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VIBRATION DAMPING SYSTEMS IN MACHINES AND MECHANISMS

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This collection is dedicated to methods of overcoming vibrations of machines and mechanisms. Methods are examined of distributing vibration energy along structures as well as methods of acoustic diagnosis of ball bearings and localizing acoustic sources.

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Engineering	Rods	Air-Cushions
Vibrations	Shock Absorbers	

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[Text] An examination is made of the propagation of vibration energy through structural elements, mainly through rod structures, and the acoustic diagnosis of ball bearings is considered as well as localization of acoustic sources.

The collection is written for scientific and engineering-technical workers.

ACTIVE VIBROPROTECTION SYSTEMS

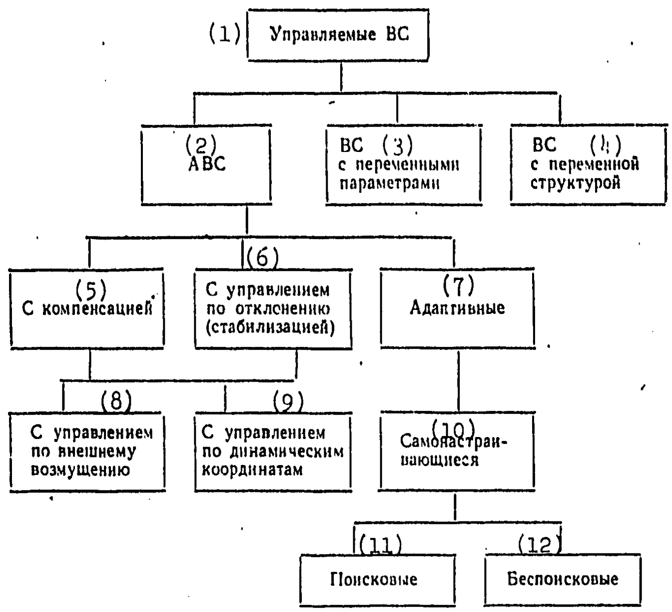
M. D. Genkin, V. V. Yablonskiy

This article examines active vibroprotection systems (AVS) as controllable systems for vibration protection and presents a classification of such systems from different viewpoints.

A controllable system for vibration protection is a system for automatically controlling the vibration of a mechanical object to reduce vibration to a predetermined level at certain points or in a region of space, in a predetermined frequency band or time region for a certain class of external actions. The object of control is a mechanism or attached structural unit, the source of information is the data on the vibration state of the object, the criterion of effectiveness of control is the magnitude of the vibration or some functional that characterizes the vibroactivity of the object in the final analysis.

As a rule, controllable vibroprotection systems (VS) require energy input from an additional external source. These systems can be divided into three groups (see the diagram).

Classification of controllable and active vibroprotection systems



KEY:
1--Controllable VS
2--AVS
3--VS with variable parameters
4--VS with variable structure
5--With compensation
6--With deflection control (by stabilization)
7--Adaptive
8--With control with respect to external perturbation
9--With dynamic coordinate control
10--Self-adjusting
11--Searching
12--Non-searching

In the first group are the AVS. In these the actuating elements act directly on the object along with the disturbing factors. Passive parameters usually remain unaltered.

In the second group are VS in which the actuating devices act on passive elements (a mass, a spring, a damper), changing their value in some way (continuously or in steps). For instance a change takes place in the distribution of unbalanced masses (autobalancing), the mass or stiffness of a dynamic antivibrator. When there is a fairly slow change in parameters, the system as a whole behaves like a passive system. Rapid changes that are comparable in velocity to a vibrational process lead to fundamentally new properties such as increased stability.

In the third group are VS with variable structure, where there is a change not only in parameters, but in the order of activation of various links.

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In systems with variable parameters and variable structure, energy losses from the outside source are determined by the mechanism of action on the parameters and structure, and do not directly involve the vibrational process itself.

Practically always the active vibroprotection is provided by the combined action of actuating (active) and passive elements of the system. In some cases the vibroprotective role of the passive elements is especially clear. For instance in the electrohydraulic vibroprotection system of Ref. 1 there is a spring that is connected in series with a hydraulic cylinder, and the AVS with force control in the shock-absorbing mechanisms of Ref. 2 also acts only in combination with fairly pliable elastic elements. Some authors [Ref. 3] call such systems hybrid in contrast to "purely active." We feel that such a division is unsound since there are no "purely active" systems (i. e. systems that are independent of passive parameters).

D. Karnopp [Ref. 4] uses the term "semi-active" for a system with an electro-hydraulic mechanism that changes the stiffness of an elastic suspension. According to the classification that we propose, this is a system with variable parameters.

In extensive use are mathematical models of AVS as systems with additional stiffness, damping or mass introduced by feedback with respect to the corresponding kinematic parameters (displacement, velocity, acceleration) [Ref. 2, 5]. On the other hand the block diagrams of passive systems are depicted as control systems [Ref. 6]. Obviously this gives no basis for considering AVS a type of systems with variable parameters.

Actually the AVS, in contrast to passive systems, provides a wider choice of reciprocal and negative inserted parameters. Moreover, the region of constancy of a given inserted parameter is limited by the passband of the feedback loop. The introduction of equivalent parameters is useful for a more graphic representation of the effectiveness of the system in the working frequency band, but is completely useless in stability analysis.

AVS with control with respect to perturbation. In the theory of vibroprotection many versions of AVS with control with respect to perturbation can be considered. The source of control is a signal proportional to the perturbing factors (force or kinematic). Usually in automation, perturbations are taken as independent of the response of the system that changes in the process of regulation. Such for instance are the kinematic perturbations of equipment on the base side, and also the vibrations of the shaft and bearings of a spring-mounted rotor machine if the components of the AVS are on a foundation or other supporting structural elements, i. e. they are "decoupled" from the source. The main goal is taken as realization of a transfer function that ensures "invariance" (independence of perturbations) of the selected dynamic coordinates. This is open-cycle control. This requires extensive information on the system.

On the basis of a given form of the continuous spectrum of random perturbations and various criteria of vibroprotection, the theory of optimum filters is used to find the optimum structure of the VS, which in a number of instances must necessarily contain active elements [Ref. 7].

Essentially different from this kind of control is vibrocompensation with control with respect to the dynamic coordinates of the object [Ref. 2, 8-15]. These coordinates in a certain frequency range can be treated as external perturbations of some part of the system. An example is provided by the dynamic forces f transmitted through elastic elements (shock absorbers) to the foundation of machines, or from the outside to the equipment to be siolated. The best protection for equipment to be isolated is provided by applying compensating (active) forces f^a to the points of action of the given perturbations; to do this, it is natural to use these perturbations themselves as the control signal. Invariance in this case should be understood as "decoupling" of the object from the source of vibrations. The ideal transfer function of the control circuit is equal to $f^a/f = -1$, i. e. it is frequency-independent, which is favorable for wide-band vibroprotection.

In contrast to control with respect to external perturbations, stability may be disrupted in a system due to feedbacks through the object. Some versions of control can be partly or completely represented as control with respect to deviation of a dynamic force [Ref. 14]. Stability can be improved by using correcting circuits, filters, or combined control (with respect to perturbation and deflection).

An AVS is proposed in Ref. 16 with control with respect to deformation of an elastic element. This coincides with control with respect to force, where the shock absorber is an elastic link without losses.

AVS with deflection control. This control principle is used in the overwhelming majority of VS with electromechanical feedback that are described in the literature. They have certain advantages: there is no requirement for complete information on the perturbations or (to a certain extent) on the characteristics of the equipment to be isolated since a slight change in the latter has little effect on damping efficiency.

Ref. 17-20 deal with general studies of feedback with respect to acceleration, velocity and displacement in simple unidirectional mechanical circuits. Research has been done [Ref. 18, 21] on the stability and other properties of AVS in arbitrary elastic systems and in shock-mounted objects with six degrees of freedom.

Ref. 22-25 examine multichannel systems of electromechanical feedback with respect to deflections as applied to rods, plates, shells and an acoustic medium.

Many papers have been devoted to the development and theory of individual devices as well as to their applications. In Ref. 26, 27 a study is done on electromagnetic vibrocompensators unified into a single element together with a vacuum tube and a feedback coil (velocity sensor). Ref. 28-36 deal with electrodynamic devices. Despite the great possibilities proved for these devices by many authors, only laboratory models have been described. Apparently the problem lies in the difficulties of making reliable and economic vibrators.

Matters are different when it comes to controllable hydraulic and pneumatic suspensions designed for damping low-frequency actions and compensating slowly changing loads. Playing a decisive role here on the one hand have been the high operational reliability and the large developed forces with speed sufficient for quasistatic conditions (although according to the latest data the frequency range has been increased to a few hundred hertz); on the other hand there have been the extensive possibilities of control provided by electromechanical sensors (there are VS with purely hydraulic control as well [Ref. 31]). Controllable hydraulic and pneumatic devices have been developed on the basis of equipment that has already been perfected, checked out and put into wide use. The first research on electrohydraulic devices was done in Ref. 1. In the USSR this research has been developed in Ref. 7, 20, 31-33 and elsewhere. A model of a VS for a human operator has been made [Ref. 7], and controllable suspensions are being developed for motor vehicles [Ref. 34]. A special group is constituted by self-leveling elastic supports for foundationless installation of machine tools, mainly pneumatic (Barry Control Co., and in the Soviet Union an original design has been worked out in Ref. 35).

In modern machines that operate on moving objects it is advisable to use pliable vibration-damping rotor suspensions in the bearings together with a self-leveling system (stabilization of rotor position relative to the stator and housing). Ref. 36, 37 describe stabilizing electromagnets placed between the stator and bearings, and controlled by a signal proportional to deflection of the rotor or change in the load on the bearings. In view of the low time constant, the low electromagnetic stiffness of the system and its insensitivity to transverse displacements, it has an advantage in speed, particularly as compared with pneumatic devices, and combines better with elastic elements (parallel connection). Calculations show that an especially appreciable vibration-damping effect is realized with simultaneous use of electromagnets as controllable vibrocompensators on the vibration frequency.[1] After solution of a number of engineering problems an electromagnetic system may be used in rotor machines.

AVS with compensation and stabilization (particulars of analysis). Mainly, we use a system of ordinary differential equations or partial differential equations with constant coefficients and with additional terms in the second member. In connection with the introduction of elements of automatic control circuits (amplifiers, filters) there is a tendency to change to structural methods that are common to the object and the control circuit. The Routh-Hurwitz criterion is used to evaluate stability only in characteristic

[1]See the article by M. D. Genkin, V. G. Yelezov, M. A. Pronina and V. V. Yablonskiy, "Active Vibroprotection System with Control with Respect to Low-frequency and Vibrational Perturbations" in this collection.

equations of up to order 4. The Bode-Nyquist criterion [Ref. 22, 30] is most widely used in more complicated cases. In Ref. 13, 14 an immitance criterion is used that includes impedances determined experimentally by the same vibrocompensators.

Most AVS are made to ensure linear operation, and nonlinearity is treated as a spurious eradicable effect. In systems for stabilizing quasistatic displacements (including in the self-leveling supports mentioned above) the nonlinearity of the responses of elastic and active elements is considerable in operation on large displacements. In the system with electromagnet of Ref. 37 the elastic elements must necessarily have a rigid response to ensure static stability of the object in the magnetic field. We should expect more and more extensive use of nonlinear methods for comprehensive evaluation of operability under working conditions. A number of authors have pointed out an improvement in stability with nonlinear operation.

Self-adjusting AVS. So far these systems have remained almost unconsidered in the literature, even though they are obviously needed, particularly in control with respect to perturbation in separate frequency bands, under conditions of variable frequency and drift of the characteristics of the controlled object.

Non-searching self-adjusting AVS. An example is a system of a fairly large number n of controllable vibrators that compensate oscillations in a girder structural element,[1] or in some region of an extended plate. For instance vibrations on a given frequency are minimized at a mean-square or maximum level that is determined from the readings of m acceleration converters. Under certain conditions we can limit ourselves to compensation of the initial vibration at n points. For this purpose we measure beforehand the natural and reciprocal compliance (matrix Y). From the vector of initial vibration X_0 we use a digital computer incorporated into the AVS to calculate the vector of the necessary compensating (active) force

$$F^a = -Y^{-1}X_0.$$

Since the matrix Y can be found and inverted beforehand on required frequencies, a comparatively small computer unit is needed for operational control of vibration. Non-searching systems provide the highest speed, which is necessary when there is a considerable change of the initial vibration in time.

Ref. 25 considers selection of the points of installation of vibrocompensators when there is a limitation on power consumption.

Searching self-adjusting AVS. The main peculiarity of the quality function of the vibroprotection system for a complex object is the presence of "local

[1]See the article by V. A. Tikhonov and V. V. Yablonskiy, "Some Problems of Vibrocompensation of Elastic Systems" in this collection.

extrema." Therefore methods of sequential variation of dynamic test forces are not well suited to searching. In Ref. 25 a computer was used to study a method of simultaneous change in all forces in proportion to residual vibration, and also a simplex method for the same model (a girder). The former method was effective for damping "pure" normal modes with a surplus of vibrators. The speed of the latter method depends considerably on the initial point of the search and on the dimensions of the simplex. This simplex must be reduced during search to achieve the required accuracy of control, and of course there is a concomitant reduction of speed. The speed is determined by the speed of vibration measurement on each step, and by the setting of appropriate rated forces on the vibrators.

Considering the feasibility of making miniature self-contained computing devices, we should consider self-adjusting AVS with perturbation control the most promising for sources of vibration that are polyharmonic in nature. No less important is self-adjustment of control circuits with feedback to improve efficiency.

Optimum control of damping of oscillations of elastic systems. Research on control of oscillating elastic systems is based on L. S. Pontryagin's maximum principle and also on dynamic programming (Bellman's principle) [Ref. 38-40]. In particular, an examination was made of control of the oscillations of a jet, a rod and a thin plate. Typical formulation of the problem: find a control, i. e. a system of forces or kinematic actions applied at certain (outside or intermediate) points of the object to reduce its free oscillations to a predetermined level in the shortest time. A similar problem is formulated for vibration dampers [Ref. 14]. Such a control is one of the methods of active vibroprotection as applied to nonstationary actions, and specifically impact actions. Practical realization of this control leads to AVS with optimum control that appreciably surpass self-adjusting AVS in their capabilities. Such systems have already been created for fairly slow processes (e. g. damping of gyroscope oscillations).

Considering what has been said, let us note the major factors that are most conducive to more extended use of AVS in technology:

1. Availability of powerful compact actuating mechanisms (hydraulic and pneumatic) that provide the necessary dynamic and static forces in the low-frequency range (from a few hertz to tens of hertz).

2. Low magnitude of vibrational forces transmitted to supports, particularly in shock-mounted mechanisms of rotor type. This enables the use of AVS with control with respect to force for additional reduction of vibrations. Actuating elements (as a rule electromagnetic vibrators) may have small overall dimensions and low power consumption under these conditions.

3. The capability for using passive vibroprotection devices (particularly elastic links) to improve the conditions of operation of active elements and enhance the total effect.

4. A wide choice of methods and means of control (force and vibration transducers, amplifiers, filters, correcting circuits), and the possibility of making compact computers.

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AN ACTIVE VIBROPROTECTION SYSTEM WITH CONTROL WITH RESPECT TO LOW-FREQUENCY AND VIBRATIONAL PERTURBATIONS

M. D. Genkin, V. G. Yelezov, M. A. Pronina, V. V. Yablonskiy

An examination is made in Ref. 1-3 of active vibroprotection systems (AVS) with control with respect to force or strain on an elastic element that connects the source of oscillation to the object that is to be isolated. These systems contain individual control channels that include a force or deformation transducer, an amplifier and an electromagnet that operates on the object to be isolated close to the point of application of the force. The transfer ratio of the vibrocompensation system has a constant value in the vibration band and is close to zero in the low-frequency region. Control

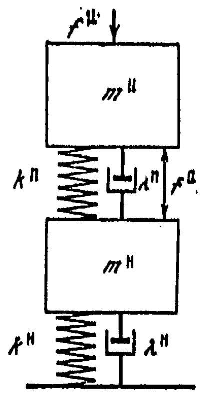

Fig. 1

is equivalent to reducing the stiffness of the elastic elements in the vibration band. On the other hand, the stabilization system of Ref. 3 has a constant transfer ratio on low frequencies and does not operate in the vibration band. Control is equivalent to increasing the stiffness of elastic elements on low frequencies. In the first case the differential of oscillations is artificially increased on relatively stiff shock absorbers. In the second case it becomes possible to use a pliable insulating suspension for mechanisms on moving objects under conditions of varying misalignments, as well as inertial loads (external low-frequency perturbations).

On the basis of simple computational examples, this paper investigates vibroprotection by simultaneous control in the vibration and low-frequency perturbation bands.

Diagram of the AVS and basic equations. The mechanical system (Fig. 1) contains mass $m^И$ excited by external force $f^И$ (source of oscillations), an elastic element with complex stiffness $k^П(1 + j\lambda^П)$, ($\lambda^П$ is the loss factor), and also mass m^H and an elastic element with complex stiffness $k^H(1 + j\lambda^H)$ -- the model of the object to be isolated -- the "load." The action of the controlled electromagnet with frame fastened to the source (mass $m^И$) and armature fastened to the object to be isolated (the rigid base) is depicted in accordance with the conventional scheme [Ref. 1, 2] by two equal and opposite active forces $f^а$ applied to mass $m^И$ and the base.

The transfer function has the following form:

$$\frac{f^а}{f} = K_f(p) = \frac{K_{fc}}{T_c p + 1} - \frac{K_{fк} T_к p}{T_к p + 1},$$

$$K_f(p) = \frac{K_{fc}}{T_c^2 p^2 + 2\zeta_c T_c p + 1} - \frac{K_{fк}(T_к^2 p^2 + 2\zeta_к T_к p)}{T_к^2 p^2 + 2\zeta_к T_к p + 1} \qquad (1)$$

for RC and LCR filters respectively.

In the frequency region ($p = j\omega$) this function can be transformed to the following form (for LCR filters):

$$K_f(\Omega) = K_{fc}\frac{1 - \Omega^2/\Omega_c^2}{(1 - \Omega^2/\Omega_c^2)^2 + \lambda_c^2\Omega^2/\Omega_c^2} + K_{fк}\frac{(1 - \Omega^2/\Omega_к^2 - \lambda_к^2)\,\Omega^2/\Omega_к^2}{(1 - \Omega^2/\Omega_к^2)^2 + \lambda_к^2\Omega^2/\Omega_к^2} -$$
$$- j\left[K_{fc}\frac{\lambda_c\Omega/\Omega_c}{(1 - \Omega^2/\Omega_c^2)^2 + \lambda_c^2\Omega^2/\Omega_c^2} + K_{fк}\frac{\lambda_к\Omega/\Omega_к}{(1 - \Omega^2/\Omega_к^2)^2 + \lambda_к^2\Omega^2/\Omega_к^2}\right], \qquad (2)$$

where $\Omega = \omega/\omega_0$; $\omega_0 = \sqrt{k^п/m^и}$; f is the force in the elastic element (control signal); K_{f_C} and K_{f_H} are the nominal coefficients of amplification of the control circuit in the bands of stabilization (when $\Omega \ll 1$) and vibrocompensation (when $\Omega \gg 1$) respectively; Ω_C, Ω_H are the normalized natural frequencies of the low-frequency and high-frequency LCR filters; λ_C, λ_H are the inductance loss factors of the filters, the reciprocals of their Q.

The forced oscillations in the AVS that operates in the object (see Fig. 1) are described by the following system of equations:

$$\begin{aligned} m''\ddot{x}'' + k''(1 + j\lambda'')(1 + K_f)(x'' - x'') &= f'', \\ m''\ddot{x}'' + k''(1 + j\lambda'')(1 + K_f)(x'' - x'') + k''(1 + j\lambda'')x'' &= 0, \end{aligned} \qquad (3)$$

where $f'' = f_0''e^{j\omega t}$; $x = x_0 e^{j(\omega t + \varphi)}$, and the transfer ratio of the control circuit K_f is defined by expression (1).

We evaluate the effectiveness of vibration damping as usual on the basis of the coefficient $A(\Omega) = f/f^{H}$ of transfer of force to the rigid base via an elastic element in a system with one degree of freedom. It is natural to take the measure of effectiveness of active vibration damping as the absolute value of the ratio of frequency responses $|A(\Omega)/A^{a}(\Omega)|$ of the passive and active systems in the vibration frequency band. For a system of wide-band vibrocompensation [Ref. 1] when $\Omega \gg 1$ (considerably above resonance) the effectiveness is equal to

$$|A/A_{\text{к}}^{a}| = B_{\text{к}} = |\bar{k}_0/\bar{k}_{\text{экв}}| = 1/(1 - K_{f\text{к}}), \qquad (4)$$

where $\bar{k}_0$ is the complex stiffness of the system without control ("regular" stiffness) that ensures resistance to low-frequency oscillations, and $\bar{k}_{\text{экв}} = \bar{k}_0(1 - K_{f\text{к}})$ is the equivalent stiffness in the vibration band.

We evaluate the effectiveness of stabilization from the reduction in displacements of the mass or deformations of the spring caused by the low-frequency actions (when $\Omega \ll 1$). The effectiveness can be expressed as

$$B_c = |\bar{k}_{\text{экн}}/\bar{k}| = 1 + K_{fc}, \qquad (5)$$

where $\bar{k}$ is the complex stiffness of the system without control, which is taken as clearly lower than the "regular" stiffness, and therefore, generally speaking, does not ensure stability to low-frequency perturbations; $\bar{k}_{\text{экн}} = \bar{k}(1 + K_{fc})$ is the equivalent stiffness in the low-frequency band.

The effectiveness of vibration damping with an AVS having control of both types can be expressed by the quantity

$$B = \bar{k}(1 + K_{fc})/\bar{k}(1 - K_{f\text{к}}) = (1 + K_{fc})/(1 - K_{f\text{к}}), \qquad (6)$$

i. e. as the product of expressions (4) and (5). Stabilization should ensure the required stiffness on low frequencies, i. e. the equality

$$|\bar{k}|(1 + K_{fc}) = |\bar{k}_0|. \qquad (7)$$

Vibrocompensation must ensure the minimum equivalent stiffness $\bar{k}(1-K_{fH})$ in the frequency band of vibration that is permissible with the existing constraints in the low-frequency band. Assuming condition (7), equation (6) characterizes the effectiveness of this AVS as compared with the "regular" passive vibration damper (with stiffness $\bar{k}_0$).

Duality of the two control systems. Let z_1, z_2 be the impedances of the arms of the simplest L-shaped low-frequency or high-frequency filter (LFF and HFF). If $z_1/(z_1+z_2)$ is the transfer function of the HFF, then $z_2/(z_1+z_2)$ is the transfer function of the LFF with the same elements but connected in reverse order. In accordance with expression (1), the equivalent stiffness $\bar{k}(1+K_f)$ that appears in equations (3) can be expressed and transformed as follows:

for vibrocompensation:

$$\bar{k}_{\text{экв}} = \bar{k}\left(1 - K_{f\text{к}}\frac{z_1}{z_1+z_2}\right) = \bar{k}(1-K_{f\text{к}})\left(1 + \frac{K_{f\text{к}}}{1-K_{f\text{к}}}\frac{z_2}{z_1+z_2}\right), \quad (8)$$

for stabilization

$$\bar{k}_{\text{экн}} = \bar{k}\left(1 + K_{fc}\frac{z_2}{z_1+z_2}\right) = \bar{k}(1+K_{fc})\left(1 - \frac{K_{fc}}{1+K_{fc}}\frac{z_1}{z_1+z_2}\right). \quad (9)$$

Comparing expressions (8) and (9) we find that the AVS with compensation is equivalent to a stabilizing AVS in which the stiffness of the elastic element is equal to $\bar{k}(1-K_{fH})$, while the transfer ratio in the low-frequency region is $K_{fH}/(1-K_{fH})$; the AVS with stabilization is equivalent to a compensating AVS in which the stiffness of the elastic element is equal to $\bar{k}(1+K_{fC})$, while the transfer ratio in the vibration band is $K_{fC}/(1+K_{fC})$. This enables us to investigate stabilizing systems by methods presented in Ref. 1, 2, but with consideration of the dependence of the stiffness of the equivalent elastic element on the transfer ratio K_{fC}.

Both kinds of control can also be represented in a form such as vibrocompensation

$$\bar{k}_{\text{экв}} = \bar{k}\left(1 + K_{fc}\frac{z_{2c}}{z_{1c}+z_{2c}} - K_{f\text{к}}\frac{z_{1\text{к}}}{z_{1\text{к}}+z_{2\text{к}}}\right) =$$
$$= \bar{k}(1+K_{fc})\left(1 - \frac{K_{fc}}{1+K_{fc}}\frac{z_{1c}}{z_{1c}+z_{2c}} - \frac{K_{f\text{к}}}{1+K_{fc}}\frac{z_{1\text{к}}}{z_{1\text{к}}+z_{2\text{к}}}\right), \quad (10)$$

where z_{iC}, z_{iH} are the elements of two different filters (LFF and HFF in the initial expression, HFF in the final expression). The equivalent transfer ratio (last two terms) has a "two-step" frequency response.

Expression (10) implies directly that if the limiting frequencies are sufficiently separated, the controls can be treated separately, each in its own band. Frequency separation is useful for reducing the mutual influence of frequency distortions, and also for getting away from resonant frequencies. If on the other hand the limiting frequencies are equal and in this connection the elements of filters z_{iC} and z_{iH} are identical, the equivalent control has the simplest form

$$\bar{k}_{\text{экв}} = \bar{k}(1 + K_{fc})\left(1 - \frac{K_{fc} + K_{fн}}{1 + K_{fc}} \frac{z_1}{z_1 + z_2}\right),$$

that is analogous to AVS that have already been studied with a single HFF in the control channel.

Object with one degree of freedom and aperiodic links. As $m^{H} \to \infty$ (massive object to be isolated) the motion of the system is described by a single equation (we consider viscous friction):

$$m^{п}\ddot{x}^{п} + 2\delta(1 + K_f)\dot{x}^{п} + k^{п}(1 + K_f)x^{п} = f^{п}, \quad 2\delta = \lambda\omega_0^2/\omega = \text{const}.$$

In the case of control of form (1) with aperiodic links in both channels, a characteristic polynomial of fourth order is obtained. Let the initial system relative to which the effectiveness is calculated have stiffness $k_0^{П}$ and natural frequency ω_0. Assigning fixed values of 2δ, T_C, T_H (in dimensionless form), we will vary the stiffness $k^{П}$ and frequency ω_0 of the object, each time selecting the value of K_{fC} in accordance with the equality $k^{П}(1 + K_{fC}) = k_0^{П}$, thus ensuring the same resistance to low-frequency perturbations as in the initial system. Using the Routh-Hurwitz criterion, we find the value of K_{fH} on the boundary of stability. The results of calculation of effectiveness B according to (6) are summarized in Table 1.

TABLE 1

ω_0	T_c	$T_н$	K_{fc}	$K_{fн}$	B, дБ
1	—	0,05	0	0,83	5,9
0,75	0,1	0,05	0,79	0,73	6,6
0,5	0,1	0,05	3	0,27	5,5

NOTE: дБ = dB

These data show that combined control gives a greater effect than each form of control separately.

Object with two degrees of freedom and oscillatory links (LCR filters). The conditions of greatest effectiveness of an AVS in an object with two degrees of freedom were investigated in accordance with system of equations (3) and control characteristic (2). With consideration of stability requirements and vibration damping requirements, the values of Ω_C, Ω_H were chosen in a range of 0.5-5. The problem of obtaining the maximum value of B was formulated. An investigation of this kind was done for shock-mounted mechanisms in which the suspensions must not be noticeably deformed over a relatively wide range of low-frequency perturbations $\Omega = 0\text{-}0.5$ (i. e. up to half of the resonant frequency), while vibration damping must be effective in the range of $\Omega \gg 5$. The ratio $\alpha^{ИН} = m^{И}/m^{H} = 0.01$ and partial frequency $\Omega_H = \sqrt{k^{н}/m^{н}}/\omega_0 = 100$ correspond to a massive and rigid foundation.

The Nyquist stability conditions as applied to the function of the reciprocal relation [Ref. 4] had the following form:

$$\operatorname{Re} T(j\Omega) = \{[\operatorname{Re} K_f(\Omega_i)(b_i - \lambda^{\text{н}}\lambda) - \operatorname{Im} K_f(\Omega_i)(\lambda^{\text{н}} + \lambda b_i)](b_i - \lambda^{\text{н}}\lambda - \Omega_i^2 a_i) + [\operatorname{Re} K_f(\Omega_i)(\lambda^{\text{н}} + \lambda b_i) + \operatorname{Im} K_f(\Omega_i)(b_i - \lambda^{\text{н}}\lambda)](\lambda^{\text{н}} + \lambda b_i - \Omega_i \lambda^{\text{н}})\}/[(\lambda^{\text{н}} + \lambda b_i - \Omega_i^2 \lambda^{\text{н}})^2 + (b_i - \lambda^{\text{н}}\lambda - \Omega_i^2 a_i)^2] > -1, \quad (11)$$

where $a_i = 1 - \Omega_i^2/\Omega_{\text{н}i}^2$; $b_i = 1 - (1 + \alpha_{\text{нн}})\Omega_i^2/\Omega_{\text{н}}^2$, $\operatorname{Re} K_f(\Omega_i)$, $\operatorname{Im} K_f(\Omega_i)$ are the values of the real and imaginary parts of the transfer ratio of the control circuit on frequency Ω_i; Ω_i is the i-th root of the equation

$$[\operatorname{Re} K_f(\Omega)(\lambda^{\text{н}} + \lambda b) + \operatorname{Im} K_f(\Omega)(b - \lambda^{\text{н}}\lambda)](b - \lambda^{\text{н}}\lambda - \Omega^2 a) - [\operatorname{Re} K_f(\Omega)(b - \lambda^{\text{н}}\lambda) - \operatorname{Im} K_f(\Omega)(\Omega^{\text{н}} + \lambda b)](\lambda^{\text{н}} + \lambda b - \Omega^2 \lambda^{\text{н}}) = 0, \quad (12)$$

$$a = 1 - \Omega^2/\Omega_{\text{н}i}^2; \quad b = 1 - (1 + a_{\text{нн}})\Omega^2/\Omega_{\text{н}}^2.$$

In solving the optimization problem, the input parameters were taken as Ω_C, λ_C, Ω_H, λ_H, K_{fC}, K_{fH}, i. e. those parameters of the control circuit that are at the disposal of the designer.

As a result of investigation of the six-dimensional space (K_{fC}, K_{fH}, λ_C, λ_H, Ω_C, Ω_H) by the method of global linear programming search with subsequent gradient descent from the 5-10 sets with greatest values of B, the sets of parameters are found for which B has the greatest value, satisfying conditions (11), (12). The sets that are best in the given sense for $\lambda = 0.1$ and 0.25 are given in Table 2, where the sets with the highest value of B in each column are found after gradient descent, and correspond to the extremum value of B. Plotted in Fig. 2 are the regions of stability (on the side with shading) in the K_{fC}-K_{fH} plane for the optimum sets that correspond to the maximum value of B at different λ.

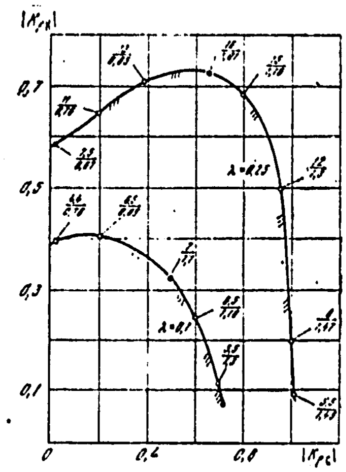

Fig. 2

The curves in Fig. 2 show the monotonic nature of the reduction in B with deviation of K_{fC} and K_{fH} from the optimum values shown on the curve by asterisks. For each of the points indicated on the boundary, the numbers show the corresponding values of the degree of damping B and the critical frequency of self-excitation of the system Ω_{KP}. The change in Ω_{KP} along both boundaries is also monotonic.

A comparison of the effectiveness of the versions of AVS considered in Ref. 1-3 with that provided by the given AVS (assuming optimum parameters of its control circuit) shows that the vibration damping effect with combined

TABLE 2

B, дБ	K/c	K/к	Ωкр	λ_c	$\lambda_к$	Ω_c	$\Omega_к$
$\lambda = 0,1$							
—	0,09	0,31	0,88	0,84	3,79	2,52	0,53
7	0,5	0,33	1,1	0,5	3,02	3,31	0,5
$\lambda = 0,25$							
—	0,37	0,52	0,94	0,84	3,79	2,52	0,53
16	0,65	0,74	1,01	0,5	0,39	2,68	0,5

NOTE: дБ = dB

control is 5 dB higher on the average than with control in the band of low-frequency perturbations or in the vibration band.

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SOME PROBLEMS OF VIBROCOMPENSATION OF ELASTIC SYSTEMS

V. A. Tikhonov, V. V. Yablonskiy

The use of additional sources of vibration energy (vibrators) in different fields of technology as elements of an active vibroprotection system leads to expansion of the possibilities of facilities for passive vibration damping of elastic structures. The problem of selecting a law of additional excitation to reduce the resultant vibration and noise of machines and mechanisms is basic in this regard, and is solved in different ways. Ref. 1, 2 describe vibration control that is accomplished by a system of automatic force control and regulation with respect to deviation with electromechanical feedback. However, the frequency band of regulation is considerably limited by the conditions of stability of automatic control in a closed arrangement.

Problems of stability are not decisive when adaptive systems with optimzlizing control are used in an open arrangement. Self-contained control of vibrators by optimizer-computing devices assumes assignment of an algorithm of adaptive search for the extremum state of the object. The theory of possible optimum control in this case has not been adequately developed. The solution with respect to L. S. Pontryagin's maximum principle for simple elastic systems has either not been found in explicit form [Ref. 3] or else requires a considerable volume of mathematical operations with a relatively simple model for realization [Ref. 4, 5]. In Ref. 6 the law of excitation of vibration sources is determined with respect to energy efficiency although the resultant solution may also be non-optimum in the generally accepted sense. The integral condition of finding the solution enables us to find effective compensation of the given field of vibrations only in local zones of the control object. The question of the minimum necessary number of discrete vibration sources remains open.

This article gives the results of computer modeling of some algorithms of vibrocompensation in linear elastic systems. Estimates are made on the necessary number of vibrators depending on the given "complexity" of the initial vibration field and the required compensation effectiveness. The possibility of force control by a reduced number of vibrators is discussed.

Formulation of the problem. Let the motion of the elastic system be described by a system of equations of the form

$$L_{\text{к}}\ddot{u}^a + L_{\text{п}}u^a + L_{\text{д}}u^a = F^a. \tag{1}$$

Here $u^a(t,x,y,z)$ is the vector function of spatial displacement of a point; $F^a(t,x,y,z) = \{f_1^a, f_2^a, \ldots, f_n^a\}$ is the vector function that assigns the distribution of forces from the controlling vibrators; $L_{\text{к}}$, $L_{\text{п}}$, $L_{\text{д}}$ are linear symmetric differential operators with given boundary conditions; $L_{\text{к}}$, $L_{\text{п}}$ are gradients of the quadratic functional of kinetic energy and potential energy of elastic deformation respectively; $L_{\text{д}}$ is a dissipative operator that is taken as equal to $L_{\text{д}} = \mu L_{\text{п}} \partial/\partial t$ for internal friction in the material. We will take the forces F^a as controlling forces, the function $F^a(t,x,y,z)$ on time segment $[t_0, t_{\text{к}}]$ being a piecewise-continuous function of time and coordinates defined in the region of permissible controls: $F^a \in D_f$.

We write the criterion of quality of vibration control as

$$Q(F^a) = \| u_{\text{к}}^a(F^a) - u^0 \|, \tag{2}$$

where the norm is taken in a functional Hilbert space in which the energy scalar product is given; u^0 is the given distribution of vibration at time t_0. In contrast to Ref. 4, the quality function here depends implicitly on the controlling forces.

The problem of optimum control of vibration is formulated as follows. Among all the admissible controls $F^a \in D_f$ that transform the elastic system from the given initial state u^0 to the required final state $u^a - u^0_K$, it is necessary to find the one such that

$$Q(F^a) = \min_{F^a \in D_f}.$$

After selecting as the initial state of the system one such that the sources of forces F^a are disconnected, the control with vibrocompensation is reduced to minimization of the norm of the resultant vibration, i. e. from the mathematical standpoint to determination of the minimum mean-square deviation in the case of linear oscillations [Ref. 7] for the vibration field excited by the controlling forces away from the initial field excited by some unknown load F.

An important class of vibration controls is harmonic control, which is often realized in practice. It is easy to show that in the general case this control is not optimum (with respect to the condition in Ref. 3). For instance for a girder of fixed cross section the optimum control found in Ref. 4 would be

$$f^a_{\text{опт}}(t, x) = \frac{1}{2\gamma} v(t, x),$$

where v(t,x) is the solution of the homogeneous equation of flexural oscillations of a girder with the corresponding boundary conditions, and condition

$$\dot{v}(t_K, x) = (2\alpha/m)\,[u^a(t_K, x) - u_0].$$

Here γ and α are the parameters of the quality functional; m is the linear mass of the girder.

Expansion of the solution in a Fourier time series leads to an expression for the control in the form

$$f^a_{\text{опт}}(t, x) = \frac{1}{2\gamma} \sum_1^\infty (C_i \cos \omega_i t + D_i \sin \omega_i t)\, X_i(x),$$

where ω_i are eigenvalues; $X_i(x)$ is the eigenform of the initial boundary value problem, and the coefficients C_i and D_i in the general case are not equal and do not vanish independently.

Restricting the class of admissible controls, let us formulate the problem of minimizing the quality function with harmonic control depending on the distribution and magnitude of forces f^a. Let us consider motion of a system in the class of harmonic oscillations with frequency ω, and controlling forces as quasistationary harmonics of the same frequency, but with different amplitude and phase on finite time intervals (transient processes are excluded from consideration): where x^0, x^a, F^a are complex functions of coordinates. Then, eliminating the time term in (1)

$$[(1 + j\lambda) L_n - \omega^2 L_K]\, x^a = f^a, \quad (3)$$

where λ is the loss factor (taken as frequency-independent), we can get the expression $x^a = K(f^a)$ (K is the influence function). It is required to minimize quality function (2), which we write for instance for one-dimensional systems (Fig. 1) as

$$Q_1 = \left[\int_0^1 |x^0(\xi) - x^a(\xi)|^2 d\xi\right]^{1/2}, \qquad (4)$$

or with respect to the criterion of the maximum of the modulus

$$Q_2 = \max_\xi |x^0(\xi) - x^a(\xi)|. \qquad (5)$$

Here ξ is the dimensionless coordinate

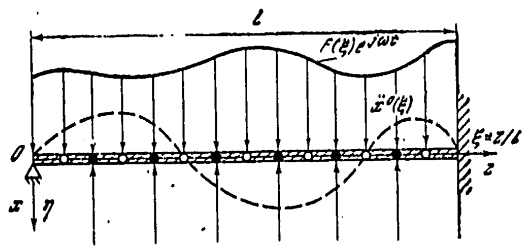


Fig. 1

Estimation of the required number of vibrators. Let us determine the number n of vibrators as sources of concentrated controlling forces $f^a = \sum_{i=1}^{n} f_i^a \delta(\xi - \xi_i)$, arbitrarily distributed with respect to length (area) such that with proper selection of the magnitudes of forces f_i^a, the norm of the function of the resultant vibration distribution $[Q_1^*]^2 = (\overline{x^0 - x^a}, x^0 - x^a) = \|x^0 - x^a\|^2$ differs from the norm of the initial distribution function $[Q_1^0]^2 = (\overline{x^0}, x^0) = \|x^0\|^2$ by no more than a set number of times (or by B decibels):

$$20\lg(Q_1^*/Q_1^0) \leqslant B_1,$$

or

$$[Q_1^*]^2/[Q_1^0]^2 \leqslant 10^{-B_1/10} = B_{1\Lambda}. \qquad (6)$$

Expanding all functions appearing in (3) in a series with respect to eigenfunctions X_l ($l = 1, 2, \ldots$) of a homogeneous operator with corresponding boundary conditions, and using the condition of orthonormalization with respect to kinetic energy

$$(L_\kappa X_l, X_m) = \delta_{lm}, \quad l, m = 1, 2, \ldots, \qquad (7)$$

where δ_{lm} is the Kronecker delta, we get the relations

$$x^0(\xi) = \sum_1^\infty \alpha_l X_l(\xi), \quad \beta_l = p_l/L_\kappa[(1 + j\lambda)\omega_l^2 - \omega^2]. \qquad (8)$$

Here β_l and p_l are the coefficients of expansion of the functions x^a and f^a respectively; ω_l are the natural frequencies; L_H is the operation of multiplication by the function.

With consideration of (7) and (8) we have

$$|Q_1^0|^2 = \left(\sum_l (\bar{\alpha}_l - \bar{\beta}_l) X_l, \sum_s (\alpha_s - \beta_s) X_s\right) = \sum_l [(\bar{\alpha}_l - \bar{\beta}_l)(\alpha_l - \beta_l)(X_l, X_l)] = \sum_l |\alpha_l - \beta_l|^2 = \sum_l \frac{|[(1+j\lambda)\omega_l^2 - \omega^2]\alpha_l - \tilde{p}_l|^2}{|(1+j\lambda)\omega_l^2 - \omega^2|^2},$$

where $\tilde{p}_l = p_l/L_H$.

Without loss of generality, let us set $|Q_1^0|^2 = \|x^0\|^2 = 1$. Then for systems with L_H = const we get

$$\tilde{p}_l = (\tilde{f}^a, X_l) = \sum_{i=1}^{n} \tilde{f}_i^a X_l(\xi_i)$$

and inequality (6) takes the form

$$\sum_{l=1}^{\infty} \frac{|[(1+j\lambda)\omega_l^2 - \omega^2]\alpha_l - \sum_{i=1}^{n} \tilde{f}_i^a X_l(\xi_i)|^2}{(\omega_l^2 - \omega^2)^2 + \lambda^2\omega^4} \leqslant B_{1a}. \qquad (9)$$

Having 2n arbitrary variants in the choice of amplitude and phase of the controlling forces, we determine the latter from the system of equations

$$\sum_{i=1}^{n} \tilde{f}_i^a X_l(\xi_i) = [(1 + j\lambda)\omega_l^2 - \omega^2]\alpha_l. \qquad (10)$$

Here the subscript l may represent an arbitrary set of n compensated modes. Then the residual sum is estimated by the inequality

$$\sum_{l=n+1}^{\infty} \left| \frac{\sum_i \tilde{f}_i^a X_l(\xi_i)}{(1+j\lambda)\omega_l^2 - \omega^2} \right|^2 \leqslant B_{1a} \qquad (11)$$

and characterizes the necessity of minimizing the work of the controlling forces on displacements of the component modes of order higher than n.

In a one-dimensional elastic system let a vibrational mode be excited on frequency ω that has no more than r nodes (points of inflection) with approximation by the polynomial

$$\Phi_r(\xi) = \sum_{l=1}^{r} c_l X_l(\xi)$$

in virtue of the fact that the system of eigenfunctions is a Chebyshev system. The mean-square approximation theory implies that with reference to (11) it is necessary to select the points of application and magnitude of $\tilde{f}_i^a(\xi)$ so that the function of best approximation $x^a(\xi)$ in the sense of mean square deviation from $x^0(\xi)$ is orthogonal to modes with number greater than n. If uniform approximation of the functions x^n to x^0 is required, the formulated problem can be reduced to a problem of interpolation. The values of the amplitude and phase of the forces f_i^n (2n values) are selected so that the resultant vibrations at the n points vanish:

$$(x^0 - x^n)\big|_{\xi=\xi_k} = 0, \quad k = 1, 2, \ldots, n,$$

or, substituting the expansions with respect to eigenfunctions

$$\sum_{l=1}^{r} a_l X_l(\xi_k) - \sum_{l=1}^{\infty} \frac{\sum_i f_i^n X_l(\xi_i)}{(1+i\lambda)\,\omega_l^2 - \omega^2} X_l(\xi_k) = 0,$$

we arrive at the equivalent problem of resolution of system of equations (10). Then we can use existing estimates of the error of interpolation by Chebyshev polynomials [Ref. 7]. Then with uniform approximation by power-law polynomials we get the estimate

$$|x^0(\xi) - L_n(\xi)| \leqslant M_{n+1}/2^{2n+1}(n+1)!,$$

where the $L_n(\xi)$ are Lagrange polynomials of order n; the interpolation points are selected as roots of the Chebyshev polynomial

$$M_{n+1} = \sup_{\xi\in(0,1);r} |a_r X_r^{(n+1)}(\xi)|.$$

Requiring that the square of the maximum error not exceed a predetermined degree of compensation, we get an estimate of the form

$$[M_{n+1}/2^{2n+1}(n+1)!]^2 \leqslant B_{1n}\|x^0\|^2.$$

In the case of uniform approximation in the class of trigonometric eigenfunctions, the least deviation from trigonometric polynomials $H_n(T)$ conforms to the estimate [Ref. 7]:

$$|x^0(\xi) - T_n(\xi)| < CM/2\pi n.$$

Here M is the Lipschitz factor:

$$|x^0(\xi_1) - x^0(\xi_2)| \leqslant M|\xi_1 - \xi_2|;$$

C is an absolute constant defined as

$$C = \frac{\pi^4\left(\frac{2}{\pi^2} + \int_0^{\pi/2} \frac{\sin^4\xi}{\xi^3}d\xi\right)}{4\int_0^{\pi/2} \frac{\sin^4\xi}{\xi^4}d\xi} = \frac{3\pi^7[\ln 2 + \mathrm{Ci}(\pi) - \mathrm{Ci}(2\pi)]}{8\{2\pi^3 - 4 + \pi^3[2\,\mathrm{Si}(2\pi) - \mathrm{Si}(\pi)]\}}.$$

By analogy with the preceding we get an estimate of the form

$$[CM/2\pi n]^2 \leqslant B_{1n}\|x^0\|^2.$$

In the case of arbitrary eigenfunctions we make reference to relation (11), which we simplify with consideration of the constraints on the controls: $|\tilde{f}_i^0| \leq \tilde{F}^0_{\text{доп}}$ $(i = 1, 2, \ldots, n)$. Using the notation

$$G_n = \sup_l \sum_{l=1}^{n} \tilde{f}_l^0 X_l(\xi_l),$$

and considering that for beam functions $|X_l(\xi_l)| \leq 3/2$ and $G_n \leq 3n\tilde{F}^0_{\text{доп}}/2$, we have the inequality

$$G_n^2 \sum_{l=n+1}^{\infty} [(\omega_l^2 - \omega^2)^2 + \lambda^2\omega^4]^{-1} < B_{1A}. \qquad (12)$$

Numerical analysis was done with respect to expression (12) transformed for vibroaccelerations:

$$\sum_{l=n+1}^{\infty} \{[\beta l)^4 - 1]^2 + \lambda^2\}^{-1} < G_n^{-2} 10^{-B_1/10}. \qquad (12a)$$

Here we use the notation $(\beta l)^4 = \omega_l^2/\omega^2$ and assume that $\beta = \pi(\omega_0/\omega)^{1/2}$ when $\lambda = 0.05$ and $\omega_0/\omega = 0.028$, the frequency of forced oscillations being between the first and second natural frequencies. The residual sum of the series reduced to 200 terms as a function of the parameter β is shown in Fig. 2 (20 terms of the series are sufficient for $\beta = 0.526$). With an increase in the frequency of excitation ω, a reduction in stiffness or an increase in the inertness of the system, i. e. with a reduction in the parameter β, a moment arrives when condition (12a) cannot be satisfied and thus achieve some compensation of vibrations.

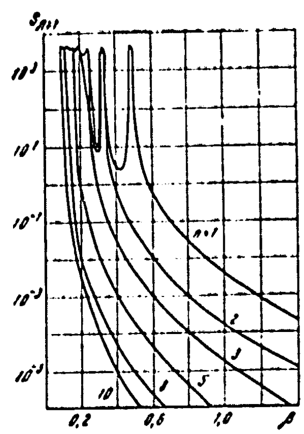


Fig. 2

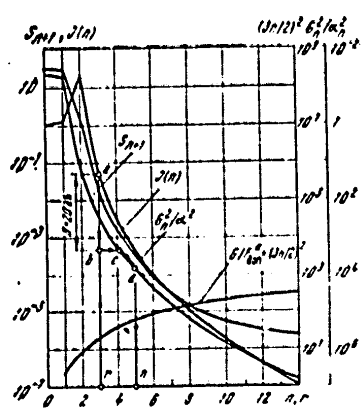


Fig. 3

An investigation was made of the feasibility of replacing the residual sum by an improper integral which, with accuracy to terms of no more than the second order of smallness of the loss factor λ is equal to

$$I(n) = \int_{n+1}^{\infty} \frac{dx}{(\beta^4 x^4 - 1)^2 + \lambda^2} = \frac{1}{16\beta\left(1 + \frac{7}{8}\lambda^2\right)}\left[\pi\left(3 + \frac{4}{\lambda} + \frac{7}{32}\lambda^2\right) + \right.$$
$$+ \frac{4}{\lambda}\operatorname{arctg}\frac{2b(n+1)}{(n+1)^2 - c} + \left(3 + \frac{7}{32}\lambda^2\right)\operatorname{arctg}\frac{2a(n+1)}{(n+1)^2 - c} +$$
$$+ \left(\frac{3}{2} + \frac{7}{64}\lambda^2\right)\ln\frac{(n+1)^2 - 2a(n+1) + c}{(n+1)^2 + 2a(n+1) + c} +$$
$$+ \frac{2}{\lambda}\ln\frac{(n+1)^2 - 2b(n+1) + c}{(n+1)^2 + 2b(n+1) + c} -$$
$$\left. - \begin{cases} \pi\left(3 + \frac{4}{\lambda} + \frac{7}{32}\lambda^2\right) & \text{when } (n+1)^2 > c \\ 0 & \text{when } (n+1)^2 < c \end{cases}\right],$$
$$a = \frac{1}{\beta}\left(1 + \frac{3}{32}\lambda^2\right), \quad b = \frac{\lambda}{4\beta}, \quad c = a^2 + b^2 = \frac{1}{\beta^2}\left(1 + \frac{\lambda^2}{4}\right).$$

Calculations show that for $\beta > 1/4$ a uniform increase is observed in the value of the integral over the residual sum (Fig. 3).

Let us note that depending on the number of the mode to be compensated

$$G_n = \sup_l \sum_{i=1}^{n} \bar{f}_i^{\,*} X_l(\xi_i) \leqslant |(1 + j\lambda)\omega_r^2 - \omega^2|\,\alpha_r.$$

The graph for the dependence of $G_n^2/\alpha_n^2 = |(\beta n)^4 - 1|^2$ on n is plotted (see Fig. 3) so that the product of the value of the given expression multiplied by S_{n+1} or I(n) for identical coordinates is equal to unity. Then to determine the number of vibrators necessary for compensation of r harmonics with a given degree of damping B, a segment B must be laid off downward from the point on the curve of G_n^2/α_n^2 with abscissa r, and the nearest point (on the increasing side) with abscissa equal to the integer n must be found from the corresponding ordinate on the curve of S_{n+1} or I(n). On Fig. 3 the moves are shown by the points a - b - c - d.

The resultant estimates are easily generalized to two-dimensional systems. For instance in the case of small flexural oscillations of a thin elastic plate, we have in (3)

$$L_n = D\Delta\Delta, \quad L_\kappa = \rho h, \quad f^*(\xi, \eta) = \sum_{i=1}^{n}\sum_{k=1}^{m} f_{ik}^*\,\delta(\xi - \xi_i)\,\delta(\eta - \eta_k).$$

where D is cylindrical stiffness; ΔΔ is the Laplace operator in dimensionless coordinates ξ and η; ρh is mass density.

Then, repeating the preceding considerations for the system of eigenfunctions $X_{l\gamma}$ orthonormalized with respect to a pair of indices, we arrive at an estimate of the form

$$\sum_{l=1}^{\infty}\sum_{\gamma=1}^{\infty} \frac{\left|[(1 + j\lambda)\omega_{l\gamma}^2 - \omega^2]\,\alpha_{l\gamma} - \sum_{i=1}^{n}\sum_{k=1}^{m} \bar{f}_{ik}^{\,*} X_{l\gamma}(\xi_i, \eta_k)\right|^2}{(\omega_{l\gamma}^2 - \omega^2)^2 + \lambda^2\omega^4} \leqslant B_{l\alpha}.$$

Let a vibrational mode be set up in the plate with r nodes relative to coordinate ξ and s nodes relative to coordinate η. Then after determining the amplitude and phase of the forces from the system of equations

$$\sum_{i=1}^{n}\sum_{k=1}^{m}\tilde{f}_{ik}^{a}X_{l\gamma}(\xi_i,\eta_k)=[(1+j\lambda)\omega_{l\gamma}^{2}-\omega^{2}]\,a_{l\gamma}\qquad(l=1,2,\ldots,n;\quad \gamma=1,2,\ldots,m)$$

under conditions that n > r and m > s it is necessary to check satisfaction of the inequalities

$$\sum_{l=n+1}^{\infty}\sum_{\gamma=m+1}^{\infty}\left|\frac{\sum_{i=1}^{n}\sum_{k=1}^{m}\tilde{f}_{ik}^{a}X_{l\gamma}(\xi_i,\eta_k)}{(1+j\lambda)\omega_{l\gamma}^{2}-\omega^{2}}\right|^{2}\leqslant B_{1\Delta}. \tag{13}$$

Relation (13), as in the case of a one-dimensional system, is the condition of minimizing the work of all forces $\tilde{f}_{ik}^{a}$ located at the points of intersection of a rectangular n × m grid on generalized displacements of modes of higher order than those characterized by the pair of numbers n and m.

Control modeling with respect to the influence matrix. Vibrocompensation in a linear elastic system assumes selection of a control F^a such that it sets up a mode close to the given X^0, but opposite in sign, i. e. the limiting condition

$$(\bar{X}^{a},X^{0})\simeq|X^{0}|^{2},$$

must be satisfied, where $F^a=\{f_k^a;\ k=1,2,\ldots,n\};\quad X^a=\{x_i^a;\ i=1,2,\ldots,m\};\ \{X^0\}=\{X^i\}$.

To determine the required vector of controlling forces, we will start from the limiting case where $X^a-X^0=0$, i. e. the case of total compensation of vibrations. Then we determine F^a from the equation

$$A\tilde{F}^{a}=\ddot{X}^{0}. \tag{14}$$

Here $A=\{a_{ik};\ i=1,2,\ldots,m;\ k=1,2,\ldots,n\}$ is a matrix (in the general case an operator-function) that in the case of harmonic oscillations is obtained as a result of formal multiplication of the elements of the influence matrix K by $-\omega^0$; the dots over the acceleration vector $\ddot{X}^0$ are a symbolic designation for amplitude (hereafter they will be omitted).

When the dimensionality of vectors X^0 and F^a coincides, i. e. the number of points of observation is equal to the number of points of unidirectional discrete control (m = n), matrix equation (14) may have a unique solution for det $A\neq 0$ and matrix rank equal to n. Usually the number of vibro-acceleration pickups installed to monitor the uniformity of compensation exceeds the number of vibrators (m > n). In the general case the system of equations may be mutually exclusive.

Let us formulate the problem as follows. It is required to select a vector F^a such that some of the equations of system (14) are satisfied identically, and the error of satisfaction of the remaining equations is determined by the condition of minimization of quality function (4). One can find an approximate solution of the entire system of equations by the method of mean squares.

Assuming that the rank of the matrix A and of the expanded matrix after the appropriate selection of points is equal to n, we write system of equations (14) in the form

$$\sum_{s=1}^{n} a_{i_k s}\tilde{f}_s^a = x_{i_k}^0, \quad \{i_k;\ k=1,2,\ldots,n\} \in \{1,2,\ldots,m\}, \tag{14a}$$

$$x_i^0 - x_i^a = x_i^0 - \sum_{s=1}^{n} a_{is}\tilde{f}_s^a, \ \{i\} = \{1,2,\ldots,m\}\setminus\{i_k\}. \tag{14b}$$

Selection of points $\{i_k\}$ -- the points where the function of the resultant distribution of vibration vanishes -- is arbitrary and is determined by the requirements of existence of a solution of equation (14a) and construction of the best approximation of x^a to x^0.

Modeling was done with application to a foundation for the shock-mounted object that consisted of two identical girders oscillating in phase. The initial vibration field was assigned either as an individual or mixed normal mode of flexural oscillations of a girder fixed and supported at the ends, or else as the result of action of a systen of concentrated forces on the shock-absorber side. The elements of the matrix were calculated for these cases by approximate computation of infinite sums of the form

$$a_{ik} = \sum_{s=1}^{\infty} \frac{X_s(\xi_i)X_s(\xi_k)}{1-\beta_s^2(1+i\lambda)}, \qquad \beta_s^2 = \frac{\omega_s^2}{\omega^2}, \qquad i=1,2,\ldots,m;$$
$$k = 1,2,\ldots,n,$$

and also in closed form in terms of Krylov functions of a complex arguemnt

$$a_{ik} = x^a(\xi_i)\big|_{\tilde{F}^a=\tilde{f}_k^a=1} = \hat{C}_{1q}S(\varkappa\xi_i) + C_{2q}T(\varkappa\xi_i) + \hat{C}_{3q}U(\varkappa\xi_i) +$$
$$+ \hat{C}_{4q}V(\varkappa\xi_i) - \varkappa V[\varkappa(\xi_i-\xi_k)], \qquad \varkappa^4 = [\beta^4(1+j\lambda)]^{-1},$$
$$\beta^4 = \omega_{0q}^2/\omega^2.$$

Here the constants $\hat{C}_{1q}$-$\hat{C}_{4q}$ were found for each segment by solving the corresponding boundary value problem.

A model was constructed for compensation of individual normal modes of the girders supported and fixed at the ends (see Fig. 1):

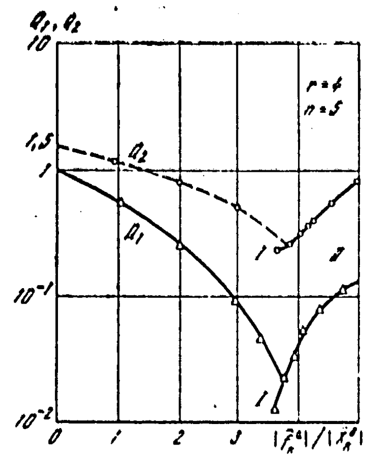
$$X_r(\xi) = \frac{\sin\beta_r\xi}{\sin\beta_r} - \frac{\operatorname{sh}\beta_r\xi}{\operatorname{sh}\beta_r}, \qquad \beta_r \approx \frac{\pi}{4}(4r+1),$$

and for mixed modes: $x_i^0 = \sum_{s=1}^{r} \dot{\alpha}_s X_s(\xi_i)$. A preliminary investigation was made of the surface of the quality function with respect to the two previously indicated criteria, preference being given among step-by-step search methods to parallel change of coordinates with a variable step

$$\Delta \tilde{F}^a = \{\Delta \mathrm{Re}\, \tilde{f}_k^a = X_r(\xi_k),\ \Delta \mathrm{Im}\, \tilde{f}_k^a = \lambda X_r(\xi_k),\ k = 1, 2, \ldots, n\}.$$

Some results of compensation of individual modes are shown in Fig. 4-6. On Fig. 4 the axis of abscissas reflects the step-by-step change in forces f^a, and the Roman numerals indicate: I--the method of influence coefficients; II--step-by-step search. A "gully" structure of the surface of the quality function is observed, i. e. there is a sharp minimum of a function of many variables, the degree of sharpness of the minimum on the surface increasing with a reduction of the number r of the mode and an increase in the number n of vibrators. The latter leads to a case of a poorly conditioned matrix A and an unstable solution I close to the global minimum of the quality function.

Fig. 4



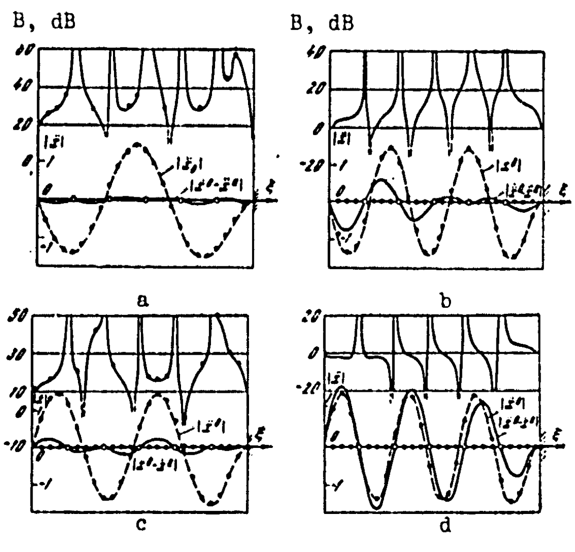


Fig. 5

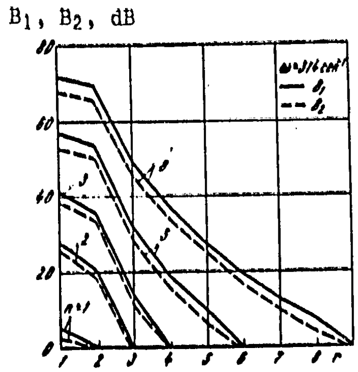
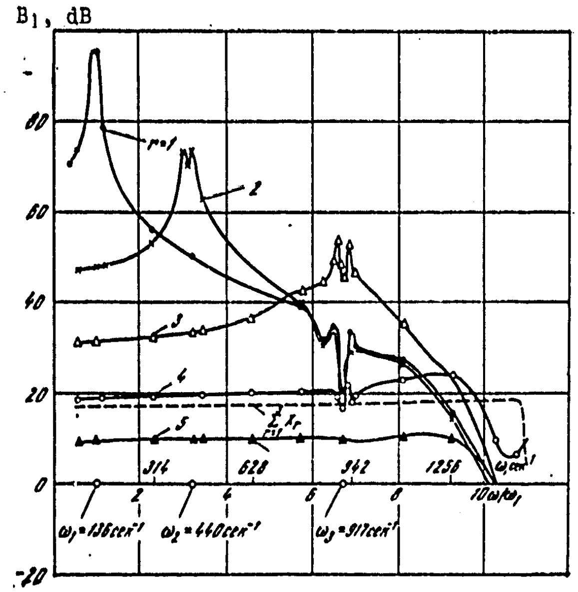


Fig. 6

Note: сек$^{-1}$ = s^{-1}



Note: сек$^{-1}$=s^{-1}

Fig. 7

The necessary number of uniformly arranged vibrators for the given degree of compensation of different modes is found from Fig. 5, 6, which corresponds to the previously given estimates. The following versions are considered in Fig. 5 for the case n = 5: a--r = 3, b--r = 4, c--r = 5, d--r = 6. The white circles denote points of installation of vibrators, the black circles denote points of observation.

The degree of compensation is shown as a function of the frequency of excitation in Fig. 7, where the first natural frequencies are shown on the axis of abscissas. On resonant frequencies and close to them an intense rise is observed in the degree of compensation of the corresponding resonant mode, and a drop for remaining modes. With an increase in frequency it is mainly higher harmonics that are excited, which leads to a reduction in the level of compensation since the latter reflects the fraction of higher uncompensated modes in the resultant vibration field. This explains the existence of a limiting frequency above which B_1 changes sign, which corresponds to excess excitation.

Force control by a reduced number of vibrators. The conventional method of vibrocompensation [Ref. 1] is based on connecting a single vibrator (or more to correspond in number to the components of the excitation force that are to be compensated) in the feedback circuit under each shock absorber, the force being developed by the vibrator $\tilde{f}_k^a$ being close in magnitude to that transmitted by the shock absorber, and opposite in phase: $f_k^a \approx -f_k$. Thus the system of compensating forces in the given case exactly reproduces the system of disturbing forces, which is ensured by setting the corresponding transfer ratio of the feedback circuit. However, in the case of a large number of shock absorbers the control system becomes too cumbersome. In particular, the number of vibrators may be much greater than the number n necessary for damping vibration of a given degree of "complexity." An investigation was made of compensation with a considerable (2-3 times) reduction in the number of vibrocompensators and retention of the same principles of control.

A computer was used to model the second method of assigning the initial field of vibrations: an examination was made of the result of excitation of a girder structural element by a system of concentrated forces transmitted to the foundation from the shock-absorber side. The forces of excitation were recalculated from the curve of vibrations (having two inflection points) taken from experimental data. An exact solution was used to calculate matrix A. It was found that the resultant initial mode is close to the first normal mode for a girder with $\omega_0/\omega = 0.117$.

In the case of force control by a smaller number of vibrators than the number of shock absorbers, the latter are connected in groups of adjacent shock absorbers with close values of the amplitudes and phases of the forces transmitted to the foundation. Corresponding to each group of adjacent shock absorbers is a single vibrator located in the middle of the group arrangement, and the control law is determined from the rule

$$\tilde{f}_k^a = -\left(\frac{\tilde{f}_{k-1}}{2} + \tilde{f}_k + \frac{\tilde{f}_{k+1}}{2}\right),$$

if the groups intersect, i. e. the same shock absorber belongs to neighboring groups and is "served" by two vibrators, and

$$\tilde{f}_k^a = -\sum_{i_k} \tilde{f}_{i_k},$$

if the groups are independent.

The solution with respect to a predetermined sum vector of the controlling force was compared with the solution by the method of influence coefficients.

It was assumed that 13 shock absorbers uniformly spaced lengthwise of the girder are broken down into groups of 2-4, depending on the number of vibrators. The results are summarized in the table.

(1) Число вибраторов n	(2) Метод коэффициентов влияния		(3) Управление по силе	
	B_1, дБ	B_2, дБ	B_1, дБ	B_2, дБ
8	77,5	69,1	38,5	38,5
6	66,6	62,8	21,1	20,1
5	62,6	57,7	27,9	27,9
4	54,9	50,7	20,4	20,3

KEY: 1--number of vibrators n
2--method of influence coefficients
3--force control
дБ = dB

Vibration control using an influence matrix provides a high degree of compensation, but this involves a larger number of vibrators than required in the case of an initial mode close to the first. The difference between the calculated vectors of controlling forces and those that are preassigned has a considerable effect with increasing n. The weak dependence of compensation on the number of vibrators in force control can be attributed to the need for more exact assignment of the forces of control with increasing n in view of the development of "gullying" of the surface of the quality function.

Thus for linear elastic systems whose natural frequencies are considerably higher than the excitation frequancy, high effectiveness of vibrocompensation can be achieved by grouping adjacent shock absorbers with little difference in amplitude and phase within the limits of the group, and by controlling vibration on the foundation with respect to the sum vector of the force. The number of vibrocompensators can be reduced to a minimum by calculating the controlling forces from an influence matrix.

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WAVE PROPAGATION OVER THIN-WALLED RODS

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One of the difficult problems of mechanics is the study of oscillations of uniform rods of arbitrary cross section. Within the framework of the linear theory of elasticity exact solutions have been found only for an elastic cylinder and a layer [Ref. 1]. The Pochhammer-Chree and Rayleigh-Lamb dispersion equations of normal waves in these structures had been derived in the last century, and have been extensively studied. These solutions have played a special part in the construction of simplified theories of calculation. The exact solution is the standard by which the approximation given by any engineering theory is evaluated.

Oscillations of strip-rods have also been fairly completely studied [Ref. 2]. The accuracy of the results found for a strip is lower than for a cylinder or a layer, and is given by the inequality

$$\lambda_t^2 \gg (2h)^2, \qquad (1)$$

where λ_t is the wavelength of a shear wave in the material; 2h is the thickness of the strip. The frequency band in which this inequality is satisfied defines the limits of applicability of the Germain-Lagrange equation of flexural oscillations of a thin plate and the dynamic equations of the plane stressed state that describe longitudinal-transverse oscillations of a strip [Ref. 3]. Nevertheless, this frequency band considerably exceeds the range of applicability of engineering theories that are valid when

$$\lambda_t \gg 2H, \qquad (2)$$

where 2H is the width of the strip (transverse dimensions of the rod). This circumstance permits us to use the solutions for a strip in the same way as a standard in comparative analysis of different approximate theories [Ref. 4].

The theory of thin elastic shells [Ref. 5] also has approximately the same accuracy. Since thin-walled rods are cylindrical thin shells, fairly exact solutions can be found for them (e. g. Ref. 5, 6) that considerably simplify the construction of simpler models [Ref. 7].

However, for rods made up of strips the results of the theory of thin shells are not applicable because of the presence of corner joints. At present such rods are calculated exclusively by approximate methods [Ref. 4, 8, 9] with accuracy bounded by inequality (2).

This article presents a theory of wave propagation over thin-walled rods formed by joining several thin strips, i. e. over rods with profiles that are made of straight-line segments. The theory accounts for all forms of motion of the strips, and it is assumed that the Germain-Lagrange equation is valid for their flexural oscillations, while the longitudinal-transverse oscillations of the strips are described by dynamic equations of the plane stressed state. Thus the limits of applicability of the results are determined by inequality (1). Moreover, longitudinal bending is disregarded. Dispersion equations of various types of normal waves are derived for rods of the most widely used cross sections (I-beam, angle iron and so on), and an investigation is made of the general properties of the resultant equations.

The method to be used for calculation is a generalization of the conventional method of dynamic stiffnesses, and is fairly clearly outlined in Ref. 10, 11. In brief, its essence reduces to the following. When a normal wave of the form $u(\xi,\eta)\exp(ikx - i\omega t)$ propagates over a rod, where k is the constant of propagation of the wave with respect to the longitudinal coordinate x, $u(\xi,\eta)$ is the displacement vector that depends on the transverse coordinates ξ and η, forces and moments of forces of reaction arise on the lines of joining of the strips that are also exponentially dependent on x. By breaking the rod down into separate strips and substituting these forces and moments for the interaction of these strips, the initial problem can be reduced to several problems on forced oscillations of the strips under the action of forces and torques that are exponentially distributed along the edges. As shown in Ref. 10, the solutions of these problems can be written in a compact form that is convenient for further computations. This can be achieved by means of the concept of linear dynamic stiffness which is the ratio of the linear density of force (torque) to displacement (angle of turn) when they are exponentially dependent on the coordinate along the line of application of the force (moment). The junction of such solutions along the lines of their joining leads to dispersion equations of normal waves in the given thin-walled rod.

1. *An elastic strip as an element of a thin-walled rod.* The linear dynamic stiffnesses of the strip that are needed for further calculations can be

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obtained by solving the homogeneous equations of the plane stressed state and Germain-Lagrange equations when exponentially distributed forces and torques act on the edges of the strip. Some of these quantities were calculated in Ref. 10. Derivation of the others is analogous, and therefore we give here only the final results.

The following right-handed orthogonal system is chosen for the strip: the x-axis is directed along the strip and coincides with its middle line, the y-axis also lies in the plane of the strip, and the z-axis is normal to the strip. The thickness of the strip is equal to 2h, and the width is 2H. The characteristics of the material of the strip: E, ν are the Young modulus and Poisson ratio, $E_1 = E/(1-\nu^2)$, $E_2 = E/2/(1+\nu)$ are the longitudinal and shear moduli of a thin plate.

The following expressions can be obtained for the linear dynamic bending stiffnesses of the strip:

symmetric excitation

$$B_{tt}^{c} = D\theta_c/H^3\delta_c, \quad B_{t\alpha}^{c} = -B_{\alpha t}^{c} = De_c/H^2\delta_c, \quad B_{\alpha\alpha}^{c} = D\gamma_c/H\delta_c,$$
$$|B^{c}| = D^2\lambda_c/H^4\delta_c, \tag{3}$$

antisymmetric excitation

$$B_{tt}^{a} = D\theta_a/H^3\delta_a, \quad B_{t\alpha}^{a} = -B_{\alpha t}^{a} = De_a/H^2\delta_a, \quad B_{\alpha\alpha}^{a} = D\gamma_a/H\delta_a,$$
$$|B^{a}| = D^2\lambda_a/H^4\delta_a, \tag{4}$$

asymmetric excitation

$$B_{tt} = D\theta/H^3\delta, \quad B_{t\alpha} = -B_{\alpha t} = De/H^2\delta, \quad B_{\alpha\alpha} = D\gamma/H\delta,$$
$$|B| = D^2\lambda_0/H^4\delta. \tag{5}$$

The indices "c" and "a" denote symmetric and antisymmetric.

Acting in the case of symmetric excitation on the edges of the strip $y = \pm H$ are distributed forces $F_z^c(+H)\exp(ikx - i\omega t) = F_z^c(-H)\exp(ikx) - i\omega t)$ and torques $M_z^c(+H)\exp(ikx - i\omega t) = -M_z^c(-H)\exp(ikx - i\omega t)$. Solution of the problem of forced oscillations gives displacement and angles of turn on the edges $w(+H) = w(-H)$ and $\alpha(H) = -\alpha(-H)$. These quantities are reciprocally related by a matrix of linear dynamic stiffnesses

$$\begin{bmatrix} F_z^c(H) \\ M_x^c(H) \end{bmatrix} = \begin{bmatrix} B_{tt}^c & B_{t\alpha}^c \\ B_{\alpha t}^c & B_{\alpha\alpha}^c \end{bmatrix} \begin{bmatrix} w(H) \\ \alpha(H) \end{bmatrix}. \tag{6}$$

From here on we omit the repeated exponential factor $\exp(ikx - i\omega t)$, leaving only the amplitude values of the quantities.

Forces $F_z^a(H) = -F_z^a(-H)$ and torques $M_x^a(H) = M_x^a(-H)$ act in the case of antisymmetric excitation on the edges of a strip. In the case of asymmetric excitation, experimentally distributed force $F_z(H)$ and torque $M_x(H)$ act on the edge $y=+H$, while edge $y=-H$ is stress-free. Formulas (4), (5) give expressions for the elements of the matrices of linear dynamic stiffnesses that are defined by matrix model (6). In these $D=2E_1h^3/3$ denotes the cylindrical stiffness of the plate, and the Greek letters as in (3) denote the first members of the dispersion equations of normal flexural waves in a strip with different boundary conditions on the edges, for which the expressions take the following form:

for symmetric waves

$$\delta_c = \alpha \operatorname{sh}\alpha \operatorname{ch}\beta - \beta \operatorname{sh}\beta \operatorname{ch}\alpha, \quad \gamma_c = 2\mu_0^2 \operatorname{ch}\alpha \operatorname{ch}\beta,$$
$$\varepsilon_c = r\alpha \operatorname{sh}\alpha \operatorname{ch}\beta - s\beta \operatorname{sh}\beta \operatorname{ch}\alpha, \quad \theta_c = -2\mu_0^2 \alpha\beta \operatorname{sh}\alpha \operatorname{sh}\beta, \quad (7)$$
$$\lambda_c = r^2\alpha \operatorname{sh}\alpha \operatorname{ch}\beta - s^2\beta \operatorname{sh}\beta \operatorname{ch}\alpha,$$

for antisymmetric waves

$$\delta_a = \alpha \operatorname{ch}\alpha \operatorname{sh}\beta - \beta \operatorname{ch}\beta \operatorname{sh}\alpha, \quad \gamma_a = 2\mu_0^2 \operatorname{sh}\alpha \operatorname{sh}\beta,$$
$$\varepsilon_a = r\alpha \operatorname{ch}\alpha \operatorname{sh}\beta - s\beta \operatorname{ch}\beta \operatorname{sh}\alpha, \quad \theta_a = -2\mu_0^2 \alpha\beta \operatorname{ch}\alpha \operatorname{ch}\beta, \quad (8)$$
$$\lambda_a = r^2 \alpha \operatorname{ch}\alpha \operatorname{sh}\beta - s^2 \beta \operatorname{ch}\beta \operatorname{sh}\alpha,$$

for asymmetric clamping

$$\delta = \delta_c\lambda_a + \delta_a\lambda_c + \gamma_c\theta_a + \gamma_a\theta_c + 2\varepsilon_c\varepsilon_a = 2\alpha\beta rs + (\alpha^2r^2 + \beta^2s^2) \operatorname{sh}2\alpha \operatorname{sh}2\beta - \alpha\beta(r^2+s^2) \operatorname{ch}2\alpha \operatorname{ch}2\beta,$$
$$\gamma = 2(\gamma_c\lambda_a + \gamma_a\lambda_c) = 2\mu_0^2(\alpha r^2 \operatorname{ch}2\alpha \operatorname{sh}2\beta - \beta s^2 \operatorname{sh}2\alpha \operatorname{ch}2\beta),$$
$$\varepsilon = 2(\varepsilon_c\lambda_a + \varepsilon_a\lambda_c) = \alpha\beta rs(r+s)(1-\operatorname{ch}2\alpha \operatorname{ch}2\beta) + (\alpha^2r^3 + \beta^2s^3) \operatorname{sh}2\alpha \operatorname{sh}2\beta,$$
$$\theta = 2(\theta_c\lambda_a + \theta_a\lambda_c) = 2\mu_0^2 \alpha\beta(\beta s^2 \operatorname{ch}2\alpha \operatorname{sh}2\beta - \alpha r^2 \operatorname{sh}2\alpha \operatorname{ch}2\beta,$$
$$\lambda_0 = 4\lambda_c\lambda_a = 2\alpha\beta r^2s^2(1-\operatorname{ch}2\alpha \operatorname{ch}2\beta) - (\alpha^2r^4 + \beta^2s^4) \operatorname{sh}2\alpha \operatorname{sh}2\beta. \quad (9)$$

In formulas (7) and (8) the boundary conditions are the same on both edges, while formula (9) corresponds to a strip that is stress-free on edge $y=-H$. The letters in (7)-(9) denote the following boundary conditions on edge $y=H$: δ--clamped edge ($w=\partial w/\partial y=0$); γ--hinged ($w=M_x=0$); ε--unrealized clamps ($w=F_z=0$, $\partial w/\partial y=M_x=0$); θ--sliding clamp ($\partial w/\partial y=F_x=0$); λ--free edge ($M_x=F_x=0$). Moreover, the following notation has been introduced: $\mu_0=k_0H$; $k_0^4=\rho\omega^2/D$, $\lambda=kH$; $\alpha^2=\lambda^2-\mu_0^2$; $\beta^2=\lambda^2+\mu_0^2$; $r=\beta^2-\nu\lambda^2$; $s=\alpha^2-\nu\lambda^2$.

Let us note that relations of the form

$$\gamma_{c,a}\theta_{c,a} + \varepsilon_{c,a}^2 = \lambda_{c,a}\delta_{c,a}$$

hold among the dispersion equations for waves of different types.

We have analogous expressions for longitudinal-transverse linear dynamic stiffnesses:

for symmetric excitation

$$C_{ll}^{c} = 2T\eta_c/\Delta_c, \quad C_{ln}^{c} = -C_{nl}^{c} = 2T\chi_c/\Delta_c, \quad C_{nn}^{c} = 2T\tau_c/\Delta_c,$$
$$|C^{c}| = 2T^2\Lambda_c/\Delta_c, \tag{10}$$

for antisymmetric excitation

$$C_{ll}^{a} = 2T\eta_a/\Delta_a, \quad C_{ln}^{a} = -C_{nl}^{a} = 2T\chi_a/\Delta_a, \quad C_{nn}^{a} = 2T\tau_a/\Delta_a,$$
$$|C^{a}| = 2T^2\Lambda_a/\Delta_a, \tag{11}$$

for asymmetric excitation

$$C_{ll} = 2T\eta/\Delta, \quad C_{ln} = -C_{nl} = 2T\chi/\Delta, \quad C_{nn} = 2T\tau/\Delta,$$
$$|C| = 2T^2\Lambda/\Delta. \tag{12}$$

The first members of expressions (10) are the elements of a matrix defined by the equality

$$\begin{bmatrix} F_x^c(+H) \\ F_y^c(+H) \end{bmatrix} = \begin{bmatrix} C_{ll}^c & C_{ln}^c \\ C_{nl}^c & C_{nn}^c \end{bmatrix} \begin{bmatrix} u_x^c(+H) \\ u_y^c(+H) \end{bmatrix},$$

where u_x, u_y are displacements along the x and y axes, while the forces on the edges are related by the expressions $F_x^a(-H) = F_y^a(H)$ and $F_x^a(-H) = -F_y^a(H)$.

Expressions (11) are obtained for external forces that satisfy the equalities $F_x^a(H) = -F_x^a(-H)$, $F_y^a(H) = F_y^a(-H)$, while expressions (12) correspond to the case $F_x(-H) = F_y(-H) = 0$. In these expressions $T = 2hE_2/H$ and the Greek letters denote the first members of the dispersion equations of longitudinal-transverse normal waves in a strip with different methods of fastening the edges:

for symmetric waves

$$\Delta_c = b \operatorname{ch} \alpha_l \operatorname{sh} \alpha_t - c \operatorname{sh} \alpha_l \operatorname{ch} \alpha_t, \quad \tau_c = -\mu_t^2\alpha_t \operatorname{ch} \alpha_l \operatorname{ch} \alpha_t,$$
$$\chi_c = i\lambda\,(b \operatorname{ch} \alpha_l \operatorname{sh} \alpha_t - a \operatorname{sh} \alpha_l \operatorname{ch} \alpha_t), \quad \eta_c = -\mu_l^2\alpha_l \operatorname{sh} \alpha_l \operatorname{sh} \alpha_t,$$
$$\Lambda_c = a^2 \operatorname{sh} \alpha_l \operatorname{ch} \alpha_t - bc \operatorname{ch} \alpha_l \operatorname{sh} \alpha_t, \tag{13}$$

for antisymmetric waves

$$\Delta_a = b \operatorname{sh} \alpha_l \operatorname{ch} \alpha_t - c \operatorname{ch} \alpha_l \operatorname{sh} \alpha_t, \quad \tau_a = -\mu_t^2\alpha_t \operatorname{sh} \alpha_l \operatorname{sh} \alpha_t,$$
$$\chi_a = i\lambda\,(b \operatorname{sh} \alpha_l \operatorname{ch} \alpha_t - a \operatorname{ch} \alpha_l \operatorname{sh} \alpha_t), \quad \eta_a = -\mu_l^2\alpha_l \operatorname{ch} \alpha_l \operatorname{ch} \alpha_t,$$
$$\Lambda_a = a^2 \operatorname{ch} \alpha_l \operatorname{sh} \alpha_t - bc \operatorname{sh} \alpha_l \operatorname{ch} \alpha_t, \tag{14}$$

for asymmetric clamping

$$\Delta = \Delta_c\Lambda_a + \Delta_a\Lambda_c + 2(\tau_c\eta_a + \tau_a\eta_c) + 4\chi_c\chi_a =$$
$$= b\,(a^2 + c^2) \operatorname{ch} 2\alpha_l \operatorname{ch} 2\alpha_t - c\,(a^2 + b^2) \operatorname{sh} 2\alpha_l \operatorname{sh} 2\alpha_t - 2abc,$$

$$\tau = 2(\tau_c\Lambda_a + \tau_a\Lambda_c) = -\mu_t^2\alpha_l(a^2 \operatorname{ch} 2\alpha_l \operatorname{sh} 2\alpha_t - bc \operatorname{sh} 2\alpha_l \operatorname{ch} 2\alpha_t),$$
$$\chi = 2(\chi_c\Lambda_a + \chi_a\Lambda_c) = i\lambda\,[ab(a + c)(\operatorname{ch} 2\alpha_l \operatorname{ch} 2\alpha_t - 1) - (a^3 + b^2c) \operatorname{sh} 2\alpha_l \operatorname{sh} 2\alpha_t],$$
$$\eta = 2(\eta_c\Lambda_a + \eta_a\Lambda_c) = -\mu_t^2\alpha_t(a^2 \operatorname{sh} 2\alpha_l \operatorname{ch} 2\alpha_t - bc \operatorname{ch} 2\alpha_l \operatorname{sh} 2\alpha_t),$$
$$\Lambda = 4\Lambda_c\Lambda_a = (a^4 + b^2c^2) \operatorname{sh} 2\alpha_l \operatorname{sh} 2\alpha_t - 2a^2bc(\operatorname{ch} 2\alpha_l \operatorname{ch} 2\alpha_t - 1). \quad (15)$$

Introduced into the formulas is the notation $\mu_l = k_lH$; $\mu_t = k_tH$, $k_l^2 = \rho\omega^2/E_2$; $k_t^2 = \rho\omega^2/E_1 = k_l^2(1 - \nu)/2$; $\alpha_l^2 = \lambda^2 - \mu_l^2$; $\alpha_t^2 = \lambda^2 - \mu_t^2$; $a = 2\lambda^2 - \mu_t^2$; $b = 2\alpha_l\alpha_t$; $c=2\lambda^2$. In formulas (13)-(15), the boundary conditions on edge $y = H$ are as follows: Δ--stationary edge ($u_x = u_y = 0$); τ--edge with transverse guides ($u_x = u_{yy} = 0$); χ--unrealized clamps ($u_x = \sigma_{xy} = 0$ and $u_y = \sigma_{yy} = 0$); η--sliding clamp ($u_y = \sigma_{xy} = 0$); Λ--stress-free edge ($\sigma_{xy} = \sigma_{yy} = 0$). The dispersion equations for longitudinal-transverse waves satisfy the relations

$$2(\tau_{c,a}\eta_{c,a} + \chi_{c,a}^2) = \Delta_{c,a}\Lambda_{c,a}.$$

It should be noted that the boundary conditions $u_x = \sigma_{xy} = 0$, $u_y = \sigma_{yy} = 0$, $w = F_z = 0$, $\partial w/\partial y = M_x = 0$ cannot be realized in practice. They themselves are satisfied on individual frequencies in strips with other boundary conditions [Ref. 2]. It can be shown that boundary value problems with such conditions contradict the Lagrange variational principle. Implied here are the formal solutions of these problems that are used for the convenience of further calculations.

The investigation of wave propagation over a rod of H cross section also requires the values of linear dynamic stiffnesses of a strip free from forces on the edges under the action of forces $F_x(0)$, $F_y(0)$, $F_z(0)$ and torque $M_x(0)$ exponentially distributed along its middle line. To derive their expressions it is convenient to represent such a strip in the form of two identical strips connected by the edges, and to use the results already found. After simple transformations, we can get the following relations for this case:

for bending vibrations:

$$\begin{bmatrix} F_z(0) \\ M_x(0) \end{bmatrix} = \begin{bmatrix} 2B_{tt} & 0 \\ 0 & 2B_{\alpha\alpha} \end{bmatrix} \begin{bmatrix} w(0) \\ \alpha(0) \end{bmatrix}. \quad (16)$$

for longitudinal-transverse vibrations

$$\begin{bmatrix} F_x(0) \\ F_y(0) \end{bmatrix} = \begin{bmatrix} 2C_{ll} & 0 \\ 0 & 2C_{nn} \end{bmatrix} \begin{bmatrix} u_x(0) \\ y_y(0) \end{bmatrix}. \quad (17)$$

where the "half-strip" linear dynamic stiffnesses B_{tt}, $B_{\alpha\alpha}$, C_{ll} and C_{nn} that correspond to asymmetric excitation are given in (5), (12). That matrices (16) and (17) are diagonal is a consequence of the symmetry of the structure: symmetric forces $F_z(0)$ and $F_x(0)$ cause only symmetric responses $w(0)$ and

$u_x(0)$, while antisymmetric actions $M_x(0)$ and $F_y(0)$ cause antisymmetric responses $\alpha(0)$ and $u_y(0)$.

Thus, all linear dynamic stiffnesses of the strip are expressed by simple relations (3)-(15) in terms of dispersion equations of normal waves in a strip with different boundary conditions on the edge. This circumstance simplifies analysis of dispersion equations of waves in composite rods made up of several strips.

The results given here can be used for calculating other homogeneous structures in which an elastic strip is a composite element [Ref. 10, 11].

2. *Dispersion equation of normal waves in a thin-walled angle-iron rod.* Let us first consider an angle-iron rod (Fig. 1). It consists of two strips secured together at the edges at a right angle. Let us assign the index "1" to quantities relating to the lower strip (wall), and the index "2" to the upper strip (flange). Let us select two systems of coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2) so that the axes x_1 and x_2 coincide with the middle lines of the strips and are directed toward the same side, while the axes y_1 and y_2 lie in the plane of the strips and are directed toward the line of joining.

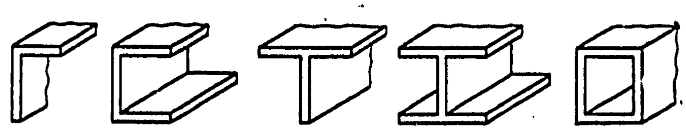

Fig. 1-5

When a normal wave of the form $u(y_1,y_2)\exp(ikx - i\omega t)$ propagates along the rod, forces of reaction arise on the line of joining of the strips that have non-zero components along all three coordinate axes, and a reactive bending moment that has one non-zero component along the x_i-axis. By taking the strips apart and substituting these reactive forces and torque for the interaction between the strips, we arrive at a problem of forced oscillations of a strip under the action of external forces and bending moments exponentially distributed along one of the edges. Their solutions are written in the form

$$\begin{bmatrix} F_z^{(i)}(H_i) \\ M_x^{(i)}(H_i) \end{bmatrix} = \begin{bmatrix} B_{ii}^{(i)} & B_{i\alpha}^{(i)} \\ B_{\alpha i}^{(i)} & B_{\alpha\alpha}^{(i)} \end{bmatrix} \begin{bmatrix} w^{(i)}(H_i) \\ \alpha^{(i)}(H_i) \end{bmatrix},$$
$$\begin{bmatrix} F_x^{(i)}(H_i) \\ F_y^{(i)}(H_i) \end{bmatrix} = \begin{bmatrix} C_{ii}^{(i)} & C_{in}^{(i)} \\ C_{ni}^{(i)} & C_{nn}^{(i)} \end{bmatrix} \begin{bmatrix} u_x^{(i)}(H_i) \\ u_y^{(i)}(H_i) \end{bmatrix}, \tag{18}$$

where $i = 1, 2$.

Linear relations exist between the forces and torques and also between the displacements and angles of turn of the strips on the line of joining. Since there are no external actions, the sum of the reactive forces and moments must obviously be equal to zero. In the selected systems of coordinates this relation is written as follows:

$$F_z^{(1)}(H_1) - F_y^{(2)}(H_2) = 0, \qquad F_x^{(1)}(H_1) + F_x^{(2)}(H_2) = 0,$$
$$M_y^{(1)}(H_1) + M_y^{(2)}(H_2) = 0, \qquad F_y^{(1)}(H_1) + F_z^{(2)}(H_2) = 0. \qquad (19)$$

The relations between the displacements and angles of turn are a consequence of the rigid joining of the strips: they should be the same for both strips. In the selected coordinates, this condition takes the following form:

$$w^{(1)}(H_1) = -u_y^{(2)}(H_2), \qquad u_x^{(1)}(H_1) = u_x^{(2)}(H_2),$$
$$\alpha^{(1)}(H_1) = \alpha^{(2)}(H_2), \qquad u_y^{(1)}(H_1) = w^{(2)}(H_2). \qquad (20)$$

Now substituting the forces and torques from (18) in equalities (19) and substituting the displacements and angle of turn relating to the first strip in accordance with (20) for the corresponding values for the second strip, we get the homogeneous system of linear equations

$$(C_{ll}^{(1)} + C_{ll}^{(2)})u_x^{(1)} + C_{ln}^{(1)}u_y^{(1)} - C_{ln}^{(2)}w^{(1)} = 0,$$
$$C_{nl}^{(1)}u_x^{(1)} + (C_{nn}^{(1)} + B_{ll}^{(2)})u_y^{(1)} + B_{l\alpha}^{(2)}\alpha^{(1)} = 0,$$
$$-C_{nl}^{(2)}u_x^{(1)} + (B_{ll}^{(1)} + C_{nn}^{(2)})w^{(1)} + B_{l\alpha}^{(1)}\alpha^{(1)} = 0,$$
$$B_{\alpha l}^{(2)}u_y^{(1)} + B_{\alpha l}^{(1)}w^{(1)} + (B_{\alpha\alpha}^{(1)} + B_{\alpha\alpha}^{(2)})\alpha^{(1)} = 0. \qquad (21)$$

In order for this system of equations to have a nontrivial solution, i. e. in order for a normal wave to propagate over the rod, its determinant must be equal to zero,

$$\begin{vmatrix} C_{ll}^{(1)} + C_{ll}^{(2)} & C_{ln}^{(1)} & -C_{ln}^{(2)} & 0 \\ C_{nl}^{(1)} & C_{nn}^{(1)} + B_{ll}^{(2)} & 0 & B_{l\alpha}^{(2)} \\ -C_{nl}^{(2)} & 0 & B_{ll}^{(1)} + C_{nn}^{(2)} & B_{l\alpha}^{(1)} \\ 0 & B_{\alpha l}^{(2)} & B_{\alpha l}^{(1)} & B_{\alpha\alpha}^{(1)} + B_{\alpha\alpha}^{(2)} \end{vmatrix} = 0. \qquad (22)$$

And this is indeed the dispersion equation of normal waves in an angle-iron rod that establishes the relation between the constant of propagation k, the angular frequency ω and the parameters of the rod.

In solving system of equations (21) in the usual way, one can find the displacements and angle of turn on the edge $y_1 = H_1$ of the first strip, and by using relations (20) -- the displacements and angle of turn on edge $y_2 = H_2$ of the second strip. On the basis of these data, one can easily find the displacements, angles of turn, stresses and all other quantities that characterize the propagation of normal waves along the rod. However, in the following discussion we will be interested mostly in the dispersion properties of normal waves, limiting ourselves to the investigation of dispersion equations of form (22), since it is the dispersion properties that determine the peculiarities of propagation of waves along infinite rods, as well as the spectral properties of natural oscillations of finite rods.

Dispersion equation (22) is the determinant of the sum of the matrices of linear dynamic stiffnesses of the individual strips relative to the forces and torques acting along the line of joining. This matrix sum is equal to the matrix of linear dynamic stiffnesses of the joining of the strips, i. e. of the entire angle-iron rod. We can convince ourselves of this if instead of (19) we assume that the sum of the reactive forces and torques is equal to the external forces and torque, and then use relations (18) and (20). Thus the dispersion equation can be interpreted as the zero value of the determinant of the matrix of linear dynamic stiffnesses of the given rod as determined on the line of joining of its component strips. This assumption is common to all composite rods considered here. It is analogous to the condition of resonance of oscillations of mechanical systems (the determinant of the matrix of dynamic stiffnesses is equal to zero), the only difference being that linear dynamic stiffnesses stand in place of the conventional dynamic stiffnesses, and instead of resonant oscillations of the system, normal waves are considered that have the capacity to propagate freely (without external forces) over an infinite composite rod.

3. *Channel-iron with identical flanges and T-rod.* The cross sections of these two types are shown in Fig. 2, 3. They are typified by a plane of mirror symmetry. It is known [Ref. 12] that in a mechanical system with such symmetry oscillations of two types exist -- symmetric and antisymmetric relative to this plane. They are independent of one another and can be studied separately.

First let us consider symmetric oscillations of channel iron with identical flanges. We select two systems of coordinates: (x_1,y_1,z_1) associated with the wall, and (x_2,y_2,z_2) associated with the upper flange; the coordinate axes are oriented in the same way as for the angle-iron rod in the preceding section. Since we are considering only symmetric movements, the oscillations of the lower flange will by the symmetric mirror image of the oscillations of the upper flange. Thus it is sufficient to consider the interaction of the wall with the upper flange alone, but keeping in mind only symmetric motions of the wall.

As in the case of the angle-iron rod, reactive forces (three components) and a reactive torque (one component) arise on the line of joining of the strips. The relation between them and the corresponding displacements for the flange in the given case is the same as in the angle-iron rod. For the wall in symmetric vibration, this relation is expressed by formulas (6). Then repeating all considerations and calculations of the preceding section we can arrive at homogeneous system (21) and dispersion equation (22) in which the linear dynamic stiffnesses of the wall calculated for excitation on the edge $y = +H$ are replaced by the corresponding linear dynamic stiffnesses of the wall with symmetric excitation.

The same results are reached by consideration of normal waves that are antisymmetric relative to the mirror plane, the only difference being that linear dynamic stiffnesses with antisymmetric excitation are taken for the wall.

Thus the dispersion equations of two modes of normal waves (symmetric and antisymmetric) in a channel iron with identical flanges take the form of a zero determinant of fourth order (22) in which the index "1" indicates the linear dynamic stiffnesses of the wall with symmetric or antisymmetric excitation. The symmetric case corresponds to longitudinal waves, while the antisymmetric case corresponds to associated flexural-torsional waves.

The T-rod consisting of two strips (see Fig. 3) also has a plane of mirror symmetry that passes through the middle plane of the wall. Therefore in this case as well we can consider symmetric and antisymmetric movements independently of one another. As before, the index "1" will be assigned to the wall, and the index "2" -- to the flange. The height of the wall is $2H_1$, and the width of the flange is $4H_2$ (double flange).

Let us select two systems of coordinates (x_1,y_1,z_1) and (x_2,y_2,z_2). We associate the first with the wall, the x_1-axis being combined with the middle line of the wall, while the y_1-axis is directed toward the flange in the plane of the strip. We associate the second coordinate system with the flange in such a way that the x_2-axis coincides with the line of joining of the wall with the flange and is directed toward the same side as the x_1-axis, while the z_2-axis coincides with the y_1-axis. The z_1 and y_2 axes are directed so that both systems are right-handed.

When a symmetric normal wave propagates along the T-rod on the line of joining of the strips, forces of reaction $F_x^{(1)}(H_1)$ and $F_y^{(1)}(H_1)$ arise that act on the first strip; $F_x^{(2)}(0)$ and $F_z^{(2)}(0)$ that act on the flange along its middle line. The third component of force and the bending moment of reaction do not arise since they cause antisymmetric motions. The sum of the forces of reaction is equal to zero:

$$F_x^{(1)}(H_1)+F_x^{(2)}(0)=0, \qquad F_y^{(1)}(H_1)+F_z^{(2)}(0)=0, \tag{23}$$

while the displacements of the strips on the line of their rigid joining should be identical:

$$u_x^{(1)}(H_1)=u_x^{(2)}(0), \qquad u_y^{(1)}(H_1)=w^{(2)}(0). \tag{24}$$

The relation between the forces of reaction and corresponding displacements for the wall is written in the usual way:

$$\begin{bmatrix} F_x^{(1)}(H_1) \\ F_y^{(1)}(H_1) \end{bmatrix} = \begin{bmatrix} C_{ll}^{(1)} & C_{ln}^{(1)} \\ C_{nl}^{(1)} & C_{nn}^{(1)} \end{bmatrix} \begin{bmatrix} u_x^{(1)}(H_1) \\ u_y^{(1)}(H_1) \end{bmatrix}. \tag{25}$$

and for the flange this relation is found from formulas (16) and (17)

$$\begin{bmatrix} F_x^{(2)}(0) \\ F_z^{(2)}(0) \end{bmatrix} = \begin{bmatrix} 2C_{ll}^{(2)} & 0 \\ 0 & 2B_{ll}^{(2)} \end{bmatrix} \begin{bmatrix} u_x^{(2)}(0) \\ w^{(2)}(0) \end{bmatrix}. \tag{26}$$

Let us recall that the flange here is treated as a doubled strip of width $4H_2$, while the linear stiffnesses $C_{ll}^{(2)}$ and $B_{ll}^{(2)}$ apply to a flange of width $2H_2$.

Substituting relations (25) and (26) in equality (23) and using (24), we get the following dispersion equation for symmetric normal waves in a T-rod:

$$\begin{vmatrix} C_{ll}^{(1)} + 2C_{ll}^{(2)} & C_{ln}^{(1)} \\ C_{nl}^{(1)} & C_{nn}^{(1)} + 2B_{ll}^{(2)} \end{vmatrix} = 0. \quad (27)$$

When a normal wave antisymmetric relative to the plane of mirror symmetry propagates along the given rod, a reaction force $F_z^{(1)}(H_1)$ and reaction torque $M_x^{(1)}(H_1)$ arise on the line of joining that act on the wall, as well as a force of reaction $F_y^{(2)}(0)$ and a reactive moment $M_x^{(2)}(0)$ that act on the central line of the flange. In the selected coordinate systems the condition of absence of resultants is written as

$$F_z^{(1)}(H_1) - F_y^{(2)}(0) = 0, \qquad M_x^{(1)}(H_1) + M_x^{(2)}(0) = 0, \quad (28)$$

and the conditions of rigid joining of the strips take the form

$$w^{(1)}(H_1) = -u_y^{(2)}(0), \qquad \alpha^{(1)}(H_1) = \alpha^{(2)}(0). \quad (29)$$

The relation between forces and displacements is expressed in terms of linear bending stiffnesses

$$\begin{bmatrix} F_z^{(1)}(H_1) \\ M_x^{(1)}(H_1) \end{bmatrix} = \begin{bmatrix} B_{ll}^{(1)} & B_{l2}^{(1)} \\ B_{2l}^{(1)} & B_{22}^{(1)} \end{bmatrix} \begin{bmatrix} w^{(1)}(H_1) \\ \alpha^{(1)}(H_1) \end{bmatrix}. \quad (30)$$

For the flange, this relation is found from formulas (16) and (17)

$$\begin{bmatrix} F_y^{(2)}(0) \\ M_x^{(2)}(0) \end{bmatrix} = \begin{bmatrix} 2C_{nn}^{(2)} & 0 \\ 0 & 2B_{22}^{(2)} \end{bmatrix} \begin{bmatrix} u_y^{(2)}(0) \\ \alpha^{(2)}(0) \end{bmatrix}. \quad (31)$$

Proceeding in the usual way, i. e. substituting relations (30) and (31) in equalities (28) and taking consideration of conditions (29), we can get the sought dispersion equation of antisymmetric normal waves in a T-rod in the form of a zero determinant of second order

$$\begin{vmatrix} B_{ll}^{(1)} + 2C_{nn}^{(2)} & B_{l2}^{(1)} \\ B_{2l}^{(1)} & B_{22}^{(1)} + 2B_{22}^{(2)} \end{vmatrix} = 0. \quad (32)$$

Equation (27) is the dispersion equation of longitudinal normal waves in the T-rod, and equation (32) is the equation for flexural-torsional normal waves.

4. *Box-beam rod and I-beam with identical flanges.* Rods of this type are shown in Fig. 4, 5. These are typified by the presence of two mutually

perpendicular planes of mirror symmetry. As a consequence of this fact, in these rods there are normal waves of four modes that exist independently of one another: longitudinal, i. e. symmetric relative to both planes of symmetry; two modes of flexural waves that are symmetric relative to one of the planes and antisymmetric relative to the other; and torsional waves that are characterized by antisymmetric motion relative to both planes of symmetry.

Let us consider first the box-beam rod (see Fig. 5). This consists of four pairwise identical strips connected in four corners. In view of the symmetry it is sufficient to consider the interaction of two strips in one of the corner joints, since the motion and interaction in the other joinings will be repeated symmetrically or antisymmetrically depending on the mode of the normal waves. Let us take two strips, e. g. the upper one and the one on the left, that form the upper left corner joint. Let us construct a coordinate system for each of these strips so that the x_1 and x_2 axes coincide with the middle lines, while the y_1 and y_2 axes lie in the plane of the strips and are directed toward the line of their intersection.

When a normal wave of any of the four modes enumerated above propagates in the angle joining, forces of reaction with three components arise as well as a reaction bending moment with one component (along the x-axis). In the selected coordinate system the relations between them, and also the relations for the displacements and angles of turn of the strips will coincide in accuracy with the analogous relations for an angle-iron rod (19) and (20). However, the relation between the forces and displacements for each of the strips is different from (18), and will be expressed in terms of symmetric and antisymmetric linear dynamic stiffnesses. The derivation of the necessary formulas and the final results repeat the computations for an angle-iron rod. The dispersion equations of the four wave modes in the box-beam rod are four zero determinants (22) in which linear stiffnesses with symmetric or antisymmetric excitation stand in place of the linear dynamic stiffnesses of the strips with asymmetric excitation. Thus in examining the longitudinal waves for both strips it is necessary to take symmetric linear stiffnesses. In the case of torsional waves all linear stiffnesses in equation (22) must be taken for antisymmetric excitation. Two cases when symmetric linear stiffnesses are taken for one of the strips in equation (22) while antisymmetric stiffnesses are taken for the other correspond to independent flexural waves in the horizontal and vertical planes.

Now let us go on to the I-beam rod (see Fig. 4). It consists of a single vertical strip (wall) and two horizontal double strips (flanges) that are taken as identical. Thanks to the presence of mirror symmetry, here as in the preceding case we can consider only one joining of the strips, e. g. the joining of the wall with the upper flange. In the lower joining everything will take place symmetrically or antisymmetrically relative to the horizontal plane of symmetry. If we take two coordinate systems -- one in the wall and the other in the flange, just as for a T-rod, the further derivation of dispersion equations here will exactly repeat the derivation done for the T-rod. The only difference is that for the wall it is necessary to take symmetric or antisymmetric linear dynamic stiffnesses. As a result we get the following dispersion equations of the four normal wave modes of the I-beam rod (for the sake of convenience the index "1" is omitted and the index "2" is replaced by a stroke):

for longitudinal waves:

$$\begin{vmatrix} C^c_{ll} + 2C'_{ll} & C^c_{ln} \\ C^c_{nl} & C^c_{nn} + 2B'_{ll} \end{vmatrix} = 0,$$

or in expanded form

$$|C^c| + C^c_{nn}(2C'_{ll}) + C^c_{ll}(2B'_{ll}) + (2C'_{ll})(2B'_{ll}) = 0, \quad (33)$$

where the first term is the determinant of the matrix of linear stiffnesses of the wall in the case of symmetric excitation by longitudinal-transverse forces;

for flexural waves in the plane of the wall

$$\begin{vmatrix} C^a_{ll} + 2C'_{ll} & C^a_{ln} \\ C^a_{nl} & C^a_{nn} + 2B'_{ll} \end{vmatrix} = 0,$$

or in expanded form

$$|C^a| + C^a_{ll}(2B'_{ll}) + C^a_{nn}(2C'_{ll}) + (2C'_{ll})(2B'_{ll}) = 0, \quad (34)$$

where $|C^a|$ is the determinant of the matrix of linear stiffnesses of the wall in the case of antisymmetric longitudinal-transverse excitation for flexural waves in the plane of the flanges

$$\begin{vmatrix} B^c_{ll} + 2C'_{nn} & B^c_{la} \\ B^c_{al} & B^c_{aa} + 2B'_{aa} \end{vmatrix} = 0,$$

or in expanded form

$$|B^c| + B^c_{ll}(2B'_{aa}) + B^c_{aa}(2C'_{nn}) + (2B'_{aa})(2C'_{nn}) = 0, \quad (35)$$

where $|B^c|$ is the determinant of the matrix of flexural linear stiffnesses of the wall in the case of symmetric excitation;

for torsional waves

$$\begin{vmatrix} B^a_{ll} + 2C'_{nn} & B^a_{la} \\ B^a_{al} & B^a_{aa} + 2B'_{aa} \end{vmatrix} = 0,$$

or in expanded form

$$|B^a| + B^a_{ll}(2B'_{aa}) + B^a_{aa}(2C'_{nn}) + (2B'_{aa})(2C'_{nn}) = 0, \quad (36)$$

where $|B^u|$ is the determinant of the matrix of flexural linear dynamic stiffnesses of the wall in the case of antisymmetric excitation on the edges.

5. *General properties of dispersion equations.* Each root of the dispersion equation gives a value of the constant of propagation of a normal wave, and therefore determines its dependence on the longitudinal coordinate of the rod. The real roots correspond to normal waves that propagate over the rod without damping, while the imaginary roots correspond to waves that are exponentially damped with respect to the x coordinate, all points of the rod fluctuating in phase. The complex roots correspond to traveling waves with amplitudes that increase or decrease exponentially with respect to x. Waves of these types have already been encountered in the study of flexural and longitudinal-transverse oscillations of a thin elastic strip [Ref. 2]. They all arise in composite thin-walled rods as well.

Dispersion equations (22), (27), (32)-(36), after substitution of the corresponding expressions for linear dynamic stiffnesses are transcendental equations in which the variable $\lambda = kH$ is squared everywhere. This means that if λ is a root of the equation, then $-\lambda$ is also a root.

The first members of the equations are real quantities, and for each of them the equalities $f(\bar{\lambda}) = \overline{f(\lambda)} = f(\lambda)$ are satisfied (the line denotes the complex conjugate). In other words, if λ is a root of the equation $f(\lambda) = 0$, then the complex conjugate $\bar{\lambda}$ will also be a root of the equation $f(\bar{\lambda}) = 0$.

Thus the real and imaginary roots of the derived dispersion equations are always met in pairs $\pm\lambda$, while the complex roots occur in groups of four -- $\pm\lambda$, $\pm\bar{\lambda}$.

We can easily go on and convince ourselves that since only rational and exponential functions of λ and $(\lambda^2 \pm \mu^2)^{1/2}$ appear in the first members of these equations, they are entire functions of first order. On the basis of general theorems of the theory of analytical functions [Ref. 13], such functions have an infinite (even) number of zeros that cannot have points of crowding in any finite part of the complex plane. This implies that in each of the rods considered above there is an infinite number of normal waves, and that the modulus of the constant of propagation increases monotonically with an increase in the number of the normal wave.

Let us now prove the following statement: on any frequency there exists a finite number of real and imaginary roots and an infinite number of complex roots of the dispersion equations investigated here.

For the proof we consider the behavior of the dimensionless constant of propagation λ of waves with high numbers whose modulus is much greater than unity, and the parameters μ_ξ^2, μ_ζ^2 and μ^2.

To do this, we expand the first members of the dispersion equations in series with respect to the small quantities $(\mu_\eta^2|2\lambda)$, $(\mu_\zeta^2|2\lambda)$, $(\mu^2|2\lambda)$ and taking only the first terms, we investigate their asymptotic behavior. The expansion is a cumbersome operation, and therefore all intermediate results are left out. As an example, we give the expansions of the linear dynamic stiffnesses of a strip with asymmetric excitation:

for flexural waves

$$\begin{aligned} &B_{tt} \approx 4D\lambda^3 (e \operatorname{sh} 4\lambda - 2\lambda)/\Delta_1 H^3, \\ &B_{ta} = -B_{at} \approx D(1-\nu)\lambda^2 [2de \operatorname{ch} 4\lambda - (4\lambda)^2]/\Delta_1 H^2, \\ &B_{aa} \approx -4D\lambda (e \operatorname{sh} 4\lambda + 4\lambda)/\Delta_1 H, \\ &|B| \approx -D^2 (1-\nu)^2 \lambda^4 [2e^2 \operatorname{ch} 4\lambda - (4\lambda)^2]/\Delta_1 H^4, \\ &\Delta_1 = 2e \operatorname{ch} 4\lambda + (4\lambda)^2; \end{aligned} \tag{37}$$

for longitudinal-transverse waves

$$\begin{aligned} &C_{tt} \approx 16hE_2\lambda \operatorname{sh} 4\lambda/(1+\nu) H\Delta_2, \\ &C_{tn} = -C_{nt} \approx 2ihE_2\lambda [2g \operatorname{ch} 4\lambda + (4\lambda)^2]/H\Delta_2, \\ &C_{nn} \approx 16hE_2\lambda \operatorname{sh} 4\lambda/(1+\nu) H\Delta_2, \\ &|C| \approx 16h^2E_2^2\lambda^2 [2 \operatorname{ch} 4\lambda - (4\lambda)^2]/H^2\Delta_2, \\ &\Delta_2 = 2f \operatorname{ch} 4\lambda + (4\lambda)^2. \end{aligned} \tag{38}$$

Here $d = (1+\nu)/(1-\nu)$, $e = (3+\nu)/(1-\nu)$, $f = (3-\nu)/(1+\nu)$, $g = (1-\nu)/(1+\nu)$. The expansions of linear dynamic stiffnesses of the strip for symmetric and antisymmetric excitations have the same form and order with respect to λ.

It is immediately clear from these formulas that at large $|\lambda|$ the flexural linear stiffnesses exceed the longitudinal-transverse stiffnesses. In other words for short waves the bending of the strips is the decisive form of motion, and the longitudinal-transverse stiffnesses in the dispersion equations can be disregarded in comparison with the flexural linear stiffnesses.

For instance let us consider dispersion equations (33) and (34) for longitudinal and flexural waves in an I-beam rod. Substituting in these equations the asymptotic formulas for linear stiffnesses and disregarding quantities $C^c_{nn}\lambda^a$ and $C^c_{ln}\lambda^a$ as having order of smallness λ^{-2} in comparison with B_{tt}, we can reduce them to the form $(C^c_{ll}{}^a + 2C^t_{ll})B^t_{tt} = 0$, or in expanded form

$$\begin{aligned} &(e' \operatorname{sh} 2\lambda' - 2\lambda')\ [a_1 \operatorname{ch} 2\lambda \operatorname{ch} 2\lambda' + a_2 \operatorname{sh} 2\lambda \operatorname{sh} 2\lambda' + \\ &+ a_3 (2\lambda')^2 \operatorname{ch} 2\lambda] = 0, \end{aligned} \tag{39}$$

where the a_j are constants, $j = 1, 2, 3$. This expression obviously decomposes into two independent equations. The first is an asymptotic dispersion equation of symmetric flexural waves in the flange that are not influenced by longitudinal-transverse movements of the wall. The second equation describes purely longitudinal waves in the wall-flange system under condition that no bending of the flanges occurs.

After carrying out the same computations on equations (35) and (36) for flexural waves of the second type and torsional waves of an I-beam rod, we can get the following asymptotic equation:

$$b_1 \operatorname{sh} 2\lambda \operatorname{ch} 2\lambda' + b_2 \operatorname{sh} 2\lambda' \operatorname{ch} 2\lambda + b_3 (2\lambda')^2 \operatorname{sh} 2\lambda = 0. \qquad (40)$$

This expression describes combined flexural oscillations (symmetric and antisymmetric) of a wall and flanges without the participation of longitudinal-transverse waves in this process. Equations of types (39) and (40) are also obtained for an I-beam rod.

For the angle-iron rod an analogous procedure leads to two independent equations: $C_{ll} + C_{ll} = 0$ and $B_{tt}|B'| + B'_{tt}|B| = 0$ which, after substituting them in (37) and (38), are reduced to equations like (39) and (40). One of these equations describes purely longitudinal waves in the angle-iron rod where bending does not occur. The second relates to purely flexural waves that are not influenced by longitudinal-transverse displacements of the strips. Equations of the same kind are obtained for a channel-iron and box-beam rod.

From this we can see that in the case of large $|\lambda|$ dispersion equations (22), (27), (32)-(36) reduce to simple equations with general form that can be represented as a linear combination of quantities $\operatorname{sh}(2\lambda \pm 2\lambda')$, $\operatorname{ch}(2\lambda \pm 2\lambda')$, $\lambda^2 \operatorname{ch} 2\lambda$ and $\lambda^2 \operatorname{sh} 2\lambda$. But such equations have only complex roots. We can convince ourselves of this by repeating the derivation of the formulas for the roots that was done in Ref. 2 for simpler equations of this type (setting $2\lambda = \zeta + i\eta$, substituting $\exp \eta/2$ for $\operatorname{sh} \eta$ and $\operatorname{ch} \eta$, disregarding all terms that vanish as $\eta \to \infty$). It turns out that after such transformations the equation has a solution only when ξ^2 and $\exp \eta$ are quantities of the same order. This implies that all its roots are complex. The imaginary parts η are proportional to the number of the root (taking them in the order of increasing absolute value), while the real parts are proportional to the logarithm of the imaginary parts.

Thus the constants of propagation of normal waves of high numbers, beginning with some λ_n, are complex quantities. But since there are no points of crowding of roots in a finite section of plane λ, only a finite number of λ_i can be situated within a circle $\lambda < |\lambda_n|$ and some of these λ_i may be real or imaginary. This proves the statement presented above.

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ON APPROXIMATE THEORIES OF FLEXURAL VIBRATIONS OF RODS

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This article presents a comparative analysis of the most widely known two-wave theories of flexural vibrations of rods [Ref. 1]. Principal emphasis is given to wave dispersion that determines the spectral properties of rods. Approximate theories are evaluated with respect to the degree of approximation of the wave dispersion in models to exact dispersion curves.

For the sake of simplicity we restrict ourselves to a thin elastic strip and its antisymmetric oscillations in its own plane that are described by dynamic equations of the plane stressed state [Ref. 2]. The solutions here [Ref. 3] are the well known Lamb waves (normal waves in an elastic layer) that like the equations themselves are valid in the frequency range where the inequality $(k_t h)^2 \ll 1$ holds, 2h being the thickness of the strip, while k_t is the wave number of shear waves in the material. The dispersion relations of the first two antisymmetric Lamb waves are used in comparative analysis as the exact dispersion curves.

To make the transition from the exact theory to the approximate theory we use the method of averaging with respect to the height of the strip-rod, and introduce the following quantities: average displacement $w = (1/2H)\int_{-H}^{H} u_y dy$, average angle of slope of the cross section $\psi = (1/I)\int_{-H}^{H} y u_x dy$, intercepting force $Q = \int_{-H}^{H} \sigma_{xy} dy$ and torque $M = \int_{-H}^{H} y\sigma_{xx} dy$, where x, y are the longitudinal and transverse coordinates, u_x, u_y are the corresponding displacements, σ_{xx}, σ_{xy} are the longitudinal and shear stresses, 2H is the height of a strip, $I = 2H^3/3$ is the moment of inertia of the cross section. Integration of equilibrium equations [Ref. 2] gives the following two equations relative to the averaged quantities:

$$2\rho H\, \partial^2 w/dt^2 = \partial Q/\partial x, \quad (1)$$

$$\rho I \partial^2\psi/\partial t^2 = \partial M/dx - Q. \quad (2)$$

In an analogous way, by integrating the Hooke relations for the plane stressed state [Ref. 2] we can get

$$Q = 2HE_2[\partial w/\partial x + u_x(H)/H], \quad (3)$$

$$M = E_1\{I\partial\psi/\partial x + 2H\nu[u_y(H) - w]\} = EI\partial\psi/\partial x + \nu\int_{-H}^{H} y\sigma_{yy} dy, \quad (4)$$

where E, ν are the Young modulus and the Poisson ratio of the material, $E_1 = E/(1-\nu^2)$, $E_2 = E/2(1+\nu)$ are the longitudinal and shear moduli of elasticity.

Equations (1)-(4) make up a complete system of equations relative to the four average quantities. They also include the two "excess" unknown quantities $u_x(H)$ and $u_y(H)$. The assumptions made in the approximate theories give additional relations among the unknowns, after which the system of equations becomes solvable. Let us now consider some specific theories.

1. The Bernoulli-Euler equation. In deriving this classical equation, the following assumptions are made [Ref. 4]: rod cross sections that are flat and perpendicular to the axis of the rod in the equilibrium state remain plane and perpendicular during bending as well; b) the longitudinal fibers into which the rod can be split up resist bending independently without influencing one another; c) the rotational inertia of a rod element is not

taken into consideration. On the basis of the third assumption the first member of (2) is equal to zero, and therefore $Q = \partial M/\partial x$. The second assumption implies that $\sigma_{yy} = 0$, and therefore from (4) we have $M = EI\partial\psi/\partial x$, and in virtue of the first assumption $\psi = -\partial w/\partial x$. Therefore, substituting $Q = \partial M/\partial x = -EI\partial^3 w/\partial x^3$ in equation (1), we get the Bernoulli-Euler equation

$$\frac{\partial^4 w}{\partial x^4} + \frac{1}{D^4}\frac{\partial^2 w}{\partial t^2} = 0, \quad (5)$$

where $D^4 = EI/2\rho H$ is a quantity proportional to the bending stiffness. Seeking the solution of the equation in the form of a free wave $w = \exp(ikx - i\omega t)$, we get the dispersion equation

$$k^4 = \omega^2/D^4 = k_0^4, \quad (6)$$

that relates the constant of wave propagation k to frequency ω. It has four roots corresponding to two direct and two reverse waves in the rod. Below we will consider only direct waves On Fig. 1, curves 3 and 4 represent dispersion equation (6). It can be easily seen that the Bernoulli-Euler equation gives a good approximation of dispersion to the exact curves 1 and 2 only on frequencies close to zero.

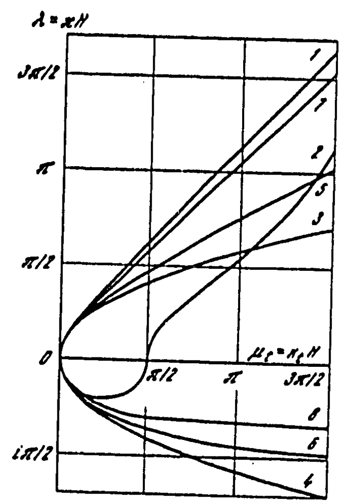

Fig. 1

2. The Rayleigh equation [Ref. 5]. In deriving this equation, we make all the assumptions of the previous section except the last. The relations $\psi = -\partial w/\partial x$, $M = EI\partial\psi/\partial x$ are satisfied here as well, but from equation (2) we get another expression $Q = \partial M/\partial x - \rho I\partial^2\psi/\partial t^2$. Substitution of this expression in (1) leads to the Rayleigh equation

$$\frac{\partial^4 w}{\partial x^4} + \frac{1}{D^4}\frac{\partial^2 w}{\partial t^2} - \frac{1}{c_l^2}\frac{\partial^4 w}{\partial x^2 \partial t^2} = 0, \quad (7)$$

where $c_l^2 = E/\rho$ is the square of the velocity of longitudinal waves in a thin rod. The dispersion equation takes the form

$$2k_{1,2}^2 = k_l^2 \pm (k_l^4 + 4k_0^4)^{1/2}, \tag{8}$$

where k_0 is the wave number of a flexural wave in a Bernoulli-Euler rod; k_l is the longitudinal wave number. Curves 5 and 6 on Fig. 1 correspond to this dispersion. As can be seen from Fig. 1, accounting for rotational inertia does not significantly improve the dispersion of waves in the rod.

3. The Bernoulli-Euler equation with consideration of shear deformations. To account for shear deformations in the Bernoulli-Euler model (without consideration of the moment of rotational inertia) we must drop the requirement for a perpendicular cross section of the line of bending while leaving the other assumptions of section 1 unchanged. In this case we can easily get an equation that differs from (7) in the coefficient of the mixed derivative, this coefficient being equal to $1/c_t^2$, where $c_t^2 = E_2/\rho$ is the square of the velocity of propagation of shear waves in a plate. The corresponding dispersion relation is derived from (8) by substituting $k_t^2 = \omega^2/c_t^2$ for k_l^2. Curves 7, 8 correspond to this relation on Fig. 1.

4. The Bresse equation [Ref. 6]. In deriving this equation it is necessary to make two assumptions: a) cross sections remain plane; b) $\sigma_{yy} = 0$, which is equivalent to the assumption of independence of deformations of individual longitudinal fibers. With consideration of these assumptions, we get the Bresse equation from (1)-(4):

$$\frac{\partial^4 w}{\partial x^4} + \frac{1}{D^4}\frac{\partial^2 w}{\partial t^2} - \left(\frac{1}{c_l^2} + \frac{1}{c_t^2}\right)\frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{1}{c_t^2 c_l^2}\frac{\partial^4 w}{\partial t^4} = 0. \tag{9}$$

Bresse was apparently the first to consider the way that bending is influenced by simultaneous shearing and rotational inertia. The third term in equation (9) reflects the influence of rotational inertia, the fourth takes account of the effect of shear deformations, and the last term, proportional to the fourth derivative with respect to time, accounts for the simultaneous action of both of these factors. The Bresse equation is a special case of the well known Timoshenko equation (with shear coefficient equal to unity), and therefore dispersion will be analyzed below. Let us note only that simultaneous accounting for shear and inertia leads to a qualitative change -- the appearance of a cutoff frequency on which the dispersion curve of the second wave passes from the imaginary region into the real region.

5. Volterra theory of plane cross sections [Ref. 7]. The only assumption in this theory is linearity of the dependence of all displacements on transverse coordinates. For bending of a strip, this assumption is equivalent to the following expressions:

$$u_x(x, y, t) = y\psi(x, t), \qquad u_y(x, y, t) = w(x, t), \tag{10}$$

From which we get the expressions for the torque $M = E_1 I \partial\psi/\partial x$ and the intercepting force $Q = 2HE_2(\partial w/\partial x + \psi)$. Together with equilibrium equations (1), (2) they form a system of four equations in four unknowns, or one Volterra equation

$$\frac{\partial^4 w}{\partial x^4} + \frac{1}{D^4}\frac{\partial^2 w}{\partial t^2} - \left(\frac{1}{c_c^2} + \frac{1}{c_t^2}\right)\frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{1}{c_t^2 c_c^2}\frac{\partial^4 w}{\partial t^4} = 0, \quad (11)$$

where $c_c^2 = E_1/\rho$ is the square of the velocity of propagation of a longitudinal wave in a thin plate. We can readily see that equation (11) can be derived from the Bresse equation (9) by replacing the Young modulus E with $E_1 = E/(1-\nu^2)$. This means that both these equations have similar dispersion relations on middle and high frequencies, but differ on low frequencies. As $\omega \to 0$ the Bresse, Bernoulli-Euler and Rayleigh equations give dispersion $k_1 \approx k_0$ and $k_2 \approx ik_0$ that coincides with the dispersion of the actual rod, and equation (11) implies that $k_1 \approx (1-\nu^2)^{1/4}k_0$, $k_2 = ik_1$. Thus the Volterra model is $(1-\nu^2)^{-1}$ times stiffer than a real rod. This is a consequence of the second equality of (10), in virtue of which this model does not permit transverse compressions and expansions.

6. Approximations of Vlasov [Ref. 8] and Ambartsumyan [Ref. 9]. The second of these theories assumes: a) $\partial u_y/\partial y = 0$ -- transverse displacements of all points of the cross section are the same; b) $\sigma_{xy}(x,y,t) = E_2\phi(x,t)f(y)$ -- tangential stresses are distributed over the cross section as a function of f(y). Using Hooke law, we get directly from this equation

$$u_x(x, y, t) = -\frac{\partial w(x,t)}{\partial x} y + \varphi(x,t)\, g(y),$$
$$u_y(x, y, t) = w(x,t), \quad g(y) = \int_0^y f(\eta)\, d\eta. \quad (12)$$

Thus in contrast to the foregoing theories, curvature of the cross sections of the rods is permitted here. From displacements (12) it is easy to calculate the stresses, and finding the averaged quantities, to get the following equation with the aid of (1), (2):

$$\frac{\partial^4 w}{\partial x^4} + \frac{1-\nu^2}{D^4}\frac{\partial^2 w}{\partial t^2} - \left(\frac{1}{c_c^2} + \frac{1}{a c_t^2}\right)\frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{1}{a c_t^2 c_c^2}\frac{\partial^4 w}{\partial t^4} = 0, \quad (13)$$

where $a = II_1/2HI_0$, I is the moment of inertia of the cross section, $I_1 = \int_{-H}^{H} f(y)\,dy;\ I_0 = \int_{-H}^{H} y g(y)\,dy.$ This equation differs from the Volterra equation in the factor a preceding c_t^2, that depends on the function f(y). When f(y) = 1/2, equations (11) and (13) coincide. For distribution of tangential stresses with respect to the parabola $f(y) = (H^{2n} - y^{2n})/2$ the coefficient a is equal to $a = (2n+3)/(2n+4)$, which gives $a = 5/6$ for quadratic distribution (n = 1).

The Vlasov is analogous. This theory is based on two assumptions relative to displacements:

$$u_x(x, y, t) = -y\varepsilon - \frac{y^3}{2H^2}\sigma^0_{xy}\frac{1}{E_2}, \qquad u_y(x, y, t) = w(x, t). \qquad (14)$$

Here $\varepsilon = -(\partial u_x/\partial y)_{y=0}$ and moreover $(\partial u_x/\partial y)_{y\to\pm H} = 0$, i. e. the cross sections are curved in such a way that after deformation they remain normal to the faces $y = \pm H$. The quantity σ_{xy} is the tangential stress on the middle line. Displacements (14) give the quadratic distribution of tangential stresses $\sigma_{xy} = \sigma_{xy}(1 - y^2/H^2)$. After the necessary transformations we can get equation (13) in which $a = 5/6$.

The coefficient a (corresponding to the Timoshenko shear coefficient), as will be shown below, considerably improves the dispersion properties of flexural waves on high frequencies. However, equation (13), like the Volterra equation, has an understated dispersion on low frequencies.

7. Approximations of Reissner [Ref. 10], Gol'denveyzer [Ref. 11] and Ambartsumyan [Ref. 12]. These theories assume: a) $\sigma_{yy} = 0$, b) $\sigma_{xy}(x,y,t) = E_2 \times \phi(x,t)f(y)$, where $f(y)$ is the distribution function of tangential stresses (in the Reissner approximation $f(y) = (H^2 - y^2)/2$). In contrast to the approximations of the preceding section, transverse deformations are assumed here, resulting in an equation that differs from the Bresse equation (9) in the coefficient a preceding c_t^2. As applied to a strip-rod, these approximations thus reduce to the Timoshenko equation that will be analyzed below.

8. The Timoshenko model [Ref. 13, 14]. In the Timoshenko theory the initial equations are equilibrium equations (1), (2) and the following expressions for the bending moment and the intercepting force:

$$M = EI\partial\psi/\partial x, \qquad Q = q2HE_2(\partial w/\partial x + \psi). \qquad (15)$$

A comparison of these relations with expressions (3) and (4) shows that the following assumptions are made here: a) $\sigma_{yy} = 0$, so that (4) implies the first relation (15), b) the cross sections remain flat, since the quantity $u_x(H)/H$ in (3) is replaced by the angle of inclination of the cross section ψ, c) the shear coefficient q is introduced. This implies that in addition to the other interpretations (see Ref. 1 and the foregoing section) the Timoshenko can be represented as a structure of the rod type with nondeformable plane cross sections that satisfies relations (15). For practical purposes it can be realized as a set of rigid plates joined by weightless elastic connections, e. g. in the form of spacers of a lighter and more pliable material that conform to conditions (15). The step of periodicity of the chain must be much less than the wavelengths of the waves being considered.

Eliminating all quantities except displacements from equations (1), (2) and (15), we get the Timoshenko equation

$$\frac{\partial^4 w}{\partial x^4} + \frac{1}{D^4}\frac{\partial^2 w}{\partial t^2} - \left(\frac{1}{c_l^2} + \frac{1}{qc_t^2}\right)\frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{1}{qc_l^2 c_t^2}\frac{\partial^4 w}{\partial t^4} = 0, \qquad (16)$$

from which we derive the dispersion equations

$$2k_{1,2}^2 = k_t^2 + \frac{k_t^2}{q} \pm \left[\left(k_t^2 - \frac{k_t^2}{q}\right)^2 + 4k_0^4\right]^{1/2}. \qquad (17)$$

Expressions (17) imply directly that on low frequencies $k_1 \approx k_0$, $k_2 \approx ik_0$, which coincides with the dispersion of the first two normal Lamb waves, while on high frequencies curves (17) approach the real asymptotes $k_1 = k_t/q^{1/2}$ and $k_2 = k_l$. The cutoff frequency of the second wave is determined from the relation $(k_tH)^2 = 3q$ and assumes the exact value for the strip at $q = \pi^2/12$. Fig. 2 shows the dispersion curves (17) for different values of the shear coefficient.

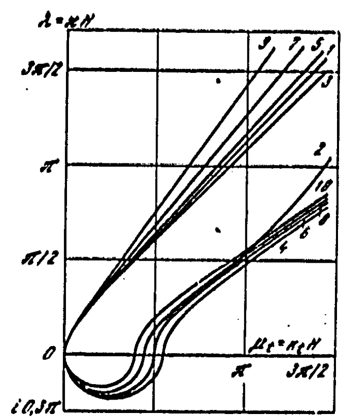

Fig. 2. Dispersion curves: 1, 2--of the first two antisymmetric Lamb waves; the other curves correspond to the Timoshenko model with different values of the shear coefficient; 3, 4--$q = 1$ (Bresse model); 5, 6--$q = \pi^2/12$; 7, 8--$q = 2/3$;9, 10--$q = 1/2$.

The choice of the coefficient q depends on the kind of problem in which the model is to be used. For instance in Ref. 15 it is proposed that the rate of propagation of the first wave in the model approach the velocity of the Rayleigh surface wave on high frequencies. In this case the dispersion of this wave coincides almost ideally with the dispersion of the first Lamb wave ($q = 0.88$ when $\nu = 1/3$). Another paper [Ref. 16] proposes that the values of q be calculated from the condition of coincidence of the cutoff frequencies of the model and the real rod (curves 5 and 6 on Fig. 2). Calculations show that this value of q gives the minimum of the absolute integral deviation of the dispersion curves of both model waves from the dispersion Lamb waves in the frequency range of $k_tH = 0\text{-}3\pi/2$. Let us note that this frequency range is the maximum possible for any two-wave model of a strip or plate since the constant of propagation of the third Lamb wave becomes real on higher frequencies. It is clear from Fig. 2 that we can get coincidence of dispersions in individual narrow sections within this range for other values of q.

The good agreement between the dispersions of both models and the exact dispersion curves enables calculation of even high natural frequencies of real rods on which the wavelength of the shear wave becomes comparable to the height of the rod ($k_t H \approx 1$).

9. Improved Timoshenko model. As implied by the preceding discussion, an essential element of the Timoshenko theory is the longitudinal coefficient q. The choice of an optimum value for this coefficient gives the best approximation with respect to dispersion among all two-wave theories with fixed coefficients in the equations. However, such a method of introducing an arbitrary coefficient is not the only one. It seems natural to introduce a greater number of arbitrary coefficients into the initial equations and to study the possibility of improving the Timoshenko equation by choosing the suitable values for these coefficients.

Let us write the expressions for the bending moment and intercepting force in more general form

$$M = pEI\partial\psi/\partial x, \qquad Q = 2HE_1(q\partial w/\partial x + s\psi), \tag{18}$$

where p, q and s are arbitrary coefficients. Replacement of expressions (15) by expressions (18) is equivalent to some change of the parameters of the elastic connections in the interpretation of the Timoshenko model given above. After substituting them in equations (1) and (2), we get the following equation:

$$\frac{\partial^4 w}{\partial x^4} + \left(\frac{s}{pq}\right)\frac{1}{D^4}\frac{\partial^2 w}{\partial t^2} - \left(\frac{1}{pc_l^2} + \frac{1}{qc_t^2}\right)\frac{\partial^4 w}{\partial x^2 \partial t^2} + + \frac{1}{pqc_l^2 c_t^2}\frac{\partial^4 w}{\partial t^4} = 0 \tag{19}$$

and the corresponding dispersion relations

$$2k_{1,2}^2 = \frac{k_l^2}{p} + \frac{k_t^2}{q} \pm \left[\left(\frac{k_l^2}{p} - \frac{k_t^2}{q}\right)^2 + 4k_0^4\frac{s}{pq}\right]^{1/2}. \tag{20}$$

On low frequencies the two roots of (20) approach the values

$$k_1 \approx k_0 (s/pq)^{1/4}, \qquad k_2 = ik_1, \tag{21}$$

and on high frequencies they approach the asymptotes $k_l/p^{1/2}$ and $k_t/q^{1/2}$. The cutoff frequency is defined by the expression

$$k_t^2 H^2 = 3s. \tag{22}$$

The special case $s = q$ is examined in Ref. 17. As we can readily see from (21), in this case $k_1 \approx k_0/p^{1/4}$, $k_2 = ik_1$, and the dispersion of the model waves on low frequencies does not coincide with the dispersion of real rods, and when $p = 1$ -- it reduces to the dispersion of the Timoshenko model.

It seems to us that correspondence between the low-frequency dispersion relations for a real rod and its model is essential, and therefore in order to choose the best values of the coefficients p, q and s we require first of all that dispersion relations (20) coincide with the corresponding relations for a real rod on low frequencies. From equations (21) we then immediately get the equality

$$s = pq. \tag{23}$$

This expression is general for all homogeneous rods, and does not depend on the shape of the cross section.

Calculations show that the divergences of wave dispersion in the real rod and the model are very sensitive to a change in cutoff frequency. In this connection, as a second condition imposed on the arbitrary coefficients we take the condition of coincidence of their cutoff frequencies, which for an arbitrary rod takes the form

$$k_t^2 r^2 = pq, \tag{24}$$

where r is the radius of inertia of the cross section, and the quantity k_t is calculated on the cutoff frequency, which for a strip-rod and a plate gives $pq = \pi^2/12$.

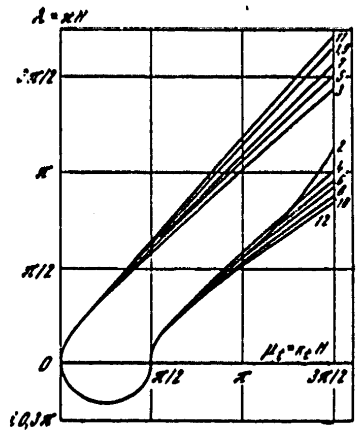

Fig. 3. Dispersion curves of the first two Lamb waves (1, 2) and waves in the refined Timoshenko model at different values of the coefficient p ($pq = \pi^2/12$): 3, 4--p = 0.62; 5, 6--p = 0.722; 7, 8--$p = \pi^2/12$; 9, 10--p = 0.94; 11, 12--p = 1 ("pure" Timoshenko model).

Fig. 3 shows the dispersion curves (20) for different values of p (the coefficients q and s were found from equalities (23) and (24)). It is clear from this figure that the Timoshenko model (p = 1) does not give the best approximation here, and the introduction of an additional correcting factor noticeably improves the dispersion of the second wave. For instance in the frequency range $k_tH = 0$-π dispersion curve 8 on Fig. 3 ($p = \pi^2/12$) practically coincides with exact curve 2. In the interval $k_tH = 0$-$3\pi/2$ the value p = 0.9

is best. The absolute integral deviation of the dispersion curves of both waves at this value is half that given by the "pure" Timoshenko model. It is shown in Ref. 18 that this improvement of dispersion properties of the model leads to a reduction of errors in calculations of high natural frequencies of real rods.

In conclusion let us note that the model that is described by equation (19) together with relations (23) and (24) is the best among the possible two-wave models with respect to dispersion properties. The incorporation of a greater number of correcting coefficients or their introduction by another method leads to distortion of low-frequency dispersion and therefore cannot be considered justified. It is also worth mentioning that a change from the Timoshenko model to an improved model can be realized by substituting the angle of turn $p\psi$ for the angle of turn ψ in expressions (15).

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INVESTIGATION OF SPACE-TIME COHERENCE OF SOME TYPES OF ACOUSTIC FIELDS

A. M. Medvedkov, A. T. Shargayev

In determining the influence that mechanisms installed on a common base and working as a group have on the overall sound field of a room, consideration is taken of the exchange of energy between mechanisms that is due to the propagation of vibrations through the base connections. This exchange shows up in the fact that signals taken off synchronously from sensors installed on the feet of the mechanisms are correlated with each other, and moreover are correlated with the signal taken off from a microphone set at the investigated point in the room.

The excitation, propagation and emission of vibrations are wave processes and therefore it is natural to expect that the properties of these processes will show up primarily in such a specific characteristic as the coherence function [Ref. 1].

We will analyze space-time coherence on the basis of a model in the form of a string that interacts with the acoustic medium. The string is excited by concentrated transverse forces that may be statistical in the general case.

Let us consider steady-state vibrations of an infinite string in a medium with consideration of the internal friction of the material of the string under the action of a force that varies harmonically with time. To do this, we introduce the complex tension

$$\tilde{T} = T(1 + i\eta), \tag{1}$$

where T is the true tension of the string, and η is the loss factor, $\eta^2 \ll 1$.

Let us take the rectangular coordinate system XYZ so that the X-axis is along the axis of the string, and the XOZ plane coincides with the plane of vibrations of the string. In this coordinate system the equation of motion of the string is written as

$$\pi r_0^2 \rho_0 \frac{\partial^2 \bar{z}}{\partial t^2} = \tilde{T} \frac{\partial^2 \bar{z}}{\partial x^2} + \bar{F}_p + F, \tag{2}$$

where r_0 is the radius of the string, ρ_0 is the density of the string, $\bar{z}(x,t) = \bar{z}(x)e^{-i\omega t}$ is the displacement of the string, $F(x,t) = \delta(x)e^{-i\omega t}$ is a concentrated force with a single amplitude that acts on the string at point x, $\bar{F}_p(x,t)$ is the reaction of the medium to the motion of the string, $\bar{z}(x)$ is the complex amplitude of displacement of the string, $\delta(x)$ is the Dirac delta function, and ω is the angular frequency of harmonic oscillations of the external force.

The pressure change in the medium that is due to vibrations of the string satisfies the wave equation that is associated with the string in the cylindrical coordinate system (axis of symmetry directed along the X-axis, angle θ read out positively from the plane of vibrations) and is written as

$$\frac{\partial^2 \bar{p}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{p}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \bar{p}}{\partial \theta^2} + \frac{\partial^2 \bar{p}}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 \bar{p}}{\partial t^2} = 0, \tag{3}$$

where $\bar{p}(r,\theta,x,t) = \bar{p}(r,\theta,x)e^{-i\omega t}$ is the pressure in the medium that is caused by string vibrations, c is the speed of sound in the medium, $\bar{p}(r,\theta,x)$ is the complex amplitude of the pressure.

From the equality of normal velocities of the string and the ambient medium, we get the following boundary condition:

$$\frac{\partial^2 \bar{z}}{\partial t^2}\cos\theta = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial r}\bigg|_{r=r_0}, \tag{4}$$

where ρ is the density of the medium.

The reaction of the medium is the sum of the projections of elements of force $\bar{p}(r_0,\theta,x)r_0 d\theta$ on the z-axis, i. e.

$$\bar{F}_p = -\int_0^{2\pi} r_0 \bar{p}(r_0,\theta,x)\cos\theta\, d\theta\, e^{-i\omega t}. \tag{5}$$

It is assumed in the given expressions for the unknown parameters that they vary in time according to a harmonic law with the frequency of the perturbing force. Therefore expressions (2)-(5) can be written in the form

$$\pi r_0^2 \rho_0 (-\omega^2)\bar{z} = \bar{T}\frac{\partial^2 \bar{z}}{\partial x^2} + \bar{F}_p + \delta(x), \qquad (6)$$

$$\frac{\partial^2 \bar{p}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{p}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \bar{p}}{\partial \theta^2} + \frac{\partial^2 \bar{p}}{\partial x^2} + k^2 \bar{p} = 0, \qquad (7)$$

$$\bar{z}\cos\theta = \frac{1}{\rho\omega^2}\frac{\partial \bar{p}}{\partial r}\Big|_{r=r_0}. \qquad (8)$$

We write the sought solution of equations (6)-(8) as follows:

$$\bar{z}(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_z(\xi)\, e^{\pm i\xi x} d\xi, \qquad (9)$$

$$\bar{p}(r,\theta,x) = \frac{\cos\theta}{2\pi}\int_{-\infty}^{\infty} S_p(\xi)\, H_1^{(1)}(\zeta r)\, e^{i\xi x} d\xi, \qquad (10)$$

where $H_1^{(1)}(\zeta r)$ is a Hankel function of the first kind.

The plus sign in the exponential function of (9) corresponds to propagation of plane waves in the positive direction ($x > 0$), and the minus sign -- in the negative direction. Considering waves that damp out at infinity, we take $\operatorname{Im}\zeta \geq 0$. The solution in formula (10) obviously satisfies the Helmholtz equation and the Sommerfeld radiation condition. After substituting (9) and (10) in equations (6) and (8), we get a system of two algebraic equations relative to the two unknowns $S_z(\xi)$ and $S_p(\xi)$, and the solutions give

$$S_z(\xi) = \left(\frac{1}{m}\right)\left[-\omega^2 + c_0^2(1+i\eta)\xi^2 + \frac{\pi\rho r_0\omega^2}{m}\frac{H_1^{(1)}(\zeta r_0)}{\zeta H_1^{(1)\prime}(\zeta r_0)}\right]^{-1}, \qquad (11)$$

$$S_p(\xi) = \rho\omega^2\left[m\zeta H_1^{(1)\prime}(\zeta r_0)\right]^{-1}\left[-\omega^2 + c_0^2(1+i\eta)\xi^2 + \frac{\pi r_0\rho\omega^2}{m}\frac{H_1^{(1)}(\zeta r_0)}{\zeta H_1^{(1)\prime}(\zeta r_0)}\right]^{-1}, \qquad (12)$$

where $m = \pi r_0^2 \rho_0$ is the linear density of the string material, $c_0^2 = T/m$ is the velocity of transverse waves in the string in a vacuum.

For small values of r_0 and low frequencies, i. e. when $\zeta r_0 \ll 1$, we can limit ourselves to the first term of the asymptotic expansion for the Hankel function and its derivative, and as a result we get

$$H_1^{(1)}(\zeta r_0)/\zeta H_1^{(1)\prime}(\zeta r_0) = -r_0. \qquad (13)$$

Let us use the notation

$$\bar{c}_0^2 = c_0^2(1+i\eta), \quad \bar{\xi}_0^2 = \xi_0^2(1+i\eta)^{-1} \simeq \xi_0^2(1-i\eta), \qquad (14)$$

$\xi_0 = \omega/c_0$ is the wave number of flexural vibrations in the string.

To investigate pressure in the far zone ($\zeta r \gg 1$) we use a conventional asymptotic representation of the Hankel function [Ref. 2]

$$H_1^{(1)}(\zeta r) = \sqrt{2/\pi\zeta r}\exp\{i[\zeta r - (3/4)\pi]\}. \tag{15}$$

Substituting (11), (12) in the expressions for displacement (9) and pressure (10), taking (13)-(15) into consideration, we get the following relations:

$$\bar{z}(x) = \frac{1}{2\pi mc_0^2}\int_{-\infty}^{\infty}\frac{\exp(i\xi x)\,d\xi}{\xi^2 - \bar{\xi}_0^2(1 + \rho/\rho_0)}, \tag{16}$$

$$\bar{p}(r, 0, x) = \frac{\cos\theta}{2\pi m\bar{c}_0^2}\int_{-\infty}^{\infty}\frac{r_0^2\rho\omega^2\sqrt{\pi\zeta/2r}\exp\{i[\zeta r + \xi x - (3/4)\pi]\}}{i[\xi^2 - \bar{\xi}_0^2(1 + \rho/\rho_0)]}\,d\xi. \tag{17}$$

We calculate integrals (16) and (17) with the use of the theory of residues and the method of steepest descents [Ref. 1]. With condition Im≥0 we get

$$\bar{z}(x) = -\frac{i\exp(-x\xi_0\sqrt{1 + \rho/\rho_0}\,\eta/2)\exp(-ix\xi_0\sqrt{1 + \rho/\rho_0})}{2mc_0^2\xi_0\sqrt{1 + \rho/\rho_0}\,(1 + i\eta/2)}, \tag{18}$$

$$\bar{p}(r, 0, x) = \frac{i\rho\omega^2 kr_0^2}{2m\bar{c}_0^2}\,\frac{\cos\psi_0}{k^2\sin^2\psi_0 - \bar{\xi}_0^2(1 + \rho/\rho_0)}\,\frac{\cos\theta\exp(ikR)}{R}, \tag{19}$$

where

$$R = r/\cos\psi_0; \qquad \mathrm{tg}\,\psi_0 = x/r. \tag{20}$$

For the wave zone (kr>>1) a considerable part is played by the small part of the path of integration that includes the point of steepest descent ψ_0. We find its location on complex plane ψ from (20) and conditions $x > 0$, $r \gg r_0$. Obviously

$$0 \leqslant \psi_0 < \pi/2. \tag{21}$$

Let us consider the vibrations of a string excited at points x_1 and x_2 by concentrated transverse forces $f_1(t)$ and $f_2(t)$ that are taken as independent steady-state random processes of second order. For the sake of definiteness we set $x_1 = 0$, $x_2 \geq 0$. Let us use the notation $F_1(\omega)$ and $F_2(\omega)$ for the Fourier transforms of forces $f_1(t)$ and $f_2(t)$, and $W_1(\omega,x)$ and $W_2(\omega,x-x_2)$ for the Fourier transformers of the accelerations of the string at point x that are caused by these forces. According to Ref. 3, the Fourier transform for the response of a linear system in the frequency region is equal to the product of the Fourier transform of the input action multiplied by the frequency response of the system, and therefore we have

$$W_1(\omega, x) = H_1(\omega, x)F_1(\omega), \tag{22}$$

$$W_2(\omega, x - x_2) = H_2(\omega, x - x_2)F_2(\omega). \tag{23}$$

The expressions for the frequency responses $H_1(\omega,x)$ and $H_2(\omega,x-x_2)$ are obtained by using (18) and the relation

$$H(\omega, x) = (i\omega)^2\,\bar{z}(x), \tag{24}$$

$$H_1(\omega, x) = \frac{i\omega\exp\left(-\frac{\omega\eta}{2}\frac{x}{c_0}\sqrt{1 + \frac{\rho}{\rho_0}}\right)\exp\left(-\frac{i\eta}{2} - i\omega\frac{x}{c_0}\sqrt{1 + \frac{\rho}{\rho_0}}\right)}{2mc_0\sqrt{1 + \rho/\rho_0}}, \tag{25}$$

$$H_2 = (\omega, x - x_2) = \frac{i\omega \exp\left[\frac{\omega\eta}{2}\left(\frac{x - x_0}{c_0}\right)\sqrt{1 + \frac{\rho}{\rho_0}}\right] \exp\left[i\omega\left(\frac{x - x_0}{c_0}\right)\sqrt{1 + \frac{\rho}{\rho_0}}\right]}{2mc_0\sqrt{1 + \rho/\rho_0}} \times \exp(-i\eta/2). \quad (26)$$

For a point x located to the left of point x_2, $x - x_2 < 0$. The total accelerations $W(x_1)$ and $W(x_2)$ at points x_1 and x_2 with consideration of (22)-(26) are written in the form

$$W(x_1) = W_1(x_1) + W_2(x_1) = \frac{i\omega \exp\left(-\frac{i\eta}{2}\right) F_1(\omega)}{2mc_0\sqrt{1 + \rho/\rho_0}} + \frac{i\omega \exp\left(-\frac{i\eta}{2} - \frac{\omega\eta\tau}{2} - i\omega\tau\right) F_2(\omega)}{2mc_0\sqrt{1 + \rho/\rho_0}}, \quad (27)$$

$$W(x_2) = W_1(x_2) + W_2(x_2) = \frac{i\omega \exp\left(-\frac{i\eta}{2} - \frac{\omega\eta\tau}{2} - i\omega\tau\right) F_1(\omega)}{2mc_0\sqrt{1 + \rho/\rho_0}} + \frac{i\omega \exp\left(-\frac{i\eta}{2}\right) F_2(\omega)}{2mc_0\sqrt{1 + \rho/\rho_0}}, \quad (28)$$

where

$$\tau = (x/c_0)\sqrt{1 + \rho/\rho_0}.$$

According to Ref. 2, the relations

$$M[F_j(\omega) F_j^*(\omega')] = G_j(\omega)\,\delta(\omega - \omega'), \quad (29)$$

$$G_j(\omega) = M[|F_j(\omega)|^2], \quad j = 1, 2, \quad (30)$$

$$M[F_1(\omega) F_2^*(\omega)] = 0, \quad (31)$$

hold for steady-state independent random processes, where $G_j(w)$ is spectral density, M is the operator of mathematical expectation, $\delta(\omega)$ is the delta function.

For the spectral densities of total accelerations at points of the string with coordinates x_1 and x_2 we have

$$S_{11}(\omega) = \frac{\omega^2 G_1(\omega)}{4m^2c_0^2(1 + \rho/\rho_0)} + \frac{\omega^2 \exp(-\omega\eta\tau)\, G_2(\omega)}{4m^2c_0^2(1 + \rho/\rho_0)}, \quad (32)$$

$$S_{22}(\omega) = \frac{\omega^2 \exp(-\omega\eta\tau)\, G_1(\omega)}{4m^2c_0^2(1 + \rho/\rho_0)} + \frac{\omega^2 G_2(\omega)}{4m^2c_0^2(1 + \rho/\rho_0)}. \quad (33)$$

Using formulas (27)-(31) for the mutual spectral density of accelerations $W(x_1)$ and $W(x_2)$ we get

$$S_{12}(\omega) = \frac{\omega^2 \exp(-\omega\eta\tau/2 + i\omega\tau)\, G_1(\omega)}{4m^2c_0^2(1 + \rho/\rho_0)} + \frac{\omega^2 \exp(-\omega\eta\tau/2 - i\omega\tau)\, G_2(\omega)}{4m^2c_0^2(1 + \rho/\rho_0)}. \quad (34)$$

By analogous arguments we find expressions for the spectral density of total pressure $S_p(\omega, r, \theta, x)$ at the investigated point $N(r, \theta, x)$

$$S_p = \frac{\rho^2 c^2 r_0^4 \cos^2\psi_{01} \cos^2\theta\, \omega^2 G_1(\omega)}{4m^2 c_0^4 [\sin^2\psi_{01} - (c^2/c_0^2)(1+\rho/\rho_0)]^2 R_1^2} + \frac{\rho^2 c^2 r_0^4 \cos^2\psi_{02} \cos^2\theta\, \omega^2 G_2(\omega)}{4m^2 c_0^4 [\sin^2\psi_{02} - (c^2/c_0^2)(1+\rho/\rho_0)]^2 R_2^2} \qquad (35)$$

and for the mutual spectral density of the total acceleration at point x and the total pressure at a point of field N

$$S_{1p} = \frac{\rho c r_0^2 \omega^2 \cos\theta}{4m^2 c_0^3 \sqrt{1+\frac{\rho}{\rho_0}}} \times \left\{ \frac{\cos\psi_{01} \exp(-i\omega\tau_1) G_1(\omega)}{\left[\sin^2\psi_{01} - \frac{c^2}{c_0^2}\left(1+\frac{\rho}{\rho_0}\right)\right] R_1} + \frac{\cos\psi_{02} \exp[-i\omega(\tau+\tau_2)] G_2(\omega)}{\left[\sin^2\psi_{02} + \frac{c^2}{c_0^2}\left(1+\frac{\rho}{\rho_0}\right)\right] R_2} \right\}, \qquad (36)$$

where

$$r_1 = R_1/c;\; r_2 = R_2/c;\; r = (x/c_0)\sqrt{1+\rho/\rho_0};\; \operatorname{tg}\psi_{01} = x/r;$$
$$\operatorname{tg}\psi_{02} = (x_2 - x)/r; \qquad R_j = r/\cos\psi_{0j} \qquad (j = 1, 2).$$

In formulas (35), (36) we set $\eta = 0$ to simplify calculations. The coherence function of total accelerations at points of the string x_1 and x_2 with consideration of formulas (32)-(34) is written as

$$\gamma_{12}^2(\omega) = \frac{|\exp(i\omega\tau) G_1(\omega) + \exp(-i\omega\tau) G_2(\omega)|^2}{[\exp(\omega\eta\tau) G_1(\omega) + G_2(\omega)][G_1(\omega) + \exp(\omega\eta\tau) G_2(\omega)]}. \qquad (37)$$

After simple transformations this formula can be written in the following way:

$$\gamma_{12}^2(\omega) = \frac{G_1^2(\omega) + G_2^2(\omega) + 2G_1(\omega) G_2(\omega)\cos\omega\tau}{[\exp(\omega\eta\tau) G_1(\omega) + G_2(\omega)][G_1(\omega) + \exp(\omega\eta\tau) G_2(\omega)]}. \qquad (38)$$

This formula implies that the coherence function of total accelerations of the string at points of application of forces $f_1(t)$ and $f_2(t)$ will decrease with an increase in the damping factor η in the string. The numerator of the coherence function includes an interference term $\cos 2\omega\tau$ that is a consequence of the wave nature of the investigated oscillatory processes and leads to a periodic change in the coherence function. In particular, as $\eta \to 0$ and $G_1 = G_2$ we have $\gamma_{12}^2(\omega) = \cos^2\omega\tau$, i. e. the coherence function varies as the square of the cosine of $\omega\tau$.

The expression for the coherence function that relates the total acceleration at an investigated point of the string to the total pressure at the investigated point of the field $N(\tau, \theta, x)$ can be obtained from formulas (32), (35), (36).

Disregarding attenuation and considering that in the wave zone $x_2/r<<1$, $\psi_{02} \simeq \pi - \psi_{01}$, we have

$$\gamma_{1N}^2(\omega) = \frac{G_1^2(\omega) + G_2^2(\omega) + 2G_1(\omega)G_2(\omega)\cos[\omega(\tau_1 - \tau_2 - \tau)]}{[G_1(\omega) + G_2(\omega)]^2}.$$

If R_0 denotes the radius vector that joins the middle of the string segment between the points of application of forces $f_1(t)$ and $f_2(t)$ to point N of the field, and θ denotes the angle that this radius vector makes with the positive direction of the axis of the string, then when $x_2/r<<1$, we have

$$\theta \simeq \pi/2 - \psi_{01},$$

$$R_1 = R_0 + \frac{x_2}{2}\cos\theta, \qquad R_2 = R_0 - \frac{x_2}{2}\cos\theta,$$

$$\tau_1 = \frac{R_0}{c} + \frac{x_2}{2c}\cos\theta, \qquad \tau_2 = \frac{R_0}{c} - \frac{x_2}{2c}\cos\theta.$$

With consideration of these formulas, the coherence function can be written as

$$\gamma_{1N}^2(\omega) = \frac{\left|G_1(\omega)\exp\left\{-i\omega\left[\left(\frac{x_2}{2c}\right)\cos\theta - \frac{x_2}{2c_0}\sqrt{1+\frac{\rho}{\rho_0}}\right]\right\} + G_2(\omega)\exp\left\{i\omega\left[\frac{x_2}{2c}\cos\theta - \frac{x_2}{2c_0}\sqrt{1+\frac{\rho}{\rho_0}}\right]\right\}\right|^2}{[G_1(\omega) + G_2(\omega)]^2}.$$

When $G_1(\omega) = G_2(\omega)$ the coherence function is written as

$$\gamma_{1N}^2(\omega) = A\cos^2\psi,$$

$$A = |\exp\{-i\omega[(2/c)R_0 + (x_2/c_0)\sqrt{1+\rho/\rho_0}]\}|^2 = 1,$$

$$\psi = \omega\left(\frac{x_2}{2c}\cos\theta - \frac{x_2}{2c_0}\sqrt{1+\frac{\rho}{\rho_0}}\right) = \frac{\pi x_2}{\lambda}\left(\cos\theta - \frac{c}{c_0}\sqrt{1+\frac{\rho}{\rho_0}}\right).$$

Obviously the coherence function takes on its maximum value when $\psi = 0$, i. e. when

$$\cos\theta = \frac{c}{c_0}\sqrt{1+\frac{\rho}{\rho_0}} = \sin\psi_{01}.$$

Thus the coherence function reaches the maximum value when the coordinate of a point on the path of steepest descent Γ on the real axis is equal to the angle that the radius vector drawn to point N from the point of application of force $f_1(t)$ makes with the x-axis.

The result can be attributed to the fact that the field at observation point N is formed mainly from plane waves that propagate at the same angle ψ_{01}, and therefore is distinguished by high homogeneity.

The normalized value of the coherence function has a radiation pattern in the plane that passes through the axis of the string and observation point N

$$\gamma_{1N}^2/(\gamma_{1N}^2)_{max} = \cos^2\psi.$$

Fig. 1

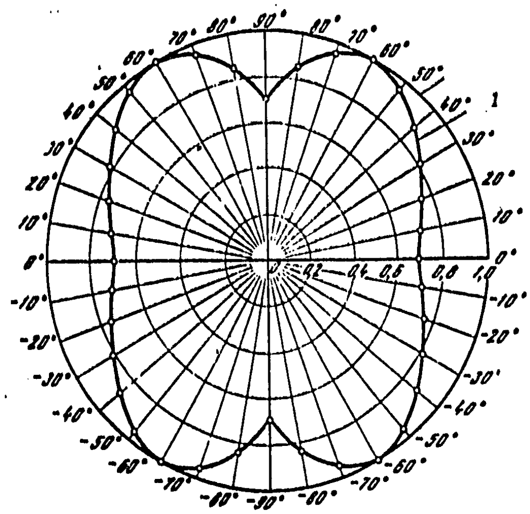
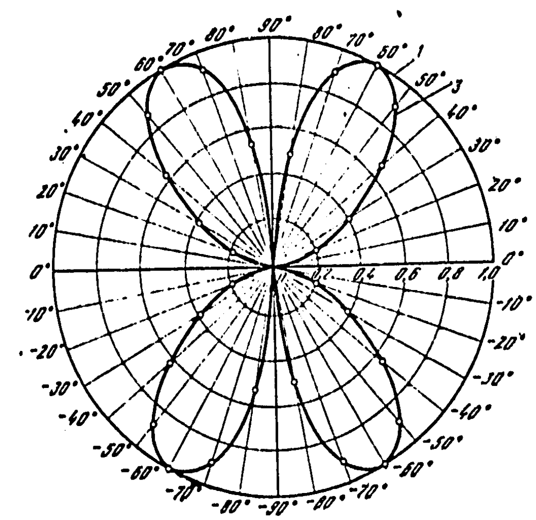


Fig. 2



The three-dimensional radiation pattern is obtained by rotating the curve for $\cos\psi$ about the OX axis.

The radiation pattern of the coherence function that relates the total acceleration at one of the points of application of the force to the total pressure in the field that is caused by vibrations of the string under the action of two random forces with different spectral densities is an analog of the radiation pattern of two monopoles [Ref. 3] with accuracy to the angle $\frac{\pi x_2}{\lambda}\frac{c}{c_0}\sqrt{1+\frac{\rho}{\rho_0}}$. For a given x_2/λ this angle depends on the ratio of the velocities of sound in the medium and the string, and also on the ratio of densities of the medium and the string material.

Shown in Fig. 1, 2 are examples of radiation patterns of the coherence function when $c/c_0 \simeq 1/2$, $\sqrt{1+\rho/\rho_0} \simeq 1$. Curve 1 corresponds to the case where $x_2/\lambda = 0$, i. e. where the points of application of forces $f_1(t)$ and $f_2(t)$ coincide the radiation pattern of the coherence function is a circle of unit radius.

Curves 2 and 3 correspond to the cases where $x_2/\lambda = 1/2$ and $x_2/\lambda = 1$ respectively. These figures show that as the distance between the points of application of the forces increases the lobes of the radiation pattern become narrower. This property of the radiation pattern of the coherence function is determined chiefly by the distance x_2/λ between the points of application of the forces, whereas the deviation of the maximum of the radiation pattern from the straight line perpendicular to the axis of the string and passing through the middle of the segment that lies between the points of application of the forces is determined by the parameters that characterize the string and the adjacent medium.

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A METHOD OF VIBRATION DAMPING IN SOME ROD STRUCTURES

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Multidimensional linear oscillatory systems enable damping of vibrations by redistribution of perturbing forces (velocities) at the input of the system while maintaining the integral characteristics of their intensity on the same level.

Fundamental problems that are associated with the possibility of implementing such a method of vibration damping were discussed for general linear systems in Ref. 1. A specific example of damping of the oscillations of a structural member (cantilever rod) by selection of the optimum distribution of perturbing forces was considered in detail in Ref. 2. In our paper we examine the possibilities of this type of vibration damping, analyze some special cases, and evaluate the effectiveness of vibration-damping rod structures that utilize the given principle.

Minimizing the flow of vibrational energy in multidimensional linear systems. The flow of vibrational energy radiated to supporting connections is expressed by a Hermitian form of the vector of complex amplitudes of generalized forces or generalized velocities at the input of the system [Ref. 1]. It is possible to minimize this form, i. e. to achieve an effect of vibration damping by matching the optimum relations among the different components of these vectors. As an example we consider a system with a harmonic force Q and torque M applied at the input. For the sake of convenience in subsequent calculations we introduce a constant of proportionality r with dimensionality of length between the force and the torque:

$$M = rQ. \qquad (1)$$

Using the notation y_{ij} (i, j = 1, 2) to denote the elements of the complex matrix of input mobility Y (reverse resistance) of the system, we write the expression of active power N at the input in terms of the complex amplitudes of force Q and torque M:

$$N = 0{,}5\ \mathrm{Re}\,\{y_{11}Q\overline{Q} + y_{12}(Q\overline{M} + \overline{Q}M) + y_{22}M\overline{M}\}. \qquad (2)$$

The problem of vibration damping in the given instance reduces to minimizing quadratic form (2) for fixed Q and constant y_{ij}, i. e. to the proper choice of the value of r. Let us note that the minimization we are considering here with fixed force Q differs from the minimization studies in Ref. 1 under the condition of constancy of the sum of the squares of the components of the input parameters. In the given case the optimum vector of the input parameters (Q, M) is no longer an eigenvector of form N. In the given problem there is no difficulty in finding the optimum vector (i. e. determining the optimum r). Substituting relation (1) in formula (2) we get

$$\frac{N}{N|_{r=0}} = \frac{g_{22}}{g_{11}}r^2 + 2\frac{g_{12}}{g_{11}}r + 1, \qquad g_{ij} = \mathrm{Re}\,y_{ij}. \qquad (3)$$

Since $g_{22} > 0$, quadratic trinomial (3) has a unique minimum that is reached when $r = r^*$

$$r^* = -g_{12}/g_{22}$$

and is equal to

$$(N/N|_{r=0})_{\min} = 1 - g_{12}^2/g_{11}g_{22}. \qquad (4)$$

As a consequence of the positive definiteness of form N, the value of (4) lies between zero and unity.

We have examined the problem of minimizing the flow of energy by redistributing perturbing forces for a two-dimensional system, i. e. a system with two inputs. The formal generalization of this analysis to the case of any number of input parameters presents no difficulties. The direct generalization of the given problem is a problem of quadratic programming with constraints of the equality type. Methods of solving this problem have been thoroughly worked out (see for instance Ref. 3).

Damping of flexural vibrations of a rod or plate. In the following discussion we will concentrate attention on development of one version of a system of the given kind, namely an elastic rod or plate in flexural vibration and attached to some load. The rod vibrations are considered within the framework of elementary beam theory. The complex amplitude of transverse oscillations u(x) is represented as a linear combination of A. N. Krylov's function of the argument kx, where the wave number k is expressed in terms of the angular frequency ω by the relation (for a plate) $k^4 = \rho h\omega^2/D$ ($D = Eh^3/12(1-\nu^2)$ is the cylindrical stiffness, ρ is density, h is thickness, ν is the Poisson ratio).

We have

$$u(x) = C_1 S(kx) + C_2 T(kx) + C_3 U(kx) + C_4 V(kx). \qquad (5)$$

The complex amplitudes of the bending moment M and the transverse force Q are defined by the formulas

$$Q = Du'''(x), \qquad M = Du''(x). \qquad (6)$$

Using the subscript l to denote the values of the quantities where $x = l$, we write the conditions on the right end of the beam:

$$Q_l = Q, \qquad M_l = rQ. \qquad (7)$$

For the sake of simplicity we will assume that the matrix of mechanical resistance of the load is diagonal. Then the conditions on the left end of the rod are written in the form

$$Q_0 = Dk^3 z_1 i\omega u_0, \qquad M_0 = Dk^2 z_2 i\omega u_0'/k, \qquad (8)$$

where the zero subscript denotes the corresponding quantities when $x = 0$. The active power radiated to the load is equal to

$$N = 0{,}5\,\mathrm{Re}\,\{Q_0 \overline{i\omega u_0} + M_0 \overline{i\omega u_0'}\}. \qquad (9)$$

Relations (8) imply that

$$N = 0{,}5 Dk^3\omega^2 (\mathrm{Re}\, z_1 |u_0|^2 + \mathrm{Re}\, z_2 |u_0'/k|^2). \qquad (10)$$

The quantities u_0, u_0'/k are related to Q_l, M_l by equations that derive from (5), (8)

$$\begin{aligned} M_l/Dk^2 &= (U - i\omega z_1 T) u_0 + (V + i\omega z_2 S) u_0'/k, \\ Q_l/Dk^3 &= (T - i\omega z_1 S) u_0 + (U + i\omega z_2 V) u_0'/k, \end{aligned} \qquad (11)$$

in which the coefficients contain A. N. Krylov's function of the argument kl. Relations (8)-(11) enable us to write the energy flow as a quadratic function of $|Q|$ in which the coefficient is a quadratic trinomial of r. After this we can easily minimize the energy flow with respect to r.

It is of interest to consider the limiting case of infinite mechanical resistance of the load to transverse force z_1. This case can be analyzed by replacing the first condition of (8) with the condition $u_0 = 0$, or passing to the limit in (11) as $z_1 \to \infty$, $u_0 \to 0$, $z_1 u_0 < \infty$. Elementary calculations show $|u_0|$ as well as N vanish at $r = r^*$:

$$r^*/l = T/k^l S. \qquad (12)$$

Let us note that the optimum choice of r^*/l (12) is independent of the turning resistance z_2.

The second important limiting case is the case of infinite turning resistance z_2. Proceeding by analogy with the foregoing, we find that N vanishes when $z = z^*$:

$$r^*/l = S/klV. \tag{13}$$

In both cases considered

$$N/N|_{r=0} = [(r/r^*) - 1]^2. \tag{14}$$

It is easy to verify that on high frequencies the influence that the mechanical impedance of the load has on the quantity r^* becomes negligible. When $kl \to \infty$, $r^* \sim 1/k$ regardless of the values of z_1, z_2. In the case where both z_1 and z_2 are large (the rod is clamped), the degree of vibration damping is evaluated not by the quantity N, but rather by the value of the coefficient of transfer K with respect to force. This coefficient is

$$K = Q_0/Q = (krV - S)/(S^2 - VT) \tag{15}$$

and vanishes at the r defined by formula (13).

We have discussed vibration damping by matching the ratio of the generalized forces at the input of the system. In some designs it may be convenient to match the ratio between the generalized displacements at the input. In this case

$$u_l = u, \qquad u_l' = u/r. \tag{16}$$

Let us give the formulas for energy flow in limiting cases:

as $z_1 \to \infty$ $\qquad N/N|_{r=\infty} = [(r^*/r) - 1]^2, \qquad r^*/l = U/klT.$

as $z_2 \to \infty$ $\qquad N/N|_{r=\infty} = [(r^*/r) - 1]^2, \qquad r^*/l = V/klU, \qquad (17)$

The preceding general examination shows that by varying the quantity r^*/l we can achieve appreciable (in limiting cases total) vibration damping. Now let us consider the use of this effect in some specific structures.

Minimizing the vibrational energy that travles through a T joint between plates. Many elements in building construction, shipbuilding and other structural elements can be modeled by an infinite rod or an infinitely long thin plate. Here we investigate the propagation of waves through structures of the type considered above in the case where the load is an infinite plate. The problem is considered plane. By using the equations of oscillations of an infinite plate excited by a force and torque, we get the relations among the complex amplitudes of the forces and displacements at the joining point O in the form

$$Q_0 = -2ia_H A_H u_0, \qquad M_0 = 2(1 + i) D_H k_H u_0'. \tag{18}$$

Here a_H is the wave number of longitudinal oscillations, k_H is the wave number of flexural vibrations of an infinite plate,

$$a_н^2 = \rho_н h_н \omega^2 / A_н; \qquad k_н^4 = \rho_н h_н \omega^2 / D_н;$$
$$D_н = E_н h_н^3 / 12(1 - \nu_н^2), \qquad A_н = E_н h_н / (1 - \nu_н^2)$$

are the flexural and longitudinal stiffnesses respectively. Comparing (18) with (8) we get

$$\omega z_1 = -\beta_1, \qquad \omega z_2 = (1 - i)\beta_2, \tag{19}$$

where

$$\beta_1 = 2a_н A_н / Dk^3, \qquad \beta_2 = 2D_н k_н / Dk.$$

Within the framework of applicability of the theory of thin plates the quantity β_1 is large. To be precise,

$$\frac{1}{\beta_1} = \frac{1}{2}\sqrt{\frac{E\rho(1-\nu_н^2)}{E_н\rho_н(1-\nu^2)}}\,\frac{h}{h_н}\,kh,$$

i. e. $1/\beta_1$ has the order of the ratio of the thickness of the plate to the wavelength.

The flow of energy through the joining is equal to

$$N = \frac{1}{2} Dk^3 \omega (\beta_1 |u_0|^2 + \beta_2 |u_0'/k|).$$

Expressing the quantities $|u_0|$, $|u_0'/k|$ in terms of $|Q|$ with the use of relations (11), (19), (1), we get the following expression for $N = N(r)$:

$$N(r) = \frac{\omega |Q|^2}{2Dk^3\beta_2}\,\frac{p_2(kr)^2 + 2p_1 kr + p_0}{d^2}. \tag{20}$$

Here

$$p_0 = \frac{\beta_2}{\beta_1}\left[\left(\frac{V}{\beta_2} + S\right)^2 + S^2\right] + \frac{U^2}{\beta_1^2} + T^2;$$
$$p_1 = -\frac{\beta_2}{\beta_1}\left[\left(\frac{V}{\beta_2} + S\right)\left(\frac{U}{\beta_2} + V\right) + SV\right] - \frac{UT}{\beta_1^2} - ST;$$
$$p_2 = \frac{\beta_2}{\beta_1}\left[\left(\frac{U}{\beta_2} + V\right)^2 + V^2\right] + \frac{T^2}{\beta_1^2} + S^2;$$
$$d^2 = \frac{1}{4}\left(E + \frac{A}{\beta_1} + \frac{D}{\beta_1\beta_2}\right)^2 + \frac{1}{4}\left(E - \frac{B}{\beta_2} - \frac{A}{\beta_1}\right)^2;$$

$A = 2(ST - UV)$, $B = 2(TU - SV)$, $D = 2(TV - U^2)$, $E = 2(S^2 - TV)$ are known combinations of A. N. Krylov's functions.

It is easy to see that $p_2 > 0$, $p_1 < 0$. Consequently the quantity $N(r)$ has a unique minimum that is reached when $r = r^*$: $r^*/l = -p_1/klp_2$.

As was noted above, the quantity β_1 is large compared with unity ($\beta_1 \sim (kh)^{-1}$). For the purpose of a qualitative analysis we can take $\beta_1 \to \infty$ in (20). This limiting case has been considered above. The result is expressed by formulas (12) and (14), and does not depend on β_2. The frequency dependence of the ratio of $N(r)$ to $N(0)$ is shown in Fig. 1. This ratio remains close to zero over a wide range of frequencies.

Let us now consider the case where the displacement at the input of the system is fixed and the angle of turn (formula (16)) is matched. By using the relations between the generalized displacements on the left and right ends of the plate

$$u_l = (S - i\beta_1 V)\,u_0 + (T + (1+i)\,\beta_2 U)\,u_0'/k,$$
$$u_l'/k = (V - i\beta_1 U)\,u_0 + (S + (1+i)\,\beta_2 T)\,u_0'/k,$$

we get

$$N/N\,|_{r=\infty} = [q_2 (kr)^{-2} + 2q_1 (kr)^{-1} + q_0]/q_0, \qquad (21)$$

where

$$q_2 = \frac{1}{\beta_1\beta_2}(T + \beta_2 U)^2 + \frac{\beta_2}{\beta_1} U^2 + \frac{S^2}{\beta_1^2} + V^2;$$
$$q_1 = -\frac{1}{\beta_1\beta_2}(S + \beta_2 T)(T + \beta_2 U) - \frac{\beta_2}{\beta_1} TU + \frac{SV}{\beta_1^2} + UV;$$
$$q_0 = \frac{1}{\beta_1\beta_2}(S + \beta_2 T)^2 + \frac{\beta_2}{\beta_1} T^2 + \frac{V^2}{\beta_1^2} + U^2.$$

The value of (21) reaches a minimum when $r = -q_2/kq_1$. A general idea of the nature of (21) can be obtained as above by examining the case $\beta_1 \to \infty$. After passing to the limit we get formulas (17).

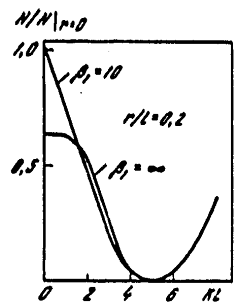


Fig. 1

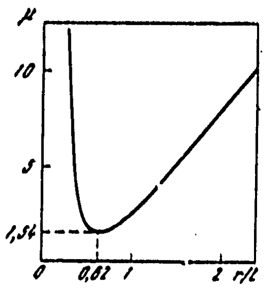
Our study shows that the vibrational energy that passes through T-joined plates depends strongly on the method of excitation. By properly choosing the coefficient of proportionality between the input force and torque, we can make this energy close to zero on any given frequency and in a fairly wide neighborhood of this frequency.

Evaluating the effectiveness of a vibration damping structure. Let us consider a vibration damping rod structure that uses the described effects. Let a vertical perturbing force $f_0 \exp i\omega t$ act on a body of mass m that rests on n identical cantilever plates of length l so that the force and torque that act on the end of each plate are related by expression (1). The degree of vibration damping is evaluated by the coefficient of transfer of the force $T = |F_0/f_0|$, where F_0 is the complex amplitude of the force that acts on the base. It is natural to evaluate the effectiveness of vibration damping by comparing this coefficient to the coefficient of transfer of a system of the same static structure in which the cantilever plates are replaced by springs. As implied by the expression for the coefficient of transfer of a unit plate (15), in the absence of damping we can make the coefficient of transfer T vanish on any predetermined frequency by matching the parameters of the plate and the value of coefficient r. The required static stiffness is ensured independently by appropriate choice of the number of plates and (or) their width. Thus the given system removes the usual contradiction between the requirement of sufficient static stiffness of the shock absorber and the requirement of vibration damping. Simple calculations show that the mass $m_д$ of the plates is related to the mass m of the object by the formula

$$m_д/m = \mu\,(\omega_0/\omega_*)^2,$$

where ω_0 is the natural frequency of the reference system; ω_* is the frequency on which damping is required; the values of the coefficient μ are taken from the graph in Fig. 2.

Fig. 2



To account for damping in the material, no fundamental difficulties are involved: instead of the Young modulus E, we must introduce the complex modulus $\tilde{E} = E(1 - i\eta)$. The damping factor η will be taken as independent of frequency. The calculations of the coefficient of transfer were done on a computer over a wide frequency range for different parameters. The table summarizes the lower limits of vibration damping over the frequency range $0.98\omega_* \le \omega \le 1.02\omega_*$, $\omega_* = 2\omega_0$ (in decibels with respect to the reference system). The results show that the use of a shock absorber of the described type in a support structure can be an effective means of vibration damping: appreciable suppression is realized in the vicinity of frequency ω*. At different r/l the frequency responses may differ considerably. The value of r/l should be selected on the basis of the nature of the perturbation, the desirable features of the frequency response and structural requirements. Considering the comparatively great

r/l	η										
	0	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09	0,10
0,1	27,9	27,7	27,0	26,1	25,0	24,0	23,1	22,2	21,4	20,1	20,0
0,15	28,9	28,7	28,0	27,0	26,0	24,9	23,9	23,0	22,2	21,4	20,6
0,2	18,3	18,0	17,3	16,4	15,4	14,4	13,5	12,7	12,0	11,3	10,7
0,25	30,4	30,1	29,5	28,5	27,5	26,4	25,4	24,5	23,6	22,8	22,0
0,3	30,3	30,8	30,1	29,2	28,1	27,1	26,1	25,1	24,2	23,4	22,7
0,35	28,1	27,8	27,1	26,2	25,2	24,1	23,1	22,1	21,3	20,5	19,7
0,4	23,7	23,5	22,8	21,8	20,8	19,7	18,8	17,8	17,0	16,2	15,5
0,45	16,6	16,4	15,7	14,8	13,8	12,8	11,9	11,1	10,3	9,6	9,0
0,5	−12,6	−8,7	−4,8	−2,6	−1,3	−0,5	0	0,4	0,7	0,8	1,0
0,55	2,9	2,8	2,5	2,2	1,9	1,7	1,5	1,4	1,3	1,2	1,1
0,6	8,9	8,6	8,0	7,2	6,4	5,6	4,8	4,2	3,7	3,2	2,9
0,65	6,6	6,3	5,8	5,1	4,3	3,7	3,1	2,6	2,2	1,9	1,6
0,7	−11,5	−9,1	−6,1	−4,1	−3,0	−2,2	−1,7	−1,4	−1,2	−1,0	−0,9
0,75	−12,0	−9,4	−6,3	−4,3	−3,1	−2,3	−1,8	−1,4	−1,2	−1,0	−0,9
0,8	9,1	8,9	8,3	7,4	6,6	5,7	5,0	4,4	3,8	3,3	2,9
0,85	14,4	14,2	13,5	12,6	11,6	10,6	9,7	8,9	8,1	7,4	6,7
0,9	17,4	17,2	16,5	15,6	14,5	13,5	12,5	11,6	10,8	10,1	9,4
0,95	19,4	19,2	18,5	17,5	16,5	15,5	14,5	13,5	12,7	11,9	11,2
1,0	20,8	20,6	19,9	18,9	17,9	16,8	15,8	14,9	14,0	13,2	12,5

weight of vibration damping systems that operate in the wave region, they should be used in frame structures. It is feasible to improve the effectiveness of vibration damping systems of the given type by using inhomogeneous rods with lumped masses and composite systems. For instance if we replace a homogeneous plate by a plate with periodically placed weights, we can reduce the transverse dimension of a vibration damping system by a factor of $(1+f)^{3/2}$, where f is the ratio of the mass of the weights to the mass of the plate. A comparison with other known designs of vibration damping supports shows that a system of the given type may be preferable, especially on frequencies close to the natural frequency of the system.

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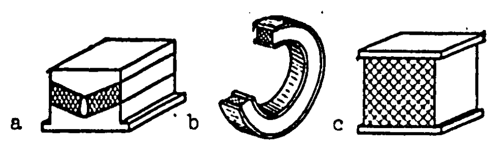
ON THE OPERATION OF RUBBER-METAL SHOCK ABSORBERS

R. I. Veytsman, N. M. Ostapishin, G. V. Tarkhanov

An investigation of rubber-metal shock absorbers in the low-frequency band of 10^{-3}-10^{-2} Hz is of interest since the question frequently arises in computational practice concerning the extent to which low-frequency dynamic characteristics can be considered as static.

Fig. 1



An investigation was made of the characteristics of shock absorbers of three types (Fig. 1): angle (a), ring (b) and a cube to which end blocks are vulcanized (c). In the frequency region up to 1 Hz the strain curves were recorded on a pulsator with harmonic change in the load relative to the predetermined average value. The static stiffness was taken as the ratio of the peak-to-peak amplitude of the load to that of the deformation on the first loading cycle on a frequency of 10^{-3}-10^{-2} Hz, while the dynamic stiffness was taken as this ratio after stabilization of the hysteresis loop. The loss factor was determined from the ratio of the area of the hysteresis loop to the amplitude of the potential energy of deformation of the shock absorber.

Stabilization of the hysteresis loop occurred after 15-20 loading cycles. The shape of the loop is close to elliptic, enabling representation of the stiffness of the shock absorber in complex form $C_0(1 + i\Delta/\pi)$, where the logarithmic decrement of vibrations Δ is equal to half the loss factor [Ref. 1].

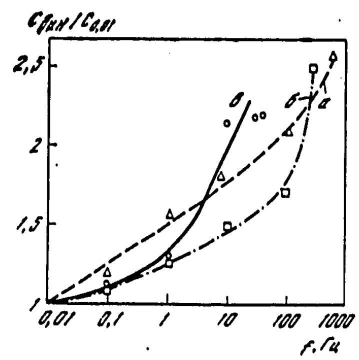
Above 1 Hz, stiffness was determined from the natural frequency, while the logarithmic decrement was determined from the width of the resonance peak of the investigated system.

On frequencies of 1-30 Hz the system consisted of a shock absorber with a weight G that was excited by a vibrator through a force sensor.

On higher frequencies the studies were done in a frame where the weight was pressed between two shock absorbers and excited by vibrators. At the same time the system was compressed by a predetermined static load.

The studies showed that in the frequency range from 10^{-3} to 10 Hz the logarithmic decrement increases smoothly from 0.2 to 0.3-0.4, and when the excitation frequency is raised to 10^3 Hz it reaches a value of 0.8-1. The logarithmic decrement increases by no more than 10-20% with an increase in the static load from zero to double the rated value.

Fig. 2



When the static load is increased from 50 to 100% of the nominal value, dynamic stiffness of the rubber-metal cubes and angle shock absorbers increases by 20-30%, while in the ring shock absorbers the stiffness remains almost unchanged.

The change in dynamic stiffness with frequency is due both to the way that the elastic modulus of the rubber depends on frequency, and to the change in the ratio of the height of the rubber mass to wavelength. The experimental values of this dependence are of particular interest in the low-frequency region, where wave effects are still weak. The relative values of dynamic stiffnesses $C_f/C_{0.01}$, where f is the excitation frequency, increase with frequency (Fig. 2): $C_f/C_{0.01} = (0.15\text{-}0.25)\lg(100f) + 1$ for $f \leq 100$. This change in stiffnesses corresponds approximately to the change in the elastic modulus of rubber with frequency [Ref. 2].

For rubber-metal rings (b) and cubes (c) the dynamic stiffness on a frequency of 0.01 Hz is approximately equal to the static value, but for an angle shock absorber (a) it exceeds the static value by a factor of 1.65. Thus the hysteresis properties of rubber-metal shock absorbers with harmonic excitation can be taken into consideration by introducing a complex modulus of elasticity beginning with frequencies of 10^{-3}-10^{-2} Hz. In this connection the modulus of stiffness and the logarithmic decrement must be determined from the steady-state strain curves after 15-20 loading cycles. The ratio between static and dynamic stiffnesses depends on the type of shock absorber.

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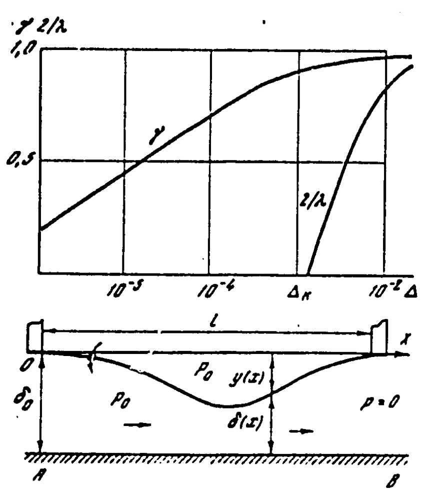
EVALUATING THE LOADING CAPACITY OF AN AIR-CUSHION SUPPORT

R. I. Veytsman

In recent years, air-cushion supports have come into use in a number of branches of engineering. The air flowing under the support raises it above the surface of the bed, enabling movement of heavy equipment installed on supports without great effort. The use of these supports requires determination of such characteristics as loading capacity, static and dynamic stability. In this article I will discuss the engineering evaluation of the loading capacity, which is the foremost characteristic to be determined in choosing a support.



The devices considered here [see for instance Ref. 1) contain an elastic annular plate or an axisymmetric shell in the lower part. Airflow takes place radially in a gap with a height that depends on the lift of the support as a whole and on the local deflection of the plate due to the difference in air pressures above the plate and in the gap beneath it. The most difficult problem is to determine the total force F acting on the plate. To determine F it is necessary to solve the interrelated problems of deformation of the plate and change of the air pressure in the gap.

The law of change in air pressure lengthwise of the gap depends on the type of flow. It will be assumed here that the air velocity is low and that flow is laminar. Without complicating the problem with questions of calculating three-dimensional flow and the theory of shells, let us consider the simplest example. We will assume that the plate is initially flat, and its width l is so much smaller than the radius that the problem can be solved as a plane problem. Flow is taken as plane-parallel. The edges of the plate are taken as fixed.

The lower part of the figure shows the computational model: in the cavity above OX over the plate, the air is under pressure p_0. The air flows through orifices close to the edge of the plate $x=0$ into gap OABX, and here it expands from $p=p_0$ at $x=0$ to $p=0$ at $x=l$. Since the size of the gap $\delta(x)$ changes smoothly, we use the law of flow of a viscous fluid between parallel walls:

$$dp/dx = -12\,\mu Q/\delta^3.$$

The deflection y of the plate is determined from the equation

$$EJd^4y/dx^4 = p_0 - p.$$

The notation used: Q--volumetric flowrate per unit of width of the flow; μ--coefficient of dynamic viscosity of air; EJ--stiffness of a unit of width of the plate; F--force acting per unit of width of the plate; R_k--concentrated force acting on a unit of width of the plate in section x_k where the plate is in contact with the support.

Let us use dimensionless variables and parameters:

$$\xi = x/l;\quad \eta = (\delta_0 - y)/\delta_0;\quad \Delta = EJ\delta_0/p_0l^4;$$
$$\lambda = 12\mu lQ/p_0\delta_0^3;\quad \beta = -EJ\delta_0^4/12\mu l^5Q;\quad \gamma = F/p_0l.$$

The sought relation between parameters Δ and γ can be found after solving the main nonlinear equation

$$\beta d^5\eta/d\xi^5 = 1/\eta^3.$$

Let us use the notation

$$J_k(\xi) = \int_0^\xi \xi^k\, d\xi/\eta^3;\quad J_k = J_k(1).$$

Using the conditions of the problem on the edge $\xi = 0$, we write the integrals of the main equation

$$\beta\eta^{IV} = J_0(\xi) = (1 - p/p_0)/\lambda, \quad (1)$$

$$\beta\eta^{III} = C_1 + \xi J_0(\xi) - J_1(\xi) = C_1 + \left(\xi - \int_0^\xi \frac{\xi p}{p_0}\, d\xi\right) \Big/ \lambda, \quad (2)$$

$$\beta\eta' = C_2\xi + \frac{1}{2} C_1\xi^2 + \frac{1}{6}\xi^3 J_0(\xi) - \frac{1}{2}\xi^2 J_1(\xi) + \frac{1}{2}\xi J_2(\xi) - \frac{1}{6} J_3(\xi), \quad (3)$$

$$\beta(\eta - 1) = \frac{1}{2} C_2\xi^2 + \frac{1}{6} C_1\xi^3 + \frac{1}{24}\xi^4 J_0(\xi) - \frac{1}{6}\xi^3 J_1(\xi) + \frac{1}{4}\xi^2 J_2(\xi) - \frac{1}{6}\xi J_3(\xi) + \frac{1}{24} J_4(\xi). \quad (4)$$

The conditions of fixing on the edge $\xi = 1$ give equations for determining the coefficients C_1 and C_2:

$$6C_2 + 3C_1 = -J_0 + 3J_1 - 3J_2 + J_3, \quad (5)$$

$$12C_2 + 4C_1 = -J_0 + 4J_1 - 6J_2 + 4J_3 - J_4. \quad (6)$$

The method of successive approximations can be used to solve the problem. After assigning the initial approximation, e. g. $\eta^{(0)} = 1$, we determine $J_0 = 1$, and find the dimensionless gradient $\lambda = 1/J_0$ from (1). The initial gap Δ is an assigned quantity, and we can determine $\beta = -\Delta/\lambda$. From (5) and (6) we find the coefficients C_1 and C_2. Now from (4) we determine the first approximation $\eta^{(1)}$. We get the next approximation in a similar way. After η has been found, the force $\gamma = 1 - J_1/J_0$ is determined from (1) and (2). This procedure can be successful only in cases where none of the approximations $\eta^{(i)}(\xi)$ vanishes anywhere on $0 \le \xi \le 1$.

Let us note the good convergence of the iteration process. For the force parameter γ we can limit ourselves to the first approximation. Two approximations are sufficient for flexures η. For hydraulic drag λ, the necessary number of approximations increases with a reduction in the parameter Δ of the gap, where Δ approaches the limiting value Δ_k. It can also be noted that the position of the throat of the channel (i. e. the value of ξ where η has a minimum) is almost independent of Δ. The quantity γ is also little dependent on Δ.

When $\Delta < \Delta_k$ the plate touches the base at $\xi = \xi_k$, and flow is blocked. The deformation of the plate in this case follows the equation $\eta^{IV} = 0$ on section $0 \le \xi \le \xi_k$, and $\eta^{IV} = -1/\Delta$ on the section $\xi_k \le \xi \le 1$. Assigning the value of ξ_k and using boundary conditions $\eta = 1$ and $\eta' = 0$ in cross sections $\xi = 0$, $\xi = 1$, and also $\eta = \eta' = 0$ in cross section $\xi = \zeta_k$, we determine η_- at $\xi \le \xi_k$, and η_+ at $\xi \ge \zeta_k$. The requirement of continuity of the moment $\eta''_- = \eta''_+$ at $\xi = \xi_k$ determines the relation between Δ and ξ_k under conditions of contact. To determine the force parameter it is necessary to account for the reaction of the support at the point of contact, which is proportional to the difference $\eta''_+ - \eta''_-$ at $\xi = \xi_k$, accordingly $\gamma = 1 - \xi_k - (\eta''_+ - \eta''_-)\Delta$.

The upper part of the figure shows γ and $^1/\lambda$ as functions of Δ. We determine the loading capacity of the support by adding the force that acts on the plate to the forces that are created by the pressure differentials to other parts of the device.

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PROBLEMS OF ACOUSTIC DIAGNOSIS

Yu. I. Bobrovnitskiy, M. D. Genkin, M. F. Dimentberg

One of the new areas of science that has been formulated in the last decade is acoustic diagnosis, i. e. diagnosis of the state of an investigated object from its noise or vibrations. The development of acoustic diagnosis has been fostered to a great extent by the development of methods of analyzing statistical signals (by using computers), and also by the development of such fields of science as identification of dynamic systems and pattern recognition. Some of the results found in these fields have been used directly in acoustic diagnosis. However, the particulars of the objects and goals of the study here require the development of new approaches and methods. It is no accident that the most significant advances in acoustic diagnosis have involved the development of specific research techniques.

The class of problems of acoustic dynamics is constantly expanding. In addition to "traditional" problems associated with checking the state of engineering facilities, these problems include many questions of investigation of the oscillatory properties of systems of various natures, including biological systems. With respect to their goals, all these problems can be divided into five interrelated types, partly considered in Ref. 1, 2. In this paper we give their characteristics and briefly review the principal methods of solution and results that have been found.

Estimates of the structural parameters of an engineering object. This is the most important and general problem from the standpoint of applications. It is also most typical for machines and mechanisms. Its purpose is the measurement (evaluation) of structural parameters (or in other words -- parameters of state, internal parameters) of the investigated object with respect to the characteristics of its acoustic signal. Sometimes the problem also includes classification of the object with the measured parameters in some state or another. The signal characteristics are termed informative diagnostic features. Naturally the structural parameters to be measured must influence sound formation within the object as otherwise their changes could have no effect on the acoustic signal or diagnostic features.

The use of noise and vibration signals to diagnose the state of machines and mechanisms on a scientific basis was apparently first described in Ref. 3, where a relation was established between errors in gear manufacture and assembly on the one hand, and the spectra of acoustic signals on the other. It was shown that the mean square levels of vibrations or noises in certain spectral regions are typical features that can be used with a certain degree of accuracy to evaluate the quality of gear manufacture and assembly. These results were then further developed and used in many fields of engineering, in particular in the motor vehicle industry to develop systems of acoustic diagnosis of transmissions and other parts of motor vehicles. Ref. 4 contains a general formulation of the problem of evaluating the structural parameters of machines and mechanisms and presents simple schemes for acoustic diagnosis of machine components. Subsequently a large number of papers were published dealing with the investigation of possibilities and the development of facilities for acoustic diagnosis of engineering objects of a variety of types. A survey of the most important of these works is given below in a discussion of the major approaches and techniques in solution of the given problem.

The general assumptions for the formulation of diagnosis in evaluating the structural parameters of an object are outlined in Ref. 4, 5. Mathematically the problem reduces to solution of the system of equations

$$P_i = f_i(a_1, a_2, \ldots, a_n), \qquad i = 1, 2, \ldots, m, \tag{1}$$

that establish the relation between the independent diagnostic feasures P_i and the independent internal parameters of the object a_j via functions f_i. Here the features P_i are known, and it is required to determine the quantities a_j.

Obviously the solution is determined by the form of the functions f_i that can be found from a theoretical examination of the mechanism of sound formation within the object, or in an experimental way. In the latter case it is convenient to represent them as polynomials of finite degree in a_j that approximate real relations (1). For instance, limiting ourselves to a linear relation and considering only the two parameters a_1, a_2, we can write

$$P_i = f_{i0} + f_{i1}a_1 + f_{i2}a_2 + R_i, \tag{2}$$

where f_{i0}, f_{i1}, f_{i2} are constant coefficients, $R_i = f_{i3}a_3 + \ldots$ is the part of P_i due to the influence of the parameters $a_3, a_4, \ldots, a_n$. The quantity R_i can be considered constant only in those cases where features P_i are independent of unconsidered parameters $a_3, a_4, \ldots, a_n$ or when these parameters remain unchanged throughout the entire course of diagnosis. Otherwise the value of R_i will vary from experiment to experiment, which is equivalent to an increase in the errors of measurements of P_i, and accordingly the calculations of parameters a_1 and a_2. Errors of diagnosis are described more completely in Ref. 6.

Let us examine in more detail two points that are of practical importance: a) how many features are needed to define parameters $a_1, \ldots, a_K$ when relations (1) or (2) are known; b) how many different states of the object with known values of $a_1, a_2, \ldots, a_k$ must be assigned so that they can be used to determine the specific form of polynomial relations (2).

The first question has no single answer. In the case where the features P_i are monotonically dependent on the parameters $a_1, a_2, \ldots, a_k$, k features are sufficient. In the presence of extrema, the number of features increases. For instance in the case of two parameters when there is one extremum we can limit ourselves to a quadratic dependence of P_i on a_1 and a_2; then the maximum number of features is five. This is equivalent to saying that relations (2) in the given instance are linear equations in five linearly independent unknowns a_1, a_2, a_1^2, a_1a_2 and a_2^2.

The answer to the second question is simpler. Experimental determination of q unknown coefficients in expansions of type (2) requires assignment of q different states of the object with unknown values of internal parameters. For quadratic polynomials of the parameters this number is $q = 1 + 2k + C_k^2$. When cubic terms are considered, the number increases to $q = 1 + 3k + k(k-1) + C_k^3 + C_k^3$ and so on. In the ideal case, one feature depends on only one parameter. In this case the least number of features and preliminary tests are needed for determining the specific form of the relations.

The selection of the informative features is the most difficult part of the given problem of acoustic diagnosis. In the case of unsuccessful choice of the features, their changes from an increase or reduction in the parameters a_j may be insufficiently large, as a result of which the incidental changes in conditions of measurements may be perceived as a change in the internal state of the object. In this case the features are said to have low informativeness or low sensitivity with respect to the given structural parameters $(\partial P_i/\partial a_j)$. The main requirement for a diagnostic feature is maximum sensitivity to one of the structural parameters and minimum sensitivity to all the others [Ref. 7].

In determining the most informative diagnostic features it is necessary to know the structure of the acoustic signal, and to do this a detailed investigation must be made of sound formation inside the object of diagnosis. Knowing the structure of the signal and the way that its different components depend on the internal parameters of the object we are able to find the best features and construct a model of the signal that is equivalent to the actual signal with respect to the given features, or to design a circuit for forming a model of the signal, i. e. to construct a model of the object of diagnosis as a sound generator. Thus a most important stage in acoustic diagnosis of the state of an object is the construction of its acoustic model of diagnosis. By such a model we mean a circuit that contains sources of random and (or) deterministic signals, and also linear and nonlinear elements, a signal being formed at the output of the circuit that is identical to the acoustic signal of the object to be simulated with respect to the set of diagnostic features. The characteristics of the sources and component elements of the model bear a one-to-one relationship with the structural parameters to be measured in the object. These parameters are measured (evaluated) by identifying the object and model with respect to closeness of diagnostic features.

The acoustic models of diagnosis used at the present time can be arbitrarily divided into two types (deterministic and statistical) depending on the kind of signals that are used.

The deterministic models have been developed more than others. Associated with these models are the most significant advances in the field of acoustic diagnosis of actual objects. The output signals are deterministic periodic functions: periodic series of pulses caused by collisions between parts, or harmonic functions associated with the rotation of parts of the machine or mechanism. The informative diagnostic features here are amplitudes, duration and times of appearance of pulses, and also frequency, amplitude and phase of harmonic signals. As a rule, the relation between these features and the internal parameters is determined on the basis of an analysis of the physical processes of sound formation without the aid of cumbersome experiments. Models with deterministic signals have been substantiated and give good practical results for comparatively low-rpm machines with a small number of internal sources of sound in which it is possible to distinguish the pulses caused by separate collisions of components. Such models are used in acoustic diagnosis of electrical machines [Ref. 8], mining machines [Ref. 9], internal combustion engines [Ref. 4], bearings [Ref. 10] and many other objects [e. g. Ref. 11-13]. Let us mention that for deterministic models there are a number of hardware realizations of diagnostic systems.

It is much more complicated to set up acoustic diagnosis in the case of high-rpm complex machines and mechanisms with a large number of internal sources. In these devices the vibrations and noises have a continuous spectrum where there are no pronounced discrete components and the individual pulses follow each other so closely that they merge and become indistinguishable. The acoustic signals in this case are random processes, and statistical models of diagnosis are needed that have output signals that are also random. For instance the signals of modern speed-reducers are random. The main sources of vibrations and noises in these mechanisms are processes of tooth engagement and the errors of gear manufacture, installation of gear trains, imbalance of shafts and the like that influence these processes. The problem of sound formation here is very complicated and is considered in many articles (e. g. Ref. 14-18) in connection with problems of acoustic diagnosis. On the basis of existing concepts a diagnostic model is presented in Ref. 19 for the simplest spur gear train with consideration of the following factors: profile errors of engagement, variable stiffness of engagement, errors of the main pitch and deformation of teeth that lead to collisions as the teeth engage. In this model the variable stiffness of engagement is represented by a step function of time with random amplitudes and random duration of the intervals of one-couple and two-couple meshing, the amount of deformation of a pair of teeth is modeled by the sum of two harmonic signals with random amplitudes and phases, and impact excitation is characterized by a series of momentary shocks with random amplitude synchronized with random moments of tooth engagement. The diagnostic model of a pair of teeth is thus represented as a linear system with random parametric, kinematic and pulse excitation. In a number of cases the characteristics of these random quantities can be selected in such a way that the output signals of the model become equivalent to the signals of an actual gear couple with respect to a whole series of diagnostic features [Ref. 20]. It should be noted that the informative features here are rather complex signal characteristics (bispectra, bivariate probability distribution functions, lines of regression, [kepstry] and so forth) that can be determined only by using high-speed digital computers. Analysis of some of them shows that in speed-reducing mechanisms there is a strong nonlinear relation among the different components of the acoustic signal [Ref. 21]. This shows that there are nonlinear elements in real objects and that it is necessary to further improve the model of diagnosis of gear meshing.

Another complicated object of diagnosis that is considered in Ref. 22 is an aircraft engine. On the basis of analysis of the fine structure of acoustic signals by probabilistic methods the authors have constructed a model of diagnosis that contains sources of random noise and sinusoidal signals as well as linear and nonlinear elements. The model is the equivalent of an engine with respect to a number of features that characterize bivariate probability distribution laws and nonlinear correlations of different signal components. This statistical model can also be classified as phenomonological since it does not reflect the physics of processes of sound formation inside the object.

In conclusion, let us mention one more phenomenological model [Ref. 23]. In this model the acoustic signal is represented as an expansion with respect to the powers of structural parameters with random coefficients. For instace for one parameter a the signal model takes the form

$$y(t) = x_0(t) + ax_1(t) + a^2 x_2(t) + \dots, \qquad (3)$$

where x_0, x_1, x_2 are steady-state random intercorrelated functions with characteristics selected from the condition of closeness of signal (3) to the actual signal with respect to certain features. If the features selected are the mean square level and components of the Laguerre spectrum of the signal, the model is amenable to simple hardware realization. Such a device was made and used to diagnose one of the parameters of a speed reducer [Ref. 24]. Since "instructing" the device requires experiments with known states of the object, which may be large in number as demonstrated above, this model (and device) can be recommended for diagnosis and continuous acoustic monitoring of large machines and mechanisms with complicated acoustic signals.

Evaluation of stability reserve, wear, reliability. The next class of problems in acoustic diagnosis consists in evaluating the stability reserve of a machine or structure based on analysis of acoustic signals. Let us assume that in the space of parameters of the system $a_1, a_2, \dots, a_n$ there exists some boundary of the region of admissible working modes such that the condition of stable operation is determined by an inequality of the form $F(a_1, \dots, a_n) > 0$. It is required to evaluate the quantity F, which it is natural to call the stability reserve, on the basis of analysis of oscillatory processes that arise in the system in the normal mode of operation; in this case the values of the parameters $a_1, \ldots, a_n$ are unknown; however, there is a dynamic model of the system that defines the form of the functional dependence of F on a_i [Ref. 25]. Of course the choice of the function F is always ambiguous (e. g. the quantity F^2 can also be considered the stability reserve), however, this circumstance is insignificant from the standpoint of most of the applications where this characteristic can be used.

Quantitative indices of the stability reserve are used first of all for comparative evaluation of the stability reserve of different systems that are obtained when certain design modifications are made. In the second place, on the basis of experience in the practical utilization of systems of the given class quantitative norms can be set up for the admissible values of the stability index. In the third place the stability reserve as a rule characterizes the degree of sensitivity of the system to external factors. Of interest in this respect for instance is Ref. 26 where a clear correlation is experimentally established between the stability reserve (with respect to rpm) of a circular saw and the quality of the saw cut. Let us also note that if it is possible to introduce special signals into the system during testing we can even do away with assigning the form of the function $F(a_1, \ldots, a_n)$ and express the stability reserve directly in terms of the parameters of the useful signal. For instance in Ref. 27 it was suggested that the system be set into self-oscillation with limited amplitude during testing by using controllable positive feedback; in this case the value of feedback gain corresponding to the limit of self-excitation of oscillations can be used as the stability reserve.

However, here we consider some examples of evaluation of the function F for a system with unknown parameters a_i, $i = 1, \ldots, n$, under conditions of normal operation in the presence of some "natural" wide-band random perturbations. The simplest example is a system model described by an ordinary differential linear equation with constant coefficients. In this case it can be assumed that $F = \alpha_m$, where α_m is the minimum among all values of the real parts α_k of the roots λ_k of the characteristic equation of the system. The quantity α_m can be evaluated by correlation or spectral analysis of the process $x(t)$ at the output of the system. If the natural frequency Ω_k of the system ($\lambda_k = \alpha_k + i\Omega_k$) are sufficiently far apart, the best estimate will apparently be given by the following procedure: $x(t)$ is passed through bandpass filters tuned to frequencies Ω_k, and the envelopes at the output of each filter are analyzed. For instance one can evaluate α_m from the normalized correlation function $r(\tau)$ of the envelope: without consideration of the influence of the filter $r(\tau) \approx \exp(-\alpha_m \tau)$ and at τ that are not excessively small, the filter has little influence. We can also evaluate α from the average period or the average number of extrema of the envelope (see the corresponding relations in Ref. 28, 29); such methods afford simple hardware realization and are convenient for fast operational analysis (without use of a digital computer). When methods of this kind are used it is important to select the bandwidth Δ of the filters in the best way: the smaller the Δ the better will be the "cutoff" of oscillations with respect to "extraneous" degrees of freedom with frequencies $\Omega_k \neq \Omega_m$; however in the case of Δ that are too small (comparable in order of magnitude with α_m) the useful signal is strongly distorted and the estimates of α_m are imprecise.

As another example [Ref. 25, 30] let us consider the model of a system described by the equation

$$\ddot{x} + 2\alpha\dot{x} + \Omega^2 x (1 + \lambda \sin 2\nu t) = \zeta(t), \qquad (4)$$

where α, λ, $|\nu - \Omega|$ are small constant quantities.

The condition of parametric stability of system (4) can be written as

$$\mu = \lambda\Omega^2/2\,[(\nu^2 - \Omega^2)^2 + 4\nu^2\alpha^2]^{1/2} < 1.$$

On the other hand, as shown in Ref. 25, 30, the probability density function $w(\phi)$ of the phase of process $x(t)$ can be represented as

$$w(\varphi) = C/[1 - \mu\cos(2\varphi - \psi)],$$

where C, ψ are constants, so that the parameter μ can be evaluated as a "coefficient of nonuniformity" of the phase

$$\mu = (w_{max} - w_{min})/(w_{max} + w_{min}).$$

The quantity $F = 1 - \mu^2$ can be taken as an index of stability reserve.

The second problem in acoustic diagnosis of this class is determination of the degree of wear of machine components in normal operation. Wear is accompanied by changes in a number of structural parameters of a component. The main difficulty of the problem is to determine the appropriate function of the parameters $F(a_1, \ldots, a_n)$ that would characterize wear. An example of solution of such a problem is given in Ref. 31, where it is proposed that the so-called generalized effective error of engagement (deviation of the transfer ratio from the nominal value) be taken as the characteristic of gear quality. This error is related in a simple way to the characteristics of the autocorrelation function of the acoustic signal.

An analogous formulation is possible for the problem of evaluating the degree of reliability (strength) of given components of engineering objects. As a rule, failure of a component is preceded by a continuous change in structural parameters and accordingly in the characteristics of the acoustic signal (for instance the intensity of ultrasonic vibrations with the formation of cracks in a material). It is obvious that the approaches described above can be applied to solution of this problem as well.

Problems of classification of states. The goal of the problem of acoustic diagnosis of this class is to use vibration or noise signals to determine what state an investigated object may be in out of several possible states, or to which of several possible objects an acoustic signal may belong. Typical examples of such problems are found in quality control of finished items (adjusted-unadjusted, suitable-unsuitable), classification of ship noises in hydroacoustics or heart noises in medicine and so forth. Problems of the quality control type are typical for machines and mechanisms. In this connection the "machine in adjustment" state can be broken up into several "substates" that are characterized for instance by a definite service life. In the limit with increasingly finer gradations we arrive at the problems considered above.

The classification problems in such a formulation are essentially problems of pattern recognition [Ref. 32], or more precisely recognition of sound patterns (the central problem in this area of science is automatic recognition of speech sounds [Ref. 33]). The usual approach in solving them is as follows. The aggregate of features of an acoustic signal (P_1, P_2,..., P_n) forms a so-called image (n-dimensional vector) in contrast to the pattern to which the state of the machine or mechanism corresponds. Compact regions correspond to patterns in the n-dimensional space of images. The problem is to define these regions on the basis of some kind of similarity of images. Frequently a standard image is put into correspondence with each pattern. Then an investigated image is compared with all standard images and classified with the pattern that has a standard closer than the others with respect to a selected measure of similarity.

In acoustic classification of the states of machines and mechanisms the mean square levels of vibrations on characteristic frequencies of the object are most frequently used as features, e. g. on the tooth frequencies of speed reducers [Ref. 9, 34, 35], or the amplitudes of periodic components in the function of autocorrelation of vibrations [Ref. 4, 36]. The recognition of states is accomplished by using the threshold values of levels (over the threshold means "machine out of adjustment") that are set after examining a sufficient number of machines in known states. More complicated characteristics of the signals and more complicated functions can also be used as the features and as the measure of similarity of images [Ref. 1, 32, 37].

Another approach that is used in solving problems of classifying states is based on recognition with respect to conditional probabilities [Ref. 32, 38]. In this procedure, a matrix of conditional probabilities $w(P_i/V_k)$ is set up during the instruction period, where P_i is some value of the i-th feature, V_k is the k-th state of the investigated object. Then the elements of this matrix are used to determine the most probable state of the object that corresponds to predetermined values of the features. This approach involves a large number of instruction experiments needed to find the elements of the matrix of conditional probabilities.

Let us also take note of methods that involve conversion of acoustic images to visual images [Ref. 33, 39]. This enables us to use algorithms developed in the theory of visual pattern recognition or to utilize more fully the capabilities of a human operator [Ref. 40].

Separation of sources of vibrations (noises). A very important problem in acoustic diagnosis is the so-called problem of separating sources of vibrations; the job is to detect the sources, and may include quantitative evaluation of the contribution made by each source to the observed vibration process. First let us consider this problem in the qualitative aspect where it is required to distinguish the predominant mechanism of excitation of vibrations.

In Ref. 41, 42 an examination was made of some problems of recognition of types of quasilinear oscillatory systems with one degree of freedom on the basis of statistical analysis of the processes in systems.

The systems examined were of the form

$$\ddot{x} + h(x, \dot{x}) + \Omega^2 x[1 + \xi(t)] = \zeta(t), \quad (5)$$

$$\ddot{x} + h(x, \dot{x}) + \Omega^2 x(1 + \lambda \cos 2\nu t) = \zeta(t). \quad (6)$$

Criteria of recognition are based on analysis of the probability density functions $w(y)$ -- the square of the amplitude $y(t)$ of process $x(t)$ -- and $w(\phi)$ -- the phase $\phi(t)$ of process $x(t)$. Let us introduce the following classification:

I. Self-oscillatory system -- system (5) having a stable periodic solution when $\xi(t) \equiv 0$, $\zeta(t) \equiv 0$.

II. System with external random excitation -- a system of type (5) in which $\langle y\rangle_\zeta \gg \langle y\rangle_\xi$, where always $x(t) \equiv 0$ when $\xi(t) \equiv 0$, $\zeta(t) \equiv 0$. Here $\langle y\rangle_\zeta$ is the average value $\langle y\rangle$ of the square of the amplitude when $\xi(t) \equiv 0$, $\langle y\rangle_\xi$ is the value of $\langle y\rangle$ when $\zeta(t) \equiv 0$.

III. System with parametric random excitation -- a system of type (5) in which $\langle y\rangle_\zeta \ll \langle y\rangle_\xi$, where always $x(t) \equiv 0$ when $\xi(t) \equiv 0$, $\zeta(t) \equiv 0$.

IV. System with periodic and random external excitation.

V. System with periodic parametric and random external excitation (6) having a stable periodic solution when $\zeta(t) \equiv 0$.

The criteria of pairwise recognition of systems of types I-V are summarized in the table. Here the amplitude criteria are denoted by Y, where Y_0 means that the function $w(y)$ decreases monotonically for all $y > 0$, Y_1 means that $w(y)$ has a finite interval of increase. The phase criteria are denoted by Φ, where the subscript associated with the Φ indicates the number of maxima of the function $w(\phi)$ on interval $[0, 2\pi)$. Each cell of the i-th line and the j-th column (i, j = I,..., V) gives the criteria according to which the hypothesis of membership in class i is valid as opposed to membership in class j. For instance, according to cell II, IV, the satisfaction of conditions Φ_0, Y_0 indicates that the system belongs to class II rather than to class IV. Satisfaction of conditions Y_1, Φ_1 shows in accordance with cell IV, II, that the system belongs to class IV rather than to class II. The biased criterion $Y_1^{(2)}$ is valid with a condition: it indicates membership to class IV only in the case of a sufficiently high relative level of the periodic component of the oscillations [Ref. 43]. The criterion Y_1 has the same bias for systems V as well (see Ref. 42, where the limit of monotonic behavior of the function w(y) is found).

Let us discuss special variants of the criteria Y as applied to systems with cubic nonlinearity

$$f(b)/\Omega = 2\alpha_1 b + \beta_1 b^3, \qquad \alpha_1 > 0, \qquad \beta_1 > 0,$$

where $f(b) = \int_0^{2\pi} h(b\sin\theta, \Omega b\cos\theta)\cos\theta\, d\theta$. The criterion $Y^{3)}$ is of this type: for the function w(y) found in an experiment two relations are constructed $[\psi_{IV}(z_{IV}) - \psi_{IV}(z_{0,IV})]\ \mathrm{sign}\ (y - y_{0,IV})$ and $[\psi_V(z_V) - \psi_V(z_{0,V})]\ \mathrm{sign}(y - y_{0,V})$ and the system is taken as belonging to class IV (or V) if relation ψ_{IV} (or ψ_V) is closer to linear. Here

$$\psi_{IV}(y) = \ln[y^{1/2} w(y)], \qquad \psi_V(y) = \ln[y^{1/2} w(y)],$$

$$z_{IV} = (y^{1/2} - y_{0,IV}^{1/2})^2\, \mathrm{sign}\,(y - y_{0,IV}),$$

$$z_V = (y - y_{0,V})^2\, \mathrm{sign}\,(y - y_{0,V}),$$

and $y_{0,IV}$, $y_{0,V}$ are points of maxima of the functions ψ_{IV} and ψ_V respectively.

As far as recognition of systems II and III is concerned, two simple rough criteria are possible: we classify a system as III if $y_m \ll \langle y \rangle$ (where y_m is the square of the most probable value of the amplitude) and if the curve of $\ln w(y)$ has positive curvature everywhere. The third criterion is that the experimental curve be approximated by the theoretical relation [Ref. 41]

$$w(y) = Ce^{-\beta y}(\varkappa + y)^{\varkappa\beta - \alpha},$$

where C, α, β, κ are constants. When $\alpha < 1$ the system belongs to class III, and when $\alpha > 1$ -- to class II (the value $\alpha = 1$ corresponds to the limit of stochastic stability of the linear system obtained from (5) when $h(x, \dot{x}) = 2\alpha\dot{x}$.

Гипотеза (1)	Альтернатива (2)				
	I	II	III	IV	V
I	—	Y_1	Y_1	Φ_0	Φ_0
II	Y_0	—	$Y^{1)}$	Y_0, Φ_0	Y_0, Φ_0
III	Y_0	$Y^{1)}$	—	Y_0, Φ_0	Y_0, Φ_0
IV	Φ_1	$Y_1^{2)}$, Φ_1	$Y_1^{2)}$, Φ_1	—	$Y^{3)}$, Φ_1
V	Φ_2	$Y_1^{2)}$, Φ_2	$Y_1^{2)}$, Φ_2	$Y^{3)}$, Φ_2	—

KEY: 1--Hypothesis
2--Alternative

Although these criteria have been derived theoretically only for systems with one degree of freedom, they can be taken as the basis for analyzing more complicated systems, checking out the possibilities for extrapolation of this kind in numerical experiments. For instance in Ref. 44 the feasibility of determining critical velocity from the realizations of random vibrations was demonstrated for the simplest model of flutter (systems with two degrees of freedom). The criteria given in the table can be used in particular for determining the predominant mechanism of transverse vibrations of rods in a longitudinal fluid flow.

The following problem [Ref. 45] can also be classified among qualitative problems of the "separation of sources." Assume that oscillations are observed in a multiple-mass system that are caused by a wide-band random load applied to one of the masses; it is required to determine this mass. It was shown in Ref. 45 that when the generalized coordinates to be measured are selected so that the matrix of generalized masses is diagonal, a frequency ω_* will be found for any $j \neq k$ such that the function $\psi_{jk} = \Phi_{jj}(\omega)/\Phi_{kk}(\omega)$ will be a monotonically decreasing function for all $\omega > \omega_*$. Here $\Phi(\omega)$ are the spectral densities of processes that correspond to the subscripts. This property can be used as a criterion for determining the number k (which has been confirmed by analog computer experiments). However, unfortunately this criterion is exceptionally sensitive to the hypothesis that there is only one source of excitation. The fact is that ω_* may be greater than the natural frequencies of the system. Therefore the representation of the system may be distorted by the presence of an additional source that is weak (compared to those being detected) but high in frequency. In this respect, it is desirable to get a more "stable" solution for the formulated problem.

Quantitative problems of separation (localization) of sources of vibrations or noise consist in determining the contributions of several simultaneously operating sources (for instance machines and mechanisms) to structural fields of vibrations or the atmospheric noise in a room (engine room, machine shop). Frequently in solving this problem a representation of the room and sources as an $(n+1)$-terminal network is used. Its n inputs on which steady-state random signals $x_i(t)$ of the sources are given are connected through linear links that have pulse transfer functions $h_i(t)$ to an adder at whose output (the $n+1$)-th terminal) a signal is formed at the observation point:

$$z(t) = \sum_{i=1}^{n} \int_{-\infty}^{t} h_i(t-\theta)\, x_i(\theta)\, d\theta + \xi(t), \qquad (7)$$

where $\xi(t)$ is noise uncorrelated with $x_i(t)$ that is caused by extraneous sources.

The problem was first solved for the case of statistically independent sources in a medium without dispersion for which the pulse transfer functions have the form $h_i(t) = h_i\delta(t - T_i)$, where the h_i are the transfer coefficients, $\delta(\ldots)$ is the Dirac function, T_i are delay times [Ref. 46]. The contributions W_i of the sources to the energy of the output signal $\overline{z^2}$ (the line denotes time-averaging wherever it occurs) are calculated in terms of the maximum values of the functions of mutual correlation of the signals $x_i(t)$ and $z(t)$:

$$W_i = h_i^2 \overline{x_i^2} = |\overline{zx_i}(T_i)|^2 / \overline{x_i^2}|.$$

For arbitrary functions $h_i(t)$ and independent $x_i(t)$ the problem is solved analogously with replacement of the correlation functions and the mean square levels by the corresponding spectral densities [Ref. 47]:

$$W_i(\omega) = |H_i(\omega)|^2 F_{ii}(\omega) = |S_{iz}(\omega)|^2 / F_{ii}(\omega),$$

where ω is angular frequency, $H_i(\omega)$ is the spectrum of the function $h_i(\omega)$, $F_{ii}(\omega)$ is the energy spectrum of signal $x_i(t)$, $S_{iz}(\omega)$ is the mutual spectral density of signals $x_i(t)$ and $z(t)$. For the case of correlated sources, the solution of the problem consists [Ref. 48] in determining the quantities $H_k(\omega)$ from the system of equations $S_{iz}(\omega) = \sum_{k=1}^{n} H_k(\omega) F_{ik}(\omega)$, where $i = 1, 2, \ldots, n$, $F_{ik}(\omega)$ is the mutual spectral density of signals $x_0(t)$ and $x_k(t)$, and in the next calculation of contributions $W_k(\omega) = |H_k(\omega)|^2 F_{kk}(\omega)$. The problem was considered in a similar formulation in Ref. 49, 50 as well.

Use of the solutions noted above for the problem of separation of sources is justified in cases where it is possible to measure the signals $x_i(t)$ directly in the sources. Most frequently in practice there is no such possibility, and it is necessary to use the readings of sensors located in the space between the sources and the point of observation as the signals $y_i(t)$ that characterize the sources. In the case of machines and mechanisms such signals are the vibrations on their frames. The computational scheme here is represented as an $(n+1)$-terminal network within which there are n independent sources with signals $x_i(t)$ that are not accessible for measurement. In passing through the linear links, these signals form input signals that are accessible for observation,

$$y_i(t) = \sum_{k=1}^{n} \int_{-\infty}^{t} h_{ik}(t-\vartheta)\, x_k(\vartheta)\, d\vartheta, \qquad i = 1, 2, \ldots, n, \qquad (8)$$

and output signal (7). Despite the independence of signals $x_i(t)$, the observed input signals (8) are statistically related due to the presence of cross connections that are characterized by the transfer functions $h_{ik}(t)$. In the absence of these connections ($h_{ik}(t) \equiv 0$ when $i \neq k$) each signal $y_i(t)$ is linearly dependent on only one $x_i(t)$, and then the given system reduces to the preceding computational scheme with independent sources [Ref. 47]. The approximate solution of the problem for the case of weak cross connections was found in Ref. 51. Corrections are given there for the solution of Ref. 46. The case of arbitrary connections is examined in Ref. 52. This work shows that knowing the autocorrelation and mutual correlation functions of signals (7) and (8) or their corresponding spectral characteristics enables us to find the contributions that the sources make to signal (7) only in special cases, e. g. for known functions $h_{ii}(t)$ [Ref. 53] in a medium without dispersion and with infinite velocity of propagation of perturbations [Ref. 54], when the sources can operate autonomously if only in unnatural conditions and so forth. However, in the general case these data are insufficient for determining the transfer functions, the characteristics of the sources, and consequently the desired contributions since the resultant system contains $2n^2 + n + 1$ unknowns and only $(n+1)^2$ equations. Introducing source-characterizing signals auxiliary to (8) such as $y_{n+1}(t)$, $y_{n+2}(t)$ and so on does not change matters since the additional spectral and correlation characteristics are linearly dependent on the original characteristics [Ref. 55, 56]. Thus to solve the problem of separation of sources in the quantitative formulation, in addition to all possible spectral and correlation characteristics of the measured signals (7) and (8) we need supplementary data on the investigated oscillatory system that are obtained without utilizing these signals. As far as we know, this problem has not been solved as yet in sufficiently general form.

Determining the dynamic characteristics of mechanical systems. Finally, an important class of problems in acoustic diagnosis consists in quantitative identification of given elements of a system, i. e. in determining the dynamic characteristics of these elements on the basis of analyzing input and output signals. For systems with lumped parameters this is a classical problem; it reduces to solution of a Wiener-Hopf integral equation relative to the unknown pulse transfer function of the system. This problem has been fairly well examined in the literature (e. g. Ref. 57); here we will discuss only certain aspects of quantitative identification that are specific to mechanical systems. In mechanical systems it frequently happens that the input excitation is inaccessible to measurement, although it can be taken as wide-band. A typical example of such a situation is vibrations of an element of an elastic structural member stimulated by turbulent pulsations of liquid or gas pressure. In the general case the problem consists in approximating the spectral density of the vibration process by an assigned function. It should be noted only that in such an identification scheme that does not utilize measurement of the input signal, severe requirements are made on the accuracy of measurements and on the duration of the analyzed realization.

When high-frequency random vibrations propagate in extensive elements of machines and structural members it is often feasible and advisable to analyze the oscillations as traveling waves. Such problems have been studied most completely for one-dimensional systems [Ref. 49]. Here it is possible to evaluate the rate of propagation of waves (with respect to the quantity τ_m corresponding to the absolute maximum r_m of the coefficient of mutual correlation $r(\tau)$ of vibrations in two points), the coefficient of attenuation (with respect to r_m), and also the coefficients of transmission and reflection of waves from identical barriers. In the case of media without dispersion such an analysis is done directly for the measured processes. For media with dispersion the analysis is done separately in narrow frequency bands by preliminary passage of the processes through band filters. In this case the group velocity of wave propagation is found from a series of values of τ_m, while the coefficients of attenuation, reflection and transmission must be evaluated with consideration of the reduction in the values of r_m due to "racing" of the wave packet within the limits of each band [Ref. 49]. As one of the numerous applications we will mention evaluation of the elastic characteristics of pipelines with flowing liquid for purposes of diagnosing cardiovascular disorders [Ref. 58].

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BALL BEARING DIAGNOSIS BY A VIBRATION METHOD

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Spectrograms of ball bearing vibration show a background of a continuous spectrum against which characteristic discrete components are observed to vary in proportion to the speed of rotation. These spectral components arise because of deviations of the races from circularity in a pattern called waviness. To evaluate the vibroactivity of waviness, a spectral method is used that enables the derivation of analytical relations between the parameters of the harmonics of waviness and the parameters of the resultant vibration: frequency and amplitude. On this basis, solutions are found for the following problems in ball bearing diagnosis:

by a vibration method, without disassembling the bearing, the technical state of the outer and inner rings is determined, which is characterized by the presence of harmonic components with high vibroactivity in the spectra of waviness of the races;

Before a bearing has been assembled, the harmonic components of waviness on the ball races of the inner and outer rings are established (by a vibration method), and a prediction is made concerning the corresponding frequencies and amplitudes of vibration of the bearing assembled from these rings;

for each harmonic component of waviness of the outer or inner ring, the admissible value of its amplitude is determined as a function of vibration requirements.

The information on the technical state of the ball races of the rings can be used to select rings for assembling low-noise ball bearings; to determine the degree of suitability of a bearing under working conditions; to calculate the vibration of bearing units that arises as a consequence of kinematic perturbation due to waviness of the ball races of the bearing.

The spectral method of evaluating waviness is to use a Fourier series to represent the deviations $\Delta\rho_k(\phi_k)$ of the ball race from circularity

$$\Delta\rho_k(\varphi_k) = a_0 + \sum_{i=1}^{\infty} a_{ki} \cos(i\varphi_k + \alpha_{ki}),$$

where $i = 2\pi/\lambda_i$ is the number of the harmonic that shows how many waves of a given angular pitch λ_i fit into the nominal circle; a_{ki} is the amplitude of the i-th harmonic of the k-th ring; α_{ki} is the phase shift; a_0 is the average value of the function $\Delta\rho_k(\phi_k)$ for the period; ϕ_k is the polar angle (for the inner ring k = 1, for the outer ring k = 2).

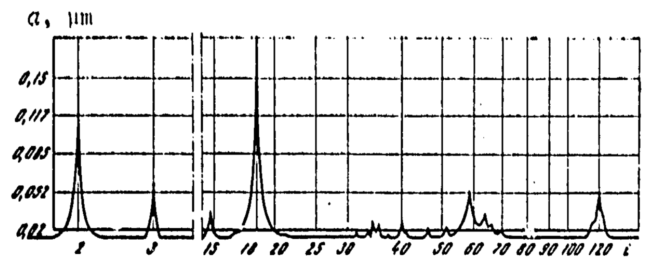

Fig. 1

The set of amplitudes a_{ki} of the Fourier series forms a spectrum of waviness of the ball race for each ring of the ball bearing. By using the electronic equipment described in Ref. 1, 2 this spectrum is automatically recorded (as a probe runs over the surface of the rotating part) in the form of a diagram (Fig. 1) on which the numbers of the harmonics are plotted along the axis of abscissas, while the corresponding amplitudes are plotted along the axis of ordinates. In this way, the harmonics of even very high numbers can be fixed with amplitudes of hundredthes of a micrometre.

The vibration frequencies in Hertz of the harmonic components of waviness are calculated by the formula

$$f_{ki} = in_k/60 \qquad (k = 1, 2), \tag{1}$$

where

$$n_1 = n[1 - 1/2(1 + r/R)]; \qquad n_2 = n/2(1 + r/R).$$

Here n_1 and n_2 are the speeds (in rpm) of the inner (n_1) and outer (n_2) rings relative to the separator); R is the radius of the ball race of the inner ring; r is the radius of the ball; n is the shaft speed.

The vibration amplitudes with respect to acceleration of the harmonic components of waviness are determined by the expression

$$W_{ki} = 4\pi^2 f_{ki}^2 a_{ki}, \tag{2}$$

which is transformed to

$$W_{ki} = \frac{\pi^2 n_k^2}{900} a_{ki} i^2, \qquad k = 1, 2. \tag{3}$$

Since the vibration acceleration in this expression is directly proportional to the quantity in parentheses, even very small amplitude components a_{ki} of the spectrum of waviness that could have been disregarded at low numbers of the harmonics can cause considerable vibration accelerations if they are related to harmonics of sufficiently high numbers. Therefore the discrete components of the spectrum of waviness should be evaluated from the standpoint of vibroactivity as a function of the products $a_{ki}i^2$ rather than with respect to the values of the amplitudes a_{ki} of these components. To illustrate this, Fig. 1 shows the spectrogram of waviness of the ball race of the inner ring of one of the ball bearings, and we give here the values of the quantities a_{ki} and $a_{ki}i^2$ for a number of harmonics of the spectrum

i	2	3	18	58	120
a_{ki}, µm	0.140	0.055	0.170	0.052	0.050
$a_{ki}i^2$, µm	0.56	0.50	55.0	175.0	720.0

As we can see, the harmonics of low numbers (i = 2, 3) that are associated with a flat spot have low vibroactivity with respect to acceleration, while the 120-th harmonic of waviness, despite low amplitude, has very high vibroactivity.

Thus the most dangerous from the standpoint of vibroactivity are the harmonics of waviness of higher numbers.

In machine building, low-noise ball bearings are often required to have a spectrum of vibration recorded with respect to acceleration at a given rotational velocity that is bounded throughout the entire frequency range by some line, e. g. a sloping straight line with equation

$$W_{max} = F(f). \tag{4}$$

This requirement also imposes limitations on the amplitudes of harmonics of different numbers. After substituting the value of (4) in expression (2), we get a formula for calculating the limiting admissible amplitude of the i-th harmonic of waviness

$$a_{ki} = F(f_{ki})/4\pi^2 f_{ki}^2. \tag{5}$$

Relations (1), (2) and (5) enable solution of all the problems formulated above for ball bearing diagnosis. To get the solutions, these relations must be represented as tables or nomograms plotted for a given bearing size, predetermined rotational velocity and vibration requirements.

As an example, Fig. 2 shows a nomogram plotted for ball bearing No 306 at a rotational velocity of n = 1500 rpm.

Straight lines 1 and 2 on the nomogram establ'·h the relation between the numbers of harmonics of waviness of the inner .1) and outer (2) rings and the frequency f of the stimulated vibration. For instance the 27-th harmonic of waviness of the outer ring causes bearing vibration on a frequency of 255 Hz, while the same harmonic of waviness on the inner ring causes vibration with a frequency of 420 Hz.

The parallel straight lines 3 on the nomogram relate the amplitudes of the harmonics of waviness, the vibration frequency (number of the harmonic) and the vibration level. For instance the 27-th harmonic of waviness of the outer ring with an amplitude of 0.1 µm on a frequency of 255 Hz causes bearing vibration of 55 dB. The reverse relation also holds: in the vibration

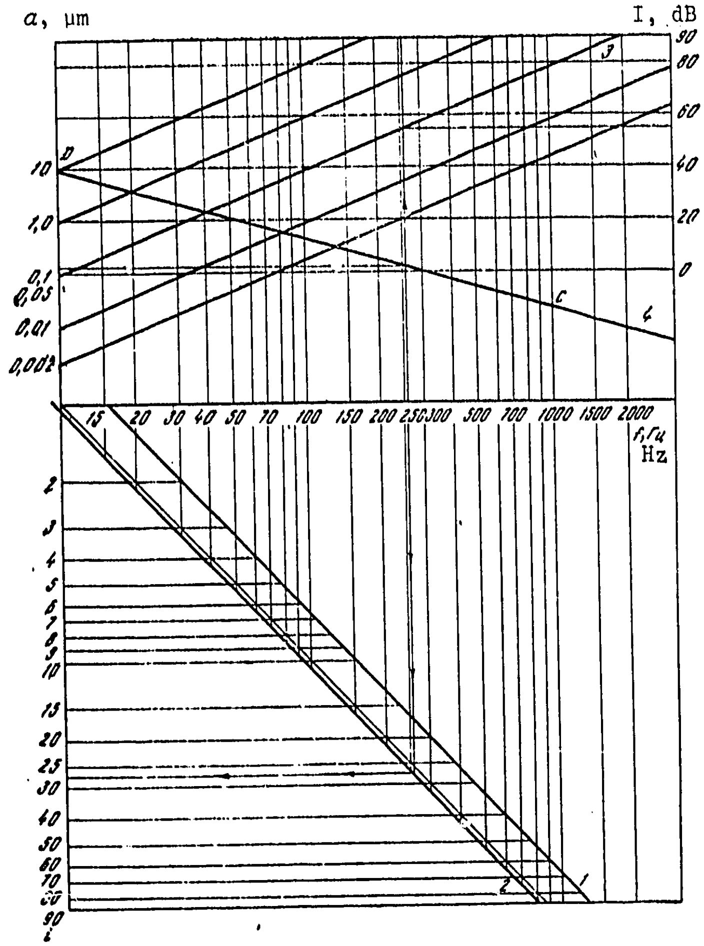

Fig. 2

spectrum of the ball bearing the 27-th harmonic of waviness of the outer ring with amplitude of 0.1 μm corresponds to a level of 55 dB on a frequency of 255 Hz if the vibration frequency changes in proportion to the speed of rotation.

Straight line 4 of the nomogram that reflects the requirements with respect to vibration imposed on the bearing shows the relation between the vibration frequency (number of the harmonic) and the limiting admissible amplitude of the harmonic of waviness. For instance in order for the vibration from the 27-th harmonic of waviness of the outer ring on a frequency of 255 Hz to stay below the admissible level, the amplitude of this harmonic must not exceed 0.11 μm.

Thus the described nomogram enables solution of different problems in the diagnosis of a ball bearing. The accuracy of the solution depends on the resolution of the analyzer and the scales of the nomogram.

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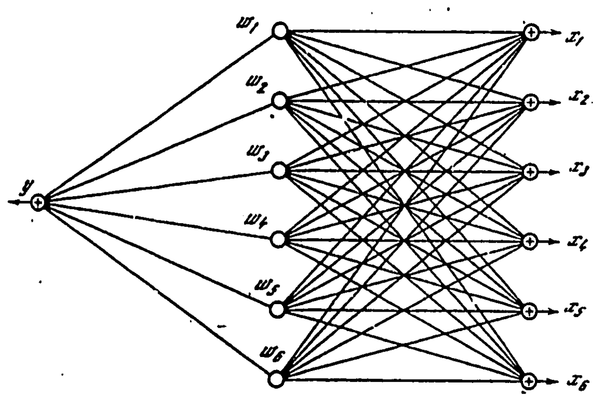
LOCALIZATION OF ACOUSTIC SOURCES

I. P. Biryukova, A. M. Medvedkov, V. V. Naumova

Determination of the contribution that individual mechanisms operating as a group make to the overall acoustic field is usually handled by appropriate analysis of synchronous multichannel recordings of signals taken from microphones set up close to the mechanisms as well as at the investigated point of the field. The signal taken from a given microphone is determined not only by the physical parameters of the acoustic field that is formed by the mechanism closest to the microphone, but also by the parameters of the acoustic fields that are induced simultaneously by the working mechanisms. In this case the signals that are usually taken as steady-state random processes of second order are statistically related or partly coherent.

Coherence in this case is understood in the sense of the definition presented in Ref. 1.



Let us introduce the following notation:

$w_i(t)$ -- component of the random vector of unobservable signals that corresponds to the i-th source (mechanism);

$x_i(t)$ -- component of the random vector of observable signals that corresponds to the i-th microphone;

$y(t)$ -- observable random signal of the microphone set up at the given point of the acoustic field;

$g_{ij}(t)$ -- impulse transfer function of the linear system that connects the i-th and j-th sources;

$h(t)$ -- impulse transfer function of the linear system that connects the i-th source to the investigated point of the acoustic field;

t -- time.

In these definitions, an unobservable signal is understood as the microphone signal determined by the corresponding parameters of the acoustic field that is formed by a given mechanism in autonomous operation, while an observable signal is understood as a signal corresponding to the superposition of acoustic fields that are formed by the combined operation of mechanisms [Ref. 2].

The Fourier transforms of the signals and the impulse transfer functions will be denoted by the corresponding capital letters.

A schematic of the formation of observable signals in the case of N statistically independent sources for $N=6$ is shown in the diagram on the preceding page, where the time dependence of the signals is not indicated. Symmetric dynamic systems are considered for which $G_{ij}(f)=G_{ji}(f)$, and it is also assumed that $G_{ii}(f)=1$, where f is frequency. This enables us to get an exact solution of the problem in the frequency region. We will look for the solution on the basis of the example of two acoustic sources; the case $N>2$ can be examined in an analogous scheme.

In accordance with the diagram we write

$$x_j(t) = \sum_{i=1}^{2} \int_0^{\infty} g_{ji}(\tau)\, w_i(t-\tau)\, d\tau, \qquad (1)$$

$$y(t) = \sum_{i=1}^{2} \int_0^{\infty} h_i(\tau)\, w_i(t-\tau)\, d\tau. \qquad (2)$$

After carrying out Fourier transformation on both members of the equations, we get

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 1 & G_{12} \\ G_{21} & 1 \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}, \qquad (3)$$

$$Y = [H_1,\ H_2] \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}. \qquad (4)$$

In these analytical expressions and the ones to follow, frequency dependence is not indicated for the sake of brevity.

Let us set up the direct product of the two-component vector [X*], the complex conjugate to vector [X], with its Hermitian conjugate $[X^*]^+$, and then carry out the operation of mathematical expectation for both members of the resultant equality and integrate it in the vicinity of the investigated frequency, and as a result [Ref. 1] we get

$$\begin{bmatrix} S_{x_1x_1} & S_{x_1x_2} \\ S_{x_2x_1} & S_{x_2x_2} \end{bmatrix} = \begin{bmatrix} 1 & G_{12} \\ G_{21} & 1 \end{bmatrix}^* \begin{bmatrix} S_{w_1w_1} & 0 \\ 0 & S_{w_2w_2} \end{bmatrix} \begin{bmatrix} 1 & G_{12} \\ G_{21} & 1 \end{bmatrix}, \qquad (5)$$

where $S_{x_ix_i}$ is the spectral density of the observable signal $x_i(t)$; $S_{x_ix_j}$ is the mutual spectral density of the observable signals $x_i(t)$ and $x_j(t)$; $S_{w_iw_i}$ is the spectral density of the unobservable signal $w_i(t)$; G_{ij} is the frequency response of the linear system that connects the i-th and j-th sources.

By analogous calculations we can get the following expression for the spectral density of signal y(t):

$$S_{yy} = [H_1 \;\; H_2]^* \begin{bmatrix} S_{w_1w_1} & 0 \\ 0 & S_{w_2w_2} \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \end{bmatrix}.$$

To study equation (5) we introduce a complete orthogonal basis system of Pauli spin matrices

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}.$$

The algebraic properties of the Pauli spin matrices are determined by the following relations [Ref. 3]:

$$\sigma_\alpha\sigma_\beta = -\sigma_\beta\sigma_\alpha = i\sigma_\gamma,$$

where α, β, γ represent a cyclic permutation in (1)-(3),

$$(\sigma_j)^2 = \sigma_0, \qquad j = 0, 1, 2, 3, \qquad (6)$$

$$\sigma_j\sigma_0 = \sigma_0\sigma_j = \sigma_j \qquad (j = 0, 1, 2, 3), \qquad (7)$$

$$Sp\,(\sigma_i\sigma_j) = 2\sigma_{ij} \qquad (i, j = 0, 1, 2, 3), \qquad (8)$$

where Sp is the Spur of the matrix; σ_{ij} is the Kronecker symbol.

In matrix equation (5) the unknown quantities are the elements of the matrix of spectral densities of the unobservable signals $[S_{w_iw_j}]$ and of the matrix of frequency responses $[G_{ij}]$.

Let us expand the matrices that appear in equation (5) with respect to the complete system of Pauli spin matrices. With consideration of equalities (6)-(8), the matrix of spectral densities of observable signals is written as

$$[S_{x_i x_j}] = \frac{1}{2}\sum_{l=0}^{3} S_l \sigma_l, \tag{9}$$

where the $S_l = Sp(\sigma_l [S_{x_i x_j}])$ are Stokes parameters.

Introducing the notation

$$G_{12} = \alpha_1 + i\alpha_2,$$

for the matrix of frequency responses we get the expression

$$[G_{ij}] = \frac{1}{2}\sum_{l=0}^{3} \beta_l \sigma_l, \tag{10}$$

where $\beta_0 = 1;\ \beta_1 = 0;\ \beta_2 = \alpha_1 + i\alpha_2;\ \beta_3 = 0.$

The matrix of spectral densities of observable signals can be presented as

$$[S_{w_i w_j}] = \frac{1}{2}\sum_{l=0}^{3} \hat{S}_l \sigma_l, \tag{11}$$

where $\hat{S}_2 = \hat{S}_3 = 0.$

Substituting expressions (9)-(11) in equation (6), we get

$$\sum_{l=0}^{3} S_l \sigma_l = \sum_{i=0}^{3} \beta_i^* \sigma_i^* \sum_{l=0}^{3} \hat{S}_l \sigma_l \sum_{k=0}^{3} \beta_k \sigma_k. \tag{12}$$

By using relations (6), (7), let us simplify the second member of equation (12), and then by equating the coefficients before spin matrices with identical indices standing in the first and second members, we reduce this equation to the following system

$$\begin{aligned} &\hat{S}_0 [1 + (\alpha_1^2 + \alpha_2^2)] = S_0, \qquad \hat{S}_1 [1 - (\alpha_1^2 + \alpha_2^2)] = S_1, \\ &2\alpha_1 \hat{S}_0 = S_2, \qquad 2\alpha_2 S_1 = S_3. \end{aligned} \tag{13}$$

Using the representation of the complex frequency response G_{12} in trigonometric form

$$G_{12} = \alpha_1 + i\alpha_2 = |G_{12}|\cos\varphi + i\,|G_{12}|\sin\varphi,$$

we get the following equations for determining the elements of frequency response from system (13) when $\alpha_1 \neq 0$ and $\alpha_2 \neq 0$:

$$\begin{aligned} &\operatorname{tg}\varphi = S_0 S_3 (1 - |G_{12}|^2)/S_1 S_2 (1 + |G_{12}|^2), \\ &|G_{12}|^4 + 2A\,|G_{12}|^2 + 1 = 0, \end{aligned} \tag{14}$$

where

$$A = (S_1^2 S_2^2 - S_0^2 S_3^2 - 2S_1^2 S_0^2)/(S_1^2 S_2^2 + S_0^2 S_3^2).$$

It can be simply shown that A satisfies the inequality $A \leq 1$. This condition implies that there is always a unique solution of equation (14) that lies in the interval between zero and unity. With the aid of the resultant solution we get the spectral densities of unobservable signals from (5)

$$S_{w_1w_1} = (S_{x_1x_1} - |G_{12}|^2 S_{x_2x_2})/(1 - |G_{12}|^2 |G_{21}|^2),$$
$$S_{w_2w_2} = (S_{x_2x_2} - |G_{12}|^2 S_{x_1x_1})/(1 - |G_{12}|^2 |G_{21}|^2).$$

To determine the contribution of the first and second sources it is necessary to find expressions for the frequency responses H_1 and H_2 of the dynamic system that connects the sources with the predetermined point of the acoustic field.

From form (3) we get the following expressions for the Fourier transform of the observable signals:

$$W_1 = (X_1 - G_{21}X_2)/(1 - G_{12}G_{21}),$$
$$W_2 = (X_2 - G_{12}X_1)/(1 - G_{12}G_{21}). \quad (15)$$

Using formulas (4), (15) for the mutual spectral density of the observable signal $w_1(t)$ and signal $y(t)$ we get the expression

$$S_{w_1y} = (S_{x_1y} - G^*_{21}S_{x_2y})/(1 - G^*_{12}G^*_{21}).$$

Taking into consideration that $H_1 = S_{w_1y}/S_{w_1w_1}$, we finally write

$$H_1 = \frac{(1 - |G_{12}|^2 |G_{21}|^2)(S_{x_1y} - S^*_{21}S_{x_2y})}{(S_{x_1x_1} - |G_{12}|^2 S_{x_2x_2})(1 - G^*_{12}G^*_{21})}, \quad (16)$$

Analogously for H_2 we get

$$H_2 = \frac{(1 - |G_{12}|^2 |G_{21}|^2)(S_{x_2y} - S^*_{12}S_{x_1y})}{(S_{x_2x_2} - |G_{21}|^2 S_{x_1x_1})(1 - G^*_{12}G^*_{21})}. \quad (17)$$

By using formulas (16), (17), frequency responses H_1 and H_2 are expressed in terms of the spectral densities of the observable signals and the computational values G_{12}.

Considering that the unobservable signals are statistically independent, the contribution of each source to the overall acoustic field is determined from the formula

$$S_{y_iy_i} = |H_i|^2 S_{w_iw_i}.$$

It is easily established that the given method of localizing acoustic sources is related to existing approximate methods, and specifically to the method of residual spectra presented in detail in Ref. 1.

In accordance with this work, we have the following expressions for the spectral densities of the observable signals:

$$S_{x_1x_1} = \sum_{j=1}^{2} |G_{1j}|^2 S_{w_jw_j}, \qquad S_{x_1x_2} = G_{12}S_{w_1w_1} + G^*_{21}S_{w_2w_2}. \quad (18)$$

With consideration of relations (18), the coherence function of observable signals is written as

$$\gamma^2_{x_1x_1} = \frac{|G_{12}S_{w_1w_1} + G^*_{21}S_{w_2w_2}|^2}{(S_{w_1w_1} + |G_{12}|^2 S_{w_2w_2})(S_{w_2w_2} + |G_{21}|^2 S_{w_1w_1})}.$$

As a result, for the residual spectrum of the observable signal $x_1(t)$ we get the expression

$$S_{x_1x_1\cdot x_2} = S_{x_1x_1}(1 - \gamma^2_{x_1x_2}) = S_{w_1w_1}\chi_1,$$

where

$$\chi_1 = \frac{1 + |G_{12}|^2|G_{21}|^2 - G_{12}G_{21} - G^*_{12}G^*_{21}}{1 + |G_{12}|^2\beta^2}, \qquad \beta = \frac{S_{w_1w_1}}{S_{w_2w_2}}.$$

In this formula $G_{12} = G_{21}$ is found from the solution of equation (14). After simple calculations it can be shown that the contribution of the source to the overall acoustic field as calculated by the method of residual spectra $S_{yy\cdot x_2}$ [Ref. 1], and the contribution $S_{yy\cdot w_2}$ determined by the proposed method are related by the expression

$$S_{yy\cdot w_2} = \chi_1^{-1} S_{yy\cdot x_2}.$$

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ON THE PROBLEM OF VIBRATION DIAGNOSIS OF TECHNOLOGICAL FLAWS IN ELECTRIC MACHINES

V. A. Avakyan

The main job in vibration diagnosis of electric machines is to establish the relation between the characteristics of the spectrum of vibrations of the housing (measurable parameters) and technological flaws (diagnosed parameters) [Ref. 1-3].

Let us consider vibrations of an electric induction motor and use the model of force factors and the model of isolated actions of flaws that were developed in Ref. 4: a force element that consists of Z_c poles (balls) and is characterized by the function $F_c(\phi)$ rotates relative to a stationary system with velocity ω_C and interacts with flaws $\Delta_B(\phi)$ of the rotor barrel and grooves of the rotating bearing rings (rotational velocity is equal to ω_B), and also with flaws $\Delta_H(\phi)$ of the stationary rings and the stator. The force action of system Z_C on a solid can be represented by the function $F(\phi_C, \phi_B)$ ($\phi_C = \omega_C t$, $\phi_B = \omega_B t$) that permits spectral representation. Obviously the Fourier expansion here should be carried out in the complex region [Ref. 5].

The force fields $F_C(\phi,\Delta)$ are nonlinear functions not only of angle ϕ, but also of the depth of action Δ, which in turn is a function of the deviation of the shape that is due to the flaws. Expanding in a series with respect to small quantities Δ_H, Δ_B, and leaving only the first terms, one can get the following expression of the complex amplitude C_k of the Fourier series for frequencies $KZ_C\omega_C$ in the case of isolated actions of flaws:

$$C_k = Z_c\delta_H(Z)f_c(\rho - Z)/2\pi = Z_c\delta_H(j(KZ_c - 1))f_c(-jKZ_c)/2\pi, \quad Z = jKZ_c + \rho, \quad \rho = -j. \tag{1}$$

Now let us consider the case of simultaneous action of numerous flaws. As the characteristic of the force field here it is necessary to take the system of all poles $F_{C\Sigma}(\phi)$ rather than the single pole $F_C(\phi)$ (flaws may be multiple -- flat spots, waviness) and to represent them in a generalized way. After a number of simple transformations we can get a generalized expression for the amplitude of complex oscillations

$$C_k = \frac{1}{2\pi}\delta(Z)f_{c\Sigma}(S - Z)\big|_{Z=jk+\rho,\, \rho=-j} = \frac{1}{2\pi}\delta_H(j(K-1))f_{c\Sigma}(-jK). \tag{2}$$

It is difficult to give preference to any one of these approaches: C_k is expressed more simply in (2) and corresponds better to the concept of harmonics in Fourier analysis (for instance we can draw the conclusion that only harmonics that are multiples of Z_C show up rather than any harmonics), but this expression is more complicated in pracitcal use since it requires a more complicated image of the force field; on the other hand, expression (1) in many instances is much more convenient (for instance the conclusion of appearance of harmonics that are multiples of Z_C follows from this expression automatically without any additional analysis). Therefore expression (1) can be used for specific analysis of versions of simple interactions, while generalized expression (2) can be used in the general case of complex interactions.

We consider a typical spectrum first on the basis of the example of interaction of an isolated flaw Δ_H and the poles F_C of fairly general form -- a sinusoidal half-wave; the flaw has width π/l, and the pole has width π/m, where l and m are any positive numbers (integers or fractions). We represent the Fourier transform of the flaw as follows:

$$\delta_H(jS) = 2\cos(\pi S/2)l^{-1}(1 - S^2)^{-1}\exp[-i\rho(\varphi_H + \pi/2l)] = A_\delta(jS)\exp[-\varphi_\delta(jS)]. \tag{3}$$

The first cofactor in (3) characterizes the modulus of the spectrum $A_\delta(S)$, and the second cofactor characterizes its phase $\phi_\delta(S)$ (which is more conveniently expressed in absolute frequencies). Let us note that (3) coincides with the expression for the same spectrum [Ref. 4] derived from other initial assumptions. The spectrum of the pole $f_c(JS)$ is expressed also by formula (3) if l is replaced by m and is assumed that the phase shift of the flaw $\phi_H=0$. As implied by expression (1), the width of the vibration spectrum is determined by the more extended element (pole or flaw), while the scale of its amplitudes is determined by the product of dimensions $(1/lm)$. It is easily shown that the shapes of the elements (rectangular, triangular, bell-shaped, trapezoidal) do not have such an appreciable effect on the spectrum: the product of the equivalent values of its width multiplied by the dimension of an element is approximately constant for different shapes.

The phase spectrum of complex oscillations is not their essential characteristic; it is much more important to determine the position of the axes of the ellipses of vibration, i. e. the characteristics that can be determined in vibrodiagnosis. It can be shown that the axis of the ellipse is directed along the axis of the flaw independently of the dimensions and for all harmonics. Calculation also shows that this direction is independent of the shape of a flaw as well. In the general case the question of the direction of the major or minor axis of the ellipse with respect to the axis of the flaw requires more precise definition since some amplitudes (see formula (3)) may take on negative values. The same thing applies to the case of a zero value of one of the amplitudes, which corresponds to degeneration of the ellipse to a circle. In this case of course we must change to analysis on other harmonics.

The validity of the theoretical models has been repeatedly verified experimentally. Diagnostic studies of the deterministic components of the vibration spectra of machines were done with a narrow-band spectral analyzer of heterodyne type with passband of 6 ± 2 Hz at 3 dB attenuation. For exact measurement of the frequencies of the discrete components of the spectrum and comparison with the calculated values, the investigated signal was combined with the signal from a tone generator, and then it was measured by a digital frequency meter. This brought the frequencies of the detected components into correspondence with the calculated values within 0.5-1%. The vibration ellipses were studied from the Lissajou figures displayed on the screen of an electronic oscilloscope by using signals from mutually perpendicular vibration sensors.

An example of a study of a vibration ellipse on the frequency of rotation can be found in Ref. 1, where it is shown that in the case of one-sided tightening of a bearing cover that leads to misalignment of the outer ring of the bearing, the circular vibration field is deformed into an ellipse, the major axis being directed along the axis of tightening (flaw).

An example of determining the nature of a flaw on the basis of measurement of machine vibration is described in Ref. 6, which gives the results of an experimental evaluation of the nonuniformity of an air gap with accuracy that is fairly high for practice.

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SOFTWARE SYSTEM FOR CHECKING AND DIAGNOSING ELECTRIC MACHINES

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The creation of an automated system for quality control of production in sectors with mass production such as the production of electric machines in a general industrial series cannot be realized without using automatic test equipment controlled by a computer. For effective operation of the entire system it is necessary that the automatic test equipment must operate in real time. In this case the program software is complicated by the fact that it must contain controlling, support and applied programs.

In this paper an examination is made of the problems of software for applied jobs, i. e. applied programs. The principal applied problems associated with checking and diagnosis of electric machines are: checking the overall vibration velocity (or vibration acceleration) in certain points of the machine; analysis of the vibration state of the machine and recognition of the sources of significant components of vibrations; determination of the nonuniformity of the air gap; insulation strength tests; taking the loading characteristic and determining the nominal values from this characteristic and the like.

In connection with such a considerable variety of problems, the applied programs are implemented as blocks, both the number and content of the blocks themselves being capable of continued growth.

Except for the last of the jobs indicated above, all applied problems are based on spectral analysis of vibrations, specifically the detection of deterministic and random components and classification of the latter for the purpose of vibrodiagnosis of the flaws of an electric machine. Therefore this paper describes a complex of algorithms intended for realizing and detecting sources of significant (in the sense of Ref. 1) vibration components.

The complex operates with a limited amount (only 512 readings) of input data since it is time-oriented for information input to the Minsk-32 digital computer via the Nairi-1 digital computer.

Let us denote the vibrations of the housing of an electric machine by x(t) and y(t). We will treat these processes as the real and imaginary parts of a complex process $z(t) = x(t) + iy(t)$ and study its spectrum (Fourier transform) in the complex plane in the form of vibration ellipses on which the flaw detection technique developed in Ref. 1 is based. A vibration ellipse on a given frequency is a hodograph of the vector-sum of two rotating (forward and reverse) vectors whose moduli are defined in terms of the complex Fourier coefficients of the positive Z(m) and negative Z(-m) frequencies of the corresponding harmonic. The semimajor axis БПО of the vibration ellipse on the given frequency is $|Z(m)| + |Z(-m)|$, the semiminor axis is $МПО(m) = ||Z(m)| - |Z(-m)||$, and the angle of inclination of the major axis of the ellipse

$$\varphi_m = [\arg(Z(m)) + \arg(Z(-m))]/2.$$

Obviously when one of the vectors Z is missing, the ellipse degenerates into a circle, and when Z(m) and Z(-m) are close to the moduli the ellipse degenerates into a line. Such a representation of the results of analysis is associated with the fact that the ellipses are informative for diagnosis since they enable evaluation of the nature of a flaw (rotating or pulsating, i. e. a circle or an ellipse-line), its position (from the inclination of the major axis) and dimension (from a "harmonic series" of ellipses) [Ref. 2]. The ellipses also characterize the degree of coherence (phase ratios) of the vibrations along the X and Y axes.

Besides, to get the Fourier image of the functions Z(t) on a digital computer it is convenient to use a fast Fourier transformation algorithm that is designed for a complex form of input data [Ref. 3, 4], i. e. it gives a gain (in time) of calculations as compared with the fast Fourier transformation of two harmonic oscillations.

Deterministic vibration ellipses are used for frequency recognition of flaws that show up on frequencies determined by the corresponding kinematic equations [Ref. 2]. Recognition is realized by comparing the calculated frequencies of the assumed flaws with the frequencies of the components found in the vibration spectrum in a predetermined 5% interval.

It should be noted here that while ellipses are quite informative for deterministic vibrations, they are physically unrealistic for random processes in the general case as there is no constant rate of turn of the vectors. However, random vibrations are quite extensive, and there should be a spectral model for them. On this basis, and retaining uniformity of approaches, we will also study random vibrations in the complex plane and use an elliptical model for them.

Let us note that for the two major forms of random vibrations -- narrow-band, i. e. passing through the resonance of the electric machine, and coherent, i. e. those for which the minor axis is negligibly small in power compared with the major axis -- the elliptical models correspond to physical reality and there is no need for mutual spectral approaches.

Let us express the Fourier images of the processes with respect to axes X and Y in terms of the Fourier coefficients of positive and negative frequencies by the following formulas [Ref. 4]:

$$X(m) = [Z(m) + Z^*(N-m)]/2,$$
$$Y(m) = [Z(m) - Z^*(N-m)]/2j, \quad m = \overline{0,\ N-1}, \qquad (1)$$

where $Z^*(N-m)$ is the conjugate value of the Fourier coefficient of the $(N-m)$-th harmonic, i. e. of the negative m-th harmonic of the complex process $(Z(N-m) = Z(-m))$, N is the number of readings of the analog process $Z(t)$.

Considering that for a complex process in the general case $|Z(m)| \neq |Z(-m)|$, let us use the notation $Z(m) = a + jb$, $Z(N-m) = c + jd$. Bearing in mind that $a^2 + b^2 = |Z(m)|^2$, and $c^2 + d^2 = |Z(N-m)|^2$, we easily get from (1)

$$2[|X(m)|^2 + |Y(m)|^2] = |Z(m)|^2 + |Z(-m)|^2.$$

The resultant equation implies that statistical averaging for $|Z(m)|^2$ and $|Z(-m)|^2$ does not differ from statistical averaging of $|X(m)|^2$ and $|Y(m)|^2$.

The angle ϕ_m at which the vectors $Z(m)$ and $Z(-m)$ meet (the angle of inclination of the ellipse) should be statistically averaged for classification of random ellipses. The classification is done on the basis of the quantity

$$S = \sum_{i=1}^{l} \text{МПО}_i^2(m) \Big/ \sum_{i=1}^{l} \text{БПО}_i^2(m),$$

where $\text{МПО}_i^2(m)$ is the square of the semiminor axis of the ellipse of the m-th harmonic obtained on the i-th segment of realization; l is the number of segments of realization.

The quantity S characterizes the resultant average ellipse obtained from ellipses on the same frequency.

There is no rigorous mathematical basis for classifying ellipses with respect to the parameter S, and therefore a classification is given below that is based on evaluation of the geometric parameters of ellipses of powers.

If $S \geq 0.5$, the process on the given frequency is treated as circular, whereas if $S > 0.9$ -- the process is classified as partly circular, and for cases where $0.5 \leq S \leq 0.9$ the process is approximately circular. On the other hand if $S < 0.5$ the ellipses are taken as lines, and the process on the given frequency is classified with respect to the variance of the angle of inclination of the major axis of the average ellipse. When narrow ellipses have sharply pronounced directionality, i. e. they are concentrated in a very narrow angle range ($\sigma(\phi) \leq 25°$), the process on the given frequency is classified as coherent; if the ellipses are completely narrow, i. e. $S \leq 0.25$, the process is taken as partly coherent, and if $0.25 < S < 0.5$ -- approximately coherent.

In the case where the angles of inclination of the axes of narrow ellipses are spread over a wider range ($\sigma(\phi) > 25°$), the process is classified as uncorrelated in space, and in the range of $25° < \sigma(\phi) \leq 40°$ the process is taken as approximately uncorrelated in space, while at $\sigma(\phi) > 40°$ it is purely uncorrelated in space. Such a classification of random ellipses gives an idea of random vibrations with respect to orthogonal directions in real machines.

The complex of algorithms developed is intended for analyzing vibrations of electric motors and hand-held tools (up to three mechanical transmissions).

The results of the analysis are put out in the form of a table showing the following information: the number of the harmonic, the (central) frequency or band (in Hz) of the isolated component, the type of diagnosed flaw and the deviation from the design frequency (%), the equivalent residual vibrostability, i. e. the square root of the residual power (mm/s), the ratio МПО/БПО; $|Z(m)|$ and $|Z(-m)|$ (mm/s); mathematical expectation and mean square value of the angle of inclination of the resultant ellipse, and the type of process on the given frequency according to the accepted classification.

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