second. In the northern part of the region of the trade winds ($\sim$ to the north of Parallel 10 north) the motion is descending, and in the southern part (to the south of Parallel 10 north), it is ascending.

In concluding this short review of the general circulation of the atmosphere let us point to the fact that the presence of the tropopause must have essential significance in the development of atmospheric circulation. The tropopause, of which we can schematically conceive as of a discontinuity surface of the first temperature derivative, is a surface dividing the tropospheric

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mass of air from the stratospheric mass. The disposition of this surface of separation is undoubtedly tied in with the general circulation of the atmosphere and with the details of this circulation. The processes of regeneration and diffusion of the tropopause are also tied in with the general circulation of the atmosphere. Thus, with the development of a cyclone, the tropopause descends, with the stratospheric air following it. In its descent the air is warmed adiabatically and its radiation equilibrium is disturbed. The mass of stratospheric air which has descended begins to cool, with the result that the tropopause is regenerated at a greater height and the descended stratospheric air becomes assimilated with the tropospheric. Inversely, with the development of anticyclones there occurs regeneration of the tropopause at a lower level while the tropospheric air ascends into the stratosphere.

Section 3. The Geographical Distribution of Solar and Terrestrial Radiation.

In Chapter VII the mean thermal balance for the earth as a whole was analyzed. The actual values for solar radiation coming into the atmosphere and also the values for terrestrial radiation, at various latitudes and at various seasons of the year, depart essentially from the mean values cited in Chapter VII.

The computation of the magnitude of solar radiation entering at the boundary of the atmosphere is a problem rather astronomical than meteorological. This problem was resolved by Milankovich. Without citing the tabulated data, we will analyze the graph constructed on the basis of the latter (Figure 143).

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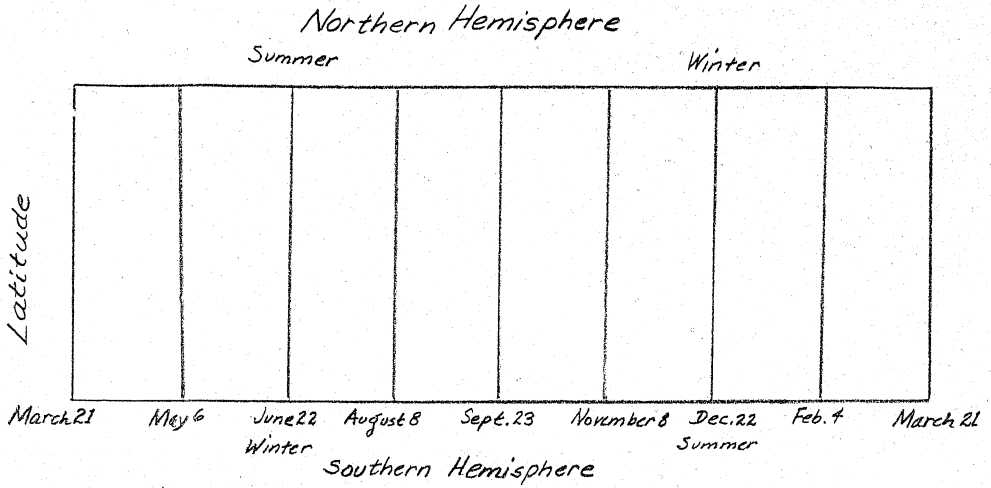


Figure 143. Geographical distribution of solar radiation in the absence of atmosphere over one square centimeter of horizontal surface in a diurnal period (24 hours)

The above graph characterizes the amount of energy entering upon one square centimeter of horizontal surface over a diurnal period at various latitudes and various seasons of the year, assuming there is no atmosphere. Geographical latitudes are plotted along the y-axis and along the x-axis is plotted the time of the year expressed in solar longitude (upper scale) and the respective dates (lower scale). During the summer solstice, the North Pole receives the maximum amount of energy per 24 hours since at this point the sun irradiates the surface of the earth and the atmosphere for 24 hours. The secondary maximum takes place at Parallel 40 north and is expressed by a mild bulging out of the isoline 1000 calories/per square centimeter over a diurnal period. In the Polar region of the southern hemisphere during the summer solstice, no solar radiation enters the atmosphere since the sun at that time is below the horizon. During the winter the distribution of the

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solar radiation inflow is inverted. The radiation received by the southern hemisphere during the southern summer is greater than the radiation received by the northern hemisphere during the northern summer, since the earth is nearer the sun during the northern winter than it is during the northern summer.

Figure 144 shows the amount of direct solar radiation received over a diurnal period by every square centimeter of the earth's surface, if the coefficient of transparency is assumed to be 0.7.

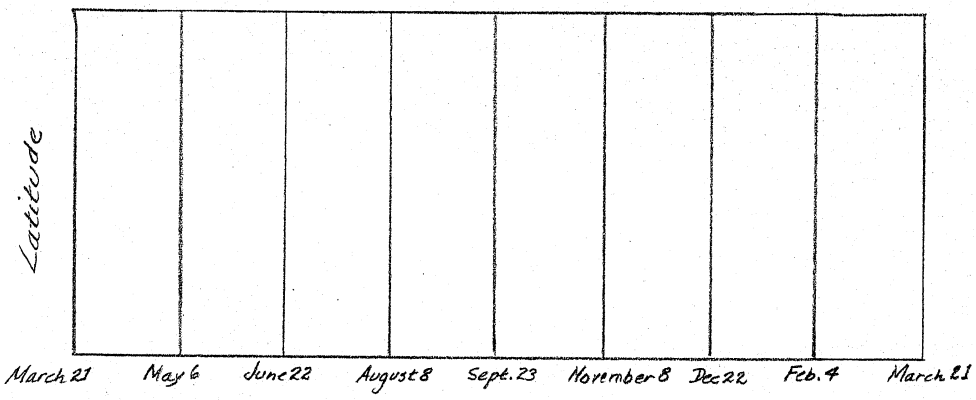


Figure 144. Geographical distribution of solar radiation, with coefficient of transparency equal to 0.7, over a diurnal period, for each square centimeter of horizontal surface.

In this case the amount of energy received by the earth's surface is considerably less than if there were no atmosphere. The diminishing of direct solar radiation is particularly great in the polar regions where the sun is low over the horizon. In the low latitudes, where the zenith distance of the sun is smaller, the degree of weakening of solar radiation is also smaller. Therefore,

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the summer maximum in this case is not at the Pole, but at the place where, as indicated above, the secondary maximum occurs, assuming that there is no atmosphere, i.e. at Parallel 40 north.

The computation of the geographical distribution of terrestrial radiation is considerably more difficult. Simpson suggested a method for the computation of outgoing radiation from the earth and from the atmosphere, which method was based on his simplified diagram of the absorption spectrum of long-wave radiation in water vapor and carbon dioxide (See Section 3, Chapter VII).

The results derived by Simpson are cited in Table 51.

Latitude	Solar (incoming) Radiation (cal/cm^2 min)	Terrestrial (outgoing) Radiation (cal/cm² min)	
		as per Simpson	As per Albrecht
0	0.339	0.271	0.305
10	0.334	0.282	0.304
20	0.320	0.284	0.299
30	0.297	0.284	0.288
40	0.267	0.282	0.276
50	0.232	0.277	0.263
60	0.193	0.273	0.250
70	0.160	0.260	0.230
80	0.144	0.252	0.225
90	0.140	0.252	0.255
Mean	0.271	0.271	

Table 51. Incoming and outgoing radiation.

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The second column shows the incoming into the atmosphere solar radiation, with the albedo (light reflecting power) excluded. The third column contains data obtained by Simpson for terrestrial (outgoing) radiation. The mean values for outgoing and incoming radiation coincide. According to Simpson, terrestrial radiation changes little with latitude, which fact is tied in with the assumption he made about the constancy of the temperature of the clouds at all latitudes. With solid cloud cover terrestrial radiation will increase in the direction of the pole, in keeping with the higher temperature values of the stratosphere in the polar regions. Already beginning with Parallel 35 north, terrestrial radiation is beginning to predominate over solar radiation. In order to compensate for the loss of radiation energy in the higher latitudes, there must be a continuous transfer of heat from the low into the high latitudes, with the atmospheric currents. This circumstance is decisive for the general circulation of the atmosphere. Simpson also pointed out that terrestrial radiation is distributed rather uniformly not only in space, but also in time, without manifesting any substantial changes during the year.

However, it is necessary to correct somewhat the results derived by Simpson, since the absorption of energy by water vapor proceeds at a weaker rate than was accepted by Simpson, as per Gettner (See Section 3, Chapter VII). Such a divergence of the assumptions by Simpson with actual conditions have no essential significance for the lower atmospheric layers, while in the case of the stratosphere it leads to essentially different results. In addition, it turns out that in the stratosphere there isn't any vapor even in the

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amount equivalent to 0.3 millimeters of precipitation. Therefore, radiation in the region of full absorption by water vapor must occur not only in the stratosphere, but also at least partially in the upper troposphere. Since the decrease in water vapor content with height occurs in conformity with the law of exponents, the maximum radiation will occur in the lowest part of the contemplated layer.

This circumstance was noticed by Albrecht, who pointed out that "emission layers", i.e. layers with the maximum radiation, are entirely in the upper troposphere, and not in the stratosphere, with the temperatures of these emission layers differing from each other to a smaller degree than the temperatures of the stratosphere at various latitudes. Therefore, according to Albrecht, terrestrial radiation changes with latitude more sharply than was indicated by Simpson. According to Simpson, the cold surface of the earth, at the pole, radiates together with the warm stratosphere and at the equator the warm surface of the earth radiates together with the cold stratosphere. As a result of this, there occurs some compensation leading to the smoothing of the differentials in the summary radiation at various latitudes. According to Albrecht, there is no such compensation and terrestrial radiation diminishes toward the Pole together with the temperature of the earth's surface. That is why terrestrial radiation changes more abruptly with latitude, in Albrecht's computation, than in Simpson's.

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Section 4. Theorems on Circulation

In the preceding section it was shown that in the low latitudes the influx of solar radiation predominates considerably over terrestrial and atmospheric radiation into universal space, and inversely, in the high latitudes terrestrial and atmospheric radiation into space predominates considerably over the influx of solar radiation. Therefore, there must exist in the atmosphere a continuous transfer of heat from the low latitudes to the high latitudes, together with the air currents.

In connection with this, it becomes natural to pose the question as to whether the transfer of heat from the low to the high latitudes constitutes some stationary closed circulation. Motions in the atmosphere are very frequently presented diagrammatically as closed circulations, although, strictly speaking, the presence of such circulations in the atmosphere has not yet been proved.

In this section we will analyze several general theorems on circulation of velocity.

In the mechanics of a solid body, a system of material points is studied as one piece. Similarly, in hydrodynamics, it becomes possible to study the motion of an aggregate of particles lying on a closed curve, and not each particle individually. An important part in studying the motion of a liquid is played by the concept of circulation of velocity.

Let us visualize an arbitrary closed curve s, described at a given moment of time in a liquid that is in motion. Let us compute

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the tangential v_s and the normal v_n velocity components for each particle of the liquid, which lies at the given moment on the curve s. It is obvious that the normal velocity component is not tied in with the shifting of the liquid particle along the curve s. Only the tangential velocity component is tied in with such shifting. We will determine the direction of the shifting by the direction of the tangential velocity component of the particle, considering it positive when it coincides with the a priori given positive direction of the circuit of the curve. As the positive direction of the circuit of the curve we will accept such a direction as will leave the surface, delimited by the curve, on the left. Let us ascertain the general trend of the motion of all particles of the liquid, lying on the outline s with relation to this outline. This general tendency of the motion of all particles of the liquid is determined not only by the direction of the tangential velocity component, but also by its magnitude. Therefore, the evaluation of this tendency for an individual particle is expressed by the scalar product (Figure 145)

$$(v, dr) = v\cos(v, dr)\,ds = u\,dx + v\,dy + w\,dz, \qquad (1)$$

where r is the radius-vector of the particle, and $ds = |dr|$ is the minor element of the arc of the outline taken near this particle. Summating the values of the scalar product (1) along the entire curve and passing over to the limit, when $dr \to 0$, we derive a curvilinear integral along a closed outline

$$\Gamma = \oint_s (v, dr) = \oint v_s\,ds = \oint (u\,dx + v\,dy + w\,dz) \qquad (2)$$

$$[\Gamma] = [L^2 T^{-1}],$$

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which characterizes the general trend of the motion with relation to the outline of all particles lying on this outline at a given moment of time. Integral Γ is called the circulation of velocity v along a closed outline S . A circulation of velocity is positive if the general tendency of the motion along the outline coincides with the positive direction of the outline circuit, and vice versa. When there is no tendency in the particles to shift along the outline, the circulation equals zero.

Let us prove that a circulation of velocity along an arbitrary closed curve S , which is delimiting an arbitrary surface σ , equals the algebraic sum of circulations along all curves, which are delimiting parts of this surface. Let us visualize curve S delimiting an arbirary surface σ (Figure 146).

Figure 145. Circulation of velocity along an outline.

Figure 146. Circulation of a velocity along a closed outline is equal to the sum of elementary circulations.

In dividing surface σ into parts: $\Delta\sigma_1, \Delta\sigma_2, \ldots, \Delta\sigma_n$,

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we compose the sum of circulations of velocity along the outlines delimiting the surfaces $\Delta\Gamma_1 \ldots \Delta\Gamma_n$. It is obvious that, in summation, integrals $\int v_s\, ds$ taken along the inner segments of the outline, will mutually cancel themselves out since each such integral will occur twice and with opposite signs, in conformity with the direction of the circuit of the inner parts of the surface the direction of which is indicated by arrows.

Circulation of velocity is tied in with the vortex of velocity by the correlationship

$$\oint_s (v, dr) = \int_\sigma (\mathrm{rot}\, v, n)\, d\sigma, \qquad (3)$$

which expresses the Stokes theorem and shows that a circulation of velocity along an arbitrary closed curve s is equal to the current of the vortex of velocity through any surface, which is delimited by this curve. From (3) it follows that the circulation of velocity characterizes specifically the vortex motion of a liquid, since for an irrotational motion, the circulation of velocity equals zero. We analyzed the circulation of velocity along a motionless geometrical outline drawn in a moving liquid. We will not analyze a liquid outline, which is at all times formed by the same liquid particles and which can shift and undergo deformation without breaking so that the connection between particles remains undisturbed. A necessary and adequate condition for the existence of such a liquid outline is the continuity of functions.

$$x = x(a,b,c,t);\quad y = y(a,b,c,t);\quad z = z(a,b,c,t),$$

which express the laws of motion of a liquid particle in Lagrangian

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variations. The circulation of velocity along a liquid outline s will, of course, be expressed by the same formula as the circulation of velocity along a geometrical outline, the difference being only in the fact that s is now assumed to be a closed line, which at all moments of time consists of the same liquid particles. It is obvious that, with the course of time , the circulation along the liquid outline, generally speaking, varies. Let us consider the variation of circulation in a unit of time. To accomplish this, we differentiate both parts of (2) for time:

$$\frac{d\Gamma}{dt} = \frac{d}{dt}\oint(v, dr) = \oint\left(\frac{dv}{dt}, dr\right) + \oint\left(v, \frac{d}{dt}(dr)\right).$$

Since what is involved is the variation of circulation along a liquid closed curve, consisting at all times of the same particles, it is necessary to take an individual derivative. The first integral of the right part represents the circulation of acceleration along the same liquid outline. The second integral equals zero. Indeed:

$$\frac{d}{dt}(dr) = d\left(\frac{dr}{dt}\right) = dv;$$

therefore:

$$\oint\left(v, \frac{d}{dt}(dr)\right) = \oint(v, dr) = \oint d\left(\frac{v^2}{2}\right) = 0.$$

We arrived at the Thomson theorem, to the effect that the variation in a unit of time of the circulation of velocity along a closed liquid outline, equals the circulation of acceleration along the same outline:

$$\frac{d\Gamma}{dt} = \oint\left(\frac{dv}{dt}, dr\right) = \oint\left(\frac{dv}{dt}dx + \frac{dv}{dt}dy + \frac{dv}{dt}dz\right), \tag{14}$$

$$\left[\frac{d\Gamma}{dt}\right] = \left[L^2 T^{-2}\right].$$

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Substituting into equation (4), in place of the acceleration components, their values from the equation of motion of an ideal atmosphere:

$$\left.\begin{aligned} \frac{du}{dt} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\omega \sin\varphi v, \\ \frac{dv}{dt} &= -\frac{1}{\rho}\frac{\partial p}{\partial y} - 2\omega \sin\varphi u, \\ \frac{dw}{dt} &= -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + 2\omega \cos\varphi u. \end{aligned}\right\} \qquad (5)$$

Then we derive:

$$\frac{d\Gamma}{dt} = -\oint \frac{1}{\rho}\left(\frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz\right) - \oint g\,dz -$$

$$-2\omega \oint \left[\sin\varphi(u\,dy - v\,dx) + \cos\varphi u\right] dz.$$

The first integral, entering into equation (5), can obviously be written down as : $\oint \frac{dp}{\rho}$, where the variation in pressure is taken along the contour s.

The second integral becomes zero, since the curvilinear integral along the closed outline from the full differential equals zero.

Let us further assume that the outline, along which the integrals are computed, is not very large so that the variation of latitude φ within the limits of this outline can be disregarded. Then $\sin\varphi$ can be carried out beyond the sign of the integral. Let us ascertain the geometrical significance of the integral $\oint(u\,dy - v\,dx + \cot\varphi\, u\,dz)$. Let us project outline s to a horizontal plane. We designate the area, delimited by the horizontal projection of the outline, by F. It is

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Substituting into equation (4), in place of the acceleration components, their values from the equation of motion of an ideal atmosphere:

$$\left.\begin{aligned} \frac{du}{dt} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\omega \sin\varphi v, \\ \frac{dv}{dt} &= -\frac{1}{\rho}\frac{\partial p}{\partial y} - 2\omega \sin\varphi u, \\ \frac{dw}{dt} &= -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + 2\omega \cos\varphi u. \end{aligned}\right\} \qquad (5)$$

Then we derive:

$$\frac{d\Gamma}{dt} = -\oint \frac{1}{\rho}\left(\frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz\right) - \oint g\,dz - 2\omega \oint \left[\sin\varphi(u\,dy - v\,dx) + \cos\varphi u\right]dz.$$

The first integral, entering into equation (5), can obviously be written down as : $\oint \frac{dp}{\rho}$, where the variation in pressure is taken along the contour s.

The second integral becomes zero, since the curvilinear integral along the closed outline from the full differential equals zero.

Let us further assume that the outline, along which the integrals are computed, is not very large so that the variation of latitude φ within the limits of this outline can be disregarded. Then $\sin\varphi$ can be carried out beyond the sign of the integral. Let us ascertain the geometrical significance of the integral $\oint(u\,dy - v\,dx + \cot\varphi\, u\,dz)$. Let us project outline s to a horizontal plane. We designate the area, delimited by the horizontal projection of the outline, by F. It is

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obvious that the variation of area F with time is tied in with the motion of the elements of curve s. Thus, with the shifting of the element ds of curve s in the direction x, area F, in a unit of time, changes by magnitude udy, and with the shifting of this element in the direction y, the area changes by magnitude (-vdx). Consequently,

$$\oint (u dy - v dx + \cot\varphi\, u dz) = \frac{dF}{dt},$$

and the variation of circulation in a unit of time, will be expressed by:

$$\frac{d\Gamma}{dt} = -\oint \frac{dp}{\rho} - 2\omega \sin\varphi \frac{dF}{dt}. \qquad (6)$$

Formula (6) expresses the basic theorem of circulation, as advanced by V. B'yerkness.

Formula (6) shows that, if the liquid is barotropic, the variation in the circulation is stipulated only by the rotation of the earth. In a stationary system of coordinates, the circulation of velocity in a barotropic liquid does not change with time. In such a limited form, the theorem of circulation for a non-compressible liquid was first proved by Helmholz.

In a baroclinical atmosphere, the change in circulation, and consequently, the change in the intensity of the vortex, tied in with the former, is effected at the expense of the baroclinicity of the atmosphere.

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The theorem of circulation is expressed by prognostic equation, since it becomes possible to compute the future change in circulation with time by the distribution of density and pressure prevailing at the given contemplated moment. As soon as the motion of the liquid outline begins, the circulation begins to change further also under the effect of the earth's rotation, which is indicated by the second term of the right-hand part of equation (6).

The change in circulation, assuming the earth is stationary, under the effect of baroclinicity of the atmosphere, can be represented in a different form with the aid of elementary transpositions.

Since $\frac{1}{\rho} = \alpha$, then

$$\frac{d\Gamma}{dt} = -\oint \alpha dp. \tag{7}$$

Remembering that $\alpha p = RT$ we derive

$$\frac{d\Gamma}{dt} = \oint p d\alpha, \tag{8}$$

since $\oint dT = 0$.

Eliminating the specific volume with the aid of the equation of state, we find:

$$\frac{d\Gamma}{dt} = -R\oint T d\ln p. \tag{9}$$

Interpolating the potential temperature, in accordance with (8) of section 16, Chapter II, we derive:

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$$\frac{d\Gamma}{dt} = -c_p \oint T d\ln\theta. \qquad (10)$$

Interpolating, finally, the entropy, in accordance with equality $d\varphi = c_p d\ln\theta$, we derive:

$$\frac{d\Gamma}{dt} = -\oint T d\varphi. \qquad (11)$$

The computations of the magnitude of $\frac{d\Gamma}{dt}$ become simplified if the liquid outline, by which the integration is conducted, is selected in a special manner. Let us construct a system of isobaric surfaces for the values of p, differing from each other by one, and a system of isosteric surfaces for the values of α, also differing from each other by one. Then the entire space will be divided into unitary isobaro-isosteric solenoids. Let us consider a quadrangular liquid outline, formed by two isobars and two isosteres (Figure 147)

Figure 147. Change in circulation of velocity along a liquid outline is expressed by the number of isobaro-isosteric solenoids encompassed by this outline.

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The number of unitary solenoids, encompassed by this outline equals mn. Let us prove that in this case

$$\frac{d\Gamma}{dt} = m \cdot n. \qquad (12)$$

Indeed:

$$\frac{d\Gamma}{dt} = \oint_{12341} p\,d\alpha = \int_{3}^{4} p\,d\alpha + \int_{1}^{2} p\,d\alpha =$$

$$= (p-m)[\alpha-(\alpha+n)] + p(\alpha+n-\alpha) = m \cdot n.$$

Correlationship (12) remains true for an arbitrary liquid outline so that the change of circulation along an arbitrary liquid outline equals the number of unitary isobaro-isosteric solenoids encompassed by this outline. The change of circulation occurs in the direction from the pressure gradient to the specific volume gradient. If the liquid outline is sufficiently large, it may occur that it will encompass the positive as well as the negative unitary solenoids, around which the directions of circulation are opposite to each other. In this case, the change of the circulation of velocity along a liquid outline equals the difference between the number of the unitary positive and the number of the unitary negative isobaro-isosteric solenoids.

By the same argumentation it can be shown that the change of circulation of velocity along a liquid outline is equal to the difference between the number of the unitary positive and the unitary negative isothermo-isentropic solenoids. The change of circulation occurs in the direction from the entropy gradient to the temperature gradient.

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It is convenient to use another outline for the computation of the change of circulation, the outline consisting of two vertical lines and two isobars (Figure 148).

Figure 148. The change of circulation of velocity along a liquid outline is expressed by the difference of the relative geopotentials of the isobaric surfaces.

In this case:

$$\frac{d\Gamma}{dt} = -\int_{12341} \frac{dp}{\rho} = -\int_{2}^{3} \frac{dp}{\rho} - \int_{4}^{1} \frac{dp}{\rho}. \qquad (13)$$

It may be assumed that, along the verticals, the basic equation of statics $dp = -g\rho\, dz$ is fulfilled; then:

$$\frac{d\Gamma}{dt} = g(h_2 - h_1) \qquad (14)$$

or

$$\frac{d\Gamma}{dt} = \Phi_2 - \Phi_1, \qquad (15)$$

where Φ is the relative geopotential of the isobaric surface p over surface p_0.

Let us compute the change of the circulation of velocity along the same outline, consisting of two isobars and two vertical lines, with

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the aid of formula (9):

$$\frac{d\Gamma}{dt} = -R\oint_{12341} T\,d\ln p = -R\int_{2}^{3} T\,d\ln p - R\int_{4}^{1} T\,d\ln p.$$

Designating the mean temperatures along the vertical by T_2 and T_1, respectively we derive:

$$\frac{d\Gamma}{dt} = R\ln\frac{p_0}{p}(T_2 - T_1). \qquad (16)$$

The theorems in circulation can be used for the explanation of such local circulations as breezes, mountain-valley winds, and others. In these cases, the compensating currents at the upper levels are less sharply pronounced than the ones below. Nevertheless, due to considerations of continuity, such circulation may be considered as a closed one.

In breezes, the air over land is heated more intensively by day, and cooled more intensively by night, while the surface of the water, due to intermixing with the deep layers, practically retains a constant temperature. Therefore, the slope of the isosteric surfaces changes from day to night, resulting in a change of direction of this closed circulation (Figure 149).

Figure 149. (a) Night breeze, (b) day breeze.

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By the same token, the air temperature at a mountain slope rises more intensively during the day, and diminishes more intensively at night. Therefore, a wind blowing up the slope develops during the day while at night there is a downslope wind.

Section 5. Application of the Theorems of Circulation to the General Circulation of the Atmosphere.

It was already mentioned above that, with our knowledge still limited, it is impossible even to construct rigid equations which would describe the general circulation of the atmosphere. We are, therefore, forced to limit ourselves to a priori coarse diagrams, which are very far from the actual atmospheric circulation, in the hope that these coarse diagrams will help explain at least partially the most important features of atmospheric circulation.

Let us first assume that the earth is stationary, the underlying surface - homogeneous, and the air is heated at the equator and cooled at the poles. In this case, the general circulation of the atmosphere consists in that in each hemisphere there is present one meridional "wheel" of circulation: the heated air rises at the equator and, at the upper layers, flows off toward the poles, where it descends. Its place is taken by the cold air flowing toward the equator from the poles.

If the atmosphere were an ideal liquid, such a circulation would keep on developing further. If we take an outline (Figure 150), consisting of 2 isobars p_0 and p and two verticals located, respectively, at the pole and at the equator, the change of circulation along this outline can be computed by using the formula

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$$\frac{d\Gamma}{dt} = R\left(T_{eq} - T_{po}\right) \ln \frac{p_o}{p}$$

where T_{eq} and T_{po} are, respectively, the mean values for the temperatures of the columns of air at the equator and pole. Assuming that $T_{eq} - T_{po} = 40^{\circ}$, p_o = 1000 millibars, and p = 300 millibars (which corresponds to the level of the cirrus clouds ~ 9000 meters), we derive

$$\frac{d\Gamma}{dt} = 138 \times 10^{6} \ cm^2/sec^2.$$

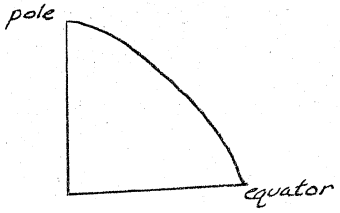


Figure 150. Liquid outline for the computation of the change of circulation of velocity, assuming a stationary earth, in the atmosphere, assuming the absence of friction.

Since the change of circulation along a liquid outline equals the circulation of the acceleration along the same outline, then, if we accept the length of the outline as equal to half of the earth's circumference, i.e., 2 x 10^9 centimeters for the mean meridional acceleration, we derive:

$$\frac{dv}{dt} = 6.9 \times 10^{-2} \ cm/sec^2.$$

Considering the mean acceleration as continuous, it is easy to compute that already after 6 hours, circulation attains considerable velocity, and after 24 hours, it attains hurricane velocity. (See Table 52)

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TABLE 52

Theoretical velocity of the general circulation without friction.

Time	1 second	3 hours	6 hours	24 hours
Mean meridional velocity in meters per second	6.9×10^{-4}	7.4	14.9	59.6
Mean meridional shift in kilometers	0.345×10^{-6}	40.4	160.5	2580
Mean zonal velocity at latitude 30 degrees in meters per second	2.52×10^{-8}	2.94	11.7	188
Opposite acceleration at latitude 30 degrees in centimeters per second^2	1.84×10^{-10}	2.14×10^{-2}	8.51×10^{-2}	1.37

Computing the path, which an air particle describes during this process, we satisfy ourselves to the effect that already after 67 hours this air particle will return to its original positon, having completed a closed path, which is coincident with the curve of integration.

However, an essential correction must be interpolated into these computations. With the development of such considerable velocities, the difference in the air temperature at the pole and at the equator would

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tend to decrease rapidly; therefore, the change of circulation would be much smaller. Incidentally, were there no other factors, the circulation would continue to become intensified; but since there are no ideal liquids in nature, the viscosity of the liquid must always be taken into consideration in such evaluations. In the presence of viscosity the entire amount of radiation energy will not by far change into kinetic energy. Part of the kinetic energy will, due to viscosity, be dissipated, being converted into internal and potential energies. Thus, assuming a stationary earth, a steady closed circulation would finally establish itself.

But, with a rotating earth, such a circulation is impossible. Due to the effect of the Coriolis force, the air currents will not move in the direction of the pressure gradient, and with uniform motion, they will be perpendicular to the latter. In so doing, all the currents will tend to assume a latitudinal direction, departing to the right of the meridian in the northern hemisphere, and to the left of the meridian in the southern hemisphere. Thus, under the effect of the earth's rotation, the air currents over a supposedly homogeneous underlying surface, tend to become purely zonal.

Section 6. The Transfer of Heat from Equator to the Pole

Helmholz, Jeffries, and other scientists, proceeding from different considerations, proved that a purely zonal circulation in the atmosphere is impossible even though the surface of the earth were perfectly homogeneous, that is, if there were no continents and oceans.

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Therefore, the result of the general circulation must be the transfer of heat in the atmosphere from the equator to the pole. In the opposite case, the temperature differential, as between the pole and the equator, would tend to increase infinitely, while in reality, this differential varies very little even from season to season.

The transfer of heat in a meridional direction can be effected by two processes: the advection of air masses having various temperatures, and the macroturbulent exchange of air masses.

If two mutually opposite air currents have a meridional component of velocity and a different mean temperature a heat current stipulated by advection, the value of which can be easily computed, will be directed along the meridian.

Heat can be transferred from the equator to the pole also by the so-called macroturbulent exchange of air masses.

In the latter case, the general circulation of the atmosphere may be looked upon as a turbulent exchange on a grand scale. From this viewpoint, the elements of turbulence, included in the basic current directed along the latitude, are the individual cyclones and anticyclones moving in various directions.

The indicated analogy between the general circulation and the turbulent exchange of air masses is, of course, a very crude one. The basic postulate in the theory of turbulence is the requirement that the number of the elements of turbulence be adequately large, while in this case we are dealing with individual vortices, the number of which

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is a priori known to be small. Nevertheless, the analogy turns out to be useful.

Defant studied the transfer of heat for extratropical latitudes, where the meridional components of velocity are small, and where therefore, the flow of heat is predominantly tied in with turbulent exchange.

The formula determining the meridional heat current Q, stipulated by macroturbulence, can be constructed upon the analogy with formula (3) from Section 4, Chapter XV, for the computation of the vertical heat current, which was induced by turbulence, namely

$$Q = -c_p \cdot A \cdot \frac{d\theta}{R d\varphi}, \qquad (1)$$

where c_p is the specific heat capacity at constant pressure, A is the coefficient of macroturbulent exchange, θ is the temperature, R is the earth's radius, and φ is the latitude.

The order of magnitude for A, as evaluated by Defant, equals 4×10^{17} grams per centimeter second. This magnitude is $10^5 - 10^6$ times greater than the coefficient of vertical turbulent exchange, and 10^2 times greater than the coefficient of lateral intermixing, or in other words, the coefficient of horizontal exchange. Magnitude A can be determined by formula

$$A = \frac{\Sigma m l}{F t} \qquad (2)$$

which was derived for the vertical turbulent exchange. In this

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formula, m is the mass of the individual vortex (cyclone or anticyclone), F is the area of the ring encompassing the earth at the latitudinal circle, l is the path of the vortex up to where it crosses the above ring and t is the time of averaging.

The value of A may be obtained, without reference to formula (2), indirectly with the aid of formula (1), if the heat flow and the horizontal temperature gradient in the direction of the meridian are known.

Ongstrem made a theoretical study of the transfer of heat at all latitudes, including the tropical, and his results coincide well with direct temperature determinations taken at various latitudes. But, as per preceding text, the trade winds and the antitrades predominate in the tropical latitudes. There are no anticyclones in these latitudes, and tropical cyclones are rare. Therefore, the flow of heat in the tropical latitudes is tied in with advection, while in the extra-tropical latitudes this flow is tied in with turbulence.

V. V. Shuleykin advanced explanations for the fact that Ongstrem, without differentiating tropical and extratropical latitudes, arrived at a theoretical distribution of temperature along the entire meridian that was quite close to observational values. Both/ turbulent exchange and advection induce a meridional flow of heat, which is in direct ratio to the temperature gradient component in this direction. Consequently, the first as well as the second process may be formally expressed with the aid of identical equations, in which the meridional component of temperature gradient is multiplied by some conditional coefficient of

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heat conductivity, which has the same order of magnitude, both in the case of heat advection and in the case of turbulent exchange, even though the origin and physical meaning of the two phenomena are different.

Section 7. The Transfer of Heat from the Ocean to the Continent.

The atmosphere may be considered as a thermodynamic machine, operating between a heater and a refrigerator. The source of energy of the general circulation is the sun. In the tropical latitudes, the solar radiation current, entering the atmosphere, is considerably in excess of the outflow energy radiation by the atmosphere and the terrestrial surface. Therefore the tropical latitudes can be considered the heater for this thermodynamic machine (the atmosphere). On the other hand, in both polar regions, the radiation of energy into interplanetary space by the atmosphere and the terrestrial surface is greater than its inflow in the form of solar radiation. Therefore, the polar regions are the refrigerators for this thermodynamic machine (the atmosphere).

The tropical heater and both polar refrigerators operate throughout the year without change of sign. But, alongside of these, there are in the atmosphere heaters and refrigerators, operating only several months a year and changing signs in transition from winter to summer and vice versa. The continents in the summer function as heaters, and in the winter as refrigerators. The vast areas of the oceans with warm currents function as powerful heaters during the winter. During the summer, in most cases, the surface of the ocean acts as a refrigerator.

The important part played by these seasonal heaters and refrigerators in the general circulation of the atmosphere, is underscored

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by academician V. V. Shuleykin.

The constantly operating heaters and refrigerators stipulate the interzonal and zonal circulations, while the seasonal heaters and refrigerators generate the monsoon circulation.

In 1935, V. V. Shuleykin computed for the first time the flow of heat from ocean to continent, an operation not even seriously attempted heretofore. Defant, Ongstrem and other scientists were making averaging computations for the air temperatures along the entire parallel. In doing so, they of course excluded completely the effect of the thermal currents from ocean to continent. However, the effect of such currents is tremendous.

V. V. Shuleykin emphasizes that these thermal currents may play a greater part than the meridional thermal currents in the formation of the climate of many areas. The thermal current from the Atlantic Ocean in the winter induces the generation of tremendous latitudinal temperature gradients on the continent of Eurasia. Let us recall that the mean January temperature in Murmansk is minus 10 degrees Centigrade, and Verkhoyansk (lying even somewhat to the south of Murmansk), this mean temperature is minus 49 degrees Centigrade, thus making the mean temperature differential of two points on the same parallel equal to 39 degrees Centigrade. It is characteristic that the mean January temperature in Yalta is plus 4 degrees Centigrade, i. e., only 14 degrees above the mean January temperature in Murmansk, even though Murmansk lies 25 degrees to the north of Yalta.

Without going into details of the method employed by V. V. Shuleykin for the computation of the thermal currents from ocean to

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continent, we shall point out briefly the basic stages of these computations.

V. V. Shuleykin proceeds from the analysis of isothermal charts. The mean horizontal temperature gradient determines the summary thermal current, consisting of the meridional thermal current and the thermal current stipulated by the effect of the ocean. In order to compute the thermal current/ induced by the effect of the ocean, it is sufficient to deduct the vector-meridional current from the summary current. Turning to the field of temperatures, it is consequently/ necessary to deduct from the factual field of mean temperatures/ the temperature field which would have existed, assuming the absence of continents and oceans over a supposedly homogeneous terrestrial surface.

In thus finding the differential of the temperature fields, V. V. Shuleykin derived the isanomalic charts, that is the isolines of departures of temperatures from a temperature field/ which would exist under conditions of a supposedly homogeneous underlying surface. These isanomalic charts specifically illustrate the effect of the heat exchange between ocean and continent upon the distribution of temperature.

The January isanomal for Murmansk is plus 11 degrees Centigrade, for Verkhoyansk it is isanomal minus 25 degrees, for Yalta it is isanomal zero degrees. The July isanomal for Verkhoyansk is plus 7 degrees.

Shuleykin then proceeds to the determination of the conditional coefficient of heat conductivity, accepting it as the value of the

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product AH, where H is the height of the active layer, i.e., the atmospheric layer, which participates in the heat exchange between ocean and continent, and A is a constant numerical coefficient, which has the dimensionality of the coefficient of heat conductivity. Shuleykin finds that

$$AH = 1.28 \times 10^{20} \frac{\text{calories}}{\text{year degrees}}$$

The conditional coefficient of heat conductivity shows how many calories pass over the period of a year through a strip having a width of one centimeter and a height H, if the temperature gradient is one degree per centimeter.

Proceeding from isanomalic charts and from the values of the conditional coefficient of heat conductivity, V. V. Shuleykin calculated that through each running centimeter of the coastline, on the average, during the year, penetrate to the territory of the USSR 4.1 x 10^{12} calories. Since this amount of heat is lost by the sea, it becomes possible to evaluate the amount of heat entering the atmosphere through one square centimeter of sea surface. This amount of heat turns out to be equal to 41000 calories a year per square centimeter.

In still another thesis, Shuleykin, analyzing the thermal cycle of the Kara Sea, determined that this sea gives off, in the process of heat exchange with the atmosphere and by radiation, 48,000 calories per year per square centimeter. If radiation loss is excluded from this number, somewhat less than 41,000 calories will remain. Nevertheless, the order of magnitude of this value, determined by V. V. Shuleykin by various methods, is the same.

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The analysis of thermal currents from ocean to continent by Shuleykin provides the explanation for many important meteorological phenomena. [redacted] bring out the causes for the generation of the so-called "cold pole" in the area of Verkhoyansk. Further, proceeding from the analysis of perennial means to the analysis of individual years, Shuleykin and his pupils succeeded in determining the close connection between the abnormally warm winters of Eurasia and the abnormally high temperatures of the Gulfstream. By the same method, it was proved that none of the other continents are subject to the effect of the oceans to as high a degree as Eurasia.

The analysis by V. V. Shuleykin also brought out the tremendous part played by the monsoon currents, which are generated in the ocean-continent system. They pointed out with particular emphasis that this important mechanism of the general circulation is so essential that none of the theories of the general circulation can be considered complete, without the inclusion of the monsoon currents generated in the system ocean-continent, as its most important link.

Section 8. The Mathematical Theory of General Circulation, by N. E. Kochin.

The problem of formulating a theory of the general circulation of the atmosphere is one of the basic problems of dynamic meteorology. The several attempts at it are as yet far from accomplishment. The maximum success in this direction was attained by N. E. Kochin and E. N. Blinova, and the basic content of their theories will be given in this and the following sections.

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We repeatedly mentioned the difficulties encountered in the solution of the basic practical problem of dynamic meteorology, which problem is the pre-computation of the weather. The construction of a complete and rigid theory of the general circulation of the atmosphere encounters the same difficulties.

Kochin simplifies the problem considerably by leaving out of consideration all problems tied in with the inflow of heat, accepting the temperature distribution in the contemplated part of the atmosphere as fixed, without bothering to ascertain what brought about such a distribution. By the given temperature distribution, the distribution of pressure is sought, and by the latter the motion of the atmosphere is determined.

On the other hand, Kochin made a detailed analysis of the effect of turbulent viscosity upon the general circulation of the atmosphere, by carrying into dynamic meteorology the methods of the theory of the boundary layer.

In Section 9 of Chapter IX, we analyzed the simplifications, which can be interpolated, according to Kochin, into the equations of atmospheric motion, proceeding from the theory of the boundary layer.

Let us first turn to the case of steady zonal circulation of the atmosphere. The equations of the atmospheric motion for this case, in accordance with Kochin, can be written down in spherical coordinates $r, \theta = \frac{\pi}{2} - \varphi, \psi$, in the following manner;

$$\mu \frac{\partial^2 v_\theta}{\partial r^2} + 2\omega\rho \cos\theta v_\psi = \frac{1}{r}\frac{\partial p}{\partial \theta}, \qquad (1)$$

$$\mu \frac{\partial^2 v_\psi}{\partial r^2} - 2\omega\rho \cos\theta \cdot v_\theta = 0, \qquad (2)$$

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$$\frac{\partial p}{\partial r} + g\rho = 0, \tag{3}$$

$$v_r = -\frac{1}{\rho r^2 \sin\theta} \int_a^r \frac{\partial(\rho v_\theta r \sin\theta)}{\partial\theta} dr, \tag{4}$$

where v_θ, v_ψ, v_r are components of wind velocity, a is the radius of the earth, μ is the coefficient of virtual viscosity.

By the use of these equations, all the elements of the steady zonal circulation can be determined, if the following are given:

(1) The temperature distribution in the entire atmosphere $T(r, \theta)$;

(2) The pressure distribution on the earth's surface

$$p_0(\theta) = p(a, \theta)$$

(3) Viscosity distribution in the entire atmosphere.

First of all, the pressure $p(r, \theta)$ at any point in the atmosphere can be determined by the barometric formula

$$p(r, \theta) = p_0(\theta) \cdot e^{-\int \frac{g \cdot dz}{RT(r, \theta)}} \tag{5}$$

When pressure p and temperature T are known, it is possible to determine density by the equation of state:

$$\rho(r, \theta) = \frac{p(r, \theta)}{RT(r, \theta)} \tag{6}$$

Assuming there were no viscosity, then to the just determined pressure distribution would correspond a gradient wind directed along the parallels, the velocity of which would be determined by formula

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$$v_\psi = \frac{1}{2\omega\rho\cos\theta} \cdot \frac{1}{a} \frac{\partial p}{\partial \theta}. \qquad (7)$$

However, in the presence of viscosity, in addition to the zonal velocity component v_ψ, there appears a meridional component v_θ. Both velocity components can be obtained for the entire thickness of the atmosphere by way of integrating the system of equations (1) and (2). These equations can be substituted by one, by interpolating a complex velocity;

$$v_\theta + i v_\psi = u.$$

Multiplying equation (2) by i and adding the result to equation (1), we obtain:

$$\mu \frac{\partial^2 u}{\partial r^2} - 2\omega\rho\cos\theta \, iu = \frac{1}{r} \frac{\partial p}{\partial \theta}. \qquad (8)$$

In integrating equation (8), two arbitrary complex constants are obtained. They are determined from the boundary conditions

$$v_\theta = v_\psi = 0 \quad \text{when } r = a \qquad (9)$$

$$v_\theta, v_\psi \text{ finite when } r \longrightarrow \infty. \qquad (10)$$

The vertical velocity component v_r can be determined with an adequate degree of preceision from equation (4).

What was said above with relation to zonal circulation may be extended to the more general case of an arbitrary circulation of the atmosphere. If the temperature distribution in the entire atmosphere $T(r, \theta, \psi)$, and the pressure distribution at the earth's surface $p(a, \theta, \psi)$ are given, and if the coefficient of turbulence μ

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has a constant value, the distribution of pressure and velocity in the entire atmosphere may be determined.

The problem, as in the case of steady zonal circulation, divides into 3 stages.

(1) The pressure distribution is determined by the barometric formula

$$p(r,\theta,t)=p(a,\theta,t)e^{-\frac{g}{R}\int_a^r \frac{dz}{T(r,\theta,\psi)}} \tag{11}$$

(2) By the two equations

$$\mu\frac{\partial^2 v_\theta}{\partial r^2}+2\omega\rho\cos\theta\cdot v_\psi=\frac{1}{r}\frac{\partial p}{\partial\theta}, \tag{12}$$

$$\mu\frac{\partial^2 v_\psi}{\partial r^2}-2\omega\rho\cos\theta\cdot v_\theta=\frac{1}{r}\frac{\partial p}{\partial\psi}\cdot\frac{1}{\sin\theta} \tag{13}$$

the horizontal wind field is determined.

(3) Following this, the vertical velocity component is determined from the equation of continuity

$$v_r=-\frac{1}{\rho r^2\sin\theta}\int_a^r\left[\frac{\partial}{\partial\theta}(\rho v_\theta r\sin\theta)+\frac{\partial\rho v_\psi^2}{\partial\psi}\right]dr. \tag{14}$$

Finally, in the very general case of non-zonal and non-steady circulation, with a variable coefficient of turbulence μ, Kochin reduces the problem to the determination of:

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(1) pressure by the barometric formula

$$p(r,\theta,\psi,t)=p(a,\theta,\psi,t)\cdot e^{-\frac{g}{R}\int_a^r \frac{dz}{T(r,\theta,\psi,t)}} \qquad (15)$$

velocity $u = v_\theta + i v_\psi$ from equation

$$\rho\frac{\partial u}{\partial t} = -2i\omega\rho\cos\theta u + \frac{\partial}{\partial r}\mu\frac{\partial u}{\partial r} - \frac{i}{r\sin\theta}\frac{\partial p}{\partial\psi} - \frac{1}{z}\frac{\partial p}{\partial\theta} \qquad (16)$$

and (2) vertical velocity v_r from equality

$$v_r = -\frac{1}{\rho r^2\sin\theta}\int_a^r\left[\frac{\partial\rho z v_\theta\sin\theta}{\partial\theta} + \frac{\partial\rho r v_\varphi}{\partial\psi} + \frac{\partial\rho r^2\sin\theta}{\partial t}\right]dz. \qquad (17)$$

If the pressure gradient were not to change with height, i.e., if the atmosphere were considered as a non-compressible liquid, the velocity field would then be described by the Okkerblom equations. Within the limits of the boundary layer, the complex velocity would satisfy these equations, and beyond it, the wind would be a gradient one. But since the atmosphere has the characteristics of a baroclinical liquid, the pressure gradient will change with height, and consequently, virtual viscosity will play a part in the atmosphere, wherever the pressure gradient changes with height.

It may, therefore, occur that the thickness δ of the boundary layer will turn out to be many times greater than $\delta=\sqrt{\frac{\mu}{\omega\rho}}$ (See Section 9, Chapter IX), since the effect of turbulent viscosity may extend several kilometers, i.e., actually through the entire troposphere. This remarkable result derived by Kochin is contradictory to the then prevailing views, such as the thesis by Jeffries, in which the effect of viscosity

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pertains only to a layer 300 meters in depth. On the basis of this, Jeffries made a series of erroneous deductions, such as the insistence that no steady zonal circulation is possible in the terrestrial atmosphere.

In 1936, Kochin constructed a model of steady zonal circulation, proceeding from the constant value of $\mu = 2$ m/m. sec. Analogous computations were performed by Gel'bke for a variable value of μ. Dorodnitsyn has shown that a given distribution of pressure at the terrestrial surface is not a condition sine qua non for the Kochin method of computation, and that it can be substituted by another condition. He assumed that the atmosphere, up to some adequately great height H, is involved in the contemplated motion, and in addition, he took into account the general lag of the atmosphere with relation to the rotation of the earth.

In 1938, Dorodnitsyn, Izvekov, and Shvets repeated the computations for the construction of the Kochin model of zonal circulation, proceeding from a more perfect system of equations and basing themselves on new data for temperature and pressure distribution. They succeeded in proving that, in the case of zonal circulation, there are present in the atmosphere 3 basic "rings" of a distinctly pronounced circulation in a vertical plane. The tropical and the polar rings are of thermal origin. They "operate" as simple thermodynamic machines. The heaters are below at high pressure, the refrigerators are above at low pressure. The middle ring of the temperate zones is of dynamic origin. At the belt of high pressure (in the horse latitudes), the meridional currents diverge, and the loss of mass is compensated by descending currents. In the low pressure belt (in the subarctic) the convergin currents are compensated by ascending currents.

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Section 9. The Hydrodynamic Theory of Pressure Waves and Atmosphere Centers of Action, by E. N. Blinova

E. N. Blinova analyzed the problem of long term forecasting of pressure, temperature, and wind, proceeding from the equations of hydromechanics.

The results which she obtained constitute a valuable contribution in the construction of a mathematical theory of the general circulation of the atmosphere.

The earth is considered as a globe having a radius a. The gravitational force and the Coriolis force act upon the atmosphere. The force of friction is assumed not to exist. As the initial equation, the Fridman equation (16) from Section 1, Chapter IX is utilized. It determines the change in the vortex vector of a given particle of liquid. In order to derive this equation, we take Euler's equation in its vectorial form

$$\frac{dv}{dt} = -\frac{1}{\rho}\,\mathrm{grad}\,p + \mathrm{grad}\,\Phi + 2[\omega, v]. \qquad (1)$$

Let us remove the vortex from both parts of this equation. Assuming $\Omega = \mathrm{rot}\ v$, we derive the following equation, which is the Fridman equation:

$$\frac{d\Omega}{dt} - (\Omega, \mathrm{grad})v + \Omega\,\mathrm{div}\,v = -2\,\mathrm{rot}[\omega, v] - \frac{1}{\rho T}[\mathrm{grad}\,T\ \mathrm{grad}\,p] \qquad (2)$$

Converting to coordinate equations, Blinova interpolates the following spherical coordinates: r - the distance from the center of the earth, θ - the co-latitude ($\theta = 0$ at the north pole, and $\theta = \pi$ at the south pole), and λ - the longitude of location.

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The problem considers the motion of the atmosphere as consisting of the basic motion, which is the zonal circulation, and the perturbations of this motion; therefore, the component velocities along axes θ and λ, and pressure can be taken as

$$v_\theta = v_\theta(\theta,\lambda,t);\ v_\lambda = \bar{v}_\lambda(\theta) + v_\lambda(\theta,\lambda,t);\ P = \bar{p}(\theta) + p(\theta,\lambda,t). \quad (3)$$

Let us note that here, as well as in the further procedure, all values are averaged with relation to height. All the elements of perturbation v_θ, v_λ, p are considered small.

Assuming that

$$\bar{v}_\lambda = a \cdot a \cdot \sin\theta \quad (4)$$

where a is the angular velocity of the air in its motion relative to the earth, but not dependent on θ. The ratio a/ω is called the circulation index.

Considering the wind as a gradient wind, we obtain the following formula for pressure

$$\bar{p} = p_0 + \rho a^2 \alpha\omega \sin^2\theta. \quad (5)$$

The temperature formula is taken as follows

$$T = \bar{T}(\theta) + T'(\theta,\lambda) + T''(\theta,\lambda,t), \quad (6)$$

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where $\overline{T}+T'$ is the mean (monthly or seasonal) temperature distribution, stipulated by radiation, turbulence, and disposition of continents and oceans. This distribution is considered as fixed, with the zonal temperature given by the formula which is analogous with formula (5) for zonal pressure:

$$\overline{T}(\theta) = T_0 + M \sin^2\theta, \tag{7}$$

where T_0 and M are constant magnitudes, T'' is the departure from steady temperature distribution. Magnitudes T' and T'', which express the perturbations of the zonal temperature, are considered small.

Assuming that there are no vertical velocities, and disregarding the variations in density, Blinova interpolates the stream function ψ, proceeding from the equation of continuity

$$v_\theta = -\frac{\partial \psi}{a \sin\theta \partial\lambda}; \quad v_\lambda = -\frac{\partial \psi}{a \partial\theta} \tag{8}$$

It is obvious that the stream function ψ for a small perturbation may be presented as the sum of 2 stream functions

$$\psi(\theta,\lambda,t) = \psi'(\theta,\lambda) + \psi''(\theta,\lambda,t). \tag{9}$$

Here $\psi'(\theta,\lambda)$ is a stream function, which is not dependent on time, and it expresses the steady part of the perturbation of zonal circulation. This function is stipulated by the presence of the steady perturbation of the zonal temperature distribution, expressed by term $T'(\theta,\lambda)$ in equality (6), and $\psi''(\theta,\lambda)$ is the stream function expressing the non-steady perturbation of zonal circulation.

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By the same token, the pressure perturbation $p(\theta,\lambda,t)$ is taken as the sum of two components: $p'(\theta,\lambda)$, which is the steady perturbation of zonal pressure distribution, and $p''(\theta,\lambda,t)$, which is the non-steady perturbation:

$$p(\theta,\lambda,t)=p'(\theta,\lambda)+p''(\theta,\lambda,t) \tag{10}$$

By projecting the Fridman equation (2) onto axis r, by substituting the velocity components with the respective derivatives of the stream function, and by disregarding the small terms, Blinova derives the following basic equation of the problem

$$\left(\frac{\partial}{\partial t}+\alpha\frac{\partial}{\partial\lambda}\right)\left[\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}+\frac{1}{\sin\theta}\frac{\partial^2\psi}{\partial\lambda^2}\right)\right]+2(\alpha+\omega)\sin\theta\frac{\partial\psi}{\partial\lambda}=$$

$$=\frac{1}{\rho T}\left[\frac{\partial\bar{p}}{\partial\theta}\cdot\frac{\partial(T'+T'')}{\partial\lambda}-\frac{\partial\bar{T}}{\partial\theta}\cdot\frac{\partial p}{\partial\lambda}\right]. \tag{11}$$

She simplifies somewhat this equation by disregarding the terms containing T'' as compared with the terms containing T', and the terms $\frac{\partial\bar{T}}{\partial\theta}\cdot\frac{\partial p}{\partial\lambda}$ as compared with $\frac{\partial\bar{p}}{\partial\theta}\cdot\frac{\partial T'}{\partial\lambda}$, justifying this disregarding by subsequent computations. The, equation (11) assumes the form

$$\left(\frac{\partial}{\partial t}+\alpha\frac{\partial}{\partial\lambda}\right)\cdot\left[\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{1}{\sin\theta}\frac{\partial^2\psi}{\partial\lambda^2}\right]+2(\alpha+\omega)\sin\theta\frac{\partial\psi}{\partial\lambda}=$$

$$=a^2\alpha\omega\frac{\sin 2\theta}{T}\frac{\partial T'}{\partial\lambda}. \tag{12}$$

The equation of motion, taken for axis λ, has the form of

$$\frac{\partial p}{\rho\sin\theta\,\partial\lambda}=-\left(\frac{\partial}{\partial t}+\alpha\frac{\partial}{\partial\lambda}\right)\frac{\partial\psi}{\partial\theta}+2(\alpha+\omega)\cot\theta\frac{\partial\psi}{\partial\lambda} \tag{13}$$

Finally, Blinova takes the equation of heat inflow in the form of

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$$\frac{\partial T''}{\partial t} + a\frac{\partial T'}{\partial \lambda} = \frac{1}{a^2 \sin\theta} \cdot \frac{\partial \bar{T}}{\partial \theta} \cdot \frac{\partial \psi''}{\partial \lambda}. \qquad (14)$$

Thus, the system of equations (12), (13) and (14) contain 3 unkown functions : ψ, p, T' .

The steady part of the solution for ψ' of equation (12) is sought in the form of an infinite series by spherical functions:

$$\psi'(\theta,\lambda) = \sum_{n=1}^{\infty} \sum_{m=1}^{n} (C_n^m \cos m\lambda + C'^m_n \sin m\lambda) \cdot P_n^m(\cos\theta), \qquad (15)$$

with the coefficients C_n^m and C'^m_n determined by formulas

$$C_n^m = \frac{2a^2\omega}{\tilde{T}} \frac{\frac{n-m}{2n-1} \cdot \delta_{n-1}^m + \frac{m+n+1}{2n+3} \cdot \delta_{n+1}^m}{2\frac{\alpha+\omega}{\alpha} - n(n+1)},$$

$$C'^m_n = \frac{2a^2\omega}{\tilde{T}} \frac{\frac{n-m}{2n-1} \cdot \delta'^m_{n-1} + \frac{m+n+1}{2n+3} \cdot \delta'^m_{n+1}}{2\frac{\alpha+\omega}{\alpha} - n(n+1)} \qquad (16)$$

where δ_n^m and δ'^m_n are the coefficients of expansion of T' into a series by spherical functions, $\tilde{T}$ is the mean temperature value for the entire atmosphere.

The steady perturbation of the zonal temperature distribution is found from equation (13):

$$p'(\theta,\lambda) = 2\rho(a+\omega) \sum_{n=1}^{\infty} \sum_{m=1}^{n} \left\{ \left[C_{n-1}^m \frac{(n-m)(n-1)}{(2n-1)\cdot n} + \right.\right.$$

$$\left. + C_{n+1}^m \frac{(n+2)(n+m+1)}{(n+1)(2n+3)} \right] \cos m\lambda + \left[C'^m_{n-1} \frac{(n-m)(n-1)}{2n-1} + \qquad (17)$$

$$\left.\left. + C'^m_{n+1} \frac{(n+2)(n+m+1)}{(n+1)(2n+3)} \right] \sin m\lambda \right\} P_n^m(\cos\theta),$$

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The non-steady part of the solution of equation (12) is sought in the form of

$$\psi''(\theta,\lambda,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{n}\left[D_n^m\cos(m\lambda+\sigma t)+D'^m_n\sin(m\lambda+\sigma t)\right]P_n^m(\cos\theta), \quad (18)$$

where

$$\sigma=\frac{2(\alpha+\omega)m}{n(n+1)}-\alpha m, \quad (19)$$

and D_n^m, D'^m_n are the unknown coefficients to be determined. For the non-steady pressure perturbation $p''(\theta,\lambda,t)$ from equation (13) is derived an expression, analogous with (17), only $m\lambda$ is substituted by $(m\lambda+\sigma t)$, and the constants C_n^m and C'^m_n by D_n^m and D'^m_n. Such representation of the non-steady part of pressure perturbation is utilized by Blinova for the determination of the coefficients D_n^m and D'^m_n.

Thus, we derive:

$$A_n^m-2\rho(\alpha+\omega)\left[D_{n-1}^m\frac{(n-m)(n-1)}{(2n-1)\cdot n}+D_{n+1}^m\frac{(n+2)(n+m+1)}{(n+3)(2n+3)}\right]=0,$$

$$A'^m_n-2\rho(\alpha+\omega)\left[D'^m_{n-1}\frac{(n-m)(n-1)}{(2n-1)\cdot n}+D'^m_{n+1}\frac{(n+2)(n+m+1)}{(n+1)(2n+3)}\right]=0, \quad (20)$$

where A_n^m and A'^m_n are the coefficients of expansion, $\psi''(\theta,\lambda,t)$ is the series of spherical functions, with $t=0$.

Blinova seeks a solution, which is symmetrical with relation to the equator. To accomplish this, it is necessary, in expressions (17) and (20), to take all the n's by the same even count as the m's. Then, all coefficients D_n^m and D'^m_n will, in sequence, be determined

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with the aid of equations (20).

Expression (17) determines the location and intensity of the centers of action in the atmosphere. An analogous expression for p" makes it possible to compute the pressure distribution by its given initial distribution.

With the aid of education (14), the non-steady temperature perturbation is determined:

$$T''(\theta,\lambda,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{n}\left\{\left[B_n^m\cos m(\lambda-\alpha t)+B'^m_n\sin m(\lambda-\alpha t)\right]P_n^m-\right.$$

$$-2N_n\left[D_n^m\sin\left(m\lambda+\frac{\sigma-m\alpha}{2}t\right)-\right.$$

$$\left.\left.-D'^m_n\cos\left(m\lambda+\frac{\sigma-m\alpha}{2}t\right)\right]\left[(n+m)P^m_{n-1}+\right.\right. \qquad (21)$$

$$\left.\left.+(n-m+1)P^m_{n+1}\right]\cdot\sin\frac{\sigma+m\alpha}{2}\cdot t\right\},$$

where

$$N_n=\frac{M}{a^2}\frac{n(n+1)}{(2n+1)(\omega+\alpha)}, \qquad (22)$$

and B_n^m and B'^m_n are the coefficients of expansion of $T''(\theta,\lambda,t)$ into a series by the spherical functions of the expression, with

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t = o.

These comutations by Blinova made it possible to derive theoretically the existence of the Icelandic and Aleution depre sions, and the Azorian, Pacific, Siberian and Canadian areas of high pressure.

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CHAPTER XIX

THE THEORY OF PERTURBATIONS OF ATMOSPHERIC MOTIONS

Section 1. The Application of the Theory of Small Oscillations to Atmospheric Motions.

In the preceding chapters we discussed mainly the steady motions of the atmosphere. However, the actual atmospheric motions are by no means steady. The study of steady motions makes it possible to obtain merely a general concept of the background, upon which it is necessary to superpose some other motions specifically instrumental in the creation of the entire range of the actual atmospheric motions.

The mathematical study of most of the problems of dynamic meteorology, tied in with the research into the non-steady atmospheric motions, encounters insuperable difficulties. Let us admit for a minute that the problem is simplified by the introduction of crude assumptions such as, to the effect, that the atmosphere is non-compressible, that there is no inflow of heat, that turbulence and viscosity are absent. Then the problem reduces itself to the solution of a system of equations of hydrodynamics for an ideal and non-compressible liquid. However, the precise integration of even such a simplified system of equations is impossible. This is due to the fact that the equations of hydrodynamics contain non-linear terms

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z},\quad u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z},\quad u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z},$$

and, as yet there is no method for the precise integration of a system of non-linear equations.

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However, there is one fruitful method, which makes it possible to reduce the equations of atmospheric motions to a linear form, or as the expression goes, to linearize these equations. This is the so-called method of small oscillations, or the method of small perturbations. The essence of this method is in that the motion under study is considered as a sum of two motions - a certain steady motion and a small perturbation superposed upon it. Amongst the problems solvable by the method of small perturbations, is one of the most important problems of dynamic meteorology, namely, the origin and development of extratropical cyclones. In accordance with the latest viewpoint, extratropical cyclones develop as a wave disturbance, which is generated on the surface of separation between two air masses having different temperatures and velocities. If the generated disturbance increases with time, the initial wave disturbance is gradually transformed into a vortex disturbance, or a cyclone. Thus the problem of extratropical cyclogenesis is reduced to the problem of stability of a surface of separation.

In this case, the basic steady atmospheric motion can be considered to be geostrophic motion of two air masses, separated by a stable surface, the slope of which is determined by the Margules formula.

Upon this steady motion are superposed small perturbations, and conditions are studied, in the presence of which these perturbations lead to small velocity oscillations about the steady geostrophic motion, and to small oscillations of the surface of separation about the stable location, which is determinable by the Margules formula. If these conditions are not fulfilled, the perturbation increases with time, and

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further application of the method of small oscillations becomes impossible.

It is natural that the final criterion of the applicability of the derived solutions is the comparison of the theoretical results obtained with the actual conditions in the atmosphere.

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Section 2. Differential Equations of Small Perturbations.

Let us now derive the differential equations of small perturbations for an arbitrary basic motion. These equations, due to their community, possess symmetry which is usually lost in individual equations.

We will derive the equations of small perturbations for the rectangular (Descartes) system of coordinates.

We will designate the functions relating to the basic state by capital letters, and the functions relating to perturbation by corresponding small letters.

If the Euler method is applied, the basic state of the air is to be determined by the following functions:

$$U, V, W, P, Q, T, F \qquad (1)$$

U, V, W - are the velocity components of the basic motion, P is pressure, Q is density, T is the temperature of an air particle, F is the function of the coordinates of the point and time, which function when made to equal zero, is the equation of the outer or inner boundary surface.

The functions U, V, W, P, Q, T, F are considered as known and are subject to only one condition: they are to satisfy the 3 equations of motion:

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$$\left.\begin{aligned}
&\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}+W\frac{\partial U}{\partial z}=-\frac{1}{Q}\frac{\partial P}{\partial x}+X-2\omega_y W+2\omega_z V,\\
&\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}+W\frac{\partial V}{\partial z}=-\frac{1}{Q}\frac{\partial P}{\partial y}+Y-2\omega_z U+2\omega_x W,\\
&\frac{\partial W}{\partial t}+U\frac{\partial W}{\partial x}+V\frac{\partial W}{\partial y}+W\frac{\partial W}{\partial z}=-\frac{1}{Q}\frac{\partial P}{\partial z}+Z-2\omega_x V+2\omega_y U,
\end{aligned}\right\} \quad (2)$$

where X, Y, Z are components of mass forces;

they are to satisfy the equation of continuity

$$\frac{\partial Q}{\partial t}+U\frac{\partial Q}{\partial x}+V\frac{\partial Q}{\partial y}+W\frac{\partial Q}{\partial z}+Q\left(\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}+\frac{\partial W}{\partial z}\right)=0, \quad (3)$$

they are to satisfy the equation of state

$$P=QRT, \quad (4)$$

they are to satisfy the equation of the inflow of heat, which we take in the form of the equation of the polytrope

$$\frac{1}{P}\left(\frac{\partial P}{\partial t}+U\frac{\partial P}{\partial x}+V\frac{\partial P}{\partial y}+W\frac{\partial P}{\partial z}\right)-$$

$$-\frac{K}{Q}\left(\frac{\partial Q}{\partial t}+U\frac{\partial Q}{\partial y}+V\frac{\partial Q}{\partial y}+W\frac{\partial Q}{\partial z}\right)=0, \quad (5)$$

They are also to satisfy the boundary conditions: the kinematic condition

$$F(x,y,z,t)=0 \quad (6)$$

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or, if the boundary surface is an outer surface in its differential form:

$$\frac{\partial F}{\partial t} + U\frac{\partial F}{\partial x} + V\frac{\partial F}{\partial y} + W\frac{\partial F}{\partial z} = 0. \tag{7}$$

If the boundary surface is an inner surface, the boundary condition assumes the formulation

$$\left.\begin{aligned} &\frac{\partial F}{\partial t} + U\frac{\partial F}{\partial x} + V\frac{\partial F}{\partial y} + W\frac{\partial F}{\partial z} = 0, \\ &\frac{\partial F}{\partial t} + U'\frac{\partial F}{\partial x} + V'\frac{\partial F}{\partial y} + W'\frac{\partial F}{\partial z} = 0. \end{aligned}\right\} \tag{7'}$$

Such a formulation of the boundary condition follows from the fact that it must at all times consist of the same liquid particles. Finally, the contemplated functions must satisfy the dynamic condition, which, in the case of a free surface, is formulated as:

$$\frac{\partial P}{\partial t} + U\frac{\partial P}{\partial x} + V\frac{\partial P}{\partial y} + W\frac{\partial P}{\partial z} = 0, \tag{8}$$

and on the inner boundary surface the surface separating two liquids:

$$P - P' = 0 \text{ when } F(x, y, z, t) = 0, \tag{9}$$

or in differential form:

$$\left.\begin{aligned} &\frac{\partial (P-P')}{\partial t} + U\frac{\partial (P-P')}{\partial x} + V\frac{\partial (P-P')}{\partial y} + W\frac{\partial (P-P')}{\partial z} = 0, \\ &\frac{\partial (P-P')}{\partial t} + U'\frac{\partial (P-P')}{\partial x} + V'\frac{\partial (P-P')}{\partial y} + W'\frac{\partial (P-P')}{\partial z} = 0. \end{aligned}\right\} \tag{10}$$

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Let us now consider a motion which is very little different from the basic motion and is characterized by the following functions:

$$\left.\begin{aligned} &U(x,y,z,t)+u(x,y,z,t),\\ &V(x,y,z,t)+v(x,y,z,t),\\ &W(x,y,z,t)+w(x,y,z,t),\\ &P(x,y,z,t)+p(x,y,z,t),\\ &Q(x,y,z,t)+q(x,y,z,t),\\ &T(x,y,z,t)+\tau(x,y,z,t),\\ &F(x,y,z,t)+f(x,y,z,t), \end{aligned}\right\} \qquad (11)$$

which also must satisfy all the above indicated equations.

Since functions U, V, W, P, Q, T, F are known, the small perturbations expressed by the additional terms

$$v(x,y,z,t)\begin{cases} u(x,y,z,t); & q(x,y,z,t); & p(x,y,z,t);\\ v(x,y,z,t); & \tau(x,y,z,t); & f(x,y,z,t),\\ w(x,y,z,t); \end{cases} \qquad (12)$$

are fully determined.

To derive the equations of small perturbations, we substitute functions (11) into equations (2) - (10), assuming thereby that

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(1) Equations (2) - (10) characterize both the basic motion as well as the motion derived by way of superposing the small perturbations upon the basic motion;

(2) The perturbations superposed upon the basic motion are so small that magnitudes (12) and their derivatives are infinitesimals of the first order and that, consequently, their non-linear combinations can be disregarded.

This method, however, can be substituted by a more convenient one.

Let us form variations of all the equations of the basic state with radiation to the variables of Table (1) [Table 11?] considering variation δ independent of the differentiation operations in these equations. Then after the completed computations we will substitute the variations of the variables of Table (1) [Table 11?] with the corresponding magnitudes from Table (12) so that:

$$\delta U = u, \quad \delta V = v \quad etc.$$

We then derive the following system of equations of small perturbations, including also the boundary conditions:

Equations of motion

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$$\left.\begin{aligned}
&\frac{\partial u}{\partial t}+U\frac{\partial u}{\partial x}+V\frac{\partial u}{\partial y}+W\frac{\partial u}{\partial z}+u\frac{\partial U}{\partial x}+v\frac{\partial U}{\partial y}+w\frac{\partial U}{\partial z}=\\
&\quad=-\frac{1}{Q}\frac{\partial p}{\partial x}+\frac{1}{Q^2}\frac{\partial P}{\partial x}q-2\omega_y w+2\omega_z v,\\
&\frac{\partial v}{\partial t}+U\frac{\partial v}{\partial x}+V\frac{\partial v}{\partial y}+W\frac{\partial v}{\partial z}+u\frac{\partial V}{\partial x}+v\frac{\partial V}{\partial y}+w\frac{\partial V}{\partial z}=\\
&\quad=-\frac{1}{Q}\frac{\partial p}{\partial y}+\frac{1}{Q^2}\frac{\partial P}{\partial y}q-2\omega_z u+2\omega_x w,\\
&\frac{\partial w}{\partial t}+U\frac{\partial w}{\partial x}+V\frac{\partial w}{\partial y}+W\frac{\partial w}{\partial z}+u\frac{\partial W}{\partial x}+v\frac{\partial W}{\partial y}+w\frac{\partial W}{\partial z}=\\
&\quad=-\frac{1}{Q}\frac{\partial p}{\partial z}+\frac{1}{Q^2}\frac{\partial P}{\partial z}q-2\omega_x v+2\omega_y u.
\end{aligned}\right\}\quad(13)$$

Equation of continuity

$$\frac{\partial q}{\partial t}+\frac{\partial(qU)}{\partial x}+\frac{\partial(qV)}{\partial y}+\frac{\partial(qW)}{\partial z}+\frac{\partial(uQ)}{\partial x}+\frac{\partial(vQ)}{\partial y}+\frac{\partial(wQ)}{\partial z}=0. \quad(14)$$

Equation of state

$$p=RqT+RQ\tau. \quad(15)$$

Equation of the polytrope

$$\frac{1}{p}\left(\frac{\partial p}{\partial t}+U\frac{\partial p}{\partial x}+V\frac{\partial p}{\partial y}+W\frac{\partial p}{\partial z}+u\frac{\partial P}{\partial x}+v\frac{\partial P}{\partial y}+w\frac{\partial P}{\partial z}\right)-$$
$$-\frac{1}{p^2}\left(\frac{\partial P}{\partial t}+U\frac{\partial P}{\partial x}+V\frac{\partial P}{\partial y}+W\frac{\partial P}{\partial z}\right)p=\frac{\varkappa}{Q}\left(\frac{\partial Q}{\partial t}+\right. \quad(16)$$
$$+U\frac{\partial q}{\partial x}+V\frac{\partial q}{\partial y}+W\frac{\partial q}{\partial z}+u\frac{\partial Q}{\partial x}+v\frac{\partial Q}{\partial y}+w\frac{\partial Q}{\partial z}\Big)-\frac{\varkappa}{Q^2}\left(\frac{\partial Q}{\partial t}+U\frac{\partial Q}{\partial x}+V\frac{\partial Q}{\partial y}+\right.$$

The kinematic boundary condition

$$\left.+W\frac{\partial Q}{\partial z}\right)q.$$

$$F(x,y,z,t)+f(x,y,z,t)=0 \quad(17)$$

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or in its differential form:

$$\left.\begin{aligned} &\frac{\partial f}{\partial t}+U\frac{\partial f}{\partial x}+V\frac{\partial f}{\partial y}+W\frac{\partial f}{\partial z}+u\frac{\partial F}{\partial x}+v\frac{\partial F}{\partial y}+w\frac{\partial F}{\partial z}=0, \\ &\frac{\partial f}{\partial t}+U'\frac{\partial f}{\partial x}+V'\frac{\partial f}{\partial y}+W'\frac{\partial f}{\partial z}+u'\frac{\partial F}{\partial x}+v'\frac{\partial F}{\partial y}+w'\frac{\partial F}{\partial z}=0. \end{aligned}\right\} \tag{18}$$

The dynamic condition on the free boundary surface:

$$\frac{\partial p}{\partial t}+U'\frac{\partial p}{\partial x}+V\frac{\partial p}{\partial y}+W\frac{\partial p}{\partial z}+u\frac{\partial P}{\partial x}+v\frac{\partial P}{\partial y}+w\frac{\partial P}{\partial z}=0 \tag{19}$$

$$\text{when } F+f=0.$$

The dynamic boundary condition on the inner boundary surface:

$$\begin{aligned} &P+p-P'-p'=0 \\ &\text{when } F+f=0 \end{aligned} \tag{20}$$

or in its differentiated form:

$$\left.\begin{aligned} &\frac{\partial(p-p')}{\partial t}+U\frac{\partial(p+p')}{\partial x}+V\frac{\partial(p-p')}{\partial y}+W\frac{\partial(p-p')}{\partial z}+ \\ &\quad+u\frac{\partial(P-P')}{\partial x}+v\frac{\partial(P-P')}{\partial y}+w\frac{\partial(P-P')}{\partial z}=0, \\ &\frac{\partial(p-p')}{\partial t}+U'\frac{\partial(p-p')}{\partial x}+V'\frac{\partial(p-p')}{\partial y}+W'\frac{\partial(p-p')}{\partial z}+ \\ &\quad+u'\frac{\partial(P-P')}{\partial x}+v'\frac{\partial(P-P')}{\partial y}+w'\frac{\partial(P-P')}{\partial z}=0. \end{aligned}\right\} \tag{21}$$

The equations of small perturbations (13) - (21) may appear to be even more unwieldy than the initial equations (2) - (11),

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but there is a cardinal difference between them, the difference being that in the initial differential equations functions U, V, W, P, Q, T, F are unknowns, i.e. these equations are non-linear. But in the equations of small perturbations, U, V, W, P, Q, T, F are considered already as known functions. Therefore, equations (13) - (21) are linear with relation to the unknown functions u, v, w, p, q, τ, f. Thus, equations (13) - (21) are simpler than equations (2) - (11).

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Section 3. Wave Motions in the Atmosphere

Wave motions are characterized by the fact that the material particles perform small oscillations about the condition of equilibrium, or about the basic motion. Simultaneously, there occurs a strong transference of certain elements of the wave, such as the front of the wave, the form of the free surface of the wave and the like.

A classical example of the wave motion are the waves forming on a surface of water. Let us imagine that, under the effect of some momentary forces, the horizontal form of a free surface of water is disturbed. After the action of these momentary forces has stopped, the presence of the connection between the particles of water will cause the initial perturbation to spread, being imported to the more distant particles of water. In a very short time the surface of the water will assume a wave like form.

Since oscillations of maximum intensity are performed by the particles lying on the free surface, such waves are called surface waves. The causes, which tend to extinguish the initial perturbation, are the force of gravity, viscosity, surface tension. If the wave motion develops at the expense of the potential energy of position the waves are called gravitational waves. Gravitational waves may evolve not only on the free surface of a liquid, but also on a surface of discontinuity inside a liquid. Such internal gravitational waves are of particularly great significance in meteorology.

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Resilient waves, during the formation of which the potential energy of compressed or rarefied air is transformed into kinetic energy, may also propagate in the atmosphere. Such as these are sound waves and waves engendered by explosions. Since resilient waves have no significance for meteorology we will not discuss them. Inertia waves, which are propagated only by inertia and are not accompanied by transformation of kinetic energy into other varieties of energy, are of great significance in a series of important problems of dynamic meteorology.

A classical example of wave motions in the atmosphere are the waves evolved on a surface of separation of two air layers, which are moving at different velocities and have different densities. Under favorable conditions, these waves become visible. Namely, when the particles, rising in the area of the crest of the wave, attain the condensation level, and the particles descending in the area of the trough recede from the state of saturation, well known undulating cloud formations appear.

The fact that such undulating cloud formations are evolved simultaneously over a vast territory attests to the possible presence of a wave motion also in the case, when no undulating clouds are present. The upper boundary of the continuous cloud cover usually looks like the surface of the agitated sea.

Nevertheless, the above are not the only form of wave motion in the atmosphere.

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In the terrestrial atmosphere as well as in the ocean, occurs the phenomenon of high tides, which are induced mainly by the attraction of the moon, and to a lesser degree, by the attraction of the sun.

But the most meteorologically important form of wave motions in the atmosphere are waves evolving on frontal surface separating two air masses. This variety of waves is particularly important, since according to latest concepts, the evolution and further development of extratropical cyclones is tied in with them. However, the difficulties encountered in the study of wave perturbations on sloping frontal surfaces, are so great that as yet they were not overcome. Only the problem of stability of wave motions on frontal surfaces was solved.

Section 4. Gravitational waves on the Free Surface of a Non-Compressible Liquid.

Let us first analyze the simples case of wave motion on the free surface of an h-thick layer of a non-compressible liquid, which extends along horizontal lines in all directions ad infinitum. The effect of the earth's rotation is not considered.

Assuming that the basic state of the liquid is a homogeneous rectilinear motion taking place under the effect of only one external force, which is the force of gravity.

Let us place the origin of the coordinates on the free surface of this layer, with the x-axis directed parallel to the basic motion of the liquid, and the z-axis vertically up.

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Then, the characteristics of the basic motion will be as follows:

$$U=\text{const}, \quad V=W=0, \quad Q=\text{const}.$$

Since the only external force is the force of gravity:

$$X=Y=0; \quad Z=-g.$$

Then the equations of the basic motion can be written down as:

$$\frac{\partial P}{\partial x}=\frac{\partial P}{\partial y}=0; \quad \frac{\partial P}{\partial z}=-g\rho; \quad \frac{\partial U}{\partial x}=0. \tag{1}$$

Integrating the third equation, which is the equation of statics, we derive:

$$P=-g\rho z+\varphi(t). \tag{2}$$

As far as the small perturbations are concerned, it is natural to assume that they are periodical functions of time, and their absolute magnitudes are minors of the first order. In addition, we will assume that the motion under study is a plane motion, so that it will be characterized by the following functions:

$$U+u, \quad v=0; \quad w, \quad P+p, \quad Q.$$

Then, the equations of the small perturbations will assume the form

$$\frac{\partial u}{\partial t}+U\frac{\partial u}{\partial x}=-\frac{1}{\rho}\frac{\partial p}{\partial x}; \quad \frac{\partial w}{\partial t}+U\frac{\partial w}{\partial x}=-\frac{1}{\rho}\frac{\partial p}{\partial z}; \tag{3}$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0.$$

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We now formulate the boundary conditions of the problem. At the bottom, near the rigid boundary it should be:

$$w = 0 \quad \text{when} \quad z = -h. \tag{4}$$

On the free surface, the pressure must change continuously, so that by designating as p_0 the atmospheric pressure on the non-perturbed surface $z(x) = 0$, we derive:

$$P = p_0; \tag{5}$$

for the free surface, in the presence of small perturbations, the equation of the surface being

$$z(x, t) = \zeta(x, t) \tag{6}$$

we derive:

$$P + p = p_0. \tag{7}$$

Taking into account (2), we derive for the non-perturbed surface:

$$\varphi(t) = p_0$$

and for the perturbed surface:

$$-g\rho\zeta + \varphi(t) + p = p_0.$$

Eliminating p_0, we derive the following condition:

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$$p = g\rho\zeta \quad \text{when} \quad z = 0. \tag{8}$$

Magnitude ζ is a new unknown function, which expresses the configuration of the perturbed surface.

Differentiating the equation of surface by t, we derive a correlationship, which ties in ζ with the vertical velocity of the particle:

$$\frac{dz}{dt} = \frac{d\zeta}{dt} = \frac{\partial\zeta}{\partial t} + \frac{\partial\zeta}{\partial x}\frac{dx}{dt}.$$

But

$$\frac{dz}{dt} = w; \quad \frac{dx}{dt} = U + u.$$

Therefore, discarding the minor term of the second order $u \cdot \frac{\partial\zeta}{\partial x}$, we derive:

$$w = \frac{\partial\zeta}{\partial t} + U\frac{\partial\zeta}{\partial x}. \tag{9}$$

Usually, when differential equations are integrated, initial conditions are to be formulated in addition to the formulation of boundary conditions. But in our case, this formulation may be omitted, since we are analyzing periodic oscillations. In this case we will seek such values for u, w, p, which contain time only in the aspect of a factor, which is a periodical trigonometric function of time, such as

$$\cos(\sigma t), \quad \sin(\sigma t)$$

where $\sigma = 2\pi/\tau$ is the oscillation frequency and τ is the oscillation period.

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Additionally, in the simplest case the configuration can be described by a function, which contains the x-coordinate of the free surface in the form of a periodical trigonometric function cos kx, sin kx, where $k=2\pi/\lambda$ is the wave number, λ is the wave length. Thus, it is natural to seek the solution of the problem in such a form, where time t and the x-coordinate are contained under the sign of the following trigonometric functions:

$$\cos(\sigma t - kx), \quad \sin(\sigma t - kx).$$

However, the problem can be solved in a simpler and more general manner, if we turn to the complex functions and recall the formula, which ties in the exponential function with the trigonometric functions:

$$e^{i(\sigma t - kx)} = \cos(\sigma t - kx) + i \sin(\sigma t - kx).$$

Since all the equations of the small perturbations turned out to be linear, we will, in place of seeking the solution for u, w, p, ζ first seek the solutions in the form of complex functions

$$u = u_1 + iu_2; \quad w = w_1 + iw_2; \quad p = p_1 + ip_2; \quad \zeta = \zeta_1 + i\zeta_2;$$

the substantial, as well as the imaginary parts of the complex functions constitute the solution of our problem.

Now then, we will seek the solution in the following form:

$$\left.\begin{aligned} u(x,z,t) &= \bar{u}(z) \cdot e^{i(\sigma t - kx)}, \\ w(x,z,t) &= \bar{w}(z) \cdot e^{i(\sigma t - kx)}, \\ p(x,z,t) &= \bar{p}(z) \cdot e^{i(\sigma t - kx)}, \\ \zeta(x,t) &= \bar{\zeta} \cdot e^{i(\sigma t - kx)} \end{aligned}\right\} \qquad (10)$$

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Substituting (10) into (3) we derive the equations of the problem in the following form:

$$\left.\begin{aligned} i(\sigma-\kappa U)\cdot u &= \frac{i\kappa}{\rho}\cdot p, \\ i(\sigma-\kappa U)\cdot w &= -\frac{1}{\rho}\frac{\partial p}{\partial z}, \\ i\kappa u &= \frac{\partial w}{\partial z}. \end{aligned}\right\} \tag{11}$$

The boundary condition on the free surface will be written down as:

$$\left.\begin{aligned} p(x,0,t) &= g\rho\zeta(x,t), \\ w(x,0,t) &= i(\sigma-\kappa U)\zeta(x,t). \end{aligned}\right\} \tag{12}$$

Strictly speaking, under these boundary conditions, it would be proper to use expressions $p(x,\zeta,t)$ and $w(x,\zeta,t)$, but since ζ is infinitesimally small, zero can be used in its place.

Differentiating the first equation (11) by x and, the second one by z, and taking into account the third equation, we derive:

$$\kappa^2 p = \frac{\partial^2 p}{\partial z^2},$$

or, reducing by the common factor $e^{i(\sigma t-\kappa x)}$ for the determination of p(z), we derive the following linear homogeneous equation with constant coefficients:

$$\frac{d^2\bar{p}}{dz^2} - \kappa^2\bar{p} = 0. \tag{12'}$$

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The general solution of this equation is

$$\bar{p}(z) = C_1 e^{\kappa z} + C_2 e^{-\kappa z}, \tag{13}$$

Consequently,

$$p = (C_1 e^{\kappa z} + C_2 e^{-\kappa z}) e^{i(\sigma t - \kappa x)}. \tag{14}$$

Proceeding further, we find from (11):

$$\left.\begin{aligned} u &= \frac{\kappa}{\rho(\sigma - \kappa U)} (C_1 e^{\kappa z} + C_2 e^{-\kappa z}) e^{i(\sigma t - \kappa x)}, \\ w &= i \frac{\kappa}{\rho(\sigma - \kappa U)} (C_1 e^{\kappa z} - C_2 e^{-\kappa z}) e^{i(\sigma t - \kappa x)}. \end{aligned}\right\} \tag{15}$$

We shall now satisfy the boundary conditions.

Since, when $z = -h \quad w = 0$ then: $C_1 e^{-\kappa h} - C_2 e^{+\kappa h} = 0$,

whence:

$$C_1 e^{-\kappa h} = C_2 e^{+\kappa h} = \frac{C}{2},$$

where C is an arbitrary constant. Thus

$$C_1 e^{\kappa z} + C_2 e^{-\kappa z} = \frac{C}{2} [e^{\kappa(z+h)} + e^{-\kappa(z+h)}],$$

$$C_1 e^{\kappa z} - C_2 e^{-\kappa z} = \frac{C}{2} [e^{\kappa(z+h)} - e^{-\kappa(z+h)}].$$

But the expressions within the square brackets are, respectively, the hyperbolic cosine ch(z+h) and the hyperbolic sine sh(z+h).

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Consequently, the final solution of the problem will be:

$$\left.\begin{aligned} u &= \frac{C\kappa}{\rho(\sigma-\kappa U)}\, ch\,\kappa(z+h)\, e^{i(\sigma t-\kappa x)}, \\ w &= i\frac{C\kappa}{\rho(\sigma-\kappa U)}\, sh\,\kappa(z+h)\, e^{i(\sigma t-\kappa x)}, \\ p &= C\cdot ch\,\kappa(z+h)\, e^{i(\sigma t-\kappa x)}. \end{aligned}\right\} \qquad (16)$$

For the configuration of the wave, we derive the following two expressions

$$\left.\begin{aligned} \zeta(x,t) &= \frac{1}{g\rho}\cdot p(x,0,t) = \frac{C}{g\rho} ch\,\kappa h\, e^{i(\sigma t-\kappa x)}, \\ \zeta(x,t) &= \frac{w(x,0,t)}{i(\sigma-\kappa U)} = \frac{C\kappa}{\rho(\sigma-\kappa U)^2}\cdot sh\,\kappa h\cdot e^{i(\sigma t-\kappa x)}. \end{aligned}\right\} \qquad (17)$$

It is obvious that both these expressions must be identical. Therefore, designating the wave amplitude by A:

$$A = \frac{C}{g\rho} ch\,\kappa h = \frac{C\kappa}{\rho(\sigma-\kappa U)^2} sh\,\kappa h, \qquad (18)$$

and equalizing the right-hand parts of (17), we derive:

$$(\sigma-\kappa U)^2 = g\kappa\, th\,\kappa h, \qquad (19)$$

where th is the hyperbolic tangent.

$$th\,\gamma = \frac{e^{\gamma}-e^{-\gamma}}{e^{\gamma}+e^{-\gamma}}.$$

Dealing with the substantial part only, we derive the solution

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of the problem in the following form:

$$\left.\begin{aligned} u &= A(\sigma - \varkappa U)\frac{\operatorname{ch}\varkappa(h+z)}{\operatorname{sh}\varkappa h}\cos(\sigma t - \varkappa x), \\ w &= -A(\sigma - \varkappa U)\frac{\operatorname{sh}\varkappa(h+z)}{\operatorname{sh}\varkappa h}\cdot\sin(\sigma t - \varkappa x), \\ p &= A g\rho\cdot\frac{\operatorname{ch}\varkappa(h+z)}{\operatorname{ch}\varkappa h}\cdot\cos(\sigma t - \varkappa x), \\ \zeta &= A\cdot\cos(\sigma t - \varkappa x) = A\cos\sigma\left(t - \frac{\varkappa}{\sigma}x\right). \end{aligned}\right\} \qquad (20)$$

Thus, the configuration of the wave is a cosinusoid, which is shifting at velocity

$$c = \frac{\sigma}{\varkappa} \qquad (21)$$

in the direction of the positive x-axis.

Let us now express the velocity of the shifting of the wave with the aid of equation (19), substituting, in place of $\varkappa = \frac{2\pi}{\lambda}$,

$$c = \frac{\sigma}{\varkappa} = U \pm \sqrt{\frac{g\lambda}{2\pi}\operatorname{th}\frac{2\pi h}{\lambda}}. \qquad (22)$$

Now then, the velocity of the wave propagation is determined by two terms. The first term U indicates that the wave, on the whole, is shifting together with the current at the current velocity U. This is the transfer or "convection member". If $U=0$ the velocity of wave propagation is determined by the second member. The second

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