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# Translation

## Electromagnetic Effects on Communications

By

M.N. Mikhaylov, L.D. Razumov and S.A. Sokolov

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# ELECTROMAGNETIC EFFECTS ON COMMUNICATIONS

Moscow ELEKTROMAGNITNYYE VLIYANIYA NA SOORUZHENIYA SVYAZI in Russian 1979 signed to press 15 Jun 79 pp 1-263

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## CONTENTS

CONTENTS (Continued)

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[Text] A study is made of the theory of the effect of the electromagnetic fields created by high-voltage electric power transmission lines, the catenary systems of electric railways and lightning discharges on communication lines and equipment and also the theory of the galvanic effect of stray currents arising in the ground (water) in the presence of magnetic storms and with energy transmission over single-water lines. General formulas are presented for calculating the voltages and currents occurring in the cable cores and lines under the effect of electromagnetic fields, and the formulas are given for calculating the circuit parameters and the coefficients of electric and magnetic coupling. The principles of shielding theory are presented.

The book is designed for engineering and technical workers dealing with the problem of quantitative estimation of the magnitudes of the extraneous voltages and currents in the circuits and communication channels arising from the electromagnetic effect of different sources.

FOREWORD

The basic areas of development of the national economy of the USSR in 1976-1980 adopted by the 25th Congress of the CPSU define the further development of electrification, transport and communication means, improvement of the efficiency of social production and production quality. The Tenth Five-Year Plan provides for continued work on the creation of a United Automated Communications Network for the Country (YeASS), an increase in the extent of the long distance telephone channels by 1.6 times, the number of telephones in the cities and rural areas by 1.4 times, the number of newspaper strips transmitted by phototelegraphic means, by 2 times, and so on. The requirements are being raised on noise immunity and protection of the circuits and channels of the transmission systems from the effect of external electromagnetic fields of different sources, including the high-voltage lines -- electric power transmission lines and electric railways, lightning strikes and radio stations.

The problems of protecting the communication lines from the effect of external electromagnetic fields have become highly urgent in recent years in connection with fast rates of electrification, the construction of hundreds of new powerful electric power plants, new high and superhigh-voltage, long electric power transmission lines. The convergence of communication lines with high-voltage lines is taking place over longer and longer stretches. At the present time there is no main cable which does not converge with an electric power transmission line or electric railway. Inasmuch as there are high-intensity electromagnetic fields near overhead electric power transmission lines and the contact networks of electric railways, on mutual convergence of the communication lines and the high-voltage lines in the networks and channels of wire transmission systems, prolonged (for normal operation of the high-voltage lines) and short-term (in the case of emergency operating conditions of the overhead electric power transmission lines) extraneous voltages and currents can occur in the wire transmission system circuits and channels. These voltages and currents have both a dangerous effect on the service personnel, the communication lines and equipment and an interfering effect on the transmission of electrocommunication signals, lowering their quality and reliability.

In the next 15 years significant expansion of the construction of electric power plants, electric power transmission lines, electric railways and cable communication lines (mains, zonal and local) is planned. Therefore the solution of the problems of electromagnetic compatibility of high-voltage lines and communication lines is especially important.

The interfering and dangerous voltages and currents in the communication line wires and cores can also occur during discharges of atmospheric electricity during thunderstorm activity. When lightning currents enter the wires of overhead and cable lines directly, the lines are damaged along with individual elements of the equipment leading to prolonged interruptions and idle time of the communications. Usually such damage to the communication lines brings significant economic losses to the national economy, and it must be prevented, especially on the long distance cable lines with symmetric and coaxial circuits on which hundreds and thousands of electrocommunication channels are organized.

The introduction of multichannel transmission systems over cable lines with semiconductor equipment sensitive to current overloads and long-distance remote feed circuits, the appearance of new types of cables in plastic outer sheathings, the construction of high-voltage electric power transmission lines have posed new problems for the theory of the effect of external electromagnetic fields on communication lines.

In this book a discussion is presented of the principles of the theory of the effect of electric and magnetic fields of high-voltage lines, lightning discharges and magnetic storms on the overhead and cable communication circuits. A study is made of the theory of the effect through third circuits; a detailed discussion is presented of the theory of the effect on cables with grounded metal sheathings and metal sheathings insulated from the ground; formulas are obtained for calculating the electric field intensity created by high and superhigh voltage electric power transmission lines.

The given book is a supplement to the monograph by the same authors published in 1978 ZASHCHITA SOORUZHENIY SVYAZI OT OPASNYKH I MESHAYUSHCHIKH VLIYANIY [Protection of Communications from Dangerous and Interfering Effects], which was devoted to the theory and practice of protecting overhead and cable communication lines and the equipment included in them from the dangerous and interfering effects of external electromagnetic fields.

The book is designed for engineering and technical workers involved in the design, construction, operation and maintenance of communication structures. It can serve as a textbook for postgraduates and students at the institutions of higher learning who are studying the problems of the effect of external electromagnetic fields on overhead and cable communication lines.

Chapters 1, 2, 3, 6, 9 and 10 were written by M. I. Mikhaylov and L. D. Razumov; Chapter 4 was written by M. I. Mikhaylov; Chapters 5 and 8 were written by M. I. Mikhaylov and S. A. Sokolov; Chapter 7 was written by L. D. Razumov.

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Authors

# CHAPTER 1. SOURCES OF EXTERNAL ELECTROMAGNETIC FIELDS

## 1.1. Brief Information on the Development of Electrification and Wire Communications in the USSR

The production of electric power and electrification in our country is developing at exceptionally high rates. In 1975 the generation of electric power reached 1038 billion kilowatt-hours (370 times more than in 1913), and the power of the electric power plants increased to 218 million kilowatts. By the end of 1980, the production of electric power is to be increased to 1340-1380 billion kilowatt-hours [1]. In connection with the uniting of the electric networks into the Integrated Power System of the country, powerful high and superhigh voltage electric power transmission lines are being built.

The extent of the electric power transmission lines with a voltage from 35 to 800 kv was 440,000 km in 1970, of which 63,000 km were 220-330 kv overhead lines, and 12,000 km were 500 kv overhead lines. The length of the distributing networks on a voltage of up to 20 kv exceeded 3 million km in 1975. The construction of the triple phase, 750 kv overhead lines is being realized, the construction of the 1150 kv overhead lines is being planned [2]. The first 800-kv DC electric power transmission lines in the world have been built in the USSR, and a 1500-kv DC overhead line is to be built. The dynamics of the development of electric power engineering in the USSR are illustrated in Fig 1.1.

Simultaneously with the development of electric power production, the growth rates of the electric railways are increasing. In 1970 about 40,000 km of railroads have been electrified, which amounts to 28.8% of the total railroad network of the country. The AC electric railways have reached 15,000 km at this time. By the end of the Tenth Five-Year Plan another 2500 km of railroads are to be electrified [3].

Simultaneously with electrification of the country, communication lines are being built at high rates. The length of the long distance communication channels has increased by more than 6 times in the last 12 years. The coaxial cables have received predominant development. For example, in 1970 the production of coaxial cables increased by 5.4 times by comparison with 1958, and symmetric cables, by 1.5 times. In the Tenth Five-Year Plan, further significant growth of cable production is planned: main communication lines by 1.44 times, intrazonal communication cables by 1.5 times, municipal telephone communication cables by 1.29 times [4]. At the present time the main communications over symmetric cables are being realized primarily by the six-ten-channel transmission systems, and over the coaxial cables, using the K-300 and K-1920 systems.

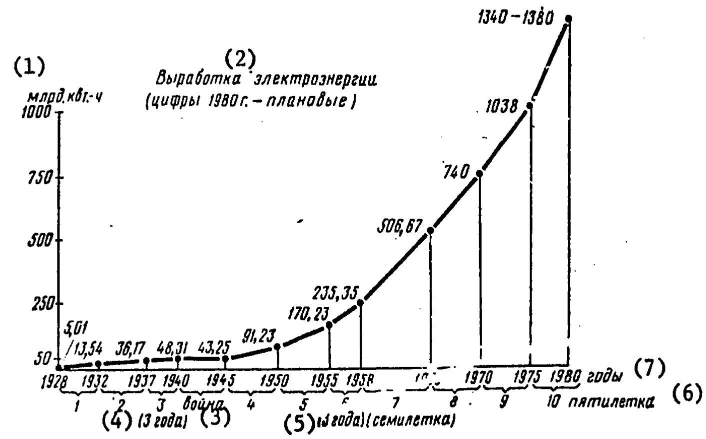



Figure 1. Graph of the development of power engineering in the USSR

Key:
1. Billions oc kilowatt-hours
2. Generatior of electric power (1980 figures -- planned)
3. War
4. 3 years
5. (3 years) (7-year plan)
6. Five-year plan
7. years

The creation of the United Automated Communication Network (YeASS) imposes increased requirements on the noiseproofness and protection of the electrocommunications networks and channels from the effect of external electromagnetic fields of various sources.

Therefore, the problem of the proper consideration of this effect and the development of economically substantiated protection measures have great urgency.

## 1.2. Electric and Magnetic Fields

**Basic Principles.** The existing sources of electric power can create currents and voltage which are constant with respect to magnitude and direction, constant with respect to direction but variable with respect to magnitude, periodically variable with respect to magnitude and direction, and of a pulsed nature (monopolar and bipolar) in the electric networks. The variable currents and voltages of different form create variable electric and magnetic fields in the space around the electric circuit. The nature of the variation of these fields is similar to the nature of variation of the electric voltage and current in the investigated circuit. If an AC voltage which varies, for example, according to a sinusoidal law, is applied to the circuit, then the current in the circuit and the magnetic and electric fields around the circuit wires will also vary variably by this law (see Fig 1.2). When a pulsating current flows through a conductor, the magnetic and electric fields formed around this conductor will also be pulsating (see Fig 1.3).

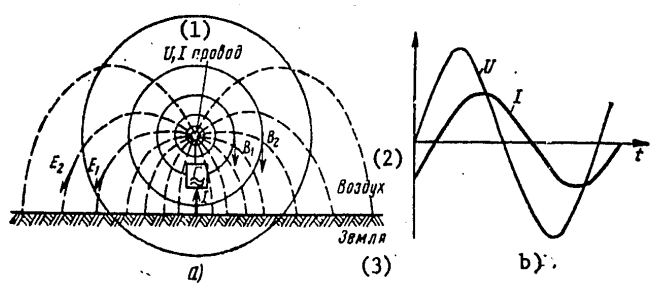



Figure 1.2. Intensity of the electric field (E) and magnetic induction of the magnetic field (B) in the space near a wire (a) and sinusoidal voltage and current curves in the wires (b)

Key:
1. U, I wire
2. Air
3. Ground

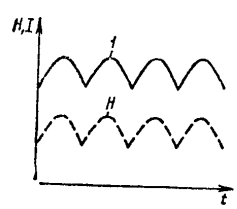



Figure 1.3. Curves of a pulsating current (1) and the intensity of its magnetic field (H)

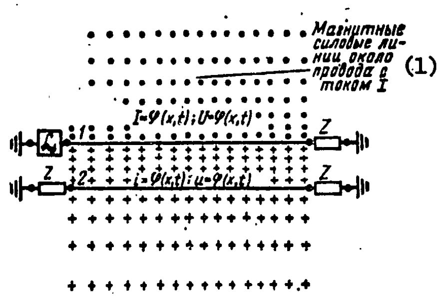



Figure 1.4. Overhead lines affecting (1) and subjected to the effect of (2) an electromagnetic field.

Key:

1. Magnetic lines of force near a line with current I

If another line is placed (see Fig 1.4) near the line with AC voltage $U=\phi(x,t)$ and current $I=\phi(x,t)$, then under the effect of the electric and magnetic induction in the second wire the so-called induced voltages and currents occur: $u=\phi(x,t)$ and $i=\phi(x,t)$. In the case where a wire with a source of electric power in it forms a circuit through the ground (a single wire circuit), in addition to the induced voltages and currents, currents and voltages of galvanic coupling (Fig 1.5) occur in a parallel second wire closed through the ground. In other words, the current in the first wire, returning through the ground, can branch from the ground into the second wire at the point where it is grounded.

The first system of wires to which the voltages are applied and through which currents flow from any source of electric power are called the influencing wires or circuits. The second system of wires in which voltages and currents arise is called the wires or circuits subjected to the electromagnetic effect.

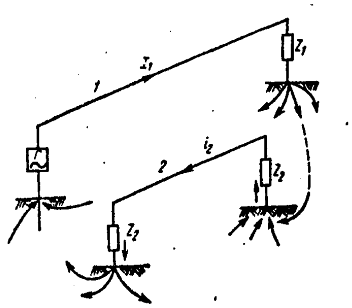


Figure 1.5. Galvanic effect between single-wire circuits

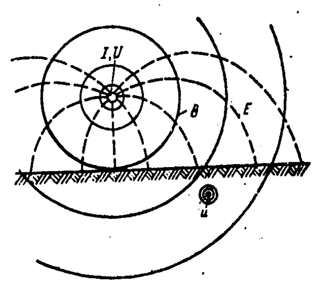


Figure 1.6. Electromagnetic field of a single-wire overhead circuit and underground cable subjected to the effect of a magnetic field

The interfering systems include the following: the electric power transmission lines, the traction networks of electric railways, radio stations which emit high-frequency electromagnetic energy, the lightning discharge channels, and so on. The systems of wires subjected to the effect of the indicated sources include any electric networks for any purpose. In this book a study is made only of the circuits subjected to the effect, through which electric power of extraordinarily small magnitude is transmitted by comparison with the power transmitted over the wires of the interfering systems. In this case, it is possible to ignore the reaction of the wires subjected to the effect. Such circuits include the overhead and cable communication line circuits.

The overhead communication line circuits are influenced by the electric and magnetic fields of the interfering lines. On underground cable circuits or cables with metal sheathings suspended on supports, are affected only by the magnetic fields of the interfering lines, for the lines of force of the electric fields are shielded by the surface of the ground or the metal sheathing of the cable (Fig 1.6). In the case where a cable in a nonmetallic sheathing and without shielding is hung from the supports of an overhead line the circuits of this cable are subjected to the effect of the magnetic and electric fields (Fig 1.7).

The voltages and currents induced by the external electromagnetic fields are extraneous for the communication circuits. Mixing with the transmitted working currents and voltages, these currents distort them, which has a negative effect on the transmission quality. Under known conditions the magnitude of the induced voltages and currents can reach very large values which are dangerous for the insulation of the communication wires and equipment. The extraneous voltages and currents occurring in the communication circuits are called dangerous or interfering depending on the effect which they have.

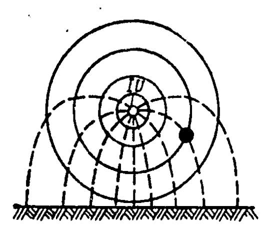


Figure 1.7. Electromagnetic field of a single-wire overhead circuit and overhead cable without metal sheathing subjected to the effect of electric and magnetic fields

For overhead and cable lines subjected to the effect of external electromagnetic fields, let us consider the following values:

For overhead lines:

The induced voltages in each wire of the communication line with respect to ground;

The potential difference between the wires of the two-wire circuit ($U_{12}$);

Induced currents in the wires of the single-wire and double-wire circuits;

For cable lines:

The induced voltages between the core and the metal sheathing of the cable and also between the core and ground;

Induced voltages between the metal sheathing and ground;

Induced current in the metal sheathing of the cables;

Induced current in the cable cores;

Potential difference between the cores of a symmetric pair or between the outer and inner conductors of the coaxial pair.

Brief Information About the Electromagnetic Field. The electromagnetic field is the basic form of matter. It is characterized by four vector values: the intensity of the electric field $\overline{E}$, the electric induction $\overline{D}$, the intensity of the magnetic field $\overline{H}$, the magnetic induction $\overline{B}$. To define a field in some region in space means to indicate these field vectors at any point of it. The electromagnetic field components -- electric (vectors $\overline{E}$, $\overline{D}$) and magnetic (vectors $\overline{H}$, $\overline{B}$) -- are mutually dependent. For a field in a vacuum the following expressions are valid:

$$\overline{D}=\varepsilon_0\overline{E},\ \overline{B}=\mu_0\overline{H}, \qquad (1.1)\ (1.2)$$

where $\varepsilon_0$ is the electric constant equal to $\varepsilon_0=(1/36\pi)\ 10^{-9}\approx 8.85\times 10^{-12}$ farads/m; $\mu_0$ is the magnetic constant equal to $\mu_0=4\pi\cdot 10^{-7}\approx 1.257\cdot 10^{-6}$ g/m; H is the magnetic field intensity, amps/m; D is the electric induction, coulombs/$m^2$.

The interrelation of the electromagnetic field vectors is defined by the properties of the medium. The theory of an electromagnetic variable field is based on the following four principles:

The total current law establishing the relation between the magnetic field intensity and the current exciting this field,

$$i_n=\oint Hdl. \qquad (1.3)$$

The concept of the "total current" includes not only the conductivity current, the density of which $\delta_{comb}=\sigma E$, but also the bias current $\delta_{bias}=\varepsilon_0(\partial E/\partial t)$. From this law it follows that any variation of the electric field in the time $|\partial E/\partial t|$ causes the appearance of a magnetic field (H);

The law of electromagnetic induction establishing the relation between the emf induced in the circuit and magnetic flux intersecting this circuit

$$e=-\frac{\partial \Phi}{\partial t} \;\&\; \oint E dl=-\frac{\partial \Phi}{\partial t}. \qquad (1.4)$$

From (1.4) it follows that any variation of the magnetic field in time $\partial\Phi/\partial$ is accompanied by the appearance of an electric field (E);

The Ostrograd-Gauss theorem establishing the relation between the vector flux of the electric field intensity through the closed surface and the electric charge located inside the given surface,

$$N=Q/\varepsilon_0. \qquad (1.5)$$

If Q=0, then N=0, and then the electric field can occur only as a result of variation of the magnetic field. In this case, the emf in the closed circuit is

$$e=\oint E dl=-\frac{\partial \Phi}{\partial t}\neq 0, \qquad (1.6)$$

and the emf of the electrostatic field in the closed circuit is

$$e=\oint E dl=0.$$

This difference is caused by the fact that the electrostatic field is a potential field, and the electric field obtained as a result of variation of the magnetic field has a vortex nature;

The cause of continuity of the magnetic flux expressed in the fact that the total magnetic induction flux through any closed surface is equal to zero. This is explained by the eddy nature of the magnetic field.

## 1.3. Sources of Electromagnetic Fields of Different Frequencies

Triple-Phase High-Voltage AC Lines. The set of electric power plants, substations and electric power receivers connected to each other by the electric network lines is called the power system. Part of the power system consisting of the generators, distributing devices, step-up and step-down substations, the electric network lines and the electric power receivers is called the electric system. The distance between the power engineering and electrical systems consists in the fact that the electric system does not include the thermal or hydraulic parts of the system, that is, the part pertaining to the primary motors and their power supply.

The electric system consists in the station and linear parts. The linear part of the electric system, including the substations and lines of different voltages, is called the electric network. The latter is used to transmit electric power from the place of its production (the electric power plant) to the user locations and for distribution of the electric power among the users. The lines connecting the electric power plant to the step-down substations are called high-voltage lines (VL). In addition, any high-voltage part of the rayon electric system network is called a high-voltage line.

An example diagram of the electric system [5] is shown in Fig 1.8. From the powerful rayon electric power plant (the rayon hydroelectric power plant [GES]) the AC electric power is transmitted through the step-up substation of the 220 kv overhead line (or 500, 750 kv line) and the step-down substation to the 110 kv rayon ring network. This network is also fed through the 110 kv line and the 6/110 kv step-up substation from the rayon thermoelectric power plant located near the peat shows. Inside the 110 kv rayon ring network there are step-down 110/10 kv substations servicing a large industrial region, in the center of which there will be a heat and electric power plant (TETs) operating on imported fuel and supplying the users of the industrial area located near the plant with electrical and thermal power. From the rayon 110 kv network through a step-down substation, the 35-kv rayon ring network is fed, from which, in turn, the local 6 kv networks with step-down transformers for the 380-220 volt distributing networks is fed.

In the lower part of the diagram, the local electric power plant of comparatively low power with distributing network of 6 kv directly departing from the station buses (to the right) connected to the system is concentrated. The feed network which provides electric power to the feed stations in the 6 kv distribution network (on the left) is connected to the same buses. The step-down transformers in the 6 kv network feed the 380/220 volt distribution networks. The step-up and step-down transformer substations of the triple-phase electrical systems can have connections of the transformer windings by the system shown in Fig 1.9.

If the star-connected line windings of a transformer have a neutral point insulated from the ground, then the overhead line connected to such transformers is called a symmetric-voltage line with insulated neutral. If the neutral points of the line windings of the transformers are connected to ground, then the overhead line connected to such transformers is called a symmetric high-voltage line with rounded neutral. Finally, in the case where the neutral point of the line windings of the transformer is connected to the ground through an arc-extinguishing coil, the overhead line connected to such a transformer is called a symmetric high-voltage

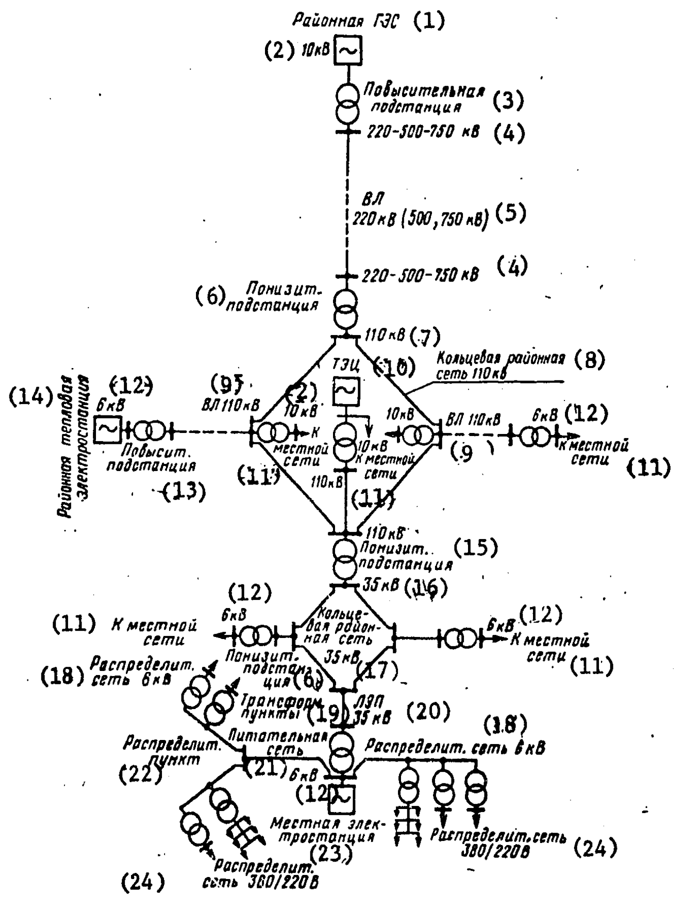



Figure 1.8. Example diagram of the electrical system

Key:

1. Rayon hydroelectric power plant
2. 10 kv
3. Step-up substation
4. 220-500-750 kv
5. 220 kv overhead line (500, 750 kv)
6. Step-down substation
7. 110 kv
8. 110 kv ring rayon network
9. 110 kv overhead line
10. Heat and electric power plant
11. To the local network
12. 6 kv
13. Step-up substation
14. Rayon heat and electric power plant
15. 110 kv step-down substation
16. 35 kv
17. 35 kv ring rayon network
18. 6 kv distribution network
19. Transformer station
20. 35 kv overhead electric power transmission line
21. Feed network
22. Distribution station
23. Local electric power plant
24. 380/220 v distribution network

line with compensated neutral. The high-voltage lines in which the ground is used as one of the operating lines are called the lines of asymmetric electric systems.

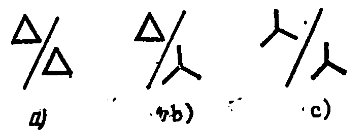



Figure 1.9. Diagram of the connection of the power transformer windings: a) triangle-triangle; b) triangle-star; c) star-star

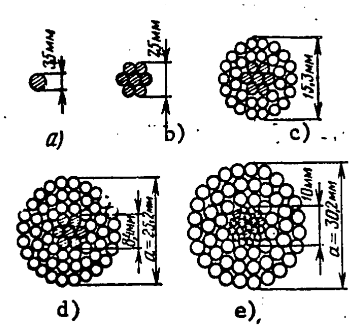



Figure 1.10. Wires for overhead high-voltage lines: a) single-wire 10 $mm^2$ in cross section (M-10); b) seven-wire 35 $mm^2$ in cross section (M-35); c) multiwire 120 $mm^2$ in cross section (AS-120); d) multiwire 332 $mm^2$ in cross section (ASO-332) made up of 54 aluminum wires d=2.8 mm and 7 steel wires d=2.8 mm; e) multiwire 480 $mm^2$ in cross section (ASO-480) made up of 54 aluminum wires d=3.37 mm and 19 steel wires d=2 mm

The rayon and distribution electric networks are divided into overhead and underground. The overhead networks are made using bare wires suspended on insulators mounted on supports. The wires of the overhead high-voltage lines can be: single-wire -- copper or steel; multiwire, made up of 7, 12, 19 or 37 individual copper or aluminum wires twisted together; multiwire steel-aluminum (AS and ASO); in each such conductor the core is made up of 3, 7, 19, and 25 twisted steel wires, around which there are lays of aluminum wire. Certain types of conductors for overhead electric power transmission lines are shown in Fig 1.10. The structural design and the dimensions of the supports for the overhead lines depend on the magnitude of the operating voltage. The types of intermediate supports for the high-voltage lines for various voltages are shown in Fig 1.11. The basic dimensions of the wire suspensions on the overhead lines for triple-phase current from 3 to 750 kv are presented in Table 1.1.

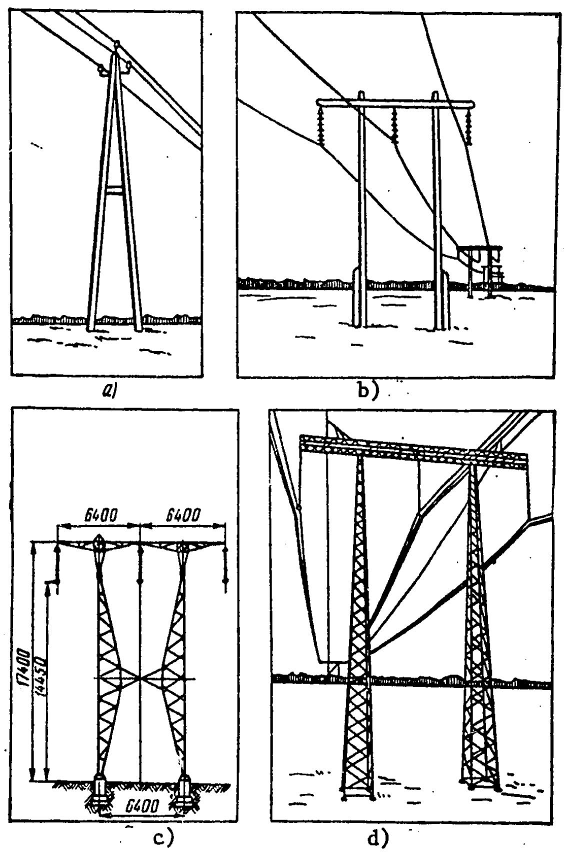



Figure 1.11. Intermediate supports of overhead high-voltage lines: a) wooden for the 10 kv overhead lines with pin insulators; b) wooden for 110 kv overhead lines; c) metal for 220 kv overhead lines; d) metal for 500 kv overhead lines

Table 1.1.

Overall Dimensions of the Wire Suspension of Electric Power Transmission Lines

| Линейное напряжение, кВ (1) | Частота, Гц (2) | Расстояние между соседними фазовыми проводами, м (3) | Расстояние между проводами в расщепленной фазе, м (4) | Высота подвеса, м (5) макс. (6) | мин. (7) | Высота опоры до точки подвеса гирлянды, м (8) | Высота гирлянды, м (9) |
|---|---|---|---|---|---|---|---|
| 3 | 50 | 1,0 | — | — | 6 | — | — |
| 6 | 50 | 1,2 | — | — | 7 | — | — |
| 10 | 50 | 1,5 | — | — | 7 | — | — |
| 35 | 50 | 3,0 | — | — | 7 | — | — |
| 110 | 50 | 4,0 | — | — | 7 | — | — |
| 154 | 50 | 5,5 | — | — | 7,5 | — | — |
| 220 | 50 | 7,5—8,5 | — | 22 | 8 | 25 | 3,0 |
| 500 | 50 | 12,5 | 0,5 | 22 | 8 | 27 | 5,0 |
| 750 | 50 | 17,5 | 0,6 | 28 | 12 | 35 | 7,0 |
| 800 | Постоянного тока (10) | 11,0 | 0,4 | 17 | 8 | 22 | 5,0 |
| 1500 | То же (11) | 22,0 | 0,6 | 20 | 9 | 30 | 10 |

Key:
1. Line voltage, kv
2. Frequency, hertz
3. Spacing between adjacent phase lines, meters
4. Spacing between lines in split phase, meters
5. Height of suspension, meters
6. Maximum
7. Minimum
8. Height of support to the point of suspension of the insulator chain, meters
9. Height of chain, meters
10. DC current
11. The same

The underground networks are made from cables laid directly in the ground or in a corridor. The high-voltage cables for electric networks are made with insulation of the phase conductors for operating voltages of 1, 3.3, 10, 35, 110, 220, and 500 kv.

The operating voltages and currents in all of the high-voltage transmission lines contain harmonics, the frequencies of which are in the frequency range from 0.1 to 150 kilohertz and higher. The harmonics have the highest amplitudes in the voice-frequency band (0.1 to 3.5 kilohertz). These harmonics reach especially high values in the three-phase AC transmission lines which feed the devices with rectifiers (electric traction DC substations, the AC electric locomotives with rectifying converters, powerful radios, electromelting furnaces, and so on). The studies made of the three-phase transmission lines with voltages of 35, 110, 220, 330, and 500 kv by the TsNIIS Institute for various load currents indicate that the curves for the line or phase voltages and currents and also the curves for the zero-sequence currents in the systems with grounded neutrals contain high-amplitude harmonics in the frequency band from 100 to 3500 hertz.

For an example in Fig 1.12 oscillograms are presented of the line voltage of the triple-phase overhead line feeding the DC traction substations and the zero sequence currents in the overhead lines with grounded neutral. The example operating values of the harmonic components of the voltages and currents (phase and zero sequences) for the 500 kv overhead lines are presented in Figures 1.13a and b; for the 220 kv overhead line, in Fig 1.14a and b; for the 110 kv overhead line in Fig 1.15a and b.

For normal operation of the overhead lines, the harmonic components of the voltage and current curves of the overhead lines are sources of the occurrence of interfering voltages and currents in the circuits and communications channels. The currents and voltages of the basic frequency of the overhead line under defined emergency conditions are sources of currents of dangerous effects in the communication circuits.

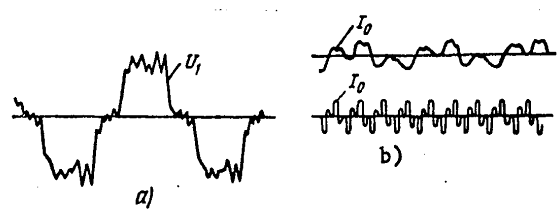



Figure 1.12 Oscillograms of the line voltage of a triple-phase 110 kv overhead line feeding the DC traction substations (a) and the zero sequence currents in the grounded neutral of the triple-phase high-voltage 110 kv overhead transformer (b)

High-Voltage DC Lines. The transmission of high electric power over a distance of more than 1000 km will become more advantageous if it is realized by superhigh voltage direct current. The construction of the conversion substations and the DC overhead lines with a working voltage between the conductors of 400 to 1500 kv is in general significantly cheaper than the construction of the triple-phase super-high voltage AC transmission lines. The super-high voltage direct current is obtained at the electric power plants by conversion of the triple-phase industrial frequency alternating current using multiphase rectifiers. The rectifying current at the output of the rectifying unit is a pulsating current containing harmonics of different frequencies. At the distributing substations the direct current is again converted to triple-phase with the help of inverters which, just as the rectifiers, are sources of currents containing harmonics of different frequencies.

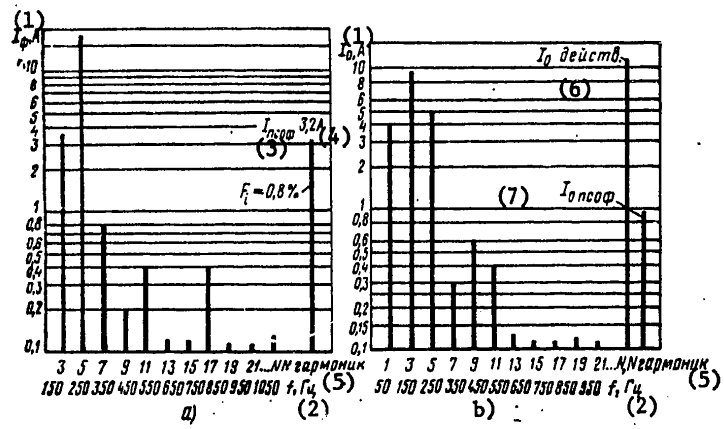



Figure 1.13. Harmonic components and psophometric value of the phase current (a) and the zero sequency current (b) of the 500 kv overhead line for $I_\phi$=400 amps

Key:

1. amps
2. hertz
3. psoph
4. 3.2 amps
5. harmonics
6. $I_0$ effective
7. $I_0$ psoph

The current harmonic frequencies in a high-voltage DC line are in the frequency range from 0.3 to 150 kilohertz and higher. Inasmuch as the DC lines are constructed for a working voltage between conductors from 400 kv and higher, the voltage harmonics in such lines have amplitudes of several thousand volts, and the current harmonics reach several tens of amperes (especially in the voice frequency range).

In Fig 1.16 the effective values are presented for the harmonic voltages in the "wire-ground" network of the DC 400 kv overhead line in the frequency band of 0.3 to 6 kilohertz. From the graph it is obvious that the absolute values of the harmonic amplitudes that are multiples of the number of rectification phases m and the basic frequency of the rectified alternating current f (f=50 hertz, m=6), in the "conductor-ground" loop in the 0.3-6 kilohertz band vary from hundreds to tens of thousands of volts. These figures indicate that the high voltage DC lines are the most dangerous sources of interference for the low-frequency telephone channels and the sound broadcast channels. The values of the harmonic voltages in the frequency band of 12-24 kilohertz in the 400 kv overhead lines will vary from hundreds of volts to hundreds of millivolts. The harmonic components of higher frequencies (24-36 kilohertz) have effective values from several tens of millivolts to several hundreds of millivolts. The indicated values of the harmonic in the 12-36 kilohertz band are sufficiently serious sources of interference in the transmission channels,

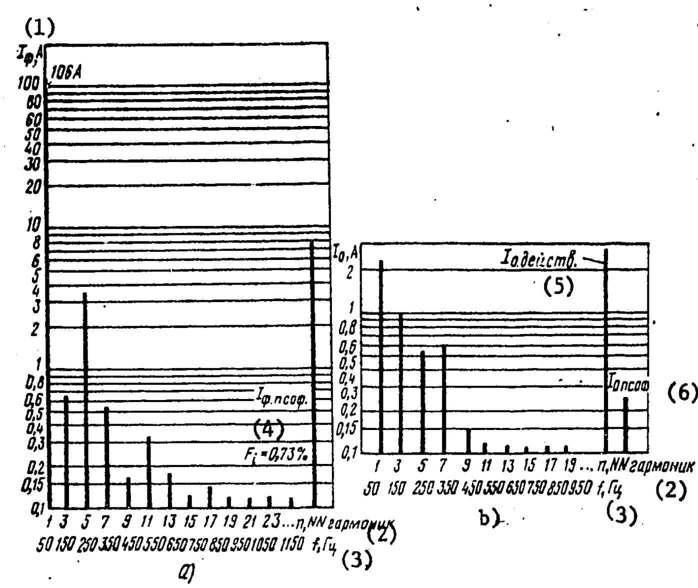



Figure 1.14. Harmonic components and psophometric value of the phase current (a) and zero sequence current (b) of the 220 kv overhead line for $I_{\phi}$=106 amps

Key:

1. amps
2. harmonics
3. f, hertz
4. $I_{\phi}$ psoph
5. $I_0$ eff
6. $I_0$ psoph

especially in the triple-channel transmission systems using the overhead communication lines (V-3). The harmonics in the frequency band of 36-150 kilohertz in the 400 kv overhead lines have such significant amplitudes that it is possible to neglect them as interference sources from the indicated overhead lines.

Inasmuch as at the present time DC overhead lines are being introduced with a working voltage of 1500 kv, the magnitudes of the current and voltage harmonics in such overhead lines are approximately doubling by comparison with the 400 kv overhead lines and, consequently, the interference in the low-frequency telephone channels is increasing as much, respectively. The voltage and current harmonics under the corresponding conditions of convergence of the overhead lines with communication lines have significant interfering effect on the transmission channels in the low and high frequency ranges.

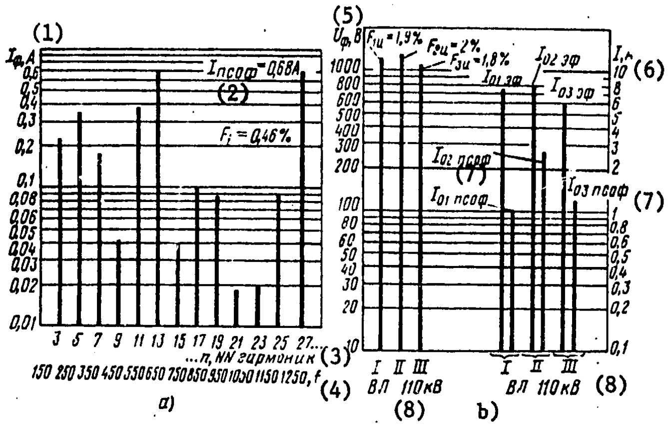



Figure 1.15. Harmonic components and psophometric value of the phase current of the 110 kv overhead lines for $I_\phi$=150 amps (a) and psophometric values of the phase voltages and currents of the zero sequence of the 110 kv overhead line for traction load (b)

Key:

1. amps
2. $I_{psoph}$=0.68 amps
3. harmonics
4. f, hertz
5. $U_\phi$, volts
6. I, amps
7. psoph
8. 110 kv overhead line

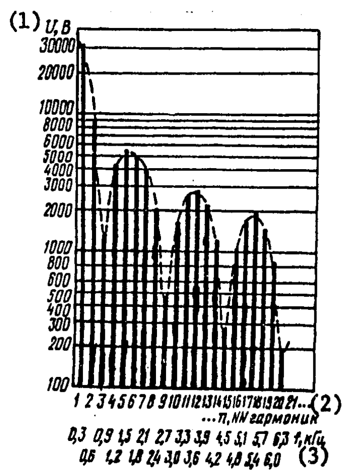



Figure 1.16. Values of the harmonic components of rectified voltage in the "conductor-ground" circuit of the DC 400 kv overhead line

Key:

1. U, volts
2. harmonics
3. f, hertz

If the energy is transmitted over a single-wire line (a "conductor-ground" system), then only half of the rectifiers operate at the station, that is, a six-phase rectification system is created in which the harmonics with frequencies that are multiples of 300 occur. When tı ısmitting power over a two-wire line (the "wire-wire" system), a 12-phase rectification system is created in which the harmonic currents with frequencies that are multiples of 600 appear on the line. In this case, when grounding the mid-points of the systems on the transmitting and receiving ends, currents will circulate in each conductor of the line by the "conductor-ground" circuit with frequencies that are multiples of 300. The magnitudes of these currents will be determined obviously just as when working with lines in the "conductor-ground" mode.

The system for transmitting the harmonic currents in the DC power transmission line with the rectifier (on the left) and inverter (on the right) on in the presence of smoothing filters is presented in Fig 1.17. At any

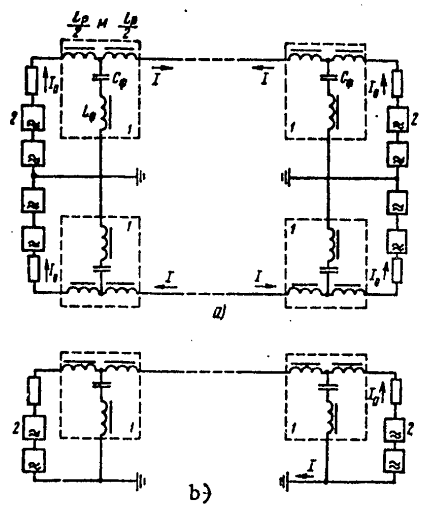


Figure 1.17. Diagram of overhead DC lines operating:
a) by the "wire-wire" system; b) by the "wire-ground" system

point of the DC line the harmonic currents from the rectifier and the inverter are added. The most dangerous case of interference msut be considered to be the one where at any point of the line like harmonics are added arithmetically, for the phase shift between them is $2\pi n$. In Fig 1.17 $L_p$, $C_\phi$, $L_\phi$ are the parameters of the filter elements (1); $I_0$ are the harmonic currents with frequencies that are multiples of 300 hertz; I are the harmonic currents with frequencies that are multiples of 600 hertz; 2 — equivalent harmonic generators.

High-Frequency Channels of the Transmission Systems Based on AC and DC Overhead Lines. The high-voltage electric power transmission lines from 6 kv and up, in addition to direct purpose, are used to transmit telemetry and signals between various points along the overhead line in the power systems. The high-frequency channels in the overhead lines are created in the range from 40 to 500 kilohertz, and they provide for transmitting telemetry, telemetering signals, telephone conversations, and also in protection circuits for the input-output substations [6]. The circuits for connecting the high-frequency installations to the conductors of the overhead lines at the transmitting and receiving stations are illustrated in Fig 1.18. The transceivers ($\Pi_1$ and $\Pi_2$) with carrier frequencies $f_1$ and $f_2$ are connected to the conductors of the high-voltage line through the filters $\Phi_1$ and $\Phi_2$ and capacitors $C_1$ and $C_2$. The chokes $L_1$ and $L_2$ block the path of high-frequency currents in the direction of the power equipment.

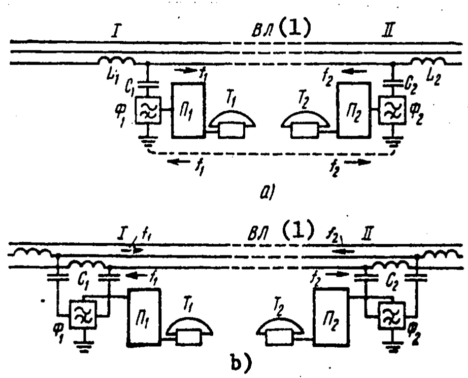



Figure 1.18. Connection of the high-frequency communications stations to the conductors of the triple-phase high-frequency line: a) by the "phase-ground" system; b) by the "phase-phase" system

Key:

1. Overhead line

The high-frequency current transmitters included in the conductors of the DC and AC overhead lines have high power (10 to 100 watts and higher) and, consequently, they are sources of interference for the channels of the transmission systems and, in particular, on the overhead lines. It is necessary to consider the effect of the high-frequency channels of the high-voltage lines on the channels of the transmission systems of the overhead and cable lines in the case where the power of the high-frequency stations of the overhead lines will exceed 5 watts and if the overhead communication lines multiplexed by the V-3 and V-12 transmission systems and the single-quad cable lines, usually having light metal covers with low protective effect, will be subjected to its effect.

DC and AC Electric Railways. In the Soviet Union the electric railways operate on direct and single-phase alternating current of industrial frequency [1.7]. In the DC electric railroads, depending on their purpose, the following voltages are used: 600-800 volts on the municipal railways (streetcars, subways); 3300 volts (on the longest 6600 volts) on the railroad trunks. The voltage is fed to the buses of the traction substations from the 6-phase mercury or semiconductor (silicon) rectifiers that convert the triple-phase AC voltage to DC. An oscillogram of the pulsation of the rectified voltage U picked up at one of the substations of a DC electric railroad is shown in Fig 1.19. The harmonic components of the pulsation curve have a frequency of $f_k$=50 mk, where m is the number of phases of the rectifier of the electric traction substation, k is the order number of the harmonic (1,2,3,...). In reality the pulsation curve for the rectified voltage, in addition to the theoretical components that are multiples of 300 hertz, always contain other harmonics which are not multiples of 300 hertz. The latter occur as a result of asymmetry of the phase voltages of the triple-phase current overhead lines feeding the traction substation. In Fig 1.20, for example, data are present from measuring the harmonics of the buses of the traction substations during normal operation of the electric railroad. From the chart it is obvious that voltage of the contact network, in addition to the harmonics with frequencies of $f_k$=50 mk, contain harmonics with a frequency $f_{(k+1)}=50(m\pm 1)k$.

For electrification by direct current, stray currents are created in the ground which cause corrosion of the underground metal pipes and also lead and aluminum sheathings and steel armor of underground cables. In order to protect the underground structures from corrosion special, expensive measures must be taken. This is one of the deficiencies of the DC railroads.

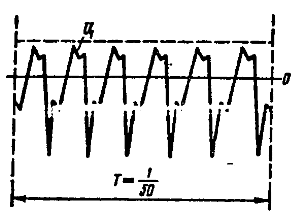



Figure 1.19. Voltage pulsation curve for the buses of the DC electric railroad substation measured before the reactor for U=3300 volts

Beginning in 1958, the electrification system for the railroads in the Soviet Union using alternating single-phase current of industrial frequency and having significant technical-economic advantages over the DC system has been widely introduced. For electrification of the railroads with alternating current the voltage of the contact network is increased to 25 kv (instead of 3 kv for direct current), which makes it possible to increase the distance between the traction substations to 40-60 km, decrease the cross section of the catenary-system conductors and, consequently, cut the copper consumption and cut the initial expenditures on setting up the electric power supply in half.

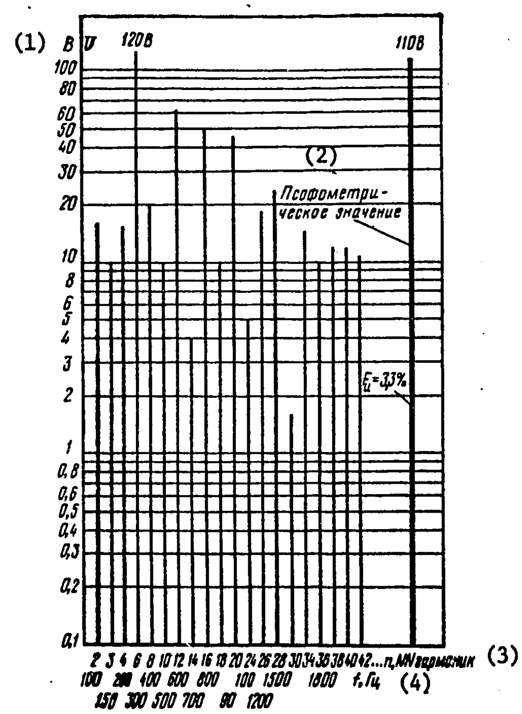



Figure 1.20. Values of the harmonic components of the rectified voltage and the traction substation of a DC electric railroad for U=3300 volts measured before the reactor

Key:

1. volts
2. Psophometric value
3. Harmonics
4. hertz

When supplying power to the AC-DC electric locomotives the voltage curves of the AC contact network, as a rule, contain a large number of harmonics in the frequency range from 0.1 to 150 kilohertz. The basic cause of the distortions of the sinusoidal shape of the voltage and current of the contact network is the presence of rectifiers in the electric locomotive to supply the electric motors of the electric locomotive (the greatest distortion is created by the thyristor rectifiers). Fig 1.21 shows oscillograms of $U_k$ and $I_k$ in the contact conductor of an AC electric railroad in supplying power to AC-DC electric locomotives. Fig 1.22 shows the values of the individual harmonics coming from the electric locomotive

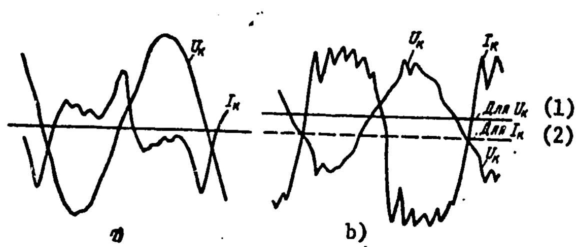



Figure 1.21. Oscillograms of the voltage $U_k$ and the current $I_k$ in the contact conductor of a single-phase AC electric railroad with AC-DC electric locomotives

Key:

1. For $U_k$
2. For $I_k$

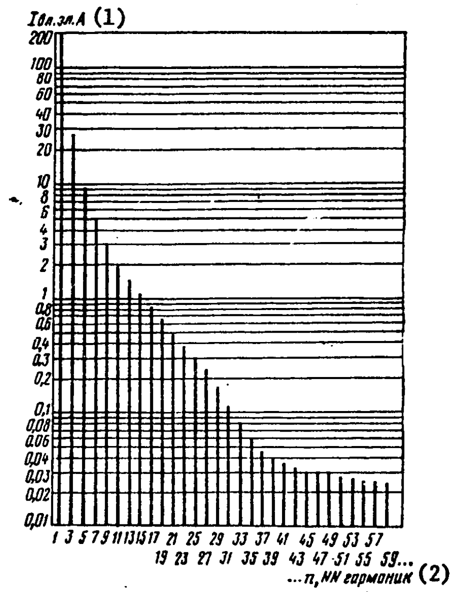



Figure 1.22. Values of the harmonic components of the current in a contact network for AC-DC electric locomotives

Key:

1. $I_{V1.el}$, amps; 2. harmonics

to the contact network. Since the harmonics of the current voltages in the DC and AC contact networks reach significant values, it is possible to conclude that the catenary systems of the electric railroads of the indicated systems are a serious source of interference for the overhead and cable communication line circuits.

In addition, the AC electric railroads have a significant deficiency consisting in the extraordinarily dangerous magnetic and galvanic effects on the communication lines. The protection measures to decrease the effect require expenditures which noticeably reduce the cost benefits of electrifying the railroads with alternating current. One of the methods of reducing this interference is conversion to new power supply systems for the traction network on a voltage of 2x25 kv [11].

The basic elements of electric railroads are the traction substation, the catenary and rail networks. The traction substations are used for conversion of high-voltage alternating current to direct or alternating current with the voltage taken for supplying power to the rolling stock (electric locomotives). The catenary system is a structure by means of which the electric power is transmitted from the substation to the electric locomotive. The catenary system is made up of conductors (contact conductor, bearing cable, amplifying lines), supporting structures (supports, brackets, guys) and various auxiliary devices. The contact (trolley) wires are made shaped (with two longitudinal chutes to grasp the conductor with clamps). The transverse cross section s of the contact conductors usually is taken equal to 85 and 100 $m^2$. The material for the contact conductors is solid-drawn electrolytic copper or bronze. The ohmic resistance (R) of the copper wires at +20°C is the following: R=0.179 ohm/km for s=100 $mm^2$; R=0.211 ohm/km for s=85 $mm^2$; R=0.275 ohm/km for s=65 $mm^2$. As the bearing cable of the chain suspension, copper, bronze, bimetal, steel and combined multiple-core wires twisted from 7, 9 or 37 strands are used.

For the catenary system of the DC electric railroads, it is most expedient to use a copper bearing cable, for in this case the number of amplifying wires is reduced, and in some cases, there is no need for them at all. The MG-120 type wire has found the broadest application of the bearing cable. In cases where the cross section of the contact wire and the cable turns out to be insufficient, an increase in cross section of the contact network wires is achieved by adding amplifying wires (feeders), which are usually suspended on the same supports. The amplifying wires are connected to the contact wire using transverse connectors. Material for the amplifying wires is copper or aluminum. The copper amplifying wires are constructed the same as the bearing cables; the MG-95 and MG-120 and also the M-150 and M-185 aluminum wires have found the broadest application. For connecting the individual wires of the contact suspension to each other, a bare, flexible copper wire 10 $mm^2$ in cross section (MGG-10) is used.

The traction substations that feed the catenary system are connected either parallel to it (parallel two-way feed) or separately (one-way feed).

The DC traction substations are connected every 10-15 km, and the AC, single-phase, 25 kv traction substation, every 40-60 km. At each DC traction substation equipment (transformers) are installed to step down the fed voltage of the triple-phase line (35-110 kv) to the operating voltage of the traction substation with subsequent conversion by means of the six-phase triple-phase current rectifiers to rectified direct current for feeding the contact network. A simplified feed circuit for the traction substation from the triple-phase, 110 kv line with voltage step down to 3.3 kv and conversion of the alternating current using the RV mercury rectifiers to direct current is presented in Fig 1.23. The DC electric railroads have an interfering and dangerous electromagnetic and galvanic effect on the cable communication lines.

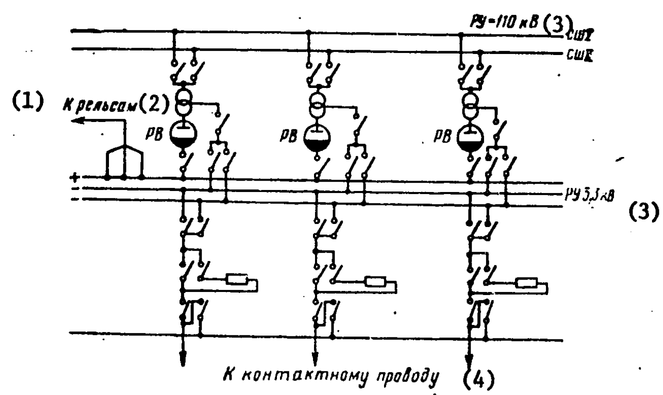



Figure 1.23. System for supplying power to the traction substation of the contact network of a 3.3 kv, DC railroad

Key:

1. To the rails
2. RV
3. kilovolts
4. To the contact wire

On the AC traction substations, equipment is installed to step down the voltage of the high-voltage feed line to the operating voltage in the contact network. The simplified diagram of the power system for a traction substation and single-phase AC catenary system is presented in Fig 1.24. The TDTG transformer indicated in this system has three windings with a voltage of 110, 27.5 and 6 kv. The AC electric railroads are a powerful source of dangerous and interfering effects on cable communication lines.

Statistical Data on the Conditions of Convergence of the Main Cables with the High-Voltage Lines. In the given item the results are presented from examining the conditions of convergence of the main communication cables with the electric power transmission lines with a voltage from 110 kv and up and the AC electric railroads [8]. In the process of inspecting the planning assignments, the length of the route was determined (in the same kilometers) for which the width of the convergence between the cable and the interfering line is within given limits. The integral distribution curves for the width of convergence obtained as a result of the inspection are presented in Fig 1.25. The convergence length of the cables with the overhead electric power transmission lines increases from year to year; the values obtained (Fig 1.25a) can be used in the near future to estimate the convergence in the future.

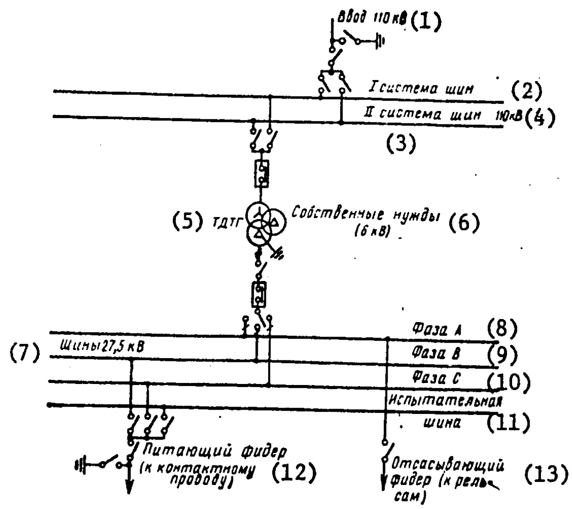



Figure 1.24. System for supplying power to the traction substation and single-phase AC contact network

Key:

1. 110 kv input
2. I bus system
3. II bus system
4. 110 kv
5. TDTG
6. In-house needs (6 kv)
7. Buses 27.5 kv
8. Phase A
9. Phase B
10. Phase C
11. Test bus
12. Supply feeder (to the contact wire)
13. Discharge feeder (to the rails)

The data on the convergence of the main cables with the AC electric railroads on the whole throughout the USSR are presented in Fig 1.25b. These data take into account the prospects for the next 10 or 15 years. In addition to the convergence width, from the indicated planning assignments, the specific resistance of the ground along the routes of the main cables was determined. Here the length of the route was established (in the kilometers for which the specific resistance of the ground is within the given limits). As a result, the integral distribution of the specific resistance of the ground was obtained (Fig 1.25c).

The convergence width of the cable with the high-voltage line and the specific resistance of the ground can be considered as random variables. The determination of their values on each kilometer of the cable route is equivalent to performing experiment on a random variable. The number of kilometers of the route with the given value, for example, width of convergence, is in such an interpretation the number of favorable events, and the ratio of the length of the cable route on which the convergence width has the given value to the total length of the trunk is the frequency of the event: the appearance of a given value of the convergence width. As is known, the frequency of the event for a large number of experiments approaches the proability of the event. Since a large number of long currents were inspected, the frequency of the appearance of the given value of the convergence width or the specific resistance to the ground can be equated to the probability.

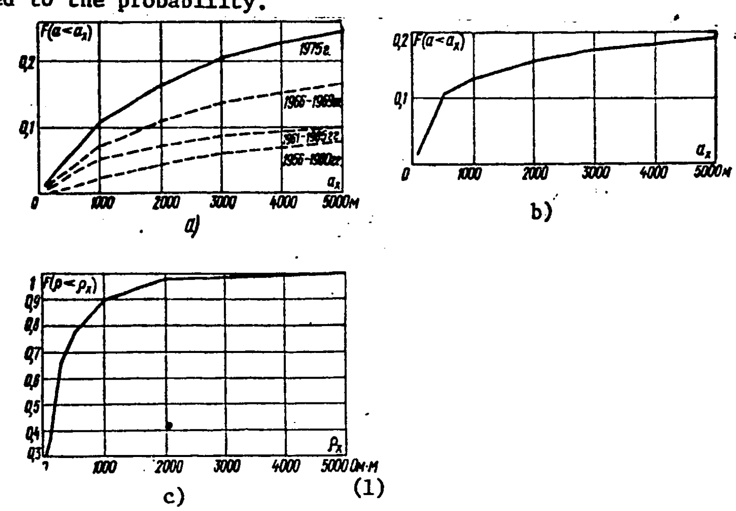



Figure 1.25. Statistical distribution functions (F) of the convergence width of the long distance cables:
a) with electric power transmission lines; b) with AC electric railroads; c) distribution of the specific resistance of the ground along the cable mains

Key:
1. ohm-m

Thus, from the data obtained, the probability of one value of the convergence width or the specific resistance of the ground or another can be found. The data obtained with respect to the convergence width and the specific resistance of the ground can be considered as some simple statistical sets of random variables or the primary statistical material.

For this set the statistical distribution function of the random variable can be constructed as was done in probability theory, that is, the frequency of such an event for which the given random variable X is less than the given value of x. In our case this is equivalent to determining the frequency (probability) that the convergence width (or the specific resistance of the ground) will be less than the given value, that is,

$$F(a) = p(a < a_x),$$

where F(a) is the statistical distribution function of the convergence width a; p is the frequency (probability) of the event; $a_x$ is the given value of the convergence width. It is possible analogously to determine

$$F(\rho_{з}^{(1)}) = P(\rho_{з}^{(1)} < \rho_{з\,x}),$$

Key: 1. g

where $\rho_{gx}$ is the given value of the specific resistance of the ground (the statistical distribution functions are seen in Fig 1.25a-c).

In order to find the value of the statistical distribution function for the given convergence width $a=a_x$, the number of kilometers of the cable routes having a convergence with $a<a_x$ was calculated, and it was divided by the total length of the mains. The curve pertaining to the entire territory of the USSR takes into account the convergence length for a convergence width $a<a_x$ with respect to all mains and the total length of the mains. The statistical distribution function of the specific resistance of the ground was constructed analogously.

The magnitudes of the convergence width and the specific resistance of the ground characterize the conditions of convergence of the cable line with the high voltage line. Thus, when using the data obtained it is possible to talk about the probability of certain convergence conditions for the examined sections of the network. The convergence width of the cable with the interfering line and the specific resistance of the ground are independent random variables. This means that if we are given defined calculated values of the convergence width and the specific resistance of the ground, then the probability of the event in which both random variables will be improved by the convergence conditions, that is, their convergence width will be less and the specific resistance of the ground will be higher than calculated is equal to the product of the probability of the two values.

Of course, the probabilities obtained for the individual specific main can also fail to be observed, but on the whole throughout the network the probability of the appearance of certain conditions of convergence must correspond to the constructed curves.

Atmospheric Electricity. Lightning Discharges. It is known that the mantle of air around the earth (the atmosphere) is made up of several layers: the troposphere with the upper boundary from 7 to 18 km above the earth, the stratosphere extending to an altitude of 7-18 to 80 km and the ionosphere which extends to an altitude of up to 900 km. The ionosphere has good electrical conductivity, and it is like one plate of an enormous spherical capacitor, the second plate of which is the spherical surface of the earth. The role of the dielectric is played by the poorly conducting air medium between them. The magnitude of the dielectric charge of the investigated spherical capacitor is reckoned at $5 \cdot 10^5$ coulombs. Here the upper plate (the ionosphere) is charged positively, and the earth's surface, negatively. The electric field intensity between the ionosphere and the earth is nonuniform as a result of different density of the air; for the earth's surface it will be about 120 v/m in the case of a clear sky.

Under the effect of the existing potential difference between the ionosphere and the ground, a current, the total magnitude of which reaches 1800 amps, flows constantly through the atmosphere, which has a poor, but still finite conductivity.

The intensity of the electric field in the atmosphere does not remain constant; at different points of the earth it is different, and to a high degree it depends on the presence of charged clouds above the surface of the earth. The charged clouds are a source of a formation of an additional electric field between the clouds and the surface of the earth. The total magnitude of the intensity of the electric field at the surface of the earth for slowly moving charged clouds can increase to 1000 volts/m, 5000 volts/meter and higher. For extraordinary magnitudes of the potential difference between the cloud and the earth (exceeding $10^9$ volts) electric discharge -- lightning -- takes place.

The clouds are accumulations of either drops of water (low clouds -- cumulus) or ice crystals (cirrus). Both are formed on condensation of water vapor in the air. Condensation is a consequence of the cooling of the mass of moist air below the temperature of the so-called "dewpoint" for which the vapor becomes saturated. For condensation of the water vapor it is necessary that nuclei be found in the mass of air around which the drops form. These nuclei are dust particles in the air or ions. The drops formed on the dust particles are neutral, and the drops, the centers of which are ions, receive positive or negative charges. However, the magnitude of each charge is insufficient for the formation of even such a small quantity of electricity as is transported by ordinary raindrops. The magnitudes of these charges are certainly insufficient for the formation o flightning currents.

In addition, the ionized drops are distributed in the cloud randomly: any part of the cloud contains both positive and negative drops. Therefore, the cloud as a whole can be considered as a body in the neutral state. The electrical phenomena can be manifested here only in the case where the electrized particles are divided into two groups: one with positive and the other with negative charges; each group of charges must be concentrated in a separate part of the cloud. In other words, it is necessary that the cloud be polarized.

The formation of lightning is always connected with the presence of a powerful ascending flow of warm moist air which when it reaches a defined height, releases the moisture contained in it in the form of the finest drops of water or ice crystals. The vertical flows of warm air are formed either during intense heating of the earth's surface by sun rays, especially at the locations having prominences -- hills (thermal thunderstorms) or when a mass of warm air encounters cold, denser hill (frontal thunderstorms). The presence of intense air flows in the clouds having high velocity leads to constant fractionization of the water drops; in this case separation of the charges takes place; a negative charge is formed on the fine drops, and a positive charge on the larger ones. In the lower regions of the cloud heavier drops are accumulated with positive charges; the lighter water mist is carried to the upper part of the cloud with negative charges (Fig 1.26).

The gradual accumulation of electric charges in the upper and lower parts of the cloud is the cause of the formation of an electric field of increasing intensity around them. This field occurs inside the cloud and around it to the surface of the earth. When the potential gradient of any point of the cloud reaches the critical value for the air at atmospheric pressure (according to Wilson, $3 \cdot 10^5$ volts/meter), a breakdown or leader discharge takes place.

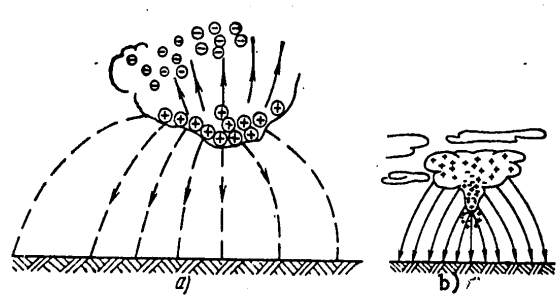



Figure 1.26. Charge distribution in a polarized cloud (a) and electric field of a charged cloud (b)

Depending on the polarization of the cloud, positive or negative ions are discharged in the direction of the electric field, and under its effect they acquire enormous velocity. The impact of these ions against the air molecules determines the ionization of the latter; ions of the same size as the cloud are repelled and carried away in the direction opposite to the cloud, and the ions with opposite sign are attracted to it. The process of ionization of the nearby layers of air brings the charge of the cloud toward the ground; the intensity of the electric field increases, and the ionization continues as a consequence of the lightning strike. The leader charge moves forward in the direction of the ground in steps at time intervals of about 100 microseconds.

The discharge beginning near a cloud (1 in Fig 1.27) is propagated at a velocity on the order of $5 \cdot 10^4$ km/sec in the direction of the ground. However, this discharge does not reach the ground, but is halted at a distance of 50 to 60 meters from the cloud. After some time (approximately 10 microseconds) a new discharge (2 in Fig 1.27) begins along the path traveled by the first discharge with the velocity on the same order. This new discharge moves toward the ground at a distance of 100 to 120 meters from the cloud. After completion of this second discharge along the path traveled by it, the third discharge begins to move (3), removing itself by 50-60 meters from the cloud, and so on. The new discharges (4 and 5) occur until the last discharge (8) reaches the ground.

As we see, the process of the formation of the lightning channel from the cloud to the ground takes place stepwise and requires a time interval on the order of thousands of microseconds for its completion. The number of pulses depends on the distance between the charged cloud and the ground. This preliminary lightning discharge is called the leading discharge or leader, and it is accompanied by a comparatively small light effect.

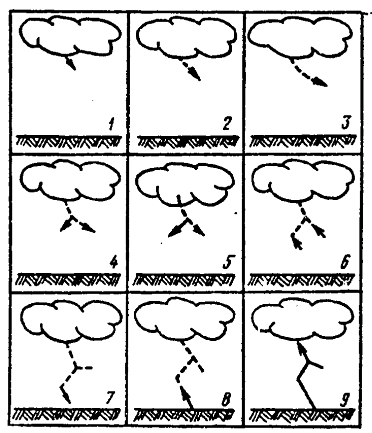



Figure 1.27. Schematic of the cloud discharge process

After the leader reaches the ground a brightly glowing discharge begins to be propagated from its surface toward the cloud with an average velocity of $3.5 \cdot 10^4$ km/sec. This is called the primary discharge (9 in Fig 1.27). This primary discharge can occur only on the surface of the ground toward the cloud. Since the conductivity of the cloud is very small by comparison with the conductivity of the ground, the potential of the center of the charge decreases very quickly in the cloud. This often serves as the cause of the occurrence of subsequent primary discharges from the ground to the cloud. The formation of subsequent primary discharges differs somewhat from the formation of the first discharge. The mechanism of lightning discharge recorded using a rotating camera is illustrated in Fig 1.28a. The current variation during repeated lightning discharges is illustrated in Fig 1.28b. For example, a second discharge which, just as the first, is made up of a leader running from the cloud to the ground and the primary discharge running in the opposite direction after the leader reaches the ground, differs from the first in that in it the leader does not advance in steps, but with a more or less uniform velocity approximately equal to $2 \cdot 10^3$ km/sec; this repeated leader is called the

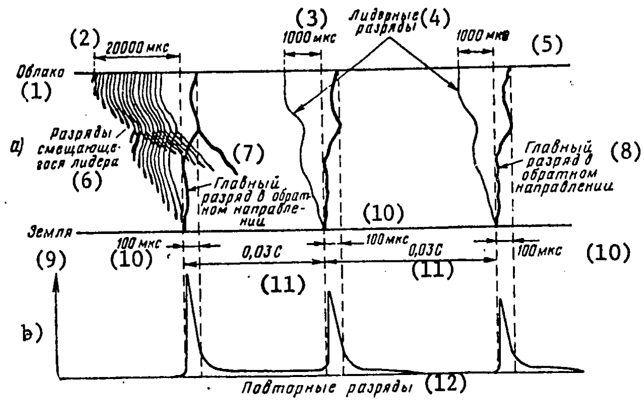



Figure 1.28. Mechanism of lightning discharge recorded using a rotating camera (a) and the current variation during repeated lightning discharges (b)

Key:

1. Clouds
2. 20000 microseconds
3. 1000 microseconds
4. Leader discharges
5. 1000 microseconds
6. Discharges of the shifting leader
7. Primary discharge in the opposite direction
8. Primary discharge in the opposite direction
9. Ground
10. 100 microseconds
11. 0.03 seconds
12. Repeated discharges

through leader. The number of repeated discharges along one horned path (channel) can fluctuate from 2 to 10 or more. If the time interval between two primary discharges following each other is relatively large and the ionization of the channel decreases to a value less than critical, the repeated leader loses the capacity for continuous movement and begins to be propagated in steps.

The most dangerous types of lightning discharge which damage communication installations are streak lightning which is discharge of the cloud in the form of a bright narrow streak, white, light blue or bright red in color from hundreds of meters to many kilometers long. The path of such lightning usually is zigzag. The streak lightning is characterized by the following data [40]: the magnitude of the charge on the thundercloud, the potential of the thundercloud, the magnitude of the lightning current, the diameter of the lightning channel, the number of repeated lightning strikes over one channel, the time intervals between repeated lightning strikes, the magnitude of the electric field gradient and the surface of the earth before discharge of the cloud. Among the enumerated parameters, the most important include the magnitude of the current inthe lightning channel, and the number of repeated strikes on one channel. The magnitude of the current in the lightning channel varies within broad limits -- from several kiloamperes to hundreds of kiloamperes. The observations have established that lightning with high current magnitude occurs only rarely. Thus, lightning with a current of 180 kiloamps occurs in cases, the number of which is equal to 0.1% of the total number of observed lightning strikes; the number of cases of lightning strikes with a current of 80 kiloamps is 4%.

Fig 1.29 shows the number of lightning strikes (expressed in percentages) for which the lightning current does not exceed the values indicated on the y-axis. The number of repeated lightning strikes following each other on one channel is characterized by the curves shown in Fig 1.30. In the majority of cases (from 25 to 80%) the number of repeated strikes in one channel fluctuates from 2 to 5, and only in the small number of cases (5%) can the number of strikes in one channel reach 10. From these curves and other observation results it follows that the lightning discharge of a cloud is characterized by the following mean values:

| | |
|---|---|
| Thundercloud charge, coulombs | 10-100 |
| Cloud potential, volts | $10^8$-$10^9$ |
| Cloud discharge current, amps | $10^4$-$2\cdot10^5$ |
| Discharge time, microseconds | 5-100 |
| Average length of lightning, km | 1-3 |
| Diameter of lightning channel, cm | 3-60 |
| Time interval between individual repeated discharges, sec | 0.001-0.5 |
| Number of repeated discharges | 1-30 |
| Pressure of ionized air in the lightning channel as a result of the effect of electromagnetic forces, Pa | $2\cdot10^5$ |
| Air temperature in the lightning channel, °C | to 1500 |
| Conductivity of the lightning channel, Siemens/m significantly greater than the conductivity of copper, that is, more than | $57\cdot10^6$ |
| Internal resistance of the lightning current source, including the resistance of the lightning channel, ohms | to 5000 |
| Resistance of the lightning channel during establishment of the primary discharge, ohms | 100 |

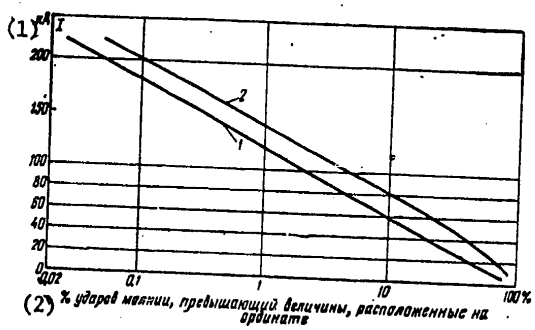



Figure 1.29. Probable percentage distribution of peak values of the lightning discharge current:
1 -- values measured on the 10 kv overhead electric power transmission lines; 2 -- lightning strikes to underground cables

Key:

1. kiloamps
2. % of lightning strikes exceeding the values on the y-axis

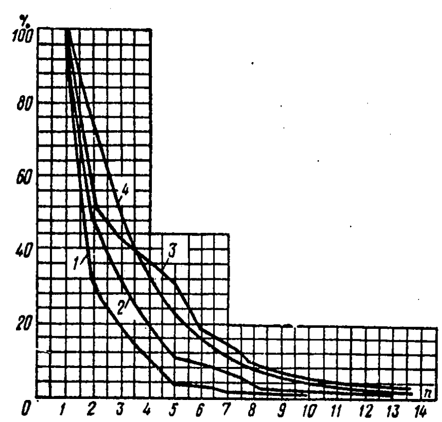



Figure 1.30. Curves for the probability of the percentage distribution of the number of pulses (repeated strikes) in one lightning channel according to the research data:
1 -- MacIcron; 2 -- Cato; 3 -- Shanlanda; 4 - Stekol'nikova

As the measurements demonstrated, the lightning discharge current is a pulse with fast build-up of the current magnitude from 0 to the maximum (the wave front) and comparatively slow decay (the tail of the wave). Inasmuch as on the oscillograms the beginning of the wave and the time of the maximum are difficult to determine exactly, as the wave front length $\tau_\phi$ it is not the time from zero to the peak that is used, but the provisional value determined by construction (Fig 1.31). For determination of $\tau_\phi$, a straight line is drawn through the points of the wave with the y-axes $0.1U_{max}$ and $0.9U_{max}$. The intersection of the straight line with the zero line and the horizontal straight line drawn through the amplitude level also determines the length of the wave front (it is expressed in microseconds). The wave length $\tau_w$ is the time passing from the provisional beginning to the time when the wave current becomes equal to half the amplitude. The lightning current pulse is characterized by the ratio of the wave front to the wave length, that is, $\tau_f/\tau_w$ (frequently $\tau_f$ is denoted by $\tau_1$, and $\tau_w$ by $\tau_2$).

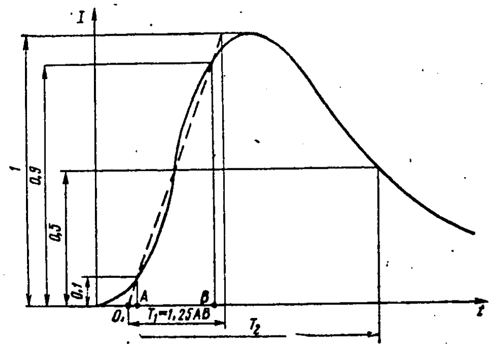



Figure 1.31. Shape of the lightning current wave

According to the observation data for the lightning strike to ground the values of the wave fronts fluctuate from 1 to 15 microseconds, and the wave length reaches 10-100 microseconds; for lightning strike to underground cables the current wave fronts reach 15-60 microseconds, and the current wave length reaches 100-1000 microseconds. The curve for the lightning current pulse with a lightning strike to flat ground has a defined charge Q=10-50 coulombs (on the average 25 coulombs), and for lightning strikes in the mountains, a charge Q=30-100 coulombs (on the average 60 coulombs). For lightning discharges to television towers the charge reaches 160 coulombs. For lightning strikes to ground, damage to the lines is possible if the cloud discharges occur directly in the lines or near them. When the lightning currents hit a support or the wires of overhead lines, destruction (splitting) of the support, fusion and breaking of the wires take place.

For lightning current strikes to ground near overhead communication lines, high voltages occur in the wires which have a destructive effect on the insulation of the equipment included in the overhead line circuits. When the lightning currents hit the overhead communication line itself, the wires, insulators, line supports and included equipment are destroyed. The cable underground lines are also subjected to damage when the lightning currents hit the ground near the point of laying the communication cables or when the lightning currents hit the cables themselves.

A large number of cases of damage to the interurban cables by lightning current hitting the cable sheathings during discharge of the cloud to ground near the cable path have been recorded. A large number of cases have been noted where the lightning currents reach the sheathing from tree roots which have been struck by lightning [9].

In order to determine the nature of the cable damage in the case of lightning strikes and to make some generalizations, studies were made of materials in which the damage was more or less described in detail. In addition, statistical data were used on lightning damage to cable long distance networks, both Soviet and foreign, which were assembled by the MKKTT [International Telephone and Telegraph Consultative Committee] from the networks in Sweden, Belgium, Denmark, England, Italy, The Netherlands, Czechoslovakia. Analyzing the data, it is possible to make the following generalizations.

1. The cables in which lightning damage has been detected had lead sheathing from 1.5 to 3.0 mm thick and an outside diameter from 18 to 68 mm.

2. The cables laid in ground with a specific resistance of more than 500 ohm-m are damaged more frequently. However, cases are known where cables have been damaged which were laid in the ground with a specific resistance of more than 50 ohm-m. Primarily the cables laid in clay, sandy, rocky, granite and peat soils are damaged.

3. Cables with small diameter of the lead sheathing are subjected to more serious damage than cables with large diameter of this sheathing.

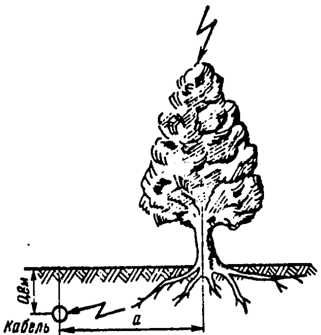
4. Cables are most frequently damaged in the regions with a number of days with thunderstorms of 15 to 60 or more per year. Serious damage has been observed in regions with poor conductivity of the ground even with a significantly smaller number of thunderstorm days per year (5-10).

5. The cases where lightning strikes the cable directly are quite frequent. Holes and breaking-up of the ground occur over the cable itself (according to the observations in the USSR and the United States) or furrows are formed which run to the cable in the ground (according to observations in a number of European countries, the USSR and the United States). In the case of direct lightning strikes of overhead lines, the supports are damaged, and the cables are melted.

6. Many cases where lightning currents have reached cables through nearby trees have been noted. The lightning strikes the tree (Fig 1.32) and runs through its roots to the ground and then to the cable; such cases have been observed in the USSR and other countries. The distance a between the tree and the cable which is jumped by the lightning electric arc increases with an increase in the specific resistance of the ground.

7. Lightning damage to cables occurs in the majority of cases simultaneously at several points along the length of the cable at a distance from several tens of meters to several tens of kilometers (30-36) from each other.

The smaller the cable diameter, the longer the length of the section that is damaged.



(1)

Figure 1.32. Lightning currents to the cable when lightning strikes a tree

Key:

1. Cable

8. The input cables of long distance mains and also the municipal cables connected to the overhead lines are damaged from lightning striking the cable supports or to the wires of the overhead lines near a cable support.

9. At the point where the lightning strikes the cable in the majority of cases the following damage occurs: indentation and bending of the cable under the effect of the external force, melting of the lead in the indentation and breaking of the strip armor, burning of the jute braiding, melting of the cable cores and charring of the paper tape (Fig 1.33a, b, c).

10. At the points in the cables remote from the place where the lightning has struck, in the majority of cases the following damage occurs: breakdowns of the insulation between the strands of the upper lay and the lead sheathing and breakdown of the insulation between the cores, burning of the insulation of the upper lay and melting of the lead sheathing from the inside, magnetization of the loading [Pupin] coils, breakdown to the housing and between the turns of the loading coils, breakdown of the symmetrizing capacitors.

Let us consider the effect of the lightning discharge on the most widespread types of cables: symmetric and coaxial cables in metal and plastic sheathings. Let us represent the cable in the form of three lines: a bold-faced line indicating the cable sheathing and two fine lines which indicate two cores of the symmetric circuit of the cable or the inside wire and the outside tube of the coaxial cable (Fig 1.34). In the case of partial penetration of the lightning current to the symmetric cable on discharge of the cloud to ground (Fig 1.34a), part of the current of the lightning channel only reaches the cable sheathing and, spreading over it from right to left, gradually diminishes. In the cable cores voltages are induced with respect

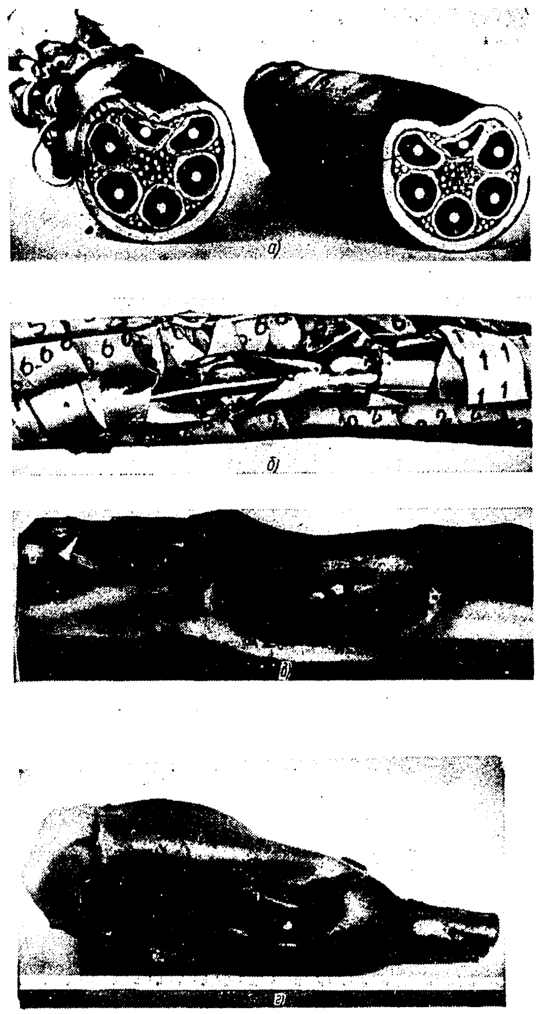


Figure 1.33. Lightning current damage to a cable:
a) denting of the sheathing and outer wires of the coaxial pairs;
b) melting and damage to the outer wires of the coaxial pairs;
c) denting and burning of the lead sheathing; d) damage to the lead coupling

to the grounded sheathing and currents induced in the cores. These voltages and currents, under normal conditions, can be dangerous to the insulation of the cable cores and for the included equipment. When burning and breaking down the insulation between the sheathing and the cable core (Fig 1.34b), part of the lightning current goes into the cable cores. In this case the voltage on the cable core with respect to the sheathing at the point where the lightning strikes becomes equal to zero, and at points remote from the point where the lightning struck, it increases to dangerous values. The magnitude of the currents in the cores at the point where the lightning strikes is maximal, and it gradually decreases on going away from this location.

Fig 1.35 shows the case of partial hitting of the coaxial cable by the lightning during discharge of a cloud to the ground. Fig 1.35a shows the case of the lightning current only entering the cable sheathing, Fig 1.35b, the sheathing and the outer tube of the coaxial pair, and Fig 1.35c, the sheathing, the outer and inner conductors of the cable.

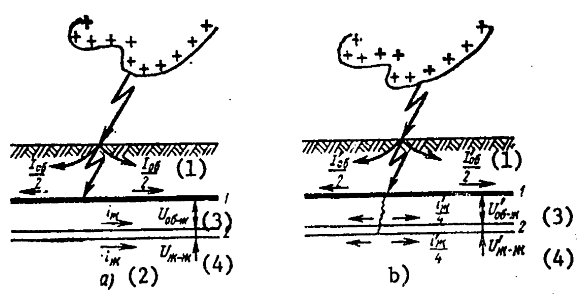



Figure 1.34. Lightning current strikes a symmetric cable:
a) only the cable sheathing; b) the cable sheathing and core

Key:
1. sh
2. c
3. sh–c
4. c–c

In the case of a lightning discharge between clouds (Fig 1.36), voltages and currents are induced in the cores and in the sheathings of symmetric and coaxial cables parallel to the lightning channel under the effect of the variable magnetic field of the lightning channel. The theory of the effect of a lightning discharge on a cable (see Fig 8) permits determination of the magnitude of the potential difference between the sheathing and the cores of the symmetric cable and also between the sheathing and the conductors of the coaxial pair. In addition, it is possible to determine the magnitude of the induced currents along the sheathing and the conductors of the cable.

Along with the damage on the communication lines during the effect of lightning discharge, damage occurs to the equipment connected to the circuits of the overhead and cable communication lines: burning and breakdown of the insulation of the entrance fittings (the line transformers), the filter elements (capacitors, chokes, resistances), the amplifier elements (electron tubes, transistors, and so on), in the terminal and intermediate stations, as a result of which the transmission is disrupted and communication idle time occurs.

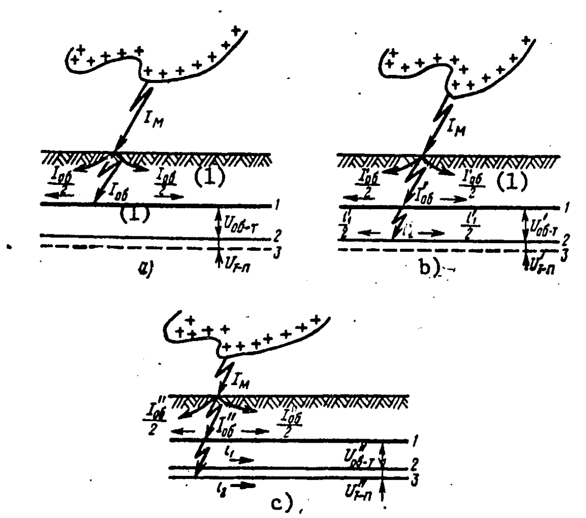



Figure 1.35. Striking of the coaxial cable by lightning:
a) the current only enters the cable sheathing; b) the sheathing and the outer conductor of the cable; c) the sheathing, the outer and inner conductors
1 -- sheathing; 2, 3 -- outer and inner conductors

Key:
1. sh

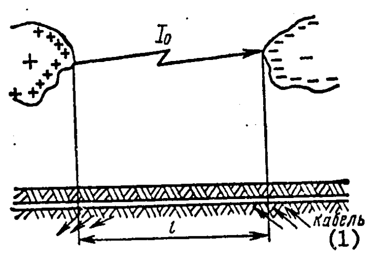



Figure 1.36. Currents of induced currents in sheathing and cable cores on occurrence of discharge currents between clouds

Key: 1. cable

1.4. Sources of Stray Currents in the Ground

The earth's crust is made up of several layers of rock (minerals), water and gases filling the cavities in the layers. The crust is a semiconductor material, the specific conductivity of which at various points of the earth is determined by the nature of the rock and the depth of its occurrence. The water of the seas and oceans is also a conducting medium. On occurrence of the electric potential differences between two points in the ground or water, electric currents occur which flow from the place with higher potential of positive sign in the directions to the places with potentials of negative sign. In the ground or in water such currents are called "stray currents." As a rule, these currents are distinguished with respect to magnitude at different points on the ground or water surface, in different layers and at different depth.

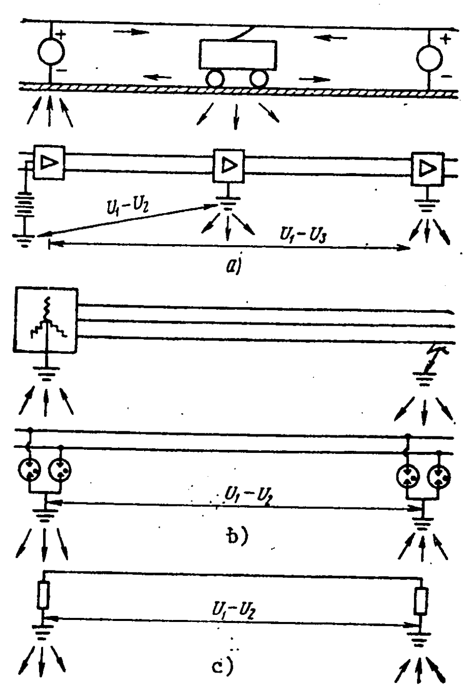



Figure 1.37. Examples of location of the grounds of single-wire circuits in the field of stray currents: a) in the remote feed circuit of the communication cable amplifiers by the "conductor-ground" systems; b) for dischargers installed on the overhead line; c) stray currents in the circuits of the single-conductor telegraph lines

The basic causes of the occurrence of the potential difference at different points of the earth's surface and the appearance of stray currents are the following: the electric power transmission lines using the "ground" as the return conductor; DC and AC electric railways (including streetcars) in which the rails and the ground are used as the return conductor [10]; magnetic storms or powerful disturbances of the magnetic field at the earth's surface. In addition, sea currents with defined velocity in the magnetic field of the earth constitute causes of occurrence of stray currents.

The communication circuits of the overhead and cable lines can be subjected to magnetic and galvanic effects of stray currents in the ground and water arising from the above-indicated causes. As is known, the magnetic effect of the stray currents on the communication circuits reaches significant values only if the stray current is variable, changing direction several times a second, that is, having low frequency (for example, 50 hertz). With slow variation of the directions of current in the ground or in the water and small values of the current, the magnetic effect is weak.

The galvanic effect of the stray currents on the communication circuits is characterized by the fact that the side currents get into the communication wires as a result of direct (galvanic) connection of the conductor to the ground or water. The single-wire communication circuit, for example, remote feed circuit of the amplifiers with respect to the "two wire-ground" system (Fig 1.37a), telegraph (Fig 1.37c), experiences the effect of stray currents directly through the resistances of the ground at the ends of the circuit; the two-wire communication circuit, through the ground resistances of the dischargers (Fig 1.37b) in the case where the potentials of the ground loops for the dischargers reach the ignition voltages of the dischargers.

# CHAPTER 2. EFFECT OF HIGH-VOLTAGE LINES ON SINGLE-WIRE CIRCUITS OF OVERHEAD LINES

## 2.1. General Principles of the Theory of the Effect of High-Voltage Lines

The theory of the effect of high-voltage lines on communication lines is based on the principles of general theory of the effect between electric circuits developed in the works of both Soviet [12-18] and foreign [19-24] scientists. Hereafter, on the basis of the specific nature of the physical and structural conditions the development of the theory of the effect of the high-current lines has proceeded somewhat specially from the development of the general theory of the effect between the communication circuits. In recent years an effort has been made to combine the two theories. In particular, in [10] by analogy with the general theory, the method of calculating the effect of high-voltage lines by the transient damping on the near and far ends of the single-wire and two-wire unshielded communication circuits is used. In [25-28], from the general point of view a study is made of the effect between different circuits and the shielding. Here the generalized theory of the effect is given in two aspects: separately for the energy transitions between the circuits and for the electromagnetic couplings considering the specific nature of the interfering circuit and the circuits subjected to the effect.

In the given book the effect of the high-voltage lines on communication lines is also considered from the point of view of the general theory of the effect between the communication circuits. In particular, the effect on the near and far ends of the circuit and through the bunched conductors, and so on is taken into account. Here the majority of formulas for calculating the effect of the high-voltage lines on the communication lines for known values of the electromagnetic couplings can be obtained using the methods of the general theory. When investigating the effect of the high-voltage lines on the overhead and cable communication lines it is necessary to consider the following peculiarities:

a) The significant difference in transmission levels over the interfering circuit (the first) and the circuits subjected to the effect (second);

b) Significant difference of the parameters of the indicated circuits;

c) The presence of segments of the second circuit located outside the convergence with the interfering line;

d) Nonuniformity of certain basic parameters, for example, nonidenticalness of the spacing between the lines for a complex convergence route;

e) Arbitrariness of the load resistance of the circuits;

f) Uniformity of the interfering field, that is, independence of the intensity of the interfering electromagnetic field with regard to the coordinate (for a frequency of 50 hertz and the first harmonics);

g) Absence of coupling caused by the electric field under the effect on the cable in the metal sheathing (or underground);

h) The presence of intermediate connections to the ground on the metal sheathings of the cables, and so on.

The coupling parameters between the circuits have specific nature. Under the effect of the coupling between the circuits usually induced voltages are determined only in the beginning or end of the circuit. Under the effect of the high-voltage lines it is necessary to know the voltages and currents also induced at intermediate points along the line. For example, when considering the effect on remote feed circuits (DP) it is important to know the magnitudes of the currents induced in the intermediate points of the remote feed circuit, for at these points the currents that arise will be the largest.

The problem with respect to calculating the effect of the high-voltage lines on the communication circuits consists in the fact that knowing the mutual arrangement of the lines, the specific resistance of the ground and the interfering currents and voltages, we must find the electric values (voltages, currents, longitudinal emf's) induced in the various circuits at any point along the communication line.

As is known from the general theory of the electromagnetic effect between communication circuits, electromagnetic coupling between them is characterized by two parameters: the conductivity of the electric coupling (the electric coupling) $Y_{12}$ which is numerically equal to the ratio of the current $\dot{I}_{2e}$ induced in the circuit subjected to the effect to the voltage $\dot{U}_1$ in the interfering circuit, and it has the dimensionality of Siemens/m (Siemens/km); the magnetic coupling resistance (magnetic coupling) $Z_{12}$ which is numerically equal to the ratio of the emf $\dot{E}_2$ induced in the circuit subjected to the effect to the current $\dot{I}_1$ in the interfering circuit, ans it has the dimensionality of ohms/meter (ohms/km). In the presence of an arbitrary number (in the general case n) of mutually influencing circuits, the solution to the problem reduces to the integration of the system of telegraphic equations:

$$\left.\begin{aligned}&\frac{d\dot{U}_i}{dx}+\sum_{k=1}^{n}\dot{I}_k Z_{ki}=0,\\&\frac{d\dot{I}_i}{dx}+\dot{U}_i Y_{ii}=\sum_{k=1}^{n}(\dot{U}_k-\dot{U}_i)Y_{ki},\end{aligned}\right\} \quad (2.1)$$

where $\dot{I}_i$, $\dot{U}_i$ are the current and voltage in the circuit i; $\dot{I}_k$, $\dot{U}_k$ are the same in the circuit k; $Z_{ki}$ is the magnetic coupling resistance (magnetic coupling) between the circuits k and i; $Y_{ki}$ is the electric coupling conductivity (the electric coupling) between the circuits k and i; $Y_{ii}$ is the natural complex conductivity per unit length of the circuit i; $Z_{ii}$ is the natural complex resistance per unit length of the circuit i.

In the most general form, that is, for an arbitrary number of circuits on the line, arbitrary parameters of each circuit, arbitrary load resistances of the circuits, arbitrary conditions of convergence with the interfering line, the solution of the indicated problem is connected with mathematical difficulties. However, this solution is also not required inasmuch as such general conditions are not encountered in practice. The number of circuits on the communication line is limited; the wires on the overhead line or the cable cores are identical or they are identical with respect to groups. Usually all of the communication line circuits in the convergence section have identical length of the section subjected to the effect or the latter are identical with respect to groups; the convergence of the communication line with a high voltage line is represented with a sufficient degree of accuracy in the form of a set of rectilinear segments, the mutual effect of some of the circuits can be neglected, and it is possible not to consider the return effect of the communication circuit on high-voltage lines.

The solution of the stated problem has been a subject of theoretical research for a long time. Historically there have been two methods of solving this problem: the method of integration of the transient currents in the convergence section and the method of differential equations, beginning with the known boundary conditions. According to the first method the emf induced in an elementary segment of the line is determined; by the emf and the input resistances of the line from the investigated elementary section the magnitude of the current induced in the circuit is found in each direction, and then by the propagation laws, the current and voltage are determined at any point of the line. Summing the currents which arrive from all elementary sections on the convergence length, it is possible to find the resultant current in the circuit. Usually this method is used to determine the currents and voltages at the end or at the beginning of the line. The second method is more universal. It permits comparatively simple determination of the induced voltages and currents at any point of the circuit subjected to the effect.

In this chapter the stated problem is solved by the second method. From the procedural point of view the authors considered it expedient to discuss the solution, successively going from the simple to the more complex cases.

## 2.2. Characteristics of the Interfering Circuits and Wire Communications Circuits

The interfering high-voltage circuits can be divided into two groups: symmetric and asymmetric. The symmetric circuits are considered to be the circuits, the wires of which have identical voltages and currents with respect to magnitude, 180° out of phase in two-wire (single-phase) circuits and 120° out of phase in three-phase circuits. The symmetric circuits have identical primary and secondary parameters and do not have residual voltages with respect to ground and equalizing currents in the ground (the so-called zero sequence). The circuits will be considered asymmetric, the wires of which have different primary parameters, and the voltages and currents in them are not equal to each other. In the general case the phase shift angles in the two-wire asymmetric circuit are not equal to 180°, and in the three-phase circuit they are not equal to 120°. The asymmetric circuits have a residual voltage in the current of the zero sequence.

If the residual voltage with respect to ground is equal to the operating voltage, and the current in the ground is equal to the operating current of the investigated circuit, the latter is entirely asymmetric. If the indicated equalities are not observed, the circuit is partially asymmetric. The following are among the asymmetric electric power transmission circuits:

The single-phase current circuit using the ground or a rail and the ground as the return conductor (Fig 2.1a).

The triple-phase current circuit, the transmission over which is realized by the "two-wire-ground" system (Fig 2.1b).

The triple-phase current circuit, the transmission over which is realized with a grounded neutral under emergency conditions (when one of the phases is shorted to the ground) (Fig 2.1c).

The high-voltage DC line operating through the ground (see Fig 1.17b).

All of the two-wire (single-phase) and three-wire (three-phase) electric power transmission circuits and also the two-wire communication circuits are in the group of partially asymmetric circuits, for as a rule the wires of these circuits do not have absolutely identical parameters.

All of the interfering lines carrying electric power except the coaxial lines create imbalanced electric and magnetic fields in the surrounding space, as a result of which they are sources of outside induced voltages and currents in the communication circuits in the zone of the fields. The circuits having a high degree of asymmetry obviously will have the greatest effect, for the intensity of the electromagnetic fields will be maximal (under other equal conditions) near the completely asymmetric circuit and minimal near the symmetric circuit.

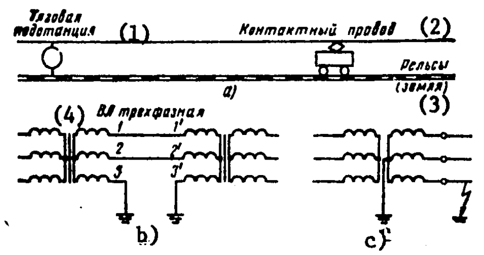



Figure 2.1. Entirely asymmetric circuits: a) single-phase current; b) triple-phase current operating by the "two-wire-ground" system; c) three-phase current with the grounded neutral in the case of short circuiting of the phase conductor to ground

Key:

1. Traction substation
2. Compact wire
3. Rails (ground)
4. Three-phase overhead line

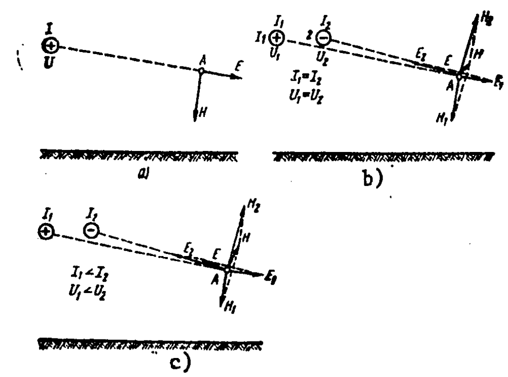



Figure 2.2. Vector diagrams of the electric field intensity (E) and magnetic field intensity (H) as a function of the completely asymmetric (a), symmetric two-wire (b) and asymmetric two-wire (c) lines

For comparison of the degree of effect of asymmetric and symmetric circuits on the communication wires Fig 2.2 shows the comparative values of the resultant vectors of the magnetic and electric field intensity at the points in space located at identical distances from the interfering lines transmitting identical powers of the energy. As is obvious, the intensity of the electromagnetic field will be maximal near the completely asymmetric circuit and minimal near the symmetric circuit. The greater the magnitude of the field intensity, the more intense the effect on the communication circuits will be. Thus, the adjacent, completely asymmetric circuits are the most dangerous for the communication circuits.

2.3. General Equations of Magnetic and Electric Effect Between Single-Wire Circuits

Let us consider the parallel convergence between two single-wire circuits (Fig 2.3). For generality of the solution let us assume that the convergence length $\ell_{II}$ is less than the length of the entire circuit subjected to the effect, that is, that in this circuit there are sections located outside convergence. The origin of the coordinates (x=0) will be taken at the beginning of convergence. Under real conditions, the interfering (let us denote it by the index 1) circuit can be the "contact wire of an electric railroad-ground" circuit or the "phase-ground" circuit of an electric power transmission line; the circuit subjected to the effect (we shall designate it by the index 2) can be the "wire-ground" circuit of the overhead line, the "sheathing-ground" circuit of a cable line, the "core-ground" circuit of a cable without metal sheathing. When the AC generator is connected to the circuit 1, voltages and currents will be induced in the second circuit as a result of the presence of electric and magnetic coupling. Let us introduce the following notation: $R_1$, $L_1$, $C_1$, $G_1$, $\gamma_1$, $z_{B1}$ are the active resistance (ohm/m), inductance (G/m), capacitance (farads/m), conductivity (siemens/m), the propagation coefficient (1/m) and the wave resistance (ohms) of the first (interfering) circuit;

$R_2$, $L_2$, $C_2$, $G_2$, $\gamma_2$, $z_{B2}$ are the same parameters of the circuit subjected to the effect;

$Z_{12}=\omega M_{12}=R_{12}+i\omega L_{12}$ is the magnetic coupling resistance (the magnetic coupling) between the single-wire circuits 1 and 2, ohms/meter;

$M_{12}$ is the mutual inductance between the single-wire circuits, g/m;

$R_{12}$ is the active component of the magnetic coupling, ohm/m;

$\omega L_{12}$ is the reactive component of the magnetic coupling, ohm/m;

$Y_{12}=G_{12}+i\omega C_{12}$ is the electric coupling conductivity (the electric coupling) between the single-wire circuits, siemens/m;

$G_{12}$ is the active component of the electric coupling, siemens/m;

$\omega C_{12}$ is the reactive component of the electric coupling (capacitive conductivity between circuits), siemens/m;

$\omega=2\pi f$ is the angular frequency of the interfering current;

$u_1$, $i_1$ are the instantaneous values of the voltage and current at an arbitrary point x of the interfering circuit, volts, amps;

$u_2$, $i_2$ are the instantaneous values of the voltage and current at an arbitrary point x of the circuit subjected to the effect, volts, amps;

$Z_{2H}$, $Z_{2k}$ are the load resistances at the beginning and end of the circuit 2, respectively;

$Z_{1k}$ is the mode resistance at the end of the interfering circuit;

$\ell_{II}$ is the convergence length;

$\ell_I$, $\ell_{III}$ are the lengths of the sections of the circuit 2 located outside convergence with the interfering circuits;

$\ell=\ell_I+\ell_{II}+\ell_{IIII}$ is the length of the circuit 2.

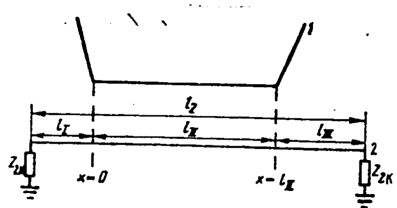



Figure 2.3. Parallel convergence between single-wire circuits: 1 -- interfering circuit; 2 -- circuit subjected to interference

When deriving the equations we assume that the parameters of all of the circuits do not depend on the coordinates of the investigated coil along the circuit and that it is possible to neglect the inverse effect from the circuit 2 on the interfering circuit 1. We shall consider that the circuits are loaded at the ends by arbitrary, different resistances. On an element of length dx of the interfering circuit we have

$$\left.\begin{aligned} u_1-(u_1+du_1)&=R_1i_1dx+L_1\frac{di_1}{dt}dx,\\ i_1-(i_1+di_1)&=G_1u_1dx+C_1\frac{du_1}{dt}dx.\end{aligned}\right\} \qquad (2.2)$$

Analogously, on an element of length of the circuits subjected to the effect in the convergence section

$$\left.\begin{aligned} u_2-(u_2+du_2)&=R_2i_2dx+L_2\frac{di_2}{dt}dx+R_{12}i_1dx+L_{12}\frac{di_1}{dt}dx,\\ i_2-(i_2+di_2)&=G_2u_2dx+C_2\frac{du_2}{dt}dx+G_{12}(u_2-u_1)dx+\\ &\quad+C_{12}\frac{d}{dt}(u_2-u_1)dx.\end{aligned}\right\} \qquad (2.3)$$

In the sections located outside convergence of the circuits subjected to the effect, the system (2.2) is valid. On going from the instantaneous values of the voltage and current to the existing complex values of $\dot{U}_1$, $\dot{I}_1$, $\dot{U}_2$, $\dot{I}_2$ and considering that $d\dot{U}/dt=i\omega\dot{U}$, $d\dot{I}/dt=i\omega\dot{I}$, $R_1+i\omega L_1=Z_1$, $R_2+i\omega L_2=Z_2$; $G_1+i\omega C_1=Y_1$, $G_2+i\omega C_2=Y_2$, we obtain the following systems of equations:

In the interfering circuit

$$\frac{d\dot{U}_1}{dx}+Z_1\dot{I}_1=0; \quad \frac{d\dot{I}_1}{dx}+Y_1\dot{U}_1=0; \tag{2.4}$$

in the convergence section of the circuit subjected to the effect, considering that $\dot{U}_2 \ll \dot{U}_1$,

$$\frac{d\dot{U}_{2\mathrm{II}}}{dx}+Z_2\dot{I}_{2\mathrm{II}}=-Z_{12}\dot{I}_1; \quad \frac{d\dot{I}_{2\mathrm{II}}}{dx}+Y_2\dot{U}_{2\mathrm{II}}=Y_{12}\dot{U}_1; \tag{2.5}$$

in the sections of the second circuit located outside convergence,

$$\frac{d\dot{U}_{2\mathrm{I}\,(\mathrm{III})}}{dx}+Z_2\dot{I}_{2\mathrm{I}\,(\mathrm{III})}=0; \quad \frac{d\dot{I}_{2\mathrm{I}\,(\mathrm{III})}}{dx}+Y_2\dot{U}_{2\mathrm{I}\,(\mathrm{III})}=0. \tag{2.6}$$

Let us find the expressions for the voltages and currents for the interfering circuit. The solution of system (2.4) is well known from the general theory of communications over wires, and it has the form:

$$\left.\begin{aligned} \dot{U}_1(x) &= \dot{U}_{10}\,\mathrm{ch}\,\gamma_1 x-\dot{I}_{10}z_{\text{в1}}\,\mathrm{sh}\,\gamma_1 x,\\ \dot{I}_1(x) &= \dot{I}_{10}\,\mathrm{ch}\,\gamma_1 x-\frac{1}{z_{\text{в1}}}\dot{U}_{10}\,\mathrm{sh}\,\gamma_1 x, \end{aligned}\right\} \tag{2.7}$$

where $\dot{U}_{10}$ and $\dot{I}_{10}$ are the voltage and current of the beginning of the circuit.

From these equations we obtain the expression for the input impedance of a single-wire circuit at any point of the line for an arbitrary load resistance $Z_{\text{load}}$ at the end:

$$\underset{(1)}{Z_{\text{вх}}}=z_{\text{в1}}\,\frac{Z^{(2)}_{\text{нагр}}/z_{\text{в1}}+\mathrm{th}\,\gamma_1 l_k}{1+(Z_{\text{нагр}}/z_{\text{в1}})\,\mathrm{th}\,\gamma_1 l_k}, \tag{2.8}$$

Key: 1. inp; 2. load

where $\ell_k$ is the distance from the point of the line at which the input impedance is defined to the end of the circuit.

Let us express the input resistance in terms of the reflection coefficient $p=(Z_{\text{load}}-z_{B1})/(Z_{\text{load}}+z_{B1})$:

$$Z_{\text{вх}}=z_{\text{в1}}\,\mathrm{cth}\,(\gamma_1 l_k-\ln\sqrt{p_1}). \tag{2.9}$$

On the basis of the expressions obtained we can write

$$\dot U_1(0)=\dot I_1(0)Z_{\text{вх}}=\dot I_1(0)z_{\text{в}1}\,\mathrm{cth}(\gamma_1 l_k-\ln\sqrt{p_1}). \tag{2.10}$$

Substituting (2.10) in (2.7), we find

$$\left.\begin{aligned}\dot U_1(x)&=\dot U_1(0)\frac{\mathrm{ch}[\gamma_1(l_{II}-x)-\ln\sqrt{p_1}]}{\mathrm{ch}(\gamma_1 l_{II}-\ln\sqrt{p_1})}=\dot I_1(0)z_{\text{в}1}\times\\&\times\frac{\mathrm{ch}[\gamma_1(l_{II}-x)-\ln\sqrt{p_1}]}{\mathrm{sh}(\gamma_1 l_{II}-\ln\sqrt{p_1})},\\\dot I_1(x)&=\dot I_1(0)\frac{\mathrm{sh}[\gamma_1(l_{II}-x)-\ln\sqrt{p_1}]}{\mathrm{sh}(\gamma_1 l_{II}-\ln\sqrt{p_1})}=\frac{\dot U_1(0)}{z_{\text{в}1}}\times\\&\times\frac{\mathrm{sh}[\gamma_1(l_{II}-x)-\ln\sqrt{p_1}]}{\mathrm{ch}(\gamma_1 l_{II}-\ln\sqrt{p_1})},\end{aligned}\right\} \tag{2.11}$$

where $p_1=(Z_{inp\ 1}-z_{B1})/(Z_{inp\ 1}+z_{B1})$; $Z_{inp\ 1}$ is the input resistance of the interfering circuit from the end of convergence in the direction of the end of the circuit defined by (2.9).

Let us find the expressions and currents in the second circuit. Let us represent the convergence section in the form of the equivalent circuit shown in Fig 2.4, where

$$Z_I=z_{\text{в}2}\,\mathrm{cth}(\gamma_2 l_I-\ln\sqrt{p_{2\text{н}}});$$
$$Z_{III}=z_{\text{в}2}\,\mathrm{cth}(\gamma_2 l_{III}-\ln\sqrt{p_{2\text{к}}});$$
$$p_{2\text{н}}=\frac{Z_{2\text{н}}-z_{\text{в}2}}{Z_{2\text{н}}+z_{\text{в}2}};\quad p_{2\text{к}}=\frac{Z_{2\text{к}}-z_{\text{в}2}}{Z_{2\text{к}}+z_{\text{в}2}}.$$

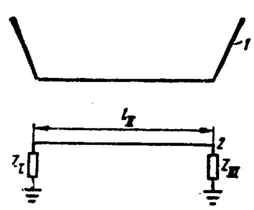



Figure 2.4. Equivalent convergence circuit

Substituting (2.10) and (2.11) in (2.5), we obtain

$$\left.\begin{aligned}\frac{d\dot U_{2II}}{dx}+Z_2\dot I_{2II}&=-\dot I_1(0)\,Z_{12}\frac{\operatorname{sh}[\gamma_1(l_{II}-x)-\ln\sqrt{p_1}]}{\operatorname{sh}(\gamma_1 l_{II}-\ln\sqrt{p_1})},\\ \frac{d\dot I_{2II}}{dx}+Y_2\dot U_{2II}&=\dot U_1(0)\,Y_{12}\frac{\operatorname{ch}[\gamma_1(l_{II}-x)-\ln\sqrt{p_1}]}{\operatorname{ch}(\gamma_1 l_{II}-\ln\sqrt{p_1})}.\end{aligned}\right\}\tag{2.12}$$

The general solution of the nonuniform system of equations (2.12) has the form

$$\left.\begin{aligned}\dot U_{2II}(x)&=C_1e^{\gamma_2x}+C_2e^{-\gamma_2x}+\frac{\dot U_1(0)\,N''_{12}\operatorname{ch}[\gamma_1(l_{II}-x)-\ln\sqrt{p_1}]}{(\gamma_2^2-\gamma_1^2)\operatorname{ch}(\gamma_1 l_{II}-\ln\sqrt{p_1})},\\ \dot I_{2II}(x)&=\frac{1}{z_{B2}}(-C_1e^{\gamma_2x}+C_2e^{-\gamma_2x})+\\ &+\frac{\dot U_1(0)\,N'_{12}\operatorname{sh}[\gamma_1(l_{II}-x)-\ln\sqrt{p_1}]}{z_{B2}(\gamma_2^2-\gamma_1^2)\operatorname{ch}(\gamma_1 l_{II}-\ln\sqrt{p_1})},\end{aligned}\right\}\tag{2.13}$$

where $C_1$ and $C_2$ are the integration constants;

$$N'_{12}=Y_{12}z_{B2}\gamma_1-\gamma_2Z_{12}/z_{B1};\quad N''_{12}=Y_{12}z_{B2}\gamma_1-\gamma_1Z_{12}/z_{B1}.\tag{2.14}$$

For determination of the integration constants let us compile the boundary conditions:

$$\left.\begin{aligned}\dot U_{2II}(0)&=-\dot I_{2II}(0)Z_I=-\dot I_{2II}(0)z_{B2}\operatorname{cth}(\gamma_2 l_I-\ln\sqrt{p_{2H}})\\ &\text{for } x=0,\\ \dot U_{2II}(l_{II})&=\dot I_{2II}(l_{II})Z_{III}=\dot I_{2II}(l_{II})z_{B2}\operatorname{cth}(\gamma_2 l_{III}-\ln\sqrt{p_{2K}})\\ &\text{for } x=l_{II}.\end{aligned}\right\}\tag{2.15}$$

From the first condition we have

$$\begin{aligned}C_1+C_2+\frac{\dot U_1(0)\,N''_{12}}{\gamma_2^2-\gamma_1^2}&=(C_1-C_2)\operatorname{cth}(\gamma_2 l_I-\ln\sqrt{p_{2H}})-\\ &-\frac{\dot U_1(0)\,N'_{12}\operatorname{th}(\gamma_1 l_{II}-\ln\sqrt{p_1})}{(\gamma_2^2-\gamma_1^2)\operatorname{th}(\gamma_2 l_I-\ln\sqrt{p_{2H}})};\end{aligned}\tag{2.16}$$

from the second condition

$$\left.\begin{aligned}C_1e^{\gamma_2l_{II}}+C_2e^{-\gamma_2l_{II}}+\frac{\dot U_1(0)\,N''_{12}\operatorname{ch}\ln\sqrt{p_1}}{(\gamma_2^2-\gamma_1^2)\operatorname{ch}(\gamma_1 l_{II}-\ln\sqrt{p_1})}&=\\ =\left(C_2e^{-\gamma_2l_{II}}-C_1e^{\gamma_2l_{II}}\right)\operatorname{cth}(\gamma_2 l_{III}-\ln\sqrt{p_{2K}})&-\\ -\frac{\dot U_1(0)\,N'_{12}\operatorname{sh}\ln\sqrt{p_1}\operatorname{cth}(\gamma_2 l_{III}-\ln\sqrt{p_{2K}})}{(\gamma_2^2-\gamma_1^2)\operatorname{ch}(\gamma_1 l_{II}-\ln\sqrt{p_1})}&.\end{aligned}\right\}\tag{2.17}$$

Solving this system with rezpect to $C_1$ and $C_2$ and substituting the expressions obtained in (2.13), we find

$$\begin{aligned}\dot{U}_{2\text{II}}(x) = &-\frac{\dot{U}_1(0)}{\gamma_2^2-\gamma_1^2}\Bigg[\frac{\operatorname{ch}(\gamma_2 l_\text{I}-\ln\sqrt{p_{2\text{н}}})\,[N'_{12}\operatorname{th}(\gamma_1 l_\text{II}-\ln\sqrt{p_1})+N''_{12}\operatorname{th}(\gamma_2 l_\text{I}-\ln\sqrt{p_{2\text{н}}})]}{\operatorname{sh}(\gamma_2 l-\ln\sqrt{p_{2\text{н}}p_{2\text{к}}})}\operatorname{ch}[\gamma_2(l_\text{II}+l_\text{III}-x)-\ln\sqrt{p_{2\text{к}}}]+\\ &+\frac{\operatorname{sh}(\gamma_2 l_\text{III}-\ln\sqrt{p_{2\text{к}}})\,[N'_{12}\operatorname{sh}\ln\sqrt{p_1}\operatorname{cth}(\gamma_2 l_\text{III}-\ln\sqrt{p_{2\text{к}}})+N''_{12}\operatorname{ch}\ln\sqrt{p_1}]}{\operatorname{sh}(\gamma_2 l-\ln\sqrt{p_{2\text{н}}p_{2\text{к}}})\operatorname{ch}(\gamma_1 l_\text{II}-\ln\sqrt{p_1})}\operatorname{ch}[\gamma_2(l_\text{I}+x)-\ln\sqrt{p_{2\text{н}}}]-\\ &-\frac{\operatorname{ch}[\gamma_1(l_\text{II}-x)-\ln\sqrt{p_1}]}{\operatorname{ch}(\gamma_1 l_\text{II}-\ln\sqrt{p_1})}N''_{12}\Bigg];\end{aligned} \tag{2.18}$$

$$\begin{aligned}\dot{I}_{2\text{II}}(x) = &\frac{\dot{U}_1(0)}{z_{\text{в}2}(\gamma_2^2-\gamma_1^2)}\Bigg[\frac{\operatorname{sh}[\gamma_1(l_\text{II}-x)-\ln\sqrt{p_1}]}{\operatorname{ch}(\gamma_1 l_\text{II}-\ln\sqrt{p_1})}N'_{12}+\\ &+\frac{\operatorname{sh}(\gamma_2 l_\text{III}-\ln\sqrt{p_{2\text{к}}})}{\operatorname{ch}(\gamma_1 l_\text{II}-\ln\sqrt{p_1})\operatorname{sh}(\gamma_2 l-\ln\sqrt{p_{2\text{н}}p_{2\text{к}}})}[N'_{12}\operatorname{sh}\ln\sqrt{p_1}\times\\ &\times\operatorname{cth}(\gamma_2 l_\text{III}-\ln\sqrt{p_{2\text{к}}})+N''_{12}\operatorname{ch}\ln\sqrt{p_1}]\operatorname{sh}[\gamma_2(l_\text{I}+x)-\ln\sqrt{p_{2\text{н}}}]-\\ &-\frac{\operatorname{ch}(\gamma_2 l_\text{I}-\ln\sqrt{p_1})}{\operatorname{sh}(\gamma_2 l-\ln\sqrt{p_{2\text{н}}p_{2\text{к}}})}[N'_{12}\operatorname{th}(\gamma_1 l_\text{II}-\ln\sqrt{p_1})+\\ &+N''_{12}\operatorname{th}(\gamma_2 l_\text{I}-\ln\sqrt{p_{2\text{н}}})]\operatorname{sh}[\gamma_2(l_\text{II}+l_\text{III}-x)-\ln\sqrt{p_{2\text{к}}}]\Bigg].\end{aligned} \tag{2.19}$$

The voltages and currents induced on the sections of the second circuit located outside convergence are determined from the equations of propagation and are equal to the following:

In the first section

$$\dot{U}_{2\text{I}}(x)=\dot{U}_{2\text{II}}(0)\frac{\operatorname{ch}[\gamma_2(l_\text{I}+x)-\ln\sqrt{p_{2\text{н}}}]}{\operatorname{ch}(\gamma_2 l_\text{I}-\ln\sqrt{p_{2\text{н}}})}, \tag{2.20}$$

$$\dot{I}_{2\text{I}}(x)=\dot{I}_{2\text{II}}(0)\frac{\operatorname{sh}[\gamma_2(l_\text{I}+x)-\ln\sqrt{p_{2\text{н}}}]}{\operatorname{sh}(\gamma_2 l_\text{I}-\ln\sqrt{p_{2\text{н}}})}; \tag{2.21}$$

In the third section

$$\dot{U}_{2\text{III}}(x)=\dot{U}_{2\text{II}}(l_\text{II})\frac{\operatorname{ch}[\gamma_2(l_\text{II}+l_\text{III}-x)-\ln\sqrt{p_{2\text{к}}}]}{\operatorname{ch}(\gamma_2 l_\text{III}-\ln\sqrt{p_{2\text{к}}})}, \tag{2.22}$$

$$\dot{I}_{2III}(x)=\dot{I}_{2II}(l_{II})\frac{\operatorname{sh}\left[\gamma_2(l_{II}+l_{III}-x)-\ln\sqrt{p_{2K}}\right]}{\operatorname{sh}(\gamma_2 l_{III}-\ln\sqrt{p_{2K}})}. \tag{2.23}$$

The formulas obtained take into account the direct magnetic and electric effects between the single-wire circuits for arbitrary load resistances of both circuits. From these formulas expressions can be found for all possible special cases investigated below. Here it is necessary to consider that the hyperbolic functions entering into the formulas of the argument containing the reflection coefficients can be represented in the form

$$\operatorname{sh}(a-\ln\sqrt{p})=\frac{e^a-p\,e^{-a}}{2\sqrt{p}};\quad \operatorname{sh}\ln\sqrt{p}=\frac{p-1}{2\sqrt{p}};$$

$$\operatorname{ch}(a-\ln\sqrt{p})=\frac{e^a+p\,e^{-a}}{2\sqrt{p}};\quad \operatorname{ch}\ln\sqrt{p}=\frac{p+1}{2\sqrt{p}}.$$

Since in the numerator and the denominator of the basic formulas of the effect obtained above the arguments containing p enter to identical degree, then $\sqrt{p}$ in the case of representation of hyperbolic functions is cancelled in the indicated way. For insulation of the circuit $p=1$ and $\operatorname{sh}(a-\ln\sqrt{p})=\operatorname{sh} a$; $\operatorname{ch}(a-\ln\sqrt{p})=\operatorname{ch} a$; $\operatorname{sh}\ln\sqrt{p}=0$; $\operatorname{ch}\ln\sqrt{p}=1$. For grounding of the circuit $p=-1$, $\operatorname{sh}(a-\ln\sqrt{p})=\operatorname{ch} a/\sqrt{-1}$; $\operatorname{ch}(a-\ln\sqrt{p})=\operatorname{sh} a/\sqrt{-1}$; $\operatorname{sh}\ln\sqrt{p}=-1/\sqrt{p}$; $\operatorname{ch}\ln\sqrt{p}=0$. On closure of the circuit to the wave impedance $p=0$. Considering that $\sqrt{p}$ in the denominator of the basic formulas of the effect is cancelled with $\sqrt{p}$ in the numerator, we find: $\operatorname{sh}(a-\ln\sqrt{p})=\operatorname{ch}(a-\ln\sqrt{p})=e^a/2$; $\operatorname{sh}\ln\sqrt{p}=-1/2$; $\operatorname{ch}\ln\sqrt{p}=1/2$.

## 2.4. Equations of Effect for Closure of the Interfering Circuit to Resistances Equal to the Phase Impedance

In a number of cases, especially for high frequencies, for example, under the effect of the high frequency channel through an overhead electric power transmission line, it is possible to consider that the interfering circuit is loaded on the convergence ends by wave impedances. Here $p_1=0$ and in the formulas (2.18) and (2.19) it is necessary to set: $\operatorname{sh}(\gamma_1 l_{II}-\ln\sqrt{p_1})=e^{\gamma_1 l_{II}}/2$; $\operatorname{ch}(\gamma_1 l_{II}-\ln\sqrt{p_1})=e^{\gamma_1 l_{II}}/2$; $\operatorname{th}(\gamma_1 l_{II}-\ln\sqrt{p_1})=1$; $\operatorname{sh}\ln\sqrt{p_1}=-1/2$; $\operatorname{ch}\ln\sqrt{p_1}=1/2$. Then for the induced voltages and currents in the convergence section we obtain

$$\begin{aligned}\dot{U}_{2II}(x)=-\frac{\dot{U}_1(0)}{\gamma_2^2-\gamma_1^2}\Bigg[&\frac{\operatorname{ch}(\gamma_2 l_I-\ln\sqrt{p_{2H}})\left[N'_{12}+N''_{12}\operatorname{th}(\gamma_2 l_I-\ln\sqrt{p_{2H}})\right]}{\operatorname{sh}(\gamma_2 l-\ln\sqrt{p_{2H}p_{2K}})}\times\\ &\times\operatorname{ch}\left[\gamma_2(l_{II}+l_{III}-x)-\ln\sqrt{p_{2K}}\right]-\\ &-\frac{\operatorname{sh}(\gamma_2 l_{III}-\ln\sqrt{p_{2K}})\left[N'_{12}\operatorname{cth}(\gamma_2 l_{III}-\ln\sqrt{p_{2K}})-N''_{12}\right]e^{-\gamma_1 l_{II}}}{\operatorname{sh}(\gamma_2 l-\ln\sqrt{p_{2H}p_{2K}})}\times\\ &\times\operatorname{ch}\left[\gamma_2(l_I+x)-\ln\sqrt{p_{2H}}\right]-N''_{12}e^{-\gamma_1 x}\Bigg],\end{aligned} \tag{2.24}$$

Key: 1. b; 2. e.

$$\dot{I}_{2II}(x) = \frac{\dot{U}_1(0)}{z_{2B}(\gamma_2^2-\gamma_1^2)}\left[e^{-\gamma_1 x}N'_{12} - \frac{\operatorname{sh}(\gamma_2 l_{III} - \ln\sqrt{p_{2K}}) \times \rightarrow}{\operatorname{sh}(\gamma_2 l - \sqrt{p_{2H}p_{2K}})}\right.$$
$$\frac{\rightarrow \times [N'_{12}\operatorname{cth}(\gamma_2 l_{III} - \ln\sqrt{p_{2K}}) - N''_{12}]\, e^{-\gamma_1 l_{II}}}{} \operatorname{sh}[\gamma_2(l_I + x) - \ln\sqrt{p_{2H}}] -$$
$$-\frac{\operatorname{ch}(\gamma_2 l_I - \ln\sqrt{p_{2H}})\,[N'_{12} + N''_{12}\operatorname{th}(\gamma_2 l_I - \ln\sqrt{p_{2H}})}{\operatorname{sh}(\gamma_2 l - \ln\sqrt{p_{2H}p_{2K}})} \times$$
$$\left.\times \operatorname{sh}[\gamma_2(l_{II} + l_{III} - x) - \ln\sqrt{p_{2K}}]\right]. \tag{2.25}$$

Let us investigate the expressions obtained. For simplification we shall consider the magnetic and electric effects separately.

Substituting the values of $N'_{12}$ and $N''_{12}$ from (2.14) in (2.24) and (2.25) and grouping the terms containing $Z_{12}$, we obtain the formulas defining the voltage and the current caused by the magnetic effect (the indexes "2" for the reflection coefficients are omitted):

$$\dot{U}_{2IIM}(x) = \frac{\dot{I}_1(0)\,Z_{12}}{\gamma_2^2-\gamma_1^2}\left\{\frac{1}{\operatorname{sh}(\gamma_2 l - \ln\sqrt{p_H p_K})}\left[\operatorname{ch}[\gamma_2(l_I + x) -\right.\right.$$
$$-\ln\sqrt{p_H}]\,[\gamma_1\operatorname{sh}(\gamma_2 l_{III} - \ln\sqrt{p_K}) - \gamma_2\operatorname{ch}(\gamma_2 l_{III} - \ln\sqrt{p_K})]\, e^{-\gamma_1 l_{II}} + \tag{2.26}$$
$$+\operatorname{ch}[\gamma_2(l_{II} + l_{III} - x) - \ln\sqrt{p_K}]\,[\gamma_2\operatorname{ch}(\gamma_2 l_I - \ln\sqrt{p_H}) +$$
$$\left.\left.+\gamma_1\operatorname{sh}(\gamma_2 l_I - \ln\sqrt{p_H})]\right] - \gamma_2 e^{-\gamma_1 x}\right\},$$

$$\dot{I}_{2IIM}(x) = -\frac{\dot{I}_1(0)\,Z_{12}}{z_{2B}(\gamma_2^2-\gamma_1^2)}\left\{\gamma_2 e^{-\gamma_1 x} - \frac{1}{\operatorname{sh}(\gamma_2 l - \ln\sqrt{p_H p_K})}\left[\operatorname{sh}[\gamma_2(l_I +\right.\right.$$
$$+x) - \ln\sqrt{p_H}]\, e^{-\gamma_1 l_{II}}[\gamma_2\operatorname{ch}(\gamma_2 l_{III} - \ln\sqrt{p_K}) - \gamma_1\operatorname{sh}(\gamma_2 l_{III} - \ln\sqrt{p_K})] +$$
$$+\operatorname{sh}[\gamma_2(l_{II} + l_{III} - x) - \ln\sqrt{p_K}]\,[\gamma_2\operatorname{ch}(\gamma_2 l_I - \ln\sqrt{p_H}) +$$
$$\left.\left.+\gamma_1\operatorname{sh}(\gamma_2 l_I - \ln\sqrt{p_H})]\right]\right\}. \tag{2.27}$$

Analogously, the voltage and current caused by the electric effect are:

$$\dot{U}_{2IIE}(x) = -\frac{\dot{U}_1(0)\,Y_{12}\,\gamma_2}{Y_2(\gamma_2^2-\gamma_1^2)}\left\{\frac{1}{\operatorname{sh}(\gamma_2 l - \ln\sqrt{p_H p_K})}\left[\operatorname{ch}[\gamma_2(l_{II} +\right.\right.$$
$$+l_{III} - x) - \ln\sqrt{p_K}]\,[\gamma_1\operatorname{ch}(\gamma_2 l_I - \ln\sqrt{p_H}) + \gamma_2\operatorname{sh}(\gamma_2 l_I - \ln\sqrt{p_H})] - \tag{2.28}$$
$$-\operatorname{ch}[\gamma_2(l_I + x) - \ln\sqrt{p_H}]\, e^{-\gamma_1 l_{II}}[\gamma_1\operatorname{ch}(\gamma_2 l_{III} - \ln\sqrt{p_K}) -$$
$$\left.\left.-\gamma_2\operatorname{sh}(\gamma_2 l_{III} - \ln\sqrt{p_K})]\right] - \gamma_2 e^{-\gamma_1 l_{II}}\right\},$$

$$\dot{I}_{2IIэ}(x)=\frac{\dot{U}_1(0)\,Y_{12}}{\gamma_2^2-\gamma_1^2}\Big\{\gamma_1\,e^{\gamma_1 x}-\frac{1}{\mathrm{sh}\,(\gamma_2 l-\ln\sqrt{p_н p_к})}[\mathrm{sh}\,[\gamma_2(l_I+x)-\ln\sqrt{p_н}]\times$$

$$\times\,e^{-\gamma_1 l_{II}}[\gamma_1\,\mathrm{ch}\,(\gamma_2 l_{III}-\ln\sqrt{p_к})-\gamma_2\,\mathrm{sh}\,(\gamma_2 l_{III}-\ln\sqrt{p_к})]+\mathrm{sh}\,[\gamma_2(l_{II}+$$

$$+l_{III}-x)-\ln\sqrt{p_к}]\,[\gamma_1\,\mathrm{ch}\,(\gamma_2 l_I-\ln\sqrt{p_н})+\gamma_2\,\mathrm{sh}\,(\gamma_2 l_I-\ln\sqrt{p_н})]]\Big\}. \qquad (2.29)$$

From these formulas it is obvious that $\dot{U}_{2IIм}(x)=-\frac{1}{Y_2}\frac{d\dot{I}_{2IIм}}{dx}$; $\dot{I}_{2IIэ}(x)=\frac{1}{Z_2}\frac{d\dot{U}_{2IIэ}}{dx}$.

The formulas (2.26)-(2.29) jointly with (2.20)-(2.23) permit determination of the magnetic and the electric effects of a single-layer high-voltage line loaded at the ends on wave impedances, on the parallel-laid overhead single-wire circuit with arbitrary load resistances.

Let us consider the case of loading of the circuits subjected to the effect by wave impedances. The calculation formulas can be obtained from (2.26)-(2.29) for $p_{beginning}=p_{end}=0$. Making this substitution, we obtain the following expressions:

$$\dot{U}_{2IIм}(x)=\frac{\dot{I}_1(0)\,Z_{12}}{2(\gamma_2^2-\gamma_1^2)}\,[(\gamma_1+\gamma_2)\,e^{-\gamma_2 x}+(\gamma_1-\gamma_2)\,e^{-(\gamma_1+\gamma_2)\,l_{II}+\gamma_2 x}-$$

$$-2\gamma_1\,e^{-\gamma_1 x}]; \qquad (2.30)$$

$$\dot{I}_{2IIм}(x)=-\frac{\dot{I}_1(0)\,Z_{12}}{2(\gamma_2^2-\gamma_1^2)\,z_{в2}}\left[2\gamma_2\,e^{-\gamma_1 x}+(\gamma_1-\gamma_2)\,e^{-(\gamma_1+\gamma_2)\,l_{II}+\gamma_2 x}-\right.$$

$$\left.-(\gamma_1+\gamma_2)\,e^{-\gamma_2 x}\right]; \qquad (2.31)$$

$$\dot{U}_{2IIэ}(x)=-\frac{\dot{U}_1(0)\,Y_{12}\,\gamma_2}{2(\gamma_2^2-\gamma_1^2)\,Y_2}\left[e^{-\gamma_2 x}(\gamma_1+\gamma_2)-e^{-(\gamma_1+\gamma_2)l_{II}+\gamma_2 x}\times\right.$$

$$\left.\times(\gamma_1-\gamma_2)-2\gamma_2\,e^{-\gamma_1 x}\right]; \qquad (2.32)$$

$$\dot{I}_{2IIэ}(x)=\frac{\dot{U}_1(0)\,Y_{12}}{2(\gamma_2^2-\gamma_1^2)}\left[2\gamma_1\,e^{-\gamma_1 x}-e^{-\gamma_2 x}(\gamma_1+\gamma_2)-e^{-(\gamma_1+\gamma_2)\,l_{II}+\gamma_2 x}\times\right.$$

$$\left.\times(\gamma_1-\gamma_2)\right]. \qquad (2.33)$$

Inasmuch as the circuit is loaded on its wave impedances, formulas (2.30)-(2.33) can be used also in calculating the induced voltages and currents in the sections located outside convergence, substituting the corresponding values of x in them. The formulas (2.30)-(2.33) permit determination of the induced voltages and currents at any point of the circuit. For example, for $\ell=\ell_{II}$ the voltages and currents induced at the beginning of the circuit (on the near end) are equal to

$$\dot{U}_{\underset{(1)}{2II\text{м.н}}}=\frac{\dot{I}_1(0)\,Z_{12}}{2(\gamma_1+\gamma_2)}\left(1-e^{-(\gamma_1+\gamma_2)\,l_{II}}\right);$$

$$\dot{I}_{\underset{(1)}{2II\text{м.н}}}=-\frac{\dot{I}_1(0)\,Z_{12}}{2(\gamma_1+\gamma_2)\,z_{в2}}\left(1-e^{-(\gamma_1+\gamma_2)\,l_{II}}\right); \tag{2.34}$$

$$\dot{U}_{2II\text{э.н}}=\frac{\dot{U}_1(0)\,Y_{12}z_{в2}}{2(\gamma_1+\gamma_2)}\left(1-e^{-(\gamma_1+\gamma_2)\,l_{II}}\right);$$

$$\dot{I}_{\underset{(1)}{2II\text{э.н}}}=-\frac{\dot{U}_1(0)\,Y_{12}}{2(\gamma_1+\gamma_2)}\left(1-e^{-(\gamma_1+\gamma_2)\,l_{II}}\right); \tag{2.35}$$

Key: 1. beginning

The voltages and currents at the end of the circuit (on the far end)

$$\dot{U}_{\underset{(1)}{2II\text{м.к}}}=-\frac{\dot{I}_1(0)\,Z_{12}\,e^{-\gamma_2 l_{II}}}{2(\gamma_1-\gamma_2)}\left(1-e^{-(\gamma_1-\gamma_2)\,l_{II}}\right);$$

$$\dot{I}_{2II\text{м.к}}=-\frac{\dot{I}_1(0)\,Z_{12}\,e^{-\gamma_2 l_{II}}}{2z_{в2}(\gamma_1-\gamma_2)}\left(1-e^{-(\gamma_1-\gamma_2)\,l_{II}}\right); \tag{2.36}$$

$$\dot{U}_{\overset{(1)}{\underset{(1)}{2II\text{э.к}}}}=\frac{\dot{U}_1(0)\,Y_{12}z_{в2}\,e^{-\gamma_2 l_{II}}}{2(\gamma_1-\gamma_2)}\left(1-e^{-(\gamma_1-\gamma_2)\,l_{II}}\right);$$

$$\dot{I}_{2II\text{э.к}}=\frac{\dot{U}_1(0)\,Y_{12}\,e^{-\gamma_2 l_{II}}}{2(\gamma_1-\gamma_2)}\left(1-e^{-(\gamma_1-\gamma_2)\,l_{II}}\right). \tag{2.37}$$

Key: 1. end

As is obvious from the expressions obtained, the voltages and currents induced at the beginning of the circuit are larger than the voltages and currents occurring at the end of the circuit. In addition, the voltages caused by the magnetic effect have different signs on different ends, and the currents of the magnetic effect are identical; for an electrical effect the currents have different signs. The total voltage at the beginning of the circuit caused by the magnetic and electric fields of the interfering line

$$\dot{U}_{2II\text{н}}=\dot{U}_1(0)\,z_{в2}\left(Y_{12}+\frac{Z_{12}}{z_{в1}z_{в2}}\right)\frac{1-e^{-(\gamma_1+\gamma_2)\,l_{II}}}{2(\gamma_1+\gamma_2)}, \tag{2.38}$$

or, denoting $\frac{Z_{12}}{z_{в1}z_{в2}}+Y_{12}=N_\text{н}$, we obtain

$$\dot{U}_{2II\text{н}}=\dot{U}_1(0)\,z_{в2}N_\text{н}\frac{1-e^{-(\gamma_1+\gamma_2)\,l_{II}}}{2(\gamma_1+\gamma_2)}. \tag{2.39}$$

Analogously for the voltage induced at the end of the circuit, denoting $Y_{12}-\frac{Z_{12}}{z_{B2}z_{B2}}=N_{K}$, we shall have

$$\dot{U}_{2IIK}=\dot{U}_1(0)z_{B2}N_K e^{-\gamma_2 l_{II}}\frac{1-e^{-(\gamma_1-\gamma_2)l_{II}}}{2(\gamma_1-\gamma_2)}. \quad (2.40)$$

Similarly, defining the currents induced at the beginning and end of the circuit, we obtain

$$\dot{I}_{2IIH}=-\dot{U}_1(0)N_H\frac{1-e^{-(\gamma_1+\gamma_2)l_{II}}}{2(\gamma_1+\gamma_2)}, \quad (2.41)$$

$$\dot{I}_{2IIK}=\dot{U}_1(0)N_K e^{-\gamma_2 l_{II}}\frac{1-e^{-(\gamma_1-\gamma_2)l_{II}}}{2(\gamma_1-\gamma_2)}. \quad (2.42)$$

As is known, in communication theory the effect of one circuit on the other is assumed to be expressed in terms of the transient damping representing the logarithm of the ratio of the power transmitted in the interfering circuit to the power occurring in the circuit subjected to the effect. The transient damping on the near end

$$A_H=\frac{1}{2}\ln\left|\frac{I_{1H}^2 z_{B1}}{I_{2IIH}^2 z_{B2}}\right|=\ln\left|\frac{I_{1H}}{I_{2IIH}}\sqrt{\frac{z_{B1}}{z_{B2}}}\right|=$$
$$=\ln\left|\frac{2(\gamma_1+\gamma_2)}{N_H\sqrt{z_{B1}z_{B2}}\,[1-e^{-(\gamma_1+\gamma_2)l_{II}}]}\right|$$

and on the far end

$$A_K=\frac{1}{2}\ln\left|\frac{I_{1K}^2 z_{B1}}{I_{2IIK}^2 z_{B2}}\right|=\ln\left|\frac{I_{1K}}{I_{2IIK}}\sqrt{\frac{z_{B1}}{z_{B2}}}\right|=$$
$$=\ln\left|\frac{2(\gamma_1-\gamma_2)}{N_K e^{-\gamma_2 l_{II}}\sqrt{z_{B1}z_{B2}}\,[1-e^{-(\gamma_1-\gamma_2)l_{II}}]}\right|.$$

For large $l_{II}$ $1-e^{-(\gamma_1+\gamma_2)}\approx 1$,

$$A_H=A'_H=\ln\left|\frac{2(\gamma_1+\gamma_2)}{N_H\sqrt{z_{B1}z_{B2}}}\right|$$

and, consequently $A_H=A'_H-\ln|1-e^{-(\gamma_1+\gamma_2)l_{II}}|$. The value of $A'_b$ in communications theory is called the transient damping between electrically long circuits.

As is known, the effect is also estimated by the magnitude of the protection. Under the effect of a high-voltage line on the communication circuit the protection of the latter on the near end is expressed by the formula $A_{p.b}=(P_2-\alpha_2\ell)-(P_1-A_b)$, and on the far end, by the formula $A_{p.e}=(P_2-\alpha_2\ell)-(P_1-A_e)$, where $P_1$ is the level of transmission of the interfering circuit (the high-voltage line); $P_2$ is the level of transmission in the communications circuit. In the given case it is necessary to consider that both ends of the communication circuit can be receiving.

Now let us consider the effect on an insulated single-wire circuit. Substituting in (2.26)-(2.29) $p_b = p_e = 1$, we obtain the following expressions:

$$\dot{U}_{2II\text{м}}(x) = \frac{\dot{I}_1(0) Z_{12}}{\gamma_2^2-\gamma_1^2}\left\{\frac{1}{\operatorname{sh}\gamma_2 l}\left[\operatorname{ch}\gamma_2(l_I+x)(\gamma_1\operatorname{sh}\gamma_2 l_{III} - \gamma_2\operatorname{ch}\gamma_2 l_{III})e^{-\gamma_1 l_{II}} + \operatorname{ch}\gamma_2(l_{II}+l_{III}-x)(\gamma_2\operatorname{ch}\gamma_2 l_I + \gamma_1\operatorname{sh}\gamma_2 l_I)\right] - \gamma_1 e^{-\gamma_1 x}\right\}; \quad (2.43)$$

$$\dot{I}_{2II\text{м}}(x) = -\frac{\dot{I}_1(0) Z_{12}}{z_{22}(\gamma_2^2-\gamma_1^2)}\left\{\gamma_2 e^{-\gamma_1 x} - \frac{1}{\operatorname{sh}\gamma_2 l}\left[\operatorname{sh}\gamma_2(l_I+x)\times(\gamma_2\operatorname{ch}\gamma_2 l_{III} - \gamma_1\operatorname{sh}\gamma_2 l_{III})e^{-\gamma_1 l_{II}} + \operatorname{sh}\gamma_2(l_{II}+l_{III}-x)\times(\gamma_2\operatorname{ch}\gamma_2 l_I + \gamma_1\operatorname{sh}\gamma_2 l_I)\right]\right\}; \quad (2.44)$$

$$\dot{U}_{2II\text{э}}(x) = -\frac{\dot{U}_1(0) Y_{12} z_{22}}{\gamma_2^2-\gamma_1^2}\left\{\frac{1}{\operatorname{sh}\gamma_2 l}\left[\operatorname{ch}\gamma_2(l_{II}+l_{III}-x)(\gamma_1\operatorname{ch}\gamma_2 l_I + \gamma_2\operatorname{sh}\gamma_2 l_I) - \operatorname{ch}\gamma_2(l_I+x)(\gamma_1\operatorname{ch}\gamma_2 l_{III} - \gamma_2\operatorname{sh}\gamma_2 l_{III})e^{-\gamma_1 l_{II}}\right] - \gamma_2 e^{-\gamma_1 x}\right\}; \quad (2.45)$$

$$\dot{I}_{2II\text{э}} = \frac{\dot{U}_1(0) Y_{12}}{\gamma_2^2-\gamma_1^2}\left\{\gamma_1 e^{-\gamma_1 x} - \frac{1}{\operatorname{sh}\gamma_2 l}\left[\operatorname{sh}\gamma_2(l_{II}+l_{III}-x)(\gamma_1\operatorname{ch}\gamma_2 l_I + \gamma_2\operatorname{sh}\gamma_2 l_I) + \operatorname{sh}\gamma_2(l_I+x)(\gamma_1\operatorname{ch}\gamma_2 l_{III} - \gamma_2\operatorname{sh}\gamma_2 l_{III})e^{-\gamma_1 l_{II}}\right]\right\}. \quad (2.46)$$

In the sections located outside convergence, the formulas (2.20)-(2.23) are effective for $p_b = p_e = 1$. Then

$$\dot{U}_{2I}(x) = \dot{U}_{2II}(0)\frac{\operatorname{ch}\gamma_2(l_I+x)}{\operatorname{ch}\gamma_2 l_I}, \quad (2.47)$$

$$\dot{I}_{2I}(x) = \dot{I}_{2II}(0)\frac{\operatorname{sh}\gamma_2(l_I+x)}{\operatorname{sh}\gamma_2 l_I}, \quad (2.48)$$

$$\dot{U}_{2III}(x) = \dot{U}_{2II}(l_{II})\frac{\operatorname{ch}\gamma_2(l_{II}+l_{III}-x)}{\operatorname{ch}\gamma_2 l_{III}}, \quad (2.49)$$

$$\dot{I}_{2III}(x)=\dot{I}_{2II}(l_{II})\frac{\operatorname{sh}\gamma_2(l_{II}+l_{III}-x)}{\operatorname{sh}\gamma_2 l_{III}}. \tag{2.50}$$

As is easy to see, for $x=-\ell_I$, that is, at the beginning of the circuit and for $x=\ell_{II}+\ell_{III}$, that is, at the end of the circuit, the induced currents are equal to zero, and the voltages

$$\dot{U}_{2I\text{н}}=\dot{U}_{2II}(0)/\operatorname{ch}\gamma_2 l_I;\quad \dot{U}_{2III\text{к}}=\dot{U}_{2II}(l_{II})/\operatorname{ch}\gamma_2 l_{III}.$$

Since the modulus of the hyperbolic cosine curve as a function of the complex argument can be both greater than and less than one, depending on the lengths of the sections $\ell_I$ and $\ell_{III}$ the voltages at the beginning or the end of the circuit can turn out to be both greater than and less than the values induced on the ends of the convergence section. In the absence of sections located outside convergence with the interfering line, that is, for $\ell_I=\ell_{III}=0$, the formulas (2.43)-(2.46) are simplified significantly:

$$\dot{U}_{2II\text{м}}(x)=\frac{\dot{I}_1(0)Z_{12}}{\gamma_2^2-\gamma_1^2}\left\{\frac{\gamma_2}{\operatorname{sh}\gamma_2 l_{II}}\left[\operatorname{ch}\gamma_2(l_{II}-x)-e^{-\gamma_1 l_{II}}\operatorname{ch}\gamma_2 x\right]-\right.$$
$$\left.-\gamma_1 e^{-\gamma_1 x}\right\}, \tag{2.51}$$

$$\dot{I}_{2II\text{м}}(x)=-\frac{\dot{I}_1(0)Z_{12}\gamma_2}{z_{B2}(\gamma_2^2-\gamma_1^2)}\left\{e^{-\gamma_1 x}-\frac{\gamma_1}{\operatorname{sh}\gamma_2 l_{II}}\left[\operatorname{ch}\gamma_2(l_{II}-x)-e^{-\gamma_1 l_{II}}\times\right.\right.$$
$$\left.\left.\times\operatorname{ch}\gamma_2 x\right]\right\}. \tag{2.52}$$

$$\dot{U}_{2II\text{э}}(x)=\frac{\dot{U}_1(0)Y_{12}z_{B2}}{\gamma_2^2-\gamma_1^2}\left\{\gamma_2 e^{-\gamma_1 x}-\frac{\gamma_1}{\operatorname{sh}\gamma_2 l_{II}}\left[\operatorname{ch}\gamma_2(l_{II}-x)-e^{-\gamma_1 l_{II}}\times\right.\right.$$
$$\left.\left.\times\operatorname{ch}\gamma_2 x\right]\right\}. \tag{2.53}$$

$$\dot{I}_{2II\text{э}}(x)=\frac{\dot{U}_1(0)Y_{12}\gamma_1}{\gamma_2^2-\gamma_1^2}\left\{e^{-\gamma_1 x}-\frac{1}{\operatorname{sh}\gamma_2 l_{II}}\left[\operatorname{sh}\gamma_2 x\, e^{-\gamma_1 l_{II}}+\operatorname{sh}\gamma_2(l_{II}-x)\right]\right\}; \tag{2.54}$$

At the beginning of the circuit (x=0)

$$\dot{U}_{\underset{(1)}{2II\text{м.н}}}=-\frac{\dot{I}_1(0)Z_{12}}{\gamma_2^2-\gamma_1^2}\left[\gamma_1-\frac{\gamma_2}{\operatorname{sh}\gamma_2 l_{II}}\left(\operatorname{ch}\gamma_2 l_{II}-e^{-\gamma_1 l_{II}}\right)\right]. \tag{2.55}$$

Key: 1. b

$$\dot{U}_{2\text{IIэ.н}} = \frac{\dot{U}_1(0)\,Y_{12}z_{в2}}{\gamma_2^2-\gamma_1^2}\left[\gamma_2 - \frac{\gamma_2}{\operatorname{sh}\gamma_2 l_{II}}\left(\operatorname{ch}\gamma_2 l_{II} - e^{-\gamma_1 l_{II}}\right)\right], \qquad (2.56)$$

at the end of the circuit ($x=\ell_{II}$)

$$\dot{U}_{2\text{IIм.к}} = \frac{\dot{I}_1(0)\,Z_{12}}{(\gamma_2^2-\gamma_1^2)\operatorname{sh}\gamma_2 l_{II}}\left[\gamma_2 - e^{-\gamma_1 l_{II}}\left(\gamma_2\operatorname{ch}\gamma_2 l_{II} + \gamma_1\operatorname{sh}\gamma_2 l_{II}\right)\right], \qquad (2.57)$$

$$\dot{U}_{2\text{IIэ.к}} = -\frac{\dot{U}_1(0)\,Y_{12}z_{в2}}{(\gamma_2^2-\gamma_1^2)\operatorname{sh}\gamma_2 l_{II}}\left[\gamma_1 - e^{-\gamma_1 l_{II}}\left(\gamma_2\operatorname{sh}\gamma_2 l_{II} + \gamma_1\operatorname{ch}\gamma_2 l_{II}\right)\right]; \qquad (2.58)$$

(1) (1)

Key: 1. e

In the middle of the convergence section ($x=\ell_{II}/2$)

$$\dot{U}_{2\text{IIм}}\left(\frac{l_{II}}{2}\right) = \frac{\dot{I}_1(0)\,Z_{12}\,e^{-\gamma_1\frac{l_{II}}{2}}\left(\gamma_2\operatorname{sh}\gamma_1\frac{l_{II}}{2} - \gamma_1\operatorname{sh}\gamma_2\frac{l_{II}}{2}\right)}{(\gamma_2^2-\gamma_1^2)\operatorname{sh}\gamma_2\frac{l_{II}}{2}}, \quad (2.59) \qquad (2.59)$$

$$\dot{U}_{2\text{IIэ}}\left(\frac{l_{II}}{2}\right) = \frac{\dot{U}_1(0)\,Y_{12}z_{в2}\,e^{-\gamma_1\frac{l_{II}}{2}}\left(\gamma_2\operatorname{sh}\gamma_2\frac{l_{II}}{2} - \gamma_1\operatorname{sh}\gamma_1\frac{l_{II}}{2}\right)}{(\gamma_2^2-\gamma_1^2)\operatorname{sh}\gamma_2\frac{l_{II}}{2}}. \quad (2.60) \qquad (2.60)$$

In the given case, just as when loading the circuit on wave impedances, the induced voltages at the beginning and end of the circuit are different. The voltage at the beginning is greater than the voltage at the end.

Let us find the formula for the case where the circuit subjected to the effect is closed at the ends to the ground; the resistances of the grounds are neglected. Substituting in (2.26)-(2.29) $p_b=p_e=-1$, we obtain

$$\dot{U}_{2\text{IIм}}(x) = \frac{\dot{I}_1(0)\,Z_{12}}{\gamma_2^2-\gamma_1^2}\left\{\frac{1}{\operatorname{sh}\gamma_2 l}\left[\operatorname{sh}\gamma_2(l_I+x)(\gamma_1\operatorname{ch}\gamma_2 l_{III} - \gamma_2\operatorname{sh}\gamma_2 l_{III})e^{-\gamma_1 l_{II}} + \operatorname{sh}\gamma_2(l_{II}+l_{III}-x)(\gamma_2\operatorname{sh}\gamma_2 l_I + \gamma_1\operatorname{ch}\gamma_2 l_I)\right] - \gamma_1 e^{-\gamma_1 x}\right\}, \qquad (2.61)$$

$$\dot{I}_{2\text{IIм}}(x) = -\frac{\dot{I}_1(0)\,Z_{12}}{z_{в2}(\gamma_2^2-\gamma_1^2)}\left\{\gamma_2 e^{-\gamma_1 x} - \frac{1}{\operatorname{sh}\gamma_2 l}\left[\operatorname{ch}\gamma_2(l_I+x)(\gamma_2\operatorname{sh}\gamma_2 l_{III} - \gamma_1\operatorname{ch}\gamma_2 l_{III})e^{-\gamma_1 l_{II}} + \operatorname{ch}\gamma_2(l_{II}+l_{III}-x)(\gamma_2\operatorname{sh}\gamma_2 l_I + \gamma_1\operatorname{ch}\gamma_2 l_I)\right]\right\}, \qquad (2.62)$$

$$\dot{U}_{2IIз}(x) = -\frac{\dot{U}_1(0)\,Y_{12}z_{в2}}{\gamma_2^2-\gamma_1^2}\left\{\frac{1}{\operatorname{sh}\gamma_2 l}\left[\operatorname{sh}\gamma_2(l_{II}+l_{III}-x)(\gamma_1\operatorname{sh}\gamma_2 l_I+\right.\right.$$
$$\left.+\gamma_2\operatorname{ch}\gamma_2 l_I)-\operatorname{sh}\gamma_2(l_I+x)(\gamma_1\operatorname{sh}\gamma_2 l_{III}-\gamma_2\operatorname{ch}\gamma_2 l_{III})e^{-\gamma_1 l_{II}}\right]-$$
$$\left.-\gamma_2 e^{-\gamma_1 x}\right\}, \tag{2.63}$$

$$\dot{I}_{2IIз}(x) = \frac{\dot{U}_1(0)\,Y_{12}}{\gamma_2^2-\gamma_1^2}\left\{\gamma_1 e^{-\gamma_1 x}-\frac{1}{\operatorname{sh}\gamma_2 l}\left[\operatorname{ch}\gamma_2(l_I+x)(\gamma_1\operatorname{sh}\gamma_2 l_{III}-\right.\right.$$
$$\left.\left.-\gamma_2\operatorname{ch}\gamma_2 l_{III})e^{-\gamma_1 l_{II}}+\operatorname{ch}\gamma_2(l_{II}+l_{III}-x)(\gamma_1\operatorname{sh}\gamma_2 l_I+\gamma_2\operatorname{ch}\gamma_2 l_I)\right]\right\}. \tag{2.64}$$

In the sections outside convergence, the formulas (2.20)-(2.23) are effective for $p_{2b}=p_{2e}=-1$:

$$\dot{U}_{2I}(x)=\dot{U}_{2II}(0)\frac{\operatorname{sh}\gamma_2(l_I+x)}{\operatorname{sh}\gamma_2 l_I}, \tag{2.65}$$

$$\dot{I}_{2I}(x)=\dot{I}_{2II}(0)\frac{\operatorname{ch}\gamma_2(l_I+x)}{\operatorname{ch}\gamma_2 l_I}, \tag{2.66}$$

$$\dot{U}_{2III}(x)=\dot{U}_{2III}(l_{II})\frac{\operatorname{sh}\gamma_2(l_{II}+l_{III}-x)}{\operatorname{sh}\gamma_2 l_{III}}, \tag{2.67}$$

$$\dot{I}_{2III}(x)=\dot{I}_{2II}(l_{II})\frac{\operatorname{ch}\gamma_2(l_{II}+l_{III}-x)}{\operatorname{ch}\gamma_2 l_{III}}. \tag{2.68}$$

At the end of the circuit the voltages are equal to zero, and the currents are defined by the formula

$$\dot{I}_{\underset{(1)}{2Iн}}=\frac{\dot{I}_{2II}(0)}{\operatorname{ch}\gamma_2 l_I}, \tag{2.69}$$

$$\dot{I}_{\underset{(2)}{2IIIк}}=\frac{\dot{I}_{2II}(l_{II})}{\operatorname{ch}\gamma_2 l_{III}}. \tag{2.70}$$

Key: 1. b; 2. e

As is obvious, the currents flowing at the begining or the end can be larger than the currents flowing in the circuit at the ends of the convergence section.

## 2.5. Equations of the Effect Between the Single-Wire Circuits for a Negligibly Small Variation of Currents and Voltages in the Interfering Circuit

General Formulas. The formulas of the effect obtained in the preceding item indicate the variation of the current and voltage along the interfering line. For low frequencies the propagation coefficient $\gamma_1$ of the actually existing single-wire circuits of overhead electric power transmission lines and electric railways is so small that it is possible to neglect the current and voltage variation along the line in the convergence section without great error. This means that the magnitude of the propagation coefficient $\gamma_1$ can be taken equal to zero. The formulas pertaining to this case can be found from the formulas of the preceding items under the condition that $\gamma_1=0$. The magnetic and electric fields along such a line also will not depend on the coordinate x. Then such fields will be called uniform. Setting $\gamma_1=0$ in (2.26)-(2.29) and considering that $\dot{I}_1(x)=\text{const}=\dot{I}_1$, and $U_1(x)=\text{const}=U_1$, we obtain

$$\dot{U}_{2\text{IIм}}(x)=\frac{\dot{I}_1 Z_{12}\left[\operatorname{ch}\left(\gamma_2 l_{\text{I}}-\ln\sqrt{p_{\text{н}}}\right)\operatorname{ch}\left[\gamma_2(l_{\text{II}}+l_{\text{III}}-x)-\ln\sqrt{p_{\text{к}}}\right]-\operatorname{ch}\left(\gamma_2 l_{\text{III}}-\ln\sqrt{p_{\text{к}}}\right)\operatorname{ch}\left[\gamma_2(l_{\text{I}}+x)-\ln\sqrt{p_{\text{н}}}\right]\right]}{\gamma_2\operatorname{sh}\left(\gamma_2 l-\ln\sqrt{p_{\text{н}}p_{\text{к}}}\right)}, \tag{2.71}$$

$$\dot{I}_{2\text{IIм}}(x)=-\frac{\dot{I}_1 Z_{12}}{Z_2}\left[1-\frac{\operatorname{ch}\left(\gamma_2 l_{\text{I}}-\ln\sqrt{p_{\text{н}}}\right)\operatorname{sh}\left[\gamma_2(l_{\text{II}}+l_{\text{III}}-x)-\ln\sqrt{p_{\text{к}}}\right]+\operatorname{ch}\left(\gamma_2 l_{\text{III}}-\ln\sqrt{p_{\text{к}}}\right)\operatorname{sh}\left[\gamma_2(l_{\text{I}}+x)-\ln\sqrt{p_{\text{н}}}\right]}{\operatorname{sh}\left(\gamma_2 l-\ln\sqrt{p_{\text{н}}p_{\text{к}}}\right)}\right], \tag{2.72}$$

$$\dot{U}_{2\text{IIэ}}(x)=\frac{\dot{U}_1 Y_{12}}{Y_2}\left[1-\frac{\operatorname{sh}\left(\gamma_2 l_{\text{I}}-\ln\sqrt{p_{\text{н}}}\right)\operatorname{ch}\left[\gamma_2(l_{\text{II}}+l_{\text{III}}-x)-\ln\sqrt{p_{\text{к}}}\right]+\operatorname{sh}\left(\gamma_2 l_{\text{III}}-\ln\sqrt{p_{\text{к}}}\right)\operatorname{ch}\left[\gamma_2(l_{\text{I}}+x)-\ln\sqrt{p_{\text{н}}}\right]}{\operatorname{sh}\left(\gamma_2 l-\ln\sqrt{p_{\text{н}}p_{\text{к}}}\right)}\right], \tag{2.73}$$

$$\dot{I}_{2\text{IIэ}}(x)=-\frac{\dot{U}_1 Y_{12}\left[\operatorname{sh}\left(\gamma_2 l_{\text{I}}-\ln\sqrt{p_{\text{н}}}\right)\operatorname{sh}\left[\gamma_2(l_{\text{II}}+l_{\text{III}}-x)-\ln\sqrt{p_{\text{к}}}\right]-\operatorname{sh}\left(\gamma_2 l_{\text{III}}-\ln\sqrt{p_{\text{к}}}\right)\operatorname{sh}\left(\gamma_2(l+x)-\ln\sqrt{p_{\text{н}}}\right]\right]}{\gamma_2\operatorname{sh}\left(\gamma_2 l-\ln\sqrt{p_{\text{н}}p_{\text{к}}}\right)}. \tag{2.74}$$

In a number of cases, the values of the average current and the average voltage induced in the convergence section for magnetic and electric effects are of interest. The average current is

$$\dot{I}_{\underset{(1)}{2\text{IIм.ср}}} = \frac{1}{l_{II}} \int_0^{l_{II}} \dot{I}_{2\text{IIм}}(x)\,dx = -\frac{\dot{I}_1 Z_{12}}{Z_2} \times \left[ 1 - \frac{2\,\text{sh}\,\gamma_2 \frac{l_{II}}{2} \left[ \text{ch}\left(\gamma_2 l_I - \ln\sqrt{p_\text{н}}\right) \text{sh}\left[\gamma_2\left(\frac{l_{II}}{2} + l_{III}\right) - \ln\sqrt{p_\text{к}}\right] + \text{ch}\left(\gamma_2 l_{III} - \ln\sqrt{p_\text{к}}\right) \text{sh}\left[\gamma_2\left(\frac{l_{II}}{2} + l_I\right) - \ln\sqrt{p_\text{н}}\right]\right]}{\gamma_2 l_{II}\, \text{sh}\left(\gamma_2 l - \ln\sqrt{p_\text{н} p_\text{к}}\right)} \right]. \quad (2.75)$$

Key: 1. ave

The average voltage

$$\dot{U}_{2\text{IIэ.ср}} = \frac{1}{l_{II}} \int_0^{l_{II}} \dot{U}_{2\text{IIэ}}(x)\,dx = \frac{\dot{U}_1 Y_{12}}{Y_2} \left[ 1 - \frac{2\,\text{sh}\,\gamma_2 \frac{l_{II}}{2} \left[ \text{sh}\left(\gamma_2 l_I - \ln\sqrt{p_\text{н}}\right) \times \text{ch}\left[\gamma_2\left(l_{II}/2 + l_{III}\right) - \ln\sqrt{p_\text{к}}\right] + \text{sh}\left(\gamma_2 l_{III} - \ln\sqrt{p_\text{к}}\right) \text{ch}\left[\gamma_2\left(l_{II}/2 + l_I\right) - \ln\sqrt{p_\text{н}}\right]\right]}{\gamma_2 l_{II}\, \text{sh}\left(\gamma_2 l - \ln\sqrt{p_\text{н} p_\text{к}}\right)} \right]. \quad (2.76)$$

Formulas (2.71)-(2.76) jointly with (2.20)-(2.23) permit determination of the effect of the single-wire, high-voltage line on a parallel laid overhead single-wire circuit with arbitrary load resistances at the ends. From these formulas the expressions can be obtained which consider not the general loading conditions, but different special cases (the circuit insulation, the load on the wave impedances, and so on) by direct substitution of the corresponding reflection coefficients in them.

Variation of the Induced Voltages and Currents Along the Circuit Insulated or Grounded at the Ends. Let us consider the variation of the induced voltages and currents on the circuit subjected to the effect for various load resistances. Let us assume that the circuit is subjected to the effect over the entire extent and the sections outside convergence are absent, $\ell_I = \ell_{III} = 0$. If $\ell_I \neq \ell_{III} \neq 0$, then the voltage and current distribution in the convergence section is equivalent to the case where they are absent, but under the condition that the load resistances are equal to the input resistances of the circuit from the beginning and end of convergence in the direction of the beginning and end of the line.

Let us find the voltage distribution in the circuit insulated at the ends for the magnetic effect:

At the beginning of the circuit, that is, for x=0,

$$\dot{U}_{2IIм}(0)=\dot{U}_{2IIм.н}=\frac{\dot{I}_1 Z_{12}}{\gamma_2 \operatorname{ch}\gamma_2 \frac{l_{II}}{2}}\operatorname{sh}\gamma_2\frac{l_{II}}{2}=\frac{\dot{I}_1 Z_{12}}{\gamma_2}\operatorname{th}\gamma_2\frac{l_{II}}{2},$$

in the middle of the circuit, $x=\ell_{II}/2$,

$$\dot{U}_{2IIм}=0,$$

at the end of the circuit, that is, for $x=\ell_{II}$,

$$\dot{U}_{2IIм}(l_{II})=\dot{U}_{2IIм.к}=-\frac{\dot{I}_1 Z_{12}}{\gamma_2}\operatorname{th}\gamma_2\frac{l_{II}}{2}.$$

These equations indicate that in the single-wire circuit insulated at the ends and located over the entire extent in a uniform external magnetic field, the induced voltage with respect to the ground is distributed in such a way that in the middle it is equal to zero, and on the ends of the circuit the voltages are identical with respect to magnitude, but at each given point in time are opposite with respect to sign. It is interesting to remember that for nonuniform interfering field the induced voltage on the near end is greater than the voltage on the far end [see (2.55) and (2.57)].

With an increase in the argument the hyperbolic tangent, varying, approaches one in wave fashion (only if the angle of the argument does not approach 90°); therefore, as $\gamma_2(\ell_{II}/2)$ increases, that is, with an increase in length or the propagation coefficient of the circuit subjected to the effect, the induced voltage will decrease. For small values of the argument where $\gamma_2(\ell_{II}/2)<<1$, that is, on short lines or for small propagation coefficients, th $\gamma_2(\ell_{II}/2)\approx\gamma_2(\ell_{II}/2)$. In this case $\dot{U}_{2IIm}(0)=\frac{1}{2}\dot{I}_1 Z_{12} l_{II}$, $\dot{U}_{2IIм}(l_{II})=-\frac{1}{2}\dot{I}_1 Z_{12} l_{II}$.

The potential difference induced between the ends of the single-wire circuit is $\dot{U}_{2IIm}(\ell_{II})-\dot{U}_{2IIm}(0)=-\dot{I}_1 Z_{12}\ell_{II}$. The last expression, as follows from the theory of mutually connected electric circuits, is the induced emf

$$\dot{E}=-\dot{I}_1 Z_{12} l_{II}: \tag{2.77}$$

This emf is called the longitudinal electromotive force, for it operates along the wire. From (2.77) it follows that the longitudinal emf induced in the wire in a unit length is defined by the expression $\dot{\mathcal{E}}=-\dot{I}_1 Z_{12}$ volts/m. The voltage with respect to ground at the end of the circuit is $\dot{E}/2$. The curves for the variation of the induced voltage along the wire insulated at the ends are presented in Fig 2.5 as a function of the values of the parameter $\gamma_2\ell_{II}$ for the magnetic effect.

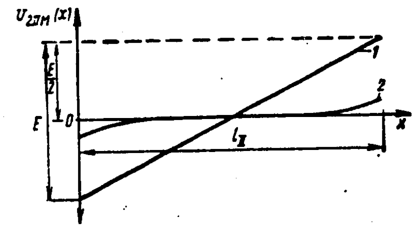



Figure 2.5. Curves for the variation of the induced voltage along an insulated communications wire under magnetic effect: 1 -- for small values of $\gamma_2 \ell_{II}$ ($\gamma_2 \ell_{II} \ll 1$); 2 -- for large values of $\gamma_2 \ell_{II}$ ($\gamma_2 \ell_{II} \gg 1$)

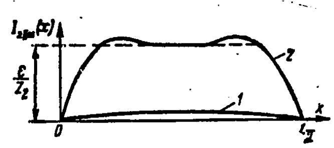



Figure 2.6. Variation of the induced current along the insulated wire under magnetic effect: 1 -- for small values of $\gamma_2 \ell_{II}$ ($\gamma_2 \ell_{II} \ll 1$); 2 -- for large values of $\gamma_2 \ell_{II}$ ($\gamma_2 \ell_{II} \gg 1$)

Let us consider the variation of the induced current under the magnetic effect. At the beginning and end of the insulated circuit, as follows from the general formula (2.72), the current is equal to zero. In the middle of the circuit for $x = \ell_{II}/2$

$$\dot I_{2IIм}(l_{II}/2) = -\frac{\dot I_1 Z_{12}}{2}\left[1 - \frac{1}{\mathrm{ch}\,\gamma_2 \frac{l_{II}}{2}}\right] = -\frac{\dot I_1 Z_{12}\,\mathrm{th}\,\gamma_2 \frac{l_{II}}{2}\,\mathrm{th}\,\gamma_2 \frac{l_{II}}{4}}{Z_2} \qquad (2.78)$$

For large values of the argument

$$\mathrm{ch}\,\gamma_2 \frac{l_{II}}{2} = \frac{e^{\gamma_2 l_{II}/2}}{2} \text{ and } \dot I_{2IIм}\left(\frac{l_{II}}{2}\right) = -\frac{\dot I_1 Z_{12}}{Z_2}\left(1 - 2e^{-\gamma_2 l_{II}/2}\right).$$

For small values of the argument ($\gamma_2 \ell_{II}/2 \ll 1$)

$$\dot I_{2IIм}(l_{II}/2) = -\frac{\dot I_1 Z_{12}\gamma_2^2 \frac{l_{II}}{2}\frac{l_{II}}{4}}{Z_2} = -\dot I_1 Z_{12} Y_{12}\frac{l_{II}^2}{8} = \frac{1}{8}\dot E Y_2 l_{II}. \qquad (2.79)$$

As is obvious from the expressions obtained, the current induced in the middle of the circuit for large values of the propagation coefficient of the single-wire circuit or long length of the circuit approaches the value of $\dot{\varepsilon}/Z_2$. For small values of $\gamma_2 \ell_{II}$ the magnitude of the induced current is appreciably decreased, and it is determined by the conduction current. The curves for the current variation are presented in Fig 2.6.

Let us find the voltage and current distribution in the insulated circuit under the electrical effect. From formula (2.73) it follows that the induced voltage for the electric effect and for $\ell_I = \ell_{III} = 0$ does not depend on the coordinate x and is equal to the interfering voltage multiplied by the ratio of the mutual conductivity to the natural conductivity of the circuit:

$$\dot{U}_{2II\text{э}} = \dot{U}_1 \frac{Y_{12}}{Y_2}. \tag{2.80}$$

Since only overhead lines are subjected to the electrical effect, for which $G_{12} \ll \omega C_{12}$ and $G_2 \ll \omega C_2$, then

$$\dot{U}_{2II\text{э}} = \dot{U}_1 \frac{C_{12}}{C_2}. \tag{2.81}$$

The current induced in the wire under the electrical effect, as follows from the formula (2.74), for $\ell_I = \ell_{III} = 0$ over the entire extent of the circuit is equal to zero.

Let us find the voltage and current distribution in the single-wire circuit grounded at the ends under the magnetic effect (let us neglect the ground resistances). As follows from the formula (2.71), the induced voltage of the entire length of the circuit is equal to zero. For nonuniform interfering field the equality to zero is not observed and the induced voltage is equal to zero only at the ends of the circuit. The current induced in the circuit when it is grounded at the ends is

$$\dot{I}_{2II\text{м}}(x) = -\frac{\dot{I}_1 Z_{12}}{Z_2} = \frac{\dot{\varepsilon}}{Z_2}. \tag{2.82}$$

Multiplying the numerator and the denominator by $\ell_{II}$, we find that

$$\dot{I}_{2II\text{м}} = \frac{\dot{E}}{Z_2 l_{II}}. \tag{2.83}$$

Formula (2.83) shows that in the single-wire circuit ideally (with zero resistances) closed at the end to ground located in a uniform external magnetic field, the induced current is equal to the induced emf divided by the total resistance of the circuit, which is identical along the entire length both with respect to amplitude and with respect to phase.

Let us consider the electrical effect on the single-wire circuit grounded at the convergence ends. The induced voltage in the grounded circuit under the electrical effect varies along the line just as the induced current in the insulated circuit under magnetic effect (see Fig 2.6); it is equal to zero at the ends of the circuit and it is maximal at the center. For $x = \ell_{II}/2$

$$\dot{U}_{2II\text{э}} = \frac{\dot{U}_1 Y_{12}}{Y_2}\left(1 - \frac{1}{\operatorname{ch}\gamma_2 \frac{l_{II}}{2}}\right) = \frac{\dot{U}_1 Y_{12} \operatorname{th}\gamma_2 \frac{l_{II}}{2} \operatorname{th}\gamma_2 \frac{l_{II}}{4}}{Y_2}. \tag{2.84}$$

Let us find the variation of the induced current under the electrical effect. The current induced in the grounded single-wire circuit under the electrical effect has the same nature of variation along the line as the voltage induced in the insulated circuit under the magnetic effect (see Fig 2.5). At the beginning and end of the circuit the currents are identical with respect to absolute value, but they are different with respect to direction. In the middle of the circuit the current is equal to zero. The magnitude of the current induced at the beginning of the circuit is

$$\underset{(1)}{\dot{I}_{2II\text{э.н}}} = -\frac{\dot{U}_1 Y_{12}}{\gamma_2} \operatorname{th}\gamma_2 \frac{l_{II}}{2}, \tag{2.85}$$

Key: 1. b
and at the end of the circuit

$$\underset{(1)}{\dot{I}_{2II\text{э.к}}} = \frac{\dot{U}_1 Y_{12}}{\gamma_2} \operatorname{th}\gamma_2 \frac{l_{II}}{2}. \tag{2.86}$$

Key: 1. e

For small values of $\gamma_2(\ell_{II}/2)$, considering that th $\gamma_2(\ell_{II}/2) \simeq \gamma_2(\ell_{II}/2)$, we obtain

$$\underset{(1)}{\dot{I}_{2II\text{э.н}}} = -\dot{U}_1 Y_{12} \frac{l_{II}}{2}; \quad \underset{(2)}{\dot{I}_{2II\text{э.к}}} = \dot{U}_1 Y_{12} \frac{l_{II}}{2}. \tag{2.87}$$

Key: 1. b; 2. e

Since the value of $\dot{U}_1 Y_{12} \ell_{II}$ is in accordance with the definition presented above for an electric coupling between circuits the current transmitted in the convergence section from the interfering circuit to the circuit subjected to effect, the equations (2.87) indicate that half of the current going over to the circuit subjected to the effect from the interfering line flows through each ground.

Thus, the total current of the electric effect transmitted from the interfering line to ground through the terminal ground is equal to the total conduction current of the electric coupling.

Variation of the Induced Voltages and Currents Along a Single-Wire Circuit Insulated at the Beginning and Close to Ground at the End. From the general formula (2.71), substituting $\ell_I = \ell_{III} = 0$ and the corresponding values of x, we find

$$\dot{U}_{2II\text{M.}\underset{(1)}{\text{н}}} = \frac{\dot{I}_1 Z_{12}}{\gamma_2} \operatorname{th} \gamma_2 l_{II} \quad \text{for } x = 0, \tag{2.88}$$

Key: 1. b

$$\dot{U}_{2II\text{M}}(l_{II}/2) = \frac{\dot{I}_1 Z_{12}}{\gamma_2} \frac{\operatorname{sh} \gamma_2 \frac{l_{II}}{2}}{\operatorname{ch} \gamma_2 l_{II}} \text{ for } x = l_{II}/2, \tag{2.89}$$

$$\dot{U}_{2II\text{M.}\underset{(1)}{\text{к}}} = 0 \quad \text{for } x = l_{II}.$$

Key: 1. e

For large values of $\gamma_2\ell_{II}$, considering that th $\gamma_2\ell_{II} \approx 1$, $\dot{U}_{2II\text{M.b}} = \dot{I}_1 Z_{12}/\gamma_2$,

$$\dot{U}_{2II\text{M}}(l_{II}/2) = \frac{\dot{I}_1 Z_{12}}{\gamma_2} e^{-\gamma_2 \frac{l_{II}}{2}}.$$

for small values of $\gamma_2\ell_{II}$

$$\dot{U}_{2II\text{M.}\underset{(1)}{\text{н}}} = -\dot{e}\, l_{II} = -\dot{E} = i\omega M_{12} \dot{I}_1 l_{II}, \tag{2.90}$$

$$\dot{U}_{2II\text{M}}(l_{II}/2) \approx -\dot{E}/2.$$

Key: 1. b

Thus, for short length of the circuit or a small propagation coefficient the voltage on the insulated end with grounding of the opposite end is equal to the total induced longitudinal emf. The voltage in the circuit increases on going away from the end proportionally to the length. The curves for the variation of the induced voltage in the investigated case are presented in Fig 2.7.

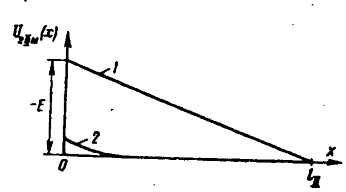



Figure 2.7. Variation of the induced voltage along the wire close to the ground at the end and insulated at the beginning under magnetic effect: 1 -- for small values of $\gamma_2\ell_{II}$($\gamma_2\ell_{II}$<<1); 2 — for large values of $\gamma_2\ell_{II}$($\gamma_2\ell_{II}$>>1)

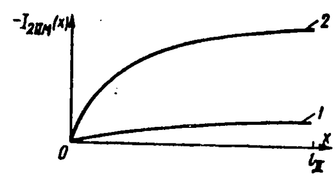



Figure 2.8. Curves for the variation of the induced current along the wire insulated at the beginning and close to ground at the end under magnetic effect: 1 -- for small values of $\gamma_2 \ell_{II} (\gamma_2 \ell_{II} << 1)$; 2 -- for large values of $\gamma_2 \ell_{II} (\gamma_2 \ell_{II} >> 1)$

Let us find the induced currents at different points of the circuit. The current at the beginning must be equal to zero. At the end of the circuit $(x = \ell_{II})$

$$\dot{I}_{2II\text{м.к}} = -\frac{\dot{I}_1 Z_{12}}{Z_2}\left(1 - \frac{1}{\operatorname{ch}\gamma_2 l_{II}}\right) = -\frac{\dot{I}_1 Z_{12} \operatorname{th}\gamma_2 l_{II} \operatorname{th}\gamma_2 l_{II}/2}{Z_2}. \quad (2.91)$$

in the middle of the circuit

$$\dot{I}_{2II\text{м}}\left(\frac{l_{II}}{2}\right) = -\frac{\dot{I}_1 Z_{12}}{Z_2}\left(1 - \frac{\operatorname{ch}\gamma_2 l_{II}/2}{\operatorname{ch}\gamma_2 l_{II}}\right) = -\frac{2\dot{I}_1 Z_{12} \operatorname{sh}\frac{3}{4}\gamma_2 l_{II} \operatorname{sh}\gamma_2 l_{II}/4}{Z_2 \operatorname{ch}\gamma_2 l_{II}}. \quad (2.92)$$

For large values of $\gamma_2 \ell_{II}/2$

$$\dot{I}_{2II\text{м}}(l_{II}) = -\frac{\dot{I}_1 Z_{12}}{Z_2}\left(1 - 2e^{-\gamma_2 l_{II}}\right),$$

that is, the magnitude of the current approaches the value determined on grounding the circuit on both sides. For small values of $\gamma_2 \ell_{II}$

$$\dot{I}_{2II\text{м}}(l_{II}) = \frac{1}{2}\dot{E}Y_2 l_{II}. \quad (2.93)$$

In other words, the current in the last case is defined by the capacitive leakage of the wire to ground. The curves for the variation of the current along the wire insulated at the beginning and close to ground at the end is shown in Fig 2.8.

Let us consider the electrical effect. The voltage with respect to ground for insulation of the wire at the beginning and grounding of it at the end will be defined by the formulas:

At the beginning of the circuit

$$\dot{U}_{2IIэ.н} = \frac{\dot{U}_1 Y_{12}}{Y_2}\left(1 - \frac{1}{\mathrm{ch}\,\gamma_2 l_{II}}\right), \tag{2.94}$$

At the end of the circuit the voltage is equal to zero. For small values of $\gamma_2 \ell_{II}$

$$\dot{U}_{2IIэ.н} = \frac{\dot{U}_1 Y_{12}}{Y_2}\,\frac{1}{2}\,\gamma_2^2 l_{II}^2 = \frac{1}{2}\dot{U}_1 Y_{12} l_{II} Z_2 l_{II}. \tag{2.95}$$

Let us find the magnitude of the induced current for the electrical effect. At the beginning of the circuit the current must be equal to zero, for the circuit is insulated. As x increases the current will increase. At the end of the circuit

$$\dot{I}_{2IIэ.к} = \frac{\dot{U}_1 Y_{12}}{\gamma_2}\,\mathrm{th}\,\gamma_2 l_{II}. \tag{2.96}$$

For small values of $\gamma_2 \ell_{II}$

$$\dot{I}_{2IIэ.к} = \dot{U} Y_{12} l_{II}. \tag{2.97}$$

Formula (2.97) indicates that the entire current transmitted to the circuit subjected to effect in the convergence section leaks to the ground through the terminal ground.

## 2.6. Effect on Bunched Conductors

The effect on the single-wire communications circuit was investigated above under the assumption that adjacent circuits are absent. The formulas obtained above are basic in the theory of the effect between the electric circuits; the expressions for other, more complex cases reduce to them. Usually not one, but several circuits are suspended at a relatively short distance from each other. In this case it is necessary to consider the effect of the adjacent, so-called "third" circuits on the investigated circuit. In the presence of a large number of third circuits the solution of the problems with respect to determining the induced voltages and currents reduces to integration of the system of generalized telegraph equations presented in §2.1. If the mutually interfering circuits are designated by indexes from 1 to n, then for voltages and currents induced in an arbitrary k-th circuit, we have the following system of differential equations:

$$\left.\begin{aligned}&\frac{d\dot{U}_k}{dx}+\sum_{i=1}^{n}\dot{I}_i Z_{ik}=-\dot{I}_0 Z_{0k},\\&\frac{d\dot{I}_k}{dx}+\dot{U}_k Y_{kk}=\dot{U}_0 Y_{0k}+\sum_{i=1}^{n}(\dot{U}_i-\dot{U}_k)Y_{ik},\end{aligned}\right\} \tag{2.98}$$

where $\dot{I}_0$ and $\dot{U}_0$ are the interfering current and voltage; $Z_{0k}$, $Y_{0k}$ are the magnetic coupling resistance and conductivity of the electrical coupling between the interfering and the k-th circuits; $Z_{ik}$ is the magnetic coupling resistance between the i-th and the k-th circuits ($Z_{kk}$ is the natural resistance of the k-th circuit); $Y_{ik}$ is the electrical coupling conductivity between the i-th and the k-th circuits ($Y_{kk}$ is the natural total conductivity of the k-th circuit).

The solution of system (2.98) in general form is awkward, and it requires the application of computer engineering. Significant simplification can be achieved for the case where all of the wires have identical parameters, they are identically loaded and they are located approximately at equal distances from the interfering line and from each other. Then on the basis of the indicated causes it is possible to consider that the currents and voltages induced in the wires are approximately identical, that is, $\dot{U}_1=\dot{U}_2=\ldots=\dot{U}_k\ldots,\ \dot{I}_1=\dot{I}_2=\dot{I}_3=\ldots=\dot{I}_k\ldots$ Also considering that

$Z_{11}+Z_{21}+Z_{31}+Z_{n1}=Z_{12}+Z_{22}+\ldots+Z_{n2}=\ldots\sum^{n} Z_{ik},$ we obtain from (2.98)

for all wires of the identical system of equations:

$$\left.\begin{aligned}&\frac{d\dot{U}_k}{dx}+\dot{I}_k Z_{\text{equiv}}=-\dot{I}_0 Z_{0k},\\&\frac{d\dot{I}_k}{dx}+\dot{U}_k \dot{Y}_k=\dot{U}_0 Y_{0k},\end{aligned}\right\} \tag{2.99}$$

Key: 1. equiv

where $Z_{\text{equiv}}=\sum_{i=1}^{n} Z_{ik}$. Equations (2.99) are an ordinary system of the type (2.5) in which the resistance of the conductor is replaced by the equivalent value. The calculation of the effect, consequently, can be made by the above-obtained formulas.

The presence of adjacent conductors increases the resistance of the investigated conductor and the propagation coefficient. Increasing the resistance leads to a decrease in the current induced in the conductor. A deficiency of the solution obtained is the possibility of its application only in the case of identical state of all of the conductors. If this condition is not satisfied, then significant simplification of the solution of the generalized telegraph equations can be achieved by representation of all the adjacent circuits in the form of the aggregate bunched conductors. In this case it is sufficient to consider three circuits: the first (interfering), second (subjected to the interference) and third, which is the aggregate bunched conductors. The effect of the adjacent circuits is taken into account as the effect through the third circuit.

## 2.7. Equations of Magnetic and Electrical Effect Through the Third Circuit[1]

The effect through the third circuit consists in series transition of power from the first (interfering) circuit to the third and from the third to the circuit subjected to the effect (the second). This double energy transfer is described by significantly more complex mathematical expressions. The third circuits can be not only adjacent communication circuits, but also metal sheathings of cables, metal pipes, shielding cables, and so on. Using the theory of the interference through the third circuits, some shielding problems, the influences on two-wire circuits, and so on can be investigated. When deriving the formulas let us make the following assumptions:

The parameters of all circuits are different;

The load resistances included at the ends of the circuit are arbitrary;

The inverse effect from the second and third circuits on the first is absent;

The inverse effect from the second circuit on the third is absent;

If the third proposition is entirely obvious, then with respect to the latter it is necessary to make a stipulation. It is valid only in cases where the third circuit has parameters differing significantly from the second circuit parameters or if the effect of the first circuit on the third is appreciably greater than the direct effect of the first on the second. Otherwise this proposition can introduce significant error. For example, if two circuits having identical or very close parameters are suspended in direct proximity to each other, then it is natural to assume that they have identical effect on each other, for the induced voltages and currents in both circuits will be approximately identical. In such cases the proposition of the absence of effect with the second circuit on the third introduces significant error, and it is necessary either to solve the generalized telegraph equations or to use the approximate solution given in §2.6. The possible converging systems for the three circuits (first, second and third) are shown in Fig 2.9. Let us introduce additional notation:

$\dot{U}_{3i}(x)$, $\dot{I}_{3i}(x)$ are the voltage and current in the third circuit in the convergence sections with the second circuit at the point with the coordinate $x$ ($i$=I, II, III); $Z_3$, $Y_3$, $\gamma_3$, $z_{B3}$ is the complex longitudinal resistance, the complex conductivity; the propagation coefficient and the wave resistance of the third circuit;

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[1]Item 2.7 was written by the results of an experiment performed jointly with V. O. Shvartsman.

$p_{3b}$, $p_{3e}$ are the reflection coefficient at the beginning and end of the third circuit; $\ell_{3i}$ is the length of the i-th section of the third circuit;

$Y_{13}$, $Z_{13}$, $Y_{32}$, $Z_{32}$ are the electric coupling conductivity and the magnetic coupling resistance between the corresponding circuits (1 and 3; 3 and 2) in the mutual convergence sections;

$$l_3 = l_{3I} + l_{3II} + l_{3III},$$

$$N'_{13} = z_{B1} z_{B3} Y_{13} \gamma_1 - Z_{13} \gamma_3; \quad N''_{13} = z_{B1} z_{B3} Y_{13} \gamma_3 - Z_{13} \gamma_1;$$

$$N'_{32} = z_{B2} z_{B3} Y_{32} \gamma_3 - Z_{32} \gamma_2; \quad N''_{32} = z_{B2} z_{B3} Y_{32} \gamma_2 - Z_{32} \gamma_3;$$

$$N'_{132} = z_{B1} z_{B3} Y_{13} \gamma_1 N'_{32} - Z_{13} \gamma_2 N''_{32};$$

$$N''_{132} = z_{B1} z_{B3} Y_{13} \gamma_2 N'_{32} - Z_{13} \gamma_1 N''_{32};$$

$$\varphi(\pm N'', \gamma_m, l_n, p_m, \pm N', \gamma_q, l_r, p_q) = \pm N'' \operatorname{ch}(\gamma_m l_n - \ln \sqrt{p_m}) \times$$
$$\times \operatorname{sh}(\gamma_q l_r - \ln \sqrt{p_q}) \pm N' \operatorname{sh}(\gamma_m l_n - \ln \sqrt{p_m}) \times$$
$$\times \operatorname{ch}(\gamma_q l_r - \ln \sqrt{p_q}).$$

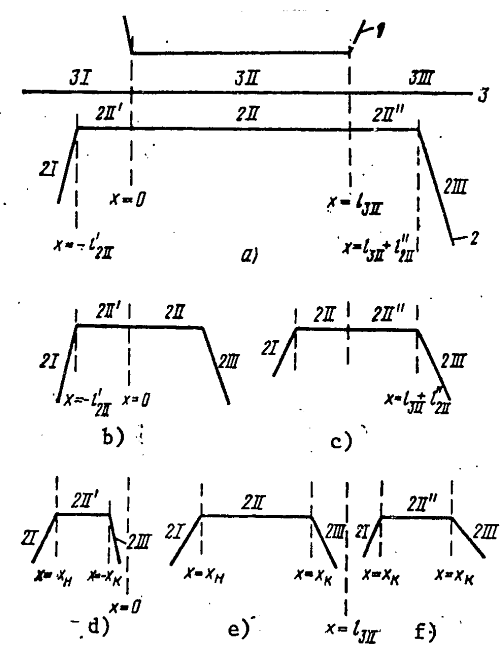


Figure 2.9. Diagrams of the arrangement of the circuits (2I, 2II, 2II', 2II", 3I, 3II, 3III -- numbers of the circuit sections)

The voltages and the currents induced in the second circuit as a result of the effect of currents and voltages occurring in the first circuit satisfy the following system of equations in each i-th section

$$\frac{d\dot{U}_{2i}}{dx}+\dot{I}_{2i}Z_2=-\dot{I}_{3i}Z_{32};\quad \frac{d\dot{I}_{2i}}{dx}+\dot{U}_{2i}Y_2=\dot{U}_{3i}Y_{32}. \tag{2.100}$$

The system of differential equations (2.100) differs from the system (2.5) solved above by the fact that in the righthand side the interfering currents and voltages are the result of the effect of the first circuit on the third. The values of $\dot{I}_{3i}$ and $\dot{U}_{3i}$ are determined in §2.3 and 2.5 and in [29]. Solving the system (2.100) just as (2.5), that is, using the method of variation of constants to find the partial solution and determining the integration constants of the general solution from the boundary conditions, we obtain the formulas for the voltages and currents induced in the second circuit through the third circuit in each of the sections of their mutual convergence. The formulas for the voltages occurring during convergence indicated in Fig 2.9a are presented in Table 2.1. The voltage formulas for the other cases of convergence are not presented here because of saving space. The current formulas are also not presented for the same reasons.

The voltages in the sections of the second circuit located outside convergence with the first and third circuits, that is, in the sections 2I and 2III, are determined by the following general formulas:

$$\dot{U}_{2I}(x'_{н})=\dot{U}_{2i}(x_{нi})\frac{\mathrm{ch}\left[\gamma_2(l_{2I}-x'_{н})-\ln\sqrt{p_{2н}}\right]}{\mathrm{ch}\left(\gamma_2 l_{2I}-\ln\sqrt{p_{2н}}\right)}, \tag{2.101}$$

$$\dot{U}_{2III}(x'_{к})=\dot{U}_{2i}(x_{кi})\frac{\mathrm{ch}\left[\gamma_2(l_{2III}-x'_{к})-\ln\sqrt{p_{2к}}\right]}{\mathrm{ch}\left(\gamma_2 l_{2III}-\ln\sqrt{p_{2к}}\right)}, \tag{2.102}$$

where $x'_b$ and $x'_e$ are the current distances from the beginning and end of convergence of the second circuit with the third circuit; $x_{bi}$, $x_{ei}$ are the coordinates of the beginning and end of convergence of the second circuit with the third circuit (for example, for the system in Fig 2.9a $x_{bi}=-\ell'_{2II}$, $x_{ei}=\ell_{2II}+\ell''_{2II}$); $U_{2i}(x_{bi})$ is the voltage induced at the beginning and end of convergence of the second circuit with the third (for example, for the system in Fig 2.9a, $U_{2i}(x_{bi})=U_{2II}(-\ell'_{2II})$).

The formulas in Table 2.1 are the most general formulas for the convergence circuits in Fig 2.9. For example, from the formula for the voltage induced in the section 2II of system 2.9a, for $\ell_{2I}=\ell_{2III}=0$, $\ell''_{2II}=\ell_{3III}=0$ and $\ell'_{2II}=\ell_{3I}$ we obtain the formulas presented in [29]; for $\ell_{2I}=\ell'_{2II}=\ell''_{2II}=\ell_{2III}=0$ and loading of all the circuits on wave impedances we obtain the formulas presented in [16].

Table 2.1

## Formulas for the Effect Through a Third Circuit

| Схема сближения (1) | Определяемая величина (2) | Формула (3) | Номер формулы (4) |
| --- | --- | --- | --- |
| 1 | 2 | 3 | 4 |
| Рис. 2.9 *a* (5) | $\dot U_{2II'}(x)$ | $\frac{1}{z_{в3}(\gamma_3^2-\gamma_2^2)\,\mathrm{sh}\,(\gamma_3 l_3-\ln\sqrt{p_{3н}p_{2к}})}\left\{\frac{\dot U_3(0)\,\varphi\,(N''_{32},\ \gamma_3,\ l_{3I}-l'_{2II},\ p_{3н},\ -N'_{32},\ \gamma_3,\ l_{2I},\ p_{3н})}{\mathrm{ch}\,(\gamma_3 l_{3I}-\ln\sqrt{p_{3н}})}\right.$ (7) $\times$<br>$\times\mathrm{ch}\,[\gamma_3(l_{2II}+l''_{2II}+l_{2III}-x)-\ln\sqrt{p_{2к}}]+$ (6)<br>$+\left[\frac{\dot U_3(l_{3II})\,\varphi\,(N''_{32},\ \gamma_3,\ l_{3III}-l''_{2II},\ p_{2к},\ -N'_{32},\ \gamma_3,\ l_{2III},\ p_{2к})}{\mathrm{ch}\,(\gamma_3 l_{3III}-\ln\sqrt{p_{2к}})}\right.-$ (6)<br>$-\frac{\dot U_1(0)\,[\varphi\,(N'_{132},\ \gamma_2,\ l_{3II}+l''_{2II}+l_{3III},\ p_{2к},\ -N''_{132},\ \gamma_1,\ l_{3II},\ p_1)+}{}$ →<br>→ $\frac{+\varphi\,(-N'_{132},\ \gamma_2,\ l''_{2II}+l_{3III},\ p_{2к},\ +N''_{132},\ p_1)]}{z_{в1}(\gamma_2^2-\gamma_1^2)\,\mathrm{ch}\,(\gamma_1 l_{3II}-\ln\sqrt{p_1})}\left.\right]\mathrm{ch}\,[\gamma_3(l_{3I}+l'_{2II}+x)-\ln\sqrt{p_{3н}}]\left.\right\}-$ (7)<br>$-\frac{\dot U_3(0)\,N''_{32}\,\mathrm{ch}\,[\gamma_3(l_{3I}+x)-\ln\sqrt{p_{3н}}]}{z_{в3}(\gamma_3^2-\gamma_2^2)\,\mathrm{ch}\,(\gamma_3 l_{3I}-\ln\sqrt{p_{3н}})}$ | 1 |

Key:
1. Convergence system
2. Defined value
3. Formula
4. Formula number
5. Fig 2.9a
6. e
7. b

Table 2.1 [continued]

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| Рис. 2.9ы (5) | $\dot{U}_{2II}(x)$ | $\begin{aligned} &\frac{1}{z_{\text{в}3}(\gamma_3^2-\gamma_2^2)\,\mathrm{sh}(\gamma_3 l_3-\ln\sqrt{p_{2\text{н}}p_{2\text{к}}})}\left\{\left[\frac{\dot{U}_3(0)\,\varphi(N''_{32},\ \gamma_3,\ l_{3I}-l'_{2II},\ p_{3\text{н}},\ -N'_{32},\ \gamma_3\ l_{2I},\ p_{2\text{н}})}{\mathrm{ch}(\gamma_3 l_{3I}-\ln\sqrt{p_{3\text{н}}})} \quad (7)\right.\right.\\ &\left.-\frac{U_1(0)\,\varphi(N'_{132},\ \gamma_3,\ l_{3I}+l'_{2II},\ p_{2\text{н}},\ +N''_{132},\ \gamma_1,\ l_{3II},\ p_1)}{z_{\text{в}1}(\gamma_2^2-\gamma_1^2)\,\mathrm{ch}(\gamma_1 l_{3II}-\ln\sqrt{p_1})}\right]\times\\ &\times\mathrm{ch}\,[\gamma_3(l_{3II}+l''_{2II}+l_{2III}-x)-\ln\sqrt{p_{2\text{к}}}]+\\ &+\left[\frac{\dot{U}_3(l_{3II})\,\varphi(N''_{32},\ \gamma_3,\ l_{3III}-l''_{2II},\ p_{3\text{к}},\ (6)\ -N'_{32},\ \gamma_3,\ l_{2III},\ p_{2\text{к}})}{\mathrm{ch}(\gamma_3 l_{3III}-\ln\sqrt{p_{3\text{к}}})}-\right.\\ &\left.-\frac{\dot{U}_1(0)\,\varphi(-N'_{132},\ \gamma_3,\ l''_{2II}+l_{2III},\ p_{2\text{к}},\ +N''_{132},\ p_1)}{z_{\text{в}1}(\gamma_2^2-\gamma_1^2)\,\mathrm{ch}(\gamma_1 l_{3II}-\ln\sqrt{p_1})}\right]\times\\ &\left.\times\mathrm{ch}\,[\gamma_3(l_{2I}+l'_{2II}+x)-\ln\sqrt{p_{2\text{н}}}]\right\}-\frac{\dot{U}_{3II}(x)\,N''_{32}}{z_{\text{в}3}(\gamma_3^2-\gamma_2^2)}+\\ &+\frac{\dot{U}_1(0)\,N''_{132}\,\mathrm{ch}\,[\gamma_1(l_{3II}-x)-\ln\sqrt{p_1}]}{z_{\text{в}1}z_{\text{в}3}(\gamma_2^2-\gamma_1^2)(\gamma_3^2-\gamma_2^2)\,\mathrm{ch}(\gamma_1 l_{3II}-\ln\sqrt{p_1})}. \end{aligned}$ | 2 |

Table 2.1 [continued]

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| Рис. 2.9 *а* | $\dot U_{2II''}(x)$ | $\frac{1}{z_{в3}(\gamma_3^2-\gamma_2^2)\,\mathrm{sh}\,(\gamma_2 l_2-\ln\sqrt{p_{2н}p_{2к}})}\left\{\left[\frac{\dot U_3(0)\,\varphi(N''_{32},\ \gamma_3,\ l_{3I}-l'_{2II},\ p_{3н},\ -N'_{32},\ \gamma_3,\ l_{3I},\ p_{3н})}{\mathrm{ch}\,(\gamma_3 l_{3I}-\ln\sqrt{p_{3н}})}-\right.\right.$ $-\frac{\dot U_1(0)\,[\varphi(N'_{132},\ \gamma_1,\ l_{3I}+l'_{2II},\ p_{3н}\ +N''_{132},\ \gamma_1,\ l_{3II},\ p_1)+\varphi(-N'_{132},\ \gamma_1,\ l_{2II}+}{z_{в1}(\gamma_2^2-\gamma_1^2)\,\mathrm{ch}\,(\gamma_1 l_{3II}-\ln\sqrt{p_1})}\rightarrow$ $\left.\rightarrow\frac{+\,l'_{2II}+l_{2I},\ p_{3н},\ -N''_{132},\ p_1)]}{}\right]\mathrm{ch}\,[\gamma_2(l_{3II}+l'_{2II}+l_{3III}-x)-\ln\sqrt{p_{2к}}]+$ $+\frac{\dot U_3(l_{3II})\,\varphi(N''_{32},\gamma_3,\ l_{3III}-l''_{2II},p_{3к},\ -N'_{32},\ \gamma_3,\ l_{3III},\ p_{3к})}{\mathrm{ch}\,(\gamma_3 l_{3III}-\ln\sqrt{p_{3к}})}\,\mathrm{ch}\,[\gamma_2(l_{3I}+l'_{2II}+$ $\left.+x)-\ln\sqrt{p_{2н}}]\right\}-\frac{\dot U_3(l_{3II})\,N''_{32}\,\mathrm{ch}\,[\gamma_3(l_{3II}+l_{3III}-x)-\ln\sqrt{p_{3к}}]}{z_{в3}(\gamma_3^2-\gamma_2^2)\,\mathrm{ch}\,(\gamma_3 l_{3III}-\ln\sqrt{p_{3к}})}$ | 3 |

The formulas for the convergence systems in Fig 2.9d and f are of interest. They make it possible to determine the indirect effect of the first circuit on the second in the absence of a direct mutual convergence between them. The formulas presented in Table 2.1 can be simplified significantly, for example, for the special cases where the load resistances in the second and third circuits are identical, the length of the circuits are identical, $\gamma l=0$, and so on.

## 2.8. Determination of the Electrical Effect Using the Maxwell Equations

The formulas obtained above for the electrical effect in general form are awkward. The formulas for the electrical effect can be obtained by a simpler method used in electrostatics with the help of the Maxwell equations which relate the potentials and the charges of the conductors.

Let us consider the general case of suspension of n conductors on an overhead communication line subjected to the effect of a high-voltage line wire. Let us designate the interfering wire with the index I, the communications wires with the indexes 1, 2,...,n; $U_I$, $q_I$ are the potential and the charge per unit length of the interfering wire (volts, coulombs/km); $U_1, U_2,\ldots,U_n$; $q_1, q_2,\ldots,q_n$ are the potentials and charges of the corresponding wires of the communications line (volts, coulombs/km). The Maxwell equations for a system of n charged conductors have the form:

$$\left.\begin{aligned} U_I &= q_I\alpha_I + q_1\alpha_{1I} + q_2\alpha_{2I} + \cdots + q_n\alpha_{nI}, \\ U_1 &= q_I\alpha_{I1} + q_1\alpha_{11} + q_2\alpha_{21} + \cdots + q_n\alpha_{n1}, \\ U_2 &= q_I\alpha_{I2} + q_1\alpha_{12} + q_2\alpha_{22} + \cdots + q_n\alpha_{n2}, \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ U_n &= q_I\alpha_{In} + q_1\alpha_{1n} + q_2\alpha_{2n} + \cdots + q_n\alpha_{nn}, \end{aligned}\right\} \quad (2.103)$$

where $\alpha_I$, $\alpha_{I1}$ ... $\alpha_{11}$; $\alpha_{ik}$ are the potential coefficients; $\alpha_I$ is the natural potential coefficient of the interfering wire equal to $\alpha_I=18\cdot10^6 \ln (D_{II}/r_I)$, km/farads; $D_{II}$ is the distance from the interfering wire to its mirror image with respect to the ground; $r_I$ is the radius of the interfering wire; $\alpha_{11}$, $\alpha_{22}$, ..., $\alpha_{nn}$ are the natural potential coefficients of the communication wires; $\alpha_{ii}=18\cdot10^6 \ln (D_{ii}/r_i)$, km/farads; $D_{ii}$ is the distance between the i-th communications wire and its mirror image with respect to the earth's surface (twice the height of suspension); $r_i$ is the radius of the i-th wire; $\alpha_{Ii}$ is the mutual potential coefficient between the interfering wire and the i-th wire subjected to the effect: $\alpha_{Ii}=18\cdot10^6 \ln (D_{Ii}/a_{Ii})$, km/farad; $D_{Ii}$ is the distance from the interfering wire to the mirror image of the wire i; $a_{Ii}$ is the distance between the interfering conductor and the i-th conductor; $\alpha_{ik}$ is the mutual potential coefficient between the i-th and the k-th wires: $\alpha_{ik}=18\cdot10^6 \ln (D_{ik}/a_{ik})$, km/farad; $D_{ik}$ is the distance between the wire i and the mirror image of the wire k; $a_{ik}$ is the distance between the wires i and k. It is obvious that $\alpha_{Ii}=\alpha_{iI}$, $\alpha_{ik}=\alpha_{ki}$.

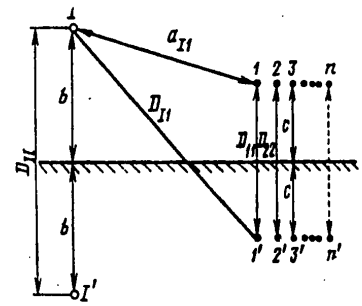

Figure 2.10. Determination of the potential coefficient

In the electric field of the wires located above ground, nothing changes if instead of the ground on the other side of its surface we place wires with opposite charges at the same distance (the mirror image); with small radius of the wires by comparison with the spacing between wires the electrical axes of the charges can be considered to coincide with the geometric axes of the wires. In Fig 2.10, 1, 2, ..., n denote the wires, and 1', 1', 2',...,n' denote their mirror images. The spacings between the wires are denoted there also.

Let us solve the Maxwell equations for three cases:

1) All wires of the communications line are insulated;

2) All wires of the communications line are grounded;

3) On the communications line there are m grounded wires and the rest are insulated.

When determining the electrical effect it is necessary to consider that under the effect of the interfering circuit in the wires subjected to the effect the charges are separated, that is, on opposite sides of the wire there are two equal charges with different signs. The variation of the effect of the electric field causes variation of the induced charges, but the separation remains. If the wire subjected to the effect is insulated from the ground, then as a result of its small diameter the electric fields of the induced charges in the outer space are compensated, and on the adjacent wires the potential and charge distribution will not change. The charges on the insulated wires in the Maxwell equations can be taken equal to zero.

If the wire is grounded, then the charge of the same sign as the charge on the interfering wire flows into the ground, and on the wire subjected to the effect, the charge of opposite sign remains in the bound state (the wire potential will be equal to zero). This induced charge will influence the charge distribution in the adjacent wires. The potentials of the grounded wires in the Maxwell equations must be taken equal to zero.

All Wires on the Communications Line Are Insulated. In order to simplify the solution let us assume that the parameters of all wires of the communications line are identical; the wires are located approximately at equal distances from each other and are suspended at identical height. Let us also assume that the spacing between the wires 1, 2, ..., n is significantly less than the distance from the communications line to the interfering wire. The last proposition gives the basis for considering that $q_1 = q_2 = \dots = q_n = q_A$. The assumptions made make it possible to introduce some mean (arithmetic mean) potential coefficients which pertain to all wires of the communication line. Let us find these mean values:

$$\bar{\alpha}_{I1} = \bar{\alpha}_{II} = \frac{1}{n}(\alpha_{I1} + \alpha_{I2} + \dots + \alpha_{In}),$$

$$\bar{\alpha}_{11} = \frac{1}{n}(\alpha_{11} + \alpha_{22} + \dots + \alpha_{nn}),$$

$$\bar{\alpha}_{12} = \frac{1}{n-1}(\alpha_{12} + \alpha_{13} + \dots + \alpha_{1n}) = \frac{1}{n-1}(\alpha_{21} + \alpha_{23} + \dots + \alpha_{2n}) = \dots$$

In this case $U_1 = U_2 = \dots = U_n = U_A$, and the system (2.103) is converted to a system of two equations which are valid for any k-th communications wire:

$$\left.\begin{aligned} U_I &= q_I \alpha_I + n q_A \bar{\alpha}_{I1}, \\ U_A &= q_I \bar{\alpha}_{I1} + q_A \bar{\alpha}_{11} + (n-1) q_A \bar{\alpha}_{12}. \end{aligned}\right\} \tag{2.104}$$

Since the wires are insulated, $q_A = 0$, and on each of the wires $U_A = q_I \bar{\alpha}_{I1}$. Considering that $q_I = U_I/\alpha_I$, we obtain

$$U_A = U_I \frac{\alpha_{I1}}{\alpha_I}. \tag{2.105}$$

From (2.105) it follows that the voltage induced on the insulated conductors under the electrical effect does not depend on the number of adjacent communication wires. Let us find

$$\bar{\alpha}_{I1} = \bar{\alpha}_{IA} = 18 \cdot 10^6 \ln \frac{D_{IA}}{a_{IA}} = 18 \cdot 10^6 \ln \sqrt{\frac{a^2 + (b+c)^2}{a_2 + (b-c)^2}}.$$

Hence,

$$\bar{\alpha}_{IA} = 9 \cdot 10^6 \ln \frac{a^2 + b^2 + c^2 + 2bc}{a^2 + b^2 + c^2 - 2bc} = 9 \cdot 10^6 \ln \frac{1 + \dfrac{2bc}{(a^2 + b^2 + c^2)}}{1 - \dfrac{2bc}{(a^2 + b^2 + c^2)}}.$$

For $b \gtrless c$ and $2bc \lessgtr b^2 + c^2$, we have $2bc/a^2 + b^2 + c^2 < 1$. The logarithm of the type $\ln(1+X/1-X)$ for $X < 1$ can, as is known, be expanded in the series

$$\ln \frac{1+x}{1-x} = 2\left(X + \frac{X^3}{3} + \frac{X^5}{5} + \dots\right).$$

In the given case $X=2bc/(a^2+b^2+c^2)$; therefore, for $a \geqslant 10$ meters it is possible with sufficient degree of accuracy to assume (limiting ourselves only to the first term of the expansion of the logarithm) that

$$\ln\frac{1+X}{1-X} \approx 2X, \quad \bar{\alpha}_{1A} = 36\cdot 10^6 \frac{bc}{a^2+b^2+c^2}.$$

Substituting $\bar{\alpha}_{1A}$ in (2.105) and considering that $\alpha_I = 18\cdot 10^6 \ln(D_{II}/r_I)$, we obtain

$$\dot{U}_A = \frac{36\cdot 10^6\, bc\, \dot{U}_1}{18\cdot 10^6 \ln\frac{D_{II}}{r_I}(a^2+b^2+c^2)} = \frac{2\dot{U}_1 bc}{\ln\frac{D_{II}}{r_I}(a^2+b^2+c^2)}.$$

Let us denote $2/\ln(D_{II}/r_I)=k_1$, then

$$\dot{U}_A = k_1 \dot{U}_1 \frac{bc}{a^2+b^2+c^2}. \tag{2.106}$$

For the single-wire line in practice $\ln(D_{II}/r_I) \approx 8.5$ to 9. For example, for a suspension height of 8 meters and $r_I=0.004$ m, $\ln(D_{II}/r_I)=8.3$ and $k_1=0.24$.

If not one but several wires under one interfering potential are affected, then it is necessary to take the average distance from the equivalent wire as $D_{II}$, and by $r_I$ it is necessary to mean the equivalent radius of the system of interfering wires defined by the formula $r_{equiv}=\sqrt[n]{r_{mean}(a_{mean})^{n-1}}$, meters, where $r_{mean}$ is the mean radius of the wires, meters; $a_{mean}$ is the mean distance between the wires, meters; $a_{mean}=\sqrt[n-1]{a_{I2}a_{I3}\cdots a_{In}}$, where $a_{I2}, a_{I3}\ldots a_{In}$ is the distance between the conductor I and the remaining (2,3,..., n) wires. For example, for a single railroad track without additional wires the spacing between the contact wire and the bearing cable will be 1.6 meters, and between the contact wire and the ground, 8 meters. Both wires are interfering, and the average distance between the equivalent wire and the ground is $8+(1.6/2)=8.80$ m, $a_{mean}=1.6$ m, $r_{mean}=0.005$ m. Then

$$k_1 = \frac{2}{\ln\frac{D_{II}}{r_{\text{экв}(1)}}} = \frac{2}{\ln\frac{17,6}{\sqrt{0,005\cdot 1,6}}} = \frac{2}{5,28} = 0,38 \approx 0,4.$$

Key: 1. equiv

It is possible analogously to find the coefficients $k_1$ for other interfering systems.

All the Wires On the Communication Line Are Grounded. Let us propose, as before, that the parameters of all the wires subjected to the effect are identical and the wires are identically arranged; let us assume, as before, that $q_1=q_2=\ldots=q_n=a_A$. Then we obtain the system (2.104). Since all the wires are grounded, $\dot{U}_A=0$, and we obtain the system:

$$\left.\begin{aligned} \dot{U}_I &= q_I\alpha_I + nq_A\bar{\alpha}_{II}, \\ 0 &= q_I\bar{\alpha}_{II} + q_A(\bar{\alpha}_{11} + (n-1)\bar{\alpha}_{12}). \end{aligned}\right\} \tag{2.107}$$

Let us find $q_I$ and $q_A$. From the first equation

$$q_I = \frac{\dot{U}_I - nq_A\bar{\alpha}_{II}}{\alpha_I}.$$

Substituting the value of $q_I$ in the second equation of the system, we have:

$$\begin{gathered} \frac{\dot{U}_I - nq_A\bar{\alpha}_{II}}{\alpha_I}\bar{\alpha}_{II} + q_A(\bar{\alpha}_{11} + (n-1)\bar{\alpha}_{12}) = 0, \\ \dot{U}_I\bar{\alpha}_{II} - nq_A\bar{\alpha}_{II}^2 + q_A\alpha_I(\bar{\alpha}_{11} + (n-1)\bar{\alpha}_{12}) = 0, \\ q_A = -\frac{\dot{U}_I\bar{\alpha}_{II}}{\alpha_I(\bar{\alpha}_{11} + (n-1)\bar{\alpha}_{12}) - n\bar{\alpha}_{II}^2}. \end{gathered} \tag{2.108}$$

For spacing between the interfering wire and the communication line of more than 10 meters, it is possible without great error to neglect the last term in the denominator even for n=10-20. Then

$$q_A = -\frac{\dot{U}_I\bar{\alpha}_{II}}{\alpha_I(\bar{\alpha}_{11} + (n-1)\bar{\alpha}_{12})}.$$

By the induced charge in the wire it is possible to find the current which flows in the ground. The total charge on the wire $Q_A=a_A\ell$, where $\ell$ is the length of the wire subjected to the effect. The discharge current is equal to the derivative with respect to time, that is, $I_A=(\partial/\partial t)Q_A$. Assuming that $\dot{U}_I$, and, consequently, $q_A$ vary by a sinusoidal law, that is, $\dot{q}_A=q_Ae^{i\omega t}$, we obtain

$$\dot{I}_A = -\frac{\partial}{\partial t}\dot{q}_A l = -i\omega lq_A e^{i\omega t},$$

or

$$|\dot{I}_A| = \omega l\dot{U}_I\frac{\bar{\alpha}_{II}}{\alpha_I(\bar{\alpha}_{11} + (n-1)\bar{\alpha}_{12})}.$$

Then

$$\frac{\dot{U}_I\bar{\alpha}_{II}}{\alpha_I} = k_1\dot{U}_I\frac{bc}{a^2+b^2+c^2},$$

then

$$|\dot{I}_A| = \frac{k_1\dot{U}_1\,\omega\, lbc}{[\bar{\alpha}_{11} + (n-1)\bar{\alpha}_{12}](a^2+b^2+c^2)}. \qquad (2.109)$$

The natural potential coefficient $\alpha_{11}$ is on the average equal approximately to $(8.5\text{-}9)\cdot 18\cdot 10^6$, and the mutual potential coefficient between the communications wires located on one line, $(2.5\text{-}3)\cdot 18\cdot 10^6$. Taking the average values of $8.75\cdot 18\cdot 10^6$ and $2.75\cdot 18\cdot 10^6$, we obtain
$\bar{\alpha}_{11} + (n-1)\bar{\alpha}_{12} = 18\cdot 10^6[8{,}75 - 2{,}75 + 2{,}75n] \approx 54\cdot 10^6(n+2)$.
Substituting this value, we obtain

$$|\dot{I}_A| = \frac{k_1\dot{U}_1\,\omega\, lbc}{54\,(n+2)\,(a^2+b^2+c^2)}\,10^{-6}\text{, amps}$$

Let us denote $k_2 = k_1\omega/54$. Then

$$|\dot{I}_A| = k_2\dot{U}_1\,\frac{l}{n+2}\,\frac{bc}{a^2+b^2+c^2}\,10^{-3}\text{, mA,} \qquad (2.110)$$

where n is the number of grounded wires in addition to the investigated one.

On the Communications Line of m Grounded Wires and n-m Insulated Wires. Let us denote the grounded wires by the indexes 1, 2,...,m; the insulated wires are A, B, ..., Just as before, let us propose that the parameters of all of the grounded wires are identical, and the wires are identically arranged with respect to the interfering wire and the ground; the spacings between the grounded wires are identical. The same assumption will be made with respect to the insulated wires. The charges on the grounded wires are identical, that is, $q_1 = q_2 = \ldots = q_m$; the total charges on the insulated wires are equal to zero ($q_A = q_B = \ldots = 0$); the voltages on the grounded wires are equal to zero. Introducing the mean values of the potential coefficients from the general system of Maxwell equations we obtain three equations pertaining to the interfering wire, any of the grounded wires any of the insulated wires:

$$\left.\begin{aligned} \dot{U}_I &= q_I\,\alpha_I + q_1 m\bar{\alpha}_{I1}, \\ 0 &= q_I\,\bar{\alpha}_{I1} + q_1(m-1)\,\bar{\alpha}_{12} + q_1\,\bar{\alpha}_{11}, \\ \dot{U}_A &= q_I\,\bar{\alpha}_{IA} + q_1 m\,\bar{\alpha}_{1A}, \end{aligned}\right\} \qquad (2.11)$$

where

$$\bar{\alpha}_{I1}=\frac{1}{m}(\alpha_{I1}+\alpha_{I2}+\cdots+\alpha_{Im});$$

$$\bar{\alpha}_{12}=\frac{1}{m-1}(\alpha_{12}+\alpha_{13}+\cdots+\alpha_{1m})=\frac{1}{m-1}(\alpha_{21}+\alpha_{23}+\cdots+$$
$$+\alpha_{2m})=\cdots;$$

$$\bar{\alpha}_{11}=\frac{1}{m}(\alpha_{11}+\alpha_{22}+\cdots+\alpha_{mm});\quad \bar{\alpha}_{IA}=\frac{1}{n-m}(\alpha_{IA}+\alpha_{IБ}+\cdots);$$

$$_{\mathrm{A}}=\frac{1}{m}(\alpha_{1\mathrm{A}}+\alpha_{2\mathrm{A}}+\cdots+\alpha_{m\mathrm{A}})=\frac{1}{m}(\alpha_{\;Б}+\alpha_{2Б}+\cdots+\alpha_{mБ})=\ldots$$

For determination of $\dot{U}_A$, it is necessary to find $q_1$ and $q_2$ from the first two equations of system (2.111). The system of the first two equations does not differentiate from the system (2.107) under the condition of replacing n by m and $q_A$ by $q_1$. Hence, neglecting the term $n\bar{\alpha}^2_{I1}$ in the denominator (2.108), we obtain

$$q_1=-\frac{\dot{U}_I\bar{\alpha}_{I1}}{\alpha_I[\bar{\alpha}_{11}+(m-1)\bar{\alpha}_{12}]},$$

$$q_I=\frac{1}{\alpha_I}[\dot{U}_I-q_1 m\bar{\alpha}_{I1}]=\frac{1}{\alpha_I}\left\{\dot{U}_I+\frac{\dot{U}_I\bar{\alpha}_{I1}m\bar{\alpha}_{I1}}{\alpha_I[\bar{\alpha}_{11}+(m-1)\bar{\alpha}_{12}]}\right\}\approx\frac{\dot{U}_I}{\alpha_I}.$$

Substituting $q_1$ and $q_I$ in the third equation, we find

$$\dot{U}_A=\frac{\dot{U}_I}{\alpha_I}\bar{\alpha}_{IA}-\frac{\dot{U}_I\bar{\alpha}_{I1}m\bar{\alpha}_{1A}}{\alpha_I[\bar{\alpha}_{11}+(m-1)\bar{\alpha}_{12}]}=\frac{\dot{U}_I\bar{\alpha}_{IA}}{\alpha_I}\times$$
$$\times\left\{1-\frac{m\bar{\alpha}_{I1}\bar{\alpha}_{1A}}{\bar{\alpha}_{IA}[\bar{\alpha}_{11}+(m-1)\bar{\alpha}_{12}]}\right\}. \tag{2.112}$$

The expression in front of the parenthesis is the potential induced on the insulated wire in the absence of grounded adjacent wires, that is, for m=0, and the expression in parentheses indicates the presence of grounded wires decreasing the electrical effect on the insulated wire. Thus, this expression is the shielding coefficient of the grounded wires, and formula (2.112) can be represented in the form

$$\dot{U}_A=\dot{U}_{A\,\text{из}}S_3, \tag{2.113}$$
(1)

Key: 1. ins

where

$$S_э = 1 - \frac{m\bar{\alpha}_{11}\bar{\alpha}_{1A}}{\alpha_{1A}[\bar{\alpha}_{11} + (m-1)\bar{\alpha}_{12}]}. \tag{2.114}$$

$S_e$ is always less than 1, varying from 1 for m=0 to $1-(\bar{\alpha}_{I1}\bar{\alpha}_{1A}/\bar{\alpha}_{IA}\bar{\alpha}_{12})$ for m=∞. In the latter case if $\bar{\alpha}_{I1}=\bar{\alpha}_{IA}$ and $\bar{\alpha}_{1A}=\bar{\alpha}_{12}$, then the expression in parentheses is equal to zero and, consequently, total shielding occurs. Formula (2.114) makes it possible to find the shielding effect of any number of grounded wires located on a column line arbitrarily with respect to the insulated communications wire subjected to the effect. Since $\bar{\alpha}_{11}+(m-1)\bar{\alpha}_{12} \approx 3(m+2)18 \cdot 10^6$ the shielding factor

$$S_э \approx 1 - \frac{m\bar{\alpha}_{11}\bar{\alpha}_{1A}}{54 \cdot 10^6 (m+2)\bar{\alpha}_{1A}}.$$

If $\bar{\alpha}_{I1}=\bar{\alpha}_{IA}$, that is, the grounded and shielded conductors are arranged identically with respect to the interfering wire, then

$$S_э \doteq 1 - \frac{m\bar{\alpha}_{1A}}{54 \cdot 10^6 (m+2)}.$$

The current in any of the grounded conductors is defined by (2.109) for n=m.

## 2.9. Galvanic Effect on Single-Wire Circuits

Sources of Galvanic Effect. As was pointed out above, the galvanic effect is caused by currents falling in the ground. In addition to magnetic forms, current sources in the ground are asymmetric powerful current installations which include ground-return electric power transmission lines (single-phase "wire-ground," triple-phase "two-wire-ground," DC and AC electric railways). The calculations show that for parallel convergence between the high-voltage lines and communication lines the magnetic effect, as a rule, is appreciably greater than the galvanic effect. In a number of cases, in particular, at the intersections with location at one of the grounds over the single-wire communications circuit in direct proximity to the ground of the substation, the voltages caused by the galvanic effe can turn out to be greater than the voltage from the magnetic effect.

Effect of High-Voltage Lines. Let us consider the convergence of the single-wire communications circuit with the electric power transmission line using the "wire-ground" circuit. The standard case of flow of a current through such a system is the single-phase short-circuiting of the overhead electric power transmission lines. The potential of any point M on the surface of the earth created by currents flowing into the ground from a spot ground, as is known, is equal to $U_g=I_g\rho_g/2\pi a$, volts, where $I_g$ is the ground current, amps; a is the spacing between the point on the ground M and the ground, meters. Then the potential difference ΔU between the grounds of a single wire circuit (Fig 2.11a) can be defined by the formula

$$\Delta U=\frac{I\rho_{\text{з}}^{(1)}}{2\pi}\left(\frac{1}{a_1}-\frac{1}{a_2}-\frac{1}{a_3}+\frac{1}{a_4}\right),\ \text{volts.}$$

Key: 1. g

If one of the grounds of the communications circuit lies outside the propagation zone of the currents of the overhead electric power transmission lines, then for the case shown in Fig 2.11b,

$$\Delta U=\frac{I\rho_{\text{з}}}{2\pi}\left(\frac{1}{a_1}-\frac{1}{a_3}\right),\ \text{volts.}$$

and for the case depicted in Fig 2.11c,

$$\Delta U=\frac{I\rho_{\text{з}}^{(1)}}{2\pi a_1},\ \text{volts}$$

Key: 1. g

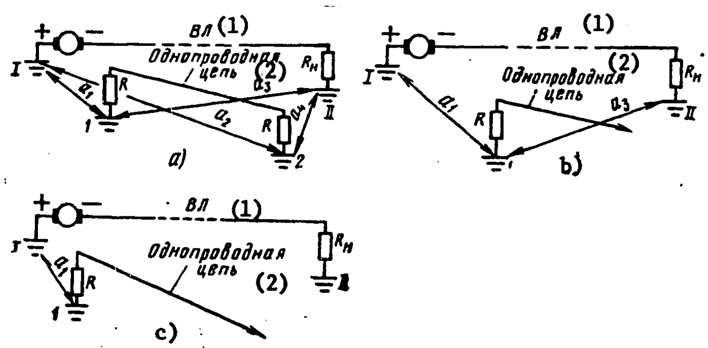


Figure 2.11. Location of grounds 1 and 2 of single-wire circuits and operating ground of the transmitting (I) and receiving (II) ends of the overhead transmission line using the "wire-ground" system

Key: 1. overhead line; 2. single-wire circuit

If the grounds of the overhead electric power transmission lines are located between the ground of the single-wire circuit, then the conditions of mutual arrangement of the ground can exist for which in the single-wire circuit no potential difference caused by the galvanic effect will occur (for example, if the ground of the overhead electric power transmission line is at identical distances from the grounds of the single-wire communications circuit). In general form the magnitude of the galvanic effect can in the single-wire circuit be defined by the formula

$$I_{\text{п}}=\frac{\Delta U}{Z_{\text{общ}}^{(1)}},\ \text{A,} \tag{2.115}$$

Key: 1. total

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30 APRIL 1980 M. N. MIKHAYLOV, L. D. RAZUMOV AND S. A. SOKOLOV 2 OF 4

where $Z_{total}$ is the total resistance of the single-wire circuit, equipment and ground, ohms.

Effect of DC Electric Railways. The DC electric railways are electric systems in which the return current returns partially through the rails and partially through the ground. In the stretch between two traction substations the current in the rails will be maximal at the points of contact of the wheels of the electric locomotive with the rails even near the traction substation, and it will be minimal between the electric locomotives and the traction substation. Correspondingly, the magnitude of the current in the ground varies: at the point located near the traction substations or near the electric locomotives, they will be minimal. Under actual conditions, depending on the location of the moving electric locomotives in the section both the magnitude and direction of the currents in the ground can vary.

Flowing in the ground, the electric traction currents (stray currents) create different potentials at different points of the ground. If both grounds of a single-wire circuit or one of them is located in the zone of transmission of the stray currents, then under the effect of the potential difference of the ground in the circuit currents arise (the interference currents from the galvanic effect).

For determining the magnitude of the interfering current it is necessary to consider two cases of location of the grounds of a single-wire circuit:

The communication line is located along the electrified section of the railway and both grounds (for example, in the attended and unattended repeater stations) are in the zone of propagation of the stray currents;

The communication line is located perpendicular to the direction of the roadbed of the electrified section of the railway. Only one ground of the single-wire circuit is in the propagation zone of the stray currents.

In the first case, the magnitude and sign of the potential of each ground of the single-wire circuit will depend on the graph of the movement of the electric trains, the specific conductivity of the ground, the method of feeding the contact circuit (two-way or one-way) and the location of the circuit ground with respect to the railroad bed. In the conductor joining two grounds, under the effect of the potential difference an equalizing current arises, the magnitude of which can be determined by (2.115), where $\Delta U$ must be understood as the potential difference of the points in the ground in which the grounds of a single-wire circuit are located, that is, $\Delta U = U_1 - U_2$. Since the potentials $U_1$ and $U_2$ in this formula can have values from $+U_{1\,max}$ to $-U_{1\,max}$ and from $+U_{2\,max}$ to $-U_{2\,max}$, the potential difference can assume different values: $(-U_1-U_2)$ $(+U_1-U_2)$, $(-U_1+U_2)$, $(U_1+U_2)$; here the absolute values of $U_1$ and $U_2$ can fluctuate from 0 to $U_{max}$. In practice, the nearby traction substations basically influence the magnitude of the ground potentials of the single-wire circuits. Consequently, the interfering currents in the single-wire circuit can have different magnitude and change their direction. In order to confirm what was stated in Fig 2.12 the curves are presented for the variation of the interfering voltages (ΔU) obtained by the TsNIIS Institute for the single-wire circuit of different length running along the electrified railroad; both grounds are located in the propagation zone of the stray electric currents.

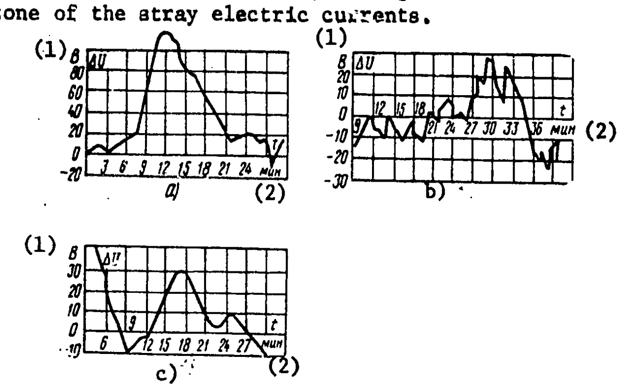



Figure 2.12. Variation of the interfering voltage in the single-wire circuit for location of both grounds in the stray current zone with a wire length of:
a) 69 km; b) 146 km; c) 53 km

Key:
1. volts
2. minute

If one of the circuit grounds is located near the electrified section of the railroad, and the other is outside the zone of effect of the electrified railroad ($U_2$=0), the magnitude of the currents in the circuit will vary directly proportionally to the potential difference ($\pm U_1$-0), that is, ($\pm U_1$). The performed measurements demonstrated that the interference voltage in the single-wire circuits as a result of the galvanic effect of the electric railways can reach 10-60 volts for normal operation of the railroad and 250-300 volts for disconnection of the traction substation closest to the terminal station to which the remote feed circuit is connected. In direct proximity to the rails the potential difference between the points along the ground remote from each other can reach 700 volts or more. Considering that in the current sources of the remote feed circuits, depending on the transition system, the working voltages of 250, 350-400 volts are used, the above-indicated interference voltages are not permitted.

The potentials of the ground points created by the stray currents of the electric railroad can be defined by theoretical means. For an infinitely long railroad and a single load as is schematically demonstrated in Fig 2.13, we obtain the following expressions for the potential U(x,y) of point A of the ground with the coordinates x,y [21]:

$$U(x,\ y)=\frac{I\rho_3\gamma_p}{2\pi}\Omega(\gamma_p x,\ \gamma_p y). \qquad (2.116)$$

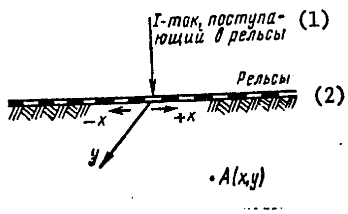



Figure 2.13. Calculation of potentials at point A located in the stray current field

Key:
1. I -- current going to the rails
2. Rails

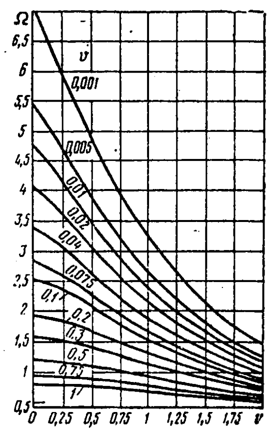



Figure 2.14. Dependence of the function Ω(u,v) on u lying within the limits of 0-2 for different values of v (0.001-1)

The maximum potential occurs for x=0, so that

$$U_{\text{макс}} = U(0,\ y) = \frac{I\rho_{з}\gamma_{р}}{2\pi}\,\Omega(0,\ \gamma_{р}\,y), \tag{2.117}$$

Key: 1. max; 2. g

where $\rho_g$ is the specific resistance of the ground, ohm-m; Ω is a special function taking into account the dependence of the potentials of the ground points on x and y. The values of this function are presented in Fig 2.14.

Let us consider the pro· ·rties of the function Ω in more detail. The function Ω(u,v) is posit.ve for all finite values of the argument, and it has a finite value. For $v \to \infty$ and finite u and also for $u \to \infty$ and finite v the function $\Omega(u,v) \to 0$. For $v \to 0$ and any u $\Omega(u,v) \to \infty$. At all points except the points for $v \to 0$, the function Ω(u,v) is continuous. Ω(u,v) is an even function of the arguments u and v. For a fixed finite value of v, the function Ω has a unique maximum at the point u=0; at all remaining points Ω(u,v) decreases continuously. For large values of the arguments v and finite u the function $\Omega(u,v) \to 1/v$, and for large values of the argument u and finite v, $\Omega(u,v) \to 1/u$. For $u \geq 10$ and $v \neq 0$

$$\Omega(u,\ v)=\frac{1}{u}\left(1+\frac{2!}{u^2}+\frac{4!}{u^4}+\ \cdots\right);$$

for u=0

$$\Omega(0,\ v)=\frac{\pi}{2}\left[S_0(v)-Y_0(v)\right],$$

where $S_0$ is the Strouve function; $Y_0$ is the Neuman function [21]. For $v \ll 1$

$$\Omega(0,\ v)\approx K_0(v),$$

where $K_0$ is the second type, zero-order Bessel function; $\gamma_{rail}=\sqrt{\frac{r_{rail}}{R_{trans.rail}}}$ is the propagation coefficient of the current in the rails; $r_{rail}$ is the longitudinal resistance of DC rail, ohm/m; $R_{trans.rail}$ is the transient resistance between the rails and the ground, ohm-m.

Replacing the rails by the equivalent cylinder, the transient resistance between the rails and the ground can be found from the expression

$$\underset{(1)}{R_{\text{пер.р}}}=\underset{(2)}{R_{\text{нз.р}}}+\frac{\overset{(3)}{\rho_з}}{\pi}\ln\frac{1.12}{\underset{(4)}{\gamma_р}a}, \tag{2.118}$$

Key: 1. trans.rail; 2. ins.rail; 3.g; 4. rail

where $R_{ins.rail}$ is the resistance of the insulating (ballast) layer in the roadbed of the railroad, ohm-m; a is the equivalent radius of the rail, meters.

If the ties are considered as electrodes from which the current flows into the ground, then the rail track can be represented in the form of a solid strip with width equal to the width of the upper structure of the railroad track. The equivalent radius of this plate [21] is equal to 1/4 of its width, that is, a=(1/4)b, where b is the width of the upper structure of the railroad track. For a single track railroad, a=1 meter; for a double-track railroad, a=2 meters.

The formula (2.116) was obtained under the assumption that the return pole of the current source is at infinity. As applied to railroad tracks, this formula gives the solution for the case of one load (the electric locomotive) located far from the traction substations; here it is proposed that the contact wire is connected to the positive pole of the current source. Accordingly, in order to consider the effect of the return pole of the current source, that is, the traction substation, it is possible to use the method of superposition. For the case of one traction substation and one electric locomotive with a distance between them of $\ell$, the resultant potential U'(x,y) of the ground point will be determined from the expression

$$U'(x,\ y)=U(x,\ y)-U(|l-x|,\ y)$$

or

$$U'(x,\ y)=\frac{I\overset{(1)}{\rho_з}\gamma_p}{2\pi}[\Omega(\gamma_p x,\ \gamma_p y)-\Omega(\gamma_p|l-x|,\ \gamma_p y)]. \quad (2.119)$$

Key: 1. g

Analogously, it is possible to determine the potential of the investigated ground point for any number of electric locomotives in the section. As the electric locomotives move, the potentials of the ground points will vary as a result of variation of the spacing between the electric locomotives and the traction substation. It is obvious that from the point of view of protection from interference it is necessary to consider the maximum possible potential difference between the ground points at which the grounds of the remote feed circuit are located. This maximum possible potential difference will depend primarily on the location of the grounds of the single-wire circuit with respect to the railroad bed. Let us consider several typical cases.

1. The single-wire circuit of an overhead or cable line is intersected by an electric railroad near the traction substation. One of the grounds of the circuit is located near the railroad bed. In this case the potential at the location of the second ground in practice can be taken equal to zero, and the maximum potential at the location of the first ground can be defined by the formula (2.116). Then

$$\Delta U=\frac{\overset{(1)}{I_{т.п}}\overset{(2)}{\rho_з}\gamma_p}{2\pi}\underset{(3)}{\Omega}(\underset{(4)}{\gamma_p} x,\ \gamma_p y), \quad (2.120)$$

Key: 1. traction substation; 2. g; 3. rail

where $I_{traction\ substation}$ is the maximum current of the traction substation.

2. The single-wire circuit of the overhead or cable line intersects an electric railroad in the middle between the traction substations. One of the grounds is located near the railroad bed. In this case the maximum grounding potential is caused by the load currents from the electric locomotives passing by, and it can be defined by (2.116) considering the effect of the currents of the adjacent locomotives. The potential of the second ground can be taken equal to zero. Then

$$\Delta U = \frac{\overset{(1)}{\rho_з}\gamma_р}{2\pi} \sum_{k=1}^{\overset{(2)}{n}} I_{\underset{(3)}{эл.к}} \Omega(\gamma_{\underset{(4)}{р}} |x_k|, \; \gamma_р y), \tag{2.121}$$

Key: 1. g; 2. rail; 3. k-th electric locomotive; 4. rail

where $I_{k\text{-th electric locomotive}}$ is the current consumed by the k-th electric locomotive; $x_k$ is the distance along the x-axis from the first ground to the k-the electric locomotive; for an electric locomotive passing the point of intersection $X_k=0$; n is the number of electric locomotives which must be considered when determining $\Delta U$.

3. The single-wire circuit of an overhead or cable line is arranged parallel to the electric railroad. The first ground is located opposite the traction substation, the spacing between the grounds of the single-wire circuit is equal to half the distance between the traction substations. In this case the potential created at the location of the first ground will be opposite in sign with respect to the potential created by the load currents of the electric locomotives at the location of the second ground. With some approximation it is possible to consider that the load current of the traction substation is equal to the total load current consumed by the electric locomotive near the second ground. Then the potential difference occurring in the single-wire circuit will be equal to twice the potential created at the location of the first ground, that is,

$$\Delta U = \frac{I_{т.п}\rho_з\gamma_р}{\pi}\Omega(0, \; \gamma_р y). \tag{2.122}$$

The investigated case is the worst with respect to interference in the single-wire circuit.

4. The single-wire circuit is arranged parallel to the electric railroad. Both grounds of the remote feed circuit are located opposite the adjacent traction substations. In this case the potential difference occurring between the grounds will be determined at each given point by the difference in load currents of the traction substations. Considering, however, that at certain points in time one of the traction substations can be completely loaded and the other not loaded at all, the potential difference for the calculations must be determined by (2.116) for this worst case.

Fig 2.15 shows the theoretically calculated boundary of the potential difference $\Delta U$ between the grounds of a single-wire circuit as a function of the specific resistance of the ground for the case where one of the grounds is located near a DC electric railroad bed near the traction substation, and the other ground is outside the stray current zone. The calculation is performed for single track and double track railroads with a load current of the traction substation of 1500 and 3000 amps respectively. The spacing between the first ground of the single-wire circuit and the electric railroad rails is equal to 100 meters, the x-coordinate [formula (2.120)] is assumed

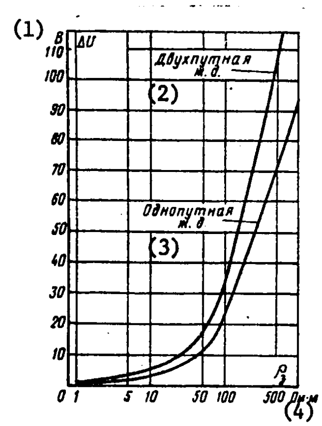



Figure 2.15. Potential difference ΔU between the ground of a single-wire circuit as a function of the specific resistance of the ground $\rho_g$ for the case where one ground is located near the DC electric railroad near the traction substation, and the other is outside the stray current zone

Key:
1. volts
2. Double-railroad track
3. Single railroad track
4. ohm-m

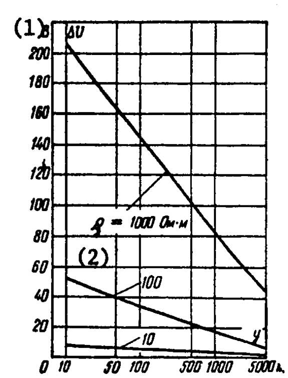



Figure 2.16. Potential difference ΔU between the grounds of the single-wire circuit as a function of the distance (y) between the first ground and the double-track DC railroad bed for different values of the specific resistance of the ground (the second ground is outside the stray current zone)

Key: 1. volts; 2. $\rho_g$=1000 ohm-m

equal to zero. As is obvious from Fig 2.15, the potential difference with large specific resistance of the ground reaches a quite significant value (100 to 150 volts), which was observed in practice.

Fig 2.16 shows the dependence of the potential difference calculated for the same case of location of the ground and caused by the galvanic effect for a distance y between the first ground and the double-track DC railroad bed and different values of the specific resistance of the ground for a traction load current of 3000 amps.

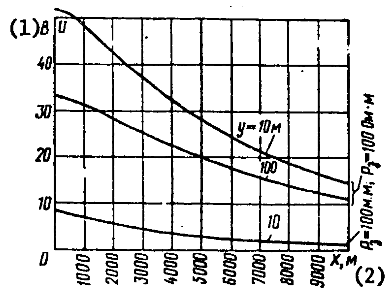



Figure 2.17. Curves for the potential U of the ground point as a function of the distance x between this point and the traction substation (or electric locomotive) for different values of $\rho_g$ and the distances y to the rails

Key:

1. volts
2. $\rho_g$=10 ohms-meters; $\rho_g$=100 ohm-meters

Fig 2.17 shows the potential at the investigated ground point as a function of the distance x between this point and the location of the traction substation (or the electric locomotive) for different values of the specific resistance of the ground ρ and diferent distances y from the same point to the rails. The calculation was made for a load current of 3000 amps (approximately to the double-track electric railroad).

The presented calculations indicate that the potential difference at different points of the ground caused by the effect of the stray electric traction currents can reach significant values entirely inadmissible with respect to interference in the single-wire communication circuits.

Effect of AC Electric Railroads. The AC electric railroads, just as the DC electric railroads, are systems in which the return current returns through the rails and partially through the grounds. However, in the given case, in contrast to the DC electric traction it is necessary to consider the magnetic effect of the contact network on the rails. At each point in time, a current will be induced in the rails which is approximately opposite with respect to phase to the current coming directly to the rails. The current in the rails will be closed through the ground and at various points of the ground it will create potentials which are opposite with respect to sign to the potentials created by the current coming directly to the rails from the wheels of the electric locomotive.

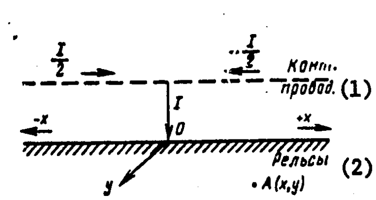



Figure 2.18. Derivation of the formula for the potential of point A of the ground near an AC electric railroad

Key:
1. Contact wire
2. Rails

Let us consider the case of location of the point A of the ground near a DC electric railroad (Fig 2.18). In accordance with what has been discussed above, the potential at any point of the ground can be represented in the form of the sum of two components: the potential $\dot{U}_{g.g}$ created by the currents coming directly through the electric locomotive to the rails and from the rails to the ground, and the potential $U_{g.m}$ created by the currents induced as a result of the magnetic effect of the contact network and flowing from the rails to the ground. Thus,

$$\underset{(1)}{\dot{U}_{\text{г}}}=\underset{(2)}{\dot{U}_{\text{г.г}}}+\underset{(3)}{\dot{U}_{\text{г.м}}}. \qquad (2.123)$$

Key: 1. g; 2. g.g; 3. g.m

The value of $\dot{U}_{g.g}$ is determined analogously to the preceding by the formula

$$\underset{(1)}{\dot{U}_{\text{г.г}}}=\frac{\dot{I}\underset{(3)}{\rho_{\text{з}}}\gamma_{\text{р}}}{2\pi}\Omega(\gamma_{\text{р}}x,\ \gamma_{\text{р}}y). \qquad (2.124)$$

$$\underset{(2)}{\dot{U}_{\text{г.м}}}=-\frac{\dot{I}\rho_{\text{з}}Z_{1\text{р}}\gamma_{\text{р}}}{2\pi \underset{(4)}{Z_{\text{р}}}}\Omega(\gamma_{\text{р}}x,\ \gamma_{\text{р}}y), \qquad (2.125)$$

Key: 1. g.g; 2. g.m; 3. g; 4. rail

where $Z_{1\ rail}$ is the mutual resistance between the contact wire and the rails, ohm/m; $Z_{rail}$ is the total resistance of the rails, ohm/m; $\gamma_{rail}=\sqrt{Z_{rail}/R_{trans.\ rail}}$, 1/m.

The transient resistance between the rails and the ground can be determined with a sufficient degree of accuracy from the formula

$$\underset{(1)}{R_{\text{пер.р}}} = \underset{(2)}{R_{\text{из.р}}} + \underset{(3)}{(\rho_з/\pi)} \ln \frac{1,12}{a\sqrt{|Z_p|/R_{\text{пер.р}}}} \cdot \quad (1)$$

Key: 1. trans. rail; 2. ins. rail; 3. g

Substituting (2.124) and (2.125) in (2.123), we obtain

$$\dot{U}_г = \frac{\dot{I}\rho_з\gamma_p}{2\pi}\,\Omega(\gamma_p x,\ \gamma_p y)\left[1 - \frac{Z_p}{Z_p}\right].$$

The expression in brackets is the ideal coefficient of shielding effect of the rails $S_{\text{rails ideal}}$. Hence

$$\underset{(1)}{\dot{U}_г} = \frac{\dot{I}\,\overset{(2)}{\rho_з\gamma_p}\,\overset{(3)}{S_{\text{р.ид}}}}{2\pi}\,\Omega(\gamma_p x,\ \gamma_p x). \qquad (2.126)$$

Key: 1. g; 2. rail; 3. rail ideal

The dependence of the function $\Omega(u,v)$ for $u=|u|e^{i\phi_u}$ and $v=|v|e^{i\phi_v}$ on $|u|$ for different values of $|v|$ for two values of the angles $\phi_u=\phi_v=30°$ and $\phi_u=\phi_v=45°$ is presented in Fig 2.19a and b.

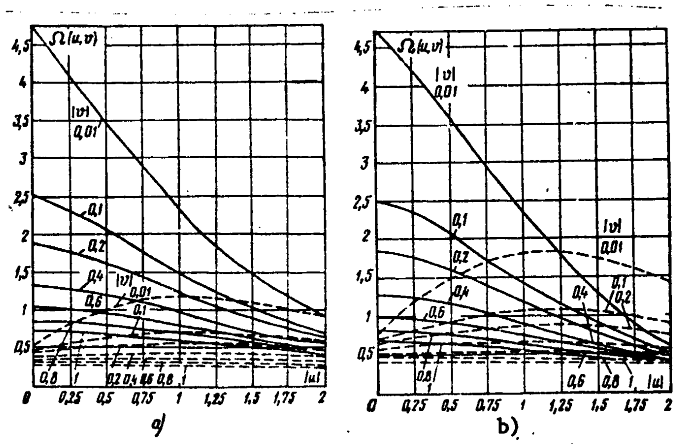



Figure 2.19. Dependence of the function $\Omega(u,v)$:
a) for $\phi_u=\phi_v=30°$; b) for $\phi_u=\phi_v=45°$ ——— active component (>0);
- - - imaginary component (<0)

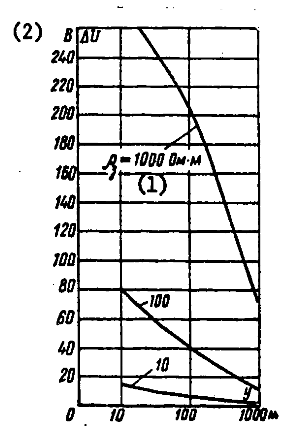



Figure 2.20. Potential difference ΔU between the grounds of the single-wire circuit as a function of the distance y between the first ground and the double-track electric railroad bed

Key:
1. $\rho_g$=1000 ohm-m
2. volts

All of the above-presented formulas for different cases of convergence of the single-wire circuit with the DC electric railroad are valid also for the AC electric railroad under the condition of introduction of the coefficient $S_{rail\ ideal}$ into the righthand sides of the formulas.

Fig 2.20 shows the dependence of the potential difference ΔU on the distance y between the first ground and the double-track AC electric railroad bed for various values of the specific resistance $\rho_g$ of the ground (the second ground of the single-wire circuit is located outside the zone of propagation of the stray currents). These curves are calculated for the traction load current of 1000 amps. From a comparison of the values of ΔU it is obvious that for small values, higher potential differences are created. This is explained by significantly greater (by 6 to 10 times) differences in the propagation constant of an alternating current in the rails. At the same time for large distances y (near 1000 meters) the potential difference created by the DC electric traction is greater than for AC electric traction, which is explained by the nature of variation of the function Ω and the presence in (2.126) of the cofactor $S_{rail\ ideal}$.

# CHAPTER 3. EFFECT OF ENTIRELY ASYMMETRIC CIRCUITS ON THE TWO-WIRE OVERHEAD LINE CIRCUITS

## 3.1. Equations in General Form

Let us consider the convergence of a single-wire interfering circuit and the double-wire telephone circuit of an overhead line subjected to the effect (see Fig 3.1). For simplification of the derivations, let us assume that the length of the circuit is equal to the convergence length. Let us introduce additional notations: $\dot{U}_a(x)$, $\dot{U}_b(x)$ are the voltages with respect to ground at any point x of the wires a and b of the telephone circuit, volts;

$\dot{I}_a(x)$, $\dot{I}_b(x)$ are currents in the wires a and b of the telephone circuit, amps;

$Y_{ab}=G_{ab}+i\omega C_{ab}$ is the electrical coupling between the wires a and b of the telephone circuit, siemens/km;

$Z_{ab}=R_{ab}+i\omega L_{ab}$ is the magnetic coupling between the wires of the telephone circuit, ohm/km;

$Y_{1a}=G_{1a}+i\omega C_{1a}$, $Y_{1b}=G_{1b}+i\omega C_{1b}$ is the electric coupling between the interfering circuit and each wire of the telephone circuit, siemens/km;

$Z_{1a}=R_{1a}+i\omega L_{1a}$, $Z_{1b}=R_{1b}+i\omega L_{1b}$ is the magnetic coupling between the interfering circuit and each wire of the telephone circuit, ohm/km.

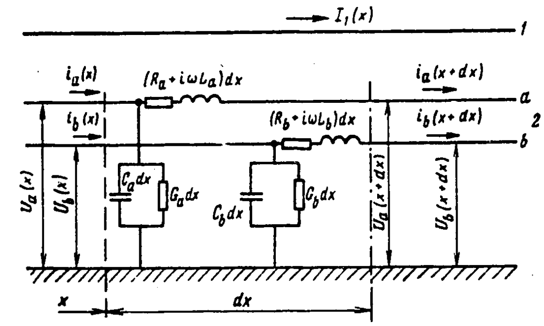



Figure 3.1. Convergence of the single-wire interfering line (1) with a two-wire telephone circuit (2)

The linear differential equations of voltage and current in each wire of the telephone circuit have the form

$$\left.\begin{aligned}
-\frac{d\dot U_a}{dx} &= [R_a - R_{ab} + i\omega(L_a - L_{ab})]\dot I_a + \dot I_1(R_{1a} + i\omega L_{1a}),\\
-\frac{d\dot U_b}{dx} &= [R_b - R_{ab} + i\omega(L_b - L_{ab})]\dot I_b + I_1(R_{1b} + i\omega L_{1b}),\\
-\frac{d\dot I_a}{dx} &= (G_a + i\omega C_a)\dot U_a + (\dot U_a - \dot U_b)(G_{ab} + i\omega C_{ab}) +\\
&\qquad + (\dot U_a - \dot U_1)(G_{1a} + i\omega C_{1a}),\\
-\frac{d\dot I_b}{dx} &= (G_b + i\omega C_b)\dot U_b + (\dot U_b - \dot U_a)(G_{ba} + i\omega C_{ba}) +\\
&\qquad + (\dot U_b - U_1)(G_{1b} + i\omega C_{1b}),
\end{aligned}\right\} \quad (3.1)$$

where $\dot U_1$ and $\dot I_1$ are the voltage and current at any point of the interfering circuit.

When analyzing the induced voltages and currents in the two-wire circuit it is expedient to use the terminology of the method of symmetric components. As is known, the method of symmetric components can be used when investigating any n-phase system where the number of symmetric components is equal to the number of phases (in the triple-phase system there are three: zero, direct and return). The two-wire communication circuit can be provisionally called two-phase by analogy with three-phase with respect to number of wires. In this system the two components are zero and direct. The currents and voltages of the zero sequence are equal in the two wires and have the same direction. The current in the ground is equal to twice the value of the current in the wire. The voltages of the zero sequence between the wire and the ground are equal to each other for the two wires. The currents of the direct sequence in both wires are equal to each other, but they have directions opposite to each other. The voltage of the direct sequence in each wire is equal to half the voltage between the wires. The currents and voltages of the direct sequence in the given analysis are the interference currents and voltages in the two-wire telephone circuit which also requires definition.

Taking this into account, let us compile the differential equations for the voltages and currents induced in the asymmetric telephone circuit, expressing the latter in terms of the symmetric components of the direct and the zero sequence:

$$\dot{U}_A = \frac{1}{2}(\dot{U}_a + \dot{U}_b),\ \dot{I}_A = \frac{1}{2}(\dot{I}_a + \dot{I}_b), \tag{3.2}$$

where $\dot{U}_a = \dot{U}_A - \Delta\dot{U};\ \dot{U}_b = \dot{U}_A + \Delta\dot{U};\ \dot{I}_a = \dot{I}_A - \Delta\dot{I};\ \dot{I}_b = \dot{I}_A + \Delta\dot{I}.$

Then let us derive the arithmetic mean values of all of the primary parameters of the communication circuit, representing them in the following form:

$$\left.\begin{aligned} R_A &= \frac{1}{2}(R_a + R_b);\ R_a = R_A - \Delta R;\ R_b = R_A + \Delta R;\\ L_A &= \frac{1}{2}(L_a + L_b);\ L_a = L_A - \Delta L;\ L_b = L_A + \Delta L;\\ C_A &= \frac{1}{2}(C_a + C_b);\ C_a = C_A - \Delta C;\ C_b = C_A + \Delta C;\\ G_A &= \frac{1}{2}(G_a + G_b);\ G_a = G_A - \Delta G;\ G_b = G_A + \Delta G. \end{aligned}\right\} \tag{3.3}$$

In exactly the same way the total mutual conductivities between the circuits and the total mutual resistances between them are represented in the following equations:

$$\left.\begin{aligned} &Y_{1A} = \frac{1}{2}(Y_{1a} + Y_{1b});\\ &Y_{1a} = Y_{1A} - \Delta Y_{1A} = G_{1A} - \Delta G_{1A} + i\omega(C_{1A} - \Delta C_{1A});\\ &Y_{1b} = Y_{1A} + \Delta Y_{1A} = \Delta G_{1A} + G_{1A} + i\omega(C_{1A} + \Delta C_{1A});\\ &Z_{1A} = \frac{1}{2}(Z_{1a} + Z_{1b});\\ &Z_{1a} = Z_{1A} - \Delta Z_{1A} = R_{1A} - \Delta R_{1A} + i\omega(L_{1A} - \Delta L_{1A});\\ &Z_{1b} = Z_{1A} + \Delta Z_{1A} = R_{1A} + \Delta R_{1A} + i\omega(L_{1A} + \Delta L_{1A}). \end{aligned}\right\} \tag{3.4}$$

In these expressions the values of the indexes a and b denote significant values of the voltages, currents and the parameters of the telephone circuit; the values of the index A denote the components of the zero sequence; the components of the direct sequence in each wire are denoted by $\pm\Delta\dot{U}$, $\pm\Delta\dot{I}$. Replacing significant values of all of the variables entering into the equations of system (3.1) by the symmetric components presented in (3.3) and (3.4) and subtracting the second equation of the system (3.1) from the first equation of this system and the fourth from the third, we obtain

$$-\frac{d(2\Delta\dot{U})}{dx}=2\Delta\dot{I}(R_A-R_{ab}+i\omega(L_A-L_{ab})+2\dot{I}_A(\Delta R+ +i\omega\Delta L)+2\dot{I}_1(\Delta R_{1A}+i\omega L_{1A}). \tag{3.5}$$

$$-\frac{d(2\Delta\dot{I})}{dx}=2\Delta\dot{U}[G_A+2(G_{ab}+i\omega C_{ab})+i\omega C_A+G_{1A}+i\omega C_{1A}]+ +2\dot{U}_A(\Delta G+i\omega\Delta C+\Delta G_{1A}+i\omega C_{1A})-2\dot{U}_1(\Delta G_{1A}+i\omega\Delta C_{1A}). \tag{3.6}$$

Let us introduce the additional notation:

$\Delta G+i\omega\Delta C=\rho$ is the asymmetry of the conductivity of the telephone circuit wires with respect to the ground, siemens/km;

$2(\Delta R+i\omega\Delta L)=\xi$ is the asymmetry of the resistance of the telephone circuit wires, ohms/km;

$2(\Delta R_{1A}+i\omega\Delta L_{1A})=2\Delta Z_{1A}=Z_{1T}$ is the magnetic coupling between the high voltage wire and the two-wire telephone circuit, ohms/km;

$\Delta G_{1A}+i\omega C_{1A}=Y_{1T}$ is the electric coupling between the high voltage wire and the two-wire telephone circuit, siemens/km;

$Z_T$, $Y_T$ are the parameters of the two-wire telephone circuit which are expressed by the equations

$$\left.\begin{aligned}&Z_T=2[R_A+R_{ab}+i\omega(L_A-L_{ab})];\\&Y_T=\frac{1}{2}[G_A+G_{1A}+i\omega(C_A+C_{1A})]+G_{ab}+i\omega C_{ab};\\&2\Delta\dot{U}=\dot{U}_T;\ \frac{d\dot{U}_T}{dx}=\dot{U}'_T;\ \Delta\dot{I}=\dot{I}_T;\ \frac{d\dot{I}_T}{dx}=\dot{I}'_T.\end{aligned}\right\} \tag{3.7}$$

Introducing this notation into (3.5) and (3.6), we obtain

$$\left.\begin{aligned}&\dot{U}'_T+\dot{I}_TZ_T+\dot{I}_A\xi+\dot{I}_1Z_{1T}=0,\\&\dot{I}'_T+\dot{U}_TY_T+\dot{U}_A(\rho+Y_{1T})-U_1Y_{1T}=0.\end{aligned}\right\} \tag{3.8}$$

Since these equations include the values characterizing the asymmetry of the telephone circuit with respect to the interfering line and with respect to the ground, each of the values of $\dot{U}_T$ and $\dot{I}_T$ can be represented in the form of the sum of two components: $\dot{U}_T=\dot{U}_{T1}+\dot{U}_{T2}$, $\dot{I}_T=\dot{I}_{T1}+\dot{I}_{T2}$. Let us call the components $\dot{U}_{T1}$ and $\dot{I}_{T1}$ the interference voltage and current occurring in the telephone circuit as a result of asymmetry of its wires with respect to the interfering wire and the components $\dot{U}_{T2}$ and $\dot{I}_{T2}$, the voltage and current of the interference in the same circuit occurring as a result of asymmetry of the total resistance and the total conductivity of the telephone circuit wires with respect to ground. It is most convenient to determine the indicated components of the voltage and current separately, using the method of superposition. For this purpose, when determining the components $\dot{U}_{T1}$ and $\dot{I}_{T1}$ we assume that the factors causing the appearance of the interference voltage $\dot{U}_{T2}$ and current $\dot{I}_{T2}$ are absent; when determining the components $\dot{U}_{T2}$ and $\dot{I}_{T2}$ we assume that there are no factors causing the appearance of the voltage $\dot{U}_{T1}$ and the current $\dot{I}_{T1}$. Then the general system (3.8) can be represented in the form of two systems of equations, namely:

1) For $\Delta R=0$, $\Delta G=0$, $\Delta L=0$, $\Delta C=0$ and in the presence of asymmetry of the circuit wires with respect to the interfering wire ($\dot{Y}_{1T}\neq 0$, $Z_{1T}\neq 0$) we obtain the first system of equations:

$$\left.\begin{aligned} \dot{U}'_{\text{т}1}+\dot{I}_{\text{т}1}Z_{\text{т}}+\dot{I}_1 Z_{1\text{т}}=0, \\ \dot{I}'_{\text{т}1}+\dot{U}_{\text{т}1}Y_{\text{т}}+(\dot{U}_{\text{A}}-\dot{U}_1)\,Y_{1\text{т}}=0; \end{aligned}\right\} \qquad (3.9)$$

2) For $Y_{1T}=0$ and $Z_{1T}=0$ and in the presence of asymmetry in the wire resistance and in their conductivity with respect to the ground, that is, for $\Delta R\neq 0$, $\Delta L\neq 0$, $\Delta C\neq 0$, $\Delta G\neq 0$, we obtain the second system of equations:

$$\left.\begin{aligned} \dot{U}'_{\text{т}2}+\dot{I}_{\text{т}2}Z_{\text{т}}+\dot{I}_{\text{A}}\,\varepsilon=0, \\ \dot{I}'_{\text{т}2}+\dot{U}_{\text{т}2}Y_{\text{т}}+\dot{U}_{\text{A}}\,\rho=0. \end{aligned}\right\} \qquad (3.10)$$

The systems of equations (3.9) and (3.10) will be solved individually.

3.2. Determination of the Interference Voltage Components Caused by Asymmetry of Arrangement of the Telephone Circuit Wires with Respect to the Interfering Wire

In order to determine the first components $\dot{U}_{T1}$ and $\dot{I}_{T1}$ of the interference voltage and current in the telephone circuit it is necessary to solve the system of equations (3.9). Since the value of $\dot{U}_A$ (the voltage component of the zero series induced in the telephone circuit) is small by comparison with the magnitude of the interfering voltage in the interfering wire $U_1$, in equation (3.9) it can be neglected. Then we obtain

$$\left.\begin{aligned} \dot{U}'_{\text{т}1}+\dot{I}_{\text{т}1}Z_{\text{т}}+\dot{I}_1 Z_{1\text{т}}=0, \\ \dot{I}'_{\text{т}1}+\dot{U}_{\text{т}1}Y_{\text{т}}-\dot{U}_1 Y_{1\text{т}}=0. \end{aligned}\right\} \qquad (3.11)$$

Let us determine the component $\dot{U}_{T1}$ for the uncrossed circuit. The system (3.11) is analogous to the above-solved system of differential equations (2.5) in which the parameters of the effect between the single-wire circuits $Z_{12}$ and $Y_{12}$ are replaced by the parameters of the effect between the single-wire and two-wire circuits $Z_{1T}$ and $Y_{1T}$, and the voltages and currents in the single-wire circuit subjected to the effect $\dot{U}_{2II}$ and $\dot{I}_{2II}$ are replaced by analogous values in two-wire circuits $\dot{U}_{T1}$ and $\dot{I}_{T1}$. Introducing the propagation coefficient in the two-wire circuit $\gamma_T=\sqrt{Z_T Y_T}$ and the reflection coefficients $p_{t.b}$ at the beginning and $p_{t.e}$ at the end of the two-wire circuit, on the basis of the solutions of (2.18) and (2.19), considering that $\ell_I=\ell_{III}=0$, we obtain

$$
\begin{aligned}
\dot U_{\text{т}1}(x) = {} & -\frac{\dot U_1(0)}{\gamma_{\text{т}}^2-\gamma_1^2}\left[\frac{\overset{(1)}{\mathrm{ch}\ln\sqrt{p_{\text{т.н}}}}\,[N'_{1\text{т}}\,\mathrm{th}\,(\gamma_1 l_{II}-\ln\sqrt{p_1}) \to}{\mathrm{sh}\,(\gamma_{\text{т}} l_{II}-\ln\sqrt{p_{\text{т.н}}p_{\text{т.к}}})}\right. \\
& \frac{\to -N''_{1\text{т}}\,\mathrm{th}\,(\ln\sqrt{p_{\text{т.н}}})]}{}\,\mathrm{ch}\left[\gamma_{\text{т}}(l_{II}-x)-\ln\sqrt{p_{\text{т.к}}}\right]+ \\
& +\frac{\overset{(2)}{\mathrm{sh}\ln\sqrt{p_{\text{т.к}}}}\,[N'_{1\text{т}}\,\mathrm{sh}\ln\sqrt{p_1}\,\mathrm{cth}\,(\ln\sqrt{p_{\text{т.к}}})-N''_{1\text{т}}\,\mathrm{ch}\ln\sqrt{p_1}]}{\mathrm{sh}\,(\gamma_{\text{т}} l_{II}-\ln\sqrt{p_{\text{т.к}}p_{\text{т.н}}})\,\mathrm{ch}\,(\gamma_1 l_{II}-\ln\sqrt{p_1})}\times \\
& \left.\times\,\mathrm{ch}\left(\gamma_{\text{т}} x-\ln\sqrt{p_{\text{т.н}}}\right)-\frac{N''_{1\text{т}}\,\mathrm{ch}\,[\gamma_1(l_{II}-x)-\ln\sqrt{p_1}]}{\mathrm{ch}\,(\gamma_1 l_{II}-\ln\sqrt{p_1})}\right];
\end{aligned}
\tag{3.12}
$$

$$
\begin{aligned}
\dot I_{\text{т}1}(x) = {} & \frac{\dot U_1(0)}{z_{\text{в.т}}(\gamma_{\text{т}}^2-\gamma_1^2)}\left[\frac{\mathrm{sh}\,[\gamma_1(l_{II}-x)-\ln\sqrt{p_1}]\,N'_{1\text{т}}}{\mathrm{ch}\,(\gamma_1 l_{II}-\ln\sqrt{p_1})}+\right. \\
& +\frac{\mathrm{sh}\ln\sqrt{p_{\text{т.к}}}\,[N'_{1\text{т}}\,\mathrm{sh}\ln\sqrt{p_1}\,\mathrm{cth}\,(\ln\sqrt{p_{\text{т.к}}})-N''_{1\text{т}}\,\mathrm{ch}\ln\sqrt{p_1}]}{\mathrm{ch}\,(\gamma_1 l_{II}-\ln\sqrt{p_1})\,\mathrm{sh}\,(\gamma_{\text{т}} l_{II}-\ln\sqrt{p_{\text{т.н}}p_{\text{т.к}}})}\times \\
& \times\,\mathrm{sh}\left(\gamma_{\text{т}} x-\ln\sqrt{p_{\text{т.н}}}\right)-\frac{\mathrm{ch}\ln\sqrt{p_{\text{т.н}}}\,[N'_{1\text{т}}\,\mathrm{th}\,(\gamma_1 l_{II}-\ln\sqrt{p_1}) \to}{\mathrm{sh}\,(\gamma_{\text{т}} l_{II}-\ln\sqrt{p_{\text{т.н}}p_{\text{т.к}}})} \\
& \left.\frac{\to -N''_{1\text{т}}\,\mathrm{th}\,\ln\sqrt{p_{\text{т.н}}}]\,\mathrm{sh}\,[\gamma_{\text{т}}(l_{II}-x)-\ln\sqrt{p_{\text{т.к}}}]}{}\right],
\end{aligned}
\tag{3.13}
$$

Key: 1. b; 2. e

where $N'_{1\text{т}}=Y_{1\text{т}}z_{\text{в.т}}\gamma_1-\frac{Z_{1\text{т}}}{z_{\text{в}1}}\gamma_{\text{т}}$; $N''_{1\text{т}}=Y_{1\text{т}}z_{\text{в.т}}\gamma_{\text{т}}-\frac{Z_{1\text{т}}}{z_{\text{в}1}}\gamma_1$.

The formulas (3.12) and (3.13) are the general formulas of the effect of the single-wire circuit on a two-wire, uncrossed circuit as a result of asymmetry of arrangement of the circuit wires with respect to the interfering wire, and they take into account the arbitrary load resistances in both circuits.

Under actual conditions the two-wire circuit, as a rule, is loaded at the ends by wave impedance. If it is assumed here that the interfering circuit is also loaded under a wave impedance we obtain comparatively simple formulas by which usually the voltage induced in the two-wire circuit is calculated. For the indicated load conditions

$$
\dot U_{\text{т}1}(x)=-\frac{\dot U_1(0)}{2(\gamma_{\text{т}}^2-\gamma_1^2)}\left[(N'_{1\text{т}}+N''_{1\text{т}})\,e^{-\gamma_{\text{т}}x}+(N'_{1\text{т}}-N''_{1\text{т}})\,e^{-\gamma_{\text{т}}(l_{II}-x)-\gamma_1 l_{II}}-2N''_{1\text{т}}\,e^{-\gamma_1 x}\right].
\tag{3.14}
$$

For x=0, that is, at the beginning of the circuit

$$
\dot U_{\text{т}1\text{н}}=\frac{\dot U_1(0)}{2(\gamma_1+\gamma_{\text{т}})}z_{\text{в.т}}\left(\frac{Z_{1\text{т}}}{z_{\text{в}1}z_{\text{в.т}}}+Y_{1\text{т}}\right)\left[1-e^{-(\gamma_1+\gamma_{\text{т}})l_{II}}\right].
\tag{3.15}
$$

The expression (3.15) can be represented in the form of the sum of two components taking into account the magnetic and electric effects separately

$$\dot{U}_{\text{т1н}} = \dot{U}_{\text{т1 н.м}} + \dot{U}_{\text{т1 н.э}},$$

where

$$\dot{U}_{\text{т1 н.м}} = \frac{\dot{I}_1(0)\, Z_{1\text{т}}}{2(\gamma_1+\gamma_\text{т})}\left[1-e^{-(\gamma_1+\gamma_\text{т})\, l_{II}}\right], \tag{3.16}$$

$$\dot{U}_{\text{т1 н.э}} = \frac{\dot{U}_1(0)\, Y_{1\text{т}} z_{\text{в.т}}}{2(\gamma_1+\gamma_\text{т})}\left[1-e^{-(\gamma_1+\gamma_\text{т})\, l_{II}}\right). \tag{3.17}$$

For $x=\ell_{II}$, that is, at the end of the circuit

$$\dot{U}_{\text{т1 к}} = \frac{\dot{U}_1(0)\, z_{\text{в.т}}\, e^{-\gamma_\text{т} l_{II}}}{2(\gamma_1-\gamma_\text{т})}\left(\frac{Z_{1\text{т}}}{z_{\text{в1}} z_{\text{в.т}}} - Y_{1\text{т}}\right)\left[1-e^{-(\gamma_1-\gamma_\text{т})\, l_{II}}\right], \tag{3.18}$$

or $\dot{U}_{\text{т1к}} = \dot{U}_{\text{т1к.м}} + \dot{U}_{\text{т1к.э}}$,

where

$$\dot{U}_{\text{т1 к.м}} = \frac{\dot{I}_1(0)\, Z_{1\text{т}}}{2(\gamma_1-\gamma_\text{т})}\, e^{-\gamma_\text{т} l_{II}}\left[1-e^{-(\gamma_1-\gamma_\text{т})\, l_{II}}\right], \tag{3.19}$$

$$\dot{U}_{\text{т1 к.э}} = -\frac{\dot{U}_1(0)\, Y_{1\text{т}}\, z_{\text{в.т}}}{2(\gamma_1-\gamma_\text{т})}\, e^{-\gamma_\text{т} l_{II}}\left[1-e^{-(\gamma_1-\gamma_\text{т})\, l_{II}}\right]. \tag{3.20}$$

The effect caused by asymmetry of the arrangement of the circuit wires subjected to the effect can also be expressed in terms of the transient damping, that is, in terms of

$$A = \ln\left(\frac{\overset{(1)}{\dot{U}_{1\text{н}}}}{\dot{U}_{\text{т1.н}}}\sqrt{\frac{\overset{(2)}{z_{\text{в.т}}}}{z_{\text{в1}}}}\right). \tag{3.21}$$

Key: 1. b; 2. w

where $\dot{U}_{1b}$ is the voltage at the beginning of the interfering circuit, the wave impedance of which is equal to $z_{w1}$; $\dot{U}_{T1b}$ is the induced voltage at the beginning of the telephone circuit, the wave impedance of which is $z_{w.t}$.

The expressions for the transient damping at the near end between the interfering single-wire circuit and the two-wire, uncrossed circuit subjected to the effect can be obtained by substituting the values of $U_{T1b}$, $U_{T1b.e}$, $U_{T1b.m}$ from (3.15)-(3.17) in (3.21). After this substitution we obtain:

For the electromagnetic effect

$$A_{\text{т1н}} = \ln\left|\frac{2(\gamma_1+\gamma_{\text{т}})}{\left(Y_{1\text{т}}+\frac{Z_{1\text{т}}}{z_{\text{в1}}z_{\text{в.т}}}\right)\sqrt{z_{\text{в1}}z_{\text{в.т}}}\left[1-e^{-(\gamma_1+\gamma_{\text{т}})\,l_{\text{II}}}\right]}\right|,$$

for the magnetic effect only

$$\dot{A}_{\text{т1 н.м}} = \ln\left|\frac{2(\gamma_1+\gamma_{\text{т}})\sqrt{z_{\text{в1}}z_{\text{в.т}}}}{Z_{1\text{т}}\left[1-e^{-(\gamma_1+\gamma_{\text{т}})\,l_{\text{II}}}\right]}\right|,$$

for the electrical effect

$$A_{\text{т1 н.э}} = \ln\left|\frac{2(\gamma_1+\gamma_{\text{т}})}{Y_{1\text{т}}\sqrt{z_{\text{в1}}z_{\text{в.т}}}\left[1-e^{-(\gamma_1+\gamma_{\text{т}})\,l_{\text{II}}}\right]}\right|.$$

3.3. Determination of the Interference Current and Voltage Components Caused by Asymmetry of the Electric Parameters of Telephone Circuit Wires

The voltage $\dot{U}_{T2}$ and current $\dot{I}_{T2}$ caused by asymmetry of the electrical parameters of telephone circuit wires will be obtained from the solution of the above-described system of equations (3.10). Comparing the system of equations (3.10) with the system (2.100), we see their complete identity. The voltage $\dot{U}_A$ and current $\dot{I}_A$ entering into the system (3.10) are values induced in the single-wire circuit which in the given case plays the role of a third circuit. The voltage $\dot{U}_3$ and the current $\dot{I}_3$ entering into the system (2.100) are also the result of the effect on the single-wire (third) circuit. The values of $\xi$ and $\rho$ characterizing the longitudinal and transverse asymmetry of the wires of a two-wire circuit are analogous to the resistance and conductivity of the coupling ($Z_{32}$, $Y_{32}$) between the third and second circuits. Thus, the voltage $\dot{U}_{T2}$ and the current $\dot{I}_{T2}$ occur as a result of double conversion of the energy: from the interfering line to the single-wire circuit -- the first transition as a result of which the voltage $\dot{U}_A$ and the current $\dot{I}_A$ are induced; from the single-wire circuit as a result of longitudinal ($\xi$) and transverse ($\rho$) asymmetries of the wires to the telephone circuit -- the second transition. The single-wire circuit plays the role of a third circuit, the convergence length of which with the second, that is, with the telephone circuit, is equal to the length of the circuits themselves. The solution of system (3.10) is entirely analogous to the above-given solution of the system (2.100), and it depends on the state of the single-wire circuit.

As was indicated above, usually the two-wire circuit is closed by its wave resistances. The single-wire circuit is insulated on the ends either from the ground or is closed to the arbitrary resistances. The latter occurs in the case of using the "wire-ground" or "wire-wire" circuits with transmission of the remote feed or telegraph communications through the "two-wire-ground" circuit. For example, on transmission over a remote feed circuit the wires are connected to the fittings through protective LC-filters, the input resistance of which at voice frequencies is from several hundreds to thousands of ohms.

The formulas for calculating the component $\dot{U}_{T2}$ and $\dot{I}_{T2}$ can be obtained directly from the formulas for the interference through the third circuit presented in §2.7 if the voltage $\dot{U}_{2II}$ and the current $\dot{I}_{2II}$ are replaced by the voltage $\dot{U}_{T2}$ and the current $\dot{I}_{T2}$; the voltage $\dot{U}_{3II}$ and the current $\dot{I}_{3II}$ are replaced by the voltage $\dot{U}_A$ and the current $\dot{I}_A$, the magnetic coupling resistance $Z_{32}$ is replaced by the asymmetry of the total resistances $\xi$, the electric coupling conductivity $Y_{32}$ is replaced by asymmetry of the total conductivity taken with opposite sign $-\rho$.

Let us find the expression for $\dot{U}_{T2}$ and $\dot{I}_{T2}$ for the case where the single-wire circuit is loaded on the ends with arbitrary resistances. The two-wire circuit is closed to the wave impedance. For this purpose we use the formula (2) of Table 2.1 in which it is necessary to set:

$$l_{III}=l'_{2II}=l''_{2II}=l_{2III}=0;\ \ p_{2H}=p_{2K}=0;\ \ p_{3H}=p_{3K}=p_A;$$
$$l_{2II}=l_{3II}=l_{II};\quad z_{B3}=z_{BA};\ \ \gamma_3=\gamma_A;\ \ \gamma_2=\gamma_T;\ \ z_{B2}=z_{B.T};\quad N'_{32}=N'_{AT}=$$
$$=-z_{BA}z_{B.T}\rho\gamma_A-\xi\gamma_T;\quad N''_{32}=N''_{AT}=-z_{BA}z_{B.T}\rho\gamma_T-\xi\gamma_A;\quad N'_{132}=N'_{1AT}=$$
$$=z_{B1}z_{BA}Y_{1A}\gamma_1N'_{AT}-z_{1A}\gamma_TN''_{AT};\qquad N''_{132}=N''_{1AT}=z_{B1}z_{BA}Y_{1A}\gamma_TN'_{AT}-$$
$$-Z_{1A}\gamma_1N''_{AT};\qquad N'_{13}=N'_{1A}=z_{B1}z_{BA}Y_{1A}\gamma_1-Z_{1A}\gamma_A;\qquad N''_{13}=N''_{1A}=$$
$$=z_{B1}z_{BA}Y_{1A}\gamma_A-z_{1A}\gamma_1.$$

Making the indicated substitutions, we obtain

$$\dot{U}_{T2}(x)=\frac{1}{2z_{BA}(\gamma_A^2-\gamma_T^2)}\Big\{N'_{AT}z_{bA}\big(\dot{I}_A(0)e^{-\gamma_Tx}-\dot{I}_A(l_{II})e^{-\gamma_T(l_{II}-x)}\big)+$$
$$+N''_{AT}\big(\dot{U}_A(0)e^{-\gamma_Tx}+\dot{U}_A(l_{II})e^{-\gamma_T(l_{II}-x)}\big)-$$
$$\frac{\dot{U}_1(0)}{z_{B1}(\gamma_T^2-\gamma_1^2)\operatorname{ch}(\gamma_1l_{II}-\ln\sqrt{p_1})}\Big[e^{-\gamma_Tx}\,[N'_{1AT}\operatorname{sh}(\gamma_1l_{II}-\ln\sqrt{p_1})+$$
$$+N''_{1AT}\operatorname{ch}(\gamma_1l_{II}-\ln\sqrt{p_1})]+e^{-\gamma_T(l_{II}-x)}\,[N'_{1AT}\operatorname{sh}\ln\sqrt{p_1}+$$
$$+N''_{1AT}\operatorname{ch}\ln\sqrt{p_1}]\Big]\Big\}-\frac{\dot{U}_A(x)N''_{AT}}{z_{BA}(\gamma_A^2-\gamma_T^2)}+$$
$$+\frac{\dot{U}_1(0)N''_{1AT}\operatorname{ch}[\gamma_1(l_{II}-x)-\ln\sqrt{p_1}]}{z_{B1}z_{BA}(\gamma_T^2-\gamma_1^2)(\gamma_A^2-\gamma_T^2)\operatorname{ch}(\gamma_1l_{II}-\ln\sqrt{p_1})}. \tag{3.22}$$

If the interfering line is loaded on its wave impedance, then it is necessary to set $\operatorname{sh}(\gamma_1l_{II}-\ln\sqrt{p_1})=\operatorname{ch}(\gamma_1l_{II}-\ln\sqrt{p_1})=$
$=e^{\gamma_1l_{II}}/2;\ \operatorname{sh}\ln\sqrt{p_1}=-1/2;\ \operatorname{ch}\ln\sqrt{p_1}=1/2.$
Then

$$\dot{U}_{\text{т}s}(x)=\frac{1}{2z_{\text{вA}}(\gamma_A^2-\gamma_\text{т}^2)}\Big\{N'_{A\text{т}}z_{\text{вA}}\big(\dot{I}_A(0)e^{-\gamma_\text{т}x}-\dot{I}_A(l_{II})e^{-\gamma_\text{т}(l_{II}-x)}\big)+$$
$$+N''_{A\text{т}}\Big[\dot{U}_A(0)e^{-\gamma_\text{т}x}+\dot{U}_A(l_{II})e^{-\gamma_\text{т}(l_{II}-x)}\Big]-\frac{\dot{U}_1(0)}{z_{\text{в}1}(\gamma_\text{т}^2-\gamma_1^2)}\times$$
$$\times\Big[e^{-\gamma_\text{т}x}(N'_{1A\text{т}}+N''_{1A\text{т}})+e^{-\gamma_\text{т}(l_{II}-x)-\gamma_1 l_{II}}(N''_{1A\text{т}}-N'_{1A\text{т}})\Big]\Big\}-$$
$$-\frac{\dot{U}_A(x)N''_{A\text{т}}}{z_{\text{вA}}(\gamma_A^2-\gamma_\text{т}^2)}+\frac{\dot{U}_1(0)N''_{1A\text{т}}e^{-\gamma_1 x}}{z_{\text{вA}}z_{\text{в}1}(\gamma_\text{т}^2-\gamma_1^2)(\gamma_A^2-\gamma_\text{т}^2)}.$$

The voltage occurring at the beginning of the circuit, that is, for x=0,

$$\dot{U}_{\text{т}2\text{н}}=\frac{1}{2z_{\text{вA}}(\gamma_A^2-\gamma_\text{т}^2)}\Big\{N'_{A\text{т}}z_{\text{вA}}\Big[\dot{I}_A(0)-\dot{I}_A(l_{II})e^{-\gamma_\text{т}l_{II}}\Big]+$$
$$+N''_{A\text{т}}\Big[\dot{U}_A(0)+\dot{U}_A(l_{II})e^{-\gamma_\text{т}l_{II}}\Big]-\frac{\dot{U}_1(0)}{z_{\text{в}1}(\gamma_\text{т}^2-\gamma_1^2)}\Big[N'_{1A\text{т}}+N''_{1A\text{т}}+$$
$$+e^{-(\gamma_\text{т}+\gamma_1)l_{II}}(N''_{1A\text{т}}-N'_{1A\text{т}})\Big]\Big\}-\frac{U_A(0)N''_{A\text{т}}}{z_{\text{вA}}(\gamma_A^2-\gamma_\text{т}^2)}+$$
$$+\frac{\dot{U}_1(0)N''_{1A\text{т}}}{z_{\text{в}1}z_{\text{вA}}(\gamma_A^2-\gamma_\text{т}^2)(\gamma_\text{т}^2-\gamma_1^2)};\qquad(3.23)$$

and at the end of the circuit

$$\dot{U}_{\text{т}2\text{к}}=\frac{1}{2z_{\text{вA}}(\gamma_A^2-\gamma_\text{т}^2)}\Big\{N'_{A\text{т}}z_{\text{вA}}\Big[\dot{I}_A(0)e^{-\gamma_\text{т}l_{II}}-\dot{I}_A(l_{II})\Big]+$$
$$+N''_{A\text{т}}\Big[\dot{U}_A(0)e^{-\gamma_\text{т}l_{II}}+\dot{U}_A(l_{II})\Big]-\frac{\dot{U}_1(0)}{z_{\text{в}1}(\gamma_\text{т}^2-\gamma_1^2)}\Big[(N'_{1A\text{т}}+N''_{1A\text{т}})\times$$
$$\times e^{-\gamma_\text{т}l_{II}}+(N''_{1A\text{т}}-N'_{1A\text{т}})e^{-\gamma_1 l_{II}}\Big]\Big\}-\frac{\dot{U}_A(l_{II})N''_{A\text{т}}}{z_{\text{вA}}(\gamma_A^2-\gamma_\text{т}^2)}+$$
$$+\frac{\dot{U}_1(0)N''_{1A\text{т}}e^{-\gamma_1 l_{II}}}{z_{\text{в}1}z_{\text{вA}}(\gamma_\text{т}^2-\gamma_1^2)(\gamma_A^2-\gamma_\text{т}^2)}.\qquad(3.24)$$

The formulas (3.23) and (3.24) pertain to the arbitrary load resistances of the wires making up the two-wire circuit.

Let us consider the case of insulation of the wires where the currents at the beginning and end of the single-wire circuit are equal to zero, that is, $\dot{I}_A(0)=\dot{I}_A(\ell_{II})=0$, and from (3.23) and (3.24) after certain transformations, considering that $\dot{U}_A(0)=\dot{U}_{Ab}$, and $\dot{U}_A(\ell_{II})=\dot{U}_{Ae}$, we obtain

$$\dot{U}_{\text{т2н}} = U_{\text{Ан}}\left\{\frac{1}{2z_{\text{вА}}(\gamma_A^2-\gamma_\text{т}^2)}\left[N'_{A\text{т}}\left(\frac{\dot{U}_{\text{Ак}}}{\dot{U}_{\text{Ан}}}e^{-\gamma_\text{т} l_{II}}-1\right)+\right.\right.$$
$$\left.\left.+\frac{\dot{U}_1(0)(N''_{1A\text{т}}-N'_{1A\text{т}})}{\dot{U}_{\text{Ан}}z_{\text{в1}}(\gamma_\text{т}^2-\gamma_1^2)}\left[1-e^{-(\gamma_\text{т}+\gamma_1)l_{II}}\right]\right]\right\}; \tag{3.25}$$

$$U_{\text{т2к}} = U_{\text{Ак}}\left\{\frac{1}{2z_{\text{вА}}(\gamma_A^2-\gamma_\text{т}^2)}\left[N''_{A\text{т}}\left(\frac{\dot{U}_{\text{Ан}}}{\dot{U}_{\text{Ак}}}e^{-\gamma_\text{т} l_{II}}-1\right)+\right.\right.$$
$$\left.\left.+\frac{\dot{U}_1(0)(N''_{1A\text{т}}+N'_{1A\text{т}})e^{-\gamma_1 l_{II}}}{\dot{U}_{\text{Ак}}z_{\text{в1}}(\gamma_\text{т}^2-\gamma_1^2)}\left[1-e^{-(\gamma_\text{т}-\gamma_1)l_{II}}\right]\right]\right\}. \tag{3.26}$$

In these expressions the voltages on the wires are defined by the formula

$$U_{\text{Ан}} = -\frac{\dot{U}_1(0)z_{\text{вА}}Y_{1\text{А}}}{(\gamma_A^2-\gamma_1^2)\,\text{sh}\,\gamma_\text{А} l_{II}}\left[\gamma_1\left(\text{ch}\,\gamma_\text{А} l_{II}-e^{-\gamma_1 l_{II}}\right)-\gamma_\text{А}\,\text{sh}\,\gamma_\text{А} l_{II}\right]-$$
$$-\frac{\dot{I}_1(0)Z_{1\text{А}}}{(\gamma_A^2-\gamma_1^2)\,\text{sh}\,\gamma_\text{А} l_{II}}\left[\gamma_1\,\text{sh}\,\gamma_\text{А} l_{II}-\gamma_\text{А}\left(\text{ch}\,\gamma_\text{А} l_{II}-e^{-\gamma_1 l_{II}}\right)\right],$$

where the first term is caused by the electrical effect and is equal to the following (after small transformations)

$$\dot{U}_{A\,\text{н.э}} = \frac{\dot{U}_1(0)z_{\text{вА}}Y_{1\text{А}}}{\gamma_\text{А}^2-\gamma_1^2}\left(\gamma_\text{А}-\gamma_1\frac{\text{ch}\,\gamma_\text{А} l_{II}-e^{-\gamma_1 l_{II}}}{\text{sh}\,\gamma_\text{А} l_{II}}\right), \tag{3.27}$$

the second term is caused by the magnetic effect and is equal to

$$\dot{U}_{A\,\text{н.м}} = -\frac{\dot{I}_1(0)Z_{1\text{А}}}{\gamma_A^2-\gamma_1^2}\left(\gamma_1-\gamma_\text{А}\frac{\text{ch}\,\gamma_\text{А} l_{II}-e^{-\gamma_1 l_{II}}}{\text{sh}\,\gamma_\text{А} l_{II}}\right), \tag{3.28}$$

$$\dot{U}_{\text{А к}} = -\frac{\dot{U}_1(0)z_{\text{вА}}Y_{1\text{А}}}{(\gamma_A^2-\gamma_1^2)\,\text{sh}\,\gamma_\text{А} l_{II}}\left[\gamma_1\left(1-\text{ch}\,\gamma_\text{А} l_{II}e^{-\gamma_1 l_{II}}\right)-\right.$$
$$\left.-\gamma_\text{А}\text{sh}\gamma_\text{А} l_{II}\right]-\frac{\dot{I}_1(0)Z_{1\text{А}}}{(\gamma_A^2-\gamma_1^2)\,\text{sh}\,\gamma_\text{А} l_{II}}\left[\gamma_1\,\text{sh}\,\gamma_\text{А} l_{II}-\gamma_\text{А}\left(1-\text{ch}\,\gamma_\text{А} l_{II}e^{-\gamma_1 l_{II}}\right)\right],$$

where the first term is caused by the electrical effect and is equal to

$$\dot{U}_{\text{А к.э}} = \frac{\dot{U}_1(0)z_{\text{вА}}Y_{1\text{А}}}{\gamma_A^2-\gamma_1^2}\left[\gamma_\text{А}-\gamma_1\frac{1-\text{ch}\,\gamma_\text{А} l_{II}e^{-\gamma_1 l_{II}}}{\text{sh}\,\gamma_\text{А} l_{II}}\right]. \tag{3.29}$$

the second term is caused by the magnetic effect and is equal to

$$\dot{U}_{A\,\text{к.м}} = -\frac{\dot{I}_1(0)Z_{1\text{А}}}{\gamma_A^2-\gamma_1^2}\left[\gamma_1-\gamma_\text{А}\frac{1-\text{ch}\gamma_\text{А} l_{II}e^{-\gamma_1 l_{II}}}{\text{sh}\gamma_\text{А} l_{II}}\right]. \tag{3.30}$$

Let us represent the expressions (3.25) and (3.26) in the form of two components caused by the electrical and magnetic effects. The voltages caused only by the electrical effect can be obtained if we set $Z_{1A}=0$ in (3.25) and (3.26). Then

at the beginning of the circuit

$$\dot{U}_{\text{т}2\,\text{н.э}}=\dot{U}_{\text{A}\,\text{н.э}}\left\{\frac{N''_{A\text{т}}}{2z_{\text{вA}}\left(\gamma_A^2-\gamma_\text{т}^2\right)}\left[\frac{\gamma_\text{A}\,\text{sh}\,\gamma_\text{A}\,l_{II}-\gamma_1\left(1-\text{ch}\,\gamma_\text{A}\,l_{II}e^{-\gamma_1 l_{II}}\right)}{\gamma_\text{A}\,\text{sh}\,\gamma_\text{A}\,l_{II}-\gamma_1\left(\text{ch}\,\gamma_\text{A}\,l_{II}-e^{-\gamma_1 l_{II}}\right)}\times\right.\right.$$
$$\left.\times e^{-\gamma_\text{т} l_{II}}-1\right]+\frac{N'_{A\text{т}}\,\text{sh}\,\gamma_\text{A}\,l_{II}\left(\gamma_\text{A}^2-\gamma_1^2\right)\left[1-e^{-(\gamma_1+\gamma_\text{т})\,l_{II}}\right]}{2z_{\text{вA}}\left(\gamma_\text{A}^2-\gamma_\text{т}^2\right)\left(\gamma_\text{т}+\gamma_1\right)\left[\gamma_\text{A}\,\text{sh}\,\gamma_\text{A}\,l_{II}-\rightarrow\right.}\qquad(3.31)$$
$$\left.\frac{}{\left.\rightarrow-\gamma_1\left(\text{ch}\,\gamma_\text{A}\,l_{II}-e^{-\gamma_1 l_{II}}\right)\right]}\right\},$$

at the end of the circuit

$$\dot{U}_{\text{т}2\,\text{к.э}}=\dot{U}_{\text{A}\,\text{э.к}}\left\{\frac{N''_{\text{Aт}}}{2z_{\text{вA}}\left(\gamma_\text{A}^2-\gamma_\text{т}^2\right)}\left[\frac{\gamma_\text{A}\,\text{sh}\,\gamma_\text{A}\,l_{II}-\gamma_1\left(\text{ch}\,\gamma_\text{A}\,l_{II}-e^{-\gamma_1 l_{II}}\right)}{\gamma_\text{A}\,\text{sh}\,\gamma_\text{A}\,l_{II}-\gamma_1\left(1-\text{ch}\,\gamma_\text{A}\,l_{II}\,e^{-\gamma_1 l_{II}}\right)}\times\right.\right.$$
$$\left.\left.\times e^{-\gamma_\text{т} l_{II}}-1\right]+\frac{N'_{A\text{т}}\left(\gamma_\text{A}^2-\gamma_1^2\right)\text{sh}\,\gamma_\text{A}\,l_{II}\,e^{-\gamma_1 l_{II}}\left(1-e^{-(\gamma_\text{т}-\gamma_1)\,l_{II}}\right)}{2z_{\text{вA}}\left(\gamma_\text{т}-\gamma_1\right)\left[\gamma_\text{A}\,\text{sh}\,\gamma_\text{A}\,l_{II}-\gamma_1\left(1-\text{ch}\,\gamma_\text{A}\,l_{II}\,e^{-\gamma_1 l_{II}}\right)\right]}\right\}.\qquad(3.32)$$

The voltages caused only by the magnetic effect can be obtained if we set $Y_{1A}=0$ in (3.25) and (3.26). Then

at the beginning of the circuit

$$\dot{U}_{\text{т}2\,\text{н.м}}=\dot{U}_{A\,\text{н.м}}\left\{\frac{N''_{A\text{т}}}{2z_{\text{вA}}\left(\gamma_A^2-\gamma_\text{т}^2\right)}\left[\frac{\gamma_1\,\text{sh}\,\gamma_\text{A}\,l_{II}-\gamma_\text{A}\left(1-\text{ch}\,\gamma_\text{A}\,l_{II}\,e^{-\gamma_1 l_{II}}\right)}{\gamma_1\,\text{sh}\,\gamma_\text{A}\,l_{II}-\gamma_\text{A}\left(\text{ch}\,\gamma_\text{A}\,l_{II}-e^{-\gamma_1 l_{II}}\right)}\times\right.\right.$$
$$\left.\left.\times e^{-\gamma_\text{т} l_{II}}-1-\frac{\text{sh}\,\gamma_\text{A}\,l_{II}\left(\gamma_\text{A}^2-\gamma_1^2\right)\left[1-e^{-(\gamma_\text{т}+\gamma_1)\,l_{II}}\right]}{\left(\gamma_\text{т}+\gamma_1\right)\left[\gamma_1\,\text{sh}\,\gamma_\text{A}\,l_{II}-\gamma_\text{A}\left(\text{ch}\,\gamma_\text{A}\,l_{II}-e^{-\gamma_1 l_{II}}\right)\right]}\right]\right\},\qquad(3.33)$$

at the end of the circuit

$$\dot{U}_{т2\,н.к} = \dot{U}_{A\,к.м} \left\{ \frac{N''_{Aт}}{2z_{вA}\,(\gamma_A^2 - \gamma_т^2)} \left[ \frac{\gamma_1 \operatorname{sh} \gamma_A\, l_{II} - \gamma_A \left( \operatorname{ch} \gamma_A\, l_{II} - e^{-\gamma_1 l_{II}} \right)}{\gamma_1 \operatorname{sh} \gamma_A\, l_{II} - \gamma_A \left( 1 - e^{-\gamma_1 l_{II}} \operatorname{ch} \gamma_A\, l_{II} \right)} \times \right.\right.$$

$$\left.\left. \times\, e^{-\gamma_т l_{II}} - 1 + \frac{(\gamma_A^2 - \gamma_1^2) \operatorname{sh} \gamma_A\, l_{II}\, e^{-\gamma_1 l_{II}} \left[ 1 - e^{-(\gamma_т - \gamma_1)\, l_{II}} \right]}{(\gamma_т - \gamma_1) \left[ \gamma_1 \operatorname{sh} \gamma_A\, l_{II} - \gamma_A \left( 1 - \operatorname{ch} \gamma_A\, l_{II}\, e^{-\gamma_1 l_{II}} \right) \right]} \right] \right\}. \qquad (3.34)$$

Let us consider the equations (3.31)-(3.34) in more detail. The formulas for the voltages induced at the beginning or end of the single-wire circuit insulated at the ends under magnetic and electrical effects appear in front of the braces in these equations. The complex expressions in these braces which depend on the asymmetry (transverse ρ and longitudinal ξ) of the telephone circuit, the primary and secondary parameters of the two-wire and the single-wire circuits, the propagation constant of the interfering line and the line length characterize the process of double energy transfer from the interfering circuit to the two-wire circuit as a result of asymmetry of the latter. These expressions can be denoted by certain coefficients η characterizing the degree of sensitivity of the telephone circuit to interference. Indeed, they do not have dimensionality; they depend directly proportionately on the asymmetry of the two-wire circuit, and they are equal to the ratio of the voltage in the two-wire circuit to the average voltage induced on the circuit wires with respect to the ground. Thus, we can write

$$\dot{U}_{т2\,\underset{(1)}{н}.\underset{(2)}{э}} = \dot{U}_{A\,э.н}\, \eta_{из.э.н}, \quad (3.35) \qquad \dot{U}_{т2\,к.\underset{(3)}{э}} = \dot{U}_{A\,э.к}\, \eta_{из.э.к}, \qquad (3.36)$$

Key: 1. ins; 2. b; 3. e

$$\dot{U}_{т2\,н.м} = U_{A\,м.н}\, \eta_{из.м.н}, \quad (3.37) \qquad U_{т2\,к.м} = \dot{U}_{A\,м.к}\, \eta_{из.м.к}, \qquad (3.38)$$

where $\eta_{из.э(м)н(к)}$ are the sensitivity coefficients of the circuit for insulation of the wires making up the two-wire circuit under electrical and magnetic effects defined by the expressions included in the braces in (3.30)-(3.34).

Now let us find the voltages induced in the two-wire circuit for closure of the "wire-ground" circuit to the wave resistances. In this case $I_{Aн} = -U_{Aн}/z_{вA}$, $I_{Aк} = U_{Aк}/z_{вA}$. After this substitution in (3.23), and a number of transformations we have

$$\dot{U}_{т2\,н} = \dot{U}_{Aн} \left\{ \frac{1}{2z_{вA}\,(\gamma_A^2 - \gamma_т^2)} \left[ \frac{\dot{U}_{Aк}}{\dot{U}_{Aн}}\, e^{-\gamma_т l_{II}} (N''_{Aт} - N'_{Aт}) - \right.\right. \qquad (3.39)$$

$$\left.\left. - (N''_{Aт} + N'_{Aт}) - \frac{\dot{U}_1(0)\,(N'_{1Aт} - N''_{1Aт})}{\dot{U}_{Aн} z_{в1}\,(\gamma_т^2 - \gamma_1^2)} \left[ 1 - e^{-(\gamma_1 + \gamma_т)\, l_{II}} \right] \right] \right\},$$

$$\dot{U}_{T2\,\text{к}} = \dot{U}_{A\,\text{к}} \left\{ \frac{1}{2z_{\text{вA}}(\gamma_A^2 - \gamma_T^2)} \left[ \frac{\dot{U}_{A\text{н}}}{\dot{U}_{A\text{к}}} e^{-\gamma_T l_{II}} (N''_{AT} - N'_{AT}) - (N''_{AT} + N'_{AT}) - \right.\right.$$

$$\left.\left. - \frac{\dot{U}_1(0)(N''_{1AT} + N'_{1AT}) e^{-\gamma_T l_{II}}}{\dot{U}_{A\text{к}} z_{\text{в}1}(\gamma_T^2 - \gamma_1^2)} \right] 1 - e^{-(\gamma_1 - \gamma_T) l_{II}} \right] \right\}. \qquad (3.40)$$

In these expressions the voltages on the wires are

$$\dot{U}_{A\,\text{н.э}} = \frac{\dot{U}_1(0) Y_{1A} z_{\text{вA}}}{2(\gamma_A + \gamma_1)} \left[1 - e^{-(\gamma_1 + \gamma_A) l_{II}}\right], \qquad (3.41)$$

$$\dot{U}_{A\,\text{н.м}} = \frac{\dot{I}_1(0) Z_{1A}}{2(\gamma_A + \gamma_1)} \left[1 - e^{-(\gamma_1 + \gamma_A) l_{II}}\right], \qquad (3.42)$$

$$\dot{U}_{A\,\text{к.э}} = \frac{\dot{U}_1(0) Y_{1A} z_{\text{вA}} e^{-\gamma_A l_{II}}}{2(\gamma_1 - \gamma_A)} \left[1 - e^{-(\gamma_1 - \gamma_A) l_{II}}\right], \qquad (3.43)$$

$$\dot{U}_{A\,\text{к.м}} = -\frac{\dot{I}_1(0) Z_{1A} e^{-\gamma_A l_{II}}}{2(\gamma_1 - \gamma_A)} \left[1 - e^{-(\gamma_1 - \gamma_A) l_{II}}\right]. \qquad (3.44)$$

Just as before let us represent the equations obtained in the form of two components caused by the electric and magnetic effects. From (3.39) and (3.40), substituting the values of $\dot{U}_{A\text{b}}$, $\dot{U}_{Ae}$, $N'_{1AT}$, $N''_{1AT}$, after some transformations we obtain:

The voltage caused only by the electrical effect

$$\dot{U}_{T2\,\text{н.э}} = \dot{U}_{A\,\text{н.э}} \left\{ \frac{1}{2z_{\text{вA}}(\gamma_A^2 - \gamma_T^2)} \left[ \frac{(\gamma_A + \gamma_1) e^{-\gamma_A l_{II}} \left[1 - e^{-(\gamma_1 - \gamma_A) l_{II}}\right]}{(\gamma_1 - \gamma_A)\left[1 - e^{-(\gamma_1 + \gamma_A) l_{II}}\right]} \times \right.\right.$$

$$\times e^{-\gamma_T l_{II}} (N''_{AT} - N'_{AT}) - (N''_{AT} + N'_{AT}) + \qquad (3.45)$$

$$\left.\left. + \frac{2N'_{AT}(\gamma_1 + \gamma_A)\left[1 - e^{-(\gamma_T + \gamma_1) l_{II}}\right]}{(\gamma_T + \gamma_1)\left[1 - e^{-(\gamma_1 + \gamma_A) l_{II}}\right]} \right] \right\},$$

$$\dot{U}_{T2\,\text{к.э}} = \dot{U}_{A\,\text{к.э}} \left\{ \frac{1}{2z_{\text{вA}}(\gamma_A^2 - \gamma_T^2)} \left[ \frac{(\gamma_1 - \gamma_A)\left[1 - e^{-(\gamma_1 + \gamma_A) l_{II}}\right] \times \rightarrow}{(\gamma_1 + \gamma_A) e^{-(\gamma_A - \gamma_T) l_{II}} \times \rightarrow} \right.\right.$$

$$\frac{\rightarrow \times (N''_{AT} - N'_{AT})}{\rightarrow \times \left[1 - e^{-(\gamma_1 - \gamma_A) l_{II}}\right]} - (N''_{AT} + N'_{AT}) + \frac{2N'_{AT}(\gamma_A - \gamma_1) e^{-(\gamma_T - \gamma_A) l_{II}}}{(\gamma_T - \gamma_1)\left[1 - e^{-(\gamma_1 - \gamma_A) l_{II}}\right]} \times \qquad (3.46)$$

$$\left.\left. \times \left[1 - e^{-(\gamma_1 - \gamma_T) l_{II}}\right] \right] \right\}.$$

the voltage caused only by the magnetic effect

$$\dot{U}_{\text{т2 н.м}}=\dot{U}_{A\,\text{н.м}}\left\{\frac{1}{2z_{\text{вA}}(\gamma_A^2-\gamma_\text{т}^2)}\left[\frac{(\gamma_A+\gamma_1)\,e^{-(\gamma_A+\gamma_\text{т})\,l_{II}}\times\left[1-e^{-(\gamma_1-\gamma_A)\,l_{II}}\right]}{(\gamma_A-\gamma_1)\left[1-e^{-(\gamma_A+\gamma_1)\,l_{II}}\right]}\times(N''_{A\text{т}}-N'_{A\text{т}})-(N''_{A\text{т}}+N'_{A\text{т}})+\frac{2N''_{A\text{т}}(\gamma_A+\gamma_1)\left[1-e^{-(\gamma_A+\gamma_1)\,l_{II}}\right]}{(\gamma_\text{т}+\gamma_1)\left[1-e^{-(\gamma_A+\gamma_1)\,l_{II}}\right]}\right]\right\}, \tag{3.47}$$

$$\dot{U}_{\text{т2 к.м}}=\dot{U}_{A\,\text{к.м}}\left\{\frac{1}{2z_{\text{вA}}(\gamma_A^2-\gamma_\text{т}^2)}\left[\frac{(\gamma_A-\gamma_1)\left[1-e^{-(\gamma_1+\gamma_A)\,l_{II}}\right]\times(N''_{A\text{т}}-N'_{A\text{т}})}{(\gamma_1+\gamma_A)\,e^{-(\gamma_A-\gamma_\text{т})\,l_{II}}\times\left[1-e^{-(\gamma_1-\gamma_A)\,l_{II}}\right]}-(N''_{A\text{т}}+N'_{A\text{т}})+\frac{2N''_{A\text{т}}(\gamma_A-\gamma_1)\,e^{-(\gamma_\text{т}-\gamma_A)\,l_{II}}\left[1-e^{-(\gamma_1-\gamma_\text{т})\,l_{II}}\right]}{(\gamma_\text{т}-\gamma_1)\left[1-e^{-(\gamma_1-\gamma_A)\,l_{II}}\right]}\right]\right\}. \tag{3.48}$$

The formulas (3.45)-(3.48) differ from (3.31)-(3.34) by the expressions in braces characterizing the transition of the induced energy from the single-wire circuit closed to the wave impedances to the two-wire also closed to its wave impedances. Analogously to the preceding, the expressions in the braces will be denoted by the coefficients $\eta$ with indexes indicating the load resistances of the single-wire circuit, the type of effect and the beginning or end of the circuit ($\eta_{w.e.b}$, $\eta_{w.M.b}$, $\eta_{w.e.e}$, $\eta_{w.m.e}$). Thus, the formulas obtained, beginning with (3.23) and so on, permit calculation of the voltages occurring in the two-wire circuit caused by longitudinal and transverse asymmetry of the circuit.

The determination of the induced voltages, although awkward, is entirely possible. In practice the accuracy of the calculation by these formulas is low. This is explained by the fact that the values of the asymmetry of the total resistances $\xi$ and the total conductivities $\rho$ of the circuit entering into them depend on the number of factors which are not subject to exact consideration, and they are determined by the type of line, the state of the circuit, the weather, and so on. In addition, the expressions presented above in terms of the sensitivity of coefficients depend not only on the parameters of the circuit itself and its length, but also on the parameters of the interfering circuit and on the ratio of the length of the circuit and the convergence length.

Under the effect of the contact circuit of an electric railroad, as a result of movement of the electric locomotive the length of the interfering section section varies with time, and, consequently, the sensitivity coefficient of the circuit will also vary. Thus, we can talk about only the approximate calculation for some of the idealized conditions, for example, for the provisional location of the electric trains in the section, defined weather conditions, and so on. This indicates the necessity for using the statistical method of calculating voltages in the two-wire circuit based on the application of probability theory.

Considering what has been discussed, it appears expedient to use experimental data on the magnitudes of the sensitivity coefficients when calculating the voltages in the two-wire circuit. Here it is necessary to use some of the mean statistical values determined for different convergence conditions, weather, lengths, and so on. Knowing the previously obtained experimental values of the sensitivity coefficient, the magnitude of the voltages induced in the two-wire circuit is determined by multiplying the value of the coefficient times the voltage occurring in the single-wire circuit.

The most reliable method of measuring the sensitivity coefficient of the circuit consists in using the actual electric power transmission line or contact wire of the electric railroad. The measurement system is illustrated in Fig 3.2. Taking the ratio $\dot{U}_{T2b}/\dot{U}_{Ab}=n_b$ and $\dot{U}_{T2e}/\dot{U}_{Ae}=n_e$, we obtain the sensitivity coefficients of the circuit at the beginning and end of the line for different state of the single-wire circuit. The sensitivity coefficient determined by the systems in Fig 3.2 indicates both the magnetic and electric effect and is exactly equal for the given conditions of convergence to the values of the expressions in the braces of the formulas (3.25) and (3.26) for the system 3.2a, the formulas (3.39) and (3.40) for the system 3.2b.

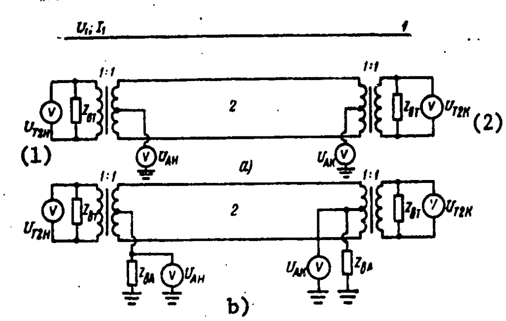



Figure 3.2. Systems for measuring the sensitivity coefficients of the circuit (2) to interference using the interfering line (1): a) for insulation of the circuit wire from the ground; b) for loading the wires on wave impedances

Key:

1. b
2. e

The approximate values of the sensitivity coefficients can be obtained by simulating the electrical or magnetic effect directly on the circuits themselves. The measurement systems are presented in Fig 3.3a-d (the measuring generator is placed approximately in the middle of the circuit). The sensitivity coefficients measured by these systems are approximately equal to the values in the braces of the expressions (3.31)-(3.34) (for insulation of the wires of the two-wire circuit) and (3.45)-(3.48) (for closure of the wires to the wave impedances), but without considering the propagation coefficient in the interfering line. Calculating the voltage induced in the telephone circuit with respect to the sensitivity coefficient found by the systems in Fig 3.3a-d, an error is permitted, the value of which increases with an increase in frequency, for the higher the frequency, the greater the role played by the propagation coefficient of the interfering line. In addition, the error occurs as a result of divergence in the circuit lengths. Even for a large number of measurements it is possible to encompass all of the combinations of circuit and convergence lengths encountered in practice. Therefore the results of the calculations even when using the experimentally obtained sensitivity coefficients of the circuits must be considered approximate. These calculations offer the possibility of estimating the expected values of the induced interference voltages, as experience shows usually with some engineering margin.

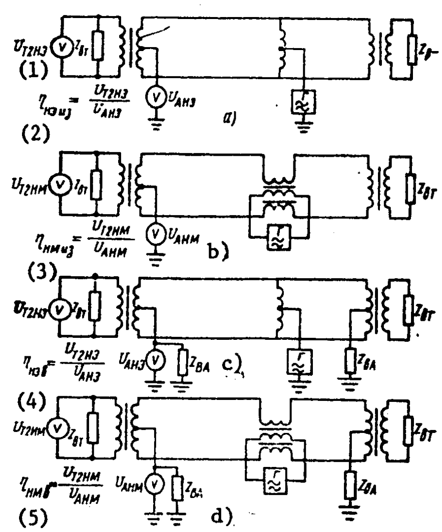



Figure 3.3. Systems for measuring the sensitivity coefficients of a two-wire circuit to interference: a) and b) for electrical and magnetic effects and wires insulated from the ground; c) and d) the same, for closure of the "wire-ground" circuits by wave impedances

Key:

1. b
2. be ins
3. bM ins
4. be w
5. bM w

The results of the measurements by the systems in Fig 3.3 in the low-frequency band indicate [30] that the sensitivity coefficients for the electrical and magnetic effect are close to each other and approximately correspond to the asymmetry of the telephone circuit in the presence of alternating current measured at the beginning of the circuit by the system in Fig 3.4, that is, $\eta_{b,e} \approx \eta_{bM} = \eta_b$. From the presented analysis it follows that the induced voltage at the beginning of the two-wire uncrossed communication circuit is made up of the geometric sum of four components, two of which are caused by the electrical effect and two by the magnetic effect, that is,

$$\dot U_{\text{н}} = \dot U_{\text{т1 н.э}} + \dot U_{\text{т2 н.э}} + \dot U_{\text{т1 н.м}} + U_{\text{т2 н.м}}. \qquad (3.49)$$

The effect of the single-wire circuit on the two-wire telephone circuit caused by asymmetry of the electrical parameters of the wires of this circuit, analogously to the preceding, can be expressed in terms of the transient damping which for this case is

$$A_{\text{н}} = \frac{1}{2} \ln \left| \frac{\dot U_1^2 z_{\text{в.т}}}{\dot U^2_{\text{т2 н}} z_{\text{в1}}} \right| = \ln \left| \frac{\dot U_1}{\dot U_{\text{т2 н}}} \right| + \frac{1}{2} \ln \frac{z_{\text{в.т}}}{z_{\text{в1}}}.$$

Considering that in general form $\dot U_{T2b} = \dot U_{Ab}\eta$, we obtain

$$\underset{(1)}{A_{\text{н}}} = \ln \left| \frac{\dot U_1}{\dot U_{\text{Aн}}} \right| + \ln \frac{1}{\eta} + \frac{1}{2} \ln \frac{\overset{(2)}{z_{\text{в.т}}}}{z_{\text{в1}}}. \qquad (3.50)$$

Key: 1. b; 2. w

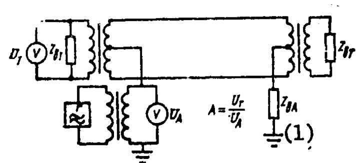


Figure 3.4. System for measuring the asymmetry of the circuit with alternating current
Key:
1. w

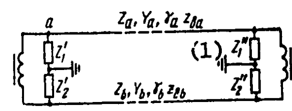


Figure 3.5. Determination of voltages in the two-wire circuit considering asymmetry of the wave equipment
Key:
1. w

In this equation the first term is the transient damping between the single-wire interfering and single-wire communication circuit without correction for the wave impedance; the second term is the damping of the energy on making the transition from the single-wire circuit to the two-wire circuit. The third term takes into account the difference of wave impedances of the single-wire and two-wire circuits.

3.4. Determination of the Voltage in the Two-Wire Circuit Caused by Asymmetry of the Introductory Equipment

It was proposed above that the interference in the two-wire circuit occurs as a result of asymmetry of the parameters of the circuit wires. The possible asymmetry of the equipment was not taken into account. However, the voltage induced in a two-wire circuit is caused not only by asymmetry of the circuit itself, but also asymmetry of the equipment with respect to the ground. The asymmetry of the introductory equipment can be ignored only in cases where the magnitude of the damping of the asymmetry of it by 8.0 to 13 decibels or more is higher than the damping of the asymmetry of the circuit itself. In practice, the asymmetry of the introductory equipment must be taken into account when calculating the interference in the circuits over which the remote feed of the repeater stations is transmitted and also in the circuits with directly included asymmetric filters or other analogous devices.

Let us consider the circuit shown in Fig 3.5. In this system $Z'_1$, $Z'_2$, $Z''_1$, $Z''_2$ are the equivalent resistances between each input terminal of the equipment and the ground; $Z_{wa}$, $Z_{wb}$, $Z_a$, $Z_b$, $Y_a$, $Y_b$ $\gamma_a$, $\gamma_b$ are the total longitudinal resistances, the total conductivities of the insulation, the propagation coefficients on the wave impedances of each wire (a and b) forming a two-wire circuit. Let us propose that the voltage in the communication channel occurs only as a result of asymmetry of the equipment. Let us denote this voltage by $\dot{U}_{Ta}$. We shall consider that the wire parameters are identical, that is, $Z_a=Z_b=Z_A$, $Y_a=Y_b=Y_A$; $z_{Ba}=z_{Bb}=z_{BA}$; $\gamma_a=\gamma_b=\gamma_A$.

Let the investigated circuit of length $\ell$ be subjected to the effect of a high-voltage line over its entire length. Then the current through the resistance $Z'_1$ (Fig 3.5) on the basis of the theorem of an equivalent generator

$$\dot{I}'_1=\frac{\dot{U}_{a0}}{Z'_1+Z_{\text{вх}\,a}\,(1)}, \tag{3.51}$$

Key: 1. inp

where $\dot{U}_{a0}$ is the voltage with respect to the ground occurring at point a for insulation of the first wire from the ground, that is, for disconnection of the resistance $Z'_1$; $Z_{inp\ a}$ is the input impedance of the "first wire-ground" from the point a in the direction of the opposite end. The magnitude of this input impedance can be determined by the known formula

$$Z_{(1)}=z_{BA}\,\mathrm{cth}\left(\gamma_A\,l-\ln\sqrt{p''_1}\right),$$

Key: 1. inp a

where $p''_1=(Z''_1-z_{wA})/(Z''_1+z_{wA})$.

The magnitude of the voltage $\dot{U}_{a0}$ can be determined by one of the formulas obtained above. The current through the resistance $Z'_2$ is

$$\dot{I}'_2 = \frac{\dot{U}_{w0}^{(1)}}{Z'_2 + Z_{inp\,b}^{(2)}}, \tag{3.52}$$

Key: 1. w; 2. inp

where $\dot{U}_{b0}$ is the voltage with respect to ground occurring at the point b for insulation of the second wire, that is, for disconnection of the resistance $Z'_2$; $Z_{inp\ b}$ is the input resistance of the "second wire-ground" circuit from the point b in the direction of the opposite end equal to

$$Z_{inp\,b}^{(1)} = z_{b\,A}\,\mathrm{cth}\left(\gamma_A l - \ln\sqrt{p''_2}\right),$$

Key: 1. inp b

where $p''_2 = (Z''_2 - z_{wA})/(Z''_2 + z_{wA})$. The potential at the point a is equal to $\dot{U}_a = \dot{I}_1'Z'_1$, and at the point b, $\dot{U}_b = \dot{I}'_2Z'_2$. The potential difference between the points a and b

$$\dot{U}_a - \dot{U}_b = \frac{\dot{U}_{a0}Z'_1}{Z'_1 + Z_{inp\,a}} - \frac{\dot{U}_{b0}Z'_2}{Z'_2 + Z_{inp\,b}}. \tag{3.53}$$

If at the point a and b the two-wire circuit is loaded on a wave impedance, then the voltage on it, that is, the interference voltage, will be approximately equal to half the potential difference between the points a and b.

$$\dot{U}_{Ta} = \frac{1}{2}(\dot{U}_a - \dot{U}_b) = \frac{1}{2}\left(\frac{\dot{U}_{a0}Z'_1}{Z'_1 + Z_{inp\,a}} - \frac{\dot{U}_{b0}Z'_2}{Z'_2 + Z_{inp\,b}}\right). \tag{3.54}$$

Let us transform the expression (3.53). First of all it is possible with sufficient accuracy to assume that $\dot{U}_{a0} = \dot{U}_{b0} = \dot{U}_{A0} = (\dot{U}_{a0} + \dot{U}_{b0})/2$. Then

$$\dot{U}_{Ta} = \frac{1}{2}\dot{U}_{A0}\left(\frac{Z'_1}{Z'_1 + Z_{inp\,a}} - \frac{Z'_2}{Z'_2 + Z_{inp\,b}}\right) = \frac{\dot{U}_{A0}}{2}A' = \frac{1}{2}\dot{U}_{A0}\,e^{-A_a}, \tag{3.55}$$

where $A_a = \ln(1/A')$. The expression A' is a value which depends on the asymmetry of the equipment. Indeed, the more the resistances $Z'_1$ and $Z'_2$ differ from each other, the larger this value is. In the limiting case if $Z'_1 = \infty$, and $Z'_2 = 0$, then $\dot{U}_{Ta} = \dot{U}_{A0}/2$. Let us assume that $Z_{inp\ a} = Z'_2 + \Delta_1 Z$, $Z_{inp\ b} = Z'_1 + \Delta_2 Z$, where $\Delta_1 Z$ and $\Delta_2 Z$ are values equal to $\Delta_1 Z = Z_{inp\ a} - Z'_2$, $\Delta_2 Z = Z_{inp\ b} - Z'_1$. Then

$$A' = \frac{Z'_1}{Z'_1 + Z'_2 + \Delta_1 Z} - \frac{Z'_2}{Z'_2 + Z'_1 + \Delta_2 Z}.$$

or

$$A'=\frac{Z_1'}{(Z_1'+Z_2')(1+\Delta_1 Z)/(Z_1'+Z_2')}-\frac{Z_2'}{(Z_1'+Z_2')(1+\Delta_2 Z)/(Z_1'+Z_2')}. \tag{3.56}$$

The values of the ratios $\Delta_1 Z/(Z'_1+Z'_2)$ and $\Delta_2 Z/(Z'_1+Z'_2)$ are always less than one. If in the first approximation these ratios are neglected by comparison with one, the expression for the asymmetry of the equipment is significantly simplified, and it will have the form

$$A'\approx A=\frac{Z_1'-Z_2'}{Z_1'+Z_2'}. \tag{3.57}$$

In reality, the asymmetry will be somewhat less than calculated by (3.57), for the values greater than one are excluded from the denominators of the two terms of expression (3.56). Consequently, when determining the asymmetry of the equipment in accordance with (3.57) calculated by (3.55) the values of the interference are obtained somewhat larger than expected in reality. Thus, more exactly

$$A'=\frac{Z_1'-Z_2'}{Z_1'+Z_2'}\,\frac{1}{k_a}=\frac{A}{k_a}, \tag{3.58}$$

where $k_a$ is the correction factor greater than one. For the calculation this correction factor can be neglected if the magnitude of the voltage in the two-wire circuit caused by asymmetry of the input equipment is expressed in terms of the voltage $\dot U_{Ab}$ induced on the circuit wires with respect to the ground. The value of this voltage

$$\dot U_{A\,н}=\frac{1}{2}(\dot U_a+\dot U_b)=\frac{1}{2}\left(\frac{\dot U_{a0}Z_1'}{Z_1'+Z_{вх\,a}}+\frac{\dot U_{b0}Z_2'}{Z_2'+Z_{вх\,b}}\right)= \\ =\frac{\dot U_{A0}}{2k_a}\,\frac{Z_1'+Z_2'}{Z_1'+Z_2'}=\frac{U_{A0}}{2k_a}. \tag{3.59}$$

Hence

$$\dot U_{\tau a}=\dot U_{A\,н}\frac{Z_1'-Z_2'}{Z_1'+Z_2'}=\dot U_{A\,н}A. \tag{3.60}$$

The component of the interference voltage in the two-wire circuit caused by asymmetry of the circuit itself, as was demonstrated above, is

$$\dot U_{\tau 2\,\underset{(1)}{н}}=\dot U_{\tau\,\underset{(2)}{ц}}=\dot U_{A\,н}\eta. \tag{3.61}$$

Key: 1. b; 2. circuit

The total voltage in the two-wire circuit, considering that the phase angles of the components are not known exactly and are random variables, can be defined by the quadratic addition law:

$$\dot{U}_{т.сум} = \sqrt{(\dot{U}_{та})^2 + (\dot{U}_{т.ц})^2} = \dot{U}_{A\,м}\sqrt{A^2+\eta^2}. \qquad (3.62)$$

On comparison of (3.61) and (3.62) it is obvious that for A=η the voltage caused by asymmetry of the equipment and is equal to the voltage caused by asymmetry of the circuit.

The interference in the low-frequency telephone channel with repeaters or convergence with any high-voltage line is created as a result of the effect of the harmonic components of the current and voltage of the interfering line. The resultant interference voltage in the circuit is

$$\underset{(1)}{\dot{U}_{т.рез}} = \sqrt{\sum_{k=1}^{n}(\dot{U}_{тk}p_k)^2},$$

Key: 1. resultant

where $\dot{U}_{Tk}$ is the interference voltage caused by the effect of the k-th harmonic component of the current; $p_k$ is the coefficient of the acoustic effect for the frequency of the k-th harmonic component.

Since the interference norm is given at the point with relative level as -6.9 decibels, the interference voltage $\dot{U}_{t.\ res}$ and $\dot{U}_{Tk}$ must be determined at this point. The voltage occurring at the point with relative level of -6.9 decibels for the frequency of the k-th harmonic component depends on the gain of the terminal repeater, the magnitude of the filter damping; if they are included at the beginning of the receiving channel, the values of the attenuation usually included at the input of the attenuation network:

$$\dot{U}_{тk} = \dot{U}_{т(k)}\,10^{1/20\,(S_k - a_k - a_y)},$$

where $\dot{U}_{T(k)}$ is the voltage occurring at the end of the circuit with the frequency of the k-th harmonic component of the interfering current defined by (3.62); $S_k$ is the gain of the repeater for the frequency of the k-th harmonic component, decibels; $a_k$ is the attenuation of the filters for the frequency of the k-th harmonic component included in the channel, for example, the filter K=0.3, in the operator channel of the K-60P equipment; $a_y$ is the attenuation of the attenuation network, for example, in the operator channel of the K-60P equipment $a_y$=11.3 decibels.

Since the calculation of the interference voltage on the frequency of each harmonic component is quite tedious, it is possible to use the calculation method on one frequency, for example, 800 hertz, with introduction of a correction factor.

## 3.5. Effect of the Two-Phase and Three-Phase High-Voltage Overhead Lines on the Overhead Communication Line Circuits

When investigating the effect on the communication circuit it is expedient to represent each multiwire high-voltage line of n wires as being made up of two interfering circuits: completely symmetric and completely asymmetric ("n wires-ground"). For example, it is possible to represent the two-phase interfering circuit by the superposition method just as the two-wire circuit subjected to the effect, in the form of two circuits: the two-wire symmetric circuit with applied interfering voltage $\dot{U}$ and interfering current $\dot{I}$ (the direct sequency current) and completely asymmetric with respect to the "two-wires-ground" system to each wire of which an identical interfering voltage $U_0$ is applied with respect to the ground and an identical interfering common current $\dot{I}_0$ is applied in the common conductors running in one direction (the zero sequence current).

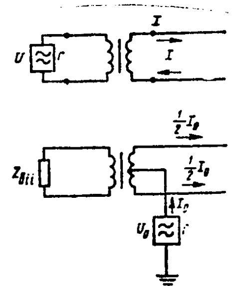



Figure 3.6. Diagrams of the transmission of interfering currents in a two-wire, asymmetric circuit

It is also possible to represent the three-phase interfering circuit as made up of two interfering circuits: symmetric triple-phase with identical line voltages and phase currents 120° out of phase (the direct sequence current) and entirely asymmetric with respect to the "three-phase-ground" system with voltage $U_0$ between each wire and the ground and a common current in the three wires $\dot{I}_0$ (the zero sequence current).

By their fields (electric and magnetic) the indicated high-voltage circuits induce side currents and voltages in the communication circuits. The calculation formulas for determining these currents and voltages under the effect of each component of the circuit of a multiwire line are obtained from solution of the above-investigated general differential equations. The calculation of the effect of the multiwire high-voltage lines on the single-wire communication circuit reduces to determining the effect of two circuits on it: the entirely asymmetric circuit ("two wires-ground" or "three-wire-ground") and the symmetric circuit (two-phase or three-phase).

The determination of the components of the induced voltage caused by the effect of these circuits is made by the formula obtained from the solution of the differential equations (§2.3). For example, the first component caused by the effect of an entirely asymmetric circuit with equality of the lengths of the interfering line and the line subjected to the effect are calculated by (§2.4):

For the case where the circuit subjected to the effect is closed to the wave impedance

$$\dot{U}_{A\,\text{н}1}=\frac{1}{2}\dot{U}_{i\ \text{о.н}}z_{\text{в}A}\left(Y_{iA}+\frac{Z_{iA}}{z_{\text{в}\,i}z_{\text{в}A}}\right)\frac{1-e^{-(\gamma_i+\gamma_A)\,l}}{\gamma_i+\gamma_A}, \tag{3.63}$$

For the case where the circuit subjected to the effect is insulated at the ends

$$\dot{U}_{A\,\text{н}1}=\frac{\dot{U}_{i\ \text{о.н}}\,z_{\text{в}A}}{\gamma_i^2-\gamma_A^2}\left[\frac{Z_{iA}}{z_{\text{в}\,i}z_{\text{в}A}}\left(\gamma_i+\gamma_A\frac{e^{-\gamma_i l}-\operatorname{ch}\gamma_A l}{\operatorname{sh}\gamma_A l}\right)+\right.$$
$$\left.+Y_{iA}\left(\gamma_A+\gamma_i\frac{e^{-\gamma_i l}-\operatorname{ch}\gamma_A l}{\operatorname{sh}\gamma_A l}\right)\right]. \tag{3.64}$$

The second component caused by the effect of the symmetric two-phase or three-phase circuit is calculated by the following formulas for equality of the circuit lengths:

For the case where the circuit subjected to the effect is closed to the wave impedance

$$\dot{U}_{A\,\text{н}2}=\frac{1}{2}\,\frac{\dot{U}_{ii\,\text{н}}\,z_{\text{в}A}}{\gamma_{ii}+\gamma_A}\left(Y_{ii\,A}+\frac{Z_{ii\,A}}{z_{\text{в}\,ii}z_{\text{в}A}}\right)\left[1-e^{-(\gamma_{ii}+\gamma_A)\,l}\right], \tag{3.65}$$

For the case where the circuit subjected to the effect is insulated at the ends

$$\dot{U}_{A\,\text{н}2}=\frac{\dot{U}_{ii\,\text{н}}\,z_{\text{в}A}}{\gamma_{ii}^2-\gamma_A^2}\left[\frac{Z_{ii\,A}}{z_{\text{в}\,ii}z_{\text{в}A}}\left(\gamma_{ii}+\gamma_A\frac{e^{\gamma_{ii} l}-\operatorname{ch}\gamma_A l}{\operatorname{sh}\gamma_A l}\right)+\right.$$
$$\left.+Y_{ii\,A}\left(\gamma_A+\gamma_{ii}\frac{e^{-\gamma_{ii} l}-\operatorname{ch}\gamma_A l}{\operatorname{sh}\gamma_A l}\right)\right]. \tag{3.66}$$

In these formulas:

The index i pertains to the asymmetric interfering circuit and is equal to 2 for the two-phase interfering circuit or 3 for the three-phase interfering circuit;

$\dot{U}_{i\ \text{res.b}}$ is the residual voltage with respect to ground operating at the beginning of the interfering asymmetric circuit ($\dot{U}_{2\ \text{res.b}}$ in the two-phase circuit, $\dot{U}_{3\ \text{res.b}}$ in the three-phase circuit); in practice the magnitude of this voltage reaches 5% of the operating voltage;

$Y_{iA}$ and $Z_{iA}$ are the electric and magnetic couplings respectively between the asymmetric interfering circuit ("two wires-ground" or "three wires-ground") and the single-wire circuits subjected to the effect (A); $z_{wi}$ and $\gamma_i$ are the wave impedance and the propagation coefficient of the above-indicated asymmetric interfering circuit respectively;

The index ii pertains to the symmetric interfering circuit and is equal to 22 for the two-phase circuit or 33 for the three-phase circuit;

$\dot{U}_{iib}$ is the operating voltage between the conductors at the beginning of the interfering symmetric circuit;

$Y_{iiA}$ and $Z_{iiA}$ are the electric and magnetic couplings between the symmetric interfering circuit and the single-wire circuit, respectively;

$z_{wii}$ and $\gamma_{ii}$ are the wave impedance and the propagation coefficient of the symmetric interfering circuit respectively.

The calculation of the effect of the multiwire high-voltage line on the two-wire communication circuit reduces to determination of the four components caused by the effect between the following circuits:

Between the asymmetric interfering and two-wire symmetric circuits subjected to the effect (Fig 3.7a) in which the asymmetry coefficient of the interfering circuit is designated $\eta_1$);

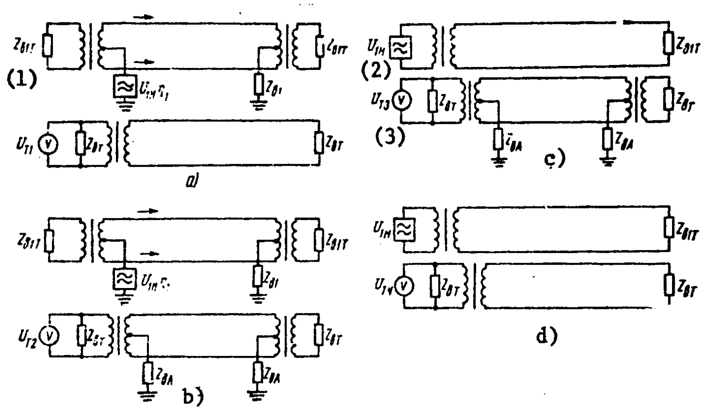



Figure 3.7. Diagrams for the determination of the effect between two-wire incompletely symmetric circuits

Key:
1. w
2. b
3. g

Between the asymmetric interfering and asymmetric circuit subjected to the effect (the cases of closure of the midpoints of the line transformers to the matched resistances of the corresponding "wire-ground" circuits are illustrated in Fig 3.7 b and c);

Between the symmetric interfering circuit and the asymmetric circuit subjected to the effect (Fig 3.7c);

Between the symmetric interfering circuit and the symmetric circuit subjected to the effect (Fig 3.7d).

The calculation of the voltage components caused by the effect between the indicated circuits is made by the formula analogous to the preceding ones. The first component of the voltage in the two-wire communication circuit caused by the effect between the asymmetric interfering circuit and the two-wire symmetric circuit is defined by the formula

$$\dot{U}_{\text{т.н}1}=\frac{1}{2}\dot{U}_{i\,\text{о.н}}\,z_{\text{в.т}}\left(Y_{i\,\text{т}}+\frac{Z_{i\,\text{т}}}{z_{\text{в}\,i}z_{\text{в.т}}}\right)\left(1-e^{-(\gamma_i+\gamma_{\text{т}})\,l}\right)\frac{1}{\gamma_i+\gamma_{\text{т}}}.$$

The second component caused by the effect between the asymmetric interfering circuit and asymmetric circuit subjected to the effect is defined by the formula $\dot{U}_{\text{т.н}2}=\dot{U}_{\text{Аи}1}\eta$, where $U_{Ab1}$ is defined by (3.63) or (3.64). The third component caused by the effect between the symmetric interfering circuit and the asymmetric circuit subjected to the effect is defined by the formula $\dot{U}_{\text{т.н}3}=\dot{U}_{\text{Аи}2}\eta$, where $\dot{U}_{Ab2}$ is calculated by (3.65) or (3.66).

The fourth component caused by the effect between the symmetric interfering circuit and the symmetric circuit subjected to the effect is defined by the formula

$$\dot{U}_{\text{т.н}4}=\frac{1}{2}\dot{U}_{ii\,\text{н}}z_{\text{в.т}}\left(Y_{ii\,\text{т}}+\frac{Z_{ii\,\text{т}}}{z_{\text{в}\,ii}z_{\text{в.т}}}\right)\frac{1-e^{-(\gamma_{ii}+\gamma_{\text{т}})\,l}}{\gamma_{ii}+\gamma_{\text{т}}},$$

where $Y_{iiT}$ and $Z_{iiT}$ are the electric and magnetic couplings between the symmetric multiwire interfering circuit and the two-wire symmetric circuit.

## CHAPTER 4. EFFECT OF THE ELECTRIC FIELD OF A SUPERHIGH VOLTAGE LINE ON THE ORGANISM OF MAN

Until recently it was considered that the triple-phase overhead line with close convergence and intersection with the overhead communications line can have a harmful effect on the health of the service personnel of the communications line only in case where the man is in direct contact with the communications line and induced current passes through the body from the effect of the electromagnetic field of the overhead current to this communications line. It was considered that in the absence of direct contact with the wires of the communications line subjected to the effect of overhead line, the man is not subjected to any harmful effect.

Medical studies have established [31] that the high-intensity electric field is dangerous for the organism of man. Such a field is created by the overhead 330, 500, 750 kv lines and higher. A man under the overhead line (or near them) at the indicated voltages is subjected to the effect of an electric field and, depending on the duration of the flow of the capacitive currents created by the field through his body, functional disturbances of the cardiovascular and central nervous systems can occur [32].

It has been established experimentally that the capacitive current of up to 50 microamperes passing through the body of man (from head to foot) has no noticeable effect on the organism of man. For a current of 80 microamps, weakly expressed functional disorders are noted. These disorders increase together with an increase in the current. The currents flowing through the body of man to the ground caused by the effect of the electric field of overhead lines are proportional to the potential or the intensity of the field at the location of the man.

The Institute of Protection of Labor of the Ministry of Public Health jointly with the ORGRES of the USSR Ministry of Power Engineering established standards for the admissible time for service personnel to be in the electric field of overhead lines on the basis of the performed research, depending on the field intensity at the location of the man [33]. A man can spend an unlimited amount of time in an electric field without harm if the intensity of the electric field of the overhead line at the point where he is located does not exceed 5 kv/m. For an intensity of 10 kv/m the man can be in the electric field for no more than 3 hours. With an intensity of E=15 kv/m the permissible time in the field is no more than 1.5 hours; for E=20 kv/m, no more than 10 minutes, and for E=25 kv/m, the man can be in the field a total of 5 minutes. Failure to observe the indicated hygienic normatives can cause functional disorders of the cardiovascular system and other systems of the vital activity of man.

Accordingly, it is of interest to determine the danger zones near electric power transmission lines with an operating line voltage from 330 kv up in which at the height of the man the fields can occur with an intensity of more than 5 kv/meter. In order to determine the danger zone it is necessary to write the maximum equations for the potentials of an electric field created by the wires of a three-phase line in the air near the electric power transmission line.

Fig 4.1 shows the section of the three-phase overhead line and mirror images of the wires. In the same figure the notation is given for all of the required variables entering into the equations for determination of the potential at the point P (x,y).

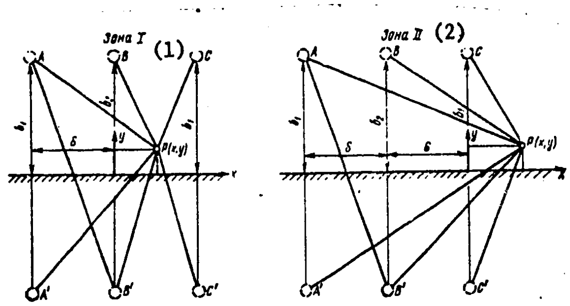



Figure 4.1. Section of a three-phase overhead line for calculating the potential and the intensity of an electric field at the point P(x,y)

Key:
1. Zone I
2. Zone II

The electric field potentials near an overhead line at any point P(x,y) will be defined as the sume of the potentials created at the investigated point by each phase conductor individually, that is, $U_P = \dot{U}_{1\phi} + \dot{U}_{2\phi} + \dot{U}_{3\phi}$. Determing each of these potentials considering the phase shift and the presence of the residual voltage, we obtain

$$U_P = \frac{U_\Phi\left[-\alpha_{P1} + \frac{1}{2}(\alpha_{P2}+\alpha_{P3}) + i\,\frac{\sqrt{3}}{2}(\alpha_{P2}-\alpha_{P3})\right]}{\bar{\alpha}_{11}-\bar{\alpha}_{12}} + \frac{U_0(\alpha_{P1}+\alpha_{P2}+\alpha_{P3})}{\bar{\alpha}_{11}+2\bar{\alpha}_{12}}, \quad (4.1)$$

where $\dot{U}_\phi$ is the phase voltage of a three-phase transmission line; $\dot{U}_0$ is the residual voltage of the three-phase electric power transmission line; $\alpha_{P1}$, $\alpha_{P2}$, $\alpha_{P3}$, $\bar{\alpha}_{11}$, $\bar{\alpha}_{12}$ are the potential coefficients defined by the formula

$$\alpha_{P1}=k\ln\frac{A'P}{AP},\quad \alpha_{P2}=k\ln\frac{B'P}{BP},\quad \alpha_{P3}=k\ln\frac{C'P}{CP},$$

$$\bar{\alpha}_{11}=k\ln\frac{2b_1}{r_1},\quad k=18\cdot 10^{6},\quad \bar{\alpha}_{12}=\frac{\alpha_{AB}+\alpha_{BC}+\alpha_{AC}}{3},$$

$$\alpha_{AB}=k\ln\frac{AB'}{\delta},\quad \alpha_{BC}=k\ln\frac{BC'}{\delta},\quad \alpha_{AC}=k\ln\frac{AC'}{2\delta},$$

$r_1$ is the radius of the phase conductor, meters.

Assuming that $\dot{U}_0=0$, the value of the modulus of the complex intensity of the electric field under the wires and near the wires of overhead lines at the point P(x,y) can be defined by the formula

$$E_P=\frac{C\dot{U}_\Phi}{2\pi\varepsilon}\sqrt{\left|\frac{\partial\dot{U}_P}{\partial x}\right|^2+\left|\frac{\partial\dot{U}_P}{\partial y}\right|^2}. \tag{4.2}$$

In order to determine the maximum distances from the overhead lines for which man is not subjected to dangerous electric field effects, let us find the magnitude of the intensity at the points located in zone II (see Fig 4.1). For the second zone the field beyond the projections of the edge phases

$$\dot{U}_P=\frac{C\dot{U}_\Phi}{2\pi\varepsilon}[\ln\sqrt{(b_1+y)^2+(2\delta+x)^2}-\ln\sqrt{(b_1-y)^2+(2\delta+x)^2}+$$
$$+(0{,}5+i0{,}866\sqrt{(b_1-y)^2+(x+\delta)^2}-\ln\sqrt{(b_1+y)^2+(\delta+x)^2}, \tag{4.3}$$

$$-\frac{\partial U_P}{\partial x}=\frac{CU_\Phi}{2\pi\varepsilon}\Bigg[(2\delta+x)\left(\frac{1}{(b_1+y)^2+(2\delta+x)^2}-\frac{1}{(b_1-y)^2+(2\delta+x)^2}\right)+$$
$$+(0{,}5+i0{,}856)(\delta+x)\left(\frac{1}{(b_1-y)^2+(\delta+x)^2}-\frac{1}{(b_1+y)^2+(\delta+x)^2}\right)+$$
$$+(0{,}5-i0{,}866)x\left(\frac{1}{(b_1-y)^2+x^2}-\frac{1}{(b_1+y)^2+x^2}\right)\Bigg], \tag{4.4}$$

$$-\frac{\partial U_P}{\partial y}=-\frac{CU_\Phi}{2\pi\varepsilon}\Bigg[\left(\frac{b_1+y}{(b_1+y)^2+(2\delta+x)^2}+\frac{b_1-y}{(b_1+y)^2+x^2}\right)+$$
$$+(0{,}5+i0{,}866)\left(\frac{b_1-y}{(b_1-y)^2+(\delta+x)^2}+\frac{b_1+x}{(b_1+y)^2+(\delta+x)^2}\right)-$$
$$-(0{,}5-i0{,}866)\left(\frac{b_1-y}{(b_1-y)^2+x^2}+\frac{b_1+y}{(b_1+y)^2+x^2}\right)\Bigg], \tag{4.5}$$

where C is the capacitance of the "phase-ground" circuit, farads/m.

For calculation of the electric field intensity at different points above the earth's surface under the overhead wires and at different distances from the axis of the overhead line it is possible to use the values of the parameters of the supports and the wires of the overhead line with line voltages of 330, 500, 750, 1150 kv presented in Table 4.1.

Substituting the numerical values of the parameters of the effect (see Table 4.1) in the calculation formulas along the span and in the plane perpendicular to the route of the overhead electric power transmission lines for different distances x from the axis of the power transmission lines, we obtain the field intensities for the points located at a height h=1.8 m above the surface of the ground for the 330, 500, 750 and 1150 kv overhead lines with a span length of 400 meters.

Table 4.1

Parameters of the Supports and Wires in a Span of an Overhead Line for Various Line Voltages

| | | | | |
|---|---|---|---|---|
| Line voltage, kv | 330 | 500 | 750 | 1150 |
| Phase voltage, kv | 190 | 289 | 434 | 665 |
| Distance between phase wires $\delta$, meters | 9.1/18.2 | 12.5/25 | 17.5/35 | 26.1/52.2 |
| No of wires in the phase n, pieces | 2 | 3 | 4 | 8 |
| Equivalent radius of a phase r, m | 0.074 | 0.13 | 0.27 | 0.42 |
| Height of suspension of the wire, m | 22 | 22 | 23 | 24 |
| Overall dimensions of the line, m in the middle of the span | 7.5 | 8 | 9 | 11 |
| Average height of the suspension of the wires, m | 12.5 | 13 | 13.7 | 15.3 |
| Equivalent capacitance of the "phase-ground" circuit, C, farads/meter | $12.3 \cdot 10^{-12}$ | $12.1 \cdot 10^{-12}$ | $13.4 \cdot 10^{-12}$ | $14.1 \cdot 10$ |

The results of calculating the values of the electric field intensity of the overhead power transmission lines along the span at the points located at the height of h=7.8 meters from the ground surface at different distances (x) from the axis of the overhead line in the perpendicular directions are presented in the form of the curves $E=\phi(x)$ depicted in Fig 4.2 a-d. From these curves it is obvious that the most dangerous for man are the places in the middle of the span where the overhead line wires have minimum height of suspension, and the less dangerous places located closer to the supports of the overhead line, inasmuch as at these places the dimensions of the wires are maximal. If we consider the field intensity of 5 kv/meters and less as the admissible safe intensity for human health, then from the curves $E=\phi(x)$ it follows that near the 330 kv overhead line the safe zone is beyond the limits of 5-6 meters from the projection of the edge wire on the surface of

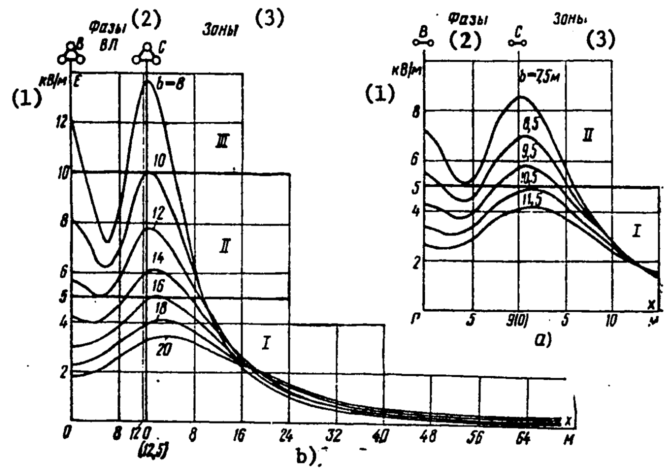
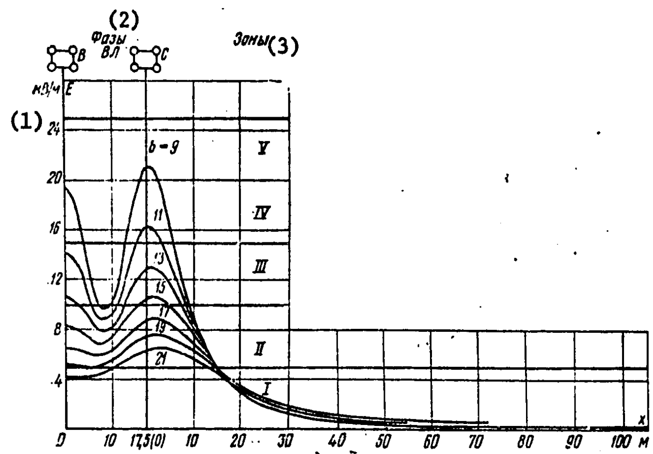


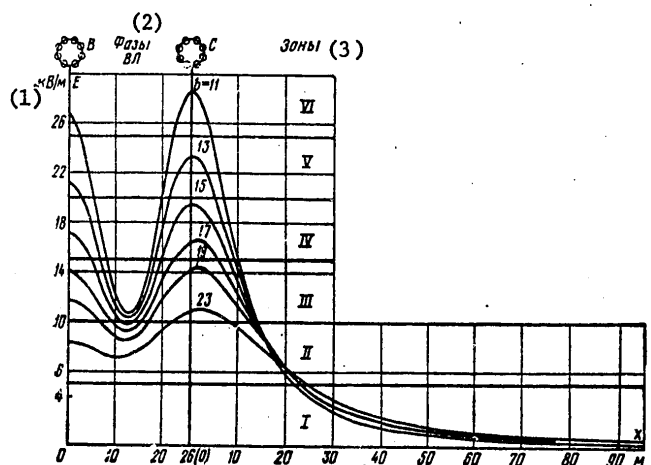
Figure 4.2. Curves for the electric field intensity under the wires of overhead lines: a) overhead lines -- 330 kv (for a distance between phases of 9 meters and overall dimensions of 7.5 meters); b) 500 kv (for a spacing between phases of 12.5 meters and dimensions of 8 meters)

Key:
1. kv/m
2. Phases of overhead lines
3. Zones

the ground in the middle of the span; near the 500 kv overhead power lines, the area beyond 10 meters; near 750 kv overhead lines, beyond 15 meters; near 1150 kv lines, beyond 25 meters.




c)




d)

Figure 4.2 [continued]. c) 750 kv (for spacing between the phases of 17.5 meters and dimensions of 9 meters); d) 1150 kv (for spacing between phases of 26 meters and dimensions of 11 meters)

Key:
1. kv/m
2. Phases of overhead lines
3. Zones

## CHAPTER 5. EFFECT OF CLOUD DISCHARGE ON OVERHEAD COMMUNICATION LINES

### 5.1. Inductive Effect on the Communication Lines

On the wires of an overhead communication line, overvoltages can occur either as a result of induction from the discharge current of the cloud on the ground or discharged between the clouds or as a result of direct discharge of the cloud to the communication line (a lightning strike to elements of the line — wires, supports). These effects of the lightning discharge to a communication line differ sharply from each other both in quantitative and qualitative respects and, consequently, the requirements on the protective devices must be different in the various cases.

Let us first consider the first type of effect — the inductive effect of a lightning discharge on a communications line. Let a cloud with a negative charge -Q be located above the communication line. Then as a result of electric induction on the surface of the ground, positive charges appear. An electric field occurs between the cloud and the ground. The communications wire suspended parallel to the surface of the earth at a height h is located along some equipotential surface; at the side closest to the cloud, the wire charge is positive, and on the opposite side, negative. If the investigated wire is well insulated on the ground, then the total charge of the wire is zero. At the time of discharge of the cloud to the ground with the disappearance of the electric field, both charges on the wire are neutralized, and the potential of the wire becomes equal to zero.

If the length of the communications wire, as occurs in the majority of cases, greatly exceeds the length of the charged cloud and the wire has insufficiently good insulation with respect to the cloud, the negative charge of the wire drifts to the right and left of the section affected by the cloud and it gradually leaks to the ground. The positive charge connected with the cloud remains on the wire. The wire potential will be zero until discharge of the cloud begins (Fig 5.1). For discharge of the cloud to the ground, the charge of the cloud and the electric field between the cloud and the ground disappear. The charge on the wire, remaining free, now (after the cloud discharge) begins to move to the right and to the left from the point of its occurrence. The speed of the charge, as is known, is defined by the line parameters (this speed is close to the speed of light).

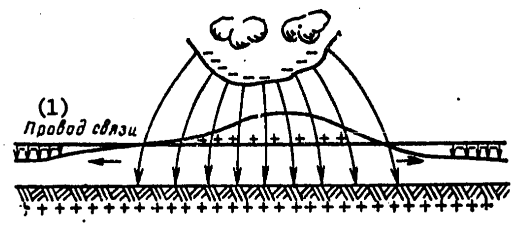



Figure 5.1. Distribution of induced discharges on a communications wire in the electric field of a charged cloud

Key:

1. Communications wire

At the time of discharge of the cloud on the wire, there is some potential distribution which is caused by the magnitude of the charge on each element of the conductor and its capacitance with respect to ground. Here, a pair of identical voltage waves and a pair of current waves occur on the wire which move at a speed of $1/\sqrt{LC}$ (for a line without losses) in opposite directions.

The initial charged distribution along the wire for specific conditions of location of the cloud, its size and the charge distribution can be determined analytically. The charge distribution along the wire obtained up to the beginning of discharge has the nature of a bell-curve with maximum under the center of the cloud. For quantitative determination of the magnitude of the voltage induced on the wire with lightning discharge it is necessary to consider the following factors: the charge distribution of the cloud and its magnitude, the relative location of the cloud and the wire (the height of the thundercloud above the earth), the electric field intensity between the cloud and the earth, the nature and the duration of the cloud discharge.

The external electric field creates vertical and horizontal (longitudinal) components of the intensity. Under its effect for a lightning discharge in the wires, the transient process occurs which lasts even after the lightning discharge is stopped (Fig 5.2). The differential equation for the line potential will have the form:

$$\frac{\partial^2 U}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2 U}{\partial t^2}=\frac{\partial E_x}{\partial x}. \quad (5,1)$$

The equation is solved by the ordinary method of solving partial differential equations:

$$U(x, t)=\frac{c}{2}\int_0^t E_x[\tau, x-c(t-\tau)]\,d\tau-\frac{c}{2}\int_0^t E_x[\tau, x+c(t-\tau)]\times$$
$$\times\, d\tau+\varphi(x, t), \qquad (5.2)$$

where $\phi(x,t)$ is the scalar potential of the external electric field at the given point, and the sum of the integrals is a traveling voltage wave over the line.

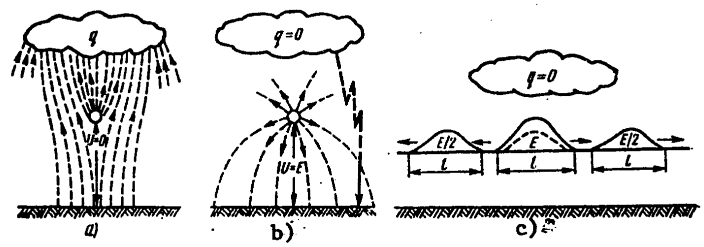


Figure 5.2. Formation of stray voltage and current waves in the communications wire during discharge of a cloud to ground

Under the effect of a variable field during lightning discharge the charges on the line wires are divided into bound and free charges. The bound charges are held by an external field; they insure, as Razevig noted [34], the preservation of the zero potential of the wire in the steady state. The free charges leak in the form of traveling waves in both directions from the point of occurrence (opposite to the point where the lightning struck). If the lightning strike with an amplitude of $I_M$ took place at a distance b from a line of height h, then as a result of integrating expression (5.2) for x=0 the traveling wave has the form

$$U(0, t)=\frac{60I_M h}{b}\,\frac{\beta}{(\beta ct/b)^2+1}\left[\frac{ct}{b}-\frac{1}{\sqrt{(\beta ct/b)^2+1}}\right],$$

where $\beta=v/c<1$, v is the rate of development of the return discharge, which is connected with the amplitude of the lightning current. According to Norinder and Lundholm

$$\beta=\frac{v}{c}=\frac{1}{\sqrt{1+\frac{ac}{30I_M}}}, \qquad (5.3)$$

where a=30,000 microcoulombs-ohm/m is the Tepler spark constant.

In Table 5.1 the lightning current amplitude is presented as a function of the return discharge rate.

Table 5.1

Relation Between the Lightning Current Amplitude and the Return Discharge Rate

| Амплитуда тока молнии, кА(1) | 2 | 5 | 10 | 20 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|---|
| $\beta = v/c$ | 0,047 | 0,074 | 0,105 | 0,147 | 0,228 | 0,316 | 0,427 |

Key:
1. Lightning current amplitude, kiloamps

The solution for the induced voltage from the magnetic component of the field at the point close to the point where the lightning strikes gives the following expression:

$$U_{\text{и.м}}(0,\ t) = \frac{60 I_{\text{м}} h}{b} \frac{\beta}{\sqrt{(\beta ct/b)^2 + 1 - \beta^2}}, \quad (5.4)$$

where $I_M$ is the lightning current amplitude; h is the height of the line supports; $\beta = v/c$ is the rate of development of the lightning discharge; c is the speed of light.

In the general case for a square lightning current wave it is possible to consider that the amplitude of the induced voltages defined by the equality $U_{i.M} = 30\ I_M h/b$. If the current front has a linear steepness a, that is, the current varies according to the law $i = at$ to its amplitude, the electric and magnetic components of the voltage induced in the line at the point closest to the point where the lightning struck has the form:

$$\underset{(1)}{U_{\text{и.э}}}(0,\ t) = \frac{60ah}{\beta c} \ln \frac{\sqrt{(\beta ct/b)^2 + 1 - \beta^2} - \beta^2 \frac{ct}{b}}{1 - \beta^2}, \quad (5.5)$$

$$\underset{(2)}{U_{\text{и.м}}}(0,\ t) = \frac{60ah}{c} \ln \frac{\beta \frac{ct}{b} + \sqrt{(\beta ct/b)^2 + 1 - \beta^2}}{1 + \beta} \quad (5.6)$$

Key: 1. i.e; 2. i.M

for $t \leq \tau_\phi + (b/c)$, where $\tau_\phi$ is the front duration. For a time $t > \tau_\phi + b/c$ the solution is found by superposing two oblique forms of the current corresponding to the buildup and decay of it.

Fig 5.3 shows the shape of the total induced voltage wave in the line calculated according to (5.5) and (5.6) for the case h=5 m, a=30 kiloamps/microseconds, $\tau_\phi$=4 microseconds, $I_M$=120 kiloamps.

The distribution of the bound charge along the wire varies from point to point of the wire in accordance with the intensity of the electrostatic field of the cloud. When the cloud discharges, the bound charge is freed with the same rate with which the cloud discharges, and it causes stray voltage and current waves propagated to the right and left of the point of occurrence of the charge (see Fig 5.2).

On high-voltage transmission lines, induced voltages have been recorded with an amplitude up to 500000 volts.

In order to estimate the effective values of the voltages occurring on communication wire, in the USSR measurements have been made of the induced voltages during lightning discharges on clidonographs. The maximum peak voltage on the clidonograms was 40,000 volts, and the minimum was 6000 volts.

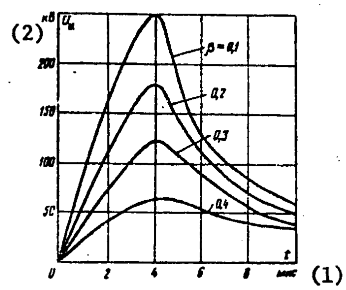



Figure 5.3. Total induced voltage in the line for h=5 m, a=30 kiloamps/microsecond, $\tau_\phi$=4 microseconds, $I_M$=120 kiloamps, $\beta$=V/c, where V is the inverse discharge rate

Key:
1. $\tau$, microseconds
2. $U_i$, kilovolts

For large voltages on the communication wires, discharge takes place from the wire along the surface of the insulators (TF-1 and TF-2) to the hooks and the supports. The surface discharge on wet supports begins for voltages of about 50-60 kv. For dry supports the induced voltages on the wires can reach 100 kv.

Thus, it is possible to consider it established that for lightning discharge to ground the maximum magnitude of the induced overvoltage wave which can be propagated over the communication wires with wet insulators and wires does not exceed 50 kv.

In order to avoid danger arising in the communications circuits from the inductive effect of the lightning discharges, it is necessary to protect the terminal devices (the telegraph-telephone office equipment, the subscriber equipment, cables) connected to the wires of the overhead line using the corresponding devices. The latter must withstand charge power of the wires, which according to the statistical observations [34], sharply exceeds 8-10 joules on each wire.

5.2. Cases of Occurrence of Inductive Overvoltages on the Wires of Overhead Lines

The number of cases where the voltage induced on the line exceeds the amplitude $U_i$, can be found from the integral

$$n = LNq \int_{b_{\text{мин}}\,(2)}^{b_{\text{макс}}\,(1)} \Delta n(b)\, db, \tag{5.7}$$

Key: 1. max; 2. min

where L is the line length; N is the number of thunderstorm days per year; q is the average number of discharges per $km^2$ of the earth's surface for 1 thunderstorm day (according to the data of the USSR Academy of Sciences $q \simeq 0.1$); $b_{max}$ is the maximum distance determined by the maximum value of the current, the probability of which is meaningful to consider; $b_{min}$ is the minimum distance from the line to the point struck by the lightning, $b_{min}$=2-3h. For smaller distances, the lightning strike will occur to the line itself. If $b_{min} \to 0$, then we obtain the expression for the total number of overvoltages in the line.

The probability that the lightning current amplitude will exceed the value $I_M$ is well known. It is determined from the formula (LPI)

$$P(I) = 10^{-\frac{I_M}{60}} = e^{-kI}. \tag{5.8}$$

It was established above that the lightning strike at a distance b from the line will induce a voltage $U_M$ only in the case where $I_M \geq U_M b/30h$. Inasmuch as the probability of such currents is equal to P(I), $\Delta n(b)$ will have the same value for the lightning strike at the given distance b. Substituting the value of $\Delta n(b)$ in the integral (5.7), we find that

$$n = LNq \int_{b_{\text{мин}}\,(2)}^{b_{\text{макс}}\,(1)} 10^{-\frac{U_M b}{30h60}}\, db = LNq \int_{b_{\text{мин}}\,(2)}^{\infty} e^{-\frac{kU_M b}{30h}}\, db.$$

Key: 1. max; 2. min

Integrating, we obtain (setting $b_{max} \to \infty$)

$$n = LNq \frac{1800}{U_M} h \cdot 10^{-\frac{U_M b_{\text{мин}}\,(1)}{1800h}}. \tag{5.9}$$

Key: 1. min

Substituting the values of L=100 km, q=0.1 strikes/km$^2$ per day, let us transform the formula

$$n = \frac{18Nh}{U_M} 10^{-\frac{U_M b_{\text{мин}}}{1800h}} \text{ (1)},$$

Key: 1. min

where h is the height, and $U_M$ is the voltage, kv.

Setting $b_{min}=0$, we find that the total number of cases of appearance of voltages on the line (100 km per year) with an amplitude $U_M$ is given by the expression $n=18\ Nh/U_M$.

5.3. Magnitude of the Overvoltages Induced in the Wires Considering Corona

During movement along the communications wires the overvoltage wave undergoes distortions and damping. When developing the protection measures it is necessary to know the magnitude of the incident voltage wave at the input of the protected project. The inexactness with respect to the probable magnitude of the damping and the distortion can lead to significant overestimation of the possible overvoltage at the input to the telegraph or telephone office and on the terminals of the subscriber equipment or, on the contrary, to underestimation of its value. The studies performed on the electric power transmission lines indicate that along with the electrical parameters of the wires, the damping and the distortion of the waves are significantly influenced by a corona of the wires during overvoltages, for the parameters of the overhead line circuit change significantly during corona [35].

The essence of the corona phenomenon is ionization of the air near the wire under high voltage (see Chapter 1). The ionization of the air is accompanied by a characteristic crack and a very noticeable blue glow. This phenomenon occurs for some critical voltage on the wires. For the two-wire circuit the critical voltage (in kilovolts) for which the corona phenomenon begins to occur on the wires can be determined for normal air pressure and temperature of t=25°C by the Peak formula

$$U_{\text{кр (1)}} = 10,5d \ln \frac{2D}{d}, \qquad (5.10)$$

Key: 1. cr

where d is the wire diameter, meters; D is the spacing between the circuit wires, meters.

For communication wires with induced voltage wave during a lightning discharge it is necessary to substitute a value of 2h -- the distance from the investigated communications wire to its mirror image with respect to the earth's surface, in place of D in the Peak formula, that is,

$$U_{\text{кр}} = 10{,}5d \ln \frac{4h}{d}.$$
(1)

Key: 1. cr

Research has established that the critical corona voltage depends on the state of the wire surface and atmosphere conditions. These factors can be taken into account by introducing the three coefficients $m_0$, $m_\pi$, $\delta$ into the Peak formula. Then we obtain

$$U_{\text{кр}} = 10{,}5 m_0 m_\text{п} \delta d \ln \frac{4h}{d}, \tag{5.11}$$
(1)

Key: 1. cr

where $m_0$ is the coefficient taking into account the state of the wire surface (for new wires $m_0=1$, and for wires in the air for a prolonged period of time, $m_0=0.98$ to $0.93$); $m_\pi$ is the coefficient taking into account the state of the weather (for dry weather $m_\pi=1$, and for fog or rain it varies within the limits from 0.8 to 0.9); $\delta$ is the relative density of the air determined from the equality $\delta=3.9b/273+t$ [b is the barometric pressure in torrs (mm of mercury); t is the air temperature, °C].

The values of the voltages $U_{cr}$ calculated by (5.10) for which the corona phenomenon appears on the communication wires are presented in Table 5.2.

As is obvious from Table 5.2, the values of the critical voltages on the communication wires of any diameter are within the limits which can be induced on the wires for lightning strikes to ground. For direct lightning strikes the voltages on the communication wires can reach hundreds of thousands and even millions of volts, so that in this case the corona phenomenon will always occur. From the point of view of protection of the communication installations the corona phenomenon is a positive factor, for during corona the losses in the line increase and, consequently, the energy going to the input to the telegraph or telephone station decreases significantly. The existing theory of the damping and deformation of the stray waves for a voltage above critical is based on a number of arbitrary assumptions and cannot be used for quantitative evaluation of the phenomena.

For practical calculations of the decrease in amplitude of the stray waves in the electric power transmission lines, usually the Faust-Mendger formula is used:

$$U_l = U_0 \frac{1}{kU_0 l + 1}, \tag{5.12}$$

where $U_\ell$ is the wave amplitude at a point removed by $\ell$ km from the point of occurrence of the wave; $U_0$ is the wave amplitude at the point of its occurrence; k is the numerical coefficient obtained experimentally and depending on the parameters of the electric power transmission lines, the shape and polarity of the wave.

Table 5.2

Critical Voltages on Overhead Wires

| (1) Коэффициенты, учитывающие состояние провода, погоды | (2) Напряжение $U_{кр}$(кВ) при диаметре провода $d$, см | | | |
|---|---|---|---|---|
| | 0,3 | 0,35 | 0,4 | 0,5 |
| $m_0=1$, $m_н=1$, $\delta=1$ | 28,2 | 29,2 | 36,5 | 44,4 |
| $m_0=0{,}93$, $m_н=1$, $\delta=1$ | 26,2 | 27,0 | 33,6 | 41,2 |
| $m_0=0{,}93$, $m_н=0{,}85$, $\delta=1$ | 22,0 | 22,9 | 28,5 | 35,0 |

Key:
1. Coefficients taking into account the state of the wire, weather
2. Voltage $U_{cr}$(kv) for a wire diameter d, cm

In addition, the studies performed by the Khar'kov Electrotechnical Institute [36, 37] demonstrated that the coefficient k also depends on the amplitude of the initial wave; the value of k varies within the limits from $1.5\cdot10^{-4}$ to $5\cdot10^{-4}$. The formula (5.11) for the communications lines can give only approximate results, for the parameters of the communications line differ from the parameters of the transmission lines as a result of the large number of wires suspended on them.

The preliminary data which were obtained when determining the induced potentials on the communication wires during lightning discharges to ground indicate the presence of significant dampings on the line. Thus, it was noted that the wave with a voltage of 40,000 volts on steel wire (d=4 mm) dropped to 2500 volts after 500 meters.

## 5.4. Effect of Direct Lightning Strikes on Overhead Communication Lines

**Magnitudes of the Overvoltages in the Communication Lines.** The discharge of a thundercloud to the ground sometimes occurs through a communication line. In this case the lightning can strike either the wires of the line between the supports or the support directly. In case the lightning reaches the wires in the span between the supports the current is divided into two parts, and it is propagated in the form of stray waves through the line to the right and left of the point where the strike occurred at a speed approximately equal to $100\cdot10^3$ to $150\cdot10^3$ km/sec. The lightning strike in the line can be considered as a case of wave transfer from the circuit (the lightning channel) with one wave impedance to the single-wire communications circuit with another wave impedance. The voltage U of the stray wave traveling through the wires from the point where the lightning strikes will be determined from the equation

$$\left.\begin{aligned} U &= \frac{2Z_{в.л}U_0}{Z_{в.л}^{(1)}+2Z_{в.м}},\\ U_0 &= IZ_{в.м}^{(2)} \end{aligned}\right\} \qquad (5.13)$$

Key: 1. overhead line; 2. lightning channel where $Z_{overhead\ line}$ is the wave impedance of bunched conductors of overhead line; $Z_{lightning\ channel}$ is the wave impedance of the lightning channel; I is the lightning current.

The value of U can reach a value which is sufficient for establishment of the current destroying the support. If the wave impedance of the lightning channel is taken as $Z_{lightning\ channel}=200$ ohms [37], the wave impedance of the bunched conductors $Z_{lightning\ channel}=300$ ohms and $I=4000$ amps, the voltage on the wires with respect to ground will be $6.88 \cdot 10^6$ volts. The currents of the direct strike and the overvoltages of this magnitude cause destruction of the line structures. The maximum electric strength of the communications line on the wooden supports according to the empirical formula of the LPI Institute is equal to $U_p=550 \sqrt[3]{h}$ where h is the height of the support.

Effect on the Support. The studies [38, 39] indicate that for splitting dry posts with current pulses it is necessary to apply a voltage from 200 to 300 kv per m of length of the post. It follows from this that in order to split a post 6 meters long a voltage from $1.2 \cdot 10^6$ to $1.8 \cdot 10^6$ volts is used. Observations have established that before rain the splitting of the posts takes place more often than after the rain. During splitting either more or less long splinters are torn out of the post or the post itself breaks into several parts along the fibers.

At the present time scientists do not have a united opinion with respect to the cause of this damage. When a lightning current passes through the capillaries of the tree filled with air and moisture, intensified gas and vapor formation occurs. Under the effect of very high temperatures in the lightning channel even in the absence of moisture the tree can be converted to vapor and gas,the effect of which leads to destruction of the tree. I. S. Stekol'nikov [40] explains the splitting of a post by the effect of the electric forces of the fibers on each other. On transmission of the current through the fibers of the tree, electrodynamic forces develop which are aimed at the axis of the post and the electrostatic forces directed in the opposite direction causing repulsion of the fibers. On transmission of the lightning current these forces are partially mutually equalized. After the current stops, the electrodynamic forces disappear, and only the electric forces remain, under the effect of which the fibers repel each other with great force.

Effect on Line Insulators. On the appearance of an electric arc between the conductors and supports, significant thermal and mechanical stresses occur in the insulators. The electric voltage in the porcelain is explained by the occurrence of current waves having large amplitudes and steepness of the front. Inasmuch as there are air sections on the surface of the insulator, the current transmission is retarded, as a result of which breakdown and rupture occur.

The mechanical stresses are explained by powerful heating of the insulator at the point of occurrence of the voltage arc.

The effect of the current directly on the wire at the point where it is attached to the insulator can cause pieces of the porcelain to break off. The damage occurs predominantly at the neck of the insulators near the binder where the voltage of the electric field between the binder and the hook is the highest.

Effect on Communication Wires. As a result of the effect of powerful lightning currents, burning and fusion of the wires take place. The statistical observations performed both in the USSR [41] and abroad [38] indicate that for lightning strikes the line wires break both in the span and near the insulators. Several dozen cases of damage to overhead communication lines near supports protected by lightning rods are recorded annually. The most characteristic for the overhead lines are the following types of damage: melting of the copper wire communication lines in the case of a direct lightning strike with high force; burning along the wires — copper and steel — from a direct lightning strike; burning of the line wires and destruction of the insulators from a direct lightning strike. Most frequently the wires are burned to the nearest post protected by a lightning rod. For especially strong discharges the wires can melt for several spans on both sides of the lightning strike. As a result of burning through the bronze wire breaks down into pieces 3-5 cm long over a significant section. Since the amount of heat required to melt the wire is determined quite precisely, there is a possibility of concluding the magnitude of the lightning currents which occurred over the damaged wires. The method of calculating the heating of the wire during transmission of electric power is presented in [41].

The relation between the current $I_M$ (in amperes) melting the wire, the material and the geometric parameters of the conductor and also the transmission time of the current through the wire is expressed by the formula $I_M = kS/\sqrt{\tau}$, where S is the wire cross section, $mm^2$; $\tau$ is the current transmission time, seconds; k is the numerical coefficient which depends on the material of the wire.

The average values of the coefficient k obtained by calculation, and corrected by the experimental data, are presented in Table 5.3.

Table 5.3

Values of the Numerical Coefficient Taking Into Account the Wire Material

| Wire material | Value of the coefficient k for bringing the wire to the following temperatures | | |
|---|---|---|---|
| | Incandescent | Melting | Evaporation |
| Copper | 220 000 | 305 000 | 327 000 |
| Iron | 90 000 | 115 000 | 438 000 |
| Aluminum | 128 000 | 200 000 | 230 000 |

The presented formula has great practical significance. This formula is used to calculate the grounding devices both in high voltage engineering and in communication engineering, when designing the protection of linear communications structures from direct lightning strikes. When lightning strikes in the majority of cases partial rupture of the wires takes place at the point struck; here an electric arc crater is formed in which part of the metal melts. Accordingly, the operating cross section of the wire and its rupture strength decrease. If the rupture strength of the wire is less than its tensile strength between posts, it breaks. In the winter at a temperature of up to −20°C the communication wires of the overhead lines have a load equal to 1/3 of their tensile strength. In the summer at a temperature of +20°C, the tension of the wires with sag of up to 30 km is half the tensile strength so that there is a sixfold rupture margin of safety. Therefore the overhead wires break as a result of their melting where the lightning strikes only when the cross section of the unmelted part of the wire will be less than 1/3 or 1/6 of the rated cross section of the wire respectively.

For a lightning strike to the sheathing of a suspended communications cable (cable lead-ins or inserts), holes can appear in its lead sheathing. For a lightning current of $50.10^3$ amps and a transmission time of 100 microseconds, for example, 50 $mm^3$ of lead will melt. For a thickness of the cable shielding of 2 mm, a hole 6 mm in diameter is formed. As a result of pressure in the crater and under the effect of the current, the melted lead frequently gets into the cable between its sheathing and the cores.

The mechanical tension of the wire increases from a decrease in cross section, but also under the effect of the electrodynamic forces which occur at the point where the lightning strikes. Let us propose that a lightning current I for a lightning channel diameter d is directed perpendicular to the wire. Since the current tries to maintain its initial direction, as a result of bunching of the electric field lines at the bend in the wire electrodynamic forces occur which have an influence on its mechanical effect in the form of a strong impact (impetus). The force F acting per unit length of the wire is defined in newtons per meter by the formula

$$F = 10^{-6} B \frac{I}{2}, \tag{5.14}$$

where B is the magnetic induction of the magnetic field. This value, as is known, can be determined from the equality $B=0.796\ \mu_0 10^5 (I/x)=0.1\ I/x$; $\mu_0$ is the magnetic permeability of the vacuum ($\mu_0=4\pi\cdot 10^{-7}$ henries/m).

Substituting B in (5.14), we obtain $F\simeq 0.5\cdot 10^{-7}\ I^2/x$. The force acting on a section of the line of length $\ell$, reckoning from the point that the lightning struck, is

$$F_0 = \int_{x=0,5d}^{l} F dx \approx 0,5\cdot 10^{-7} I^2 \ln \frac{2l}{d}.$$

The effect of the force of the internal magnetic field in the channel itself cannot be considered as a result of its small magnitude. For lightning current of I=50,000 amps and the channel diameter d=0.5 cm the pressure directly at the point where the lightning struck will exceed 200 N per unit length of wire. Therefore for a lightning strike to an overhead wire, a mechanical stress occurs in it which is several times greater than normal, which can lead to rupture of the wire.

## 5.5. Overvoltages on the Entrance Cables and Cable Inserts to Overhead Lines

The entrance of overhead communication lines to the repeater stations in the majority of cases is made by cables. When the overhead lines, for example, cross rivers and railroads at the points of intersection with the overhead lines often cable inserts are used, the length of which, depending on the local conditions, can vary from several tens of meters to several kilometers. Usually the entrance cables and the cable inserts are laid in the ground (underground cables), and they are suspended very rarely on the wooden supports (the overhead cables).

For direct lightning strikes to an overhead line and also for lightning strikes to ground near an overhead line the magnitude of the voltage in the cores of the cable inserts and the cable lead-ins will differ from the overvoltages occurring on the overhead wires.

Let us consider what the magnitude of the voltage will be on the right end of the cable if a square overvoltage wave $U_{AB}$ occurring in the overhead wire (Fig 5.4a and b) approaches the left end of the cable insert or cable entrance. At the point of connection of the overhead line to the cable line (point B) the voltage wave $U_{AB}$ is refracted and reflected at the points B and C. Let us determine the resultant wave $U_{CD}$ after the cable insert, again reaching the overhead line CD and the resultant voltage wave $U_{Cg}$ on the terminals of the communications equipment for a cable entrance. The resultant voltage wave going to the overhead line CD (from the right hand side of the cable insert) will be defined [37] by the sum of the series

$$U_{CD}=\alpha_{AB}\alpha_{BC}U_{AB}(t)+\alpha_{AB}p_{CB}p_{BC}\alpha_{BC}U_{AB}(t-2t_0)+ \\ +\alpha_{AB}p_{CB}^2p_{BC}^2\alpha_{CB}U_{AB}(t-4t_0)+\cdots+\alpha_{AB}p_{CB}^np_{BC}^n\times \\ \times\alpha_{BC}U_{AB}(t-2nt_0), \qquad (5.15)$$

where $\alpha_{AB}=2z_{w2}/(z_{w2}+z_{w1})$ and $\alpha_{BC}=2z_{w1}/(z_{w1}+z_{w2})$ are the refraction coefficients of the voltage wave at the points B and C; $z_{w1}$ and $z_{w2}$ are the wave impedance of the overhead line circuit and cable line circuit respectively; $p_{BC}=p_{CB}=(z_{w1}-z_{w2})/(z_{w1}+z_{w2})$ are the reflection coefficients at the points B and C; $t_0=\ell/v=\sqrt{L_2C_2}$ is the time of transmission of the wave through the cable insert of length $\ell$, the capacitance $C_2$ and the inductance $L_2$ for a wave propagation rate through the cable v; $t/2t_0-1\leq n<t/2t_0$ is the number of successive wave reflections.

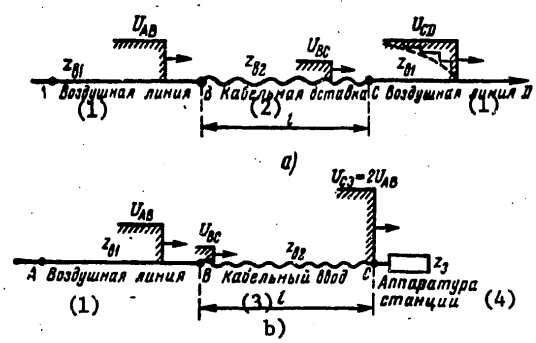



Figure 5.4. Calculation of overvoltages on the overhead line: a) with the cable insert; b) with cable entrance

Key:
1. Overhead line
2. Cable insert
3. Cable entrance
4. Station equipment

If we take the time of appearance of the first wave at point C as the time origin, then the value of $U_{CD}$ after the n-th reflection is determined from the equation

$$U_{CD}=\alpha_{AB}\alpha_{BC}U_{AB}\sum_{k=0}^{n}p^{2k}=A[1-p^{2(n+1)}]U_{AB}, \tag{5.16}$$

where

$$A=\frac{\alpha_{AB}\alpha_{BC}}{1-p^2}=\frac{2z_{B1}2z_{B2}^{(1)}}{(z_{B1}+z_{B2})^2-(z_{B1}-z_{B2})^2}=1.$$

Key: 1. w

Thus, the step wave with gradually increasing voltage goes to the overhead line CD (after the cable insert)

$$U_{CD}=\left[1-\left(\frac{z_{B1}-z_{B2}}{z_{B1}+z_{B2}}\right)^{2n+2}\right]U_{AB}, \tag{5.17}$$
(1)

Key: 1. w

where $n\leq t/2t_0<n+1$.

By the end of the steady state process (for $n\to\infty$) the voltage on the second end of the cable insert strives to be equal to the voltage of the incident wave at the first end of this insert. Approximately replacing the step nature of build-up of the voltage wave on the right end of the cable insert by a smooth nature of build-up of the wave, it is possible to represent the expression (5.17) in the form

$$U_{CD} \approx U_{AB}\left(1-e^{-\frac{t}{T}}\right), \tag{5.18}$$

where
$$T = t_0/\ln\frac{z_{w1}+z_{w2}}{z_{w1}-z_{w2}}.$$

Since $z_{w1} \gg z_{w2}$, representing the expression $\ln(z_{w1}+z_{w2})/(z_{w1}-z_{w2})$ in the form of a series and replacing $t_0$ by the cable parameters, we obtain

$$T = \frac{\sqrt{L_2C_2}}{2\left[\frac{z_{w2}}{z_{w1}}+\frac{1}{3}\left(\frac{z_{w2}}{z_{w1}}\right)^3+\ldots\right]} \approx \frac{z_{w1}C_2}{2}. \tag{5.19}$$

It follows from this that the shape of the wave in the overhead line with wave impedance ($z_{w1}$) after passage through the cable insert ($z_{w2} \ll z_{w1}$) will be approximately the same as in the case of replacement of the cable by a concentrated capacitance. In the case of a cable entrance where the cable cores are connected to the telegraph or telephone equipment with input impedance with respect to ground ($Z_g$), the resultant voltage $U_{Cg}$ at the station end of the cable entrance can be defined by (5.16) for substitution in it of the following corresponding coefficients of reflection and refraction of the wave for the end of the cable entrance: $\alpha_{AB}=2z_{w2}/(z_{w2}+z_{w1})$, $\alpha_{BC}=2Z_g/(Z_g+z_{w2})$, $p_{BC}=(z_{w1}-z_{w2})/(z_{w1}+z_{w2})$, $p_{BC}=(Z_g-z_{w2})/(Z_g+z_{w2})$.

If the winding of the line transformer or the communications equipment is connected at the end of the two-wire telephone circuit, for each wire of the circuit the load with respect to ground at the station end of the cable will be $Z_g=\infty$, for each wire of the telephone circuit is open with respect to ground. In this case the refraction and reflection coefficients will be correspondingly: $\alpha_{BC}=2$, $p_{CB}=1$, and the resultant voltage on the station end of the cable entrance will approach twice the voltage of the incident wave on the overhead line at the limit, that is, $U_{Cg} \to 2U_{AB}$.

Actually, considering the above-indicated conditions, it is possible to write equations (5.15) in the following form:

$$\left.\begin{aligned} U_{CD} &= \alpha_{AB}2U_{AB}(t)+\alpha_{AB}p_{BC}2U_{AB}(t-2t_0)+\alpha_{AB}p_{BC}^2 2U_{AB}(t-4t_0)+ \\ &\quad +\ldots+\alpha_{AB}p_{BC}^n 2U_{AB}(t-2nt_0), \\ \text{or} \quad U_{CD} &= \alpha_{AB}2U_{AB}\sum_k^n p_{BC}^k = 2U_{AB}A\left[1-p_{BC}^{(n+1)}\right], \end{aligned}\right\} \tag{5.20}$$

where

$$\dot{A}=\frac{2\alpha_{AB}}{1-p_{BC}}=\frac{2z_{B2}}{(z_{B1}+z_{B2})\left(1-\frac{z_{B1}-z_{B2}}{z_{B1}+z_{B2}}\right)}=1.$$

Thus, on the station end of the cable core the voltage gradually increases in accordance with the equation

$$U_{Cg}=2U_{AB}\left[1-\left(\frac{z_{B1}-z_{B2}}{z_{B1}+z_{B2}}\right)^{n+1}\right], \tag{5.21}$$

where $n \leq t/2t_0 < n+1$, For $n \to \infty$ $U_{Cg} \simeq 2U_{AB}$.

The conclusion drawn with respect to the overvoltages in the input cable is valid only for the cases of comparatively short cables where it is possible to neglect the wave attenuation. In addition, the magnitude of the resultant voltage on the station end of the entrance cable will also depend on the length and shape of the incident wave.

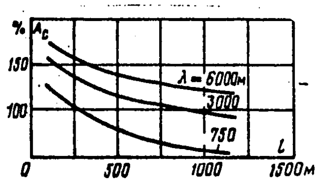



Figure 5.5. Ratio $A_c$ (in percentages) of the maximum voltage on the station end of the entrance cable to the maximum voltage of the incident wave as a function of the length of the entrance cable $\ell$ for different wave lengths $\lambda$

As an example, in Table 5.4 the results are presented from calculating the maximum voltage on the station end of the entrance cable considering the attenuation and the shape of the incident wave as a function of the length of this cable for three incident waves of different length. The values presented in this table were obtained for the following initial data: wave impedance of the wire of the overhead line $z_{w1}$=150 ohms; wave impedance of the core of the entrance cable $z_{w2}$=50 ohms; attenuation in the cable cores for the wave length given in Table 2.7, respectively: $a_1$=0.18 decibels; $a_2$=0.47 decibels; $a_3$=1.65 decibels; a wave propagation rate through the cable v on the average equal to $150 \cdot 10^6$ m/sec.

The relation for the maximum voltage $U_{Cg}$ on the station end of the entrance cable as a function of its length $\ell$ and the length of the incident wave $\lambda_w$ is presented graphically in Fig 5.5 according to the data in Table 5.4.

Table 5.4

Maximum Voltage on the Station End of the Entrance Cable

| Cable length, $\ell$, meters | Maximum voltage on the station end of the entrance cable (in percentages) of the maximum voltage of the incident wave for | | |
|---|---|---|---|
| | $\tau$=5 microseconds, $\lambda_W$=750 meters | $\tau$=20 microseconds, $\lambda_W$=3000 meters | $\tau$=50 microseconds, $\lambda_W$=7500 meters |
| 75 | 134 | 163 | 180 |
| 150 | 118 | 148 | 168 |
| 300 | 97 | 130 | 156 |
| 600 | 71 | 113 | 138 |
| 1200 | 50 | 92 | 122 |

From the data in Table 5.4 and in Fig 5.5 it is obvious that the voltage occurring on the station end of the entrance cable is greater the less the cable length and the greater the length of the incident wave by comparison with the length of this cable. The presented calculations indicate that on the station end of the entrance cable, in particular, for short length, voltages can occur which are almost twice the voltage occurring on the overhead line. Thus, the cable insulation and the insulation of the station equipment are in danger of breakdown.

# CHAPTER 6. EFFECT OF MAGNETIC STORMS AND CURRENTS IN THE GROUND ON COMMUNICATION LINES

## 6.1. Occurrence of Magnetic Storms

Many years of observations have established that the occurrence of magnetic storms on the earth's surface is closely connected with the phenomena observed on the surface of the sun. On the sun from time to time sections appear with reduced brightness which appear as dark spots against the background of the brighter surface. Such spots on the sun sometimes appear in groups in which they have no outlines and boundaries; sometimes the spots appear separately from each other. The number of spots in the visible part of the solar surface does not change from day to day. The number of spots can be determined by the formula $R=k(10g+S)$, where g is the number of groups of sun spots; S is the number of clearly distinguishable separate spots in groups; k is a constant defined by the observer and instruments. At the Zürich Observatory (Switzerland) $k=1$.

In Fig 6.1 data are presented on the number of sun spots obtained by the same laboratory in October 1955 [42]. As is obvious, the average number of spots in a month is 59; on the 16th, 17th and 18th of October there were 0, and on the 29th of October, there was a maximum of 124.

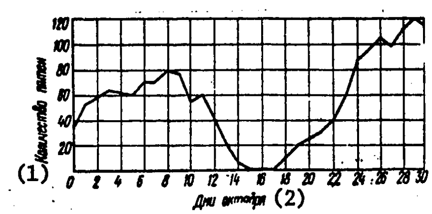



Figure 6.1. Number of spots on the sun during different days in October 1955

Key:
1. No of spots
2. Days in October

Table 6.1

Number of Spots on the Sun

| 1946 | | | | | | 1947 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Jan | Mar | May | Jul | Sep | Nov | Jan | Mar | May | Jul | Sep | Nov |
| 50 | 80 | 70 | 120 | 90 | 130 | 120 | 130 | 150 | 200 | 160 | 120 |

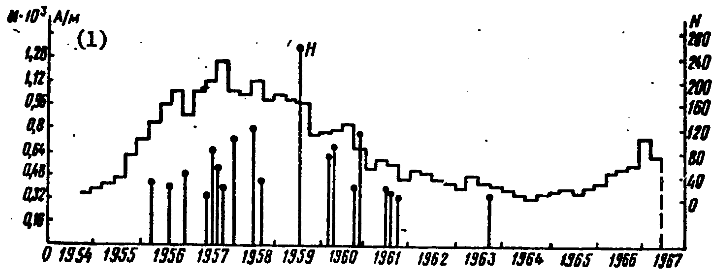


Figure 6.2. Number of spots with flares on the sun (N) and intensity of the magnetic field (H) by years for the period from 1954 to 1965

Key:

1. amps/meter

In Fig 6.2 the mean values are presented for the number of spots on the sun (N) and the magnetic field intensity of the earth (H) for every 3 months in the period from 1954 to 1967. As is obvious, the mean monthly number of spots does not remain constant by month or by years.

Many years of observations have demonstrated that the interval between two adjacent maxima of spots fluctuated from 17 to 7 years, and between two minima, from 14 to 9 years. The intervals between the maximum and minimum numbers can vary from 11 to 3 years, and the intervals between the minimum and the following maximum numbers, from 7 to 3 years. Finally, from the average values it becomes clear that in general for a decrease in the curve from maximum to minimum a little more time is required than for it to rise from the minimum to the maximum.

If the data on the number of spots by years from the period from 1850 to 1970 is represented in the form of a curve $N=\phi(T)$, it is possible to note that the interval between the maximum values on the average is 11 years.

The temperature of the dark spots periodically appearing on the surface of the sun is approximately equal to 1500°K, which is below the temperature of the parts of the photosphere surrounding them (6000°K). It is assumed that out of a pair of sun spots one is the location at which the strong magnetic field enters, and the other, the location from which it exits. For several days after the formation of sun spots two spots of irregular shape move separately from each other in a longitudinal direction and enlarge in size. When both spots reach a maximum magnitude, one of them begins to decay rapidly and disappears, and the other spot exists for a longer time; its average duration is four times greater than the time of existence of the first spot. The time of existence and the dimensions of the sunspots are subject to large variation. There are spots which exist for only a few hours or days, but there are spots that exist for several weeks and months.

The largest groups of spots were photographed in 1947 and in 1957 during the years of maximum spot formation. The large spots appear appreciably more rapidly than the small ones. Studies of 2000 sun spots at the Central Observatory in Europe (Zürich) demonstrated that on the average 60% of the observed spots occurred in pairs, and 40% were single.

On formation of sun spots with powerful magnetic fluxes the gas atoms in the solar atmosphere acquire a definitely directional movement under the effect of magnetic lines of force. This gives rise to an enormous cluster of positively charged ions in one region and negatively charged electrons in another. At the time of sudden change in intensity of the magnetic field, equalization of the electric charges takes place similar to a lightning discharge in the earth's atmosphere. In any case, extraordinarily intense lightning or a flare exceeding the brightness of ordinary sunlight occurs on the surface of the sun.

Thus, the formation of sun spots is accompanied by flares on the surface of the sun. The flares can be divided into four types (four classes): 1, 2, 3, 4. The data characterizing each type (class) of flare on the sun are presented in Table 6.2.

Table 6.2

Characteristics of Solar Flares

| Type (class) of flare | Size of spot, millionth of the solid angle $2\pi/10^6$ | Average duration of flare, minutes |
|---|---|---|
| 1 | 100-300 | 17 |
| 2 | 300-750 | 29 |
| 3 | 750-1200 | 62 |
| 4 | >1200 | 180 |

Several minutes after the occurrence of a flare its intensity will be maximum, and then it slowly disappears. The flares occur primarily in the central part of a group of large sun spots. During flares radiation of four types occurs: ultraviolet, corpuscular, radiowave and cosmic radiation. The radiation of the first two types causes variation of the earth's magnetic field.

### 6.2. Variation of the Earth's Magnetic Field During Ultraviolet Radiation

The ultraviolet radiation during flares after a short time interval (8 min), reaching the earth's surface, has a quite strong effect on its magnetic field (for the ordinary state of the sun the ultraviolet radiation is much weaker).

The ultraviolet radiation causes ionization of the air in layers located at high altitude from the surface of the sun (on the order of 100 km), and they become electrically conducting. The ionization is greatest during the daytime. In these good-conducting layers, strong currents occur induced by the magnetic field of the earth on rotation of it around its axis. Under the effect of these currents during the day the intensity and direction of the magnetic field changes at the investigated point of the earth. At night these changes stop inasmuch as the ultraviolet radiation stops.

For flares on the sun, the magnetic field changes sharply on the surface of the earth. This is explained by the fact that for the flares the ultraviolet radiation in the direction of the earth increases sharply, as a result of which the ionization of the upper layers of the earth's atmosphere increases significantly, and, consequently, their conductivity increases.

From what has been stated it follows that first the magnetic field of the earth changes significantly under the effect of the ultraviolet radiation only for flares on the sun and, secondly, these changes take place only in that part of the earth which is illuminated by the sun.

### 6.3. Variation of the Magnetic Field of the Earth in the Case of Corpuscular Radiation

The variations of the magnetic field caused by the corpuscular radiation occur both on the hemisphere lighted by the sun and on the darkness of the earth, that is, in the daytime and at night. This is explained by the following. On approaching the earth, the front of the corpuscular flux is subjected to the effect of the rotating magnetic field of the earth. Here the ions and electrons uniformly distributed in the flux will be separated on approaching the earth so that the basic part of the ions will pass through one side of the earth, and the basic part of the electrons, through the other side of the earth. As a result, the earth will be surrounded by two currents circulating in opposite directions, one of which is made up of negative charges and the other, positive charges. These currents jointly make up the common magnetizing current, the direction of which will be opposite to the direction of rotation of the earth. As a result of this, the state of the magnetic field is disturbed at each point of the earth's surface both in the daytime and at night. These disturbances are the most intense during flares on the sun.

The variations of the earth's magnetic field as a result of ordinary diurnal deviations are on the order of $(10-20)\cdot10^{-4}$ tesla; as a result of ultraviolet radiation during flares they reach $(20-40)\cdot10^{-4}$ tesla; as a result of corpuscular radiation during flares the deviation can increase by 25 times by comparison with the variation from ultraviolet radiation, that is, reach 0.1 tesla and more.

The variations of the magnetic field as a result of corpuscular radiation are also magnetic storms on the earth. The corpuscular theory of magnetic storms at the present time is generally recognized. According to this theory the disturbances of the magnetic field in the ionosphere and the aurora polaris are caused by penetration of charged particles (corpuscles) leaving the sun into the earth's atmosphere. The particles are emitted by the active regions of the solar surface in the form of narrow, radially directed beams. The speed of the corpuscular flux is 1000 to 3000 km/sec. The flux is made up of particles of both signs and on the whole is neutral. This corpuscular flux is the cause of the occurrence of the annular current of the ionosphere around the earth in the plane of the geomagnetic equator. The interaction of this current with the earth's magnetic field causes magnetic storms and currents in the earth and in the seas and oceans.

### 6.4. Potential Difference Between Different Points of the Earth During Magnetic Storms and the Effect of it on Communications Circuits

As the observations performed in various countries (the USSR, Sweden, Norway, Canada, the United States, and so on) indicate, during magnetic storms electric currents occur on the surface of the earth and also in the seas and oceans. The currents in the ground (water) during magnetic storms flow predominantly from east to west and from west to east. At geomagnetic latitudes from 45° to 67° of the northern and southern hemispheres (Fig 6.3) the magnitude of these currents increases in accordance with the magnetic disturbance coefficients curve $f=\phi(\alpha°)$ (Fig 6.4), it decreases slowly in the direction of the North and South Poles (after a geomagnetic latitude of 67°), and it increases sharply at the geomagnetic equator [43].

If the direction of the communications line coincides with the direction of the stray currents occurring during magnetic storms, then on closure of the communications wire to ground (water) part of the stray current of the earth (water) enters it. This current will have an interfering (sometimes dangerous) effect on the operation of the transmission systems.

It has been noted that the magnitude of the potential difference between two points of the earth selected in the direction of the geomagnetic latitudes, that is, the direction of flow of the currents of magnetic storms is approximately proportional to the distance between these points ($\ell_{12}$), the specific resistance of the earth ($\rho_g$) in the regions of occurrence of the currents and the magnitude of the currents in the ground $I_g$ occurring during magnetic storms. In other words, if two points of the earth are joined by an electric power transmission line through grounded neutral points of transformers, then the voltmeter connected to the line with high internal resistance records magnitude of the voltage drop in the ground between the indicated points of the earth $U_1$-$U_2$ which is determined by the formula

$$U_1 - U_2 \approx \xi \rho_g l_{12} I_g \cos \beta, \qquad (6.1)$$

Key: 1. g

where $\rho_g$, $\ell_{12}$ are the values for each specific case obtained during measurements; $\xi$ is the numerical coefficient considering the local conditions; $\beta$ is the angle between the direction of the current in the ground and the direction of the communication line.

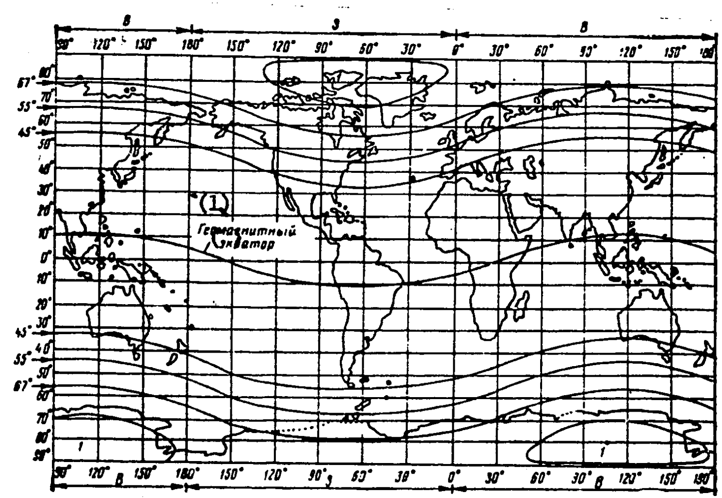



Figure 6.3. Map of the earth with the geomagnetic equator and geomagnetic latitudes (45°, 55°, 67°, 80°): 1-1 -- poles of the magnetic axis

Key: 1. geomagnetic equator

From (6.1) it is also possible to determine the magnitude of the current in the ground caused by a magnetic storm $I_g=(U_1-U_2)/\xi\rho_g\ell_{12}\cos\beta$. The presented law for the underground communication cables, the overhead communication line and the electric power transmission lines is applicable also for underwater cables if the magnetic storms of identical intensity occur simultaneously at all points of the cable, that is, within the limits of up to 1000 km [43].

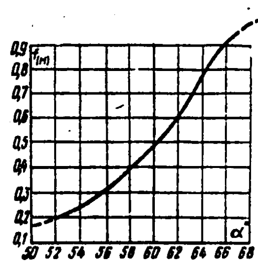


Figure 6.4. Magnitude of the coefficient of magnetic disturbances as a function of the geomagnetic latitude

The potential difference ($U_1-U_2=U_{12}$) occurring during a magnetic storm between two points of the earth at a distance $\ell_{12}$ from each other and reduced to 1 km is called the potential gradient of the earth: $G=U_{12}/\ell_{12}$, volts/km. The potential gradient is defined as the value of the foreign currents and voltages in communication lines. Therefore when designing means of protecting communication circuits from the effect of magnetic storms it is necessary to know the limits of variation of the potential gradients of the ground.

The magnitudes of the gradients of the ground observed at different points during a magnetic storm occurring on 10-12 February 1958 are presented in Table 6.3.

Table 6.3

Voltages Occurring in Communication Lines During the Magnetic Storm of 10-12 February 1958

| Country | Between which points | Type and length of line, km | Measured value, volts | Gradient, volt/km |
|---|---|---|---|---|
| Sweden | Stockholm-Södertälge | Cable, 33 | 100 | 3 |
| | Vernamo | Experimental overhead, 1 | 7.2 | 7.2 |
| | Vernamo | The same | 3.5-6 | 3.5-6 |
| | Stockholm-Göteborg | Cable, 430 | 80-200 | - |
| USSR | Petrozabodsk-Kem' | Overhead, 450 | 500 | 1.1 |
| | Ufa-Sim | The same, 150 | 70 | 0.5 |

From Table 6.3 it follows that the magnitude of the earth's potential gradient during magnetic storms fluctuates within the broad limits from fractions of a volt to several tens of volts per kilometer and depends on the intensity of the storm itself, the geomagnetic latitude, the specific resistance of the earth and the angle between the direction of the currents in the ground and the direction of the communication line. In the territory of the USSR, the potential gradient of the current (G, volts/km) during strong magnetic storms usually does not exceed 2-3 volts/km.

The currents in the wires of the communication lines during magnetic storms can occur if conditions are created on the communications lines for the formation of a single-wire "point on the earth-wire-second point on the earth" circuit. Fig 6.5 shows the circuits of the overhead and cable communication lines and the strong current lines in which during magnetic storms foreign currents can occur which lead to disturbance of the basic transmission and to damage to the equipment and elements of the line structures. Such circuits are as follows: all single-wire circuits (telegraph, signal circuits, the remote feed circuits using the "wire (two wires)-ground" system [wire-return system]; the two-wire circuits used as single-wire with grounding of the midpoint of the line transformers; two-wire circuits on both ends of which protective dischargers with ignition voltage from 250 to 1200 volts are included between the wires and the ground; the triple-phase transmission lines with grounded midpoints of the line windings of the three-phase transformers.

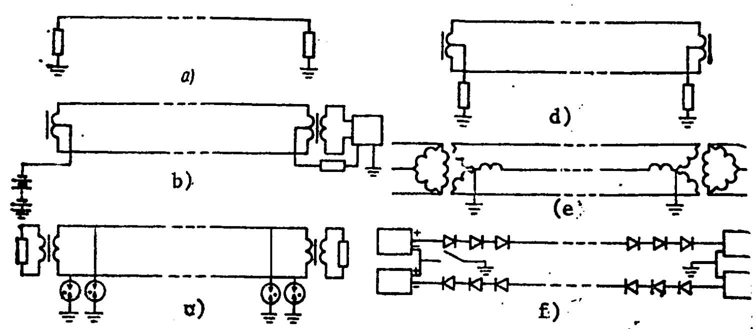



**Figure 6.5. Circuits in which the formation of interfering currents is possible during magnetic storms:**
a) signal or telegraph single-wire; b) two-wire by the "two wires-ground" system; c) telephone protected by dischargers; d) telegraph by the Picard system; e) powerful triple-phase overhead line with grounded neutral; f) marine cable

The magnitude of the currents in the communication wires occurring between two points under the effect of a potential difference of $U_1$-$U_2$ is determined by the formula $I=(U_1-U_2)/(R\ell+2Z)$, where $R\ell$ is the total resistance to direct current of a circuit of length $\ell$, ohms; Z is the resistance of the terminal devices closed to ground, ohms.

In order to have an idea about the harmful effect of the stray currents occurring during magnetic storms, on the overhead and cable line communication installations and also the electric power supply installations, let us present several examples described in the literature. In Sweden during a magnetic storm, a fire of enormous proportions occurred at one of the telephone offices which destroyed a significant part of the equipment. A large number of line protectors and carbon protectors installed on the communication lines of the country were damaged. In the northern part of the United States during the magnetic storm on 24 March 1940, numerous cases of burning out of the protectors were recorded on the overhead telephone and telegraph lines along with failures of the carbon protectors and other equipment. As a result of the high potentials of the ground in the high-voltage network in the system with grounded neutral, overheating of the windings of two transformers at two power substations was recorded. In the regions where damage was noted both on the telephone and telegraph networks, just as on the electric power supply networks, the specific resistance of the ground had a higher value (1000 to 5000 ohm-meter).

During the magnetic storm recorded on 10-12 February 1958, damage was observed simultaneously on the overhead and cable communication lines in Sweden, England and the USSR. In Sweden on all communication lines more than 80 km long the carbon protectors ignited with a breakdown voltage of 1000 to 1100 volts. This means that the electric field gradient in the ground at the time of response of the protectors was 25 to 27.5 volts/km. In the copper wires of the overhead communication lines d=3-4 mm the currents through the carbon protectors reached 5-10 amps during the magnetic storm. The arcs between the carbon plates of the protectors under prolonged effect of the current of such magnitudes heats the carbon protectors and makes them incandescent, as a result of which fires can occur. The fires occurring on 11 February 1958 at a number of stations (Carlsbad, Malunga, Lit, and so on) at telegraph and telephone offices occurred for this reason. In some cable telegraph channels the filaments of the rheostat tubes were burned by the ground currents. Up to 260 carbon protectors and several hundred cutouts were damaged. In several sections of railroad the railroad signal system was destroyed. During the same magnetic storm on 10-12 February 1958 all of the telephone circuits on the Kola Peninsula between Murmansk and Leningrad, Petrozavodsk and Kandalaksha, between Petrozavodsk and other stations failed; rumbling, crackling and other noise were heard in the circuits and communication channels on a voltage up to 100 millivolts; on many of the circuits the cutouts burned out. The telephone and telegraph communications were disrupted for 2 days [44]. From the presented data it follows that the foreign currents occurring in the power lines and in communication lines during magnetic storms can cause the electric substations to fail, completely destroy the telephone and telegraph communications over all transmission channels both in the low and the high frequency regions and cause damage to the equipment.

# CHAPTER 7. EFFECT OF HIGH-VOLTAGE LINES ON COMMUNICATION CABLES

## 7.1. Effect on Cables without Metal Coverings

Overhead cables without metal coverings (sheathings, armor), for example, of the PRVPM type, just as overhead electric power transmission lines, are subject to electric and magnetic effects. The formulas obtained in Chapters 2 and 3 pertain fully to overhead cables without metal coverings.

Underground cables without metal coverings are only subject to magnetic effects. In order to calculate the voltages and currents induced in the cable cores, it is necessary to use the formulas of the magnetic effect obtained above.

## 7.2. Effect on the Metal Covers of Cables in the Absence of Intermediate Grounds

Let us determine the voltages on the metal covers[1]. For the investigation of the theory of the effect of high-voltage lines on communications cables let us introduce the following notation:

$\dot{U}_{(c-sh)n}(x)$, $\dot{I}_{(c-sh)n}(x)$, $\dot{E}_{(c-sh)n}(x)$ are the voltage (volts), current (amps) and the longitudinal vmf per unit length (volts/meter) induced in the "core-sheathing" circuit at the point with the coordinate x in the n segment of the cable (n = I, II, III, see Figure 2.1);

$\dot{U}_{(c-g)n}(x)$, $\dot{I}_{(c-g)n}(x)$, $\dot{E}_{(c-g)n}(x)$ are the same with respect to the cable ground in the metal sheathing;

$\dot{U}_{cn}(x)$, $\dot{I}_{cn}(x)$, $\dot{E}_{cn}(x)$ are the same in the "core-ground" circuit of the cable without metal sheathing;

$\dot{U}_{shn}(x)$, $\dot{I}_{shn}(x)$, $\dot{E}_{shn}(x)$ are the same in the "sheathing-ground" circuit;

[1] The term "metal sheathing" or simply "sheathing" will be used hereafter instead of the term "metal covers."

$\dot{J}_{sh} = -\frac{d\dot{I}_{sh}}{dx}$ is the linear leakage current density from the metal sheathing of the cable to ground, amps/meter;

$\dot{U}_{sh-g}(x)$ is the voltage on the sheathing with respect to the near point of the ground, volts;

$\gamma_{c-sh}$, $\gamma_{c-g}$, $\gamma_{sh}$, $z_{w(c-sh)}$, $z_{w(c-g)}$, $z_{w-sh}$ are the propagation coefficients and wave impedances of the circuits respectively: "core-sheathing," "core-ground" and "sheathing-ground," 1/m, ohms;

$Z_{c-sh}$, $Z_{c-g}$, $Z_{sh}$ are the longitudinal resistances of the circuits, ohms/m;

$Y_{c-sh}$, $Y_{c-g}$, $Y_{sh}$ are the conductances of the circuits, Sm/m;

$Z_{sh.b}$, $Z_{sh.e}$, $Z_b$, $Z_e$ are the load impedances at the beginning and end of the "sheathing-ground," "core-sheathing" or the "core-ground" circuits, ohms;

$p_{sh.b}$, $p_{sh.e}$, $p_b$, $p_e$ are the reflection coefficients at the beginning and end of the "sheathing-ground" circuit, at the beginning and end of the "core-sheathing" or "core-ground" circuits;

$Z_{coup}$ is the coupling resistance of the metal sheathing, oh-m;

$Z_{1\ sh}$, $Z_{1\ c}$ are the magnetic coupling resistance between the influencing line and the sheathing (core) of the cable, ohm-m;

$\rho_g$ is the specific resistance of the ground, ohm-m.

The "metal sheathing-ground" circuit is the usual single-line circuit. The influence on this circuit in the absence of intermediate grounds was investigated above in Chapter 2. When calculating the effect on this circuit it is necessary to distinguish two cases:

1) The metal covers of the cable are well-insulated from the ground [the cables in an outer plastic hose having high insulation resistance (5000 to 10,000 ohms-km and more)];

2) The metal covers of the cable have uniformly distributed good electrical contact with ground (cables with outer jute covering, for example, type TB, TZB, MKSB, and so on; cables laid in corridors, for example, TG, TZG, and so on).

In the first case the voltages and currents induced in the metal covers can be found by the corresponding formulas (§ 2.3-2.5). Here the voltage with respect to the remote and near currents (at a distance of approximately 1 meter from the cable or above it) are in practice identical. In the second case the voltage with respect to the near ground point can differ significantly from the voltage with respect to the remote current. It is approximately equal to the leakage current with metal covers multiplied by the transient resistance between the metal covers and the near ground point. In general form (considering the resistance of the possible outer covering) this voltage is equal to the following in volts

$$\underset{(1)}{\dot{U}_{об\text{-}з}} = \underset{(2)}{\dot{J}_{об}}\left[\underset{(3)}{R_{из}} + \frac{\overset{(4)}{\rho_з}}{\pi}\ln\frac{\delta}{\sqrt{\underset{(2)}{D_{об}}h}}\right], \qquad (7.1)$$

Key: 1. sh-g 3. ins
2. sh 4. g

where $R_{ins}$ is the resistance of the outer covering of the cable, ohm-m; $\delta$ is the distance from the cable to the near ground point with respect to which the voltage is determined, meters (the maximum value of $\delta$ can be taken equal to 1.2 meters); $D_{sh}$ is the outside diameter of the metal coverings of the cable, meters; h is the depth of laying the cable, meters.

As is obvious from (7.1), for determination of the voltage induced with respect to the near points of the ground, it is necessary to know the leakage current density. The magnitude of the leakage current is equal to the derivative of the longitudinal current taken with opposite sign. In the presence of a magnetic effect the induced voltage (with respect to the remote ground point)

$$\underset{(a)}{\dot{U}_{об}(x)} = -\frac{1}{\underset{(a)}{Y_{об}}}\frac{dI_{об}}{dx} = J_{об}\frac{1}{\underset{(a)}{Y_{об}}}.$$

Key: a. sh

Hence, we find that

$$\underset{(1)}{\dot{U}_{об\text{-}з}(x)} = \underset{(2)}{\dot{U}_{об}(x)}\,Y_{об}\left[\underset{(3)}{R_{из}} + \frac{\rho_з}{\pi}\ln\frac{\delta}{\sqrt{\underset{(2)}{D_{об}}h}}\right]. \qquad (7.2)$$

Key: 1. sh-g 3. ins
2. sh

The conductivity between the metal sheathing of the underground cable and the remote ground point (siems per meter) is determined by the formula [21]

$$\underset{(a)}{Y_{об}} = \frac{1}{\underset{(b)}{R_{пер.об}}} = \left[\underset{(c)}{R_{из}} + \frac{\overset{(e)}{\rho_з}}{\pi}\ln\frac{1,12}{\underset{(d)}{\gamma_{об}}\sqrt{\underset{(d)}{D_{об}}h}}\right]^{-1}$$

Key: a. sh c. ins e. g
b. trans.sh d. sh

Then the voltage with respect to the near ground point will be

$$\underset{(1)}{\dot{U}_{об\text{-}з}} = \underset{(2)}{\dot{U}_{об}}(x) \frac{R_{из} + \frac{\rho_з}{\pi} \ln \frac{\delta}{\sqrt{D_{об} h}}}{\underset{(3)}{R_{из}} + \frac{\rho_{\overset{(4)}{з}}}{\pi} \ln \frac{1,12}{\underset{(2)}{\gamma_{об}} \sqrt{\underset{(2)}{D_{об}} h}}} . \quad (7.3)$$

Key: 1. sh-g 3. ins
2. sh 4. g

The resistance of the jute covering is insignificant so that

$$\underset{(1)}{\dot{U}_{об\text{-}з}}(x) = \underset{(2)}{\dot{U}_{об}}(x) \frac{\ln \frac{\delta}{\sqrt{D_{об} h}}}{\ln \frac{1,12}{\gamma_{об} \sqrt{D_{об} h}}}, \text{ volts.}$$

Key: 1. sh-g 2. sh

Let us determine the voltages and currents occurring in the metal sheathings as a result of the galvanic effect. Let us consider the cases where a cable, the metal coverings of which do not have special insulating coating, come close to an AC electric railroad. In the case of parallel approach of the cable to the electric railroad the voltage caused by the galvanic effect, as a rule, is appreciably less than the voltage occurring as a result of the magnetic effect. However, in the case of perpendicular crossing of the cable over the electric railroad where the magnetic effect on the metal sheathing can be neglected, the galvanic effect is predominant, and it must be determined.

The diagram of perpendicular intersection of a cable with an AC electric railroad track is shown in Figure 7.1. The potentials of the ground points located along the line perpendicular to the electric railroad, without considering the effect of the currents leaking out of the cable, are equal to the following [21]:

$$\underset{(1)}{\dot{U}_з}(0, y) = \frac{\dot{I}_0}{2\pi} \gamma_р \rho_з S_р \Omega(0, \underset{(2)}{\gamma_р} y), \quad (7.4)$$

Key: 1. g 2. r

where $\Omega(0, \gamma_r y)$ is a special function (see § 2.9).

Correspondingly, the current field intensity along the cable axis

$$\dot{E}_y = -\frac{\partial \dot{U}_з}{\partial y} = -\frac{\dot{I}_0}{2\pi} \gamma_р \underset{(1)}{\rho_з} \underset{(2)}{S_р} \frac{\partial}{\partial y} \Omega(0, \gamma_р y), \quad (7.5)$$

Key: 1. g. 2. r

where $\gamma_r$ is the propagation coefficient of the "rail-ground" circuit (1/m); $S_r$ is the coefficient of the shielding effect of the rails; since for small values of $\gamma_r y$ (to 0.1)

$$\Omega(0,\ \gamma_p y) \approx K_0(\gamma_p y) = \ln\frac{1.12}{\gamma_p y},\ \text{then}\ \dot{E}_y = \frac{\dot{I}_0}{2\pi}\gamma_p \rho_з S_p \frac{1}{y}. \qquad (7.6)$$

(a) (b)

Key: a. r b. g

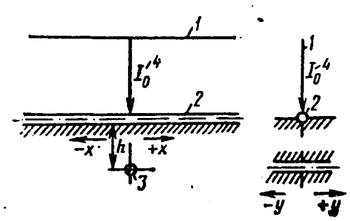


Figure 7.1. Diagram of the intersection of the cable with the electric railroad: 1 -- contact line; 2 -- rails; 3 -- cable; $I_0$ -- current coming to the rails.

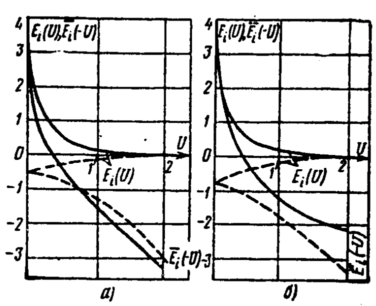


Figure 7.2. Integral exponential functions of the argument $\gamma_{sh} y = |\gamma_{sh} y| e^{i\phi} = |U| e^{i\phi}$: a) for $\phi = 30°$; b) for $\phi = 45°$ ——— real components (>0); ---- imaginary component (<0).

The current occurring in the metal sheating of the cable from the applied distributed longitudinal intensity $E_y$ is determined from the following equation:

$$\frac{d^2\dot{I}_{об}}{dy^2} - \gamma_{об}^2 I_{об} = -\frac{\dot{E}_y}{R_{пер.об}}. \qquad (7.7)$$

(1) (2)

Key: 1. sh 2. trans.sh

As a result of the solution of (7.7), we obtain

$$\dot{I}_{\text{об}} = \frac{\dot{I}_0 \gamma_p \rho_з S_p}{4\pi\gamma_{\text{об}} R_{\text{пер.об}}} \left[ e^{-\gamma_{\text{об}} y} \int_{-\infty}^{\gamma_{\text{об}} y} e^{s} \frac{1}{s} ds + e^{\gamma_{\text{об}} y} \int_{\gamma_{\text{об}} y}^{\infty} e^{-s} \frac{1}{s} ds \right]. \quad (7.8)$$

Key: a. sh  b. trans.sh  c. g  d. r

Let us denote the exponential integral

$$\int_{-\infty}^{\gamma_{\text{об}} y} e^{s} \frac{1}{s} ds = -\overline{\text{Ei}}(-\gamma_{\text{об}} y)$$

Key: a. sh

and the integral

$$\int_{\gamma_{\text{об}} y}^{\infty} e^{-s} \frac{1}{s} ds = \text{Ei}(\gamma_{\text{об}} y).$$

The values of these integrals for two values of the angles of the propagation coefficient are presented in Figure 7.2. Thus,

$$\dot{I}_{\text{об}} = \frac{\dot{I}_0 \gamma_p \rho_з S_p}{4\pi R_{\text{пер.об}} \gamma_{\text{об}}} [e^{\gamma_{\text{об}} y} \text{Ei}(\gamma_{\text{об}} y) - e^{-\gamma_{\text{об}} y} \overline{\text{Ei}}(-\gamma_{\text{об}} y)]. \quad (7.9)$$

The leakage current density

$$\dot{j}_{\text{об}} = -\frac{\dot{I}_0 \gamma_p \rho_з S_p}{4\pi R_{\text{пер.об}}} [e^{\gamma_{\text{об}} y} \text{Ei}(\gamma_{\text{об}} y) + e^{-\gamma_{\text{об}} y} \overline{\text{Ei}}(-\gamma_{\text{об}} y)]. \quad (7.10)$$

For small values of y where the depth of laying the cable h is commensurate with y, the functions $\overline{\text{Ei}}$ and Ei must be taken not from the argument $\gamma_{sh} y$, but from the argument $\gamma_{sh} d$, where $d = \sqrt{y^2 + h^2}$ is the distance from the investigated point to the point of intersection. In this case if the current goes to the rails not at the intersection point, but at a distance $x_1$ from it, then for small values the problem is solved analogously. The current occurring in the metal sheathing of the cable will be

$$\dot{I}_{\text{об}} = \frac{\dot{I}_0 \gamma_p \rho_з S_p}{4\pi R_{\text{пер.об}}} e^{-\gamma_p x_1} [e^{\gamma_{\text{об}} y} \text{Ei}(\gamma_{\text{об}} y) - e^{-\gamma_{\text{об}} y} \overline{\text{Ei}}(-\gamma_{\text{об}} y)] \quad (7.11)$$

Key: 1. sh  2. trans.sh

and the linear leakage current density

$$\dot{j}_{об} = -\frac{\dot{I}_0 \gamma_p \rho_з S_p}{4\pi R_{пер.об}} e^{-\gamma_p x_1} [e^{\gamma_{об} y} \mathrm{Ei}(\gamma_{об} y) + e^{-\gamma_{об} y} \overline{\mathrm{Ei}}(-\gamma_{об} y)]. \qquad (7.12)$$

It is possible to use formulas (7.11) and (7.12) to values of $|\gamma_r y| = |\gamma_r x_1| = 0.2$. The cable sheathing potential with respect to the near ground point can be determined by (7.1).

Let us consider the case where a cable approaches the electric power transmission line. The electric power transmission lines have a galvanic effect on the cables laid in a row both for parallel approach of them and on intersection at the time of short circuiting of the phase to ground. In this case the ground points near the electric power plant or substation and also near the short circuit point on the line acquire high potentials with respect to the remote ground points. The short circuit currents also create potentials and currents in the underground metal pipes laid nearby. However, in the case of parallel approach of a cable to an overhead electric power line, the dangerous voltage caused by the magnetic effect is appreciably greater than the voltage caused by the galvanic effect. Therefore the galvanic effect of the overhead electric power line on the metal sheathing of the cable must be calculated only on intersection of the cable with the path of the overhead electric power line perpendicular or close to it.

Let us consider the diagram shown in Figure 7-3. The current $\dot{I}_0$ is a short circuit current going to ground through the ground circuit of the substation or at the point of grounding of the phase on the line. The potential of the metal sheathing of the cable at any point where the coordinate x with respect to the remote ground point created by the current $\dot{I}_0$ is [21]

$$\underset{(a)}{\dot{U}_{об}} = \frac{\dot{I}_0}{2\pi} \gamma_{об} \underset{(b)}{\rho_з} \Omega(\gamma_{об} x, \gamma_{об} a_1). \qquad (7.13)$$

Key: a. sh b. g

The current in the sheathing at any point with the coordinate

$$\dot{I}_{об} = \pm \frac{\dot{I}_{об}}{2\pi} \rho_з \psi(\gamma_{об} x, \gamma_{об} a_1). \qquad (7.14)$$

The leakage current from the sheathing at any point with the coordinate x

$$\underset{(a)}{\dot{j}_{об}} = \frac{\dot{I}_0 \rho_з \text{ (b)}}{2\pi \underset{(c)}{R_{пер.об}}} \left[\frac{1}{\sqrt{x^2 + a_1^2}} - \gamma_{об} \Omega(\gamma_{об} x, \gamma_{об} a_1)\right], \qquad (7.15)$$

Key: a. sh b. g c. trans.sh

where $\Omega$ and $\psi$ are special functions shown in Figures 2.19 and 7.4. The potential with respect to the near ground points can be determined by (7.1).

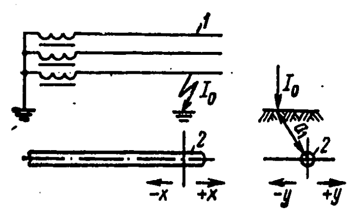


Figure 7.3. Calculation of the galvanic effect of an overhead line on the metal coverings of a communication cable: 1 -- overhead line; 2 -- cable; $I_0$ -- single-phase short circuit current.

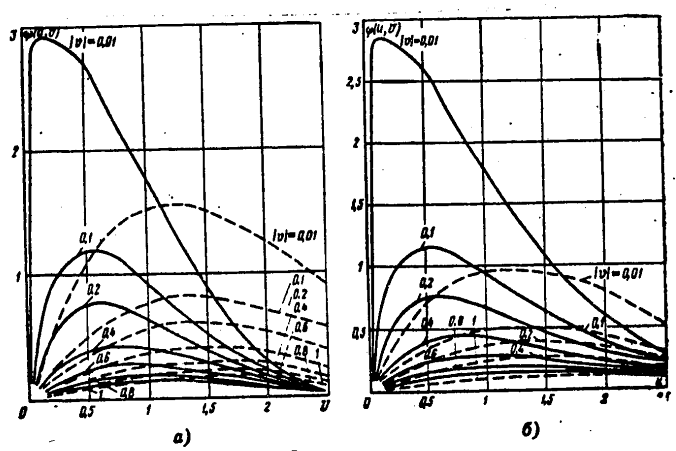


Figure 7.4. Dependence of the function $\phi(u, v)$ on $|u|$ for various values of $|v|$: a) for $\phi_u = \phi_v = 45°$; b) for $\phi_u = \phi_v = 30°$, —— real component (>0); --- imaginary component (<0).

7.3. Effect on the Metal Sheathing of Cables in the Presence of Intermediate Grounds

The formulas for the effect on single-wire circuits obtained in item 2.3 and 2.5 pertain to the case where the circuit, in particular, the "metal coverings (sheathing)-ground" circuit, has grounds of arbitrary resistance only with respect to ends of the cable. Of course, this conditions is not always satisfied. As a rule, the cable sheathing in plastic outer covering has intermediate grounds. Here two cases are possible: the cable sheathing in the investigated section as one or two arbitrarily located grounds, the resistances of which are arbitrary; the cable sheathing was grounded more or less uniformly with the help of relatively frequently arranged grounds with predominately the same resistance. Both cases are encountered in practice.

Determination of Induced Voltages and Currents in Metal Sheathing in the Presence of Intermediate Grounds at the Ends of an Approach Section. Let us consider the approach of a cable in a plastic outer covering to a high voltage line in the presence of four grounds of arbitrary resistance located on the ends of the cable and on the ends of the approach section (Figure 7.5).

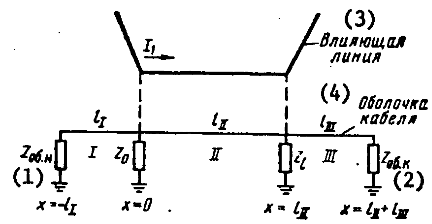



Figure 7.5. Diagram of the approach of a cable to a high voltage line in the presence of intermediate grounds on the sheathing located at the ends of the approach section.

Key: 1. sh.b 3. affecting line
2. sh.e 4. cable sheathing

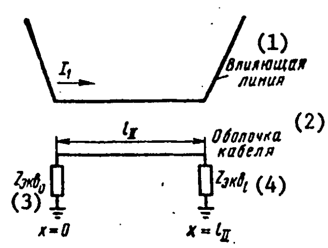



Figure 7.6. Equivalent diagram of the approach section of the cable to the high voltage line.

Key: 1. affecting line 3. $\text{equiv}_0$
2. cable sheathing 4. $\text{equiv}_\ell$

Let us denote the resistances between the sheathing and the ground included at the ends of the approach section by $Z_0$ at the beginning of the approach section for $x = 0$ and $Z_\ell$ at the end of the approach section for $x = \ell_{II}$.

Then the approach section of the sheathing turns out to be loaded on the resistances: at the point $x = 0$, $Z_{\text{equiv }0} = Z_{\text{inp.b}}Z_0/(Z_{\text{inp.b}}+Z_0)$ and at the point $x = \ell_{II}$, $Z_{\text{equiv }\ell} = Z_{\text{inp.e}}Z_\ell/(Z_{\text{inp.e}}+Z_\ell)$, where $Z_{\text{inp.b}} = z_{\text{w.sh}}\ \text{cth}\ (\gamma_{sh}\ell_I - \ln\sqrt{\rho_{\text{sh.b}}})$ is the input impedance of section I (left) of the "sheathing-ground" circuit (from the point $x = 0$ in the direction of the beginning of the cable); $Z_{inp.e} = z_{w.sh}$ cth $(\gamma_{sh}\ell_{III} - \ln\sqrt{p_{sh.e}})$ is the input impedance of section III (right) of the "sheathing-ground" circuit (from the point $x = \ell_{II}$ in the direction of the end of the cable). The equivalent diagram of the approach section of the cable with the high voltage line is shown in Figure 7.6.

The reflection coefficients at the beginning ($p_{sh0}$) and end ($p_{sh\,\ell}$) of the equivalent approach section have the form

$$\underset{(a)}{p_{об\,0}} = \frac{Z_{экв\,0} - z_{в.об}}{\underset{(b)}{Z_{экв\,0}} + \underset{(c)}{z_{в.об}}}; \quad p_{об\,l} = \frac{Z_{экв\,l} - z_{в.об}}{Z_{экв\,l} + z_{в.об}}.$$

Key: a. sh b. equiv c. w.sh

For determination of the current and voltage on the equivalent approach section we can now use the expressions obtained in item 2.3-2.5. Setting $x = 0$ and denoting $k_0 = Z_0/(Z_{inp.b}+Z_0)$ and $k_\ell = Z_\ell/(Z_{inp.e}+Z_\ell)$, we obtain:

in the first section

$$\underset{(a)}{\dot{I}_{об\,I}(x)} = \frac{2\dot{I}_1 Z_{1об} \operatorname{sh}\ln\sqrt{p_{об\,0}}\,\operatorname{sh}\gamma_{об}\frac{l_{II}}{2}\operatorname{sh}\left(\gamma_{об}\frac{l_{II}}{2} - \ln\sqrt{p_{об\,l}}\right)}{Z_{об}\operatorname{sh}\left(\gamma_{об}l_{II} - \ln\sqrt{p_{об\,0}p_{об\,l}}\right)} \times$$
$$\times k_0 \frac{\operatorname{sh}\left[\gamma_{об}(l_I + x) - \ln\sqrt{p_{об.н}}\right]}{\operatorname{sh}\left(\gamma_{об}l_I - \ln\sqrt{\underset{(b)}{p_{об.н}}}\right)}; \tag{7.16}$$

Key: a. sh b. sh.b

$$\dot{U}_{об\,I}(x) = -\frac{2\dot{I}_1 Z_{1об} \operatorname{sh}\ln\sqrt{p_{об\,0}}\,\operatorname{sh}\gamma_{об}\frac{l_{II}}{2}\operatorname{sh}\left(\gamma_{об}\frac{l_{II}}{2} - \ln\sqrt{p_{об\,l}}\right)}{\gamma_{об}\operatorname{sh}\left(\gamma_{об}l_{II} - \ln\sqrt{p_{об\,0}p_{об\,l}}\right)} \times$$
$$\times k_0 \frac{\operatorname{ch}\left[\gamma_{об}(l_I + x) - \ln\sqrt{p_{об.н}}\right]}{\operatorname{sh}\left(\gamma_{об}l_I - \ln\sqrt{p_{об.н}}\right)}; \tag{7.17}$$

in the approach section

$$\dot{I}_{об\,II}(x) = -\frac{\dot{I}_1 Z_{1об}}{Z_{об}}\left[1 - \frac{\operatorname{ch}\ln\sqrt{p_{об\,0}}\operatorname{sh}\left[\gamma_{об}(l_{II} - x) - \ln\sqrt{p_{об\,l}}\right] + \rightarrow}{\operatorname{sh}\left(\gamma_{об}l_{II} - \ln\sqrt{p_{об\,0}p_{об\,l}}\right)}\right.$$
$$\left.\frac{\rightarrow + \operatorname{ch}\ln\sqrt{p_{об\,l}}\operatorname{sh}\left(\gamma_{об}x - \ln\sqrt{p_{об\,0}}\right)}{}\right], \tag{7.18}$$

$$\dot{U}_{об\,II}(x) = \frac{\dot{I}_1 Z_{1об}}{\gamma_{об}} \frac{\operatorname{ch}\ln\sqrt{p_{об\,0}}\operatorname{ch}\left[\gamma_{об}(l_{II} - x) - \ln\sqrt{p_{об\,l}}\right] - \rightarrow}{\operatorname{sh}\left(\gamma_{об}l_{II} - \ln\sqrt{p_{об\,0}p_{об\,l}}\right)}$$
$$\frac{\rightarrow - \operatorname{ch}\ln\sqrt{p_{об\,l}}\operatorname{ch}\left(\gamma_{об}x - \ln\sqrt{p_{об\,0}}\right)}{}; \tag{7.19}$$

in the third section

$$\dot{I}_{\text{об III}}(x)=\frac{2\dot{I}_1 Z_{1\text{об}} \operatorname{sh} \ln \sqrt{p_{\text{об } l}} \operatorname{sh} \gamma_{\text{об}} \frac{l_{II}}{2} \operatorname{sh}\left(\gamma_{\text{об}} \frac{l_{II}}{2} - \ln \sqrt{p_{\text{об} \bullet}}\right) \times \operatorname{sh}\left[\gamma_{\text{об}}\left(l_{II}+l_{III}-x\right) - \ln \sqrt{p_{\text{об.к}}}\right]}{Z_{\text{об}} \operatorname{sh}\left(\gamma_{\text{об}} l_{II} - \ln \sqrt{p_{\text{об} \bullet} p_{\text{об } l}}\right) \operatorname{sh}\left(\gamma_{\text{об}} l_{III} - \ln \sqrt{p_{\text{об.к}}}\right)} k_l; \quad (7.20)$$

$$\dot{U}_{\text{об III}}(x)=\frac{2\dot{I}_1 Z_{1\text{об}} \operatorname{sh} \ln \sqrt{p_{\text{об } l}} \operatorname{sh} \gamma_{\text{об}} \frac{l_{II}}{2} \operatorname{sh}\left(\frac{\gamma_{\text{об}} l_{II}}{2} - \ln \sqrt{p_{\text{об} \bullet}}\right) \times \operatorname{ch}\left[\gamma_{\text{об}}\left(l_{II}+l_{III}-x\right) - \ln \sqrt{p_{\text{об.к}}}\right]}{\gamma_{\text{об}} \operatorname{sh}\left(\gamma_{\text{об}} l_{II} - \ln \sqrt{p_{\text{об} \bullet} p_{\text{об } l}}\right) \operatorname{sh}\left(\gamma_{\text{об}} l_{III} - \ln \sqrt{p_{\text{об.к}}}\right)} k_l. \quad (7.21)$$

Correspondingly, the mean currents with respect to the sections are equal to:

$$\dot{I}_{\text{об I ср}}{}^{(a)}=\frac{4\dot{I}_1 Z_{1\text{об}} \operatorname{sh} \ln \sqrt{p_{\text{об} \bullet}} \operatorname{sh} \gamma_{\text{об}} \frac{l_{II}}{2} \operatorname{sh}\left(\gamma_{\text{об}} \frac{l_{II}}{2} - \ln \sqrt{p_{\text{об } l}}\right) \times \operatorname{sh} \gamma_{\text{об}} \frac{l_I}{2} \operatorname{sh}\left(\gamma_{\text{об}} \frac{l_I}{2} - \ln \sqrt{p_{\text{об.н}}}\right)}{Z_{\text{об}} \gamma_{\text{об}} l_I \operatorname{sh}\left(\gamma_{\text{об}} l_{II} - \ln \sqrt{p_{\text{об} \bullet} p_{\text{об } l}}\right) \operatorname{sh}\left(\gamma_{\text{об}} l_I - \ln \sqrt{p_{\text{об.н}}}\right)} k_{\bullet},$$

$$\dot{I}_{\text{об II ср}}=-\frac{\dot{I}_1 Z_{1\text{об}}}{Z_{\text{об}}}\left[1-\frac{2 \operatorname{sh} \gamma_{\text{об}} \frac{l_{II}}{2}\left[\operatorname{ch} \ln \sqrt{p_{\text{об} \bullet}} \operatorname{sh}\left(\gamma_{\text{об}} \frac{l_{II}}{2} - \ln \sqrt{p_{\text{об } l}}\right) + \operatorname{ch} \ln \sqrt{p_{\text{об } l}} \operatorname{sh}\left(\gamma_{\text{об}} \frac{l_{II}}{2} - \ln \sqrt{p_{\text{об} \bullet}}\right)\right]}{\gamma_{\text{об}} l_{II} \operatorname{sh}\left(\gamma_{\text{об}} l_{II} - \ln \sqrt{p_{\text{об} \bullet} p_{\text{об } l}}\right)}\right],$$

$$\dot{I}_{\text{об III ср}}=\frac{4\dot{I}_1 Z_{1\text{об}} \operatorname{sh} \ln \sqrt{p_{\text{об } l}} \operatorname{sh} \gamma_{\text{об}} \frac{l_{II}}{2} \operatorname{sh}\left(\gamma_{\text{об}} \frac{l_{II}}{2} - \ln \sqrt{p_{\text{об} \bullet}}\right) \times \operatorname{sh} \gamma_{\text{об}} \frac{l_{III}}{2} \operatorname{sh}\left(\gamma_{\text{об}} \frac{l_{III}}{2} - \ln \sqrt{p_{\text{об.к}}}\right)}{Z_{\text{об}} \gamma_{\text{об}} l_{III} \operatorname{sh}\left(\gamma_{\text{об}} l_{II} - \ln \sqrt{p_{\text{об} \bullet} p_{\text{об } l}}\right) \operatorname{sh}\left(\gamma_{\text{об}} l_{III} - \ln \sqrt{p_{\text{об.к}}}\right)}{}^{(b)} k_l.$$

Key: a. mean b. sh.e

The currents through the ground $Z_0$

$$\dot{I}_{Z0} = -\dot{I}_{\text{об II}}(0) \frac{Z_{\text{вх.н}}{}^{(a)}}{Z_{\text{вх.н}} + Z_{\bullet}}. \quad (7.22)$$

Key: a. inp. e

For the selected positive direction of the current in the sheathing (in the direction of an increase in the coordinate x) the current $\dot{I}_{Z0}$ enters the cable sheathing. The current through the ground $Z_\ell$

$$\dot{I}_{Zl} = \dot{I}_{\text{об II}}(l_{II}) \frac{Z_{\text{вх.к}}}{Z_{\text{вх.к}} + Z_l}. \quad (7.23)$$
(a)

Key: a. inp. e

For the selected positive direction of the current in the sheathing the current $\dot{I}_{Z\ell}$ will be provisionally considered positive.

Determination of the Voltages and Currents in the Metal Sheathing in the Presence of Uniformly Arranged Grounds. Most frequently in practice the conditions of multiple grounding are encountered under which all of the grounds have approximately identical resistance to leakage and are located at the same distance from each other. Let us consider such conditions, but for great generality let us assume that the resistances of the grounds connected to the ends of the cable and at the ends of the approach section (Figure 7.7), are distinguished from the resistances of the intermediate grounds. The distance between adjacent grounds (length of section) will be denoted s; the number of sections in the segments $k_1 = \ell_I/s$; $k_2 = \ell_{II}/s$; $k_3 = \ell_{III}/s$. If the cable has an outer plastic covering, then at both frequencies (when it is possible to neglect the distributed conductivity of the "sheathing-ground" circuit and is fed $I_1(x) = \text{const}$) the fact on the multiply and uniformly grounded sheathing can be determined by the methods based on solving the equations in finite differences.

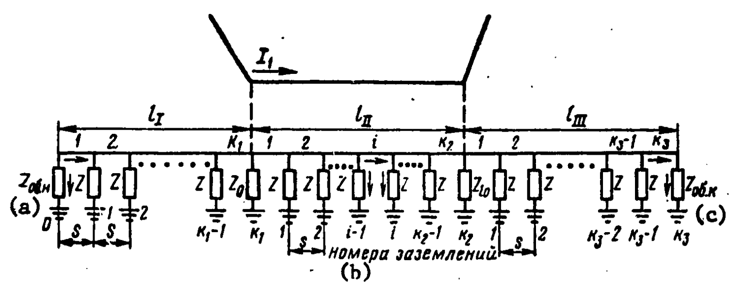



Figure 7.7. Diagram of the approach of a cable to an interfering line in the presence on the sheathing of uniformly arranged identical grounds.

Key: a. sh.d b. ground numbers c. sh.e

Let us represent the equivalent diagram of the approach section as shown in Figure 7.8. Here $Z_{\text{inp.b}}$ and $Z_{\text{inp.e}}$ are the input resistances of the "sheathing-ground" circuit; $z_0$, $z_\ell$ are the provisional resistances giving terminal loads of $Z_0$ and $Z_\ell$ parallel to the resistance Z, that is, $Z_0 = Zz_0/(Z+z_0)$;

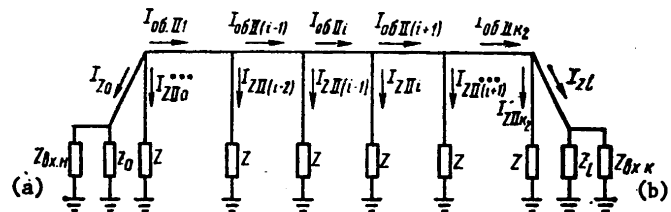


Figure 7.8. Equivalent diagram of the approach section in the presence of multiple grounding

Key: a. inp.b b. inp.e

$Z_\ell = Zz_\ell/(Z+z_\ell)$. For the currents induced in the cable sheathings in the approach section, we can write [22] the difference equations:

for longitudinal currents in the sheathing

$$\dot I_{\text{об}\,(i-1)} - 2\dot I_{\text{об}\,i}\left(1+\frac{Z_{\text{об}}s}{2Z}\right) + \dot I_{\text{об}\,(i+1)} = -\frac{\dot E}{Z}; \tag{7.24}$$

(a)
Key: a. sh

for the currents through the ground

$$\dot I_{Z\,(i-1)} - 2\dot I_{Zi}\left(1+\frac{Z_{\text{об}}s}{2Z}\right) + \dot I_{Z\,(i+1)} = 0. \tag{7.25}$$

In equation (7.24), $\dot E = \dot I_1 Z_{1\,\text{sh}}s$ is the emf induced in the cable sheathing on the length of the section, volts. The general solutions of the nonuniform (7.24) and uniform (7.25) equations have the form

$$\dot I_{\text{об}\,i} = \frac{A}{1-e^{\alpha}}e^{\alpha i} + \frac{B}{1-e^{-\alpha}}e^{-\alpha i} + \frac{E}{Z_{\text{об}}s}, \tag{7.26}$$

$$\dot I_{Zi} = Ae^{\alpha i} + Be^{-\alpha i}, \tag{7.27}$$

where A and B are constants defined by the boundary conditions; $\alpha$ is the coefficient which is a value analogous to the transfer coefficient of one section and obtained from the expression ch $\alpha = 1 + (Z_{\text{sh}}s/2Z)$.

Finding the constants A and B from the boundary conditions at the ends of the cable and the approach sections, substituting them in (7.26) and (7.27), we obtain[1]:

[1] A more detailed behavior of the solution is presented in [45].

in the first section

$$\dot{I}_{\text{об I } i} = \dot{I}_{\text{об II } 1} \frac{Z}{Z+Z_{\text{об}0}} \frac{z_0}{z_0+Z_{\text{вх.н}}} \frac{Z \operatorname{sh}\alpha(i-1) - (Z_{\text{об.н}}+Z)\operatorname{sh}\alpha i}{Z \operatorname{sh}(k_1-1) - (Z_{\text{об.н}}+Z)\operatorname{sh}\alpha k_1}, \tag{7.28}$$

(a) (b) (c)

Key: a. sh b. inp.b c. sh.b

$$\dot{I}_{Z\,\text{I}\,i} = 2I_{\text{об II } 1} \frac{Z}{Z+Z_{\text{об}0}} \frac{z_0 \operatorname{sh}\frac{\alpha}{2}}{z_0+Z_{\text{вх.н}}} \frac{(Z_{\text{об.н}}+Z)\operatorname{ch}\alpha(i+1/2) - Z\operatorname{ch}\alpha(i-1/2)}{Z\operatorname{sh}\alpha(k_1-1) - (Z_{\text{об.н}}+Z)\operatorname{sh}\alpha k_1}; \tag{7.29}$$

in the second section

$$\dot{I}_{\text{об II } i} = \frac{\dot{E}}{Z_{\text{об}\,s}} \left\{ 1 - \frac{2\left[Z\operatorname{sh}\frac{\alpha}{2}\left[Z_{\text{об}\,0}\operatorname{ch}\alpha\left(k_2+\frac{1}{2}-i\right) + Z_{\text{об}\,i} \times \right.\right.}{Z_{\text{об}\,0}Z_{\text{об}\,i}\operatorname{sh}\alpha(k_2+1) + 2(Z_{\text{об}\,0}+} \right.$$

$$\left. \frac{\left.\left. \times \operatorname{ch}\alpha\left(i-\frac{1}{2}\right)\right] + Z_{\text{об}\,0}Z_{\text{об}\,i}\operatorname{sh}\alpha\frac{k_2+1}{2}\operatorname{ch}\alpha\left(\frac{1}{2}(k_2+1)-i\right)\right]}{+Z_{\text{об}\,i})Z\operatorname{sh}\frac{\alpha}{2}\operatorname{ch}\alpha\left(k_2+\frac{1}{2}\right) + ZZ_{\text{об}\,s}\operatorname{sh}\alpha k_2} \right\}, \tag{7.30}$$

(a)

Key: a. sh

$$\dot{I}_{Z\,\text{II}\,i} = -\frac{4\dot{E}\operatorname{sh}\frac{\alpha}{2}}{Z_{\text{об}\,s}} \frac{Z_{\text{об}\,0}Z_{\text{об}\,i}\operatorname{sh}\alpha\left(\frac{k_2}{2}-i\right)\operatorname{sh}\frac{\alpha}{2}(k_2+1) +}{Z_{\text{об}\,0}Z_{\text{об}\,i}\operatorname{sh}\alpha(k_2+1) +}$$

$$\frac{+Z\operatorname{sh}\frac{\alpha}{2}[Z_{\text{об}\,0}\operatorname{sh}\alpha(k_2-i) + Z_{\text{об}\,i}\operatorname{sh}\alpha i]}{+2(Z_{\text{об}\,0}+Z_{\text{об}\,i})Z\operatorname{sh}\frac{\alpha}{2}\operatorname{ch}\alpha\left(k_2+\frac{1}{2}\right) + ZZ_{\text{об}\,s}\operatorname{sh}\alpha k_2}; \tag{7.31}$$

in the third section

$$\dot{I}_{\text{об III } i} = \dot{I}_{\text{об II } k2} \frac{Z}{Z+Z_{\text{об}\,l}} \frac{z_l}{z_l+Z_{\text{вх.к}}} \times$$

$$\times \frac{(Z_{\text{об.к}}+Z)\operatorname{sh}\alpha(k_3+1-i) - Z\operatorname{sh}\alpha(k_3-i)}{(Z_{\text{об.к}}+Z)\operatorname{sh}\alpha k_3 - Z\operatorname{sh}\alpha(k_3-1)}, \tag{7.32}$$

(a)

Key: a. inp.e

$$\dot{I}_{z\,III\,i} = 2I_{об\,II\,к2}\frac{Z}{Z+z_{об\,l}}\frac{z_l\,\mathrm{sh}\frac{\alpha}{2}}{z_l+Z_{вх.к}}\times$$
$$\times\frac{(Z_{об.\,}+Z)\,\mathrm{ch}\,\alpha\left(k_3+\frac{1}{2}-i\right)-Z\,\mathrm{ch}\,\alpha\left(k_3-\frac{1}{2}-i\right)}{(Z_{об.к}+Z)\,\mathrm{sh}\,\alpha k_3-Z\,\mathrm{sh}\,\alpha\,(k_3-1)},\qquad(7.33)$$

Key: a. sh,e  b. inp.e

where $Z_{sh\ 0} = z_0 Z_{inp.b}/(z_0+Z_{inp.b})$, $Z_{sh\ \ell} = z_\ell Z_{inp.e}/(z_\ell + Z_{inp.e})$.

The input impedances $Z_{inp.b}$ and $Z_{inp.e}$ for five or more sections in segments I and II, respectively, can be taken equal to the wave impedance of the section $z_w = \sqrt{Z_{sh}s(Z+Z_{sh}s)}$. With a decreased number of sections the input impedances must be calculated by the formulas of circuit theory.

Let us find the average currents induced in the cable sheathing in the first section,

$$\dot{I}_{об\,I\,ср}=\frac{1}{k_1}\sum_{i=1}^{k_1}\dot{I}_{об\,I\,i}=\dot{I}_{об\,II\,1}\frac{Zz_0\left[Z\,\mathrm{sh}\frac{\alpha}{2}(k_1-1)-(Z_{об.н}+Z)\times\right.}{(Z_{об\,0}+Z)(z_0+Z_{вх.н})k_1[Z\,\mathrm{sh}\,\alpha\,(k_1-1)-}$$
$$\frac{\left.\times\,\mathrm{sh}\frac{\alpha}{2}(k_1+1)\right]\mathrm{sh}\frac{\alpha}{2}k_1}{-(Z_{об.н}+Z)\,\mathrm{sh}\,\alpha k_1]\,\mathrm{sh}\frac{\alpha}{2}};\qquad(7.34)$$

Key: a. sh I mean  b. inp.b  c. sh.b

in the second section

$$\dot{I}_{об\,II\,ср}=\frac{1}{k_2}\sum_{i=1}^{k_2}\dot{I}_{об\,II\,i}=\frac{\dot{E}}{Z_{об}s}\left\{1-\frac{2Z_{об\,0}Z_{об\,l}\,\mathrm{sh}\,\alpha\frac{k_2+1}{2}\,\mathrm{sh}\frac{\alpha}{2}k_2/\mathrm{sh}\frac{\alpha}{2}+Z\,\mathrm{sh}\,\alpha k_2(Z_{об\,0}+Z_{об\,l})}{k_2[Z_{об\,0}Z_{об\,l}\,\mathrm{sh}\,\alpha\,(k_2+1)+2(Z_{об\,0}+Z_{об\,l})Z\,\mathrm{sh}\frac{\alpha}{2}\times\mathrm{ch}\,\alpha\left(k_2+\frac{1}{2}\right)+2Z_{об}s\,\mathrm{sh}\,\alpha k_2}\right\};\qquad(7.35)$$

in the third section

$$\dot{I}_{об\,III\,ср}=\frac{1}{k_3}\sum_{i=1}^{k_3}\dot{I}_{об\,III\,i}=\dot{I}_{об\,II\,k2}\frac{Zz_l\left[(Z_{об.к}+Z)\,\mathrm{sh}\frac{\alpha}{2}(k_3+1)-\right.}{(Z+Z_{об\,l})(z_l+Z_{вх.к})k_3\times}$$

(b)

$$\frac{\to - Z\,\mathrm{sh}\,\frac{\alpha}{2}(k_з - 1)\Big]\,\mathrm{sh}\,\frac{\alpha}{2}k_з}{\times\,[(Z_{\underset{(a)}{об.к}} + Z)\,\mathrm{sh}\,\alpha k_з - Z\,\mathrm{sh}\,\alpha(k_з - 1)]\,\mathrm{sh}\,\frac{\alpha}{2}}. \quad (7.36)$$

Key: a. sh.e b. inp.e

It is possible to calculate the effect in the case of uniform arrangement of identical grounds by the formulas of item 2.3 under the condition of replacement of the concentrated grounds by the equivalent transient resistance $R_{trans.equiv}$ between the metal sheathing and ground. The formula for determining $R_{trans.equiv}$ will be obtained by selection in [46, 47]:

$$R_{\underset{(a)}{пер.экв}} = \frac{R_з s \sqrt{nR_з}}{\sqrt{nR_{\underset{(b)}{з}}} + 0.14\sqrt{s}}, \quad (7.37)$$

Key: a. trans.equiv
b. g

where $R_g$ is the resistance of the grounds, ohms, s is the spacing between the ground, meters; n is the number of sections between the grounds.

For spacing between the grounds $\sqrt{nR_g} >> 0.14\sqrt{s}$, $R_{trans} = R_g s$ ohm-meter.

The calculations of the induced currents in aluminum sheathing of MKSAShp cable by (7.28)-(7.32) and by (7.16)-(7.20) with the introduction of the equivalent transient resistance defined by (7.37), demonstrated that for $\ell_{II}$ = 20 km the results differ by 5-10% for s = 1-2 km and in the sections I and II reach 20-30% for s = 4-5 km.

In order to discover the dependence of the induced current in the sheathing on various factors, calculations were performed, the results of which are partially represented in Figures 7.9 and 7.10. In Figure 7.9 the relation is presented for the current modulus along the sheathing of the cable at various distances between the grounds, and in Figure 7.10, the analogous current function through the grounds. The calculation was made for f = 50 hertz, $\ell_I$ = 0, $\ell_{II}$ = 20 and $\ell_{III}$ = 10 km, $\dot{E}_{sh}$ = 10 v/km. The performed calculations also demonstrated that the effect of the terminal grounds on the current magnitude in the sheathing is appreciably stronger than the intermediate grounds. This indicates the possibility of an increase in the resistances to leakage of the intermediate grounds under the condition of insurance of small resistances to leakage of the terminal grounds.

### 7.4. Effect on Cable Cores in the Absence of Intermediate Grounds on the Metal Coverings

Primary Relations. In general form the symmetric communications cable is a multiwire system, in which in addition to the two-wire circuits it is

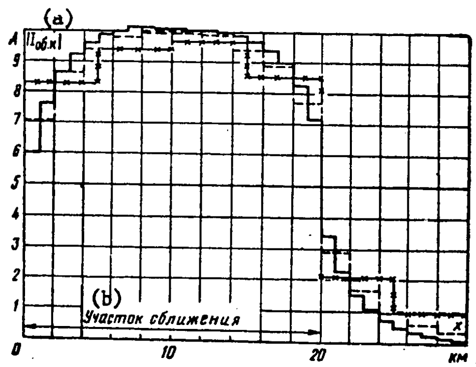



Figure 7.9. Currents induced in an aluminum, multiply grounded sheathing for Z = 5 ohm, $Z_{sh.b} = Z_{sh.e} = Z_{\ell} = 2$ ohms: —— s = = 1 km; --- s = 2 km; -x-x- s = 5 km.

Key: a. $|I_{sh.e}|$ b. approach section

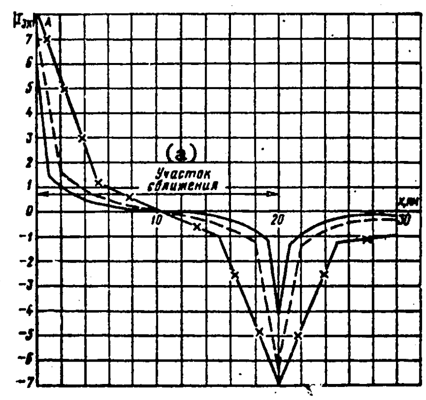



Figure 7.10. Currents flowing through the grounds of a multiply grounded aluminum sheathing for $Z_{sh.b} = Z_{sh.e} = Z_{\ell} = 2$ ohms. Z = 5 ohms: —— s = 1 km; --- s = 2 km; -x-x- s = 5 km.

Key: a. approach section

necessary to consider three types of asymmetric circuits: "core-sheathing," "core-ground" "sheathing-ground" (Figure 7.11). From the point of view of the general theory, two circuits -- "core-sheathing" and "core-ground" -- are basic (subject to the effect), and the "sheathing-ground" circuit plays the role of the third circuit. The effect on the "core-sheathing" circuit of the symmetric cable is realized as a result of double energy transfer:

from the high voltage line to the "sheathing-ground" circuit and from the latter, just as from the third circuit, to the "core-sheathing" circuit. The effect on the "core-ground" circuit also is the result of two transfers: directly from the high voltage line and through the sheathing as the third circuit.

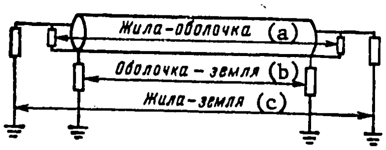



Figure 7.11. Asymmetric circuits in the cable.

Key: a. core-sheathing c. core-ground
b. sheathing-ground

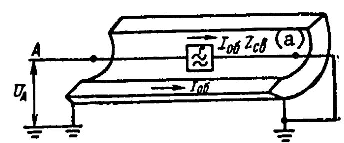



Figure 7.12. Determination of the mutual resistance between the "sheathing-ground" and "core-ground" circuits.

Key: a. $I_{sh}Z_{coup}$

The effect on the coaxial pair with external metal sheathing occurs as a result of triple energy transfer: from the high voltage line to the "sheating-ground" circuit, from it to the intermediate "outer conductor-sheathing" circuit and then to the coaxial pair. Here the third circuit is the "sheathing-ground," and the fourth circuit is the "outer conductor-sheathing."

The distinguishing feature of the metal sheathing as the third circuit is the fact that the coupling parameters between the interfering line and the sheathing and between the interfering line and the "core-ground" circuit are in practice identical, whereas in the general case these parameters are not equal. The quantitatively indicated energy transfers from the "sheathing-ground" circuit to the "core-sheathing" circuit or to the intermediate "outer conductor-sheathing" circuit are determined by the coupling resistance of the sheathing.

By the coupling resistance of the sheathing we mean, just as in the general theory of the effect between coaxial circuits, the ratio of the intensity of the electric field $\dot{E}_a$ at the inner surface of the sheathing to the current $\dot{I}_{sh}$ flowing in the "sheathing-ground" circuit, that is,

$$Z_{св} = \frac{\dot{E}_a}{\dot{I}_{об}}. \tag{7.38}$$

(1) (2)

Key: 1. coup 2. sh

The electric field intensity on the inner surface of the sheathing is the induced emf active in the "core-sheathing" circuit. Thus, the longitudinal emf $E_{c-sh}$ occurring in the "core-sheathing" circuit under the effect of the external magnetic field is

$$\dot{E}_{ж\text{-}об}(x) = \dot{I}_{об}(x) Z_{св}. \tag{7.39}$$

(1) (2) (3)

Key: 1. c-sh 2. sh 3. coup

On insulation of the sheathing, the current induced in it is minimal, and the emf occurring in the "core-sheathing" circuit assumes the smallest value. The smaller the resistance of the sheathing grounds, the greater the current and emf in the "core-sheathing" circuit.

The cases of interest where the currents in the interfering circuit do not depend on the length, that is, where the interfering magnetic field is uniform. The current induced in the sheathing in this case can be represented, as follows from item 2.5, by the general expression

$$\dot{I}_{об}(x) = \frac{\dot{E}_{об}}{Z_{об}} \varphi(\gamma_{об} l), \tag{7.40}$$

where $\dot{E}_{sh}$ is the emf induced in the sheathing per unit length; $\phi(\gamma_{sh}\ell)$ is the function taking into account the length of the circuit subjected to the effect, its parameters and the load resistances. For ideal grounding the ends of the approach section $\phi(\gamma_{sh}\ell) = 1$ and the induced current

$$\dot{I}_{об} = \frac{\dot{E}_{об}}{Z_{об}}. \tag{7.41}$$

Substituting (7.41) in (7.39), we find that the emf induced in the "core-sheathing" circuit is

$$\dot{E}_{ж\text{-}об} = \dot{E}_{об} \frac{Z_{св}}{Z_{об}}. \tag{7.42}$$

(a) (b) (c)

Key; a. c-sh b. sh c. coup

Inasmuch as the cable sheathing and core inside the sheathing are encompassed in practice by the same magnetic flux, the coupling parameters between the interfering line and the sheathing and the "core-ground" circuit are identical; consequently, the emf's induced in the "core-ground" and "sheathing-ground" circuits are equal, that is,

$$\underset{(1)}{\dot{E}_{\text{об}}}(x) \approx \underset{(2)}{\dot{E}_{\text{ж}}}(x). \tag{7.43}$$

Key: 1. sh 2. c

Then (7.42) can be rewritten in the form

$$\underset{(1)}{\dot{E}_{\text{ж-об}}} = \underset{(2)}{\dot{E}_{\text{ж}}} \frac{Z_{\text{св}}{}^{(3)}}{Z_{\text{об}}{}^{(4)}}, \tag{7.44}$$

Key: 1. c-sh 2. c 3. coup 4. sh

that is, for ideal grounding of the sheathing and a uniform interfering magnetic field the emf induced in the "core-sheathing" circuit is equal to the emf induced in the core in the absence of metal sheathing multiplied by the ratio $Z_{coup}/Z_{sh}$. This ratio is the coefficient of protective effect (KZD) of the ideal grounded metal sheathing of the cable located in a uniform interfering magnetic field

$$\underset{(1)}{S_{\text{ид}}} = \frac{Z_{\text{св}}{}^{(2)}}{Z_{\text{об}}{}^{(3)}}. \tag{7.45}$$

Key: 1. id 2. coup 3. sh

Thus, under the indicated conditions $\dot{E}_{c-sh} = \dot{E}_c S_{id}$.

Let us find the emf which arises in the "core-ground" circuit. This emf decreases as a result of the back effect of the current induced in the sheathing:

$$\underset{(1)}{\dot{E}_{\text{ж-з}}}(x) = \underset{(2)}{\dot{E}_{\text{ж}}}(x) - \underset{(3)}{\dot{I}_{\text{об}}}(x)\, \underset{(4)}{Z_{\text{об-ж}}}. \tag{7.46}$$

Key: 1. c-g 2. c 3. sh 4. sh-c

The coupling resistance $Q_{sh-c}$ between the "sheathing-ground" and the "core-ground" circuits is equal to the following, as follows from Figure 7.12:

$$\underset{(1)}{Z_{\text{об-ж}}} = \underset{(2)}{Z_{\text{об}}} - \underset{(3)}{Z_{\text{св}}}. \tag{7.47}$$

Key: 1. sh-c 2. sh 3. coup

The magnitude of the voltage arising between the point A (Figure 7.12) and ground with a cable length equal to one,

$$\dot{U}_{\text{A}} = \dot{E}_{\text{об-ж}} = \dot{I}_{\text{об}} Z_{\text{об}} - \dot{I}_{\text{об}} Z_{\text{св}}.$$

Hence,

$$Z_{\text{об}\cdot\text{ж}} = Z_{\text{об}} - Z_{\text{св}} = \dot{E}_{\text{об}\cdot\text{ж}}/\dot{I}_{\text{об}}.$$
(1) (2) (3)

Key: 1. sh-c 2. sh 3. coup

Thus, the longitudinal emf occurring in the "core-ground" circuit per unit length of cable is

$$\dot{E}_{\text{ж}\cdot\text{з}}(x) = \dot{E}_{\text{ж}}(x) - \dot{I}_{\text{об}}(x)\,(Z_{\text{об}} - Z_{\text{св}}). \qquad (7.48)$$
(1) (2) (3) (4)

Key: 1. c-g 2. c 3. sh 4. coup

Taking $Z_{sh}$ outside the parentheses, we have

$$\dot{E}_{\text{ж}\cdot\text{з}}(x) = \dot{E}_{\text{ж}}(x) - \dot{I}_{\text{об}}(x)\,Z_{\text{об}}\,(1 - S_{\text{ид}}). \qquad (7.49)$$
(1) (2) (3) (4)

Key: 1. c-g 2. c 3. sh 4. id

For a uniform interfering field and ideal grounding of the sheathing, considering (7.41) and (7.43), we obtain

$$\dot{E}_{\text{ж}\cdot\text{з}}(x) = \dot{E}_{\text{ж}} S_{\text{ид}}. \qquad (7.50)$$
(1) (2) (3)

Key: 1. c-g 2. c 3. id

Thus, for ideal grounding of the metal sheathing of the cable in a uniform external magnetic field, the emf's induced in the "core-sheathing" and "core-ground" circuits are identical.

Ieffect on the "Core-Sheathing" Circuit. Let us consider the general case of approach of the cable to the interfering line (see Figure 2.3). The voltages and currents induced in the "core-sheathing" circuit in each section of the cable satisfy the system of equations:

$$\frac{d\dot{U}_{(\text{ж}\cdot\text{об})\,n}}{dx} + Z_{\text{ж}\cdot\text{об}}\,\dot{I}_{(\text{ж}\cdot\text{об})\,n}(x) = \dot{E}_{(\text{ж}\cdot\text{об})\,n}(x); \quad \frac{d\dot{I}_{(\text{ж}\cdot\text{об})\,n}(x)}{dx} + Y_{\text{ж}\cdot\text{об}}\,\dot{U}_{(\text{ж}\cdot\text{об})\,n} = 0. \qquad (7.51)$$
(1)

Key: 1. c-sh

Solving by the method of variation of constants and considering that $\dot{E}_{c-sh} = \dot{I}_{sh} Z_{coup}$, we obtain

$$\left.\begin{aligned}
&\dot U_{(\text{ж-об})\,\text{I}}(x)=A_1 e^{\gamma_{\text{ж-об}}x}+B_1 e^{-\gamma_{\text{ж-об}}(x)}-\frac{Z_{\text{св}}Y_{\text{об}}}{\gamma^2_{\text{об}}-\gamma^2_{\text{ж-об}}}\dot U_{\text{об I}}(x),\\
&\text{(a)}\\
&\dot I_{(\text{ж-об})\,\text{I}}(x)=\frac{1}{z_{\text{в (ж-об)}}}\left(-A_1 e^{\gamma_{\text{ж-об}}x}+B_1 e^{-\gamma_{\text{ж-об}}x}\right)-\\
&\qquad-\frac{Z_{\text{св}}\gamma^2_{\text{ж-об}}}{Z_{\text{ж-об}}\left(\gamma^2_{\text{об}}-\gamma^2_{\text{ж-об}}\right)}\dot I_{\text{об I}}(x),
\end{aligned}\right\} \quad (7.52)$$

$$\left.\begin{aligned}
&\dot U_{(\text{ж-об})\,\text{II}}(x)=A_2 e^{\gamma_{\text{ж-об}}x}+B_2 e^{-\gamma_{\text{ж-об}}x}-\frac{Z_{\text{св}}Y_{\text{об}}}{\gamma^2_{\text{об}}-\gamma^2_{\text{ж-об}}}\dot U_{\text{об II}}(x)-\\
&\text{(b)}\\
&-\frac{\dot I_1(0)\,Z_{1\text{об}}Z_{\text{св}}\gamma^2_{\text{об}}\gamma_1\,\text{ch}\left[\left(\gamma_1(l_{\text{II}}-x)-\ln\sqrt{p_1}\right]\right.}{Z_{\text{об}}\left(\gamma^2_{\text{об}}-\gamma^2_{\text{ж-об}}\right)\left(\gamma^2_{\text{ж-об}}-\gamma_1^2\right)\text{sh}\left(\gamma_1 l_{\text{II}}-\ln\sqrt{p_1}\right)},\\
&\dot I_{(\text{ж-об})\,\text{II}}(x)=\frac{1}{z_{\text{в (ж-об)}}}\left(-A_2 e^{\gamma_{\text{ж-об}}x}+B_2 e^{-\gamma_{\text{ж-об}}x}\right)-\\
&\qquad\text{(c)}\ -\frac{Z_{\text{св}}Y_{\text{об}}}{z_{\text{ж-об}}\left(\gamma^2_{\text{об}}-\gamma^2_{\text{ж-об}}\right)}\dot I_{\text{об II}}(x)-\\
&-\frac{\dot I_1(0)\,Z_{1\text{об}}Z_{\text{св}}\gamma^2_{\text{ж-об}}\gamma^2_{\text{об}}\,\text{sh}\left[\gamma_1(l_{\text{II}}-x)-\ln\sqrt{p_1}\right]}{Z_{\text{ж-об}}Z_{\text{об}}\left(\gamma^2_{\text{об}}-\gamma^2_{\text{ж-об}}\right)\left(\gamma^2_{\text{ж-об}}-\gamma_1^2\right)\text{sh}\left(\gamma_1 l_{\text{II}}-\ln\sqrt{p_1}\right)},
\end{aligned}\right\} \quad (7.53)$$

$$\left.\begin{aligned}
&\dot U_{(\text{ж-об})\,\text{III}}(x)=A_3 e^{\gamma_{\text{ж-об}}x}+B_3 e^{-\gamma_{\text{ж-об}}x}-\frac{Z_{\text{св}}Y_{\text{об}}}{\left(\gamma^2_{\text{об}}-\gamma^2_{\text{ж-об}}\right)}\dot U_{\text{об III}},\\
&\dot I_{(\text{ж-об})\,\text{III}}(x)=\frac{1}{z_{\text{в (ж-об)}}}\left(-A_3 e^{\gamma_{\text{ж-об}}x}+B_3 e^{-\gamma_{\text{ж-об}}x}\right)-\\
&\qquad-\frac{Z_{\text{св}}Y_{\text{об}}}{\gamma^2_{\text{об}}-\gamma^2_{\text{ж-об}}}\dot U_{\text{об III}}(x).\\
&\qquad\qquad\text{(d)}
\end{aligned}\right\} \quad (7.54)$$

Key: a. c-sh b. coup c. w d. sh

For determination of the integration constants $A_1$-$A_3$ and $B_1$-$B_3$ let us write the boundary conditions:

for $x = -\ell_I$

$$\dot U_{(\text{ж-об})\,\text{I}}(-l_{\text{I}})=-\dot I_{(\text{ж-об})\,\text{I}}(-l_{\text{I}})Z_{\text{н}}=\dot I_{(\text{ж-об})\,\text{I}}(-l_{\text{I}})z_{\text{в (ж-об)}}\,\text{cth}\ln\sqrt{p_{\text{н}}};$$

for $x=l_{\text{II}}+l_{\text{III}}$ (2)

$$\dot U_{(\text{ж-об})\,\text{III}}(l_{\text{II}}+l_{\text{III}})=\dot I_{(\text{ж-об})\,\text{III}}(l_{\text{II}}+l_{\text{III}})Z_{\text{к}}=$$

$$=-\dot I_{(\text{ж-об})\,\text{III}}(l_{\text{II}}+l_{\text{III}})z_{\text{в (ж-об)}}\,\text{cth}\ln\sqrt{p_{\text{к}}};$$

for $x=0$ (3)

$$\dot I_{(\text{ж-об})\,\text{I}}(0)=\dot I_{(\text{ж-об})\,\text{II}}(0);\quad \dot U_{(\text{ж-об})\,\text{I}}(0)=\dot U_{(\text{ж-об})\,\text{II}}(0);$$

for $x=l_{\text{II}}$

$$\dot I_{(\text{ж-об})\,\text{II}}(l_{\text{II}})=\dot I_{(\text{ж-об})\,\text{III}}(l_{\text{II}});\quad \dot U_{(\text{ж-об})\,\text{II}}(l_{\text{II}})=U_{(\text{ж-об})\,\text{III}}(l_{\text{II}}).$$

(1)

Key: 1. c-sh 2. b 3. e

Determining the six integration constants from the boundary conditions and substituting them in (7.52)-(7.54), after a number of transformations we find that in each section of the cable

$$\dot{U}_{(\text{ж-об})\,n}(x) = \frac{S_{\text{нд}}\,\gamma_{\text{об}}^2}{\gamma_{\text{об}}^2 - \gamma_{\text{ж-об}}^2}\left[\dot{U}_{\text{ж}\,n}(x) - \dot{U}_{\text{об}\,n}(x)\right] + S_{\text{нд}}\dot{\Phi}_U(x), \tag{7.55}$$

$$\dot{I}_{(\text{ж-об})\,n}(x) = \frac{S_{\text{нд}}\,\gamma_{\text{об}}^2}{\gamma_{\text{об}}^2 - \gamma_{\text{ж-об}}^2}\left[\dot{I}_{\text{ж}\,n}(x) - \frac{Z_{\text{об}}\,\gamma_{\text{ж-об}}^2}{Z_{\text{ж-об}}\,\gamma_{\text{об}}^2}\dot{I}_{\text{об}\,n}\right] + S_{\text{нд}}\dot{\Phi}_I(x), \tag{7.56}$$

where $\dot{U}_{c\,n}(x)\,\dot{I}_{c\,n}(x)$ are determined by the formulas item 2.3-2.5 pertaining to the magnetic effect; for $\gamma_2 = \gamma_{c\text{-}sh}$, $p_{2b(e)} = p_{b(e)}$; $\dot{U}_{shn}(x)$, $\dot{I}_{shn}(x)$, the same; for $\gamma_2 = \gamma_{sh}$, $p_{2b(e)} = p_{sh.b(e)}$; $\dot{\Phi}_U(x)$, $\dot{\Phi}_I(x)$ are the values arising from the effect of the currents flowing in the "sheathing-ground" circuit on the "core-sheathing" circuit (volts, amperes);

$$\begin{aligned}\dot{\Phi}_U(x) = {} & \frac{Z_{\text{об}}}{(\gamma_{\text{об}}^2 - \gamma_{\text{ж-об}}^2)\,\text{sh}\,(\gamma_{\text{ж-об}}\,l - \ln\sqrt{p_{\text{н}}p_{\text{к}}})}\left[\frac{\dot{I}_{\text{об}}(-l_{\text{I}})}{\text{sh}\,(-\ln\sqrt{p_{\text{об,н}}})}\,\times\right. \\ & \times \underset{(c)}{\varphi_{\text{н}}}\,\text{ch}\,[\gamma_{\text{ж-об}}(l_{\text{II}} + l_{\text{III}} - x) - \ln\sqrt{p_{\text{к}}}] - \frac{\dot{I}_{\text{об}}(l_{\text{II}} + l_{\text{III}})}{\text{sh}\,(-\ln\sqrt{p_{\text{об.к}}})}\,\times \\ & \left.\times\,\varphi_{\text{к}}\,\text{ch}\,[\gamma_{\text{ж}}(l_{\text{I}} + x) - \ln\sqrt{p_{\text{н}}}]\right],\end{aligned} \tag{7.57}$$

$$\begin{aligned}\dot{\Phi}_I(x) = {} & \frac{Z_{\text{об}}}{(\gamma_{\text{об}}^2 - \gamma_{\text{ж-об}}^2)\,z_{\text{в}\,(\text{ж-об})}\,\text{sh}\,(\gamma_{\text{ж-об}}\,l - \ln\sqrt{p_{\text{н}}p_{\text{к}}})}\,\times \\ & \times\left[\frac{\dot{I}_{\text{об}}(-l_{\text{I}})}{\text{sh}\,(-\ln\sqrt{p_{\text{об.н}}})}\,\overset{(b)}{\varphi_{\text{н}}}\,\text{sh}\,[\gamma_{\text{ж-об}}(l_{\text{II}} + l_{\text{III}} - x) - \ln\sqrt{p_{\text{к}}}] = \right. \\ & \left. + \frac{\dot{I}_{\text{об}}(l_{\text{II}} + \overset{(e)}{l_{\text{III}}})}{\text{sh}\,-\ln(\sqrt{\underset{(f)}{p_{\text{об.к}}}})}\,\underset{(d)}{\varphi_{\text{к}}}\,\text{sh}\,[\gamma_{\text{ж-об}}(l_{\text{I}} + x) - \ln\sqrt{p_{\text{н}}}]\right],\end{aligned} \tag{7.58}$$

$$\begin{aligned}\varphi_{\text{н}\,(\text{к})} = {} & \underset{(a)}{\gamma_{\text{об}}}\,\text{ch}\ln\sqrt{p_{\text{об.н}\,(\text{к})}}\,\text{sh}\ln\sqrt{p_{\text{н}\,(\text{к})}} - \gamma_{\text{ж}}\,\text{sh}\ln\sqrt{\underset{(g)}{p_{\text{об,н}\,(\text{к})}}}\,\times \\ & \times\,\text{ch}\ln\sqrt{p_{\text{н}\,(\text{к})}}.\end{aligned} \tag{7.59}$$

Key: a. sh b. c-sh c. b d. e

e. sh.b f. sh.e g. b(e)

As is obvious from the expressions obtained here, the voltages and currents induced in the "core-sheathing" circuit are determined by the effect of the high-voltage line on the "core-ground" circuit (under the condition that the metal sheathing is absent), on the "sheathing-ground" circuit and the additional effect of the currents flowing in the sheathing on the core. The formulas are comparatively simple with respect to notation, but the calculations by them in the general case are very tedious considering the propagation coefficient in the interfering line. Less involved calculations are obtained in the following individual special cases only: $p_b = p_e = 1$, $\ell_I = \ell_{III} = 0$, $\gamma_1 = 0$.

The basic formulas for calculating the voltages and currents for $\gamma_1 = 0$ are presented in Table 7.1 (the formulas for the currents in the sections outside the approach are not presented for reasons of economy of space). Using these formulas, it is possible to consider the following special cases: the sheathing is insulated, grounded or closed to the wave impedances, the core is insulated, grounded, closed to the identical or diffe- .t resistanc.s. For example, taking the length of the sections of the cab.e located outsid the approach to the interfering line equal to zero, that is, setting $\ell_I = \ell_{III} = 0$, we obtain

$$\begin{aligned}
\dot U_{(\text{ж-об})\,II}(x) = \frac{I_1 Z_{1\text{об}} S_{\text{нд}}}{\gamma_{\text{об}}^2 - \gamma_{\text{ж-об}}^2} \Bigg\{ & \frac{\gamma_{\text{об}}^2 \left[\text{ch}\ln\sqrt{p_{\text{н}}}\,\text{ch}\left[\gamma_{\text{ж-об}}(l_{II} - x) - \ln\sqrt{p_{\text{к}}}\right] - \text{ch}\ln\sqrt{p_{\text{к}}}\,\text{ch}\left(\gamma_{\text{ж-об}}x - \ln\sqrt{p_{\text{н}}}\right)\right]}{\gamma_{\text{ж-об}}\,\text{sh}\left(\gamma_{\text{ж-об}} l_{II} - \ln\sqrt{p_{\text{н}} p_{\text{к}}}\right)} - \\
& - \frac{\gamma_{\text{об}}\left[\text{ch}\ln\sqrt{p_{\text{об.н}}}\,\text{ch}\left[\gamma_{\text{об}}(l_{II} - x) - \ln\sqrt{p_{\text{об.к}}}\right] - \text{ch}\ln\sqrt{p_{\text{об.к}}}\,\text{ch}\left(\gamma_{\text{об}}x - \ln\sqrt{p_{\text{об.н}}}\right)\right]}{\text{sh}\left(\gamma_{\text{об}} l_{II} - \ln\sqrt{p_{\text{об.н}} p_{\text{об.к}}}\right)} - \\
& - \frac{2\,\text{sh}\,\gamma_{\text{об}}\frac{l_{II}}{2}\left[\text{sh}\left(\gamma_{\text{об}}\frac{l_{II}}{2} - \ln\sqrt{p_{\text{об.к}}}\right)\varphi_{\text{н}}\,\text{ch}\left[\gamma_{\text{ж-об}}(l_{II} - x) - \ln\sqrt{p_{\text{к}}}\right] - \text{sh}\left(\gamma_{\text{об}}\frac{l_{II}}{2} - \ln\sqrt{p_{\text{об.н}}}\right)\varphi_{\text{к}}\,\text{ch}\left(\gamma_{\text{ж-об}}x - \ln\sqrt{p_{\text{н}}}\right)\right]}{\text{sh}\left(\gamma_{\text{ж-об}} l_{II} - \ln\sqrt{p_{\text{н}} p_{\text{к}}}\right)\text{sh}\left(\gamma_{\text{об}} l_{II} - \ln\sqrt{p_{\text{об.н}} p_{\text{об.к}}}\right)} \Bigg\}; \qquad (7.60)
\end{aligned}$$

$$\begin{aligned}
I_{(\text{ж-об})\,II}(x) = -\frac{I_1 Z_{1\text{об}} S_{\text{нд}}}{Z_{\text{ж-об}}\left(\gamma_{\text{об}}^2 - \gamma_{\text{ж-об}}^2\right)} \Bigg\{ & \gamma_{\text{об}}^2\left[1 - \frac{\text{ch}\ln\sqrt{p_{\text{н}}}\,\text{sh}\left[\gamma_{\text{ж-об}}(l_{II} - x) - \ln\sqrt{p_{\text{к}}}\right] + \text{ch}\ln\sqrt{p_{\text{к}}}\,\text{sh}\left[\gamma_{\text{ж-об}}x - \ln\sqrt{p_{\text{н}}}\right]}{\text{sh}\left(\gamma_{\text{ж-об}} l_{II} - \ln\sqrt{p_{\text{н}} p_{\text{к}}}\right)}\right] - \\
& - \gamma_{\text{ж-об}}^2\left[1 - \frac{\text{ch}\ln\sqrt{p_{\text{об.н}}}\,\text{sh}\left[\gamma_{\text{об}}(l_{II} - x) - \ln\sqrt{p_{\text{об.к}}}\right] + \text{ch}\ln\sqrt{p_{\text{об.к}}}\,\text{sh}\left(\gamma_{\text{об}}x - \ln\sqrt{p_{\text{об.н}}}\right)}{\text{sh}\left(\gamma_{\text{об}} l_{II} - \ln\sqrt{p_{\text{об.н}} p_{\text{об.к}}}\right)}\right] + \\
& + \frac{2\gamma_{\text{ж-об}}\,\text{sh}\,\gamma_{\text{об}}\frac{l_{II}}{2}}{\text{sh}\left(\gamma_{\text{об}} l_{II} - \ln\sqrt{p_{\text{об.н}} p_{\text{об.к}}}\right)\text{sh}\left(\gamma_{\text{ж-об}} l_{II} - \ln\sqrt{p_{\text{н}} p_{\text{к}}}\right)}\Bigg[\varphi_{\text{н}}\,\text{sh}\left(\gamma_{\text{об}}\frac{l_{II}}{2} - \ln\sqrt{p_{\text{об.к}}}\right)\text{sh}\left[\gamma_{\text{ж-об}}(l_{II} - x) - \ln\sqrt{p_{\text{к}}}\right] + \\
& + \varphi_{\text{к}}\,\text{sh}\left(\gamma_{\text{об}}\frac{l_{II}}{2} - \ln\sqrt{p_{\text{об.н}}}\right)\text{sh}\left(\gamma_{\text{ж-об}}x - \ln\sqrt{p_{\text{н}}}\right)\Bigg]\Bigg\}. \qquad (7.61)
\end{aligned}$$

Table 7.1. General formulas for the effect on the "core-sheathing" circuit for $l_{c\text{-}sh} = 0$ ($y_{c\text{-}sh} = y_c$)

| Value | Formula |
|---|---|
| $\dot U_{(\text{ж-об})\,\text{I}}(x)$ (a) | $\frac{2 l_1 Z_{1\,\text{об}} S_{\text{нж}}}{\gamma_{\text{об}}^2-\gamma_{\text{ж}}^2}\left\{\frac{\gamma_{\text{об}}^2 \operatorname{sh}\left[\gamma_{\text{ж}}\left(\frac{l_{II}}{2}+l_{III}\right)-\ln\sqrt{p_{\text{н}}}\right]\operatorname{sh}\gamma_{\text{ж}}\frac{l_{II}}{2}\operatorname{ch}\left[\gamma_{\text{ж}}(l_I+x)-\ln\sqrt{p_{\text{н}}}\right]}{\gamma_{\text{ж}}\operatorname{sh}(\gamma_{\text{ж}}l-\ln\sqrt{p_{\text{н}}p_{\text{к}}})} - \frac{\gamma_{\text{об}}\operatorname{sh}\left[\gamma_{\text{об}}\left(\frac{l_{II}}{2}+l_{III}\right)-\ln\sqrt{p_{\text{об.н}}}\right]\operatorname{sh}\gamma_{\text{об}}\frac{l_{II}}{2}\operatorname{ch}\left[\gamma_{\text{об}}(l_I+x)-\ln\sqrt{p_{\text{об.н}}}\right]}{\operatorname{sh}(\gamma_{\text{об}}l-\ln\sqrt{p_{\text{об.н}}p_{\text{об.к}}})} - A(x)\right\}$ |
| $\dot U_{(\text{ж-об})\,\text{II}}(x)$ | $\frac{l_1 Z_{1\,\text{об}} S_{\text{нж}}}{\gamma_{\text{об}}^2-\gamma_{\text{ж}}^2}\left\{\frac{\gamma_{\text{об}}^2}{\gamma_{\text{ж}}\operatorname{sh}(\gamma_{\text{ж}}l-\ln\sqrt{p_{\text{н}}p_{\text{к}}})}\left[\operatorname{ch}(\gamma_{\text{ж}}l_I-\ln\sqrt{p_{\text{к}}})\operatorname{ch}\left[\gamma_{\text{ж}}(l_{II}+l_{III}-x)-\ln\sqrt{p_{\text{н}}}\right] - \operatorname{ch}(\gamma_{\text{ж}}l_{III}-\ln\sqrt{p_{\text{н}}})\operatorname{ch}\left[\gamma_{\text{ж}}(l_I+x)-\ln\sqrt{p_{\text{к}}}\right]\right] - \frac{\gamma_{\text{об}}}{\operatorname{sh}(\gamma_{\text{об}}l-\ln\sqrt{p_{\text{об.н}}p_{\text{об.к}}})}\left[\operatorname{ch}(\gamma_{\text{об}}l_I-\ln\sqrt{p_{\text{об.к}}})\operatorname{ch}\left[\gamma_{\text{об}}(l_{II}+l_{III}-x)-\ln\sqrt{p_{\text{об.н}}}\right] - \operatorname{ch}(\gamma_{\text{об}}l_{III}-\ln\sqrt{p_{\text{об.н}}})\operatorname{ch}\left[\gamma_{\text{об}}(l_I+x)-\ln\sqrt{p_{\text{об.к}}}\right]\right] - 2A(x)\right\}$ |
| $\dot I_{(\text{ж-об})\,\text{II}}(x)$ | $-\frac{l_1 Z_{1\,\text{об}} S_{\text{нж}}}{Z_{\text{ж-об}}(\gamma_{\text{об}}^2-\gamma_{\text{ж}}^2)}\left\{\gamma_{\text{об}}^2\left[1-\frac{\operatorname{ch}(\gamma_{\text{ж}}l_I-\ln\sqrt{p_{\text{к}}})\operatorname{sh}\left[\gamma_{\text{ж}}(l_{II}+l_{III}-x)-\ln\sqrt{p_{\text{н}}}\right]+\operatorname{ch}(\gamma_{\text{ж}}l_{III}-\ln\sqrt{p_{\text{н}}})\operatorname{sh}\left[\gamma_{\text{ж}}(l_I+x)-\ln\sqrt{p_{\text{к}}}\right]}{\operatorname{sh}(\gamma_{\text{ж}}l-\ln\sqrt{p_{\text{н}}p_{\text{к}}})}\right] - \gamma_{\text{ж}}^2\left[1-\frac{\operatorname{ch}(\gamma_{\text{об}}l_I-\ln\sqrt{p_{\text{об.к}}})\operatorname{sh}\left[\gamma_{\text{об}}(l_{II}+l_{III}-x)-\ln\sqrt{p_{\text{об.н}}}\right]+\operatorname{ch}(\gamma_{\text{об}}l_{III}-\ln\sqrt{p_{\text{об.н}}})\operatorname{sh}\left[\gamma_{\text{об}}(l_I+x)-\ln\sqrt{p_{\text{об.к}}}\right]}{\operatorname{sh}(\gamma_{\text{об}}l-\ln\sqrt{p_{\text{об.н}}p_{\text{об.к}}})}\right]+2Б(x)\right\}$ |

Table 7.1 (continued)

| Value | Formula |
|---|---|
| $\dot{U}_{(ж.об)\,III}(x)$ | $-\frac{l_1 Z_{1\,об} S_{вх}}{\gamma_{об}^2-\gamma_ж^2}\left\{\frac{\gamma_{об}^2 \operatorname{sh}\left[\gamma_ж\left(\frac{l_{II}}{2}+l_I\right)-\ln\sqrt{p_н}\right]\operatorname{sh}\gamma_ж\frac{l_{II}}{2}\operatorname{ch}\left[\gamma_ж(l_{II}+l_{III}-x)-\ln\sqrt{p_к}\right]}{\gamma_ж\operatorname{sh}(\gamma_ж l-\ln\sqrt{p_н p_к})}-\frac{\gamma_{об}\operatorname{sh}\left[\gamma_{об}\left(\frac{l_{II}}{2}+l_I\right)-\ln\sqrt{p_{об.н}}\right]\operatorname{sh}\gamma_{об}\frac{l_{II}}{2}\operatorname{ch}\left[\gamma_{об}(l_{II}+l_{III}-x)-\ln\sqrt{p_{об.к}}\right]}{\operatorname{sh}(\gamma_{об} l-\ln\sqrt{p_{об.н}p_{об.к}})}+A(x)\right\}$ |
| $A(x)$ | $\frac{\operatorname{sh}\gamma_{об}\frac{l_{II}}{2}\left[\operatorname{sh}\left[\gamma_{об}\left(\frac{l_{II}}{2}+l_{III}\right)-\ln\sqrt{p_{об.к}}\right]\varphi_н\operatorname{ch}\left[\gamma_ж(l_{II}+l_{III}-x)-\ln\sqrt{p_к}\right]-\operatorname{sh}\left[\gamma_{об}\left(\frac{l_{II}}{2}+l_I\right)-\ln\sqrt{p_{об.н}}\right]\varphi_к\operatorname{ch}\left[\gamma_ж(l_I+x)-\ln\sqrt{p_н}\right]\right]}{\operatorname{sh}(\gamma_ж l-\ln\sqrt{p_н p_к})\operatorname{sh}(\gamma_{об} l-\ln\sqrt{p_{об.н}p_{об.к}})}$ |
| $Б(x)$ | $\frac{\gamma_ж\operatorname{sh}\gamma_{об}\frac{l_{II}}{2}\left[\operatorname{sh}\left[\gamma_{об}\left(\frac{l_{II}}{2}+l_{III}\right)-\ln\sqrt{p_{об.к}}\right]\varphi_н\operatorname{sh}\left[\gamma_ж(l_{II}+l_{III}-x)-\ln\sqrt{p_к}\right]+\operatorname{sh}\left[\gamma_{об}\left(\frac{l_{II}}{2}+l_I\right)-\ln\sqrt{p_{об.н}}\right]\varphi_к\operatorname{sh}\left[\gamma_ж(l_I+x)-\ln\sqrt{p_н}\right]\right]}{\operatorname{sh}(\gamma_ж l-\ln\sqrt{p_н p_к})\operatorname{sh}(\gamma_{об} l-\ln\sqrt{p_{об.н}p_{об.к}})}$ |

Key: a. c-sh b. sh c. c d. sh-b e. sh-e f. e

If in these formulas we set $p_{sh.b} = p_{sh.e} = -1$, that is, it is assumed that the sheathing is ideally grounded by the ends of the approached section, then the expressions will be obtained in which consideration of the presence of the metal sheathing is done by simple multiplication of $\dot{U}_c$ or $\dot{I}_c$ by the idealized coefficient of protective effect.

With a complex approach route, two methods of solution are possible: either by the mean resistance of the magnetic coupling over the entire approach route with subsequent calculation pertaining to the parallel approach; or superposition of the induced values from each approach section. The calculations demonstrated that it is possible to use the first method with an error not exceeding 2-3%.

Effect in the "Core-Ground" Circuit. The following emf is induced in the "core-ground" circuit

$$\underset{(a)}{\dot{E}_{ж\cdot з}}(x) = \underset{(b)}{\dot{E}_{ж}}(x) - \underset{(c)}{\dot{I}_{об}}(x)(Z_{об} - \underset{(d)}{Z_{cв}}). \tag{7.62}$$

Key: a. c-g b. c c. sh d. coup

In this equality $\dot{I}_{sh}(x)Z_{coup} = \dot{E}_{c.sh}(x)$, $\dot{I}_{sh}(x)Z_{sh} = \dot{E}_{c.sh}(x)/S_{id}$. Thus,

$$\dot{E}_{ж\cdot з}(x) = \dot{E}_{ж}(x) - \frac{\dot{E}_{ж\cdot об}(x)}{S_{ид}} + \dot{E}_{ж\cdot об}(x),$$

or

$$\dot{E}_{ж\cdot з}(x) = \underset{(a)}{\dot{E}_{ж\cdot об}}(x)\left(1 - \frac{1}{\underset{(b)}{S_{ид}}}\right) + \dot{E}_{ж}(x). \tag{7.63}$$

Key: a. c-sh b. id

In contrast to the "core-sheathing" circuit, in the "core-ground" circuit the induced emf decreases with an increase in current in the sheathing. When the sheathing is insulated; the maximum emf occurs in the "core-ground" circuit. In order to lower the emf in the "core-ground" circuit the sheathing must be grounded. The minimum emf is induced for ideal grounding of the sheathing at the ends of the approach section.

For determination of the voltages and currents occurring in the "core-ground" circuit, let us write the system of equations:

$$\frac{d\dot{U}_{(ж\cdot з)\,n}(x)}{dx} + \dot{I}_{(ж\cdot з)\,n}(x)\,Z_{ж\cdot з} = \dot{E}_{(ж\cdot з)\,n}(x);$$

$$\frac{d\dot{I}_{(ж\cdot з)\,n}(x)}{dx} + Y_{ж\cdot з}\underset{(a)}{\dot{U}_{(ж\cdot з)\,n}}(x) = 0. \tag{7.64}$$

Key: a. c-g

As a result of the solution of this system, analogously to the preceding one after a number of transformations we obtain the following expressions for voltages and currents induced in the n-th section of the cable:

$$\underset{(a)}{\dot U_{(\text{ж-з})\,n}}(x)=\frac{S_{\text{ид}}\gamma_{\text{об}}^2-\gamma_{\text{ж-з}}^2}{\gamma_{\text{об}}^2-\gamma_{\text{ж-з}}^2}\dot U_{\text{ж}\,n}(x)+(1-\underset{(b)}{S_{\text{ид}}})\times \left[\underset{(c)}{\dot U_{\text{об}\,n}}(x)\frac{\gamma_{\text{об}}^2}{\gamma_{\text{об}}^2-\gamma_{\text{ж-з}}^2}-\dot\Phi_U(x)\right], \qquad (7.65)$$

Key: a. c-g  b. id  c. sh

$$\dot I_{(\text{ж-з})\,n}(x)=\frac{S_{\text{ид}}\gamma_{\text{об}}^2-\gamma_{\text{ж-з}}^2}{\gamma_{\text{об}}^2-\gamma_{\text{ж-з}}^2}\dot I_{\text{ж}\,n}(x)+(1-S_{\text{ид}})\times \left[\dot I_{\text{об}\,n}(x)\frac{Z_{\text{об}}\gamma_{\text{ж-з}}^2}{Z_{\text{ж-з}}(\gamma_{\text{об}}^2-\gamma_{\text{ж-з}}^2)}-\dot\Phi_I(x)\right]. \qquad (7.66)$$

where $\dot\Phi_{\dot U}$, $\dot\Phi_{\dot I}$ are determined by (7.57) and (7.58) on replacement of the parameters of the "core-sheathing" circuit by the circuit parameters "core-ground"; $\dot U_{cn}$, $\dot I_{cn}$ are determined by the formulas item 2.3-2.5 pertaining to the magnetic effect, for $\gamma_2 = \gamma_{c-g}$, $p_{2b(e)} = p_{b(e)}$. By means of simple transformations it is possible to show that

$$\left.\begin{aligned}\dot U_{(\text{ж-з})\,n}(x)&=\dot U_{(\text{ж-об})\,n}(x)\left(1-\frac{1}{S_{\text{ид}}}\right)+\dot U_{\text{ж}\,n}(x),\\ \underset{(a)}{\dot I_{(\text{ж-з})\,n}}(x)&=\underset{(b)}{\dot I_{(\text{ж-об})\,n}}(x)\left(1-\frac{1}{\underset{(c)}{S_{\text{ид}}}}\right)+\dot I_{\text{ж}\,n}(x).\end{aligned}\right\} \qquad (7.67)$$

Key: a. c-g  b. c-sh  c. id  d. c n

where $\dot U_{(c-sh)n}$ and $\dot I_{(c-sh)n}(x)$ are defined by (7.55), (7.56), in which the parameters pertaining to the "core-sheathing" circuit are replaced by the parameters of the "core-ground" circuit. As is obvious from (7.67) for $S_{id}\to 1$, $U_{c-g}\to\dot U_{cn}$; for $S_{id} \ll 1$ the first times are equal to $-U_{c-sh}/S_{id}$ and $-I_{c-sh}/S_{id}$. The potential difference between the core and the provisional near ground point determined by the current leakage from the cable sheathing is $\dot U_{c-g}(x) = \dot U_{(c-sh)}(x) + \dot U_{sh}(x)$.

Calculation of Effect Considering Adjacent C[illegible]. A study was made above of the effect on the cable cores without considering the mutual effect between cores. Although at low frequencies this effect does not have such great significance as at high frequencies, consideration of it permits better evaluation of the effect of the induced voltages and currents.

Let us consider a cable containing a large number of cores. In all the cores when the cable approaches the interfering line, emf's and mutually interfering currents are induced. For low frequencies it is possible to assume that the induced emf's are identical in all cores. If in this case all the cores are arranged identically with respect to the sheathing and relative to each other, it is possible to consider that the voltages and currents in the cores are also identical. In this case the solution is simplified significantly.

For multiple-lay cables this assumption cannot be made; therefore for simplification we shall consider all adjacent cores as an aggregate bunch, the parameters of which differ significantly from the parameters of the investigated core. This aggregate bunch is an additional intermediate circuit which is under the same metal sheathing with the basic circuit subject to the effect. Then the result of the effect on the basic circuit can be represented in the form of the sum of two components: the result of the direct effect and the result of the effect through the intermediate circuit. If, as was indicated above, the "sheathing-ground" circuit is the third circuit, then the bunch can be considered as the fourth circuit.

The performed studies demonstrated that consideration of the bunched cores in general form leads to highly awkward expressions. It is simpler to consider the bunch if it is assumed that the cable sheathing is ideally grounded ($p_{sh.b} = p_{sh.e} = -1$).

The resultant voltage in the core caused both by the direct effect ($\dot{U}_{dir}$) and by the effect through the bunch is (since the sheathing is grounded, $\dot{U}_{c-g} = \dot{U}_{c-sh} = \dot{U}_c$): $\dot{U}_{c.res}(x) = \dot{U}_{c.dir}(x) + \dot{U}_{c.bu}(x)$, where $U_{c.bu}(x)$ is the voltage induced in the cable core through the bunch. Hence, the coefficient of protective effect of the bunch $S_{bu}(x) = \dot{U}_{c.res}(x)/\dot{U}_{c.dir} = 1 + (\dot{U}_{c.bu}(x)/\dot{U}_{c.dir}(x))$. For $x = 0$, that is, at the beginning of the cable, $S_{bu}(0) = 1 + (U_{c.bu}(0)/U(0)_{c.dir})$.

Assuming that there is no inverse effect from the direction of the basic circuit on the bunched cores, we obtained values of the coefficient of useful effect of the bunch $S_{bu}(0)$ for different conditions of its loading (see Table 7.2).

The following additional notation is introduced in Table 7.2: $Z_{bu}$, $Y_{bu}$, $\gamma_{bu}$ are the total resisatnce, the total conductivity and the propagation coefficient of the bunched cores; $K'_{bu.c} = Z_{bu.c} - (Y_{bu.c})/Y_{bu}\, Z_c$; $K''_{bu.c} = Z_{bu.c} - (Y_{bu.c})/Y_c\, Z_{bu}$; $Z_{bu.c}$, $Y_{bu.c}$ are the resistance of the magnetic coupling and the conductivity of the electrical coupling between the bunch and the investigated core of the cable.

As follows from the formulas in Table 7.2 for a short cable length with $\gamma_{bu}\ell_{II} \ll 1$ the shielding coefficient of the insulated bunch approaches one. The shielding coefficient of the grounded bunch does not depend on the length.

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30 APRIL 1980 M.N. MIKHAYLOV, L.D. RAZUMOV AND S.A. SOKOLOV 3 OF 4

Table 7.2. Forumulas for the coefficients $S_{bu}(0)$ of the bunched cores in the cable with grounded sheathing

| Condition of the bunch | $S_{bu}(0)$ |
|---|---|
| Insulated | $1-\frac{Y_{\text{п}}K'^{(a)}_{\text{п.ж}}}{\gamma_{\text{п}}^2-\gamma_{\text{ж}}^2}\left(1-\frac{\gamma_{\text{ж}}\,\text{th}\,\gamma_{\text{п}}\,l_{II}/2}{\gamma_{\text{п}}\,\text{th}\,\gamma_{\text{ж}}\,l_{II}/2}\right)$ |
| Grounded | $1-\frac{Z_{\text{п.ж}}}{Z_{\text{п}}}$ |
| Loaded by wave impedances at the ends of the section | $1-\frac{Y_{\text{п}}^{(b)}}{\gamma_{\text{п}}^2-\gamma_{\text{ж}}^{2\,(c)}}\left\{K'_{\text{п.ж}}\left[1-\frac{\gamma_{\text{ж}}\left(1-e^{-\gamma_{\text{п}}l_{II}}\right)}{2\gamma_{\text{п}}\,\text{th}\,\gamma_{\text{ж}}\frac{l_{II}}{2}}\right]-K''_{\text{п.ж}}\frac{\gamma_{\text{ж}}^2\left(1-e^{-\gamma_{\text{п}}l_{II}}\right)}{2\gamma_{\text{п}}^2}\right\}$ |

Key: a. bu.c b. bu c. c

The shielding effect of the bunch of insulated adjacent cores at a frequency of 50 hertz in a segment with a total length of about 100 km was investigated experimentally by A. A. Snarskiy [48]. A bunch of 24 (6 quads) cores insulated at the ends decreased the voltage induced in the core at the beginning of the cable by 7.3%, which corresponds to a coefficient of protective effect of 0.927. For cables with a capacity of 14 × 4, a coefficient of 0.9 is recommended, and for a cable with a capacity of 7 × 4, a coefficient of 0.95.

Effect on Coaxial Cables with Outer Metal Coverings. Let us consider a coaxial pair with outer metal sheathing (Figure 7.13). The following circuits exist here in general form: the "inner conductor-outer conductor" coaxial pair, the "inner conductor-sheathing" coaxial pair, the "inner conductor-ground" coaxial pair, "outer conductor-sheathing" coaxial pair, "outer conductor-ground" and "sheathing-ground" coaxial pairs. In the presence of an external magnetic field voltages and currents will be induced in all of the enumerated circuits. The voltages and currents induced in the coaxial pair and the outer conductor are of the greatest interest.

The voltages induced in the coaxial pair itself constitute a danger to the insulation of the pair and the input fittings. The current induced in the inner conductor flowing through the feed circuits of the intermediate repeaters causes interference in the channels, nonlinear distortions, spurious modulation, and it also can damage the transistorized equipment. The voltage induced between any conductor of the coaxial pair and the outer metal sheathing is dangerous to the insulation of the corresponding input fittings.

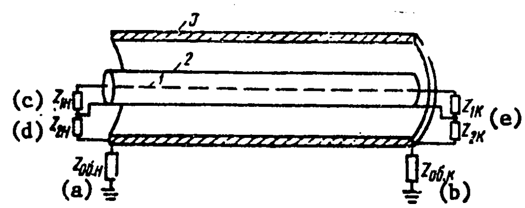


Figure 7.13. Schematic of the circuits of a coaxial cable.

Key: a. sh.b b. sh.e c. 1e d. 2e e. ...b

Let us consider the effect on the coaxial pair. The current in the outer conductor induces voltages and currents in the coaxial pair itself through the coupling resistance of the conductor. This current can flow both in the "outer conductor-sheathing" circuit and in the "outer conductor-ground" circuit. Thus, the effect on the coaxial pair can be considered as a double transition through intermediate circuits along two paths: the first path is from the "sheathing-ground" circuit through the coupling systems of the sheathing to the "outer conductor-sheathing" circuit (the first junction) and from the "outer conductor-sheathing" circuit through the coupling resistance of the outer conductor to the coaxial pair (the second junction); the second path is the direct effect on the "outer conductor-ground" circuit (the first junction) and from the "outer conductor-ground" circuit to the coaxial pair (second junction).

In practice the outer conductors of the coaxial pairs are either completely insulated from the sheathing (for example, in the K-120 system) or they are connected directly to the sheathing (for example, in the K-1920 system). Accordingly, it is possible to consider that the current in the outer conductor will flow only through the "outer conductor-sheathing" circuits, and the effect on the coaxial pair takes place along the first path, that is, through the two intermediate circuits: "sheathing-ground" and "outer conductor-sheathing." The formulas for calculating the effect on the coaxial pairs are obtained in [49] and are not presented here. The presence of the double transfer of energy through the intermediate circuits leads to significant complication of the calculated formulas. In the case of ideal grounding of the metal sheathing the process of the energy transfer to the coaxial pair is significantly simplified. In this case the first of the above-indicated transfers through the intermediate circuit is considered a simple multiplication by $S_{id\ 1}$, that is, the ideal coefficient of protective effect of the outer metal sheathing. The second transfer (from the "outer conductor-sheathing" circuit to the coaxial pair) is considered just as the effect on the "core-sheathing" circuit of the "sheathing-ground" circuit in a symmetric cable.

Analysis shows that on grounding of the sheathing of the coaxial cable, the induced values in the coaxial pair in any section can be determined by the



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formulas pertaining to the "core-sheathing" circuit of the symmetric cable multiplied by the coefficient $\alpha = Z_{coup2}/Z_{out-sh}$, where $Z_{coup2}$ is the coupling resistance of the outer conductor; $Z_{out-sh}$ is the total longitudinal resistance of the "outer conductor-sheathing" circuit. Here $\gamma_{c-sh}$, $Z_{c-sh}$, $Z_{sh}$, $\gamma_{sh}$, $S_{id}$, $p_{sh}$ in the formulas must be replaced respectively by the analogous values pertaining to a coaxial cable: $\gamma_{c.c}$, $Z_{c.c}$ are the propagation coefficient and the total longitudinal resistance of the coaxial circuit; $Z_{out-sh}$, $\gamma_{out-sh}$ are the total longitudinal resistance and propagation coefficients of the "outer conductor-sheathing" circuits; $p_{out-sh}$ is the reflection coefficient at the beginning and end of the "outer conductor-sheathing" circuit.

The main coaxial cables of the majority of types have not one, but several coaxial pairs. The presence of adjacent coaxial pairs if the outer conductors are insulated from each other and from the sheathing has no effect (in the low-frequency band) on the distribution and magnitude of the induced currents in the investigated pair. In the case of connection of the outer conductors of all of the coaxial pairs to the sheathing, which usually occurs in practice, induced currents arise, the interaction of which must be taken into account. The simplest solution is obtained if it is proposed that all of the coaxial pairs are identical, they are loaded identically and they are arranged identically with respect to the sheathing and with respect to each other.

In real structures, for example, in the KMB-4 cable, this proposition is completely valid. When investigating the effect on the 2,6/9,4 pairs of the KMB-8/6 cable it is possible not to consider the effect of the small-diameter pairs, for their resistance is appreciably greater, and for investigation of the effect on the 1,2/4,6 pairs it is possible to consider all pairs identical. Under these conditions the calculation of the effect can be made by the formulas for a single pair, replacing the resistance $Z_{out-sh}$ by the equivalent resistance defined by the formula

$$\underset{(a)}{Z_{\text{экв}}} = \underset{(b)}{Z_{\text{вн·об}}} + Z_{1i} + Z_{2i} + \cdots,$$

Key: a. equiv b. out-sh

where $Q_{1i}$, $Z_{2i}$ ... are the resistances of the magnetic coupling between the first, second, ... and i-th "outer conductor-sheathing" circuits.

## 7.5. Calculation of the Effect in the Case of Several Cables Laid Parallel

In cases where two or more (in the general case n) cables are subjected to the effect of a high-voltage line, it is necessary to consider the mutual effect between them. It is natural to propose that all of the circuits entering into the multicable system ("core-sheathing," "core-ground" "sheathing-ground") influence each other, but the effect of the currents flowing in the "core-sheathing" circuit is insignificant by comparison with the effect of the currents flowing in the "sheathing-ground" circuit. This is caused first of all by the lower magnitude of the current flowing through the cores and, secondly, the additional shielding of the sheathing. Inasmuch as the cables are laid in the ground, in addition to magnetic effects they also have a galvanic effect on each other. If the cable sheathings are insulated from the ground, then the galvanic effect can be neglected inasmuch as the leakage currents from the sheathings are small. If the cables do not have an insulated coating, then it is necessary to consider both types of effects.

The voltages and currents in the sheathing of the kth cable satisfy the following system of equations which take into account the mutual magnetic and galvanic effects:

$$\frac{d\dot U_{\text{об}\,k}}{dx}+\sum_{i=1}^{n}\dot I_{\underset{(a)}{\text{об}}\,i}Z_{ik}=-\dot I_{\underset{(b)}{\text{вл}}}Z_{0k};\quad \frac{d\dot I_{\text{об}\,k}}{dx}+Y_{\text{об}\,k}\Bigl(\dot U_{\text{об}\,k}+\sum_{i=1}^{n}\frac{d\dot I_{\text{об}\,i}}{dx}R_{\underset{(c)}{\text{пер}}\,ik}\Bigr)=0, \tag{7.68}$$

Key: a. sh b. effect c. trans

where $Z_{ik}$ are the magnetic coupling resistances between the "sheathing-ground" circuits of the ith and kth cables ($Z_{kk}$ is the natural resistance of the "sheathing-ground" circuit of the kth cable), ohms/m; $R_{trans\ ik}$ is the transient resistance through the ground between the sheathings of the ith and the kth cables ($R_{transkk} = 0$), ohm-m; $Z_{ok}$ are the resistances of the magnetic coupling between the interfering line and the "sheathing-ground" circuit of the kth cable, ohm/m.

In practice the spacings between the cables laid parallel are always much less than the spacing between them and the interfering lines so that it is possible to set $Z_{01} = Z_{02} = Z_{03} = \ldots = Z_{0k}$. For simplification let us propose that all the cables and also the spacings between them are identical and equal to the geometric mean of all of the mutual spacings, that is,

$$Z_{\text{об}1}=Z_{\text{об}2}=\ldots=Z_{\text{об}\,k}=Z_{\text{об}};\ Y_{\text{об}1}=Y_{\text{об}2}=\ldots Y_{\text{об}\,k}=Y_{\text{об}};$$
$$Z_{12}=Z_{21}=Z_{13}=Z_{31}=\ldots=Z_{ik}=Z_{\text{об-об}}=i\omega M_{\text{об-об}};\ \text{(a)}$$
$$R_{\text{пер}12}=R_{\text{пер}21}=R_{\text{пер}13}=R_{\text{пер}31}=\ldots R_{\underset{(b)}{\text{пер}}\,ik}=R_{\text{пер.об-об}}.$$

Key: a. sh b. trans

The transient resistance between the sheathings can be defined by the following approximate formula



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$$\underset{(a)}{R_{\text{пер об-об}}} = \frac{\rho_3}{\pi} K_0 \left[\gamma_{\text{об}} \sqrt[4]{(a^2+R^2)(a^2+4h^2)}\right],$$

Key: a. trans sh.sh

where a is the average spacing between the cables, meters; R is the average outside radius of the sheathings, meters; h is the depth of burial, meters.

If the cable sheathings are loaded on the ends by identical resistances, the voltages and currents in them are also identical. Then the first group of equations of system (7.68) taking into account the magnetic coupling is converted into one equation which is valid for any cable in the bunch:

$$\frac{d\dot{U}_{\text{об}}}{dx} + \dot{I}_{\text{об}} Z_{\text{об}} + \dot{I}_{\text{об}} Z_{\text{об-об}} (n-1) = -\dot{I}_{\text{вл}} Z_{1\cdot\text{об}},$$

or

$$\frac{d\dot{U}_{\text{об}}}{dx} + \dot{I}_{\text{об}} [Z_{\text{об}} + Z_{\text{об-об}} (n-1)] = -\underset{(a)}{\dot{I}_{\text{вл}}} Z_{1\cdot\text{об}}.$$

Key: a. effect

The second group of equations taking into account the galvanic coupling is also converted to one equation which is valid for any cable:

$$\frac{d\dot{I}_{\text{об}}}{dx} + \underset{(a)}{\dot{U}_{\text{об}} Y_{\text{об}}} + \frac{d\dot{I}_{\text{об}}}{dx} Y_{\text{об}} \underset{(b)}{R_{\text{пер.об-об}}} (n-1) = 0,$$

Key: a. sh b. trans.sh.sh

or

$$\frac{d\dot{I}_{\text{об}}}{dx} + \dot{U}_{\text{об}} \left(\frac{1}{Y_{\text{об}}} + R_{\text{пер.об-об}} (n-1)\right)^{-1} = 0.$$

Thus, the above investigated system of two equations is obtained in which the presence of the adjacent cables in the bunch is taken into account, introducing the equivalent parameters:

$$Z_{\text{экв.об}} = Z_{\text{об}} + (n-1) Z_{\text{об-об}}, \quad \underset{(a)}{Y_{\text{экв.об}}} = \left[\frac{1}{Y_{\text{об}}} + R_{\text{пер.об-об}} (n-1)\right]^{-1}.$$

Key: a. equiv.sh

The calculation of the effect consequently can be made by the formulas which pertain to a single cable, but with consideration of the equivalent parameters.

As was demonstrated, the effect on the "core-sheathing" circuit is determined by the current induced in the "sheathing-ground" circuit. Inasmuch as the equivalent resistance of the cable sheathing in the bunch increases, the current decreases and, consequently, the effect on the "core-sheathing" circuit diminishes. The magnitude of the ideal coefficient of usual effect of the sheathing also changes, for the ratio $Z_{coup}/Z_{equiv.sh}$ becomes less. Here, the shielding effect of adjacent cables is manifested. The same effect is also obtained under the effect on the "core-ground" circuit.

### 7.6. Effect on Cable Cores with Multiply Grounded Sheathings

Let us consider a cable subjected to the effect of a high voltage line, and let us assume that the sheathing is grounded in the approach section at the arbitrary point M (Figure 7.14) where the grounding resistance is Z. Let us denote the current flowing through the ground $\dot I_z$. This current creates voltages and currents in the "core-sheathing" circuit which can be represented as follows:

in the cable sections located to the left of the ground,

$$\left.\begin{aligned}
\dot I_{(\text{ж-об})\,z}(x) &= \frac{\dot I_z Z_{\text{об}}\,\gamma^2_{\text{ж-об}} S_{\text{ид}} \operatorname{ch}\left(\gamma_{\text{ж-об}}\, l_{z\text{-к}} - \ln\sqrt{p_{\text{к}}}\right)\times \operatorname{sh}\left[\gamma_{\text{ж-об}}\left(l_{\text{I}}+x\right)-\ln\sqrt{p_{\text{н}}}\right]}{Z_{\text{ж-об}}\left(\gamma^2_{\text{об}}-\gamma^2_{\text{ж-об}}\right)\operatorname{sh}\left(\gamma_{\text{ж-об}}\, l-\ln\sqrt{p_{\text{н}}p_{\text{к}}}\right)},\\
\dot U_{(\text{ж-об})\,z}(x) &= -\frac{\dot I_z Z_{\text{об}}\,\gamma_{\text{ж-об}} S_{\text{ид}} \operatorname{ch}\left(\gamma_{\text{ж-об}}\, l_{z\text{-к}} - \ln\sqrt{p_{\text{к}}}\right)\times \operatorname{ch}\left[\gamma_{\text{ж-об}}\left(l_{\text{I}}+x\right)-\ln\sqrt{p_{\text{н}}}\right]}{\left(\gamma^2_{\text{об}}-\gamma^2_{\text{ж-об}}\right)\operatorname{sh}\left(\gamma_{\text{ж-об}}\, l-\ln\sqrt{p_{\text{н}}p_{\text{к}}}\right)};
\end{aligned}\right\} \tag{7.69}$$

Key: a. c-sh b. id c. e d. b

in the cable sections located to the right of the ground,

$$\left.\begin{aligned}
\dot I_{(\text{ж-об})\,z} &= -\frac{\dot I_z Z_{\text{об}}\,\gamma^2_{\text{ж-об}} S_{\text{ид}} \operatorname{ch}\left(\gamma_{\text{ж-об}}\, l_{z\text{-н}} - \ln\sqrt{p_{\text{н}}}\right)\times \operatorname{sh}\left[\gamma_{\text{ж-об}}\left(l_{\text{II}}+l_{\text{III}}-x\right)-\ln\sqrt{p_{\text{к}}}\right]}{Z_{\text{ж-об}}\left(\gamma^2_{\text{об}}-\gamma^2_{\text{ж-об}}\right)\operatorname{sh}\left(\gamma_{\text{ж-об}}\, l-\ln\sqrt{p_{\text{н}}p_{\text{к}}}\right)},\\
\dot U_{(\text{ж-об})\,z}(x) &= -\frac{\dot I_z Z_{\text{об}}\,\gamma_{\text{ж-об}} S_{\text{ид}} \operatorname{ch}\left(\gamma_{\text{ж-об}}\, l_{z\text{-н}} - \ln\sqrt{p_{\text{н}}}\right)\times \operatorname{ch}\left[\gamma_{\text{ж-об}}\left(l_{\text{II}}+l_{\text{III}}-x\right)-\ln\sqrt{p_{\text{н}}}\right]}{\left(\gamma^2_{\text{об}}-\gamma^2_{\text{ж-об}}\right)\operatorname{sh}\left(\gamma_{\text{ж-об}}\, l-\ln\sqrt{p_{\text{н}}p_{\text{к}}}\right)},
\end{aligned}\right\} \tag{7.70}$$

where $\ell_{z-e}$, $\ell_{z-b}$ are the distances from the ground to the end and the beginning of the cable respectively.

On derivation of these formulas the direction of the current $\dot{I}_z$ is provisionally taken as positive (the current flows out of the cable sheathing). If the direction of the current is the opposite, then the signs of the variables change. Formulas (7.69) and (7.70) make it possible to consider any number of arbitrarily arranged grounds on all sections of the cable by the superposition method.

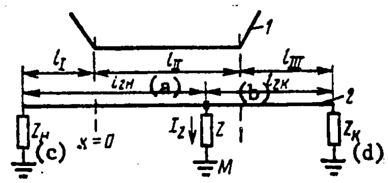

Figure 7.14. Approach of a cable to an interfering line in the presence of one arbitrarily arranged intermediate ground on the sheathing: 1 -- interfering line; 2 -- communication cable.
Key: a. zb b. ze c. e d. b

In the case of multiple grounding and uniform arrangment of the grounds on the sheathing, the calculation of the effect on the cable cores can be made by the formulas of item 7.4, replacing the discrete grounds by a uniformly distributed transient resistance defined by (7.37). As the calulations indicated, it is possible to use this method in the "core-sheathing" circuit with a spacing between grounds of no more than 2 km and ground resistances of no more than 5 ohms; in the "core-ground" circuit, for grounding resistances to 20-30 ohms independently of the spacing between grounds.

## 7.7. Effect on the Two-Wire Circuit of a Cable Line

Effect of an Asymmetric Overhead Line. As demonstrated above, the effect on the two-wire circuits is caused by asymmetry of the arrangment of the circuit conductors with respect to the interfering wire and asymmetry of the electrical parameters of the wires and the input fittings. On cables the spacing between cores is always incommensurably smaller than the distance to the interfering conductor; therefore it is possible to neglect the asymmetry of arrangment of the conductors.

When investigating the effect on cable lines, it is sufficient to consider only the asymmetry, and accordingly, it is possible to represent the voltage induced in the two-wire circuit just as in overhead lines, in the form of the sum of the squares of two components: $U_T = \sqrt{U_{t.c}^2 + U_{tA}^2}$. The first of these components depends on the voltage induced in the "core-sheathing" circuit and the asymmetry of the core parameters with respect to the sheathing. The magnitude of this component can be determined by the formula $U_{t.c} = U_{c-sh}, \eta$, where $\eta$ is the sensitivity coefficient of the circuit to the interference. The second term depends on the voltage induced in the "core-ground" circuit, and the VKO[1] asymmetry. The magnitude of this component $U_{TA} = U_{c-g}, A$, where A is the VKO asymmetry coefficient of the equipment.

[1]Overhead cable equipment.

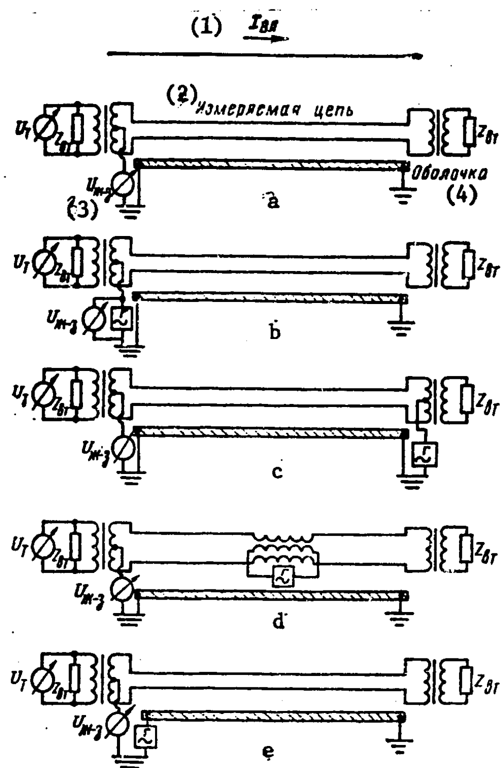


Figure 7.15. Systems for measuring the sensitivity coefficient of cable circuits $\eta = U_T/U_{c-g}$: when using the interfering line (a); when simulating the effect by including an outside generator: at the midpoint of the measured circuit on the near end (b); at the midpoint of the measured circuit on the far end (c); in both lines longitudinally in the middle of the circuit (d); in the "sheathing-ground" circuit (e).

Key: 1. effect 3. c-g
2. measured circuit 4. sheathing

Although in general form $U_{c-sh} \neq U_{c-g}$, with good grounding of the metal covers (no more than 3-5 ohms) and also for all cables with jute outside covering it is possible with sufficient accuracy to set $U_{c-sh} \approx U_{c-g}$. Then $U_T = U_{c-g}\sqrt{\eta^2 + A^2}$. Usually an effort is made to make the asymmetry of the equipment appreciably lower (asymmetry higher) than the asymmetry of the line so that, as a rule, $A \ll \eta$. In this case we obtain the simple expression: $U_T = U_{c-g}\eta$.

The systems for measuring the sensitivity coefficient of the two-wire cable circuit to interference with grounded sheathing are presented in Figure 7.15.

The studies [30] demonstrated that it is necessary to use the system for circuits of short length (to 10 km for noncoil-loaded and to 25-30 km for coil-loaded) in Figure 7.15b as the basic system for measuring the sensitivity coefficient with artifical excitation of the longitudinal emf's, and for the longer circuit, the system in Figure 7.15c. The results of the measurements by these systems under the indicated conditions are close to the actual values of the sensitivity coefficients measured on the same circuits by the system in Figure 7.15a.

The VKO asymmetry is determined as was indicated above for the overhead line equipment.

Effect of Two-Phase and Three-Phase Overhead Lines. When determining the effect of multiwire high-voltage lines on a two-wire cable network it is possible in practice to consider the energy transfer only to the asymmetric cable circuit ("core-sheathing," "core-ground") and from it to the two-wire circuit by introduction of the sensitivity coefficients of the circuit and the VKO asymmetry. The asymmetric cable network is affected by two circuits: the asymmetric "two wires-ground" interfering circuit or "three wires-ground" interfering circuit and the symmetric interfering circuit (two-phase or three-phase). The effect of both the asymmetric and symmetric circuits is defined by the same formulas, but with different magnetic coupling coefficients (mutual induction) between the circuits, The formulas for calculations of the coefficients of mutual induction between the various circuits are presented in Chapter 9.

# CHAPTER 8. VOLTAGE BETWEEN THE CABLE CORE AND SHEATHING WHEN THE CABLE IS DIRECTLY STRUCK BY LIGHTNING

## 8.1. Formation of an Electric Arc Between the Point Struck by Lightning and the Cable

The formation of an electric arc between the cable and a point struck by lightning on the ground was investigated by Ye. D. Zunde, who, as a result of absence of experimental data, took the breakdown voltage of the ground equal to: $E_{breakdown} = 250$ kv/meter for $\rho \leq 100$ ohm-meter and $E_{breakdown} = 500$ kv/m for $\rho \geq 1000$ ohm-m. Beginning with this proposition, the expressions were obtained for determining the distance jumped by the electric arc as a function of the specific resistance of the ground. The experimental data available at the present time have made it possible to define these formulas more precisely [50]. Considering the point struck by the lightning as a point ground, we find that the field intensity on the surface of a hemisphere of radius r (with its center at the point struck by the lightning) is equal to

$$E = J\rho = \frac{I_M}{2\pi r^2}\rho, \tag{8.1}$$

where $I_M$ is the lightning current amplitude; $\rho$ is the specific resistance of the ground; J is the current density.

A spark zone is formed around the strike point, the radius of which $r_0$ is determined by the maximum breakdown voltage $E_{breakdown\ max}$ measured in a uniform field. Let us denote the intensity $E_{breakdown\ max} = E_0$; then from (8.1) we obtain

$$r_0 = \sqrt{\frac{I_M \rho}{2\pi E_0}}.$$

For sand with a specific resistance of $\rho = 150$ ohm-m and with predischarge time $t_{pr} = 2$-$4$ microseconds, $E_0 = 14$ kv/cm. Setting the lightning current amplitude equal to 220 kiloamps, we find that in sand at $\rho = 100$ ohm-m, $r_0 =$ $= 1.58$ m, and for $\rho = 1000$ ohm-m, $r_0 = 5$ m.

The dimensions of the hemispherical "ground" at the point struck by lightning are determined by the intensity $E = E_0$. The individual streamers can be propagated beyond the limits of the spark zone if the potential difference is sufficient for breakdown with an average breakdown intensity of the field $E_{average}$.

The potential difference between the surface of the spark zone and the surface of the hemisphere of radius $r_1$ is

$$U_{01} = \frac{I\rho}{2\pi r_0} - \frac{I\rho}{2\pi r_1} = \frac{I\rho}{2\pi}\left(\frac{1}{r_0} - \frac{1}{r_1}\right). \tag{8.2}$$

For an average field intensity $E_{ave}$, this difference can be represented in the form

$$U_{01} = \underset{(a)}{E_{cp}}(r_1 - r_0). \tag{8.3}$$

Key: a. average

If $E_{average}$ is the average breakdown intensity of the ground, then $r_1 - r_0$ is the distance to which the streamers can be propagated with uniform structure of the ground. Equating (8.3) to (8.2), we find that

$$\frac{I\rho}{2\pi}\,\frac{r_1 - r_0}{r_1 r_0} = \underset{(a)}{E_{cp}}(r_1 - r_0).$$

Key: a. average

Considering (8.1), we obtain

$$r_1 = r_0 \frac{E_0}{\underset{(a)}{E_{cp}}}. \tag{8.4}$$

Key: a. average

For sand the magnitude of the average breakdown intensity is $E_{average} \approx 3.50$ kv/cm. Substituting the values of $E_0$ and $E_{average}$ in expression (8.4), we obtain for the current $I_M$ = 220 kiloamps for $\rho$ = 100 ohm-m, $r_1$ = 6.32 m and for $\rho$ = 1000 ohm-m, $r_1$ = 20 m.

As a result of the high conductivity of the metal sheathing of the cable, the symmetry of the electric field is disturbed, which causes a rise in the potentials at the remote points of the ground. In the first approximation these potentials are equal to zero. In reality, the sections of the cable located near the points struck by the lightning acquire some average potential between the zero potential of the remote point of the ground and the field potential at the same point in the absence of the cable. Setting the cable potential approximately equal to zero, we find that the potential difference $U_{02}$ between the spark zone in the cable is

$$U_{02} = \frac{I_M \rho}{2\pi r_0}, \tag{8.5}$$

or

$$U_{02} = E_{\text{cp}} (r_2 - r_0), \tag{8.6}$$

(a)

Key: a. average

where $r_2$ is the distance between the point struck by the lightning in the cable.

If $E_{\text{average}} = E_{\text{average breakdown}}$, then $r_2$ is the distance which can be broken down by the streamer in the presence of a cable in the ground; equating (8.5) and (8.6), we obtain the expression for determination of $r_2$:

$$r_2 = r_0 \left(1 + \frac{E_0}{E_{\text{cp.np}}}\right) = \left(1 + \frac{E_0}{E_{\text{cp}}}\right) \sqrt{\frac{I_M \rho}{2\pi E_0}}. \tag{8.7}$$

(a) (b)

Key: a. average breakdown
b. average

Substituting the values of $r_0$, $E_0$ and $E_{\text{average breakdown}}$ assumed above, we find that for $I_M$ = 220 kiloamps and $\rho$ = ohm-m, $r_2$ = 7.9 m, and for $\rho$ = 1000 ohm-m, $r_2$ = 25 m.

Expression (8.7) makes it possible to obtain a value of $r_2$ for the maximum propagation of the streamer and maximum distance jumped by the electric arc of the lightning current, and expression (8.4) makes it possible to obtain the value of $r_1$ for the minimum propagation of the streamer and minimum breakdown distance. Since $E_0$ is appreciably greater than $E_{\text{average breakdown}}$, $r_1$ and $r_2$ are close to each other. In the practical calculations it is possible to consider that when lightning strikes, breakdown can occur in the ground with respect to the direction of the cable from a distance of $r_2$.

The cases observed in practice indicate that the length of the arc between the point where the lightning strikes and the cable will be 3-6 meters, and sometimes tens of meters. In such cases, in the cable sheathing opposite the strike point a characteristic dent is observed although traces of the arcs are visible only in the first meters of the path with respect to the direction of the cable. Sometimes electric breakdown to the cable during a lightning strike in a tree occurs along the surface of the tree roots. Here the cable surface is indented not only opposite the tree, but also at some distance from it, which is explained by the location of the cable relative to the root system.

8.2. Derivations of the General Formula for Calculating the Voltage in a Cable

When the lightning currents get into a cable with metal protective covering made of lead (or aluminum) sheathing and steel bonds, voltages occur in the symmetric cable between the sheathing and cores, and in the coaxial cable, between the inside and outside conductors of the pair (that is, between the central core and the tube) and also between the cable sheathing and the outer conductor (tube) in a multipair coaxial cable if the conductors of the pairs are not connected to the sheathing.

When the lightning current gets into the cable through the "sheathing (tube)-ground" network, currents and voltages also occur in the "central core-ground" network. The process of the current and voltage propagation in these interconnected circuits is described approximately by the following telegraph equations:

$$\left.\begin{aligned}
-\frac{\partial u_1}{\partial x} &= R_1 i_1 + L_1 \frac{\partial i_1}{\partial t} + M_{12}\frac{\partial i_2}{\partial t},\\
-\frac{\partial u_2}{\partial x} &= R_2 i_2 + L_2 \frac{\partial i_2}{\partial t} + M_{21}\frac{\partial i_1}{\partial t},\\
-\frac{\partial i_1}{\partial x} &= g_1 u_1 + g_{12}(u_1-u_2) + C_{12}\frac{\partial (u_1-u_2)}{\partial t},\\
-\frac{\partial i_2}{\partial x} &= g_2 u_2 + g_{21}(u_2-u_1) = C_{21}\frac{\partial (u_2-u_1)}{\partial x},
\end{aligned}\right\} \qquad (8.8)$$

where $i_1$ and $i_2$ are the currents flowing through the "central core-ground" and the "tube-ground" circuit; $u_1$ and $u_2$ are the core and tube (sheathing) voltages with respect to an infinitely removed point of ground; $R_1$, $L_1$, $C_1$, $g_1$ are the resistance, inductance, capacitance and conductivity of the leakage of the cable core respectively with respect to ground per unit length of the cable; $R_2$, $L_2$, $C_2$ and $g_2$ are the analogous parameters of the tube (sheathing); $M_{12}$ is the mutual inductance of the "tube-ground" and the "core-ground"; $g_{12}$ is the conductivity between the core and the tubes (sheathing); $C_{12}$ is the capacitance between the core and the tubes.

Since the current in the sheathing $i_2$ is not sinusoidal, but is a pulse of arbitrary shape, the system of equations (8.8) cannot be solved by the traditional method. This system is solved most simply by the method of operation Laplace transformations. When making the transition to the operation transforms, we obtain

$$\left.\begin{aligned}-\bar{u}_1' &= Z_1\bar{i}_1 + Z_{12}\bar{i}_2^{*)},\\ -\bar{u}_2' &= Z_{12}\bar{i}_1 + Z_2\bar{i}_2,\\ -\bar{i}_1' &= (Y_1+Y_{12})\bar{u}_{12} - Y_{12}\bar{u}_2,\\ -\bar{i}_2' &= -Y_{12}\bar{u}_1 + (Y_2+Y_{12})\bar{u}_{12}.\end{aligned}\right\} \tag{8.9}$$

Considering the ground surrounding the cable as a coaxial cylinder of radius $r_3$, for the propagation parameters of the pulse through the "core-ground" and "tube-ground" circuits it is possible to obtain the following expressions:

$$Z_1 = pL_1 + R_1 = \frac{\mu p}{2\pi}\ln\frac{r_3}{r_1} + R_1,$$

$$Z_2 = pL_2 + R_2 = \frac{\mu p}{2\pi}\ln\frac{r_3}{r_2} + R_2,$$

$$Z_{12} = pM_{12}, \quad Y_1 = pC_1 + g_1 \approx 0,$$

$$Y_{12} = pC_{12} + g_{12} = p\frac{2\pi\varepsilon_{12}}{\ln\frac{r_2}{r_1}} + \operatorname{tg}\delta\, p\frac{2\pi\varepsilon_{12}}{\ln\frac{r_2}{r_1}} \approx p\frac{2\pi\varepsilon_{12}}{\ln\frac{r_2}{r_1}},$$

$$Y_2 = pC_2 + g_2 + p\frac{2\pi\varepsilon_3}{\ln\frac{r_3}{r_2}} + \frac{\pi}{\rho_3\ln\frac{r_3}{r_2}} = \frac{\pi}{\ln\frac{r_3}{r_2}}\left(2p\varepsilon_3 + \frac{1}{\rho_3}\right) \approx$$

$\approx \dfrac{\pi}{\ln\frac{r_3}{r_2}}$ — for cables, the metal sheathing of which has good contact with the ground;

$Y_2 \approx p\dfrac{2\pi\varepsilon_{\text{fl}}\,(a)}{\ln\frac{r_3}{r_2}\,(b)}$ for cables in a flexible tube,

Key: a. flexible tube b. g

where $R_1$ and $R_2$ are the core and sheathing (tube) resistances respectively; $r_1$, $r_2$, $r_3$ are the core radius, the tube radius and the equivalent radius of the ground; $\varepsilon_{12}$, $\varepsilon_g$, $\varepsilon_{\text{flexible tube}}$ are the dielectric constants of the insulation between the central core and the tube, the ground and the flexible tube respectively; tg $\delta$ is the loss tangent in the core insulation from the sheathing.

The ground radius $r_g$ corresponds to the depth of penetration of the current

*) In these equations the dash indicates the derivative with respect to x; the desired variables and coefficients depend on the parameter p.

into the ground. It can be estimated approximately by the Rudenberg formula: $r_g = 6\sqrt{\tau_w \rho_g}$, meters, where $\tau_w$ is the wavelength (the halfdecay time) of the lightning current, microseconds; $\rho_g$ is the specific resistance of the ground, ohm-m.

For the calculations of the capacitance $C_2$ in the case of the cable in a flexible tube $r_g$ can be approximately assumed equal to the outside radius of the flexible tube $r_{\text{flexible tube}}$. The expressions for the voltage between the core and the tube (sheathing) do not include the value of $r_g$. Nevertheless, the introduction of $r_g$ into the intermediate calculations will help to understand the physical aspect of the phenomena and to obtain smooth formulas.

The equations (8.9) are a system of first order uniform linear differential equations. The methods of their solution are generally known. This system of equations reduces to the fourth-order uniform linearity equation. For the current in the core $i_1$ we obtain the equation

$$\bar{i}_1^{IV} - \bar{i}_1^{II}[Y_2 Z_2 + Y_{12}(Z_1 + Z_2 - 2Z_{12})] + \bar{i}_1 Y_2 Y_{12}(Z_1 Z_2 - Z_{12}^2) = 0. \tag{8.10}$$

Let us denote $\alpha = Y_2 Z_2 + Y_{12}(Z_1 + Z_2 - 2Z_{12})$, $\beta = Y_2 Y_{12}(Z_1 Z_2 - Z^2_{12})$. Then the equation (8.10) assumes the form $i_1^{IV} - \alpha \bar{i}_1^{II} + \beta = 0$. For the propagation constants we have the characteristic equation $\gamma^4 - \alpha\gamma^2 + \beta = 0$. Its solution $\gamma = \pm\sqrt{\frac{\alpha}{2} \pm \sqrt{\frac{\alpha^2}{4} - \beta}}$. Hence,

$$\gamma_1 = \sqrt{\alpha} \approx \sqrt{Y_2 Z_2}, \quad \gamma_2 = \sqrt{\frac{\beta}{\alpha}} \approx \sqrt{Y_{12}\left(Z_1 - \frac{Z_{12}^2}{Z_2}\right)}.$$

Since in practice for any $p\alpha^2 \gg 4\beta$ and $Y_2 Z_2 \gg Y_{12}(Z_1 + Z_2 - 2Z_{12})$. Thus, the general solution of the differential equation (8.10) for the current $i_1$ has the form

$$\bar{i}_1(x, p) = A_1(p)e^{-\gamma_1 x} + A_2(p)e^{-\gamma_2 x} + A_3(p)e^{\gamma_1 x} + A_4(p)e^{\gamma_2 x}.$$

For $x \to \infty$ it is necessary that $\bar{i}_1 \to 0$; therefore $A_3 = A_4 = 0$; then we obtain

$$\bar{i}_1(x, p) = A_1(p)e^{-\gamma_1 x} + A_2(p)e^{-\gamma_2 x}.$$

From equations (8.9), by simple algebraic calculations we find that the voltage $\bar{u}_{12}$ between the central core and tube (sheathing) of the cable and the current in the tube $\bar{i}_2$ are equal to the following:

$$\bar{u}_{12}(x,\ p)=\bar{u}_2(x,\ p)-u_1(x,\ p)=\frac{\bar{i}_1'}{Y_{12}}=-\frac{A_1\gamma_1 e^{-\gamma_1 x}+A_2\gamma_2 e^{-\gamma_2 x}}{Y_{12}},$$

$$\bar{i}_2(x,\ p)=\frac{A_1\gamma_1^2 e^{-\gamma_1 x}+A_2\gamma_2 e^{-\gamma_2 x}+Y_{12}(Z_{12}-Z_1)(A_1 e^{-\gamma_1 x}+A_2 e^{-\gamma_2 x}}{Y_{12}(Z_2-Z_{12})}.$$

When determining the integration constants $A_1$ and $A_2$, it is necessary to consider the following boundary conditions:

At the point where the lightning strikes the current hits the metal sheathing and flows through it in both directions; breakdown of the insulation between the core and the sheathing does not occur;

Breakdown of the insulation of the cores from the sheathing takes place, the cores at the point where the lightning strikes turn out to be closed to the sheathing, the current is propagated both through the cores and through the sheathing.

The most important boundary condition is the first, inasmuch as it determines the maximum voltage in the cable.

8.3. Calculation of the Voltage in the Cable, the Metal Sheathing of Which has a Constant Contact with the Ground

Let the cable have only a jute cover over the metal sheathing. Inasmuch as the jute becomes wet very quickly in the ground and fails, the cable sheathing can be considered in good contact with the ground. For example, these cables include the MKSB type cable. Let us consider the first type of boundary conditions, that is, breakdown of the insulation of the cable core from the sheathing does not occur. In this case at the point where the lightning strikes (for x = 0)

$$\bar{i}_1(0,\ p)=0,\ \bar{i}_2(0,\ p)=\frac{1}{2}\bar{i}_0(p),$$

where $\bar{i}_0(p)$ is the mapping of the current $i_0(t)$ flowing in the lightning channel.

Using the boundary conditions to determine the integration constants, after simple calculations we find

$$u_{12}(x,\ p)=\frac{1}{2}\ \frac{\bar{i}_0(p)\,Z_{\text{c}\text{в}}^{(a)}}{\gamma_1^2-\gamma_2^2}(\gamma_1 e^{-\gamma_1 x}-\gamma_2 e^{-\gamma_2 x}), \qquad (8.11)$$

Key: a. coup

where $Z_{coup} = Z_2 - Z_{12}$ denotes the coupling resistance.



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The spectrum of the lightning current (the main discharge) is within the limits to 20 kilohertz, which corresponds to small values of the parameter p. For the cable, the sheathing of which has good contact with the ground, for small p it is possible to write

$$\left.\begin{aligned} &Z_{c3} \approx R_2, \\ &\text{(a)} \\ &\gamma_1 = \sqrt{Y_3 Z_3} \approx \sqrt{g_2 Z_2} = \sqrt{p g_2 L_2 \left(1 + \frac{R_2}{pL_2}\right)} \approx \\ &\qquad \approx \sqrt{p g_2 L_2} \approx \sqrt{p \frac{\mu}{2\rho_3}}, \\ &\gamma_2 \approx \sqrt{p C_{12}(R_1 + R_2)}. \end{aligned}\right\} \quad (8.12)$$

Key: a. coup

Considering (8.12) and proceeding to the originals, we obtain

$$u_{12}(x, t) = \frac{R_2}{2(\alpha_1^2 - \alpha_2^2)} [\alpha_1 g(\alpha_1 x, t) - \alpha_2 g(\alpha_2 x, t)], \quad (8.13)$$

where

$$\alpha_1 = \sqrt{\mu/2\rho_3}, \quad \alpha_2 = \sqrt{C_{12}(R_1 + R_2)},$$

$$g(\alpha x, t) = \int_0^t i(t-\tau) \frac{1}{\sqrt{\pi\tau}} e^{-\frac{\alpha^2 x^2}{4\tau}} d\tau.$$

At the point where the lightning strikes (for x = 0) formula (8.13) assumes the form

$$u_{12}(0, t) = \frac{R_2}{2[\sqrt{\mu/2\rho_3} + \sqrt{C_{12}(R_1 + R_2)}]} \int_0^t i(t-\tau) \frac{1}{\sqrt{\pi\tau}} d\tau.$$

Since $\sqrt{\mu/2\rho} \gg \sqrt{C_{12}(R_1+R_2)}$, then

$$u_{12}(0, t) = R_2 \sqrt{\rho/2\mu} \int_0^t i(t-\tau) \frac{1}{\sqrt{\pi\tau}} d\tau. \quad (8.14)$$

It is known that the lightning current has the form of pulses with a front of 1-5 microseconds and a halfdecay time of about 50 microseconds. Performing the numerical integration in (8.14) for a current wave of 3/50 at the point where the lightning strikes, for a voltage amplitude $U_M$ between the core and the sheathing of the cable not insulated from the ground, we obtain the expression

$$U_M = 2{,}1 I_M R_2 \sqrt{\rho}, \quad (8.15)$$

where $U_M$ is the voltage amplitude, volts, $I_M$ is the lightning current amplitude, kiloamps; $R_2$ is the resistance of the cable sheathing to direct current, ohms/km; ρ is the specific resistance of the ground, ohm-meter.

From (8.15) it is obvious that the voltage amplitude is directly proportional to the current amplitude of the lightning, the resistance of the cable sheathing and the square root of the specific resistance of the ground. The shape of the voltage is similar to the shape of the current, but it is elongated appreciably by comparison with it. Thus, for a current shape of 1.5/40 the voltage wave has the form 60/300. The amplitude and the shape of the voltage are significantly influenced by the current wavelength. Integrating (8.14), it is possible to show that the voltage amplitude depends on the wavelength of the current by the following law:

$$U_м = 0{,}296\, I_м R_з \sqrt{\rho \tau_{в}}, \qquad (8.16)$$

(a)

Key: a. w

where $\tau_w$ is the current wavelength of the lightning, microseconds; the remaining notation is the same as in reference (8.15).

It must be remembered that formula (8.16) is valid to values of $\tau_w \simeq$ 200-250 microseconds and that on derivation of it the duration of the current front was taken much less than the wavelength. The variation of the current front $\tau_f$ is felt little in the voltage. This is explained by the fact that for the lightning current the cable is an integrating circuit, and the output voltage of the "core-sheathing" circuit depends on the total charge flowing through the cable. Since $\tau_f \ll \tau_w$, the charge depends on the wavelength. On propagation through the cable, the voltage varies with respect to shape and it damps with respect to amplitude depending on the length of path and the specific resistance of the ground (Figure 8.1). The greater the specific resistance of the ground, the greater the voltage amplitude and the more slowly it damps. For example, for ρ = 200 ohm-m, the voltage decreases by half at a distance of 250 meters from the strike point, and for ρ = 2000 ohm-m, only after 600 meters.

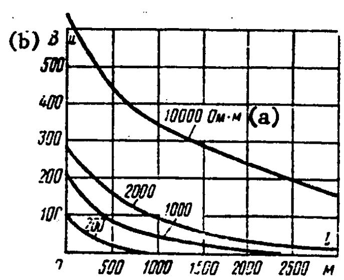


Figure 8.1. Voltage amplitude as a function of the path length and the specific ground resistance.

Key: a. ohm-m b. volts

8.4. Calculation of the Voltage if Lightning Strikes the Cable, the Metal Sheathing of Which is Insulated from the Ground by a Flexible Tube

Again, let us consider the first type of boundary conditions. In this case the conductivity of the flexible tube is negligible, and the flow of current to ground takes place through the capacitance $C_2$. Therefore for small values of p we have

$$\gamma_1 = \sqrt{pC_2R_2}, \ \gamma_2 = \sqrt{pC_{12}(R_1+R_2)}.$$

Subsituting the values of $\gamma_1$ and $\gamma_2$ in the general formula for the voltage (8.11) and proceeding to the originals for the first type of boundary condition, we find that the voltage between the core and the sheathing is described by the same expression as in (8.13):

$$u_{12}(x, t) = \frac{R_2}{2(\alpha_1^2-\alpha_2^2)}[\alpha_1 g(\alpha_1 x, t) - \alpha_2 g(\alpha_2 x, t)], \tag{8.17}$$

but the values of $\alpha$ have the form: $\alpha_1 = \sqrt{C_2R_2}, \ \alpha_2 = \sqrt{C_{12}(R_1+R_2)}.$

For x = 0 (at the point where the lightning strikes) expression (8.17) is simplified:

$$u_{12}(0, t) = \frac{R_2}{2[\sqrt{C_2R_2}+\sqrt{C_{12}(R_1+R_2)}]}\int_0^t i(t-\tau)\frac{1}{\sqrt{\pi\tau}}d\tau.$$

Performing the numerical integration for the current wave 3/50, we obtain the following expression for the voltage amplitude between the core and the sheathing (tube) at the point where the lightning strikes for the cable in the flexible hose:

$$U_M = \frac{1{,}66 I_M R_2}{\sqrt{C_2R_2}+\sqrt{C_{12}(R_1+R_2)}}, \tag{8.18}$$

where $U_M$ is the voltage amplitude, volts; $I_M$ is the current amplitude, kiloamps; $R_2$ is the sheathing resistance, ohm/km; $C_2$ is the capacitance metal sheathing (tube) with respect to ground, farads/km; $C_{12}$ is the capacitance of the central core with respect to the tube, farads/km. In the case of a symmetric cable $C_{12}$ is the capacitance of the bunched cores with respect to the sheathing.

From (8.18) it is obvious that in the presence of a flexible tube and for the same current the induced voltage is appreciably greater than the voltage in the cable, the sheathing of which is continuously grounded, for the path followed by the current through the sheathing is appreciably greater. Physically the occurrence of the voltage between the core and the sheathing can be the following. Let the current i flow through the "sheathing-ground" circuit of the cable with resistance R ohm/km and of length $\ell$. This current creates a voltage drop in the cable sheathing equal to $u = iR\ell$. The field intensity on the inside surface of the sheathing is a generator for the "sheathing-core" circuit (Figure 8.2). The larger the value of R and $\ell$, the greater the value of u for the same current. On propagation through the cable, the sheathing of which is in good contact with the ground, the current continually flows off to the ground. If the cable sheathing is insulated from the ground, the current occurs with low damping to a significant greater distance, as a result of which for the same current the magnitude of the voltage is appreciably greater. At the same time the flexible hose is not a serious obstacle for the current getting into the cable sheathing at the point where the lightning strikes itself. Indeed, the potential difference between the point where the lightning strikes and the cable can reach a value of several millions of volts, whereas the electric strength of the flexible tube is a total of 25-50 kv.

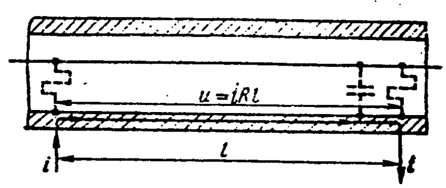


Figure 8.2. Equivalent diagram of the "sheathing-ground" and "core-ground" circuits.

Let us compare the numerical values of the voltages for a current of one kiloamp for a cable in a flexible tube and without it. Let $R_2 = 1$ ohm/km, $\rho = 400$ ohm-m, $R_1 = 16$ ohm/km, $C_2 = 1.2 \cdot 10^{-6}$ farads/km, $C_{12} = 0.5 \cdot 10^{-6}$ farads/km. Substituting these values in (8.15) and (8.18), we obtain 42 volts for the cable without the flexible tube and 412 volts for the cable in the flexible tube, respectively, that is, a difference of 10 times. By comparing formulas (8.15) and (8.18) we see that the flexible tube is equivalent to the ground with specific resistance of $\rho_e = \mu/2C_2R_2$.

As we see, the grounding of the metal sheathing of the cable has a significant effect on the magnitude of the voltage in the cable. Therefore in order to lower the voltage in the cable it is expedient to ground the metallic sheathing.

8.5. Calculating the Voltage When Lightning Strikes a Cable with Periodically Grounded Sheathing

Now let us have a cable in a flexible tube, the metal sheathing of which is connected at equal intervals of length $\ell$ to the ground with the resistance $R_g$. The concentrated grounds can be considered distributed on the length $\ell$ with leakage conductivity $g_2 = 1/R_g\ell$. If the value of $R_g$ is sufficiently small, then $g_2 >> pC_2$ (for small p), and the propagation constants assume the form: $\gamma_1 \approx \sqrt{p(L_2R_3)\ell}$, and $\gamma_2$, as before, $\gamma_2 \approx \sqrt{pC_{12}(R_1+R_2)}$.

We can consider with some approximation in the lightning frequency spectrum that $L_2 \approx$ const; therefore the voltage between the core and the sheathing is described as before by the expression (8.13)

$$u_{12}(x,\ t)=\frac{R_2}{\alpha_1^2-\alpha_2^2}\,[\alpha_1 g(\alpha_1 x,\ t)-\alpha_2 g(\alpha_2 x,\ t)],$$

but $\alpha_1$ and $\alpha_2$ have the form

$$\alpha_1=\sqrt{L_2/R_3 l},\ \alpha_2=\sqrt{C_{12}(R_1+R_2)}.$$

For x = 0 we obtain

$$u_{12}(0,\ t)=\frac{R^2}{2\,[\sqrt{L_2/R_3 l}+\sqrt{C_{12}(R_1+R_2)}}\int\limits_0^t i(t-\tau)\frac{1}{\sqrt{\pi\tau}}\,d\tau.$$

Performing the numerical integration for the current wave of 3/50 and considering that $\sqrt{L_2/R_3 l}\gg\sqrt{C_{12}(R_1+R_2)}$, for the voltage amplitude between the core and the sheathing of the cable we obtain the expression

$$U_м=37{,}3 I_м R_2\sqrt{R_3 l}, \tag{8.19}$$

where $U_M$ is the voltage amplitude, volts; $I_M$ is the current amplitude of the lightning, kiloamps; $R_2$ is the resistance of the sheathing, ohms/km; $R_3$ is the resistance of the concentrated ground, ohms; $\ell$ is the distance between grounds, km.

Let us determine the limit of applicability of formula (8.19).

1. Formula (3.94) was derived under the assumption of small values of p, that is, low values of the frequency, when $Z_{coup}$ can be considered equal to the dc resistance $Z_{coup} \approx R_2$. This is valid for an angular frequency $\omega$ of no more than 20-300 kilohertz. Otherwise it is possible to obtain a significant error (more than 20%).

2. If the damping of the current through the capacitance of the sheathing with respect to the ground is too large, then the grounding of the cable becomes meaningless. Therefore the formula (8.19) is applicable when $1/R_3\ell >> pC_2$ or $R_3\ell << 1/pC_2$.

Assuming that for a lightning frequency of $p_1 = 10$ kilohertz, $C_2 = 1 \cdot 10^{-6}$ farads/km, we find that $1/pC_2 = 10^2$ ohm-km. Formula (8.19) is applicable if $R_3\ell$ does not exceed 10% of $1/pC_2$, that is, if $R_3\ell \ll 10$ ohm-km.

## 8.6. Calculation of the Voltage Along the Communication Cable in the Case of Breakdown of the Insulation of the Cores from the Sheathing at the Point where the Lightning Strikes

A study was made above of the case where at the point where the lightning strikes the voltage $u_{12}(x, t)$ does not exceed the electric strength of the insulation of the cable core from the sheathing. If at the point where the lightning strikes there is breakdown of the insulation between the cores and the sheathing, the integration constants are found from the conditions:

$$\left.\begin{aligned} &\bar{u}_{12}(0, p) = 0, \\ &\bar{i}_1(0, p) + \bar{i}_2(0, p) = \bar{i}_0(p). \end{aligned}\right\} \tag{8.20}$$

From equation (8.9) and the boundary conditions (8.20), after algebraic calculations, we obtain

$$u_{12}(x, p) = \bar{i}_0(p) \frac{R_2 \gamma_1}{\gamma_1^2 - \gamma_2^2} \left(e^{-\gamma_1 x} - e^{-\gamma_2 x}\right). \tag{8.21}$$

Proceeding to the originals and using the theorems of convolution and obligation of transforms, we obtain

$$u_{12}(x, t) = \frac{R_2 \alpha_1}{\alpha_1^2 - \alpha_2^2} [g(\alpha_1 x, t) - g(\alpha_2 x, t)], \tag{8.22}$$

where

$$g(\alpha x, t) = \int_0^t i(t-\tau) \frac{1}{\sqrt{\pi \tau}} e^{-\frac{\alpha^2 x^2}{4\tau}} d\tau, \quad \alpha_2 = \sqrt{C_{12}(R_1 + R_2)},$$

and $\alpha_1$, depending on the type of cable, has the form: $a_1 = \sqrt{\mu/2\rho_3}$ -- for a cable, the metal sheathing of which is in continuous contact with the ground; $\alpha_1 = \sqrt{C_2 R_2}$ for a cable with flexible tube over a metal sheathing; $\alpha_1 = \sqrt{L_2/R_3}\,\ell$, for the cable in a flexible tube, the metal sheathing of which is periodically (with a spacing $\ell$) grounded to the resistance $R_3$.

Correspondingly, for the cables of these types expression (8.22) assumes the form:

$$u_{12}(x, t) \approx R_2 \sqrt{\rho_3/\mu} \int_0^t i(t-\tau) \frac{1}{\sqrt{\pi\tau}} \left[ e^{-\frac{\mu x^2}{8\rho_3 \tau}} - e^{-\frac{C_{12}(R_1+R_2) x^2}{4\tau}} \right] d\tau,$$

$$u_{12}(x,\ t)=\frac{R_2\sqrt{C_2R_2}}{C_2R_2-C_{12}(R_1+R_2)}\int_0^t i(t-\tau)\frac{1}{\sqrt{\pi\tau}}\times$$
$$\times\left[e^{-\frac{C_2Rx^2}{4\tau}}-e^{-\frac{C_{12}(R_1+R_2)x^2}{4\tau}}\right]d\tau,$$
$$u_{12}(x,\ t)\approx R_2\sqrt{R_2l/L_2}\int_0^t i(t-\tau)\frac{1}{\sqrt{\pi\tau}}\left[e^{-\frac{L_2x^2}{4R_2l\tau}}-e^{-\frac{C_{12}(R_1+R_2)x^2}{4\tau}}\right]d\tau.$$

From investigation of the formulas it is obvious that in the case of breakdown of the core insulation at the point where the lightning strikes the voltage along the cables of all three types varies as follows. For $x = 0$, $u_{12} = 0$, then the voltage increases on going away from the point $x = 0$, and at some distance from the point where the lightning strikes, it reaches a maximum. As is obvious from Figure 8.3, the voltage of the cable with continuous grounding of the sheathing, reaching a maximum, decreases insignificantly over a very long extent; over the entire length almost maximum voltage is applied to the insulation between the core and the sheathing. If at any point a place is encountered with weak insulation, then secondary breakdown takes place. In practice it was noted long ago that when lightning strikes a series od damaged places in the insulation occurs, sometimes a significant distance from each other. As a rule, the repeated damages occur in areas with weak insulation, for example, in the couplings, for their wearing is done at increased temperature, which sometimes causes partial weakening of the insulation of the cores.

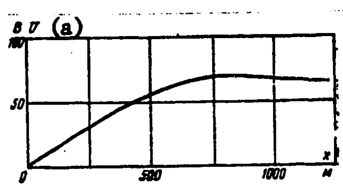


Figure 8.3. Variation of the voltage along the cable in the case of breakdown between the core and sheathing at the point where the lightning strikes ($x = 0$).

Key: a. U, volts

In order to determin the distance $x_M$ from the point where the lightning strikes at which the voltage between the core and the sheathing reaches a maximum value, it is necessary to take the mixed derivative $\partial^2 u_{12}/\partial x \partial t$ and equated to zero. However, the derivative of (8.22) is an expression that is too complex. Its solution is difficult. It is possible to simplify the problem somewhat if we find the extremum of the operation transform (8.21) with respect to x. Differentiating (8.21) with respect to x and equating the derivative to zero, we find that the magnitude of the voltage will be maximal for $x_M$ defined by the expression

$$x_M = \frac{1}{\gamma_1 - \gamma_2} \ln \frac{\gamma_1}{\gamma_2}. \tag{8.23}$$

As is obvious from (8.23), the maximum voltage between the cores and the metal sheathing is reached at a distance which depends on the parameters of the cables, the surrounding ground and the equivalent frequency of the lightning current.

For cables of the investigated types (8.23) assumes the form

$$x_{M1} = \sqrt{\frac{2\rho}{\mu p}} \ln \sqrt{\mu/2\rho C_{12}(R_1+R_2)} -$$ for a cable with continuously grounded sheathing,

$$x_{M2} = \frac{\frac{1}{2}\ln (C_2R_2/C_{12}(R_1+R_2))}{\sqrt{pC_2R_2} - \sqrt{pC_{12}(R_1+R_2)}} -$$ for a cable in a flexible tube,

$$x_{M3} = \sqrt{R_3 l/pL_2} \ln \sqrt{L_2/R_3 l C_{12}(R_1+R_2)} -$$ for a cable in a tube with periodically grounded sheathing.

Being given the values of p = 10 kilohertz, ρ = 400 ohm-m, $\mu = 1.256 \cdot 10^{-6}$ g/m, $R_1$ = 16 ohm/km, $R_2$ = 1.0 ohm/km, $C_{12} = 0.5 \cdot 10^{-6}$ farads/km, $C_2 = 1.2 \cdot 10^{-6}$ ohm/km, $L_2 = 2 \cdot 10^{-3}$ g/km, $R_3$ = 10 ohm, ℓ = 1 km, we find that for the indicate types of cables x is correspondingly equal to the following: $x_{m1}$ = = 775 m, $x_{M2}$ = 11.6 km, $x_{M3}$ = 153 m.

As we see, the maximum voltage in the cable with a flexible tube is the most removed from the point where the lightning strikes. A voltage close to the maximum occurs over a very long length so that the equipment connected to the cable is under high voltage in spite of its significant removal from the point where the lightning strikes.

8.7. Calculation of the Voltage in a Coaxial Cable, the Tube of which is Insulated from the Metal Sheathing.

In practice the case frequently occurs where the coaxial cable is insulated from the metal sheathing. Usually the connection of the tube to the sheathing is made only at certain points along the line. Let us consider the single-coaxial cable having a layer of insulation over the coaxial pair and only then the conducting sheathing. The voltage will have the following form for various boundary conditions:

1) in the case where breakdown of the insulation does not occur at the point where the lightning strikes either between the sheathing and the coaxial cable or between the tube and the inside conductor,

the voltage between the sheathing and the coaxial tubes is

$$u_{00}(x, t) = \frac{R_{\text{об}}}{2(\alpha_1^2 - \alpha_2^2)} [\alpha_2^2 g(\alpha_2 x, t) - \alpha_1 g(\alpha_1 x, t)];$$

the voltage between the tube and the central core is

$$\bar{u}_{00}(x, t) = \frac{R_{\text{об}}(R - R_{\text{об}})C}{2(\alpha_1^2 - \alpha_2^2)} \left[ \frac{\alpha_2^2 g(\alpha_2 x, t) - \alpha_1 g(\alpha_1 x, t)}{\bar{\alpha}_2^2 - \alpha_1^2} - \right.$$
$$\left. - \frac{\bar{\alpha}_2 g(\bar{\alpha}_2 x, t) - \alpha_2 g(\alpha_2 x, t)}{\bar{\alpha}_2^2 - \bar{\alpha}_1^2} \right];$$

the current in the coaxial tube is

$$i_{00}(x, t) = \frac{R_{\text{об}} C}{2(\alpha_1^2 - \alpha_2^2)} [h(\alpha_2 x, t) - h(\alpha_1 x, t)];$$

the current in the core is

$$\bar{i}_{00}(x, t) = \frac{R_{\text{об}}(R - R_{\text{об}}) C \bar{C}}{2(\alpha_1^2 - \alpha_2^2)} \left[ \frac{h(\bar{\alpha}_2 x, t) - h(\alpha_1 x, t)}{\bar{\alpha}_2^2 - \alpha_1^2} - \right.$$
$$\left. - \frac{h(\bar{\alpha}_2 x, t) - h(\alpha_2 x, t)}{\bar{\alpha}_2^2 - \alpha_1^2} \right];$$

2) in the case where the sheathing and the coaxial tube are either connected at the point where the lightning strikes or breakdown of the insulation takes place between them, the insulation between the tube and the core remains undamaged,

the voltage between the sheathing and the coaxial tube

$$u_{10}(x, t) = \frac{R_{\text{об}} \alpha_1}{2(\alpha_1^2 - \alpha_2^2)} [g(\alpha_2 x, t) - g(\alpha_1 x, t)];$$

the voltage between the coaxial tube and the core

$$u_{10}(x, t) = \frac{R_{\text{об}}(R - R_{\text{об}})C}{2(\alpha_1^2 - \alpha_2^2)} \left[ \frac{\bar{\alpha}_2 g(\alpha_2 x, t) - \alpha_1 g(\alpha_1 x, t)}{\bar{\alpha}_2^2 - \alpha_1^2} - \right.$$
$$\left. - \frac{\alpha_1 \bar{\alpha}_2 g(\bar{\alpha}_2 x, t) - \alpha_1 \alpha_2 g(\alpha_2 x, t)}{\alpha_2(\bar{\alpha}_2^2 - \alpha_2^2)} \right];$$

the current in the coaxial tube

$$i_{10}(x, t) = \frac{R_{\text{об}}^{(1)}}{2(\alpha_1^2 - \alpha_2^2)} \sqrt{C/R} [\alpha_1 h(\alpha_2 x, t) - \alpha_2 h(\alpha_1 x, t)];$$

Key: 1. sh

the current in the core

$$\bar{i}_{10}(x,t)=\frac{R_{об}(R-R_{об})C\bar{C}}{2(\alpha_1^2-\alpha_2^2)}\left[\frac{h(\bar{\alpha}_2 x,\ t)-h(\alpha_1 x,\ t)}{\bar{\alpha}_2^2-\bar{\alpha}_1^2}-\right.$$
$$\left.-\frac{h(\bar{\alpha}_2 x,\ t)-h(\alpha_2 x,\ t)}{(\alpha_2/\alpha_1)(\bar{\alpha}_2^2-\alpha_2^2)}\right];$$

3) in the case where complete breakdown of the insulation takes place at the point where the lightning strikes between the sheathing and the tube and between the tube and the central core, that is, the sheathing, the coaxial tube and the core are closed among each other at one point (this usually occurs under the indentation in the sheathing at the point where the lightning strikes),

the voltage between the coaxial tube and the sheathing

$$u_{11}(x,t)=\frac{R_{об}(R-R_{об})C\alpha_1}{2(\alpha_1^2-\alpha_2^2)}\times$$
$$\times\left[\frac{g(\bar{\alpha}_1 x,\ t)-g(\alpha_1 x,\ t)}{\alpha_2^2-\alpha_1^2}-\frac{g(\bar{\alpha}_2 x,\ t)-g(\alpha_2 x,\ t)}{\bar{\alpha}_2^2-\alpha_2^2}\right];$$

the current in the core

$$\bar{i}_{11}(x,t)=\frac{R_{об}(R-R_{об})C\bar{C}}{2(\alpha_1^2-\alpha_2^2)}\alpha_1\times$$
$$\times\left[\frac{\alpha_1 h(\bar{\alpha}_2 x,\ t)-\alpha_2 h(\alpha_1 x,\ t)}{\alpha_1\bar{\alpha}_2(\bar{\alpha}_2^2-\alpha_1^2)}-\frac{\alpha_2 h(\bar{\alpha}_2 x,\ t)-\bar{\alpha}_2 h(\alpha_1 x,\ t)}{\alpha_2\bar{\alpha}_2(\bar{\alpha}_2^2-\alpha_2^2)}\right].$$

In the formulas of this item the following notation is used:

$$g(\alpha x,t)=\int_0^t i(t-\tau)\frac{1}{\sqrt{\pi\tau}}e^{-\frac{(\alpha x)^2}{4\tau}}d\tau,$$

$$h(\alpha x,t)=\int_0^t i(t-\tau)\frac{\alpha x}{2\tau\sqrt{\pi\tau}}e^{-\frac{(\alpha x)^2}{4\tau}}d\tau,$$

$$\alpha_1=\sqrt{\mu/2\rho};\quad \alpha_2=\sqrt{RC};\quad \bar{\alpha}_2=\sqrt{\bar{R}\bar{C}};$$

$R_{sh}$ is the sheathing resistance; R and C — are the resistance and the capacitance per unit length of the "coaxial tube-cable sheathing" circuit; $\bar{R}$ and $\bar{C}$ are the resistance and the capacitance per unit length of the "core-coaxial tube" circuit.

The Chilean telephone company used the indicated formulas to perform calculations of the currents and voltages in a coaxial cable for the following values of the parameters: $R_{sh}$ = 1 ohm/km; R = 7.2 ohm/km; $\bar{R}$ = 21.4 ohm/km;

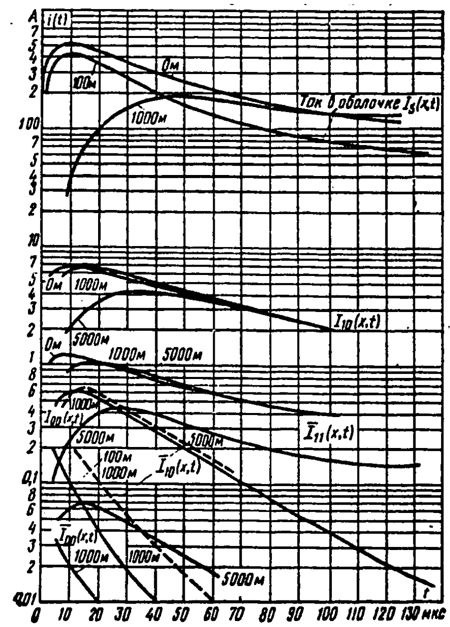


Figure 8.4. Currents in a complex coaxial cable during a lightning strike: $I_s$ -- current in the shea

$I_s$ -- current in the sheathing
I -- current in the shielding of the coaxial pair
$I_{00}$ -- current in the core of the coaxial pair
} the cable insulation is not damaged;

$I_{10}$ -- current in the shielding
I -- current in the core
} for breakdown of the the "shielding-sheathing" insulation at the point where the lightning strikes;

$I_{11}$ -- current in the core in the case of complete damage of the insulation at the point where the lightning strikes (both the "sheathing-shielding" insulation and the "shielding-core" insulation)

Key: a. current in the sheathing b. microseconds

$C = 0.1 \cdot 10^{-6}$ farads/km; $\bar{C} = 0.5 \cdot 10^{-6}$ farads/km; $i(t) = 1150\ (e^{-0.013 \cdot 10^6 t} - e^{-0.5 \cdot 10^6 t})$.

The results of the calculations are presented in Figures 8.4 and 8.5 a and b.

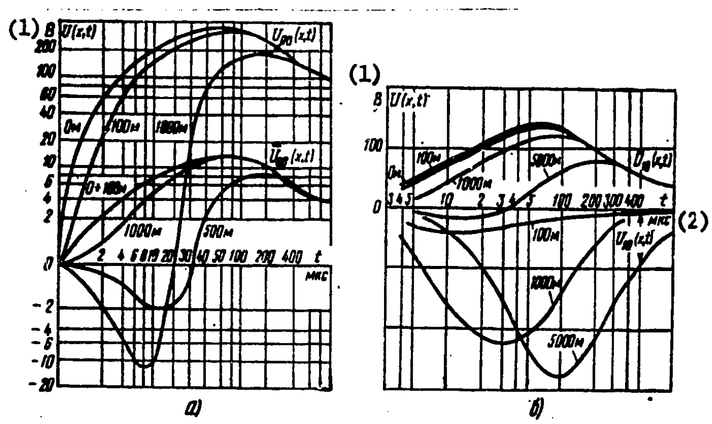

Figure 8.5. Voltage between the core and the shielding of the coaxial pair and between the shielding and the sheathing of the cable from the lightning strike ($\rho = 10^4$ ohm-meter): a) in the absence of breakdowns of the insulation; b) in the case of breakdown of the insulation at the point where the lightning strikes and the point of connection of the sheathing and the shielding to each other.

Key: 1. volts 2. microseconds

## 8.8. Probability of Lightning Damage to a Cable Laid in Open Terrain

When the lightning leader approaches the ground, charges begin to move in the latter toward the point located under the leader. At the time the leader reaches the ground and the time of beginning of the return strike, the potential of this point can be so high that an electric arc occurs between it and the cable.

The conditions of the formation of the arc were investigated in section 3.2.1 where the derivation of the formula is presented for the distance jumped by the electric arc to the cable. If the distance is large and the average field intensity in this space is less than the critical breakdown value, the arc does not burn. However, this does does not mean that the cable will not be subjected to damage under such conditions.

As has already been noted above, when the lightning current flows through the sheathing between the cores and the sheathing, a potential difference occurs which is greater the greater the amplitude of the flowing current. If this potential difference at any time turns out to be greater than the electric strength of the insulation between the cores and the sheathing, breakdown occurs and, consequently, damage to the cable. For each type of cable there is a pulse current amplitude $i_0$ such that for currents exceeding $i_0$ breakdown of the belt insulation and the insulation of the cable cores takes place. The value of $i_0$ depends on the resistance of the metal protective coverings of the cable, the pulse strength of the insulation, the specific resistance of the surrounding ground.

When lightning strikes at a distance y from the cable, $y > y_{max}$ (where $y_{max}$ is the maximum distance jumped by the arc), the current flows in all directions from the point of the strike, the electric arc to the cable does not occur, and part of the current goes to the cable. In this case the damage to the cable insulation occurs according to Sunde [21]; if the amplitude of the lightning current exceeds the value

$$i = i_0 \frac{\ln 1/\gamma r}{\ln (1/\gamma y + 1)}, \tag{8.24}$$

where $\gamma \approx 0.08\sqrt{\rho}$ is the propagation constant of the lightning current through the sheathing, 1/m; r is the cable radius, m.

As Sunde has demonstrated, within the limits of some distance $y_1 < y_{max}$ from the cable the damage can be caused only by the electric arc to the cable; nevertheless, for lightning strikes farther than $y_1$ damage is also caused without an arc by the leakage current as soon as the latter exceeds i, although the occurrence of the arc is not excluded in this case.

The probable number of occurrences of damage from lightning striking an underground cable laid in level terrain will be defined, according to Sunde, as follows. Let $q = 0.1$ 1/km$^2$-day be the number of lightning strikes per km$^2$ on one day of lightning; then the number of strikes $d\nu$ to the ground within the limits from y to y + dy on both sides of the cable will be: $d\nu = 2qSdy$, where S is the length of the cable. The striking of lightning at a distance y will cause damage when the current i exceeds some value which depends on the distance $i = f(y)$. The number of strikes, the amplitude of which is greater than i is described by the expression $P(i) = e^{-ki}$, where $k = 0.038$ 1/kiloamps. The number of strikes dn leading to damage will then be

$$dn = P(i)\, d\nu = 2qS e^{-ki} dy.$$

The total number of cases of damage as a result of long strikes to ground in a year will be determined from the expressions

$$n = 2qSN \int_0^Y e^{-ki} dy,$$

where N is the number of days of lightning per year for the given terrain; Y is the maximum distance from the cable which is meaningful to consider. The integral is conveniently divided into two:

$$n = 2qNS \int_0^Y e^{-ki} dy = 2qNS \left( \int_0^{y_1} e^{-ki} dy + \int_{y_1}^{Y} e^{-ki} dy \right). \tag{8.25}$$

where $(0, y_1)$ is the interval of distances from the cable in which an arc occurs; $(y_1, Y)$ is the interval of distances from the cable in which damage takes place before the occurrence of an arc (although the latter can occur). Integrating (8.25) by parts, we find

$$n = 2qNS\left[Y e^{-kI} + k\int_{i_0}^{i_1} y e^{-ki}\,di + k\int_{i_1}^{I} y e^{-ki}\,di\right]. \qquad (8.26)$$

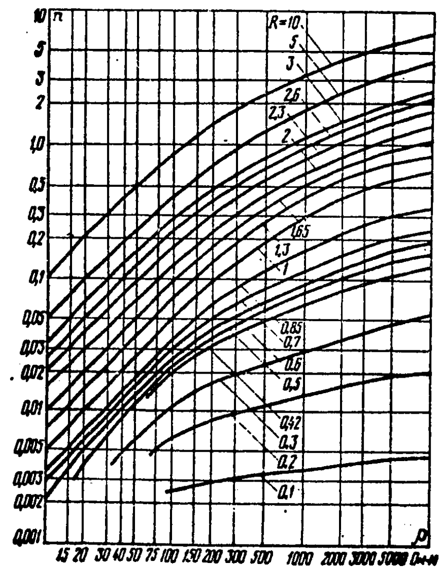


Figure 8.6. Graphs of the probability of damage of underground cables by lightning.

Key: a. ohm-m

Here, for the first integral which is in the right-hand side of expression (8.26), the dependence of y on i is expressed by the formula (8.7), and for the second integral, by the formula (8.24); i is the value of the current corresponding to the distance $y_1$ which can be found, equating (8.7) and (8.24); I is the maximum possible current amplitude of the lightning.

The integrals of equation (8.26) are taken by the numerical methods, and the corresponding tables and graphs exist for them. On the basis of the solution of the integrals (8.26), the graphs are compiled for the probable number of their occurrences of damage to the cables presented in the "Handbook on the protection of underground coupling cables from lightning strikes." At the present time in a number of advanced countries of the world the



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thunderstorm activity is estimated not by the number of days with a thunderstorm, but by the mean total duration of the thunderstorms in hours. This only requires variation of the coefficients in the formula (8.26) and, without touching on the essence of the conclusion, permits more exact determination of the probability of damage. The graph of the probability of damaged underground cable by lightning is presented in Figure 8.6 for the following values of the parameters: total thunderstorm activity T = 36 hr/year, electric strength of the insulation 3000 volts. Formula (8.26) and the graph in Figure 8.6 pertain to cables, the metal sheathing of which is in good contact with the ground (for example, the MKSB type cable).

## 8.9. Probability of Lightning Damage to Underground Cable Laid Beside a Forest

The possibility of lightning damage to an underground cable laid through a forest is the same as the cable laid in an open field inasmuch as the uniformly distributed trees, independently of their distance from the cable will orient the lightning strikes to themselves with equal probability. On passage of the cable near the edge of the forest the picture changes significantly. Indeed, trees at the edge of the forest will orient strikes toward themselves from a strip adjacent to the forest (Figure 8.7a).

According to the research data of A. A. Akopyan, et al. [3, 10], a developed lightning channel is "attracted" to objects which project above surrounding structures, and usually lightning will strike the high point. Since the development of a lightning discharge depends on a number of random circumstances, the strike can also bypass the high point, but the probability of this event is low. The higher the object, the greater the space it will "protect" from lightning strikes, and the more reliably it will do this. By the "lightning rod protection zone" usually we mean the space near the lightning rod within the boundaries of which there is low probability of lightning damage to the protected targets. The height of the lightning rod has decisive significance on the protective effect of it; its grounding is no significant role. Therefore it is possible to use ordinary trees as lightning rods. The protective zone of the lightning rod is not a completely defined value; it is characterized by the probability that lightning will penetrate the zone. It is natural that the higher the admissible probability of penetration, the greater the area can be assumed to be protected.

The experiments of A. A. Akopyan established the dimensions of the protected zone beginning with the conditions that out of 1000 strikes not one strike enters the zone. The greatest radius of the protection zone at the surface of the ground for a lightning rod $r = 1.5\ h$, where h is the height of the lightning rod. In the case of an overhead cable, this zone is decreased as a result of a more uniform electric field, and it is equal to 1.2 h. For the edge of the forest it is possible to take the width of the protection zone to be 1.5 h, for the tops of the trees are not a continuous line and create sufficient field distortion. All of the strikes which trees collect from the strip adjacent to a forest will hit an underground cable laid in direct proximity to the roots of these trees at the edge of the forest. On the

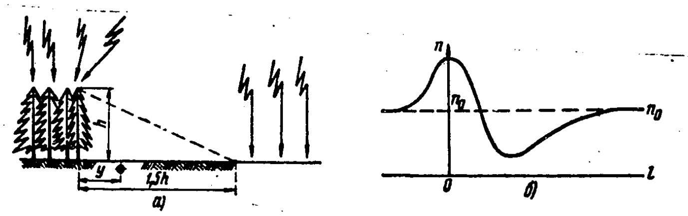


Figure 8.7. Probability of lightning damage to cables laid beside the forest: a) orientation of discharges; b) variation in probability of damage to the cable by a lightning current depending on the discharges between the cable and the forest.

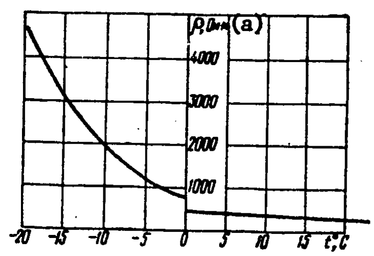


Figure 8.8. Specific resistance of clay as a function of temperature.

Key: a. ρ, ohm-m

other hand, a cable laid in the protection zone of the trees at the edge of the forest and at the same time located sufficiently far from it so as to reduce the number of lightning strikes forming an arc from the roots of the trees to the cable to a minimum will be protected from direct lightning strikes. Consequently, the number of cases of damage to an underground communications cable from lightning strikes when the cable is laid along a forest depends on the distance between the cable and the forest and can be both greater than and less than the number of cases of damage to the cable laid through open terrain (under other equal conditions).

Analogous statements can be made with respect to any prominent objects located along the path of the cable, whether overhead communications lines, the supports of overhead electric power transmission lines, individual trees, and so on. The picture will be the same qualitatively.

Now let the cable be located at a distance $y = \ell$ from the edge of the forest and let $\ell < 1.5$ h (see Figure 8.7a). The integration region will be broken down into three parts: one part corresponds to the protection zone of the forest, and the other two, to the regions lying outside the protection zone on both sides of it:

$$n=qNS\left(\int_{-l}^{1,5h-l} e^{-kl}\,dy+\int_{-l}^{-Y} e^{-kl}\,dy+\int_{1,5h-l}^{Y} e^{-kl}\,dy\right). \tag{8.27}$$

In the first integral in the right-hand side of expression (8.27), the current i does not depend on y and is completely determined by the distance $\ell$ between the forest and the cable. Indeed, if $\ell < y_1$, then for a lightning strike at the edge of the forest the cable will be damaged if the arc of lightning current is capable of jumping the distance $\ell$ between the cable and the forest, that is, when the lightning current amplitude $i \geq \ell^2\rho/1.21\, e_0$. If $\ell > y_1$, then when lightning strikes the edge of the forest the cable will be damaged when the leakage current to the cable exceeds $i_0$. Setting $y = \ell$ in (8.24), we find the boundary amplitude of the lightning current such that the cable will be damaged in the case of lightning strikes to the edge of the forest with any large amplitude. In addition, considering that

$$\int_{-l}^{-Y} e^{-kl}\,dy=\int_{l}^{Y} e^{-kl}\,dy,$$

let us rewrite (8.27) in the following form:

$$n=qNS\left(e^{-ki_l}\,1,5h+\int_{l}^{Y} e^{-ki(y)}\,dy+\int_{1,5h-l}^{Y} e^{-ki(y)}\,dy\right), \tag{8.28}$$

where $i_\ell$ is found from (8.7) or (8.24) (depending on the relation between $\ell$ and $y_1$) for $y = \ell$.

The integrals in the right-hand side of (8.28) are analyzed analogously to the integral (8.26).

In order to determine the probable number of cases of damage to the cable located in the protection zone of the trees at the edge of the forest, expression (8.28) is used. If the cable is located outside the protection zone of the forest and at a distance $\ell$ from it (that is, $\ell > 1.5$ h), the probable number of cases of damage to the cable can be found from the expression

$$n=qNS\left[\int_{0}^{l-1,5h} e^{-kl}\,dy+1,5h\,e^{-ki_l}+\int_{l}^{Y} e^{-kl}\,dy+\int_{0}^{Y} e^{-kl}\,dy\right].$$

These integrals are taken just as in the preceding cases. In Figure 8.7b the curve is presented which indicates the probable number of cases of damage to the cable as a function of the distance between the cable and the forest (for $\rho$ = 800 ohm-m). The dotted line $n_0$ indicates the probable number of cases of damage to the same cable but located in an open field under other equal conditions. From the graph it is obvious that by proper selection of the location of the cable it is possible essentially to reduce the expected number of cases of damage from lightning strikes. When the cable is laid at some distance $\ell_{opt}$ from the forest it is possible to achieve the minimum possible number of cases of damage to the cable. The distance $\ell_{opt}$ can be found from the condition $dn/d\ell = 0$.

Differentiating the right-hand side of the equality (8.28) with respect to $\ell$, after simple transformations we have

$$1+1,5kh\frac{di_l}{dl}=e^{k\left(i_l-i_{1,5h-l}\right)}. \qquad (8.29)$$

The solution to the equation is complicated by the fact that the dependence of the current $i_\ell$ on $\ell$ is different for different $\ell$: namely, for $\ell < y_1$ it is expressed by the formula (8.7), and for $\ell > y_1$, by the formula (8.24).

The greater part of the cases of damage to the cable occur as the result of the occurrence of a lightning current arc to the cable. The probable number of cases of damage from leakage currents is low. Part of them, in turn, are accompanied by an arc to the cable. Therefore we shall consider $i_\ell = \ell^2 e_0/$ $/1.21\rho$. Equation (8.29) assumes the form

$$1+1,5h\frac{2kle_0}{1,21\rho}=e^{-\frac{ke_0}{1,21\rho}\left(2,25h^2-3hl\right)}.$$

Solving the equation graphically, it is possible with accuracy which is entirely sufficient for practical purposes, to determine the most advantageous distance from the forest at which the cable must be laid: for h = 10 m, $\ell_{opt} \simeq$ 15 meters; for h = 15 m, $\ell_{opt} \simeq$ 17.5 meters; for h = 20 m, $\ell_{opt} \simeq$ 20 meters. It must be noted that when deriving the formulas, the presence of roots on the trees was not taken into account. It is known that the pulse breakdown voltage with respect to the surface of the tree will be 200 to 300 kv/m, which, by comparison with the breakdown strength of the ground, is smaller. Accordingly, the actual distance between the tree and the cable will be less than the distance between them measured on the surface of the ground. From Figure 8.6 it is obvious that removal of the cable from the forest leads to fewer cases of damage than bringing it closer. Therefore considering what has been stated above, the optimal distances must in practice be increased by the length of the roots.

## 8.10. Probability of Lightning Damage to Cable Laid along a Cut Through a Forest

When laying a cable through a cut through the forest, we have a superposition of zones of orientation of the lightning strikes to the opposite edges of the forest. It must be considered here that the strikes directed to the left part of the cut to its center will be oriented to the left edge of the forest, and to the right part, to the right edge of the forest. The maximum width of the strip from which edge of the forest the strikes are collected will in this case be not 3 h as would first appear, but 7 h. It is assumed that the terrain is level, and the height of the trees is no less than 6 to 8 meters.

In practice the cable is laid either through a specially made cut or through a cut made when building other structures (a road, electric power transmission lines, and so on). The width of the cut will be different in all cases. If the width of the cut is less than $y_{min}$, where $y_{min}$ is the distance within the limits of which the lightning current with an amplitude $i_0$ forms an arc to the cable, then the forest can be considered solid, and the calculation of the expected number of cases of damage must be made by the usual method without considering the cuts. For $\rho$ = 1000 ohm-m, for example, $y_{min}$ = 6 m, and in this case it is indifferent whether the cable is laid in the middle of the cut or along its edge. For greater width of the cut the cable must be laid down the middle. The number of cases of damage when laying the cable in the middle of the cut is given by the expression

$$n = 2qNS\left[l\,e^{-kl_l} + \int_l^Y e^{-kl}\,dy\right], \tag{8.30}$$

where $2\ell$ is the width of the cut.

If the forest is moved back to both sides on the cable, then the cable will be damaged less and less, and at the limit the minimum number of cases of damage will occur for a cut width close to 7 h. With a further increase in width of the cut, the tree protection zone will be divided into two parts will decrease rapidly. Therefore the probable number of cases will begin to increase rapidly, the expected number of cases of damage to the cable when it is laid in the middle of the cut more than 7 h wide is given by the expression

$$n = 2qNS\left[\int_0^{l-1,5h} e^{-kl}\,dy + 1,5h\,e^{-kl_l} + \int_l^Y e^{-kl}\,dy\right],$$

where the integral is uncovered, just as before, depending on the relation between $\ell$ and $y_1$ ($\ell$ is half the width of the cut).

With an increase in the cut width above 7 h to some limit laying the cable in the middle will lead to a minimum number of cases of damage. However, beginning with some width, the effect of the forest will weaken to such a degree that the strikes in the space between the wooded areas will damage the cable more frequently than if the cable were located closer to the forest rather than in the middle of the cut. It is obvious that at this critical width of the cut, the second derivative of n with respect to $\ell$ changes sign; therefore differentiating (8.30) twice and setting the result equal to zero, we obtain the equation

$$k\left(\frac{di_l}{dl}\right)^2-\frac{d^2i_l}{dl^2}=0. \tag{8.31}$$

From (8.31), it is possible to find the maximum width of the cut 2ℓ for which laying the cable in the middle does not lead to a decrease in the number of cases of damage. However, it is necessary to consider that the maximum distance from the cable, in case of a strike within the limits of which damage is possible would be approximately 70 to 80 meters. Outside this distance, the strikes with any current amplitude do not cause damage. In addition, with an increase in width of the cut above 7 h the protection zone decreases sharply. Therefore for cuts more than 8 h wide the effect of the second part of the forest on the orientation of the lightning strikes can be neglected, and the location of the cable in the middle of the cut becomes meaningless.

## 8.11. Effect of Single Trees Located Near a Cable or High Structures on Damage to the Cable by Lightning Strikes

Damage to a cable by a lightning strike through individual trees located near a cable has been mentioned more than once in the literature. However, a quantitative evaluation has not been made. If a tree of height h is located at a distance ℓ from a cable, the number of cases of damage for the section of cable 3 h long near the tree is given by the expression

$$n=qN\pi(1,5h)^2e^{-kl_1}+2qN\int\limits_0^{1,5h}\left[dS\int e^{-kl}\,dy\right].$$

It is obvious that the limits of the internal integral are variables which depend on S and ℓ (S is the distance reckoned along the cable).

## 8.12. Effect of Overhead Lines Located Near the Cable on the Degree to Which it is Subject to Lightning Strikes

Now let us consider the effect of an overhead communication line located directly next to a cable on the probability of damage to it by lightning. The strikes in the absence of a line in strips 1.2 h wide on both sides of the line now will hit the line, and after jumping the supports the currents will form an arc to the cable, that is, all of the strikes hitting in the strip 2.4 h wide turn out to be direct strikes for the cable. The procedure for calculating the number of cases of damage to the cable is the same as in the preceding case; therefore we shall not discuss it. In order to avoid damage to this case the overhead line is equipped with spark gaps with remote grounds. The grounds are a significant distance to the side. Here the damage is reduced to a minimum.

The grounding will be located at a distance of 60 to 70 meters from the cable; then the currents falling from the ground in practice do not cause damage for any amplitudes. The adopted distance of 25 to 30 meters excludes the currents forming an arc to the cable.

If the line is not equipped with remote grounds, then the calculation formed by the method analogous to the one discussed above indicates that the minimum number of cases of damage occur with the cable locator at a distance from 0.61 h to 0.65 h from the communication line or electric power transmission line, where h is the average height of suspension of the wires. However, in this case the decrease in the probable number of cases of damage to the cable is insignificant.

### 8.13. Calculation of the Probable Number of Cases of Damage to Cables by Lightning in Permafrost Zones and with Two-Layer Structure of the Soil

The operation of long distance underground communication cables in permafrost zones has demonstrated that they are subject to damage from lightning strikes sometimes even to a higher degree than in ordinary soil in spite of the fact that in regions with frozen ground, cables are used with high mechanical strength and, as a rule, with high longitudinal conductivity of the external protective coverings.

In the structure of the soil in permafrost zones it is possible clearly to isolate two layers that differ sharply with respect to their properties: the upper layer, so-called active layer (the thickness of which during the melting period is 2-3 meters) saturated with moisture and having a low specific resistance of the soil, and the almost nonconducting permafrost layer. The periods of greatest thunderstorm activity and maximum thawing of the upper layer of the ground are shifted relative to each other. Thus, the greatest volume of thawing of the ground can occur in September, and the thunderstorm activity lasts from May to September (with a maximum in June-July). In Table 8.1 the characteristic mean monthly soil temperatures are presented.

Table 8.1. Mean monthly soil temperatures

| Depth, meters | Mean monthly soil temperature degrees, for month | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | I | II | III | IV | V | VI | VII | VIII | IX | X | XI | XII |
| 0,4 | —12,9 | —13,0 | —9,1 | —3,3 | 0,3 | 6,2 | 10,7 | 11,8 | 7,4 | 2,1 | —2,2 | —8,6 |
| 0,8 | —8,3 | —9,6 | —7,6 | —3,5 | —0,9 | —1,6 | 6,4 | 8,5 | 6,4 | 1,6 | 0,0 | —3,4 |
| 1,6 | —0,9 | —3,5 | —4,3 | —2,9 | —1,4 | —0,9 | —0,4 | 1,3 | 2,4 | 1,0 | 0,1 | 0,0 |
| 2,0 | —0,4 | —1,9 | —3,2 | —2,9 | —1,5 | —1,0 | —0,7 | 0,0 | 1,0 | 0,6 | 0,1 | 0,0 |
| 2,5 | —0,2 | —0,2 | —1,0 | —1,7 | —1,3 | —1,0 | —0,8 | —0,7 | —0,4 | —0,2 | —0,2 | —0,2 |

The total duration of the thunderstorms during the year is on the average about 50 hours (for 20 days with a thunderstorm). For comparison let us point out that, for example, in Moscow the thunderstorm duration is a total of 31 hours (for 26 days with thunderstorms).

The greatest thunderstorm activity is observed where the soil still is only beginning to thaw, the temperature at the depth of laying the cable is negative: according to Table 8.1, at a depth of 0.8 meters the soil temperature is -1.6° C in June; it is +6.4° C in July; at a depth of 1.6 meters it is -0.9 and -0.4° C, respectively. The cable laid at a depth of about 1 meter is in entirely frozen ground in May; in June-July it is somewhere near the boundary between the melting and frozen ground. The specific resistance of the frozen ground is extremely high. Figure 8.8 shows the specific resistance of clay as a function of temperature for a moisture of 15%. On transition of the temperature through zero the specific resistance varies discontinuously from 140 to 320 ohm-m, and then it increases with a decrease in temperature.

The specific resistance of the soil along with the thunderstorm activity is the basic external factor determining the degree to which the cable is subject to lightning discharge effects. As follows from a comparison of Table 8.1 with Figure 8.8, the probability of damage to the cable by lightning in May is more than twice the probability of damage to the cable in August even without consideration of the nonuniformity of distribution of the thunderstorms during the thunderstorm season.

Thus, the calculation of the lightning protection with respect to the specific resistance of the melting ground leads to serious errors in estimating the probability of damage to the cable by lightning.

By the end of the thunderstorm season when the active layer has melted, only the upper layer has good conductivity, and the lower layers in practice are nonelectrically conducting. The lightning currents leak off in the upper layer of the soil, which can cause a high damage rate to the cable located in this layer by comparison with ordinary conditions. Consequently, permafrost imposes special requirements on the lightning protection of the cables. When estimating the probable number of cases of damage to the cable in permafrost zones it is necessary to consider the following:

1) high specific resistance of the upper layers of the soil at the beginning of the thunderstorm season;

2) unfavorable relation of the specific resistances of the layers at the end of the thunderstorm seasons;

3) noncoincidence in time of the greatest thunderstorm activity and the greatest melting of the soil;

4) in some regions an increased total duration of thunderstorms.

Let us determine approximately how the potential difference between the cores and sheathing of an underground cable varies for the nearby poorly conducting layer of the soil. The potential $U_1$ of the ground points at a distance r from the lightning strike point with uniform soil with a specific resistance is determined by the formula

$$U_1 = \frac{I_m \rho_1}{2\pi r}, \tag{8.32}$$

where $I_m$ is the lightning current amplitude.

If in the case of a two-layer structure, the upper layer of the soil of thickness h has a specific resistance $\rho_1$, and the lower layer, a specific resistance $\rho_2$, then the potential of the ground points will be

$$U_2 = \frac{I_m}{2\pi r}\left[\rho_2 + (\rho_1 - \rho_2)\, e^{-kr/2h}\right], \tag{8.33}$$

where k is a function of the ratio $\rho_1/\rho_2$ (Table 8.2).

Considering in the first approximation the potential of the cable sheathing equal to the potential of the surrounding points of the ground and the potential of the cores equal to zero, by (8.32) and (8.33) it is possible roughly to estimate the increase in voltage in the cable in the case of a two-layer structure of the soil. Assuming that the lightning strikes occurs to the ground directly over the cable, that is, $r = h_1$, where $h_1$ is the depth of laying the cable, we find that

$$U_2/U_1 = \rho_2/\rho_1 + \left(1 - \frac{\rho_2}{\rho_1}\right) e^{-kh_1/2h}. \tag{8.34}$$

Table 8.2. Values of the coefficient k as a function of ratio of the specific resistances of the ground layers

| $\rho_1/\rho_2$ | 100 | 10 | 1 | 0,1 | 0,02 |
|---|---|---|---|---|---|
| $k$ | 2 | 1,84 | 1,16 | 0,4 | 0,12 |

On the average it is possible to consider the depth of laying of the cable $h_1$ = 1 meter, and the thickness of the active layer h = 2 meters. Considering these values in Table 8.3, the calculated ratios $U_2/U_1$ are presented as a function of $\rho_1/\rho_2$.

Table 8.3. Ratio of the cable potential for two-layer soil to the cable potential for uniform structure of the soil (or $\rho_1$) as a function of the ratio of the specific resistances of the layers

| $\rho_1/\rho_2$ | 100 | 10 | 1 | 0,1 | 0,02 |
|---|---|---|---|---|---|
| $U_2/U_1$ | 0,61 | 0,67 | 1,0 | 1,8 | 2,5 |

From an analysis of formula (8.34) it follows that for $\rho_1/\rho_2 \gg 1$, $U_2/U_1 \to$ $\to e^{-kh_1/2h}$, that is, for small specific resistance of the lower layer, some reduction in the induced voltage takes place, and this reduction is determined by the ratio of the depth of burying the cable and the thickness of the upper layer of the soil. If $\rho_1/\rho_2 \ll 1$, that is, then the lower layer has very poor conductivity, the leakage of the current from the cable sheathing is complicated, the lightning current in practice is all propagated in the upper layer, and a significant increase in voltage takes place. The latter case is realized both in the permafrost regions and in rocky ground.

The effect of the two-layer nature of the soil is especially strongly felt in the case of lightning strikes at a significant distance from the cable. In this case with good conductivity of the nearby lower layer the lightning currents simply do not hit the cable. In the case of poor conductivity of the lower layer even the remote lightning strikes lead to damage to the cable which does not occur for uniform structure of the soil.

It is possible approximately to consider the two-layer structure of the soil by the introduction of a correction factor m for the specific resistance of the upper layer: $\rho_{equiv} = m\rho$. Indeed, the voltage amplitude in the cable with uniform structure of the soil $\rho_1$ is calculated by the formula $U_{1M} = 2.1 \cdot I_M R \sqrt{\rho_1}$, where $I_M$ is the amplitude of the lightning current, and R is the resistance of the cable sheathing. In the case of the two-layer structure of the soil, the voltage varies as a function of the ratio $\rho_1/\rho_2$ in accordance with Table 8.3. Let us denote the ratio $U_{2M}/U_{1M} = \sqrt{m}$. Then we can write

$$U_{2M} = \sqrt{m}\, U_{1M} = \sqrt{m} \cdot 2,1\, I_M R \sqrt{\rho_1} = 2,1\, I_M R \sqrt{m \rho_1}.$$

Thus, the two-layer soil can be replaced by uniform with an equivalent specific resistance of $\rho = m\rho_1$. The magnitude of the correction factor m depends on the ratio of the specific resistances of the layers $\rho_1/\rho_2$ and the ratio of the depth of laying of the cable $h_1$ to the thickness of the upper layer. The values of m for ratios of $h_1/h$ and $\rho_1/\rho_2$ are presented in Table 8.4.

## 8.14. Maximum Lightning Current as a Function of the Specific Soil Resistance

It was already stated above that the amplitudes of the lightning currents vary within broad limits. The upper limiting value of the current is 250 kiloamps recorded on the object with low grounding resistance. In mountainous regions where the specific resistance is high, the observed values of the currents are appreciably lower. In mountainous regions a different ratio between the discharges of positive and negative polarity than in the lowlands is also observed. Generalizing, it is possible to say that the specific resistance of the soil in a thunderstorm region influences the magnitude of the currents in the ground in the case of a lightning strike. The resistance of the damaged objects is also felt in the current magnitude.

Table 8.4. Values of the correction factor m for different magnitudes of ratios $\rho_1/\rho_2$ and $h_1/h$

| $h_1/h$ | Factor of m for $\rho_1/\rho_2$ | | | | |
|---|---|---|---|---|---|
| | 100 | 10 | 1,0 | 0,1 | 0,02 |
| 0,1 | 0,81 | 0,86 | 1,0 | 1,21 | 1,96 |
| 0,2 | 0,66 | 0,73 | 1,0 | 1,96 | 2,56 |
| 0,3 | 0,365 | 0,45 | 1,0 | 3,25 | 6,25 |
| 0,8 | 0,207 | 0,285 | 1,0 | 5,3 | 10,6 |
| 1,0 | 0,141 | 0,212 | 1,0 | 6,8 | 13,0 |
| 2,0 | 0,021 | 0,059 | 1,0 | 16,0 | 42,0 |

For example, for lightning strikes to the ungrounded or poorly grounded overhead lines the lightning current is limited by the wave impedance of the circuit. In the underground cables with continuous grounding of the sheathing, the currents are appreciably higher than in the overhead lines.

A decrease in the maximum values of the currents with an increase in the specific resistance of the soil and the resistance of the grounding of the objects can have great significance for the practice of lightning protection measures. However, it is not necessary to simplify the problem unjustifiably and exaggerate the significance of this fact. For large values of the resistances of the grounds the potentials of the lightning-damaged objects can be so high that this leads to the development of electric breakdown in an unforeseen location and to damage.

Figure 8.9 shows the results of measuring the lightning current amplitude in soils with different specific resistance. The greatest magnitude of recorded currents occurs in the region with small specific resistance. Constructing the envelope curve of the maximum values, it is possible to establish that the lightning current amplitudes $I_M$ vary as a function of the specific resistance of the soil $\rho$ approximately by the square hyperbola law (in the range of variation of $\rho$ from 200 to 1100 ohm-meter) approaching a constant for large $\rho$:

$$I_M = 16 + 2\cdot 10^6/\rho^2. \tag{8.35}$$

Considering that the curve in Figure 8.8 is constructed on the basis of comparatively few data, it is expedient to introduce the safety factor

$$I_M = 1,5\,(16 + 2\cdot 10^6/\rho^2). \tag{8.36}$$

into the formula (8.35).

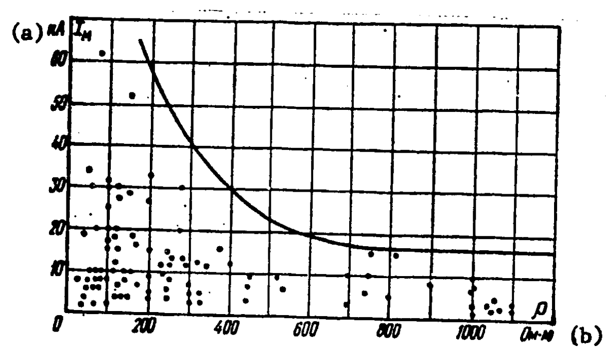


Figure 8.9. Results of measuring the lightning current amplitude in the ground with different specific resistance.

Key: a. kiloamps b. ohm-meter

The probability of damage to the cable considering the function (8.36) was calculated by the same procedure as before, but the upper integration limit with respect to the current now depends on the specific resistance of the soil. The performed calculation indicates that considering the dependence of the current amplitude on the specific resistance of the soil, the probability of damage to the cable turns out to be 30% less than for the calculation with maximum current amplitude of 250 kiloamps. The use of this derivation when calculating the lightning protection of the underground cables will permit significant reduction of the expenditures on lightning protection and it will lead to material savings. However, it is necessary to use this fact cautiously inasmuch as the good conducting and well-grounded cables lead to the inverse effect -- an increase in the current in the sheathing.

# CHAPTER 9. PARAMETERS OF THE ELECTROMAGNETIC COUPLING BETWEEN CIRCUITS OF DIFFERENT ELECTRIC SYSTEMS

## 9.1. General Principles

From the above-investigated theory of the effect between the electric circuits it follows that the induced voltages and currents in the circuit subject to the effect are characterized by the parameters of the electric and magnetic effects. The electric effect parameter is the total mutual conductivity of the electric coupling between the interfering circuit and the circuit subjected to interference, numerically equal to the ratio of the current induced in the circuit subject to effect to the voltage active in the interfering circuit, that is

$$Y_{ik} = G_{ik} + i\omega C_{ik} = \dot{I}_k/\dot{U}_i, \tag{9.1}$$

where $G_{ik}$ is the active component of the electric coupling usually called the galvanic coupling; $C_{ik}$ is the capacity coupling or the mutual capacitance between the interfering circuit and circuits subjected to interference. The index i pertains to the interfering circuit and depending on the type of interfering circuit, it can be designated by 1, 12, 123, and so on. The index k refers to the circuit subjected to the effect, and depending on the type of circuit in the given chapter it will be denoted by A for single-wire, T for double-wire telephone circuits.

The parameter of the magnetic effect $Z_{ik}$ is the total mutual resistance of the magnetic coupling or the total resistance of the mutual inductance between the interfering circuit and the circuit subject to interference numerically equal to the ratio of the emf induced in the circuit subjected to the effect to the current in the interfering circuit

$$Z_{ik} = \omega M_{ik} = R_{ik} + i\omega L_{ik} = -(\dot{E}_k/\dot{I}_i), \tag{9.2}$$

where $R_{ik}$ is the active component of the magnetic coupling; $\omega L_{ik}$ is the reactive component of the magnetic coupling,

$$|\omega M_{1k}| = \sqrt{R_{1k}^2 + \omega^2 L_{1k}^2}, \tag{9.3}$$

$M_{1k}$ is the mutual inductance or the magnetic coupling between the circuits considering the active and reactive components occurring as a result of losses in the ground.

### 9.2. Capacitive Coupling Between Single-Wire Circuits of Overhead Lines

When considering the parameter of the electrical effect between the circuits of the overhead lines both for suspension of the wires on single supports and for suspension on different supports the active component of the parameter of the electrical effect can be neglected in view of its extraordinarily small value by comparison with the reactive component. Therefore the equation (9.1) can be assumed to: $Y_{1k} \simeq i\omega C_{1k}$. Thus, determination of the parameters of the electrical effect between the different circuits of the overhead lines reduces to determination of the capacitive coupling between these circuits.

The capacitive couplings $C_{1k}$ or the partial couplings between the circuits are determined from solution of the Maxwell equations compiled for the investigated system of conductors [10]. Let the conductors 1 and A (Figure 9.1) have potentials $U_1$ and $U_A$ under the operating conditions and carry charges $q_1$ and $q_A$. Let us write the Maxwell equations for two conductors [see (2.103)]:

$$\left.\begin{aligned} \dot{U}_1 &= k(q_1\alpha_{11} + q_A\alpha_{1A}), \\ \dot{U}_A &= k(q_1\alpha_{1A} + q_A\alpha_{AA}), \end{aligned}\right\} \tag{9.4}$$

where $k = 18\cdot 10^6$; $\alpha_{11}$, $\alpha_{AA}$, $\alpha_{1A}$ are the potential coefficients.

Let us solve the system (9.4) with respect to the charges $q_1$ and $q_A$:

$$\left.\begin{aligned} q_1 &= \frac{1}{k}\left(\frac{\alpha_{AA}}{\alpha_{11}\alpha_{AA} - \alpha_{1A}^2}\dot{U}_A - \frac{\alpha_{1A}}{\alpha_{11}\alpha_{AA} - \alpha_{1A}^2}\dot{U}_1\right), \\ q_A &= \frac{1}{k}\left(\frac{\alpha_{11}}{\alpha_{11}\alpha_{AA} - \alpha_{1A}^2}\dot{U}_A - \frac{\alpha_{1A}}{\alpha_{11}\alpha_{AA} - \alpha_{1A}^2}\dot{U}_1\right). \end{aligned}\right\} \tag{9.5}$$

Equations (9.5) can be rewritten as follows:

$$\left.\begin{aligned} q_1 &= C_{11}U_1 + C_{1A}(U_1 - U_A) = U_1(C_{11} - C_{1A}) - U_A C_{1A}, \\ q_A &= C_{AA}U_A + C_{1A}(U_A - U_1) = U_A(C_{A1} + C_{AA}) - U_1 C_{A}. \end{aligned}\right\} \tag{9.6}$$

Here the value of $C_{11}$ is the part of the total charge of the system of conductors which pertains to the proportion of the first conductor (1), where all of the conductors are joined to each other and they are communicated a potential equal to one, that is, $\dot{U} = \dot{U}_A = 1$; in other words, $C_{11}$ is the part of the total capacitance of the entire system (where all of the conductors are joined together) which is caused by the presence of the first conductor (1) in the given system. The analogous value of $C_{AA}$ is caused by the presence of the conductor A. The value of $C_{1A}$ is the mutual capacitance between the first and the second conductors under the conditions of the given system. Comparing (9.5) and (9.6) between each other, we see that

$$\left.\begin{aligned} C_{11}+C_{1A} &= \frac{\alpha_{AA}}{\alpha_{11}\alpha_{AA}-\alpha_{1A}^2}\,\frac{1}{k},\\ C_{1A} &= \frac{\alpha_{1A}}{k\left(\alpha_{11}\alpha_{AA}-\alpha_{1A}^2\right)},\\ C_{AA}+C_{1A} &= \frac{1}{k}\,\frac{\alpha_{11}}{\alpha_{11}\alpha_{AA}-\alpha_{1A}^2}.\end{aligned}\right\} \quad (9.7)$$

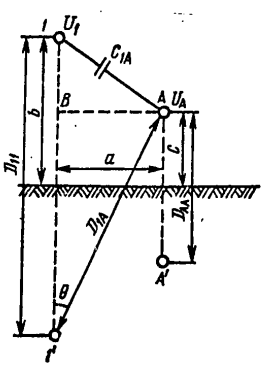


Figure 9.1. Calculation of the capacitive coupling between single-wire circuits 1 and A.

Hence, substituting the value of k and neglecting $\alpha_{1A}^2$ as a small value by comparison with $\alpha_{11}\alpha_{AA}$, we obtain

$$C_{11}=\frac{10^{-6}}{18}\,\frac{\alpha_{AA}-\alpha_{1A}}{\alpha_{11}\alpha_{AA}}.$$

In exactly the same way

$$\left.\begin{aligned} C_{AA} &= \frac{10^{-6}}{18}\,\frac{\alpha_{11}-\alpha_{1A}}{\alpha_{11}\alpha_{AA}},\\ C_{1A} &= \frac{10^{-6}}{18}\,\frac{\alpha_{1A}}{\alpha_{11}\alpha_{AA}}.\end{aligned}\right\} \quad (9.8)$$

Considering that (Figure 9.1)

$$\alpha_{11} = \ln\frac{2b}{a_{11}}, \quad \alpha_{AA} = \ln\frac{2c}{\alpha_{AA}},$$
$$\alpha_{1A} = \ln\frac{D_{1A}}{a_{1A}} = \ln\sqrt{\frac{a^2+(b+c)^2}{a^2+(b-c)^2}},$$

we find

$$C_{1A} = \frac{10^{-6}}{18}\frac{\alpha_{1A}}{\alpha_{11}\alpha_{AA}}. \tag{9.9}$$

After substitution of the values of $\alpha_{1A}$, $\alpha_{11}$, $\alpha_{AA}$, we obtain

$$C_{1A} = 27{,}8\cdot 10^{-9}\frac{\ln\frac{a^2+(b+c)^2}{a^2+(b-c)^2}}{\ln\frac{2b}{a_{11}}\ln\frac{2c}{a_{AA}}}, \tag{9.10}$$

where $a_{11}$ and $a_{AA}$ are the radii of the conductors 1 and A. For the structures encountered in practice for the interfering circuits and the circuits subjected to interference of the overhead lines, the potential coefficients $\alpha_{11}$ and $\alpha_{AA}$ are approximately identical, and on the average they are equal to 8.5 to 9. Substituting this number in (9.8), we obtain

$$C_{1A} = 0{,}77\,\alpha_{1A}\cdot 10^{-9}, \tag{9.11}$$

For $\alpha_{11} = \alpha_{AA} = 8.5$, from (9.8) we find, neglecting $\alpha_{1A}$ by comparison with $\alpha_{11}$ and $\alpha_{AA}$, that $C_{AA} = C_{11} = (10^{-6}/18)(1/8.5) = 6.5\cdot 10^{-9}$, farads/km, and for $\alpha_{11} = \alpha_{AA} = 9$ - $C_{11} = C_{AA} = 6.17\cdot 10^{-9}$ farads/km. As as is obvious, the values of $C_{11}$ and $C_{AA}$ can differ insignificantly from each other. Therefore for calculations, they can be set identical and equal to $6.3\cdot 10^{-9}$. Substituting the value of $\alpha_{1A}$ obtained in Chapter 2 in (9.11), we find

$$C_{1A} \approx 1{,}5\cdot 10^{-9}\frac{bc}{a^2+b^2+c^2}. \tag{9.12}$$

### 9.3. Capacitive Couplings Between the Triple-Phase and Single-Wire Circuits

For determination of the magnitude of the capacitive coupling between the triple-phase ac circuit and an insulated communications line we write the system of Maxwell equations

$$\left.\begin{aligned}\dot U_1 &= k(q_1\alpha_{11}+q_2\alpha_{12}+q_3\alpha_{13}),\\ \dot U_2 &= k(q_1\alpha_{21}+q_2\alpha_{22}+q_3\alpha_{23}),\\ \dot U_3 &= k(q_1\alpha_{31}+q_2\alpha_{32}+q_3\alpha_{33}),\\ U_A &= k(q_1\alpha_{A1}+q_2\alpha_{A2}+q_3\alpha_{A3}),\end{aligned}\right\} \tag{9.13}$$

where $U_1$, $U_2$, $U_3$ are the voltages of the wires of the triple-phase high-voltage circuit with respect to ground; $k = 18 \cdot 10^6$; $\alpha_{11}$, $\alpha_{22}$, $\alpha_{33}$, $\alpha_{12}$, $\alpha_{23}$, and so on are the coefficients which were calculated by the general formula $\alpha_{ik} = \ln(D_{ik}/a_{ik})$, for example, $\alpha_{11} = \ln(D_{11}/a_{11})$, where $D_{11}$ is the distance of the wire 1 from its mirror image; $a_{11}$ is the radius of the wire 1.

In equations (9.13) four values remain unknown; $\dot U_A$, $q_1$, $q_2$, $q_3$. The remaining values are as follows: $\dot U_1$, $\dot U_2$, $\dot U_3$ are given, and the coefficients $\alpha_{11}$, $\alpha_{22}$, ..., $\alpha_{12}$, and so on for each given case are determined from the mutual arrangement of the wires (Figure 9.2). For simplification of the written equations we make a number of the assumptions that follow below.

In equations (9.13) instead of the coefficients $\alpha_{12}$, $\alpha_{23}$, $\alpha_{31}$ without special error are under the above indicated conditions it is possible to substitute their arithmetic mean value $\bar\alpha_{12} = (\alpha_{12} + \alpha_{23} + \alpha_{31})/3$; the coefficients $\alpha_{11}$, $\alpha_{22}$ and $\alpha_{33}$ usually differ little from each other; therefore it is possible to substitute instead of them their arithmetic mean value, $\bar\alpha_{11} = (\alpha_{11} + \alpha_{22} + \alpha_{33})/3$. Then the first three equations of system (9.13) assume the form:

$$\left.\begin{aligned}\dot U_1 &= k(q_1\bar\alpha_{11}+q_2\bar\alpha_{12}+q_3\bar\alpha_{12}),\\ \dot U_2 &= k(q_1\bar\alpha_{12}+q_2\bar\alpha_{11}+q_3\bar\alpha_{12}),\\ \dot U_3 &= k(q_1\bar\alpha_{12}+q_2\bar\alpha_{12}+q_3\bar\alpha_{11}).\end{aligned}\right\} \tag{9.14}$$

Solving these equations with respect to $kq_1$, $kq_2$, $kq_3$, we obtain:

$$\left.\begin{aligned}kq_1 &= \frac{\dot U_1(\bar\alpha_{11}+\bar\alpha_{12})-(\dot U_2+\dot U_3)\bar\alpha_{12}}{(\bar\alpha_{11}-\bar\alpha_{12})(\bar\alpha_{11}+2\bar\alpha_{12})},\\ kq_2 &= \frac{\dot U_2(\bar\alpha_{11}+\bar\alpha_{12})-(\dot U_3+\dot U_1)\bar\alpha_{12}}{(\bar\alpha_{11}+2\bar\alpha_{12})(\bar\alpha_{11}-\bar\alpha_{12})},\\ kq_3 &= \frac{\dot U_3(\bar\alpha_{11}+\bar\alpha_{12})-(\dot U_1+\dot U_2)\bar\alpha_{12}}{(\bar\alpha_{11}-\bar\alpha_{12})(\bar\alpha_{11}+2\bar\alpha_{12})}.\end{aligned}\right\} \tag{9.15}$$

For equal voltages between the wires in the triple-phase system the voltages of the conductors of this system with respect to the ground can fail to be equal to each other both with respect to amplitude and with respect to phase.

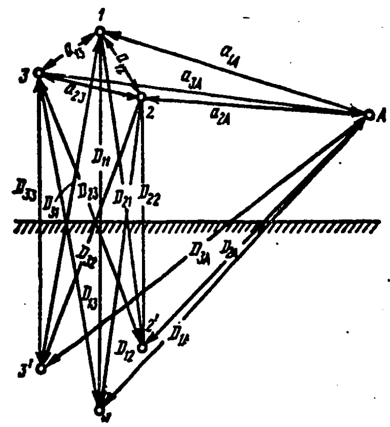


Figure 9.2. Determination of the potential on the communications line A under the electrical effect of the triple-phase overhead line.

In this general case the triple-phase system will have a residual voltage equal to the geometric sum of the voltage vectors of each conductor with respect to ground.

Indeed, let the triple-phase line have equal voltages between the conductors; then these voltages can be represented graphically by three vectors making up an equilateral triangle (Figure 9.3a). If the voltages of each conductor of a triple-phase line with respect to ground have equal amplitudes and are 120° out of phase with respect to each other, then the position of the ground potential is determined by the point zero which is located in the center of the triangle. The lines 01, 02 and 03 connecting the point 0 to the apexes of the triangle are the voltage vectors of the conductors with respect to ground. If the voltages of the conductors with respect to ground have different amplitudes or are out of phase by different angles (or both simultaneously), then the point 0 (ground potential) will not be in the center of the triangle, but can occupy any position both inside and outside it, for example, at the point $0_1$ in the figure 9.3b. In this case the voltages of the conductors with respect to ground are also represented by three vectors joining the point $0_1$ to the apexes of the triangle.

From 9.3b it is possible to see that the voltage vectors of the lines with respect to ground are the geometric difference of two vectors: for example, the vector $0_1 1$ is the geometric difference of the vector 01 and $0_1 0$; the vector $0_1 2$ is the geometric difference of the vectors 02 and $0_1 0$; the vector $0_1 3$ is the geometric difference of the vectors 03 and $0_1 0$. The geometric sum of all three voltage vectors with respect to ground obviously is the geometric

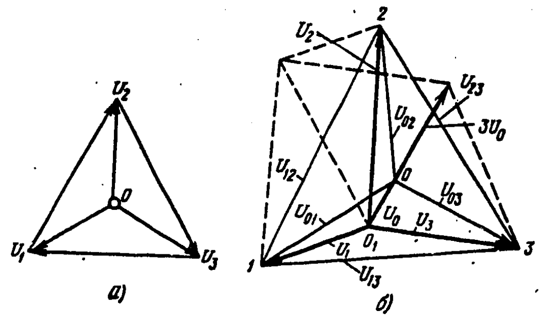


Figure 9.3. Vector diagram of the voltages of a triple-phase overhead line for phase load: a) symmetric; b) asymmetric.

sum of the components of the vectors $0_1 1 + 0_1 2 + 0_1 3 = 01 + 02 + 03 + 0_1 0 + 0_1 0 + 0_1 0$ or

$$\dot{U}_1+\dot{U}_2+\dot{U}_3=\dot{U}_{\phi1}+\dot{U}_{\phi2}+\dot{U}_{\phi3}+\dot{U}_0+\dot{U}_0+\dot{U}_0. \tag{9.16}$$

In the right hand side of the last equation the sum of the first three vectors equal to each other and 120° out of phase is zero. The sum of the remaining three vectors (equal to each other and in phase) will be equal to the triple vector $0_1 0$. Thus, it is possible to write that the geometric sum of the voltages of the conductors of a triple phase line with respect to ground is $3\dot{U}_0$:

$$\dot{U}_1+\dot{U}_2+\dot{U}_3=3\dot{U}_0. \tag{9.17}$$

This last voltage ($3\dot{U}_0$) is called the residual voltage of the three-phase system. From equation (9.17) it follows that

$$\left.\begin{aligned} \dot{U}_2+\dot{U}_3&=3\dot{U}_0-\dot{U}_1,\\ \dot{U}_3+\dot{U}_1&=3\dot{U}_0-\dot{U}_2,\\ \dot{U}_1+\dot{U}_2&=3\dot{U}_0-\dot{U}_3. \end{aligned}\right\} \tag{9.18}$$

Substituting the values of $\dot{U}_2 + \dot{U}_3$, $\dot{U}_3 + \dot{U}_1$, $\dot{U}_1 + \dot{U}_2$ from the system (9.18) in (1.15), we obtain

$$\left.\begin{aligned} kq_1&=\frac{\dot{U}_1}{\bar{\alpha}_{11}-\bar{\alpha}_{12}}-\frac{3\dot{U}_0\bar{\alpha}_{12}}{(\bar{\alpha}_{11}-\bar{\alpha}_{12})(\bar{\alpha}_{11}+2\bar{\alpha}_{12})},\\ kq_2&=\frac{\dot{U}_2}{\bar{\alpha}_{11}-\bar{\alpha}_{12}}-\frac{3\dot{U}_0\bar{\alpha}_{12}}{(\bar{\alpha}_{11}-\bar{\alpha}_{12})(\bar{\alpha}_{11}+2\bar{\alpha}_{12})},\\ kq_3&=\frac{\dot{U}_3}{\bar{\alpha}_{11}-\bar{\alpha}_{12}}-\frac{3\dot{U}_0\bar{\alpha}_{12}}{(\bar{\alpha}_{11}-\bar{\alpha}_{12})(\bar{\alpha}_{11}+2\bar{\alpha}_{12})}. \end{aligned}\right\} \tag{9.19}$$

For determination of the induced voltage on the communications lines let us take the last equation from system (9.13) and, substituting the values of $kq_1$, $kq_2$. $kq_3$ from (9.19) in it, we obtain

$$\dot{U}_A = \frac{\dot{U}_1\alpha_{A1}+\dot{U}_2\alpha_{A2}+\dot{U}_3\alpha_{A3}}{\bar{\alpha}_{11}-\bar{\alpha}_{12}} - \frac{3\dot{U}_0\bar{\alpha}_{12}(\alpha_{A1}+\alpha_{A2}+\alpha_{A3})}{(\bar{\alpha}_{11}-\alpha_{12})(\bar{\alpha}_{11}+2\bar{\alpha}_{12})}. \tag{9.20}$$

The voltage vectors of each of the three phases with respect to ground can be expressed as follows:

$$\left.\begin{aligned} \dot{U}_1 &= \dot{U}_0 - \dot{U}_\Phi, \\ \dot{U}_2 &= \dot{U}_0 + \dot{U}_\Phi\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right), \\ \dot{U}_3 &= \dot{U}_0 + \dot{U}_\Phi\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right), \end{aligned}\right\} \tag{9.21}$$

where $U_\phi$ is the phase voltage of the triple-phase, high-voltage line.

Let us replace the values of the potentials $\dot{U}_1$, $\dot{U}_2$, $\dot{U}_3$ in (9.20) by the phase voltages according to the equations of system (9.21):

$$\dot{U}_A = \frac{(\dot{U}_0-\dot{U}_\Phi)\alpha_{A1}+\left[\dot{U}_0+\dot{U}_\Phi\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\right]\alpha_{A2}+\left[\dot{U}_0+\dot{U}_\Phi\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)\right]\alpha_{A3}}{\bar{\alpha}_{11}-\bar{\alpha}_{12}} - \frac{3\dot{U}_0\bar{\alpha}_{12}(\alpha_{A1}+\alpha_{A2}+\alpha_{A3})}{(\bar{\alpha}_{11}-\bar{\alpha}_{12})(\bar{\alpha}_{11}+2\bar{\alpha}_{12})}.$$

After transformation, we obtain

$$\dot{U}_A = \frac{\dot{U}_\Phi\left[-\alpha_{A1}+\frac{1}{2}(\alpha_{A2}+\alpha_{A3})+i\frac{\sqrt{3}}{2}(\alpha_{A2}-\alpha_{A3})\right]}{\bar{\alpha}_{11}-\bar{\alpha}_{12}} + \frac{\dot{U}_0(\alpha_{A1}+\alpha_{A2}+\alpha_{A3})}{\bar{\alpha}_{11}+2\bar{\alpha}_{12}}. \tag{9.22}$$

The first term of this formula characterizes the electrical effect of a completely symmetric triple phase overhead line caused only by different arrangement of phase conductors of the overhead line with respect to the conductor subjected to the effect; the second term characterizes the electrical effect of a completely asymmetric "three wire-ground" circuit.

9.4. Capacitive Coupling Between the Entirely Symmetric Triple Phase and Single-Wire Circuit

Let us consider the first term of the right-hand side of equation (9.22), denoting it by $U_{A1}$:

$$\dot{U}_{A1}=\frac{\dot{U}_\Phi}{\bar{\alpha}_{11}-\bar{\alpha}_{12}}\left[-\alpha_{A1}+\frac{1}{2}(\alpha_{A2}+\alpha_{A3})+i\frac{\sqrt{3}}{2}(\alpha_{A2}-\alpha_{A3})\right].$$

The modulus of this expression

$$|\dot{U}_{A1}|=\frac{\dot{U}_\Phi}{\bar{\alpha}_{11}-\bar{\alpha}_{12}}\sqrt{\left[\frac{1}{2}(\alpha_{A2}+\alpha_{A3})-\alpha_{A1}\right]^2+\left[\frac{\sqrt{3}}{2}(\alpha_{A2}-\alpha_{A3})\right]^2}.$$

Substituting $\alpha_{A2}-\alpha_{A1}=h_2$ and $\alpha_{A3}-\alpha_{A1}=h_3$ here and replacing the corresponding values under the radicals by $h_2$ and $h_3$, after transformations we obtain

$$\dot{U}_{A1}=\frac{\dot{U}_\Phi}{\bar{\alpha}_{11}-\bar{\alpha}_{12}}\sqrt{h_2^2+h_3^2-h_2h_3}. \tag{9.23}$$

The wires of the triple-phase transmission line can be located either in one plane (horizontal, inclinded or vertical) or in a triangle. On location of the wires in the horizontal, inclined (Figure 9.4) or vertical planes, the equality $h_2=-h_3$, and then (9.23) can rewritten as follows:

$$\dot{U}_{A1}=\frac{\dot{U}_\Phi}{\bar{\alpha}_{11}-\bar{\alpha}_{12}}\sqrt{h_2^2+h_2^2+h_2^2}=\sqrt{3}h_2\frac{\dot{U}_\Phi}{\bar{\alpha}_{11}-\bar{\alpha}_{12}}.$$

The value of $h_2=\alpha_{A2}-\alpha_{A1}$ can be determined by the Taylor formula

$$h_2=\frac{\partial\alpha_{A1}}{\partial a}(\Delta a)_{12}+\frac{\partial\alpha_{A1}}{\partial b}(\Delta b)_{12},$$

where according to Figure 9.4 $(\Delta a)_{12}=\delta_{12}\cos\alpha$, $(\Delta b)_{12}$ is the difference with respect to height in the suspension of the conductors 1 and 2 of the triple phase line; $\delta_{12}$ is the distance between the wires 1 and 2 of the triple phase line.

Substituting the following values in the expression for $h_2$

$$\frac{\partial\alpha_{A1}}{\partial a}=-\frac{4abc}{(a^2+b^2+c^2)^2},\quad \frac{\partial\alpha_{A1}}{\partial b}=\frac{2c(a^2+c^2-b^2)}{(a^2+b^2+c^2)^2},$$

we obtain

$$h_2 = -\frac{4abc\,\delta_{12}}{(a^2+b^2+c^2)^2}\cos\alpha + \frac{2c(a^2+c^2-b^2)\,\delta_{12}\sin\alpha}{(a^2+b^2+c^2)^2} = \frac{2c\,\delta_{12}}{(a^2+b^2+c^2)^2}\,[-2ab\cos\alpha + (a^2+c^2-b^2)\sin\alpha]. \quad (9.24)$$

Replacing $\delta_{12}$ by the geometric mean distance between the three wires, we obtain

$$\delta = \sqrt[3]{\delta_{12}\,\delta_{13}\,\delta_{23}}. \quad (9.25)$$

For $\delta_{12} = \delta_{13}$ and $2\delta_{12} = \delta_{23}$ $\delta = \sqrt[3]{\delta_{12}\delta_{12}2\delta_{12}}$ $\sqrt[3]{2}\delta_{12}$. Hence $\delta_{12} = \delta/1.26$. Now substituting this value of $\delta_{12}$ in (9.24), we obtain

$$h_2 = \frac{2c\delta}{1,26\,(a^2+b^2+c^2)^2}\,[-2ab\cos\alpha + (a^2+c^2-b^2)\sin\alpha]. \quad (9.26)$$

Thus, for the given case the magnitude of the induced voltage $\dot{U}_{A1}$ after substitution of the obtained value of $h_2$ in (9.23) will be defined by the formula

$$\dot{U}_{A1} = \frac{1,6\sqrt{3}\,\dot{U}_{\Phi}\,c\,\delta}{(\overline{\alpha}_{11}-\overline{\alpha}_{12})\,(a^2+b^2+c^2)^2}\,[-2ab\cos\alpha + (a^2+c^2-b^2)\sin\alpha]. \quad (9.27)$$

The voltage on the insulated communications line subjected to the electrical effect of a symmetric triple phase line, in accordance with (3.75) will be determined from the equation

$$\dot{U}_{A1} = \frac{\dot{U}_{ll\,x}\,z_{BA}}{\gamma_{ll}^2-\gamma_A^2}\,Y_{ll\,A}\left(\gamma_A + \gamma_{ll}\,\frac{e^{-\gamma_{ll}l} - \mathrm{ch}\,\gamma_A l}{\mathrm{sh}\,\gamma_A l}\right). \quad (9.28)$$

In accordance with the conditions adopted when deriving expression (9.27) ($\gamma_{ll} = 0$), equation (9.28) assumes the form

$$\dot{U}_{A1} = \frac{\dot{U}_{ll\,x}\,z_{BA}}{\gamma_A}\,Y_{llA}.$$

Knowing that

$$Y_{llA} = G_{llA} + i\omega C_{llA} \approx i\omega C_{llA} \quad \text{and} \quad \frac{z_{BA}}{\gamma_A} = \frac{1}{G_A + i\omega C_A} \approx \frac{1}{i\omega C_A},$$

we obtain

$$\dot{U}_{A1} = \frac{\dot{U}_{iiH} C_{iiA}}{C_A}, \tag{9.29}$$

where $C_{iiA} = C_{123A}$ is the magnitude of the capacitive coupling between the triple-phase line and the communications lines; $\dot{U}_{iiH}$ is the phase voltage ($\dot{U}_{\phi}$).

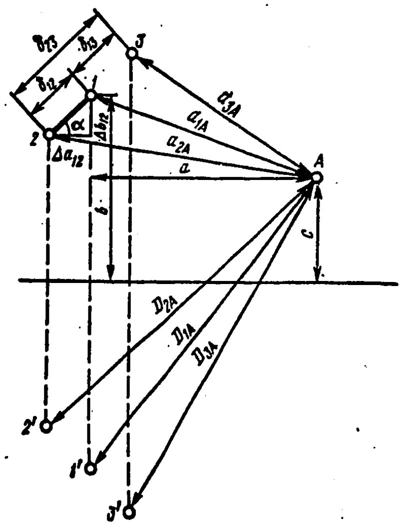


Figure 9.4. Calculation of the capacitive coupling between the triple-phase overhead line and the communications line A (with inclined arrangement of the overhead lines)

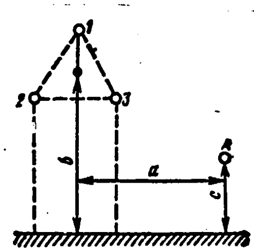


Figure 9.5. Calculation of the capacitive coupling between a triple phase overhead line and the communications line A (for triangular arrangement of the conductors of he overhead line)

Equating (9.29) and (9.27), we find that the magnitude of the capacitive coupling between the symmetric, triple phase overhead line and the communications line will be determined from the equation

$$C_{123A} = \frac{1,6\sqrt{3}\,c\,\delta}{(a^2+b^2+c^2)^2}\;\frac{(a^2-b^2+c^2)\sin\alpha - 2\,ab\cos\alpha}{\bar{\alpha}_{11}-\bar{\alpha}_{12}}\;C_A. \tag{9.30}$$

Depending on the location of the conductors of the triple phase overhead line the values of the capacitive coupling will be different. Considering that $C_A \approx 6.5\cdot10^{-9}$ farads/km and considering that on the average $\bar{\alpha}_{11} = 8.5$ and $\alpha_{12} = 2.5$, we find that

for horizontal arrangement of the conductors when the angle $\alpha = 0$,

$$C_{123A} = \frac{6\cdot10^{-9}\,abc\,\delta}{(a^2+b^2+c^2)^2};$$

for vertical arrangement of the conductors where the angle $\alpha = 90°$,

$$C_{123A} = \frac{3\cdot10^{-9}\,(a^2-b^2+c^2)\,c\,\delta}{(a^2+b^2+c^2)^2};$$

for $a > 50$ meters

$$C_{123A} = \frac{3\cdot10^{-9}\,c\,\delta}{a^2};$$

for triangular arrangement of the wires and for $\delta = \delta_{12} = \delta_{13} = \delta_{23}$ (Figure 9.5) the values of $h_2$ and $h_3$ will be determined from the equations

$$h_2 = \frac{\partial\alpha_{A1}}{\partial a}\,\frac{\delta}{2} + \frac{\partial\alpha_{A1}}{\partial b}\,\sqrt{3}\,\frac{\delta}{2}, \quad h_3 = -\frac{\partial\alpha_{A1}}{\partial a}\,\frac{\delta}{2} - \frac{\partial\alpha_{A1}}{\partial b}\,\sqrt{3}\,\frac{\delta}{2}.$$

Then

$$h_2^2+h_3^2-h_2h_3 = \frac{3\delta^2}{4}\left[\left(\frac{\partial\alpha_{A1}}{\partial a}\right)^2 + 3\left(\frac{\partial\alpha_{A1}}{\partial b}\right)^2 + 2\sqrt{3}\,\frac{\partial\alpha_{A1}}{\partial a}\,\frac{\partial\alpha_{A1}}{\partial b}\right].$$

Since $\partial\alpha_{A1}/\partial a = -4abc/(a^2+b^2+c^2)^2$, $\partial\alpha_{A1}/\partial b = 2c(a^2-b^2+c^2)/(a^2+b^2+c^2)^2$, then

$$h_2^2+h_3^2-h_2h_3 = \frac{\delta^2c^2}{(a^2+b^2+c^2)^4}\,[2\,ab-\sqrt{3}\,(a^2-b^2+c^2)]^2.$$

Thus,

$$\sqrt{h_2^2+h_3^2-h_2h_3} = \frac{\sqrt{3}\,\delta\,c}{(a^2+b^2+c^2)^2}\,\frac{2}{\sqrt{3}}\,ab - a^2 - c^2 + b^2),$$

or it is possible approximately to set

$$\sqrt{h_2^2+h_3^2-h_2h_3} \approx \frac{\sqrt{3}c\delta}{a^2+b^2+c^2}.$$

Then the magnitude of the capacitive couplings

$$C_{123A} = \frac{2\cdot 10^{-9}c\delta}{a^2+b^2+c^2}. \qquad (9.31)$$

9.5. Capacitive Coupling Between the "Triple Wire-Ground" Circuit of a Triple-Phase Line and Single-Wire Circuit

For determination of the magnitude of the capacitive coupling between the completely asymmetric triple-phase circuit ("three wires-ground" and the communications wire) let us consider the second term of the right-hand side of (9.22), denoting it by $\dot{U}_{A2}$,

$$\dot{U}_{A2} = \frac{\dot{U}_0(\alpha_{A1}+\alpha_{A2}+\alpha_{A3})}{\alpha_{11}+2\bar{\alpha}_{12}}. \qquad (9.32)$$

In this formula for simplification it is possible to replace the coefficients $\alpha_{A1}$, $\alpha_{A2}$, $\alpha_{A3}$ by their arithmetic means $\bar{\alpha}_{A1} = (\alpha_{A1} + \alpha_{A2} + \alpha_{A3})/3$. Then

$$\dot{U}_{A2} = \frac{3\dot{U}_0\bar{\alpha}_{A1}}{\bar{\alpha}_{11}+2\alpha_{12}}. \qquad (9.33)$$

The voltage induced in the communications wire of the three-conductor, completely asymmetric line with interfering voltage $\dot{U}_{10H} = \dot{U}_0$, in accordance with (3.73) will be determined from the expression

$$\dot{U}_{A\,\text{H}\,1} = \dot{U}_{A2} = \frac{U_0 z_{\text{B}\,A}}{\gamma_i^2-\gamma_A^2} Y_{iA}\left(\gamma_A + \gamma_i \frac{e^{-\gamma_i l} - \operatorname{ch}\gamma_A l}{\operatorname{sh}\gamma_A l}\right).$$

In accordance with the above adopted conditions ($\gamma_i = 0$)

$$U_{A2} \approx \frac{U_0 C_{iA}}{C_A}, \qquad (9.34)$$

where $C_{iA} = C_{3\phi 0A}$ is the magnitude of the capacitive coupling between the "three wire-ground" circuit of the triple-phase line and the communications wire. Equating (9.34) and (9.33), we find that

$$C_{3\phi 0A} = \frac{3\bar{\alpha}_{A1}}{\bar{\alpha}_{11}+2\bar{\alpha}_{12}} C_A. \qquad (9.35)$$

Since $\bar{\alpha}_{A1} \approx 2bc/(a^2+b^2+c^2)$ and $\bar{\alpha}_{11}=8{,}5$; $\bar{\alpha}_{12}=2{,}5$; $C_A=6{,}5\cdot 10^{-9}$, then (9.35) can be rewritten in the following form:

$$C_{3\Phi 0 A} = \frac{2{,}9 \cdot 10^{-9}\, bc}{a^2 + b^2 + c^2}. \qquad (9.36)$$

9.6. Capacitive Coupling Between a Triple-Phase Symmetric Line and a Two-Wire Circuit

The Maxwell equations for calculating the capacitive coupling between a triple-phase symmetric line and two-wire communications circuit will be written analogously to the equations of (9.13):

$$\left.\begin{aligned} \dot{U}_1 &= k(\alpha_{11} q_1 + \alpha_{12} q_2 + \alpha_{13} q_3), \\ \dot{U}_2 &= k(\alpha_{21} q_1 + \alpha_{22} q_2 + \alpha_{23} q_3), \\ \dot{U}_3 &= k(\alpha_{31} q_1 + \alpha_{32} q_2 + \alpha_{33} q_3), \\ \dot{U}_A &= k(\alpha_{A1} q_1 + \alpha_{A2} q_2 + \alpha_{A3} q_3), \\ \dot{U}_Б &= k(\alpha_{Б1} q_1 + \alpha_{Б2} q_2 + \alpha_{Б3} q_3). \end{aligned}\right\} \qquad (9.37)$$

Neglecting the inverse effect of the charges of the communications wires on the triple-phase line wires, for each wire the triple-phase line we obtain

$$\dot{U}_i = 18 \cdot 10^6 \sum_{k=1}^{3} \alpha_{k\,i} q_k,$$

where $k = 1, 2, 3$; that is, for each communications wire:

$$\left.\begin{aligned} \dot{U}_A &= 18 \cdot 10^6 \sum_{k=1}^{n} \alpha_{A\,k} q_k, \\ \dot{U}_Б &= 18 \cdot 10^6 \sum_{k=1}^{n} \alpha_{Б\,k} q_k. \end{aligned}\right\} \qquad (9.38)$$

Denoting $18 \cdot 10^6 \sum_{k=1}^{n} \alpha_{Ak} q_k = S_A$ and $18 \cdot 10^6 \sum_{k=1}^{n} \alpha_{Бk} q_k = S_Б$, we find that the potential difference between the communications wires

$$\dot{U}_A - \dot{U}_Б = \dot{U}_{т\,1в} = (S_A - S_Б) \frac{\omega l C_A}{2} z_{в.т}, \qquad (9.39)$$

where $l$ is the length of the communications circuit

$$S_A = \frac{\dot{U}_\Phi}{\bar{\alpha}_{11} - \bar{\alpha}_{12}} \sqrt{h_2^2 + h_3^2 - h_2 h_3} = \frac{\dot{U}_\Phi}{\bar{\alpha}_{11} - \bar{\alpha}_{12}} \sqrt{3} h_2, \qquad (9.40)$$

$$h_2 = \alpha_{A1} - \alpha_{A2}, \quad h_3 = \alpha_{A1} - \alpha_{A3}, \quad \alpha_{A1} = \frac{2bc}{a^2 + b^2 + c^2},$$

$$h_2 = \frac{\partial \alpha_{A1}}{\partial a} \Delta a_{12} + \frac{\partial \alpha_{1A}}{\partial b} \Delta b_{12} = -\frac{4abc\,\delta_1}{(a^2 + b^2 + c^2)^2} \cos\varphi +$$

$$+\frac{2c(a^2-b^2+c^2)\delta_1}{(a^2+b^2+c^2)^2}\sin\varphi=\frac{2c\delta_1}{(a^2+b^2+c^2)^2}[-2ab\cos\varphi+ +(a^2-b^2+c^2)\sin\varphi].$$

Substituting the value of $h_2$ in (9.40), we obtain

$$S_A=\frac{2\sqrt{3}c\delta_1\dot{U}_\Phi}{(\bar{\alpha}_{11}-\bar{\alpha}_{12})(a^2+b^2+c^2)^2}[-2ab\cos\varphi+(a^2-b^2+c^2)\sin\varphi]. \tag{9.41}$$

For different cases of arrangement of the wires by this formula we obtain:

for horizontal arrangement of the wires of the triple-phase line

$$S_A=\frac{\sqrt{3}}{1,89}\dot{U}_\Phi\frac{abc\delta}{(a^2+b^2+c^2)^2};$$

for vertical arrangement of the wires of the triple phase line

$$S_A=\frac{\sqrt{3}}{3,78}\dot{U}_\Phi\frac{c\delta(a^2-b^2+c^2)}{(a^2+b^2+c^2)^2};$$

for triangular arrangment of the triple phase line wires

$$S_A=\frac{\sqrt{3}}{6}\dot{U}_\Phi\frac{c\delta}{a^2+b^2+c^2}.$$

Now of the indicated cases of location of the wires it is possible to determine the difference $S_A - S_B$ by the Taylor formula, that is

$$S_A-S_B=\frac{\partial S_A}{\partial a}\Delta a-\frac{\partial S_A}{\partial c}\Delta c. \tag{9.42}$$

For different cases of location of the wires by this formula we obtain:

for horizontal arrangement of the wires of the triple phase line for the communications circuit suspended on cross pieces

$$S_A-S_B=\frac{\sqrt{3}}{1,89}\dot{U}_\Phi\frac{bc\delta a_c(3a^2-b^2-c^2)}{(a^2+b^2+c^2)^3}, \tag{9.43}$$

where $\alpha_c$ is the distance between the wires A and B;

the same thing, for the communications circuits suspended on hooks,

$$S_A-S_B=\frac{\sqrt{3}}{1,89}\dot{U}_\Phi\frac{aba_c\delta(a^2+b^2-3c^2)}{(a^2+b^2+c^2)^3}; \tag{9.44}$$

for vertical arrangement of the wires of the triple-phase line for the communications circuit suspended on cross pieces,

$$S_{\text{А}}-S_{\text{Б}}=\frac{\sqrt{3}}{3,78}\dot{U}_{\Phi}\frac{aca_{c}\delta\,(a^{2}-3b^{2}+c^{2})}{(a^{2}+b^{2}+c^{2})^{3}}; \qquad (9.45)$$

the same thing, for the communications circuits suspended on hooks

$$S_{\text{А}}-S_{\text{Б}}=\frac{\sqrt{3}}{7,56}\dot{U}_{\Phi}\frac{a_{c}\delta\,[a^{4}+4b^{2}c^{2}-(b^{2}-c^{2})^{2}]}{(a^{2}+b^{2}+c^{2})^{3}}; \qquad (9.46)$$

for the triangular arrangment of the triple phase line wires for the communications circuit suspended on cross pieces

$$S_{\text{А}}-S_{\text{Б}}=\frac{\sqrt{3}}{3,0}\dot{U}_{\Phi}\frac{aca_{c}\delta}{(a^{2}+b^{2}+c^{2})^{2}}; \qquad (9.47)$$

the same, for the communications circuit suspended on hooks

$$S_{\text{А}}-S_{\text{Б}}=\frac{\sqrt{3}}{6}\dot{U}_{\Phi}\frac{a_{c}\delta\,(a^{2}+b^{2}-c^{2})}{(a^{2}+b^{2}+c^{2})^{2}}. \qquad (9.48)$$

Thus, subsituting the values obtained for $S_A - S_B$ from (9.43)-(9.48) in (9.39), we obtain respectively:

for horizontal arrangment of the wires of the triple-phase line for the communication circuit suspended on cross pieces;

$$\dot{U}_{\text{т1н}}=\frac{\omega C_{\text{А}}l}{2}z_{\text{в.т}}\frac{\sqrt{3}}{1,89}\dot{U}_{\Phi}\frac{bca_{c}\delta\,(3a^{2}-b^{2}-c^{2})}{(a^{2}+b^{2}+c^{2})^{3}};$$

the same for the communication circuits suspended on hooks,

$$\dot{U}_{\text{т1н}}=\frac{\omega C_{\text{А}}l}{2}z_{\text{в.т}}\frac{\sqrt{3}}{1,89}\dot{U}_{\Phi}\frac{aba_{c}\delta\,(a^{2}+b^{2}-3c^{2})}{(a^{2}+b^{2}+c^{2})^{3}};$$

for the vertical arrangement of the wires of the triple-phase line for the communications circuit suspended on cross pieces,

$$\dot{U}_{\text{т1н}}=\frac{1}{2}\omega C_{\text{А}}l z_{\text{в.т}}\frac{\sqrt{3}}{3,78}\dot{U}_{\Phi}\frac{aca_{c}\delta\,(a^{2}+c^{2}-3b^{2})}{(a^{2}+b^{2}+c^{2})^{3}};$$

the same, for the communications circuit suspended on hooks,

$$\dot{U}_{\text{т1н}}=\frac{1}{2}\omega C_{\text{А}}l z_{\text{в.т}}\frac{\sqrt{3}}{7,56}\dot{U}_{\Phi}\frac{a_{c}\delta\,[a^{4}+4b^{2}c^{2}-(b^{2}-c^{2})^{2}]}{(a^{2}+b^{2}+c^{2})^{3}};$$

for the triangular arrangment of the wires of the triple-phase line for the communications circuit suspended on cross pieces

$$\dot{U}_{\text{т1н}} = \frac{1}{2}\,\omega C_{\text{A}}\, l\, z_{\text{в.т}}\, \frac{\sqrt{3}}{3,0}\, \dot{U}_{\Phi}\, \frac{aca_c\,\delta}{(a^2+b^2+c^2)^2}\ ;$$

the same for the communications circuit suspended on hooks

$$\dot{U}_{\text{т1н}} = \frac{1}{2}\,\omega C_{\text{A}}\, l\, z_{\text{в.т}}\, \frac{\sqrt{3}}{6}\, \dot{U}_{\Phi}\, \frac{a_c\,\delta\,(a^2+b^2-c^2)}{(a^2+b^2+c^2)^2}\ .$$

The capacitance of a single wire $C_A$ which will be quantitatively set equal to $6.5\cdot10^{-9}$ farads/km enters into all of the last formulas for $U_{T1H}$. The voltage in the two-wire communications circuit for the electrical effect of the triple-phase symmetric line was determined from the solution of the differential equations, as a result of which the equation (see item 3.5) was obtained. Under the adopted conditions this equation assumes the form

$$\dot{U}_{\text{т1н}} = \frac{1}{2}\, \dot{U}_{\text{11н}}\, z_{\text{в.т}} Y_{\text{11т}}\, l, \tag{9.49}$$

where $\dot{U}_{\text{11н}}=\dot{U}_{\Phi};\ Y_{\text{11т}}\approx i\omega C_{\text{123т}};\ C_{\text{123т}}$ is the magnitude of the capacitive coupling.

Equating (9.49) successively to the equations written above for the various cases of suspension of the wires of the triple-phase line and communications network, we obtain respectively:

for horizontal arrangement of the wires of the triple-phase line:

a) for the communications circuits suspended on cross pieces

$$C_{\text{123т}} = \frac{6,0\cdot10^{-9}\, bca_c\,\delta\,(3a^2-b^2-c^2)}{(a^2+b^2+c^2)^3}\ ;$$

b) for the communications circuits suspended on hooks,

$$C_{\text{123т}} = \frac{6,0\cdot10^{-9}\, aba_c\,\delta\,(a^2+b^2-3c^2)}{(a^2+b^2+c^2)^3}\ ;$$

for vertical arrangement of the triple-phase line wires:

a) for the communications circuits suspended on cross pieces

$$C_{\text{123т}} = \frac{3,0\cdot10^{-9}\, aca_c\,\delta\,(a^2+c^2-3b^2)}{(a^2+b^2+c^2)^3}\ ;$$

b) for the communications circuits suspended on hooks,

$$C_{\text{123т}} = \frac{1,5\cdot10^{-9}\, a_c\,\delta\,[a^4+4b^2c^2-(b^2-c^2)^2]}{(a^2+b^2+c^2)^3}\ ;$$

for the triangular arrangment of the wires of the triple-phase line:

a) for the communications circuits suspended on cross pieces,

$$C_{123T} = \frac{3,7 \cdot 10^{-9}\, a c a_c \delta}{(a^2+b^2+c^2)^2};$$

b) for the communications circuits suspended on hooks

$$C_{123T} = \frac{185 \cdot 10^{-9}\, a_c \delta (a^2+b^2-c^2)}{(a^2+b^2+c^2)^2}.$$

## 9.7. Capacitive Coupling Between Single-Wire and Two-Wire Circuits

Analogously to the preceding, obtaining the expression for the voltage in a two-wire circuit subjected to the effect of a single-wire circuit from the solutions of the maximum equations of electrostatics and equating this expression to the voltage determined by (3.16), we find the following formulas for determining the magnitude of the capacitive coupling between the single-wire circuit and the two-wire circuit:

for the circuit suspended on cross pieces,

$$C_{1T} = \frac{2,6 \cdot 10^{-9}\, a b c a_c}{(a^2+b^2+c^2)^2}, \tag{9.50}$$

for the circuit suspended on hooks,

$$C_{1T} = \frac{1,3 \cdot 10^{-9}\, b a_c (a^2+b^2-c^2)}{(a^2+b^2+c^2)^2}. \tag{9.51}$$

Analogously, it is possible to obtain the formulas for determining the magnitudes of the capacitive coupling between the "three-phase-ground" circuit and the two-wire circuit, between the two-wire circuits for different arrangement of the wires on the supports and also between the two-wire interfering line and the single-wire communications circuit. All of the formulas for determining the magnitudes of the capacitive coupling between the high-voltage circuits of the overhead lines and the communications circuits are presented in Table 9.1.

## 9.8. Determination of the Mutual Inductance Between Single-Wire Circuits of Overhead Lines

The total mutual resistance between the interfering circuit and the circuits subjected to interference (the magnetic coupling resistance) is an important value for determining the magnetic effect of one circuit or another. It acquires especially great significance when determining the effect between circuits using the ground as the return conductor, that is, between the single-wire circuits.

The active component of the magnetic coupling resistance $Z_{ik}$ [see (9.2)] is caused by the losses to eddy currents in the nearby metal masses and in the ground and also the direct transfer of the currents to the ground. In the case of the effect between the overhead lines, the active component caused by the presence of adjacent wires is negligibly small and is not taken into account. When suspending the sires on the same supports (the effect between communications circuits) the component caused by losses to ground is also not considered.

Mutual Inductance for Parallel Arrangement of the Wires of Single-Wire Lines. From theory it is known that the total magnetic coupling resistance between the interfering single-wire lines (1) and those subjected to the interference (2) is equal to the ratio of the electromotive force inducted in the wire subjected to the effect of the line to the current in the interfering line, that is,

$$\dot{Z}_{м\,(1-2)} = -\frac{\dot{E}_2}{\dot{I}_1} = -i\,\omega M_{12}, \tag{9.52}$$

where

$$\dot{E}_2 = -i\,\omega \dot{A}_{x1}, \tag{9.53}$$

$M_{12}$ is the magnitude of the mutual inductance between the two lines; $A_{x1}$ is the horizontal component of the vector potential of the electromagnetic field of an infinitely long overhead "conductor-ground" line in the air. Hence,

$$M_{12} = A_{x1}/\dot{I}_1. \tag{9.54}$$

A large number of papers by both Soviet and foreign authors have been devoted to the mathematical definition of $A_{x1}$, and, consequently, the definition of $M_1$. The first strict solution of the problem of the field of a vertical dipole placed at the interface of air and a conducting medium (the ground) with defined conductivity with constant physical constants of the air and the ground was obtained by Sommerfeld in 1905 [52].

In our country the studies of an ac electromagnetic field from extended sources of finite length were performed by academician V. A. Fok and professor V. A. Bursianov, who in 1926 solved the basic problem of the theoretical definition of an electromagnetic field of a conductor of finite length located on the surface of a uniform halfspace in general form [53]. The solutions for the vector potential $\bar{A}$ are presented in the expressions of the variable integrals which, however, could not be used for practical calculations.

The approximate formulas for determining the magnitudes of the mutual inductance between wires of finite length based on the laws of transmission of a direct current in the ground were presented by Campbell [54], and between the small-diameter wires of infinite length with uniform ground, by Pollaczek [19], Carson [20], Haberland [55], Shalimov [66], and so on. It is necessary to note the papers by Carson and Pollaczek, who investigated the problems of the propagation of electromagnetic waves along a conductor suspended above the surface of uniform ground with finite conductivity. In these papers the following assumptions were made:

1) the ground is a semiinfinite medium with constant finite conductivity;

2) the wire cross section is infinitely small;

3) the daming of the electric and magnetic fields in the direction of the axis of the wire is small;

4) the transverse components of the electric current in the ground are negligibly small by comparison with the longitudinal components;

5) the bias and conduction currents in the air and the bias currents in the ground are absent.

Under these assumptions Carson obtained the general formula for the mutual inductance,

$$M_{K} = i\frac{\mu_0}{\pi}\int_0^\infty [\sqrt{u^2+i}-u]\, e^{-b\alpha u}\, e^{c\alpha\sqrt{u^2+i}} \cos(a\alpha u)\, du. \tag{9.55}$$

The analogous general Pollaczek formula has the form:

$$M_{\Pi} = \frac{i\mu_0}{2\pi\alpha^2}\left\{\int_{-\infty}^{0} e^{u(ia+b)+c\sqrt{u^2+i\alpha^2}}\,[\sqrt{u^2+i\alpha^2}+u]\, du + \int_0^\infty e^{u(ia-b)+c\sqrt{u^2+i\alpha^2}}[\sqrt{u^2+i\alpha^2}-u]\, du\right\}. \tag{9.56}$$

For c > 0 Haberland obtained the expression

$$M_{X} = \frac{\mu_0}{2\pi}\ln\frac{D}{d} + \frac{i\mu_0}{\pi}\int_0^\infty [\sqrt{u^2+i}-u]\, e^{-u\alpha(b+c)} \cos(a\alpha u)\, du. \tag{9.57}$$

Beginning with (9.56) and (9.55), Pollaczek and Carson obtained approximate formulas, the numerical calculations by which give identical results. For large spacings between the lines ($\alpha a > 10$)

$$M_{K} = M_{\Pi} = -\frac{i\mu_0}{\pi a^2\alpha^2}. \tag{9.58}$$

In these formulas the folloing notation is introduced: a -- horizontal spacing between lines, meters; b -- average height of suspension of the interfering line, meters; c -- average height of suspension of the line subjected

Table 9.1. Formulas for determining the coupling capacitances between the circuits of overhead lines and communications circuits

| Magnitude of capacitive coupling | Formulas, farads/km |
|---|---|
| 1 | 2 |
| Between a single-wire overhead line and a single-wire communications circuit or between the contact conductor of an electric railroad and a single-wire communications circuit | $C_{1\text{Л}} = \frac{1{,}5 \cdot 10^{-9}\, bc}{a^2 + b^2 + c^2}$ |
| Between the "three wire-ground" overhead line circuit and a single-wire communications circuit | $C_{3\Phi.0\text{A}} = \frac{2{,}9 \cdot 10^{-9}\, bc}{a^2 + b^2 + c^2}$ |
| Between a single-wire overhead line circuit and the two-wire telephone circuit | 1) Telephone circuit on crosspieces<br>$C_{1\text{T}} = \frac{2{,}6 \cdot 10^{-9}\, abca_c}{(a^2 + b^2 + c^2)^2}$;<br>2) -telephone circuit on hooks<br>$C_{1\text{T}} = \frac{1{,}3 \cdot 10^{-9}\, ba_c (a^2 + b^2 - c^2)}{(a^2 + b^2 + c^2)^2}$ |
| Between the "three wire-ground" overhead line circuit and the two-wire telephone circuit | 1) Telephone circuit on crosspieces<br>$C_{3\Phi 0\text{T}} = \frac{5{,}0 \cdot 10^{-9}\, abca_c}{(a^2 + b^2 + c^2)^2}$;<br>2) telephone circuit on hooks<br>$C_{3\Phi 0\text{T}} = \frac{2{,}5 \cdot 10^{-9}\, ba_c (a^2 + b^2 - c^2)}{(a^2 + b^2 + c^2)^2}$ |
| Between a three-phase symmetric overhead line circuit and a single-wire telephone circuit | 1) For horizontal arrangement of the overhead line wires<br>$C_{123\text{A}} = \frac{6{,}0 \cdot 10^{-9}\, abc\,\delta}{(a^2 + b^2 + c^2)^2}$;<br>2) for vertical arrangment of the overhead line wires<br>$C_{123\text{A}} = \frac{3{,}0 \cdot 10^{-9}\, c\,\delta (a^2 - b^2 + c^2)}{(a^2 + b^2 + c^2)^2}$;<br>3) for triangular arrangement of the overhead line wires<br>$C_{123\text{A}} = \frac{2{,}0 \cdot 10^{-9}\, c\,\delta}{(a^2 + b^2 + c^2)}$ |
| Between a three-phase symmetric overhead line circuit and a two-wire telephone circuit | 1) For horizontal arrangement of the overhead line conductors: |

Table 9.1 (continued)

| Magnitude of capacitive coupling | Formulas, farads/km |
|---|---|
| 1 | 2 |
| | telephone circuit on crosspieces<br>$C_{123T}=\frac{6{,}0\cdot 10^{-9}bca_c\delta(3a^2-b^2-c^2)}{(a^2+b^2+c^2)^3};$<br>telephone circuit on hooks<br>$C_{123T}=\frac{6{,}0\cdot 10^{-9}aba_c\delta(a^2+b^2-3c^2)}{(a^2+b^2+c^2)^3};$<br>2) for vertical arrangment of the overhead line wires:<br>telephone circuit on crosspieces<br>$C_{123T}=\frac{3{,}0\cdot 10^{-9}aca_c\delta(a^2+c^2-3b^2)}{(a^2+b^2+c^2)^3};$<br>telephone circuit on hooks<br>$C_{123T}=\frac{1{,}5\cdot 10^{-9}a_c\delta[a^4-4b^2c^2-(b^2-c^2)^2]}{(a^2+b^2+c^2)^3};$<br>3) for triangular arrangment of the overhead line wires:<br>telephone circuit on crosspieces<br>$C_{123T}=\frac{3{,}7\cdot 10^{-9}ac\delta a_c}{(a^2+b^2+c^2)^2};$<br>telephone circuit on hooks<br>$C_{123T}=\frac{1{,}85\cdot 10^{-9}a_c\delta(a^2+b^2-c^2)}{(a^2+b^2+c^2)^2}.$ |
| Between a two-wire overhead line circuit and single-wire communications circuit | 1) For horizontal arrangment of the overhead line wires<br>$C_{12A}=\frac{3{,}0\cdot 10^{-9}abc\delta}{(a^2+b^2+c^2)^2};$<br>2) for vertical arrangment of the overhead line wires<br>$C_{12A}=\frac{6{,}5\cdot 10^{-9}c\delta(a^2-b^2+c^2)}{(a^2+b^2+c^2)^2}$ |
| Between two two-wire circuits | 1) For horizontal arrangement of the wires of the interfering circuits:<br>telephone circuit on crosspieces<br>$C_{13T}=\frac{10{,}0\cdot 10^{-9}bca_c\delta(3a^2-b^2-c^2)}{(a^2+b^2+c^2)^3};$ |



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Table 9.1 (continued)

| Magnitude of capacitive coupling | Formulas, farads/km |
|---|---|
| 1 | 2 |
| | telephone circuit on hooks $C_{12T} = \frac{10{,}0\cdot 10^{-9} a b a_c \delta}{(a^2+b^2+c^2)^2}$; 2) for vertical arrangment of the wires of the interfering circuit: telephone circuit on crosspieces $C_{12T} = \frac{5{,}0\cdot 10^{-9} a c \delta a_c (a^2+c^2-3b^2)}{(a^2+b^2+c^2)^3}$; telephone circuit on hooks $C_{12T} = \frac{2{,}5\cdot 10^{-9} a_c \delta [a^4+4b^2c^2-(b^2-c^2)^2]}{(a^2+b^2+c^2)^3}$. |

Note. In the case of the presence on the communications line of suspended grounded wires, the right-hand sides of the formulas must be multiplied by the coefficient 2.7 (m + 2), where m is the number of grounded wires.

to the effect (for the underground cable c < 0), meters; $\alpha = \sqrt{\mu_0 \gamma_g \omega}$; $\mu_0$ equal to $4\pi\cdot 10^{-7}$ is the magnetic constant, g/meters; $\gamma_g$ is the specific conductivity of the ground, Siemens/meter; $\omega$ is the angular frequency; D is the spacing between the conductor subjected to the interference and the mirror reflection of the interfering conductor, meters; d is the shortest distance between wires, meters; $i = \sqrt{-1}$. The Pollaczek formulas for determining the values of the mutual inductance between the single-wire circuits suspended above the ground with uniform structure have the following form:

1) for small values of the parameter $|k|\sqrt{a^2+(b+c)^2} \leqslant 0{,}5$

$$M_{1A} = \left\{2\ln\frac{12{,}66}{\sqrt{\gamma_з f_к [a^2+(b-c)^2]}} + 1 - \right.$$
$$\left. - i\left(\frac{\pi}{2} + 0{,}1187\sqrt{\gamma_з f_к}\, e^{i\frac{3}{4}\pi}(b+c)\right)\right\} 10^{-4}, \tag{9.59}$$

where $f_k$ is the frequency of the interfering current, kilohertz; |k| is the modulus of the wave number, $|k| = 0{,}028\sqrt{10\gamma_з f_к} = 0{,}0887\sqrt{\gamma_з f_к} = 0{,}28\sqrt{0{,}1\gamma_з f_к}$;

2) for average values of the parameter $0{,}5 < |k|\sqrt{a^2+(b+c)^2} \leqslant 3$

$$M_{1\text{-A}} = \left\{ 4\frac{\mathrm{kei}'(0{,}089\sqrt{\gamma_э f_к}\,a) - i\,\mathrm{ker}'(0{,}089\sqrt{\gamma_э f_к}\,a)}{0{,}089\sqrt{\gamma_э f_к}\,a} - \right.$$
$$\left. - \frac{507\,e^{-i\frac{6}{4}\pi}}{\gamma_э f_к a^2} \right\} 10^{-4}, \tag{9.60}$$

where ker |ka| and kei(ka) are the real and imaginary parts of the first order second type Kelvin function or the modified first order, second type Bessel function of the complex argument ka;

3) for large values of the parameter $|k|\sqrt{a^2+(b+c)^2}>3$

$$M_{1\text{-A}} = \left[ \ln\frac{a^2+(b+c)^2}{a^2+(b-c)^2} + \frac{10(b+c)}{a^2\sqrt{0{,}1\gamma_э f_к}} - \right.$$
$$\left. - i\left( \frac{10(b+c)}{a^2\sqrt{0{,}1\gamma_э f_к}} + \frac{510}{a^2\gamma_э f_к} \right) \right] 10^{-4}. \tag{9.61}$$

As is obvious from the presented Pollaczek formulas, for calculating the values of the mutual inductance significant time is required. In order to facilitate and accelerate the use of these formulas in Figure 9.6, the curves $M = \phi(a\ 10\sqrt{\gamma_g f})$ are presented which were calculated by the Pollaczek formulas, where f is the frequency of the interfering current, hertz. The curves presented in Figure 9.6 offer the possibility of determining the magnitude of the mutual inductance between the single-wire circuits on variation of the the parameter $a\sqrt{10\gamma_g f}$ within broad limits (from 0.01 to 100,000). Three scales are presented on the y-axis for the value of M (for the curves I and II, one scale), and on the x-axis, three scales for the parameter $a\sqrt{10\gamma_g f}$ (for the curves III and IV — one scale). These curves offer the possibility of determining the conductivity of the ground $\gamma_g$ by the known values of the mutual inductance M and the frequency of the interfering current f. For this purpose, plotting the known value of M on the y-axis, on the x-axis we find the corresponding value of the parameter A: $A = a\sqrt{10\gamma_g f}$. Then, substituting the values of a and f for which the value of M was measured in this equality, we calculate the value of $\gamma_g$.

In practice, most frequently the effect is calculated for frequencies of 50 and 800 hertz. In Figures 9.7 and 9.8, nomograms are presented for determination of the mutual inductance between the single-wire circuits at these frequencies [58]. The examples of the use of the nomograms are shown in the figures.

For acceleration of the determination of the magnitudes of the mutual inductance between the single-wire circuits for defined values of the specific resistance of the ground it is possible to use the curves $M = \phi(a)$ calculated by the Pollaczek formula and the curves $E = \phi(a)$ under the effect of the

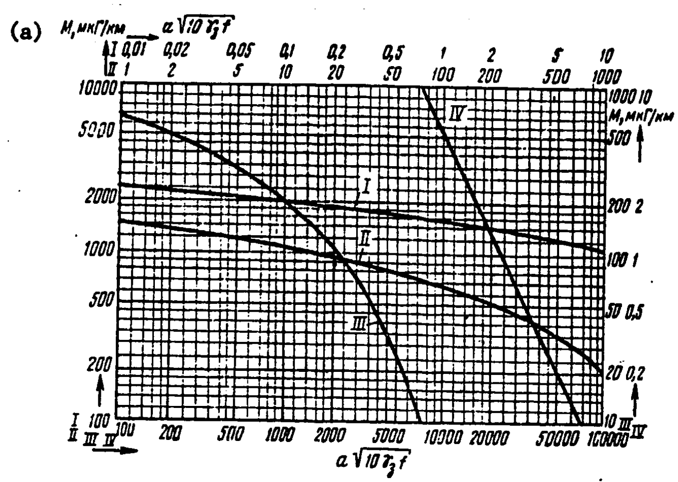



Figure 9.6. Mutual inductance between the single-wire circuits as a function of the parameters $a\sqrt{10\gamma_g f}$

Key: a. M, μg/km

current of frequency 50 hertz and magnitude 1 kiloamp (Figure 9.9). In addition, for determination of the magnitude of the mutual inductance between the single-wire circuits it is possible to recommend the approximate formula for the modulus M obtained by M. V. Kostenko by simplifying the Carson-Pollaczek formulas [59]:

$$M_{1\text{-}A} = 2\cdot 10^{-4}\ln\frac{\sqrt{a_s^2+(b+c+800\sqrt{\rho_s/f})^2}}{\sqrt{a_s^2+(b-c)^2}}, \qquad (9.62)$$

where $a_e$ is the spacing between the single-wire circuits (along the horizontal) meters; b -- the height of suspension of the interfering circuit, meters; c -- the height of suspension of the wire subjected to the effect, meters; $\rho_g$ is the specific resistance of the ground, ohm-meters; f is the frequency of the interfering current, hertz.

The error in the calculation by the approximate formula is permitted in the direction of the margin and in the majority of cases does not exceed 10%; in comparatively rare cases the error can reach up to 20%.

Mutual Induction for Oblique Convergence Sections of Single-Wire Lines. For determination of the modulus of the mutual inductance between the single-wire lines on oblique convergence sections frequently the same formulas and nomograms are used which are indicated above for parallel convergence with

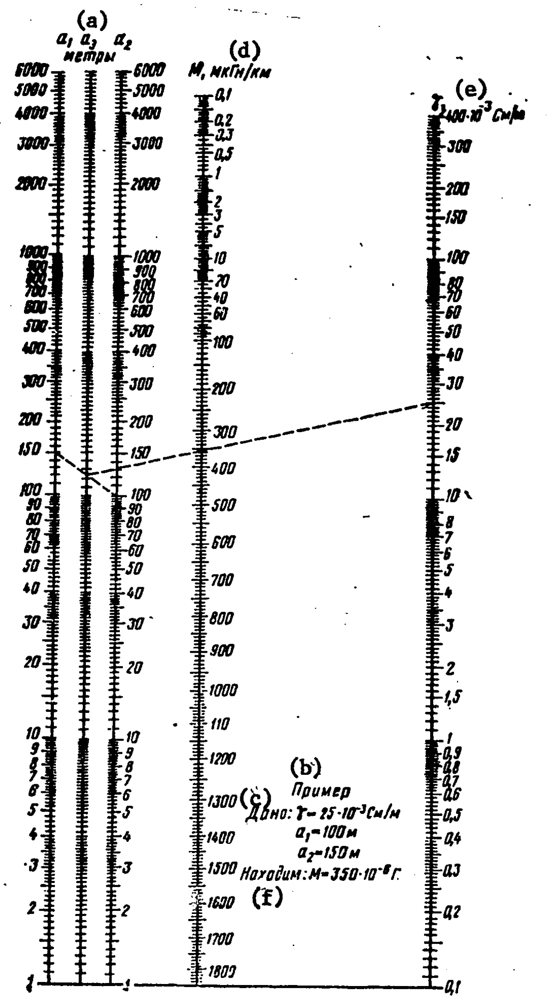



Figure 9.7. Nomogram for determining the mutual inductance between the single-wire circuits for f = 50 hertz as a function of a and $\gamma_g$.

Key:
a. meters
b. example
c. given:
d. M, μg/km
e. $\gamma_g$, Siemens/meter
f. we find: $M = 350 \cdot 10^{-6}$ g.

some equivalent spacing between lines ($a_e$). It is possible to replace the oblique conversions by parallel with equivalent spacing between the lines ($a_e$) defined by the following formulas with an error not exceeding 5-10%:

$$a_{\text{э}} = \frac{a_1 + a_2}{2} \quad \text{for} \quad a_1 \leqslant a_2 < 2a_1,$$

$$a_{\text{э}} = \sqrt{a_1 a_2} \quad \text{for} \quad a_1 \leqslant a_2 < 3a_1,$$

$$a_{\text{э}} = \frac{1}{3}(a_1 + 2a_2) \quad \text{for} \quad a_1 \leqslant a_2 < 5a_1.$$

(1)

Key: 1. e

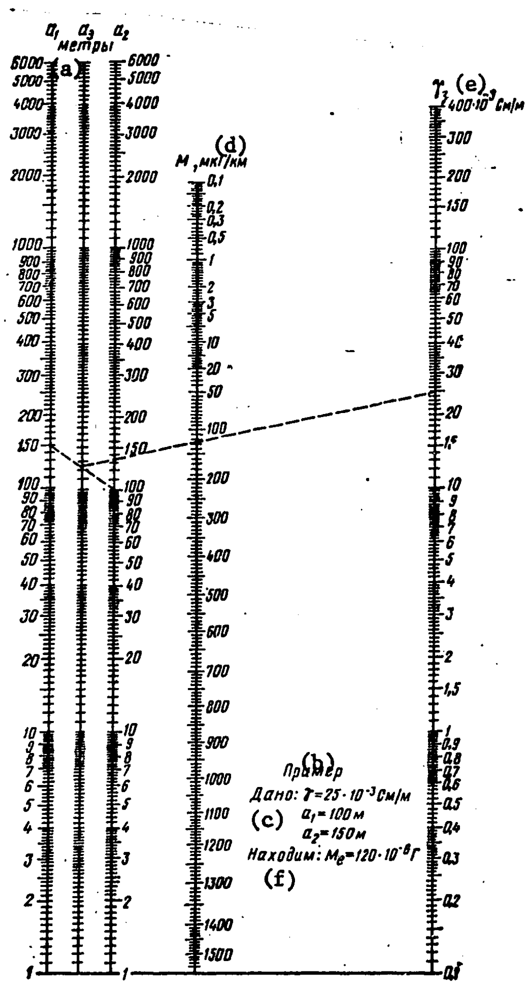



**Figure 9.8. Nomogram for determining the mutual inductance between the single-wire circuits for f = 800 herts as a function of a and $\gamma_g$.**

Key: a. meters; b. example; c. given; d. M, μg/km; e. $\gamma_g$, Siemens/meter; f. we find: $M = 120\cdot10^{-6}$ g

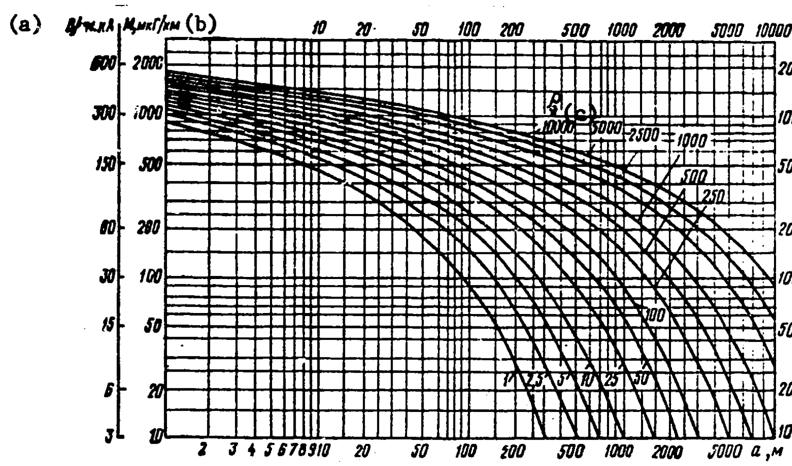



Figure 9.9. Curves for the mutual inductance between single-wire circuits calculated by the Pollaczek formula and curves for the induced emf on a length of 1 km with an interfering current of 1 kiloamp at a frequency of f = 50 hertz for regions with specific resistance of the ground of $\rho_g$ (ohm-m).

Key: a. B, km/kiloamp  c. $\rho_g$
b. M, μg/km

The presented conditions make it necessary to divide the oblique convergence section of long length into the corresponding individual sections. With this method of determining the value of M the calculation of the interference is naturally complicated.

The method is demonstrated below for a more exact determination of the mutual resistance between single-wire lines on oblique convergences of them. Here it is proposed that the conductivity of the ground and the height of suspension of the wires do not vary over the entire section of the oblique convergence.

In the oblique convergence section with limiting spacings between the lines with respect to the ends of the section $a_1$ and $a_2$, the total magnetic coupling resistance per unit length (1 km) will vary from $\omega M_1 = \phi(a_1)$ to $\omega M_2 = \phi(a_2)$. The mean value of the total magnetic coupling resistance per unit length of the oblique convergence section can be expressed by the formula

$$\omega M_{cp} = \varphi(x_{cp}) = \frac{1}{a_1 - a_2} \int_{a_1}^{a_2} \varphi(x)\, dx = \frac{1}{(a_2 - a_1)\operatorname{ctg} \alpha} \int_{a_1}^{a_2} \varphi(x) \operatorname{ctg} \alpha \, d\alpha, \qquad (9.63)$$

where x is the variable spacing between lines where $a_1 \leq x < a_2$; $\phi(x)$ is the total magnetic coupling resistance at the investigated point expressed by the Carson or Pollaczek formula; $\alpha$ is the angle between the directions of the investigated lines.

Formula (9.63) can be used on variation of the spacing between the lines. Here it is necessary to note that $\omega M_{mean}$ can be equal to zero, that is, by (9.63) it is possible to determine the mean value of the total magnetic coupling resistance also for the case of intersection of the lines.

The analytical expression of this formula in expanded form after substitution of the Carson or Pollaczek expressions in it in place of $\phi(x)$ is extremely complicated, and the analytical integration of this functions presents significant difficulties, but with sufficient accuracy for technical purposes the value of the integral $\int_{a_1}^{a_2} \varphi(x)dx$ can be found by numerical integration.

Let us imagine the investigated section of oblique convergence as separated into elementary convergence sections with insignificant difference in the spacings between the lines at the ends of the sections $dx = x_{i+1} - x_i$. For each such elementary section let us determine the numerical value of the expression under the integral sign entering into the above-presented formula, setting $\phi(x)$ equal to half the sum of the upper bound $\phi(x_{i+1})$ and lower bound $\phi(x_i)$ of the total magnetic coupling resistance in the investigated element. The sum of the numerical values of the expression under the integral sign obtained for all the elements of the given section of the oblique convergence is an approximate value of the integral entering into (9.63):

$$\int_{a_1}^{a_2} \varphi(x)\,dx \approx \sum_{i=1}^{i=n} \varphi(x)\,(x_{i1} - x_i),$$

where n is the number of elements into which the section of oblique convergence bounded by the y-axes $a_1$ and $a_2$ is divided; x is the spacing between the lines for the element $(x_{i+1} - x_i)$ corresponding to the mean value of the function $\phi(x)$ in this interval.

Varying the upper integration limit ($a_2$) in (9.63) with constant value of the lower limit ($a_1$), it is possible to obtain the relation in the numerical expression of the mean value of the total magnetic coupling resistance for the oblique convergence sections as a function of the maximum spacing between lines.

For example, in Figure 9.10 we have the value of $\omega M$ (ohm/km) as a function of the spacing between the parallel lines for the case $\gamma_g = 0.0265$ Siemens/m with a height of the interfering line of b = 10 meters and with a height of

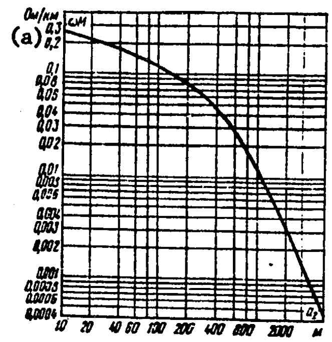


Figure 9.10. Total magnetic coupling resistance as a function of spacing between parallel single-wire circuits for $\gamma_g$ = 0.0265 Siemens/m and f = 50 hertz.

Key: a. ohm/km

the line subjected to the effect c = 6 meters. For the same conditions the above-described method can be used to calculate the mean values of the total magnetic coupling resistance for the sections of oblique convergence with different invariant minimum spacings and variable maximum spacings between lines. By the results of these calculations the curves presented in Figure 9.11 are constructed. Curve 1 expresses $\omega M$ as a function of a = $a_2$ for parallel convergence sections. The remaining curves express the mean value of $\omega M$ (ohm/km) for oblique convergence sections as a function of the maximum spacing between lines ($a_2$) for the given oblique convergence section with constant values of the minimum spacing ($a_1$) between the lines equal to 0, 10, 100 and so on meters.

When calculating the electromagnetic effect in the regions where the conductivity of the ground $\gamma_g$ = 0.0265 Siemens/m, the use of the curves (9.11) reduces to the following:

1) the point corresponding to the maximum spacing between lines for the given oblique convergence section is noted on the x-axis of these curves;

2) from the noted point, the perpendicular is drawn to the x-axis, and it is continued to the intersection with this curve on which on the right the minimum spacing between lines which the given oblique convergence section has is noted;

3) the value of the y-axis of the point of intersection of this curve and the indicated perpendicular is the mean value of the total magnetic coupling resistance per unit length (ohm/km) of the given oblique convergence section.

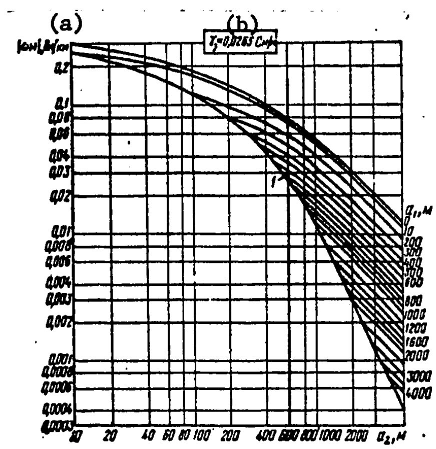



Figure 9.11. Curves for the total magnetic coupling resistance as a function of the spacing between lines on the oblique convergence sections for f = 50 hertz ($\gamma_g$ = 0.0265 Siemens/meter, b = 10 m and c = 6 m). Curve 1 corresponds to $a_1 = a_2$.

Key: a. ohm/km b. $\gamma_g$ = 0.0265 siemens/m

In accordance with what has been discussed, the longitudinal emf on the complex convergence path will be defined by the formula

$$\dot{E} = I \sum_{i=1}^{n} \omega M_i l_i. \tag{9.64}$$

For each terrain having a defined value of the specific conductivity of the ground when calculating $\omega M$ for the oblique convergence sections it is necessary to have special curves $\omega M = \phi(a)$ constructed for the given values of the ground conductivity at a frequency f. The curves for the specific conductivity of the ground $\gamma_g$ = 0.01 Siemens/meter are presented in Figure 9.12. In this figure, the maximum value of the spacing $a_2$ between the lines on the oblique convergence section is plotted on the x-axis, and the value of the active component of the total magnetic coupling resistance R or the active component (X) is plotted on the y-axis. The indicated function is presented in the form of curves for a number of values of the maximum spacing $a_1$ between lines in the oblique convergence section. The magnitude of the total resistance $\omega M = R + iX$.

The modulus of the value of $\omega M$ which must be substituted in the formula for determining the longitudinal emf will be equal to the square root of the sum of the squares of the active and reactive components, that is, $|\omega M| = \sqrt{R^2+X^2}$, where R is determined from the curve $R = \phi_1(a)$, X is determined from the curves $X = \phi_2(a)$.

From the presented curves it follows that the active components of the total magnetic coupling resistance have little significance when determining the modulus $\omega M$ for small spacings between the lines and for low conductivity of the ground. If we do not consider the active component of the total magnetic coupling resistance, then for all spacings between the lines the error when determining the value of M does not exceed the following: 5% for $\gamma_g = 0.1 \cdot 10^{-3}$ Siemens/meter, 10% for $\gamma_g = 2 \cdot 10^{-3}$ Siemens/meter, 20% for $\gamma_g = 10 \cdot 10^{-3}$ Siemens/meter, 25% for all other larger values of $\gamma_g$.

In addition to the indicated method of determining the mutual inductance in the oblique convergence connections there are several other procedures proposed, for example, by Lacey [60], and so on. The simplest formula for determining the value of M in the oblique convergence sections proposed by Kolle and recommended for practical calculations by the MKKTT [International Telegraph and Telephone Consultative Committee] has the following form:

$$M = \frac{1000}{a_2 - a_1} l \left[ G(a_2) - G(a_1) \right]. \qquad (9.65)$$

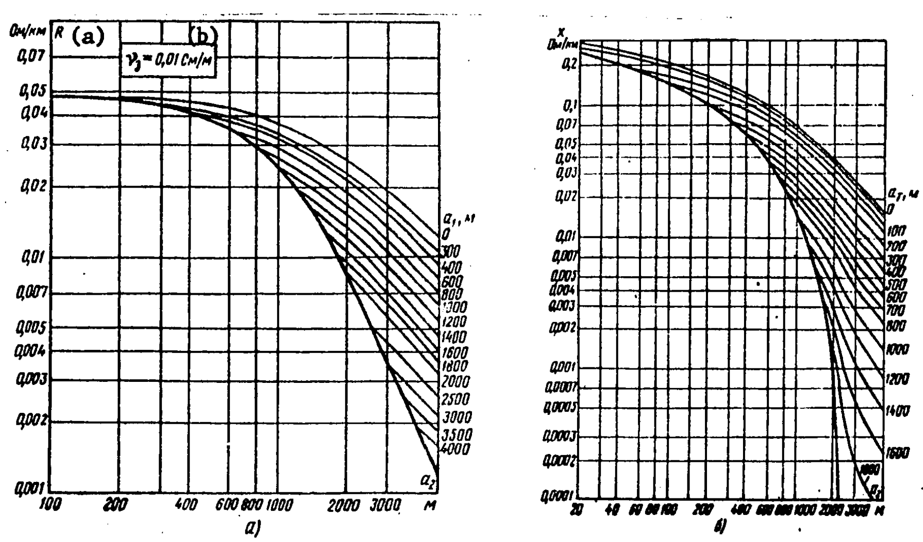



Figure 9.12. Curves for function $\omega M = \phi(a_1, a_2)$ for $\gamma_g = 0.01$ Siemens/m, f = 50 hertz: a) active components; b) reactive components.

Key: a. ohm/km b. Siemens/m

In this formula the functions $G(a_2)$ and $G(a_1)$ are determined from the equation

$$G(a) = a \ln\left(1 + \frac{R^2}{a_2}\right) + 2k \operatorname{arctg} \frac{a}{k},$$

where $\ell$ is the length of the oblique convergence section, km; $k = 774/\sqrt{\gamma_g f} \approx$ $\approx 800 \sqrt{\rho_g/f}$; f is the frequency, hertz; $\gamma_g$ is the specific conductivity of the ground, Siemens/m; $a_2$ and $a_1$ are the spacings between the lines on the ends of the oblique convergence section, meters.

Mutual Inductance between Short Single-Wire Lines. In practice it is necessary to consider the magnetic effect between short lines in two cases:

for calculations of the dangerous effect of the overhead line on the communications line sections which are located near the place where the high voltage line phase is short circuited;

for determining the specific resistance of the ground by ac current using two "conductor-ground" loops.

For determination of the longitudinal emf (E) in this case it is impossible to use the Pollaczek formulas for M, for in the first case the value of E will be somewhat high, and in the second case the defined magnitude of the specific resistance of the ground will be somewhat low. The approximate substantiation of the reduction in magnitude of the mutual inductance between the overhead line and the communications line in the convergence section adjacent to the short circuit point is presented in [59].

By the case of short-circuit on a high-voltage line fed from two sides, it is recommended that the values of the mutual inductance in the convergence section adjacent to the short-circuit point be determined approximately by the following formula:

$$M_K = M_\infty \left[1 - \frac{(D-d)(\dot{I}_1 - \dot{I}_2)}{2\dot{I}_1 \ln \frac{D}{d}}\right], \tag{9.66}$$

where $M_\infty$ is the magnitude of the mutual inductance for infinitely long lines; $D = \sqrt{a^2+(b-c+2h_0)^2}$; $d = \sqrt{a^2+(b-c)^2}$; $\dot{I}_1$ is the short circuit current in the overhead line flowing from left to right, amps; $I_2$ is the same flowing from right to left, amps; a is the spacing between the convergence line and the communications line, meters; b is the mean height of suspension of the wires of the overhead line, meters; c is the average height of suspension of the communications line, meters; $h_0 \approx 400 \sqrt{\rho_g/f}$ is the depth of the "equivalent plane" of the zero potential, meters; $\rho_g$ is the specific resistance of the ground; f = 50 hertz.

If the phase short circuit occurred on the overhead line fed from one direction, then $I_2 = 0$ and then $M_k$ will be defined by the formula

$$M_{\kappa} = M_{\infty}\left[1 - \frac{D-d}{2\ln\frac{D}{d}}\right]. \qquad (9.67)$$

The closeness of the presented formulas (9.66) and (9.67) is explained primarily by the fact that these formulas do not take into account the galvanic effect of the overhead line on the communications circuit through the grounded ends of the overhead line and the communications line. More exact formulas for determining the mutual inductance M between the single-wire circuits of limited length fully taking into account the finite effect were derived by L. G. Pozdnyakov [61].

## 9.9. Determination of the Mutual Inductance Between Single-Wire and Double-Wire Circuits

The magnitude of the mutual inductance between the circuits suspended on one column line will depend primarily on the mutual arrangement of these circuits. The effect of the ground on the magnitude of the mutual inductance coefficient in this case will be felt very little. The active component $R_{12}$ of the magnetic coupling resistance $Z_{12}$ in this case can be neglected. The magnitude of the mutual inductance between the single-wire and the double-wire circuits without considering the effect of the ground is defined as follows.

In Figure 9.13, the relative arrangement of the single-wire and double-wire circuits is shown. Let an ac sinusoidal current flow in the wire 1, the instantaneous value of which is equal to $i_1$. The magnitude of the magnetic flux penetrating the loop of the two-wire circuit is determined from the equation

$$\Phi_1 = \int_{a_{1A}}^{a_{1Б}} H_x l_p\,dx = \int_{a_{1A}}^{a_{1Б}} \frac{0,2\,i_1}{x}\, l_p\,dx = 0,2\,i_1\,l_p \ln\frac{a_{1Б}}{a_{1A}}, \qquad (9.68)$$

where $\ell_p$ is the parallel convergence length, cm.

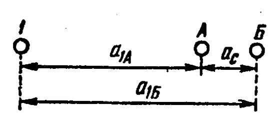



Figure 9.13. Determination of the mutual inductance between single-wire and double-wire circuits.

The emf induced in the loop formed by a two-wire circuit will be defined by the formula $e = (-d\Phi/dt)10^{-8}$. Substituting the value of $\Phi_1$, we obtain

$$e = -\frac{d}{dt}\left[0{,}2\, i\, l_p \ln \frac{a_{1Б}}{a_{1A}}\right] 10^{-8}.$$

Since $i = I\sqrt{2}\, e^{i\omega t}$, then

$$\dot{E} = -i\, 0{,}2\omega l_p \dot{I}_1 \ln \frac{a_{1Б}}{a_{1A}} 10^{-7}. \qquad (9.69)$$

Let us rewrite this equation, expressing the length $\ell_p$ in kilometers. Then we obtain

$$\dot{E} = -i\omega\, 2\, l_p \dot{I}_1 \ln \frac{a_{1Б}}{a_{1A}} 10^{-4}. \qquad (9.70)$$

Since for an overhead line, the induced emf in the double-wire circuit is defined by the formula $\dot{E} = -i\omega\ell_p \dot{I}_1 M_{1-T}$, comparing the last two expressions, we find

$$M_{1\text{-}T} = 2 \ln \frac{a_{1Б}}{a_{1A}} 10^{-4}. \qquad (9.71)$$

In the case of suspension of the interfering circuit and the circuit subjected to interference on two different lines, the return current of the interfering single-wire circuit propagated through the ground can have a noticeable effect on the magnitude of the induced emf in the two-wire circuit, that is, the magnitude of the mutual induction coefficient between the indicated circuits will depend on the magnitude of the specific conductivity of the ground through which the interfering current flows.

The magnitude of the mutual induction between the single-wire and the two-wire circuits in this case can be defined by the formula

$$M_{1\text{-}T} = \frac{\partial M_{1\text{-}A}}{\partial a} a_c \qquad (9.72)$$

for suspension of the double-wire circuit on the crosspieces and by the formula

$$M_{1\text{-}T} = \frac{\partial M_{1\text{-}A}}{\partial c} \Delta c \qquad (9.73)$$

for suspension of the double-wire circuit on hooks. Here $\Delta c = a_c \sin \alpha$, where $\alpha$ is the angle of inclination of the plane in which the wires of the two-wire circuit are located.

The indicated expressions include the value of $M_{1-A}$ defined by the Pollaczek or Carson formulas (see above).

Thus, for suspension of the two-wire circuit on crosspieces:

a) for the parameters $|k| \sqrt{a^2+(b+c)^2} \leqslant 0{,}5$, considering that

$$M_{1\text{-A}} = \left\{2\ln\frac{12{,}66}{\sqrt{\gamma_з f_к[a^2+(b-c)^2]}}+1-\right.$$
$$\left.-i\left(\frac{\pi}{2}+0{,}119\sqrt{\gamma_з f_к}\,e^{i\frac{3}{4}\pi}(b+c)\right\}10^{-4}, \qquad (9.74)$$

$\partial M_{1\text{-A}}/\partial a = 2\cdot10^{-4}a/[a^2+(b-c)^2]$ and, consequently, $M_{1\text{-T}} = 2aa_c\cdot10^{-4}/[a^2+(b-c)^2]$. For $a > 20$ m it is possible with accuracy to 5% to take the formula for $M_{1-T}$ in the form $M_{1-T} = 2a_c\cdot10^{-4}/a$;

b) for the parameter $0.5 < |ka| \leq 3$

$$M_{1\text{-A}} = \left\{\frac{\text{kei}'|ka| - i\,\text{ker}'|ka|}{|ka|} - \frac{507\,e^{-i\frac{6}{4}\pi}}{\gamma_з f_к a^2}\right\}10^{-4}.$$

Here $K_0(|ka|\sqrt{i}) = \text{ker}(|ka|) + i\,\text{kei}(|ka|)$, $K'_0(|ka|\sqrt{i}) = \text{ker}'(|ka|) + i\,\text{kei}'(|ka|)$, where $\text{ker}|ka|$ and $\text{kei}|ka|$ are the real and imaginary parts of the modified first order second type Bessel function of a complex argument. Taking the derivative with respect to a, we obtain

$$\frac{\partial M_{1\text{-A}}}{\partial a} = \left\{\frac{8}{e^{i\frac{6}{4}\pi}|ka|^3} + \frac{4}{|ka|}K_0(|ka|\sqrt{i}) + \frac{8i}{|ka|}K'_0(|ka|\sqrt{i})\right\}|k|\,10^{-4}$$

and, consequently,

$$M_{1\text{-T}} = \frac{\partial M_{1\text{-A}}}{\partial a}a_c = \left\{\frac{8}{e^{i\frac{6}{4}\pi}|ka|^3} + \frac{4}{|ka|}K_0(|ka|\sqrt{i}) + \frac{8i}{|ka|}K'_0(|ka|\sqrt{i})\right\}|k|\,a_c\,10^{-4};$$

c) for the parameter $|ka| > 3$, $M_{1-A}$ is defined by the formula

$$M_{1\text{-A}} = \left[\ln\frac{a^2+(b+c)^2}{a^2+(b-c)^2} + \frac{10(b+c)}{a^2\sqrt{0{,}1\gamma_з f_к}} - \right.$$
$$\left.-i\left(\frac{10(b+c)}{a^2\sqrt{0{,}1\gamma_з f_к}} + \frac{510}{a^2\gamma_з f_к}\right)\right]10^{-4},$$
$$\frac{\partial M_{1\text{-A}}}{\partial a} = -2\left[\frac{4abc}{a^2[a^2+2(b^2+c^2)]+(b+c)^2(b-c)^2} + \frac{10(b+c)}{a^3\sqrt{0{,}1\gamma_з f_к}} - \right.$$
$$\left.-i\left(\frac{10(b+c)}{a^3\sqrt{0{,}1\gamma_з f_к}} + \frac{510}{a^3\gamma_з f_к}\right)\right]10^{-4};$$

for a > 100 m

$$\frac{\partial M_{1\text{-A}}}{\partial a} \approx \frac{i\,1020}{a^3\gamma_з f_к}10^{-4}; \quad M_{1\text{-T}} = \frac{\partial M_{1\text{-A}}}{\partial a}a_c = i\,0{,}1\frac{a_c}{a^3\gamma_з f_к}.$$

## 9.10. Determination of the Mutual Inductance Between Two-Wire Circuits

When determining the mutual inductance between two-wire circuits suspended on one column line, the effect of the ground in which the eddy currents occur, in view of the small distance between the conductors of the circuits by comparison with their distance to the ground, can be neglected just as in the derivation of (9.73) for $M_{1-T}$. Therefore the active component of the mutual resistance is taken equal to zero. In this case the magnitude of the mutual inductance between the two-wire circuits will be determined only by their mutual arrangement. If we denote the instantaneous currents in the conductors of the interfering circuit (Figure 9.14) by $i_1$ and $i_2$, then the total magnetic flux created by these currents in the A-B loop will be expressed by the equality

$$\Phi = \Phi_1 + \Phi_2 = 0{,}2\, i_1 l_p \left( \ln \frac{a_{1Б}}{a_{1A}} - \ln \frac{a_{2Б}}{a_{2A}} \right). \quad (9.75)$$

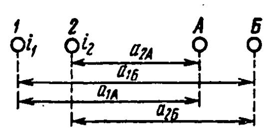



Figure 9.14. Determination of the mutual induction between the two-wire circuits.

The value of the emf in the loop of this two-wire circuit will be determined from the equation

$$e = 0{,}2\, l_p \left( \ln \frac{a_{1Б}}{a_{1A}} - \ln \frac{a_{2Б}}{a_{2A}} \right) \frac{di_1}{dt} 10^{-8}.$$

For a sinusoidal current this value will be determined from the equation

$$e = -0{,}2\, l_p \left( \ln \frac{a_{1Б}}{a_{1A}} - \ln \frac{a_{2Б}}{a_{2A}} \right) 10^{-8} \cdot i\, \omega I_1 \sqrt{2}\, e^{i \omega t}.$$

The effective value of the indicated electromotive force

$$\dot{E} = -i\, 2 \omega l_p \dot{I}_1 \ln \frac{a_{1Б}\, a_{2A}}{a_{1A}\, a_{2Б}} 10^{-9}.$$

Expressing the length in kilometers, we obtain

$$\dot{E} = -i\, 2 \omega l_p \dot{I}_1 \ln \frac{a_{1Б}\, a_{2A}}{a_{1A}\, a_{2Б}} 10^{-4}.$$

Hence the magnitude of the mutual inductance reduced to one kilometer of convergence will be defined by the expression

$$|M_{T.T}| = 2 \cdot 10^{-4} \ln \frac{a_{1B} a_{2A}}{a_{1A} a_{2B}}. \tag{9.76}$$

The magnitude of the mutual induction between two two-wire circuits suspended on separate overhead lines will be calculated considering the effect of the ground. In this case

$$M_{12T} = \left( \frac{\partial M_{1T}}{\partial a} - \frac{\partial M_{2T}}{\partial a} \right) a_c. \tag{9.77}$$

The calculated formulas for $M_{12T}$ for suspension of the wires of the circuit subjected to the effect on crosspieces, hve the form:

a) for small distances between the circuits where $|ka| \leq 0.5$,

$$M_{12T} = \frac{2 a_c \delta}{a^2} 10^{-4};$$

b) for average distances between the circuits where $0.5 < |ka| < 3$,

$$M_{12 \cdot T} = \left[ 1 - \left( \frac{a}{a+\delta} \right)^3 \right] \left[ -\frac{4bc}{a^2} - \alpha_1 + i(\alpha_1 + \alpha_2) \frac{2a_c}{a} \right] 10^{-4};$$

c) for large distances between the circuits where $|ka| > 3$,

$$M_{12 \cdot T} = \frac{24 a_c \delta}{|ka|^2 a^2} 10^{-4}.$$

## 9.11. Determination of the Mutual Inductance Between the Three-Phase Symmetric Line and the Communication Circuits

The formula for the mutual induction between the three-phase and the single-wire circuits has the form

$$M_{123 \cdot A} = \sqrt{3} \frac{\partial M_{1A}}{\partial a} \delta. \tag{9.78}$$

Using this formula, in special cases we obtain:

for small values of the parameter where $|ka| < 0.5$, $a > 20$ m

$$M_{123 \cdot A} = \sqrt{3} \frac{2\delta \cdot 10^{-4}}{a};$$

for average values of the parameter $0.5 < |ka| \leq 3$

$$M_{123\text{-}A} = \sqrt{3}\,|k|\,\delta \left\{ \frac{8}{e^{i\frac{6}{4}\pi}|ka|^3} + \frac{4}{|ka|} K_0(|ka|\sqrt{i}) + \frac{i\,8}{|ka|} K_0'(|ka|\sqrt{i}) \right\} 10^{-4};$$

for large values of the parameter $|ka| > 3$, $a > 100$ m

$$M_{123\text{-}A} = \sqrt{3}\,\delta \frac{\partial M_{1A}}{\partial a} = \sqrt{3}\,\delta \frac{1020\,i}{a^3 \gamma_3 f_K} 10^{-4} = 0{,}1765\,i \frac{\delta}{a^3 \gamma_3 f_K}.$$

The fifth investigated commission of the MKKTT [International Telephone and Telegraph Consultative Committee] for approximate calculations of the interfering effect of the triple-phase transformation line on the low frequency channel of a telephone circuit recommends determination of the magnitude of the mutual inductance between the triple-phase line and the conductor of the telephone circuit by the curve $M_{123-A} = \phi(a)$ constructed as a function of the spacing between them for f = 800 hertz for an average conductivity of the ground equal to $\gamma_g = 10 \cdot 10^{-3}$ Siemens/m (Figure 9.15).

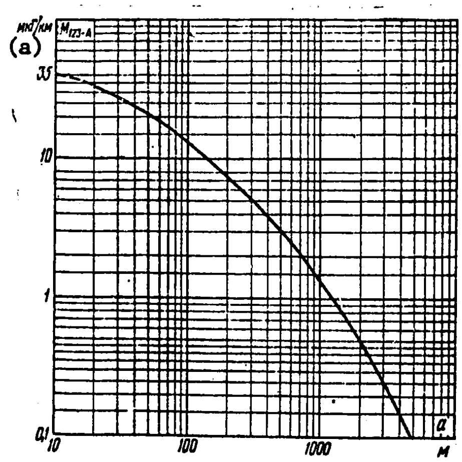



Figure 9.15. Curve for the mutual inductance between the triple-phase transmission line and the single-wire communications line as a function of the spacing between them $M_{123-A} = \phi(a)$ for f = 800 hertz.

Key: a. μg/km

The formula for the mutual induction between the triple-phase and the two-wire circuits has the form

$$M_{123\text{-}\mathrm{T}} = \sqrt{3}\,\frac{\partial^2 M_{1A}}{\partial a^2}\,\delta a_c.$$

Using this formula, in special cases we obtain:

for small values of the parameter $|ka| \le 0.5$

$$\frac{\partial^2 M_{1A}}{\partial a^2} = 2\cdot 10^{-4}\,\frac{a^2-(b-c)^2}{[a^2+(b-c)]^2};$$

$$M_{123\text{-}\mathrm{T}} = 2\sqrt{3}\cdot 10^{-4}\,\delta\,\frac{a_c}{a^2};$$

for the mean values of the parameter $0.5 < |ak| < 3$

$$M_{123\text{-}\mathrm{T}} = \sqrt{3}\,\delta a_c\,\frac{4k^2}{|ak|^2}\left\{-\frac{6}{a^2k^2} - 3K_0(|ak|\sqrt{i}) + \right.$$
$$\left. + K_1(|ak|\sqrt{i})\left(|ka| - \frac{6i}{|ka|}\right)\right\}10^{-4};$$

for large values of the parameter $|ka| > 3$

$$\frac{\partial^2 M_{1A}}{\partial a^2} = -0{,}306\,i\,\frac{1}{a^4\gamma_3 f_\kappa},$$

$$M_{123\text{-}\mathrm{T}} = -0{,}53\,i\,\frac{a_c\delta}{a^4\gamma_3 f_\kappa}.$$

In these formulas $\delta$ is the geometric mean spacing between the conductors of the triple-phase line, meters; $\delta = \sqrt[3]{\delta_{12}\delta_{23}\delta_{13}}$ ; $a_c$ is the spacing between conductors of the two-conductor circuit, meters; a is the spacing between the triple-phase line and the communications line, meters; $|k| = 0{,}28\sqrt{0{,}1\gamma_3 f_\kappa} = 0{,}089\sqrt{\gamma_3 f_\kappa} = 0{,}028\sqrt{10\gamma_3 f_\kappa}$; $f_k$ is the frequency, kilohertz; $\gamma_g$ is the specific conductivity of the ground, Siemens/m; $K_1(|ka|)$ is the modified first order second type Bessel function; $K_0(|ka|)$ is the same, zero order, second type.

### 9.12. Determination of the Active Component of the Parameter of the Magnetic Effect Between Single-Wire and DC Circuits

For close mutual arrangement of the grounds of two single-wire circuits it is necessary to consider the effect between them through the ground. The active component taking into account this effect, $R_{1A}$ will be defined as follows.

Let us take two single-wire circuits with ground at the points 1, 2, A and B (Figure 2.19b). The potentials at the point A and B of the circuits subjected to the effect will be defined by the formulas:

$$\dot U_A = \frac{\dot I\rho_3}{2\pi}\left(\frac{1}{r_{A1}} - \frac{1}{r_{A2}}\right),\quad \dot U_B = \frac{\dot I\rho_3}{2\pi}\left(\frac{1}{r_{B1}} - \frac{1}{r_{B2}}\right). \qquad (9.79)$$

The potential difference between the points A and B

$$\dot{U}_{AБ} = \dot{U}_A - \dot{U}_Б = \frac{\dot{I}\rho_з}{2\pi}\left(\frac{1}{r_{A1}} - \frac{1}{r_{A2}} - \frac{1}{r_{Б1}} + \frac{1}{r_{Б2}}\right),$$

Key: a. g

Where $\rho_g$ is the specific resistance of the ground, ohm-m; $r_{A1}$, $r_{A2}$, $r_{B1}$, $r_{B2}$ are the distances between grounds, meters.

Hence, we find that for 1 km of length of circuit subjected to the effect

$$R_{1A} = \frac{1}{l}\frac{\dot{U}_{AБ}}{\dot{I}} = \frac{\rho_з}{2\pi l}\left(\frac{1}{r_{A1}} - \frac{1}{r_{A2}} - \frac{1}{r_{Б1}} + \frac{1}{r_{Б2}}\right), \tag{9.80}$$

where $\ell$ is the length of the circuit subjected to the effect, km.

9.13. Specific Conductivity of the Ground

As is obvious from the above-presented formulas, the mutual inductance, in particular between the single-wire circuits, depends to a significant degree on the specific conductivity of the ground. For the last 30 or 40 years in the USSR and other countries a great deal of work has been done on determining the specific conductivity of the ground at different places. The greater part of the data were obtained by measuring the mutual induction between two "conductor-ground" loops and subsequent calculations by the Pollaczek and Carson formulas. The information about the specific conductivity of the ground (for a frequency of 50 hertz) for the country is presented in the map for the specific conductivity of the ground compiled in the first approximation by the TsNIIS Institute on the basis of numerous measurements and the study of the geological map of the USSR. This map with information about the specific conductivity of the ground within the boundaries of the European part of the USSR is presented in the Protection Rules [62].

The experiments have established that the apparent specific conductivity of the ground with its uniform structure at any depth remains constant, does not depend on the frequency. However, as a result of the surface effect, the depth of penetration of the current into the ground h depends on the current frequency. This dependence of the depth of penetration of the current on frequency with known conductivity of the ground is obvious from the following formula:

$$h = \frac{0,5 \cdot 10^3}{\sqrt{f\gamma_з}}, \tag{9.81}$$

Key: a. g

where f is the frequency, hertz; $\gamma_g$ is the conductivity of the ground, Siemens/m.

For layered structure of the ground with different specific conductivity of the layers, the total apparent specific conductivity varies as a function of frequency. It is known that the higher the current frequency, the less the depth of penetration into the ground as a result of the surface effect. The currents of various frequencies penetrating into the ground to different depths pass through the layers with different specific conductivity. Accordingly, the total apparent specific conductivity will be determined by the magnitudes of the conductivity of the layers of the ground through which the current of given frequency passes.

The laws of variation of the specific conductivity of the ground (siemens per meter) as a function of the current frequency are different for different geological structures of the ground. Thus, the following relations were established experimentally [63]:

1) for the vicinity of Zaporozh'ye $\gamma_g = 0.3\sqrt{f}\cdot 10^{-3}$;

2) for the vicinity of Nikopolye $\gamma_g = 0.8\sqrt{f}\cdot 10^{-3}$;

3) for the vicinity of the Donets basin $\gamma_g = (270/\sqrt{f})10^{-3}$;

4) for the vicinities of Tbilisi and Batumi $\gamma_g = (257/\sqrt{f})10^{-3}$;

5) for the vicinity of Magnitogorsk $\gamma_g = 0.37\sqrt{f}\cdot 10^{-3}$;

6) for the vicinity of Kemi $\gamma_g = 0.098\sqrt{f}\cdot 10^{-3}$;

7) for the vicinity of Apatita $\gamma_g = (47/\sqrt{f})10^{-3}$;

8) for the vicinities of Ol'denburg and Dobernya $\gamma_g = (150/\sqrt{f})10^{-3}$. In the general case it is possible to write:

$$\underset{(a)}{\gamma_з} = C_1\left(\frac{1}{f_n}\right) \quad \text{or} \quad \gamma_з = C_2\,(f^n), \tag{9.82}$$

Key: a. g

where $C_1$ and $C_2$ are certain constants; n is a positive number which can be greater than and less than one.

The first formula of system (9.82) pertains to cases where the specific conductivity in the upper layers of the ground is less than in the lower layers. The second formula pertains to cases where the specific conductivity in the upper layers is greater than in the lower layers.

The map of the specific conductivity of the ground was compiled only for a frequency of 50 hertz. For a frequency of 800 hertz or any other frequency the conductivies of the ground must be determined by (9.82) considering the values of $C_1$ and $C_2$ determined experimentally for different regions. Since the calculation of the electromagnetic effect is made by the mean apparent conductivity of the ground with a frequency of the interfering current, for terrains with layered structure of the ground where no measurements were taken, this mean value of the specific conductivity of the ground can be determined [57] by the curves for the function constructed by the formula

$$\frac{\gamma_{g0}}{\gamma_{g1}} = F(h_1 \sqrt{10\gamma_{g1} f}) \qquad (9.83)$$

(a)

Key: a. g

for the two-layered structure of the ground with different specific conductivities of each layer. These curves establish the relation between the specific conductivity of the upper horizontal layer $\gamma_{g1}$, the specific conductivity of the lower horizontal layer $\gamma_{g2}$, the apparent mean specific conductivity $\gamma_0$ (which also must be taken in the calculations of the electromagnetic effect), the depth of occurrence of the upper layer $h_1$ and the frequency f of the interfering current for different values of the ratio $\gamma_{g2}/\gamma_{g1}$.

In Figure 9.16, curves are presented for the following values of the ratio between the conductivities of the lower and upper layers: $\gamma_{g2}/\gamma_{g1} = 1/3$; 1/10; 1/30; 1/100; 1/300; 1/1000, and also $\gamma_{g2}/\gamma_{g1} = 3$; 10; 30; 100; 300; 1000.

Let us consider an example of the application of the indicated curves. Let us propose that it is necessary to determine the apparent specific conductivity of the ground for layered structure of it. The ground at the investigated point is made up of the surface layer of clay 50 meters thick located above the precambrian rock. The specific conductivity of the clay found in the given retion is $30 \cdot 10^{-3}$ siemens/m, and the precambrian rock (for example, granite, diabase, gneiss, and so on) has on the average a specific conductivity of $0.3 \cdot 10^{-3}$ siemens/m. Let us determine the apparent specific conductivity of the ground for the given region when f = 50 hertz and when f = 800 hertz. Since in the given case $\gamma_{g2}/\gamma_{g1} = 0.3 \cdot 10^{-3}/30 \cdot 10^{-3} = 1/100$, for determination of $\gamma_{g0}$ we use the third curve from the bottom in Figure 9.16a. The parameter $p = h_1 \sqrt{10 f \gamma_{g1}}$ for f = 50 hertz will be determined from the equality $p_{50} = 50\sqrt{10 \cdot 30 \cdot 10^{-3} \cdot 50} = 194$, after substitution of numerical values in it, and for f = 800 hertz, $p_{800} = 50\sqrt{10 \cdot 30 \cdot 10^{-3} \cdot 800} = 776$.

The parameter p = 194 according to the third curve from the bottom in Figure 9.16a corresponds to the relation $\gamma_{g0} = 0.074$; hence $\gamma_{g0} = 0.074\gamma_{g1}$. The parameter p = 776 by the same curve corresponding to the ratio $\gamma_{g0}/\gamma_{g1} = 0.7$; hence $\gamma_{g0} = 0.7\gamma_{g1}$. Therefore for a frequency of the interfering current

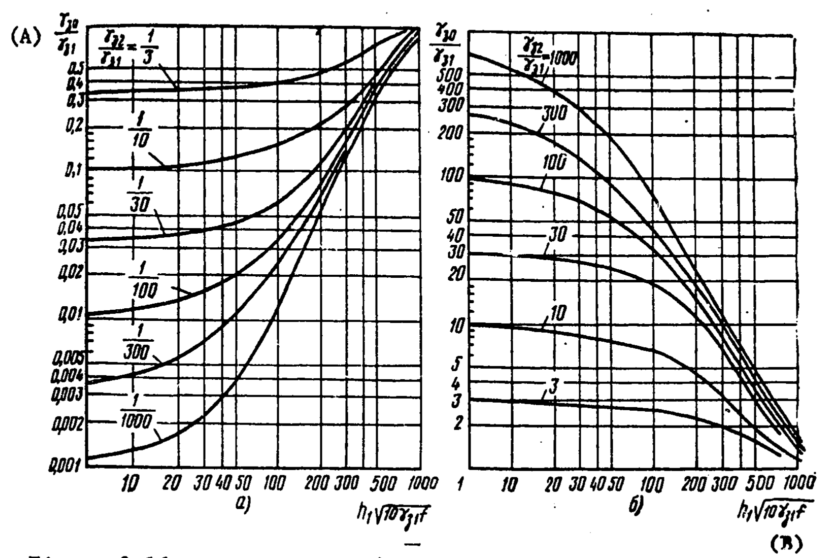



Figure 9.16. Curves for $\gamma_0/\gamma_{g1}$ as a function of the parameter $h_1 \sqrt{10\gamma_{g1}f}$: a) for $\gamma_{g2} < \gamma_{g1}$; b) for $\gamma_{g2} > \gamma_{g1}$.

Key: A. $\gamma_{g0}/\gamma_{g1}$ B. g

$f = 50$ hertz, $\gamma_{g0} = 0.074 \cdot 30 \cdot 10^{-3} = 2.2 \cdot 10^{-3}$ Siemens/m, and for a frequency of $f = 800$ hertz, $\gamma_{g0} = 0.7 \cdot 30 \cdot 10^{-3} = 21 \cdot 10^{-3}$ Siemens/m.

Consequently, for using the curves presented in Figure 9.16a and b it is necessary first to know the geological structure of the given terrain, the thickness of the upper layer and determine the specific conductivities of the rock entering into the upper and lower layers of the ground. The experiments of the TsNIIS Institute [65] confirm the possibility of practical application of curves presented in [57].

## 9.14. Measurement of the Specific Conductivity of the Ground

There are several methods of measuring the specific conductivity of the ground, but basically two are used -- induction with the application of ac current and the method of four electrodes, with the application of direct current. For the induction method, the measurements are taken either on two single-wire (the interfering circuit and the one subjected to interference) circuits or on one single-wire (the interfering circuit) circuit and the loop subjected to the effect of the circuit. The existence of the induction method of measurement using two single-wire circuits consists in the following. An insulated wire of length $\ell_1$ kilometers is laid in the measured section, both ends of which are grounded. The source (generator) of the current with a frequency of $f_1$ is connected to a section of the wire, and the magnitude of the steady-state current $\dot{I}_1$ is measured in the "conductor-ground" loop.

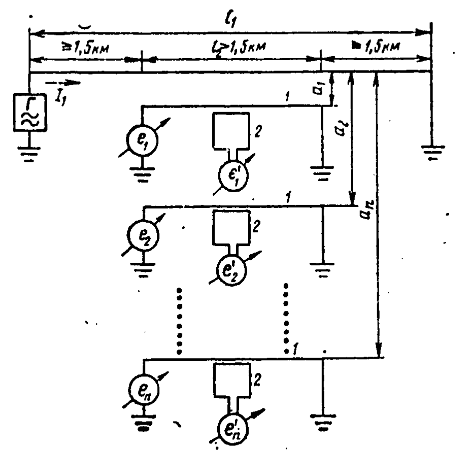



Figure 9.17. System for measuring the specific conductivity of the ground by the induction method when using single-wire circuits (1) and the loop (2).

In the parallel laid lines of length $\ell_2$ at distances of $a_1$, $a_2$, ..., $a_n$ of the interfering wire, the induced emf's $\dot{e}_1$, $\dot{e}_2$, ..., $\dot{e}_n$ are measured (see Figure 9.17). Knowing the values of the emf, the corresponding values of the coefficient of mutual inductance between the interfering wire and the other measuring wires are determined:

$$M_1 = \frac{\dot{e}_1}{2\pi f l_2 \dot{I}_1}; \quad M_2 = \frac{\dot{e}_2}{2\pi f l_2 \dot{I}_1} \dots \tag{9.84}$$

By the curves $M = \phi(a, f_1, \gamma_g)$ (see Figure 9.6) or by the corresponding nomograms (Figure 9.7) by the measured values of M, the values of the apparent specific conductivity of the ground $\gamma_{g0}$ are found for the frequency of the interfering current.

If the values of $\gamma_{g0i}$ calculated for all of the measured M for different distances between the single-wire circuits differ little from each other, this means that the geological structure of the ground is uniform at the depth of penetration of the current in the ground and that the ground is made up approximately of one rock. The experiments have established that the depth of penetration of the current in the uniform ground depends on the current frequency, the length of the interfering single-wire circuit and the conductivity of the ground. The direct and alternating low-frequency current to 50 hertz penetrated into the ground to a depth approximately equal to 1/3 of the length of the single-wire interfering circuit. The alternating current of higher frequency penetrates deep into the ground in a smaller thickness of the layer the higher the frequency and the greater the specific conductivity of the ground.

When measuring the apparent specific conductivity of the ground by the induction method using two single-wire circuits, the depth of penetration of the current into the ground is determined by (9.81) for the frequency of the interfering current. The interfering circuit must have a length equal to 3 h, and the circuit subjected to the effect, h. The values of h for different frequencies of the interfering current are different, and they fluctuate within broad limits depending on the apparent specific conductivity. When determining the conductivity of the ground in the investigated section the length of the measured lines is selected, presupposing low conductivity of the ground in the section (usually $2 \cdot 10^{-3}$ to $5 \cdot 10^{-3}$ Siemens/meter). The given method is comparatively simple, not requiring complex measurement equipment. However, it is quite difficult to use this method when performing the measurements in mountainous terrain, in populated areas and so on. In these cases, in order to facilitate the measurements, it is better to use the induction coils in the form of loops instead of the measuring lines.

The application of the measuring loop is substantiated by the following arguments. The mutual inductance m between the single-wire line and the turns of the loop is defined by the formula

$$m = ns\frac{dM}{da}, \tag{9.85}$$

where n is the number of turns; s is the area of the loop, $m^2$; M is the coefficient of mutual inductance between the single-line circuits, mg/m; a is the spacing between the interfering conductor and the frame.

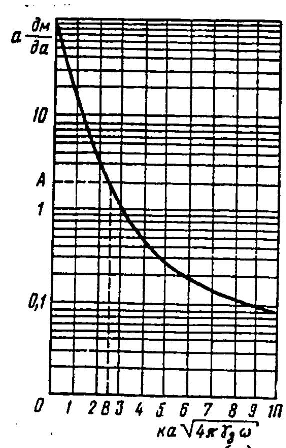
The emf induced in the frame will be

$$|\dot{e}'| = |2\pi f m \dot{I}|. \tag{9.86}$$

Comparing (9.85) and (9.86), we obtain

$$\frac{dM}{da} = \frac{|\dot{e}'|}{ns 2\pi f \dot{I}}. \tag{9.87}$$

Inasmuch as the value of dM/da is a function of the specific conductivity of the ground, after determining it from the measurement data for M, it is possible to find the magnitude of the specific conductivity of the ground at the point of location of the loop. For the given method of measurement the problem reduced to finding the first derivative of the expressions for M between the single-wire circuits with respect to the distance a between them.




(a)

Figure 9.18. Curves for a dM/da as a function of the parameter (ka)

Key: a. g

Taking the Pollaczek formulas for M and determining the derivative of these expressions by a, it is possible graphically to construct the relation of $|adM/da|$ as a function of the parameter $|ak| = a\sqrt{4\pi\gamma_g \omega}$ (Figure 9.18) [64]: $adM/da = \phi(a|k|)$. In the presence of the indicated graph in practice in order to find the magnitude adM/da, we proceeded as follows. According to the measurement data for $\dot{e}'$, $\dot{I}$, f and the loop parameters n, s, the value of dM/da is found. Multiplying this value by a, we obtain $adM/da = A$ which we find by the y-axis of the graph $A = \phi(|ka|)$. Drawing a horizontal line from point A to the intersection with the curve and dropping the perpendicular to the intersection with the x-axis, we obtain a value of B of the parameter $|ka|$. Dividing the magnitude B by a, we obtain the value of $B/a = k_1$. This value also permits determination of the desired value of

$$\gamma_з = \frac{k_1^2}{4\pi\omega}. \tag{9.88}$$

It is possible to use the graph (Figure 9.18) under the condition $0.5 \leq |ka| \leq 3$. For values of $|ka| > 3$ it is possible to obtain the desired value of $\gamma_g$ directly from the measurement data in the induced emf loop. This follows from the formula for mutual inductance with large values of $|ka|$:

$$M = \frac{i \cdot 4}{|ka|^2}. \tag{9.89}$$

Then

$$\frac{dM}{da} = \frac{i \cdot 8}{k^3 a^3}. \tag{9.90}$$

For this case the induced emf in the loop will be $\dot{e} = 2\pi f m \dot{I}$, where $m = ns(dM/da)$. Substituting the value of dM/da from (9.90) in this expression, we obtain

$$m = \mathrm{i} n s \frac{8}{k^2 a^3} \tag{9.91}$$

and

$$\dot{e} = \mathrm{i} \dot{I} \cdot 2\pi f n s \frac{8}{k^2 a^3}. \tag{9.92}$$

Substituting the value of $k^2 = 4\pi\omega\gamma_g$ in this formula, we obtain

$$\dot{e} = \mathrm{i} 2\pi f n s \dot{I} \frac{8}{4\pi \cdot 2\pi f \gamma_з a^3}.$$

From this expression it is possible to determine $\gamma_g$:

$$\gamma_з = \frac{2 \dot{I} n s}{\pi a^3 \dot{e}}. \tag{9.93}$$

Thus, knowing the current in the single-wire circuit $\dot{I}$, the structural parameters of the loop and the induced emf measured by the voltmeter for the given horizontal arrangement of the loop, it is possible to obtain the value of $\gamma_g$ by (9.93).

For measurements by the method of four electrodes, two single-wire circuits are created: exciting, which includes the dc source (or the low-frequency alternating current -- 16-14 hertz), and the measuring circuit which includes the measuring instrument. The length of the exciting circuit is taken three time the length of the measuring circuit. The current source in the form of a rotary generator and measuring device are mounted in the same apparatus (MS-08) called the "resistance meter." The ends of the exciting and measuring circuits are grounded through the electrodes 1, 2, 3, 4 located on a straight line (Figure 9.19). The instrument has two feed ($I_1$ and $I_2$) and two measuring ($E_1$ and $E_2$) terminals through which it is connected to the exciting and measuring circuits. The voltage from the measuring instrument is fed to the feed circuit on turning its switch; the instrument pointer indicates a value of the resistance R on the scale, knowing which, by the formula

$$\rho_з = 2\pi R a \tag{9.94}$$

the specific resistance or specific conductivity of the ground at the depth h is determined:

$$\gamma_з = \frac{1}{2\pi R a}; \tag{9.95}$$

where a is the distance between two adjacent electrodes. By varying the spacings between the electrodes, that is, increasing the length of the feed and measuring circuits, it is possible to obtain certain values of $\gamma_g$. If these values are approximately equal to each other, this means that the ground is uniform at different depth; if the values obtained differ from each other, then the apparent (mean) magnitude of the conductivity of the ground for the required current frequency must be determined by the master curves [65].

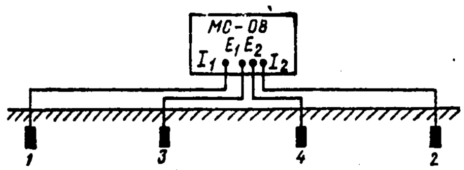



Figure 9.19. System for measuring the specific conductivity of the ground by the four-electrode method.

Recently the M-416 instrument has been used to measure the specific resistance of the ground by the four-electrode method.

## CHAPTER 10. ELECTRICAL CIRCUIT PARAMETERS

### 10.1. Primary Parameters of Single Circuits of Overhead Lines

When determining the parameters of single-wire circuits, the losses in the ground are taken into account, but the resistances of the terminal grounds are assumed to be equal to zero.

The formulas for determining the primary parameters of the circuits of the overhead lines and the rail and contact circuits of electric railways are presented in [66, 75, 76].

The calculation formulas for determining the primary parameters of the circuits of a single wire and the "two wire-ground"overhead lines are presented in Table 10.1. The derivation of these formulas is presented in the communications theory and line structure courses [15, 16, 18 and so on]; therefore only brief explanations for these formulas are presented below.

In Table 10.1 the following notation is adopted: $d_c$ is the outside diameter of the communications line, meters; $\rho_M$ is the specific electrical resistance of the wire material, ohm-meters; $R_1$ is the resistance to direct current of the inside part of the bimetallic conductor, ohm/km; $R_2$ is the reistance to direct current of the outer part of the bimetal wire, ohm/km; $k_1$, $k_2$ are the coefficients which take into account the surface effect in the uniform wire and are determined from Table 10.2 as a function of the parameter $x = 5 \cdot \sqrt{f_k \mu_r / 10R_0}$ for the uniform wire and $x = 5\sqrt{f_k \mu_{1r}/10R_1}$ for the bimetal wire, where $f_k$ is the current frequency, kilohertz; $\mu_r$ is the relative magnetic permeability of the uniform wire; $\mu_{1r}$ is the same for the inside part of the bimetal wire; k', k" are the coefficients which take into account the surface effect in the bimetal wire; they are determined from Table 10.3 as a function of $y = 2 \cdot 10^{-2} d_c \sqrt{10 f_k \mu_{2r} / \rho_{2M}}$, where $\rho_{2M}$ is the specific electrical resistance of the outer part of the bimetal wire, ohm-m; $\mu_{2r}$ is the relative magnetic permeability of the outer part of the bimetal wire; Δ is the thickness

Table 10.1. Calculated formulas for the primary parameters of the circuits of overhead communication lines

| Parameter | Calculation formulas for determining the circuit parameters | |
|---|---|---|
| | "Wire-ground" | "Two wires-ground" |
| Resistance to direct current, ohm/km | 1) For uniform wires<br>$R_0 = \rho_{\text{м}} \frac{4000}{\pi d_{\text{c}}^2}$;<br>2) for bimetal wires<br>$R_{0\text{бм}} = \frac{R_1 R_2}{R_1 + R_2}$ | $R_{0\text{т}0} = \frac{1}{2} R_0$ |
| Active resistance to alternating current, ohm/km | 1) For uniform wires<br>$R_{\text{A}} = R_0 k_1 + 2{,}51 f_{\text{к}} P$;<br>2) for bimetal wires; for<br>for $f_k \leq 10$<br>$R_{\text{Aбм}} = \frac{R_1 k_1 R_2}{R_1 k_1 + R_2} + 2{,}51 f_{\text{к}} P$;<br>for $f_k > 10$<br>$R_{\text{Aбм}} = R_2 \left(1 - \frac{\Delta}{d_{\text{c}}}\right) k' + 2{,}51 f_{\text{к}} P$ | $R_{\text{т}0} = \frac{1}{2} R_0 k_1 + 2{,}51 f_{\text{к}} P$ |
| External inductance according to Pollaczek, g/km | 1) For $c\sqrt{\gamma_{\text{з}} f_{\text{к}}} < 11$<br>$L'_{\text{A}} = \left(2 \ln \frac{8}{d_{\text{c}} \sqrt{0{,}1 \gamma_{\text{з}} f_{\text{к}}}} + 1\right) 10^{-4}$;<br>2) for $c\sqrt{\gamma_{\text{з}} f_{\text{к}}} > 14$<br>$L'_{\text{A}} = \left(2 \ln \frac{4c}{d_{\text{c}}} + \frac{5}{c\sqrt{0{,}1 \gamma_{\text{з}} f_{\text{к}}}}\right) 10^{-4}$ | 1) For $c\sqrt{\gamma_{\text{з}} f_{\text{к}}} < 2{,}8$<br>$L'_{\text{т}0} = \left(\ln \frac{320}{\gamma_{\text{з}} f_{\text{к}} d_{\text{c}} a_{\text{c}}} + 1\right) 10^{-4}$;<br>2) for $c\sqrt{\gamma_{\text{з}} f_{\text{к}}} > 20$<br>$L'_{\text{т}0} = \left(\ln \frac{8c^2}{d_{\text{c}} a_{\text{c}}} + \frac{2{,}5}{c\sqrt{0{,}1 \gamma_{\text{з}} f_{\text{к}}}} + \frac{5}{c\sqrt{0{,}1 \gamma_{\text{з}} f_{\text{к}}}}\right) 10^{-4}$ |
| External inductance according to Carson, g/km | $L'_{\text{A}} = \left(2 \ln \frac{4c}{d_{\text{c}}} + 4Q\right) 10^{-4}$ | $L'_{\text{т}0} = \left(\ln \frac{8c^2}{d_{\text{c}} a_{\text{c}}} + 4Q\right) 10^{-4}$ |
| Internal inductance, g/km | 1) for uniform wires<br>$L''_{\text{A}} = \frac{1}{2} \mu k_4 \cdot 10^{-4}$;<br>2) for bimetal wires:<br>for $f_k < 5$<br>$L''_{\text{Aбм}} = \frac{\mu_2}{2} \left[1 + \frac{R_2}{R_1 k_1 + R_2} \times \left(4 \ln \frac{d_{\text{c}}}{d_1} + \frac{\mu_1}{\mu_2} - 1\right)\right] 10^{-4}$; | $L''_{\text{т}0} = \frac{1}{2} L''_{\text{A}}$ |

Table 10.1 (continued)

| | Calculation formulas for determining the circuit parameters | |
|---|---|---|
| | "Wire-ground" | "Two wire-ground" |
| | for $f_k > 5$ <br> $L''_{A6м} = \frac{\mu_a}{2} \frac{\Delta}{d_c} k'' \cdot 10^{-4}$ | |
| Capacitance of a single circuit (in absence of icing and frost), farads/km | $C_A = \frac{72,2 \cdot 10^{-9}}{\ln \frac{4c}{d_c}}$ | $C_{тө} = \frac{72,2 \cdot 10^{-9}}{\ln \frac{4c}{d_{э.с}}}$ |
| Insulation conductivity, siemens/km | $G = 2\left[G'_0 + n' f_к \cdot 10^{-6}\right]$ | $G_{тө} = 2G_A$ |

Table 10.2. Coefficients taking into account the surface effect in a uniform wire

| $x$ | $k_1$ | $k_2$ | $x$ | $k_1$ | $k_2$ |
|---|---|---|---|---|---|
| 0,0 | 1,000 | 1,000 | 4,5 | 1,863 | 0,616 |
| 0,5 | 1,000 | 1,000 | 5,0 | 2,043 | 0,556 |
| 1,0 | 1,005 | 0,997 | 5,5 | 2,219 | 0,507 |
| 1,5 | 1,026 | 0,987 | 6,0 | 2,394 | 0,465 |
| 2,0 | 1,078 | 0,961 | 7,0 | 2,734 | 0,400 |
| 2,5 | 1,175 | 0,913 | 8,0 | 3,094 | 0,351 |
| 3,0 | 1,318 | 0,845 | 9,0 | 3,446 | 0,313 |
| 3,5 | 1,492 | 0,766 | 10,0 | 3,799 | 0,282 |
| 4,0 | 1,676 | 0,688 | >10 | $\frac{\sqrt{2}\,x+1}{4}$ | $\frac{2\sqrt{2}}{x}$ |

Table 10.3. Coefficients taking into account the surface effect in a bimetal wire

| $y$ | $k'$ | $k''$ | $y$ | $k'$ | $k''$ |
|---|---|---|---|---|---|
| 0,1 | 0,94 | 160 | 1,3 | 1,43 | 2,7 |
| 0,2 | 0,97 | 100 | 1,4 | 1,52 | 2,5 |
| 0,3 | 1,00 | 62 | 1,5 | 1,61 | 2,4 |
| 0,4 | 1,02 | 38 | 1,6 | 1,70 | 2,3 |
| 0,5 | 1,04 | 24 | 1,7 | 1,79 | 2,2 |
| 0,6 | 1,06 | 14 | 1,8 | 1,88 | 2,1 |
| 0,7 | 1,09 | 8 | 1,9 | 1,97 | 2,0 |
| 0,8 | 1,13 | 6 | 2,0 | 2,06 | 1,9 |
| 0,9 | 1,17 | 5 | 2,5 | 2,52 | 1,6 |
| 1,0 | 1,23 | 4 | 3,0 | 3,01 | 1,3 |
| 1,1 | 1,28 | 3,2 | 3,5 | 3,50 | 1,1 |
| 1,2 | 1,34 | 2,9 | 4,0 | 4,00 | 1,0 |
| | | | >4,0 | $k'=y$ | $k''=4/y$ |

of the outer part of the bimetal wire, meters; P, Q are the coefficients taking into account the active and reactive components of the losses in the ground and determined by the curves in Figure 10.1a and b as a function of $p = 0.558c\sqrt{0.1\gamma_g f_k}$; c is the mean height of suspension of the wires of the communication line, meters; $\gamma_g$ is the specific conductivity of the ground, siemens/meter; $d_1$ is the diameter of the inside part of the bimetal wire, meters; $d_{e.c} = \sqrt{2ad_c}$ is the equivalent diameter of the circuit made up of two wires, metrs; $a_c$ is the spacing between the wires of the two-wire circuit, meters; $G_0$ is the conductivity of the insulation of the overhead two-wire circuit with direct current equal to $0.01 \cdot 10^{-6}$ siemens/km in dry weather and $0.5 \cdot 10^{-6}$ siemens/km in the summer in wet weather; n' is the coefficient taking into account the increase in conductivity of the insulation for alternating current and equal to 0.05 in the summer and winter in dry weather and 0.25 in the winter in wet weather.

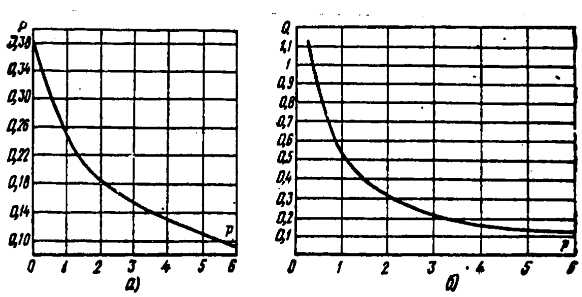



Figure 10.1. Curves for determining a) the values of the coefficient P as a function of the parameter p; b) the values of the coefficient Q as a function of the parameter p.

As is obvious from Table 10.1, the active resistance of the single-wire circuits is made up of two components: the resistances of the circuit wires and the resistances of the losses in the ground. Here it is necessary to distinguish the loss resistance in the ground from the resistance of the circuit grounds which in the formulas of the given chapter is not taken into account. The loss resistance in the ground, as follows from Table 10.1, is equal to $R_{g.\ell} = 2.51f_kP \approx \pi^2 f$, that is, it does not depend on the height of suspension of the wire above the ground and the specific resistance of the ground. More exact studies performed by M. G. Shalimov and L. V. Shagarova demonstrated [66, 67] that the loss resistance in the ground depends on the specific conductivity of the ground and height of suspension of the wire. Here the higher the conductivity, the more perceptible these relations are. In Figure 10.2 the relations calculated by the more exact formulas are presented for $R_{g.\ell}$ as a function of the height of suspension h and the specific conductivity of the ground for a frequency of 50 hertz.

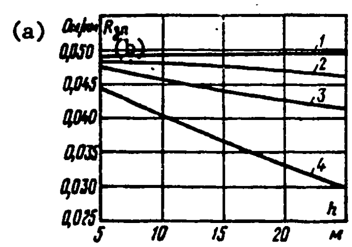



Figure 10.2. Loss resistance in the ground as a function of height of suspension of the wire (h) for a specific conductivity of the ground: 1 -- $\gamma_g = 10^{-6}$ siemens/m; 2 -- $\gamma_g =$ $= 10^{-6}$ siemens/m; 3 $\gamma_g = 0.1$ siemens/m; 4 -- $\gamma_g = 1$ siemen/m.

Key: a. ohm/km b. g

The inductance of any circuit consists of two components: the external inductivity L' defined as the ratio of the external (outside the wires) magnetic flux to the currents flowing in the conductor and creating this flux, and the inside inductance L" defined as the ratio of the internal (inside the wires) magnetic flux to the current flowing in the conductor. With an increase in frequency, as a result of the surface effect, the magnetic flux is forced out of the wire and the internal inductance diminishes. At sufficiently high frequencies the value of the internal inductance can be neglected by comparison with the external inductance which does not depend on the frequency. The internal inductance of the circuit made up of bimetal wires at frequencies above the voice spectrum in practice is equal to the internal inductance of the outer part of the conductor. The formulas for the external inductance are presented in Table 10.1 in two forms: according to Pollaczek [19] and according to Carson [20]. As the calculations show, both formulas give approximately coinciding results. The calculation by the Pollaczek formulas in a defined frequency band is less tedious and somewhat more exact, for it does not require the use of tables, the use of which naturally always introduce some error. For defined values of the variables, for example, for $14 > c\sqrt{\gamma_g f_k} \geq 11$, for the "conductor-ground" circuit the Pollaczek formulas cannot be expressed in simple form and contain the Bessel functions. In such cases, it is recommended that the Carson formulas be used. Since the maximum effect of the high voltage lines and the communications lines occur in the absence of frost and ice, the formulas for the capacitance of the overhead circuits are presented in Table 10.1 only under these conditions [68].

## 10.2. Total Resistance of the Communications Wire in Bunched Conductors

In cases where large currents are induced in the communications lines as a result of the effect of high voltage lines, for example, when calculating the placement of dischargers, it is necessary to consider the interaction of the electromagnetic fields of all of the wires among each other. As a result of this interaction of the total resistance of the communications line in the bunch of parallel conductors will differ from the total resistance of the same single wire.

Let a completely asymmetric circuit of an electric power transmission line exist with interfering current $\dot{I}_0$ and in the convergence length $\sum_1^m \ell_{ni}$ let there be several (N) communication wires suspended on one column line. Let us denote: $\dot{E}_1$, $\dot{E}_2$, ..., $\dot{E}_N$ -- the induced emf's in the communications lines; $M_{12}$, $M_{13}$, ..., $M_{1N}$ are the mutual inductances between the communications lines; $Z_{is} = \omega M_{is}$ is the total magnetic coupling resistance between the wires with order numbers i and s, where i pertains to the interfering line, and s pertains to the lines subjected to the effect; $i_1$, $i_2$, ..., $i_N$ are the induced currents in the communication wires; $L_1$, $L_2$, ..., $L_N$ are the inductances of the communication wires. The equation for the emf in each of the communication wires closed at the ends through discharges can be written in the following general form:

$$\dot{E}_s = l \sum_{i=1}^{N} Z_{is} i_i. \qquad (10.1)$$

Here in practice it is possible to make certain assumptions which permit simplification of the system of equations:

1) all of the communication wires can be divided into groups a, b, c, ... with a number of conductors in each group $n_a$, $n_b$, $n_c$ having identical cross section and made of identical material;

2) the values of the mutual inductanc between the wires of each group can be taken identical for all groups and equal to the mean value of these coefficients $M_{aa} = M_{bb} = M_{cc} = \ldots = M$;

3) it is possible to assume that the magnitude of the mutual inductance between any wire of group a and any wire of any other group, for example, b or c, is the mean constant value of $M_{ab} = M_{ac} = M_{cb} = \ldots - M$.

Then the equations of system (10.1) for any group of wires can be rewritten in the following form:

$$\left.\begin{aligned} \dot{E}_a &= l\{[R_a + i\omega L_a + i(n_a - 1)\omega M] i_a + i\omega M i_b n_b + \ldots\}, \\ \dot{E}_b &= l\{[R_b + i\omega L_b + i(n_b - 1)\omega M] i_b + i\omega M i_a n_a + \ldots \\ \dot{E}_c &= l\{[R_c + i\omega L_c + i(n_c - 1)\omega M] i_c + i\omega M i_a n_a + \ldots, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\} \qquad (10.2)$$

where $i_a$, $i_b$, $i_c$, ... are the current in each conductor of the corresponding group.

Let us divide the left and right-hand sides of each equation by $i_a$, $i_b$, $i_c$ respectively and let us denote $i_b/i_a = k_b$; $i_c/i_a = k_c$. Then equations (10.2) are obtained in the following form:

$$\left.\begin{array}{l} \dot{E}_a/i_a + Z_a l = l\,[R_a + i\omega L_a + i(n_a - 1)\omega M + i\omega n_b k_b M + \ldots], \\ \dot{E}_b/i_b = Z_b l = l\left[R_b + i\omega L_b + i(n_b - 1)\omega M + i\omega n_a \frac{1}{k_b} M + \ldots\right], \\ \dot{E}_c/i_c = Z_c l = l\left[R_c + i\omega L_c + i(n_c - 1)\omega M + i\omega n_a \frac{1}{k_c} M + \ldots\right], \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{array}\right\} \quad (10.3)$$

For determination of the coefficients $k_b$, $k_c$ which enter into these equations, let us divide one equation (for example, the first) by another other (for example, the second). We obtain

$$k_b = \frac{\dot{E}_a i_b}{\dot{E}_b i_a} = \frac{Z_a}{Z_b} =$$

$$= \frac{R_a + i\omega L_a + i(n_a - 1)\omega M + i\omega M n_b k_b + i\omega M n_c k_c + \ldots}{R_b + i\omega L_b + i(n_b - 1)\omega M + i\omega M n_a \frac{1}{k_b} + i\omega M n_c \frac{k_c}{k_b} + \ldots}$$

After transformation we obtain

$$k_b = \frac{R_a + i\omega(L_a - M)}{R_b + i\omega(L_b - M)}, \quad k_c = \frac{R_a + i\omega(L_a - M)}{R_c + i\omega(L_c - M)} \ldots .$$

For simplification of the expression for the total resistance of the wire it is possible to approximately define the coefficients $k_b$, $k_c$, ... as moduli, for example,

$$|k_b| = \frac{\sqrt{R_a^2 + \omega^2(L_a - M)^2}}{\sqrt{R_b^2 + \omega^2(L_b - M)^2}}. \quad (10.4)$$

If the values of the coefficients $k_b$, $k_c$, ... are known, then the magnitude of the total resistance of 1 km of wire in bunched conductors will be determined from (10.3), for example,

$$\left.\begin{array}{l} Z_a = \sqrt{R_a^2 + \omega^2[L_a + M(n - 1 + n_b k_b + \ldots)]^2}, \\ Z_b = \sqrt{R_b^2 + \omega^2\left[L_b + M\frac{1}{k_b}(n_a + n_b k_b + \ldots) - M\right]^2}. \end{array}\right\} \quad (10.5)$$

If a bunch of N conductors exists which is made up of several groups a, b, c where there are $n_a$, $n_b$, $n_c$ conductors in each group with identical cross

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30 APRIL 1980 M. N. MIKHAYLOV, L. D. RAZUMOV AND S. A. SOKOLOV 4 OF 4

section and made of identical material, it is possible to prove that the total resistance of 1 km of the bunched conductors will be defined by the expression

$$Z_N = \frac{R_a + i\omega\,[L_a + M\,(n_a + k_b n_b + n_c k_c + \ldots - 1)]}{n_a + k_b n_b + k_c n_c + \ldots}. \tag{10.6}$$

As we shall see, when using the formulas (10.4)-(10.6), it is necessary to determine the values of L, M, $k_b$, $k_c$, ... in advance. For L and M it is possible to recommend that the Pollaczek formulas which in simplified form for copper wires when f = 50 hertz are written as follows:

$$L = \left(2\ln\frac{17{,}9}{r_c\sqrt{0{,}1\,\gamma_3}} + 1\right)10^{-4},$$

$$M = \left(2\ln\frac{17{,}9}{a\sqrt{0{,}1\,\gamma_3}} + 1\right)10^{-4},$$

where $r_c$ is the radius of the communications wire, meters; a is the geometric mean spacing between the communication conductors, that is, $a = \sqrt[n]{a_1 a_2 a_3 \ldots a_n}$; $\gamma_g$ is the specific conductivity of the ground, siemens/meter.

10.3. Capacitance of the Communications Line Conductors

The communications line conductors have a capacitance with respect to ground and with respect to the adjacent conductors of this line. The problem with respect to determining the capacitance of the conductors reduces to confining the relation between the potentials and charges of the individual conductors of the communication line, and it is solved using the Maxwell equations which relate the charges and potentials of the conductors (see Chapter 2).

Let n conductors be suspended on the line, and let them carry charges of $q_1$, $q_2$, ..., $q_n$ from the corresponding energy sources. Then in order to determine the potentials of the communication line conductors, the system of equations of the type of (2.103) is valid:

$$\left.\begin{array}{l} U_1 = 18\cdot 10^9\,(q_1\alpha_{11} + q_2\alpha_{12} + \ldots + q_n\alpha_{1n}), \\ U_2 = 18\cdot 10^9\,(q_1\alpha_{21} + q_2\alpha_{22} + \ldots + q_n\alpha_{2n}), \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ U_n = 18\cdot 10^9\,(q_1\alpha_{n1} + q_2\alpha_{n2} + \ldots + q_n\alpha_{nn}), \end{array}\right\} \tag{10.7}$$

where $U_1$, $U_2$, ..., $U_n$ are the potentials of the corresponding conductors, volts.

In order to simplify the solution of this system let us take the coefficients $\alpha_{12}$, $\alpha_{13}$, ..., $\alpha_{1n}$, $\alpha_{23}$, $\alpha_{34}$, ..., $\alpha_{ik}$ equal to the arithmetic mean value of their sum $\bar{\alpha}_{12}$; the coefficients $\alpha_{11}$, $\alpha_{22}$, ..., $\alpha_{nn}$ are taken equal to the arithmetic mean value of $\bar{\alpha}_{11}$. Let us also assume that the charges of all of the communications conductors are identical, that is, $q_1 = q_2 = \ldots = q_n = \bar{q}_1$. Thus,

$$\left.\begin{aligned}\bar{\alpha}_{11} &= \frac{1}{n}(\alpha_{11}+\alpha_{22}+\ldots+\alpha_{nn}),\\ \bar{\alpha}_{12} &= \frac{1}{(n-1)^2}(\alpha_{12}+\alpha_{13}+\ldots+\alpha_{1n}+\alpha_{21}+\alpha_{23}+\ldots+\alpha_{2n}+\ldots).\end{aligned}\right\} \quad (10.8)$$

Substituting these average values from the Maxwell equation, we obtain a number of identical equations:

$$U_1 = 18\cdot 10^6 [\bar{q}_1\bar{\alpha}_{11} + \bar{q}_1 (n-1)\bar{\alpha}_{12}] = q_1\, 18\cdot 10^6 [\bar{\alpha}_{11} + (n-1)\bar{\alpha}_{12}],$$
$$U_2 = 18\cdot 10^6 \bar{q}_1 [\bar{\alpha}_{11} + (n-1)\bar{\alpha}_{12}] \text{ and so on.}$$

Hence,

$$\bar{q}_1 = \frac{U_1}{18\cdot 10^6 [\bar{\alpha}_{11} + (n-1)\bar{\alpha}_{12}]}.$$

Since $q_1 = C_1U_1$, the capacitance of the conductor

$$C_{11} = \frac{1}{18\cdot 10^6 [\bar{\alpha}_{11} + (n-1)\bar{\alpha}_{12}]}. \quad (10.9)$$

The coefficients $\bar{\alpha}_{11}$ and $\bar{\alpha}_{12}$ fluctuate within the following limits: $\bar{\alpha}_{11} = 8.5$ to $9.0$; $\bar{\alpha}_{12} = 2.5$ to $3.0$. Substituting these values in (10.9), we find that the mean value of the capacitance of the conductor in a bunch of identically charged conductors is: for $\bar{\alpha}_{11} = 8.5$ and $\bar{\alpha}_{12} = 2.5$

$$C_{11} = C_A = \frac{1}{18\cdot 10^6 [8{,}5 + (n-1)\,2{,}5]} \approx \frac{22{,}2\cdot 10^{-9}}{2{,}4+n}, \quad (10.10)$$

for $\bar{\alpha}_{11} = 9$ and $\bar{\alpha}_{12} = 3$

$$C_{11} = C_A \approx \frac{18\cdot 10^{-9}}{n+2}. \quad (10.11)$$

Thus, if all of the conductors of the communication line are charged from an outside source with identical charge, the capacitance of each conductor with respect to ground will depend on the number of adjacent charged conductors. The capacitance of the individual conductor in a bunch of charged conductors will be less the larger the number of conductors.

The capacitance of a single conductor, as is known, will be expressed in terms of the natural potential coefficient in the form $C_{11} = 1/18\cdot 10^6\bar{\alpha}_{11}$. Since on the average $\bar{\alpha}_{11} = 9$, then $C_{11} \approx 6.1\cdot 10^{-9}$ farads/km. The capacitance of a single conductor from (10.2) is

$$C_{11} = \frac{18 \cdot 10^{-9}}{n+2} = \frac{18 \cdot 10^{-9}}{1+2} = 6 \cdot 10^{-9} \text{ farads/km.}$$

## 10.4. Primary Parameters of the "Metal Coverings-Ground" Circuit

As was indicated above, in a cable with external metal sheathing in general form in addition to the two-conductor circuits there are three types of asymmetric circuits: "metal coverings (sheathing)-ground," "core-sheathing," "core-ground." Let us consider the "sheathing-ground" circuit. The parameters of the metal sheathings of communication cables must be known primarily to determine their shielding effect.

The "metal coverings of the cable-ground" circuit is a single-wire circuit from the electrical point of view with return of the current through the ground. This type of line, just as any single-wire circuit, has four primary parameters: the active resistance $R_{sh}$ (ohm/m), the inductance $L_{sh}$ (g/m), the capacitance $C_{sh}$ (farads/m) and the conductivity $G_{sh}$ (Siemens/meter) and also two secondary parameters: the propagation coefficient $\gamma_{sh}$ (1/m) and wave impedance $z_{w.sh}$ (ohms). The total resistance of the metal covering $Z_{sh} = R_{sh} + i\omega L_{sh}$. In the high frequency range $\omega L_{sh} >> R_{sh}$ and $Z_{sh} \approx i\omega L_{sh}$. The total conductivity $Y_{sh} = G_{sh} + i\omega C_{sh}$.

For cables not having an outside metal covering of the plastic sheathing on top, that is, for cables with jute covering, for bare cables laid in a corridor, the capacitance with respect to ground is close to zero and is entirely shunted by the active conductivity of the circuit. For such cables in all of the investigated frequency range $Y_{sh} = G_{sh} = 1/R_{trans.sh}$, where $R_{trans.sh}$ is the transient resistance between the metal coverings and ground, ohm-m. For cables with outside plastic sheathing having high resistance in the radial direction and small dielectric losses, $G_{sh} << \omega C_{sh}$ and $Y_{sh} \approx i\omega C_{sh}$.

The total resistance of the "metal conductors-ground" circuit consists of two components: the internal resistance of the metal conductors or so-called surface resistance ($Z_{sh.s}$) and the external resistance ($Z_{sh.ex}$) causes by the external magnetic flux and losses in the ground. Thus, $Z_{sh} = Z_{sh.s} + Z_{sh.ex}$. The total surface resistance is equal to the ratio of the electric field intensity along the external surface of the metal coverings to the current flowing in the "metal coverings-ground" circuit.

In general form the surface resistance of the single-layer metal sheathing of solid construction will be determined by the formula [69]

$$Z_{\text{об.п}} = \frac{1}{2\pi r_2} \sqrt{\frac{i\omega\mu}{\gamma_M}} \frac{I_0(kr_2) K_1(kr_1) + K_0(kr_2) I_1(kr_1)}{I_1(kr_2) K_1(kr_1) - I_1(kr_1) K_1(kr_2)}, \tag{10.12}$$
(a)

Key: a. sh.s

where $r_1$ and $r_2$ are the inner and outer radii of the sheathing, meters; $\mu$ is the magnetic permeability of the sheathing metal, g/meter; $\gamma_M$ is the conductivity of the sheathing metal, siemens/meter; $k = \sqrt{i\omega\mu\gamma_M}$ : $I_0$, $K_0$, $I_1$, $K_1$ are the modified cylindrical first and second type, 0 and first order Bessel functions.

Substituting the asymptotic approximations of the Bessel functions in the formula for $Z_{sh.s}$, we obtain:

for low frequencies ($kr_2 \le 0.25$)

$$Z_{\text{об.п}} = R_{\text{п.т}} \frac{1 - \frac{1}{2} k r_1^2 \ln \frac{r_2}{r_1}}{1 - \frac{k^2 r_1^2 r_2^2}{2(r_2^2 - r_1^2)} \ln \frac{r_2}{r_1}} \approx R_{\text{п.т}}, \tag{10.13}$$
(a) (b)

Key: a. sh.s b. dc

where $R_{dc}$ -- the resistance of the sheathing to direct current (formula 10.13) -- can be used in practice only for direct current);

for high frequencies ($kr_1 > 5$)

$$Z_{\text{об.п}} = \frac{1}{2\pi r_2} \sqrt{\frac{i\omega\mu}{\gamma_M}} \operatorname{cth} kt + \frac{r_1 + 7r^2}{16\pi r_1 r_2^2 \gamma_M}. \tag{10.14}$$

where $t = r_2 - r_1$ is the thickness of the sheathing. Hence,

$$R_{\text{об.п}} = \frac{1}{2 r_2} \sqrt{\frac{\mu f}{\pi \gamma_M}} \frac{\operatorname{sh} u + \sin u}{\operatorname{ch} u - \cos u} + \frac{r_1 + 3 r_2}{16 \pi r_1 r_2^2 \gamma_M},$$

$$L_{\text{об.п}} = \frac{1}{2 \pi r_2} \sqrt{\frac{\mu}{2 \omega \gamma_M}} \frac{\operatorname{sh} u - \sin u}{\operatorname{ch} u - \cos u},$$

where $u = t\sqrt{2\omega\mu\gamma_M}$. For $u > 5$ $\operatorname{sh} u \approx \operatorname{ch} u$; $\operatorname{sh} u \gg \sin u$; $\operatorname{ch} u \gg \cos u$. Then

$$R_{\text{об.п}} = \frac{1}{2 r_2} \sqrt{\frac{\mu f}{\pi \gamma_M}} + \frac{r_1 + 3r_2}{16 \pi r_1 r_2^2 \gamma_M}, \quad L_{\text{об.п}} = \frac{1}{2\pi r_2} \sqrt{\frac{\mu}{2\omega\gamma_M}}.$$
(a)

Key: a. sh.s

If in (10.14) kt $\leq$ 0.5, then cth kt $\approx$ 1/kt and then

$$Z_{\text{об.п}} = \frac{1}{2\pi r_3 t \gamma_м} + \frac{r_2 + 3r_3}{16\pi r_1 r_2^2 \gamma_м} = \frac{3r_2^2 - r_1^2 + 6r_1 r_3}{16\pi \gamma_м r_1 r_2^2 t},$$

that is, in this case the resistance is only one active component.

The total surface resistance of the bimetal sheathing consisting of two solid cylindrical tubes can be determined from the formula [16]

$$Z_{\text{об.п}} = \frac{k_2}{2\pi r_3 \gamma_{м2}} \frac{k_2 \mu_1 a \bar{f} + k_1 \mu_2 b e}{k_2 \mu_1 a d + k_1 \mu_2 b c},$$

where $k_1 = \sqrt{i\omega\mu_1\gamma_{м1}}$; $k_2 = \sqrt{i\omega\mu_2\gamma_{м2}}$;

$$a = I_0(k_1 r_2) K_1(k_1 r_1) + K_0(k_1 r_2) I_1(k_1 r_1),$$
$$b = I_1(k_1 r_2) K_1(k_1 r_1) - K_1(k_1 r_2) I_1(k_1 r_1),$$
$$c = I_1(k_2 r_3) K_0(k_2 r_2) + K_1(k_2 r_3) I_0(k_2 r_2),$$
$$d = I_1(k_2 r_3) K_1(k_2 r_2) - K_1(k_2 r_3) I_1(k_2 r_2),$$
$$e = I_0(k_2 r_3) K_0(k_2 r_2) - K_0(k_2 r_3) I_0(k_2 r_2),$$
$$\bar{f} = I_0(k_2 r_3) K_1(k_2 r_2) + K_0(k_2 r_3) I_1(k_2 r_2).$$

$\mu_1$ and $\mu_2$ are the magnetic permeabilities of the first and second layers of the bimetal sheathing, respectively; $\gamma_{M1}$ and $\gamma_{M2}$ are the conductivities of the first and second layers of the bimetal sheathing, respectively; $r_1$ is the inside radius of the first (inner) layer; $r_2$ is the outer radius of the first layer; $r_3$ is the outer radius of the second circuit.

In the voice-frequency range the surface resistance of the metal coverings can be determined by simpler formulas. The active resistance can be taken equal to the ohmic resistance (for direct current). Here the resistance of the losses to hysteresis in steel armored strips is not taken into account. However, the calculation by the formulas presented in [71] indicates that it is possible to neglect these losses for practical calculations. The ohmic resistance of a solid metal tube

$$\underset{(a)}{R_{\text{об.п.т}}} = R_0 = \frac{1}{\pi\gamma_м (r_2^2 - r_1^2)}. \tag{10.15}$$

Key: a. sh.dc

The ohmic resistance of a strip tube and the inside inductance of the steel strip armor can be determined by the formulas [71]:

$$\underset{(a)}{R_{\text{бр.п.т}}} = R_{0\,\text{бр}} = \frac{\pi D_{cp}}{a_э^2 b_э \gamma_м}, \tag{10.16}$$

Key: a. armor dc

$$L_{\text{об.п}}_{(a)} = \frac{1,5\,\pi\,\mu_r\,b_э}{a_э}\,10^{-7}, \qquad (10.17)$$

Key: a. sh.s

where the thickness of the equivalent strip is

$$b_э = nb - \frac{nb\Delta}{a_э},$$

n is the number of strips; b is the thickness of one strip; Δ is the clearance between turns of the strip (in the case of overlap Δ is negative); $a_e = a + \Delta$, a is the strip width; $D_{mean}$ is the mean parameter of the bronze, meters; $\mu_r$ is the relative magnetic permeability of the steel strip.

The internal inductance of the cylindrical sheathing made of nonmagnetic material can in practice be neglected.

The active resistance of metal covering containing several sheathings will be defined as the resistance of a parallel connection, that is, for metal coverings containing sheathing and armor, $R_{sh.s} = R_{o.armor}R_{o.sheathing}/(R_{o.armor} + R_{o.sh})$. Thus, $Z_{sh.s} = R_o + i\omega L_{sh.s}$. The external resistance of the metal coverings will be defined by the following approximate formula

$$Z_{\text{об.в}}_{(a)} = i\omega\frac{\mu_з}{2\pi}\ln\frac{1,85}{r'_м\sqrt{\gamma^2_{м.п} + \overline{\gamma}^2_з}}. \qquad (10.18)$$

Key: a. sh.er

where $\mu_g$ is the magnetic permeability of the ground which in the first approximation is equal to $4\pi\cdot10^{-7}$ g/meter; $r'_M$ is the equivalent radius of the metal coverings, $r'_м = \sqrt{r_м\sqrt{r^2_м + 4h^2_к}} \approx \sqrt{2r_м h_к}$, meters; $r_M$ is the geometric outside radius of the metal coverings, meters; $h_k$ is the depth of burial of the cable, meters; $\gamma_{M.\pi}$ is the propagation coefficient of the currents in the "metal coverings-ground" circuit, 1/m; $\gamma_{M.\pi} = \sqrt{Z_{M.\pi}/R_{trans.\pi}}$; $\overline{\gamma}_g$ is the propagation coefficient of the currents in the ground equal to $\overline{\gamma}_g = \sqrt{i\omega\mu_g\gamma_g}$; $\gamma_g$ is the specific conductivity of the ground, siemens/meter; $R_{пер.п} = R_{из.м.п} + \frac{1}{\pi\gamma_з}\ln\frac{1,12}{\gamma_{м.п}\sqrt{2r_к h_к}}$, ohm-m; $R_{из.м.п}$ is the coefficient of the outer insulating coating on the metal coverings, ohm-meter. For cables with ordinary jute covering over the armor or for bare cables in ordinary corridors

The determination of the external resistance from the expression (10.18) is connected with significant difficulties, for this expression is transcendental. As is obvious from (10.18), the values of $\gamma_{M.\pi}$ and $\bar{\gamma}_g$ are under the logarithms. This means that the large range of variation of these variables corresponds to the small range of variation of $Z_{sh.ex}$. Accordingly, for cables with ordinary external jute covering without great error it is possible to assume that $\gamma_{M.\pi} = \bar{\gamma}_g$, and then

$$\underset{(a)}{Z_{об.в}} = \left[\pi^2 f + i\,\omega \ln \frac{1,72}{D_м h_к \omega \mu_з \gamma_з}\right] 10^{-7}. \quad (10.19)$$

Key: a. sh.ex

The first, active component of this expression is caused by losses in the ground, and it depends only on the frequency. The second component -- the reactive resistance of the external inductance -- can be represented in the form $\omega L_{ext}$, where $L_{ext}$ is the external inductance of the cable sheathing. On high frequencies the surface resistance $Z_{sh.s}$ is small by comparison with $Z_{sh.ex}$.

The transient resistance between the metal coverings of the cable and ground is (for $\gamma_{M.} = \bar{\gamma}_M$);

$$\underset{(a)}{R_{пер.п}} = \frac{1}{\pi \gamma_з} \ln \frac{1,12}{|\gamma_з| \sqrt{2 r_м h_к}} - i \frac{1}{4 \gamma_з}. \quad (10.20)$$

Key: a. trans.π

In the case where the metal coverings of the cable are multiply grounded with respect to length of the transient resistance between them and the ground must be determined by the formula

$$\underset{(a)}{R_{пер}} = \frac{R'_{пер} R_{пер.п}}{R'_{пер} + R_{пер.п}}, \quad (10.21)$$

Key: a. trans

where $R'_{trans}$ is the transient resistance caused by the presence of the grounds, ohm-meter,

$$R'_{пер} = \frac{R_з l_з \sqrt{n R_з}}{\underset{(a)}{\sqrt{n R_з} + 0,14\sqrt{l_з}}}, \quad (10.22)$$

Key: a. g

$\ell_g$ is the mean spacing between adjacent grounds, meters; $R_g$ is the mean existence of one ground, ohm; n is the number of sections between grounds. If $\ell_g$ and $R_g$ along the section differ significantly from each other, then in (10.22) it is necessary to substitute the arithmetic mean value of $\ell_g$ and the value of $R_g$ determined by the total conductivity of the grounds: $R_g = (n + 1)/G_{g.total}$, where $G_{g.total} = \sum_{i=1}^{n+1}(1/R_{3i})$, $R_{3i}$ is the resistance of the ith ground. Formula (10.22) is the empirical formula; therefore the values entering into it can be substituted only in the units which are indicated in the notation, that is, $\ell_g$ in meters, $R_g$ in ohms.

In Table 10.4 values are presented which were calculated by (10.20) for the transient resistance between the metal coverings of a cable with outside jute covering and the ground. Figure 10.3 shows the curves calculated by (10.22) for the transient resistance of $R'_{trans}$ as a function of $R_g$ and $\ell_g$.

Table 10.4. Transient resistance between the metal coverings of a cable with outside jute covering and ground

| Frequency, kilohertz | Outside diameter, m | Transient resistance, ohm-meters for specific resistance of the ground, ohm-m | | | | | |
|---|---|---|---|---|---|---|---|
| | | 10 | | 100 | | 1000 | |
| | | Modulus | Angle | Modulus | Angle | Modulus | Angle |
| 0,05 | 0,02 | 23,2 | —6°12′ | 267 | —5°21′ | 3030 | —4°43′ |
| | 0,06 | 21,4 | —6°42′ | 249 | —5°44′ | 2860 | —5°00′ |
| 0,8 | 0,02 | 18,8 | —7°38′ | 224 | —6°24′ | 2600 | —5°30′ |
| | 0,06 | 17,1 | —8°25′ | 207 | —6°55′ | 2240 | —5°54′ |
| 10 | 0,02 | 14,8 | —9°42′ | 185 | —7°47′ | 2210 | —6°30′ |
| | 0,06 | 13,2 | —10°58′ | 167 | —8°38′ | 2040 | —7°03′ |
| 100 | 0,02 | 11,3 | —12°49′ | 149 | —9°42′ | 1850 | —7°47′ |
| | 0,06 | 9,52 | —15°13′ | 131 | —10°59′ | 1670 | —8°38′ |

As is obvious from this table, the transient resistance depends almost directly proportionally on the specific resistance of the ground, and with an increase in frequency, it decreases with respect to modulus and increases with respect to phase angle.

The total surface resistance of the metal coverings of the cable in a plastic outer sheathing will be defined by the same expressions as for the cable without plastic sheathing.

In formula (10.18) in the presence of a plastic outer sheathing it is possible to neglect $\gamma_{M.\pi}$ by comparison with $\gamma_g$. Then

$$Z_{об.в} = i\omega\frac{\mu_з}{2\pi}\ln\frac{1.85}{r'_м\sqrt{\omega\mu_з\gamma_з}}$$

or

$$Z_{об.в} = i\omega \frac{\mu_з}{2\pi} \ln \frac{1,85}{r'_м |\gamma_з| e^{i\,45°}} = \left[\pi^2 f + i\omega \ln \frac{3,42}{D_м h_к \omega \mu_з \gamma_з}\right] 10^{-7}. \qquad (10.23)$$

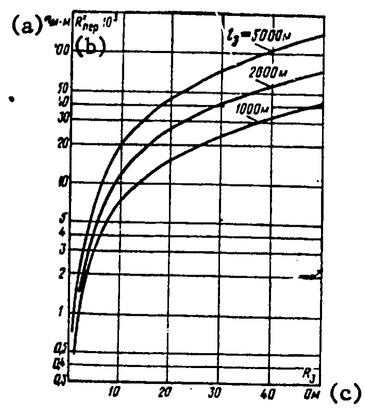


Figure 10.3. Transient resistance between the uniformly, multipally grounded sheathing and ground as function of the resistance $R_g$ of the grounds and the distance $\ell_g$ between them.

Key: a. ohm-meter b. $R'_{trans}$ c. ohm

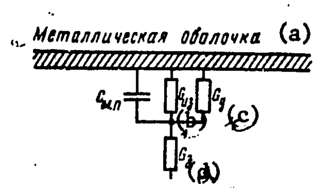


Figure 10.4. Determination of the total transient conductivity between a metal cable sheathing with outside plastic coating and ground.

Key: a. metal sheathing b. D c. g

In the given case when determining the transient conductivity between the metal cable coverings and ground it is impossible to neglect the insulation resistance of the outer coating and the capacitance between the external metal covers and the ground. The equivalent diagram for determining the total conductivity between the metal coverings of the cable with plastic insulation and ground is presented in Figure 10.4. The total conductivity

$$Y_{м.п}=\frac{1}{[G_{д}+G_{из}+i\omega C_{м.п}]^{-1}+\frac{1}{\pi\gamma_{з}}\ln\frac{1,12}{\gamma_{м.п}\sqrt{D_{м.п}h_{к}}}},\qquad(10.24)$$

(a) (b)

Key: a. D b. ins

where $G_D$ is the active conductivity caused by losses in the dielectric of the plastic insulation, siemens/m; $G_{ins}$ is the active conductivity of a plastic insulation, siemens/meter; $C_{M.\pi}$ is the capacitance between the metal coverings and the grounds, farads/meter.

The active conductivity causes by losses in the dielectric is determined bv the known formula $G_D = \omega C_{M.\pi} tg\ \delta$, where tg δ is the tangent of the dielectric losscangle in plastic insulation. The values of tg δ for different materials are presented in Table 10.5. The capacitance between the outer metal sheathing and ground can be defined by the formula

$$C_{м.п}=C_{об}=\frac{\varepsilon_r}{18\ln\frac{r_{п}}{r_2}}10^{-9},\ \text{farads/m},\qquad(10.25)$$

where $\varepsilon_r$ is the relative dielectric constant of the material of the plastic sheathing, the values of which are presented in Table 10.6; $r_D$ is the outside radius of the plastic sheathing, meters; $r_2$ is the outside radius of the metal covers, meters.

Since the active conductivity of the plastic insulation is very low (insulation resistance $R_{ins.M.\pi}$ is about 1000 Mohms-m, that is, $10^9$ ohm-m), in formula (10.24) the value of $G_{ins}$ can be neglected. For practical calculations it is also possible to neglect the value of $G_D$ by comparison with $i\omega C_{M.\pi}$. Then

$$Y_{м.п}=\frac{1}{\frac{1}{i\omega C_{м.п}}+\frac{1}{\pi\gamma_{з}}\ln\frac{1.12}{\gamma_{м.п}\sqrt{D_{м}h_{к}}}}.\qquad(10.26)$$

In the low frequency range (to 10-20 kilohertz) with mean specific resistance of the ground (to 100-200 ohm-m) the second term in the denominator of formula (10.26) can be neglected, and then $Y_{M.\pi} = i\omega C_{M.\pi}$. The values of the capacitances $C_{M.\pi}$ calculated by (10.25) as a function of the cable sizes and thickness of the insulation are presented in Table 10.7.

It is necessary to determine the transient conductivity according to (10.26) graphically. For this purpose, the right-hand side of the equality (10.26)

Table 10.5. tg δ of cable insulating materials

| Insulating materials | tg δ for current frequency, hertz | | | |
|---|---|---|---|---|
| | $10^3$ | $10^4$ | $10^5$ | $10^6$ |
| Polyvinyl chloride (flexible tubing) | 0.03-0.1 | -- | -- | -- |
| Polyethylene | 0.0004 | 0.0004 | 0.0004 | 0.0004 |
| Styroflex | 0.00035 | 0.00014 | 0.00013 | 0.00011 |
| Cable paper | 0.009 | 0.011 | 0.030 | 0.045 |

Table 10.6. Dielectric constant of cable insulating material

| Insulation material | Polyvinyl chloride (flexible tubing) | Polyethylene | Styroflex | Cable paper |
|---|---|---|---|---|
| ε | 6-7 | 2.3 | 2.4 | 2.0 |

Table 10.7. Capacitance between the metal coverings and ground

| Material of plastic sheathing | Outside radius of metal sheathing, mm | Thickness of plastic sheathing, mm | Capacitance of metal sheathing with respect to ground, nanofarads/m | Material of plastic sheathing | Outside radius of metal sheathing, mm | Thickness of plastic sheathing, mm | Capacitance of metal sheathing with respect to ground, nanofarads/m |
|---|---|---|---|---|---|---|---|
| Polyvinylchloride | 5 | 1 | 1,99 | Polyethylene | 5 | 1 | 0,7 |
| | | 2 | 1,07 | | | 2 | 0,38 |
| | 10 | 1 | 3,79 | | 10 | 1 | 1,34 |
| | | 2 | 1,99 | | | 2 | 0,7 |
| | | 3 | 1,33 | | | 3 | 0,49 |
| | 15 | 1 | 5,42 | | 15 | 1 | 1,92 |
| | | 2 | 2,95 | | | 2 | 1,04 |
| | | 3 | 1,99 | | | 3 | 0,7 |
| | 20 | 1 | 7,4 | | 20 | 1 | 2,62 |
| | | 2 | 3,79 | | | 2 | 1,34 |
| | | 3 | 2,59 | | | 3 | 0,92 |

is constructed as a function of $Y_{M.\pi}$, and the desired value is found by the point of intersection of the constructed curve with the straight line drawn from the origin of the coordinates at an angle of 45°. The values of $Y_{M.\pi}$ for cables with a capacitance of 4×4×1.2 type MKSAShp and MKSABpShp are presented in Table 10.8.

Table 10.8. Total transient conductivity between metal coverings and ground

| Frequency, kilohertz | Transient conductivity, siemens/m, with a specific ground resistance, ohm-m | | | | | |
|---|---|---|---|---|---|---|
| | 10 | | 100 | | 1000 | |
| | Modulus | Angle | Modulus | Angle | Modulus | Angle |
| 0,05 | $0{,}421\cdot10^{-6}$ | 90° | $0{,}421\cdot10^{-6}$ | 90° | $0{,}421\cdot10^{-6}$ | 90° |
| 0,8 | $6{,}73\cdot10^{-6}$ | 90° | $6{,}73\cdot10^{-6}$ | 90° | $6{,}73\cdot10^{-6}$ | 90° |
| 10,0 | $84{,}1\cdot10^{-6}$ | 90° | $84{,}1\cdot10^{-6}$ | 90° | $84{,}1\cdot10^{-6}$ | 90° |
| 100,0 | $0{,}84\cdot10^{-3}$ | 89°4′ | $0{,}81\cdot10^{-3}$ | 81°9′ | $0{,}41\cdot10^{-3}$ | 36°27′ |

Table 10.9. Primary parameters of the metal coverings of MkSABpShp and MKSSShp cables

| Current in sheathing, amps | MKSABpSHp 4×4×1.2 | | | MKSSShp 4×4×1.2 | | |
|---|---|---|---|---|---|---|
| | $R_{sh}$, ohm/km | $L_{sh}$, mg/km | $C_{sh}$, μfar./km | $R_{sh}$, ohm/km | $L_{sh}$, mg/km | $R_{sh}$, μfarads/km |
| 2 | 0,42 | 4,8 | 1,34 | 1,81 | 3,4 | 0,74 |
| 5 | 0,42 | 5,6 | 1,34 | 1,81 | 4,4 | 0,74 |
| 10 | 0,42 | 7,3 | 1,34 | 1,81 | 6,8 | 0,74 |
| 20 | 0,42 | 14,7 | 1,34 | 1,81 | 14,9 | 0,74 |
| 30 | 0,42 | 16,6 | 1,34 | 1,81 | 14,7 | 0,74 |
| 50 | 0,42 | 14,1 | 1,34 | 1,81 | 11,6 | 0,74 |
| 75 | 0,42 | 11,6 | 1,34 | 1,81 | 9,6 | 0,74 |
| 100 | 0,42 | 9,5 | 1,34 | 1,81 | 8,1 | 0,74 |

## 10.5. Primary Parameters of the "Core-Sheathing" Circuit

The "core-sheathing" circuit is analogous to the cable circuit, but with high eccentricity, that is, with large bias of the internal conductor relative to the central axis of the sheathing. The total resistance of the "core-sheathing" circuit can be represented in the form of the sum of the internal resistance of the cable core, the internal resistance of the metal coverings and the external resistance caused by the interconductor inductance:

$$Z_{c-sh} = Z_{c.int} + Z_{sh.ind} + i\omega L_{c-sh}.$$

The total internal resistance of the cable core

$$\underset{(a)}{Z_{ж.внутр}} = R_ж + i\omega L_{ж.внутр} = \frac{1}{2\pi r_a} \sqrt{\frac{i\omega\mu}{\gamma_м}} \frac{I_0(kr_a)}{I_1(kr_a)}.$$

Key: a. c. int

where $R_c = R_{c0}k_1$; $L_{c.int} = (1/2)\mu k_2 \cdot 10^{-7}$; $k_1$ and $k_2$ are the coefficients taking into account the surface effect (see Table 10.2).

The total resistance of the single-layer metal sheathing in the "core-sheathing" circuit is determined by the formula [69]

$$\underset{(a)}{Z_{об.внутр}} = R_{об.внутр} + i\omega L_{об.внутр} = \frac{1}{2\pi r_1} \sqrt{\frac{i\omega\mu}{\gamma_м}} \times \frac{I_0(kr_1)K_1(kr_2) + K_0(kr_1)I_1(kr_2)}{I_1(kr_2)K_1(kr_1) - K_1(kr_2)I_1(kr_1)}. \quad (10.27)$$

Key: a. sh.int

In order to determine the internal primary parameters of the metal coverings in the voice frequency range it is possible to use the formulas (10.15)–(10.17). The external (interconductor) inductance of the circuit with the core located in the center of the sheathing is

$$\underset{(a)}{L_{ж\text{-}об}} = 2\ln\frac{r_1}{r_a}10^{-7}. \quad (10.28)$$

Key: a. c-sh

In formulas (10.27) and (10.28), $r_a$ is the core radius; $r_1$ and $r_2$ are the internal and external radii of the metal sheathing. The capacitance of the circuit with the core located in the center of the sheathing $C_{c-sh} = \varepsilon_r \cdot 10^{-9}/18 \ln (r_1/r_a)$, and the conductivity of the insulation $G_{c.sh} = \omega C_{c.sh} \operatorname{tg} \delta$.

Since the cores of the symmetric cable are not located in the center, considering the bias, the external inductance and capacitance of the "core-sheathing" circuit can be defined by the formulas [72]:

$$(a)\ L_{ж\text{-}об.внешн} = 2\left[\ln\frac{r_1}{r_a} - \frac{\delta^2}{1-\delta^2}\right]10^{-7},$$

$$(b)\ C_{ж\text{-}об} = \frac{\varepsilon_r \cdot 10^{-9}}{18\left[\ln\frac{r_1}{r_a} - \frac{\delta^2}{1-\delta^2}\right]},$$

Key: a. c.sh.ext b. c-sh

where $\vartheta=\Delta/r_1$ is the eccentricity of the core equal to the ratio of the distance of the cable core from the center ($\Delta$) to the inside radius of the metal sheathing: $\delta = r_a/r_1$. As is obvious from these formulas, the external inductance as a result of eccentricity decreases, and the capacitance of the circuit increases. The total resistance of the "two cores-sheathing" circuit

$$Z_{(c-c)-sh} = Z_{(c-c)\,int} + Z_{sh.int} + i\omega L_{(c-c)-sh},$$

where $Z_{(c-c)\,int} = Z_{c.int}/2$; $L_{(c-c)-sh} = 2\ln(r_1/r_{a.equiv})10^{-7}$; $r_{a.equiv} = \sqrt{a_c r_c}$; $a_c$ is the mean spacing between cores. The capacitance of the "two core-sheathing" circuit $C_{(c-c)-sh} = \varepsilon_r \cdot 10^{-9}/18\ln(r_1/r_{a.equiv})$, the conductivity of the insulation $G_{(c-c)-sh} = \omega C_{(c-c)-sh}\,\mathrm{tg}\,\delta$.

As the measurements show, the capacitance of the "core-sheathing" circuit is approximately 1.87 times larger than the capacitance of the basic two-wire circuit, and the capacitance of the "two core-sheathing" circuit, that is, the capacitance of the Picard circuit is 3.46 times greater than the capacitance of the basic circuit. The more exact formulas for calculating the parameters of the asymmetric "core-sheathing" circuits of the single-quad cables in the longitudinal frequency range are obtained in [74].

## 10.6. Parameters of the "Core-Ground" Circuit

The following relations were established between the primary parameters of the "core-ground" circuit and the primary parameters of the "core-sheathing" circuit:

active resistance

$$R_{c-g} = R_{c-sh} - R_{sh} + R_g,$$

where $R_g = \pi^2/f \cdot 10^{-7}$ is the loss resistance in the ground, ohm/m;

inductance

$$L_{c-g} = L_{c-sh} + L_{sh.ext},$$

where $L_{sh.ext}$ is the external inductance of the sheathing [see (10.22) and (10.26)]:

capacitance

$$C_{c-g} = \frac{C_{c-sh}C_{sh}}{C_{c-sh} + C_{sh}},$$

where $C_{sh}$ is the capacitance between the sheathing and the ground [see (10.28)]:

in the presence of an external jute cover $C_{c-g} \approx C_{c-sh}$ and the conductivity:

$$G_{c-g} = \frac{G_{c-sh} G_{sh}}{G_{c-sh} + G_{sh}},$$

where $G_{sh} = 1/R_{trans.sh}$ is the conductivity between the sheathing and the ground [see (10.20)-(10.22) and (10.24)].

The components of the total resistance of a bunch of n identical cores can be defined by the approximate formula, replacing the bunch by the equivalent conductor [73]. The active resistance of the bunch $R_b = R_c/n$; the inductance of the bunch on return of the current through the ground

$$\text{(a)}\ L_{\text{п}} = \frac{\mu_0}{2\pi}\left[\ln\left(\frac{h}{r_{\text{п}}} + \frac{1}{4}\right) + (\mu_r - 1)\ln\frac{r_{2\,\text{бр}}}{r_{1\,\text{бр}}}\right],$$
(b)

Key: a. b          a. armor

where $r_b$ is the outside radius of the bunch, meters; $\mu_0$ is the magnetic constant ($\mu_0 = 4\pi \cdot 10^{-7}$ g/m); $\mu_r$ is the relative magnetic permeability of the material of the armored cover; $r_{1\ armor}$ and $r_{2\ armor}$ are the outside and inside radii of the armored cover, meters; h is the equivalent depth of occurrence of the concentrated current in the ground, meters; with respect to Carson $h = 1.85/\sqrt{\omega \mu_g \gamma_g}$.

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