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USSR

PROCESSING SPACE-TIME SIGNALS
(IN INFORMATION TRANSMISSION CHANNELS)

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Moscow OBRABOTKA PROSTRANSTVENNO-VREMENNYKH SIGNALOV (V KANALAKH PEREDACHI INFORMATSII) in Russian 1976 pp 1-208

[Book by D.D. Klovskiy and V.A. Soyfer, Izdatel'stvo "Svyaz'", 208 pages]

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[Translation of the book "Obrabotka Prostranstvenno-Vremennykh Signalov (V Kanalakh Peredachi Informatsii)" by D. D. Klovskiy and V. A. Soyfer, Izdatel'stvo Svyaz', Moscow, 1976, 208 pages]

[Text] Short Description

This book sets forth the general principles of constructing devices for space-time processing of signals in digital information transmission channels. The model used permits description of a broad class of real physical wave channels, including channels in the optical range. The construction of processing devices is based on measuring channel characteristics. The algorithms for processing space signals are oriented to the equipment of holographic (in the broad sense of the concept) systems.

This book is intended for a broad range of specialists working on the development and design of data processing systems and also for college students in the corresponding specializations.

Foreword

Equipment based on holographic techniques gives the engineer new means for constructing devices for space-time signal processing.

Significant contributions to solving the problems of optimal space-time processing have been made by P. A. Bakut, A. A. Kuriksha, R. Kennedy, G. Van Tris, S. Ye. Fal'kovich, and certain other Soviet and foreign authors. However, publications dealing with this subject are dispersed in many different periodicals and there are no books which set these problems forth in a systematic manner. The present book fills this gap.

This work investigates the general principles of optimal and suboptimal signal processing in time-space channels during the transmission of discrete messages.

The book has four chapters. The first chapter is devoted to the search for an acceptable statistical model to describe the signal and noise field at the output of real space-time communications channels.

The second chapter reviews the algorithms for estimating the parameters that define the model of a stochastic channel. Primary attention is devoted to optimal and suboptimal estimation of the coordinates of factorization of the channel characteristics on the basis selected. This estimation determines the most noiseproof procedure for processing the signal being analyzed. The special features of measuring the characteristics of a space-time channel using Wiener or Kalmanov filtration and the principles of constructing adaptive compensators to realize optimal filtration in channels with scattering are reviewed.

The third chapter of the book is devoted to a synthesis of the algorithms of optimal and suboptimal processing of space-time signals containing discrete messages, while the fourth chapter analyzes their noise suppression.

The first, second, and fourth chapters were written by the authors together. V. A. Soyfer wrote the third chapter and the appendices. D. D. Klovskiy performed the general editing.

The authors express their gratitude to doctor of technical sciences N. P. Khvorostenko for reviewing the book and offering a series of remarks that helped to improve it.

We request that all comments be sent to Izdatel'stvo Svyaz' at 101000, Moskva-Tsentr, Chistoprudnyy Bul'var, 2.

Basic Designations

A, B, C, D	- parameters of a function of a generalized Gaussian distribution of a modulus
$B_x(t, t' \vec{r}, \vec{r}')$	- correlation function of a random field $x(t, \vec{r})$
$B_x(\tau, \rho)$	- correlation function of a stationary homogeneous field $x(t, \vec{r})$
D_x, D_y, D_z	- geometric dimensions of the spatial domain of field analysis
E_1	- energy of the signal at position 1
F	- width of the signal spectrum

$F_{\text{кор}}$ [or F_{cor}]	- interval of correlation by frequency
$G(\omega, \vec{\omega}_g)$	- energy spectrum of a channel characteristic
$g(t, \xi, \vec{r})$	- pulse surge characteristic of a space-time filter
$h(f, t, \vec{r})$	- transfer function of a channel
$h(t, \xi, \vec{r})$	- pulse surge characteristic of a channel
$\overline{h^2}$	- mean statistical signal/noise ratio
$K(\omega, \vec{\omega_g})$	- transfer function of a coordinated space-time filter
M	- number of orthogonal signals
$M(\omega, \vec{\omega}_g)$	- function that determines a regularizing functional
m_{xk}, m_{yk}	- mean values of coordinates of factorization of a channel characteristic
N^T, N^F, N^R	- number of coordinates of characteristic factorization by independent variables of time, frequency, and space
N_0	- spectral density of white noise field output
$N(t, r)$	- noise field
p	- probability of erroneous solution
q^2	- statistical parameter of a channel
$R_x(t, t', \vec{r}, \vec{r})$	- normed correlation function of a field
$r = (x, y, z)$	- spatial variable of a field[1]
$s(t)$	- signal at input of a channel
$s_1(t)$	- signal of position 1 at input of a channel
T_c [or T_s]	- length of an element of a signal in transmission

[1] The quantity r is always a vector quantity with the exception of the particular cases specified in the text.

T	- interval of field analysis in time
$U_1(t, \vec{r})$	- field in the reception domain corresponding to position 1 of the transmitted signal
$w_N(x_1, \ldots, x_N)$	- multidimensional density of distribution of a set of random quantities
$x(t, \xi, \vec{r})$, $y(t, \xi, \vec{r})$	- quadrature components of a pulse surge characteristic of a channel
$z(t, \vec{r})$	- observed field
α	- parameter of regularization
β^2	- statistical parameter of a channel
$\Delta f_{\text{макс}}$ [or Δf_{max}]	- width of the energy spectrum of signal fade-outs in time
$\varepsilon(t, \vec{r})$	- measurement error
$\overline{\varepsilon^2}$	- mean quadratic value of measurement error
$\eta(t, \xi, \nu)$	- pulse surge characteristic of a channel in angle-place coordinates
η	- angle-place variable
χ_k	- eigen values of an integral equation
Λ	- space-time domain of field analysis
μ^T	- parameter that characterizes rate of fade-outs
ν^T, ν^T, ν^R	- degree of selectivity of a channel according to the variables of time, frequency, and space
$\xi_{\text{МАКС}}$ [or ξ_{max}]	- channel memory
$\vec{\rho}_{\text{кор}}$ [or ρ_{cor}]	- interval of correlation by space
σ^2_x, σ^2_y	- dispersions of quadrature components of a channel characteristic
$\tau_{\text{кор}}$ [or τ_{cor}]	- interval of correlation of channel parameters in time
$\phi_k(t, \xi, \vec{r})$	- eigen function of an integral equation

ϕ_p - statistical parameter of a channel

ψ_k - functionals computed by an optimal field processing device

ω - cyclical frequency

$\vec{\omega}_g$ - frequency of a spatial spectrum

The designations for the special functions correspond to those adopted in [29].

Symbols:

$\bar{x}$ - average value of random quantity x

s(t) - signal s(t) conjugate according to Hilbert

$\hat{u}(t, \vec{r})$ - estimation of field $u(t, \vec{r})$

h* - quantity complexly conjugate with h

$S(\omega, \vec{\omega}_g)$ - spectrum of signal (field) $s(t, \vec{r})$

C^N_n - number of combinations from N by n.

Introduction

The problems of optimal processing of space-time signals in data transmission channels are attracting ever-growing attention, and this is not accidental. But what does optimal space-time processing offer in comparison with techniques of spatial signal processing already known?

Above all it points out one of a number of methods of spatial processing that provides the best quality characteristics of information transmission. In the second place, if we know the algorithm of optimal processing we can always suggest a large number of suboptimal algorithms whose characteristics are close to potentially achievable ones. In the third place, the system developer will be able to compare any processing algorithm that is proposed against the best. Specifically, the techniques of spatial scattering have become widespread in channels in the short-wave and ultrashort-wave ranges. The theory of space-time processing gives sound criteria for choosing the number of scattered antennas for such channels and the shape of their diagrams (space patterns) in each particular case. In the stage of system development and design such data are extremely valuable.

For channels in the optical range the theory of processing space-time signals is the only and an objectively necessary development of the theory of processing time function-signals. The processing techniques suggested by this theory pose new problems for holographic engineering and open up new opportunities for coherent optical data transmission systems (transmission in a turbulent atmosphere, transmission beyond the limits of direct visibility, and others).

Most communications channels are classified as wave channels and to one degree or another the spatial distribution of the transmitting and receiving structures and the route of signal propagation must be taken into account.

Until recently the synthesis of receiving-transmitting antennas and pure time processing devices in transmission and reception was done independently (separately) according to various specific requirements (quality criteria). Most of the results in the theory of optimal methods of transmitting discrete messages have come on the assumption that the antennas are fixed in transmission and reception and the system is optimized only with respect to time processing of the signal.

However, the limitations inherent in a system and its potential capabilities can only be identified if we make maximum use of information on the properties of the medium of propagation and existing noise in the channel and search for optimal solutions for the design of the receiving-transmitting complex, not assuming a priori a separation of the operations of time and space processing of the signal and not fixing the type of spatial signal processing.

It may be expected that optimal space-time signal processing compared to purely time-optimal processing will be more effective where the quality of data transmission is more strongly influenced by external noise than internal equipment noise. But the influence of external noise on the quality of communications is becoming decisive as a result of advances in developing low-noise receiving-transmitting equipment for space and ground channels.

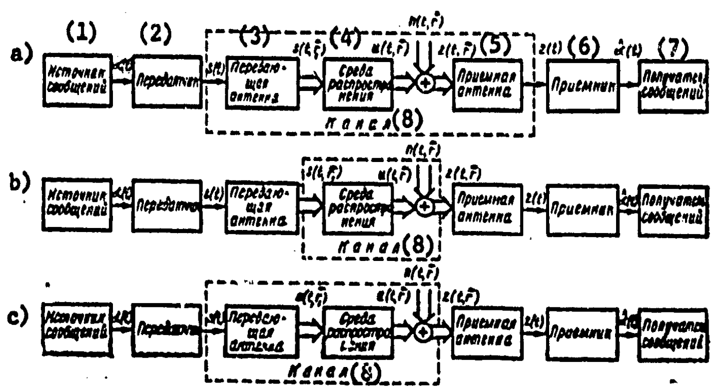
Chapter 1. Model of a Space-Time Channel

1.1 Structure of Systems for Data Transmission by Space Channels

In any data transmission system it is possible to identify, in addition to the source and recipient of messages, the following basic blocks: transmitter, channel, and receiver [51, 104]. We will consider the source of the messages and the transmitter, which includes the coding device, modulator, and transmitting antenna, to be given and then we will consider the last two blocks: the channel (medium of propagation) and receiver. We will assume here, however, that it is possible to control the operation of the transmitter by selecting an appropriate assemblage of signals used to transmit information.

Let us consider the concept of a continuous channel in more detail, because in this work it differs slightly from the traditional concept. In consideration of the problems of optimal reception of messages in the most diverse wave ranges,"channel" ordinarily means the entire transmission part of the system, from the input of the transmitter antenna to the output of the receiving antenna [30, 51, 104] (see Figure 1.1a below). In this case all the variations of channel

Figure 1.1. Models of time and space-time channels: a) time; b) space by input and output; c) space by output



Key: (1) Source of Messages [blocks directly underneath identical in meaning];
(2) Transmitter;
(3) Transmitting Antenna;
(4) Medium of Propagation;
(5) Receiving Antenna;
(6) Receiver;
(7) Recipient of Messages;
(8) Channel.

models can be classified as space-concentrated models or time models. They connect the time function-signals at input $s(t)$ and the output $z(t) = u(t) + n(t)$ of the channel [$u(t)$ is the usable signal at the output, and $n(t)$ is additive noise] by means of some operator, usually linear [40, 44]. In data transmission systems signals $s(t)$ and $z(t)$ very often should be considered vector processes of some particular dimensionality. An example is communications systems with parallel data input to the channel and separate reception.

Use of the model in Figure 1.1a makes it possible to formulate the problem of searching for optimal (from the standpoint of system effectiveness) methods of converting a message to signal $s(t)$ in transmission and signal $z(t)$ into the message on reception.

At the present time, methods of optimal and suboptimal processing of space-time field-signals in various channels are becoming widespread [18, 46, 52, 82]. In the optical range this kind of treatment and processing of processes in space and in time is the only possible one. The techniques of optimal and suboptimal signal processing in other wave bands, for example short-wave, ultrashort-wave, and hydroacoustic channels, are also space techniques. It is possible to construct a spatially distributed model of a channel that connects field $s(t, r_1)$ at the output of the transmitter antenna where $r_1 = (x_1, y_1, z_1)$ is the radius-vector of a field point in transmission, and the field $z(t, r) = u(t, r) + n(t, r,)$ at the input of the receiving antenna where $r = (x, y, z)$ is the radius vector of a field point in reception, $u(t, r)$ is the signal field at the channel output, and $n(t, r)$ is the noise field (see Figure 1.1b above).

Representation of a continuous time-space channel in the form of the model in Figure 1.1b requires significantly more a priori information than representation of a time channel in the form of the model in Figure 1.1a. In this case, however, it is possible to pose the problem of optimizing all devices for conversion of messages into a signal in transmission and conversion back into messages in reception, including the construction of optimal signal-field convertors in transmission and field-signal convertors in reception (transmitting and receiving antennas).

In this work we consider the transmitting antenna to be given, and so we will not investigate the model of a channel with space-time signals at the input and output (see Figure 1.1b) further, but rather will concentrate attention on models of a channel (see Figure 1.1c) in which the input signal is purely temporal (concentrated in space) but the output signal is a space-time signal. For simplicity we will consider the fields to be scalar.

For vector fields such as electromagnetic fields the results obtained by us can be applied to any of the scalar components. Where there is a correlation among components of the vector field a rigorous solution requires study of the total vector field (for example, by solving the corresponding vector differential equations of the field [43, 135]). However, in many situations of practical interest this correlation can be disregarded.

1.2 System Characteristics of a Space-Time Channel and Continuous Models of It

If we consider the space-time channel under analysis to be a linear system with variable parameters, it can be described by various system characteristics [40, 47, 132] (see Figure 1.2 below). Among them are the following:

$h(t, \xi, \vec{r})$ — surge characteristic of the channel, that is, the reaction of the channel at moment in time t at point in space r to a Delta pulse fed to the input at moment $t-\xi$. We consider that the intensity of the field at point r can be measured by placing an elementary antenna at this point;

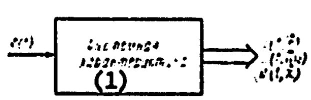
$H(t, f, \vec{r}) \leftrightarrow h(t, \xi, \vec{r})$ — transfer function of the channel, related to $h(t, \xi, r)$ by a Fourier transform by variable f;

$U(v, \xi, \vec{r}) \leftrightarrow h(t, \xi, \vec{r})$ — spectrum of channel reaction at frequency v to a Delta pulse related to $h(t, \xi, r)$ by a Fourier transform by variable b;

$$\left.\begin{aligned} \eta(t, \xi, \vec{\omega}_g) &\leftrightarrow h(t, \xi, \vec{r}) \\ \dot{H}(t, f, \vec{\omega}_g) &\leftrightarrow \dot{H}(t, f, \vec{r}) \\ V(v, \xi, \vec{\omega}_g) &\leftrightarrow \dot{U}(v, \xi, \vec{r}) \end{aligned}\right\}$$ — Fourier transform of the corresponding functions according to variables $\vec{r} \leftrightarrow \vec{\omega}_g$.

If we consider a receiving area sufficiently distant from the area where the scattered field forms, when the signal received is concentrated within narrow spatial angles ϕ, then $\eta(t, \xi, \omega_r) = \eta(t, \xi, \phi)$ has meaning as an angle-space surge characteristic [47] and defines the reaction in an antenna that performs selection of signals with angle of arrival ϕ at moment in time t to a Delta pulse fed to the channel input at moment $t-\xi$; $H(t, f, \omega_g) = H(t, f, \phi)$ has the meaning of an angle-space transfer function; $V(v, \xi, \omega_g) = V(v, \xi, \phi)$ defines at frequency v the spectrum of reaction of the channel in which the output signal is selected by angle ϕ, to a Delta pulse fed to the input at moment in time $t-\xi$.

Figure 1.2. Description of a Space-Time Channel by System Characteristics.



Key: (1) System Characteristic.

In connection with advances in optical techniques of processing nonlight signals [56, 92] and with the use of coherent optical signals to transmit information, there is one more group of system functions of a channel that should be considered. They can be introduced by using the Fresnel transform for a space variable to describe the diffraction of waves in the near zone. As an example of a system function in this group let us consider the Frenel transform of the function h(t ξ, x), which depends on one spatial Cartesian coordinate x that is defined by the relation

$$P(t,\ \xi,\ x) = \sqrt{2}\,\omega_0 \int_{-\infty}^{\infty} h(t,\ \xi,\ \chi)\, e^{i\omega_0^2 (x-\chi)^2}\, d\chi. \qquad (1.1)$$

The function $\Phi(\chi) = \sqrt{2}\,\omega_0 e^{i\omega_0^2 \chi^2}$ is called a Frenel function.

It can be seen from the definition of the Frenel transform (1.1) that it is a convolution of the initial function h(·, ·, x) with the Frenel function Φ (x). To each reading of the function h(·, ·, x) at point x there corresponds a Frenel function with a weight equal to the value of the initial function at a given point, the Frenel picture. The Frenel image is the sum of continuously displaced Frenel pictures.

The surge characteristic of a channel is related to the Frenel system function by an inverse Frenel transform

$$h(t,\ \xi,\ x) = \frac{1}{\omega_0 \sqrt{2}} \int_{-\infty}^{\infty} P(t,\ \xi,\ \chi)\, e^{-i\omega_0^2 (x-\chi)^2}\, d\chi. \qquad (1.2)$$

For the functions $f_1(x)$ and $f_2(x)$, which have the Frenel images $P_1(\chi)$ and $P_2(\chi)$, the following relations are fulfilled

$$\left.\begin{aligned} f_1(x) \otimes f_2^*(x) &= \frac{1}{2\pi} P_1(\chi) P_2^*(\chi), \\ f_1(x) \otimes f_2^*(-x) &= \frac{1}{2\pi} P_1(\chi) P_2^*(-\chi), \end{aligned}\right\} \qquad (1.3)$$

where ⊗ is the symbol of convolution of functions. It should be noted that the Frenel transform is performed directly by spatial variables and the Frenel image is treated in the same spatial coordinates as the initial function. This is the basic difference between the Frenel transform and the Fourier transform.

Using the system characteristics given above it is easy to establish the relationship between signal field u(t, r) or u(t, ϕ) at the output of the channel and the random input signal s(t). For example,

$$u(t,\ \vec{r}) = \int_{-\infty}^{\infty} s(t-\xi)\, h(t,\ \xi,\ \vec{r})\, d\xi, \qquad (1.4)$$

$$u(t,\ \varphi) = \int_{0}^{\infty} s(t-\xi)\, \eta(t,\ \xi,\ \varphi)\, d\xi, \qquad (1.5)$$

$$\dot{u}(t,\ \varphi) = \int_{-\infty}^{\infty} \dot{S}(f)\, \dot{H}(t,\ f,\ \varphi)\, e^{i2\pi ft}\, df, \qquad (1.6)$$

where S(f) is the Fourier spectrum of realization of S(t),

$$u(t, \varphi) = \int_0^{\infty} \int_{-\infty}^{\infty} s(t-\xi) V(\xi, \nu, \varphi) e^{-i2\pi\nu\xi} \, d\xi \, d\nu, \quad (1.7)$$

$$u(t, x) = \frac{1}{\omega_0 \sqrt{2}} \int_0^{\infty} \int_{-\infty}^{\infty} s(t-\xi) P(t, \xi, \chi) e^{-i\omega_0^2 (x-\chi)^2} \, d\xi \, d\chi. \quad (1.8)$$

These relationships make it possible to single out different models of a continuous space-time channel which permit physical treatment. Thus, according to (1.5) the channel can be viewed as a multipath medium of propagation (delay line with even branches) with the paths differing by both time shifts ξ and angles of arrival ϕ at the receiving place. In each path the signal is subjected to modification according to weight $\eta(t, \xi, \phi)$, which in the general case changes in time.

According to (1.7) the channel may be viewed as a multipath medium of propagation with the paths differing by time shifts ξ, frequency (Doppler) shifts v, and angles of arrival ϕ. Each path is characterized by a complex coefficient (weight) $V(\xi, v, \phi)$.

According to (1.8) the channel may be interpreted as a multipath medium of propagation in which the individual paths differ by time shifts ξ and Frenel displacements χ. In each path the signal is modified in conformity with weight $P(t, \xi, \chi)$.

With the limitations imposed on channel characteristics (finite memory, limited space of analysis of the field being received, and the like), we may move from continuous models to discrete models.

1.3 Different Mechanisms of Random Propagation of Waves in Real Space-Time Channels

In order to use the system characteristics and models of a linear space-time channel in the analysis and synthesis of communications systems they should be refined considering the properties of real wave channels. For brevity we will use this term to mean an arbitrary channel in which messages are transmitted by means of a freely propagating wave.

As experience demonstrates, the large majority of real wave channels that transmit information are random (stochastic), that is, they are media with random inhomogeneities. Therefore, system functions that describe such a channel should be viewed as random fields characterized by a particular probabilistic model. The range of questions connected with solving the problems of wave propagation given statistical inhomogeneities of the medium is a practically important branch of the theory of wave propagation. This includes the scattering of radio waves in the troposphere and ionosphere, the scattering of optical waves (body and surface), optical and radial flickering, superdistant propagation of ultrashort waves, reflection of radio waves from the surface of the sea and from irregular land surfaces (location from flying craft and location of the moon), reflection of electromagnetic and optical waves from manmade objects of complex form (flying craft), scattering of acoustic waves in the sea, and so on. The theory of propagation of waves in media with random inhomogeneities is quite well developed today, and primary credit for this goes to Soviet scientists [97, 116, 12]. Most of the problems mentioned above can be formally divided into two classes [84].

1. Wave propagation occurs in homogeneous media, but the boundary between them has random properties. This means either random irregularities of the actual boundary surface or randomly distributed sectors with different reflection and pass coefficients on this surface. The reflection of waves from the surface of the moon can be put in the first category; the second, with certain assumptions, may include reflections from flying craft. In all problems of this type one must deal with deterministic differential equations of the field and its probabilistic structure is determined through stochastic boundary conditions.

2. Propagation occurs in a medium whose properties change randomly in space. This situation occurs, for example, with long-range tropospheric propagation of ultrashort waves. In this case the actual differential equations that describe the process of wave propagation contain random variable coefficients and the boundary conditions are usually deterministic.

Of course, mixed problems with both body and surface random inhomogeneities are also possible. In the general case, inhomogeneities can change in time.

An example of the first type of problem is scattering on a statistically uneven surface, for example on surface $z = \xi(x, y, t)$ where ξ is a random two-dimensional variable field. The amplitude-phase characteristic of the surface is $\dot{A}(x, y, z, t) = A(x, y, z, t)\,e^{-i\varphi(x, y, z, t)}$.

If plane wave $e^{i(\omega t + kz)}$ strikes a surface and the boundary is such that $\phi = 0$ at it, then for wave function $\phi(x, y, z, t)$, which in semi-space $z>0$ satisfies the deterministic equation

$$\Delta\varphi - \frac{n_0^2}{c^2}\frac{\partial^2\varphi}{\partial t^2} = 0, \qquad (1.9)$$

we have the boundary condition $\phi[x, y, \xi(x, y, t)] = 0$.

An example of the second type of problem is an unlimited space (semi-space or layer) in which this equation is correct

$$\Delta\varphi - \frac{n^2(r, t)}{c^2}\frac{\partial^2\varphi}{\partial t^2} = 0, \qquad (1.10)$$

where $n(r, t)$ is a random field, an index of refraction of the medium.

The stochasticity of the differential equations given is determined by the given random fields ξ or n. Statistics on these physical fields are accumulated by direct measurement and various types of modeling.

At the present time, a great deal of factual material has been accumulated and processed concerning these characteristics for real types of channels (for example, see [32]).

During the investigation of wave channels with random inhomogeneities (with scattering), the phenomenological approach to the problem of wave propagation has been widely used. It is based on beam representations [16, 49].

These representations are based on the experimentally established fact that formation of the scattered field at the place of reception is accomplished chiefly by the so-called "luminescent" or "burning" spots which occupy a relatively small part of the scattering surface (body) of the scatterer (see Figure 1.3 below). As this figure shows schematically, the field being received may be formed by two or more areas of scattering that are dispersed in space.

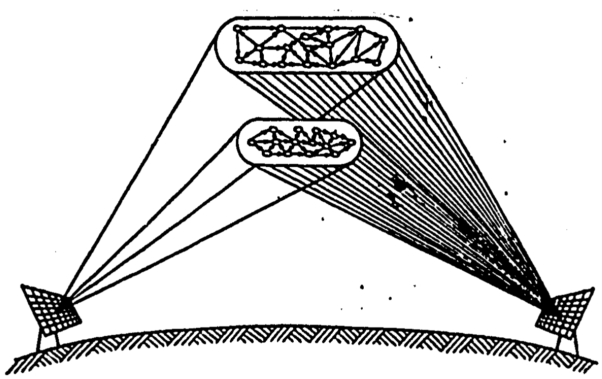


Figure 1.3. Model of the Formation of a Scattered Field at the Receiving Place

If the scattering surface or body alters its orientation and/or shape slightly over time, the luminescent spots begin to be shifted and to change their intensity. The origin of the luminescent spots conforms with the concepts of the geometric optic according to which a wave striking a body represents a bundle of beams. Each of the beams is reflected from a corresponding area, forming a reflected beam. If each beam undergoes more than one reflection before striking the receiving area, this is what is called multiple scattering. In many cases Borne's first approximation [80, 135] which disregards the effect of multiple scattering may be considered correct. It can be said that in the case of single scattering a model of parallel wave propagation occurs, while in the case of multiple scattering it is a model of sequential parallel propagation. Let us consider more closely these models of propagation and the probabilistic description of the field of signals received $u(t, \vec{r})$ or, which is the same thing, the probabilistic description of particular system characteristics of the channel with a deterministic input action, resulting from each model.

The model of parallel propagation (single scattering). The number of beams striking the receiving area, the number of "luminescent" spots, will be greater where the area of scattering is larger and more complex. If single scattering occurs, the beams reaching the reception area can be considered independent of one another, and the field at the input of the receiving antenna can be written in "adaptive" form as follows

$$u(t, r) = \sum_{k=1}^{K(t)} u_k(t, \vec{r}, \vec{v}_k), \qquad (1.11)$$

where u_k is the shape of the oscillation of random beam k and $\vec{v}_k$ is the vector of the random and deterministic parameters that characterize random beam k.

Because communications signals are narrow-band processes (the frequency band F_c that they occupy is much smaller than the average spectrum frequency f_0), (1.11) can conveniently be written as follows

$$u(t, \vec{r}) = x(t, \vec{r}) \cos \omega_0 t + y(t, \vec{r}) \sin \omega_0 t = A(t, \vec{r}) \cos[\omega_0 t - \varphi(t, \vec{r})], \qquad (1.12)$$

where

$$\left.\begin{aligned} x(t, \vec{r}) &= \sum_{k=1}^{K(t)} x_k(t, \vec{r}, \vec{v}_k), \\ y(t, \vec{r}) &= \sum_{k=1}^{K(t)} y_k(t, \vec{r}, \vec{v}_k) \end{aligned}\right\} \text{ -- quadrature components;} \qquad (1.13)$$

$$A(t, \vec{r}) = \sqrt{x^2(t, r) + y^2(t, \vec{r})} \text{ -- envelope of resultant field at point } \vec{r}; \qquad (1.14)$$

$$\varphi(t, \vec{r}) = \operatorname{arc\,tg} \frac{y(t, \vec{r})}{x(t, \vec{r})} \text{ -- phase of resultant field at point r;} \qquad (1.15)$$

$$A_k(t, \vec{r}) = \sqrt{x_k^2(t, \vec{r}, \vec{v}_k) + y_k^2(t, \vec{r}, \vec{v}_k)} \text{ -- envelope of signal by beam k;}$$

$$\varphi_k(t, \vec{r}) = \operatorname{arc\,tg} \frac{y_k(t, \vec{r}, \vec{v}_k)}{x_k(t, \vec{r}, \vec{v}_k)} \text{ -- phase of signal by beam k.}$$

It is often convenient to switch from writing the field in material form (1.12) to a composite writing:

$$\dot{u}(t,\vec{r}) = \dot{A}(t,\vec{r})\,e^{i\omega_0 t} = A(t,\vec{r})\,e^{i[\omega_0 t-\varphi(t,\vec{r})]} = u(t,\vec{r}) + i\tilde{u}(t,\vec{r}), \quad (1.16)$$

where

$$\left.\begin{array}{l}\dot{A}(t,\vec{r}) = A(t,\vec{r})\,e^{-i\varphi(t,\vec{r})} \text{ -- composite envelope;} \\ \tilde{u}(t,\vec{r}) \text{ -- field, conjugate according to Hilbert, with } u(t,\vec{r}).\end{array}\right\} \quad (1.17)$$

The number of "luminescent" points in the channels that are of greatest practical interest is very large for the most diverse wave ranges. For example, according to experimental data in [76], this number was on the order of $k = 10^3$ for reflection from the moon. The contribution of each of the components to the total process is always small, and taken together with the assumption of the independence of the components this makes it possible to use the central limiting theorem of the theory of probability and consider the field at the channel output to be Gaussian.

Statistical description of the channel in a Gaussian approximation is simplest and comes down to determining the first two statistical moments of the field, the regular component u(t, r) and the correlation function $B_u(t, t+\Delta t; r, r+\Delta r)$, or determining the mathematical expectations of quadrature components: the autocorrelation and mutual correlation functions. There are many works [32, 33, 80, and others] that deal with the determination of regular components and correlation functions of fields at the output of channels that differ physically.

Many authors of theoretical and experimental investigations describe the correlation of quadrature components of the field being received in the most diverse wave bands as a Gaussian curve by both measurements (time and space).

As we know [16, 104, 140], the random process with Gaussian correlation is singular, and this means that its entire future can be predicted by known past and present methods of linear interpolation.

Therefore, the Gaussian approximation of the correlation function cannot be considered acceptable for a fundamentally unpredictable process such as the random field of a signal at the receiving point.

Very often [49, 104] the correlation coefficient of local-stationary quadrature components of a field is approximated by the demonstrative law (for each of the variables)

$$R_x(\lambda) = R_y(\lambda) = e^{-\alpha_\lambda |\lambda|}, \quad \alpha_\lambda > 0, \quad \lambda = \Delta t, \Delta r. \qquad (1.18)$$

Approximation (1.18) does not entail singularity of the field and with a Gaussian distribution signifies that the process is a one-dimensional Markov process. A more universal function for approximation of the correlation characteristics of a slowly changing stochastic field at the output of a wave channel is the function which corresponds to a rational (bilinear) energy spectrum of random order. This spectrum may also be found, specifically with a multidimensional Markov random process. In subsequent analysis it will be necessary to have parameters that determine the intervals of correlation of the stochastic field by time τ_{cor}, by frequency F_{cor}, and by spatial coordinates $\vec{\rho} = (x_{cor}, y_{cor}, z_{cor})$. For the sake of definiteness we will consider that these parameters are determined by the equivalent rectangle method (energy criterion) [51, 64]. The question of the mutual dependence (correlation) of fluctuations of the field in time, by spectrum, and by space will be taken up below during the study of stochastic models of time-space channels based on correlation properties.

A model of sequential-parallel propagation (multiple scattering). Things are considerably more complex in the case of sequential parallel propagation than with simple parallel propagation of waves. At the present time, although there have been numerous theoretical investigations in this field [45, 67, 97], they are far from complete and experimental data are not extensive [67]. Special interest in this model of propagation, which takes account of the multiple scattering of signals, has arisen in recent years and is explained by the extensive use of such a form of wave propagation for transmission of information along optical channels both within and beyond the field of direct visibility. We also encounter multiple scattering in the transmission of signals along multistage short-wave routes. The field in the receiving area can be formally represented in this case just as it is represented in the case of single scattering, as the sum of a definite number K of components (1.11). However, the difference is that the component no longer can be considered independent. The reason is that the multiply scattered components of the total field have been formed by common "luminescent" spots (scattering points) located within the area of space that forms the signal being received.

When considering multiple scattering, a rigorous solution of wave differential equations of the field taking account of boundary effects and the random properties of the medium is extremely difficult. In this case it is simpler to obtain probabilistic characteristics of the field being received on the basis of the pheonomenalogical approach by introducing the channel model shown in Figure 1.4 below. According to this model the channel is treated as a countable (finite or infinite) set of space-time filters with characteristics $\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_L$. Each filter participates in formation of the field in the reception area both directly and with the assistance of all or some of the other filters. In the general case, the contribution to the field received at point r coming from the output of filter k may be represented as follows.

$$u_k(t, r) = \mathcal{L}_k(v)\left[1 + \sum_{p=1}^{L-1} \mathcal{L}_p^{(1)}(v) + \sum_{r=1}^{C_2^{L-1}} \mathcal{L}_r^{(2)}(v) + \ldots \right.$$
$$\left. \ldots \sum_{i=1}^{C_2^{L-1}} \mathcal{L}_i^{(l)}(v) + \ldots \right] s(t, r). \qquad (1.19)$$

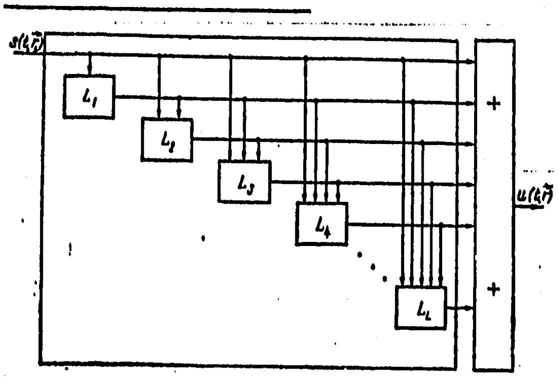


Figure 1.4. Model of a Channel with Sequential Parallel Propagation of Transmitted Signals

The characteristics $\mathcal{L}_k$ are considered linear operators in the functional Hilbert space. $\mathcal{L}_i^l$ is composition i of the number of possible, non-repeating compositions l of the partial operators without $k(l=\overline{1, \ldots, (L-1)}$; $i=\overline{1, \ldots, C_l^{L-1}})$; C_l^{L-1} is the number of combinations from L-1 by l; v is the set of parameters that determines the operator; I is the symbol of the unitary operator; s(t, r) is the input signal which excites the set of filters that contribute to the total field received at point r.

For example, when M = 3 it follows from [1.19] that

$$u_3(t, r) = [\mathcal{L}_3(v) + \mathcal{L}_1(v)\mathcal{L}_3(v) + \mathcal{L}_2(v)\mathcal{L}_3(v) + \mathcal{L}_1(v)\mathcal{L}_2(v)\mathcal{L}_3(v)]\, s(t, r). \qquad (1.20)$$

The total field at point r with sequential parallel propagation is determined by summing the components $u_k(t, r)$:

$$u(t, r) = \sum_{k=1}^{K} \mathcal{L}_k(v)\left[1 + \sum_{p=1}^{L-1} \mathcal{L}_p^{(1)}(v) + \sum_{\Omega=1}^{C_2^{L-1}} \mathcal{L}_\Omega^{(2)}(v) + \ldots \right] s(t, r). \qquad (1.21)$$

If it is possible to disregard the contribution caused by the interaction of operators (that is, if we can disregard multiple scattering), then (1.21) gives a model of the field being received considering only parallel propagation.

The effect of sequential parallel propagation in its particular manifestation apparently occurs only in channels with scattering. For example, the propagation of light waves in a transparent atmosphere can be described by a sequential mechanism [45, 67]. Fluctuations of amplitude (intensity) and phase occur in a light wave after traveling a cer-

certain distance in turbulent atmosphere. It may be assumed that the turbulent atmosphere shows itself as a certain set of sequentially arranged lenses with randomly changing properties. Such a sequential model is qualitatively corroborated by the fact that, according to experimental findings, the logarithm of the amplitude (or intensity) of a fluctuating wave often has a Gaussian probability distribution [67, 111].

Let us move on to a more detailed discussion of a probabilistic model of a linear space-time channel with sequential parallel wave propagation, treating the channel as a set of linear space-time filters participating together in shaping the field being received.

1.4. A One-Dimensional Probabilistic Model of a Channel With Sequential Parallel Propagation

The multidimensional distribution of a particular system function (or field at the output) of a channel for all characterizing variables is a complete description of a stochastic space-time channel. In practice, however, the developer sometimes does not have such complete a priori information. For the problems of optimal and suboptimal processing of the field considered in this book it is entirely sufficient to describe a probabilistic model of the channel with one-dimensional distributions and correlation characteristics. The correlation functions that correspond to the different models of space-time channels will be considered below, but here we will focus attention on one-dimensional distributions of the probabilities of sections of particular system characteristics of a channel.

If the linear operator $\mathcal{L}_k(v)$ (or the characteristics of the corresponding space-time filters) introduced in subchapter 1.3, are equated with the transfer functions $H_h(t, f, r)$, the summary operator (resulting transfer function $H(t, f, r)$) corresponding to transform (1.21) can be written as follows:

$$\dot{H}(t,f,r)=\sum_{k_1=1}^{L}\dot{H}_{k_1}(t,f,r)+\sum_{k_1>k_2=1}^{L}\dot{H}_{k_1}(t,f,r)\dot{H}_{k_2}(t,f,r)+\ldots+\sum_{k_1>k_2>\ldots>k_l=1}^{L}\dot{H}_{k_1}(t,f,r)\dot{H}_{k_2}(t,f,r)\ldots\dot{H}_{k_l}(t,f,r)+\ldots+\dot{H}_L(t,f,r)\,H_{L-1}(t,f,r)\ldots H_1(t,f,r). \qquad (1.22)$$

As can be seen from (1.22), the resulting transfer function of a channel with sequential parallel propagation of waves is determined in the general case through constituent system characteristics using a rather complex expression. Instead of (1.22) it is possible to construct other resulting system characteristics for a channel with sequential parallel propagation. For example, for the resulting transfer characteristic we can write the relation

$$h(t,\xi,r)=\sum_{k_1=1}^{L} h_{k_1}(t,\xi,r)+\sum_{k_2>k_1=1}^{L}\int_0^\infty h_{k_1}(t,\xi_1,r)\,h_{k_2}(t,\xi-\xi_1,r)\,d\xi_1+\sum_{k_3>k_2>k_1=1}^{L}\int_0^\infty\int_0^\infty h_{k_1}(t,\xi_1,r)\,h_{k_2}(t,\xi_2-\xi_1,r)\,h_{k_3}(t,\xi-\xi_2,r)\,d\xi_1\,d\xi_2+\ldots+\int_0^\infty\int_0^\infty\ldots\int_0^\infty h_1(t,\xi_1,r)\,h_2(t,\xi_2-\xi_1,r)\,h_3(t,\xi_3-\xi_2,r)\ldots h_L(t,\xi-\xi_{L-1},r)\,d\xi_1\ldots d\xi_{L-1} \quad (1.23)$$

This relation follows from (1.22) if we consider that the operations of multiplying the system functions $H_1(t, r, r)$ and $H_2(t, f, r)$ are equivalent to the operations of convoluting the corresponding transfer characteristics.

$$\dot H_1(t,f,r)\,\dot H_2(t,f,r)\longleftrightarrow\int_0^\infty h_1(t,\xi_1,r)\,h_2(t,\xi-\xi_1,r)\,d\xi_1. \quad (1.24)$$

With purely parallel propagation (disregarding multiple scattering)

$$\dot H(t,f,r)=\sum_{k=1}^{N}\dot H_k(t,f,r). \quad (1.25)$$

In the case of sequential propagation only

$$\dot H(t,f,r)=\prod_{l=1}^{L}\dot H_k(t,f,r). \quad (1.26)$$

We should give special consideration to the particular situation of sequential parallel propagation, described by the common term of the relation (1.22)

$$\dot H(t,f,r)=\sum_{k=1}^{N}\prod_{l=1}^{L_k}H_{lk}(t,f,r). \quad (1.27)$$

When substantiating the probabilistic model of fadeouts in a channel it is usually supposed [49, 80, 104, 135, 137] that the number of scatterers N forming the total signal at the point of reception is large. However, for the general situation of sequential parallel propagation this assumption is not adequate to find the limiting distribution (where $N\to\infty$) of a random quantity (1.27). The point is that probability theory does not yet have a limiting theorem for distributions of the sums of the products of random quantities. Therefore, at first we will consider the two theoretically extreme situations: purely additive (1.25) and purely multiplicative (1.26). Then we will also discuss the intermediate additive-multiplicative situation (1.27).

The purely additive situation in formation of a received field. Where $N \to \infty$ we may usually consider the conditions of the central limiting theorem to be met. This allows us to view the section (reading) of the transfer function of the channel H(t, f, r) = x(t, f, r)+iy(t, f, r) as a composite Gaussian random quantity. Its quadrature components X(t, f, r) and y(t, f, r) are, in the general case, dependent and have arbitrary (unequal) mathematical expectations m_x and m_y and non-identical dispersions $\sigma_x^2 \neq \sigma_y^2$. The channel model we are discussing is called a four-parameter or generalized Gaussian model [49, 89].

The conditions of physical feasibility of the channel impose definite limitations on the relations between the quadrature components of the transfer function x(ω) and y(ω). They can be obtained starting from the condition h(ξ) = 0 where ξ< 0 or from the equivalent condition

$$h(\xi) = h(\xi)\cdot 1(\xi), \qquad (1.28)$$

where 1(ξ) is a unitary function [92].

Now we will perform a Fourier transform on the right and left parts of the last relation. This makes it possible to convert to the relation for the transfer function of a physically feasible channel:

$$\dot H(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \dot H(\omega')\,\dot U(\omega-\omega')\,d\omega', \qquad (1.29)$$

where $U(\omega) = \pi\delta(\omega)+1/i\omega$ is the spectrum of the unitary function.

From this integral it is easy to obtain the expressions that relate the real x(ω) and imaginary y(ω) parts of the transfer function of a physically feasible channel:

$$\left.\begin{aligned} x(\omega) &= \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{y(\omega')}{\omega-\omega'}\,d\omega',\\ y(\omega) &= -\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\omega')}{\omega-\omega'}\,d\omega'.\end{aligned}\right\} \qquad (1.30)$$

The integrals in (1.30) should be considered as

$$x(\omega) = \frac{1}{\pi}\lim_{\Omega\to\infty}\int_{-\Omega}^{\Omega}\frac{y(\omega')}{\omega-\omega'}\,d\omega'.$$

For a stochastic channel the convergence of the integrals cited must be understood in the mean quadratic sense. The relations obtained allow us to state that the real and imaginary parts of the transfer function of a real channel must be interlinked by a Hilbert transform. For linear deterministic filters this result is not new (for example, see [92]).

Using the properties of the Hilbert transform it is not difficult to show that the statistical characteristics of the functions $x(\omega)$ and $y(\omega)$ must meet certain requirements. In particular, if it is assumed that the channel is homogeneous for frequency in the broad sense, then $x(\omega)$ and $y(\omega)$ are random processes with identical correlation functions that are noncoherent on coinciding frequencies.

Within the framework of a Gaussian probabilistic model of a channel these properties of $x(\omega)$ and $y(\omega)$ lead to the Rayleigh or Rice distribution of the modulus of the transfer function (amplitudes of the signal received). In the case of a channel that is inhomogeneous in frequency the correlation functions of the processes $x(\omega)$ and $y(\omega)$ have the following relation

$$B_x(\omega_1,\ \omega_2) = \frac{1}{\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B_y(\omega_1',\ \omega_2')\frac{d\omega_1'}{\omega_1-\omega_1'}\,\frac{d\omega_2'}{\omega_2-\omega_2'} \qquad (1.31)$$

and are, in the general case, nonidentical. The mutual correlation functions

$$B_{xy}(\omega_1,\ \omega_2) = -\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{B_x(\omega_1,\ \omega_2')}{\omega_2-\omega_2'}\,d\omega_2',$$

$$B_{yx}(\omega_1,\ \omega_2) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{B_y(\omega_1,\ \omega_2')}{\omega_2-\omega_2'}\,d\omega_2'$$

are nonidentical and in the general case do not become zero on coinciding frequencies; in other words, the processes $x(\omega)$ and $y(\omega)$ are not noncoherent.

Within the framework of the Gaussian probabilistic model of a channel these properties of processes lead to a generalized Gaussian four-parameter [49] or Hoyt (sub-Rayleigh) [135] distribution of the modulus of the transfer function.

It is also possible to make the opposite assertion, that the occurrence of a four-parameter or sub-Rayleigh distribution of the modulus is inevitably linked to frequency nonhomogeneity of the channel. Because many actual communications channels are inhomogeneous for frequency, it may be expected that the generalized Gaussian or sub-Rayleigh model of fluctuations will occur in many cases.

It is always possible to pass to x and y, the independent quadrature components of transfer function H, by rotating the axes of the coordinates (orthogonal transformation) [26, 49].

Relying on these results, we will hereafter assume that the real and imaginary components of a section of the transfer function are independent and have the parameters m_x, σ_x^2 and m_y, σ_y^2 respectively. Treating the pulse surge characteristic of the channel in the composite form of representation $H(t, \xi, r) = x(t, \xi, r)+iy(t, \xi, r)$, we will make similar assumptions with respect to its real and imaginary components. Thus, the quadrature components of the transfer function (or composite pulse surge characteristic) are independent and have a normal distribution:

$$\left.\begin{aligned} w_1(x) &= \frac{1}{\sqrt{2\pi\sigma_x^2}}\exp\left[-\frac{(x-m_x)^2}{2\sigma_x^2}\right],\\ w_1(y) &= \frac{1}{\sqrt{2\pi\sigma_y^2}}\exp\left[-\frac{(y-m_y)^2}{2\sigma_y^2}\right].\end{aligned}\right\} \qquad (1.32)$$

In this case the one-dimensional distribution of the modulus $\gamma=\sqrt{x^2+y^2}$ can be obtained in the form below [89]

$$w_1(\gamma) = \sum_{n=0}^{\infty}\frac{R}{n!}\sigma^{2n}\frac{\partial^{2n}}{\partial m_I^n\,\partial m_{II}^n}\times$$
$$\times\left\{\frac{\gamma}{\sigma^2}\exp\left[-\frac{\gamma^2+m_I^2+m_{II}^2}{2\sigma^2}\right]I_0\left(\frac{\gamma}{\sigma^2}\sqrt{m_I^2+m_{II}^2}\right)\right\}, \qquad (1.33)$$

where the following designations have been introduced:

$$m_I = \frac{m_x+m_y}{\sqrt{2}};\quad m_{II} = \frac{m_x-m_y}{\sqrt{2}};\quad \sigma^2 = \frac{\sigma_x^2+\sigma_y^2}{2};\quad R = \frac{\sigma_y^2-\sigma_x^2}{\sigma_y^2+\sigma_x^2}.$$

There are also other forms for writing this distribution [49, 135]. Where certain conditions are met, a number of particular cases follow from the generalized distribution (1.33):

1. The Beckman (or three-parameter [49]) distribution follows from (1.33) with a certain phasing of the regular component $m_y = 0$, $m_x \neq 0$, and $\sigma_x^2 \neq \sigma_y^2$. Let us stress that within the framework of the generalized Gaussian model the existence of a regular component of the signal being received is not necessarily linked to the hypothesis of the existence of a "regular" beam in the channel; the regular component $m_x^2 + m_y^2 \neq 0$ can also occur as a result of special characteristics of wave scattering [49, 51, 104, 125].

2. The Rice (or generalized Rayleigh) distribution is obtained from (1.33) where there is channel symmetry by dispersions of quadrature components $\sigma_x^2 = \sigma_y^2 = \sigma^2$ and $R = 0$.

3. The Hoyt (or sub-Rayleigh [49]) distribution follows from (1.33) where $\sigma_x^2 \neq \sigma_y^2$ and in the absence of a regular component $m_x = m_y = 0$.

If $\sigma_x^2 = 0$ in this case, the distribution of amplitudes is determined by the one-sided normal law [49] which corresponds to the deepest fade-outs within the framework of the four-parameter model.

4. The Rayleigh distribution is obtained from (1.33) in the absence of both asymmetry $\sigma_x^2 = \sigma_y^2 = \sigma^2$ and a regular component $m_x = m_y = 0$.

It is easy to trace the conditions under which the interference sum (1.25) within independent components $H_k(t, f, r)$ gives rise to one or another field distribution $H(t, f, r)$.

Assuming that the amplitudes and phases of the elementary components

$$\gamma_k = |\dot{H}_k| = \sqrt{x_k^2 + y_k^2} \quad \text{and} \quad \Theta_k = \operatorname{arc\,tg} \frac{y_k}{x_k}$$

are independent, it is possible to write

$$\left.\begin{aligned}
&m_x = \bar{x} = \sum_{k=1}^{L} \bar{\gamma}_k \overline{\cos\Theta_k}, \quad m_y = \bar{y} = \sum_{k=1}^{L} \bar{\gamma}_k \overline{\sin\Theta_k};\\
&\sigma_x^2 = \sum_{k=1}^{L} \left[\overline{\gamma_k^2}\, \overline{\cos^2\Theta_k} - (\bar{\gamma}_k \overline{\cos\Theta_k})^2 \right];\\
&\sigma_y^2 = \sum_{k=1}^{L} \left[\overline{\gamma_k^2}\, \overline{\sin^2\Theta_k} - (\bar{\gamma}_k \overline{\sin\Theta_k})^2 \right];\\
&B_{xy} = \overline{(x - m_x)(y - m_y)} = \frac{1}{2} \sum_{k=1}^{L} \overline{\gamma_k^2}\, \overline{\sin 2\Theta_k} +\\
&+ \sum_{\substack{m=1\\ m\neq l}}^{L} \sum_{l=1}^{L} \bar{\gamma}_m \bar{\gamma}_l \overline{\sin\Theta_m}\, \overline{\cos\Theta_l} - m_x m_y.
\end{aligned}\right\} \quad (1.34)$$

If the elementary components have identical statistics, then

$$\left.\begin{aligned}
&m_x = L\bar{\gamma}\overline{\cos\Theta}, \quad m_y = L\bar{\gamma}\cdot\overline{\sin\Theta};\\
&\sigma_x^2 = L\overline{\gamma^2}\,\overline{\cos^2\Theta} - \frac{m_x^2}{L}, \quad \sigma_y^2 = L\overline{\gamma^2}\,\overline{\sin^2\Theta} - \frac{m_y^2}{L};\\
&B_{xy} = \frac{1}{2} L\overline{\gamma^2}\,\overline{\sin 2\Theta} - \frac{m_x m_y}{L}.
\end{aligned}\right\} \quad (1.35)$$

Analyzing (1.35), it is possible to draw certain general conclusions about the possible model of the channel:

1. If the phases of the elementary components are distributed evenly in the range from $-\pi$ to $+\pi$, then $m_x = m_y = 0$, $\sigma_x^2 = \sigma_y^2$, $B_{xy} = 0$ and the scattered field is a Rayleigh vector.

2. With fluctuations in the phases of the elementary components within limits significantly exceeding π, the resulting field is also a Rayleigh vector. This conclusion follows from the fact that for the periodic functions $\sin\Theta$ and $\cos\Theta$, instead of the distribution function given within large limits it is possible to use another, reduced to the interval of periodicity [64]. If only the initial distribution of the phases of the elementary components is not a periodic function, then the convoluted distribution within the limits $[-\pi, +\pi]$ will be closer to even where the limits of the fluctuations of the phase of the elementary components are greater.

3. Where the fluctuations in phases of the elementary components are symmetrical relative to their average value, equal to zero, and the dispersion of phase fluctuations is not too great, then owing to the parity of the distribution functions from (1.35) it follows that $m_x \neq 0$, $m_y = 0$, $\sigma_x^2 \neq \sigma_y^2$ and $B_{xy} = 0$, that is, the scattered field forms a three-parameter vector [49, 135].

4. With asymmetric fluctuations of phases of the elementary components $m \neq 0$, $m \neq 0$, $\sigma_x^2 \neq \sigma_y^2$ and $B_{xy} \neq 0$, that is, the scattered field is a four-parameter vector.

Thus, with the assumptions made, the generalized Gaussian statistics of a scattered field are a consequence of asymmetry, which can be explained in the distribution of phases of elementary waves on the basis of the physical processes related to the propagation of waves in random media. If there is a regular beam at the receiving point in addition to the scattered field, it is natural that cases 1 and 4 lead to a resulting field in the form of a Rice vector, whereas cases 2 and 4 yield a resulting field in the form of a four-parameter vector.

Experimental data show that the generalized Gaussian distribution and its various particular cases cover a very large class of communications channels [49, 135]. Solving the stochastic wave equation of the field for different mechanisms of wave propagation also leads to a generalized Gaussian model and a number of its particular cases [32, 33, 49, 50, 116, 80, 124-126, 135]. In addition to parameters m_x, m_y, σ_x^2, and σ_y^2, it is convenient to introduce four other parameters which have graphic physical meaning:

$$q^2 = \frac{m_x^2 + m_y^2}{\sigma_x^2 + \sigma_y^2} \qquad (1.36)$$

— the ratio of the average powers of the regular and fluctuating parts of the transfer function or surge characteristic of the channel;

$$\beta^2 = \sigma_x^2/\sigma_y^2 \qquad (1.37)$$

— the coefficient that characterizes asymmetry by dispersions of quadrature components;

$$\varphi_p = \operatorname{arc\,tg}(m_y/m_x) \qquad (1.38)$$

— the phase angle of the regular component;

$$\overline{\gamma^2} = m_x^2 + m_y^2 + \sigma_x^2 + \sigma_y^2 \qquad (1.39)$$

— the mean square of the transfer function (surge characteristic).

For a full description of the channel it is sufficient to consider the following ranges of change in the parameters introduced:

$$0 \leqslant q^2 \leqslant \infty;\; 0 \leqslant \beta^2 \leqslant 1;\; 0 \leqslant \varphi_p \leqslant (\pi/2);\; 0 \leqslant \overline{\gamma^2} < \infty.$$

A whole series of useful formulas relating to the four-parameter distribution of the modulus is contained in the literature [89]. Let us observe that the two-parameter m-distribution of Nakagami [137] satisfactorily approximates the four-parameter distribution of amplitudes [49]. The distribution of the independent variable of the surge function $\phi = \operatorname{arc\,tg}(y/x)$ for a generalized Gaussian channel is contained in the literature [49, 64].

The purely multiplicative situation in formation of the field being received. If we write the transfer function of partial filter k in (1.26) in the form $\dot{H}_k = e^{\chi_k} e^{i\varphi_k}$, it is not difficult from (1.26) to obtain the following

$$\dot{H} = e^{\chi + i\varphi} = \gamma e^{i\varphi}. \qquad (1.40)$$

For the quantities

$$\chi = \sum_{k=1}^{L} \chi_k, \quad \varphi = \sum_{k=1}^{L} \varphi_k \qquad (1.41)$$

where $L \to \infty$, the conditions of the central limiting theorem are met, making it possible to consider them Gaussian random quantities.

With multiple scattering, in particular for a stochastic optical channel, the modulus of the transfer function $\gamma = e^{\chi}$ and its independent variable ϕ can be considered statistically independent [67, 111]. The one-dimensional distribution of modulus γ is logarithmically normal

$$w_1(\gamma) = \frac{1}{\sqrt{2\pi\sigma_\chi^2}\,\gamma} e^{-(\ln\gamma - m_\chi)^2/2\sigma_\chi^2}. \qquad (1.42)$$

Parameter σ_χ^2 (dispersion of the logarithm of modulus γ) may be related to Nakagami's parameter m [57, 137]

$$m = (\overline{\gamma^2})^2 / [\overline{\gamma^4} - (\overline{\gamma^2})^2], \qquad (1.43)$$

which, changing in the range from 0.5 to ∞, is a convenient measure of the depth of signal fadeouts (the depth of fadeouts increases with a decrease in m). The following relations are correct:

$$\sigma_\chi^2 = \frac{1}{4}\ln\left(1+\frac{1}{m}\right), \quad m = \frac{1}{e^{4\sigma_\chi^2}-1}. \qquad (1.44)$$

For small dispersions of the logarithm of amplitude $(\sigma_\chi^2 \leqslant 0.5)$, the logarithmically normal distribution (1.42) is satisfactorily approximated by Nakagami's m-distribution where m>3 and therefore also by the Rice distribution [49, 57].

For large dispersion values $(\sigma_\chi^2 > 0.5)$ the approximation shown above is unsatisfactory because under these conditions the logarithmically normal distribution, unlike the m-distribution, is characterized by a very slow decrease in probability density in the domain of large values of the independent variable.

Let us pass on to a consideration of the question of the distribution of phases for the purely multiplicative situation of formation of the received field. As can be seen from (1.41), the distribution of phases ϕ in an infinite interval is governed by the Gaussian distribution.

However, for the problem of optimal signal processing, the distribution law of phases in the segment $[-\pi, +\pi)$, that is, the distribution reduced to the interval of periodicity, is most interesting. Beginning from the result in [64], it is not difficult to show that the distribution of the random quantity ϕ in the interval of periodicity $[-\pi, +\pi]$ has the form

$$w_1(\varphi) = \frac{1}{2\pi}\left[1 + 2\sum_{r=1}^{\infty}|\Theta_1(r)|\cos r\varphi\right], \qquad (1.45)$$

where $\Theta_1(u)$ is the characteristic function of the quantity ϕ.

In this case the quantity ϕ is normally distributed. We will suppose that its average value is equal to zero. This can always be done, considering the distribution of the initial phases relative to the average phase increment. Then, from (1.45) it is not difficult to obtain

$$w_1(\varphi) = \frac{1}{2\pi}\left[1 + 2\sum_{r=1}^{\infty} e^{-\sigma_\varphi^2 r^2/2}\cos r\varphi\right] = \frac{1}{2\pi}\,\vartheta_3\left(\frac{\varphi}{2},\ e^{-\sigma_\varphi^2/2}\right). \qquad (1.46)$$

where $\vartheta_3(z, g)$ is Jacoby's Theta function [29].

From a practical point of view the most interesting values to consider are the values of the parameter $\sigma_\phi^2 \gg 1$. In this case, it follows from (1.46) that

$$w_1(\varphi) = 1/2\pi, \qquad (1.47)$$

in other words, there is an even distribution of the initial phase of the transfer function of the channel in the segment $[-\pi, +\pi]$. The even character of the distribution of the initial phase in channels with sequential wave propagation has been pointed out more than once in theoretical and experimental works [67, 97, 111].

In order to compare the probabilistic models of the channels for the purely additive and the purely multiplicative situations of formation of the field received, it is relevant to consider the distributions of the quadrature components of the transfer function in both cases. With an independent logarithmically normally distributed modulus and a uniformly distributed independent variable the joint distribution of quadrature components can be written in the following form

$$w_2(x, y) = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{2\pi\sigma_\chi^2 (x^2+y^2)}} \exp\left[-\frac{(\ln\sqrt{x^2+y^2} - m_\chi)^2}{2\sigma_\chi^2}\right]^2 \quad (1.48)$$

Beginning from (1.48), it is easy to observe that the quadrature components x and y have the same distribution laws with identical statistical parameters, for example:

$$w_1(x) = \frac{1}{2\pi} \frac{1}{\sqrt{2\pi\sigma_\chi^2}} \int_{-\infty}^{\infty} \frac{1}{x^2+y^2} \exp\left[-\frac{(\ln\sqrt{x^2+y^2} - m_\chi)^2}{2\sigma_\chi^2}\right] dy. \quad (1.49)$$

These distributions are symmetrical relative to the ordinate axis. This means that the logarithmically normal distribution of amplitudes and a uniform distribution of phases preclude the possibility of the appearance of quadrature components with non-zero mathematical expectations. For small values of the parameter σ_χ^2, the distributions of the quadrature components are bimodal and very far from the Gaussian law.

The additive-multiplicative situation of formation of the field being received. We write expression (1.27) in the following form

$$\dot{H}(t, f, r) = \sum_{k=1}^{N} \dot{H}_k(t, f, r) = x + iy = \gamma e^{i\varphi}, \quad (1.50)$$

where

$$\dot{H}_k = \gamma_k e^{i\varphi_k} = \prod_{l=1}^{L_k} \dot{H}_{lk}(t, f, r). \quad (1.51)$$

If no constraints are imposed on the set of random components H_{lk} and the quantities L_k and N, it is extremely difficult to find one-dimensional distributions of H. It may be asserted that, in principle, situations are possible that yield the most diverse distributions for H. However, it is worthwhile to undertake at least a qualitative treatment of the relations which will enable us to emphasize the special importance of the two limiting types of distributions: four-parameter and logarithmically normal. With this purpose in mind, let us consider first the case where the random composite quantities

$$\dot{H}_{lk} = x_{lk} + iy_{lk} \qquad (1.52)$$

are mutually independent and the number of factors forming H does not depend on k and is equal to $L_k = Q$. Figure 15 below shows the model of sequential parallel propagation of waves being considered, including QN linear filters with characteristics H_{lk}.

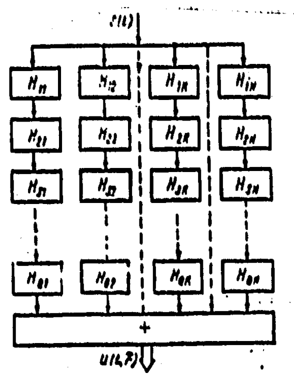


Figure 1.5. Channel with Several Independent paths of Sequential Propagation of Transmitted Signals.

As a result of the independence of the parallel paths of the model (components of H_k, see Figure 1.5 above), it is natural that where $N \to \infty$ the distribution tends toward a generalized Gaussian distribution regardless of the distribution of the components. Therefore, let us consider the case of a limited number of components N.

As the results of digital modeling show, for the model in Figure 1.5 on the condition that the parameters of the filters are random but invariant in time

$$\dot{H}_{lk}(t, f, r) = \dot{H}_{lk}(f, r), \qquad (1.53)$$

the law of distribution of the modulus H(t, f, r) in (1.50) where $N < 10$ is determined more by the multiplicative character of the relationship among components than the additive aspect.

Apparently it can be expected that as the intensity of the relations among the particular components H_k in (1.50), that is, of the signals in the parallel paths of propagation, grows stronger, the dominating role of the multiplicative aspect of the relationship will increase. As for distribution ϕ with a limited number of components N in (1.50), beginning from [67] a uniform distribution of phase may be considered typical.

When the parameters of the spatial filters of the model change randomly in time in the interval of the analysis, a lessening of the impact of the multiplicative aspect of the relationship should be expected. Indeed, if the width of the energy spectrum of time fadeouts $\Delta f_{max} = 1/\tau_{cor}$ is considered limited (τ_{cor} is the interval of time correlation), the function H_{1k} (t, f, r) may be represented by a Kotel'nikov series with uncorrelated references, which should lead to an increase in the number of components in formation of the total transfer function.

These results can also be applied in looking for the distribution of more complex additive-multiplicative formation (1.25).

Thus, it may be stated that the generalized Gaussian probabilistic model of a field is acceptable for describing a broad class of real channels with both single and multiple scattering, but in the latter case the sphere of application of this model is definitely narrower.

1.5. Statistical Models of Space-Time Channels Based on Correlation Properties

In solving the problems of optimal processing of fields, as will be demonstrated below, correlation characteristics are decisive for describing not only Gaussian but also stochastic fields of arbitrary shape.

In this connection, we should consider the classification of fields by degree of correlation in time, by frequency, and space. To do so the correlation function of any system characteristic of a channel may be investigated [47].

An exceptionally important property of the correlation functions of real space channels which makes it considerably easier to construct optimal processing diagrams is the fact that they are partially or completely factorable, that is, they are represented in the form of products of correlation functions by separate variables. In particular, a review of the correlation functions computed for a whole series of channels [32, 33, 40, 80] shows that in many cases they are spatially distinct, that is, the space correlation coefficient is factorable. As will be shown below, factorization by the space variable makes it possible to greatly simplify the algorithms of optimal processing and to separate space and time processing of fields received.

In engineering practice it is often convenient to characterize a particular space-time channel depending on the relations among the correlation intervals of the field by frequency F_{cor}, in time τ_{cor}, and by space ρ_{cor}, and among such important characteristics of a communications system as length of signals T_s, width of the spectrum of channel signals F_s, and spatial extent of field R analyzed at the receiving place.

Let us observe that the signals used to transmit information are always finite (T_s is limited). But this means, strictly speaking, that their spectrum is not limited. Nonetheless, when solving applied problems we assume that F_s is also limited.

We determine duration T and spectrum width F of the signal at the output of a channel bound to an input signal with parameters T_s and F_s by the following relations [46]:

where $\xi_{макс}$ [ξ_{max}] = $1/F_{cor}$ in the interval of signal scattering in time (channel memory) caused by the imperfectness of the frequency characteristics or the lack of a transfer characteristic from the Delta function (owing to multibeam wave propagation, nonlinearity of the phase-frequency characteristic, and the like); $\Delta f_{max} = 1/\tau_{cor}$ is the interval of signal scattering by frequency (or width of the energy spectrum of time fadeouts) caused by change in channel parameters over time and mutual displacement of the areas of signal formation and reception.

The channel memory ξ_{max} may sometimes exceed the duration of signals transmitted T_s substantially, for example in high-speed sequential transmission of messages in short samples. When there are no protective time intervals and small-base signals ($2F_sT_s \approx 2$) are used, this gives rise to intercharacter interference [49, 53].

For most radio communications channels, the interval of frequency scattering $\Delta f_{max} \ll F_s$. Where long-duration complex signals are used the correlation time $\tau_{cor} = 1/\Delta f_{max}$ may be considerably less than signal length T_s.

Let us introduce parameters that characterize the number of degrees of freedom of the stochastic field received

$$\left.\begin{aligned} \nu^T &= \ln N^T = \ln[1 + T/\tau_{кор}], \\ \nu^F &= \ln N^F = \ln[1 + F/F_{кор}], \\ \nu^R &= \ln N^R = \ln[1 + R/\rho_{кор}], \end{aligned}\right\} \quad (1.55)$$

and call them respectively the degree of channel selectivity in time, by frequency, and by space.

The quantity $N^T = [1+T/\tau_{cor}]$ determines the approximate number of non-correlated (and therefore, independent for Gaussian processes) time readings of the signal in interval T; N^F is the number of non-correlated frequency components in the spectrum of the field received. The quantity N^R has an analogous meaning.

It is apparent that the larger the number of ν^T, ν^F, and ν^R, the greater the set of possible realizations of the received field will be and the more complex the model of the channel that gave rise to them will be.

We will call a channel nonselective for given parameter P if $\nu^P \approx 0$. But if $\nu^P > 0$, then we will consider the channel selective by this parameter. Thus, if $\nu^T \approx 0$ (there is just one uncorrelated reading in interval T) or

$$T \ll \tau_{\text{пор}}, \quad (1.56)$$

we will call the channel nonselective in time. Such a channel is often called a channel with slow fadeout, or fast fadeout where condition (1.56) is not met.

Considering what has been said above, it is possible to define eight types of space-time channels by degree of selectivity (considering just one space coordinate) and group them schematically as shown in Figure 1.6 below. The simplest one of them is the channel that is nonselective by frequency, time, and space ($v^T \simeq v^F \simeq v^R \simeq 0$) and the most complex is selective for all these parameters ($v^T > 0$, $v^F > 0$, and $v^R > 0$).

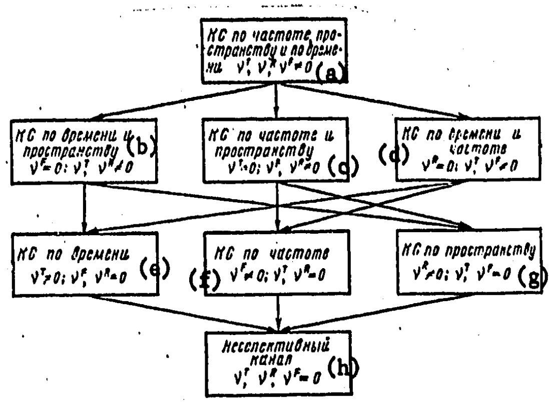


Figure 1.6. Classification of Space-Time Channels by Selectivity.

Key:
- a) Selective Channel by Frequency, Space, and Time;
- b) Selective Channel by Time and Space;
- c) Selective Channel by Frequency and Space;
- d) Selective Channel by Time and Frequency;
- e) Selective Channel by Time;
- f) Selective Channel by Frequency;
- g) Selective Channel by Space;
- h) Nonselective Channel.

1.6. Model of Spatially Distributed Additive Noise

Spatially distributed additive noise n(t, r) added linearly with the signal field u(t, r), forms the following total field, accessible for analysis

$$z(t, r) = u(t, r) + n(t, r), \quad (1.57)$$

and is generated by a set of factors such as thermal and cosmic noise, internal equipment noise recalculated for the input of the system, the signals of interfering stations, and the like. Many works [121, 124, 135] investigate the physical and statistical properties of spatially distributed noise.

Additive noise can be arbitrarily divided into two large classes according to effect on the signal received:

1. Noise that affects the entire frequency-time and space domain of signal existence;

2. Locally active noise pulsed in space and time, focused by frequency.

Numerous statistical findings illustrate that additive noise that affects the entire domain of existence of a signal field can usually be considered Gaussian or close to Gaussian. The reason is that the noise field is, as a rule, created by a large number of weakly interconnected spatially scattered sources [103].

Locally active noise is often characterized by high probabilities of large amplitudes and is not described well by the Gaussian law. It is possible in principle, having devised a certain probabilistic model of locally active noise [10], to solve the problem of synthesizing an optimal signal processing device against a background of varied additive noise. However, this device proves quite complex. It should be kept in mind that locally active noise is usually described by nonstationary processes whose statistics change continuously and are often unknown. Techniques of controlling such noise often amount to eliminating part of the signal located in the frequency-space-time area of the noise. The effectiveness of such techniques depends on the properties of the space-time signals and increases with increase in the size of the frequency-space-time domain occupied by the signal (using signals with a large base [73, 76]).

Another way of controlling locally active noise is special coding or using the ideas of adaptive compensation [133].

In this book we resolve the problem of synthesizing processing devices that are optimal in relation to noise which operates in the entire domain of existence of the space-time signal, and this noise is not necessarily treated as Gaussian. Locally active noise can be controlled by using technical measures, which take the form of certain attachments to the optimal receiving device or the form of replaceable algorithms and programs for processing signals in digital machines. In this case the problem of analyzing such algorithms and devices in various noise settings becomes timely. The present book, however, does not take up these matters.

We will assume that the noise field n(t, r) is stationary in time and space with an average value $\overline{n(t, r)} = 0$. This model conforms to many theoretical and experimental works.

The appearance of the noise correlation function is determined by the physics of the processes from which it formed and may differ greatly in different channels. However, it can be said with confidence that a component of the white noise type, whose energy spectrum is uniform in the pass band of the receiver, can always be provisionally identified from noise n(t, r). If this were not true we would have the well-known paradox [140] which asserts the possibility of error-free reception in the presence of noise. Work [16] gives an explanation of this paradox. It shows that the degeneration of the solution (singularity) owes its origin to failure to take account of white noise objectively existing in the receiving units. The white noise component guarantees the stability of the solution procedure.

The present book does not assume, in solving synthesis problems, that additive n(t, r) is Gaussian. However, in solving problems of analysis (evaluation of noise suppression capability), we will consider the Gaussian white noise field with the correlation function

$$B_n(t, t+\Delta t;\; r, r+\Delta r) = \frac{N_0}{2}\,\delta(\Delta t)\,\delta(\Delta r), \qquad (1.58)$$

where n_0 is the spectral density of noise output, to be additive noise.

This was done exclusively to achieve maximum clarity in presentation of the material and to make the properties of the stochastic channel, not additive noise, paramount. In all the material that follows additive noise n(t, r) is considered to be independent of usable signal u(t, r). This assumption is not always correct [16, 121]. However, for most data transmission systems the dependence between the signal and additive noise is not great enough to make it advisable to take the noise into account in constructing the optimal algorithm.

Let us now quickly touch on the characteristics of additive noise in the optical wave range and the associated question of the applicability of further results to processing optical fields.

We know that visible light has a dual quantum-wave nature, that is, a quasiharmonic optical wave u(t, r) with central frequency f_0 can also be treated as a stream of photons, quantums of energy of magnitude hf_0 where h is the Planck constant.

The photon arriving at the photoelectric convertor (photo detector) causes emission of an electron. This is the quantum transition. In studying questions of the statistical reception of optical signals it is customary [23, 78, 119] to consider that the fluctuation observed is a sequence of quantum transitions taking place under the effect of light, that is, that a signal at the output of the photo detector is being considered.

The basic statistical characteristic with this approach is the distribution of probabilities of the number of photoelectrons in a fixed time interval, and the receiver is a photon counter with subsequent logical processing. At the given level of technological development with a large number of series-produced energy-sensitive elements, this energy approach to the reception of optical signals is apparently natural, because there is a critical practical need for its results. However, from the standpoint of noise proof reception, the approach that considers both the amplitude and phase relations of the light field being received using a classical description, on the assumption that it is acceptable, is better in principle [45].

The vector of intensity of an electrical field, for example, contains such relations.

Having posed the problem of optimal selection of information from the vector of intensity of an electrical field by some statistical criterion, let us arrive at a definite algorithm for processing the optical signal. The distinguishing feature of such algorithms is the fact that signal processing in space, in time, and by amplitude and phase are, in the general case, inseparable and demand a space-time filter for realization.

The discovery of the holographic method of processing signals and realization of coherent optical data processing systems [56, 74, 106] in combination with the appearance of high-speed computers with elaborate software [28] make the new algorithms of space-time processing of optical (and not only optical) signals highly practical. Nonetheless, many of the algorithms for processing space-time signals in the following chapters are marked for the future, not the present, level of development of data transmission technology.

Additive noise in the optical range is subdivided into external and internal. External noise is the natural background emission caused by the chaotic emission of quantums by sources at heightened temperature or reflecting emission from other sources. The sun, moon, stars, and other planets make the primary contribution to the natural background for ground receivers. With data transmission in space the Earth is also a background source. The natural background emission is satisfactorily described by a model of a Gaussian noise field in which the spectral density of output may be considered uniform [78, 119] in the pass band of the optical receiver.

The internal noise of a "wave" optical receiver, as is also true of a radio wave receiver, is caused primarily by thermal noise and in recalculation for the input of the device is described by a model of a Gaussian "white" noise field. In the "quantum" optical receiver (especially the heterodyne type), which includes a photon counter, quantum noise is usually paramount [119]. The spectral density of Gaussian white noise at the output of the photo detector (it can be recalculated for the input also) in this case is formed of the spectral density of the natural background emission N_ϕ and the spectral density of quantum noise hf_0/η, where η is quantum efficiency, the ratio of the average number of electrons emitted or generated by the detector to the average number striking the photon detector. If $N_{0\phi}$ is large compared to hf_0/η, the "wave" and "quantum" approaches coincide fully, so additional "quantum noise" may be disregarded.

They can, with some precision, be neglected for even greater changes in hf_0/η. This makes it possible to apply to quantum receivers the primary (at least qualitative) conclusions on the influence of a stochastic channel on the characteristics of data transmission systems which are drawn below in analyzing several wave receivers that do not use the phase relations of space-time signals.

1.7. Linear Model of Signal and Noise Fields Obtained by the Method of State Variables

The models of a space-time channel and additive noise which were discussed above were based on consideration of the properties of real physical channels. However, there is another, abstract mathematical approach to construction of a model of a stochastic channel. It is suggested by general systems theory [34, 42], and is based on the concept of the space of states, which determine the behavior of the system.

Let us consider the formal construction of a linear model of a channel and noise on the basis of state equations. A linear model of an actual vector stochastic continuous-time process y(t) [19] has the following form

$$\left.\begin{aligned} \frac{d\vec{x}}{dt} &= F(t)\vec{x}(t) + G(t)\overrightarrow{v(t)} \\ \vec{y}(t) &= H(t)\vec{x}(t) \end{aligned}\right\} t \in T, \qquad (1.59)$$

where $\vec{x}(t)$ is the vector of state; F(t), G(t), and H(t) are matrices which depend, in the general case, on time. They are related in a one-to-one manner with the correlation function of the process y(t).

For a complete description of the model (1.59) it is also necessary to assign the initial and boundary conditions for the state vector x(t). It should be emphasized that this model of random functions (1.59) may have nothing in common with the physics of the processes described by these functions.

For example, suppose x(t) = h(t) is the pulse transfer characteristic of a linear stochastic channel. Then from physical considerations, x(t) is the response of a linear stochastic field, with constant parameters that describe the channel, to a deterministic action in the form of a Delta function.

But when model (1.59) is used, x(t) is the response of a linear deterministic filter with variable parameters to a stochastic action in the form of white noise v(t).

It is significant that model (1.59) permits a formal description of any stationary random process with a rational (bilinear) energy spectrum as well as a broad class of nonstationary random processes. There is a regular method of synthesizing a dynamic model according to the autocorrelation function for stationary processes [94]. To do this the energy spectrum of the process is factored and then the task of synthesizing the equivalent vector Markov process in form (1.59) is accomplished automatically.

For random functions of several variables (fields), there is at present no such regular method of synthesizing dynamic systems with distributed parameters [108]. The appearance of such a technique is being held back, in the first place, by the absence of a theorem on factorization of spectrums in the multidimensional case and, secondly, the lack of a mathematical technique for constructing a vector Markov field from a scalar model of autoregression. As a result of this, there are different approaches to describing random fields by the method of state variables. We will consider the two basic ones.

The state equation and observations corresponding to the function of two variables (time and space) may be written in the form of a vector differential equation in partial derivatives, including a spatial integral operator [4, 18, 19, 143]:

$$\left.\begin{aligned} &\frac{\partial \vec{x}(t, r)}{\partial t} = F(r)\vec{x}(t, r) + G(r)\vec{u}(t, r), \\ &[\vec{x}(t, r)] = 0, \quad t \in T, \quad r \in R, \\ &z(t, r) = H(t, r)\vec{x}(t, r) + n(t, r), \end{aligned}\right\} \quad (1.60)$$

F(r) and G(r) are linear spatial operators (integral, differential, or integral-differential). The remaining designations are analogous to those introduced above taking into account the appearance of new spatial coordinate r. Field u(t, r) is considered to be δ-correlated in time and in space. For example, a stochastic system with distributed parameters may have an equation of state that appears as follows

$$\left.\begin{aligned} &\frac{\partial x(t, r)}{\partial t} = a^2\left[\frac{\partial^2 x(t, r)}{\partial r^2} + bx(t, r)\right] + u(t, r), \; 0 < r < R, \\ &\left.\frac{\partial x(t, r)}{\partial r}\right|_{r=0} = \left[\frac{\partial x(t, r)}{\partial r} + x(t, r)\right]_{r=R} = 0, \\ &z(t, r) = x(t, r) + n(t, r). \end{aligned}\right\} \quad (1.61)$$

A specific feature of representation (1.60) is the division of operations on the field into spatial and temporal. Projecting this fact to the algorithm of processing signals represented by model (1.60), it could be expected that space and time processing in these algorithms would also be separated. However, investigations show [19] that in the general case this is not so. Extensive use of model (1.60) should be expected in later volumes of monographs [18].

The lack of mixed signal derivatives in model (1.60) limits the class of random fields which can be described exactly by this model. Specifically, this model does not cover fields with the correlation function

$$B_u[t-t', r-r'] = \sigma_u^2 \exp(-\alpha_t|t-t'| - \alpha_r|r-r'|), \qquad (1.62)$$

which have the Markov property and are sometimes called Markov fields [108]. From the practical and theoretical standpoints Markov fields, scalar and vector, are extremely interesting and merit further consideration.

The primary equation of the states of a Markov vector field can be written as follows

$$\frac{\partial \vec{x}(t, r)}{\partial t} + \frac{\partial \vec{x}(t, r)}{\partial r} + \frac{\partial^2 \vec{x}(t, r)}{\partial t\, \partial r} = F(t, r)\vec{x}(t, r) + G(t, r)\vec{u}(t, r). \qquad (1.63)$$

Matrices F(t, r) and G(t, r) in the general case depend on both space and time.

The description of signals by state variables methods is oriented to further application of Kalman filters [18, 19, 42]. Kalman filters are most adaptable for realization on digital computing machines because they make it possible to receive signal evaluations in recurrent form. Orienting to digital processing, the state equation (1.63) should be written in discrete time ($t = k\Delta t$) and with a discrete space variable ($r = l\Delta r$). The discrete representation of the state equation for a scalar Markov field is as follows

$$x(k+1, l+1) = \rho_1 x(k+1, l) + \rho_2 x(k, l+1) - \rho_1 \rho_2 x(k, l) + \sqrt{(1-\rho_1^2)(1-\rho_2^2)}\, u(k, l). \qquad (1.64)$$

For a field that contains $N^t \times N^r$ elements, k and l vary from 1 to N^t and N^r respectively. The following designations

$$\rho_2 = \exp(-\alpha_t \Delta t), \quad \rho_1 = \exp(-\alpha_r \Delta r), \qquad (1.65)$$

have been introduced in (1.64); they have the physical meaning of correlation coefficients of neighboring field readings (corresponding to the independent variables of time and space). Models (1.63) and (1.64) will be used in the following chapter to construct estimates of the transfer function of a stochastic channel.

Model (1.64) is a particular case of an autoregression model [108], which has the following appearance for stationary uniform fields

$$x(k+m,\ l+n) = \sum_{\substack{p=0 \\ (p,\ q)\neq(m,\ n)}}^{m} \sum_{q=0}^{n} g_{pq}\, x(k+p,\ l+q) + g_{mn}(k,\ l)\, u(k,\ l) \quad (1.66)$$

In the continuous variation the differential equation in partial derivatives of order (m+n) corresponds to equation (1.66).

In addition to the autoregression model, models of a sliding mean and autoregressions of a sliding mean may find broad use to describe a field in given discrete readings.

The model of a channel that forms a space-time field in the reception region is based on representation of the channel by a space-time stochastic filter with variable parameters. Channels are classified as selective and nonselective based on independent variables of time, frequency, and space according to effect on signals transmitted. Consideration of the physical processes of wave propagation shows that in the general case the observed field contains an additive-multiplicative mixture of signals that have traveled different paths of propagation. The system functions that describe the channel should be viewed as random fields. In many cases of practical importance these fields have Gaussian statistics, but the situations are common, in particular for an optical channel, where the probabilistic model of the field is very far from the Gaussian model (logarithmically normal amplitudes and uniform phases).

The classical wave description of an optical channel and the noise operating in it is completely acceptable if the algorithms for processing the fields are oriented to coherent optical systems and the number of photons that define the signal is not too large.

The approach based on the method of state variables deserves attention in the analysis of space-time channels.

Chapter 2. Measurement of the Space-Time Characteristics of a Stochastic Channel

2.1. Formulation of the Problem of Measurement of the Space-Time Characteristics of a Stochastic Channel

Numerous works by Soviet and foreign authors have been devoted to the problem of measuring the characteristics of linear channels with randomly changing parameters [46, 88, 90, 132, 139, 141]. In these works primary attention has been devoted and primary results obtained applicable to the measurement of the pulsed surge characteristic $h(\xi)$ or the transfer function $h(f)$ of a time channel with constant parameters using non-informational test signals. We should stress that in most of the prominent works the problem of measurement is solved without taking account of additive noise in the channel.

The algorithms of measurements obtained theoretically with this approach have, of course, a very particular form [139] and quantitative conclusions concerning the quality of the practical heuristic algorithms proposed have not been drawn [46]. The most basic published work [139] is non-constructive in character. This work, which uses a deterministic approach to measurement, shows that it is theoretically possible to measure a channel characteristic as precisely as one wishes, but it does not show how to realize this possibility in practice.

The results in works [146, 88, 132] are closer to practice, but they do not give attention to the question of the influence of noise on measurement, which changes the entire picture qualitatively. The work closest to the formulation in our work is [141], in which the problem of measuring a transfer function of a time channel is resolved by optimal filtration.

We have obtained and investigated algorithms for measuring the characteristics of the most general type of space-time channel in the presence of additive noise of arbitrary intensity with the use of various a priori information concerning the characteristics studied and noise in the channel. We consider measurement of channel characteristics both where special probe signals are used and with the help of informational signals. Primary attention is devoted to measuring the instantaneous values of channel characteristics.

Let us formulate the problem of measuring the instantaneous space-time characteristics of a channel. The total field at the output of the channel can be written as follows

$$z(t, r) = u(t, r, \vec{\alpha}) + n(t, r),$$
$$0 \leq t \leq T, \quad r \in \Lambda, \qquad (2.1)$$

where n(t, r) is spatially distributed additive noise; $u(t, r, \vec{\alpha})$ is the response of a linear channel to input signal $s(t, \vec{\alpha})$:

$$u(t, r, \vec{\alpha}) = \mathrm{Re} \int_0^\infty \dot{h}(t, \xi, r) \dot{s}(t-\xi, \vec{\alpha})\, d\xi. \qquad (2.2)$$

A linear space-time channel is fully described or, as they say, identified [88, 139] by any system function, in particular h(t, ξ, r). In the general case of a stochastic channel with scattering, function h(t, ξ, r) should be treated as the realization of a certain random function, that is

$$\dot{h}(t, \xi, r) \equiv \dot{h}(t, \xi, r, \omega), \qquad (2.3)$$

where ω is an element of probabilistic space Ω.
In further consideration of particular realizations of the this random function we will drop the probabilistic variable ω.

For particular realizations of signals, noise, and system functions we will assume, as follows from physical considerations, that the conditions of squared integrability are met:

$$\left.\begin{aligned}&\int_0^T \int_0^T \int_\Lambda |\dot{h}(t, \xi, r)|^2\, dt\, d\xi\, dr < \infty,\\ &\int_0^T \int_\Lambda |n(t, r)|^2\, dt\, dr < \infty, \quad \int_0^T |s(t)|^2\, dt < \infty.\end{aligned}\right\} \qquad (2.4)$$

For random functions conditions (2.4) should be met with a probability close to one.

Now let us formulate, in general form, the problem of measuring the space-time characteristics of a channel. Realizations of the function z(t, r) are accessible to observation. There is a relation that links an observed fluctuation with the transmitted signal s(t) and channel characteristics:

$$z(t, r) = \mathrm{Re} \int_0^\infty \dot{h}^*(t, \xi, r) \dot{s}(t-\xi)\, d\xi + n(t, r). \qquad (2.5)$$

The problem of measuring a space-time characteristic of a linear channel h(t, ξ, r) is reduced to solving a first-order Fredholm integral equation (2.5). We know [98, 99] that the problem is incorrect in the sense of solution stability, that is, large deviations in the solution h(t, ξ, r) follow small deviations in the values of function z(t, r).

Indeed, suppose the function $h(t, \xi, r)$ is the solution to equation (2.5). Then the function $h_1(t, \xi, r) = h(t, \xi, r) + \sin p\xi$ will be the solution to the same equation with an observed function

$$z_1(t, r) = z(t, r) + \int_0^\infty s(t-\xi)\sin p\xi s d\xi. \qquad (2.6)$$

Where p is quite large the values of the integral in the last equation may become vanishingly small while the values of the function $\sin p\xi$ are finite quantities. This means that large deviations in the solution to equation (2.5) may correspond to small deviations in the observed function. In practice, the process of measuring channel characteristics inevitably involves the effect of noise, which makes a significant contribution to fluctuation at the channel output and this also makes the problem of measuring channel characteristics incorrect.

Several works by Soviet mathematicians have been devoted to the solution of incorrect problems [8, 9, 75, 85, 98, 99]. It has been demonstrated that the problem under review may be considered correct and its solution may be stable if we seek it in the group of smooth functions. Academician A. N. Tikhonov introduced the concepts of the class of regularizable incorrect problems and developed a general solution method called the regularization method. Works [98-100] contain a detailed description of the regularization method.

It is easy to clarify the essential physical features of the regularization method by considering the situation where the function $h(t, \xi, r)$ in segment of time [0, T] can be considered invariable (channel with slow fadeouts in time). The use of the regularization method [8, 9, 99] leads to the following regularized spectrum of solution (of the channel transfer function):

$$H^\alpha(\omega, \omega_g) = \frac{1}{1 + \alpha \frac{M(\omega, \omega_g)}{|\dot{S}(\omega)|^2}} \frac{\dot{Z}(\omega, \omega_g)}{\dot{S}(\omega)}, \qquad (2.7)$$

where $M(\omega, \omega_g)$ is an even non-negative function and $S(\omega)$ and $Z(\omega, \omega_g)$ are the spectra of realizations of the input signal and observed fluctuation.

It can be seen from (2.7) that the regularized solution $H^\alpha(\omega, \omega_g)$ differs from the "classical" $H(\omega, \omega_r) = Z(\omega, \omega_g)/S(\omega)$ by the factor $\left(1 + \frac{M(\omega, \omega_g)}{|\dot{S}(\omega)|^2}\right)^{-1}$, whose values, as common sense suggests, must be small at those frequencies where the intensity of usable signal $S(\omega)$ is low. This is achieved by appropriate selection of the type of function $M(\omega, \omega_g)$ and values of the regularization parameter α.

In development of A. N. Tikhonov's regularization method, stable solutions to integral equation (2.5) considering the stochastic characteristics of the studied functions, signals, and noise are sought in the statistical set of smooth functions [75, 101]. A serious shortcoming of this method of searching for stable solutions to integral equation (2.5) is the lack of unambiguous recommendations on choosing the optimal regularization functions $M(\omega, \omega_r)$ and regularization parameters α in the particular situation. Therefore, in what follows we will use this method when, for one reason or another, it is impossible to use other, more constructive optimal or quasioptimal solutions.

Two different approaches to measuring the functions that describe a channel characteristic are possible in the search for optimal solutions:

a. measuring the coordinates of factoring the function on the chosen basis;

b. measuring the ongoing values of the function obtained by filtration.

The first approach is more convenient when measuring a channel characteristic to organize some particular type of scattered reception as a whole when engineering realization depends entirely on progress in signal processing and memory device technology.

It is advisable to use the second approach in problems oriented to the procedures of sequential (recurrent) processing of signals transmitted by stochastic channels. We should observe that as a result of the limited nature of the space from which the signal is extracted at the receiving point, the first approach is preferable for space signal processing. Let us consider the specific features of both approaches to measurement.

Measuring the coordinates of factorization. We will consider that to measure a certain function $f(u)$ of one or several variables means to measure the coordinates $\{f_k\}$ of expansion of this function into a series on a certain basis $\{\phi_k(u)\}$:

$$f(u) = \sum_k f_k \varphi_k(u), \quad u \in \Lambda. \tag{2.8}$$

This definition of measurement of a function is broad enough to cover many practical process and field measurment situations. The coordinates of expansion (2.8) f_k are linear functionals

$$f_k = \int_\Lambda f(u)\varphi_k(u)\,du. \tag{2.9}$$

Thus, the problem of measuring space-time characteristics of a channel comes down to a problem of estimating parameters, the coordinates of factorization of the measured characteristics on a certain basis. The statistical theory of estimating parameters has now been quite extensively developed [64, 102, 103]. There are effective methods that enable one to obtain good estimates of parameters both where a large amount of information on the parameters being estimated is available and in a situation of a priori uncertainty. It is now possible to formulate the problem of measuring the characteristics of a channel with scattering as a classical problem of estimating parameters. This is expansion of the function into a series on a certain basis.

$$h(t, \xi, r) = \sum_k h_k \varphi_k(t, \xi, r). \qquad (2.10)$$

Field z(t, r), which is defined by relation (2.5) and depends on set of parameters $\{h_k\}$, is accessible to observation. We must make the best estimate of parameters $\{h_k\}$, in conformity with the criterion chosen, using known values of realization of the function z(t, r) for this purpose. Applying contemporary methods of parameter estimation to the model being considered here, which is expressed by the relation (2.5), it is possible to construct a large number of algorithms for estimating the coordinates of factorization of the space-time characteristics of a channel. We will require that the estimates of parameters $\hat{h}_k$ be linear functionals of the observed oscillation:

$$\hat{h}_k = A_k \int_0^T \int_\Lambda z(t, r) \psi_k(t, r)\, dt\, dr + B_k, \qquad (2.11)$$

where A_k and B_k are coefficients and $\psi_k(t, r)$ is a weighted function.

Such measurement of channel characteristics may be called linear [139].

We will give more good arguments in favor of linear measurement of space-time characteristics of a linear channel with scattering below. For now we will just observe that for the very important case of a Gaussian field, linear estimates of coordinates of factorization are optimal in the class of all possible estimations. Thus, with this approach the problem of measuring channel characteristics has two parts: choosing the method of expanding the characteristic being measured into a series and constructing algorithms for estimating the coordinates of factorization.

Measurement by filtration. The approach based on the techniques of linear filtration is effective for solving the problems of measuring random functions, in particular for solving the problems of identification of linear stochastic filters. In the theory of linear filtration we should single out two branches that correspond to two different approaches to describing the measurement of functions and noise in a channel. If the function being measured is defined by its correlation characteristics, we arrive at a Wiener filter. In this case the optimal, by the mean quadratic criterion, linear estimate $\hat{h}(\xi, r)$ is the output magnitude of the linear system whose surge characteristic $g(t, \mu)$ is determined by the corresponding linear regression equation (for a scalar spatial variable):

$$\hat{h}(\xi, r) = \int_{-\infty}^{r} \int_{-\infty}^{\xi} z(t, \mu) g(\xi - t, r - \mu)\, dt\, d\mu. \qquad (2.12)$$

Where processes are non-stationary and the area of field analysis (in time and space) is greatly limited, it is extremely difficult or impossible to solve the problem by the Wiener filtration technique.

In this situation, another approach to linear (and also nonlinear) filtration holds great practical interest. This approach is based on descriptions of the fields of signals and noise received not through their correlation functions but rather through equations of the state of the system which generates them while excited by white noise. In principle, it would be possible to assign the equation of state of a system in the study of fields by Maxwell equations or Langevin equations [135], and on this basis it would be possible to construct the corresponding time-space Kalman type filters to estimate the parameters, and then on their basis construct corresponding devices for optimal fields processing. However, at the present time it is difficult to see the paths leading to engineering realization of these space-time devices for signal processing. A more practical way to construct Kalman type filters will be considered below.

2.2. Expansion of Space-Time Characteristics of a Channel into Series and Discrete Models of a Channel

Let us pass on to consideration of various methods of expanding space-time characteristics into series on the given basis. Such factorization makes it possible to single out a calcuable set of coordinates [64, 139] that define the channel and also introduce various discrete models of a random space-time channel.

Let us review the factorization using the example of system function $h(t, \xi, r)$.

We will represent the pulsed surge characteristic $h(t, \xi, r)$ in the form of an orthogonal factorization in a certain limited domain. This is always possible because the signals being transmitted along a communications channel may be considered, in approximate analysis, finite in time $[0, T_s]$ and in frequency $[-F_s, F_s]$. In practice the channel causes limited scattering in time and frequency, but the area of the field which is taken into account in signal processing is limited. Area Λ may be defined from the relation

$$\Lambda = [0, T] \times [0, T] \times [0, R], \qquad (2.17)$$

where $[0, T]$ is the interval of field analysis in the temporal domain, with $T > T_s$*; $[0, R]$ is the spatial interval of analysis of the field being received (for the sake of compact notation, the scalar spatial variable is considered here).

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* In principle it is possible to conduct an analysis of the signal being received in time and frequency domains that are narrower than the corresponding domains of the signals transmitted. We will not analyze this situation, however, and will focus attention on devices which extract all possible information from the field domain being analyzed.

In general form, the factorization of the pulsed surge characteristic of a channel is written as follows:

$$\dot{h}(t, \xi, r) = x(t, \xi, r) + i y(t, \xi, r) = \sum_k x_k \varphi_k(t, \xi, r) + i y_k \widetilde{\varphi}_k(t, \xi, r). \quad (2.18)$$

The real and imaginary components of the coordinates of expansion of (2.18) are linear functionals whose values are determined by the values of the concrete realization of function h(t, ξ, r) and selection of the base:

$$\left.\begin{aligned} x_k &= \int_{\Lambda} x(t, \xi, r)\varphi_k(t, \xi, r)\,dtd\xi\,dr, \\ y_k &= \int_{\Lambda} y(t, \xi, r)\widetilde{\varphi}_k(t, \xi, r)\,dtd\xi\,dr. \end{aligned}\right\} \quad (2.19)$$

The choice of functions $\phi_k(t, \xi, r)$ may be dictated by various requirements. Among them are the requirements for best approximation of the function being expanded into a series with a minimum number of series members (2.18); lack of correlation among coordinates of factorization; convenience and simplicity in practical realization of algorithms for factoring and regenerating functions, and the like. The purpose of the factorization plays a large part in the choice. This purpose is determined by the criterion of factorization quality. We will observe that in problems of optimal processing of space-time signals the requirement of minimization of mean quadratic deviation between the right and left parts of equation (2.18) is not mandatory and practical considerations may dictate a choice of type of functions ϕ_k and number of members in the expansion of (2.18) that is very far from optimal from the standpoint of approximating function h(t, ξ, r). Let us consider several concrete forms of factoring channel characteristics which are of greatest theoretical and practical interest.

Factorization by the Karunen-Loew theorem. To shorten the notation, we will assume here that the real and imaginary parts of the pulsed surge characteristic of a channel are independent and have identical correlation functions

$$B_x(t, t', \xi, \xi', r, r') = B_y(t, t', \xi, \xi', r, r') = B_h(t, t', \xi, \xi', r, r'). \quad (2.20)$$

Application of the results to the case of non-identical correlation functions of quadrature components is simple and will be done below as necessary.

According to the Karunen-Loew theorem [31] it is possible to receive non-correlated coordinates of factorization (2.18), selecting as $\varphi(t, \xi, r) = \widetilde{\varphi}(t, \xi, r)$ eigen functions of the second-order Fredholm integral equation whose nucleus is the correlation function $B_h(t, t', \xi, \xi', r, r')$. The integral equation is written in the following form

$$\varkappa\varphi(t, \xi, r) = \int\limits_{\Lambda} B_h(t, t', \xi, \xi', r, r')\varphi(t', \xi', r')\, dt'd\xi'dr', \qquad (2.21)$$

where χ is the eigen numbers of integral equation (2.21).

Using the fact that $\dot{s}(t) = s(t)+i\tilde{s}(t)$, the composite field at its output is defined by the relation

$$\vec{u}(t, r) = \int \dot{s}(t-\xi)h^*(t, \xi, \vec{r})d\xi$$

and taking factorization [2.18] into account we receive this representation of the (observed) real part of the signal at the channel output:

$$u(t, r) = \mathrm{Re}\sum_k (x_k - i y_k)\dot{s}_k(t, r) = \sum_k x_k s_k(t, r) + y_k\tilde{s}_k(t, r). \qquad (2.22)$$

The composite signal $\dot{s}_k(t, r) = s_k(t, r)+i\tilde{s}_k(t, r)$ is defined by the convolution

$$\dot{s}_k(t, r) = \int\limits_0^\infty \dot{s}(t-\xi)\varphi_k(t, \xi, r)d\xi. \qquad (2.23)$$

It is not difficult to show that if the transmitted signals are narrowband signals, then signals $s_k(t, \vec{r})$ and $\tilde{s}_k(t, \vec{r})$ are interrelated by a Hilbert transform by variable t.

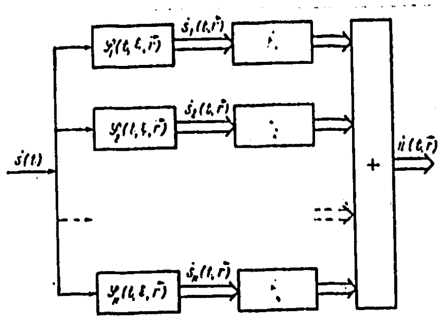

Figure 2.1. Discrete Model of a Space-Time Channel.

Formula (2.22) was received on the assumption that series (2.18) permits term-by-term integration with weight s(t), which follows from physical considerations. According to model (2.22), which is depicted schematically in Figure 2.1 above, the signal at the place of reception may be treated as the sum of space-time signals that have traveled over a set of non-correlated paths. In each of these paths the signal undergoes regular distortions introduced by a space-time filter with variable parameters and a (known) characteristic $\phi_k(t, \xi, r)$, and also receives the random amplitude factor $h_k = \sqrt{x^2_k + y^2_k}$ and the random phase shift $\phi_k = \mathrm{arc\ tg}(y_k/h_k)$, which are determined by composite coefficient $h_k = h_k e^{i\phi_k}$. The fadeouts described by model (2.22) are in the general case selected in space in time, and by frequency.

As was observed in Chapter 1, the situation of a spatially divisible correlation function of a channel where

$$B_h(t, t', \xi, \xi', r, r') = B_h^{\rm I}(t, t', \xi, \xi') B_h^{\rm II}(r, r'). \qquad (2.24)$$

has become widespread in real communications channels and is thus significant in the analysis of information transmission systems.

In this case integral equation (2.21) breaks into a system of two integral equations

$$\left.\begin{aligned} \varkappa^{\rm I} \varphi^{\rm I}(t, \xi) &= \int_0^T \int_0^T B_h^{\rm I}(t, t', \xi, \xi') \varphi^{\rm I}(t', \xi') dt' d\xi', \\ \varkappa^{\rm II} \varphi^{\rm I}(\vec{r}) &= \int_0^R B_h^{\rm II}(r, r') \varphi^{\rm II}(r') dr', \end{aligned}\right\} \qquad (2.25$$

where $\varkappa^{\rm I}\varphi^{\rm I}(t, \xi)$ and $\varkappa^{\rm II}\varphi^{\rm II}(r)$ are the eigen functions and eigen numbers respectively of integral equations (2.25). Under conditions of a spatially divisible correlation function factorization (2.18) takes the form

$$\dot{h}(t, \xi, r) = \sum_l \sum_m (x_{lm} + i\, y_{lm}) \varphi_l^{\rm II}(r) \varphi_m^{\rm I}(t, \xi). \qquad (2.26)$$

Correspondingly, the signal at the receiving point

$$u(t, r) = \mathrm{Re} \sum_l \sum_m (x_{lm} - i\, y_{lm}) \dot{s}_m(t) \varphi_l^{\rm II}(r) = \sum_l \sum_m [x_{lm} s_m(t) + y_{lm} \tilde{s}_m(t)] \varphi_l^{\rm II}(r). \qquad (2.27)$$

Signal $s_m(t) = s_m(t) + i\tilde{s}_m(t)$ is defined by the convolution

$$\dot{s}_m(t) = \int_0^\infty \dot{s}(t - \xi) \varphi_m^{\rm I}(t, \xi) d\xi. \qquad (2.28)$$

Model (2.27), which is depicted in Figure 2.2 below, makes it possible to consider the space-time signal at the receiving point as a set of non-correlated signals transmitted by a group of "antennas" with directivity diagrams (patterns) $\phi^{\rm II}{}_i(r)$, $i = 1, 2, \ldots$, with the signal corresponding to each of the "antennas" undergoing regular frequency-time distortions introduced in the channel by the assemblage of filters with variable parameters with characteristics $\phi_m(t, \xi)$ and also random amplitude and phase distortions defined by the composite coefficient $h_{lm} = x_{lm} + iy_{lm}$.

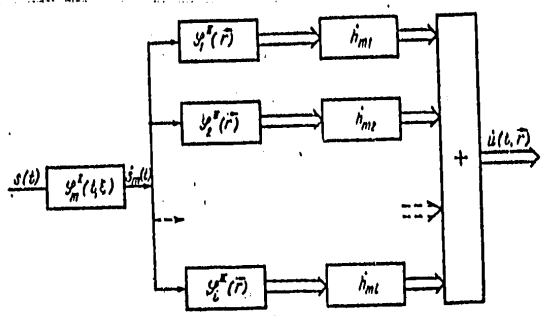

Figure 2.2. Discrete Model of a Channel with Spatially Divisible Correlation Function

In the case where just one "antenna" of the entire set of "antennas" remains, fadeouts become nonselective by space. But if one filter remains from the set of variable-parameter filters that form the signal at the receiving point, the fadeouts should be treated as nonselective in time and frequency. It is clear that the degree of selectivity of fadeouts in the given channel (with assigned physical properties) is determined both by correlation intervals ψ_{cor}, F_{cor}, and ρ_{cor} and by the domain of field analysis in time T, by frequency F, and in space R.

The model of a channel considered here, including a set of filters with variable parameters and characteristics $\phi^I_m(t, \xi)$, is inconvenient for practical use. It is difficult to realize a set of filter with variable parameters. Therefore, it is very attractive to substitute filters with constant parameters for the variable-parameter filters. The characteristic of a variable-parameter filter should be treated as a function of two variables: t and ξ. We know [129] that a function of many variables can be apprcximated as exactly as we like by a set of functions with separating variables, specifically $\varphi(t, \xi) = \sum_p \varphi_p \alpha_p(t) \beta_p(\xi)$.

This makes it possible to replace the variable-parameter filter with a set of constant-parameter filters and multipliers. For modern information retransmission technology, however, it is easiest and most convenient to replace the variable-parameter filter with a constant-parameter one whose characteristic maximally approximates that of the filter being replaced. If the criterion of closeness of the filter characteristics is taken to .imum functional be the criterion of minimum functional $\Phi = \int_0^T \int_0^t |\varphi(t, \xi) - \psi(\xi)|^2 dt\, d\xi$, then the pulsed surge characteristic of the physically feasible filter with constand parameters should be determined from this relation [139]

$$\psi(\xi) = \frac{1}{T-\xi} \int_\xi^T \varphi(t, \xi)\, d\xi. \qquad (2.29)$$

It is perfectly obvious that such an approximation is permissible only in channels whose parameters do not change too fast in time, that is, in channels with weak time selectivity.

Let us turn to the practically important channels with a divisible (factorable) function for all variables:

$$B_h(t, t', \xi, \xi', r, r') = B^I_h(t, t')\, B^{II}_h(\xi, \xi')\, B^{III}_h(r, r'). \qquad (2.30)$$

In this case a system of three integral equations corresponds to integral equation (2.22):

$$\left.\begin{aligned} \varkappa^{I}\varphi^{I}(t) &= \int_0^T B_h^{I}(t, t')\varphi^{I}(t')\,dt', \\ \varkappa^{II}\varphi^{II}(\xi) &= \int_0^T B_h^{II}(\xi, \xi')\varphi^{II}(\xi')\,d\xi', \\ \varkappa^{III}\varphi^{III}(r) &= \int_0^R B_h^{III}(r, r')\varphi^{III}(r')\,dr'. \end{aligned}\right\} \quad (2.31)$$

In this case factorization [2.18] takes the form

$$h(t, \xi, r) = \sum_l \sum_m \sum_k (x_{lmk} + i\,y_{lmk})\,\varphi_l^{III}(r)\,\varphi_m^{II}(\xi)\,\varphi_k^{I}(t). \quad (2.32)$$

At the receiving point the signal corresponding to the channel model (2.32) is written as follows:

$$u(t, r) = \mathrm{Re}\sum_l \sum_m \sum_k (x_{lmk} - i\,y_{lmk})\,\dot{s}_m(t)\,\varphi_k^{I}(t)\,\varphi_l^{III}(r) =$$

$$= \sum_l \sum_m \sum_k [x_{lmk}s_m(t) + y_{lmk}\tilde{s}_m(t)]\,\varphi_l^{III}(r)\,\varphi_k(t). \quad (2.33)$$

Signal $s_t(t) = s_m(t) + i\tilde{s}_m(t)$ is determined by the convolution

$$\dot{s}_m(t) = \int_0^\infty \dot{s}(t - \xi)\,\varphi_m^{II}(\xi)\,d\xi. \quad (2.34)$$

According to model (2.33) the space-time signal at the receiving point is formed by a group of non-correlated signals transmitted by a set of antennas, subjected in the channel to frequency distortions in a filter with constant parameters, with characteristic $\phi^{II}{}_m(\xi)$, modulated according to the known law $\phi^{I}{}_k(t)$, and having received random amplitude multipliers and phase shifts.

As already observed above, many real communications channels with scattering may be considered uniform by frequency and space, at least in analysis. The correlation function of the pulsed surge characteristic, which depends on the angular coordinate, for such channels has the following form

$$B_h(t, t', \xi, \xi', \vartheta, \vartheta') = B_h^{I}(t, t')\,\delta(\xi - \xi')\,\delta(\vartheta - \vartheta').$$

It must be considered that the linear dimensions of any real antenna are limited [70], which is what determines its discrimination $\delta\vartheta$. For example, when a slot antenna of size R is used the discrimination $\delta\vartheta = 1/R$. This means that the process $h(t, \xi, \vartheta)$ accessible to observation by a real antenna according to variable ϑ (t, ξ parameters) may be considered the result of filtration of a Delta-correlated process by an ideal low-frequency filter with a pass band $[-1/2\delta, 1/2\delta]$.

The normed correlation function of such a process is written as follows (64).

$$R^{III}(\vartheta-\vartheta')=\frac{\sin[(\vartheta-\vartheta')/\delta\vartheta]}{(\vartheta-\vartheta')/\delta\vartheta}. \qquad (2.35)$$

The eigen functions and eigen numbers $\phi^{III}(\vartheta)$ and χ^{III} corresponding to (2.35) are determined from the equation

$$\varkappa^{III}\varphi^{III}(\vartheta)=\int_{-\theta}^{0}\varphi^{III}(\vartheta')\frac{\sin[(\vartheta-\vartheta')/\delta\vartheta]}{(\vartheta-\vartheta')/\delta\vartheta}d\vartheta'. \qquad (2.36)$$

We know [70, 113] that the eigen functions of integral equations (2.36) are extracted spheroidal functions which we will designate $\phi_p(\cdot)$, p = 1, 2,... Analogously, assuming that the field of the signal is analyzed after passing ideal low-frequency filter with boundary frequency F, it is possible to write an integral equation that corresponds to factorization of the channel characteristic by variable ξ(t and ϑ parameters):

$$\varkappa^{II}\varphi^{II}(\xi)=\int_{0}^{T}\varphi^{II}(\xi')\frac{\sin 2\pi F(\xi-\xi')}{2\pi F(\xi-\xi')}d\xi', \qquad (2.37)$$

which defines the eigen numbers χ^{II} and eigen functions $\phi^{II}(\xi)$, which are extracted spheroidal wave functions.

We know [113] that where the condition $\theta / \delta\vartheta \gg 1$ is met, an approximate solution of integral equation (2.36) is the aggregate of functions

$$\varphi_l^{III}(\vartheta)=\frac{\sin(\vartheta/\delta\vartheta-l\pi)}{\vartheta/\delta\vartheta-l\pi}, \quad l=1,2,\ldots \qquad (2.38)$$

Accordingly, where condition TF>>1 is met, an approximate solution to integral equation (2.37) is the functions

$$\varphi_m^{II}(\xi)=\frac{\sin(2\pi\xi F-m\pi)}{2\pi\xi F-m\pi}, \quad m=1,2,\ldots \qquad (2.39)$$

For the antennas and signals with large bases used in practice in channels with scattering, the conditions

$$\theta/\delta\vartheta\gg 1,\ TF\gg 1 \qquad (2.40)$$

can usually be considered to be met. This makes it possible to write the factorization of the pulsed surge characteristic of a channel with uniform fadeouts in space and by frequency in the following form

$$\eta(t,\xi,\vartheta)=\sum_l\sum_m\sum_k(x_{lmk}+i\,y_{lmk})\frac{\sin(\vartheta/\delta\vartheta-l\pi)}{\vartheta/\delta\vartheta-l\pi}\,\frac{\sin(2\pi\xi F-m\pi)}{2\pi\xi F-m\pi}\varphi^{I}(t). \qquad (2.41)$$

Correspondingly, the signal at the receiving point

$$u(t,\vartheta)=\mathrm{Re}\sum_l\sum_m\sum_k(x_{lmk}-i\,y_{lmk})\,\dot{s}\left(t-\frac{m}{F_c}\right)\varphi_k^{I}(t)\frac{\sin(\vartheta/\delta\vartheta-l\pi)}{\vartheta/\delta\vartheta-l\pi}=$$

$$=\sum_l\sum_m\sum_k\left[x_{lmk}s\left(t-\frac{m}{F_c}\right)+y_{lmk}\tilde{s}\left(t-\frac{m}{F_c}\right)\right]\frac{\sin(\vartheta/\delta\vartheta-l\pi)}{\vartheta/\delta\vartheta-l\pi}\varphi_k^{I}(t). \qquad (2.42)$$

The difference between model (2.42) and model (2.33) is in establishing the concrete type of "antenna" directivity diagram and replacement of constant-parameter filters with a delay line with uniformly distributed (with an interval of 1/F) branches. Let us emphasize that the individual paths by which the signal arrives in the reception area here continue to be non-correlated. We may note certain properties of expanding the characteristics of a channel with scattering into series according to the Karunen-Loew theorem. Work [107] shows that this factorization has the following two remarkable properties: 1) it minimizes the mean quadratic error caused by retaining just a finite number of terms in an infinite factorization series (2.18); 2) it gives a larger amount of information than any other factorization concerning a function represented by a truncated series, no matter how many members of the series (2.18) are retained.

These properties make the Karunen-Loew factorization a powerful tool for measuring the characteristics of channels with scattering. The fact that the coordinates of expansion into a series in this case prove non-correlated greatly simplifies further use and analysis of the results of measurement. This is why the Karunen-Loew factorization is often called optimal. However, expanding into a series according to the Karunen-Loew theorem also has two significant shortcomings: 1) it requires a large amount of a priori information (knowledge of the correlation function of the measured characteristic) which is often lacking or very unreliable; 2) the eigen functions of factorization of channel characteristics sometimes have highly complex structures and practical realization of them in the form of space-time filters with variable parameters proves complicated.

In practice, non-optimal expansion into series has become widespread in measuring realizations of random processes and fields. Let us look at some of these non-optimal factorizations.

Some non-optimal expansions of channel characteristics into series. A very large number of expansions of function h(t, ξ, r) into series in solving the problem of measuring the function may be suggested. Obviously, not all these factorizations will be close to the "optimal" Karunen-Loew factorizations in terms of properties. The loss of optimality here results from the rejection of certain a priori information on the channel, as a result of which the system of functions selected as a basis may prove very far from the set of eigen functions of integral equations (2.21). As an example of poor selection we may refer to the attempt to expand the pulsed surge characteristic of multibeam channel h(t, ξ, r) by variable ξ into a Taylor series. Because the eigen functions of expansion here are Delta functions, it is difficult to expect good convergence of the power series and high information content in the coordinate of factorization of function h(t, ξ, r). Analysis of signal processing schemes shows that contemporary practice has many examples of successful choice of a base for measuring the characteristics of a channel with scattering, choice is dictated by the intuition and experience of the developers.

Factorization according to the V. A. Kotel'nikov theorem [60] is important for finite functions, which characteristics of a channel may be considered during analysis. For example

$$\eta(t,\xi,\vartheta)=\sum_l\sum_m\sum_k(x_{lmk}+iy_{lmk})\frac{\sin(\vartheta/\delta\vartheta-l\pi)}{\vartheta/\delta\vartheta-l\pi}\,\frac{\sin(2\pi\xi F-m\pi)}{2\pi\xi F-m\pi}\times$$
$$\times\frac{\sin(2\pi tF-k\pi)}{2\pi tF-k\pi}. \qquad (2.43)$$

The quantities x_{lmk} and y_{lmk} are uniform readings of the function $h(t, \xi, \vartheta)$ on the axes corresponding to the variables t, ξ, ϑ.

According to (2.43) the signal at the reception place is written in this form

$$u(t,\vartheta)=\mathrm{Re}\sum_l\sum_m\sum_k(x_{lmk}-iy_{lmk})\dot{s}\left(t-\frac{m}{F}\right)\frac{\sin(\vartheta/\delta\vartheta-l\pi)}{\vartheta/\delta\vartheta-l\pi}\times$$
$$\times\frac{\sin(2\pi FT-k\pi)}{2\pi FT-k\pi}=\sum_l\sum_m\sum_k\left[x_{lmk}s\left(t-\frac{m}{F}\right)+y_{lmk}\tilde{s}\left(t-\frac{m}{F}\right)\right]\times$$
$$\times\frac{\sin(\vartheta/\delta\vartheta-l\pi)}{\vartheta/\delta\vartheta-l\pi}\,\frac{\sin(2\pi FT-k\pi)}{2\pi FT-k\pi}. \qquad (2.44)$$

The physical interpretation of model (2.44) is the same as that of model (2.42).

Now let us consider some expansions into a series of function $h(t, \xi, r)$ that take account of the physics of change in the function according to individual variables in real communications channels. Considering function $h(t, \xi, r)$ along one of the axes, for example along axis t, we will designate it $h(t, \cdot)$ and designate the coordinates of its expansion into a series $x_k(\cdot)$ and $y_k(\cdot)$ respectively.

The finite function $h(t, \cdot)$ can be expanded to a Fourier series in the segment [0, T] using the periodic continuation procedure:

$$\dot{h}(t,\cdot)=\sum_k[x_k(\cdot)+iy_k(\cdot)]\exp\left(i2\pi\frac{k}{T}t\right). \qquad (2.45)$$

This model interprets signal fadeouts in time as a consequence of Doppler shifts in the frequency of the transmitted signal (times 1/T) introduced by a channel with scattering.

The dynamic properties of a channel with scattering are characterized by the behavior of function $h(\xi, \cdot)$. We may identify at least three categories of mathematical models used to approximate the pulsed surge characteristic of a dynamic object with incomplete a priori information (in the absence of complete data on the statistical characteristics of the observed signals and functions under study):

1. Approximation of the function $h(\xi, \cdot)$ by a series of orthogonal functions in finite segment [0, T] which, according to available

a priori information, contains the most significant part of the function being approximated. Chebyshev polynomials, power and trigonometric functions [77], and Delta functions may be used as orthogonal functions here. The merit of this approach lies in the possibility of getting by with a comparatively small number of members of the series when the type of factorization functions is correctly chosen. This can only be done with a definite amount of a priori information.

2. Approximation of the function $h(\xi, \cdot)$ in the segment [0, T] by a power function using the Goodman-Reswick method [130]. In this case the segment [0, T] is broken into a series of intervals of length Δ. With this approach a large amount of a priori information is not required, but it is apparent that this involves a loss of optimality in factorization compared with the first type of factorization and factorization according to the Karunen-Loe theorem.

3. Representation of the pulsed surge characteristic using the "local approximation method" which combines the advantages of the first two forms and is a generalization of them. According to the local approximation method the pulsed surge characteristic $h(\xi, \cdot)$ is written in the form

$$h(\xi, \cdot) = \sum_{p=1}^{P} \sum_{m=1}^{M} h_{pm}(\cdot)\, \varphi_{pm}(\xi), \tag{2.46}$$

where

$$\varphi_{pm}(\xi) = \begin{cases} \varphi_m(\xi) & \text{for } \Delta p < \xi \leqslant \Delta(p-1), \\ 0 & \text{for } \xi < \Delta p,\ \xi > \Delta(p-1), \end{cases} \tag{2.47}$$

p is a whole number determined by the magnitudes of segment [0, T] and interval Δ

$$P = \left[\frac{T}{\Delta}\right], \tag{2.48}$$

$\phi_m(\xi)$, m = 1, 2, ...are functions that are orthogonal on a segment of magnitude Δ.

As can be seen from (2.46), segment [0, T] is uniformly filled by intervals of magnitude Δ, in each of which the function $h(\xi)$ is approximated by the sum of orthogonal functions in the given interval.

Where the channel model (2.46) is used, the signal at the place of reception is written in this form

$$u(t, r) = \operatorname{Re} \sum_{p=1}^{P} \sum_{m=1}^{M} \dot{h}_{pm}(t, r)\, \dot{s}_{pm}(t) = \sum_{p=1}^{P} \sum_{m=1}^{M} x_{pm}(t, r)\, s_{pm}(t) + y_{pm}(t, r)\, \tilde{s}_{pm}(t), \tag{2.49}$$

where $\dot{s}_{pm}(t) = s_{pm}(t) + i\tilde{s}_{pm}(t)$ is defined by the relation

$$\dot{s}_{pm}(t) = \int_{\Delta(p-1)}^{\Delta p} s(t-\xi)\, \varphi_{pm}(\xi)\, d\xi. \tag{2.50}$$

Where the Goodman-Reswick method is used, model (2.59) is simplified

$$u(t,\ r)=\mathrm{Re}\sum_{p=1}^{P}h_p(t,\ r)\dot{s}_p(t)=\sum_{p=1}^{P}x_p(t,\ r)s_p(t)+y_p(t,\ r)\widetilde{s_p}(t), \quad (2.51)$$

where, in conformity with (2.50)

$$s_p(t)=\int\limits_{\Delta(p-1)}^{\Delta p}s(t-\xi)\,d\xi. \quad (2.52)$$

The local approximation method may also be used to represent function h(t, •).

Analogously to (2.46) and using the same designation, we obtain

$$h(t,\ \cdot)=\sum_{k=1}^{K}\sum_{p=1}^{P}h_{kp}(\cdot)\,\varphi_{kp}(t). \quad (2.53)$$

Where the Goodman-Reswick method is used, model (2.53) changes into

$$h(t,\ \cdot)=\sum_{k=1}^{K}h_k(\cdot)\,L_k(t), \quad (2.54)$$

where

$$L_k(t)=\begin{cases}1 \text{ for } \Delta(k-1)\leqslant t\leqslant\Delta k,\\ 0 \text{ for } t<\Delta(k-1),\ t>\Delta k.\end{cases} \quad (2.55)$$

Factorization (2.54) plays an important role in the construction of contemporary signal processing devices in channels with scattering. It is apparent that the magnitude of interval Δ is ultimately determined by the interval of correlation τ_{cor} of function h(t, •). However, it cannot be stated that the choice $\Delta=\tau_{cor}$ is best; further, it is possible to show [104] that this is not true and that quantity Δ should be selected to be less than the interval of correlation τ_{cor}.

By combining the various factorizations given in this section it is possible to obtain a large number of notations for the pulse surge characteristic of a channel.

We will give just one of these representations, one that is very important in practice. By substituting a Kotel'nikov function (on the time axis) for the power functions (2.55) in factorization (2.43), we obtain

$$h(t,\ \xi,\ \vartheta)=\sum_{l}\sum_{m}\sum_{k=1}^{K}(x_{lmk}+i\,y_{lmk})\frac{\sin(\vartheta/\delta\vartheta-l\pi)}{\vartheta/\delta\vartheta-l\pi}\,\frac{\sin(2\pi F\xi-m\pi)}{2\pi F\xi-m\pi}\,L_k(t). \quad (2.56)$$

Correspondingly, the field at the place of reception is determined by the relation

$$u(t,\theta)=\sum_l\sum_m\sum_u (x_{lmk}-i\,y_{lmk})\dot{s}\left(t-\frac{m}{F}\right)\frac{\sin(\theta/\delta\theta-l\pi)}{\theta/\delta\theta-l\pi}L_k(t)=$$
$$=\sum_l\sum_m\sum_k\left[x_{lmk}s\left(t-\frac{m}{F}\right)+y_{lmk}\tilde{s}\left(t-\frac{m}{F}\right)\right]\frac{\sin(\theta/\delta\theta-l\pi)}{\theta/\delta\theta-l\pi}L_k(t). \quad (2.57)$$

Model (2.57) assumes power (step-wise) changes in the parameters of a channel in time.

The specific features of the objective of changing a channel characteristic, constructing an optimal signal receiver that uses the data of this measurement, may lead to representations (models) of the functions under study that are unusual from the standpoint of traditional measurements.

A striking illustration of the preceding statement is the representation of the pulsed surge characteristic in this form

$$h(t,\xi,r)=x_p\varphi_p(t,\xi,r)+i\,y_n\varphi_n(t,\xi,r),$$
$$\text{where}\quad |x_p|=\max_k|x_k|;\ |y_n|=\max_k|y_k| \quad (2.58)$$

and, correspondingly, representation at the place of reception in the form

$$u(t,r)=x_ps_p(t,r)+y_n\tilde{s}_n(t,r). \quad (2.59)$$

According to model (2.59), the channel is considered to have one path, and this single path considered by the model has maximum values for the quantities $|x_p|$ and $|y_p|$ among all the paths of propagation existing at the given moment in time and in the given realm of space. Let us note that in the general case this path can prove "hybrid" and contain coordinate p of factorization of a real component of the pulsed surge characteristic and coordinate n of an imaginary component.

Representation of a channel by two, three, and so on highly "weighted" paths of propagation may be considered an elaboration of model (2.59), which we will call adaptive because it changes in time and in space.

From the standpoint of traditional measurement, perhaps the most unusual looking channel model is the one that makes measurements themselves unnecessary. This is the model that ignores fluctuations in the channel characteristic and assumes that a stochastic channel is fully described by its regular component, that is, that factorization of the characteristic has the following form

$$h(t,\xi,r)=\sum_k(m_{xk}+i\,m_{yk})\varphi_k(t,\xi,r). \quad (2.60)$$

It will be shown in Chapter 4 that in many situations the use of an approximation of a real stochastic channel as a model of an ideal channel is fully justified and provides high noise suppression in receiving discrete messages, close to the highest possible.

In concluding our consideration of discrete models of a channel, we will touch on the question of the properties of mean statistical characteristics of coordinates of factorization.

2.3. Second-Order Statistics of the Coordinates of Factorization of Channel Characteristics

Let us suppose that (2.18) is a Karunen-Loew factorization of random function h(t, ξ, r) which has an average value $M_1\{h(t,\xi,r)\} = M_1\{x(t,\xi,r)\} + iM_1\{y(t,\xi,r)\}$ with the correlation functions of components

$$\left.\begin{aligned} B_x(t, t', \xi, \xi', r, r') &= \sigma_x^2(t, \xi, r) R_x(t, t', \xi, \xi', r, r'), \\ B_y(t, t', \xi, \xi', r, r') &= \sigma_y^2(t, \xi, r) R_y(t, t', \xi, \xi', r, r'), \end{aligned}\right\} \quad (2.61)$$

where $\sigma^2_{x(y)}(t, \xi, r) = B_{x(y)}(t, t, \xi, \xi, r, r)$ is the dispersion of the quadrature component.

In the case of a stationary homogeneous field,

$$\left.\begin{aligned} \sigma^2_{x(y)}(t, \xi, r) &= \sigma^2_{x(y)}, \\ R_{x(y)}(t, t', \xi, \xi', r, r') &= R_{x(y)}(t-t', \xi-\xi', r-r'). \end{aligned}\right\} \quad (2.62)$$

In what follows we will chiefly be considering homogeneous stationary fields. We will consider the quadrature components of channel characteristics to be non-coherent in the generalized sense, that is, non-correlated at coinciding moments in time, at identical frequencies, and at the same points in space. This is not a severe limitation on the statistical model being used because no assumption is made concerning the equality of dispersions of quadrature components (we assume asymmetry of dispersions $\sigma^2_x \neq \sigma^2_y$).

It is not difficult to compute the mathematical expectations of the coordinates of factorization (2.18):

$$\left.\begin{aligned} m_{xk} &= M_1\{x_k\} = \int_\Lambda M_1\{x(t, \xi, r)\} \varphi_k(t, \xi, r)\, dt\, d\xi\, dr, \\ m_{yk} &= M_1\{y_k\} = \int_\Lambda M_1\{y(t, \xi, r)\} \tilde{\varphi}_k(t, \xi, r)\, dt\, d\xi\, dr. \end{aligned}\right\} \quad (2.63)$$

For a stationary homogeneous field, owing to the orthonormality of the functions ϕ and $\tilde{\phi}$, it follows from (2.63) that

$$m_{xk} = m_x, \quad m_{yk} = m_y. \quad (2.64)$$

The dispersions of coordinates of factorization (2.18) are equal to the eigen values of corresponding integral equations

where $\{\chi_k\}$ and $\{\tilde{\chi}_k\}$ are sets of eigen values of the equations:

$$\sigma^2_{xk} = D\{x_k\} = \varkappa_k, \quad \sigma^2_{yk} = D\{y_k\} = \tilde{\varkappa}_k, \quad (2.65)$$

$$\left.\begin{aligned} \varkappa\varphi(t,\ \xi,\ r) &= \int\limits_{\Lambda} B_x(t,\ t',\ \xi,\ \xi',\ r,\ r')\,\varphi(t',\ \xi',\ r')\,dt'd\xi'dr', \\ \tilde{\varkappa}\,\tilde{\varphi}(t,\ \xi,\ r) &= \int\limits_{\Lambda} B_y(t,\ t',\ \xi,\ \xi',\ r,\ r')\,\tilde{\varphi}(t',\ \xi,\ r')\,dt'd\xi'dr' \end{aligned}\right\} \quad (2.66)$$

respectively.

It is not difficult to show that the coordinates of the Karunen-Loew factorization are mutually non-correlated, and in the case of Gaussian fields they are statistically independent.

The Karunen-Loew factorization has been most fully studied for random processes described by functions of one variable, time [18, 31, 44]. However, the theory has been elaborated for the case of functions of several variables [82, 83, 89]. It is relevant here to note several properties of the eigen functions and eigen values of integral equations of type (2.66), which are characteristic for functions of several variables.

The monotonicity of eigen numbers [82, 83]. We will number the eigen numbers of equation (2.66) in order of descending size $\varkappa_1 \geqslant \varkappa_2 \geqslant \ldots \geqslant 0$. Emphasizing the dependence of eigen numbers on area Λ, we will write $\chi_k(\Lambda)$. Then the eigen value $\chi_k(\Lambda)$ is a monotonically increasing function of the dimensions of area Λ, that is

$$\varkappa_k(\Lambda) < \varkappa_k(\Lambda'), \quad \Lambda \subset \Lambda'. \qquad (2.67)$$

greatest eigen number [18, 82]. Suppose $x(t, \vec{\theta})$ is a homogeneous stationary field with the following spectral density [82]:

$$\dot{S}_x(\vec{\omega}) = \int\limits_{-\infty}^{\infty} B_x(\vec{Q})\, e^{-i2\pi\vec{Q}\vec{\omega}}\, d\vec{Q}. \qquad (2.68)$$

The greatest eigen number $\chi_{max}(\Lambda)$ satisfies the inequality

$$\varkappa_{макс}(\Lambda) \leqslant \max_{\vec{\omega}} S_x(\vec{\omega}). \qquad (2.69)$$

Asymptotic behavior with expansion of area Λ. We will consider a stationary homogeneous field in the infinite area

$$\varkappa\varphi(t,\ \xi,\ r) = \iiint\limits_{-\infty}^{\infty} B_x(t-t',\ \xi-\xi',\ r-r')\,\varphi(t',\ \xi',\ r')\,dt'd\xi'dr'. \quad (2.70)$$

The solution to the integral equation may be written in the form

$$\dot{\varphi}(t,\ \xi,\ r) = \exp\{i\,\omega_t t + i\,\omega_\xi \xi + i\,\omega_r r\}. \qquad (2.71)$$

where $-\infty \omega_t,\ \omega_\xi,\ \omega_r \leqslant \infty$.

Thus, the composite exponential function is an eigen function of integral equations (2.70). Substituting (2.71) into (2.70), we calculate the value of the eigen number

$$\varkappa = \int_{-\infty}^{\infty} B_x(\vec{Q})\, e^{-i2\pi\vec{Q}\vec{\omega}}\, d\vec{Q} = \dot{S}_x(\vec{\omega}), \qquad (2.72)$$

that is, the eigen number for the given vector frequency is equal to the value of the spectral density at this frequency. We see that the set of eigen values and eigen functions have become uncountable, that is, instead of expansion to a series (2.18) we are in fact dealing with an integral. Considerations of convenience in using eigen functions (2.71) and the simplicity of computing eigen numbers (2.72) compel us to use them for approximate expansion to a series of the sample functions of homogeneous stationary random fields given in limited area:

$$-T/2 \leqslant t \leqslant T/2;\quad -T/2 \leqslant \xi \leqslant T/2;\quad -R/2 \leqslant r \leqslant R/2.$$

In this case, it is possible to select eigen functions from the relation

$$\varphi_k(t,\ \xi,\ r) = \exp\left\{ i\,2\pi \frac{n}{T} t + i\,2\pi \frac{m}{T} \xi + i\,2\pi \frac{l}{R} r \right\}, \qquad (2.73)$$

and eigen numbers from the relation

$$\varkappa_k = \dot{S}_x\left(\frac{n}{T};\ \frac{m}{T};\ \frac{l}{R} \right). \qquad (2.74)$$

In formulas (2.73) and (2.74) the multi-index k corresponds to the triple index mni. The intermediate case of limited areas for some variables and unlimited (in practice) for others, for example

$$-\infty \leqslant t \leqslant \infty;\quad -T/2 \leqslant \xi \leqslant T/2;\quad -R/2 \leqslant r \leqslant R/2,$$

does not produce fundamentally new results: the factorizations of functions are combined.

Factorization. To shorten the notation we will consider a random field of two variables x(u, v) with correlation function
If the function of two variables $B_x(\tau, \rho)$ may be written in the form of the products of two functions of one variable (factored)

$$B_x(\tau,\ \rho) = B_x^{I}(\tau)\, B_x^{II}(\rho), \qquad (2.75)$$

Then the eigen functions and eigen numbers are also factored:

where the eigen functions ϕ^{I} and the eigen numbers χ^{I} correspond to correlation function $B^{I}_x(\tau)$ and the functions ϕ^{II} and numbers χ^{II} correspond to $B^{II}_x(\rho)$.

$$\varphi_k(u,\ v) = \varphi_l^{I}(u)\, \varphi_n^{II}(v),\quad \varkappa_k^{II} = \varkappa_l^{I} \varkappa_n^{II}. \qquad (2.76)$$

The property of factorization has already been used above. It can be demonstrated by substituting (2.76) into an integral equation of the general type of (2.66). For illustration, let us consider an example with practical meaning. Suppose the correlation function of a field is described by the expression

$$B_x(u-u',\ v-v')=\sigma_x^2\exp\{-\alpha_u|u-u'|-\alpha_v|v-v'|\}. \qquad (2.77)$$

Corresponding to the correlation function (2.77) is spectral density

$$S(\omega_u,\ \omega_v)=\sigma_x^2\frac{2\alpha_u}{\omega_u^2+\alpha_u^2}\ \frac{2\alpha_v}{\omega_v^2+\alpha_v^2}. \qquad (2.78)$$

The integral equation which determines the eigen function has the following form

$$\int_{-U}^{U}\int_{-V}^{V}\sigma_x^2\exp\{-\alpha_u|u-u'|-\alpha_v|v-v'|\}\,\varphi(u',\ v')\,du'dv'=\varkappa\varphi(u,\ v). \qquad (2.79)$$

By direct substitution it is easy to check whether the eigen numbers and eigen functions factor:

$$\varphi_k(u,\ v)=\varphi_l^{I}(u)\,\varphi_n^{II}(v),\quad \varkappa_k=\varkappa_k^{I}\varkappa_n^{II}\sigma_x^2, \qquad (2.80)$$

where [18]

$$\varphi_l^{I}(u)=\begin{cases}\dfrac{\cos b_l u}{U^{1/2}\left(1+\dfrac{\sin^{*} b_l U}{2b_l U}\right)^{1/2}}, & l-\text{uneven}\\[2ex] \dfrac{\sin b_l u}{U^{1/2}\left(1-\dfrac{\sin b_l U}{b_l U}\right)^{1/2}}, & l-\text{even}\end{cases}\qquad -U<u\leqslant U, \qquad (2.81)$$

where the values of b_l are determined from the transcendant equation

$$\left(\operatorname{tg} bU+\frac{b}{\alpha_u}\right)\left(\operatorname{tg} bU-\frac{\alpha_u}{b}\right)=0 \qquad (2.82)$$

and determine the eigen numbers

$$\varkappa_l^{I}=2\alpha_u/(\alpha_u^2+b_l^2),\ l=1,\ 2,\ \ldots \qquad (2.83)$$

The eigen numbers χ^{II} and the eigen functions $\phi^{II}(v)$ corresponding to the second variable are computed analogously. We see that the eigen functions are sinusoidal, given on part of the plane, and their frequencies are not multiples of one another. The eigen number corresponding to eigen function k is equal to the value of the energy spectrum at the frequency of sinusoid k.

2.4. Measurement of Channel Characteristics Using Test Signals (Gaussian Field)

If during conventional measurement of processes all a priori information is a multidimensional distribution of the probabilities of the functions being measured or certain numerical characteristics of this distribution, such information will be inadequate for identification of the channel. It is also necessary to have information on the input action of the channel in order, by the response to it, to judge the parameters of the channel. We should consider three different classes of input actions, signals by which the channel is measured:

1. test signals (or probes);
2. information signals corresponding to transmission of discrete messages;
3. information signals corresponding to the transmission of continuous messages.

In the first case, a signal s(t) of known form is fed into the sequence of information signals at definite periods. This test signal does not carry information and is used to measure the parameters of a stochastic channel and organize the work of the receiving unit (adaptation). Input to the channel is a deterministic process.

In the second case, a sequence of signals of known form $s_1(t)$, $1 = \overline{1, M}$ arrives at the input of the channel. Each member of the sequence carries a coresponding discrete message and is used simultaneously to extract information about the state of the channel. Input to the channel should be treated as a quasideterministic process of the type

$$s(t) = \sum_{l=1}^{M} \delta_l s_l(t), \quad 0 < t < T, \tag{2.84}$$

where δ_1 is a random quantity that assumes two possible values $\delta_1 = (1, 0)$,

$$\left.\begin{aligned} \mathcal{P}\{\delta_l = 1\} &= p(\alpha_l), \\ \mathcal{P}\{\delta_l = 0\} &= 1 - p(\alpha_l), \end{aligned}\right\} \tag{2.85}$$

where $p(a_1)$ is the a priori probability of message 1.

Finally, in the third case the process at the input of the channel is stochastic, and in the general case non-stationary.

Because we are considering the transmission of discrete information, we do not analyze the measurement of channel parameters for a stochastic input signal. We will merely observe that the starting point for measurement of the pulsed surge characteristic of the channel in this case be the following relation [64]

$$B_{xy}(t, t') = \int_{-\infty}^{\infty} h(t', \xi) B_x(t, \xi) \, d\xi, \tag{2.86}$$

where $B_x(t, t')$ is the correlation function of the input signal and $B_{xy}(t, p')$ is the mutual correlation function of the input and output processes.

In this section we will consider the measurement of channel characteristics by test signals. We assume that we have available the full complement of a priori information concerning the functions being measured. First of all let us consider the case of the Gaussian statistics of a channel.

We will review the problem of measuring the coordinates of factorization of channel characteristics from the standpoint of estimating parameters. It is possible to observe the space-time field

$$z(t, r) = u(t, r) + n(t, r), \tag{2.87}$$

where n(t, r) is spatially distributed noise and

$$u(t, r) = \sum_{k=1}^{N} x_k s_k(t, r) + y_k \widetilde{s}_k(t, r). \tag{2.88}$$

The Bayes estimate of the coordinates of expansion for a symmetrical function of losses on the assumption of unimodality of the a posteriori density of the parameter being estimated and its symmetry relative to mode should, according to the general approach of [64], be computed from the relation

$$\hat{v}_r = \underbrace{\int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty}}_{2N} v_r\, w_{2N}[\vec{v} \mid z(t, r)]\, d\vec{v}, \tag{2.89}$$

where $\vec{v} = \{v_r\} = \{x_1, x_2, \ldots, x_N, y_1, y_2, \ldots, y_N\}$ is the vector of the parameters being measured.

The a posteriori density $w_{2N}[\vec{v} \mid z(t, r)]$ is computed from the relation

$$w_{2N}[\vec{v} \mid z(t, r)] = \frac{w_{2N}(\vec{v})\, l[z(t, r) \mid \vec{v}]}{\underbrace{\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty}}_{2N} w_{2N}(\vec{v})\, l[z(t, r) \mid \vec{v}]\, d\vec{v}}, \tag{2.90}$$

where $w_{2N}(\vec{v})$ is the a priori density and $l[z(t, r)\vec{v}]$ is the plausibility relation.

Suppose that the parameters being estimated (coordinates of expansion) together represent Gaussian quantities with multidimensional density

$$w_{2N}(\vec{v}) = \frac{1}{2\pi^{N} (\det K_v)^{1/2}} \exp\left[-\frac{1}{2}(\vec{v} - \vec{\bar{v}})^T K_v^{-1} (\vec{v} - \vec{\bar{v}})\right]. \tag{2.91}$$

If additive noise is also described by a model of a Gaussian random field with a zero mean and a correlation function $B_u(t, t', r, r')$, the plausibility relation may be written in the following form

$$l[z(t, r) \mid \vec{v}] = \exp\left\{\int_0^T \int_0^R \left[\sum_{k=1}^{N} x_k v_k(t, r) + y_k \widetilde{v}_k(t, r)\right] \times \right.$$
$$\left. \times \left[z(t, r) - \sum_{l=1}^{N} \frac{x_l}{2} s_l(t, r) + \frac{y_l}{2} \widetilde{s}_l(t, r)\right] dt\, dr\right\}. \tag{2.92}$$

In this $v_k(t, r)$ and $\tilde{v}_k(t, r)$ are determined from solving the spectral equation which will be discussed below. Substituting (2.91) and (2.92) into (2.90) and performing orthogonal transforms on the integration variables (rotating the axes of the coordinates in 2N-dimensional space), it is not difficult following [64] to obtain an expression for the Bayes estimate of the coordinate of expansion in the general case under consideration. This expression is not given here because of its cumbersomeness. We will only observe that the signal processing algorithm for calculating the estimate is non-linear and the estimate of the coordinate corresponding to path of propagation k is determined by solving a system of 2N algebraic equations. Owing to its complexity the optimal Bayes estimate can hardly be recommended for practical use in devices that measure a channel. To obtain a practically useful result it is advisable to introduce a constraint oriented to simplification of the estimate algorithm and to assume that the correlation properties of the test signal allow the paths to be separated at the receiving point

$$\sum_{i=1}^{N}\sum_{k=1}^{N}{}^{i\neq k}\, x_i\; x_k\int_0^T\int_0^R s_i(t,r)\,v_k(t,r)\,dt\,dr+y_i y_k\int_0^T\int_0^R \tilde{s}_i(t,r)\,\tilde{v}_k(t,r)\,dt\,dr+$$
$$+x_i y_k\int_0^T\int_0^R \tilde{s}_i(t,r)\,v_k(t,r)\,dt\,dr+y_i x_k\int_0^T\int_0^R s_i(t,r)\,\tilde{v}_k(t,r)\,dt\,dr=0. \quad (2.93)$$

In addition, we will assume that under conditions of selective fadeouts the orthogonality of the signal and its Hilbert conjugate is preserved

$$\int_0^T\int_0^R s_k(t,r)\,\tilde{v}_k(t,r)\,dt\,dr=\int_0^T\int_0^R \tilde{s}_k(t,r)\,v_k(t,r)\,dt\,dr=0. \quad (2.94)$$

It is advisable to replace conditions (2.93) and (2.94) with a more rigid but also clearer constructive condition

$$\int_0^T\int_0^R s_i(t,r)\,v_k(t,r)\,dt\,dr=\int_0^T\int_0^R \tilde{s}_i(t,r)\,\tilde{v}_k(t,r)\,dt\,dr=0, \quad (2.95)$$

which, for the sake of brevity, we will call the condition of complete separation. It can be said that equation (2.95) defines the class of signals that permit optimal measurement of the characteristics of a space-time stochastic channel. In fact, if the test signals used do not make it possible to satisfy (2.95), the optimal estimate algorithm is so complex that technical realization of the algorithms should be placed in doubt. Even in the case of inexact satisfaction of condition (2.95), it is advisable when constructing the estimate algorithm to ignore incomplete separation of paths. Below we will compute and analyze the characteristics of estimate algorithms where conditions (2.95) are not met.

Assuming that the condition of full separation of paths (2.95) is fulfilled, from (2.89) we receive an optimal Bayes estimate of the coordinate of expansion:

$$\left.\begin{aligned}\hat{x}_k &= \frac{\sigma_{xk}^2 \int\limits_0^T \int\limits_0^R z(t, r)\, v_k(t, r)\, dt\, dr + m_{xk}}{1 + \sigma_{xk}^2 \int\limits_0^T \int\limits_0^R s_k(t, r)\, v_k(t, r)\, dt\, dr},\\ \hat{y}_k &= \frac{\sigma_{yk}^2 \int\limits_0^T \int\limits_0^R z(t, r)\, \tilde{v}_k(t, r)\, dt\, dr + m_{yk}}{1 + \sigma_{yk}^2 \int\limits_0^T \int\limits_0^R \tilde{s}_k(t, r)\, \tilde{v}_k(t, r)\, dt\, dr}.\end{aligned}\right\} \quad (2.96)$$

We see that in the case of a separation of paths at the place or reception the Bayes estimates of Gaussian coordinates in a setting of Gaussian noise are linear.

Let us go into somewhat more detail concerning the quantities and function included in optimal estimates (2.96). The functions $v_k(t, r)$ and $\tilde{v}_k(t, r)$ are solutions to the integral equations:

$$\left.\begin{aligned}&\int\limits_0^T \int\limits_0^R B_n(t, t', r, r')\, v_k(t', r')\, dt'\, dr' = s_k(t, r),\\ &\int\limits_0^T \int\limits_0^R B_n(t, t', r, r')\, \tilde{v}_k(t', r')\, dt'\, dr' = \tilde{s}_k(t, r)\end{aligned}\right\} \quad (2.97)$$

and are determined by the channel model chosen and the correlation function of noise. In the case of a "white" noise field when

$$B_n(t, t', r, r') = \frac{N_0}{2}\, \delta(t - t')\, \delta(r - r'), \quad (2.98)$$

from (2.97) we obtain

$$v_k(t, r) = \frac{2}{N_0}\, s_k(t, r);\ \tilde{v}_k(t, r) = \frac{2}{N_0}\, \tilde{s}_k(t, r). \quad (2.99)$$

In general, the form of converted reference signals $v_k(t, r)$ is more complex than the form of initial signals $s_k(t, r)$; this is also true for Hillbert transforms. In fact, even if signal $s_k(t, r)$ allows separation of space and time variables

$$s_k(t, r) = s_k(t)\, \varphi_i^{II}(r), \quad (2.100)$$

signal $v_k(t, r)$ may not permit such factorization. For reference signal $v_k(t, r)$ to permit separation of space-time variables, in addition to fulfilling condition (2.100) it is sufficient that the correlation function of the noise field be spatially divisible $B_n(t, t', r, r') = B_n^{I}(t, t')B_n^{II}(r, r')$. In this case, to each of the integral components of (2.97 there are two corresponding integral equations

$$\left.\begin{aligned}&\int_0^T B_n'(t,t')\,v_k(t')\,dt' = s_k(t),\\&\int_0^R B_n''(r,r')\,\varphi_l''(r')\,dr' = \varphi_l''(r)\end{aligned}\right\} \quad (2.101)$$

and correspondingly $$v_k(t,r) = v_n(t)\,\varphi_l''(r). \quad (2.102)$$

The index k here corresponds to the double index nl. It is not difficult to show that for narrow-band transmitted signals

$$s_k(t,r) = A_k(t,r)\cos[\omega_0 t + \varphi_k(t,r)],$$

where $_k(t, r)$ and $\phi_k(t, r)$ are slowly changing functions and signals $v_k(t, r)$ and $\tilde{v}_k(t, r)$, just like signals $s_k(t, r)$ and $\tilde{s}_k(t, r)$ are related by a Hilbert transform according to variable t. Therefore, in particular, for the case of factorization of variables

$$\tilde{v}_k(t,r) = \tilde{v}_n(t)\,\varphi_l''(r). \quad (2.103)$$

From the point of view of simplifying realization of estimate algorithms, separating time and space variables is an exceptionally powerful factor which makes it possible to separate space and time processing. The questions of realizing algorithms for space-time processing are considered in greater detail in the next chapter.

A consideration of the case of small values of intervals of noise correlation in time $\tau_{cor.n}$ and in space $\rho_{cor.n}$ in comparison with the interval of analysis is of fundamental practical interest:

$$T/\tau_{cor.n} \gg 1, \quad R/\rho_{cor.n} \gg 1. \quad (2.104)$$

Fulfillment of conditions (2.104) makes it possible [22, 109] to change in (2.96) to infinite boundaries of integration and to write

$$\left.\begin{aligned}\hat{x}_k &= \frac{\sigma_{xk}^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} z(t,r)\,v_k(t,r)\,dt\,dr + m_{xk}}{1+\sigma_{xk}^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s_k(t,r)\,v_k(t,r)\,dt\,dr}\\ \hat{y}_k &= \frac{\sigma_{yk}^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} z(t,r)\,\tilde{v}_k(t,r)\,dt\,dr + m_{yk}}{1+\sigma_{yk}^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{s}_k(t,r)\,\tilde{v}_k(t,r)\,dt\,dr}\end{aligned}\right\} \quad (2.105)$$

According to Parseval's theorem [92, 109], the conditions are met

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s_k(t, r)\,v_k(t, r)\,dt\,dr = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{s}_k(t, r)\,\tilde{v}_k(t, r)\,dt\,dr. \quad (2.106)$$

In what follows we will consider conditions (2.106) approximately fulfilled with a finite domain of integration as well. Let us introduce the designation

$$d_k = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s_k(t, r)\,v_k(t, r)\,dt\,dr. \quad (2.107)$$

In the case of white noise field $d_k = \frac{2}{N_0}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s_k^2(t, r)\ dtdr$ determines the ratio of signal energy in path of propagation k to the spectral density of noise output. It is useful to introduce the parameters

$$2h_{xk}^2 = \sigma_{xk}^2 d_k, \quad 2h_{yk} = \sigma_{yk}^2 d_k, \quad (2.108)$$

which characterize the signal/noise ratio for orthogonal components.

It can be seen from (2.105) that the basic operation that must be performed to compute the estimates of the coordinate of expansion is computation of the mutual correlation between the field accessible to observation and the reference field. This operation can be accomplished by means of a coordinated space-time filter. Because such coordinated filters are considered in detail in Chapter 3, here we will only observe that if we are dealing with a filter that has characteristic

$$g_k(t, r) = v_k(T - t,\ R - r), \quad (2.109)$$

when fields z(t, r) are fed to its input, at the output at moment in time T with spatial interval of observation R it is possible to obtain the correlation function (see Figure 2.3 below)

$$I_k(T, R) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} z(t, r)\,v_k(t, r)\,dt\,dr. \quad (2.110)$$

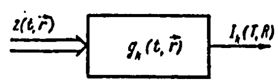

Figure 2.3. Space-Time Filter

This makes it possible to depict the estimator of the coordinate of expansion in the form shown in Figure 2.4 below

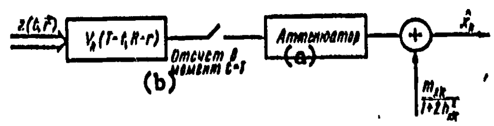

Figure 2.4. Estimator of the Coordinate of Expansion, Constructed on the Basis of a Coordinated Space-Time Filter

Key: (a) Attenuator;
(b) Reading at Moment t = T.

Let us move on to a consideration of a more general approach to measuring the coordinates of expansion of channel characteristics, setting aside the rigid assumption of the Gaussian character of the functions and noise being measured.

2.5. Linear Measurement of the Coordinates of Expansion of Channel Characteristics Using Test Signals

By assigning different laws of distribution of the coordinates of expansion of channel characteristics it is possible to receive a series of estimation algorithms from the class of Bayes algorithms. Each of them will be optimal within the framework of the probabilistic model corresponding to it. Practical use of Bayes algorithms for measurement of channel characteristics is hardly possible, however, for a number of reasons. In the first place, the probabilistic model of estimated functions and noise is usually not known exactly. Under these conditions, investigation of the stability of the Bayes algorithm in relation to the deviations of real probability distributions from the model become very important. And it may occur that with small deviations from the model the Bayes algorithm will have characteristics that are far from those expected. In the second place, as some examples show that when non-Gaussian parameters are being estimated, Bayes algorithms of estimation are extremely complex and require large amounts of a priori data. These causes make it necessary to base construction of estimation algorithms on a principle that differs from the one used with the Bayes approach. Specifically, simplicity of realization of the algorithm should be used as a quality criterion. Therefore, it is most sensible in this respect to speak of linear estimation algorithms. Moreover, in the Gaussian case Bayes estimations are linear. Inasmuch as many channels have probabilistic models of components that are close to Gaussian, as was demonstrated in the preceding chapter, it may be expected that linear estimation will prove close to optimal in a broad class of channels.

The linear method of estimation has received considerable attention lately [64, 66, 100, 117] and the use of nonlinear estimations in the tasks of processing space-time signals requires serious study, going beyond the framework of this book.

We will look for linear estimations of the coordinates of expansion of channel characteristics in the form

$$\left.\begin{aligned}\hat{x}_k &= A_k \int_0^T \int_0^R z(t, r)\,\psi_k(t, r)\,dt\,dr + B_k,\\ \hat{y}_k &= \tilde{A}_k \int_0^T \int_0^R z(t, r)\,\tilde{\psi}_k(t, r)\,dt\,dr + \tilde{B}_k.\end{aligned}\right\} \qquad (2.111)$$

To optimize estimations (2.111) we should find the optimal values of coefficients A_k and B_k (or $\tilde{A}_k$ and $\tilde{B}_k$) and the type of functions $\psi_k(t, r)$ (or $\tilde{\psi}_k(t, r)$), which furnish the extremum of the quality index. Under the conditions of the problem being solved here it is advisable to use the criterion of minimum average risk for a symmetrical loss function, that is, to look for these quantities and functions from the condition

$$\Pi(\hat{x}, x) = \overline{F(\hat{x} - x)}, \qquad (2.112)$$

where F is an even function and Π is the function of losses.

Appendix 1 shows that where there are imposed constraints (linear estimates for a symmetrical function of losses), average risk does not depend on the probabilistic laws of distribution of the estimated parameters and noise. The type of optimal linear estimate depends only on the first two statistical moments of the measured function and noise and is invariant with respect to their distribution laws. Therefore, the Bayes (linear) estimates obtained above, which are optimal within the framework of a Gaussian model, remain optimal in the class of linear estimates for any other probabilistic models of the estimated parameters and noise.

This line of reasoning presupposes meeting the condition of full separation of paths at the point of reception, for only in this case are the Bayes estimates of Gaussian coordinates linear. Using the methods of functional analysis, it may also be shown [90] that estimates

$$\left.\begin{aligned}\hat{x}_k &= \frac{\sigma_{xk}^2 \int_0^T \int_0^R z(t, r)\,v_k(t, r)\,dt\,dr + m_{xk}}{1 + 2h_{xk}^2}\\ \hat{y}_k &= \frac{\sigma_{yk}^2 \int_0^T \int_0^R z(t, r)\,\tilde{v}_k(t, r)\,dt\,dr + m_{yk}}{1 + 2h_{yk}^2}\end{aligned}\right\} \qquad (2.113)$$

are optimal in the class of all possible linear estimates even where the condition of full separation of paths (2.95) is not fulfilled.

Let us emphasize once again that the distinguishing feature of the estimates (2.113) is that they were obtained without any assumptions of a certain concrete law of distribution of the parameters and additive noise. This means that for optimal linear measurement of the pulsed surge characteristic of a channel with scattering it is sufficient to know its first two statistical moments and the correlation function of noise. It is difficult to overestimate the importance of this fact for the theory and practice of message transmission by channels with scattering.

This section has discussed the measurement of channel characteristics in isolation from the ultimate goal of measurement, which is to use estimates of the coordinates to organize the work of the receiving unit. However, the specific features of the ultimate goal have an inevitable influence on measurement algorithms. This is linked first of all to the rate of change of channel parameters in time. The estimate of characteristics must be organized in such a way that it is possible to use the information obtained by channel measurement during interval of time T, which is allocated for transmission of at least one information signal.

In general, measurement of a channel by means of test signals transmitted in series with information signals (parallel transmission of probing signals is considered below) makes sense only in channels with fadeouts that are nonselective (slow) in time when

$$\tau_{cor} \gg T. \qquad (2.114)$$

For a channel with slow fadeouts it is convenient to use representations based on the model of a delay line in time

$$h(\xi, r) = \sum_m [x(r) + i\,y(r)]\,\delta\left(\xi - \frac{m}{F}\right) \qquad (2.115)$$

and a model of the "delay line by frequency" (in angle-place coordinates).

$$H(f, \vartheta) = x(f, \vartheta) + i\,y(f, \vartheta) = \sum_m [x(\vartheta) + i\,y(\vartheta)]\,\delta\left(f - \frac{m}{T}\right). \qquad (2.116)$$

In the first case the pulsed surge characteristic is being measured; in the second it is the transfer function.

Let us consider linear measurement in the channel described by model (2.115), assuming that conditions (2.104) are met, making it possible to treat the space-time domain of observation as infinite. The signal in path of propagation k has the form

$$s_k(t, r) = s_k\left(t - \frac{m}{F}\right)\varphi_i^{H}(r), \qquad (2.117)$$

The reference field may be written in the form

$$v_k(t, r) = v_m(t)\,\varphi_i^{H}(r), \qquad (2.118)$$

and, in this way, spatial processing is separated from temporal. The reference signal $v_m(t)$ is determined by solving the integral equation

$$\int_{-\infty}^{\infty} B_n^l(t-t')\,v_m(t')\,dt' = s\left(t-\frac{m}{F}\right), \qquad (2.119)$$

which can be solved successfully in the frequency domain [22, 109]. The simplest way to realize the algorithm of processing in time is found in using a whitening filter with characteristic G(f) determined from equation

$$|G(f)|^2 N(f) = N_0/2, \qquad (2.120)$$

where N(f) is the energy spectrum of noise and $N_0/2$ is the spectral density of equivalent white noise output.

A schematic diagram of the estimator is given in Figure 2.5 below.

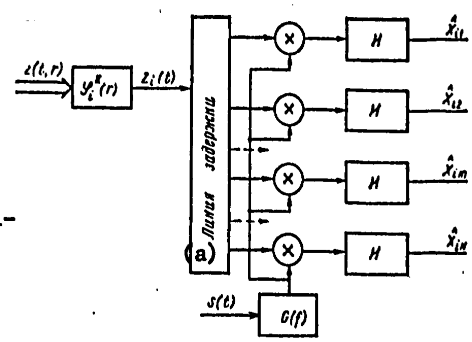

Figure 2.5. Coordinate Estimator Built on the Basis of a Delay Line.

Key: (a) Delay Line

Let us look in more detail at measuring readings of the transfer function of a channel. The dispersions of the quadrature components of a transfer function are determined by the expressions

$$\left.\begin{aligned} G_x(\omega, \omega_g) &= M_1\left\{\left[x(\omega, \omega_g) - \overline{x(\omega, \omega_g)}\right]^2\right\}, \\ G_y(\omega, \omega_g) &= M_1\left\{\left[y(\omega, \omega_g) - \overline{y(\omega, \omega_g)}\right]^2\right\}, \end{aligned}\right\} \qquad (2.121)$$

where $\overline{x(\omega, \omega_g)}$ and $\overline{y(\omega, \omega_g)}$ are average values of the quadrature components.

It is possible to obtain expressions for the optimal linear estimates and readings of the transfer function by operating in the same way as was done to obtain estimates (2.113). Adopting, for the sake of brevity, the designations $x(\omega_m, \omega_{gp}) = x(\omega, \omega_g)$ and $y(\omega_m, \omega_{gp}) = y(\omega, \omega_g)$, we write these estimates as follows:

$$\hat{x}(\omega, \omega_g) = \frac{\mathrm{Re}[\dot{S}(\omega)]\,\mathrm{Re}[\dot{Z}(\omega, \omega_g)] + \mathrm{Im}[\dot{S}(\omega)]\,\mathrm{Im}[\dot{Z}(\omega, \omega_g)]}{|\dot{S}(\omega)|^2 + \dfrac{N(\omega, \omega_g)}{G_x(\omega, \omega_g)}} + \frac{\overline{x(\omega, \omega_g)}}{1 + \dfrac{G_x(\omega, \omega_g)}{N(\omega, \omega_g)}|\dot{S}(\omega)|^2},$$

$$\left.\begin{aligned}\overset{*}{y}(\omega, \omega_g) &= \frac{\operatorname{Re}[\dot S(\omega)]\operatorname{Im}[\dot Z(\omega, \omega_g)] + \operatorname{Im}[\dot S(\omega)]\operatorname{Re}[\dot Z(\omega, \omega_g)]}{|\dot S(\omega)|^2 + \frac{N(\omega, \omega_g)}{G_y(\omega, \omega_g)}} + \\ &+ \frac{\overline{y(\omega, \omega_g)}}{1 + \frac{G_y(\omega, \omega_g)}{N(\omega, \omega_g)}|\dot S(\omega)|^2};\end{aligned}\right\} \quad (2.122)$$

As was observed in the preceding chapter, the dispersions of quadrature components of a transfer function in a homogeneous channel are identical: $G_x(\omega, \omega_g) = G_y(\omega, \omega_g) = G(\omega, \omega_g)$. In this case, combining the two equations included in (2.122), we obtain an expression for the optimal linear estimate of readings of the transfer function of a channel

$$\hat H(\omega, \omega_g) = \frac{1}{1 + \frac{N(\omega, \omega_g)}{G(\omega, \omega_g)|\dot S(\omega)|^2}} \cdot \frac{\dot Z(\omega, \omega_g)}{\dot S(\omega)} + \frac{\overline{H(\omega, \omega_g)}}{1 + \frac{G(\omega, \omega_g)|\dot S(\omega)|^2}{N(\omega, \omega_g)}}, \quad (2.123)$$

where $\overline{H(\omega, \omega_g)} = \overline{x(\omega_g)} + i\overline{y}(\omega, \omega_g)$ is the average value of the transfer function of the channel.

If the average value of the transfer function $\overline{H(\omega, \omega_g)} = 0$ it follows from (2.123) that

$$\hat H(\omega, \omega_g) = \frac{1}{1 + \frac{N(\omega, \omega_g)}{G(\omega, \omega_g)|\dot S(\omega)|^2}} \frac{\dot Z(\omega, \omega_g)}{\dot S(\omega)}. \quad (2.124)$$

Comparing (2.124) and (2.7) we see how the function and parameter of regularization were chosen for a linear estimate of the transfer function of a channel.

Expressions (2.123) and (2.124) permit a clear physical interpretation. The optimal linear estimate of the reading of a transfer function in each frequency of a time-space spectrum is the sum of the weighted a priori average $\overline{H(\omega, \omega_g)}$ and the weighted "classical" transfer function computed by observed data $Z(\omega, \omega_g)/S(\omega)$. In those spectral components where the average intensity of noise compared to the average intensity of the usable component of the observed field is large $N(\omega, \omega_g)/[G(\omega, \omega_g)|\dot S(\omega)|^2] \gg 1$, its a priori average is adopted as the estimate of the transfer function; in the opposite case where $N(\omega, \omega_g)/[G(\omega, \omega_g)|\dot S(\omega)|^2] \ll 1$ the "classical" solution computed for the observed realization, the ratio of the spectra of the input and output signals, is used as the estimate of the transfer function. If the noise intensity is large and the average value of the transfer function is small, one more physical interpretation can be given to estimates (2.123) and (2.124). Assuming that $N(\omega, \omega_g)/G(\omega, \omega_g)|\dot S(\omega)|^2 \gg 1$ in (2.123) we obtain

$$\hat H(\omega, \omega_g) \approx \frac{G(\omega, \omega_g)}{N(\omega, \omega_g)} \dot Z(\omega, \omega_g)\dot S^*(\omega) + \overline{H(\omega, \omega_g)}. \quad (2.125)$$

Entry (2.125) allows us to say that an optimal estimate of the transfer function can be obtained at the output of a linear space-time filter coordinated with the signal being transmitted.

In conclusion, let us consider characteristics of the estimates obtained.

Characteristics of optimal linear estimates. Unconditional average estimates and estimate dispersions (2.113) can be obtained easily in the form

$$\left.\begin{aligned}
M_1\{\hat{x}_h\} &= m_{xh} + \frac{1}{1+2h_{xk}^2}\sum_{p=1}^{N\,(p\neq k)} 2h_{xp}^2 m_{xp},\\
D\{\hat{x}_h\} &= \sigma_{xk}^2\left[\frac{2h_{xk}^2}{1+2h_{xk}^2} + \sum_{p=1}^{N\,(p\neq k)} \frac{2h_{xkp}^2\left(1+2h_{xkp}^2\right)}{\left(1+2h_{xk}^2\right)^2}\right],\\
M_1\{\hat{y}_h\} &= m_{yk} + \frac{1}{1+2h_{yk}^2}\sum_{p=1}^{N\,(p\neq k)} 2h^2_{ykp} m_{yp},\\
D\{\hat{y}_h\} &= \sigma_{yk}^2\left[\frac{2h_{yk}^2}{1+2h_{yk}^2} + \sum_{p=1}^{N\,(p\neq k)} \frac{2h_{ykp}^2\left(1+2h_{ykp}^2\right)}{\left(1+2h_{yk}^2\right)^2}\right].
\end{aligned}\right\} \qquad (2.126)$$

In (2.126) the following designations are introduced.

$$\left.\begin{aligned}
2h_{x\,kp}^2 &= d_{hp}\sigma_{xp}^2, \quad 2h_{ykp}^2 = d_{hp}\sigma_{yp}^2,\\
d_{hp} &= \int_0^T\int_0^R s_h(t,r)\,v_p(t,r)\,dt\,dr.
\end{aligned}\right\} \qquad (2.127)$$

Analysis (2.126) shows that where the condition of full separation of paths at the receiving point is met, the optimal linear estimates are unbiased; the dispersions of estimates tend asymptotically (where h^2_{xk}, $h^2_{yk}\to\infty$) toward the dispersions of the estimated quantities. In this case, if the result of [64] is used the conditional dispersions of estimates

$$D\{\hat{x}_h \mid x_h\}, \quad D\{\hat{y}_h \mid y_h\}$$

tend to zero. Where the conditions of complete separation of paths are not met, estimates (2.126) are biased and their dispersions do not tend toward the dispersions of the estimated quantities independently of the signal/noise ratio (the quantities h^2_{xk} and h^2_{yk}) and the conditional dispersions of estimates do not tend to zero.

As an example, let us consider the case $m_{xk} = m_x$; $\sigma^2_{xk} = \sigma^2_x$, $k = \overline{1, N}$. We assume that the quantities d_{kp} diminish as members of a geometric equation with index $q<1$ where $d_h = d$; $d_{fp} = dq^p$; $p = \overline{1, N}$.

From (2.126) we obtain an expression for the bias (displacement)

$$\left.\begin{aligned} \Delta_k^x &= \frac{2 m_x d \sigma_x^2}{1+2\sigma_x^2 d}\,\frac{1-q^N}{1-q},\\ \Delta_k^x &= \frac{2 m_x d \sigma_x^2}{1+2\sigma_x^2 d}\,\frac{q}{1-q},\quad N=\infty \end{aligned}\right\} \qquad (2.128)$$

and dispersions of the estimate

$$\left.\begin{aligned} D\{x_h\} &= \sigma_x^2\left[\frac{2\sigma_x^2 d}{1+2\sigma_x^2 d} + \frac{2\sigma_x^2 d}{(1+2\sigma_x^2 d)^2}\,\frac{1-q^N}{1-q} + \right.\\ &\left. + \frac{4\sigma_x^4 d^2}{(1+2\sigma_x^2 d)^2}\cdot\frac{1-q^{2N}}{1-q^2}\right],\\ D\{x_h\} &= \sigma_x^2\left[\frac{2\sigma_x^2 d}{1+2\sigma_x^2 d} + \frac{2\sigma_x^2 d}{(1+2\sigma_x^2 d)^2}\,\frac{q}{1-q} + \right.\\ &\left. + \frac{4\sigma_x^4 d^2}{(1+2\sigma_x^2 d)^2}\,\frac{q^2}{1-q^2}\right],\quad N=\infty \end{aligned}\right\} \qquad (2.129)$$

Where the signal/noise ratio increases ($d \to \infty$), from (2.128) and (2.129) we obtain (for $N \neq \infty$)

$$\Delta_k^x = \frac{1-q^N}{1-q}\, m_x,\quad D\{\hat{x}_h\} = \sigma_x^2\left[1 + \frac{1-q^{2N}}{1-q^2}\right]. \qquad (2.129)*$$

from which it can be seen that with poor separation of the paths at the place of reception, where the quantity q differs substantially from zero, the bias of the estimates may reach large quantities and the dispersion of estimates may be very far from the dispersion of the estimated quantity.

Characteristics of estimates of the transfer function. It is easy to see that estimates of readings of the transfer function (2.123) are unbiased. The unconditional dispersion of the estimate of the transfer function at a certain frequency $\omega = 2\pi f$ is easily computed and has the form

$$D\{\hat{H}(\omega, \omega_g)\} = G(\omega, \omega_g)\,\frac{[G(\omega, \omega_g)\,|\dot{S}(\omega)|^2]/N(\omega, \omega_g)}{1+[G(\omega, \omega_g)\,|\dot{S}(\omega)|^2]/N(\omega, \omega_g)}. \qquad (2.130)$$

Accordingly, the conditional dispersion is determined by the formula

$$D\{\hat{H}(\omega, \omega_g)\,|\,\dot{H}(\omega, \omega_g)\}\,\frac{N(\omega, \omega_g)}{|S(\omega)|^2 + \frac{N(\omega, \omega_g)}{G(\omega, \omega_g)}}. \qquad (2.131)$$

It can be seen from (2.131) that for those components of the space-time spectrum where the average intensity of the usable signal significantly

* [The misnumbering -- duplication of numbers -- is in the Russian text.]

exceeds the average intensity of noise $G(\omega, \omega_g)|\dot{S}(\omega)|^2/N(\omega, \omega_g) \gg 1$, the dispersions of estimates of readings of the transfer function are close to the dispersions of the quantities being estimated.

Now let us look at the integral characteristic of estimate quality, the mean quadratic error of estimation of the transfer function as a whole. If N^F readings of the transfer function are taken on axis ω and N^θ are taken on axis ω_g, then (average error output) the measurements are determined from the relation.

$$\overline{\varepsilon^2} = \frac{1}{N^F N_k^0} \sum_{k=-\frac{N^F}{2}}^{\frac{N^F}{2}} \sum_{p=-\frac{N^0}{2}}^{\frac{N^0}{2}} \frac{N(\omega_k, \omega_{gp})/|S(\omega_k)|^2}{1 + [N(\omega_k, \omega_{gp})/|S(\omega_k)|^2 G(\omega_k, \omega_{gp})]}. \quad (2.132)$$

The technique of "indirect" measurement of readings of the pulsed surge characteristic, through readings of the transfer function using the discrete Fourier transform, is very widespread. In recent times the Cooley-Taki [93] algorithm, which has received the name quick Fourier transform, has been used to compute the discrete Fourier transform. Computing the estimates of N readings $h(\xi_p, \omega_{gi})$ through estimates of function readings $H(\omega_k, \omega_{gi})$, using the discrete Fourier transform it is possible to perform the computation according to the formula

$$\hat{h}(\xi_p, \omega_{gi}) = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}} \hat{H}(\omega_k, \omega_{gi}) e^{-i2\pi\frac{p}{N}k}. \quad (2.133)$$

It is assumed here that readings on the axis of frequencies are arranged uniformly with interval $\Delta f = 1/2T$, while on the time axis they have interval $\Delta t = 1/2F$, in other words $\xi_p = p\Delta t$, $p = \overline{-N/2, N/2}$; $\omega_{gk} = 2\pi k\Delta f$, $k = \overline{-N/2, N/2}$.

With this method of measuring readings of the surge characteristic, they all, as can be seen from (2.133), have the same dispersions

$$D\{\hat{h}(\xi_p, \omega_{gi})\} = \sum_{k=-N/2}^{N/2} D\{\hat{H}(\omega_k, \omega_{gi})\}. \quad (2.134)$$

It follows from (2.134) that for the dispersion of estimates of readings of the surge characteristic calculated from estimates of readings of the transfer function, the following relation is fulfilled.

$$\inf_{-N/2 \leq p \leq N/2} D\{\hat{h}(\xi_p, \omega_{gi})\} > \sup_{-N/2 \leq k \leq N/2} D\{\hat{H}(\omega_k, \omega_{gi})\}. \quad (2.135)$$

Inequality (2.135) says that computation of estimates of readings of one system function of the channel through estimates of readings of the other is far from the best way to construct estimates.

In completing our consideration of linear estimates of coordinates by means of test signals, let us briefly consider the questions of measuring channel characteristics in conditions of a priori uncertainty and the associated questions of measuring the mean statistical parameters of a channel.

2.6. Incomplete A Priori Information and Measurement of the Mean Statistical Parameters of a Channel

Widespread practical introduction of the results of information transmission theory is greatly retarded by what is known as a priori uncertainty [64]. It involves the lack of complete a priori information on the processes and noise being estimated. In the case of linear measurement of channel characteristics the a priori uncertainty finds expression in lack of full information on the first two statistical moments of the functions and noise being measured, which is required to construct optimal algorithms for linear estimation (2.113). The problem of determining the average values and correlation functions of characteristics is solved theoretically and experimentally for many real channels [32, 80, 135]. It is relevant here to consider certain aspects of this problem that control the specific features of channel measurements.

Let us look first at estimates of mathematical expectation. For the sake of determinacy we will speak of measuring the average value of the transfer function. When oscillation $e^{i2\pi ft}$ is fed to the input of the channel, we have at the output a usable signal in the form

$$\dot{u}(f, t, r) = \dot{H}(f, t, r)\, e^{i2\pi ft}. \qquad (2.136)$$

We will assume that the degree of coherence of the oscillation emitted is high and that by synchronous heterodyning it is possible to isolate the transfer function H(f, t, r) itself in the receiving end, with the inevitable additive noise, of course, so that the observed field has the form

$$\dot{z}(f, t, r) = \dot{H}(f, t, r) + \dot{n}(f, t, r). \qquad (2.137)$$

To estimate the average value of the transfer function it is necessary to assume that in a certain limited domain of the space-time plane it is described by a model of a homogeneous stationary field, for example in rectangle $0 \leq t \leq T^0,\ 0 \leq r \leq R^0$. The general expression for linear estimation of the mathematical expectation of a field is written in the form

$$\overline{\dot{H}(f, t, r)} = \int_0^{T^0} \int_0^{R^0} \dot{z}(f, t, r)\, a(t, r)\, dt\, dr, \qquad (2.138)$$

where the weight function satisfies the condition

$$\int_0^{T^0} \int_0^{R^0} a(t, r)\, dt\, dr = 1. \qquad (2.139)$$

Work [20] shows that in the presence of noise it is advisable to choose a weight function that differs from the constant, that is, mean arithmetic estimates should be used

$$\overline{H(f, t, r)} = \frac{1}{T^0 R^0} \int_0^{T^0} \int_0^{R^0} Z(f, t, r)\, dt dr. \qquad (2.140)$$

Estimate (2.140) is unbiased. The dispersion of the estimate is computed from the relation

$$D\{\overline{H}\} = \frac{1}{(T^0 R^0)^2} \int_0^{T^0} \int_0^{R^0} \int_0^{T^0} \int_0^{R^0} [B_H(t-t', r-r') + B_n(t-t', r-r')]\, dt dt' dr dr'. \qquad (2.141)$$

Let us omit parameter f in formula (2.141). It is more convenient to calculate the dispersion in the frequency domain [20] from the relation

$$D\{\overline{H}\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [G_H(\omega_t, \omega_g) + G_n(\omega_t, \omega_g)]\, |\Phi(\omega_t, \omega_g)|^2 d\omega_t d\omega_g, \qquad (2.142)$$

where

$$\Phi(\omega_t, \omega_g) = \int_0^{T^0} \int_0^{R^0} e^{i\omega_t t + i\omega_g r}\, dt dr. \qquad (2.143)$$

As an example, let us consider a transfer function whose spectral density is bilinear:

$$\omega_t = 2\pi\nu.$$

$$G_H(\omega_t, \omega_g) = \sigma_H^2 \frac{\alpha_g \alpha_t}{\pi^2 (\omega_g^2 + \alpha_g^2)(\omega_t^2 + \alpha_t^2)}. \qquad (2.144)$$

We assume white noise with a spectral density

$$G_n(\omega_t, \omega_g) = \begin{cases} G_n, & -\Omega_g \leq \omega_g \leq \Omega_g, \\ & -2\pi F \leq \omega_t \leq 2\pi F, \\ 0, & \text{— outside this domain} \end{cases} \qquad (2.145)$$

Let us observe that quantities G_n and σ^2_H are dimensionless. Substituting (2.144) and (2.145) in (2.143), it is not difficult to obtain an expression for the dispersion of the estimate

$$D\{\overline{H}\} = \frac{4\sigma_H^2}{(2 + \alpha_g R^0)(2 + \alpha_t T^0)} + \frac{G_n}{2} \frac{1}{R^0 \Omega_g} \frac{1}{T^0 F}. \qquad (2.146)$$

It can be seen from (2.146) that the dispersion of the estimate of the average value of the field contains two components. One of them is determined by the characteristics of the field for which the estimate is constructed and the smaller it is, the more uncorrelated values of the function being averaged will be contained in the domain of measurement. The other component is determined by the noise statistics.

For many channels the intervals of correlation of noise in time and in space are considerably smaller than the corresponding intervals of the usable signal

$$\Omega_r \ll \alpha_g, \; F \ll \alpha_t. \qquad (2.147)$$

In this case, if the average noise output is not excessively high compared to the average output of usable signal, the dispersion of the estimate of the average value of the transfer function will be determined basically by the characteristics of the field that describes the transfer function.

Let us observe that it is possible to construct an estimate of the mathematical expectation of the field using only the spatial variable or the time variable.

$$\overline{\dot H(f,\, t,\, r)} = \frac{1}{T^{01}} \int\limits_0^{T^{01}} \dot Z(f,\, t,\, r)\, dt, \quad \overline{\dot H(f,\, t,\, r)} = \frac{1}{R^{01}} \int\limits_0^{R^{01}} \dot Z(f,\, t,\, r)\, dr. \qquad (2.148)$$

In order to maintain the quality of the estimate, it is necessary that the number of uncorrelated values of the function contained in segment $[0,\ T^{01}]$ and $[0,\ R^{01}]$ (2.148) is the same as in rectangle $[0,\ T^0]$ and $[0,\ R^0]$ (2.140).

Now let us consider the characteristics of measuring the average values of the coordinates of expansion of channel characteristics. Beginning from the relation

$$\overline{z(t,\, r)} = \int\limits_0^{\infty} \overline{h(t,\, \xi,\, r)}\, s(t,\, \xi)\, d\xi \qquad (2.149)$$

and using factorization of channel characteristics we arrive at the equation that interrelates the average value of the observed signal with the average values of the coordinates of expansion

$$\overline{z(t,\, r)} = \sum_{k=1}^{N} \bar h_k s_k(t,\, r). \qquad (2.150)$$

If the average value $\overline{Z(t,\ r)}$ is known, it is simplest to solve equation (2.150) in the case where the signals in certain paths of propagation (separation of paths) are orthogonal. In this case the estimate of the average value of coordinate of expansion k can be computed from the equation

$$\bar h_k = \frac{1}{d_k} \int\limits_0^T \int\limits_0^R \overline{z(t,\, r)}\, s_k(t,\, r)\, dt dr. \qquad (2.151)$$

It is simplest to obtain the estimate of the average value of the signal being received by repeating transmission of the test signal P times during an interval that does not exceed the interval of a stationary state

$$PT < T^0, \qquad (2.152)$$

from which we obtain the upper boundary for the number of repetitions

$$P < [T^0/T + 1]. \qquad (2.153)$$

The algorithm for obtaining the estimate of the average value of the coordinate of expansion is written in the following form

$$\overline{h}_k = \frac{1}{P} \sum_{p=1}^{P} \int_0^R \int_{(P-1)T}^{PT} z(t, r) s_k(t, r)\, dt\, dr. \qquad (2.154)$$

A great advantage of algorithm (2.154) is that it is realized using a space-time coordinated filter designed to obtain estimates of the coordinates of expansion of channel characteristics. This means that no additional apparatus is needed to study the average value of the channel. Furthermore, information on the average values of characteristics can be refined in the process of data transmission, using test signals designed for constructing ongoing estimates.

The problem of correlation analysis of random fields is very common in many different applications [35]. There is a whole series of estimation algorithms based primarily on spectral representations of observed oscillations. Here we will deal with another approach to measuring correlation characteristics, one that is more appropriate to the specific features of the problems under consideration. For convenience we will not consider the average value of the signal, assuming it hypothetically to be equal to 0. If we begin from the expansion

$$h(t, \xi, r) = \sum_{k=1}^{N} h_k \varphi_k(t, \xi, r), \qquad (2.155)$$

it is possible to approximate the estimate of the correlation function with the expression

$$\hat{B}_k(t-t', \xi-\xi', r-r') = \overline{[h(t, \xi, r) h^*(t', \xi', r')]} = \sum_{k=1}^{N} \sum_{g=1}^{N} \overline{(h_k h_g^*)} \varphi_k(t, \xi, r) \varphi_g(t', \xi', r'). \qquad (2.156)$$

The mutual correlation of coordinates of expansion can be computed by repeating transmission of the test signal just as is done during estimation of the average values of coordinates. The algorithm for computing correlation has this form

$$\overline{(h_k h_g^*)} = \frac{1}{P}\sum_{p=1}^{P}\left[\int_0^R \int_{(p-1)T}^{pT} \dot z(t, r)\,\dot s_k(t, r)\,dt\,dr\right] \times \times \left[\int_0^R \int_{(p-1)T}^{pT} \dot z(t, r)\,\dot s_p(t, r)\,dt\,dr\right]^*. \quad (2.157)$$

If function $\phi_k(t, \xi, r)$ is well chosen, then series (2.156) converges uniformly to $B_h(t, t', \xi, \xi', r, r')$ where $N \to \infty$. Of course, the choice will be more "successful" if the correlation function is known and function $\phi_k(t, \xi, r)$ is the eigen functions of the integral equation. In this case, Mercer's theorem [18] guarantees uniform convergence, but the measurements themselves make no sense. It is possible to suggest sequential refinement of the estimates of the correlation function. Initially choosing functions $\phi'_k(t, \xi, r)$ from some a priori assumptions, we construct estimate $B'_h(t, t', \xi, \xi', r, r')$ in conformity with (2.155) and (2.156). Then we approximate the estimate of one of the typical correlation functions, for which it is advisable to use functions to which bilinear spectra correspond. After determining the eigen functions corresponding to the approximated correlation function, we use them to refine the estimates (2.157). It is advisable here to use the algorithm of stochastic approximation [64] and in step (n = 1) make an estimate of the coefficient of correlation $\hat\rho_{kg}^{n+1}$ from the equation

$$\hat\rho_{kg}^{n+1} = \hat\rho_{kg}^{n} + a_n\left[\overline{(h_k h_g^*)_n} - \hat\rho_{kg}^{n}\right], \quad (2.158)$$

where $\overline{(h_k h^*_x)}$ is determined by (2.157) with functions $\phi^N_k(t, \xi, r)$ corresponding to step n.

Coefficients a_n satisfy the conditions

$$\sum_{n=1}^{\infty} a_n = \infty; \quad \sum_{n=1}^{\infty} a_n^2 < \infty. \quad (2.159)$$

Use of this procedure guarantees convergence of the estimates, but practical realization of the algorithm is complicated.

Linear estimates (2.113) have an unquestionable advantage with respect to the required amount of a priori information on the correlation function. To construct linear estimates it is sufficient to know the energy coefficients of transmission of different paths of propagation which, as follows from (2.157), can be computed from the relations

$$\hat{\sigma}^2_{hk} = \frac{1}{P}\sum_{p=1}^{P}\left|\int_0^R \int_{(p-1)T}^{pT} z(t, r)\dot{s}_k(t, r)\,dt\,dr\right|^2. \qquad (2.160)$$

Estimates of the dispersions of the quadrature components are determined accordingly by expression

$$\left.\begin{aligned}\hat{\sigma}^2_{xk} &= \frac{1}{P}\sum_{p=1}^{P}\left[\int_0^R \int_{(p-1)T}^{pT} z(t, r)\,s_k(t, r)\,dt\,dr\right]^2,\\ \hat{\sigma}^2_{yk} &= \frac{1}{P}\sum_{p=1}^{P}\left[\int_0^R \int_{(p-1)T}^{pT} z(t, r)\,\tilde{s}_k(t, r)\,dt\,dr\right]^2.\end{aligned}\right\} \qquad (2.161)$$

Estimates (2.157), (2.160), and (2.161), like the estimates of average values of coordinates, can be computed and refined during the process of information transmission without additional apparatus.

If the mathematical expectations and dispersions of transmission coefficients are not known at the moment that data transmission begins, it is advisable in constructing estimates to use the criterion of maximum plausibility. In this case it is not difficult to obtain estimates of the coordinates in the form

$$\left.\begin{aligned}\hat{x}_k^{\text{МП}} &= \frac{1}{d_k}\int_0^T\int_0^R z(t, r)\,v_k(t, r)\,dt\,dr,\\ \hat{y}_k^{\text{МП}} &= \frac{1}{d_k}\int_0^T\int_0^R z(t, r)\,\tilde{v}_k(t, r)\,dt\,dr.\end{aligned}\right\} \qquad (2.162)$$

In the case of slow fadeouts in time, the expression for the estimate of coordinates of the transfer function corresponding to (2.162) is written as follows

$$\hat{H}^{\text{МП}}(\omega_k, \omega_g) = \dot{Z}(\omega_k, \omega_g)/\dot{S}(\omega_k) \qquad (2.163)$$

and coincides with the "classical" decision in a certain frequency band of the space-time spectrum.

Estimates (2.162) and (2.163) are unbiased and asymptotically effective. It is not difficult to show that their dispersions diminish in inverse proportion to the signal/noise ratio, for example

$$D\left\{\hat{x}_k^{\text{МП}}\right\} = \frac{1}{d_k}. \qquad (2.164)$$

It can be seen from (2.164) that for small values of d_k the dispersion of estimates increases without restriction. This is a serious shortcoming of maximum-plausibility estimates. If the statistical characteristics of the measured function and noise are not known, then the statistical approach used here for obtaining estimates of coordinates is not suitable. Under these conditions, it is possible to use A. N. Tikhonov's regularization method, which guarantees the possibility of receiving stable solutions in the absence of a priori information. From a statistical standpoint this method can be called nonparametric.

For a channel with fadeouts that are slow in time the regularization method [9, 85] leads to the following regularized solution for the coordinates of expansion of the transfer function of a channel.

$$\hat{H}^{\alpha}(\omega, \omega_g) = \frac{1}{1 + \alpha \frac{M(\omega, \omega_g)}{|\dot{S}(\omega)|^2}} \frac{\dot{Z}(\omega, \omega_g)}{\dot{S}(\omega)}. \qquad (2.165)$$

In this $M(\omega, \omega_g)$ is a given, even negative function which determines the regular rising functional and satisfies the conditions:
1) $M(0, \omega_g) \geq 0$, $M(\omega, 0) \geq 0$; 2) $M(\omega, \omega) > 0$ for ω, $\omega_g \neq 0$; 3) for sufficiently large $|\omega|$, $|\omega_g|$ $M(\omega, \omega_g) \geq C > 0$.

If the spatial variable is not considered (the signal is a function of time), an analog of function (2.165) is

$$\hat{H}^{\alpha}(\omega) = \frac{1}{1 + \alpha M(\omega)/\dot{S}(\omega)^2} \frac{\dot{Z}(\omega)}{\dot{S}(\omega)}. \qquad (2.166)$$

In such problems Tikhonov-type regularizers of order p [9] are usually used, assigning a function of the type

$$M(\omega) = \omega^{2p}, \qquad (2.167)$$

which determines the set of regularizing operators and, using a certain algorithm, the value of regularization parameter a is found. In works [98, 99] it is shown that use of regularizers of type (2.167) makes it possible to obtain a stable solution to the problem.

In the situation of space-time signals under consideration, Tikhonov-type regularizers should be used, choosing

$$M(\omega, \omega_g) = \omega^{2p} \omega_g^{2l}. \qquad (2.168)$$

The other possibility, frequently employed in practice, is to choose a function $M(\omega, \omega_g)$ that affords a truncation of the spectrum of the space-time frequencies of the function under study.

$$M(\omega, \omega_g) = \begin{cases} 0, & -2\pi F \leq \omega \leq 2\pi F, \ -\Omega_g \leq \omega_g \leq \Omega_g; \\ \infty, & \omega, \omega_g \text{ -- outside the indicated domain} \end{cases} \qquad (2.169)$$

Selection of the optimal values of the regularization parameter a and the orders p and l of the regularizers in (2.168) as well as choosing the optimal values of F and Ω_g in (2.169) should be done using all available a priori information about the functions and noise under study.

2.7. Measurement of the Space-Time Characteristics of a Channel Using Information Signals

In the preceding sections we have constructed algorithms for measuring the space-time characteristics of a channel using special probing signals designed for precisely this purpose. A distinguishing feature of the probe signal is that it does not contain information about the message subject to transmission, but is only used to obtain estimates of a channel parameter in order to organize the work (for adaptation or self-adjustment) of the receiving units. In this case the necessary noise suppression is achieved at the cost of an inevitable reduction in the rate of data transmission along the communication channel. If the rate of fadeouts of channel parameters in time is small compared to the rate of data transmission, the relative decrease in rate of data transmission caused by the necessity of sending probing signals proves completely acceptable (a large number of information signals is sent for each probe) [49]. But if transmission is being done on a channel with fadeouts that are selective in time, there is one probing signal for each information signal and the rate of data transmission is cut in half. The desire to maintain the rate of information transmission on the condition of obtaining adequately high noise suppression forces us to use information signals to study the channel. This possibility has been discussed theoretically [58, 131] and realized in practice in the Reich system [131]. If during the use of probing signals to measure the channel we are always able, because we know the sign of the probing signal, to state with a probability of one that a probing signal is or is not contained in the observable oscillation (ideal classification), then when measuring a channel with information signals a different situation occurs. In this case, it is never possible to know in advance which transmitted signal (one of M possible signals) corresponds to the oscillation received at the output of a channel with scattering. All that can be said, with the a priori probability of transmitted signals P_l $l = \overline{1, M}$ is that the oscillation received corresponds to a signal in position l, that is, the sample at hand is unclassified [64].*

The situation becomes even more complicated when intercharacter interference must be taken into account [63].

A significant number of works [see the survey in 64] has been devoted to the problem of estimations of parameters by unclassified samples, but at the present time there is no one method of synthesizing estimation algorithms. We will consider the heuristic approach to the problem. This approach allows construction of a fairly straight forward algorithm for estimating parameters with an unclassified sample.

Thus, let us look at the situation when position l of a signal $s_l(t)$ being transmitted to a receiver is unknown, in other words, the channel must be studied directly by a working information sample. We will show that under these conditions measurement algorithms synthesized earlier can be modified. Estimates constructed in the preceding sections can be treated here as conditional estimates obtained on the assumption of

* [In cases of possible confusion letter "1" written as "l."]

transmission of position 1 of the signal. Thus, the following conditional estimates will be modifications of estimates (2.96) for the use of information signals:

$$\left.\begin{aligned} \hat{x}_k^l &= \frac{\sigma_{xk}^2 \int_0^T \int_0^R z(t, r) v_{lk}(t, r)\,dt\,dr + m_{xk}}{1 + 2h_{xlk}^2}, \\ \hat{y}_k^l &= \frac{\sigma_{yk}^2 \int_0^T \int_0^R z(t, r) \tilde{v}_{lk}(t, r)\,dt\,dr + m_{yk}}{1 + 2h_{ylk}^2}, \end{aligned}\right\} \quad (2.170)$$

where designations are introduced analogously to (2.94) for the signal $s_{lk}(t, r) = s_{lk}(t, r) + i\tilde{s}_{lk}(t, r)$ in path of propagation k, corresponding to position l of the transmitted signal

$$s_{lk}(t, r) = \int_0^\infty s_l(t-\xi)\,\varphi_k(t, \xi, r)\,d\xi. \quad (2.171)$$

Correspondingly, functions $v_{lk}(t, r)$ and $\tilde{v}_{lk}(t, r)$ are solutions to the integral equations

$$\left.\begin{aligned} &\int_0^T \int_0^R B_n(t, t', r, r')\, v_{lk}(t', r')\,dt'\,dr' = s_{lk}(t, r), \\ &\int_0^T \int_0^R B_n(t, t', r, r')\, \tilde{v}_{lk}(t', r')\,dt'\,dr' = \tilde{s}_{lk}(t, r). \end{aligned}\right\}, \quad (2.172)$$

Parameters h^2_{xlk} and h^2_{ylk} are determined from the relations

$$\left.\begin{aligned} 2h_{xlkgp}^2 &= \sigma_{xk}\sigma_{xp} \int_0^T \int_0^R s_{lk}(t, r)\, v_{gp}(t, r)\,dt\,dr, \\ 2h_{ylkgp}^2 &= \sigma_{yk}\sigma_{yp} \int_0^T \int_0^R \tilde{s}_{lk}(t, r)\, \tilde{v}_{gp}(t, r)\,dt\,dr. \end{aligned}\right\} \quad (2.173)$$

For coinciding positions g = 1 in identical paths, these designations are used to shorten the writing:

$$h_{ylkgp}^2 = h_{ylk}^2; \quad h_{xlkgp}^2 = h_{xlk}^2.$$

As before, the conditions of complete separation of paths at the place of reception are considered to be fulfilled for each position of the signal.

Let us consider the properties of the conditional estimates (2.170). We should distinguish two possible situations here: a) the position of the transmitted signal $s_1(t)$ has been "guessed" correctly; b) the position of the transmitted signal has been "guessed" incorrectly. The mathematical expectation and dispersions of estimates for the first case have been determined above. For the second case (a signal in position r has in fact been transmitted, but the receiving end thinks that a signal in 1 position was transmitted), the mathematical expectation and dispersion of the conditional estimate have the following appearance:

$$\left.\begin{aligned} M_1\{\hat{x}_k^l\} &= m_{xk}\frac{1+2h^2_{xlkgk}}{1+2h^2_{xlk}} + \frac{1}{1+2h^2_{xlk}}\sum_{p=1}^{p\neq k} 2h^2_{xlkgp} m_{xp},\\ D\{\hat{x}_k^l\} &= \sigma^2_{xk}\sum_{p=1}^{N}\frac{4h^4_{xlkgp}+2h^2_{xlkgp}}{(1+2h^2_{xlk})^2},\\ M_1\{\hat{y}_k^l\} &= m_{yk}\frac{1+2h^2_{ylkgn}}{1+2h^2_{ylk}} + \frac{1}{1+2h^2_{ylk}}\sum_{p=1}^{p\neq k} 2h^2_{ylkgp} m_{yp},\\ D\{\hat{y}_k^l\} &= \sigma^2_{yk}\sum_{p=1}^{N}\frac{4h^2_{ylkgp}+2h^2_{ylklp}}{(1+2h^2_{ylk})^2}. \end{aligned}\right\} \quad (2.174)$$

Analysis of (2.174) confirms the obvious fact that if position 1 of the expected signal at the receiving end does not correspond to the position of transmitted signal g, the conditional estimates (2.170) will have very poor characteristics for generally accepted indexes of the quality of estimates. They will be biased and their dispersions will not approach the dispersions of the estimated quantities with growth in the signal/noise ratio. However, if we recall that conditional estimates (2.170) were not designed to measure a channel characteristic as accurately as possible, but only to organize the work of the receiving unit, we will have to look differently at these estimations. We know that a device to distinguish M signals should have M branches, in each of which the oscillation being received is processed on the assumption that it contains a signal corresponding to this branch (for example branch l, $l = \overline{1, M}$) of the position. If conditional estimates (2.170) are constructed in each of the branches of the receiving unit, then in the branch corresponding to the position of the transmitted signal we will receive an estimate of the channel characteristic that possesses optimal properties, with characteristics (2.126), while in [m-1] branches that do not correspond to the position of the transmitted signal we will receive an estimate with characteristics (2.174). As can be seen from (2.174), where the condition of separation of paths is fulfilled, estimates (2.170) are asymptotically unbiased for small signal/noise ratios ($h^2_{xkl}\to 0$, $h^2_{ykl}\to 0$). Whereas with large signal/noise ratios their mathematical expectation and dispersion tend to zero. The latter means that zero will be adopted as the estimate of the surge characteristic in the branches not corresponding to the transmitted oscillation. It is apparent that this is not at all a bad estimate because in fact no signal for that position was transmitted on the channel. We will return to a quantitative discussion of this question below during consideration of algorithms for optimal signal discrimination.

Let us now consider certain possibilities of improving the quality of estimates during channel measurement using information signals. Suppose that M conditional estimates (2.170) have been formed in M branches of the receiver and, moreover, that the a priori probabilities of transmitted signals $p_1, p_2, \ldots, p_M$ are known. By averaging expressions (2.170) for optimal estimates of the coordinates of expansion $\hat{x}^l_k$ and $\hat{y}^l_k$, constructed on the assumption of transmission of position l of the signal along l using the set of a priori probabilities p_l, $l = \overline{1, M}$, we determine a certain unconditional estimate

$$\hat{x}_k = \sum_{l=1}^{M} p_l \hat{x}_k^l, \quad \hat{y}_k = \sum_{l=1}^{M} p_l \hat{y}_k^l. \qquad (2.175)$$

The average values and dispersions of unconditional estimations (2.175), on the assumption that the conditions of full separation of paths at the place of reception are met, are written

$$\left.\begin{aligned} M_1\{\hat{x}_k\} &= \sum_{l=1}^{M} p_l \frac{m_{xk} + m_{xk}\sum_{g=1}^{M} p_g 2h^2_{xlkgk}}{1 + 2h^2_{xlk}}, \\ D\{\hat{x}_k\} &= \sum_{l=1}^{M} p_l^2 \sigma_{xk}^2 \frac{\sum_{g=1}^{M} p_g 4h^4_{xlkgk} + 2h^2_{xlk}}{(1 + 2h^2_{xkl})^2}, \\ M_1\{\hat{y}_k\} &= \sum_{l=1}^{M} p_l \frac{m_{yk} + m_{ykg}\sum_{g=1}^{M} p_g 2h^2_{ylkgk}}{1 + 2h^2_{ykl}}, \\ D\{\hat{y}_k\} &= \sum_{l=1}^{M} p_l^2 \sigma_{yk}^2 \frac{\sum_{g=1}^{M} p_g 4h^4_{ylkgk} + 2h^2_{ylk}}{(1 + 2h^2_{ykl})^2}. \end{aligned}\right\} \qquad (2.176)$$

If signals that are orthogonal in the amplified sense for each of the paths of propagation are used to transmit information, and for them

$$\int_0^T \int_0^R s_{lk}(t, r) v_{gp}(t, r)\, dt\, dr = \int_0^T \int_0^R s_{lk}(t, r) \tilde{v}_{gp}(t, r)\, dt\, dr = 0. \qquad (2.177)$$

for any k, p with $g \neq 1$, then from (2.176) we obtain

$$\left.\begin{aligned} M_1\{\hat{x}_k\} &= m_{xk} \sum_{l=1}^{M} p_l \frac{1 + 2h_{xlk}^2 p_l}{1 + 2h_{xlk}^2}, \\ D\{\hat{x}_k\} &= \sigma_{xk}^2 \sum_{l=1}^{M} p_l^2 \frac{2h_{xlk}^2 + p_l 4h_{xlk}^4}{(1 + 2h_{xlk}^2)^2}, \\ M\{\hat{y}_k\} &= m_{yk} \sum_{l=1}^{M} p_l \frac{1 + 2h_{ylk}^2 p_l}{1 + 2h_{ylk}^2}, \\ D\{\hat{y}_k\} &= \sigma_{yk}^2 \sum_{l=1}^{M} p_l^2 \frac{2h_{ylk}^2 + p_l 4h_{ylk}^4}{(1 + 2h_{ylk}^2)^2}. \end{aligned}\right\} \qquad (2.178)$$

Analysis of (2.178) shows that where signals that are orthogonal in the amplified sense are used the estimates (2.175) are unbiased for small signal/noise ratios. As the signal/noise ratio grows, linear bias also increases, and the higher it becomes the closer the transmitted signals approach equiprobable signals. In the latter case the bias is (M-1)/M of the average value of the quantities being estimated.

Using the method from [58], the linear bias of estimates (2.178) can be eliminated by replacing estimate (2.175) with the following

$$\left.\begin{aligned} \hat{x}_k^{II} &= \sum_{l=1}^{M} \hat{x}_k^l \frac{p_l}{\sum_{l=1}^{M} p_l \dfrac{1 + 2h_{xlk}^2 p_l}{1 + 2h_{xlk}^2}}, \\ \hat{y}_k^{II} &= \sum_{l=1}^{M} \hat{y}_k^l \frac{p_l}{\sum_{l=1}^{M} p_l \dfrac{1 + 2h_{ylk}^2 p_l}{1 + 2h_{ylk}^2}}. \end{aligned}\right\} \qquad (2.179)$$

Estimates (2.179) are unbiased, but their dispersions increase in comparison with the dispersions of the estimations (2.175). The greatest increase in dispersion (M times) is observed for equiprobable signals. Where non-equiprobable signals are used the dispersions of estimations (2.179) may approach the dispersions (2.126) of the optimal estimations constructed using probing signals.

The possibilities considered for improving the quality of estimations of a channel characteristic based on adding conditional estimations have one significant shortcoming: their use significantly complicates the procedure for forming estimations. The desire to formulate estimations that resemble the conditional estimations (2.170) for simplicity of construction and the optimal estimations (2.113) for quality leads to construction of estimations using "decision feedback" [46, 49, 58].

The use of decision feedback is an effective means of improving the noise suppression of discrete systems in channels with slowly changing parameters [133] and makes it possible in practice to accomplish optimal processing of signals in channels with intercharacter interference [49, 53, 133].

In this work we are oriented to the set of signals, promising for space-time channels with scattering, that meets the condition of separation of paths, so intercharacter interference can be disregarded. However, decision feedback continues to be an effective means for insuring preliminary classification of a sample of the field under analysis until completion of the procedure for estimating the parameters of space-time channel.

In channels with slow fadeouts and a sufficient signal/noise ratio, when the receiving unit provides a low probability of erroneous reception of characters, decision feedback permits an almost ideal classification of an delayed sample of the field under analysis (teaching with an almost ideal teacher [64]).

In fact, if the decisions made by the receiver concerning characters transmitted in the interval of study of channel parameters $T_{st} = KT(k = 1, 2, 3, \ldots)$ are considered absolutely correct, this makes it possible even without a special probing signal to classify field $z(t, \overline{r})$ delayed for time T_{st}. On the other hand, if fadeouts in the channel are slow the estimates of channel characteristics obtained persist in the intervals of analysis of subsequent samples as well.

Because it is possible, in principle, to study channels by information samples in the interval $I_{st} >> T$, in the conditions stipulated above the decisive feedback can provide a higher quality estimation of channel parameters than when the channel is studied by special probing samples only. With this in mind, we will hereafter ascribe estimates (2.96), obtained on the assumption that special probing signals are transmitted, to any method of studying a channel under conditions of ideal classifications, in particular where decision feedback is used. We will observe, however, that because errors in registration of characters are inevitable (although with very low probability), classification of a field before determining estimations is still not ideal when done with decision feedback (teaching with a "real teacher" [64]).

Needless to say, the quality of estimation of channel parameters, which depends on the correctness of classification by information samples, will be better when decision feedback (teacher) is used than when it is not (when there is no teacher and the sample received is not fully classified). It is possible, in principle, to improve the quality of estimations in a system with decision feedback (classify the input field) if we use information on the a posteriori probability of transmission of particular signal positions. In this case, however, the procedure becomes much more complex [53, 58].

When speaking of the use of decision feedback to classify the field under analysis and then estimate parameters, we should keep in mind that to obtain good estimations certain limitations must be introduced on the form of the signals transmitted or their probabilistic structure. These matters are considered in greater detail in work [58].

2.8. Measurement of the Characteristics of a Stochastic Channel from the Standpoint of the Theory of Linear Filtration

Here we will take up measurement of channel characteristics based on the theory of linear Wiener and, to some degree, Kalman filtration.

Right at the start, let us state clearly the additional limitations associated with the fact that the results of measuring a characteristic of a channel must be used to organize the work of a processing unit for the purpose of extracting usable information from the field being received.

Let us suppose that by measurement a certain estimate of a channel characteristic has already been obtained, for example the transfer function $\hat{H}(f, t, r)$ in restricted domain (-f, f), (0, T), (0, R) and it must be used in the algorithm for processing the field under analysis, which contains a usable message. This can be done in an interval of analysis of length T immediately following the interval of measurement [0, T] in two cases:

a) measurement and processing correspond to the very same time interval, but these operations are spread out for an interval of length T by delay of the signal carrying information for this time;

b) the parameters of the channel change so slowly that the results of measurement at previous moments of time can be used effectively at later times (in one or several intervals of analysis of information signals).

If during the coordinate measurement considered above both these cases deserve attention and have a clear basis for realization, in the case of measurement from filtration standpoint the second is of greater practical interest today.

Having available the transfer function estimate $\hat{H}(f, t, r)$ obtained in processing the channel response to the test signal, there are two ways, in principle, to process the field u(t, r) being received against a background of additive noise n(t) during transmission of discrete messages.

In the first place, it may be done by finding the estimates of all variants of the blocks from k expected signals[1]

$$\hat{s}_i(t, r) = \int_{-\infty}^{\infty} \dot{S}_i(f) \dot{H}(f, t, r) \times e^{-i2\pi ft} df, \; i = 1, M^k$$

and compute their correlations $$\hat{F}_i = \int_0^{kT} \int_0^{R} \hat{s}_i(t, r) \times z(t, r) dt dr$$

and compare the latter (with due regard for the energies of the realizations). In the second place, it may be done by correcting the distortions introduced in the signals transmitted by stochastic channel by constructing a filter which is an additive compensator (analog or digital) whose characteristics depend on the existing estimate $\hat{H}$ (f, t, r) and makes it possible to perform compensation (or correction) according to some particular criterion [36, 46, 79, 133]. Algorithms of optimal processing will be analyzed in detail in Chapter 3.

Here we will only observe that in channels with selective fadeouts (by frequency, in time, and by space), optimal processing comes down to dispersed reception of some particular set of coordinates). Under these conditions, optimal processing does not require measurement of characteristics by filtration, but rather assumes estimates of the coordinates of expansion of the characteristic will be obtained on the chosen basis.

Considering what has been said, we will concentrate our attention on the possibilities of measuring the characteristics of a stochastic channel by filtration with subsequent use of the results to construct adaptive compensators.

Construction of a filter with a given frequency characteristic in a finite interval of time is a difficult problem, even if it is a filter with constant parameters. The problem is even more complex for parameters that vary considerably. This is why we are devoting primary attention here to the situation of slow changes in channel parameters when transfer function H(f, t, r) and, correspondingly, its estimate $\hat{H}(f, t, r)$ may be considered to be independent of time in the interval of analysis, although they change slowly from interval to interval,

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1 If intercharacter noise can be disregarded, the code is primitive, and the channel contains broad-band noise, then the by-element procedure of [53] is optimal and, therefore, we can speak of the elements, not the blocks, of the expected signals.

which is what gives rise to adaptive compensation. We should stress that the receiving of estimates of channel characteristics here is linked to construction of an adaptive compensator which can be used in transmission of both digital and analog information in a channel.

Let us look, in order, at algorithms of measurement based on the Wiener and Kalman approaches to filtration of signals against a noise background.

Wiener filtration of channel characteristics. Formalistically, when solving the problem of measuring the characteristics of a linear channel we are dealing with a convolution-type integral equation of the first kind

$$z(t, r) = \int_{-\infty}^{\infty} h(t, \xi, r) s(t - \xi) d\xi + n(t, r). \qquad (2.180)$$

The difference between equation (2.180) and the "classical" Fredholm equation of the first kind [38] is that the observed function and measured characteristic are functions of two variables (time and space) while the input signal is the function of a single variable (time). This feature is not limiting in the sense of using the results of the theory of linear filtration, but it leaves an inevitable imprint on the algorithms for computing estimates. Because we want to use the results of the Wiener theory, we will assume that the observed oscillation Z(t, r), measured characteristic h(ξ, r), and noise n(t, r) are realizations of stationary homogeneous fields and, moreover, that the measured characteristic and noise are not correlated. We will designate the spectral densities ($\overrightarrow{\omega k}$ — densities [84]) of the processes h(ξ, r) and n(t, r) with $G(\omega, \omega_g)$ and $N(\omega, \omega_g)$ respectively.

A distinction should be made between two filtration regimes [16]: filtration with delay (theoretically infinite) and filtration in real time. In the former case we assume that realization of the function z(t, r) can be entered and stored in the memory of a certain device for a certain time (theoretically infinite) and then subjected to processing by an optimal linear filtration algorithm. In the second case, the processing is done in real time, at the rate of signal reception, by a linear filter. Unlike the first, the second case demands fulfillment of the condition of physical feasibility expressed in the requirement that the response of the optimal filter must not outstrip actions at its input. As we know [16], despite the apparent difference in characteristics of filters working in both regimes, they are similar. Here we will consider primarily filtration with delay. Mathematically speaking, the problem is formulated as follows. Accessible for processing we have a mixture of usable signal, the field u(t, r), and noise

$$z(t, r) = u(t, r) + n(t, r). \qquad (2.181)$$

This mixture is processed by a linear space-time filter with constant parameters and characteristic g(ξ, r). At the output of the filter we receive the result

$$\hat{h}(t, r) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(\xi, r') z(t-\xi, r-r')\, d\xi\, dr'. \quad (2.182)$$

It is necessary to choose the characteristic of the linear optimal filter in such a way that the mean quadratic error of estimation is the minimum possible

$$\overline{\varepsilon^2} = M_1\{|\hat{h}(t, r) - h(t, r)|^2\} \to \min. \quad (2.183)$$

Function h(t, r), according to (2.180) and (2.181), may be treated as a linear transformation of usable signal u(t, r) by a linear filter whose characteristic is inverse to the spectrum of the transmitted signal

$$h(t, r) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\dot{U}(\omega, r)}{\dot{S}(\omega)} e^{i\omega t}\, d\omega, \quad (2.184)$$

where $U(\omega, r)$ and $S(\omega)$ are the spectra of realizations of u(t, r) and s(t) respectively.

Substituting (2.182) and (2.184) in (2.183), we obtain the equation for finding the characteristic of the optimal linear filter

$$\min \overline{\varepsilon^2} = \min M_1 \left\{ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(\xi, r') z(t-t', r-r')\, d\xi\, dr' - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\dot{U}(\omega, r)}{\dot{S}(\omega)} e^{i\omega t}\, d\omega \right\}. \quad (2.185)$$

The minimum in (2.185) is sought for all possible characteristics of linear filter $g(\xi, r)$. This problem is solved by the standard procedure and yields the following optimal estimate of the transfer function of a stochastic channel:

$$\hat{H}(\omega, \omega_g) = \frac{1}{1 + \dfrac{N(\omega, \omega_g)}{G(\omega, \omega_g)\,|\dot{S}(\omega)|^2}} \, \frac{\dot{Z}(\omega, \omega_g)}{\dot{S}(\omega)}, \quad (2.186)$$

where $Z(\omega, \omega_g)$ is the spectrum of realization of the observed field.

The estimate of the pulsed surge characteristic $\hat{h}(t, r)$ may be computed through estimate (2.186)

$$\hat{h}(t, r) = \frac{1}{2\pi} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hat{H}(\omega, \omega_g) e^{i(\omega t + \omega_g r)}\, d\omega\, d\omega_g. \quad (2.187)$$

Estimates (2.186) and (2.187) can be obtained at the output of a space-time filter with the transfer function

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$$\dot{K}(\omega, \omega_g) = \frac{\dot{S}^*(\omega)}{|\dot{S}(\omega)|^2 + \frac{N(\omega, \omega_g)}{G(\omega, \omega_g)}}. \qquad (2.188)$$

The expression for the mean quadratic error of estimations (2.186) and (2.187) has this form

$$\overline{\varepsilon^2} = \frac{1}{\pi^2}\int_0^\infty\int_0^\infty E(\omega, \omega_g)\, d\omega\, d\omega_g, \quad E(\omega, \omega_g) = \frac{N(\omega, \omega_g)}{|\dot{S}(\omega)|^2 + \frac{N(\omega, \omega_g)}{G(\omega, \omega_g)}}, \qquad (2.189)$$

where $\Gamma(\omega, \omega_g)$ is the energy spectrum of error of the Wiener estimation.

We will deal with questions of realizing algorithms of optimal filtration below.

The technique for solving a convolution-type integral equation of the first kind (2.180) based on the approach from the standpoint of linear filtration is very particular. The approach based on the ideas of regularization should be considered more common [8, 9, 85, 86].

Let us consider application of the method of regularization to a convolution-type equation in the multidimensional (two-dimensional) case, writing an equation of a somewhat more general type than (2.180)

$$z(t, r) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(t, \xi, r')\, s(t-\xi, r-r')\, d\xi\, dr' + n(t, r). \qquad (2.190)$$

Equation (2.180) is a particular case of (2.190) where $s(t, r) = s(t)\phi(r)$. This generalization will be useful below. Application of the method of regularization leads to the following regularized solution relative to the transfer function of a channel

$$\hat{\dot{H}}^\alpha(\omega, \omega_g) = \frac{1}{1 + \alpha\frac{M(\omega, \omega_g)}{|\dot{S}(\omega, \omega_g)|^2}}\,\frac{\dot{Z}(\omega, \omega_g)}{\dot{S}(\omega, \omega_g)}, \qquad (2.191)$$

where α is the parameter of regularization; $M(\omega, \omega_g)$ is an assigned, even, non-negative function that meets the conditions: a) $M(0, 0) > 0$; b) $M(\omega, \omega_g) > 0$ for $\omega, \omega_g \neq 0$; c) $M(\omega, \omega_g) > C > 0$ for $|\omega| \to \infty, |\omega_g| \to \infty$.

It is demonstrated for the broadest class of functions that where Tikhonov-type regularizers are used

$$M(\omega, \omega_g) = \omega^{2p}\,\omega_g^{2l} \qquad (2.192)$$

there exists (in some metric) the relationship between the error of the regularized solution and the error of observation for which the sequence of approximated regularized solutions converges toward the exact solution.

In the particular case when the measured, observed functions and noise are described by models of stationary random processes and error is estimated in the mean quadratic metric, the optimal regularizing function that insures minimum error is determined by solving the problem of optimal Wiener filtration and has the form

$$M(\omega, \omega_g) = N(\omega, \omega_g)/G(\omega, \omega_g). \qquad (2.193)$$

Computing the mean quadratic error of solution (estimation) (2.191) for function $M(\omega, \omega_g)$ of arbitrary type, we obtain

$$\left.\begin{aligned} D\{\hat{H}^\alpha\} &\doteq \frac{1}{\pi^2}\int_0^\infty\int_0^\infty E^\alpha(\omega, \omega_g)\,d\omega\,d\omega_g, \\ E^\alpha(\omega, \omega_g) &= \frac{|\dot{S}(\omega, \omega_g)|^2 N(\omega, \omega_g) + [\alpha M(\omega, \omega_g)]^2 G(\omega, \omega_g)}{[|\dot{S}(\omega, \omega_g)|^2 + \alpha M(\omega, \omega_g)]^2}, \end{aligned}\right\} \qquad (2.194)$$

where $E^\alpha(\omega, \omega_g)$ is the energy spectrum of error of a Tikhonov estimate. Relating the results to equation (2.180), we replace $S(\omega, \omega_g)$ in formulas (2.191) and (2.194) with $S(\omega)$.

Let us see what kind of device there must be to compute the estimate of the transfer function of a channel (2.191) using a Tikhonov-type regularizer (2.192). The estimate may be written as follows

$$\hat{H}^\alpha(\omega, \omega_g) = \frac{1}{|\dot{S}(\omega, \omega_g)|^2 + \alpha\omega^{2p}\omega_g^{2l}}\dot{S}^*(\omega, \omega_g)\dot{Z}(\omega, \omega_g). \qquad (2.195)$$

The linear filter that computes the estimate may be represented as a series of two filters: one coordinated with the transmitted signal, with a frequency characteristic that is conjugate with the spectrum of the transmitted signal

$$\dot{K}_1(\omega, \omega_g) = \dot{S}^*(\omega, \omega_g), \qquad (2.196)$$

and one that does not introduce phase distortions and has the characteristic

$$K_2(\omega, \omega_g) = \frac{1}{|\dot{S}(\omega, \omega_g)|^2 + \alpha\omega^{2p}\omega_g^{2l}}. \qquad (2.197)$$

Assuming that the very widespread condition

$$|\dot{S}(\omega, \omega_g)|^2 = A = \text{const}, \qquad (2.198)$$

is met, we write the following

$$K_2(\omega, \omega_g) = \frac{1}{A}\cdot\frac{1}{1 + \frac{\alpha}{A}\omega^{2p}\omega_g^{2l}}. \qquad (2.199)$$

It is simple to realize a filter with this characteristic as a system with feedback (see Figure 2.6 below). It is stable where $\alpha > 0$.

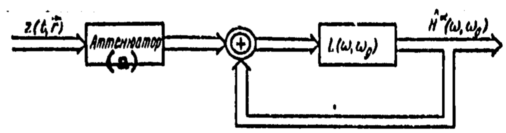


Figure 2.6. Realization of an Evaluation Filter Based on a Linear System with Feedback.

Key: a) Attenuator.

Characteristic K (ω, ω_g) is related to the characteristic of open system $L(\omega, \omega_g)$ by the relation

$$K_2(\omega, \omega_g) = \frac{L(\omega, \omega_g)}{1 + L(\omega, \omega_g)}. \quad (2.200)$$

Comparing (2.199) and (2.200) it is possible to write the characteristic of a filter encompassed by the feedback circuit:

$$L(\omega, \omega_g) = \frac{A}{\alpha} \frac{1}{\omega^{2p}} \frac{1}{\omega_g^{2l}}. \quad (2.201)$$

A filter with characteristic (2.200) performs 2p-multiple integration of field Z(t, r) in time and 2l-multiple integration by space. The fact that time processing and space processing can be separated here in filter $K_2(\omega, \omega_g)$ is important. The possibility of partially separating time and space processing is an advantage of regularized estimations (2.197) compared to Wiener estimates (2.186).

One other advantage of estimations (2.197) is that they make it possible to get by with less a priori information. To substantiate this statement let us consider an example of practical interest, estimating the transfer function of an isotropic channel.

We will consider only space filtration, assuming that time filtration is done separately or that it is unnecessary, as is the case, for example, in a channel with smooth fadeouts by frequency. On the basis of (2.195) assuming, in this case, that $\omega_g = \sqrt{\omega_x^2 + \omega_y^2}$, we write

$$\hat{H}^{\alpha}(\omega_g) = \frac{1}{|S(\omega_g)|^2 + \alpha\omega_g^{2l}}. \quad (2.202)$$

We will presuppose that the energy spectra of isotropic fields are described by bilinear functions and have the asymptotics

$$N(\omega_g) \approx N_0/\omega_g^{2n}, \quad G(\omega_g) \approx G_0/\omega_g^{2p}. \quad (2.203)$$

Relative to the square of the modulus of the signal $|S(\omega_g)|^2$ we will also suppose that its asymptotics are described by the power function

$$|\dot{S}(\omega_g)|^2 \approx S_0/\omega_g^{2s}. \qquad (2.204)$$

At first glance supposition (2.204) contradicts the model being used by assuming that the probing signal does not at all depend on the spatial coordinate. In fact, however, it is always necessary to consider that real signals have a practically limited spatial spectrum. This circumstance is reflected by condition (2.204). There is no detailed information about functions $N(\omega_g)$, $G(\omega_g)$, and $|S(\omega_g)|^2$; the only things known are the asymptotics (2.203) and (2.204). This is a very common situation in practice. Under these conditions the problem of optimization of regularizer order l and regularization parameter α can be formulated and solved analogously to the procedure employed in [8]. The optimal (in the sense of minimum mean quadratic error) value of the parameter of regularization α^{opt} is determined from the relation

$$\alpha^{opt} = S_0 \left\{ \frac{N_0}{S_0 G_0} \frac{\mu(1-\mu)}{\gamma(1-\gamma)} \frac{\sin\pi\gamma}{\sin\pi\mu} \right\} \frac{1}{\gamma+\mu}, \qquad (2.205)$$

where

$$\left. \begin{aligned} \mu &= \frac{2s-2n+1}{2(s+l)}, \quad 0<\mu<1; \\ \gamma &= \frac{2p-1}{2(s+l)}, \quad 0<\gamma. \end{aligned} \right\} \qquad (2.206)$$

Investigating mean quadratic error (2.194) at the extremum, it may be shown that it reaches its minimum where

$$l = 2p - 2n. \qquad (2.207)$$

The last deduction becomes clear after a comparison of (2.193) and (2.203). The values of the parameter of regularization α^{opt} calculated according to formula (2.205) are shown in Figure 2.7 below.

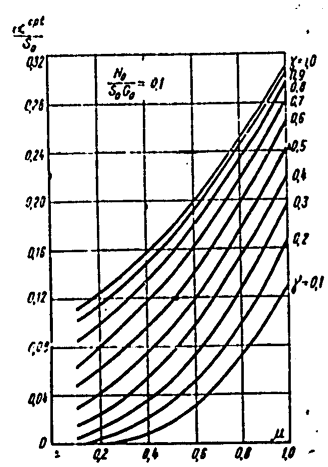


Figure 2.7. Values of the Normed Regularization Parameter.

In completing our discussion of Wiener filtration, we should point out that questions of constructing filters that work in real time have been taken up, for example, in works [16, 18].

A specific feature of the space-time filtration considered here is that it may be necessary to reject a real-time regime (to introduce delay), primarily because of the need for space processing. This eliminates the problem of using future (in time) values of the observed oscillation and makes it possible to use the oscillation as a whole to carry out filtration in time. The question of whether real time or introducing delay is a better filtration procedure will be decided by progress in the techniques of space-time filtration (holographic filters) and delay systems (delay lines).

Kalman filtration of channel characteristics. A generalization of the Kalman theory of filtration for the case of systems with distributed parameters may be used to solve the problem of estimating the space-time characteristic $h(\xi, r)$ [19, 143]. An analysis of the results of these works shows that the optimal linear estimate $\hat{h}(\xi, r)$ is the output signal of a nonstationary dynamic system with distributed parameters encompassed by a feedback line.

It is relevant here to ask what kind of processing device will realize the idea of adaptive compensation and be synthesized from the standpoint of Kalman filtration. It is obvious that it will contain a nonstationary dynamic system with distributed parameters of variable (adaptive reorganization depending on estimates $\hat{h}(t, r)$ received) structure. Realization of such a device in a continuous variation is hard to consider feasible in the foreseeable future. The clear way to realizing Kalman filtration algorithms lies in the digital variation. In its essence the Kalman theory is most adapted to digital computer technology.

To clarify the essential features of processing the general type of spatially distributed signals, let us consider an example based on the results of [108]. We will suppose that the spatial transfer function (real) $H(\omega_x, \omega_y)$ of a channel is observed for realization $Z(\omega_x, \omega_y)$, observed in the plane of the spatial spectrum of the signal, so that

$$Z(\omega_x, \omega_y) = S(\omega_x, \omega_y) H(\omega_x, \omega_y) + n(\omega_x, \omega_y), \quad (2.208)$$

where $S(\omega_x, \omega_y)$ is the spectrum of the probing signal and $n(\omega_x, \omega_y)$ is the spectrum of realization of white noise. Then, in addition to continuous recording of fields, we will use discrete recording, assuming that discretization with a uniform interval $\Delta\omega_x = \phi\omega_y$ has been carried out along axes ω_x and ω_y. The discrete variation of entry (2.208) is

$$Z(k, l) = S(k, l) H(k, l) + n(k, l). \quad (2.209)$$

The meaning of the designations is obvious.

We will assume that the correlation function of the field being measured is bi-exponential

$$B_H(\Omega_x, \Omega_y) = \sigma_H^2 \exp\{-\alpha_x |\Omega_x| - \alpha_y |\Omega_y|\}. \quad (2.210)$$

In this case the discrete field is described by a model of a stationary source

$$H(k+1,\ l+1) = \rho_1 H(k+1,\ l) + \rho_2 H(k,\ l+1) - \rho_1\rho_2 H(k,\ l) + \sqrt{(1-\rho_1^2)(1-\rho_2^2)}\, n(k,\ l), \qquad (2.211)$$

where N(k, 1) are uncorrelated noise readings with dispersion $\sigma^2{}_n$;

$$\rho_1 = \exp(-\alpha_x|\Delta\omega_x|);\quad \rho_2 = \exp(-\alpha_y|\Delta\omega_y|).$$

From (2.211) it can be seen that each element of the field is predictable using three neighboring elements. In the continuous variation a second-order differential equation in partial derivatives corresponds to entry (2.211).

The optimal Kalman estimate of the transfer function can be obtained in recurrent form

$$\hat{H}(k+1,\ l+1) = \rho_1\hat{H}(k+1,\ l) + \rho_2\hat{H}(k,\ l+1) - \rho_1\rho_2\hat{H}(k,\ l) + P(k,\ l)[Z(k,\ l) - S(k,\ l)\hat{H}(k,\ l)]. \qquad (2.212)$$

Function P(k, 1) is computed through the correlation function of filtration error. In the continuous variation the differential equation for an optimal Kalman estimate of the transfer function may be obtained in the form

$$\frac{\partial\hat{H}(\omega_x,\ \omega_y)}{\partial\omega_x} + \frac{\partial\hat{H}(\omega_x,\ \omega_y)}{\partial\omega_y} + \frac{\partial^2\hat{H}(\omega_x,\ \omega_y)}{\partial\omega_x\,\partial\omega_y} = -k_\alpha\hat{H}(\omega_x,\ \omega_y) + P(\omega_x,\ \omega_y)[Z(\omega_x,\ \omega_y) - S(\omega_x,\ \omega_y)\hat{H}(\omega_x,\ \omega_y)], \qquad (2.213)$$

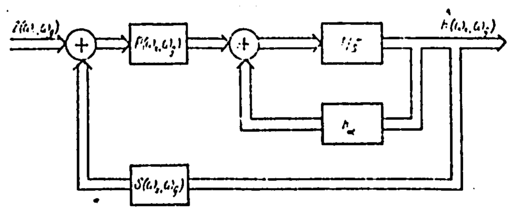
where

$$k_\alpha = \alpha_x + \alpha_y - \alpha_x\alpha_y. \qquad (2.214)$$

Figure 2.8 shows a space-time filter that accomplishes the computation of estimate (2.213). An operator to differentiate the function of two variables is introduced in the figure:

$$\vec{s} = \frac{\partial}{\partial\omega_x} + \frac{\partial}{\partial\omega_y} + \frac{\partial^2}{\partial\omega_x\,\partial\omega_y}. \qquad (2.215)$$

Figure 2.8. Space-Time Dynamic System That Computes the Kalman Estimate.



The function $P(\omega_x, \omega_y)$ is determined after solving the second-order differential equation in partial derivatives relative to the dispersion $D_{\hat{H}}(\omega_x, \omega_y)$ of the estimate (dispersion equation [18]). This equation does not contain the observed signal. It is not possible to obtain an exact analytic solution to it, but there are powerful numerical solution techniques. The dispersion of a Kalman estimate, unlike the dispersion of the Wiener estimate (2.189), is a variable quantity that depends generally on the time and space coordinates, but in the example under consideration only on the space coordinates. When the intervals of analysis tend to infinity, the dispersion of Kalman estimation diminishes monotonically, tending to the dispersion of the Wiener estimate, and in the given case is determined by the relation

$$D_{\hat{H}}(\infty, \infty) \equiv \overline{\varepsilon^2} = \frac{1}{\pi^2}\int_0^\infty\int_0^\infty \frac{N_0}{|\dot{S}(\chi, \eta)|^2 + \dfrac{N_0}{G(\chi, \eta)}}\, d\chi\, d\eta, \qquad (2.216)$$

where, for the example under consideration

$$G(\chi, \eta) = \frac{2\alpha_x}{\chi^2 + \alpha_x^2}\,\frac{2\alpha_y}{\eta + \alpha_y^2}. \qquad (2.217)$$

To check the monotonicity of decrease in the dispersion of the Kalman estimation depending on increase in the interval of analysis, algorithm (2.212) was modeled on a computer for different values of the parameters of the function being measured. The machine experiment confirmed that the decrease of dispersion is monotonic and showed that a smooth regime is achieved sooner (for smaller sizes of the analysis interval) when the correlation between adjacent readings of the measured function is less.

A very interesting and constructive approach to the filtration of space-time fields is the approach based on the assumption of the parametric dependence of equations of state on the spatial independent variable. This approach is outlined in article [19] and will, it appears, receive detailed treatment in later volumes of monograph [18].

A channel characteristic may be represented in the form of expansion into a series with divided space and time variables.

$$h(\xi, r) = \sum_i h_i(\xi)\, \varphi_i(r). \qquad (2.218)$$

This representation makes it possible to separate time and space processing. Assuming that space processing has been performed, let us consider processing the signal that is a function of time.

The specific feature of such consideration, in distinction from [18], is that the signal we are seeking, the channel characteristic, is linked to the usable signal included in the observed oscillation by means of a linear operation (filtration):

$$u(t) = \int_0^\infty h(\xi) s(t-\xi) d\xi. \qquad (2.219)$$

We will drop the index of spatial path i. Formally speaking, to use the elaborated theory of Kalman filtration we may consider that "signal" h(t) has been transmitted before passing through the "filter" with characteristic $s(\xi)$, as shown in Figure 2.9 below.

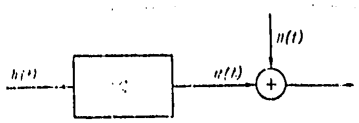


Figure 2.9. Model Yielding a Modified Vector of State.

Introducing a modified vector of state [18] and writing modified equations for it, we arrive at standard Kalman-Bussey equations of observation relative to the modified vector.

In concluding this part, let us observe that, of course, there is a possibility of applying the results obtained above to the case of measurement (filtration) of characteristics by means of information signals, not test signals. In this case all the estimates constructed here should contain a signal in position 1 and be treated as conditional. However, such a generalization is hardly advisable. This is because, when we are oriented to construction of an adaptive compensator, we usually use a test signal or perform an ideal classification of the field under analysis by means of decision feedback. This is precisely how existing systems of adaptive compensation are built [36, 79, 133].

But if we are dealing with an optimal processing (reception) unit, it is not wise to construct estimates from the filtration standpoint because coordinate estimates are more convenient. We must note that estimates obtained from a filtration standpoint have a major advantage over coordinate estimates because simpler signals that do not necessarily meet the conditions of path separation at the place of reception can be used in their construction. In fact, the condition of path separation has not figured in this section at all. This does not mean, however, that any signal we like can be used to obtain estimates of the transfer function of a channel from a filtration standpoint.

By analyzing estimation algorithms we can achieve perfectly clear formulations of the requirements made of probing signals. First of all they should have a sufficiently broad space-time spectrum that enables them to encompass all the frequencies under consideration. In the second place, they must have uniformity of the amplitude characteristic of signals in the assigned band:

$$|\hat{S}(\omega, \omega_g)| = \text{const}, \quad -2\pi F \leq \omega \leq 2\pi F; \quad -\Omega_g \leq \omega_g \leq \Omega_g. \qquad (2.220)$$

The latter requirement makes it possible to greatly simplify filtration algorithms and improve their characteristics.

2.9. Adaptive Compensators for a Space-Time Channel

A rigorous theory of adaptive compensators (equalizers) of a channel must be constructed from the standpoint of modern control theory [34]. In this case the compensating linear filter should be treated as a dynamic object of control to which the control action is fed from a block for estimation of the characteristics of the channel (see Figure 2.10 below). Such consideration would make it possible to investigate the dynamic regime of adaptive compensation.

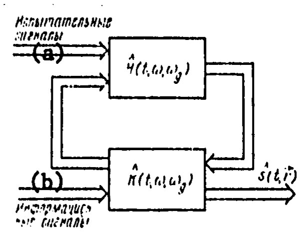


Figure 2.10. Adaptive Compensator for a Space-Time Stochastic Channel.

Key: a) Test Signal;
b) Information Signals.

Let us consider a simplified analysis of adaptive compensation, breaking the problem into two parts: measurement of the channel by means of a test signal, and actual compensation of distortions introduced in information signals by the channel. As estimates of a slowly changing channel characteristic the adaptive compensator uses an estimate constructed from the standpoint of the theory of linear filtration.

We will consider in order two types of compensators of a space-time channel. The first type of compensator is designed to restore the transmitted signal-time function and is a space-time filter at whose output an estimate of the transmitted signal $\hat{s}_1(t)$ is obtained. The second compensator is a combined type in which space processing is performed separately from time processing. This compensator has spatial dispersion. Time processing involves compensating for frequency distortions of the signal in spatial path i of propagation $i = \overline{1, N^I}$. Because they have been studied less, we will devote more attention to the first type of compensators, even though from a practical standpoint compensators of the second type are better understood and more convenient at the current level of development of technology. The theory of compensators for signal-time functions has been quite extensively developed [36, 48, 79].

If we have an estimate of the characteristic of a stochastic channel, for example $\hat{h}(\xi, r)$, it is possible to use it to restore the shape of the transmitted signal at position 1, beginning from the relation

$$z(t, r) = \mathrm{Re}\int_0^\infty h(\xi, r)\, s(t-\xi)\, d\xi + n(t, r). \qquad (2.221)$$

Let us consider the solution to this problem from the standpoint of the Wiener theory of filtration (assuming infinite delays). The difference between this and the problem considered in the previous section is that the nucleus of the equation whose solution we are seeking is not given exactly. Let us suppose that the true channel characteristic $h(\xi, r)$ is composed of its inexact estimate $\hat{h}(\xi, r)$ and random error $h^0(\xi, r)$ so that

$$h(\xi, r) = \hat{h}(\xi, r) + h^0(\xi, r). \qquad (2.222)$$

Let us assume that the error of estimate $h^0(\xi, r)$ is described by a stationary homogeneous function with a zero average and energy spectrum $E(\omega, \omega_g)$ (2.189), determined by the type of algorithm used to estimate the channel characteristic. The inexactness of measurement of the channel characteristic can be reduced to the appearance of supplementary additive noise

$$\varepsilon_l(t, r) = \mathrm{Re}\int_0^\infty h^0(t, \xi)\, s_l(t-\xi)\, d\xi. \qquad (2.223)$$

We will employ the method of randomization of the signals being transmitted [36], in other words we will consider the sequence of signals transmitted to be a random stationary process with an energy spectrum

$$G_s(\omega) = \frac{1}{M}\sum_{l=1}^{M} p_l\, |\dot{S}_l(\omega)|^2, \qquad (2.224)$$

where p_l is the probability of appearance of a signal at position 1.

The method of randomization makes it possible, in particular, to escape the dependence of supplementary additive noise on the signal being transmitted and to consider it an uncorrelated (with the signal) stationary homogeneous random field with energy spectrum

$$N_\varepsilon(\omega, \omega_g) = E(\omega, \omega_g)\, G_s(\omega). \qquad (2.225)$$

The summary (total) additive noise

$$\nu(t, r) = \varepsilon(t, r) + n(t, r) \qquad (2.226)$$

thus has the energy spectrum

$$N_\nu(\omega, \omega_g) = N(\omega, \omega_g) + N_\varepsilon(\omega, \omega_g) \qquad (2.227)$$

and does not depend on the usable signal.

Using the approach described in the preceding section, we arrive at the following form of optimal Wiener estimation of the transfer function of an adaptive compensator for a space-time channel:

$$\hat{K}(\omega, \omega_g) = \frac{\hat{H}^*(\omega, \omega_g)}{\left|\hat{H}(\omega, \omega_g)\right|^2 + \frac{N(\omega, \omega_g)}{G_s(\omega)} E(\omega, \omega_g)}. \qquad (2.228)$$

The filter with characteristic (2.228) is a space-time filter. That is why the signal at the output of this filter, when field z(t, r) is fed to its input, will be space-time

$$\hat{S}(\omega, \omega_g) = \dot{Z}(\omega, \omega_g)\hat{K}(\omega, \omega_g) \qquad (2.229)$$

or

$$s(t, r) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{S}(\omega, \omega_g)\, e^{-i\omega t - i\omega_g r}\, d\omega\, d\omega_g.$$

Further processing in the transmission of digital information is usually predetermined and involves computation of the correlation between the expected 1-position $s_1(t)$ and the output signal of the compensating filter (2.229). In this case we are interested only in the space-averaged signal at the output of the compensator, which is obtained by integrating (2.229) by the spatial coordinate

$$\hat{s}(t) = \int_0^R \hat{s}(t, r)\, dr. \qquad (2.230)$$

But the situation changes if the spatial independent variable of signals $s_1(t, r)$, which carry information, is taken into account. In the general case, processing at the output of the compensator is space-time processing and involves computing the correlations

$$\hat{F}_l = \int_0^T\int_0^R \hat{s}(t, r)\, s_l(t, r)\, dt\, dr, \quad l = \overline{1, M}. \qquad (2.231)$$

It is apparent that, in most cases, we can consider that the dependence on spatial coordinates is the same for all positions of the signals being transmitted, in other words

$$s_l(t, r) = s_l(t)\,\varphi(r), \quad l = \overline{1, M}. \qquad (2.232)$$

Then processing at the output of the adaptive compensator is greatly simplified and amounts to sequential operations of averaging with weight in space

$$\hat{s}(t) = \int_0^R \hat{s}(t, r)\,\varphi(r)\, dr \qquad (2.233)$$

and computing the correlation in time

$$F_l = \int_0^T \hat{s}(t)\, s_l(t)\, dt, \quad l = \overline{1, M}. \qquad (2.234)$$

The mean quadratic error of the signal $\hat{s}(t, r)$ restored by the compensator with characteristic (2.228) is determined by the expression

$$\left.\begin{aligned} \overline{\delta^2} &= \frac{1}{\pi^2} \int_0^\infty \int_0^\infty \overline{\delta^2(\omega, \omega_g)}\, d\omega\, d\omega_g, \\ \overline{\delta^2(\omega, \omega_g)} &= \frac{N(\omega, \omega_g) + N_e(\omega, \omega_g)}{\left|\hat{H}(\omega, \omega_g)\right|^2 + \frac{N(\omega, \omega_g)}{G_s(\omega)} + E(\omega, \omega_g)}. \end{aligned}\right\} \qquad (2.235)$$

The mean quadratic error (2.235) must be treated as conditional, for fixed channel parameters, because it contains the random function $|\hat{H}(\omega, \omega_r)|$. The numerator of estimate (2.228) also contains this random function, which makes it more difficult to realize the estimate. For practical considerations, we may suggest replacing estimate (2.228) with the suboptimal

$$\hat{K}(\omega, \omega_g) = \frac{\hat{H}^*(\omega, \omega_g)}{G(\omega, \omega_g) + \frac{N(\omega, \omega_g)}{G_s(\omega)} + 2E(\omega, \omega_g)}, \qquad (2.236)$$

where instead of random function $|\hat{H}(\omega, \omega_g)|^2$ we use its average value

$$M\left\{\left|\hat{H}(\omega, \omega_g)\right|^2\right\} = G(\omega, \omega_g) + E(\omega, \omega_g), \qquad (2.237)$$

Suboptimal estimate (2.236) will be closer to optimal as the error of measurement of the transfer function is reduced and the amplitude fluctuation in the channel weakens (the smaller the spread of $|H(\omega, \omega_g)|$ is relative to its average value).

Let us look further at the adaptive compensator whose characteristic is chosen using the regularization method. It has the form

$$K^\alpha(\omega, \omega_g) = \frac{\hat{H}^*(\omega, \omega_g)}{\left|\hat{H}(\omega, \omega_g)\right|^2 + \alpha M(\omega, \omega_g)}. \qquad (2.238)$$

The mean quadratic error of the signal restored by the compensator with characteristic (2.238) is determined by the expression

$$\left.\begin{aligned} &D\{\hat{s}^{\alpha}\} = \frac{1}{\pi^2}\int_0^{\infty}\int_0^{\infty} E^{\alpha}(\omega, \omega_g)\, d\omega\, d\omega_g, \\ &E^{\alpha}(\omega, \omega_g) = \frac{|\hat{H}(\omega, \omega_g)|^2 [N(\omega, \omega_g) + N_{\varepsilon}(\omega, \omega_g)]}{\left[|\hat{H}(\omega, \omega_g)|^2 + \alpha M(\omega, \omega_g)\right]^2} + \\ &\quad + \frac{[\alpha M(\omega, \omega_g)]^2 G_s(\omega)}{\left[|\hat{H}(\omega, \omega_g)|^2 + \alpha M(\omega, \omega_g)\right]^2}. \end{aligned}\right\} \qquad (2.239)$$

Another possibility is computing the estimate from a simpler relation than (2.38):

$$K^{\alpha}(\omega, \omega_g) = \frac{\hat{H}^*(\omega, \omega_g)}{G(\omega, \omega_g) + E(\omega, \omega_g) + \alpha M(\omega, \omega_g)}. \qquad (2.240)$$

Considerations relative to selecting functions $M(\omega, \omega_g)$ and the values of the regularization parameter α included in estimates (2.238) and (2.240) remain the same as in the preceding section.

Let us move on to the compensator which realizes the Kalman algorithm for filtration of a space signal (a continuation of the example in the preceding section).

The model of the signal observed in the plane of the spatial spectrum has this form

$$Z(\omega_x, \omega_y) = \hat{H}(\omega_x, \omega_y) S(\omega_x, \omega_y) + n_{\varepsilon}(\omega_x, \omega_y) + n(\omega_x, \omega_y), \qquad (2.241)$$

where

$$n_{\varepsilon}(\omega_x, \omega_y) = \mathring{H}(\omega_x, \omega_y) S(\omega_x, \omega_y) \qquad (2.242)$$

is noise related to inexactness in estimating the transfer function of the channel. In discrete form the model of observation is written as follows

$$Z(k, l) = \hat{H}(k, l) S(k, l) + n_{\varepsilon}(k, l) + n(k, l). \qquad (2.243)$$

A special feature of this problem, as we see, is the presence of colored noise $n_{\varepsilon}(\omega_x, \omega_y)$ related to the inexactness of measurement. There is a corresponding generalization of the Kalman theory of filtration [18]. Equation (2.242) may be written in the form

$$\left.\begin{aligned} Z(\omega_x, \omega_y) &= C(\omega_x, \omega_y) X(\omega_x, \omega_y) + n(\omega_x, \omega_y), \\ Z(k, l) &= C(k, l) X(k, l) + n(k, l). \end{aligned}\right\} \qquad (2.244)$$

In equation (2.244) there is a vector function

$$\left.\begin{aligned} X(\omega_x,\ \omega_y) &= \begin{bmatrix} S(\omega_x,\ \omega_y) \\ n_\varepsilon(\omega_x,\ \omega_y) \end{bmatrix}, \\ X(k,\ l) &= \begin{bmatrix} S(k,\ l) \\ n_\varepsilon(k,\ l) \end{bmatrix}, \end{aligned}\right\} \quad (2.245)$$

corresponding to the variables of state of the system that models the field being measured and non-white noise. Matrix $C(\omega_y,\ \omega_y)$ is called modulated

$$\left.\begin{aligned} C(\omega_x,\ \omega_y) &= [\hat{H}(\omega_x,\ \omega_y),\ 1], \\ C(k,\ l) &= [\hat{H}(k,\ l),\ 1]. \end{aligned}\right\} \quad (2.246)$$

We assume that the correlation function of colored noise has the form

$$B_\varepsilon(\Omega_x,\ \Omega_y) = \sigma_\varepsilon^2 e^{-\beta_x|\Omega_x|-\beta_y|\Omega_y|}, \quad (2.247)$$

and we designate

$$\mu_1 = e^{-\beta_x|\Delta\omega_x|},\quad \mu_2 = e^{-\beta_y|\Delta\omega_y|}. \quad (2.248)$$

The estimate equation in continuous form is written as follows

$$\frac{\partial \hat{x}(\omega_x,\ \omega_y)}{\partial \omega_x} + \frac{\partial \hat{x}(\omega_x,\omega_y)}{\partial \omega_y} + \frac{\partial^2 \hat{x}(\omega_x,\ \omega_y)}{\partial \omega_x \partial \omega_y} =$$
$$= F(\omega_x,\ \omega_y)\hat{x}(\omega_x,\ \omega_y) + P(\omega_x,\ \omega_y)[Z(\omega_x,\ \omega_y) - C(\omega_x,\ \omega_y)\hat{x}(\omega_x,\ \omega_y)], \quad (2.249)$$

where these desginations are introduced

$$F(\omega_x,\ \omega_y) = \begin{bmatrix} -k_\alpha & 0 \\ 0 & -k_\beta \end{bmatrix},\quad k_\alpha = \alpha_x + \alpha_y - \alpha_x\alpha_y;$$
$$k_\beta = \beta_x + \beta_y - \beta_x\beta_y. \quad (2.250)$$

The vector equation (2.249) can be written in the form of a system of two scalar equations that give estimates of the signal and colored noise.

$$\left.\begin{aligned} &\frac{\partial \hat{S}(\omega_x,\omega_y)}{\partial \omega_x} + \frac{\partial \hat{S}(\omega_x,\omega_y)}{\partial \omega_y} + \frac{\partial^2 \hat{S}(\omega_x,\ \omega_y)}{\partial \omega_x \partial \omega_y} = \\ &= -k_\alpha \hat{S}(\omega_x,\ \omega_y) + P_{11}(\omega_x,\ \omega_y)[Z(\omega_x,\ \omega_y) - \\ &- \hat{S}(\omega_x,\ \omega_y)\hat{H}(\omega_x,\ \omega_y) - \hat{n}_\varepsilon(\omega_x,\ \omega_y)], \\ &\frac{\partial \hat{n}_\varepsilon(\omega_x,\ \omega_y)}{\partial \omega_x} + \frac{\partial \hat{n}_\varepsilon(\omega_x,\ \omega_y)}{\partial \omega_y} + \frac{\partial^2 \hat{n}_\varepsilon(\omega_x,\ \omega_y)}{\partial \omega_x \partial \omega_y} = \\ &= -k_\beta \hat{n}_\varepsilon(\omega_x,\ \omega_y) + P_{21}(\omega_x,\ \omega_y)[Z(\omega_x,\ \omega_y) - \\ &- \hat{S}(\omega_x,\ \omega_y)\hat{H}(\omega_x,\ \omega_y) - \hat{n}_\varepsilon(\omega_x,\ \omega_y)]. \end{aligned}\right\} \quad (2.251)$$

The space-time filter that computes estimate (2.251) is shown in Figure 2.11 below.

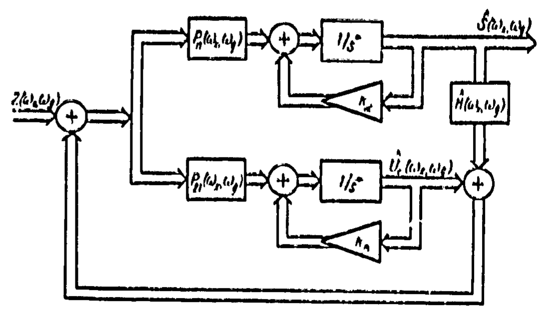


Figure 2.11. Kalman Compensator for a Space-Time Channel.

The functions $P_{11}(\omega_x, \omega_y)$ and $P_{21}(\omega_x, \omega_y)$ are determined by solving a system of two second-order differential equations (dispersed equation) that do not contain the observed signal. In the discrete variation, the equations of estimate (2.251) have the form

$$\left.\begin{aligned} &\hat{S}(k+1, l+1) = \rho_1 \hat{S}(k+1, l) + \rho_2 \hat{S}(k, l+1) - \rho_1 \rho_2 \hat{S}(k, l) + \\ &+ P_{11}(k, l)[Z(k, l) - \hat{H}(k, l)\hat{S}(k, l) - \hat{n}_e(k, l)], \\ &\hat{n}_e(k+1, l+1) = \mu_1 \hat{n}_e(k+1, l) + \mu_2 \hat{n}_e(k, l+1) - \\ &- \mu_1 \mu_2 \hat{n}_e(k, l) + P_{21}(k, l)[Z(k, l) - \hat{H}(k, l)\hat{S}(k, l) - \\ &\qquad - \hat{n}_e(k, l)] \end{aligned}\right\} \tag{2.252}$$

and may be realized easily on a digital computer.

Let us sum up certain results concerning the two methods of estimating characteristics (by coordinate and filtration), which are projected respectively to optimal processing (dispersion) and additive compensation. The algorithms for coordinate estimation are relatively simple to accomplish and make it possible to obtain very high quality indexes as long as the conditions of separation of paths at the place of reception are fulfilled and, as a result, there is no intercharacter noise. The algorithms for estimates from the standpoint of filtration are invariant with respect to fulfillment of the conditions of path separation and, therefore, permit the use of a broader class of channel signals.

Optimal filtration algorithms are difficult to realize and demand complex hardware (adaptive space-time filters). It is comparatively simple to realize suboptimal algorithms for filtration of characteristics, in particular algorithms with separate processing in space and in time.

The quality characteristics of these suboptimal algorithms which process the field received in series may, however, be considerably worse than quality characteristics for optimal processing of the field received as a whole (the coordinate method). In connection with these circumstances, one is led to the idea of combined processing of the field being received using relatively simple signals in transmission: partially additive compensation of channel distortions with subsequent processing from the standpoint of dispersed reception (the coordinate method). A corresponding structure of the processing system for a space-time field is shown in Figure 2.12 below.

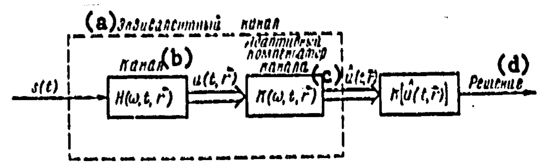


Figure 2.12. Two-Stage Processing of a Space-Time Signal.

Key:
- a) Equivalent Channel;
- b) Channel;
- c) Adaptive Channel Compensator;
- d) Solution.

In this case the adaptive compensator has a relatively simple problem: eliminate intercharacter interference, in other words, somewhat improve the channel and make subsequent processing by a multichannel dispersed reception device easier.

It is obvious that the two-stage processing procedure is very promising for stochastic channels with a high degree of selectivity.

The problem of measuring the space-time characteristics (identification) of a stochastic channel has been considered in detail. Two approaches to measurement were used: estimating the coordinates of expansion of a characteristic in a selected system of coordinate functions and measurements from the standpoint of linear filtration series.

It has been shown that linear estimates of the coordinates of expansion are invariant with respect to the probabilistic distribution laws of the functions and noise being measured and are determined by the average values and dispersions of the quantities estimated. The type of linear estimates and feasibility of their practical realization are determined by the choice of a discrete model of the channel (coordinate functions). The Karunen-Loew expansion is optimal, but it leads to complex discrete models of the channel and, accordingly, to estimation algorithms that are difficult to realize. Realization of the algorithms requires the presence of space-time filters with variable parameters. Simplifications of realization are possible where the variables of the coordinate functions are factored. Linear estimates are unbiased and effective where the conditions of separation of paths at the place of reception are met. When this condition is not fulfilled the characteristics of linear estimates are much worse.

It is advisable to use measurement by Wiener and Kalman filtration with an orientation to further processing in the form of adaptive compensation for channel distortions. During filtration of space-time fields, it does not appear to be compulsory to receive estimates in real time. The condition of separate processing in space and in time is more important. In the absence of a priori data on the statistical moments of the characteristics being measured, it is advisable to use the method of regularization of uncorrected problems and Tikhonov regularizers. The optimal characteristic of an equalizing space-time filter (adaptive compensator) can also be synthesized from the standpoint of linear filtration theory. This affords the best (in the mean quadratic sense) restoration of the signal. It is advisable to perform adaptive compensation of channel distortions as the first stage in processing a space-time field. The second stage should be optimal processing, which is based on decision theory.

Chapter 3. Processing Space-Time Signals Containing Discrete Messages

3.1. Statement of the Problem of Optimal Reception of Messages in a Stochastic Channel

The theory of optimal space-time processing of signals began its development comparatively recently, even though the techniques of space processing were widely used in practice long ago. This situation where practice is ahead of theory has been encountered numerous times in various fields of science and technology. Moreover, it is practice that must put the problems to theory. However, the engineer usually not just raises a new (even for theory) problem, but also tries to solve it, making use of his own experience, knowledge, and intuition. When these three components are sufficient, the engineering solution that results proves close to optimal. But where is the guarantee that the solution proposed by the engineer is in fact best? Only a rigorous theory enables us to identify the exact structure of the optimal system, evaluate the potential of it or any other suboptimal system, and offer a whole series of new solutions suitable in situations so complex that finding them by the heuristic method, although possible, is highly unlikely.

In this book we are considering only by-element methods of receiving discrete messages because reception as a whole [53] is difficult to realize for space-time channels today.

While investigating the algorithm of by-element reception with arbitrary channel memory, we will disregard intercharacter interference, orienting ourselves to that set of signals which satisfactorily insures separation of the paths in the field being received.

We will devote special attention to investigation of the algorithms that perform independent processing of individual characters and do not use channel parameter estimates obtained in earlier intervals of analysis. However, to some degree the opposite situation is also investigated (use of reliable estimates of channel parameters obtained in preceding intervals under conditions of ideal classification).

It is advisable to base construction of algorithms for optimal signal processing in a stochastic channel on the principle of statistical self-adjustment (adaptation) [63]. The essential feature of the method of statistical self-adjustment is replacement of unknown parameters that determine the signal at the place of reception and are included in the decision-making rule with certain estimates of these parameters. Within the framework of a linear stochastic channel under consideration here the coordinates of expansion of a channel characteristic are such parameters. Thus, the method of statistical self-adjustment suggests the following way to construct algorithms for optimal signal processing. At first the channel is considered provisionally ideal, that is, its characteristic is assumed to be known exactly at the receiving end. Under these conditions, as we know from fundamental works [60, 68, 95], linear correlation processing is optimal (in the cases of both Gaussian and non-Gaussian noise). Strictly sepaking, this result is correct for the non-Gaussian case with certain limitations [95]. Then, using the principle of adaptation, the coordinates of expansion of the channel characteristic which are included in the optimal processing algorithm should be replaced by their estimates. The processing algorithm obtained as a result of this operation can be nonlinear, as will be seen below.

When organizing optimal space-time processing it is necessary to estimate the coordinates of expansion of the channel characteristics for those paths of propagation which are to be used for dispersed reception. After estimating the amplitude multiplier and phase shift in path of propagation k and knowing the regular signal distortions in this path, which can be determined, for example, in (2.10) as a function of $\psi_k(t, \xi, r)$, it is not difficult to construct an estimate of the signal at the place of reception, which corresponds to any position 1 of M possible transmitted positions:

$$\hat{s}_k^l(t, r) = \mathrm{Re}\,\hat{\eta}_k \int_0^\infty s_l(\xi)\, \varphi_k(t, \xi, r)\, d\xi. \qquad (3.1)$$

Construction of an optimal receiver in channels with selective fadeouts using a large number of branches for dispersion in space (constructing a multidimensional space-time filter) is not an easy job, even though engineering practice today does have some experience in the construction of devices for multiple dispersed reception (coherators [109]). The realization of a multidimensional space-time filter becomes more realistic if we move to the digital variation based on a high-speed general purpose or specialized computer [28]. The second possible way is to use coherent optical systems of signal processing [56]. The questions of realizing algorithms for space-time signal processing will be discussed in greater detail below.

In conformity with the statement of the problem of channel measurements in Chapter 2, we will consider devices for signal processing both using probing messages and where channel characteristics are measured by information signals. Because, as observed above, use of the principle of statistical self-adjustment makes it possible to break the job of constructing an optimal processing algorithm into stages, it is wise to begin our consideration from the first stage, optimal reception of a known signal against a background of spatially distributed noise.

3.2. Optimal Processing of Space-Time Signals in a Deterministic Channel. The Coordinated Space-Time Filter

The space-time filter accessible to observation is the sum of usable signal and additive noise:

$$z(t, r) = u_l(t, r) + n(t, r). \qquad (3.2)$$

In our case, space-time signals $u_l(t, r)$ are generated by transmitted signals of known form $s_l(t)$, $l = 1, \ldots, M$, in conformity with the model of a linear channel

$$u_l(t, r) = \mathrm{Re}\int_0^\infty s_l^*(\xi)\, h(t, \xi, r)\, d\xi. \qquad (3.3)$$

The channel characteristic $h(t, \xi, r)$ is considered to be known exactly. This allows us to consider signals $u_l(t, r)$, $l = 1, \ldots, \overline{M}$, corresponding to each of the M possible transmitted signal positions, to be known also.

If additive noise $n(t, r)$ is a white Gaussian noise field with spectral density N_0, then analogously to [62] it is not difficult to obtain the optimal processing algorithm for space-time signals. The decision in favor of l-position signal s (t) is made in the case where the following system of inequalities is fulfilled

$$F_l - B_l > \ln c_{gl} + F_g - B_g, \qquad (3.4)$$
$$g = \overline{1, M}; \quad g \neq l.$$

Functional F determines the magnitude of the correlation between expected signal in l position $u_l(t, r)$ and the field accessible to observation

$$F_l = \frac{2}{N_0}\int_0^T\int_0^R z(t, r)\, u_l(t, r)\, dt\, dr. \qquad (3.5)$$

The operation that defines the right part (3.5) should be performed on the input oscillation at each of the space-time intervals of observation. Quantity B_l determines the ratio of the energy of the space-time signal at the place of reception in the spectral density of noise output:

$$B_l = \frac{1}{N_0}\int_0^T\int_0^R u_l^2(t, r)\, dt\, dr. \qquad (3.6)$$

Quantity c_{gl} is determined by the chosen reception criterion.

Where the criterion of an ideal observer is used in the case of equiprobable transmitted signals $c_{gl} = 1$.

Because signal $u_l(t, r)$ may be treated approximately as a function with limited duration in time (O, T) and in space (O, R), the algorithm may be realized on the basis of coordinated space-time filters with pulsed surge characteristics

$$g_l(\xi, \chi) = \begin{cases} u_l(T-\xi, R-\varkappa) & 0 \leqslant \xi \leqslant T, \quad 0 \leqslant \chi \leqslant R; \\ 0 \text{ for } \xi \text{ and } \chi \text{ outside of } [0, T] \text{ and } [0, R] \text{ respectively.} \end{cases} \tag{3.7}$$

The output voltage of filter 1 at moment in time T at the point with coordinate R is proportional to the quantity F_1. Figure 3.1 shows a schematic diagram of a device built on the basis of coordinated filters.

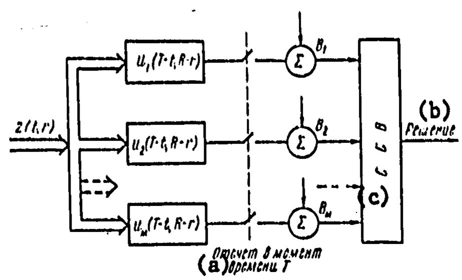


Figure 3.1. Optimal Processing in a Channel with Additive Noise

Key:
a) Reading at Moment in Time T;
b) Solution (Decision);
c) Comparison and Decision Circuit.

Let us stop in somewhat greater detail on the structure of the coordinated space-time filter. Linear filters coordinated with pulsed signals that are time functions have been studied carefully in the literature [22, 65]. The discovery of the holographic method stimulated the development of methods of coordinated filtration of space signals which are immobile images [61, 71, 74, 96]. In principle, however, coordinated spatial filtration is possible not only in the optical range but also in the radio wave range. Specifically, an antenna with a synthesized aperture [7, 70] may be considered a coordinated space filter. Let us turn to a formal description of a coordinated space-time filter.

We will consider input signal $F_1(t, r)$ of a linear filter with pulsed surge characteristic $g_1(\xi, \chi)$ when signal $z(t, r)$ is fed to its input:

$$F_l(t, r) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} z(t-\xi, r-\chi)\, g_l(\xi, \chi)\, d\xi, d\chi. \tag{3.8}$$

In the case where

$$g_l(\xi, \chi) = u_l(T-\xi, R-\chi). \tag{3.9}$$

signal (3.8) assumes the form

$$F_l(t, r) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} z(t-\xi, r-\chi)\, u_l(T-\xi, R-\chi)\, d\xi\, d\chi.$$

At moment in time T at the point with coordinate R the output voltage reaches magnitude

$$F_l(T,\ R) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} z(t,\ r)\, u_l(t,\ r)\, dt\, dr = F_l. \qquad (3.10)$$

Beginning from (3.10) the expression for $F_1(T, R)$ may be written in the form

$$F_l(T,\ R) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \dot{Z}(\omega,\ \omega_g)\, \dot{U}_l(\omega,\ \omega_g)\, d\omega\, d\omega_g, \qquad (3.11)$$

where $Z(\omega, \omega_g)$ and $U_1(\omega, \omega_g)$ are the spectra of realization of $v(t, r)$ and $u_1(t, r)$ respectively. Thus, quantity F_1, determined by equation (3.5), can be obtained with a precision to the level of the constant multiplier at the output of the coordinated linear filter. The requirement of limited duration of function $u_1(t, r)$ in time is essential so that the coordinated filter will be physically feasible. The limited duration of analyzed signals in space is always insured by the finite dimensions of the receiving antenna. Let us determine the transfer function of the coordinated space-time filter:

$$G_l(\omega,\ \omega_g) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u_l(T-\xi,\ R-\chi)\, e^{-i\omega\xi - i\omega_g\chi}\, d\xi\, d\chi = e^{-i\omega T - i\omega_g R}\, \overset{*}{\dot{U}}_l(\omega,\ \omega_g), \qquad (3.12)$$

where $\overset{*}{U}_1(\omega, \omega_g)$ is the conjugate space-time spectrum at the output of the channel corresponding to position 1 of the transmitted signal, and T and R determine the time delay and displacement in space respectively.

For the channel described by the model of a filter with constant parameters with characteristic $H(\omega, r)$, the structure of an optimal space-time processing device can be greatly simplified. In this case an analog of relation (3.3) is $\dot{U}_l(\omega,\ \omega_g) = \dot{H}(\omega,\ \omega_g)\, \dot{S}_l(\omega).$ (3.13)

The transfer function of a filter coordinated with space-time signal $u_1(t, r)$ should now satisfy the relation

$$G_l(\omega,\ \omega_r) = \dot{U}_l(\omega,\ \omega_g)\, e^{-i\omega T - i\omega_g R} = H^*(\omega,\ \omega_g)\, S_l^*(\omega)\, e^{-i\omega T - i\omega_g R}. \qquad (3.14)$$

A filter with characteristic (3.14) can be represented as a series of two filters. One of them is a spatial filter with a characteristic that does not depend on transmitted signals $H^*(\omega, \omega_g)e^{-i\omega T_1 - i\omega_g R}$; the other does not contain a spatial variable and is determined by the transfer function $S^*(\omega)\, e^{-i\omega T_2}$, and the relation $T_1 + T_2 = T$ is met. The space filter is the same for all M branches and may be connected in at the input of the optimal receiving unit. Figure 3.2 below shows the schematic diagram of such a device. The role of integrator at the output of the space filter is understandable from relation (3.11).

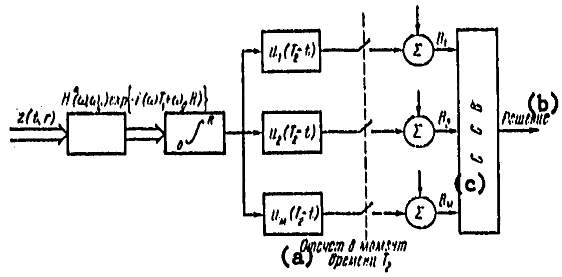


Figure 3.2. Coordinated Space-Time Filter for a Channel with Smooth Fade-outs in Time.

Key: a) Reading at Moment in Time T ;
b) Solution (Decision);
c) Comparison and Decision Circuit.

An unquestionable advantage of this method of field processing is that space processing and time processing are separated here. We should note that, despite the apparent similarity, this method of reception does not coincide with the method of adaptive compensation for distortion in the channel which was considered above.

Relying on well-known results [41, 95] for time signals, it may be observed that rule (3.4) preserves its optimal (or close to optimal) properties even in the case of non-Gaussian Delta-correlated noise with a zero mean and symmetrical distribution function.

It is not difficult to generalize the problem of optimal reception of a precisely known signal and to solve it for what is called "colored" [16, 109] noise with the correlation function $B_n(t, t', r, r')$. There are two ways to solve this problem: a "whitening" filter added at the input of the receiving device [22]; and forming a special reference signal that takes account of the correlation properties of noise [64, 109]. The second method is more rigorous, although the results of both cases are identical in all situations of practical interest.*

The rule of processing signals in colored noise is still determined by the system of inequalities (3.4), but the quantities F_1 and B_1 must be determined from the relations

$$\left.\begin{aligned} F_l &= \int_0^T\int_0^R z(t, r)\,\nu_l(t, r)\,dt\,dr, \\ B_l &= \int_0^T\int_0^R u_l(t, r)\,\nu_l(t, r)\,dt\,dr. \end{aligned}\right\} \tag{3.15}$$

Function $v_1(t, r)$ is the solution of the integral equation

$$\int_0^T\int_0^R B_n(t, t', r, r')\,\nu_l(t', r')\,dt'\,dr' = u_l(t, r). \tag{3.16}$$

* It should be remarked that the transfer process at the output of a whitening filter can cause significant intercharacter interference if no constraints are placed on the set of signals used.

The solution to the problem can also be obtained by passing signals z(t, r) and u (t, r) through "whitening" spatial filters with subsequent correlation processing. If the colored noise has a spectral density of $N(\omega, \omega_g)$ the square of the modulus of the frequency characteristic $K(\omega, \omega_g)$ of the whitening filter must satisfy the relation

$$|\dot{K}(\omega, \omega_g)|^2 = \frac{N_0}{2} \frac{1}{\dot{N}(\omega, \omega_g)}. \qquad (3.17)$$

If colored noise n(t, r) is fed to the input of this filter, at its output "white" noise with a spectral density of output of N_0 is formed. Figure 3.3 below depicts the schematic diagram of a correlation receiver with filtration of input and reference signals.

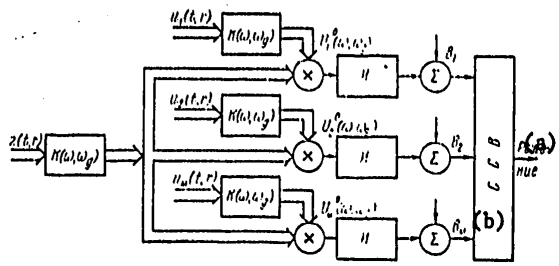


Figure 3.3. Processing a Field with Filtration of the Observed Reference Field.

Key: a) Solution (Decision);
b) Comparison and Decision Circuit.

The second possible variation of construction of an optimal device for processing space-time signals with colored noise is to build it on the basis of coordinated filters.

As Figure 3.3 shows, the spectrum of the reference signal $u^o{}_1(t, r)$ in branch 1 is formed according to the relation

$$\dot{U}_l^0(\omega, \omega_g) = \dot{U}_l(\omega, \omega_g)\dot{K}(\omega, \omega_g), \qquad (3.18)$$

where $U_1(\omega, \omega_g)$ is the spectrum of signal $U_1(t, r)$. The transfer function of a filter coordinated with $U^o{}_1(t, r)$ should satisfy the relation

$$G_l(\omega, \omega_g) = [\dot{U}_l^0(\omega, \omega_g)]^* e^{-i\omega T - i\omega_g R} = \dot{U}_l^*(\omega, \omega_g)\dot{K}^*(\omega, \omega_g) e^{-i\omega T - i\omega_g R}. \qquad (3.19)$$

It follows from (3.19) that each filter 1 of M coordinated filters is a series combination of filter $K^*(f, \omega_g)e^{-i\omega T_1 - i\omega_g R_1}$, which does not depend on the ordinal number of the branch to which it is connected (on the position of the signal), and filter $U_1(\omega, \omega_r)e^{-i\omega T_2 - i\omega_g R_2}$, which is coordinated with expected 1-position signal $U_1(t, r)$ where $T = T_1 + T_2$, $R = R_1 + R_2$. The first filter is identical for all M branches and it is advisable to make it general by connection at the input of the receiving unit.

Series of two filters at the input of the receiving device, the whitening filter depicted in Figure 3.3 and the coordinate filter depicted in Figure 3.3 and the coordinate filter mentioned above, define the spatial filter with transfer function

$$\dot{K}(\omega, \omega_g)\, \dot{K}^*(\omega, \omega_g)\, e^{-i\omega T_1 - i\omega_g R_1} = \frac{N_0}{2}\, \frac{1}{N(\omega, \omega_g)}\, e^{-i\omega T_1 - i\omega_g R_1}. \qquad (3.20)$$

Figure 3.4 below shows a schematic diagram of the optimal receiver built with coordinated spatial filters.

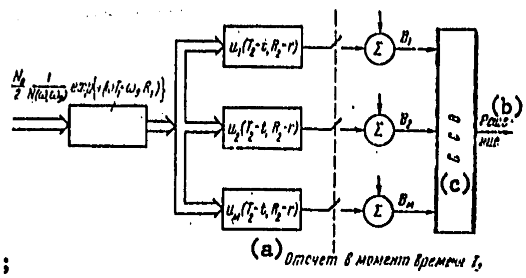


Figure 3.4. Optimal Signal Processing Against a Background of Colored Noise.

Key: a) Reading at Moment in Time T_2;
b) Solution (Decision);
c) Comparison and Decision Circuit.

Let us consider how rule (3.4) looks during the use of three concrete, widespread systems of signals.

For a system of M equiprobable signals of equal energy (in particular for a system of orthogonal signals in the amplified sense [104]), the decision rule for the criterion of an ideal observer takes the form

$$F_l > F_g, \quad g = \overline{1, M}; \quad g \neq l. \qquad (3.21)$$

Such a receiver is, evidently, most convenient for realization on the basis of coordinated filters. If two opposite signals $s_1(t)$ and $s_2(t) = -s_1(t)$ are used to transmit information the optimal processing device must decide in favor of signal $s_1(t)$ where the inequality

$$F_1 > 0 \qquad (3.22)$$

is met and in favor of signal $s_2(t)$ where the the opposite inequality is fulfilled.

But if the problem being solved is the problem of optimal detection, according to the Neuman-Pearson criterion, of signal s(t) with a known arrival time, it is not difficult from (3.4) to receive the following rule for choosing a decision:

$$F > \Omega. \qquad (3.23)$$

Threshold Ω is determined by working characteristics with a given level of probability of a false alarm.

In conclusion, let us observe that the algorithms for processing space-time signals in angular coordinates have an appearance entirely analogous to those we have considered.

For example, the quantities F_1 and B_1 in processing field $z(t,\vartheta)$ in the distant zone at angles of approach ϑ should be computed from relations that coincide with (3.8) by structure

$$\left.\begin{aligned} F_l &= \int_0^T \int_{-\Theta}^{\Theta} z(t,\vartheta)\,\vartheta_l(t,\vartheta)\,dt\,d\vartheta, \\ B_l &= \int_0^T \int_{-\Theta}^{\Theta} u_l(t,\vartheta)\,\vartheta_l(t,\vartheta)\,dt\,d\vartheta, \end{aligned}\right\} \quad (3.24)$$

where functions $B_1(t,\vartheta)$ are determined by solving the integral equation.

$$\int_0^T \int_{-\Theta}^{\Theta} B_n(t,t',\vartheta,\vartheta')\,\vartheta_l(t',\vartheta')\,dt'\,d\vartheta' = u_l(t,\vartheta). \quad (3.25)$$

3.3. Receiving Messages under Conditions of an Ideally Classified Sample by which the Channel Is Studied.

We observed in subchapter 2.7 that ideal classification of the field being analyzed is a condition that guarantees optimal estimates of the parameters of a stochastic channel. To insure this condition we may use special tests (probing) signals or information signals may be used with the traces of manipulation removed by decision feedback. The SIIP [53], Adapticom [79], and others are examples of functioning modems with test signals. In such communications systems the probing position of signal s(t) does not carry information about the message being transmitted, but is used only to measure the parameters of a stochastic channel and organize the work of the receiving unit , which is, therefore, adaptive. Ongoing estimates of the unknown parameters of the channel are computed and the receiver adapts itself primarily for those realizations of the input oscillation which are generated by the test signal, even though, in principle, information signals can also be used to some degree for this purpose. The form and duration of test signals may, in principle, differ from the form and duration of information signals.

We must consider the two possible cases of using probing signals separately. These are nonselective and selective change of channel parameters in time. The first is quite well treated in works [46, 49, 53]. If the parameters of a channel change slowly, that is, if condition (1.73) is fulfilled so that they are practically invariant in a time interval of duration KT, in order to measure the parameters of the channel it is sufficient to transmit one test signal of duration T for K information signals (see Figure 3.5 below).

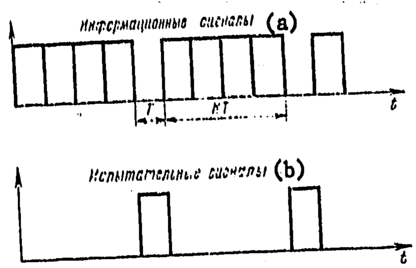


Figure 3.5. Transmission of Information and Test Signals in Time: a) Sequential; b) Parallel

Key: a) Information Signals;
b) Test Signals.

As the rate of change in channel parameters increases the number of information packages K supported by one test pulse must be reduced. There is a variation of the system in which test and information packages are alternated [59].

Despite the viewpoint found in [46], test signals can also be used in the situation of fadeouts that are selective in time when the relation $\tau_{cor} < T$ is fulfilled. Under these conditions the probing signal should be emitted continuously, parallel with emission of the information signals.

Parallel transmission of test signals is evidently most justified for the use of parallel modems (multifrequency systems [53]). V. I. Siforov's scheme [87] designed for work in a channel with fadeouts that are selective in time but nonselective by frequency realizes a particular case of parallel transmission of a test signal. In this scheme the test signal is a continuously emitted sinusoid of definite frequency. The tract of processing information signals can be adjusted according to the channel's reaction to this signal.

It should be emphasized that the use of test signals specially designed for studying the parameters of a communication channel and carrying no usable information has three disadvantages. In the first place, some of the power of the transmitter is, from the point of view of the source of messages and customer, wasted. In the second place, the potential rate of information transmission is lessened because it is necessary to send probing signals occasionally instead of information signals (this consideration remains in force for parallel transmission of probing signals also). In the third place, if a system is oriented to estimation of channel parameters by test signals only, the quality of estimations will not be good enough in some channels and as a result the system will not have the required noise suppression.

These shortcomings are particularly noticeable in channels with relatively rapid changes in channel parameters in time. Thus, if one test signal is sent in sequence with every two working signals for probing purposes, the channel loses one-third of its information transmitting power. But the average energy of the transmitter should be distributed between test and information signals in a ratio no smaller than 1:1 at the least.

These difficulties may compel the system developer to reject the idea of using only test signals to estimate channel parameters. However, the developer of a contemporary data transmission system can no longer completely discard study of communication channel parameters and adaptation because this contradicts the spirit of the times and the whole accumulated experience of communications engineering.

The answer to this situation is to construct processing devices that adapt themselves directly to the parameters of the communication channel according to information packages of signals, using decision feedback if possible [21, 53, 58]. If the probability of erroneous reception of characters is very low and $\tau_{cor} >> T$, such feedback can provide an ideal classification of the sample by which channel parameters are studied; in other words, it makes it possible to remove the traces of manipulation in this sample. The quality of estimates of channel parameters will be determined by the full energy of the demanipulated signal E^n, which participates in the measurement and may be higher than where parameters are measured only by probing samples, where accumulation is limited.

In principle, it is possible to combine measurement of channel parameters by information samples using decision feedback (the primary element) with measurement by infrequently transmitted test signals (the emergency element which assumes the primary functions of measurement where waves are carrying poorly) to improve the quality of estimates [53, 133].

Now let us turn directly to algorithms for optimal reception of discrete messages under conditions of ideal classification of the field being analyzed. Using the principle of statistical self-adjustment (adaptation) yields an algorithm of reception against a background of white noise that coincides in form with algorithm (3.4) for reception in an ideal channel

$$\hat{F}_l - \hat{B}_l > \ln C_{gl} + \hat{F}_g - \hat{B}_g, \quad g = \overline{1, M}; \quad g \neq l. \tag{3.26}$$

Functionals $\hat{F}_l$ determine the magnitude of the correlation between the estimate of the expected l-position signal $\hat{u}_l(t, r)$ and the field accessible to observation

$$\hat{F}_l = \frac{2}{N_0} \int_0^T \int_0^R z(t, r) \hat{u}_l(t, r)\, dt\, dr. \tag{3.27}$$

The quantities $\hat{B}_l$, $l = \overline{1, M}$ determine the ratio of the energy of estimation of signal l at the place of reception to the spectral density of noise output

$$\hat{B}_l = \frac{1}{N_0}\int_0^T\int_0^R \hat{u}_l^2(t,\ r)\,dt\,dr. \tag{3.28}$$

Let us stress that in the case of reception with adaptation, quantities $\hat{F}_1$ and $\hat{B}_1$ should be computed by the input oscillation at each of the space-time intervals of observation. The general appearance of the schematic diagram of an adaptive device that distinguishes M signals is shown in Figure 3.6. The device operates on the following algorithm.

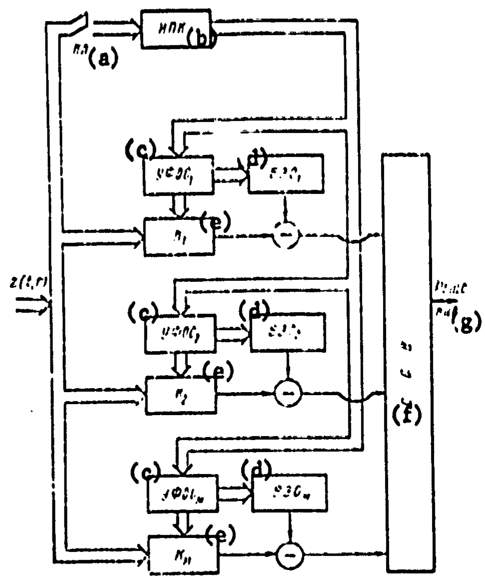


Figure 3.6. Adaptive Device that Distinguishes M Signals.

Key:
a) Switch;
b) Channel Parameter Measurement Block (IPK);
c) Device for Formation of Signal Estimate [sub-character indicates signal position];
d) Computer of Estimation Energy [sub-character indicates signal position];
e) Correlator [sub-character indicates signal position];
f) Comparison and Decision Circuit;
g) Solution (Decision).

In the channel parameter measurement (IPK) block the pulsed surge (or some other) characteristic of the channel is measured by the algorithms set forth in Chapter 2. The estimate of the characteristic obtained in this block, for example H(t, ξ, r) is used in the device for forming the signal estimate (UFOS) at position 1 in conformity with the rule

$$\hat{u}_l(t,\ r) = \int_0^\infty h(t,\ \xi,\ r)\,s_l(\xi)\,d\xi. \tag{3.29}$$

This estimate, in its turn, arrives at correlator K_1 of the estimate of the 1-position signal and input oscillation, which computes the quantities

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$\hat{F}_1$, and at VEO_1, the computer of the energy $\hat{B}_1$ of the estimate of the signal in position 1. The output signal of each of the M branches of the receiving unit, representing the difference $\hat{F}_1 - \hat{B}_1$, goes to the comparison and decision circuit (SSV). The switch at the input of the IPK is necessary with the sequential method of transmitting test signals where the measurer of channel parameters must proce the input mixture only in the time intervals allocated for transmission of test signals. The working cycle of the switch and of the comparison sampling circuits is controlled by a synchronization unit which is not shown in Figure 3.6.

The receiver whose schematic diagram is shown in Figure 3.6 differs from the receiver of space-time signals in an ideal channel shown in Figure 3.1 primarily in having the channel parameter measurement block (IPK) and the device for forming estimates of expected signals ($UFOS_l$), $l = \overline{1, M}$. The block for measurement of the parameters of a stochastic channel is described in detail in Chapter 2. At the output of this block, in the general case, is a vector of estimates of the coordinates of expansion of the components of the channel characteristics, for example

$$\hat{\vec{y}} = \{\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_N\}; \ \hat{\vec{x}} = \{\hat{x}_1, \hat{x}_2 \ldots \hat{x}_N\}.$$

The quantities that form this vector must be used in the $UFOS_1$ to construct an estimate of the 1-position signal at the place of reception. Taking account of relation (3.29) and the finite dimensionality of the vector of coordinate estimates, this estimate may be written in the form of a sum

$$\hat{u}_l(t, r) = \sum_{k=1}^{N} \hat{x}_k s_{lk}(t, r) + \hat{y}_k \tilde{s}_{lk}(t, r). \tag{3.30}$$

The regular signal $s_{1k}(t, r)$ in position 1 on path of propagation k is defined by the relation

$$s_{lk}(t, r) = \int_{-\infty}^{\infty} \varphi_k(t, \xi, r) s_l(t-\xi) d\xi. \tag{3.31}$$

Signals $s_{kl}(t, r)$, $k = 1, 2, \ldots, N$, $l = 1, 2, \ldots, M$ in two ways. For one, it is possible to compute all ($N \times M$) possible copies of the signal in advance and construct the corresponding number of generators of space-time oscillations that give the corresponding functions at their outputs. Figure 3.7 shows a schematic diagram of a device that forms an estimate of the 1-position signal using signal generators corresponding to different paths of propagation. Generator G_{1k} produces $s_{1k}(t, r)$ in position 1 of path of propagation k. At $\pi/2$ the phase shifter performs a Hilbert transform on signal $s_{1k}(t, r)$ and the oscillation $\tilde{s}_{1k}(t, r)$ is received at its input. The reference

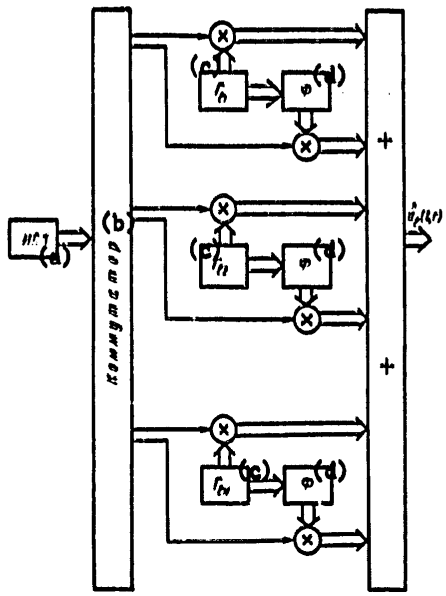

Figure 3.7. Device for Forming an Estimate of the L-Position Signal.

Key: a) Channel Parameter Measurement Block;
b) Commutator;
c) Generator [sub-characters designate signal];
d) Phase Shifter.

signals $s_{lk}(t, r)$ and $\tilde{s}_{lk}(t, r)$ are multiplied by the estimates of the quadrature components of the channel characteristic in path of propagation k, by quantities $\hat{x}_k$ and $\hat{y}_k$ respectively. Quantities $\hat{x}_k$ and $\hat{y}_k$ are distributed by the commutator. In this case the multipliers are simply amplifiers with variable amplification factors. The sum of the signals at the output of these two amplifiers represents an estimate of the random (as a result of change in channel parameters) signal in position l of path of propagation k

$$\hat{u}_{lk}(t, r) = \hat{x}_k s_{lk}(t, r) + \hat{y}_k \tilde{s}_{lk}(t, r). \qquad (3.32)$$

Next the output oscillations of all N branches corresponding to N paths of propagation may be added in the summator to form the estimate of the l-position signal at the receiving point $\hat{u}_l(t, r)$. However, this operation is not essential if further processing of signals is done according to a circuit with N × M channels, not just M channels, in other words, correlator l must consist of N correlators that realize the operations

$$\hat{F}_{lk} = \frac{2}{N_0}\int_0^T\int_0^R \hat{u}_{lk}(t, r) z(t, r)\, dt\, dr, \quad k = \overline{1, N}. \qquad (3.33)$$

Accordingly, the VEO_1 device for computing an estimate of the energy of signal 1 at the point of reception, shown in Figure 3.6, must consist of N subchannels, in each of which the quantity below is computed.

$$\hat{B}_{lk} = \frac{1}{N_0} \int_0^T \int_0^R \hat{u}_{lk}^2 (t, r)\, dt\, dr, \quad k = \overline{1, N}. \qquad (3.34)$$

This method for receiving signals $s_{lk}(t, r)$, $k = \overline{1, N}$, $l = \overline{1, M}$ has the following shortcomings.

1. The difficulty of generating oscillations of a given form. For some types of functions $\phi_k(t, \xi, r)$ which determine the necessary form of the signal (3.31) it may prove simply impossible to construct generators of signals $s_{lk}(t, r)$ on the basis of known electrical or optical elements and devices. This applies particularly to the case where the eigen functions of integral equations of the (2.21) type are used as function $\phi_k(t, \xi, r)$, $k = \overline{1, N}$. In this case, the only thing possible is to realize the algorithm for signal processing in a digital variation based on a general-purpose or specialized computer.

2. The impossibility of reorganizing the circuit with a transition to a new system of signals or a new channel model. The digital variation, of course, does not have this weakness. The second possible way of receiving signals $s_{lk}(t, r)$, $l = \overline{1, M}$, $k = \overline{1, N}$, which do not require construction of special generators, is partially or completely free of these shortcomings. The reference signal in position 1 of path of propagation k may be produced at the output of a filter with variable parameters and characteristics $\phi_k(t, \xi, r)$ in accordance with relation (3.31). Generally speaking, under this approach the number of filters N with variable parameters should correspond to each position of the signal; in other words, the total number of filters is N × M and may prove very large when multiposition systems are used in complex channels. Despite the fact that each group of N different filters repeats M times for different signal positions, in the general case it is impossible to get by with one set of N filters. Of course, the full potential of the unit described containing N × M filters is not used here. With an individual set of filters for each of the M signal positions it is possible to construct a receiving device that takes account of the specific features of distortion by the channel of each particular position of the transmitted signals. For a certain position 1, for example, not all N paths but only $N_l < N$ paths of propagation may be significant. This must be taken into account in building the receiver.

In particular cases of great practical interest there is a possibility of creating M × N reference oscillations on the basis of one set including N different filters. This possibility exists, for example, if the spectra of the signals of different positions at the output of the communications channel (at a certain point R) do not overlap (see Figure 3.8 below). Then it is possible to feed the summary signal $s(t) = \sum_{l=1}^{M} s_l(t)$ to the input of each of N filters $\phi_k(t, \xi, r)$, and at the output separate the signals corresponding to the different positions by means of pass-band filters (see Figure 3.9 below).

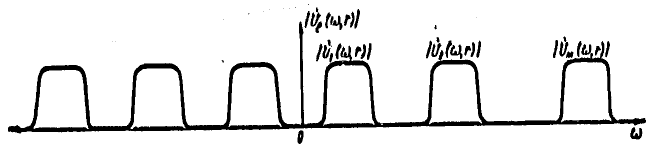

Figure 3.8. Signals with Spectra that Do Not Overlap at the Output of the Channel.

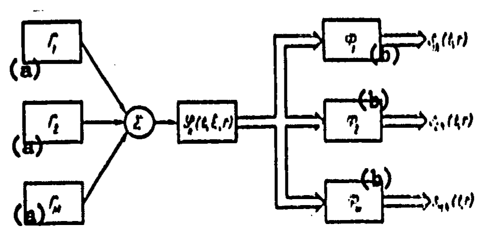

Figure 3.9. Generator of Reference Signals with Non-Overlapping Spectra.

Key: a) Generator;
b) Filter;
c) Summator.

When complex (in particular composite [2, 73]) signals are used for data transmission in actual communication channels, the necessary number of sets of N filters each can always be reduced. A complex signal $s_l(t)$ of arbitrary position l may be represented by the known [22, 109] relation

$$s_l(t) = \sum_{j=1}^{L} s_{lj}\alpha_j(t), \quad l = \overline{1, M}, \tag{3.35}$$

where $\alpha_j(t)$ are elementary signals that form the orthogonal basis and s_{lj} are the coordinates of expansion of a complex l-position signal on the given base.

Elementary signals most often find application in the form of pulses shifted in time

$$\alpha_j(t) = g(t - j\tau_\alpha), \quad j = \overline{1, L}, \tag{3.36}$$

where the pulse of unitary energy

$$g(t) = \begin{cases} \sqrt{1/\tau_\alpha}, & 0 \leqslant t \leqslant \tau_\alpha; \\ 0, & \text{for other } t. \end{cases} \tag{3.37}$$

Using expansion (3.35), it is possible to represent the l-position signal in path of propagation k in the form

$$s_{lk}(t, r) = \sum_{j=1}^{L} s_{lj}\alpha_{jk}(t, r), \qquad (3.38)$$

where the elementary signal at the channel output $\alpha_{jk}(t, r)$ is determined by the relation

$$\alpha_{jk}(t, r) = \int_{-\infty}^{\infty} \alpha_j(t-\xi)\varphi_k(t, \xi, r)\,d\xi \qquad (3.39)$$

and may be obtained at the output of a linear filter with characteristic $\phi_k(t, \xi, r)$. In general, it is necessary to have L sets of N filters to obtain L elementary reference signals $\alpha_1(t, r)$, $\alpha_2(t, r)$, ..., $\alpha_L(t, r)$. Where $L < M$ the gain in number of filters is obvious. It may prove advisable to compute elementary reference signals $\alpha_{jk}(t, r)$ where $L > M$ also, if the properties of the elementary input signals $\alpha_j(t)$ are taken into account.

In the important case where the elementary input signals are functions of type (3.36) and the channel memory ξ_{max} is less than the duration T of the signal and exceeds the duration of the elementary signal by m times

$$\xi_{макс} = m\tau_{\alpha}, \quad m = \overline{1, (L-1)}, \qquad (3.40)$$

the number of filters may be reduced to the quantity

$$L_{мин} = m + 1. \qquad (3.41)$$

In this case, each filter is used for a time interval T_s of duration

$$R = [L/L_{мин}] \qquad (3.42)$$

times. For example, where m = 1 (channel memory is equal to the length of the elementary signal) and L = 11 (the 11-position Barker code) it is possible to get by with just two sets of N filters apiece to compute elementary reference signals $\alpha_{jk}(t, r)$ from (3.39). Each of these filters will be switched on at least six times in the interval T_1 (see Figure 3.10 below). Computation of the input and output signals of the filters should be accomplished by a special unit.

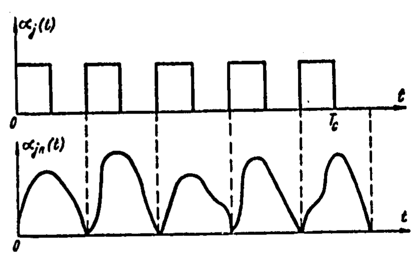

Figure 3.10. Computation of Elementary Reference Signals.

Let us observe also that it is by no means obligatory, after computing LN elementary reference signals $\alpha_{jk}(t, r)$, to construct from them MN reference signals $s_{lk}(t, r)$ corresponding to the transmitted positions. The entire linear part of the receiver may be constructed in a calculation for elementary reference signals $\alpha_{jk}(t, r)$, and where L < M this makes it possible to reduce the number of positions of the expected signals (and the number of corresponding branches of the receiver).

The subsequent computation of the quantities of the correlation coefficient and energy estimates of the transmitted signals by the receiving device is elementary and can be assigned to the comparison and decision circuit.

In conclusion, let us note that the presence of colored Gaussian noise does not introduce any specific differences in the algorithm for optimal processing of signals in a stochastic channel when compared with reception in an ideal channel. The quantities $\hat{F}_1$ and $\hat{B}_1$ should be computed by the receiver from the relations

$$\left.\begin{aligned}\hat{F}_l &= \int_0^T \int_0^R z(t, r)\, \hat{v}_l(t, r)\, dt\, dr,\\ \hat{B}_l &= \int_0^T \int_0^R \hat{u}(t, r)\, \hat{v}_l(t, r)\, dt\, dr.\end{aligned}\right\} \quad (3.43)$$

Function $\hat{v}_1$ in the general case is determined by solving the integral equation

$$\int_0^T \int_0^R B_n(t, t', r, r')\, v_l(t', r')\, dt'\, dr' = \hat{u}_l(t, r). \quad (3.44)$$

Just as in the ideal channel, it is advisable to use whitening filters.

If noise in a channel is non-Gaussian, the integrals in (3.27), (3.33), and (3.43) should be understood in the Ito sense in a number of cases [19, 95]. From an engineering point of view it is significant that integration in the Ito sense is done successfully on both a digital basis and an analog basis.

3.4. Reception of Messages in Conditions of an Unclassified Sample by which the Channel Is Studied and the Use of A Priori Data

Now let us consider the adaptive algorithm for receiving discrete messages using estimates of channel parameters based on information signals under conditions of a completely unclassified sample.

Beginning from the fact that there is a certain estimate of the channel characteristic and using the principle of adaptation, it is not difficult to write an algorithm for receiving M signals in a form coinciding with (3.26):

$$\hat{F}_l - \hat{B}_l > \ln C_{gl} + \hat{F}_g - \hat{B}_g, \quad g = \overline{1, M}; \quad g \neq l, \tag{3.45}$$

where, as before, in the case of "white" noise r = 1 M; r ≠ 1

$$\left.\begin{aligned} \hat{F}_l &= \frac{2}{N_0} \int_0^T \int_0^R z(t, r)\, \hat{u}_l(t, r)\, dt\, dr, \\ \hat{B}_l &= \frac{1}{N_0} \int_0^T \int_0^R \hat{u}_l^2(t, r)\, dt\, dr. \end{aligned}\right| \tag{3.46}$$

However, the estimates included in (3.46) for the M signals expected at the place of reception are now constructed on the basis of information samples of signals.

It turns out that, despite the external identicality of the approaches and algorithms (3.26) and (3.45), the receivers that adapt themselves on the basis of working (information) signal samples may have a structure that is completely different from the structure of receivers that use test signals or other means that provide an ideal classification of the sample by which the channel is studied. Let us look at specific algorithms for constructing the estimates of channel characteristics on the basis of information signals which were synthesized in subchapter 2.7 and see how they are used in a discrete message receiver.

The estimates of expected signals $\hat{u}_l(t, r)$, $l = \overline{1, M}$ included in (3.46) are determined by estimates of the coordinates of expansion of the channel characteristic. Let us assume at the start that to construct an estimate of an l-position signal at the place of reception we use conditional estimates (2.170) of the coordinates of expansion of channel characteristics obtained on the assumption of transmission of an l-position signal:

$$\left.\begin{aligned} \hat{x}_k^l &= \frac{\sigma_{xk}^2 \dfrac{2}{N_0} \displaystyle\int_0^T \int_0^R z(t, r)\, s_{lk}(t, r)\, dt\, dr + m_{xk}}{1 + \sigma_{xk}^2 \dfrac{2}{N_0} \displaystyle\int_0^T \int_0^R s_{lk}^2(t, r)\, dt\, dr}, \\ \hat{y}_k^l &= \frac{\sigma_{yk}^2 \dfrac{2}{N_0} \displaystyle\int_0^T \int_0^R z(t, r)\, \tilde{s}_{lk}(t, r)\, dt\, dr + m_{yk}}{1 + \sigma_{yk}^2 \dfrac{2}{N_0} \displaystyle\int_0^T \int_0^R \tilde{s}_{lk}^2(t, r)\, dt\, dr}. \end{aligned}\right\} \tag{3.47}$$

Estimates (3.47) are obtained on the assumption that there is a priori information on the channel for the case of a "white" noise field.

The estimates $\hat{u}_l(t, r)$, $l = \overline{1, N}$ included in (3.46) may be written, taking account of this remark, in the following form

$$\hat{u}_l(t, r) = \sum_{k=1}^{N} \hat{x}_k^l s_{lk}(t, r) + \hat{y}_k^l \tilde{s}_{lk}(t, r). \qquad (3.48)$$

Figure 3.11 presents a schematic diagram of a receiver that realizes algorithm (3.46) and uses conditional estimates (3.47).

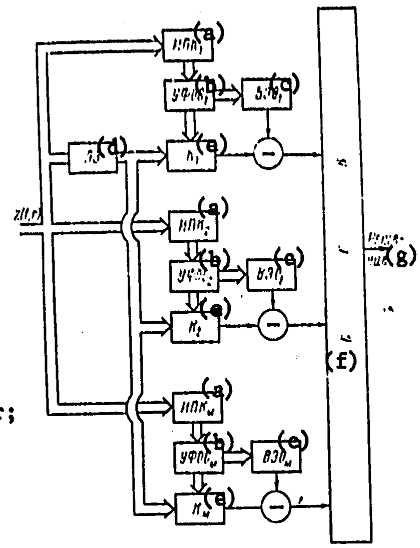

Figure 3.11. Processing Device that Adapts on the Basis of Information Signals.

Key:
- a) Channel Parameter Measurement Block;
- b) Device for Forming Signal Estimate;
- c) Estimate Energy Computer;
- d) Delay Line;
- e) Commutator;
- f) Comparison and Decision Circuit;
- g) Decision (Solution).

The circuit represented in Figure 3.11 differs from that in Figure 3.6 because it has a channel parameter measurement block in each of M branches corresponding to the transmitted signal positions and because it has a delay line, which delays the input oscillation by T. The functioning of the unit shown in Figure 3.11 does not require detailed commentary. The input oscillation z(t, r) arrives simultaneously at M channel parameter measurement block and at the input of the delay line. In the measurement blocks the conditional coordinates of estimate (3.47) are computed during time T. By moment in time T the device for forming the estimate of the l-position signal forms estimate $\hat{u}_l(t, r)$. This estimate is fed to the correlator where, in the time interval (T, 2T), the correlation of the resulting estimate and the copy of the input signal delayed for time T is computed. The work of the estimate energy computation unit does not differ in principle from the analogous device in Figure 3.6 described above. The synchronization block is not shown in Figure 3.11.

A weakness of the algorithm which is realized by the circuit in Figure 3.11 is the need to delay the input signal for a long time interval. It is difficult to insure the necessary stability in a device that contains a delay line.

There is a more convenient way to realize algorithm (3.46), using conditional estimates of expected signals (3.46). After substituting the expressions for the estimates of coordinates (3.47) in (3.46), we can write the estimate of position 1 of the expected signal in the form

$$\hat{u}_l(t, r) = \sum_{k=1}^{N} \frac{2h_{xlk}^2 \Phi_{lk} + m_{xk}}{1 + 2h_{xlk}^2} s_{lk}(t, r) + \frac{2h_{ylk}^2 \widetilde{\Phi}_{lk} + m_{yk}}{1 + 2h_{ylk}^2} \widetilde{s}_{lk}(t, r), \quad (3.49)$$

where, analogously to Chapter 2, these designations are introduced

$$h_{xlk}^2 = \frac{E\nu_{lk}}{N_0}\sigma_{xk}^2; \quad h_{ylk}^2 = \frac{E\nu_{lk}}{N_0}\sigma_{yk}^2; \quad \nu_{lk} = \frac{1}{E_l}\int_0^T\int_0^R s_{lk}^2(t, r)\,dt\,dr. \quad (3.50)$$

Using (3.49) in (3.46), we will write a modified algorithm for computation of the quantity:

$$\begin{Bmatrix}\Phi_{lk}\\ \widetilde{\Phi}_{lk}\end{Bmatrix} = \frac{1}{E_l \nu_{lk}}\int_0^T\int_0^R z(t, r)\begin{Bmatrix}s_{lk}(t, r)\\ \widetilde{s}_{lk}(t, r)\end{Bmatrix}dt\,dr.$$

$$\hat{F}_l = \frac{2E_l}{N_0}\sum_{k=1}^{N}\frac{2h_{xlk}^2 \nu_{lk}\Phi_{lk}^2 + m_{xk}\nu_{lk}\Phi_{lk}}{1 + 2h_{xlk}^2} + \frac{2h_{ylk}^2 \nu_{lk}\widetilde{\Phi}_{lk}^2 + m_{yk}\nu_{lk}\widetilde{\Phi}_{lk}}{1 + 2h_{ylk}^2}. \quad (3.51)$$

It is not difficult to extract the expressions that determine the algorithm for computing quantity $\hat{B}_1$:

$$\hat{B}_l = \frac{E_l}{N_0}\sum_{k=1}^{N}\left[\left(\frac{2h_{xlk}^2\Phi_{lk} + m_{xk}}{1 + 2h_{xlk}^2}\right)^2 + \left(\frac{2h_{ylk}^2\widetilde{\Phi}_{lk} + m_{yk}}{1 + 2h_{ylk}^2}\right)^2\right]\nu_{lk}, \quad (3.52)$$

Relation (3.51), which determines the magnitude of the correlation between the estimate of the 1-position signal at the point of reception and the input oscillation, can be written in a form that permits a more graphic physical interpretation:

$$\hat{F}_l = \frac{2E_l}{N_0}\sum_{k=1}^{N}\frac{2h_{xlk}^2\nu_{lk}}{1 + 2h_{xlk}^2}\psi_{lk}^2 + \frac{2h_{ylk}^2\nu_{lk}}{1 + 2h_{ylk}^2}\widetilde{\psi}_{lk}^2 + \frac{2}{N_0}\int_0^T\int_0^R z(t, r)\,\overline{s_l(t, r)}\,dt\,dr. \quad (3.53)$$

The quantities ψ_{1k} are the values of the correlations between signal $s_{1k}(t, r)$ at position 1 in path of propagation k and the centered input field $\overset{\circ}{z}(t, r) = z(t, r) - \overline{s_1(t, r)}$, that is

$$\begin{Bmatrix} \psi_{lk} \\ \tilde{\psi}_{lk} \end{Bmatrix} = \frac{1}{E\nu_{lk}} \int_0^T \int_0^{R_0} \overset{\circ}{z}(t, r) \begin{Bmatrix} s_{lk}(t, r) \\ \tilde{s}_{lk}(t, r) \end{Bmatrix} dt\, dr. \qquad (3.54)$$

The average value of the expected 1-position signal is represented in the form

$$\overline{s_l(t, r)} = \sum_{k=1}^{N} m_{xk} s_{lk}(t, r) + m_{yk} \tilde{s}_{lk}(t, r). \qquad (3.55)$$

Analysis of (3.53) shows that to compute the quantity $\hat{F}_1$ it is necessary to have a device with a linear part and a nonlinear part. Linear processing of the input oscillation involves computation of the correlation between the average value of the expected signal and the input mixture. Nonlinear quadratic processing may be realized by an N-channel scheme. The correlation between the centered input signal and the reference signal that is formed is computed in each channel. The reference signal may be obtained at the output by any of the means described in the preceding section. In an ideal channel only the linear part should operate; in the absence of a regular component, only the quadratic part should work.

Figure 3.12 gives a schematic diagram of a device that performs quadratic processing. As follows from (3.52), quantity $\hat{B}_1$ determines the "floating" threshold level in branch 1 of the receiver. Receivers with adaptive thresholds have been treated in many works [63, 64]. We will note just one circumstance. By joining (3.52) and (3.51) in algorithm (3.45), it is possible to obtain an algorithm for the functioning of a linear quadratic receiver with a constant threshold level in each of M branches. In this case the linear part of the algorithm of such a receiver coincides with the linear part of formula (3.53). The expressions (3.45) and (3.53) differ only by the values of coefficients for the quantities ψ^2_{1k} and $\tilde{\psi}^2_{1k}$. For most of the signal systems used in practice and considered hereafter the algorithms of optimal reception and decisions made do not depend on the energy magnitudes B_l, $l = \overline{1, m}$ signals at the reception point. This situation, as was demonstrated above, occurs for systems with orthogonal signals of equal energy, for a system with opposite signals, and in the problem of detection according to the Neuman-Pearson criterion. In view of this circumstance, the questions of computing the magnitudes of the thresholds $\hat{B}_1$ are not treated in detail here.

Let us now turn to the more widespread and practically important case of a generalized Gaussian model of a channel and Gaussian noise. It was observed in Chapter 2 that in the "Gaussian case" estimates (3.47) are optimal Bayes estimates. It is natural to raise the question, will this algorithm (3.45) with estimates (3.47) be an optimal Bayes algorithm for reception in a generalized Gaussian channel in a setting

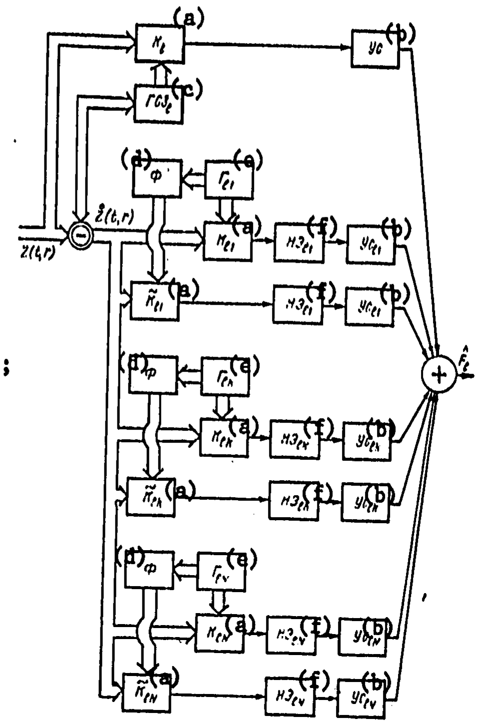

Figure 3.12. Quadratic Processing of an Observed Field.

Key:
- a) Correlator [throughout figure sub-characters refer to signals];
- b) Amplifier;
- c) GSZ [expansion unknown];
- d) Filter;
- e) Generator;
- f) NE [expansion unknown].

of Gaussian noise. It is not difficult to synthesize the optimal Bayes algorithm in conditions of separation of paths:

$$H_l - P_l > H_g - P_g + \ln c_{gl}. \qquad (3.56)$$

The quantity P_l determines the threshold level of branch l and is determined from the relation

$$P_l = \sum_{k=1}^{N} \ln (1 + 2h^2_{xlk}) (1 + 2h^2_{ylk}). \qquad (3.57)$$

The quantities H_l, $l = \overline{1, m}$ are computed by the receiver in each space-time interval of observation from the relation

$$H_l = \frac{2E_l}{N_0} \sum_{k=1}^{N} \frac{2h^2_{xlk} \nu_{lk}}{1 + 2h^2_{xlk}} \psi^2_{lk} + \frac{2h^2_{ylk} \nu_{lk}}{1 + 2h^2_{ylk}} \tilde{\psi}^2_{lk} + \frac{4}{N_0} \int_0^T \int_0^R z(t, r) \overline{s_l(t, r)}\, dt\, dr. \qquad (3.58)$$

Another representation of quantity H_1 is also possible and makes it possible to give it a more graphic physical interpretation:

$$H_l = \frac{2}{N_0}\int_0^T\int_0^R \hat{\overset{\circ}{u}}_l(t, r)\, z(t, r)\, dt\, dr + \frac{4}{N_0}\int_0^T\int_0^R \overline{s_l(t, r)}\, z(t, r)\, dt\, dr. \quad (3.59)$$

The conditional estimate of the fluctuating part of the input oscillation is determined here by the relation

$$\hat{\overset{\circ}{u}}_l(t, r) = \sum_{k=1}^{N} \frac{2h^2_{xlk}\psi_{lk}}{1+2h^2_{xlk}} s_{lk}(t, r) + \frac{2h^2_{ylk}\tilde{\psi}_{lk}}{1+2h^2_{ylk}} \tilde{s}_{lk}(t, r). \quad (3.60)$$

A comparison of algorithms (3.45) and (3.56) shows that in the general case they do not coincide. The fact that thresholds (3.52) and (3.57) are completely different is not important for those systems of signals discussed above. It is significant that, despite the great similarity, quantities $\hat{F}_1$ and H_1 are also computed by different rules. The quantity $\hat{F}_1$, as has been noted several times, is the correlation between the estimate of the input signal corresponding to position 1 and the observed mixture. The quantity H_1, as follows from (3.59), also contains correlations of the estimate of the fluctuating part of the expected signal with the centered input field and the deterministic component of the expected signal with the field at the input. However, the weights with which these quantities are summed to form the quantity H_1 are different. The weight of the second function of the correlation that depends on the average value of the signal is chosen to be two times larger. It is precisely the presence of this weighted coefficient that constitutes the difference between the optimal Bayes receiver (3.45) and estimates (3.47) in the generalized Gaussian channel with additive Gaussian noise. It is clear that in a channel without a regular component, for example in a sub-Rayleigh channel, the algorithms of both receivers under consideration coincide. When opposite signals are used both algorithms are also identical.

In a generalized Gaussian channel with Gaussian noise, the noise suppression of the Bayes receiver is unquestionably greater. However, as will be shown in Chapter 4, even completely ignoring the linear part of the Bayes receiver (assigning it a zero weight for noncoherent processing) reduces the noise suppression of reception only slightly where orthogonal signals are used. In a non-Gaussian channel with a regular component and non-Gaussian noise it is generally impossible to say which of the two algorithms will have better noise suppression. Nonetheless, it may be expected that the difference in the noise suppression of the algorithms will be insignificant and will decrease as the ratio q^2 of the average output of the regular component of the signal to the fluctuating component decreases.

In subchapter 2.7 methods of improving the properties of estimating channel parameters under conditions of a completely unclassified sample were considered. Specifically, there was a discussion of the procedure (2.175) of weighted summation of conditional estimates of parameters (3.47) and a modification of this technique that makes it possible to eliminate the linear bias of the estimates. The material in this section forces us to take a critical approach to this procedure. Thus, in a generalized Gaussian channel an optimal Bayes receiver whose algorithm has been rigorously synthesized does not use "improved" estimates; it uses conditional ones. The use of "improved" estimates in this case inevitably worsens the characteristics of the receiver (if only the algorithms of reception with "improved" and conditional estimates do not prove identical, as occurs in the detection problem, and correspondingly the characteristics remain unchanged). In the non-Gaussian case it is difficult to present a rigorous substantiation of such a strong statement, but qualitatively the picture is quite clear. Where equiprobable signals are used the "improved" estimate of the expected l-position signal is written in the form

$$\hat{u}_l(t, r) = \frac{1}{M} \sum_{r=1}^{M} \sum_{k=1}^{N} \hat{x}_k^r s_{lk}(t, r) + \hat{y}_k^r \tilde{s}_{lk}(t, r). \tag{3.61}$$

Suppose that the transmitted signals have the same energy and form an orthogonal system. Using (3.61) in algorithm (3.45), it is not difficult to show that "improving" the estimates in this case leads to an equivalent problem of reception using a test signal, and the energies of the test and information signals are equal while the intensity of the equivalent additive noise is M times greater than the intensity of the noise actually existing in the channel. We will overlook the fact that with equal energies for the test and information signals the inexactness of estimates of channel parameters will inevitably influence the noise suppression of reception. Let us assume that the channel is measured perfectly and approximates ideal measurement in each interval of analysis. The equivalent signal/noise ratio at the input of the receiver under consideration is 10 lg M db less than the signal/noise ratio actually occurring at the input of the receiver. Practice convinces us that with such a large reserve it is always possible to propose a large number of suboptimal schemes, and among them the scheme with "improved" estimates will almost certainly not be the best for either noise suppression or convenience of practical realization.

It is not difficult to generalize the algorithms obtained here for the case of non-white noise. The quantities $\hat{F}_1$ and $\hat{B}_1$, which determine (3.45), are computed for non-white noise from the relations

$$\left.\begin{aligned} \hat{F}_l &= \int_0^T \int_0^R z(t, r)\, \hat{v}_l(t, r)\, dt\,dr, \\ \hat{B}_l &= \int_0^T \int_0^R u_l(t, r)\, \hat{v}_l(t, r)\, dt\,dr. \end{aligned}\right\} \tag{3.62}$$

Functions $\hat{v}_l(t, r)$ are, generally speaking, the solution to integral equation

$$\int_0^T \int_0^R B_n(t, t', r, r') \hat{v}_l(t', r')\, dt'\, dr' = u_l(t, r) \qquad (3.63)$$

and in the given case should be computed from the relations

$$\hat{v}_l(t, r) = \sum_{k=1}^{N} \hat{x}_k^l v_{lk}(t, r) + \hat{y}_k^l \tilde{v}_{lk}(t, r), \qquad (3.64)$$

where the conditional linear estimates of the coordinates against a background of non-white noise, according to (2.113), have the appearance

$$\left.\begin{aligned} \hat{x}_k^l &= \frac{\sigma_{xk}^2 \int_0^T \int_0^R z(t, r) v_{lk}(t, r)\, dt dr + m_{xk}}{1 + \sigma_{xk}^2 \int_0^T \int_0^R s_{lk}(t, r) v_{lk}(t, r)\, dt dr}, \\ \hat{y}_k^l &= \frac{\sigma_{yk}^2 \int_0^T \int_0^R z(t, r) \tilde{v}_{lk}(t, r)\, dt dr + m_{yk}}{1 + \sigma_{yk}^3 \int_0^T \int_0^R \tilde{s}_{lk}(t, r) \tilde{v}_{lk}(t, r)\, dt dr}. \end{aligned}\right\} \qquad (3.65)$$

Using (3.65) in (3.64), we receive

$$\hat{v}_l(t, r) = \sum_{k=1}^{N} \frac{2\sigma_{xk}^2 \Phi_{lk} + m_{xk}}{1 + 2h_{xlk}^2} v_{lk}(t, r) + \frac{2\sigma_{yk}^2 \tilde{\Phi}_{lk} + m_{yk}}{1 + 2h_{ylk}^2} \tilde{v}_{lk}(t, r), \qquad (3.66)$$

where

$$2h_{xlk}^2 = \sigma_{xk}^2 E_{lk};\quad 2h_{ylk}^2 = \sigma_{yk}^2 E_{lk};\quad E_{lk} = \int_0^T \int_0^R s_{lk}(t, r) v_{lk}(t, r)\, dt dr =$$

$$= \int_0^T \int_0^R \tilde{s}_{lk}(t, r) \tilde{v}_{lk}(t, r)\, dt\, dr; \qquad (3.67)$$

$$\begin{Bmatrix} \Phi_{lk} \\ \tilde{\Phi}_{lk} \end{Bmatrix} = \int_0^T \int_0^R z(t, r) \begin{Bmatrix} v_{lk}(t, r) \\ \tilde{v}_{lk}(t, r) \end{Bmatrix} dt dr.$$

Substituting (3.66) in (3.62), we write the algorithm of linear quadratic processing of the signals against a background of colored noise.

$$\left.\begin{aligned}\hat{F}_l &= \sum_{k=1}^{N} \frac{\sigma_{xk}^2 \Phi_{lk}^2 + m_{xk} \Phi_{lk}}{1 + 2h_{xlk}^2} + \frac{\sigma_{yk}^2 \tilde{\Phi}_{lk}^2 + m_{yk} \tilde{\Phi}_{lk}}{1 + 2h_{ylk}^2}, \\ \hat{B}_l &= \sum_{k=1}^{N} \left[\left(\frac{\sigma_{xk}^2 \Phi_{lk} + m_{xk}}{1 + 2h_{xlk}^2}\right)^2 + \left(\frac{\sigma_{yk}^2 \tilde{\Phi}_{lk} + m_{yk}}{1 + 2h_{ylk}^2}\right)^2\right] E_{lk}.\end{aligned}\right\} \quad (3.68)$$

It can be shown that, as in the case of white noise, the algorithm of signal processing coincides with the algorithm of optimal Bayes processing in a generalized Gaussian channel with a precision level to the weighted coefficients for the linear and quadratic parts. In the optimal Bayes algorithm, as in the case of white noise, this ratio is 2:1.

The algorithms written here for optimal processing of space-time signals look good on paper, but the developers of an information transmission system want to see behind them, above all to find the qualities that make them easier to realize. The chief difficulty in realizing these algorithms is implementing the operations of space-time filtration which are included in the algorithms both explicitly and implicitly.

The characteristics of space-time filters are determined by the functions $\phi_k(t, \xi, r)$, which describe the channel. The appearance of these functions which, generally speaking, may vary greatly, largely determines the noise depression of the reception algorithm containing these functions. The optimal Bayes reception algorithm in a generalized Gaussian channel imposes rigid limitations on the type of function $\phi_k(t, \xi, r)$. They must be eigen functions of integral equation (2.21). Any deviations in the form of functions $\phi_k(t, \xi, r)$ will cause algorithm (3.56) to cease to be optimal and its noise suppression will be lowered owing to the occurrence of correlations among particular branches of dispersion. In a non-Gaussian channel the lack of correlation among paths, which is insured by expansion of the characteristics according to the Karunen-Loew theorem and a corresponding choice of functions $\phi_k(t, \xi, r)$, is no longer a sufficient, but merely a necessary condition for the optimality of reception algorithm (3.45). However, this does not change the essence of the matter: function $\phi_k(t, \xi, r)$ should be selected from the solution to integral equation (2.21).

During the consideration of discrete models of channels in Chapter 2, the important place occupied by the channel with spatially divisible correlation functions was noted. The eigen function for such channels can be represented in the form

$$\varphi_k(t, \xi, r) = \varphi_m^{I}(t, \xi)\, \varphi_l^{II}(r)$$

or

$$\varphi_k(t, \xi, \vartheta) = \varphi_m^{I}(t, \xi)\, \varphi_l^{II}(\vartheta). \quad (3.69)$$

The separation of time and space variables significantly simplifies realization of processing algorithms. Indeed, the 1-position signal in path of propagation k (the index k corresponds to the double index im) is written as follows

$$s_{lk}(t, r) = \varphi_l^{11}(r) s_{lm}(t). \qquad (3.70)$$

where signal $s_{1m}(t)$ may be obtained at the output of a filter that does not include spatial variables

$$s_{lm}(t) = \int_0^\infty s_l(\xi) \varphi_m^l(t, \xi) d\xi. \qquad (3.71)$$

Correspondingly, the quantities Φ_{lk}, which are included in the processing algorithm (3.45), are computed from the relation

$$\Phi_{lk} = \frac{1}{E_l \nu_{ml}} \int_0^T \int_0^R z(t, r) \varphi_l^{11}(r) s_{lm}(t)\, dt dr \qquad (3.72)$$

and may be obtained at the output of two elements connected in series:

a) an antenna that performs spatial filtration described by the relation

$$z_l(t) = \int_0^R z(t, r) \varphi_l^{11}(r) dr$$

or

$$z_l(t) = \int_{-\theta}^{\theta} z(t, \vartheta) \varphi_l^{11}(\vartheta) d\vartheta;$$

b) a filter that performs time processing of signals,

$$\Phi_{lk} = \frac{1}{E_l \nu_{ml}} \int_0^T z_l(t) s_{lm}(t) dt.$$

A channel that is homogeneous in space can be considered as an example.

It was shown in Chapter 2 that elongate spheroidal wave functions are eigen functions that determine the directivity diagrams of receiving antennas, and where the condition of a narrow spatial spectrum (for example, performed in a short wave radio channel) is met, the eigen functions are the functions $\varphi_l^{11}(\vartheta) = \frac{\sin(\vartheta/\delta\vartheta - l\pi)}{\vartheta/\delta\vartheta - l\pi}$.

Narrowly directed antennas with similar directivity diagrams have been extensively used in practice for a long time [7]. There are regular methods of synthesizing such antennas [7, 70]. It is interesting that the optimality of antennas with directivity diagrams in the form of elongate spheroidal wave functions was substantiated from a deterministic standpoint according to the criterion of minimum minor (side) lobes and maximum concentration of energy [7]. Here too the much more powerful statistical criterion of minimum probability of error in reception of discrete messages was used. The fact that optimal space processing according to the Bayes criterion in this particular case coincides with the widely used and practically proven technique of signal processing with narrowly directed antennas permits us to draw two basic conclusions. In the first place, this indirectly confirms the mathematical model of a space-time channel forming the basis of the processing algorithms and, in the second place, it makes it possible to recommend for practical use a whole series of optimal algorithms proposed theoretically here and still not realized for processing space-time signals. Optimal time processing of signals at the output of space filters (antenna) is greatly simplified in the case of factorization of functions $\phi^{I}_{m}(t, \xi)$ which are included in (3.71):

$$\varphi^{I}_{m}(t, \xi) = \varphi^{III}_{p}(\xi)\,\varphi^{IV}_{k}(t).$$

Signal $s_{1m}(t)$ may be treated as the result of filtration by a filter with constant parameters and multiplication by the time function:

$$s_{lm}(t) = \varphi^{IV}_{k}(t) \int_{0}^{\infty} s_{l}(\xi)\,\varphi^{III}_{m}(\xi)\,d\xi.$$

But under these conditions the appearance of the eigen functions $\phi^{III}_{m}(\xi)$, which determine the characteristics of the filters, may be so complex that realization of these filters proves simply impossible using known electrical elements. There are two realistic ways to synthesize filters with the given characteristics: a switch to the optical domain using holographic methods; and digital methods. These matters are discussed below. The approximate filter realization method, which is specified by the frequency characteristic, involves using delay lines with regular amplification coefficients in different branches. This method at present is finding its greatest practical application in systems for processing signals at the output of a stochastic channel. In the case of a channel that is homogeneous for frequency the eigen functions $\phi^{III}_{m}(\xi)$ are Delta functions and the delay lines are a strictly optimal filter.

3.5. Suboptimal Processing of Signals in the Absence of A Priori Data

The algorithms synthesized in the preceding section can be classed as optimal because they use optimal estimates of channel parameters and their structure is optimal in conformity with the principle of statistical self-adjustment. The algorithm of procedure (3.56) is the optimal Bayes algorithm in a generalized Gaussian channel. Where there is an optimal algorithm it is always possible to propose a large number of suboptimal (quasioptimal) algorithms that are similar to it in both structure and characteristics. Here we will construct suboptimal algorithms for signal processing that differ from optimal ones by selection of the functions $\phi_k(t, \xi, r)$, which determine the elements of the receiver, and by the form of estimates of the coordinates of expansion of the channel characteristics. Functions $\phi_k(t, \xi, r)$ here are selected not on the condition of insuring maximum noise suppression, but from several less rigorous, closer-to-practice considerations that take account of a very important factor: convenience of realization of filters whose characteristics are determined by the type of these functions. The suboptimal linear estimates obtained in Chapter 2 which require less a priori data than optimal linear estimates will be considered coordinate estimates.

The processing alogorithm continues to be defined generally by relation (3.45). In our consideration of suboptimal processing algorithms we will assume that space processing and time processing signals are separate. Space processing is accomplished by a set of space filters with assigned characteristics $\phi^2{}_i(r)$ (antennas with given directivity diagrams). We will not concentrate attention here on questions of substantiating a choice and realizing such filters. It is clear that in order to insure maximum noise suppression the type of function $\phi^2{}_i(r)$ must approximate the eigen functions of the Karunen-Loew integral equation as closely as possible. Practical considerations may make it necessary to select completely different values for these characteristics. It is apparent that situations often occur where the characteristics of space filters (antenna directivity diagrams) used in practice and convenient in all respects coincide with or are close to optimal ones. For example, the narrowly directed antennas with directivity diagrams of the type $\sin \vartheta/\vartheta$, which are used in systems of spatial dispersion in the shortwave and ultrashortwave bands, are optimal in a broad class of homogeneous space channels. In the range of optical waves, receivers with heterodyne photodetectors and direct detection are widely used [78, 119]. The optical receiver with direct detection is usually a lattice (set) of photodetectors at whose output electrical signals, proportional outputs, are summed with definite weights. Spatial processing here is reduced to a minimum and involves selecting spatial sectors of the input field which are assumed to be independent. Despite its simplicity, this algorithm of space-time processing is good enough, as study of noise suppression shows, and not just for the optical wave range. The explanation for this is that the described algorithm is a space-time generalization of J. Kostas' well-known idea [59] which permits the processing of time signals with fadeouts that are selective in time. The space-time variant of this idea goes like this. A space-time signal of duration T occupying area of space R is broken into P × Q "subsignals". Each subsignal has a duration $T_1 = T/Q$ and occupies the domain in space $R_1 = R/P$. The length T_1 of the subsignal and the quantity R_1 of the subarea are chosen to be less than the interval of correlation of channel parameters in time and in space respectively. This makes it possible to achieve reception dispersed in time and in space on P × Q branches with noncoherent addition (which is also accomplished by the optical receiver with direct detection). The optical-range heterodyne receiver usually contains a lattice of detectors also. Linear space processing, which precedes detection, is done by means of heterodyning, introducing a reference field from a coherent source with the composite envelope $U_h(t, r) = U_h(r)e^{i\pi 2fkt}$. The usable signal at the output of photo detector i (i = 1, 2, ...) of the heterodyne receiver is the result of the spatial filtration of a certain part of the spatially distributed optimal input signal

$$z_i(t) \approx \operatorname{Re} \int_{A_i} Z(t, r) U_h(r) dr,$$

where Z(t, r) is the composite envelope of the input field and A_i is the area of photo detector i. If the heterodyne field has been well chosen, space processing of the field will be close to optimal. Space processing of acoustic waves is accomplished by means of lattices consisting of elementary wave receivers [69], in other words, analogously to the processing of optical waves in a receiver with direct detection. The use of heterodyning appears promising, but practical difficulties may arise in connection with the construction of generators of an assigned form of acoustic waves. Detailed information on methods and devices for space processing of signals in different wave ranges may be found in the literature [56, 62, 69, 74, 103, 110, 112, 120]. Certain particular questions are considered in more detail below. We will stop for now on suboptimal methods of time processing of signals in channels with fadeouts that are selective in time and frequency. We consider that space (optimal or suboptimal) processing has already been performed and electrical signal v(t) is observed at the output of space processing unit i (we will drop the index i to simplify the notation). At the present time, the problem of noise-proof signal processing (time functions) in stochastic channels has been largely worked out theoretically and to a lesser degree resolved in practice. Orienting ourselves to signal processing systems actually in operation, it may be observed that processing methods based on representation of the channel by a model of a delay line with uniform or nonuniform branches, in other words, on approximation of the characteristic of the receiving filter by a lattice of Delta functions (or functions of readings) has become most widespread. This makes it possible to avoid the necessity of constructing generators of intricately formed reference signals and to use copies of transmitted signals $s_l(t, \,)$, $l = \overline{1, M}$, shifted relative to the moment of transmission; in other words, signals $s_{lm}(t) = s_l(t - \xi_m)$, $l = \overline{1, m}$, $m = \overline{1, N}$. As a rule, delay lines with uniform branches when $\xi_m = m/F_1$ are considered. The use of delay lines recommended itself as an effective means of controling signal fadeout by frequency in actual communication channels. Moreover, in channels that are homogeneous for frequency the delay line is also an optimal filter. However, other methods of realizing the ideas of dispersed reception by frequency based on representation of the channel by a model that differs from the delay line model must not be discarded. In particular, when using the method of local approximation of the surge characteristic or the Goodman-Reswick technique, when the approximating functions $\phi_{pm}(\xi)$ (2.46) and (2.47) are well chosen, it may prove convenient to use weighted segments of the spectra of the transmitted signals as reference signals. For example, an approximate realization of the Goodman-Reswick technique will be forming reference signals $s_{lk}(t)$ by passing the transmitted signals through a set of filters whose transfer functions $\Phi_i(f)$ approximate the functions $\sin \Delta f/\Delta f$ (at least the primary lobe) and are shifted in frequency by a quantity that is a multiple of $2/\Delta$ (see Figure 3.13 below).

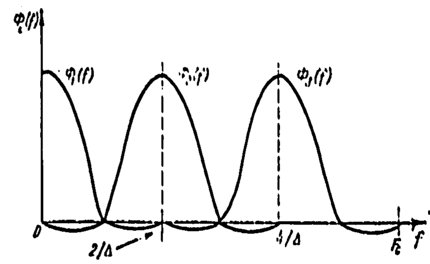

Figure 3.13. Possible Realization of the Goodman-Reswick Technique in the Frequency Domain.

The realization of such filters may prove no more complex than realization of a delay line with good characteristics. The experience of automatic control system developers, who use these methods extensively to identify dynamic objects, is good evidence of this.

Examples of realization of frequency-dispersed reception by this technique can be found in data transmission engineering [6]. As has already been observed, until recently the model of a channel with time-selective fadeouts had limited application because of the short duration of transmitted signals compared to the length of the interval of correlation of channel parameters in time. However, the use of noise-like signals in a channel such as an optical channel [119] or a space radio channel [76] forces us to take account of random changes in channel parameters during a time interval of length T. If the selectivity of the channel in time is ignored, the channel (with a fixed frequency and spatial coordinate) is represented as a model of an amplifier with a random amplification factor γ_k which is constant for time interval [kT, (k+1)T]. The factor may change quite abruptly from interval to interval, assuming independent values. Figure 3.14a below gives an example of a true change in the amplification function $\gamma(t)$ and its model. If we begin from the propositions of the theory of function approximation, in this case we have an approximation by a zero-order polynomial in segments of fixed length. If the model of slow fadeouts in time reflects the properties of a real channel, the error of this approximation may be kept small. But if the function $\gamma(t)$ changes significantly during a time interval of length T (the interval of approximation), error will, with a probability close to one, take on greater values. The first thing that comes to mind is that the interval of approximation can be reduced, that is, the interval of length T can be broken into Q subintervals of length $T_1 = T/Q$ (see Figure 3.14b). The Kostas scheme described above may be approached in exactly this way. The idea of approximating functions with polynomials in segments may be laid as the foundation for constructing a whole class of algorithms for processing signals in channels with time-selective fadeouts. For example, it is possible to use piecewise linear approximation of changes of channel parameters in time. The error of approximation in this case may be significantly less than in the case of

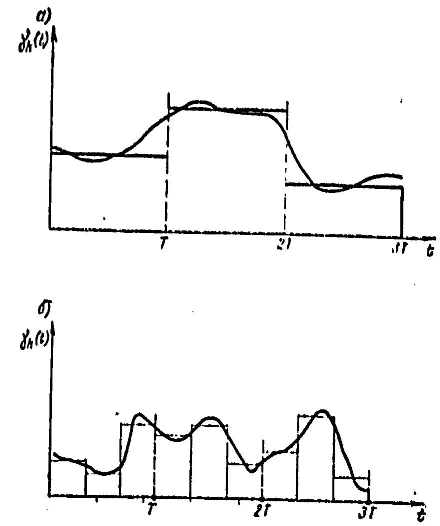

Figure 3.14. Illustration of Approximation of a Transmission Factor: a) for the Entire Interval of Signal Activity; b) in Part of the Interval of Signal Length.

approximation by zero-order polynomials, but technical realization of devices for signal processing is much more complex and requires construction of generators of linearly changing voltage (sawtooth) with a controllable inclination factor. In the current stage of technological development the Kostas' scheme seems most attractive with respect to simplicity of realization. Using the results of Chapter 2, it is easy to suggest the whole series of algorithms for suboptimal processing based on different channel models. It makes sense to do this, with an eye to a concrete communication channel and available equipment. However, let us move on to a consideration of suboptimal estimates of the coordinates of expansion of channel characteristics for a certain orthogonal system of functions, assuming that this system has already been selected. Analysis of current signal processing devices convinces that developers usually base them on white noise as a model of fluctuation noise. This is despite the large number of theoretical works which consider a model of colored noise, despite the fact that it is fundamentally clear what needs to be done in case the noise is not white, and even despite the relative simplicity of building a circuit with a "whitening" filter. It is not difficult to understand the principle of this disregard for the true correlation properties of additive noise. The first reason is that, as noted above, the white noise component is invariably present in the observed mixture. The question concerns the ratio of intensities of the "white" and "non-white" components. The second reason for failure to consider the coloration of additive noise is that, for various ratios of the intensities of colored and white noise, a scheme figured for white noise produces virtually the same noise suppression as an optimal one for non-white noise. It must be remarked, however, that this comparison is usually made within the framework of a Gaussian model of additive noise. In the case of non-Gaussian noise these conclusions often prove incorrect and then it is unquestionably necessary to take account of the coloration of noise and use an optimal whitening filter.

When a model of white noise is used the estimates of maximum plausibility of the coordinates of expansion of the pulse surge characteristic of the channel have this form

$$\left.\begin{aligned}\hat{x}_k^l &= \frac{\int\limits_0^T\int\limits_0^R z(t, r)\, s_{lk}(t, r)\, dt\,dr}{\int\limits_0^T\int\limits_0^R s_{lk}^2(t, r)\, dt\,dr},\\ \hat{y}_k^l &= \frac{\int\limits_0^T\int\limits_0^R z(t, r)\, \tilde{s}_{lk}(t, r)\, dt\,dr}{\int\limits_0^T\int\limits_0^R \tilde{s}_{lk}^2(t, r)\, dt\,dr}.\end{aligned}\right\} \qquad (3.73)$$

If the space-time channel model described above which yields a Kostas' space-time scheme is used, the estimates of the coordinates of expansion of the channel characteristic (3.73) assume the form

$$\left.\begin{aligned}\hat{x}_{kl}^l &= \frac{1}{R_1}\int\limits_{kT_1}^{(k+1)T_1}\int\limits_{lR_1}^{(l+1)R_1} z(t, r)\, s_l(t)\, dt\,dr \Big/ \int\limits_{kT_1}^{(k+1)T_1} s_l^2(t)\, dt,\\ \hat{y}_{kl}^l &= \frac{1}{R_1}\int\limits_{kT_1}^{(k+1)T_1}\int\limits_{lR_1}^{(l+1)R_1} z(t, r)\, \tilde{s}_l(t)\, dt\,dr \Big/ \int\limits_{kT_1}^{(k+1)T_1} \tilde{s}_l^2(t)\, dt.\end{aligned}\right\} \qquad (3.74)$$

but if spatial processing has already been performed and signal Z(t), the function of time in a certain spatial path i (we omit the index i), is being considered, these estimates assume the form

$$\left.\begin{aligned}\hat{x}_k^l &= \frac{1}{E_{lk}}\int\limits_0^T z(t)\, s_{lk}(t)\, dt,\\ \hat{y}_k^l &= \frac{1}{E_{lk}}\int\limits_0^T z(t)\, \tilde{s}_{lk}(t)\, dt,\end{aligned}\right\} \qquad (3.75)$$

where

$$E_{lk} = \int\limits_0^T s_{lk}^2(t)\, dt = \int\limits_0^T \tilde{s}_{lk}^2(t)\, dt. \qquad (3.76)$$

Function $s_{lk}(t)$ and its Hilbert conjugate $\tilde{s}_{lk}(t)$ is a regular l-position signal $l = \overline{1, M}$ in frequency-time path of propagation k. For the given system of signals in transmission, the type of functions $s_{lk}(t)$ and $\tilde{s}_{lk}(t)$ is determined by the choice of the channel model. For example, if the channel is described by a delay line model with uniform branches for nonselective time fadeouts, the functions $s_{lk}(t)$ are determined from the relation

$$s_{lk}(t)=s_l(t-k/F_c),\quad l=\overline{1,M},\quad k=\overline{1,M}. \qquad (3.77)$$

but if, in addition, selectivity of fadeout in time is present and accounted for, and the method of local approximation (which yields the Kostas scheme) is used to measure the channel characteristic, then estimates (3.74) assume the form

$$\left.\begin{aligned}\hat{x}^l_{km}&=\int\limits_{mT_1}^{(m+1)T_1} z(t)\,s_l(t-k/F_c)\,dt\Bigg/\int\limits_{mT_1}^{(m+1)T_1} s_l^2(t-k/F_c)\,dt,\\ \hat{y}^l_{km}&=\int\limits_{mT_1}^{(m+1)T_1} z(t)\,\tilde{s}_l(t-k/F_c)\,dt\Bigg/\int\limits_{mT_1}^{(m+1)T_1} \tilde{s}_l^2(t-k/F_c)\,dt.\end{aligned}\right\} \qquad (3.78)$$

Let us determine concretely the structure of a receiver working according to algorithm (3.45) in the case of using certain suboptimal estimates considered in this section for the more widespread signal systems. As already noted above, for a system of signals of equal energy (at the receiving end) and also in the detection problem by the Neuman-Pearson criterion, there is no need to compute the quantities of the energy of the signals being received $\hat{B}_l$, $l=\overline{1,M}$; it is enough to determine only the magnitudes of the correlations of the oscillation received and estimates of the expected signal, that is, the quantities $\hat{F}_l$. In the case of "white" noise, these quantities have the form (3.46), while in the absence of space processing

$$\hat{F}_l=\frac{2}{N_0}\int\limits_0^T z(t)\,\hat{u}_l(t)dt,\quad l=1,M. \qquad (3.79)$$

In this case the estimates of the estimated signals for all M positions are constructed on the basis of suboptimal estimates of channel characteristics. Let us note that the multiplier before the integral in (3.79) and (3.46) has been kept only to preserve the dimensionality of quantity $\hat{F}_l$ in different algorithms. A knowledge of the spectral density of the "white" noise output for the signal systems under consideration here is in no way essential during construction of a processing algorithm because the quantities $\hat{F}_l$ with different indexes are subsequently compared among themselves. If the estimates of the coordinates of expansion of the space-time characteristic of a channel are constructed according to algorithm (3.74) and the algorithm for processing signals is given in general form by the relation (3.45), it is not difficult to establish the final algorithm concretely, giving it the form

$$\hat{F}_l=\frac{2}{N_0}\sum_{k=1}^{Q}\sum_{i=1}^{P}\Psi_{lki}^2+\tilde{\Psi}_{lki}^2, \qquad (3.80)$$

where the following designations have been introduced.

$$\left\{\begin{matrix}\psi_{lkl} \\ \tilde{\psi}_{lkl}\end{matrix}\right\} = \frac{1}{E_p \nu_{lk} R_1} \int\limits_{k\Gamma_1}^{(k+1)\Gamma_1} \int\limits_{lR_1}^{(l+1)R_1} z(t, r) \left\{\begin{matrix} s_l(t) \\ \tilde{s}_l(t)\end{matrix}\right\} dt dr. \qquad (3.81)$$

The signal processing algorithm (3.80) is fairly simple and does not need explanation. The simplicity of performing processing operations in space and time is an unquestioned advantage of it.

It is perfectly obvious that the cost of this simplicity is a lessening of noise suppression in the reception of discrete messages. It is possible to diminish the loss of noise suppression by introducing more complex spatial processing, but practical considerations always prompt us to separate spatial from temporal processing. Let us consider a few algorithms for suboptimal processing of time function-signals, assuming that suboptimal spatial processing has already been accomplished.

If the delay line model is used to describe the filtering properties of a channel and fadeouts in time are considered nonselective, then it is not difficult, using (3.75) and (3.77) in (3.79), to write a suboptimal algorithm for processing the signal received in a form that allows a graphic physical interpretation. Correlation estimate $\hat{F}_1$ should be computed according to the observed oscillation for different signal positions, l = 1, 2, ..., M from the relation

$$\hat{F}_l = \frac{2}{N_0} \sum_{k=1}^{N} \psi_{lk}^2 + \tilde{\psi}_{lk}^2, \qquad (3.82)$$

where the following designations are introduced

$$\left\{\begin{matrix}\psi_{lk} \\ \tilde{\psi}_{lk}\end{matrix}\right\} = \frac{1}{E_l \nu_{lk}} \int\limits_0^T z(t) \left\{\begin{matrix} s_l\left(t - \frac{k}{F_c}\right) \\ \tilde{s}_l\left(t - \frac{k}{F_c}\right)\end{matrix}\right\} dt. \qquad (3.83)$$

Where broad-band signals are used, the quantities v_{1k} may be considered identical for all values of k. Then, as follows from (3.82), correlation F_1 can be treated as a quantity proportional to the sum of the squares of the envelopes at the output of the filters, coordinated with the copies of the transmitted 1-position signal that are delayed for a time interval that is a multiple of $1/F_s$. A schematic diagram of branch 1 of an optimal signal processing device is shown in Figure 3.15 below.

Let us observe that the scheme depicted in Figure 3.15 can get by with one squarer because it must be used for each of the channels at different moments in time which are multiples of 1/F . It is not difficult to see that (3.a2) realizes the algorithm of quadratic summation, which is widespread in practice [49].

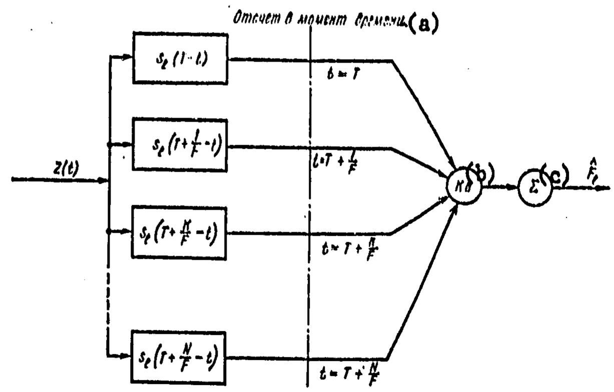

Figure 3.15. Suboptimal Processing Based on a Delay Line.

Key: a) Reading at Moment in Time;
b) Squarer;
c) Summator.

A modification of algorithm (3. 2) can be obtained by setting up a delay line with amplification factors that change stepwise in time in each branch as a model of the channel. In this case the suboptimal estimates of the coordinates of expansion of the pulse surge characteristic of the channel will be estimates (3.77), and the signal processing algorithm assumes the form

$$\hat{F}_l = \frac{2}{N_0}\sum_{k=1}^{N}\sum_{m=1}^{Q} \psi^2_{lmk} + \tilde{\psi}^2_{lmk}, \qquad (3.84)$$

in which these designations have been introduced

$$\left\{\begin{matrix}\psi^2_{lmk}\\ \tilde{\psi}^2_{lmk}\end{matrix}\right\} = \frac{1}{E\,\nu_{lk}}\int_{mT_1}^{(m+1)T_1} z(t)\left\{\begin{matrix}s_l\left(t-\frac{k}{F_c}\right)\\ \tilde{s}_l\left(t-\frac{k}{F_c}\right)\end{matrix}\right\}dt. \qquad (3.85)$$

It should be remembered that the processing algorithm (3.84) will be "good" as long as it is possible to separate all NQ paths at the place of reception. But if the conditions of path separation are violated, algorithm (3.84) loses its suboptimal qualities. This puts restrictions on the signals being transmitted. Analysis of (3.85) makes it possible to formulate the qualitative requirements for these signals. To insure fulfillment of the condition of separation of paths at the receiving point, it is sufficient if the conditions of separation of N paths caused by the filtering properties of the channel (passing through a delay line with branches) are met for a segment (part) of the signal with length T_1. Detailed analysis shows that the complex composite signals used today can, in principle. meet this requirement approximately. However, this does not preclude searching for optimal composite signals for a concrete channel with a given degree of fade-out selectivity in time and frequency. This is a problem of great practical interest, but beyond the scope of this book. It may be solved by development and generalization of works [17, 46]. The main thing is to use the modern systems approach based on the requirement of insuring maximum efficiency in the whole data transmission system in solving the problem of signal synthesis.

From a practical point of view, the channel models and corresponding estimates and algorithms of signal processing based on the adaptive, single-path model of a multipath channel considered in Chapter 2 (2.58) are interesting. In this case the path with the maximum transfer coefficients is selected as the path for each space-time interval of analysis. The corresponding signal processing algorithms will belong to the class of algorithms with autoselection [6, 49, 104]. For discrete signal discrimination devices with automatic selection of the most powerful path, the processing algorithm is given in general form by the relation

$$\hat{F}_l > \hat{F}_g, \quad g = \overline{1, M}; \quad g \neq l, \qquad (3.86)$$

where F_l is the magnitude of the correlation between the observed oscillation and the estimate of the l-position signal in the path of propagation that has the greatest intensity

$$\hat{F}_l = \frac{2}{N_0}\int_0^T\int_0^R z(t, r)\,[\hat{x}_k s_{lk}(t, r) + \hat{y}_p s_{lp}(t, r)]\,dt\,dr. \qquad (3.87)$$

It is assumed that x_k has the greatest modulus of all N estimations $\hat{x}_1$, $\hat{x}_2$, ..., $\hat{x}_N$ and that $\hat{y}_p$ is greatest among the estimates $\hat{y}_1$, $\hat{y}_2$, ..., $\hat{y}_N$. The structure of an optimal processing device with autoselection is depicted in Figure 3.16 below.

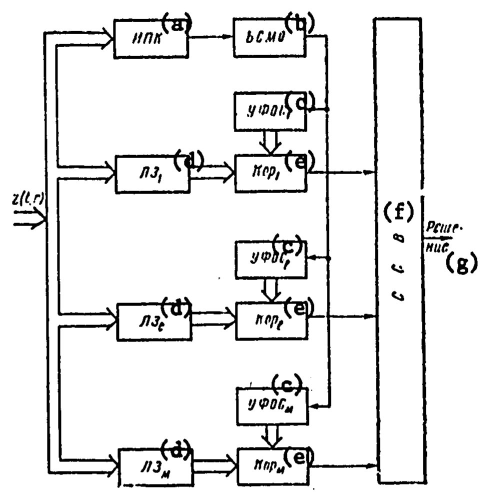

Figure 3.16. Optimal Processing with Selection of the Most Powerful Path.

Key:
- a) Channel Parameter Measurement Block;
- b) Estimate Modulus Comparison Block;
- c) Signal Estimate Forming Unit;
- d) Delay Line;
- e) Correlator;
- f) Comparison and Decision Circuit;
- g) Decision (Solution).

The channel parameter measurement block should construct unconditional estimates of the coordinates of expansion of the channel's pulsed surge characteristic. It is obvious that this requires the use of a test signal, a circuit with decision feedback, or one of the methods of obtaining unconditional estimates from conditional estimates as described in Chapter 2. The block for comparison of modules selects the estimates of quadrature components of paths that have the greatest intensity in the given interval of analysis $\hat{x}_N$ and $\hat{y}_p$ (in the particular case k = p). Then they are used to form estimates of all signal positions in the path of propagation with maximum intensity. The correlations of these M estimates with the input field are computed in M space-time correlators and then compared among themselves in the comparison and decision circuit. The delay lines, which delay the input signal by T, are needed only when information signals are used to form estimates of channel parameters. When a test signal is used the need for delay lines disappears. A great shortcoming of this algorithm is the fact that it assumes the use of unconditional estimates of channel parameters. The simplest way to receive unconditional estimates is to use test signals. A modification of the SIIP system that automatically selects the beam of maximum intensity in a shortwave channel during channel measurement by test signal is described in [53].

In the absence of test signals or decision feedback the idea of autoselection can be realized in a form that does not require the construction of unconditional estimates. In this case, the algorithm for computation of the quantity $\hat{F}_1$ included in (3.86) has this form

$$\hat{F}_l = \frac{2}{N_0}\int_0^T\int_0^R z(t, r)[\hat{x}_k^l s_{lk}(t, r) + \hat{y}_p^l \tilde{s}_{lp}(t, r)]dtdr, \qquad (3.88)$$

that is, the difference from (3.87) is that this expression uses conditional estimates of channel characteristics instead of unconditional ones. Then we may set aside autoselection "by quadrature" and use, for signal reception, the one physical path of propagation which has the greatest modulus of surge characteristic $\hat{\gamma}_k = \sqrt{\hat{x}^2_k + \hat{y}^2_k}$ of all N moduli $\hat{\gamma}_1, \hat{\gamma}_2, \ldots, \hat{\gamma}_N$ in the given interval of analysis. In this case it follows (3.88) that

$$\hat{F}_l = \frac{2}{N_0}\int_0^T\int_0^R z(t, r)[\hat{x}_k^l s_{lk}(t, r) + \hat{y}_k^l \tilde{s}_{lk}(t, r)]\, dtdr. \qquad (3.89)$$

If estimates of maximum plausibility are used for x and y , then, substituting (3.73) in (3.89), it is not difficult to obtain the algorithm for computing the quantity

$$\hat{F}_l = \max_{p=\overline{1,N}}\{\psi_{lp}^2 + \tilde{\psi}_{lp}^2\} = \psi_{lk}^2 + \tilde{\psi}_{lk}^2. \qquad (3.90)$$

Quantity $\hat{F}_1$ can be obtained as the square of the envelope at the output of the space-time filter that is coordinated with the 1-position signal in path k, the path of greatest intensity in the given interval of analysis of the input oscillation.

The signal processing algorithms considered here are suboptimal for a system of signals that are orthogonal in the amplified sense. These algorithms may be used in the detection problem or to distinguish signals of arbitrary form, but they are completely unsuitable for a system of opposite signals. In fact, analysis of all the algorithms in this section indicates that they assume quadratic processing of the oscillations received. It is plain that information contained in the sign (phase) of the signal transmitted in a communication channel is lost with this kind of processing. Linear processing of the input oscillation must be present in the algorithm for receiving opposite signals. It is natural for receiving opposite signals to use a model of a channel that takes account of regular phase distortions of transmitted signals. In this case suboptimal algorithms may completely ignore chaotic changes in a stochastic channel, which leads to the deterministic model of a stochastic channel described in Chapter 2, formula (2.60). The quantity $\hat{F}_1$ is now a correlation between the average value of expected 1-position signal and the input oscillation and is computed from the relation

$$\hat{F}_l = \frac{2}{N_0}\sum_{k=1}^{N} m_{xk}\int_0^T\int_0^R z(t, r)s_{lk}(t, r)\,dt\,dr + m_{yk}\int_0^T\int_0^R z(t, r)\tilde{s}_{lk}(t,r)\,dt\,dr. \quad (3.91)$$

In the absence of fluctuations in channel parameters, the suboptimal algorithm (3.91) becomes optimal.

Let us note that the linear reception algorithm (3.91) is, of course, suitable for complex signals as well as simple opposite signals [76]. It is possible to modify algorithm (3.91) if the output of the regular component of some path, for example, path k, greatly surpasses the others. In this case it is advisable to use precisely this path and construct a correlation between the observed oscillation and expected signals passing along this path, that is, compute the quantity $\hat{F}_1$ from the relation

$$\hat{F}_l = \frac{2m_{xk}}{N_0}\int_0^T\int_0^R z(t, r)s_{lk}(t, r)\,dt\,dr + \frac{2m_{yk}}{N_0}\int_0^T\int_0^R z(t, r)\tilde{s}_{lk}(t, r)\,dt\,dr. \quad (3.92)$$

Processing can be even further simplified by completely rejecting information about the channel (or where no such information is available) and using the reception algorithm where the quantities are computed from the relation

$$\hat{F}_l = \frac{2}{N_0}\int_0^T\int_0^R z(t, r)\overline{s_l(t, r)}\,dt\,dr. \qquad (3.93)$$

The advisability of using suboptimal algorithms in a particular channel can only be clarified after comparison of their noise suppression.

3.6. Some Ways to Realize Algorithms for Space-Time Signal Processing

It can be seen from all the material in chapters 2 and 3 that optimal space-time signal processing, both for estimating parameters and receiving discrete messages as the key operation, includes linear space-time filtration of the input field. Certain random one-dimensional quantities are computed as a result of this filtration, and then they are processed according to definite rules. The processing of one-dimensional quantities, no matter how many of them there may be, is comparatively simple. In fact, general-purpose or specialized computers built with integrated circuitry are capable of processing large arrays of data by fairly complex algorithms at high speed.

Let us look now at ways to realize devices for filtration of space-time signals. We will consider methods afforded by contemporary technology for computing the quantities ψ_k which are included in all algorithms of optimal signal processing

$$\psi_k = \int_0^T\int_0^R z(t, r)s_k(t, r)\,dt\,dr. \qquad (3.94)$$

As already observed above, owing to the finiteness of the signals transmitted and organization of the dimensions of receiving antennas, operations of the (3.94) type can be accomplished on the basis of a space-time coordinated filter.

The problem of constructing a space-time filter with a given pulsed surge characteristic of arbitrary type

$$g_k(t, r) = \begin{cases} s_k(T-\xi, R-\chi), & 0\leqslant\xi\leqslant T;\ 0\leqslant\chi\leqslant R; \\ 0, & \xi \text{ outside of } [0, T] \\ & \chi, \text{ outside of } [0, R] \end{cases} \qquad (3.95)$$

cannot be classified as simple at the current level of technological development, but it is soluble. Because the form of function $s_k(t, r)$, and therefore also of $g_k(t, r)$, is determined by the choice of a channel model through the relation

$$s_k(t, r) = \int_0^\infty s(t-\xi)\eta_k(t, \xi, r)\,d\xi. \qquad (3.96)$$

it may be stated that the characteristic of a coordinated filter is determined by the type of functions $\phi_k(t, \xi, r)$. First let us consider the case of the spatially divisible functions

$$\phi_k(t, \xi, r) = \phi_l(r)\phi_p(t, \xi). \qquad (3.97)$$

Where this condition is met, quantities ψ_k are computed from the relation

$$\psi_k = \int_0^T dt \int_0^R z(t, r)\phi_l(r)\, dr \int_0^t s(t-\xi)\phi_p(t, \xi)\, d\xi, \qquad (3.98)$$

with space and time processing done separately. It is interesting here to look at space processing because methods of realizing time signal processing in stochastic channels have been covered quite thoroughly in the literature [46, 55, 65].

The operation of spatial filtration, that is, the computation

$$f(t) = \int_0^R z(t, r)\phi(r)\, dr \qquad (3.99)$$

(here and in what follows we omit the index i) using current signal processing equipment is accomplished by various methods. Among them are the following*: a) synthesis of antennas; b) use of coherent optical systems (optical holography); c) use of digital computers (digital holography).

Despite the fact that the three methods have much in common (they are all sometimes called holographic), there are unquestionably differences resulting from the physics of the processes. Let us consider these three methods of spatial signal filtration in order.

Synthesis of antennas. Function f(t) (3.99) may be obtained at the output of antenna with directivity diagram $\phi(r)$. For determinacy we will consider a one-dimensional antenna with a linear aperture of magnitude R located in a distant zone so that the spatial coordinate is a generalized angular coordinate

$$r = \frac{2\pi R}{\lambda}\sin\theta, \qquad (3.100)$$

where θ is the elevation angle and λ is the wavelength. Synthesis of an antenna [7, 70] means selecting the amplitude-phase distribution of current $P(\xi)$ in the aperture to insure receiving directivity diagram ϕ $\phi(r)$ with the assigned properties.

Calculation of the parameters of a design that makes it possible to regenerate amplitude-phase distribution $T(\xi)$ is an independent problem.

In the formulation of this work, the optimal diagram $\phi(r)$ should be an eigen function of the second-order Fredholm integral equation

* The techniques of non-coherent signal processing using photodetectors are not considered here. They are studied in detail in [78, 110, 119].

$$\int_0^R \varphi(r') B_h(r, r') dr' = \varkappa \varphi(r), \qquad (3.101)$$

where χ are eigen numbers. The form of eigen functions $\phi(r)$ may be quite complex, but [70] demonstrates that for any given continuous function $\phi(r)$ and arbitrarily small $\varepsilon > 0$, there is a directivity diagram $\phi(r)$ for an antenna with aperture R of any length, even the smallest, that differs from $\phi(r)$ by not more than ε

$$|\varphi(r) - \varphi_\varepsilon(r)| \leqslant \varepsilon. \qquad (3.102)$$

Thus, the problem of synthesizing an optimal antenna may be solved for any stochastic channel. Let us look at the most common method of synthesizing the antenna directivity diagram, the partial diagrams method. This method assumes approximation of an intricate diagram by the sum of simple diagrams corresponding to the simplest emitters. We will show how the partial diagrams, which are most widely used in practice, can be applied to synthesize optimal antennas in a stochastic channel. Introducing the normed variable v, we pass to an aperture of magnitude 2π. We will represent the current of linear antenna I(ξ) in the interval $-\pi \leqslant \xi \leqslant \pi$ with a Fourier series

$$I(\xi) = \sum_{n=-\infty}^{\infty} a_n e^{i n \xi}. \qquad (3.103)$$

This makes it possible to write the directivity diagram in the form

$$\varphi(v) = \frac{1}{2\pi} \int_{-\pi}^{\pi} I(\xi) e^{i \xi v} d\xi = \sum_{n=-\infty}^{\infty} a_n \frac{\sin \pi (v-n)}{\pi (v-n)}. \qquad (3.104)$$

We will show that the partial diagram corresponding to the harmonic distribution of current in the antenna

$$\varphi_n(v) = \sin \pi (v-n)/\pi (v-n), \qquad (3.105)$$

is optimal in a broad class of spatially homogeneous channels. Indeed, a channel which is homogeneous for the spatial variable is Delta correlated for the angular variable, that is, the normed correlation function of its characteristic has the form

$$B_h(v, v') = \sin \pi (v-v')/\pi (v-v'). \qquad (3.106)$$

It is not difficult to show that when the condition $2R/\lambda >> 1$ is met (which always occurs in practice), functions (3.105) are eigen functions of integral equations (3.101) with nucleus (3.106). In the general case of a spatially non-homogeneous channel the derivative function $\phi(v)$ may be approximated by a truncated series (3.104) containing 2N partial Kotel'nikov functions.

Let us consider one more example. We will assume that the normed correlation function of the characteristic for the generalized angular coordinate is approximated by the expression

$$B_h(v, v') = e^{-\alpha |v'-v|}. \qquad (3.107)$$

We know from [81] that the eigen functions of integral equation (3.101) with nucleus (3.107) have the form

$$\varphi_n(v) = \sqrt{\frac{2}{2\pi + \varkappa_n}} \sin\left(\omega_n v + \frac{n\pi}{2}\right). \qquad (3.108)$$

where $\chi_n = 2\alpha/(\alpha^2 + \omega_n^2)$, and ω, ω ... are the positive roots of equation $\mathrm{tg}\, 2\pi\omega = -2\alpha\omega/(\alpha^2 - \omega^2)$. For determinacy we will consider the case of uneven values of n, when it follows from (3.108) that

$$\varphi_n(\vartheta) = \sqrt{2/(2\pi + \chi_n)} \cos \omega_n \vartheta. \qquad (3.109)$$

It is not difficult to note that the optimal antenna (3.109) belongs to the series of Dol'f-Chebyshev optimal antennas which are widely used in practice. Indeed, the directivity diagram of a Dol'f-Chebyshev antenna lattice of 2N equidistant directed emitters has the form [7]

$$F_{2N}(\vartheta) = T_{2N-1}\left(a \cos \frac{\pi R}{\lambda} \sin \vartheta\right), \qquad (3.110)$$

where $T_n(x)$ is a Chebyshev polynomial, a is a scalar multiplier, and R is the distance between emitters.

The directivity diagram (3.109) follows from (3.110) as a particular case where $2N = 2$, $a = \sqrt{2/(2\pi + \chi_n)}$. The emitters should be placed at a distance

$$R = \lambda\omega_n/\pi. \qquad (3.111)$$

from one another.

Physical holography. The discovery of holography and the appearance of holographic methods of processing wave fields offer an opportunity to realize space filters with practically arbitrary characteristics [56, 74, 92]. The holographic method is most effectively used in the optical wave range, in laser communication channels [78], and for processing acoustic fields in acoustics [69, 123]. The many advantages of coherent optical and acoustic systems of data processing make it necessary in many cases to convert radio waves to light or sound waves for further processing. For the sake of determinacy, we will speak in what follows of optical holography, which has received the widest distribution. However, it is relevant to note that acoustic holography with its extremely accessible wave sources is becoming very popular.

Let us consider a coherent optical system of spatial processing of an optical signal based on the use of the Fourier transform. If we have a hologram (Fourier image) of the function $\phi(\vec{r}) = \phi(x, y)$, which is included in equation (3.99), that is, the function

$$\Phi(\omega_x, \omega_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \varphi(x, y) e^{-i\omega_x x - i\omega_y y}\, dx dy, \qquad (3.112)$$

then the input field z(x, y) (we will omit the independent variable t) can be processed by the classical scheme shown in Figure 3.17 below. Physically speaking, the spectrum analyzers are lenses with identical focus distances. A transparency with function (3.112) entered on it should be placed in the focal plane before the two lenses. It is not difficult to show [56] that at the output of the second spectrum analyzer in the focal plane of the second lens, at the point with the coordinate R(x', y'), at moment in time t we obtain quantity f, which is defined by relation (3.99).

Figure 3.17. Optimal Space Processing by Holographic System

Key: (a) Spectrum Analyzer; (b) Spatial Filter.

New prospects for application of holographic techniques for processing space-time signals in stochastic channels appear with the use of an ultrasonic field excited by a piezoelectric element in a transparent medium as the hologram [56]. By applying an electric oscillation to the piezo element it is possible to obtain a hologram that varies in time and to perform both spatial and temporal filtration. Let us look at this in slightly more detail. Suppose we use as a modulating screen (hologram) a flat layer of liquid in which a generator (piezoelectric convertor) excites mobile ultrasonic waves. Acted upon by this generator, the index of refraction of the liquid changes according to the law

$$\dot{n}(t, r) = n_0 + \Delta n(t, r). \qquad (3.113)$$

It is clear that the equivalent transfer function corresponding to the index of refraction (3.113) changes in time and in space in a definite manner. Where the layer of liquid is fairly thin

$$n(t, r) = n_0 + \Delta_n s(x, y, t), \qquad (3.114)$$

where Δ_n is the index of modulation and s(x, y, t) is the generator signal (input signal of the convertor).

Under these conditions, the transfer function proves to be a purely phase function [56] and has the form

$$\dot{H}(f, x, y, t) = \exp[-i\alpha s(x, y, t)], \qquad (3.115)$$

where α is the index of phase modulation determined by the wave length (frequency) of ultrasonic oscillations of the generator and the properties of the piezoelectric transducer.

It is perfectly clear that in the general case we cannot obtain a space-time filter with the assigned transfer function using just one source of ultrasound. However, this problem can be approximately solved by using several (P) sources that realize filters with the transfer functions

$$\dot{H}_p(f, x, y, t) = \exp[-i\alpha_p s_p(x, y, t)], \quad p = \overline{1, P}. \qquad (3.116)$$

Connecting these filters in parallel with corresponding selection of their characteristics can give us an equivalent filter with the required transfer function

$$\dot{H}(f, x, y, t) = \sum_{p=1}^{P} \exp[-i\alpha_p s_p(x, y, t)]. \qquad (3.117)$$

Unfortunately, the question of investigating the features of constructing and using non-stationary space-time optical filters has not yet received sufficient elaboration. Therefore, the corresponding discussion in this section is not so much a description of the procedure for realizing non-stationary filters as it is suggestions for formulation of this complex problem.

Digital holography. It is commonly known that digital holography means analysis and synthesis of wave fields using computers. From the standpoint of the problems solved here, there are two ways to use the methods of digital holography:

1. synthesis of artificial holograms [37, 61, 66, 134];

2. spatial digital filtration of signals [28].

The artificial hologram is the result of a graphic computer-written representation of a certain analytic or tabular relationship that describes a hologram. For example, suppose the hologram to be synthesized is described by the function $\Phi(\omega_x, \omega_y)$, which is a Fourier transform of the function $\phi(x, y)$. The problem is to obtain a plate with the representation $\Phi(\omega_x, \omega_y)$. The arsenal of peripheral units for contemporary computers includes highly precise plotters with high resolution. The procedure for making an artificial Fourier hologram by computer is as follows. The function $\phi(x, y)$ is fed to the computer. Its finite length $|X_{max}| = R_x/2$; $|Y_{max}| = R_y/2$ (with $R_x = R_y = R$) and its finite resolution $\Delta x = \Delta y = 1/\omega_{max}$ are taken into account here. Using the Kotel'nikov theorem for the function $\phi(x, y)$, given in the square $R \times R$, we represent its Fourier transform as a series

$$\Phi(\omega_x, \omega_y) = \sum_{n=1}^{N} \sum_{m=1}^{N} \Phi\left(\frac{n}{R}; \frac{m}{R}\right) \frac{\sin(R\omega_x - n)}{R\omega_x - n} \frac{\sin(R\omega_y - m)}{R\omega_y - m}. \quad (3.118)$$

The number of N^2 readings of function $\Phi(\omega_x, \omega_y)$ can be obtained from N^2 equidimensional readings of function $\phi(x, y)$ taken with an interval $2\pi/\omega_{max}$, that is, from a matrix with dimensions $N \times N$

$$\Phi = \{\varphi(k/\omega_{макс}, p/\omega_{макс})\}_{N \times N}, \quad (3.119)$$

where

$$N = [R/\Delta x + 1], \quad (3.120)$$

by means of the algorithm for a quick Fourier transform [93]. The use of this algorithm to compute the Fourier image of the function of one variable requires [N/2 log N] times less machine time than the conventional discrete Fourier transform; for the function of two variables the corresponding multiple is $[N^2/4 \log N]$.

The set of N^2 spectrum readings of $\Phi(n/R, m/R)$ included in (3.118) fully defines the artificial hologram, which thus consists of N^2 cells. The most difficult thing in making artificial holograms is to obtain semitones. This can be done by using special representation units [61]. Artificial hologram methods based on the transition from semitone representation to black-white (binary) have also become widespread [37, 66, 134]. Binary artificial holograms are synthesized at high speed, but they are inferior to semitone artificial holograms.

Figure 3.18 (next page) gives two examples of fragments of binary programs synthesized with a BESM-4 computer interlinked to a Vektor graphic plotter.

It is quite simple to transfer the artificial hologram to photographic film or a photographic plate with the current level of development of photography. The use of artificial holograms offers new ways to realize optimal signal processing algorithms in stochastic channels. In the future one may imagine a device for processing space signals that has a broad set of presynthesized holograms corresponding to different types of correlation functions of the stochastic channel which make it possible to perform optimal filtration of fields. The "library" of artificial holograms can be supplemented as different communication channels are studied and mastered. The reorganization of the processing device necessary with the transition to a new type of channel or change in the properties of the channel being used can be accomplished easily when it is organized by such "modules."

The idea of using general-purpose computers to build signal processing units arose immediately after the appearance of the first computers. This is a tempting path which just a short time ago seemed unacceptable for most data transmission channels because of the limited capacities of computers [48], but today it no longer seems so unrealistic. Third-generation electronic machines built with integrated circuitry have high speeds (on the order of 10^6-10^7 operations a second), large memory volume (millions of bits), and well-developed systems of external units. The availability among these external units of a range of analog-digital convertors brings computers closer to the problems of signal processing in real time. The great capacities of modern computers for signal processing are also largely a result of their high degree of software development. The BPF algorithms mentioned above are unquestionably among the most effective signal processing algorithms. There is now available a description of the functional systems for data transmission whose receiving parts are specialized computers that perform quick Fourier transforms [28].

Contemporary surveys of computer development note that there is a trend toward increase in the relative cost of software compared to the cost of hardware; the ratio of these costs at the present time is roughly 6:4.

It is apparent that something similar should be expected in signal processing technology. The hardware should become increasingly compact, inexpensive, and universal, while its software will be refined and play an ever-growing part.

It is advisable to base the construction of processing algorithms on the principle of adaptation, according to which unknown functions are replaced by estimates of them. Use of this principle and estimates of channel characteristics obtained in Chapter 2 made it possible to

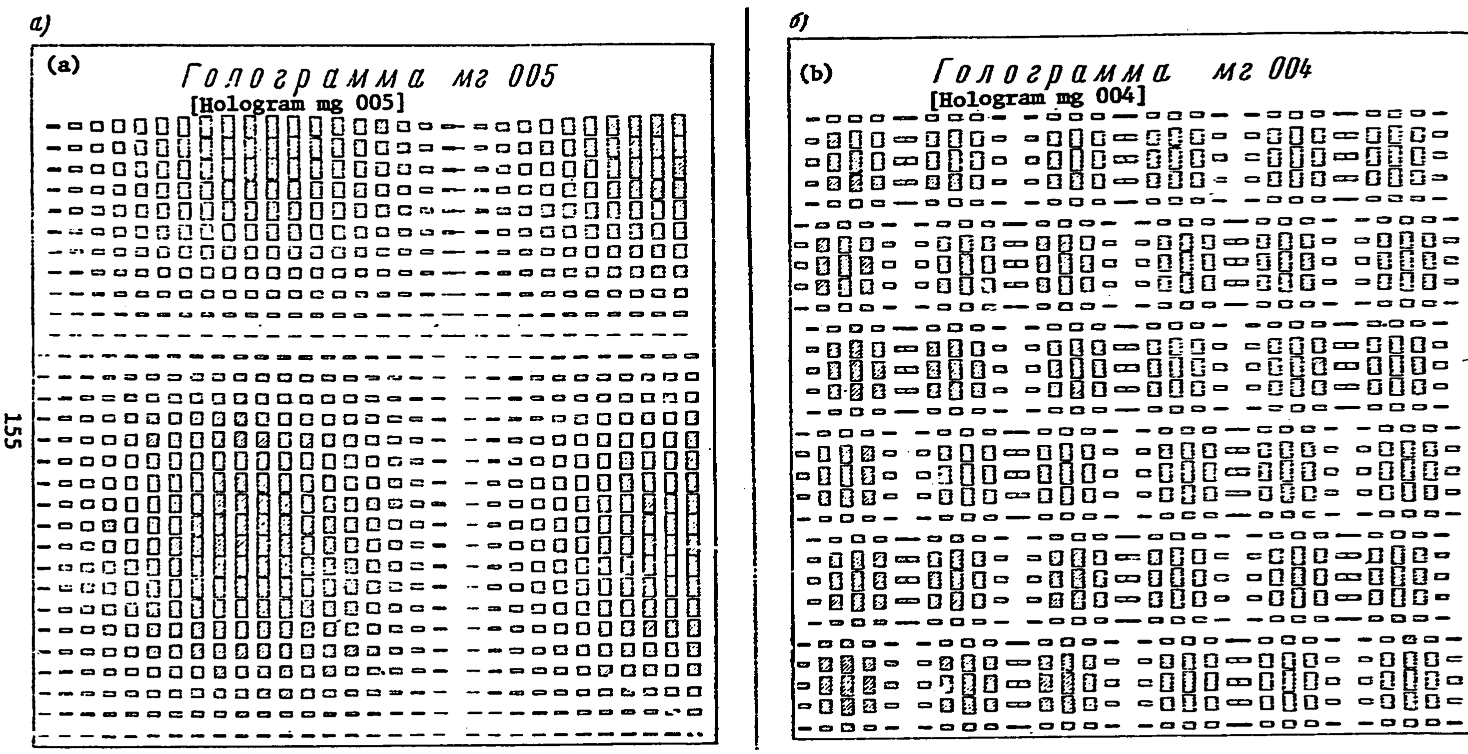

Figure 3.18. Fragments of Binary Holograms Corresponding to the First Two Eigen Functions of Integral Equation (2.79) where U = V = 180 and $a^{-1}{}_u = a^{-1}{}_v = 5$: a) $\phi_1(u, v)$; b) $\phi_2(u, v)$.

synthesize a range of field processing algorithms. With a test signal or ideal decision feedback, field processing is done by a canonical adaptive scheme that contains a channel parameter estimate block and linear correlators (the number of them is determined by the information format of the signals). With an unclassified sample it is possible to consider two different structures of the processing device. The first is adaptive and differs from the one just described in that an estimate block is installed in each branch corresponding to a position of the transmitted signals (in this case conditional estimates of the coordinates are computed). The second structure is non-adaptive and is realized by a linear quadratic scheme in which estimates of channel parameters are present inplicitly.

It was demonstrated that field processing algorithms built with a statistical substructure and using linear channel estimates coincide, in many interesting practical situations, with Bayes algorithms for processing Gaussian fields in a setting of Gaussian noise.

A number of suboptimal processing algorithms based on different discrete models of channels were proposed. It was shown that suboptimal algorithms coincide with certain algorithms already implemented in practice. There are three ways to realize space-time filters, which constitute the foundation of all processing algorithms: antenna synthesis, optical and acoustic holography, and digital holography. A consideration of practically interesting examples shows that optimal algorithms of space processing have in many cases an absolutely clear basis for realization supported by the modern arsenal of available analog and digital signal processing equipment.

Chapter 4. Analysis of Algorithms for Space-Time Signal Processing

4.1. Quality Characteristics of Data Transmission Systems and Their Determination

The preceding chapters have dealt with the problem of modeling and synthesizing systems that transmit information on a space-time channel. In this chapter we consider the problem of analysis, which is equivalent to the problem of evaluating the characteristics of synthesized algorithms for a given assemblage of signals and fixed channel properties.

If we take a systems approach to the problem of determining the characteristics of alogirithms and signal processing devices, the criterion for comparing different systems should be the value of some (usually quite complex) target function that is determined by a large number of variables.

With all the diversity of types of target functions used for problems of transmitting discrete messages, they are united by the fact that every one includes the characteristic of noise suppression, which is determined above all by the probability of erroneous reception of the transmitted signal.

For channels with memory, needless to say, this characteristic must be supplemented by a characteristic that takes account of the grouping of errors, and when considering the non-stationary state of a channel in prolonged intervals of time, frequency, and space, a communications reliability characteristic is also needed (an integral function of the distribution of the probability of error).

The problem of computing the probability of error and communications reliability is often the stumbling block during engineering analysis of functioning systems or systems being designed. And this is not just a matter of computational difficulties, although even today with the fairly high level of development of computer means it is not possible to solve every problem of computing the probability of error, grouping of error, and communications reliability in acceptable time periods with an assigned level of precision if we try to solve them by a "frontal attack." The chief difficulty is that while a system developer solving the problem of synthesis can, by using various clever tricks, bypass the question of the probabilistic laws of distribution of signals and noise (see the articles on non-parametric statistics in problems of signal processing [64]), when solving the problem of analysis this issue cannot be avoided. It is essential to adopt some particular probabilistic model of the channel and additive noise. In Chapter 2 we constructed algorithms for linear measurement of a channel characteristic that were invariant to the probabilistic laws of signals and noise. In Chapter 3 use of the principle of adaptation made it possible to apply these estimates to synthesize algorithms for processing, at the output of a stochastic channel, signals containing discrete messages. It is possible to broaden the results to the case of processing signals containing analog information. All of these algorithms are invariant to probabilistic laws, although it is perfectly clear that no probabilistic model can be used in analysis of their characteristics. In Chapter 1 we presented a fairly detailed consideration of probabilistic models of a linear stochastic channel with series-parallel signal passage. These models are capable of describing a very broad class of communication channels.

In this chapter attention will be focused on a generalized Gaussian (four-parameter) probabilistic model of a channel and its particular cases (the Beckman, Rice, Rayleigh, and sub-Rayleigh models). The primary role is not given to the generalized Gaussian model by accident. In the first place, it is apparent that it is the most acceptable model for a broad class of linear channels; the boundary theorems of probability theory give the grounds for this conclusion. In the second place, even in those channels whose physics do not correspond to the conditions of the central boundary theorem, the generalized Gaussian model may prove a good approximation of the one-dimensional distribution of the amplitudes and phases of the signal at the receiving point (it is precisely one-dimensional distributions that will be required to analyze the probability of error). Good approximation features can be expected from the generalized Gaussian law because it is a four-parameter law, that is, it depends explicitly on four parameters. The presence of four independent parameters makes it possible to approximate practically any distribution function of the given class successfully with a given degree of precision. An example might be the function of distributions of amplitudes or vector phases on a plane.

In the third place, as any investigator dealing with probabilistic calculations knows very well, it is the Gaussian assumption that usually makes it possible to carry a problem through to the end, or in any case until engineering recommendations and conclusions are received. However, it would be incorrect to recommend use of the generalized Gaussian model in situations when the Gaussian assumption contradicts the logic of things, for example, in an optical channel with a clearly marked sequential distribution of signals. Therefore, in what follows we will also consider probabilistic models of a channel that correspond to the essentially non-additive character of formation of a dispersed signal (logarithmically normal distribution of amplitudes, uniform distribution of phases).

Under these conditions it is not always possible to treat the problem of computing the probability of error without introducing additional constraints on the model of the channel.

In addition, it is not always possible to obtain a convenient calculation formula and we must simply point out numerical results obtained by calculations using complex formulas or the Monte Carlo method [14, 15, 27].

White Gaussian noise is treated as a model of additive noise throughout this chapter. A generalization to the case of "colored" noise could be made without too much effort, but it would require sacrificing the clarity of final results from the standpoint of studying the effect of a stochastic channel on the probability of error. Evaluating the noise suppression of a system given non-Gaussian local noise, for example with pulsed and focused noise, is beyond the framework of this work.

In view of the complexity of the space-time stochastic channels under consideration, we will restrict our evaluation of the quality of algorithms synthesized to finding analytic relations only for the probability of erroneous reception of the transmitted signal.

The probability of error in optimal and suboptimal algorithms will be determined in what follows, unless stipulated otherwise, on the assumption that the individual paths along which the signal travels to the reception point are statistically independent. Within the framework of the generalized Gaussian model this means that we are using a channel model based on a Karunen-Loew expansion. In this case the final expressions for the probability of error include the eigen numbers σ^2_{xk} and σ^2_{yk} of integral equation (2.21). The formulas for probability of error do not change in the case of correlated paths of propagation, only the quantities σ^2_{xk} and σ^2_{yk}. This question is considered in more detail in work [46].

The quality of optimal and suboptimal algorithms is analyzed for the use of a test signal and without a test signal in both Gaussian and (to a lesser extent) non-Gaussian channels, transmitting information by various signal systems that meet the requirement of separation of paths at the receiving point.

We should observe that engineering practice is increasingly calling for the development of a device (or program) that might be called a digital analyzer of the quality of data transmission systems. This analyzer is a general-purpose or special computer whose memory contains all possible operators for converting a signal in transmission, models of space-time channels and additive noise, and operators for processing received signals. The speed of the analyzer should allow operational production of graphic output data, for example curves of the dependence of communications reliability on permissible probability of error, which determine the quality characteristics of the systems being analyzed with the given conversions of signals in transmission and reception, in the channel, and in the given noise sources.

Discussion of the foundation for building this device (or program) beyond the framework of the present book.

4.2. The Probability of Error Under Conditions of an Ideal Classification.

First let us compute the lower boundary of probability of error in data transmission systems by binary signals $s_1(t)$ and $s_2(t)$ on the assumption that the sample used to study the channel is ideally classified. When an ideal estimation of the parameters of a stochastic channel in the interval of decision-making is done and the estimates obtained are stored without change, then the estimates $\hat{u}_0(t, r)$ and $\hat{u}_2(t, r)$ included in the linear algorithms (3.45) are exactly equal to the corresponding signals at the output of the channel. The conditional probability of error for fixed values of the parameters of stochastic channel $\vec{h}$ is determined then from the relation

$$p\{\text{ош} \mid \vec{h}\} \geqslant \frac{1}{2}\left\{1 - \Phi\left[\sqrt{\frac{1}{N_0}\int\limits_0^T\int\limits_0^R\left[\hat{u}_1(t, r) - \hat{u}_2(t, r)\right]^2 dt\,dr}\right]\right\}, \quad (4.1)$$

where $\Phi(u) = \frac{2}{\sqrt{2\pi}}\int\limits_0^u e^{-t^2/2}\,dt$ is the Kramp function. The unconditional probability of error is determined by averaging (4.1) by the set of random parameters*

$$p \geqslant \int\limits_{-\infty}^{\infty} \ldots \int\limits_{-\infty}^{\infty} p\{\text{ош} \mid \vec{h}\}\, w_{2N}(\vec{h})\, d\vec{h}, \quad (4.2)$$

First let us consider a channel with generalized Gaussian statistics on the assumption that the conditions of beam separation are met. In this case $\vec{h}$ represents a 2N-dimensional Gaussian vector with independent components. The integration is performed by the procedure described in [89] and yields the expression

$$p = \frac{1}{\pi}\int\limits_0^\infty \frac{dt}{1+t^2}\prod_{k=1}^{N}\frac{\exp\left\{-\frac{1+t^2}{2}\overline{h_k^2}\,\frac{q_k^2}{1+q_k^2}\left[\frac{\cos^2\varphi_{p\,k}}{1+\frac{2\overline{h_k^2}\,\beta_k^2\lambda_{12}}{(1+\beta_k^2)(1+q_k^2)}} + \rightarrow\right.\right.}{\sqrt{\left[1+(1+t^2)\frac{\lambda_{12}\overline{h_k^2}\,\beta_k^2}{(1+\beta_k^2)(1+q_k^2)}\right]\times\rightarrow}}$$

* This formula is also correct when considering intercharacter interference.

$$\frac{\left. \left. + \frac{\sin^2 \varphi_{pk}}{1 + \dfrac{2h_k^2 \lambda_{12}}{(1+\beta_k^2)(1+q_k^2)}} \right] \right\}}{\times \left[1 + (1+l^2) \dfrac{\lambda_{12} h_k^2}{(1+\beta_k^2)(1+q_k^2)} \right]} . \qquad (4.3)$$

Let us recall the designations introduced earlier

$$h_k^2 = 2d_k(\sigma_{xk}^2 + \sigma_{yk}^2 + m_{xk}^2 + m_{yk}^2); \; q_k^2 = \frac{m_{xk}^2 + m_{yk}^2}{\sigma_{xk}^2 + \sigma_{yk}^2}; \; \beta_k^2 = \frac{\sigma_{xk}^2}{\sigma_{yk}^2};$$

$$\varphi_{pk} = \operatorname{arc\,tg} \frac{m_{yk}}{m_{xk}} .$$

It was assumed in obtaining (4.3) that the signals have identical energy $E_1 = E_2, = E$ and, moreover, $d_{lk} = d_k$, $l = 1, 2$. The parameter λ_{12} is determined by the mutual correlation properties of signals $s_1(t)$ and $s_2(t)$:

$$\lambda_{12} = \frac{1}{E} \int_0^T s_1(t) s_2(t) \, dt. \qquad (4.4)$$

For the opposite signals $\lambda_{12} = -1$; for orthogonal signals $\lambda_{12} = 0$. Formula (4.3) may be used for calculations of the lower boundary of probability of error in the case of transmission by a system with a "passive pause" also, where $s_1(t) \equiv 0$. In this case $\lambda_{12} = 1/2$ should be put in (4.3).

Analysis of formula (4.3) shows that, where other conditions are equal, the use of opposite signals provides the lowest probability of error in data transmission. This result seems entirely natural because the device under consideration in fact works by the V. A. Kotel'nikov algorithm in an ideal channel for which the optimality of opposite signals has long been known. It should not be forgotten, however, that (4.3) is only the lower boundary of probability of error in a system with a test signal and minimizing the lower boundary (where $\lambda_{12} = -1$) does not, in general, signify minimization of the actual probability of error. This important issue needs further investigation.

Expression (4.3) is very convenient for numerical calculations on the computer and makes it possible to receive a large number of particular results. We will cite a few of them that hold practical interest.

For example, for a Rayleigh channel the formula for the lower boundary of probability of error, as follows from (4.3), takes the form

$$p \geq \frac{1}{2} \left[1 - \sum_{k=1}^{N} \frac{1}{\sqrt{1 - \dfrac{1}{\lambda_{12} h_k^2}}} \prod_{n=1}^{N}{}^{n \neq k} \frac{1}{1 - \dfrac{h_n^2}{h_k^2}} \right] . \qquad (4.5)$$

while for a channel with discrete multiple beams (homogeneous by frequency) and smooth fadeouts in time, following (4.3)

$$
p \geqslant \frac{1}{\pi}\int\limits_0^\infty \frac{dt}{1+t^2}\prod_{m=1}^{N^F}
\frac{\exp\left\{-\frac{(1+t^2)}{2}\,\overline{h_m^2}\,N^R\,\frac{q_m^2}{1+q_m^2}\right\}\times\rightarrow}
{\left[1+(1+t^2)\,\frac{\lambda_{12}\,\overline{h_m^2}\,\beta_m^2}{(1+\beta_m^2)(1+q_m^2)}\right]^{\frac{N^R}{2}}\times\rightarrow}
$$

$$
\frac{\rightarrow\times\left[\dfrac{\cos^2\varphi_{pm}}{1+(1+t^2)\,\dfrac{\lambda_{12}\,\overline{h_m^2}\,\beta_m^2}{(1+\beta_m^2)(1+q_m^2)}}+\dfrac{\sin^2\varphi_{pm}}{1+(1+t^2)\,\dfrac{\lambda_{12}\,\overline{h_m^2}}{(1+\beta_m^2)(1+q_m^2)}}\right]\Bigg\}}
{\rightarrow\left[1+(1+t^2)\,\dfrac{\lambda_{12}\,\overline{h_m^2}}{(1+\beta_m^2)(1+q_m^2)}\right]^{\frac{N^R}{2}}}. \tag{4.6}
$$

Boundary (4.6) corresponds to the case of identical spatial correlation functions in each of N^F beams. If N^F beams are "identical on the average," that is $\gamma^2_m = \gamma^2$; $q^2_m = q^2$; $B^2_m = B^2$; $\varphi_{pm} = \varphi_p$, $m = \overline{1, N^F}$, it follows from (4.6) that

$$
p \geqslant \frac{1}{\pi}\int\limits_0^\infty \frac{dt}{1+t^2}\,
\frac{\exp\left\{-\frac{(1+t^2)}{2}\,N^F N^R\,\overline{h^2}\,\frac{q^2}{1+q^2}\right\}\times\rightarrow}
{\left[1+(1+t^2)\,\frac{\lambda_{12}\,\overline{h^2}\,\beta^2}{(1+\beta^2)(1+q^2)}\right]^{\frac{N^F N^R}{2}}\times\rightarrow}
$$

$$
\frac{\rightarrow\times\left[\dfrac{\cos^2\varphi_p}{1+(1+t^2)\,\dfrac{\lambda_{12}\,\overline{h^2}\,\beta^2}{(1+\beta^2)(1+q^2)}}+\dfrac{\sin^2\varphi_p}{1+(1+t^2)\,\dfrac{\lambda_{12}\,\overline{h^2}}{(1+\beta^2)(1+q^2)}}\right]\Bigg\}}
{\rightarrow\left[1+\dfrac{(1+t^2)}{2}\,\dfrac{\lambda_{12}\,\overline{h^2}}{(1+\beta^2)(1+q^2)}\right]^{\frac{N^F N^R}{2}}}. \tag{4.7}
$$

Figure 4.1 below shows a set of graphs of the lower boundary of probability of error computed according to formula (4.7) for a system with a test signal.

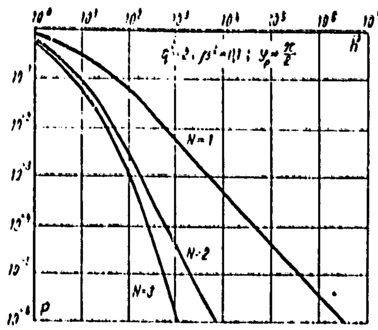

Figure 4.1. Lower boundary of probability of error in a system with a test signal.

It is interesting to consider the domain of high signal/noise ratios. In this domain the lower boundaries approach the true values of probability of error. Assuming in (4.3) that $\overline{h^2}_k$, $B^2_k \overline{h^2}_k \gg 1$, we obtain as a result of integration

$$P \geq \frac{C_{2N-1}^{N}}{\prod_{k=1}^{N} \frac{2\overline{h_k^2}\beta_k}{(1+\beta_k^2)(1+q_k^2)} \exp\left[\frac{q_k^2(1+\beta_k^2)}{2\beta_k^2}(\cos^2\varphi_{pk} + \beta_k^2\sin^2\varphi_{pk})\right]}. \quad (4.8)$$

Analysis of the resulting formula and Figure 4.1 allows us to draw conclusions concerning the effect of the mean statistical parameters of the channel on noise suppression. Specifically, it follows from (4.8) that in the absence of a regular component in the channel ($q^2_k = 0$, $k = \overline{1, N}$) the presence of asymmetry ($B^2_k \neq 1$, $k = \overline{1, N}$) always leads to an increase in the probability of error, in other words, the sub-Rayleigh channel is always worse than the Rayleigh channel. If there is a regular component things are different. The channel with asymmetry for dispersions of quadrature components can provide lower probability of error than a symmetrical (Rice) channel in the case where a weakly fluctuating component ($B^2_k \ll 1$, $k = \overline{1, N}$) has a powerful regular constituent part ($\phi_{pk} = 0$, $k = \overline{1, N}$). Under these conditions dispersed reception is insured on N branches of dispersion with an asymptotic decrease in the probability of error (where $E/N_0 \to \infty$) inversely proportional to the N-th degree of the signal/noise ratio independent of the values of the mean statistical parameters of fadeouts.

Appendix 2 shows that such an asymptotic dependence of the lower boundary of probability of error on the signal/noise ratio (where $E/N_0 \to \infty$) does not depend on the probability model of fadeouts. It follows from formulas (4.1) and П.2.5) that, for any probabilistic law of fluctuations of channel parameters that meets definite constraints for the lower boundary of probability of error under conditions of an ideal classification of the input field, this relation is correct.

$$p > C_N^{2N-1} \prod_{k=1}^{N} \frac{\pi}{d_k} w_1(x_k = 0)\, w_1(y_k = 0), \qquad (4.9)$$

Thus, the dependence of the probability of error on the signal/noise ratio in asymptotics (where $E/N_0 \to \infty$) of the type

$$p \approx 1/(E/N_0)^N, \; (\beta_k^2 \neq 0) \qquad (4.10)$$

is stable relative to the probabilistic model of fadeouts. It will be shown later that the dependence (4.10) is also correct when studying a channel according to information signals with a non-ideal classification.

As shown in Appendix 2, for a non-Gaussian (logarithmically normal by amplitudes, uniform by phases) channel, from (4.9)

$$p > C_N^{2N-1} \prod_{k=1}^{N} \frac{1}{4\pi \overline{h_k^2}} e^{3\sigma_{xk}^2}, \qquad (4.11)$$

where $\sigma_{xk}^2 = \overline{(\ln \gamma_k - \overline{\ln \gamma_k})^2}$.

The parameters $\overline{\gamma^2}_k$ and $\sigma^2{}_{xk}$ should be fixed for mutual comparison of results related to Gaussian and logarithmically normal models.

It can be seen from (4.11) that as $\sigma^2{}_{xk}$ increases (with an increase in the depth of fadeouts), the probability of error grows. Comparing (4.11) with the corresponding formula for a Rayleigh channel, which follows from (4.8), we conclude that for values $\sigma_{xk}^2 \leqslant \frac{1}{3} \ln 4\pi$ the channel with logarithmically normal amplitued is better than the Rayleigh channel. In the domain $\sigma_{xk}^2 > \frac{1}{3} \ln 4\pi$ the opposite is true.

Let us move on to a consideration of the upper boundary of probability of error. To determine the upper boundary it is convenient to write an algorithm to distinguish two signals in the form

$$\int_0^T \int_0^R [z(t, r) - \hat{u}_1(t, r)]^2\, dt dr < \int_0^T \int_0^R [z(t, r) - \hat{u}_2(t, r)]^2\, dt dr. \qquad (4.12)$$

It is not difficult to show that on the assumption that the l-position signal s (t), l = 1, 2 is transmitted, the probability of an erroneous decision is determined by the probability of fulfillment of the inequality

$$\int_0^T\int_0^R [\hat{u}_1(t, r) - \hat{u}_2(t, r)] [e_l(t, r) + n(t, r)] dt dr < \frac{1}{2}\int_0^T\int_0^R [\hat{u}_1(t, r) - \hat{u}_2(t, r)]^2 dt dr, \quad (4.13)$$

where $e_l(t, r)$ expresses the error of estimation of the l-position signal caused by inaccuracy in measuring the channel characteristic:

$$e_l(t, r) = \mathrm{Re}\int_0^\infty [h(t, \xi, r) - \hat{h}(t, \xi, r)] s_l(\xi) d\xi.$$

Entry (4.13) enables us to draw qualitative conclusions on the effect of inaccuracy in measuring a channel characteristic for noise suppression. It is not difficult to note that the effect of inaccurate measurement on the characteristics of an algorithm finds expression in the appearance of additional additive noise correlated with the signal.

The linear, unbiased estimates synthesized in Chapter 2 are used to measure channel characteristics. Then, for Gaussian additive noise n(t, r) the additional noise $e_1(t, r)$ is also Gaussian with a zero mean and correlation function $B_{e1}(t, t', r, r')$.

It is perfectly obvious that the probability of fulfillment of inequality (4.13), which is the probability of error, will be greater if additional noise $e_1(t, r)$ is white noise, which statistically does not depend on the usable signal, and has a spectral density of output of:

$$D_{0l} = \frac{B_{l1}(0,0) + B_{l2}(0,0)}{2F\Omega_g} \quad (4.14)$$

This circumstance makes it possible to write the upper boundary of the probability of error in the form

$$p_0(\text{ош}|\vec{h}) \leqslant \frac{1}{2}\left\{1 - \Phi\left[\sqrt{\frac{1}{N_0 + D_0}\int_0^T\int_0^R [\hat{u}_1(t, r) - \hat{u}_2(t, r)]^2 dt dr}\right]\right\}. \quad (4.15)$$

If optimal linear estimates are computed to study a channel, then (see Chapter 2) the mathematical expectations and dispersions of estimates coincide with the mathematical expectations and dispersions of the quantities being estimated

$$M\{\hat{x}_k\} = m_{xk};\ M\{\hat{y}_k\} = m_{yk};$$

$$D\{\hat{x}_k\} = \sigma^2_{xk};\ D\{\hat{y}_k\} = \sigma^2_{yk}.$$

It is clear from this that the upper boundaries of probability of error are determined by the same expressions as the lower boundaries, which were obtained above, except that instead of parameter N_0, the parameter $(N_0 + D_0)$ is involved, which leads to a decrease in the signal/noise ratio of $(1_0 + d_0/n)$ times. For the good linear estimates constructed in Chapter 2 in the domain of large signal/noise ratios, the following relation is correct

$$D_0 = \frac{N_0}{E^{u}} E_l, \qquad (4.16)$$

where E^{M} $[E^{i}]$ is the energy of the signal being used to study the channel.

Assuming that signals of equal energy are used ($E_l = E$, $l = 1, 2$), we conclude that the inaccuracy of optimal linear estimates leads to a decrease of $(1 = E/E)$ times in the signal/noise ratio $\overline{h^2}$. This fact enables us to conclude that the effect of the inaccuracy of measuring characteristics of a stochastic channel on noise suppression and reception of discrete messages can be eliminated in practice by studying the channel with a signal whose energy is 10 times greater than the energy of the information samples. Let us note that a completely analogous conclusion concerning the ratio of energies E^i and E was drawn by [46] for a Rayleigh channel as a result of finding exact formulas for the probability of error. The exact values of the quantity D_0 for different correlation functions of the channel should be computed from the relations in Chapter 2. Their use makes it possible to construct graphs of the probability of error for a broad class of channels with an arbitrary probabilistic model of fluctuations of parameters.

In concluding this section we will show graphically what gives optimal spatial processing the advantage over non-optimal (primitive) processing.

Let us consider a channel with smooth fadeouts in time and frequency but selective (homogeneous) fadeouts in space. The transfer function of such a channel depends entirely on the spatial frequency $H(\omega, t, \omega_r) = H(\omega_r)$.

Let us make a comparative analysis of the two schemes. The first computes and employs N^R readings of the transfer function (2.124). The second performs primitive spatial processing of signals and is constructed on the assumption that the channel is described by a model of an amplifier with a random amplification factor $H(\omega, t, \omega_r) = H$.

In the first case, the estimate of the signal in position 1 is computed in the form

$$\hat{u}_l(t, r) = \mathrm{Re}\left\{ s_l(t) \sum_{p=1}^{N^R} \hat{H}_p e^{i p \Delta\omega_r r} \right\}$$

where $\hat{H}_p$ is the estimate of reading p of the transfer function and $\Delta\omega_r$ is the distance between readings on the axis of spatial frequencies.

The expression of the lower boundary of probability of error for this example follows from (4.3): substituting $N = N^R$ in (4.3) for optimal processing and $N = 1$ for primitive processing. It is clear from this that the superiority of optimal spatial processing to non-optimal (primitive) increases as the degree of selectivity of spatial fadeouts increases.

Now let us evaluate the effect of inaccurate measurement on the probability of error in optimal and primitive space processing. First we will make a general remark. It follows from (4.14) that the intensity of additional additive noise caused by the inaccuracy of measurement is equal to the mean square of error of measurement $\overline{e^2}$. The optimal linear estimates minimize this quantity. Any other estimates give larger values of $\overline{e^2}$, and therefore there is additional noise of great intensity. Thus, the effect of inaccuracy of measurement on the probability of error will be greater in non-optimal processing than in optimal. Of course, it should not be forgotten that this conclusion is drawn for the upper boundary of probability of error, not for probability itself. The question of how close probability of error is to its upper boundary demands additional investigation in each particular case.

Let us return to the example under consideration. With optimal processing the mean square of the error of measurement of the 1-position signal on the basis of (2.123) is written in the form

$$\overline{e_l^2} = \frac{1}{\pi}\int_0^{2\pi F} |\dot{S}_l(\omega)|^2 \; \frac{1}{N^R} \sum_{p=-N^R/2}^{N^R/2} \frac{\dot{N}(\omega,\ \omega_{rp})}{|\dot{S}^{\text{и}}(\omega)|^2 + \dfrac{\dot{N}(\omega,\ \omega_{rp})}{G(\omega,\ \omega_{rp})}}\, d\omega,$$

where $S^{\text{и}}(\omega)$ is the spectrum of the signal by which a channel lying in band [-F, F] is measured.

For primitive spatial processing on the basis of (2.164) we have

$$\overline{e_l^2} = \frac{1}{\pi}\int_0^{2\pi F} |\dot{S}_l(\omega)|^2 \frac{\dot{N}(\omega,\ \omega_{r0})}{|S^{\text{и}}(\omega)|^2}\, d\omega.$$

In writing the last formula we assumed, for the sake of determinacy, that the primitive scheme estimates the transfer function at 0 spatial frequency. Let us make a comparison of the two schemes in the case, most advantageous for the optimal scheme, of equidimensional energy and amplitude spectra $N(\omega,\ \omega_r) = N_0;\ G(\omega,\ \omega_r) = G_0;\ |\dot{S}^{\text{и}}(\omega)|^2 = E^{\text{и}};\ |\dot{S}_l(\omega)|^2 = E,\ l = 1,\ 2.$

Under these conditions, the intensity of supplementary additive noise in primitive processing is $\left(1 + \frac{N_0}{E^{\text{и}} G_0}\right)$ times greater than in optimal processing.

The expression of the lower boundary of probability of error for this example follows from (4.3): substituting $N = N^R$ in (4.3) for optimal processing and $N = 1$ for primitive processing. It is clear from this that the superiority of optimal spatial processing to non-optimal (primitive) increases as the degree of selectivity of spatial fadeouts increases.

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where $S^{\text{M}}(\omega)$ is the spectrum of the signal by which a channel lying in band $[-F, F]$ is measured.

For primitive spatial processing on the basis of (2.164) we have

$$\overline{e_l^2} = \frac{1}{\pi}\int_0^{2\pi F} |\dot{S}_l(\omega)|^2 \frac{\dot{N}(\omega, \omega_{r0})}{|S^{\text{M}}(\omega)|^2}\, d\omega.$$

In writing the last formula we assumed, for the sake of determinacy, that the primitive scheme estimates the transfer function at 0 spatial frequency. Let us make a comparison of the two schemes in the case, most advantageous for the optimal scheme, of equidimensional energy and amplitude spectra $N(\omega, \omega_r) = N_0$; $G(\omega, \omega_r) = G_0$; $|\dot{S}^{\text{M}}(\omega)|^2 = E^{\text{M}}$; $|\dot{S}_l(\omega)|^2 = E$, $l = 1, 2$.

Under these conditions, the intensity of supplementary additive noise in primitive processing is $\left(1 + \dfrac{N_0}{E^{\text{M}} G_0}\right)$ times greater than in optimal processing.

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4.3 Characteristics of Devices for Processing Space-Time Signals in a Generalized Gaussian Channel (Smooth Fadeouts)

To bring the results of calculating noise suppression more closely in line with the physics of real channels it is advisable to consider separately the particular case of smooth fadeouts. In this case the channel is represented by a model of a series combination of a deterministic space-filter and an amplifier with a random, complex amplification factor.

Correspondingly, the optimal processing device for signals in each transmitted position is a one-channel device. The processing algorithm for space-time signals in a channel with smooth fadeouts follows as a particular case from the general algorithms of Chapter 3 where N = 1. Here we will consider the characteristics of space-time processing devices in a channel with generalized Gaussian statistics.

Because we are investigating a generalized Gaussian model of a channel, it is advisable to compute the noise suppression of the optimal algorithm in this channel (3.56). The probability of error computed in this case should be treated as the lower boundary of probability of error for the given channel. Algorithm (3.45) is invariant to the statistics of fadeouts in a generalized Gaussian channel with a regular signal component; of course, it is inferior to the optimal Bayes algorithm (3.56). However, the corresponding energy loss is a fraction of a decibel and in practice both algorithms provide the same probability of error. We will make our comparison of optimal algorithms against the non-coherent processing algorithms which have become widespread and follow from the formula (3.82) where N = 1 for the problem of detection and discrimination of M-orthogonal signals and against linear algorithm (3.91) in considering the problem of distinguishing two opposite signals.

We will consider three types of message-carrying signals separately: signals corresponding to the detection problem (dual signals); signals in a group of number M which are orthogonal in the amplified sense [104]; opposite signals.

In addition, we will solve the problem of computing noise suppression for dual signals of arbitrary form and on this basis find the optimal dual system of signals in a generalized Gaussian channel.

The working characteristics of an optimal detector. Two types of errors are possible in a detector that works according to algorithm (3.45): the false alarm and missing a signal. To determine the probabilities of these errors it is convenient to convert expression (3.12) to the form

$$V^2 + \tilde{V}^2 \gtrless \omega. \quad (4.17)$$

The quantities V and $\tilde{V}$ are normal and independent. When the incoming oscillation has a usable signal these quantities have the following parameters

$$\left.\begin{aligned} &M_1(V) = m_x \sqrt{\frac{2E}{N_0}\,\frac{1+2h_x^2}{2h_x^2}}, \quad M_1(\tilde{V}) = m_y \sqrt{\frac{2E}{N_0}\,\frac{1+2h_y^2}{2h_y^2}}, \\ &D(V) = 2h_x^2, \quad D(\tilde{V}) = 2h_y^2. \end{aligned}\right\} \quad (4.18)$$

In the absence of a usable signal

$$\left.\begin{aligned} &M_1(V) = m_x\sqrt{\frac{2E}{N_0}\frac{1}{2h_x^2(1+2h_x^2)}},\quad M_1(\tilde{V}) = m_y\sqrt{\frac{2E}{N_0}\frac{1}{2h_y^2(1+2h_y^2)}},\\ &D(V) = \frac{2h_x^2}{1+2h_x^2},\quad D(V) = \frac{2h_y^2}{1+2h_y^2}.\end{aligned}\right\} \quad (4.19)$$

The probabilities of a false alarm and missing a signal should be calculated according to the formulas [89] for integral function F(A, B, C, D). In the general case we may write

$$p_{\text{лт}} = 1 - F(A,\ B^{(-)},\ C^{(-)},\ D^{(-)});\quad p_{\text{пр}} = F(A,\ B^{(+)},\ C^{(+)},\ D^{(+)}). \quad (4.20)$$

The parameters present in formulas (4.20) are written as follows

$$\left.\begin{aligned} &[B^{(-)}]^2 = \beta^2\,\frac{1+\dfrac{2\overline{h^2}}{(1+\beta^2)(1+q^2)}}{1+\dfrac{2\beta^2\overline{h^2}}{(1+\beta^2)(1+q^2)}},\\ &[C^{(-)}]^2 = \frac{q^2(1+q^2)\left(1+\dfrac{1}{\beta^2}\right)\left[\cos^2\varphi_p + \dfrac{(1+q^2)(1+\beta^2)+2\beta^2\overline{h^2}}{(1+q^2)(1+\beta^2)+2\overline{h^2}}\sin^2\varphi_p\right]}{2\overline{h^2}\left[1+\dfrac{1}{\beta^2}\,\dfrac{(1+q^2)(1+\beta^2)+2\overline{h^2}\beta^2}{(1+q^2)(1+\beta^2)+2\overline{h^2}}\right]},\\ &D^{(-)} = \operatorname{arc\,tg}\left\{\operatorname{tg}\left[\varphi_p\beta\sqrt{\frac{1+2\dfrac{\beta^2\overline{h^2}}{(1+q^2)(1+\beta^2)}}{1+\dfrac{2\overline{h^2}}{(1+q^2)(1+\beta^2)}}}\right]\right\},\\ &[B^{(+)}]^2 = \beta^2,\quad [C^{(+)}]^2 = q^2\left\{1+\frac{(1+\beta^2)(1+q^2)}{2\beta^2 h^2}\left[\cos^2\varphi_p + \right.\right.\\ &\qquad\left.\left. + \frac{(1+q^2)(1+\beta^2)+2\overline{h^2}}{(1+q^2)(1+\beta^2)+2\beta^2\overline{h^2}}\sin^2\varphi_p\right]\right\},\\ &D^{(+)} = \operatorname{arc\,tg}\left\{\operatorname{tg}\left[\varphi_p\beta\sqrt{\frac{1+\dfrac{2\overline{h^2}}{(1+\beta^2)(1+q^2)}}{1+\dfrac{2h^2\beta^2}{(1+\beta^2)(1+q^2)}}}\right]\right\}.\end{aligned}\right\} \quad (4.21)$$

Parameter $\overline{h^2}$ is expressed by the formula

$$\overline{h^2} = \frac{E}{N_0}\left(m_x^2 + m_y^2 + \sigma_x^2 + \sigma_y^2\right). \quad (4.22)$$

As follows from formula (4.21), where $h_2 \gg 1$ the following inequalities are fulfilled: $B^{(-)} = 1$ and $C^{(-)} \approx 0$. The probability of a false alarm in this case is determined by the relation

$$p_{\text{лт}} = \exp(-\omega), \quad (4.23)$$

from which it is not difficult to compute the threshold

$$\omega_{\text{ст}} = -\ln p_{\text{лт}}. \qquad (4.24)$$

Using results from [89], the expression for the probability of missing a signal can be obtained in the form

$$p_{\text{пр}} = \frac{\ln(1/p_{\text{лт}})(1+\beta^2)(1+q^2)}{2\overline{h^2}\beta\exp\left[\frac{q^2(1+\beta^2)}{2\beta^2}(\cos^2\varphi_p + \beta^2\sin^2\varphi_p)\right]}. \qquad (4.25)$$

We will analyze the characteristics after consideration of the quality indexes of a non-coherent detector.

The working characteristics of a non-coherent detector. We will determine the probabilities of errors in a detector working according to algorithm (3.82). In the absence of a usable signal V and $\tilde{V}$ are distributed normally with a zero mean and dispersions equal to $EN_0/2$. Therefore, the modulus $G = \sqrt{V^2+\tilde{V}^2}$ is distributed according to the Rayleigh law. The probability of exceeding the threshold in the absence of a signal (the probability of a false alarm) is determined as follows

$$p_{\text{лт}} = \exp(-\omega), \qquad (4.26)$$

from which it is possible to find threshold level ω which insures that the given probability of a false alarm will not be exceeded:

$$\omega = -\ln p_{\text{лт}}. \qquad (4.27)$$

In the presence of a usable signal at the input of the receiver V and $\tilde{V}$ will be distributed normally as before with parameters $M\{\tilde{V}\} = M_xE$, $M\{V\} = M_yE$:

$$D\{V\} = \frac{EN_0}{2}(1+2h_x^2);\; D\{\tilde{V}\} = \frac{EN_0}{2}(1+h_y^2).$$

In this case the modulus has a four-parameter distribution. The probability of correct detection (or missing the target) is determined by means of the integral function

$$p_{\text{пр}} = F(A,\ B,\ C,\ D). \qquad (4.28)$$

In the case under consideration A, B, C, and D are determined by the formulas:

$$A = \sqrt{\frac{2\ln 1/p_{\text{лт}}}{1+\frac{2\overline{h^2}}{(1+\beta^2)(1+q^2)}}};\; B = \frac{1+\frac{2\overline{h^2}\beta^2}{(1+\beta^2)(1+q^2)}}{1+\frac{2\overline{h^2}}{(1+\beta^2)(1+q^2)}};$$

$$C^2 = \frac{\overline{h^2}q^2}{1+q^2};\; D = \varphi_p.$$

In calculations of noise suppression the formulas in [89] may be very useful.

Analogously to (4.25) for the domain of small errors and small q^2 we may use the approximate notation

$$p_{np} \approx \frac{\ln(1/p_{лт})(1+\beta^2)(1+q^2)}{2\pi^2 \beta \exp\left[\frac{q^2(1+\beta^2)}{2\beta^2}(\cos^2\varphi_p + \beta^2 \sin^2\varphi_p)\right]}. \quad (4.29)$$

It is easy to observe that expression (4.25) and (4.29) coincide. We will analyze the effect of channel parameters on the working characteristics of a non-coherent detector and then optimal detector in the stipulated conditions. It is apparent from (4.15) and (4.24) that the probability of missing a signal decreases exponentially with growth in q^2.

Where the channel transfer coefficient does not have a regular part a deepening of asymmetry by orthogonal component (decrease in B^2) increases the probability of a miss. The existence of asymmetry can provide a gain in the probability of a miss if a weakly fluctuating component of the transfer coefficient ($B^2 << 1$)has an essential average ($q^2 > 0$, $\phi_p = 0$). The working characteristics of detectors figured by the formulas and tables in [11, 49, and 89] are represented in Figure 4.2 below (the dotted lines show the curves of optimal detection and the solid lines are non-coherent detection). A comparison shows that

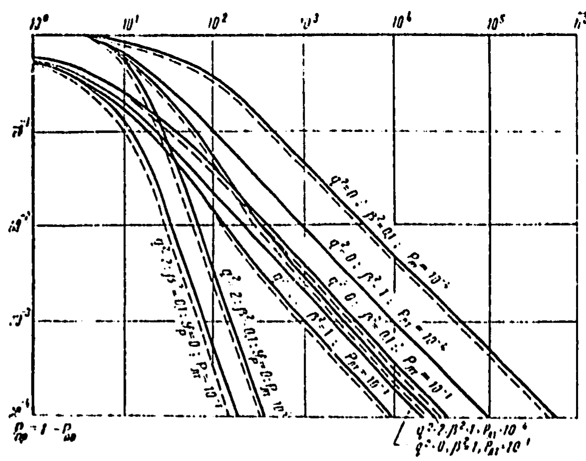

Figure 4.2. Working Characteristics of a Non-Coherent Detector and an Optimal Detector in a Channel with Smooth Fade-outs. (Solid line is non-coherent processing; broken line is optimal processing.)

the loss of information on phase does not reduce the quality of signal detection significantly. The energy loss is zero decibels for a Rayleigh distribution of amplitudes because the true distribution of phases is uniform and reaches a maximum (about 1.4 decibels) for the ideal channel (phase distribution is a Delta function). In the intermediate domain of change in parameters, the energy loss is almost intangible (fractions of a decibel), but it increases slightly with greater asymmetry in good channels ($q^2 >> 0$; $\phi_p = 0$).

The slight decrease in noise suppression combined with simplification of practical realization and the receiver's invariance in relation to channel parameters makes the non-coherent method of simple detection in a channel with smooth fadeouts preferable to the strictly optimal method.

The probability of error in a system of M-orthogonal signals with optimal processing. To calculate the probabilities of errors it is convenient to convert algorithm (3.56) to the form

$$G_l > G_g,\ g=\overline{1,\ M};\ g \neq l, \tag{4.30}$$

where

$$G_l = V_l^2 + \tilde{V}_l^2.$$

The quantities v_1 and $\tilde{V}_1$ are distributed normally. They are mutually independent owing to the orthogonality of the signals in the amplified sense, and have these parameters

$$\left.\begin{aligned}
&M_1\{V_l\} = m_x \sqrt{\frac{2E}{N_0}\,\frac{1+2h_x^2}{2h_x^2}},\quad M_1\{\tilde{V}_l\} = m_y \sqrt{\frac{2E}{N_0}\,\frac{1+2h_y^2}{2h_y^2}},\\
&D\{V_l\} = 2h_x^2,\quad D\{\tilde{V}_l\} = 2h_y^2.\\
&M_1\{V_g\} = m_x \sqrt{\frac{2E}{N_0}\,\frac{1}{2h_x^2(1+2h_x^2)}},\\
&M_1\{\tilde{V}_g\} = m_y \sqrt{\frac{2E}{N_0}\,\frac{1}{2h_y^2(1+2h_y^2)}},\\
&D\{V_g\} = \frac{2h_x^2}{1+2h_x^2};\quad D\{\tilde{V}_g\} = \frac{2h_y^2}{1+2h_y^2}.
\end{aligned}\right\} \tag{4.31}$$

The probability of error is found from the relation

$$p = M_1\{1-[1-F(G_l)]^{M-1}\}. \tag{4.32}$$

In this formula the quantity $F(G_1)$ is the probability that the random quantity G_g will exceed a certain random level G_1. The averaging in (4.32) is done according to G_1.

Observing that in the general case the distribution of moduli of G_1 and G_g is a generalized Gaussian distribution, we may, following [89], write the formula for the probability of error in a binary system (M = 2) as follows

$$p = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} \frac{R_1^n}{n!}\,\frac{R_2^k}{k!}\,\frac{\partial^{2n}}{\partial a_1^n\,\partial b_1^n}\,\frac{\partial^{2k}}{\partial a_2^k\,\partial b_2^k}\left\{Q\left[\sqrt{\frac{a_2^2+b_2^2}{1+r^2}};\ \sqrt{\frac{r^2(a_1^2+b_1^2)}{1+r^2}}\right] - \frac{1}{1+r^2}\exp\left[-\frac{r^2(a_1^2+b_1^2)+a_2^2+b_2^2}{2(1+r^2)}\right]\times\right.$$
$$\left.\times I_0\left[\frac{\sqrt{r^2(a_1^2+b_1^2)(a_2^2+b_2^2)}}{1+r^2}\right]\right\}, \tag{4.33}$$

where

$$r^2 = 1/(1+2h_x^2);$$

$$a_p^2 = \frac{(M_1\{V_p\} - M_1\{\widetilde{V}_p\})^2}{D\{V_p\} + D\{\widetilde{V}_p\}}; \quad b_p^2 = \frac{(M_1\{V_p\} + M_1\{\widetilde{V}_p\})^2}{D\{V_p\} + D\{\widetilde{V}_p\}};$$

$$R_p = \frac{D\{V_p\} - D\{\widetilde{V}_p\}}{D\{V_p\} + D\{\widetilde{V}_p\}}, \quad p = 1, 2.$$

Familiar [49] particular cases follow from formula (4.33). Analyzing parameters (4.31), of the quantity G which is in the right part of formula (4.30), it is not difficult to observe that the distribution of the right part becomes an x^2 distribution with two degrees of freedom when the condition h^2_y, $h^2_x >> 1$ is met. This makes it possible to obtain [89] an approximate formula for the probability of error

$$p = \sum_{k=1}^{M-1} (-1)^{k+1} C_k^{M-1} \frac{\exp\left\{-\frac{kq^2(1+\beta^2)}{2\beta^2}\left[\frac{1+\frac{2\bar{h}^2}{(1+\beta^2)(1+q^2)}}{1+k\frac{2\bar{h}^2}{(1+\beta^2)(1+q^2)}} \times \beta^2\sin^2\varphi_p + \frac{1+\frac{2\beta^2\bar{h}^2}{(1+\beta^2)(1+q^2)}}{1+k\frac{2\bar{h}^2\beta^2}{(1+\beta^2)(1+q^2)}}\cos^2\varphi_p\right]\right\}}{\sqrt{\left[1+k\frac{2\bar{h}^2}{(1+\beta^2)(1+q^2)}\right]\left[1+k\frac{2\bar{h}^2\beta^2}{(1+\beta^2)(1+q^2)}\right]}}. \quad (4.34)$$

Calculations show that for small values of $q^2 (q^2 \leqslant 3)$, it is possible to use formula (4.34) for practically any $\overline{h^2} \geqslant 5$. The explanation for this is that the true distribution of the right part of (4.30) (G_g) in this case is very close to the approximating x^2 distribution.

For the most interesting domain of small errors it is not difficult from (4.34) to obtain

$$p = \frac{(1+\beta^2)(1+q^2)}{2\bar{h}^2\beta}\exp\left[-\frac{q^2(1+\beta^2)}{2\beta^2}(\cos^2\varphi_p + \beta^2\sin^2\varphi_p)\right]\sum_{k=1}^{M-1}\frac{1}{k}. \quad (4.35)$$

It is interesting to observe that an expression analogous to (4.35) can be obtained for the domain of small errors in a non-coherent receiver of signals of the type under consideration. It follows easily from the general expression obtained in [49]

$$p=\sum_{k=1}^{M-1}(-1)^{k+1}C_k^{M-1}\frac{\exp\left\{-\frac{kq^2\overline{h^2}}{1+q^2}\left[\frac{\cos^2\varphi_p}{1+k\left(1+\frac{\overline{h^2}\beta^2}{(1+\beta^2)(1+q^2)}\right)}+\frac{\sin^2\varphi_p}{1+k\left(1+\frac{2\overline{h^2}}{(1+\beta^2)(1+q^2)}\right)}\right]\right\}}{\sqrt{\left[1+k\left(1+\frac{2\overline{h^2}\beta}{(1+\beta^2)(1+q^2)}\right)\right]\times\left[1+k\left(1+\frac{2\overline{h^2}}{(1+\beta^2)(1+q^2)}\right)\right]}}. \quad (4.36)$$

This confirms once again the fact observed in [42] that optimal and sub-optimal (non-coherent) receivers for a system of orthogonal signals of equal energy may provide similar noise suppression. It follows from (4.36) where M = 2 and the values of the parameter q^2 are limited, that the nature of decrease in the probability of error depending on h^2 is approximated by a power function of the type

$$p\approx\left(\frac{1}{\overline{h^2}}\right)^g;\ g\geqslant 0.5, \quad (4.37)$$

where g = 0.5 for $B^2 = q^2 = 0$ (truncated normal fadeouts), $0.5 < g < 1$ for $0 < b^2 < 1$, $q^2 = 0$ (sub-Rayleigh fadeouts), g = 1 for $B^2 = 1$, $q^2 = 0$ (Rayleigh fadeouts), and g > 1 for $q^2 > 0$ (four-parameter fadeout).

For large values of M it is convenient to use the asymptotic ($M\to\infty$) formula [16] that reduces the discrimination problem formally to a problem of detection

$$[1-F(G_l)]^{M-1}=\begin{cases}0, & G_l\leqslant G_0;\\ 1, & G_l>G_0.\end{cases}$$

The level G_o for an optimal system may be determined from the condition

$$F(G_0,\ B^{(-)},\ C^{(-)},\ D^{(-)})=2^{-1/(M-1)}, \quad (4.38)$$

parameters $B^{(-)}$, $C^{(-)}$, $D^{(-)}$ are determined by formula (4.21). For a non-coherent processing system the level G_o is given by the relation

$$e^{-G_0}=1-2^{-1/(M-1)}. \quad (4.39)$$

Correspondingly, the expression for the probabilities of errors in an optimal system takes the form

$$p=F(G_0;\ B^{(+)};\ C^{(+)};\ D^{(+)}) \quad (4.40)$$

[the parameters $B^{(+)}$, $C^{(+)}$, $D^{(+)}$ are given by formula (4.21)] and the expression for the probability of error in non-coherent processing is

$$p=F\left[\sqrt{\frac{2\ln\frac{M-1}{\ln 2}}{1+\frac{2\overline{h^2}}{(1+\beta^2)(1+q^2)}}};\ \sqrt{\frac{1+\frac{2\beta^2\overline{h^2}}{(1+\beta^2)(1+q^2)}}{1+\frac{2\overline{h^2}}{(1+\beta^2)(1+q^2)}}};\right.$$

$$\sqrt{\frac{\bar{h}^2 q^2}{1+q^2+\bar{h}^2};\ \varphi_p}\ \Bigg]. \qquad (4.41)$$

In the domain of small errors formula (4.35) can be used to calculate the noise suppression of both systems.

Figures 4.3 and 4.4 below show curves of the probability of error constructed from the formulas obtained above. From the figures and formulas it can be seen that the probability of error depends significantly on the statistical parameters of the channel. The energy loss of the worst channel considered, the sub-Rayleigh channel where $B^2 = 0.1$, is about 30 decibels compared to the ideal channel where M = 2 and $r = 10^{-4}$. The magnitude of M has little effect on this relation. Where the channel is

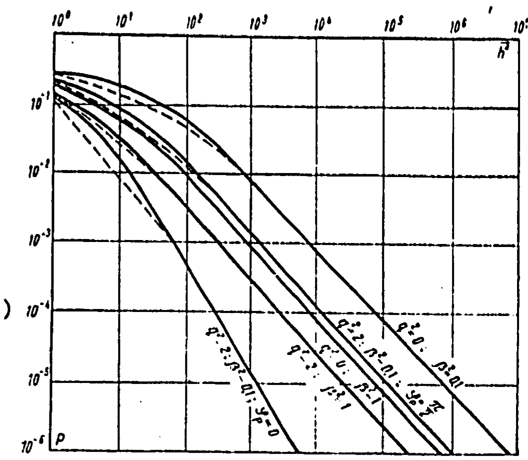

Figure 4.3. Probability of Error for Optimal and Non-Coherent Discrimination (M = 2). (Solid line is non-coherent processing and broken line is optimal processing.)

asymmetrical for orthogonal components, there may be an energy gain compared to the case where asymmetry is absent if the weakly fluctuating component ($B^2 << 1$) has a clearly expressed regular part ($q^2 >> 0$, $\phi_p = 0$). The sum included in formula (4.35) where M >> 1 may be approximated by the simple expression

$$\sum_{k=1}^{M-1} \frac{1}{k} = \ln M. \qquad (4.42)$$

This allows us to state that growth in the position format of M leads to a decrease of ln M times in the equivalent signal/noise ratio compared to a binary system and ln M/ln M' times in comparison with the positional M'.

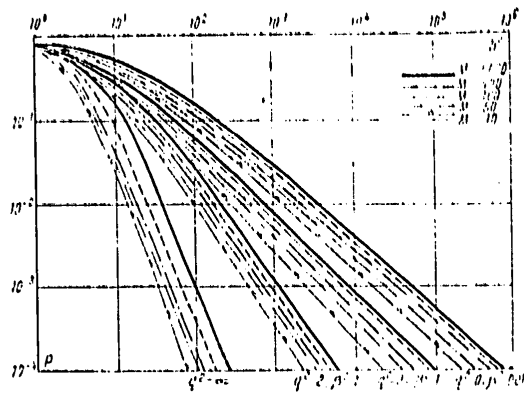

Figure 4.4. The Probability of Error with Non-Coherent Discrimination of Orthogonal Signals.

Optimal binary system of signals in a channel with smooth fadeouts. In the preceding sections the noise suppression of algorithms for concrete signal systems has been determined.

Another formulation of the problem is also interesting for data transmission [60]: find the system of signals which, with the imposed constraints, affords minimum probability of error. We will call this signal system optimal. In our search for the optimal signal system we will impose the constraints dictated by considerations of convenience in realization and economy of input. Specifically, we will require that the system be binary and that signals $s_1(t)$ and $s_2(t)$ be equiprobable and have identical energies $E_1 = E_2 = E$. In this case the mutual correlation coefficients of the signals will also be the parameters that should be used to find the optimum (minimum probability of error):

$$\left.\begin{aligned} \lambda &= \frac{1}{E}\int_0^T s_1(t)\, s_2(t)\, dt = \frac{1}{E}\int_0^T \tilde{s}_1(t)\, \tilde{s}_2(t)\, dt, \\ \tilde{\lambda} &= \frac{1}{E}\int_0^T s_1(t)\, \tilde{s}_2(t)\, dt = \frac{1}{E}\int_0^T \tilde{s}_1(t)\, s_2(t)\, dt, \\ \bar{\lambda}^2 &= \lambda^2 + \tilde{\lambda}^2. \end{aligned}\right\} \qquad (4.43)$$

To calculate noise suppression under the stipulated conditions, it is convenient to put algorithm (3.45) in this form

$$D > 0;\ D = V_1^2 + V_2^2 - V_3^2 - V_4^2, \qquad (4.44)$$

where B is the quadratic form of Gaussian variables.

The characteristic function of the quadratic form of Gaussian variables is known [114].

$$\theta_D(iu) = \frac{\exp\left[-\frac{1}{2}\tilde{M}K^{-1}\{I-(I-i2uKQ)^{1/2}\bar{M}\}\right]}{|I-i2uKQ|^{1/2}}. \qquad (4.45)$$

In this case $\bar{M} = \{\bar{V}\}$ is the matrix-column of average values of variables; K is the matrix of covariations of quantities V_k; I is the unitary matrix of the same order as K; Q is the matrix in quadratic form.

The statistical parameters of the variables V_i can be computed easily [89].

In the general case it is more convenient to calculate the probability of error using characteristic function (4.45) following the technique described in [89]. The calculated curves are shown in Figure 4.5 below.

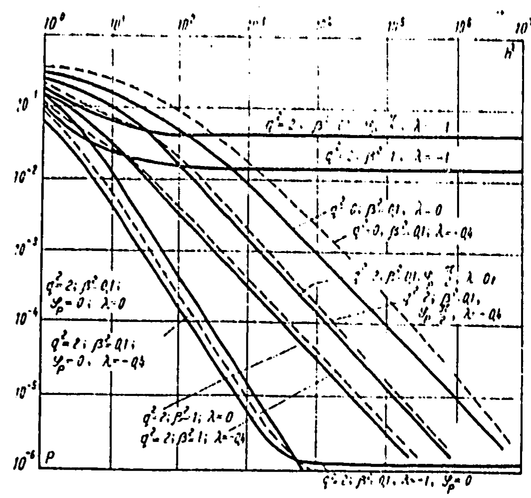

Figure 4.5. The Probability of Error in a Binary System of Non-Orthogonal Signals.

If there is no asymmetry by orthogonal components ($B^2 = 1$), we arrive at the known [142] result

$$p = Q(ac,\ bc) - \frac{1}{2}b\exp\left(-\frac{a^2c^2+b^2c^2}{2}\right)I_0(abc^2), \qquad (4.46)$$

$$\left.\begin{aligned} &a = 1 - \frac{\mu\sqrt{1-\overline{\lambda}^2}}{\sqrt{1-\mu^2\overline{\lambda}^2}}, \quad b = 1 + \frac{\mu\sqrt{1-\overline{\lambda}^2}}{\sqrt{1-\mu^2\overline{\lambda}^2}}, \\ &c = \sqrt{\frac{q^2\left([2\overline{h}^2/(1+q^2)](1-\overline{\lambda}^2) - 2(1-\lambda^2)\right)}{4\overline{h}^2(1-\overline{\lambda}^2)/(1+q^2)}}, \quad \mu = \frac{\overline{h}^2}{1+\overline{h}^2+q^2}. \end{aligned}\right\} \quad (4.47)$$

To find the optimal form of the signals we will investigate (4.46) to the extremum according to λ and $\overline{\lambda^2}$. The necessary conditions for the existence of the minimum can be written as follows:

$$\frac{\partial p}{\partial ac}\,\frac{\partial ac}{\partial \overline{\lambda}^2} = 0; \quad \frac{\partial p}{\partial ac}\,\frac{\partial ac}{\partial \lambda} = 0. \quad (4.48)$$

Having observed that

$$\exp\left(-\frac{a^2c^2+b^2c^2}{2}\right) I_0(abc^2) = -\frac{1}{ac}\,\frac{\partial Q(ac,\,bc)}{\partial ac}, \quad (4.49)$$

we will write the necessary conditions of the extremum as

$$\left.\begin{aligned} &\frac{\partial p}{\partial ac} = \frac{b}{2cc}\,\frac{\partial^2 Q(ac,\,bc)}{\partial (ac)^2} + \left(1 - \frac{b}{2a^2c^2}\right)\frac{\partial Q(ac,\,bc)}{\partial ac} = 0, \\ &\frac{\partial ac}{\partial \lambda} = 0, \quad \frac{\partial ac}{\partial \overline{\lambda}^2} = 0. \end{aligned}\right\} \quad (4.50)$$

The differential equation standing first in (4.50) is solved in an elementary manner, but it yields values of ac and bc which can never be achieved because they are outside the domain of definition of these quantities. By studying (4.50), it is not difficult to show that the coefficient of mutual correlation of λ takes two optimal values with the existence of a regular component in the channel $\lambda = -1$ and $\lambda = 0$ ($\overline{\lambda^2} = 0$) and one value $\lambda = 0$ ($\overline{\lambda^2} = 0$) in the absence of a regular component. Calculations show that this situation persists in a channel with asymmetry by orthogonal components also.

The investigation we have made allows us to state that in a channel with smooth fadeouts and a regular component, the system with opposite signals is optimal up to certain threshold values of the signal/noise ration $\overline{h^2_{\text{пор}}}$, but for larger values the system with orthogonal signals is optimal.

The threshold value $\overline{h^2_{\text{пор}}}$ can be determined from the condition of equality of probabilities of error in these two systems.

$$p(\lambda = -1) = p(\lambda = 0), \quad (4.51)$$

The expression for the probability of error in a system with opposite signals has this form [49].

$$p = \frac{1}{2}\left\{1-\Phi\left[\sqrt{\frac{2q^2\bar{h}^2\left[1+\frac{2q^2}{(1+\beta^2)(1+q^2)}(\cos^2\varphi_p+\beta^2\sin^2\varphi_p)\right]}{(1+q^2)\left[1+\frac{2\bar{h}^2\beta^2}{(1+q^2)(1+\beta^2)}\right]\left[1+\frac{2\bar{h}^2}{(1+q^2)(1+\beta^2)}\right]}}\right]\right\}. \quad (4.52)$$

The probability of error in a system of orthogonal signals was determined above (4.33). A comparative analysis of formulas (4.33) and (4.52) shows that in good channels a system with opposite signals remains optimal throughout the domain of error that is of practical interest. A significant feature of the system with opposite signals is that it has an irreducible (as $\overline{h^2}$ increases) probability of error whose value is determined by the channel parameters [49]. This makes it possible to determine the threshold value $\overline{h^2}_{\text{пор}}$ in channels that are far from ideal but have a regular component, using the relation

$$\overline{h}^2_{\text{пор}} = \frac{(1+\beta^2)(1+q^2)\exp\left[-\frac{q^2(1+\beta^2)}{2\beta^2}(\cos^2\varphi_p+\beta^2\sin^2\varphi_p)\right]}{\beta\left\{1-\Phi\left[\sqrt{q^2\frac{1+\beta^2}{\beta^2}(\cos^2\varphi_p+\beta^2\sin^2\varphi_p)}\right]\right\}}. \quad (4.53)$$

The range of variation in threshold value $\overline{h^2_{\text{пор}}}$ is very great. For example, in a Rice channel ($B^2 = 1$, $\overline{q^2} = 2$) $\overline{h^2}_{\text{пор}} = 10$, and in a generalized channel with good statistics ($q^2 = 2$, $\phi_p = 0$, $B^2 = 0.1$) $h^2_{\text{пор}} = 5\cdot10^3$. A rationally designed communications system should change the appearance of the signals used depending on the state of mean statistical parameters of the channel.

4.4. Characteristics of Detection of Space-Time Signals (Generalized Gaussian Statistics)

Let us determine the probability of a false alarm and missed signal for the algorithms of optimal and suboptimal space-time signal processing.

Optimal processing. To calculate the working characteristics, we will represent algorithm (3.45) in the form

$$G = \sum_{k=1}^{N} V_k^2 + \tilde{V}_k^2 \geqslant \omega. \quad (4.54)$$

The quantities $\tilde{V}_k$, V_k, $k = \overline{1, N}$ have Gaussian distributions and are statistically independent. When there is a usable signal in the observed oscillations these quantities have the parameters:

$$\left.\begin{aligned} M_1\{V_k\} &= m_{xk}\sqrt{2d_k}\sqrt{\frac{1+2h_{xk}^2}{2h_{xk}^2}}, \\ M_1\{\tilde{V}_k\} &= m_{yk}\sqrt{2d_k}\sqrt{\frac{1+2h_{yk}^2}{2h_{yk}^2}}, \\ D\{V_k\} &= 2h_{xk}^2;\ D\{\tilde{V}_k\} = 2h_{yk}^2. \end{aligned}\right\} \quad (4.55)$$

In the absence of a usable signal the parameters will be

$$\left.\begin{aligned} M_1\{V_k\} &= m_{xk}\sqrt{2d_k}\,\frac{1}{2h_{xk}^2}\sqrt{\frac{2h_{xk}^2}{1+2h_{xk}^2}},\\ M_1\{\tilde{V}_k\} &= m_{yk}\sqrt{2d_k}\,\frac{1}{2h_{yk}^2}\sqrt{\frac{2h_{yk}^2}{1+2h_{yk}^2}},\\ D\{V_k\} &= \frac{2h_{xk}^2}{1+2h_{xk}^2},\ D\{\tilde{V}_k\} = \frac{2h_{yk}^2}{1+2h_{yk}^2}. \end{aligned}\right\} \quad (4.56)$$

In the general case, it is convenient to calculate the probability of a false alarm and missed signal using a characteristic function, which is easy to compute for both the presence of a usable signal $\Theta_G^{(+)}$ (i u) and for the absence of a usable signal $\Theta_G^{(-)}$ (i u). These characteristic functions are determined by the identical expressions:

$$\Theta_G(\mathrm{i}\,u) = \prod_{k=1}^{N} \frac{\exp\left[\dfrac{\mathrm{i}\,uM_1\{V_k\}}{1-\mathrm{i}\,2uD\{V_k\}} + \dfrac{\mathrm{i}\,uM_1\{\tilde{V}_k\}}{1-\mathrm{i}\,2uD\{\tilde{V}_k\}}\right]}{\sqrt{[1-\mathrm{i}\,2uD\{V_k\}][1-\mathrm{i}\,2uD\{\tilde{V}_k\}]}}, \quad (4.57)$$

in which the value of parameters from (4.55) should be substituted to compute $\Theta_G^{(+)}$ (i u) and the parameters from (4.56) to compute $\Theta_G^{(-)}$(i u).

The probabilities are expressed through the corresponding characteristic functions

$$\left.\begin{aligned} p_{\text{лт}} &= 1-\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\Theta_G^{(-)}(\mathrm{i}\,u)}{\mathrm{i}\,u}\,e^{\mathrm{i}u\omega}\,du,\\ p_{\text{пр}} &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\Theta_G^{(+)}(\mathrm{i}\,u)}{\mathrm{i}\,u}\,e^{-\mathrm{i}u\omega}\,du. \end{aligned}\right\} \quad (4.58)$$

It is convenient to make the numerical calculations for formulas (4.58) using the methodology and algorithms of [89].

Let us consider the domain of small errors. Analyzing the expressions included in (4.56), we observe that where h^2_{xk}, h^2_{yk} >>1 the mathematical expectations of quantities V_k and $\tilde{V}_k$ become close to 0 and their dispersions tend toward 1. Thus, the distribution of the random quantity G in the absence of a usable signal contracts toward an χ^2-distribution with 2N degrees of freedom. This makes it possible to express the probability of a false alarm by a known [109] relation

$$p_{\text{лт}} = \frac{1}{(N-1)!}\,\Gamma(\omega, N) = e^{-\omega}\sum_{g=0}^{N-1}\frac{\omega^g}{g!}. \quad (4.59)$$

The threshold level ω_0 that is optimal according to the Neuman-Pearson criterion should be computed from equation (4.59). Looking at expressions (4.55), we may observe that in the domain h^2_x, $h^2_y >> 1$ the dispersions of quantities V_k and $\tilde{V}_k$ assume larger values. This makes it possible [89] to obtain simple expressions for the probability of missing a signal

$$p_{up} = \frac{\omega_0^N}{N!} \prod_{k=1}^{N} \frac{(1+\beta_k^2)(1+q_k^2)}{2h_k^2 \beta_k} \exp\left[- \frac{q_k^2 (1+\beta_k^2)}{2\beta_k^2} \times \left(\cos^2 \varphi_{p\,k} + \beta_k^2 \sin^2 \varphi_{p\,k}\right)\right]. \quad (4.60)$$

A whole series of particular results can be obtained from formulas (4.59) and (4.60) with different assumptions about the model of fluctuations of channel parameters.

As a numerical example let us consider a spatial model of a channel with fadeouts that are non-selective in time and frequency. The number N in formulas (4.59) and (4.60) is determined from the relation $N = N^R = [R/P_{cor} + 1]$ (one spatial coordinate r is being considered). The surge characteristic in this case is represented in the form

$$h(t, \xi, r) = g(r)\,\delta(\xi), \quad (4.61)$$

where g(r) is a random complex-valued function of the spatial coordinate.

We assume that the real and imaginary parts of function g(r) have normed correlation functions of the type

$$R_g(r - r') = \exp\left(-\frac{|r - r'|}{\rho_{кор}}\right). \quad (4.62)$$

The results of calculations of the probability of error using formulas (4.58)-(4.60) for the channel model under consideration are shown in Figure 4.6 below.

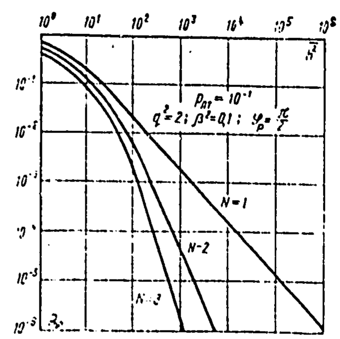

Figure 4.6. Working Characteristics of Optimal Detection in a Channel With Space-Selective Fadeouts.

Let us consider one more spatial model, a homogeneous channel in terms of space with smooth fadeouts by frequency and selective fadeouts in time. In this case, as was shown in Chapter 3, optimal spatial processing is accomplished by narrowly directed antennas with directivity diagrams of the type $\sin \vartheta/\vartheta$. The number of such antennas is N^R. Optimal time processing is accomplished by a multichannel scheme with N^T channels. Thus, in this case $N = N N^R$ in formulas (4.58)-(4.60).

It follows from formulas (4.59) and (4.60) that

$$\left.\begin{aligned} p_{\text{пт}} &= \frac{1}{(N^T N^R - 1)!} \Gamma(\omega, N^T N^R) = e^{-\omega} \sum_{g=0}^{N^T N^R - 1} \frac{\omega^g}{g!}, \\ p_{\text{пр}} &= \frac{\omega_0^{N^T N^R}}{(N^T N^R)!} \prod_{i=1}^{N^R} \prod_{k=1}^{N^T} \frac{(1+\beta_{ik}^2)(1+q_{ik}^2)}{2\overline{h_{ik}^2}\,\beta_{ik}} \times \\ &\times \exp\left[-\frac{q_{ik}^2 (1+\beta_{ik}^2)}{2\beta_{ik}^2} (\cos^2 \varphi_{p\,ik} + \beta_{ik}^2 \sin^2 \varphi_{p\,ik})\right]. \end{aligned}\right\} \quad (4.63)$$

Simple reasoning allows us to reach the conclusion that identical expressions for probability of error may correspond to mathematical channel models that differ from a physical point of view. This occurs when the values of parameters of processes of fluctuation that differ from a physical point of view coincide in the models (for example, the nature of signal fadeouts in time for one channel model may be identical to the nature of fadeouts in space for another model). In particular, formulas (4.63) will describe the probability of error for a channel that is homogeneous by frequency with smooth spatial fadeouts (in this case the parameter $N^R \times N^S$ should be used in these formulas). This feature, which may be called the reversibility feature, is typical not only of detection units but of all other units for processing space-time signals in channels with selective fadeout.

Suboptimal processing. Generally speaking, it is more complex to investigate suboptimal algorithms in a stochastic channel than optimal ones. The chief reason is the possible appearance of a statistical relationship between particular paths. The second difficulty is that there is just one optimal algorithm, but a set of suboptimal algorithms may be proposed and this gives the investigator of suboptimal algorithms the difficult job of selecting the object of investigation. Here we will consider several algorithms for processing space signals that have worked well in practice or show definite promise. The analysis of suboptimal schemes is done in those channels where they actually provide processing that is close to optimal according to definite variables (space, frequency, or time). If a unit realizes spatial scattering, for example, by means of narrowly directed antennas, the spatial paths are considered independent in analysis if the opposite is not stipulated. This does not at all mean that the general formulas for error probability obtained in this section cannot be used in the case of statistically dependent paths. In fact, when investigating the probability of error (during both detection and discrimination of signals), we are always dealing with the quadratic form of random quantities

$$Q = \sum_{k,\,p} a_{kp} V_k U_p, \qquad (4.64)$$

where a_{kp} are coefficients.

We know [5] that there is always a possibility of reducing the quadratic form to a canonical type

$$Q = \sum_k \alpha_k V_k^2 + \beta_k U_k^2, \qquad (4.65)$$

where a_k and B_k are coefficients corresponding to a new system of coordinate and U and V are variables corresponding to the transformed system of coordinates.

Lack of correlation among the variables of the transforms in form (4.65) is achieved by an appropriate choice of conversion from (4.64) to (4.65). In the Gaussian case, lack of correlation is identical to statistical independence. Therefore, in the particular case, the probability of quadratic form (4.64) exceeding a certain level through which the probability of error is calculated in any system may be computed through the corresponding probability for form (4.65) after substituting the concrete parameter values. Applied to correlated paths of propagation, some computations of probability of error are contained in works [6, 46, 104].

Let us pass on to a consideration of specific algorithms. We will consider a detector that contains a set of narrowly directed antennas (spatial processing), a delay line (processing by frequency), and a filter coordinated with the transmitted signal (processing in time) and working on the algorithm

$$G = \sum_{k=1}^{N^F} \sum_{i=1}^{N^\theta} V_{ik}^2 + \tilde{V}_{ik}^2 \geqslant \omega, \qquad (4.66)$$

where

$$\begin{Bmatrix} V_{ik} \\ \tilde{V}_{ik} \end{Bmatrix} = \int_{-\theta}^{\theta} \frac{(\sin \vartheta/\delta\vartheta - i\,\pi)}{\vartheta/\delta\vartheta - i\,\pi}\, d\vartheta \int_0^T z(t,\,\vartheta) \begin{Bmatrix} s\left(t - \frac{k}{F}\right) \\ \tilde{s}\left(t - \frac{k}{F}\right) \end{Bmatrix} dt.$$

The variables V_{ik} and $\tilde{V}_{ik}$ are Gaussian independent quantities. When the oscillation being analyzed contains a usable signal these quantities have the parameters

$$\left.\begin{aligned} &M_1\{V_{ik}\} = m_{xik} \sqrt{2d_k};\ M_1\{\tilde{V}_{ik}\} = m_{yik} \sqrt{2d_k}, \\ &D\{V_{ik}\} = 1 + 2h_{xik}^2 \mu^T;\ D\{\tilde{V}_{ik}\} = 1 + 2h_{yik}^2 \mu^T. \end{aligned}\right\} \qquad (4.67)$$

If there is no usable signal

$$M_1\{V_{ik}\} = M_1\{\widetilde{V}_{ik}\} = 0;\quad D\{V_{ik}\} = D\{\widetilde{V}_{ik}\} = 1. \qquad (4.68)$$

Parameter μ^T in formula (4.67) is determined from the relation [105]

$$\mu^T = \frac{2}{T}\int_0^T \left(1 - \frac{\tau}{T}\right) B_h(\tau)\, d\tau, \qquad (4.69)$$

where $B_h(\tau)$
is a normed correlation function of the channel characteristic according to the time variable.

From (4.69) it is not difficult to show that $0 < \mu^T < 1$. Parameter μ^T characterizes the rate of fadeout. The greater this rate is, the smaller quantity μ^T will be. For the exponential correlation function $R_h(\tau) = \exp(-|\tau|/\tau_{\text{кор}})$ the expression for μ^T takes the form

$$\mu^T = \frac{2}{(T/\tau_{\text{кор}})^2}\left[T/\tau_{\text{кор}} - 1 + e^{-T/\tau_{\text{кор}}}\right]. \qquad (4.70)$$

It can be seen from (4.68) that in the absence of a usable signal the quantity G has an χ^2- distribution with $2N^F\Lambda^\Theta$ degrees of freedom. This makes it possible to write the expression for the probability of a false alarm as

$$p_{\text{лт}} = \frac{1}{(N^F N^\Theta - 1)!}\,\Gamma(\omega, N^F N^\Theta - 1) = e^{-\omega}\sum_{g=0}^{N^F N^\Theta - 1} \frac{\omega^g}{g!}. \qquad (4.71)$$

The numerical calculations of the probability of missing a signal should be done in the same way as done in the case of the optimal scheme for formula (4.58), using the parameters (4.67).

Let us consider the case of identical, on the average, paths of propagation

$$m_{xik} = m_x,\ m_{yik} = m_y,\ i = \overline{1, N^\Theta};$$
$$\sigma^2_{xik} = \sigma^2_x;\ \sigma^2_{yik} = \sigma^2_y,\ k = \overline{1, N^T}.$$

In the presence of a usable signal the random quantity G has a non-central semisymmetrical χ^2 -distribution with $2N^FN^\Theta$ degrees of freedom [89]. The probability of a miss for even values of N^FN^Θ is determined from the relation

$$p_{\text{пр}} = F\left(\omega\sqrt{\frac{2}{1+B^2}};\ B;\ \sqrt{\frac{a^2+b^2}{2}};\ \operatorname{arc\,tg}\varphi_p\right) - \sum_{n=0}^{\infty}\frac{R^n}{n!}\,\frac{\partial^{2n}}{\partial a^n \partial b^n}\times$$

$$\times\left\{\exp\left(-\frac{a^2+b^2+2\frac{\omega^2}{1+B^2}}{2}\right)\sum_{r=1}^{\frac{N^F N^\Theta}{2}-1}\left(\frac{\omega\sqrt{2}}{\sqrt{(1+B^2)(a^2+b^2)}}\right)^r\times\right.$$
$$\times I_0\left(\frac{\omega\sqrt{a^2+b^2}}{\sqrt{1+B^2}}\right), \qquad (4.72)$$

where a series of parameters depending on quantities (4.67) has been introduced:

$$B^2=D(V)/D(\widetilde{V});\; R=\frac{D(V)-D(\widetilde{V})}{D(V)+D(\widetilde{V})};$$
$$a=\frac{\sqrt{N^F N^\Theta M_1^2(V)}-\sqrt{N^F N^\Theta M_1^2(\widetilde{V})}}{\sqrt{D(V)+D(\widetilde{V})}};$$
$$b=\frac{\sqrt{N^F N^\Theta M_1^2(V)}+\sqrt{N^F N^\Theta M_1^2(\widetilde{V})}}{\sqrt{D(V)+D(\widetilde{V})}}. \qquad (4.73)$$

It is convenient to use formula (4.72) for calculations where $R \ll 1$.

Let us consider the domain of small errors. Using the same methodology as used for the optimal scheme, we obtain the expression for the probability of a miss in the form

$$p_{np}=\frac{\omega^{N^F N^\Theta-1}}{(N^F N^\Theta-1)!\,(\mu^T)^{N^F N^\Theta}}\prod_{k=1}^{N^F}\prod_{i=1}^{N^\Theta}\frac{(1+\beta_{ik}^2)(1+q_{ik}^2)}{2\beta_{ik}h_{ik}^2}\exp\times$$
$$\times\left[-\frac{q_{ik}^2(1+\beta_{ik}^2)}{2\beta_{ik}^2\mu^T}\left(\cos^2\varphi_{p\,k}+\beta_k^2\sin^2\varphi_{p\,k}\right)\right]. \qquad (4.74)$$

A comparison of formulas (4.74) and (4.60) shows that the superiority of the optimal scheme to the suboptimal one we have considered begins to tell at a high rate of fadeouts $T/\tau_{cor} \gg 1$.

Under conditions of a high rate of fadeouts, the optimal detector provides N^T times greater multiplicity of scattering than the suboptimal one. But if the rate of fadeouts is low $T/\tau_{cor} \approx 1$, the optimal detector has no apparent advantages in noise suppression over the suboptimal one, but it is much more complex to realize and demand a much greater volume of a prior data for construction. When developing systems for detection of spatial signals (in particular in radio astronomy), it would be very useful to use data on the rate of fluctuations of reflected signals to substantiate the choice of the particular method of space-time processing.

It can be seen from expression (4.74) that in the absence of a regular component ($q^2_{ik} = 0$, $i = \overline{1, N^\Theta}$, $k = \overline{1, N^T}$) the selectivity of fadeouts in time leads to an increase in the probability of a miss in the suboptimal detector. The loss in error probability caused by channel selectivity in time in the domain of small errors is $10N^F N^\Theta \lg 1/\mu^T$ decibels and increases as the number of branches of dispersion by frequency and in space increases. Let us recall that in the optimal detector channel selectivity in time always improves the quality of the detector regardless of the statistics of fluctuations.

In channels with a regular component time selectivity may result in an improvement in the working characteristics of a suboptimal detector. It is not difficult to show from (4.74) that the gain from time selectivity with respect to probability of a miss appears when the condition

$$\sum_{i=1}^{N^\Theta}\sum_{k=1}^{N^F} \frac{q^2_{ik}(1+\beta^2_{ik})}{2\beta^2_{ik}}(\cos^2\varphi_{p\,k} + \beta^2_{ik}\sin^2\varphi_{p\,k}) > \frac{N^F N^\Theta \mu^T}{1-\mu^T}\ln\left(\frac{1}{\mu^T}\right) \quad (4.75)$$

is met, and amounts to

$$4.4\sum_{i=1}^{N^\Theta}\sum_{k=1}^{N^F} \frac{q^2_{ik}(1+\beta^2_{ik})}{2\beta^2_{ik}}(\cos^2\varphi_{p\,k} + \beta^2_{ik}\sin^2\varphi_{p\,k}) + 10\frac{N^F N^\Theta \mu^T}{1-\mu^T}\lg\frac{1}{\mu^T} \quad (4.76)$$

decibels; in good channels it may reach considerable magnitude.

Let us consider one more algorithm of suboptimal detection that realizes partial dispersion and ignores selectivity in time and in space. In explicit form the algorithm is written as follows

$$G = \sum_{k=1}^{N^F} V_k^2 + \tilde{V}_k^2 > \omega, \quad (4.77)$$

where

$$\left\{\begin{matrix} V_k \\ \tilde{V}_k \end{matrix}\right\} = \int_0^R\int_0^T z(t, r)\left\{\begin{matrix} s\left(t - \frac{k}{F_c}\right) \\ \tilde{s}\left(t - \frac{k}{F_c}\right) \end{matrix}\right\} dt dr.$$

It is not difficult to show that in this case the probability of a false alarm, which defines threshold ω, is written with the formula

$$p_{\text{fa}} = \frac{1}{(N^F-1)!}\Gamma(\omega, N^F - 1) = e^{-\omega}\sum_{p=0}^{N^F-1}\frac{\omega^p}{p!}. \quad (4.78)$$

The expression for the probability of a miss for the domain of small errors has the form

$$p_{\text{np}} = \frac{\omega^{N^F}}{N^F \mid \mu^{RT}} \prod_{k=1}^{N^F} \frac{(1+\beta_k^2)(1+q_k^2)}{2\beta_k \overline{h}_k^2} \exp\left[-\frac{q^2(1+\beta_k^2)}{2\beta_k^2 \mu^{RT}} \times \right.$$
$$\left. \times \left(\cos^2 \varphi_{p\,k} + \beta_k^2 \sin^2 \varphi_{p\,k}\right)\right], \quad (4.79)$$

where the Parameter

$$\mu^{RT} = \frac{2}{T} \frac{2}{R} \int_0^T \int_0^R \left(1 - \frac{\tau}{T}\right)\left(1 - \frac{\rho}{R}\right) B_h(\tau, \rho)\, d\tau\, d\rho. \quad (4.80)$$

The normed correlation function of the channel by the time and space variable $B_h(\tau, \rho)$ may be spatially divisible. Then the parameter

$$\mu^{RT} = \mu^R \mu^T. \quad (4.81)$$

The parameter μ^T was defined above (4.69); μ^R is determined in similar fashion.

In conclusion, let us consider an algorithm that is a spatial analog of time processing by the Kostas scheme. To make it more graphic we will consider a channel with fadeouts that are non-selective in frequency and time but selective in space. The processing algorithm is determined by the expression

$$G = \sum_{k=1}^{N^R} V_k + \widetilde{V}_k^2 \geq \omega, \quad (4.82)$$

where $\left\{ \begin{matrix} V_k \\ \widetilde{V}_k \end{matrix} \right\} = \int_{k\Delta r}^{(k+1)\Delta r} \int_0^T z(t, r) \left\{ \begin{matrix} s(t) \\ \tilde{s}(t) \end{matrix} \right\} dt dr;\ \Delta r = R/N^R.$

Where there is a usable signal in the input oscillation the parameters of quantities V_k and $\widetilde{V}_k$ will be:

$$\left.\begin{aligned} M_1\{V_k\} &= m_x \sqrt{\frac{2E}{N^R N_0}},\ M\{\widetilde{V}_k\} = m_y \sqrt{\frac{2E}{N^R N_0}}; \\ D\{V_k\} &= 1 + \frac{2\sigma_x^2 E}{N^R N_0},\ D\{\widetilde{V}_k\} = 1 + \frac{2\sigma_y^2 E}{N^R N_0}, \end{aligned}\right\} \quad (4.83)$$

where

$$\mu^R = \frac{2N^R}{R} \int_0^{\Delta r} \left(1 - \frac{\rho}{\Delta r}\right) B_h(\rho)\, d\rho. \quad (4.84)$$

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In the absence of a usable signal the quantity G has an χ^2-distribution with $2N^R$ degrees of freedom.

Considering the terms in (4.82) to be independent (this is approximately fulfilled where $\Delta r \geqslant \rho_{cor}$), we come to the conclusion that the probability of a miss is determined by an expression analogous to (4.72). For calculations it is essential to substitute the parameter N^R in (4.72) instead of the product $N^F N^\Theta$, and use parameter (4.83) in (4.73).

Analogously to (4.74), for the domain of small errors we obtain

$$P_{ur} = \frac{\omega^{N^R-1}}{(N^R-1)!\,\mu^{N^R}} \cdot \frac{1}{\left(\frac{2\bar{h}_2}{N^R}\right)^{N^R}} \exp\left[-\frac{q^2(1+\beta^2)}{2\beta^2\mu^R}(\cos^2\varphi_p + \beta_e \sin^2\varphi_p)\right], \quad (4.85)$$

where $\bar{h}^2 = \frac{E}{N_0}(m_x^2 + m_y^2 + \sigma_x^2 + \sigma_y^2)$.

In an analogous way it is possible [110] to formulate the optimal problem and determine the value of N^R which insures minimum probability of error, but with a rigorous approach this investigation encounters serious difficulties. For this reason, it is wise to choose N^R according to the obvious considerations adopted above $N^R = [R/\rho_{cor} + 1)$.

Formula (4.85), in particular, corresponds to a device for spatial processing of optical signals built in the form of a lattice of N^R photo detectors.

4.5. The Probability of Error in Discrimination of Orthogonal Signals (Generalized Gaussian Statistics)

We will consider data transmission using M signals which are orthogonal in the amplified sense under conditions of selective fadeouts. For such signals these relations must be fulfilled:

$$\int_0^T\int_0^R s_{lk}(t, r)\, s_{gk}(t, r)\, dt\,dr = \int_0^T\int_0^R s_{lk}(t, r)\, \tilde{s}_{gk}(t, r)\, dt\,dr = 0; \quad (4.86)$$

$$k = \overline{1, N};\ g,\ l = \overline{1, M};\ g \neq l.$$

Let us look in turn at optimal and suboptimal processing algorithms.

Optimal processing. To calculate the probability of error we will write algorithm (3.56) in the form

$$G_l > G_g,\ g = \overline{1, M},\ g \neq l, \quad (4.87)$$

where

$$G_l = \sum_{k=1}^{N} V_{lk}^2 + \tilde{V}_{gk}^2. \quad (4.88)$$

The random quantities g_l, $l = 1, -M$ are the quadratic forms of Gaussian variables. The components of the quadratic form are statistically independent. Assuming that the oscillation under analysis contains an l-position signal, it is not difficult to compute the mathematical expectations and dispersions of quadratic-form variables

$$\left.\begin{aligned}
&M_1\{V_{lk}\} = m_{xk}\sqrt{2d_{lk}}\sqrt{\frac{1+2h^2_{xlk}}{2h^2_{xlk}}},\\
&M_1\{\tilde{V}_{lk}\} = m_{yk}\sqrt{2d_{lk}}\sqrt{\frac{1+2h^2_{ylk}}{2h^2_{ylk}}},\\
&D\{V_{lk}\} = 2h^2_{xlk};\ D\{\tilde{V}_{lk}\} = 2h^2_{ylk},\\
&M_1\{V_{gk}\} = m_{xgk}\sqrt{2d_{gk}}\frac{1}{2h^2_{xgk}}\sqrt{\frac{1+2h^2_{xgk}}{2h^2_{xgk}}},\\
&M_1\{V_{gk}\} = m_{ygk}\sqrt{2d_{gk}}\frac{1}{2h^2_{ygk}}\sqrt{\frac{1+2h^2_{ygk}}{2h^2_{ygk}}},\\
&D\{V_{gk}\} = \frac{2h^2_{xgk}}{1+2h^2_{xgk}};\ D\{\tilde{V}_{gk}\} = \frac{2h^2_{ygk}}{1+2h^2_{ygk}}.
\end{aligned}\right\} \tag{4.89}$$

In the general case, the probability of error should be calculated by numerical methods using a characteristic function of the quadratic form of Gaussian variables.

To simplify the formulas, we will assume in what follows that the energies of different signals in identical paths are identical $\overline{h^2}_{gk} = \overline{h^2}_{lk} = \overline{h^2}_k$, $g, l = \overline{1, M}$. For a binary system of signals ($M = 2$) in a Rayleigh channel it is not difficult to obtain

$$p = 1 - \sum_{k=1}^{N} \frac{(\overline{h_k^2}+1)^{2N-1}}{(\overline{h_k^2}+2)\prod_{m=1}^{N}{}^{m\neq k}(\overline{h_k^2}-\overline{h_m^2})}. \tag{4.90}$$

The graphs of probability of error calculated in formula (4.90) for a channel with smooth fadeouts in time and by frequency and selective fadeouts in space, described by a process with exponential correlation, are shown in Figure 4.7 below.

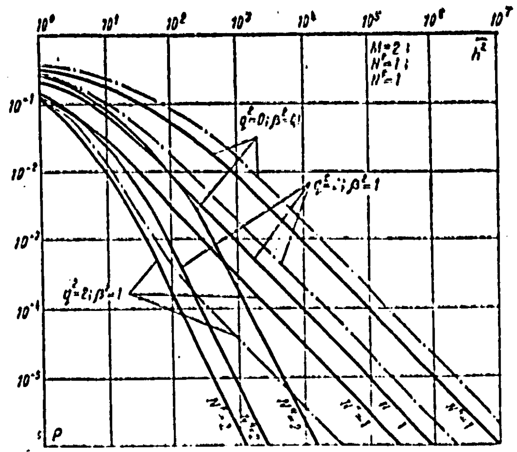

Figure 4.7. The Probability of Error in Discrimination of Orthogonal Signals (M = 2) in a Channel with Space-Selective Fadeouts. ——— Optimal Processing; — — Non-Coherent Processing $N^R = 2$.

Analyzing (4.89), we see that when the signal/noise ratio grows h^2_{xk}, $h^2_{yk} \to \infty$, the average values of the quadratic-form components disappear and the dispersions are evened out, that is, the distribution of quantity G becomes an χ^2-distribution with 2N degrees of freedom. Using the result from [89], we will write the formula for the probability of error in the indicated domain of values of the signal/noise ratio as follows

$$p = \prod_{k=1}^{N} \frac{(1+\beta_k^2)(1+q_k^2)}{2\overline{h_k^2}\beta_k} \exp\left[-\frac{q_k^2(1+\beta_k^2)}{2\beta_k}(\cos^2\varphi_{pk} + \beta_k^2 \sin\varphi_{pk})\right] \times$$
$$\times \sum_{n=1}^{M-1} (-1)^{n+1} C_n^{M-1} \sum_{g=0}^{n(N-1)} c_g n^{-(g+N)} \frac{(N+g-1)!}{(N-1)}, \qquad (4.91)$$

where the coefficients c_g are determined in [132].

A number of interesting formulas for calculating the probability of error in particular cases follow from formula (4.91). For example, for a binary system (M = 2) the expression for the probability of error has the form

$$p = \frac{C_N^{2N-1}}{\prod_{k=1}^{N} \frac{2\overline{h_k^2}\beta_k}{(1+\beta_k^2)(1+q_k^2)} \exp\left[q_k^2 \frac{(1+\beta_k^2)}{2\beta_k^2}(\cos^2\varphi_{pk} + \beta_k^2 \sin^2\varphi_{pk})\right]}. \qquad (4.92)$$

A simple comparison of the formulas obtained in this section for probability of error during signal discrimination and the characteristics of the optimal detector shows that the structure of the formulas is the same. Therefore, all qualitative conclusions drawn above concerning the effect of a stochastic channel on the characteristics of optimal detection also apply to the case of discriminating M signals. The formulas for the probability of error in discriminating M signals for particular channel models may be written just as those for the problem of detection.

Suboptimal processing during signal discrimination. Let us consider the characteristics of a unit for processing during signal discrimination. The unit is composed of a set of narrowly directed antennas, a delay line, and a set of filters coordinated with the transmitted signals, working on algorithm (4.87), and computing the quantities

$$G_l = \sum_{k=1}^{N^F} \sum_{i=1}^{N^\Theta} V_{lik}^2 + \widetilde{V}_{lik}^2, \qquad (4.93)$$

where

$$\begin{Bmatrix} V_{lik} \\ \widetilde{V}_{lik} \end{Bmatrix} = \int_{-\Theta}^{\Theta} \frac{\sin(\vartheta/\delta\vartheta - i\pi)}{\vartheta/\delta\vartheta - i\pi} d\vartheta \int_0^T z(t, \vartheta) \begin{Bmatrix} s_l\left(t - \frac{k}{F}\right) \\ \widetilde{s}_l\left(t - \frac{k}{F}\right) \end{Bmatrix} dt. \qquad (4.94)$$

The variables of quadratic forms G_1 and G_g are mutually independent Gaussian random quantities with the parameters

$$\left.\begin{aligned} &M_1\{V_{lik}\} = m_{xlk}\sqrt{\frac{2\overline{E}}{N_0}},\ M_1\{\widetilde{V}_{lik}\} = m_{ylk}\sqrt{\frac{2\overline{E}}{N_0}}, \\ &D\{V_{lik}\} = 1 + 2h_{xik}^2\mu^T,\ D\{\widetilde{V}_{lik}\} = 1 + 2h_{yik}^2\mu^T, \\ &M_1\{V_{gik}\} = M_1\{\widetilde{V}_{gik}\} = 0,\ D\{V_{gik}\} = D\{\widetilde{V}_{gik}\} = 1. \end{aligned}\right\} \qquad (4.95)$$

Noting that the quantity G_g is distributed according to the χ^2 law, we will use the result from [89] and obtain an expression for the probability of error in discrimination of M signals in the following form

$$p = \sum_{n=1}^{M-1} (-1)^{n+1} C_n^{M-1} \sum_{k=0}^{n(N^F N^\Theta - 1)} (-1)^k n^{-k} \times$$

$$\times \left\{ \frac{\partial^k}{\partial \rho^k} \prod_{m=1}^{N^F} \prod_{i=1}^{N^\Theta} \frac{\exp\left\{ \dfrac{-n\rho \dfrac{q_{im}^2}{1+q_{im}^2} \overline{h_{im}^2} \cos^2 \varphi_{p\,im}}{1 + n\rho\left[1 + \dfrac{2\overline{h_{im}^2}\beta_{im}^2\mu^T}{(1+\beta_{im}^2)(1+q_{im}^2)}\right]} \rightarrow \right.}{\sqrt{\left\{1 + n\rho\left[1 + \dfrac{2\overline{h_{im}^2}\beta_{im}^2\mu^T}{(1+\beta_{im}^2)(1+q_{im}^2)}\right]\right\}}} \times \rightarrow \right.$$

$$\left.\left.\left. + \frac{\dfrac{n\rho \dfrac{q_{im}^2}{1+q_{im}^2}\overline{h_{im}^2}\sin^2\varphi_{\rho\, im}}{1+n\rho\left[1+\dfrac{2\overline{h_{im}^2}\mu^T}{(1+\beta_{im}^2)(1-\varepsilon_{im}^2)}\right]}}{\times\left\{1+n\rho\left[1+\dfrac{2\overline{h_{im}^2}\mu^T}{(1+\beta_{im}^2)(1-\varepsilon_{im}^2)}\right]\right\}}\right\}\right\}\right|_{\rho=1}, \qquad (4.96)$$

where the quantity μ^T is determined by formula (4.96). If there is no asymmetry by orthogonal components in any of the paths ($B^2_{im} = 1$, $i = \overline{1, N^\Theta}$; $m = \overline{1, N^F}$) and the paths are identical on the average (which is, of course, an idealization), it is not difficult from formula (4.96) to obtain

$$p = \sum_{n=1}^{M-1} (-1)^{n+1} C_n^{M-1}\left[\frac{1}{1+n+\dfrac{n\overline{h}^2\mu^T}{1+q^2}}\right]^{N^F N^\Theta} \times$$

$$\times \exp\left\{-\frac{nN^F N^\Theta q^2\overline{h}^2}{(1+n)(1+q^2)+n\overline{h}^2\mu^T}\right\}\sum_{k=0}^{n(N^F N^\Theta-1)} c_k \frac{\Gamma(N^F N^\Theta+k)}{\Gamma(N^F N^\Theta)} \times$$

$$\times\left[\frac{1+\dfrac{\overline{h}^2\mu^T}{1+q^2}}{1+n+n\dfrac{\overline{h}^2\mu^T}{1+q^2}}\right] {}_1F_1\left[-k,\ N^F N^\Theta,\right.$$

$$\left.\frac{-N^F N^\Theta q^2\overline{h}^2}{(1+q^2+\overline{h}^2\mu^T)[(1+n)(1+q^2)+n\overline{h}^2\mu^T]}\right],$$

where ${}_1F_1(a, B, \gamma)$ is a degenerated hypergeometric function.

For large M calculations formulas (4.96) and (4.97) become complicated. In this case it is advisable to use asymptotic formula (4.37), which makes it possible to reduce the problem of discrimination formally to a detection problem. In this case the probability of error should be calculated by the formulas given in the preceding section for probability of a missed signal. The threshold ω included in (4.37) for calculating the probability of error in a system of M orthogonal signals should be determined from the equation

$$\frac{1}{(N^F N^\Theta - 1)!}\,\Gamma(\omega,\ N^F N^\Theta - 1) = 2^{-1/(M-1)}$$

or

$$e^{-\omega} \sum_{g=0}^{N^F N^\Theta - 1} \frac{\omega^g}{r!} = 1 - 2^{-1/(M-1)}\,. \qquad (4.98)$$

Where the number of paths is sufficiently large it is advisable to use Graham-Charlier series to compute the probability of error [64, 109].

For large signal/noise ratios, it follows from formula 4.96 that

$$p = \prod_{m=1}^{N^F} \prod_{l=1}^{N^\Theta} \frac{(1+\beta_{lm}^2)(1+q_{lm}^2)}{2\overline{h_{lm}^2}\,\beta_{lm}\mu^T} \exp\left[-\frac{q_{lm}^2(1+\beta_{lm}^2)}{2\beta_{lm}^2\mu^T} \times \right.$$
$$\left. \times \left(\cos^2\varphi_{p\,ml} + \beta_{lm}^2 \sin^2\varphi_{p\,lm}\right)\right] \sum_{n=1}^{M-1} (-1)^{n+1} C_n^{M-1} \sum_{k=0}^{n(N^F N^\Theta - 1)} c_k n^{-(k+N^F N^T)} \times$$
$$\times \frac{(N^F N^\Theta - k - 1)!}{(N^F N^\Theta - 1)!}\,. \qquad (4.99)$$

Expression (4.99) is very close in structure to the formula for the probability of error in optimal processing (4.92) and for the probability of a miss in the optimal (4.60) and suboptimal (4.74) detectors. Everything that has been said about the effect of channel parameters on the probabilities can be repeated for the probability of error under consideration here with optimal discrimination of M signals. Specifically, the algorithm (4.93) under consideration affords an energy gain resulting from time-selective fadeouts where the channel has a regular component and condition (4.75) is met. The values of the energy gain are determined by formula (4.76).

For small signal/noise ratios the selectivity of fadeouts in time leads to an energy loss, as can be seen from (4.96), but its values are low in channels with a regular component. A comparison of the formulas for the probability of error shows that in channels with time selectivity optimal processing has a great advantage over the processing we are now considering, which does not take account of the selective nature of fadeouts in time. This advantage increases as the probability of error decreases and depends on the statistical properties of the channel, reaching its maximum value in channels that are close to Rayleigh channels. Thus, in a Rayleigh channel with fadeouts that are non-selective in space and frequency, the corresponding energy gain where $p = 10^{-4}$, $M = 2$, $N^T = 3$ (the exponential correlation function $B_h(\tau)$) is 15 decibels; in a Rice channel where $q^2 = 2$ it is six decibels, and in a sub-Rayleigh channel where $B^2 = 0.1$ it is just five decibels. The physical explanation is that selectivity of fadeouts in time plays a very small role in good channels ($q^2 >> 1$). Optimal processing approaches linear, whose superiority to non-coherent (suboptimal) processing in an ideal channel $q^2 = \infty$ for the signal system under consideration is on the order of three decibels. In poor channels $q^2 = 0$ and $b^2 << 1$ an optimal processing device, as follows from algorithm (3.45), actually processes one quadrature component of the signal because the second almost always takes a zero value. But the component which is being processed also has a high probability of assuming zero values during an interval of analysis of length T. In this case it becomes ineffective to "grab" and sum noncorrelated segments of the observed field, as is done in the optimal processing device, for virtually all segments will carry close to zero energy. It is sufficient to process the signal through an entire interval of analysis of duration T. Some curves of the probability of error in optimal and suboptimal processing are given in Figure 4.7 above.

As already noted above, space, time, and frequency variables are equivalent within the framework of the approach adopted here to constructing field processing algorithms. Therefore, it is possible to suggest a number of algorithms for suboptimal processing of fields that are close to the one under consideration and constructed by ignoring fadeout selectivity for one of the variables (time in the algorithm under consideration) and considering selectivity of fadeouts in the others (for example, space and frequency).

The analysis of characteristics conducted above can easily be transferred from the suboptimal algorithm considered to other algorithms of the same class.

A conclusion common to all suboptimal algorithms of this class is that as the power of the selectivity considered (number of branches of dispersion) increases the energy gain of the optimal algorithm over the suboptimal ones decreases in the domain of large error $\overline{(h^2)}$ and increases in the domain of small errors (large $\overline{h^2}$).

Let us go on to consider a suboptimal processing algorithm that realizes the Kostas idea. For a channel that is non-selective by frequency and time but has fadeouts that are selective in space, the random quantities $V_{1k} = \tilde{V}_{1k}$, which are included in processing algorithm (4.87), have the parameters:

$$\left.\begin{aligned} M_1\{V_{lk}\} = m_x\sqrt{2E/N^R N_0},\ M_1\{\tilde{V}_{lk}\} = m_y\sqrt{2E/N^R N_0}; \\ D\{V_{lk}\} = 1 + \frac{2\sigma_x^2 E\mu^R}{N^R N_0},\ D\{\tilde{V}_{lk}\} = 1 + \frac{2\sigma_y^2 E\mu^R}{N^R N_0}, \end{aligned}\right\} \quad (4.100)$$

where μ^R is determined by formula (4.84).

The quadratic form G_g included in (4.87) has an χ^2-distribution. The expression for the probability of error is written by a formula analogous to (4.96) where μ^T is replaced by μ^R and N^Θ by N^R in this formula and $N^F = 1$, $h^2{}_1 = h^2/N^R$. For the domain of small errors it is not difficult to obtain (M = 2)

$$p = \prod_{l=1}^{N^R} \frac{(1+\beta_l^2)(1+q_l^2)\,N^R}{2\beta_l \overline{h}^2 \mu^R} \exp\left[-\frac{q_l^2(1+\beta_l^2)}{2\beta_l \mu^R}(\cos^2\varphi_{p\,l} + \beta_l^2 \sin^2\varphi_{p\,l})\right]. \quad (4.101)$$

Let us look further at the probability of error in a signal processing device whose working algorithm is based on an adaptive single-path model of a multibeam channel (selection of the maximum transfer coefficient).

For a device to discriminate M signals that realizes algorithm (3.86), we will determine the probability of error by averaging the expression for the probability of error in a channel without fadeouts by the maximum quadrature components of the transfer coefficient x_k and y_p. This can be done by disregarding the inexactness of estimates of the maximum values of the quadrature components of channel characteristics. Assuming all N paths to be identical on the average, we write the densities of distributions of probabilities of maximum values of quadrature components as follows

$$\left.\begin{aligned} w_1\left(x_k^{\text{макс}}\right) &= w(x_k)\left[\int_{-x_k}^{x_k} w_1(x_l)\,dx_l\right]^{N-1}, \\ w_1\left(y_p^{\text{макс}}\right) &= w_1(y_p)\left[\int_{-y_p}^{y_p} w_1(y_l)\,dy_l\right]^{N-1}. \end{aligned}\right\} \quad (4.102)$$

The quantities x_k and y_p included in (4.102) where $k = \overline{1, N}$ and $p = \overline{1, N}$ included in (4.102) are distributed normally with parameters (M_x, σ^2_x) and (m_y, σ^2_y) respectively. The expression for the probability of error is written in the form

$$p = \sum_{g=1}^{M-1} \frac{(-1)^{g+1} C_g^{M-1}}{1+g} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left\{-\frac{gE\left[\left(x_k^{\text{макс}}\right)^2 + \left(y_p^{\text{макс}}\right)^2\right]}{(1+g)N_0}\right\} \times$$
$$\times\, w_1\left(x_k^{\text{макс}}\right) w_1\left(y_p^{\text{макс}}\right) dx_k^{\text{макс}}\, dy_p^{\text{макс}}. \quad (4.103)$$

In the general case, the probability of error should be calculated according to formula (4.103) using computer computation methods. For an example let us consider a channel without a regular component $m_x = m_y = 0$, $N = 2$. From (4.103) it is not difficult to obtain

$$p = \sum_{g=1}^{M-1} \frac{(-1)^{g+1} C_g^{M-1}\left[1 - \frac{2}{\pi}\operatorname{arc\,tg}\sqrt{1 + \frac{2k\overline{h}^2\beta^2}{(1+k)(1+\beta^2)(1+q^2)}}\right]}{\sqrt{1 + \frac{2k\overline{h}^2\beta^2}{(1+k)(1+\beta^2)(1+q^2)}}} \times$$
$$\times \frac{\left[1 - \frac{2}{\pi}\operatorname{arc\,tg}\sqrt{1 + \frac{2k\overline{h}^2}{(1+k)(1+\beta^2)(1+q^2)}}\right]}{\sqrt{1 + \frac{2k\overline{h}^2}{(1+k)(1+\beta^2)(1+q^2)}}}. \quad (4.104)$$

For the binary system M = 2 we have, from (4.104)

$$p=\frac{1-\frac{2}{\pi}\operatorname{arc\,tg}\sqrt{1+\frac{2\bar{h}^2\beta^2}{1+\beta^2}}}{\sqrt{1+\frac{2\bar{h}^2\beta^2}{1+\beta^2}}}\;\frac{1-\frac{2}{\pi}\operatorname{arc\,tg}\sqrt{1+\frac{2\bar{h}^2}{1+\beta^2}}}{\sqrt{1+\frac{2\bar{h}^2}{1+\beta^2}}}. \quad (4.105)$$

To investigate the domain of small errors we will use the expansion from [29] in (4.104)

$$1-\frac{2}{\pi}\operatorname{arc\,tg}U=\frac{2}{\pi}\sum_{k=0}^{\infty}(-1)^k\frac{1}{(2k+1)U^{2k+1}}, \quad (4.106)$$

keeping only its first member. From (4.104), the following asymptotic formula follows (where $h^2 \to \infty$)

$$p=\frac{(1+\beta^2)^2}{\beta^2(2\bar{h}^2)^2}\sum_{g=1}^{M-1}(-1)^{g+1}\frac{g+1}{g^2}C_g^{M-1}. \quad (4.107)$$

Comparing expression (4.107) with the expression for probability of error in the optimal processing device (4.91), it is not difficult to observe that the nature of decrease in the probability of error depending on the signal/noise ratio in both cases is the same: the probability of error decreases in inverse proportion to the N degree of the signal/noise ratio. It can be shown that the energy loss of the suboptimal scheme under consideration compared to the optimal scheme is not great (no more than three decibels) and decreases as the number of paths N increases and the intensity of the regular components is reduced.

4.6. Noise Suppression of a Binary System of Opposite Signals (Generalized Gaussian Statistics)

Data transmission by broad-band opposite signals has become widespread in ground and space channels [76]. Here let us consider the potential of this system of signals in space-time processing and compare it with the potential of linear suboptimal schemes used in practice.

Optimal processing. To compute the probability of error it is convenient to convert algorithm (3.56) to the form

$$l=\sum_{k=1}^{N}V_k+\tilde{V}_k>0. \quad (4.108)$$

The Gaussian random quantities V_k, $\tilde{V}_k$, $k=\overline{1, N}$ are mutually independent and have the parameters

$$M_1\{V_k\}=D\{V_k\}=\frac{2E\nu_k}{N_0}\,\frac{m_{xk}^2}{1+2h_{xk}^2};$$

$$M_1\{\tilde{V}_k\}=D\{\tilde{V}_k\}=\frac{2E\nu_k}{N_0}\,\frac{m_{yk}^2}{1+2h_{yk}^2}.$$

In this case the Gaussian quantity I has the parameters

$$M_1(I) = D(I) = \sum_{k=1}^{N} \frac{2E\nu_k}{N_0}\left(\frac{m_{xk}^2}{1+2h_{xk}^2} + \frac{m_{yk}^2}{1+2h_{yk}^2}\right). \qquad (4.109)$$

The expression for the probability of error in a processing device working on algorithm (4.108) is described quite simply as the function of distribution of the linear form of the independent Gaussian variables and has the form

$$p = \frac{1}{2}\left\{1-\Phi\left[\sqrt{\sum_{k=1}^{N} 2\overline{h_k^2}\,\frac{q_k^2}{1+q_k^2}\left[\frac{\cos^2\varphi_{pk}}{1+\frac{2h_k^2\beta_k^2}{(1+q_k^2)(1+\beta_k^2)}} + \frac{\sin^2\varphi_{pk}}{1+\frac{2h^2}{(1+\beta_k^2)(1+q_k^2)}}\right]}\right]\right\}. \qquad (4.110)$$

It can be seen from formula (4.110) that if the regular component is missing from the field received $q^2{}_k = 0$, $k = \overline{1, N}$, the system with opposite signals becomes unworkable because the probability of error is 1/2 for any signal/noise ratio. Where there are paths of propagation with asymmetry in dispersions of quadrature components $B^2{}_k \neq 1$, the probability of error depends strongly on the phases ϕ_{pk} of the regular components in these paths. If there is no asymmetry (Rice fadeouts), the phases of the regular components do not affect the probability of error. Analysis of expression (4.110) shows that a typical feature of the system with opposite signals is a minimum probability of error that cannot be reduced with growth in the signal/noise ratio. Assuming $h^2{}_k = \infty$ in (4.110) we obtain this value for the irreducible probability of error:

$$p^{\infty} = \frac{1}{2}\left\{1-\Phi\left[\sqrt{\sum_{k=1}^{N}\frac{q_k^2(1+\beta_k^2)}{\beta_k^2}\left(\cos^2\varphi_{pk} + \beta_k^2\sin^2\varphi_{pk}\right)}\right]\right\}. \qquad (4.111)$$

Calculations show that the values of p^{∞} in channels with selective fadeouts and fairly good statistics (high values $q^2{}_k$, $k = \overline{1, N}$) may be very, very small. For example, in the channel described by the delay line model with two branches $N^S = 2$ with smooth fadeouts in time $N^T = 1$ and selective fadeouts in space $N^R = 2$ (exponential correlation function) with Rice statistics $q^2{}_k = 2$, $k = 1.2$, the limiting probability of error is of the order 10^{-6}. This level of noise suppression can already be reached approximately where $\overline{h^2} \simeq 10$ where it is possible to switch approximately to (4.111) to calculate the probability of error from 4.110). As the degree of channel selectivity increases the values of the limiting probability of error dropped sharply and may reach vanishing small values even for a weakly expressed regular component of the transfer function of a stochastic channel. These properties of a system with opposite signals leave no doubt of the advisability of transmitting information by opposite signals in stochastic channels. It is a major advantage that optimal processing of opposite signals is linear. Certain curves of probability of error calculated from formula (4.110) are shown in Figure 8 below. The dot-dash line shows the characteristics of a system with a test pulse (lower boundary) calculated according to formula (4.3).

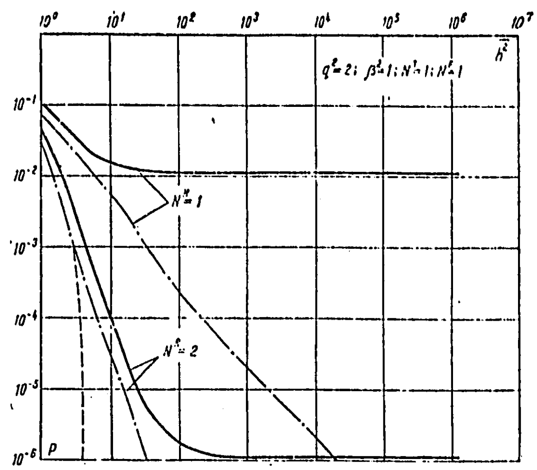

Figure 8. Probability of Error in Discrimination of Two Opposite Signals in a Channel with Smooth Fadeouts: ——— Without a Test Signal; — — with a Test Signal; ----- Ideal Channel.

Suboptimal processing. Let us determine the probability of error when opposite signals are processed by algorithm (3.91). The probability of error is determined by the probability of fulfillment of the inequality

$$\frac{2}{N_0}\sum_{k=1}^{N}\int_0^T\int_0^R z(t,\ r)\,[m_{xk}s_{1k}(t,\ r)+m_{yk}\widetilde{s}_{1k}(t,\ r]\,dt\,dr>0 \qquad (4.112)$$

on the assumption that signal $s_2(t)$ is contained in the observed oscillation. This probability is easily found in the form

$$P=\frac{1}{2}\left\{1-\Phi\left[\sqrt{\sum_{k=1}^{N}2h_k^2\frac{q_k^2}{1+q_k^2}\,\frac{1}{1+\dfrac{2\overline{h_k^2}\left(\beta_k^2\cos^2\varphi_{P\,k}+\sin^2\varphi_{P\,k}\right)}{\left(1+\beta_k^2\right)\left(1+q_k^2\right)}}}\right]\right\}. \qquad (4.113)$$

Comparing (4.113) and (4.110), we see that the errors coincide in a channel without asymmetry of expressions for probabilities of error. This is natural because algorithm (3.91) is optimal in the given case.

If there is asymmetry, of course, the optimal algorithm is superior to the suboptimal one and this superiority increases as the asymmetry grows. The value of the limiting probability of error obtained from (4.113) where $h^2{}_k \to \infty$ has the form

$$P^{\infty} = \frac{1}{2}\left\{1 - \Phi\left[\sqrt{\sum_{k=1}^{N} q_k^2 (1 + \beta_k^2) \frac{1}{\beta_k^2 \cos^2 \varphi_{p\,k} + \sin^2 \varphi_{p\,k}}}\right]\right\}. \quad (4.114)$$

The independent variable of limiting probability of error in the optimal processing device exceeds the corresponding value of the independent variable in (4.114) by the quantity

$$\eta = 10 \lg \frac{\sum_{k=1}^{N} \frac{q_k^2 (1 + \beta_k^2)}{\beta_k^2} (\cos^2 \varphi_{p\,k} + \beta_k^2 \sin^2 \varphi_{p\,k})}{\sum_{k=1}^{N} q_k^2 (1 + \beta_k^2) \frac{1}{\beta_k^2 \cos^2 \varphi_{p\,k} + \sin^2 \varphi_{p\,k}}}. \quad (4.115)$$

The degree of superiority of the optimal algorithm to suboptimal ones can be seen most graphically by considering a channel with identical average paths of propagation. For such a channel it follows from (4.115) that

$$\eta = 10 \lg \frac{1}{\beta^2} (\cos^2 \varphi_p + \beta^2 \sin^2 \varphi_p)(\beta^2 \cos^2 \varphi_p + \sin^2 \varphi_p). \quad (4.116)$$

Analysis of formula (4.116) shows that optimal processing permits a sharp improvement in noise suppression in channels with non-zero regular parts of both quadrature components ($\phi_p \neq 0$, $\phi_p \neq \pi/2$).

To complete our consideration of the characteristics of optimal and suboptimal algorithms for processing space-time signals in channels with generalized Gaussian statistics, let us review the advantages afforded by optimal processing. The benefits of such processing compared to suboptimal processing come chiefly from the fact that it makes it possible to organize N-multiple accumulation in a channel with selectivity of degree N.

The non-correlated nature of all N branches which give rise to statistical independence in the Gaussian case under consideration is achieved by selecting a channel model based on the Karunen-Loew expansion. Using any other coordinate functions of a discrete channel model leads to the appearance of dependence among the N branches of the receiving device and the efficiency of accumulation will be lower. An alternative in suboptimal processing is to choose the number of independent branches $N' < N$, which is often done in practice.

A second advantage of optimal processing is the fact that it permits the best processing of signals in each of the N branches (this question was investigated in adequate detail during consideration of a channel with smooth fadeouts).

Speaking only of spatial processing, we may say that the optimal algorithm points out the forms of antenna directivity diagrams that allow organization of accumulation for N independent branches.

Let us use examples to estimate the effect of non-optimality of antenna directivity diagrams on noise suppression for a channel with smooth fadeouts in time and frequency, but selective in space.

Suppose the discrete model of the channel has the form

$$\dot{h}(r) = \sum_{p=1}^{N^R} \dot{h}_p \varphi_p(r), \qquad (4.117)$$

where the functions $\{\phi_p(r)\}$ form an orthonormalized system but are not Karunen-Loew functions.

The magnitudes of covariance of expansion coordinates (separately for each quadrature component) are determined by the relations

$$\left. \begin{aligned} B_{kp}^x &= \int_0^R \int_0^R B_x(r-r')\varphi_k(r)\varphi_p(r')\,dr\,dr'; \\ B_{kp}^y &= \int_0^R \int_0^R B_y(r-r')\varphi_k(r)\varphi_p(r')\,dr\,dr'; \end{aligned} \right\} \qquad (4.118)$$

$$k = \overline{1, N^R};\; p = \overline{1, N^R}.$$

We assume the quadrature components are non-coherent: $B_{kp}^{xy} = 0$, k, p, 1, N^R.

Suppose, for example, that coordinates h_p in expansion (4.117) are equidistant (distance of Δr) readings for the space variable r (which corresponds to reception at narrowly directed antennas with a diagram of the type $\sin \vartheta/\vartheta$). Then the magnitudes of covariation of the coordinates with different indexes are determined, on the basis of (4.118), by the relations: $B_{kp}^x = B_x[(k-p)\Delta r];\; B_{kp}^y = B_y[(k-p)\Delta r].$ (4.119)

Thus, for the exponential correlation functions:

$$B_{kp}^x = \sigma_x^2 \exp\left(-\frac{\Delta r}{\rho_{\text{кор}}}|k-p|\right);\; B_{kp}^y = \sigma_y^2 \exp\left(-\frac{\Delta r}{\rho_{\text{кор}}}|k-p|\right). \qquad (4.120)$$

The existence of a correlation among individual branches affects the probability of error differently depending on the system of signals and statistics of fadeout in the channel [6, 46].

Let us review some examples of practical interest.

1. For a Rayleigh channel with two identical, on the average, paths (N = 2) where orthogonal signals are used (M = 2), we write, on the basis of [6], the following

$$p = \frac{2}{(\overline{h^2})^2(1-|\dot{R}|^2)+4\overline{h^2}+4}, \quad (4.121)$$

where

$$|\dot{R}| = \sqrt{(B^x_{kp}/B^x_{kk})^2+(B^y_{kp}/B^y_{kk})^2}. \quad (4.122)$$

It can be seen from (4.121) that the correlations of signals in the branches are reflected in the domain of large signal/noise ratios. If the correlated coefficients are described by expression (4.120), when $\Delta r/p_{cor} = 0.5$, from (4.122) we will receive $|R| = 0.85$. Under these conditions the energy loss owing to non-optimal special processing where $\overline{h^2} \gg 1$ will be about 2.5 decibels.

2. Let us consider a system of opposite signals working on algorithm (4.108). Where there is a correlation among particular components, in addition to parameters (4.109) the covariation quantities below will also characterize the linear form (4.108):

$$\left.\begin{aligned} B\{V_kV_p\} &= \frac{2\frac{E}{N_0}\sqrt{\nu_k\nu_p}\,B^x_{kp}}{\sqrt{(1+2\overline{h^2_{xk}})(1+2\overline{h^2_{xp}})}}, \\ B\{\widetilde{V}_k\widetilde{V}_p\} &= \frac{2\frac{E}{N_0}\sqrt{\nu_k\nu_p}\,B^y_{kp}}{\sqrt{(1+2\overline{h^2_{yk}})(1+2\overline{h^2_{yp}})}}, \quad k \neq p. \end{aligned}\right\} \quad (4.123)$$

The probability of error will still be determined by the probability of fulfillment of inequality (4.108). Quantity I is Gaussian with an average value and dispersion equal to, respectively:

$$\left.\begin{aligned} M_1\{I\} &= \sum_{k=1}^{N^R}\frac{2E\,\nu_k}{N_0}\left(\frac{m^2_{xk}}{1+2\overline{h^2_{xk}}}+\frac{m^2_{yk}}{1+2\overline{h^2_{yk}}}\right), \\ D\{I\} &= \sum_{k=1}^{N^R}\frac{2E\,\nu_k}{N_0}\left(\frac{m^2_{xk}}{1+2\overline{h^2_{xk}}}+\frac{m^2_{yk}}{1+2\overline{h^2_{yk}}}\right)+2\sum_{p<k}\frac{2E\sqrt{\nu_k\nu_p}}{N_0}\times \\ &\times\left(\frac{B^x_{kp}\,m_{xk}m_{xp}}{\sqrt{(1+2\overline{h^2_{xp}})(1+2\overline{h^2_{xk}})}}+\frac{B^y_{kp}\,m_{yk}m_{yp}}{\sqrt{(1+2\overline{h^2_{yk}})(1+2\overline{h^2_{yp}})}}\right). \end{aligned}\right\} \quad (4.124)$$

The quadratic form in (4.124) is negatively determinate, which follows from the property of the correlation function [64]. Therefore, dispersion (4.124) is always at least as great as dispersion (4.109).

From this it is clear that the probability of error in the case under consideration is always greater than it is in optimal spatial processing. The general expression for the probability of error with non-optimal processing is written in the form

$$p=\frac{1}{2}\left\{1-\Phi\left[\frac{M_1\{I\}}{\sqrt{D\{I\}}}\right]\right\}, \qquad (4.125)$$

where $M_1\{I\}$ and $D\{I\}$ are determined by formulas (4.124).

It is easy to determine the limiting probability of error in the form

$$p^{\infty}=\frac{1}{2}\left\{1-\Phi\left[\frac{\sum_{k=1}^{N^R}\frac{q_k^2(1+\beta_k^2)(\cos^2\varphi_{pk}+\beta_k^2\sin^2\varphi_{pk})}{\beta_k^2}}{\sqrt{\sum_{k=1}^{N^R}\frac{q_k^2(1+\beta_k^2)}{\beta_k^2}(\cos^2\varphi_{pk}+\beta_k^2\sin^2\varphi_{pk})+\ldots}}\right.\right.$$

$$\frac{}{\ldots\times 2\sum_{p<k=1}^{N^R}\left[B_{kp}^{x}\sqrt{\left[\frac{q_k^2(1+\beta_k^2)}{\beta_k^2}\cos^2\varphi_{pk}\right]\left[\frac{q_p^2(1+\beta_p^2)}{\beta_p^2}\cos^2\varphi_{pp}\right]}+\ldots\right.}$$

$$\left.\left.\frac{}{\ldots B_{kp}^{y}\sqrt{[q_k^2(1+\beta_k^2)\sin^2\varphi_{pk}][q_p^2(1+\beta_p^2)\sin^2\varphi_{pp}]}}\right]\right\}. \qquad (4.126)$$

It can be seen from (4.126) that the value of the limiting probabilities of error in the case we are considering, of non-optimal forms of antenna directivity diagrams, increases compared to the case considered earlier of optimal diagrams. For example, in a Rice channel with two identical, on the average, paths with correlation coefficients (4.120) where $\Delta r/p_{cor} = 0.5$, the independent variable of the Kramp function (4.126) is 1.85 times less than in (4.111). In this case the limiting probability of error where $q^2 = 4$ is not 10^{-6} as the case in optimal processing, but rather 10^{-4}, thus increasing 100 times.

4.7. Characteristics of Devices for Processing Signals in Channels with Non-Gaussian Statistics under Conditions of a Non-Classified Sample Used to Study the Channel

During analysis of signal processing devices, and this is also true for synthesis, departing from the Gaussian model of fadeouts makes consideration much more complicated. In particular, it is necessary to assume statistically independent individual paths to obtain visible results. Whereas it was possible, as noted above, in the case of Gaussian statistics to remove the assumption of non-correlatedness (independence) and the formulas for probability of error obtained on

The assumption of independent paths would keep their form in the case of dependent paths (only the parameters were changed), in the non-Gaussian case matters are different. Therefore, the formulas in this section are more specific than in the preceding section.

As a one-dimensional probabilistic model of a channel here we consider the logarithmically normal distribution of amplitudes and uniform distribution of phases that yield bimodal distributions of quadrature components. Let us recall that the distributions of quadrature components are, in the given case, identical with zero mathematical expectations $M_{xk} = M_{yk} = 0$, $k = \overline{1, N}$ and identical dispersions $\sigma^2_{xk} = \sigma^2_{yk} = \sigma^2_k$, $k = \overline{1, N}$.

We will be interested in characteristics of the processing algorithms synthesized in Chapter 3 in channels with such statistics. We must observe first of all that the "spectrum of algorithms" is narrower in this case. Specifically, we must eliminate from consideration the linear processing algorithms that use average values of transfer coefficients because they are not workable in the channel under consideration, that is, for a channel with logarithmically normal-uniform statistics it is impossible to transmit information by opposite signals with independent reception of individual signals.

The optimal processing algorithm (3.45) is a purely quadrature algorithm (it does not have a linear part). According to (3.53), the quantity $\hat{F}_1$ should be computed from the relation

$$\hat{F}_l = \frac{2E_l}{N_0} \sum_{k=1}^{N} \frac{\overline{h^2_{lk}}\, v_{lk}}{1 + 2\overline{h^2_{lk}}} (\Psi^2_{lk} + \tilde{\Psi}^2_{lk}). \tag{4.127}$$

where, in the given case

$$\begin{Bmatrix} \Psi_{lk} \\ \tilde{\Psi}_{lk} \end{Bmatrix} = \frac{1}{d_{lk}} \int_0^T \int_0^R z(t, r) \begin{Bmatrix} s_{lk}(t, r) \\ \tilde{s}_{lk}(t, r) \end{Bmatrix} dt\,dr. \tag{4.128}$$

We will consider here only discrimination of M signals of identical energy (the characteristics of detection have similar analytic expressions to those found in the case of Gaussian statistics).

To calculate the probability of error algorithm (3.45) is transformed to

$$G_l > G_g,\ g = \overline{1, M},\ g \neq l, \tag{4.129}$$

where

$$G_l = \sum_{k=1}^{N} \Psi^2_{lk} + \tilde{\Psi}^2_{lk}.$$

The dispersions of variables of the quadrature form G_1 and G_g have the form

$$\left.\begin{aligned} D(V_{lk}) &= D(\tilde{V}_{lk}) = \overline{h^2_{lk}},\\ D(V_{gk}) &= D(\tilde{V}_{gk}) = \frac{\overline{h^2_{gk}}}{1+\overline{h^2_{gk}}}. \end{aligned}\right\} \quad (4.130)$$

Noting that where $h^2_{gk} \to \infty$, $g = \overline{1, N}$ the quadrature form G_g is distributed according to the law χ^2 and 2N degrees of freedom and using the result of [89], we write an asymptotic expression for the probability of error

$$P = \prod_{k=1}^{N} \frac{1}{4\pi \overline{h_k^2}} e^{3\sigma^2_{\chi k}} \sum_{k=1}^{M-1} (-1)^{k+1} C_k^{M-1} \sum_{g=0}^{k(N-1)} c_g n^{-(g+N)} \frac{(N+g-1)!}{(N-1)!}. \quad (4.131)$$

For the binary system M = 2 it follows from (4.131) that

$$P = C_N^{N-1} \prod_{k=1}^{N} \frac{1}{4\pi \overline{h_k^2}} e^{3\sigma^2_{\chi k}}. \quad (4.132)$$

Assuming $\sigma^2_{\chi k} = \frac{1}{4} \ln \frac{1+m_k}{m_k}$ it is possible to investigate the degree of increase in the probability of error depending on the increase in the depth of fadeout (decrease of N_k, $k = \overline{1, N}$). Comparing expressions (4.131) and (4.92), we come to the conclusion that logarithmically normal statistics yield higher probabilities of error than Rayleigh statistics where the conditions $\sigma^2_{\chi k} \geq \frac{1}{3} \ln 4\pi$. are fulfilled.

Let us go on with our consideration of suboptimal processing that does not take account of the mean statistical parameters of channel fluctuations, assuming that the quantities $\hat{F}_1$ included in (3.45) are determined from the relation

$$\hat{F}_l = \frac{2E_l}{N_0} \sum_{k=1}^{N} \psi^2_{lk} + \psi^2_{lk}. \quad (4.133)$$

The algorithm for discrimination of M signals may be written in the form (4.119), and the dispersions of the variables of the quadrature forms G_1 and G_g are determined by the formulas

$$D(V_{lk}) = D(\tilde{V}_{lk}) = 1 + 2\overline{h^2_{lk}};\ D(V_{gk}) = D(\tilde{V}_{gk}) = 1. \quad (4.134)$$

It is not difficult to show that in the domain of high signal/noise ratios $\overline{h^2} \to \infty$, the expression for the probability of error in the suboptimal device under consideration coincides with the corresponding expression for an optimal device (4.131). This makes it possible to apply the conclusion that the characteristics of coherent and noncoherent discrimination of signals are identical for large signal/noise ratios to a non-Gaussian channel.

For arbitrary signal/noise ratios the probabilities of error should be determined by computer calculation. We will give the formula for probability of error for a single-path channel

$$p = \sum_{k=1}^{M-1} \frac{(-1)^{k+1} C_k^M}{1+k} \frac{1}{\sqrt{2\pi\sigma_\chi^2}} \int_0^\infty \exp\left[-\frac{k\bar{h}^2}{k+1}\gamma^2 - \frac{(\ln\gamma + \sigma_\chi^2)^2}{2\sigma_\chi^2}\right] \frac{d\gamma}{\gamma}. \quad (4.135)$$

Figure 4.9 shows a graph calculated according to formula (4.135) where M = 2. The dotted line shows the corresponding curve for a channel with Gaussian statistics (a Rayleigh channel), and the dot-dash line is for a channel without fadeouts. Comparing them allows us to estimate how much the characteristics of discrimination change with the change from Gaussian to non-Gaussian statistics.

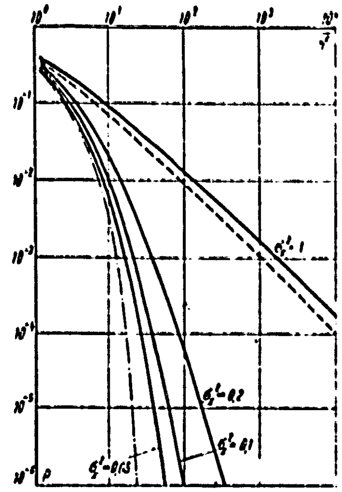

Figure 4.9. Probability of Error when Discriminating Orthogonal Signals (N = 2) in a Non-Gaussian Channel: ----- Rayleigh Channel; —·— Channel Without Fadeouts.

The analysis we have made shows that where the values of the parameter ϕ_x^2 change, logarithmically normal fadeouts cover a broad class of channels from channels close to the ideal (where $\phi_x^2 \to 0$) to channels of the sub-Rayleigh type $\left(\sigma_\chi^2 > \frac{1}{3}\ln 4\pi\right)$.

For small values of ϕ_x^2, expanding the function $\exp\left[-\frac{\bar{h}^2}{2}\gamma_\chi^2\right]$ in (4.135) into a Taylor series relative to point $(-\phi_x^2)$, it is possible to receive a convenient calculation formula for the probability of error (M = 2)

$$p = \frac{1}{2}\exp\left(-\frac{\bar{h}^2}{2}e^{-2\sigma_\chi^2}\right). \quad (4.136)$$

It can be seen from (4.126) that the nature of decrease in the probability of error relative to the signal/noise ratio is exponential, which means that the channel is close to ideal.

In conclusion, let us consider suboptimal processing based on replacement of the model of a multipass channel by a single-pass adaptive model (autoselection). We will assume that the quantity $\hat{F}_1$ included in the processing algorithm is computed from the relation

$$\hat{F}_l = \max_k \left(\eta_{lk}^2 + \tilde{\eta}_{lk}^2 \right), \qquad (4.137)$$

that is, the path with maximum power of the transfer coefficient $|h_k|^2$ is selected as the working path. The distribution of the modulus of the transfer coefficient $|h_k|$ we assume to be logarithmically normal, and the paths are taken to be identical on the average. Neglecting the inexactness of estimates of the power of the transfer coefficients of particular paths, we will determine the probability of errors from the relation

$$P = \sum_{k=1}^{M-1} \frac{(-1)^{k+1} C_{M-1}^{k}}{1+k} \int_0^\infty \exp\left[-\frac{k}{k+1} \bar{h}^2 \gamma^2 \right] w_1(\gamma)\, d\gamma, \qquad (4.138)$$

$$w_1(\gamma) = \frac{1}{\sqrt{2\pi\sigma_\chi^2}\,\gamma} \exp\left[-\frac{(\ln \gamma - \sigma_\chi^2)^2}{2\sigma_\chi^2} \right] \left[\Phi\left(\frac{\ln \gamma + \sigma_\chi^2}{\sigma_\chi} \right) \right]^{N-1}. \qquad (4.139)$$

Calculations by formula (4.138) show that the scheme with autoselection in a channel with non-Gaussian statistics, just as with the optimal scheme, insures a decrease in the probability of error inversely proportional to the N degree of this ratio for large signal/noise ratios and large ϕ_χ^2. For small values of the parameter $\sigma_\chi^2 \left[\sigma_\chi^2 \ll \frac{1}{3} \ln 4\pi \right]$ the curves of probability of error are exponential (the channel is close to ideal) and the effectiveness of autoselection is low.

* * *

We have analyzed the quality of optimal and suboptimal algorithms for processing fields carrying digital information. The generalized Gaussian probabilistic model of a channel, as the most widespread in practice and, moreover, the one with the best approximating capabilities in highly diverse situations, was used most.

We investigated different channels with non-selective and selective fadeouts. The problems of detection and discrimination of signals were considered separately. It was demonstrated that the probability of error depends significantly on the statistics of fadeouts in the channel. The best channel will be the one in which weakly fluctuating quadrature components have clearly expressed regular components (an asymmetric channel close to ideal).

A typical feature of the binary system with opposite signals during independent reception of characters is the existence of a limiting probability of error that is irreducible with growth in the signal/noise ratio. However, the values of this limiting probability are very small in good channels. For example, in a non-selective channel where $q_2 = 2$, $B^2 = 0$, and $\phi_p = 0$ we have $p^\infty = 10^{-6}$. An optimal binary system of signals was constructed in a channel with non-selective fadeouts and we determined the threshold signal/noise ratio at which the system of opposite signals loses its optimal features and the system of orthogonal signals acquires them.

It was shown with concrete examples that the use of optimal directivity diagrams makes it possible to greatly improve noise suppression in comparison with the methods of spatial processing of signals used extensively in practice at the present time.

The use of suboptimal algorithms shows that where the model of the channel is intelligently chosen, they insure error probability values that are almost as good as those of optimal algorithms and they are much simpler to realize. The system with test signals is more effective in a channel with fadeouts that are smooth in time where it is capable of insuring an energy gain of up to three decibels compared to a system without test signals.

We reviewed certain modifications of the ideas of autoselection with application to space-time signals and demonstrated that the use of this procedure for suboptimal processing is justified in many cases (both for Gaussian and non-Gaussian fadeout statistics).

Our investigation of the asymptotic behavior of error probabilities (where $E/N_0 \to \infty$) showed that independently of channel statistics, consideration and use of channel selectivity in time, space, and frequency makes it possible to achieve an accumulation effect. In this case, the probability of error diminishes as a quantity inverse to the signal/noise ratio to the extent of the selectivity in question $(E/N_0)^N$.

Conclusion

This book reviewed the general principles of constructing optimal and suboptimal signal processing devices in stochastic space-time communication channels.

Optimal processing in this case was based primarily on obtaining renewable estimates of the coordinates of channel characteristics and was oriented in its realization part to the technology of space-time filtration accomplished by both the classical techniques of dispersed reception and by techniques based on holographic principles.

It should be kept in mind that practical realization of many of the processing algorithms investigated in this book depends greatly on the progress of integrated technology that characterizes the development of electronics in our day.

In concluding this book the authors acknowledge that many questions of interest in constructing effective digital information transmission systems in stochastic space-time channels proved to be outside our framework. Among them are optimization of the communications system as a whole by finding optimal space-time processing operators not only in reception but also in transmission; the effectiveness of use of a feedback channel in space-time channels; selecting codes with due regard for the specific features of the space-time channel; assessing the difficulties of realization and noise suppression of systems for transmission of discrete messages by means of simple signals that do not satisfy the conditions of separation of paths; application of decision feedback in constructing optimal and suboptimal signal processing devices in a space-time channel; investigation of the prospects for non-linear filtration in processing space-time signals; processing for specific distributions of noise fields, and others.

The authors hope that their book will stimulate the interest of a broad range of specialists in the problems of space-time signal processing, including interest in solving the problems we have formulated here.

Appendix 1

The linear estimate of coordinate x is sought in the form (2.11)

$$\hat{x} = A \int_0^T \int_0^R z(t, r)\,\psi(t, r)\,dt\,dr + B \qquad (\Pi.1.1)$$

or in symbolic (operator) form

$$\hat{x} = A(Z, \psi) + B. \qquad (\Pi.1.2)$$

All further reasoning relies on the results of work [75]. The expression for the conditional risk function with a fixed state of the estimated (centered) parameter in operator form is written as follows

$$r(\overset{0}{x}, \psi) = (R\psi, \psi) + (\varphi - S^*\psi, z). \qquad (\Pi.1.3)$$

Operator $R\psi$ is determined by the correlation function of noise $B_n(t, t', r, r')$:

$$R\psi = \int_0^T \int_0^R B_n(t, t', r, r')\,\psi(t', r')\,dt'dr'. \qquad (\Pi.1.4)$$

Operator S and, correspondingly, its conjugate operator S* are determined by the spectrum of the transmitted signal. The expression for average risk can be written, averaging (П.1.3) the parameter being estimated by the distribution of probabilities $w_1(x)$:

$$r(\psi) = \int r(\overset{0}{x}, \psi)\,w_1(\overset{0}{x})\,d\overset{0}{x}. \qquad (\Pi.1.5)$$

Substituting (П.1.3) in (П.1.5) we obtain

$$r(\psi) = (R\psi, \psi) + (\Phi_x(\varphi - S^*\psi), (\varphi - S^*\psi)), \qquad (\Pi.1.6)$$

where Φ_x is a self-conjugated, negatively determinate linear operator defined by the correlation function $B_x(t, t', r, r', \xi, \xi')$:

$$(\Phi_x \varphi, \varphi) = \int_{\lambda} (\varphi, \overset{0}{x})^2 w_1(\overset{0}{x})\, d\overset{0}{x} = \int\int_{\lambda\lambda} \varphi(t, \xi, r)\, \varphi(t', \xi', r')\, B_x(t, t', \xi, \xi', r, r') \times$$
$$\times\, dt\,dt'\,d\xi\,d\xi'\,dr\,dr'. \qquad (\Pi.1.7)$$

It can be shown that the minimum average risk is achieved where

$$\psi = (R + S\Phi_x S^*)^{-1} S\Phi_x \varphi. \qquad (\Pi.1.8$$

Using (Π.1.8) in (Π.1.1) and adding the known mean m_x we obtain the optimal linear estimate in the form

$$\hat{x} = ((R + S\Phi_x S^*)^{-1} S\Phi_x \varphi,\ z) + Rm_x (R + S\Phi_x S^*)^{-1}. \qquad (\Pi.1.9)$$

Moving from the operator form of writing to conventional form it is not difficult to see that the optimal linear estimate coincides with the Bayes estimate of a Gaussian coordinate in a setting of Gaussian noise (2.96).

The linear estimate (Π.1.9) was obtained without constraints on the form of signals transmitted.

Appendix 2

We will find the average value of the function

$$F(x) = \exp\left[-\sum_{k=1}^{N} d_k x_k^2\right], \quad d_k > 0. \qquad (\Pi.2.1)$$

Suppose quantities x_k, $k = \overline{1, N}$ in (Π.2.1) are distributed by arbitrary laws and are statistically independent. The average value we are seeking is written in the form

$$\overline{F} = \prod_{k=1}^{N} \overline{I}_k, \qquad (\Pi.2.2)$$

where

$$\overline{I}_k = \int_{-\infty}^{\infty} e^{-d_k x_k^2} w_1(x_k)\, dx_k. \qquad (\Pi.2.3)$$

We will consider the domain $d_k \gg 1$. Using the asymptotic formula from [17] to estimate an integral of type (Π.2.3), assuming that the necessary conditions are fulfilled, we obtain

$$\overline{I}_k = \sqrt{\frac{\pi}{d_k}}\, w_1(x_k = 0) \qquad (\Pi.2.4)$$

and correspondingly

$$\overline{F} = \prod_{k=1}^{N} \sqrt{\frac{\pi}{d_k}}\, w_1(x_k = 0). \qquad (\Pi.2.5)$$

We will consider two examples.

1. The quantities x_k, $k = \overline{1, N}$ are Gaussian with parameters M_k and σ^2_k. Then

$$w_1(x_k = 0) = \frac{1}{\sqrt{2\pi\sigma_k^2}}\, e^{-m_k^2/2\sigma_k^2} \qquad (\Pi.2.6)$$

and

$$\overline{F} = \frac{1}{2^N} \prod_{k=1}^{N} \frac{1}{\sqrt{\sigma_k^2 d_k}} e^{-m_k^2/2\sigma_k^2}. \qquad (\Pi.2.7)$$

2. The quantities x_k, $k = \overline{1, N}$ are distributed according to bimodal laws (1.49) with parameters μ, $\sigma_{x}{}^{2}{}_{k}$. Then

$$w_1 (x_k = 0) = \frac{1}{2\pi} e^{-\mu_k + \frac{\sigma_{\chi k}^2}{2}}. \qquad (\Pi.2.8)$$

and

$$\overline{F} = \prod_{k=1}^{N} \frac{1}{2} \sqrt{\frac{1}{\pi d_k}} e^{-\mu_k + \frac{\sigma_{\chi k}^2}{2}}. \qquad (\Pi.2.9)$$

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