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Translation

PENETRATION

(Penetration of Compressible

Continuous Media by Solid States)

By

A.Y. Sagomonyan

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Contents

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

ANNOTATION

[Text] This book discusses the problems of the penetration of liquid and soils by solid states and also penetration by solid states having high relative velocities on impact. The penetrating bodies are assumed to be absolutely solid, elastic and in the form of elastic shells containing a liquid. When investigating the problems of the penetration of soils by solid states, a model plastic compressible continuous medium is introduced, on the basis of which detailed solutions of urgent modern problems defining all of the dynamic parameters of the movement of the soil and the penetrating body are presented. In addition, the solutions of penetration problems on impact of a solid deformable body of small dimensions with a deformable body of large dimensions are discussed. Various models for analytical investitation of the penetration problem are investigated as a function of the magnitude of the relative velocity at which the bodies meet.

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

INTRODUCTION

In this book the results are presented from studies of the modern problems of penetration of a liquid, soils and metallic obstacles by a solid state. The study of the problems connected with the penetration of various continuous media by solid states began long ago. However, until recently these studies pertained to a limited group of problems and basically were of an empirical nature. They led to several useful formulas which, however, did not give a representation of the dynamics of the penetration process itself. This is explained by the fact that for a long time the penetration problem was of interest for a narrow group of researchers, and, above all, there were significant mathematical difficulties in the theoretical study of the subject, indeterminacy of the mechanical properties of the media, and absence of reliable measuring devices in the experiments.

In connection with the occurence of new technical problems in various branches of modern practice in recent years the interest of a broad group of scientists and engineers in the problems connected with the penetration of various media by solid states has increased significantly. In addition, new possibilities have come up which are promoting progress in the study of the penetration problem. These include the following: the availability of high-speed computers which shorten the numerical calculation procedure, achievements in the development of the general methods of studying the motion of continuous media promoting the analytical investigation of the problem, achievements in improving the measuring equipment permitting the reliability of the experimental results to be increased.

This book takes up the analytical solution of penetration problems. It consists of three chapters. The first chapter is primarily on the study of the problems of the penetration of a compressible liquid by solid states. Among the results of the solutions of the problems of penetration of bodies into an incompressible liquid presented in this chapter which are of independent interest, many are used for logical relations and comparison with the results of the solutions of these problems considering the compressibility of the liquid. If the penetration rate is high or the penetration of a sufficiently blunt body is considered, then the compressibility of the liquid must be considered to obtain reliable results. These problems are of interest for entry of rockets and missiles into water, landing of spacecraft and seaplanes and other practical problems. It appears to us that this chapter contains analytical solutions of the most interesting problems of the penetration of a compressible liquid by solid states at the present time.

On the basis of the plastic compressible continuous medium simulating many types of soils, a method has been developed to study the penetration of soils by solid

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

states permitting substantial expansion of the class of solved problems. The studies of the problems of penetration of soils by solid states are made in the second chapter. The solutions obtained here make it possible to determine all of the dynamic parameters of movement during the process of penetration of the body into the soil. In addition to scientific interests, the results of the studies in this chapter will be useful to engineers involved with the problems of impact and penetration of bodies into soils.

In the last, third, chapter of the book the phenomenon of penetration is considered as the result of collision of a metal hammerhead with a metal obstacle under supersonic relative velocity conditions. The urgency of the problem of the interaction of solid states meeting at very high relative velocities is obvious. In this chapter a discussion is presented of the existing approximate analytical methods of solution. As a rule, they are contained in journal articles. In all of the chapters where it is possible, the results of the theoretical studies are compared with the results of experimental measurements.

The first chapter of the book was prepared jointly with I. P. Khlyzov, the second chapter, jointly with M. L. Gartsshteyn and V. I. Noskov. V. I. Noskov found the solutions to the problems of items 7, 9, 10 and 11 in Chapter 2. Besides these comrades, the following people assisted in preparing the manuscript: O. N. Goman, V. A. Yeroshin, A. N. Mar'yamov, V. V. Paruchikov. I should like to express my appreciation and gratitude to all of the mentioned comrades. The author will be grateful to everyone who wishes to send comments and suggestions.

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

CHAPTER 1. PENETRATION OF AN IDEAL LIQUID BY SOLID STATES

This chapter takes up the problems of the penetration of solid states into an ideal liquid. These problems are of interest with regard to the entry of missiles and rockets into water, water landings of spacecraft, seaplanes, and so on. Until comparatively recently the study of the penetration phenomenon was carried out on a model of an incompressible ideal liquid. It is possible to familiarize oneself with the basic results of these studies in the monographs [1, 2, 3].

If the speed of the penetrating body is high or the head of the body is sufficiently blunt, then in order to obtain reliable results it is necessary to consider the compressibility and wave nature of movement of a liquid.

In this chapter basically the results are presented from studies of the problems of penetration of solid states into initially still compressible ideal liquid occupying the lower halfspace. The studies are made considering the effect of the movement of air over the surface of the liquid during the processes of approaching the water and penetration of the body. In all problems the initial period of penetration preceding the appearance of a cavity is considered, where the penetrating body has been incompletely submerged in the liquid. During this time period, the pressures acting on the penetrating body reach maximum values by which the dynamic strength calculation is made.

Many items in this chapter are devoted to the study of self-similar problems of penetration in the linear and nonlinear statement. Therefore at the beginning of the chapter the derivation of the basic equations of self-similar motion of a compressible liquid and the characteristics of these equations are presented. Then comes a section on the problems of penetration of an incompressible liquid by solid states. Many of the solutions of the problems of this section which have independent significance are compared with the solutions of analogous problems of penetration of a compressible liquid by solid states. The section contains an original solution to the problement of ricochet of a plate from the surface of an incompressible liquid and investigation of the motion of a thin body from depth in the direction of the free surface. The last problem does not directly pertain to the class of penetration problems, but it is close to them with respect to mathematical statement and method of solution.

The problems of penetration of solid states into a compressible liquid are the subjects of two sections of the first chapter. In the first of them the prob- of the penetration of thin axisymmetric and flat bodies are investigated. Subsonic and supersonic penetration and also penetration at a rate equal to the speed of sound in the liquid are investigated.

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

The studies of this section demonstrate that up to high subsonic penetration velocities of thin sharp bodies, the effect of compressibility on the resistance is insignificant. In the next section on penetration of a compressible liquid, blunt solid states are considered. The correct determination of the resistance to penetration of blunt bodies, as a rule, requires consideration of the compressibility of the liquid. For example, when a cylindrical body with a flat front tip hits the surface of an incompressible liquid with any initial velocity, an instantaneous change in momentum of the liquid by a finite amount takes place. The force acting on the body at the time of impact is of a pulse nature in this case.

In reality, the disturbances in a medium are propagated with finite velocity, as a result of which the change in momentum of the liquid and velocity of the penetrating body is continuous, and the force acting on the body at the time of beginning of penetration is finite. When the disturbance waves travel far from the body, the disturbances in the medium asymptotically approach the disturbances at the corresponding point in time arising after impact on the surface of an incompressible liquid. Thus, the theory of an incompressible liquid replaces the effect of the finite force in the initial period of penetration by a pulse force which can be determined by the integral characteristic -- the loss of momentum at the time of impact. Consideration of the compressibility of the liquid permits determination of the finite pressures and force acting on the body in the initial penetration period.

On impact penetration of a compressible liquid by blunt bodies with arbitrary subsonic velocity, the displacement rate of the generatrix of the body with respect to the free surface can turn out to be close to and greater than the speed of sound in a liquid. In this case it is necessary to consider the compressibility of the medium. The terms used above "impact against the surface of the liquid" and "impact entry" are also encountered when discussing the solution of various problems in the first chapter. They are frequently used by many authors obviously to emphasize the sharp nature of the change in the parameters of motion during the initial penetration period. However, these terms do not introduce any vagueness into the definition of penetration as the process of submersion of the body in the liquid through its free surface.

All of the cases of penetration of a liquid by solid states investigated in this chapter are formulated as mathematical problems with specific initial and boundary conditions. In view of the extraordinary complexity of the analytical investigation of the essentially nonstationary problem of penetration into a compressible liquid, the penetrating bodies investigated in this chapter have a simple geometric shape. This explains the small number of penetration problems in which deformation of the penetrating body and the wave nature of its stressed state are considered.

In the required cases, the study is performed considering the effect of the lift of the free surface of the compressible liquid on the dynamic penetration process. The studies of the penetration into a compressible liquid in the first chapter, as a rule, are performed on the basis of the linearized equations of motion.

For a weakly compressible liquid such as water, when investigating a broad class of penetration problems, this approximation is entirely justified. However, in modern practice there are cases where a blunt solid state meets a free liquid surface with high relative velocity. In such problems, during the initial period of penetration, a shock wave will be propagated from the point of impact into the depths of the

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

liquid after which the movement will be described by nonlinear equations. This difficult problem has not as yet been investigated.

In this chapter the problems of penetration of a compressible liquid by a blunt wedge and a blunt cone are investigated in the nonlinear statement. It is proposed that the edge of the wedge (and the generatrix of the cone, respectively) is shifted along the free surface of the liquid at supersonic velocity. The study of these problems is based on the analytical apparatus presented in the first item. The results obtained here are of definite practical interest, and the statement of the problems themselves can attract the attention of researchers to this difficult, but prospective modern problem. This chapter also contains a section on the study of the effect of viscosity on the penetration process.

§ 1. Equations of Self-Similar Motion of a Liquid

Let the adiabatic motion of a nonviscous compressible liquid with axial symmetry be considered in the absence of mass forces. Let us direct the Ox axis of the stationary orthogonal coordinates along the axis of symmetry, and let us place the Oy axis in the plane of the meridian. In these coordinates the equations of motion, continuity and conservation of entropy of a particle are written as follows:

$$\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x},$$

$$\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y},$$

$$\frac{\partial \rho}{\partial t} + \rho\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) + v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + \frac{\rho \cdot v_y}{y} = 0,$$

$$\frac{\partial S}{\partial t} + v_x \frac{\partial S}{\partial x} + v_y \frac{\partial S}{\partial y} = 0, \qquad (1.1)$$

where v_x, v_y are the velocity components of the particles, ρ is the density of the liquid, p is the pressure, S is the entropy, t is time. Hereafter it is proposed that the speed, pressure and all other parameters of motion are uniform 0-order functions with respect to x, y and the time t. This means that the indicated parameters will be functions of the ratios

$$\xi = x/t, \quad \eta = y/t.$$

The currents having this property will be called self-similar. Usually by self-similar we mean the motion of a liquid, the parameters of which are functions of the ratios

$$x/t^{\alpha}, \quad y/t^{\alpha},$$

where α is a constant.

Let us transform the equation (1.1), proceeding to the new coordinates ξ, η. Using the obvious expressions

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$\frac{\partial}{\partial t} = -\frac{x}{t^2}\cdot\frac{\partial}{\partial \xi} - \frac{y}{t^2}\cdot\frac{\partial}{\partial \eta}; \quad \frac{\partial}{\partial x} = \frac{1}{t}\frac{\partial}{\partial \xi}, \quad \frac{\partial}{\partial y} = \frac{1}{t}\frac{\partial}{\partial \eta},$$

it is easy to find the form of the equations (1.1) in the new variables:

$$\begin{gathered}
(v_x - \xi)\frac{\partial v_x}{\partial \xi} + (v_y - \eta)\frac{\partial v_x}{\partial \eta} = -\frac{1}{\rho}\frac{\partial p}{\partial \xi},\\
(v_x - \xi)\frac{\partial v_y}{\partial \xi} + (v_y - \eta)\frac{\partial v_y}{\partial \eta} = -\frac{1}{\rho}\frac{\partial p}{\partial \eta},\\
(v_x - \xi)\frac{\partial \rho}{\partial \xi} + (v_y - \eta)\frac{\partial \rho}{\partial \eta} + \rho\left(\frac{\partial v_x}{\partial \xi} + \frac{\partial v_y}{\partial \eta}\right) + \frac{\rho v_y}{\eta} = 0,\\
(v_x - \xi)\frac{\partial S}{\partial \xi} + (v_y - \eta)\frac{\partial S}{\partial \eta} = 0.
\end{gathered} \tag{1.2}$$

In the investigated case the vorticity $\bar{\omega}(x, y, t)$ can be represented in the form

$$\bar{\omega}(x, y, t) = \frac{1}{t}\,\omega(\xi, \eta), \tag{1.3}$$

where

$$\omega(\xi, \eta) = \frac{\partial v_y}{\partial \xi} - \frac{\partial v_x}{\partial \eta}. \tag{1.4}$$

For convenience of further calculations let us introduce the notation:

$$U = v_x - \xi, \quad V = v_y - \eta, \quad W^2 = U^2 + V^2.$$

In this notation equations (1.2) and (1.4) are written as follows:

$$\begin{gathered}
U + U\frac{\partial U}{\partial \xi} + V\frac{\partial U}{\partial \eta} = -\frac{1}{\rho}\cdot\frac{\partial p}{\partial \xi},\\
V + U\frac{\partial V}{\partial \xi} + V\frac{\partial V}{\partial \eta} = -\frac{1}{\rho}\cdot\frac{\partial p}{\partial \eta},\\
\frac{\partial(\rho U)}{\partial \xi} + \frac{\partial(\rho V)}{\partial \eta} + \frac{\rho V}{\eta} + 3\rho = 0,\\
U\frac{\partial S}{\partial \xi} + V\frac{\partial S}{\partial \eta} = 0,\\
\frac{\partial V}{\partial \xi} - \frac{\partial U}{\partial \eta} = \omega(\xi, \eta).
\end{gathered} \tag{15}$$

Let us introduce the expression for the "vorticity" $\omega(\xi, \eta)$ in the plane (ξ, η) with respect to formula (1.4) into the first two equations of system (1.5). As a result, we obtain

$$U - V\omega + \frac{\partial}{\partial \xi}\left(\frac{W^2}{2}\right) = -\frac{1}{\rho}\cdot\frac{\partial p}{\partial \xi},$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$V + U\omega + \frac{\partial}{\partial\eta}\left(\frac{W^2}{2}\right) = -\frac{1}{\rho}\cdot\frac{\partial p}{\partial\eta}.$$

Multiplying the first of these relations by the differential $d\xi$, the second by $d\eta$ and adding the expressions obtained, we have

$$\omega(Ud\eta - Vd\xi) + Ud\xi + Vd\eta + d\frac{W^2}{2} + \frac{dp}{\rho} = 0. \quad (1.6)$$

This relation along the "current line" defined by the equation

$$\frac{d\xi}{U} = \frac{d\eta}{V}, \quad (1.7)$$

assumes the form

$$d\frac{W^2}{2} + Ud\xi + Vd\eta + \frac{dp}{\rho} = 0. \quad (18)$$

The next to the last equation of system (1.5) indicates that along the line (1.7) the entropy S is constant, and the pressure differential is represented by the formula

$$dp = a^2 d\rho,$$

where a is the speed of sound in the liquid.

It is easy to check that the "current line" (1.7) is a characteristic of the system of equations (1.5) with the relation (1.8) along it. The other characteristics of the system (1.5) can be obtained, for example, from investigating the Cauchy problem. By excluding the pressure, density and entropy from the first four equations of system (1.5) and the obvious relations

$$\frac{\partial p}{\partial\xi} = a^2\frac{\partial\rho}{\partial\xi} + \frac{\partial p}{\partial S}\cdot\frac{\partial S}{\partial\xi}; \quad \frac{\partial p}{\partial\eta} = a^2\frac{\partial\rho}{\partial\eta} + \frac{\partial p}{\partial S}\cdot\frac{\partial S}{\partial\eta}$$

we come to the following system of two equations:

$$\begin{gathered}(a^2 - U^2)\frac{\partial U}{\partial\xi} - UV\left(\frac{\partial U}{\partial\eta} + \frac{\partial V}{\partial\xi}\right) + (a^2 - V^2)\frac{\partial V}{\partial\eta} + \frac{a^2V}{\eta} + \\ + 3a^2 - W^2 = 0, \\ \frac{\partial V}{\partial\xi} - \frac{\partial U}{\partial\eta} = \omega. \end{gathered} \quad (1.9)$$

This system can be reduced to one equation of the type

$$(a^2 - U^2)\frac{\partial U}{\partial\xi} - 2UV\frac{\partial U}{\partial\eta} + (a^2 - V^2)\frac{\partial V}{\partial\eta} = \omega UV - \frac{a^2V}{\eta} + W^2 - 3a^2. \quad (1.10)$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

In the Cauchy problem along the line L in the plane (ξ, η) values of the desired functions of the system of equations are given. Then in our case for determination of the derivatives of U and V with respect to ξ and η along the line L, in addition to the system (1.9) we shall have two expressions

$$dU = \frac{\partial U}{\partial \xi} d\xi + \frac{\partial U}{\partial \eta} d\eta, \qquad dV = \frac{\partial V}{\partial \xi} d\xi + \frac{\partial V}{\partial \eta} d\eta. \tag{1.11}$$

Let us multiply the first of these expressions by the factor λ and add it to the second. As a result, by using the second equation of the system (1.9) we obtain

$$\lambda d\xi \frac{\partial U}{\partial \xi} + (\lambda d\eta + d\xi) \frac{\partial U}{\partial \eta} + d\eta \frac{\partial V}{\partial \eta} = dV + \lambda dU - \omega d\xi. \tag{1.12}$$

As is known, the characteristic expressions are obtained from the condition of impossibility of a unique determination of the derivatives from the system (1.9) and (1.11). This means that a linear relation must exist between the coefficients of the equations (1.10) and (1.12), which leads to the equalities

$$\frac{\lambda d\xi}{a^2 - U^2} = \frac{\lambda d\eta + d\xi}{-2UV} = \frac{d\eta}{a^2 - V^2} = \frac{dV + \lambda dU - \omega d\xi}{\omega UV - \frac{a^2 V}{\eta} + W^2 - 3a^2}. \tag{1.13}$$

From the equalities (1.13) two systems of characteristics are obtained in the plane (ξ, η), and two conditions along them, respectively:

$$\left(\frac{d\eta}{d\xi}\right)_{1,2} = \eta'_{1,2} = \frac{-UV \pm a\sqrt{W^2 - a^2}}{a^2 - U^2}, \tag{1.14}$$

$$dU + \eta'_{2,1} dV + \frac{d\xi}{a^2 - U^2}\left[\frac{a^2 V}{\eta} - W^2 + 3a^2\right] = \mp \omega \frac{a\sqrt{W^2 - a^2}}{a^2 - U^2} d\xi. \tag{1.15}$$

The equation of the third characteristic of the system (1.5) and the corresponding condition along it, according to the above-presented facts, are written in the form

$$\left(\frac{d\eta}{d\xi}\right)_3 = \eta'_3 = \frac{V}{U}, \tag{1.16}$$

$$d\frac{W^2}{2} + U d\xi + V d\eta + \frac{dp}{\rho} = 0, \quad dp = a^2 d\rho. \tag{1.17}$$

From (1.14) and (1.15) it follows that the first two systems of characteristics will be real under the condition

$$(v_x - \xi)^2 + (v_y - \eta)^2 \geqslant a^2. \tag{1.18}$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

Using equation (1.6) which is valid in any direction it is easy to obtain the following expressions along the characteristics of the first and second families of $\eta'_{1,2}$, respectively:

$$\omega d\xi = \pm \frac{\sqrt{W^2-a^2}}{a}\,d\xi + \frac{a^2-U^2}{a\,(aV \mp U\sqrt{W^2-a^2})}\left[d\,\frac{W^2}{2} + \frac{dp}{\rho}\right]. \tag{1.19}$$

If we substitute this expression for the vorticity ω in the right hand side of the conditions (1.15), after some transformations along the characteristics $\eta'_{1,2}$, we obtain

$$VdU - UdV \pm \frac{(U+V\eta'_{1,2})\,d\xi}{\sqrt{M^2-1}}\left(\frac{V}{\eta}+2\right) \pm \sqrt{M^2-1}\,\frac{dp}{\rho} = 0, \tag{1.20}$$

where

$$M = \frac{W}{a}.$$

In the case of plane-parallel flow the equations of the characteristics (1.14) in the plane (ξ, η) retain their form, and instead of expressions (1.15) and (1.20), we have

$$dU + \eta'_{2,1}\,dV + \frac{d\xi}{a^2-U^2}\,[2a^2 - W^2] = \mp\,\omega\,\frac{a\sqrt{W^2-a^2}}{a^2-U^2}\,d\xi,$$

$$VdU - UdV \pm \frac{(U+V\eta'_{1,2})}{\sqrt{M^2-1}}\,d\xi \pm \sqrt{M^2-1}\,\frac{dp}{\rho} = 0.$$

The equation of the third characteristic (1.16) and the conditions along it (1.17) also retain their form in this case.

Hereafter, when solving the problems, the expression for the displacement rate of the breakdown surface in the self-similar motion is needed. If $F_1(x, y, t)$ is the breakdown surface, then, as is known [4], the displacement rate D of this surface is expressed by the formula

$$D = \frac{-\partial F_1/\partial t}{\sqrt{(\partial F_1/\partial x)^2 + (\partial F_1/\partial y)^2}}. \tag{1.21}$$

In the case of self-similar motion the velocity D is a uniform function of zero order and, consequently, the equation of the breakdown surface will be a uniform first-order function

$$F_1(x, y, t) = tF(\xi, \eta). \tag{1.22}$$

Using (1.22), formula (1.21) assumes the form

$$D = \frac{\xi}{G}\cdot\frac{\partial F}{\partial \xi} + \frac{\eta}{G}\cdot\frac{\partial F}{\partial \eta}, \quad G = \sqrt{\left(\frac{\partial F}{\partial \xi}\right)^2 + \left(\frac{\partial F}{\partial \eta}\right)^2}. \tag{1.23}$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

Obviously, the expressions

$$\frac{1}{G}\cdot\frac{\partial F}{\partial\xi},\quad \frac{1}{G}\cdot\frac{\partial F}{\partial\eta}$$

are directional cosines of the normal to the breakdown surface with the coordinate axes $O\xi$, $O\eta$.

The expressions (1.20), (1.17) along the characteristics (1.14), (1.16) together with the boundary conditions and the conditions on the breakdown surface, including formula (1.23) permit "step-by-step" determination of the current field and the sections of the breakdown surface in the regions where the condition (1.18) is satisfied. The value of the "vorticity" ω is determined by the formula (1.6), and along the characteristics $\eta'_{1.2}$, by the formulas (1.19). Of course, for the calculations relations (1.20) can be replaced by the expressions (1.15) equivalent to them. The procedure for determining the parameters using the presented characteristics is analogous to the procedure for determining the parameters in a supersonic steady state eddy movement. Therefore we shall not describe the method of performing the required operations for calculating these parameters.

Let the flow be irrotational ($\omega = 0$). From the definition itself, it follows that for self-similar currents the velocity potentials $\bar{\varphi}(x, y, t)$ can be represented in the form

$$\bar{\varphi}(x, y, t) = t\varphi(\xi, \eta). \tag{1.24}$$

Let us introduce the auxiliary function $\Phi(\xi, \eta)$ into the investigation:

$$\Phi(\xi, \eta) = \varphi(\xi, \eta) - \frac{1}{2}(\xi^2 + \eta^2).$$

It is easy to check that

$$v_x = \frac{\partial\varphi}{\partial\xi},\quad v_y = \frac{\partial\varphi}{\partial\eta},\quad U = \frac{\partial\Phi}{\partial\xi},\quad V = \frac{\partial\Phi}{\partial\eta}.$$

According to (1.10), the function Φ satisfies the equation

$$(a^2 - U^2)\frac{\partial^2\Phi}{\partial\xi^2} - 2UV\frac{\partial^2\Phi}{\partial\xi\partial\eta} + (a^2 - V^2)\frac{\partial^2\Phi}{\partial\eta^2} + \frac{a^2V}{\eta} + 3a^2 - W^2 = 0. \tag{1.25}$$

The characteristics of the equation (1.25) describing the potential movement in the plane (ξ, η) are defined by the equations (1.14). The conditions along these characteristics in the case of potential motion will be obtained from the expressions (1.15) setting $\omega = 0$ in the right-hand side:

$$dU + \eta'_{2,1}\,dV + \frac{d\xi}{a^2 - U^2}\left[\frac{a^2V}{\eta} - W^2 + 3a^2\right] = 0.$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

Let us return to the Lagrange integral:

$$\frac{\partial \varphi}{\partial t}+\frac{v^2}{2}+\int\frac{dp}{\rho}=f(t);\quad (v^2=v_x^2+v_y^2). \tag{1.26}$$

For the self-similar motion that on the basis of (1.24) this equation is transformed into the form

$$\varphi(\xi,\ \eta)-\xi\frac{\partial\varphi}{\partial\xi}-\eta\frac{\partial\varphi}{\partial\eta}+\frac{v^2}{2}+\int\frac{dp}{\rho}=f(t).$$

In this equation the expression on the left is the function of ξ, η; on the right is the time function. It is impossible by any combination of variables ξ, η to obtain the time dimension. Hence, it follows that for self-similar flows the time function f(t) in the right-hand side of the Lagrange integral is equal to a constant:

$$\varphi(\xi,\ \eta)-\xi\frac{\partial\varphi}{\partial\xi}-\eta\frac{\partial\varphi}{\partial\eta}+\frac{v^2}{2}+\int\frac{dp}{\rho}=\text{const}.$$

Introducing the function $\Phi(\xi, \eta)$ into this equation, we obtain

$$\Phi(\xi,\ \eta)+\frac{W^2}{2}+\int\frac{dp}{\rho}=\text{const}. \tag{1.27}$$

It is clear that expression (1.8) is the differential of the integral (1.27), and now it occurs in any direction and not only along the "current line" (1.7) as happened in the general case of rotational motion.

Let us proceed to the derivation of the linearized equations of self-similar motion. The linearization of the equations of motion of a compressible liquid is based on the assumption of smallness of the disturbed motion. Here the variations of the parameters of motion and their derivatives are considered small so that in the equations of motion and continuity only the linear terms are retained.

Then after linearization, the system (1.1) assumes the form (for convenience hereafter the coordinates are given the subscript "0"):

$$\frac{\partial v_x}{\partial t}=-\frac{1}{\rho}\cdot\frac{\partial p}{\partial x_0},\qquad \frac{\partial v_y}{\partial t}=-\frac{1}{\rho}\cdot\frac{\partial p}{\partial y_0},$$
$$\frac{\partial \rho}{\partial t}+\rho\left(\frac{\partial v_x}{\partial x_0}+\frac{\partial v_y}{\partial y_0}\right)+\frac{\rho v_y}{y_0}=0,\qquad \frac{\partial p}{\partial t}=a^2\frac{\partial \rho}{\partial t}. \tag{1.28}$$

More strictly, the system (1.28) can be obtained as follows [5]. Let us propose that in the problem there is the small parameter δ. Then if we distort the parameters of motion in the form of expansions with respect to powers of δ, and substitute these expansions in the equations of system (1.1) and equate the coefficients for identical powers of δ, then in the first approximation these equations assume the form of (1.28).

There are three cases where it is possible to introduce the small parameter δ and, consequently, to construct the linear theory.

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

1. The motion after a wave of weak intensity (a sound wave) when $v/a = o(\delta)$ and the body has an arbitrary shape. For an ideal liquid this condition is equivalent to the following:

$$(p-p_0)/p_0 = o(\delta)$$

(here p_0 is the pressure in the medium at rest, v, p is the speed of the particle and the pressure after the wave, respectively).

2. The wave intensity is arbitrary, but the body introduces a small disturbance into the basic flow; in this case the small parameter is the relative thickness of the body.

3. The wave of arbitrary intensity encounters almost a vertical wall with a slope $\theta = \pi/2 - o(\delta)$; in this case the effect of the distorted part of the wall can be considered as a small reflected discontinuity disturbance.

Thus, let any of these cases occur and we obtain the equations (1.28). In these equations with the accuracy adopted above, the density ρ entering in by a factor is assumed to be constant and equal to its initial value. Excluding the velocity components and the density from the system, we obtain the pressure equation:

$$\frac{\partial^2 p}{\partial x_0^2} + \frac{\partial^2 p}{\partial y_0^2} + \frac{1}{y_0} \cdot \frac{\partial p}{\partial y_0} = \frac{1}{a^2} \cdot \frac{\partial^2 p}{\partial t^2}. \qquad (1.29)$$

It is necessary to note that the equation (1.29) is obtained without the assumption of potentialness of the flow, and the speed of sound a with the above-adopted accuracy was assumed constant. For two-dimensional motion, the pressure is determined from the equation

$$\frac{\partial^2 p}{\partial x_0^2} + \frac{\partial^2 p}{\partial y_0^2} = \frac{1}{a^2} \cdot \frac{\partial^2 p}{\partial t^2}. \qquad (1.30)$$

The Lagrange integral (1.26) for the potential motion after linearization assumes the form

$$\frac{\partial \bar{\varphi}}{\partial t} + \frac{p}{\rho} = \text{const.} \qquad (1.31)$$

Differentiating this equation with respect to t for a constant value of the density and excluding the derivative of the pressure, with the help of the two last equations of system (1.28) we obtain the equation defining the velocity potential:

$$\frac{\partial^2 \bar{\varphi}}{\partial x_0^2} + \frac{\partial^2 \bar{\varphi}}{\partial y_0^2} + \frac{1}{y_0} \cdot \frac{\partial \bar{\varphi}}{\partial y_0} = \frac{1}{a^2} \cdot \frac{\partial^2 \bar{\varphi}}{\partial t^2}. \qquad (1.32)$$

In the case of planar motion, the potential satisfies the equation

$$\frac{\partial^2 \bar{\varphi}}{\partial x_0^2} + \frac{\partial^2 \bar{\varphi}}{\partial y_0^2} = \frac{1}{a^2} \frac{\partial^2 \bar{\varphi}}{\partial t^2}. \qquad (1.33)$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

From the equation (1.33) it follows that any velocity component of planar motion also satisfies the wave equation. For example, for v_y we have

$$\frac{\partial^2 v_y}{\partial x_0^2} + \frac{\partial^2 v_y}{\partial y_0^2} = \frac{1}{a^2} \cdot \frac{\partial^2 v_y}{\partial t^2}. \qquad (1.34)$$

In the linear self-similar problems it is more convenient to introduce dimensionless coordinates:

$$x = \frac{x_0}{at}, \qquad y = \frac{y_0}{at}. \qquad (1.35)$$

In these coordinates the equations (1.32) and (1.33), correspondingly, assume the form

$$(1-x^2)\frac{\partial^2 \varphi}{\partial x^2} - 2xy\frac{\partial^2 \varphi}{\partial x \partial y} + (1-y^2)\frac{\partial^2 \varphi}{\partial y^2} + \frac{1}{y}\cdot\frac{\partial \varphi}{\partial y} = 0, \qquad (1.36)$$

$$(1-x^2)\frac{\partial^2 \varphi}{\partial x^2} - 2xy\frac{\partial^2 \varphi}{\partial x \partial y} + (1-y^2)\frac{\partial^2 \varphi}{\partial y^2} = 0, \qquad (1.37)$$

where $\phi(x, y)$ is a function related to the velocity potential $\bar{\phi}$ by the formula $\bar{\phi} = a^2 t\phi$. If we introduce this function into the Lagrange integral (1.31), we obtain

$$\varphi(x, y) - x\frac{\partial \varphi}{\partial x} - y\frac{\partial \varphi}{\partial y} + \frac{p}{\rho a^2} = \text{const}, \qquad (1.38)$$

where

$$\frac{\partial \varphi}{\partial x} = \frac{v_x}{a}, \quad \frac{\partial \varphi}{\partial y} = \frac{v_y}{a}.$$

Correspondingly, equations (1.29) and (1.30) in the coordinates of (1.35) are written in the form

$$(1-x^2)\frac{\partial^2 p}{\partial x^2} - 2xy\frac{\partial^2 p}{\partial x \partial y} + (1-y^2)\frac{\partial^2 p}{\partial y^2} - 2x\frac{\partial p}{\partial x} - 2y\frac{\partial p}{\partial y} + \frac{1}{y}\frac{\partial p}{\partial y} = 0, \qquad (1.39)$$

$$(1-x^2)\frac{\partial^2 p}{\partial x^2} - 2xy\frac{\partial^2 p}{\partial x \partial y} + (1-y^2)\frac{\partial^2 p}{\partial y^2} - 2x\frac{\partial p}{\partial x} - 2y\frac{\partial p}{\partial y} = 0. \qquad (1.40)$$

Equations (1.34) are also reduced to the form (1.40).

The characteristics of the equations (1.36), (1.37) and (1.39), (1.40) in the (x, y) plane are the same. They are defined by the equations

$$y'_{1,2} = \frac{xy \pm \sqrt{x^2 + y^2 - 1}}{x^2 - 1}. \qquad (1.41)$$

Thus, the indicated equations are of hyperbolic type outside a circle:

$$x^2 + y^2 = 1. \qquad (1.42)$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

By direct statement it is possible to check that the integrals of equations (1.41) will be straight lines:

$$y_{1,2}=\frac{C_{1,2}^2(1+x)+(1-x)}{2C_{1,2}}, \tag{1.43}$$

where $C_{1,2}=\frac{y_0\pm\sqrt{x_0^2+y_0^2-1}}{1+x_0}$ (x_0, y_0 are the coordinates of the point in the plane (x, y), through which a given pair of characteristics passes). It is easy to check that the characteristics of (1.43) are tangents to the circle (1.42).

In the case of axisymmetric motion for the equation (1.36) we obtain the system of characteristics:

$$y_1=\frac{C_1^2(1+x)+(1-x)}{2C_1},\quad dv_x+\frac{C_2^2-1}{2C_2}dv_y+\frac{dx}{1-x^2}\cdot\frac{v_y}{y}=0,$$

$$y_2=\frac{C_2^2(1+x)+(1-x)}{2C_2},\quad dv_x+\frac{C_1^2-1}{2C_1}dv_y+\frac{dx}{1-x^2}\cdot\frac{v_y}{y}=0. \tag{1.44}$$

The system of characteristics for the equation of two-dimensional potential motion (1.37) will be obtained from (1.44) if we drop the last term in the equations of the characteristics in the plane (v_x, v_y).

It is well known that in the region $x^2 + y^2 < 1$ the equation (1.40) using the Chaplygin transformation

$$x=r\cos\theta,\quad y=r\sin\theta,\quad r=\frac{2\varepsilon}{1+\varepsilon^2} \tag{1.45}$$

reduces to the Laplace equation in the region $\varepsilon < 1$:

$$\varepsilon\frac{\partial}{\partial\varepsilon}\left(\varepsilon\frac{\partial p}{\partial\varepsilon}\right)+\frac{\partial^2 p}{\partial\theta^2}=0. \tag{1.46}$$

Thus, in the plane (ε, θ) the pressure p is a harmonic function, and it can be represented as the real part of an analytical function

$$\zeta(\tau)=p(\varepsilon,\theta)+if(\varepsilon,\theta),\quad \tau=\varepsilon e^{i\theta}. \tag{1.47}$$

In this statement the determination of the pressure reduces to a boundary problem for a function of a complex variable. Any velocity component of the planar motion in a liquid in the variables ε, θ will also satisfy the Laplace equation.

For example, for v_y we shall have

$$\varepsilon\frac{\partial}{\partial\varepsilon}\left(\varepsilon\frac{\partial v_y}{\partial\varepsilon}\right)+\frac{\partial^2 v_y}{\partial\theta^2}=0. \tag{1.48}$$

Consequently, analogously to (1.47) it is possible to write

$$\zeta(\tau)=v_y(\varepsilon,\theta)+if(\varepsilon,\theta). \tag{1.49}$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

If $\zeta(\tau)$ is defined, then another velocity component v_x will be found by quadrature. The Cauchy-Riemann conditions for the function (1.49) are written as follows:

$$\frac{\partial v_y}{\varepsilon \partial \theta} = -\frac{\partial f}{\partial \varepsilon}, \quad \frac{\partial v_y}{\partial \varepsilon} = \frac{1}{\varepsilon} \cdot \frac{\partial f}{\partial \theta}. \tag{1.50}$$

From the condition of absence of vorticity it follows that

$$\frac{\partial v_x}{\partial y} = \frac{\partial v_y}{\partial x}. \tag{1.51}$$

Let us rewrite equation (1.37) in the form

$$(1 - x^2)\frac{\partial v_x}{\partial x} - 2xy\frac{\partial v_y}{\partial x} + (1 - y^2)\frac{\partial v_y}{\partial y} = 0. \tag{1.52}$$

Using expressions (1.50)-(1.52) the complete differential

$$dv_x = \frac{\partial v_x}{\partial x}dx + \frac{\partial v_x}{\partial y}dy$$

is easily reduced to the form

$$dv_x = \frac{1}{1 - x^2}\left(xydv_y + \frac{1 - \varepsilon^2}{1 + \varepsilon^2}df\right), \tag{1.53}$$

that is, it is expressed in terms of the complete differentials of the real and imaginary parts of the functions $\zeta(\tau)$.

Analogously, if the analytical function $W(z) = v_x + if$ is found, then

$$dv_y = \frac{1}{1 - y^2}\left(xydv_x - \frac{1 - \varepsilon^2}{1 + \varepsilon^2}df\right). \tag{1.54}$$

Penetration of an Ideal Incompressible Liquid by Solid States

§ 2. Penetration of an Incompressible Liquid by a Wedge

Let a rigid wedge symmetrically penetrate an ideal liquid occupying all the lower halfspace. The velocity of the wedge v_0 is constant, it is directed vertically downward, normal to the horizontal surface of the liquid (see Figure 1.1). The most general statement of this problem is presented in reference [1]. Let us take the origin of the cartesian coordinate system at the point of contact of the apex of the wedge at the free surface at the time of beginning of the penetration. The Ox axis will be directed along the horizontal surface to the right, the Oy axis will be vertically downward in the direction of the penetration velocity. The problem obviously is self-similar, and the motion is potential. The velocity potential $\phi(x, y, t)$, just as the velocity of the liquid at infinity, is equal to zero. On the faces of the wedge obviously we have the boundary condition

$$\frac{\partial \varphi}{\partial n} = v_0 \cos \beta,$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

where β is the slope of the face of the wedge with the Ox axis, n is the outer normal to the face of the wedge. On the free surface, during the entire time of movement the pressure remains constant and equal to the atmosphere. The shape of the free surface is unknown in advance and is subject to definition during the course of solving the problem.

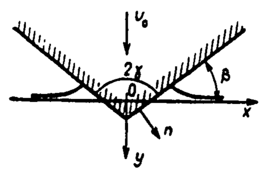

Figure 1.1

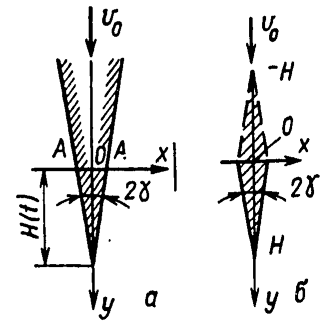

Figure 1.2

The problem reduces to determining the velocity potential from the Laplace equation for the above-indicated boundary conditions. After determining the velocity potential, the pressure in the liquid is calculated using the Cauchy-Lagrange integral. The exact solution of the problem stated here has not been found. Mathematical difficulties of the analytical solution of the problem are related to determining the shape of the free surface. In the basic work [1], a procedure is indicated for solving the problem by the method of successive approximations, and the result of a specific calculation is presented. Many researchers have engaged in studying this problem. The results of all these studies are discussed in considerable detail in the monograph [2], and they are not discussed here.

Let us proceed with the investigation of this problem in the two limiting cases where the angle β is close to zero or a right angle.

a) Penetration of an Incompressible Liquid by a Thin Wedge.

Let a wedge with a small apex angle 2γ penetrate a liquid which is initially at rest and which occupies the lower halfspace. The initial penetration rate v_0 is directed vertically downward, perpendicular to the free surface. The 0y axis is directed along the penetration rate into the liquid, the ox axis along the horizontal surface to the right. The origin of the coordinates is placed at the point of contact of the apex of the wedge with the free surface of the liquid at the time $t = 0$. It is possible to demonstrate [1] that for very small angles $\gamma(\gamma \to 0)$, with the exception of insignificant regions adjacent to the points AA of the face of the wedge (Figure 1.2), the slope of the free surface is infinitely small; therefore in the investigated linearized problem, this surface is assumed to be horizontal after the penetration period.

Let us take the problem of determining the complex potential W(z) of the motion:

$$W(z) = \varphi + i\psi = f(z), \quad z = y + ix, \qquad (2.1)$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$\frac{dW}{dz} = v_y - iv_x, \quad v_x = \frac{\partial\varphi}{\partial x}, \quad v_y = \frac{\partial\varphi}{\partial y}. \tag{2.2}$$

The boundary conditions on the faces of the wedge are shifted to the segment OH of the Oy axis, where H(t) is the depth of penetration. In this segment the boundary conditions are written as follows:

$$v_x = \pm \dot{H}(t)\cdot\gamma. \tag{2.3}$$

The dot over the H denotes differentiation with respect to t. On the free surface, the velocity potential ϕ and the velocity component v_x are equal to zero. By the principle of symmetry, let us continue the function dW/dz of formula (2.2) to the upper halfplane of the plane z = y + ix (see Figure 1.2). Let us construct this function in the entire plane xOy with a section of the segment (-H, H) of the Oy axis. In this segment the velocity component v_x, that is, the imaginary part of the function dW/dz has a discontinuity (the discontinuity is denoted by brackets): in the OH segment

$$[v_x] = 2\dot{H}\gamma,$$

in the HO segment

$$[v_x] = -2\dot{H}\gamma. \tag{2.4}$$

Denoting the limiting values of the function dW/dz on approaching the axis from the right and the left, respectively, by the signs (+) and (-), we have:

$$\left(\frac{dW}{dz}\right)_+ - \left(\frac{dW}{dz}\right)_- = \begin{cases} 2\dot{H}\gamma i & \text{on } -HO, \\ -2\dot{H}\gamma i & \text{on } OH. \end{cases} \tag{2.5}$$

According to the Sokhotskiy formula [6], we have

$$-\frac{dW}{dz} = \frac{1}{2\pi i}\int_0^H \frac{2\dot{H}\gamma i dy}{y-z} - \frac{1}{2\pi i}\int_{-H}^0 \frac{2\dot{H}\gamma i dy}{y-z}.$$

Hence, after performing the integration

$$-\frac{dW}{dz} = \frac{\dot{H}\gamma}{\pi}\ln\left[1-\left(\frac{H}{z}\right)^2\right]. \tag{2.6}$$

Here the branch of the logarithm is selected which in the segment $0 \le z \le H$ of the y axis on both sides gives

$$\operatorname{Im}\frac{dW}{dz} = \pm\dot{H}\gamma.$$

From equation (2.6), the complex potential is defined by quadrature. Calculating this integral, we obtain:

$$-W(z) = \frac{\dot{H}\gamma}{\pi}\left\{-2z\ln z + 4z + z\ln(z^2 - H^2) - H\ln\frac{z-H}{z+H}\right\}.$$

On the Oy axis the velocity potential ϕ has the form;

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

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$$\varphi = \frac{\dot{H}\gamma}{\pi}\left[2y\ln y - 4y - y\ln(H^2 - y^2) + H\ln\frac{H-y}{H+y}\right], \quad 0 \leqslant y \leqslant H. \tag{2.7}$$

The pressure along the face of the wedge is defined by the linearized Cauchy-Lagrange formula

$$\frac{\Delta p}{\rho} = \frac{p - p_0}{\rho} = -\frac{\partial\varphi}{\partial t} = -\frac{\dot{H}^2\gamma}{\pi}\ln\frac{H-y}{H+y} - -\frac{\ddot{H}\gamma}{\pi}\left[2y\ln y - 4y - y\ln(H^2 - y^2) + H\ln\frac{H-y}{H+y}\right] \quad 0 \leqslant y \leqslant H. \tag{2.8}$$

At a constant velocity $\dot{H} = v_0$, $\ddot{H} = 0$

$$\frac{\Delta p}{\rho} = \frac{p - p_0}{\rho} = -\frac{v_0^2\gamma}{\pi}\ln\frac{v_0t - y}{v_0t + y}, \quad 0 \leqslant y \leqslant v_0t.$$

The vertical force of resistance to penetration is

$$F = 2\int_0^H \Delta p\gamma dy = +\frac{2\rho\gamma^2}{\pi}(2\ln 2\cdot H\dot{H}^2 + 3,\ 4H^2\ddot{H}). \tag{2.9}$$

For a constant penetration rate $\dot{H} = v_0$, $\ddot{H} = 0$, we obtain

$$F = \frac{2}{\pi}\rho\gamma^2 v_0^3 t\cdot 2\ln 2. \tag{2.10}$$

The investigated problem was solved for the first time in reference [1], where the following formula was obtained for the force (in our notation)

$$F_1 = 1{,}78\rho v_0^3 t\gamma^2. \tag{2.11}$$

This force is twice the force defined by formula (2.10). The difference is explained by the fact that in reference [1] when calculating the force F, the momentum of the liquid particles on the free surface was not taken into account. Let us return to this problem in the section devoted to the penetration of a compressible liquid by solid states. Comparing formulas (2.10) and 2.11), we have

$$k = \frac{F}{F_1} = \frac{\frac{4\ln 2}{\pi}}{1{,}78} = 0{,}495 \approx 0{,}5.$$

b) The penetration of an incompressible liquid by a blunt wedge considering the lift of the free surface.

Let a very blunt symmetric wedge ($\beta \to 0$) penetrate an incompressible liquid. The penetration rate v_0 is perpendicular to the flat free boundary of the initially quiet liquid (Figure 1.3,a). The problem is solved in the linear statement: the boundary conditions are removed to the horizontal surface of the liquid and linearized by

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

the Cauchy-Lagrance integral. Let us take the origin of the cartesian system of coordinates xOy at the point of contact of the apex of the wedge with the liquid surface, the Ox axis is directed along the free surface to the right, and the Oy axis into the liquid, in the direction of the penetration rate. The velocity potentials satisfies the Laplace equations

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=0. \tag{2.12}$$

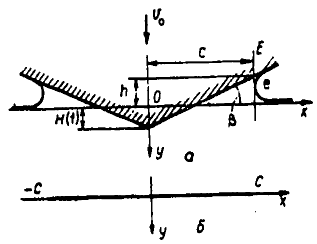

Figure 1.3.

The pressure is determined from the linearized Lagrange equation

$$\Delta p=p-p_0=-\rho\frac{\partial\varphi}{\partial t}. \tag{2.13}$$

The problem is solved for the following boundary and initial conditions (Figure 1.3,b).

On the wetted surface of the wedge (-c, c) on the Ox axis at some point in time t for $-c \leq x \leq c$ we have

$$\frac{\partial\varphi}{\partial n}=\frac{\partial\varphi}{\partial y}=v_0. \tag{2.14}$$

On the rest of the Ox axis, the velocity potential is zero:

$$\varphi=0 \text{ for } x<-c, \quad x>c. \tag{2.15}$$

At the initial point in time t = 0

$$\varphi=\frac{\partial\varphi}{\partial t}=0. \tag{2.16}$$

Along with the velocity potential ϕ in accordance with the continuity equation it is possible to introduce the harmonic current function ψ conjugate to ϕ and investigate the analytical function W in the complex plane z = x + iy, where

$$W=\varphi+i\psi; \qquad \frac{dW}{dz}=v_x-iv_y. \tag{2.17}$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

The problem stated in this way coincides in its mathematical formulation with the problem of impact of a plate 2c wide against the surface of the liquid. The solution of this problem is well known [2]. In our notation it has the form

$$W = \varphi + i\psi = iv_0\{\sqrt{z^2 - c^2} - z\}. \tag{2.18}$$

Hence, along the wetted part (-c, c) for the velocity potential we obtain

$$\varphi = -v_0\sqrt{c^2 - x^2}, \quad |x| < c. \tag{2.19}$$

The speed of the fluid on the free boundary according to (2.18) is defined by the formula

$$v_x = 0,$$
$$v_y = -v_0\left\{\frac{1}{\sqrt{1 - \frac{c^2}{x^2}}} - 1\right\}; \qquad |x| > c. \tag{2.20}$$

Let us note that in the solution obtained the velocity v_0 can be both constant and a time function. The case of the constant penetration rate will be considered further. If we do not consider the lift of the free surface and the increase in wetted surface of the wedge connected with this when determining the wetted segment of the wedge, we have

$$c = v_0 t \operatorname{ctg}\beta. \tag{2.21}$$

In the investigated self-similar problem the propagation rate of the ends of the wetted part of the edge with respect to the free surface $\dot{c}$ considering the lift of this surface is constant [1]. We shall limit ourselves to an investigation on the right-hand side of the movement of the wedge during symmetric penetration (see Figure 1.3,a). When determining the point E (the edge of the wetted segment of the face of the wedge) the foam part on the free surface is neglected [2]. This part lies above the deflectionpoint of the free surface and, therefore, it is considered that the deflection point coincides with the point E on the face of the wedge. The liquid particle e on the free surface which at the time t after the beginning of motion is at the point E of the face of the wedge obviously has the coordinate $x_e = \dot{c}t$. From Figure 1.3,a it is clear that

$$h + H = c\cdot\operatorname{tg}\beta; \quad H = v_0 t, \qquad h = \left|\int_0^t v_y(x_e, c)\,dt\right|. \tag{2.22}$$

The last integral is calculated simply:

$$h = \left|v_0\int_0^t \frac{d\tau}{\sqrt{1 - \frac{\tau^2}{t^2}}} - v_0 t\right| = v_0 t\frac{\pi}{2} - v_0 t.$$

Substituting this solution in the first equality (2.22), we obtain

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$$c = \frac{\pi}{2} v_0 (\text{ctg}\,\beta) t. \qquad (2.23)$$

Consequently, the velocity $\dot{c}$ is

$$\dot{c} = \frac{\pi}{2} v_0 \,\text{ctg}\,\beta. \qquad (2.24)$$

Thus, as a result of lift of the free surface the wetted part of the wedge on the x axis occupies the segment

$$2c = \pi v_0 \,\text{ctg}\,\beta\, t.$$

Without considering the variation of the free surface from (2.21) it follows that

$$2c = 2v_0 \,\text{ctg}\,\beta\, t.$$

If c is taken from the formula (2.21), according to (2.13) and (2.19) the pressure along the wetted part is determined from the expression

$$\Delta p = p - p_0 = \frac{\rho v_0^2 \,\text{ctg}\,\beta}{\sqrt{1 - \frac{x^2}{v_0^2 t^2 \,\text{ctg}^2\,\beta}}}. \qquad (2.25)$$

Here the total force acting on the wedge is

$$F = \pi \rho v_0^3 t \,\text{ctg}^2\,\beta. \qquad (2.26)$$

Correspondingly, when considering the variation in shape of the free surface we have

for the pressure

$$\Delta p = p - p_0 = \frac{\pi}{2} \frac{\rho v_0^2 \,\text{ctg}\,\beta}{\sqrt{1 - \left(\frac{x}{\dot{c}t}\right)^2}}, \qquad (2.27)$$

or

$$\Delta p = p - p_0 = \rho v_0 \frac{\dot{c}}{\sqrt{1 - \left(\frac{x}{\dot{c}t}\right)^2}},$$

for the force

$$F = \pi \rho v_0 \dot{c}^2 t, \qquad \dot{c} = \frac{\pi}{2} v_0 \,\text{ctg}\,\beta. \qquad (2.28)$$

By analyzing some of his results of numerical solutions, Wagner proposed the following approximation formula for calculating the resistance force [1, 2] (for the angle $0 < \beta < \pi/2$):

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$F = \pi\rho v_0^2 H \left[\frac{\pi}{2\beta} - 1\right]^2 \tag{2.29}$$

§ 3. Penetration of an Incompressible Liquid by a Cone

Let a cone of arbitrary apex angle penetrate a liquid initially at rest, occupying the entire lower halfspace. The speed of the cone v_0 is constant and directed into the depth of the liquid perpendicular to the plane of the free boundary of the liquid at rest. The period where the cone has not completely submerged in the liquid is considered. In this statement the problem is self-similar, and the motion of the liquid is potential. The parameters of motion of the liquid depend on the coordinates ξ, ζ:

$$\xi = \frac{x}{t}; \quad \zeta = \frac{z}{t},$$

where t is the time, x, z are the cartesian coordinates in the meridional plane of the investigated axisymmetric problem. The origin of the coordinates is placed at the point of contact of the apex of the cone with the surface of the liquid at the time of beginning of penetration. The 0z axis is directed vertically downward, the 0x axis is along the horizontal surface of the liquid to the right (Figure 1.4). The pressure is constant on the free surface. On the generatrix of the cone we have the boundary conditions

$$\frac{\partial\varphi}{\partial n} = v_0 \cos\beta,$$

where ϕ is the velocity potential, n is the external normal to the generatrix of the cone, β is the angle of inclination of the generatrix to the 0x axis. Just as in the case of penetration of a wedge of arbitrary apex angle with constant velocity, the solution of the investigated problem here cannot be obtained analytically. By

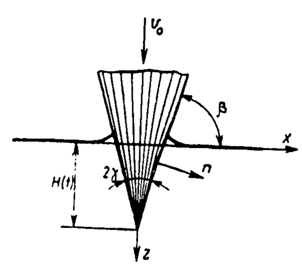

Figure 1.4.

the method of successive approximations presented in reference [1] the solution can be constructed numerically. This method can be used successively to construct the shape of the free surface and determine the force of resistance to penetration. A detailed study of this problem, including some important theorems pertaining to the geometry and kinematics of self-similar flow and the properties of the free surface connected with can be found in references [1, 2]. Here studies are made of the cases of penetration of a cone with a small apex angle and a blunt cone.

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

a) Penetration of an Incompressible Liquid by a Thin Cone. A study is made of the vertical penetration of an ideal liquid occupying the lower halfspace by a thin cone with a small apex angle. Before the beginning of penetration the liquid is at rest, the initial penetration velocity v_0 is perpendicular to the plane of the free surface of the liquid. The motion of the liquid arising on penetration by the cone will be small, and it is possible to demonstrate that with the exception of small regions in the vicinity of the points of intersection of the cone, the free surface of the liquid will differ little from the initial undisturbed surface [1, 7]; consequently, the problem can be considered in the linear statement and the boundary condition at the free surface taken down to its initial plane. The origin of the cartesian coordinate system will be placed at the point of contact of the apex of the cone with a free surface at the time of beginning of penetration t = 0.

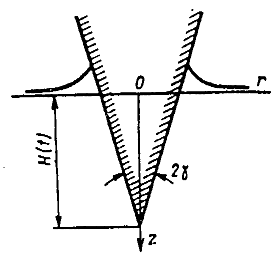

Figure 1.5.

The Oz axis will be directed vertically downward, and the Ox and Oy axes will be placed in the plane of the free surface of the liquid before beginning of penetration. The problem has axial symmetry. At some point in time t in the meridional plane the picture of the motion is illustrated in Figure 1.5. The velocity potential $\phi(x, y, z, t)$ satisfies the Laplace equation, that is, it is a harmonic function, and on the basis of the linear statement its value on the free surface z = 0 is equal to zero during the entire penetration time. Then the velocity potential, on the basis of the principle of symmetry, can be continued unevenly to the upper half-plane and represented in the form [8]

$$\varphi = -\frac{1}{4\pi}\int_{-H}^{H}\frac{q(\xi)\,d\xi}{\sqrt{(\xi-z)^2+r^2}}, \qquad r^2 = x^2 + y^2. \tag{3.1}$$

Here H(t) denotes the depth of penetration. On the basis of smallness of the angle γ, the boundary condition on the cone is written as follows:

$$\frac{\partial\varphi}{\partial r} = \dot{H}\gamma, \tag{3.2}$$

where $\dot{H}$ is the penetration rate. The problem reduces to determining the unknown function q(z) under the integral (3.1) from the boundary condition (3.2).

On the generatrix of a thin cone from formula (3.1) we have [7]

$$\left(\frac{\partial\varphi}{\partial r}\right)_{r\to 0} \to \frac{1}{2\pi}\cdot\frac{q(z)}{r}. \tag{3.3}$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

Thus, within the framework of the linear approximation it is possible to write

$$\frac{1}{2\pi r} q(z) = \begin{cases} \dot{H}\gamma, & 0 \leqslant z \leqslant H, \\ -\dot{H}\gamma, & -H \leqslant z \leqslant 0. \end{cases} \quad (3.4)$$

Noting that the radius r on the cone is related to the angle γ by the formula $r = (H - z)\gamma$, from the conditions (3.3) and (3.4) for force intensity q(z) we obtain

$$q(z) = \begin{cases} 0, & z > H, \\ 2\pi\dot{H}\gamma^2 (H - z), & 0 < z < H, \\ -2\pi\dot{H}\gamma^2 (H + z), & -H < z < 0, \\ 0, & z < H. \end{cases}$$

On the basis of (3.1), the velocity potential in final form will be represented by the formula [7]

$$\varphi = -\frac{1}{2}\gamma^2\dot{H}\int_0^H \frac{(H-\xi)\,d\xi}{\sqrt{(\xi - z)^2 + r^2}} + \frac{\gamma^2\dot{H}}{2}\int_{-H}^0 \frac{(H+\xi)\,d\xi}{\sqrt{(\xi - z)^2 + r^2}}. \quad (3.5)$$

The excess pressure is determined by the linearized Cauchy-Lagrange equation[1]:

$$p = -\rho\frac{\partial\varphi}{\partial t}. \quad (3.6)$$

From formula (3.5) on the surface of a cone we obtain

$$-\frac{\partial\varphi}{\partial t} = \frac{1}{2}\gamma^2\ddot{H}\left\{-2(H-z)\ln\gamma + 2z + (H-z)\ln 4 + H\ln\frac{z^2}{H^2 - z^2} + z\ln\frac{H-z}{H+z}\right\} + \frac{1}{2}\gamma^2\dot{H}^2\left[2(\ln 2 - \ln\gamma) + \ln\frac{z^2}{H^2 - z^2}\right], \quad (3.7)$$

where $\ddot{H}$ is the acceleration of the penetrating cone.

The analysis of the expression in the right-hand side of (3.7) demonstrates that the excess pressure p is equal to zero not on the free surface, but at the point under it at a shallow depth on the order of γ [7]. Therefore for a thin cone the total force can be obtained with high accuracy by integrating the pressure over the entire wetted surface:

$$F = 2\pi\gamma^2\int_0^H p(H - z)\,dz = m[\alpha H^2\dot{H}^2 + \lambda H^3\ddot{H}], \quad (3.8)$$

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[1]Everywhere that confusion will not be introduced by it, the excess pressure $p - p_0$ will be denoted by p.

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

where m, α and λ are constants defined below.

The pressure under the integral is defined using formulas (3.6) and (3.7). The law of motion of the cone of mass m will be defined by the formula (the weight of the cone will be neglected)

$$m\ddot{H} = -F. \qquad (3.9)$$

The results of the calculation give the following law of variation of the velocity and acceleration of the cone as a function of the depth of penetration H:

$$\dot{H} = \frac{v_0}{(1+\lambda H^3)^{\alpha/3\lambda}}, \qquad \ddot{H} = -\frac{\alpha v_0^2 H^2}{(1+\lambda H^3)^{2\alpha/3\lambda+1}}. \qquad (3.10)$$

The depth of penetration for which the acceleration of the cone reaches the maximum will be defined by the formula

$$H_{max} = \sqrt[3]{\frac{2}{2\alpha+\lambda}}. \qquad (3.11)$$

In formulas (3.8), (3.10) and (3.11) the constants α and λ have the following approximate values:

$$\alpha = -\frac{\pi\rho\gamma^4}{m}(\ln 2 + \ln\gamma);$$
$$\lambda = -\frac{\pi\rho\gamma^4}{m}\left(\frac{2}{3}\ln 2 + \frac{2}{3}\ln\gamma + \frac{1}{3}\right). \qquad (3.12)$$

Let the penetration rate be constant and equal to v_0. According to (3.6) and (3.7) the excess pressure along the generatrix of the cone in this case will be given by the formula

$$p = \rho v_0^2\gamma^2\left[2(\ln 2 - \ln\gamma) + \ln\frac{z^2}{H^2 - z^2}\right]. \qquad (3.13)$$

The force acting on the penetrating cone in the vertical direction will be defined by the formula (3.8) in which the pressure is taken from (3.13). As a result, for the force F we have:

$$F = \alpha_1 H^2 v_0^2, \qquad (3.14)$$

where the constant α_1 has the value

$$\alpha_1 = m\alpha = -\pi\rho\gamma^4(\ln 2 + \ln\gamma). \qquad (3.15)$$

The speed of the liquid on the free surface z = 0 will be found by differentiation with respect to z of the velocity potential (3.5):

$$v_z = \frac{\partial\varphi}{\partial z} = -\gamma^2\frac{\dot{H}}{2}\left[\frac{2H}{r} - 2\ln\frac{H+\sqrt{H^2+r^2}}{r}\right]. \qquad (3.16)$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

Let the penetrating body be a cone with a small apex angle 2γ which at a height of H_Γ becomes a thin cylinder of circular cross section with a radius $H_\Gamma\gamma$. It is obvious that up to a depth of $H \le H_\Gamma$ the solution to the problem coincides with the solution for an infinite cone presented above. For a depth of penetration $H > H_\Gamma$ the velocity potential will satisfy the Laplace equation, and the boundary conditions will assume the form:

$$\frac{\partial\varphi}{\partial r} = \begin{cases} 0, & 0 < z < H - H_\Gamma, \\ \dot{H}\gamma, & H - H_\Gamma < z < H, \\ 0, & z > H_\Gamma. \end{cases} \qquad (3.17)$$

Using the principle of symmetry, it is also possible here to continue the solution to the upper halfplane of the plane z, r and distribute the sources over the axis of the vertically penetrating body (along the Oz axis). It is easy to see that the distribution of the sources over the conical part of the body is identical with the distribution for an infinite cone, and the plane of the sources q on the cylindrical part of the body will be equal to zero. Therefore the velocity potential assumes the form

$$\varphi(r, z, t) = \frac{1}{2}\gamma^2\dot{H}\left\{\int_{-H}^{H_\Gamma - H}\frac{(H+\xi)\,d\xi}{\sqrt{(\xi-z)^2+r^2}} - \int_{-H_\Gamma+H}^{H}\frac{(H-\xi)\,d\xi}{\sqrt{(\xi-z)^2+r^2}}\right\}. \qquad (3.18)$$

The quadratures in the right-hand side of (3.18) are taken simply, and the velocity potential will be expressed in terms of finite functions. The excess pressure will be determined from the linearized Cauchy-Lagrange integral:

$$p = -\rho\frac{\partial\varphi}{\partial t}.$$

The calculations demonstrated the presence of negative pressures (rarefaction) on the surface of the cylindrical part of the body and in some region of the conical part adjacent to its base [8]. The maximum acceleration will always be achieved before the beginning of penetration of the cylindrical part, and for real products when $H = H_\Gamma$. In Figure 1.6 graphs are presented for the variation of the dimensionless velocity $\dot{H}/v_0$ and the ratio $\ddot{H}/v_0^2$ for a cone with an apex angle $2\gamma = 13°$ and weight mg = 20 kg penetrating water.

b) Penetration of an Incompressible Liquid by a Blunt Cone Considering the Lift of the Free Surface.

Let a very blunt cone with apex angle 2γ symmetrically penetrate an incompressible ideal liquid initially at rest. The liquid occupies the lower halfspace, the initial velocity of the cone v_0 is directed perpendicular to the plane of the free surface of the liquid.

The angle of inclination β of the generatrix of the cone to the free surface is small. In the meridional plane r, z, the penetration picture is shown in Figure 1.7.

The origin of the coordinates is taken at the point of contact at the apex of the cone of the free surface at the time of beginning of penetration, the Oz axis is directed vertically downward into the liquid, the r-axis, along the initial

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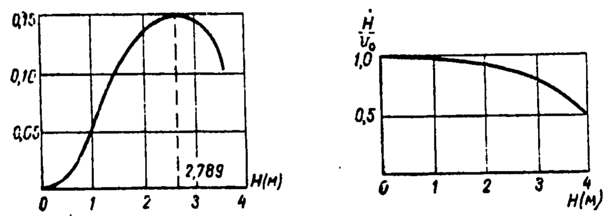

Figure 1.6.

horizontal free surface to the right. Just as in the case of penetration of a blunt wedge, the free surface of the liquid outside the spray jet is slightly inclined to the r axis, and the boundary conditions on the generatrix of the cone and the free surface are carried over to this axis. The problem is solved in the linear statement and it is formulated as follows. Let us find the velocity potential ϕ satisfying the Laplace equation

$$\frac{\partial^2\varphi}{\partial r^2}+\frac{\partial^2\varphi}{\partial z^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}=0, \tag{3.19}$$

the zero initial values

$$t=0,\ \varphi=0,\ \frac{\partial\varphi}{\partial t}=0$$

and the boundary conditions

$$z=0,\ \varphi(r,0,t)=0,\ H\operatorname{ctg}\beta<r<\infty,$$
$$\frac{\partial\varphi(r,0,t)}{\partial z}=\dot{H}(t),\ 0<r<H\operatorname{ctg}\beta, \tag{3.20}$$

where H(t) is the depth of penetration, the dot over the H denotes differentiation with respect to time t, that is, $\dot{H}$ is the penetration rate. The boundary conditions of (3.20) coincide with the boundary conditions in the problem of impact entry of a disc of radius H ctgβ into an incompressible liquid. The solution of this problem

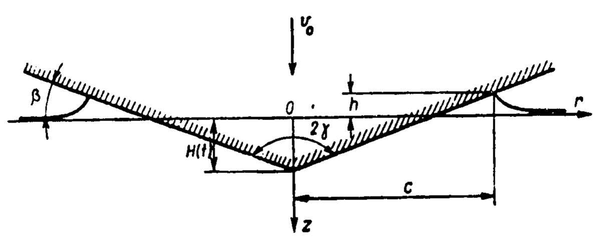

Figure 1.7.

is presented in reference [2]. Obviously, on the basis of uniqueness of the solution of the Laplace equation here it is possible to use the indicated solution. The solution of the Laplace equation which satisfies the boundary conditions (3.20) will be represented in the form

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$\varphi(r, z, t) = \int_0^\infty A(\lambda) J_0(\lambda r) e^{-\lambda z} d\lambda, \quad (3.21)$$

where $J_0(x)$ is the zero-order Bessel function. Substituting the solution of (3.21) in the boundary conditions (3.20), for determination of $A(\lambda)$ we obtain the pair of dual integral equations [7]:

$$\int_0^\infty A_1(\eta) J_0(\eta, \xi) d\eta = 0, \quad \xi > 1;$$

$$\int_0^\infty A_1(\eta) J_0(\eta, \xi) d\eta = \frac{\dot{H}H^2}{\mathrm{tg}^2\beta}, \quad \xi < 1, \quad (3.22)$$

here the following notation is introduced;

$$\lambda \frac{H}{\xi} = \eta, \quad \frac{r}{H} = \xi, \quad A_1(\eta) = A(\lambda).$$

The solution of equations (3.22) is known [7, 9]:

$$A_1(\eta) = -\sqrt{\frac{2}{\pi}} \frac{\dot{H}H^2}{\mathrm{tg}^2\beta} \sqrt{\eta} \int_0^1 \xi^{3/2} J_{1/2}(\xi, \eta) d\xi.$$

Substituting the Bessel function $J_{1/2}(x)$ defined by the formula

$$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x,$$

and the above expression under the integral sign, after calculation we obtain

$$A_1(\eta) = -\frac{H^2\dot{H}}{\mathrm{tg}^2\beta}\left(\frac{1}{\eta}\cos\eta - \frac{\sin\eta}{\eta^2}\right).$$

Now the potential ϕ according to (3.22) will be represented in the form

$$\frac{\varphi(r, z, t)}{H\dot{H}\,\mathrm{ctg}\,\beta} = -\int_0^\infty \frac{1}{\eta}\left(\cos\eta - \frac{\sin\eta}{\eta}\right) J_0(\xi, \eta)\, e^{-\frac{\eta z}{H\,\mathrm{ctg}\beta}} d\eta. \quad (3.23)$$

At the boundary z = 0, this expression assumes the form

$$\frac{\varphi(r, 0, t)}{H\dot{H}\,\mathrm{ctg}\,\beta} = -\int_0^\infty \frac{1}{\eta}\left(\cos\eta - \frac{\sin\eta}{\eta}\right) J_0(\xi, \eta) d\eta. \quad (3.24)$$

After several calculations this expression for the potential will be written as follows [8, 7]:

$$\varphi(r, 0, t) = \begin{cases} 0, & H\,\mathrm{ctg}\,\beta < r < \infty, \\ -\frac{2\dot{H}}{\pi}\sqrt{H^2\,\mathrm{ctg}^2\beta - r^2}, & 0 \leqslant r \leqslant H\,\mathrm{ctg}\,\beta. \end{cases} \quad (3.25)$$

The excess pressure along the wetted part of the generatrices of the cone will be defined by the linearized Lagrange formula,

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

For the investigated problem this formula gives

$$p = \frac{2\rho}{\pi}\left\{\frac{H\dot{H}^2 \operatorname{ctg}^2\beta}{\sqrt{H^2 \operatorname{ctg}^2\beta - r^2}} + \ddot{H}\sqrt{H^2 \operatorname{ctg}^2\beta - r^2}\right\}, \tag{3.26}$$

where $\ddot{H}$ is the acceleration of the penetrating cone. The force F acting on the cone with penetration is defined by the formula

$$F = 2\pi \int_0^{H\operatorname{ctg}\beta} pr\,dr,$$

where the pressure p in the expression under the integral sign is given by the expression (3.26). After calculation of the integral we obtain

$$F = 4\rho \operatorname{ctg}^3\beta H^2\left(\dot{H}^2 + \frac{H}{3}\ddot{H}\right). \tag{3.27}$$

For constant penetration rate ($\dot{H} = v_0$) for excess pressure from (3.26) we have

$$p = \frac{2\rho}{\pi}\,\frac{v_0^2 \operatorname{ctg}\beta}{\sqrt{1 - \dfrac{r^2}{v_0^2 t^2 \operatorname{ctg}^2\beta}}}. \tag{3.28}$$

The force acting on the cone in this case will be

$$F = 4\rho \operatorname{ctg}^3\beta v_0^4 t^2. \tag{3.29}$$

Note. The velocity potential on a disc of radius R, according to the above-indicated analogy, will be obtained from the expression of the potential (3.25) by replacing the values of Hctgβ by R and $\dot{H}$ by V_1 in it:

$$\varphi = \begin{cases} 0, & R < r < \infty; \\ -\dfrac{2V_1}{\pi}\sqrt{R^2 - r^2}, & 0 \leqslant r \leqslant R, \end{cases}$$

where V_1 is the velocity of the disc at the time t = 0 after normal impact of the disc against the surface of an incompressible liquid. The force of resistance F_1 acting on the disc at this time is

$$F_1 = 2\pi \int_0^R \rho\varphi r\,dr = \frac{4}{3}\rho V_1 R^3. \tag{3.30}$$

Let v_0 be the impact velocity of a disc of mass m. On the basis of the theorem of momentum, we have:

$$mv_0 - mV_1 = \frac{4}{3}\rho V_1 R^3.$$

Hence, the velocity V_1 will be defined:

$$V_1 = \frac{v_0}{1 + \dfrac{4}{3}\,\dfrac{\rho R^3}{m}}. \tag{3.31}$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

Thus, the apparent mass on impact of the disc against the liquid will be

$$M = \frac{4}{3}\rho R^3.$$

In the solution obtained above, the increase in surface of the wetted part of the cone as a result of lift of the liquid surface during the penetration process was not considered. The effect of the lift of the free surface on the magnitude of the wetted surface of the cone is determined by the same method as in the problem of penetration of a blunt wedge. Therfore there is no necessity for another detailed discussion of it. We shall limit ourselves to investigation of the penetration by a cone with constant velocity v_0. In this case the problem will be self-similar even in the investigated linear statement, the wetted part of the cone in the plane z = 0 will be a circle of radius c, where

$$c = \dot{c}t. \tag{3.32}$$

The variation rate of the radius $\dot{c}$ is a constant and subject to determination. It is determined from the equality

$$v_0 t + h = \dot{c}t \operatorname{tg} \beta. \tag{3.33}$$

The value of h in equation (3.33) is defined by the integral

$$h = \left| \int_0^t v_z \cdot d\tau \right|. \tag{3.34}$$

where v_z is the velocity of the particle on the free surface rising to the generatrix of the cone at the time t.

The velocity potential of the liquid when considering the lift of the free boundary in the investigated linear statement will be determined from the Laplace equation in terms of the boundary conditions (3.20) on replacement of the value of Hctgβ in them by the radius c. Therefore the value of this potential will be obtained from the expressions for the potential in the formulas (3.23) and (3.25) after the above-indicated substitution. As a result, for the velocity potential on the wetted part of the cone and the vertical velocity component on the free boundary we obtain:

$$\varphi = \begin{cases} 0, & c < r < \infty, \\ -\frac{2v_0}{\pi}\sqrt{c^2 - r^2}, & 0 \leqslant r \leqslant c; \end{cases}$$

$$v_z = \begin{cases} v_0, & r < c \\ \frac{2v_0}{\pi}\left(\arcsin\frac{c}{r} - \frac{c}{\sqrt{r^2 - c^2}}\right), & c < r. \end{cases} \tag{3.35}$$

The particle of liquid on the free surface which after the beginning of penetration at the time t arrives at contact with the generatrix of the cone has the coordinate, before the beginning of motion equal to $\dot{c}t$ (Figure 1.7). Then on the basis of (3.35) the integral (3.34) will be written as follows:

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$h = \left| \frac{2v_0}{\pi} \int_0^t \left[\arcsin \frac{\dot{c}\tau}{\dot{c}t} - \frac{\dot{c}\tau}{\sqrt{\dot{c}^2t^2 - \dot{c}^2\tau^2}} \right] d\tau \right|.$$

Calculating the integral, we shall have

$$h = \frac{2v_0}{\pi} t \left(2 - \frac{\pi}{2}\right).$$

After substitution of this value in the equality (3.33) for the rate of expansion $\dot{c}$ of the wetted surface of the cone in the plane z = 0 we obtain the formula

$$\dot{c} = \frac{4}{\pi} v_0 \operatorname{ctg} \beta, \tag{3.36}$$

Consequently, the radius of expansion of this surface at the time t will be determined from the equality

$$c = \frac{4}{\pi} v_0 (\operatorname{ctg} \beta) t. \tag{3.37}$$

For excess pressure along the generatrices of the cone we now obtain

$$p = \frac{2}{\pi} \cdot \frac{\rho v_0^2 \operatorname{ctg} \beta}{\sqrt{1 - \frac{r^2}{c^2}}} \cdot \frac{4}{\pi}.$$

The expression for the resistance force has the form

$$F = 4\rho v_0^4 \operatorname{ctg}^3 \beta\, t^2 \left(\frac{4}{\pi}\right)^3. \tag{3.38}$$

In reference [7] it is demonstrated that the penetration law expressed by formula (3.10) is observed qualitatively on penetration by a cone with finite halfapex angle γ if $\gamma < 1$. In this case the constants α and λ in formulas (3.10) are determined experimentally, for example, by the values of the penetration rate $\dot{H}$ at two given depths H_1 and H_2.

§ 4. Movement of a Thin Cone in a Liquid of Finite Depth

a) Vertical Penetration of a Liquid of Finite Depth by a Cone. Let us consider a thin cone of mass m with apex angle 2γ which vertically penetrates an incompressible liquid with depth h [10]. At the time of contact with the free surface the speed of the cone is v_0. Let us assume that the viscosity and gravitational forces can be neglected. Let us place the origin of the coordinates at the point of contact with the free surface, let us direct the x, y axes along the free surface and the Oz axis vertically downward (Figure 1.8). The pressure on the free surface is constant and equal to p_0. The problem of penetration by a cone reduces to solving the Laplace equation

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0 \tag{4.1}$$

with the boundary conditions

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$$\varphi = 0 \text{ for } z = 0; \tag{4.2}$$

$$\frac{\partial\varphi}{\partial z} = 0 \text{ for } z = h; \tag{4.3}$$

$$\frac{\partial\varphi}{\partial n} = v_n \text{ on the cone.}$$

The velocity potential of the disturbed motion will be found in the form

$$\varphi = -\frac{1}{4\pi}\int_s \frac{q(\xi, \eta, \zeta, t)}{R}\,dS,$$

where R is the distance from the point (ξ, η, ζ) of the surface S to the point (x, y, z) at which we determine the velocity potential. The surface S (see Figure 1.9) will be constructed in such a way as to satisfy the conditions (4.2), (4.3). Let us first satisfy the condition $\phi = 0$ for z = 0, for which we take the wetted surface of the cone as the surface S_0 and its mirror image with respect to the plane z = 0, where q(x, y, z, t) is continued unevenly. In order to satisfy the condition $\partial\phi/\partial n =$ 0 for z = h which still has not been satisfied, let us add the surface S_1 which is the mirror image of S_0 with respect to z = h to the surface S_0; let us complete construction of the function q(x, y, z, t) with respect to z = h evenly. As a result, we violate the boundary condition $\phi = 0$ for z = 0, but this deviation will be less than that which occurred for z = h. Continuing this process, we obtain the surface $S = S_0 + S_1 \ldots + S_n$, which at the limit for $n \to \infty$ precisely satisfies the boundary conditions (4.2), (4.3) (for the velocity potential $\phi \to 0$ for $R \to \infty$), where the function q(x, y, z, t) will be periodic with the period z = 4h. Let us present the solution of this problem for a thin cone. If γ is a small value, then the boundary condition on the cone assumes the form

$$\frac{\partial\varphi}{\partial r} = \dot{H}(t)\,\gamma, \tag{4.4}$$

where H(t) is the cone penetration law. The velocity potential of the disturbed motion of the liquid will be as follows:

$$\varphi = \frac{-1}{4\pi}\sum_{n=-\infty}^{n=\infty}(-1)^n\int_{-H}^{H}\frac{q(\zeta)\,d\zeta}{\sqrt{(2nh+\zeta-z)^2+r^2}}.$$

However, on the cone we also have

$$\frac{\partial\varphi}{\partial r} = \frac{1}{4\pi}\sum_{n=-\infty}^{n=\infty}(-1)^n\int_{-H}^{H}\frac{rq(\zeta)\,d\zeta}{[(2nh+\zeta-z)^2+r^2]^{3/2}}.$$

Hence, for $r \to 0$ we obtain

$$r\frac{\partial\varphi}{\partial r} \to \frac{q(z)}{2\pi}.$$

Thus, we have the approximate formula:

$$q(z) = 2\pi\dot{H}(t)\,\gamma r.$$

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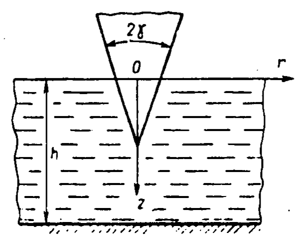

Figure 1.8.

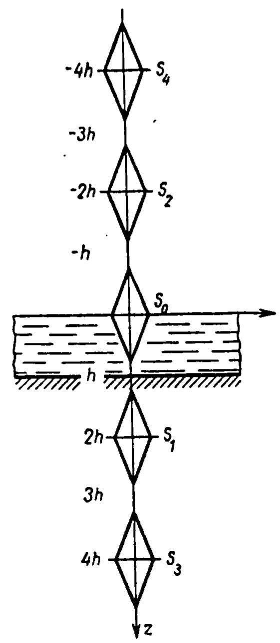

Figure 1.9.

Substituting the value of r on the cone, we arrive at the expression q(z):

$$q(z)=\begin{cases}-2\pi\dot H(t)(H+z)\gamma^2 & \text{for } -H\leqslant z\leqslant 0,\\ 2\pi\dot H(t)(H-z)\gamma^2 & \text{for } 0<z\leqslant H.\end{cases}$$

Thus, the potential of the disturbed motion of the liquid is given by the following expression:

$$\varphi=\frac{1}{2}\dot H(t)\gamma^2\sum_{n=-\infty}^{n=\infty}(-1)^n\left\{\int_{-H}^{0}\frac{(H+\zeta)\,d\zeta}{\sqrt{(2nh+\zeta-z)^2+r^2}}-\int_{0}^{H}\frac{(H-\zeta)\,d\zeta}{\sqrt{(2nh+\zeta-z)^2+r^2}}\right\}. \tag{4.5}$$

Differentiating expression (4.5) with respect to t we obtain

$$\frac{\partial\varphi}{\partial t}=\frac{1}{2}\ddot H(t)\gamma^2\sum_{n=-\infty}^{n=\infty}(-1)^n\left\{\int_{-H}^{0}\frac{(H+\zeta)\,d\zeta}{\sqrt{(2nh+\zeta-z)^2+r^2}}-\right.$$

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$-\int_0^H \frac{(H-\zeta)\,d\zeta}{\sqrt{(2nh+\zeta-z)^2+r^2}}\Bigg\}+$$

$$+\frac{1}{2}\dot{H}^2(t)\,\gamma^2\sum_{n=-\infty}^{n=\infty}(-1)^n\left\{\int_{-H}^{0}\frac{d\zeta}{\sqrt{(2nh+\zeta-z)^2+r^2}}-\right.$$

$$\left.-\int_0^H\frac{d\zeta}{\sqrt{(2nh+\zeta-z)^2+r^2}}\right\}.$$

Let us demonstrate that the series of derivatives converges absolutely and uniformly. For this purpose let us estimate the behavior of the nth term of the series for $n \to \infty$ (analogously, for $n \to -\infty$):

$$|a_n| = \left|\int_{-H}^{0}\frac{d\zeta}{\sqrt{(2nh+\zeta-z)^2+r^2}}-\int_0^H\frac{d\zeta}{\sqrt{(2nh+\zeta-z)^2+r^2}}\right| =$$

$$=\frac{1}{2nh}\left|\int_{-H}^{0}\frac{d\zeta}{\sqrt{\left(1+\frac{\zeta-z}{2nh}\right)^2+\frac{r^2}{4n^2h^2}}}-\right.$$

$$\left.-\int_0^H\frac{d\zeta}{\sqrt{\left(1+\frac{\zeta-z}{2nh}\right)^2+\frac{r^2}{4n^2h^2}}}\right|,$$

since the first term is larger than the second and they are both larger than zero, by increasing the first and decreasing the second, we have

$$|a_n| \leqslant \frac{h}{2nh-h-z-r}-\frac{h}{2nh+h+z+r}=$$

$$=\frac{1}{2h}\cdot\frac{h+z+r}{\left[1-\frac{(h+z+r)^2}{4h^2n^2}\right]}\cdot\frac{1}{n^2}.$$

The value of

$$\frac{\frac{1}{2h}(h+z+r)}{1-\left(\frac{h+z+r}{2nh}\right)^2}\leqslant M,$$

therefore $|a_n| \leq M\cdot(1/n^2)$ in this series, and, consequently, also the series (4.5) converges absolutely and uniformly, and the operation of term-by-term differentiation is entirely regular. For a cone that becomes a cylinder, with a length of the conical part equal to H_Γ, the integrals are taken within the limits from $-H$ to $-H + H_\Gamma$ and from $H - H_\Gamma$ to H, respectively.

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

Let us define the law of motion of the cone. The force acting on the cone is equal to

$$F = 2\pi\gamma^2 \int_0^H (p - p_0)(H - z)\,dz.$$

The pressure gradient will be determined from the partially linearized Lagrange integral

$$\Delta p = p - p_0 = -\rho \frac{\partial\varphi}{\partial t} - \frac{1}{2}\rho\left(\frac{\partial\varphi}{\partial r}\right)^2.$$

Using expression (4.5), we obtain the following expression for the force F:

$$-F = (\lambda_1 + \beta_1 H^3) H^3 \ddot{H} + (\alpha_1 + 3\beta_1 H^3) H^2 \dot{H}^2,$$

where

$$\lambda_1 = \frac{2}{3}\pi\rho\gamma^4\left(\ln\gamma + \ln 2 + \frac{1}{2}\right),$$

$$\alpha_1 = \pi\rho\gamma^4(\ln\gamma + \ln 2 + 1),$$

$$\beta_1 = \frac{\pi\rho\gamma^4}{36h^3}\sum_{n=1}^{n=\infty}(-1)^n\frac{1}{n^3}.$$

The equation of translational motion of a cone (its natural weight is not considered) will have the form:

$$\ddot{H}[1 + (\lambda + \beta H^3)H^3] + (\alpha + 3\beta H^3)H^2\dot{H}^2 = 0, \tag{4.6}$$

where

$$\alpha = -\frac{\alpha_1}{m}, \quad \lambda = -\frac{\lambda_1}{m}, \quad \beta = -\frac{\beta_1}{m}.$$

Integrating equation (4.6), we obtain

$$\dot{H} = \frac{v_0}{(1 + \lambda H^3 + \beta H^6)^{\frac{\alpha}{3\lambda}}}, \tag{4.7}$$

$$\ddot{H} = -\frac{\alpha v_0^2(\lambda + 2\beta H^3)H^3}{\lambda(1 + \lambda H^3 + \beta H^6)^{\frac{2\alpha}{3\lambda}+1}}. \tag{4.8}$$

Expressions (4.7) and (4.8) for $h \to \infty$ become the solutions (3.10) of this chapter. From expression (4.7) it follows that on penetration of a liquid of finite depth, the cone loses velocity faster than penetration into the halfspace.

b) Movement of a Cone in a Liquid of Finite Depth in the Direction of the Free Surface. Let a thin cone of mass m with apex angle 2γ move under the effect of the force F_0 in an ideal incompressible liquid of depth h vertically upward in the direction of the free surface. The initial speed of the cone is v_0. The pressure on the free surface will be considered constant and equal to p_0. Let us place the origin of the stationary coordinate system at the point of intersection of the axis of the cone with the bottom, let us direct the (x, y) axes along the bottom surface and the z axis, vertically upward. The velocity potential of the disturbed motion of the liquid $\phi(x, y, z, t)$ will satisfy the Laplace equation (4.1).

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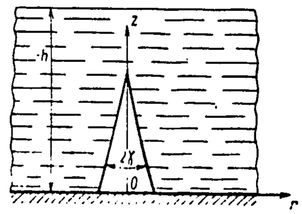

Figure 1.10.

We shall consider the free surface to vary little, and the boundary conditions on it $\phi = 0$ will be taken down to the plane $z = h$. From the condition of impermeability of the bottom we obtain:

$$\frac{\partial\varphi}{\partial z} = 0 \quad \text{for} \quad z = 0.$$

On the cone the normal velocity component of the liquid $\partial\phi/\partial n$ is equal to the projection of the velocity of the points of the cone v_n on the normal. Thus, the boundary conditions of equation (4.1) assume the form

$$\varphi = 0 \quad \text{for} \quad z = h; \tag{4.9}$$

$$\frac{\partial\varphi}{\partial z} = 0 \quad \text{for} \quad z = 0, \tag{4.10}$$

$\partial\phi/\partial n = v_n$ or (for a thin cone)

$$\partial\varphi/\partial r = \dot{H}(t)\gamma. \tag{4.11}$$

It is easy to demonstrate by arguments analogous to the arguments of § 4a that the solution of the equation (4.1) satisfying the conditions (4.9), (4.10) will be as follows for a thin cone:

$$\varphi = -\frac{1}{4\pi}\sum_{n=0}^{n=\infty}(-1)^n\left\{\int_{-H}^{H} q(\zeta)\left[\frac{1}{\sqrt{(2nh-\zeta+z)^2+r^2}} - \frac{1}{\sqrt{[2(n+1)h+\zeta-z]^2+r^2}}\right]d\zeta\right\}, \tag{4.12}$$

where H(t) is the distance between the bottom and the apex of the cone. The distribution density of the sources will be obtained from the boundary condition (4.11)

$$q(z) = \begin{cases} 2\pi\dot{H}(t)(H+z)\gamma^2 & \text{for} \quad -H \leqslant z \leqslant 0, \\ 2\pi\dot{H}(t)(H-z)\gamma^2 & \text{for} \quad 0 < z \leqslant H. \end{cases}$$

The final form of the potential of the disturbed motion of the liquid will be as follows:

$$\varphi = -\frac{1}{2}\dot{H}(t)\gamma^2\sum_{n=0}^{n=\infty}(-1)^n\left\{\int_{-H}^{0}(H+\zeta)\times\right.$$

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$$\times\left[\frac{1}{\sqrt{(2nh-\zeta+z)^2+r^2}}-\frac{1}{\sqrt{[2(n+1)h+\zeta-z]^2+r^2}}\right]d\zeta+$$

$$+\int_0^H (H-\zeta)\left[\frac{1}{\sqrt{(2nh-\zeta+z)^2+r^2}}-\frac{1}{\sqrt{[2(n+1)h+\zeta-z]^2+r^2}}\right]d\zeta\Big\}. \qquad (4.13)$$

For a cone becoming a cylinder, with a length of the conical part equal to H_Γ, the integrals are taken, respectively, within the limits from $-H$ to $-H+H_\Gamma$ and from $H-H_\Gamma$ to H. Let us define the law of motion of the cone. The force acting on the cone on the part of the liquid is

$$F=2\pi\gamma^2\int_0^H (p-p_0)(H-z)\,dz.$$

Using the Lagrange integral and expression (4.13) we arrive at the expression:

$$F=(\lambda_1+\beta_1 H)H^3\ddot{H}+(\alpha_1+2\beta_1 H)H^2\dot{H}^2,$$

where the coefficients are given by the following expressions with high accuracy:

$$\alpha_1=-\pi\rho\gamma^4\left(\ln\gamma-3\ln 2+\frac{5}{2}\right),$$

$$\lambda_1=-\frac{2}{3}\pi\rho\gamma^4\left(\ln\gamma-3\ln 2+\frac{3}{2}\right),$$

$$\beta_1=-\frac{\pi\rho\gamma^4}{h}\left[\frac{1}{4}-\sum_{n=1}^{n=\infty}\frac{(-1)^n}{4n(n+1)}\right].$$

The equation of motion of the cone assumes the form

$$m\ddot{H}=F_0-F-mg,$$

or

$$\ddot{H}(1+\lambda H^3-\beta H^4)+(\alpha-2\beta H)H^2\dot{H}^2=\frac{F_0-mg}{m}=F_1, \qquad (4.14)$$

where

$$\alpha=\frac{\alpha_1}{m},\quad \lambda=\frac{\lambda_1}{m},\quad \beta=-\frac{\beta_1}{m}.$$

The solution of the equation (4.14) under the initial conditions $t=0$,

$$H(0)=0,\quad \dot{H}(0)=v_0$$

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

has the form

$$\dot{H}=\sqrt{e^{-2\int\limits_0^H \frac{(\alpha-2\beta x)x^2}{1+\lambda x^3-\beta x^4}dx}}\times$$

$$\times\sqrt{\left[v_0^2+2\int\limits_0^H \frac{F_1(x)}{1+\lambda x^3-\beta x^4}e^{2\int\limits_0^H \frac{y^2(\alpha-2\beta y)dy}{1+\lambda y^3-\beta y^4}}dx\right]}.$$

From expression (4.14) it is easy to determine the thrust F_0 for which the column will move with a constant velocity v_0:

$$F_1=v_0^2(\alpha-2\beta H)H^2, \quad \text{or} \quad F_0=m[v_0^2(\alpha-2\beta H)H^2+g]. \tag{4.15}$$

As is obvious from expression (4.15), for movement of a cone in a liquid of finite depth in view of the effect of the free surface the force F_0 is less than for movement of it in an infinitely deep liquid (for the latter case $\beta = 0$). When the depth of the liquid approaches infinity, the law of motion of the cone in the half-space can be written as:

$$\dot{H}=\sqrt{\frac{1}{(1+\lambda H^3)^{\frac{2\alpha}{3\lambda}}}\left[v_0^2+2\int\limits_0^H \frac{F_1(x)\,dx}{(1+\lambda x^3)^{1-\frac{2\alpha}{3\lambda}}}\right]}.$$

c) Motion of a Cone in the Case of Waves on the Liquid. Let us consider thecase where there are progressive plane waves of low amplitude on the free surface of the liquid. Along with the cartesion coordinate system (x, y, z) let us consider the cylindrical system (r, z, ψ) for which the z-axis coincides with the z-axis of the cartesion system, and ψ is the angle between the x and r axes. For determinacy let us consider that the waves are propagated in the positive direction of the x-axis. The velocity potential of the general motion of the liquid will be represented in the form

$$\varphi=\varphi_0+\varphi_1+\varphi_2,$$

where ϕ_0 is the velocity potential of the liquid during vertical movement of the

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cone to the free surface of a liquid initially at rest which is defined by formula (4.12). The potential of the wave motion of the liquid ϕ_1 which satisfies the boundary conditions

$$\frac{\partial\varphi_1}{\partial z} = \frac{\sigma^2}{g}\varphi_1 \text{ for } z = h; \quad \frac{\partial\varphi_1}{\partial z} = 0 \quad \text{for } z = 0,$$

has the form [11]

$$\varphi_1 = \frac{\alpha g}{\sigma}\,\frac{\operatorname{ch} kz}{\operatorname{ch} kh}\sin(kr\cos\psi - \sigma t),$$

where α is the wave amplitude, c is the wave velocity, λ is the wavelength, $\lambda = 2\pi/k$, $\sigma^2 = gk\,\mathrm{th}\,kh$, $c = \sigma/k$. Since on the cone $\partial\phi/\partial n = v_n$, then for determination of the additional potential ϕ_2 we have the following boundary conditions:

$$\varphi_2 = 0 \quad \text{for} \quad z = h,$$

$$\frac{\partial\varphi_2}{\partial r} = -\frac{\alpha g}{c}\cos\psi\cos\sigma t\,\frac{\operatorname{ch} kz}{\operatorname{ch} kh} \quad \text{on the cone},$$

$$\frac{\partial\varphi_2}{\partial z} = 0 \quad \text{for} \quad z = 0.$$

Using the results of § 4a, we obtain the value of the potential ϕ_2:

$$\varphi_2 = \frac{\alpha g\gamma}{2c}\,\frac{\cos\psi\cos\sigma t}{\operatorname{ch} kh}\sum_{n=0}^{n=\infty}(-1)^n \times$$

$$\times\left\{\int_{-H}^{0}(H+\zeta)\operatorname{ch} k\zeta\left[\frac{1}{\sqrt{(2nh-\zeta+z)^2+r^2}}-\right.\right.$$

$$\left.-\frac{1}{\sqrt{[2(n+1)h+\zeta-z]^2+r^2}}\right]d\zeta+$$

$$+\int_{0}^{H}(H-\zeta)\operatorname{ch} k\zeta\left[\frac{1}{\sqrt{(2nh-\zeta+z)^2+r^2}}-\right.$$

$$\left.\left.-\frac{1}{\sqrt{[2(n+1)h+\zeta-z]^2+r^2}}\right]d\zeta\right\}.$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

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§5. Ricochet of a Plate from the Surface of an Ideal Incompressible Liquid

Let us consider the two-dimensional problem of the incidence of a slightly bent plate of mass m on the surface of an ideal incompressible liquid. Let us assume that the submersion of the plate is insignificant, and the free surface of the liquid differs little from its undisturbed level. Therefore the boundary conditions on the wetted surface of the plate will be taken down to the horizontal plane coinciding with the initially undisturbed level of the free surface. In addition, under the boundary condition of the free surface we shall neglect the ponderability of the liquid and the square of the velocity of absolute motion. We shall consider that the motion of the liquid begins with a state of rest [12]. Let O_1 be the origin of the stationary coordinate system coinciding with the point of contact of the trailing edge of the plate with the free surface at the initial point in time.

For determination of the liquid flow in the lower halfplane let us introduce the moving coordinate system (ξ, η) with the origin at the center of the wetted surface of the plane, let us direct the ξ axis along the undisturbed level of the free surface, the η-axis, vertically upward (Figure 1.11). Under the given assumptions, the magnitude of the complex-conjugate velocity $dW/d\zeta=U-iV$ in the moving coordinate system will be [13]

$$\frac{dW}{d\zeta} = iv_1\left(1 - \frac{\zeta}{\sqrt{\zeta^2 - c^2}}\right) - \frac{1}{2\pi i\sqrt{\zeta^2 - c^2}}\int_0^x \frac{\omega(s)\sqrt{\xi_0^2 - c^2}}{\xi_0 - \zeta}\,ds; \tag{5.1}$$

$$\xi_0 = s - x - c, \quad v_1 = -(v + \dot{\beta}c) + V_x\beta,$$

where V_x and v are the horizontal and vertical components of the plate velocity, 2c is the magnitude of the wetted surface, β is the angle of attack, $\dot{\beta}$ is the angular velocity of the plate with respect to the rear edge and x is the path traveled by the plate from the time of contact with the free surface of the liquid. The unknown function $\omega(x)$ entering into equation (5.1) and the function known in the theory of nonstationary motion of a profile as the distribution density of the eddies trailing off the rear edge characterizes the magnitude of the horizontal component of the velocity of the liquid particles at the free surface behind the plate, and it is determined from the first type Volterra integral equation:

$$f_1(x) = \int_0^x \omega(s)\sqrt{\frac{2c + x - s}{x - s}}\,ds, \tag{5.2}$$

$$\left(f_1(x) = -2\pi c v_2,\ v_2 = v_1 + \frac{1}{2}\dot{\beta}c\right).$$

The buoyancy acting on the plate is [13]

$$Y = \frac{d}{dt}\left(\frac{\pi\rho c^2}{2}v_1\right) + \pi\rho c\left(V_x + \frac{dc}{dt}\right)v_2 + \frac{\rho c}{2}\left(V_x + 2\frac{dc}{dt}\right)\int_0^x \frac{\omega(s)\,ds}{\sqrt{(2c + x - s)(x - s)}}, \tag{5.3}$$

where ρ is the liquid density.

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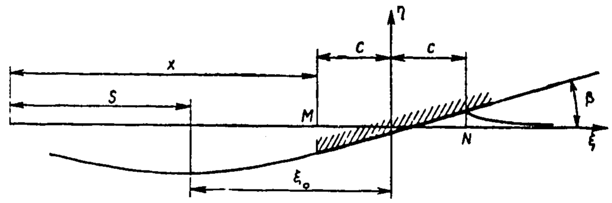

Figure 1.11

For determination of ω(x), we reduce the integral equation (5.2) to a system of ordinary differential equations [12]. For this purpose let us expand $\sqrt{2c+x-s}$ in a series with respect to powers of s/(2c+x), let us integrate it term by term and introduce the notation

$$y_n = \frac{1}{(2c+x)^n}\int_0^x \frac{s^n\omega(s)\,ds}{\sqrt{x-s}}.$$

Then equation (5.2) reduces to the following system of equations:

$$\dot{y}_n = \frac{1}{2c+x}\left\{\left[\frac{1}{2}+(n-1)(1+2\dot{c})\frac{x}{2c+x}\right]y_{n-1}+\right.$$
$$\left.+x\dot{y}_{n-1}-n(1+2\dot{c})y_n\right\},$$
$$f_1(x)=\sqrt{2c+x}\left[y_0-\frac{1}{2}y_1-\frac{1}{8}y_2-\dots-\frac{(2k-3)!!}{(2k)!!}-\dots\right] \quad (5.4)$$

with the initial conditions

$$x=0,\quad y_1(0)=\dots=y_n(0)=\dots=0.$$

Knowing the solution of system (5.4), for example, knowing $y_0(x)$, let us determine the unknown function ω(x) from the integral equation

$$y_0(x)=\int_0^x \frac{\omega(s)\,ds}{\sqrt{x-s}}.$$

However, this is the Abel integral equation, and its solution is

$$\omega(x)=\frac{1}{\pi}\left[\frac{y_0(0)}{\sqrt{x}}+\int_0^x\frac{\dot{y}_0(t)\,dt}{\sqrt{x-t}}\right].$$

Thus, for determination of ω(x) it is necessary to solve an infinite system of ordinary differential equations. In order to find the approximate solution ω(x) of equation (5.2) in the expression of the kernel in a series it is possible to limit ourselves to n terms. Then the corresponding system of ordinary differential equations will also contain n equations.

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

Let us prove convergence of this method. Without limiting the generality of the proof, let us set $2c=1$ for simplicity (nonstationary movement of the profile of unit length). Let $\omega(x)$ be the solution of equation (5.2) for the function $f_1(x)$, continuous together with the derivative for $0\leq x\leq x_0$, and $\omega_n(x)$ be the solution of the corresponding approximate equation where we shall limit ourselves to n terms in the expansion of the kernel in a series. Let us subtract one equation from the other and denote the difference $\omega-\omega_n$ by Ψ_n. Then with respect to $\Psi_n(x)$ we arrive at the equation analogous to (5.2):

$$\varepsilon_n(x)=\int_0^x \Psi_n(s)\sqrt{\frac{1+x-s}{x-s}}\,ds,$$

$$\left(\varepsilon_n(x)=\sqrt{1+x}\int_0^x \sum_{k=n+1}^{\infty} c_k\left(\frac{s}{1+x}\right)^k\frac{\omega_n(s)\,ds}{\sqrt{x-s}}\right). \tag{5.5}$$

Let us show that if $\omega_n(x)$ is continuous together with the derivative, then $\varepsilon_n(x)$ and $\dot{\varepsilon}_n(x)$ approach zero for $n\to\infty$. From the definition of $\varepsilon_n(k)$ and continuity of $\omega_n(x)$ we have:

$$|\varepsilon_n(x)|<M\sum_{k=n+1}^{k=\infty} c_k, \quad \left(\sqrt{1+x}\int_0^x\left(\frac{s}{1+x}\right)^n\frac{|\omega_n(s)|\,ds}{\sqrt{x-s}}\leq M\right).$$

However, the sum $c_{n+1}+c_{n+2}+c_{n+3}+\ldots$ is the remainder of the converging numerical series, that is, the sum approaches zero. Differentiating $\varepsilon_n(x)$, we obtain

$$\dot{\varepsilon}_n(x)=\sqrt{1+x}\int_0^x\sum_{k=n+1}^{k=\infty} c_k\left(\frac{s}{1+x}\right)^k\frac{\dot{\omega}_n(s)\,ds}{\sqrt{x-s}}+$$

$$+\frac{(n+1)\,c_{n+1}}{\sqrt{1+x}}\int_0^x\left(\frac{s}{1+x}\right)^n\frac{\omega_n(s)\,ds}{\sqrt{x-s}}.$$

For differentiation of $\varepsilon_n(x)$ we use the formula

$$\frac{d}{dt}\int_0^x\frac{f(s)\,ds}{\sqrt{x-s}}=\frac{f(0)}{\sqrt{x}}+\int_0^x\frac{f'(x)\,ds}{\sqrt{x-s}},$$

which is valid for any function $f(s)$ continuous together with the derivative in the segment $0\leq s\leq x$.

For proof of the fact that $\dot{\varepsilon}_n(x)\to 0$ for $n\to\infty$, it is sufficient to show that $(n+1)c_{n+1}\to 0$ for $n\to\infty$, for the remaining terms approach zero just as in the preceding case. It is possible to show that

$$(n+1)c_{n+1}=\frac{1}{2}\,\frac{(2n-1)!!}{(2n)!!}=\frac{(2n)!!}{2^{n+1}(n!)^2}.$$

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Applying the Sterling formula, we finally find that $(n+1)c_{n+1} \sim 1/2\sqrt{\pi n}$ for $n\to\infty$, that is, it approaches zero for $n\to\infty$. In equation (5.5) let us replace x by t, let us multiply the two sides times $(x-t)^{-1/2}dt$ and integrate within the limits from 0 to x. Now if we vary the order of integration from the right and differentiate, we obtain

$$f_n(x) = \Psi_n(x) + \frac{1}{\pi}\int_0^x \Psi_n(s) K(x, s)\, ds,$$

$$f_n(x) = \frac{1}{\pi}\int_0^x \frac{\dot{\varepsilon}_n(t)\, dt}{\sqrt{x-t}}, \quad K(x, s) = \frac{E(k)}{\sqrt{1+x-s}} - \frac{F(k)-E(k)}{(x-s)\sqrt{1+x-s}}, \tag{5.6}$$

$$k^2 = \frac{x-s}{1+x-s}.$$

Here E, F are the total elliptic integrals of the first and second types. Equation (5.6) is a second type Volterra integral equation with continuous kernel, and for it the estimate

$$|\Psi_n(x)| \leqslant |f_n(x)| + \frac{K_0}{\pi}\exp\frac{K_0 x_0}{\pi}\int_0^x |f_n(s)|\, ds;$$
$$|K(x, s)| \leqslant K_0 \tag{5.7}$$

is valid. The value of $\dot{\varepsilon}_n(x)\to 0$ for $n\to\infty$; therefore $f_n(x)$ also approaches zero for $n\to\infty$, and proceeding to the limit in the inequality (5.7), we find that $|\omega(x)-\omega_n(x)|\to 0$ for $n\to\infty$.

Now let us proceed to the solution of the problem of incidence of a flat plate of mass m on the surface of an ideal incompressible liquid. The horizontal velocity component of the plate at the time of contact with the free surface will be set equal to V_{x0}, and the vertical component, v_0; the angle of the plate with the undisturbed level of the free surface will be considered small and equal to β. The depth of submersion of the rear edge will be denoted by h(x). Let us consider the plate quite long and its movement in the liquid will be considered until the upper edge is above the free surface. The problems of ricochet of a plate from a free surface of the liquid and landing on the free surface will be considered under the following assumptions: 1) horizontal component of the plate velocity is constant; 2) the angle of the plate with the undisturbed level of the free surface does not change; 3) the plate is acted on by gravitational force and the buoyancy of the nonsteady planing (hydrostatic forces are not considered, for their effect is insignificant).

In view of the constancy of the horizontal velocity component, as the independent variable we shall take the path traveled by the plate from the time of contact with the free surface and replace the differentiation with respect to time by differentiation with respect to path: $d/dt \to V_{x0} d/dx$.

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

Under these assumptions, the movement of the plate in the vertical plane can be described by the following system of integral differential equations:

$$mV_{x0}^2 \frac{d^2h}{dx^2} = Y(x) - mg, \quad -2\pi c v_2 = \int_0^x \omega(s) \sqrt{\frac{2c+x-s}{x-s}}\, ds \tag{5.8}$$

with the initial conditions

$$x = 0, \quad h(0) = 0, \quad h'(0) = -\frac{v_0}{V_{x0}}.$$

The magnitude of the wetted surface of the plate 2c is determined by integration of the vertical velocity component of the liquid at the free surface in front of the plate. From expression (5.1) let us determine the vertical velocity component of the liquid on the ξ axis at some fixed point ξ^* ($0<c<\xi^*$, $\eta=0$). Converting by means of the equality $\xi^*=s^*-x-c$ to the stationary coordinate system, we find:

$$V(s^*, x) = \frac{1}{\sqrt{(s^*-x-2c)(s^*-x)}} \Big\{ v_1 [(s^*-x-c) -$$

$$- \sqrt{(s^*-x-2c)(s^*-x)}] + \frac{1}{2\pi} \int_0^x \omega(s) \frac{\sqrt{(2c+x-s)(x-s)}}{s-s^*} ds \Big\}$$

$$(0 < 2c + x < s^*). \tag{5.9}$$

Let us expand the expressions $\sqrt{2c+x-s}$ and $(s-s^*)^{-1}$ in uniformly converging series with respect to powers of $s/(2c+x)$ and s/s^*, let us multiply them and place the product under the integral sign. Performing term-by-term integration which is possible in view of the uniform convergence of the series, which is the product, and considering that

$$\frac{1}{(2c+x)^{n+1}} \int_0^x s^n \omega(s) \sqrt{x-s}\, ds = -\left(y_{n+1} - \frac{x}{2c+x} y_n \right),$$

$$n = 1, 2, 3, \ldots,$$

we obtain

$$\int_0^x \omega(s) \frac{\sqrt{(2c+x-s)(x-s)}}{s-s^*} ds = \frac{(2c+x)^{3/2}}{s^*} \sum_{k=0}^{\infty} b_k \left(y_{k+1} - \frac{x}{2c+x} y_k \right).$$

$$b_0 = 1, \ldots, b_n = \frac{2c+x}{s^*} b_{n-1} - \frac{(2n-3)!!}{(2n)!!}.$$

Substituting the value of the integral in expression (5.9) and performing the integration, we obtain the amount of lift of the liquid ahead of the plate at some fixed point s^*:

$$\eta(s^*, x) = \frac{1}{V_x} \int_0^x V(s^*, \tau)\, d\tau.$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

Let us find the magnitude of the wetted surface of the plate (the formation of a wake in the vicinity of the leading edge is not considered). At the leading edge of the plate s*=2c+x; consequently, the lift of the liquid is

$$\eta = \frac{1}{V_x}\int_0^x V(2c+x, \tau)\,d\tau. \tag{5.10}$$

On the other hand, the amount of lift of the liquid at the leading edge can be expressed in terms of the wetted length of the plate

$$\eta = 2c\beta + h,$$

where h is the depth of submersion of the rear edge.

Equating the values of η found by two different methods, for the wetted surface we obtain

$$2c = \frac{1}{\beta}(\eta - h),$$

where η is calculated by the formula (5.10).

Analogously, knowing the values of the vertical and horizontal components of the liquid velocity at the free surface we define the shape of the free surface after the plate:

$$\xi(s^*, x) = \frac{1}{V_x}\int_{s^*}^x U[s^* + \xi(s^*, \tau)]\,d\tau,$$

$$\eta(s^*, x) = h(s^*) + \frac{1}{V_x}\int_{s^*}^x V[s^* + \xi(s^*, \tau), \tau]\,d\tau,$$

where

$$U(s, \tau) = \begin{cases} \frac{1}{2}\omega(s), & 0 < s < x, \\ 0, & s < 0, \end{cases}$$

$$V(s_0, \tau) = \frac{1}{\sqrt{(2c+\tau-s_0)(\tau-s_0)}}\Big\{v_1[(c+\tau-s_0) - $$

$$- \sqrt{(2c+\tau-s_0)(\tau-s_0)}] - \frac{1}{2\pi}\int_0^\tau \omega(s)\frac{\sqrt{(2c+\tau-s)(\tau-s)}}{s-s_0}\,ds\Big\}.$$

Here ξ(s*, x) is the shift of the liquid particle of the free surface with the coordinate s* after the plate in the horizontal direction, η(s*, x) is the y-axis of the same particle. The integral in the expression for the vertical velocity is taken in the sense of the main value. The system of parameters of the problem, namely V_{x0}, v_0, β, m, ρ, g, permits three dimensionless combinations

$$\alpha = \frac{v_0}{V_{x0}},\ \beta,\ \gamma = \frac{V_{x0}}{\sqrt{\left(\frac{m}{\rho}\right)^{\frac{1}{2}} g}}.$$

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which also determine the nature of motion of the plate. Let us introduce the dimensionless combinations

$$x = \bar{x}\sqrt{\frac{m}{\rho}},\ h = \bar{h}\sqrt{\frac{m}{\rho}},\ v = \bar{v}V_{x0},\ \omega = \bar{\omega}\pi V_{x0},\ \ldots$$

Now replacing the integral equation in the system (5.8) by the system of ordinary differential equations (5.4), we arrive at the following system of equations describing the motion of the plate (the bars over the dimensionless variables will be omitted):

$$\begin{gathered} h' = v, \\ v' = \frac{2\lambda(\eta - h)}{1 + \lambda(\eta - h)^2}(\beta + \eta' - v)\left[\delta + 2(\beta - v) - \frac{1}{\gamma^2[1 + \lambda(\eta - h)^2]}\right], \\ y_n' = \frac{1}{2c + x}\left\{\left[\frac{1}{2} + (n - 1)(1 + 2c')\frac{x}{2c + x}\right]y_{n-1} + \right. \\ \left. + x y_{n-1}' - n(1 + 2c')y_n\right\}, \\ n = 1, 2, 3, \ldots, \end{gathered} \tag{5.11}$$

with the initial conditions

$$x = 0,\ h(0) = 0,\ v(0) = -\alpha,\ y_1(0) = \ldots = y_n(0) = \ldots = 0,$$

where

$$\lambda = \frac{\pi}{8\beta^2},\ y_0 = -\frac{2c(\beta - v)}{\sqrt{2c + x}} + \frac{1}{2}y_1 + \frac{1}{8}y_2 + \ldots + \frac{(2k - 3)!!}{(2k)!!}y_k + \ldots,$$

$$\delta = \frac{1}{\sqrt{2c + x}}\left[y_0 + \frac{1}{2}y_1 + \frac{1}{8}y_2 + \ldots + \frac{(2k - 1)!!}{(2k)!!}y_k + \ldots\right].$$

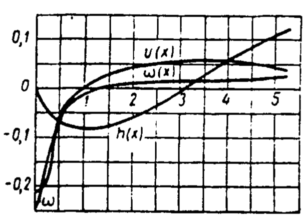

Figure 1.12

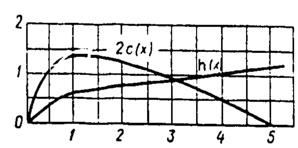

Figure 1.13

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When solving the system (5.11) for the given values of α, β, γ two cases are possible: either α, β and γ are such that ricochet of the plate from the free surface of the liquid takes place or after landing on the water, completing several oscillations, the motion goes into the planing regime (multiple deflections of the plate from the free surface are not considered). Let us present the results of numerical integration of system (5.11) for the case $\alpha=0.2$; $\beta=0.1$; $\gamma=6$ (the angle of approach to the water is about 12°, the trim angle is 6°, γ is the dimensionless velocity). The calculations were performed for n=10 (with an increase in the number of equations the results do not in practice change).

The dimensionless values of the depth of submersion of the rear edge of the plate h(x), the vertical velocity v(x) and the distribution density of the vortexes are constructed in Figure 1.12 as a function of the dimensionless path traveled by the plate. At the beginning of submersion the motion is close to self-similar touchdown of the planing step with the given values of α, β and γ. For $x\approx3$, the y-axis of the trailing edge becomes positive, but ricochet still has not occurred, and the motion continues above the undisturbed level of the free surface, which takes place as a result of lifting of the level of the liquid surface in front of the plate and corresponding to planing with so-called negative draft.

At subsequent points in time, the movement of the plate upward continues, and at $x\approx5.25$ ricochet occurs. The graphs of the magnitude of the wetted surface of the plate and the lift of the liquid at the leading edge are presented in Fig 1.13 as a function of the dimensionless path traveled by the plate from the time of contact with the free surface of the liquid.

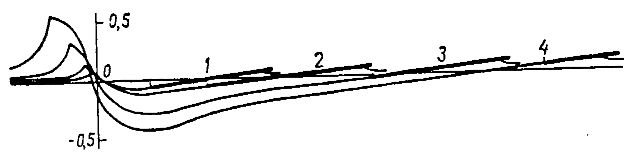

Figure 1.14

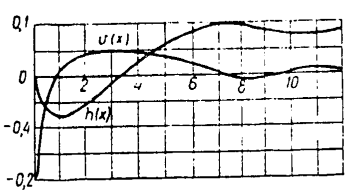

Figure 1.15

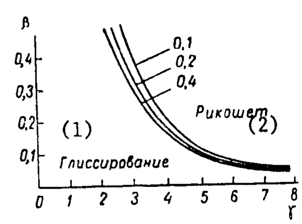

Figure 1.16

Key:
1. Planing
2. Ricochet

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The family of curves representing the shape of the free surface for several successive positions of the plate is presented in Figure 1.14. Figure 1.15 shows the results of touchdown of the planing step with conversion to planing for $\alpha=0.2$, $\beta=0.1$, $\gamma=4$. From the graphs it follows that the fluctuations in depth of submersion of the trailing edge h(x) and the vertical component of the velocity v(x) quickly damp, and the movement goes into the planing regime with negative draft.

When investigating this class of problems, it is of interest to determine the ricochet boundary, that is, the relation between α, β, and γ which on surfacing at the time of a decrease in the magnitude of the wetted surface to zero gives a vertical velocity equal to zero. Knowing the position of the ricochet boundary, it is possible in each individual case, without solving the problem, to determine whether a smooth landing of the plate on the water with transition to planing takes place or there will be ricochet.

For determination of the ricochet boundary, a series of calculations were made by the numerical integration of system (5.11). The results of the calculations are presented in Figure 1.16 where a family of curves corresponding to various angles of incidence of the plate are constructed in the plane (β, γ) (the tangents of the angle of incidence $\alpha=V_{x0}/v_0$ are indicated on the curves).

The calculations show that the tendency toward ricochet increases with an increase in the horizontal component of the velocity, with an increase in the angle of incidence (within the framework of linearity) and with a decrease in weight of the plate.

Penetration of a Compressible Liquid by Thin Bodies

§6. Statement of the Problem in Equations of Motion

We shall assume that the liquid is ideal, weightless, it occupies the lower half-space, and it is initially in the state of rest.

By a thin three-dimensional body we mean a body in which the ratio of the dimensions of the transverse cross section to the length δ is small: $\delta \ll 1$. For a thin flat body we mean a body having small thickness by comparison with its length and width.

If a thin body moves at a low angle of attack $\alpha \sim \delta$, the disturbances which it introduces into the initial state of the liquid will be on the order of δ (at individual singularities and even on lines, this condition can be violated). The body moving in a compressible medium causes a shock wave. For a thin body, the intensity of the shock wave is on the order of δ (by intensity we mean the relative pressure or density gradient on the wave), and the variation in entropy, as is known, is on the order of δ^3, that is, with a high degree of accuracy the entropy over the entire flow field can be considered constant S=const. Let us note that for such slightly compressible media as water, even in the case of powerful shock waves the entropy varies insignificantly.

According to the Lagrange theorem in the liquid initially at rest, the vortexes can occur only as a result of viscosity, nonpotentialness of the force field

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and nonbarotropicity. By assumption the liquid is ideal, and the mass forces are absent; therefore the only cause which in our case could lead to vortex formation is the nonbarotropicity caused by passage of the shock waves. However, as was noted above, on penetration by thin bodies the entropy of the liquid does not change and, consequently, its movement is potential.

Let us select the stationary cartesian coordination system x, y, z, directing the 0z axis inside the medium, and the 0x and 0y axes, along the undisturbed free surface. Considering what has been stated, the equations of motion can be written in the form

$$\frac{\partial \bar{v}}{\partial t} + \operatorname{grad} \frac{v^2}{2} = -\frac{1}{\rho} \operatorname{grad} p, \tag{6.1}$$

$$\frac{\partial \rho}{\partial t} + \bar{v} \operatorname{grad} p + \rho a^2 \operatorname{div} \bar{v} = 0, \tag{6.2}$$

$$\rho = \rho(p),\ a^2 = \frac{dp}{d\rho},\ \bar{v} = \operatorname{grad} \varphi.$$

From the first equation we have the Cauchy-Lagrange integral

$$\int_{p_0}^{p} \frac{dp}{\rho(p)} + \frac{\partial \varphi}{\partial t} + \frac{\bar{v}^2}{2} = 0, \tag{6.3}$$

p_0 is the initial pressure equal to the pressure on the free surface. Excluding the continuity (6.2) of ρ and p from the equation, using (6.1) and (6.3), we obtain the equations for the potential

$$a^2 \Delta \varphi - \frac{\partial^2 \varphi}{\partial t^2} - 2\bar{v} \frac{\partial \bar{v}}{\partial t} - \bar{v} \operatorname{grad} \frac{v^2}{2} = 0. \tag{6.4}$$

As has already been noted, the velocity components are of no higher order than δ. Assuming that their first derivatives are of the same order, we see that the last two terms of equation (6.4) are of a higher order of smallness than the first two. Retaining only these main terms, we obtain the linearized equation

$$a^2 \Delta \varphi = \frac{\partial^2 \varphi}{\partial t^2}, \tag{6.5}$$

where it is necessary to take the speed of sound in the undisturbed liquid as a with the assumed degree of accuracy. On the basis of the proposals that have been made, the initial data are $\phi = \partial\phi/\partial t = 0$.

Linearizing the Cauchy-Lagrange integral, we obtain

$$p = -\rho \frac{\partial \varphi}{\partial t}, \tag{6.6}$$

where here and hereafter p denotes the difference between the true and the initial pressure.

Let us return to the boundary conditions. During the process of motion, the free surface varies, and its shape is unknown in advance. For determination of the equation of the free surface $z = \xi(x,y,t)$ we have the condition

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$\frac{dz}{dt} = \frac{\partial\varphi}{\partial z} = \frac{\partial\zeta}{\partial t} + \frac{\partial\zeta}{\partial x}\frac{dx}{dt} + \frac{\partial\zeta}{\partial y}\frac{dy}{dt} = \\ = \frac{\partial\zeta}{\partial t} + \frac{\partial\zeta}{\partial x}\frac{\partial\varphi}{\partial x} + \frac{\partial\zeta}{\partial y}\cdot\frac{\partial\varphi}{\partial y}. \tag{6.7}$$

Experience shows that on penetration of a liquid by thin bodies, the free surface (especially in the initial stage) is deflected little from the undisturbed level, that is, $\xi\sim\delta$. Assuming also that the slope of the free surface remains small, from (6.7) we obtain

$$\frac{\partial\zeta}{\partial t} = \frac{\partial\varphi}{\partial z}. \tag{6.8}$$

The condition (6.8) is satisfied for $z=\xi(x,y,t)$, but it is sufficient to consider with the assumed degree of accuracy that it is satisfied for $z=0$. Condition (6.8) is used for determining the form of the free surface ξ if the potential is already known. In addition to the kinematic condition (6.8) on the free surface $z=\xi(x,y,t)$ the dynamic condition of equality of the pressure $p=p_0$, that is, $\partial\phi/\partial t=0$, is satisfied or, considering the initial data, $\phi=0$. As a result of smallness of ξ the last condition can be brought down to the surface $z=0$. Finally, for the potential when $z=0$ we obtain the condition

$$\varphi(0, x, y, t) = 0. \tag{6.9}$$

The condition of impermeability is satisfied on the surface of the moving body

$$\frac{\partial\varphi}{\partial n} = v_n, \tag{6.10}$$

where v_n is the projection of the velocity on the external normal to the surface of the body. Hereafter, the condition will be specified in each individual case.

Let us note that at the point of intersection of the surface of the body with the free surface the boundary conditions (6.9) and (6.10) do not agree with each other which means this point will be a singularity for the solution of the problem. In the small vicinity of this point, a significant lift of the liquid takes place by comparison with the undisturbed level.

The tip of the body will also be a singularity for the solution.

§7. Vertical Submersion of the Thin Solid of Revolution without an Angle of Attack. Penetration by a Cone

The origin of the cylindrical system of coordinates $Ozr\theta$ will be selected at the point of contact of the body with the liquid, and the time will be reckoned from the contact time. The z-axis is directed along the axis of the body into the liquid, and the r-axis, along the surface. The law of motion will be denoted by $H(t)$. The penetration of the body with velocity $v_0<a$ and $v_0>a$ is illustrated in Figures 1.17, a and 1.17, b. For vertical submersion without an angle of attack, the flow that arises is axisymmetric, that is, it satisfies the equation

$$\frac{\partial^2\varphi}{\partial z^2} + \frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} = \frac{1}{a^2}\frac{\partial^2\varphi}{\partial t^2}. \tag{7.1}$$

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The initial conditions for the function $\phi(z,r,t)$ are zero

$$\varphi(z, r, 0) = \frac{\partial \varphi}{\partial t}(z, r, 0) = 0,$$

the condition on the free surface z=0

$$\varphi(0, r, t) = 0. \tag{7.2}$$

The equation of the surface of the body is conveniently given in the form r=f(x) where x is reckoned from the leading edge inside the body so that in the selected coordinate system we have

$$r = f[H(t) - z], \quad 0 < z < H(t).$$

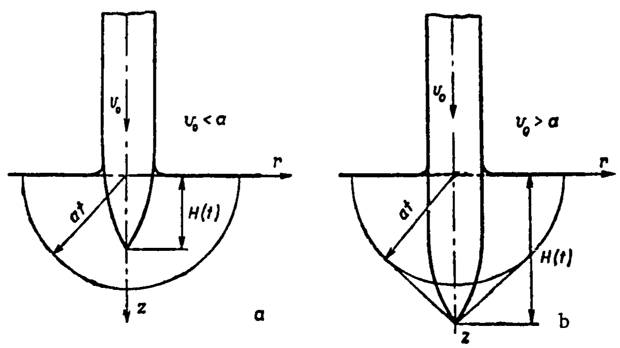

Figure 1.17

The boundary condition on the submerged part of the body has the form:

$$\frac{\partial \varphi}{\partial n} \sim \frac{\partial \varphi}{\partial r} = \dot{H}(t) f'[H(t) - z], \quad 0 < z < H(t).$$

On the rest of the Oz axis

$$\frac{\partial \varphi}{\partial r} = 0.$$

Thus, equation (7.1) must be solved under the boundary conditions

$$\frac{\partial \varphi}{\partial r} = \begin{cases} \dot{H}(t) f'[H(t) - z], & 0 < z < H(t), \\ 0, & z > H(t). \end{cases} \tag{7.3}$$

Condition (7.2) offers the possibility of continuing the function ϕ unevenly into the halfspace z< 0:

$$\varphi(-z, r, t) = -\varphi(z, r, t).$$

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Here the "reflected" body in the halfplane z<0 will have the equation

$$r = f[H(t) + z], \quad -H(t) < z < 0,$$

with the boundary condition

$$\frac{\partial \varphi}{\partial r} = \begin{cases} -\dot{H}(t) f'[H(t)+z], & -H(t) < z < 0, \\ 0, & z < -H(t). \end{cases} \tag{7.4}$$

The solution of the problem will be found in the form of the potential of the delaying sources distributed along the z-axis with density q(z, t) [14],

$$\varphi(z, r, t) = -\frac{1}{4\pi} \int_{-\infty}^{\infty} \frac{q(\xi, t')}{R} d\xi, \tag{7.5}$$

where $R=\sqrt{(\xi-z)^2+r^2}$, and t' is the "delay" time t'=t-R/A.

Expression (7.5) satisfies equation (7.1) for any quite smooth function q(z,t). For satisfaction of the initial conditions it is necessary to set q=0 for t<0. If we select the function q(z,t) so as to satisfy the boundary conditions (7.3) and (7.4), then we automatically satisfy the condition (7.2). Thus, for determination of q(z,t) we obtain the integral equation

$$-\frac{1}{4\pi} \frac{\partial}{\partial r} \int_{-\infty}^{\infty} \frac{q(\xi, t')}{R} d\xi = \frac{\partial \varphi}{\partial r}, \tag{7.6}$$

where it is necessary to substitute the value of $\partial\phi/\partial r$ from the boundary conditions (7.3) and (7.4) in the righthand side. If the shape of the body f(x) and the law of motion H(t) are given, the righthand side of the equation (7.6) is known.

However, the law of motion is unknown. For determination of it, it is possible to use Newton's law

$$m\ddot{H}(t) = F_0 - F,$$

where m is the mass of the body, F_0 is the given external force, F is the force of resistance of the liquid which is defined by the formula

$$F = 2\pi \int_0^{H(t)} f[H(t) - z] f'[H(t) - z]\, p dz.$$

Therefore in the investigated case Newton's law is written in the form

$$m\ddot{H}(t) = F_0 - \frac{\rho}{2} \int_0^{H(t)} \left\{ f[H(t) - z] f'[H(t) - z] \frac{\partial}{\partial t} \int_{-\infty}^{\infty} \frac{q(\xi, t')}{R} d\xi \right\} dz. \tag{7.7}$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

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For determination of two unknown functions q(z,t) and H(t) we have the system of integrodifferential equations (7.6) and (7.7).

We shall consider that the law of motion H(t) is known; then it is necessary to solve the equation (7.6). Let us now present (7.5) in the form

$$\varphi = -\frac{1}{4\pi}\int\limits_{-\psi_1(t)}^{\psi_2(t)}\left[\frac{q(z,t)}{R}+\frac{q(\xi,t')-q(z,t')}{R}+\frac{q(z,t')-q(z,t)}{R}\right]d\xi,$$

where Ψ_1 and Ψ_2 denote the integration limits (finite values for any t). The first integral of this equation gives

$$-\frac{1}{4\pi}q(z,t)\{-2\ln r+\ln[\psi_2-z+\sqrt{(\psi_2-z)^2+r^2}]\times$$
$$\times[\psi_1+z+\sqrt{(\psi_1+z)^2+r^2}]\}.$$

If q(z,t) is differentiable, then the remaining integrals are bounded for r=0, for, for example, according to the Lagrange theorem

$$q(\xi,t')-q(z,t')=q_z[z+\alpha(\xi-z),t'](\xi-z),$$

where $0<\alpha<1$ and

$$\left.\frac{q(\xi,t')-q(z,t')}{\sqrt{(\xi-z)^2+r^2}}\right|_{r\to 0}=q_z[z+\alpha(\xi-z),t'].$$

Hence, it follows that for $r\to 0$, ϕ has the form

$$\varphi|_{r\to 0}=\frac{1}{2\pi}q(z,t)\ln r+o(q), \tag{7.8}$$

where o(q) denotes a value on the order of q, and

$$\left.\frac{\partial\varphi}{\partial r}\right|_{r\to 0}=\frac{1}{2\pi}\frac{q(z,t)}{r}+o(q). \tag{7.9}$$

This condition differs advantageously from the condition (7.6) in that it is local. Comparing the last expression with the boundary conditions (7.3) and (7.4), we obtain

$$q(z,t)=\begin{cases}2\pi\dot H(t)f[H(t)-z]f'[H(t)-z], & 0<z<H(t),\\ -2\pi\dot H(t)f[H(t)+z]f'[H(t)+z], & -H(t)<z<0,\\ 0, & |z|>H(t).\end{cases} \tag{7.10}$$

As is obvious from (7.9), the intensity of the sources at the point z at the time t with the error $o(\delta^3)$ will only be determined by the component of the

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

normal velocity at this point at the time t. Thus, the solution of the problem will be

$$\varphi=-\frac{1}{2}\int_{0}^{\infty}\frac{1}{R}\dot{H}(t')f[H(t')-\xi]f'[H(t')-\xi]\,d\xi+$$
$$+\frac{1}{2}\int_{-\infty}^{0}\frac{1}{R}\dot{H}(t')f[H(t')+\xi]f'[H(t')+\xi]\,d\xi.$$

The boundaries of the actual region of integration are determined from the condition that at them the function under the integral sign vanishes. For the first integral this will be at $H(t')-\xi=0$, and for the second, at $H(t')+\xi=0$ (let us remember that $f(0)=0$). Consequently, the integration limits are roots of the equations

$$\xi_2=H\left[t-\frac{1}{a}\sqrt{(\xi_2-z)^2+r^2}\right],$$
$$\xi_1=-H\left[t-\frac{1}{a}\sqrt{(\xi_1-z)^2+r^2}\right], \tag{7.11}$$

in which t, z and r are the parameters. In the general case the region of integration will consist of several individual pieces, but if the function $H(t)$ is monotonic, it is easy to see that in equations (7.11) there are only two roots and the region of integration will consist of one continuous interval between these groups. Geometrically, the roots of equations (7.11) denote the coordinates of the points of intersection on the plane (ξ, t') of a semi-hyperbola

$$t'=t-\frac{1}{a}\sqrt{(\xi-z)^2+r^2} \tag{7.12}$$

with the curves $\xi=H(t')$ and $\xi=-H(t')$ which are the trajectory of the leading edge and its mapping. The limits have a simple physical meaning: they separate the sources, the disturbance of which has successfully reached the point (z, r) at the time t from the rest. It is simplest to determine the region of integration for movement of a body with constant velocity $\dot{H}(t)=v_0$. In this case the limits ξ_2 and ξ_1 are found from the quadratic equations:

$$\xi_2=v_0\left[t-\frac{1}{a}\sqrt{(\xi_2-z)^2+r^2}\right], \tag{7.13}$$

$$\xi_1=-v_0\left[t-\frac{1}{a}\sqrt{(\xi_1-z)^2+r^2}\right]. \tag{7.14}$$

If $z^2+r^2<a^2t^2$, then for subsonic and supersonic movement, each of these equations has one root having the physical meaning

$$\xi_2=\frac{1}{M^2-1}\left[M^2z-v_0t+M\sqrt{(v_0t-z)^2-(M^2-1)r^2}\right], \tag{7.15}$$

$$\xi_1=\frac{1}{M^2-1}\left[-(M^2z+v_0t)+M\sqrt{(v_0t+z^2)-(M^2-1)r^2}\right]. \tag{7.16}$$

Here $M=v_0/a$.

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

The expression for the potential in this case will be

$$\varphi = -\frac{v_0}{2}\int\limits_0^{\xi_2}\frac{1}{R}f[H(t')-\xi]f'[H(t')-\xi]\,d\xi + \frac{v_0}{2}\int\limits_{\xi_1}^{0}\frac{1}{R}f[H(t')+\xi]f'[H(t')+\xi]\,d\xi. \tag{7.17}$$

If $z^2+r^2>a^2t^2$, then in the case of subsonic movement equations (7.13) and (7.14) do not have real roots, and $\phi=0$.

For $v_0>a$ equation (7.14) has no roots, and (7.13) can have two roots, one or none. For the roots of equation (7.13) we now have

$$\xi_{1,2} = \frac{1}{M^2-1}\left[M^2z - v_0t \pm M\sqrt{(v_0t-z)^2-(M^2-1)r^2}\right], \tag{7.18}$$

where both roots are positive and the larger one ξ_2 corresponds to the "+" sign. Hence, it follows that for $(v_0t-z)^2>(M^2-1)r^2$ and $z^2+r^2>a^2t^2$ the potential ϕ has the expression:

$$\varphi = -\frac{v_0}{2}\int\limits_{\xi_1}^{\xi_2}\frac{1}{R}f[H(t')-\xi]f'[H(t)-\xi]\,d\xi. \tag{7.19}$$

The condition of multiplicity of roots of (7.18) has the form

$$(M^2-1)r^2 = (v_0t-z)^2.$$

This is the equation of the Mach cone with half-apex angle $\alpha = \operatorname{arctg}\frac{1}{\sqrt{M^2-1}}$. On the Mach cone $\phi=0$.

As an example let us consider submersion with constant velocity of a thin cone with half-apex angle γ.

1. For subsonic movement $v_0<a$, from (7.17) for excess pressure, we obtain

$$p = \frac{1}{2}\rho\gamma^2v_0^2\left[\int\limits_0^{\xi_2}\frac{d\xi}{\sqrt{(\xi-z)^2+r^2}} - \int\limits_{\xi_1}^{0}\frac{d\xi}{\sqrt{(\xi-z)^2+r^2}}\right], \tag{7.20}$$

where ξ_2, ξ_1 are defined by (7.15) and (7.16).

For $r\to 0$ we have

$$\xi_2 = v_0\frac{at+z}{a+v_0}, \quad -\xi_1 = v_0\frac{at-z}{a+v_0}$$

and from (7.20) for the pressure near the surface of the body we obtain

$$p = \frac{1}{2}\rho v_0^2\gamma^2\ln\frac{(z+\sqrt{z^2+r^2})^2(\xi_2-z)}{r^2(z-\xi_1)}.$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

On the surface of a moving cone $r=\gamma(v_0t-z)$ the excess pressure is

$$p=\frac{1}{2}\rho v_0^2\gamma^2\ln\frac{[z+\sqrt{z^2+\gamma^2(v_0t-z)^2}]^2}{\gamma^2(v_0^2t^2-z^2)}\cong\frac{1}{2}\rho v_0^2\gamma^2\ln\frac{4z^2}{\gamma^2(v_0^2t^2-z^2)}, \qquad (7.21)$$

for the force of resistance we obtain

$$F=2\pi\gamma^2\int_0^{v_0t}(v_0t-z)\,p\,dz=\pi\rho v_0^4\gamma^4t^2\ln\frac{1}{2\gamma}. \qquad (7.22)$$

Formulas (7.21) and (7.22) show that for subsonic penetration by a thin cone the pressure distribution along the generatrix and the force of resistance do not depend on the M number and coincide with the corresponding values on penetration of an incompressible liquid.

If we considered the quadratic term in the Cauchy-Lagrange equation $\frac{1}{2}\left(\frac{\partial\varphi}{\partial r}\right)^2$, then instead of formula (7.21) we would obtain

$$p=\frac{1}{2}\rho v_0^2\gamma^2\left[\ln\frac{4z^2}{\gamma^2(v_0^2t^2-z^2)}-1\right], \qquad (7.23)$$

and instead of (7.22)

$$F=\pi\rho v_0^4\gamma^4t^2\left(\ln\frac{1}{2\gamma}-\frac{1}{2}\right).$$

As is obvious, consideration of the quadratic term leads to a decrease in the resistance.

2. For supersonic penetration, the pressure distribution in the section of the surface of the cone $z<at$ is expressed by the same formula (7.21). For the section $at<z<v_0t$ it is necessary to use the formula (7.19); for $r\to 0$ the value of the limits from (7.18):

$$\xi_1=v_0\frac{z-at}{v_0-a},\quad \xi_2=v_0\frac{z+at}{v_0+a}$$

and the pressure on the body is

$$p=\frac{1}{2}\rho v_0^2\gamma^2\int_{\xi_1}^{\xi_2}\frac{d\xi}{\sqrt{(\xi-z)^2+r^2}}=\frac{1}{2}\rho v_0^2\gamma^2\ln\frac{4}{\gamma^2(M^2-1)}. \qquad (7.24)$$

This value is exactly equal to the pressure on the cone during stationary movement in an unbounded medium. This is obvious, for the free surface influences only the section of the cone which is inside the sphere $z^2+r^2<a^2t^2$.

For the force of resistance we obtain

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$F = \pi\rho v_0^2\gamma^4 \left\{\left[\int_0^{a} (v_0 t - z)\ln\frac{4z^2}{\gamma^2(v_0^2t^2 - z^2)}dz\right] + \right.$$
$$+\left[\int_{at}^{v_0 t} (v_0 t - z)\ln\frac{4}{\gamma^2(M^2-1)}dz\right]\Bigg\} = \pi\rho v_0^4\gamma^4 t^2 \left\{\left[\frac{2M-1}{2M^2}\ln\frac{4}{\gamma^2} - \right.\right.$$
$$-\ln\frac{1}{M} - 2\ln(M+1) + \frac{(M-1)^2}{2M^2}\ln(M^2-1)\Bigg] +$$
$$+\left[\frac{(M-1)^2}{2M^2}\ln\frac{4}{\gamma^2(M^2-1)}\right]\Bigg\} = \pi\rho v_0^4\gamma^4 t^2\left(\ln\frac{1}{2\gamma} + \ln\frac{4M}{(1+M)^2}\right). \quad (7.25)$$

Retaining the quadratic term in the Cauchy-Lagrange integral, we would obtain the previous formula (7.23) for $0<z<at$; for $at<z<v_0 t$

$$p = \frac{1}{2}\rho v_0^2\gamma^2\left(\ln\frac{4}{\gamma^2(M^2-1)} - 1\right)$$

and

$$F = \pi\rho v_0^4\gamma^4 t^2\left(\ln\frac{1}{2\gamma} + \ln\frac{4M}{(1+M)^2} - \frac{1}{2}\right).$$

For M=1, the value of $4M/(1+M)^2=1$ and with an increase in the Mach number, it decreases; therefore the coefficient of the force of resistance of the cone

$$C = \frac{2F}{\rho v_0^2\pi(v_0\gamma t)^2} = 2\gamma^2\left(\ln\frac{1}{2\gamma} + \ln\frac{4M}{(1+M)^2}\right)$$

decreases with an increase in the M number. For M=1 the formula (7.25) coincides with (7.22).

§8. Inclined Penetration by a Thin Body at an Angle of Attack. Inclined Penetration by Cone

Let us consider the inclined entry of a sharp thin body into a compressible liquid halfspace at an angle θ to the free surface (the angle between the initial velocity and the horizontal surface) (Figure 1.18) [15].

Let us select the origin O_1 of the stationary system of coordinates at the point of contact of the body with the liquid, and let us direct the O_1z_1 axis at an angle θ into the liquid. The law of motion of the body along the z_1 axis will be denoted by $z_1=H(t)$. We shall consider that in addition to the movement along the O_1z_1 axis (the basic movement) the body undergoes small transverse and rotational motions. For simplicity we shall assume that the transverse and rotational motions take place in the same plane, and we shall select it at the coordinate plane $O_1z_1y_1$. The Ox_1 axis is located on the free surface and supplements the coordinate system to the right.

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

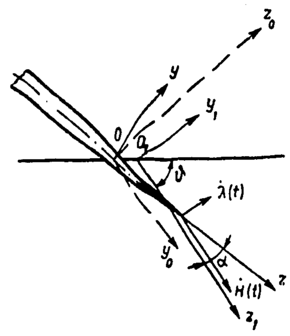

Figure 1.18

The law of transverse movements of the leading edge of the body will be denoted $y_1=\lambda(t)$, and the angle between the axis of the body and the O_1z_1 axis, in terms of $\alpha(t)$. Within the framework of the linearized theory, the values of $\alpha(t)$, $\lambda(t)$ and their derivatives must be on the order of δ.

The formulation of the boundary condition on the body in the system $O_1z_1y_1x_1$ is inconvenient, for during the process of motion the axis of the body does not coincide with the O_1z_1 axis. Therefore let us also introduce the moving coordinate system Ozyx, directing the Oz axis along the axis of the body and selecting the origin at the point of intersection of the axis of the body with the free surface (Figure 1.18). These systems are related by the expressions

$$\begin{gathered} z = z_1\cos\alpha + (y_1 - \lambda + H\operatorname{tg}\alpha)\sin\alpha = z_1 + \alpha y_1; \\ y = -z_1\sin\alpha + (y_1 - \lambda + H\operatorname{tg}\alpha)\cos\alpha = (H - z_1)\alpha + y_1 - \alpha; \\ x = x_1. \end{gathered} \tag{8.1}$$

Now in equation (6.5) which is valid for the stationary system $O_1z_1y_1x_1$, let us proceed to the moving system Ozyx. Using (8.1), we obtain

$$\begin{gathered} \frac{\partial^2\varphi}{\partial x_1^2} = \frac{\partial^2\varphi}{\partial x^2}, \\ \frac{\partial^2\varphi}{\partial y_1^2} = \frac{\partial^2\varphi}{\partial y^2} + 2\frac{\partial^2\varphi}{\partial y\partial z}\alpha + \frac{\partial^2\varphi}{\partial z^2}\alpha^2, \\ \frac{\partial^2\varphi}{\partial z_1^2} = \frac{\partial^2\varphi}{\partial y^2}\alpha^2 - 2\frac{\partial^2\varphi}{\partial y\partial z}\alpha + \frac{\partial^2\varphi}{\partial z^2}, \end{gathered}$$

that is, with accuracy to values of higher order

$$\frac{\partial^2\varphi}{\partial x_1^2} + \frac{\partial^2\varphi}{\partial y_1^2} + \frac{\partial^2\varphi}{\partial z_1^2} = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2}.$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

Then, denoting the derivative with respect to t in the Oxyz system in terms of $\partial'/\partial t$, we have

$$\frac{\partial\varphi}{\partial t}=\frac{\partial'\varphi}{\partial t}+\frac{\partial\varphi}{\partial y}[(H-z_1)\dot{\alpha}+\dot{H}\alpha-\dot{\lambda}]+\frac{\partial\varphi}{\partial z}(y_1\dot{\alpha}).$$

However, since by assumption the velocity components are small, with the adopted degree of accuracy

$$\frac{\partial\varphi}{\partial t}=\frac{\partial'\varphi}{\partial t}$$

and analogously

$$\frac{\partial^2\varphi}{\partial t^2}=\frac{\partial'^2\varphi}{\partial t^2}. \qquad (8.2)$$

Thus, in the coordinate system Oxyz, the equation (6.5) retains its form. From (8.2) it follows that the linearized Cauchy-Lagrange integral also remains invariant.

In the Ozyx system, the law of motion along the z axis of the leading edge remains as before $z=H(t)$, and the equation of the surface of the body will be $r=f[H(t)-z]$. However, now the transport velocity of the system itself is superposed on the velocity of the surface with respect to the Ozyx system. For components of the transport velocity from (8.1), we obtain the expressions:

$$v_{lx}=0;\ v_{ly}=(z-H)\dot{\alpha}-\dot{H}\alpha+\dot{\lambda},\ v_{lz}=-y\dot{\alpha}.$$

Let us introduce the polar system (r, θ) into the Oyx plane, reckoning the angle θ from the positive y-axis. In the adopted approximation, the normal to the body coincides with r, and the projection of the relative velocity of the surface points on the normal is $\dot{H}f'[H(t)-z]$. The boundary condition on the body will have the form:

$$\frac{\partial\varphi}{\partial r}=\dot{H}f'[H(t)-z]+v_{ln}=\dot{H}f'[H(t)-z]+$$
$$+[(z-H)\dot{\alpha}-\dot{H}\alpha+\dot{\lambda}]\cos\theta. \qquad (8.3)$$

As for the remaining conditions, they remain as before: the initial conditions are 0, the condition on the free surface $\phi=0$ is also satisfied on the undisturbed level, that is, for $y=z\,\mathrm{tg}\,\theta$. On the basis of linearity we shall find the solution of the problem in the form $\phi=\phi_1+\phi_2$, where ϕ_1 on the body satisfies the condition

$$\frac{\partial\varphi_1}{\partial r}=\dot{H}f'[H(t)-z],\quad 0<z<H(t), \qquad (8.5)$$

and ϕ_2, the condition

$$\frac{\partial\varphi_2}{\partial r}=[(z-H)\dot{\alpha}-\dot{H}\alpha+\dot{\lambda}]\cos\theta,\quad 0<z<H(t). \qquad (8.6)$$

The function ϕ_1 corresponds to the inclined penetration of a body moving with the velocity $\dot{H}(t)$ along its axis. The function ϕ_2 corresponds to an asymmetric

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

flow field as a result of the presence of an instantaneous angle of attack (the term $-H\dot{\alpha}\cos\theta$), the transverse translational motion (the term $\dot{\lambda}\cos\theta$) and as a result of rotational velocities with angular velocity $\dot{\alpha}$ (the term $(z-H)\dot{\alpha}\cos\theta$ under the condition (8.6)). Thus, the general problem of penetration is divided into a number of partial problems.

1. Let us consider the problem for determining ϕ_1. Although the condition on the body (8.5) coincides with the analogous condition of §7, the flow in the investigated case will not be axisymmetric as a result of asymmetric arrangement of the body with respect to the free boundary. Let us continue the function ϕ_1 to the upper halfspace so that at the symmetric points with respect to the free surface it will assume values opposite in sign. With this continuation the Oz and Oy axes mirror the free surface. The mapped system will be denoted $Oz_0y_0x_0$. The relation between the systems is as follows:

$$\begin{gathered} z_0 = z\cos 2\vartheta + y\sin 2\vartheta, \\ y_0 = z\sin 2\vartheta - y\cos 2\vartheta, \\ x_0 = x,\ r_0^2 = x_0^2 + y_0^2 = r^2\sin^2\theta + (z\sin 2\vartheta - r\cos\theta\cos 2\vartheta)^2. \end{gathered} \tag{8.7}$$

The "mapped" body will have the equation $r_0=f[H(t)-z_0]$, $0<z_0<H(t)$.

Let us distribute the sources with the intensity $q(z,t)$ along the axis of the body $0<z<H(t)$ and intensity $q_0(z_0, t)$ with respect to the axis of the "mapped" body $0<z_0<H(t)$. From expression (7.9) which is valid also in this case, we find

$$\begin{gathered} q(z, t) = 2\pi\dot{H}f[H(t)-z]f'[H(t)-z],\quad 0<z<H(t), \\ q_0(z_0, t) = -2\pi\dot{H}f[H(t)-z_0]f'[H(t)-z_0],\quad 0<z_0<H(t), \end{gathered} \tag{8.8}$$

on the rest of the z axis and z_0, we set $q=q_0=0$. For the potential ϕ_1 we obtain

$$\varphi_1 = -\frac{1}{4\pi}\int_0^{\xi_2}\frac{1}{R}q\left(\xi, t-\frac{1}{a}R\right)d\xi - \frac{1}{4\pi}\int_0^{\xi_1}\frac{1}{R_0}q_0\left(\xi, t-\frac{1}{a}R_0\right)d\xi, \tag{8.9}$$

where

$$R = \sqrt{(\xi-z)^2+r^2},\quad R_0 = \sqrt{(\xi-z_0)^2+r_0^2},$$

and the integration limits are found from the equations:

$$\xi_2 = H\left(t-\frac{1}{a}\sqrt{(z-\xi_2)^2+r^2}\right),\quad \xi_1 = H\left(t-\frac{1}{a}\sqrt{(\xi_0-\xi_1)^2+r_0^2}\right). \tag{8.10}$$

Formula (8.9) is suitable over the entire region for subsonic movement and inside the sphere $z^2+y^2+x^2\leq a^2t^2$ for supersonic movement. Outside the sphere $z^2+y^2+x^2>a^2t^2$ for supersonic movement, the effect of the free boundary is not felt, and the formula (7.19) is valid there. From (8.8) it follows that $q_0(z_0,t)=-q(z_0,t)$, and from (8.10) it follows that $\xi_1(z_0,q_0)=\xi_2(z,r)$. Therefore it is possible to write (8.9) in the form

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$\varphi_1 = \varphi_1^*(z, r, t) - \varphi_1^*(z_0, r_0, t), \tag{8.11}$$

where
$$\varphi_1^*(z, r, t) = -\frac{1}{4\pi}\int_0^{\xi_2}\frac{1}{R}\,q\left(\xi, t-\frac{1}{a}R\right)d\xi.$$

This formula makes it possible to draw some qualitative conclusions: namely, since the potential everywhere is on the order of $o(q)\sim o(\delta^2)$, except the location of the sources where the estimate (7.8) is valid for it, in the vicinity of the body $r\to 0$, $z>0$ we have

$$\varphi_1^*(z_0, r_0, t) = \varphi_1^*(z\cos 2\vartheta, z\sin 2\vartheta, t) + o(\delta^3),$$

where the value of $o(\delta^3)$ depends on θ. Inasmuch as the value of $\phi_1^*(z,r,t)$ in general does not depend on θ, it follows that the effect of the asymmetry of the streamlined flow on the pressure distribution with respect to the body is on the order of $o(\delta^3)$. This asymmetry of the pressure causes the appearance of a side force and a moment with a value of $o(\delta^4)$. The axial force, just as for vertical penetration, will be on the order of $o(\delta^4 \ln\delta)$, and it will depend on the angle θ. Thus, both forces and the moment with inclined penetration without an angle of attack turn out to be in practice of the same order. As was demonstrated below, the side force and the moment as a result of the angle θ are an order less than the analogous values as a result of the angle α. Therefore the side force and the moment as a result of θ can be neglected and we can limit ourselves to the calculation only of the force of resistance.

As an example, let us consider the motion of a cone with constant velocity $\dot{H}=v_0$ and the half-apex angle γ. We have

$$q = 2\pi v_0\gamma^2(v_0 t - z),$$

$$\xi_2 = \frac{1}{M^2-1}\left[M^2 z - v_0 t + M\sqrt{(v_0 t - z)^2 - (M^2-1)r^2}\right],$$

$$\xi_1 = \frac{1}{M^2-1}\left[M^2 z_0 - v_0 t + M\sqrt{(v_0 t - z_0)^2 - (M^2-1)r_0^2}\right].$$

Using expression (3.9), for the pressure we obtain the formula:

$$p_1 = -\rho\frac{\partial\varphi_1}{\partial t} = \frac{1}{2}\rho v_0^2\gamma^2\left[\int_0^{\xi_2}\frac{d\xi}{\sqrt{(\xi-z)^2+r^2}} - \int_0^{\xi_1}\frac{d\xi}{\sqrt{(\xi-z_0)^2+r_0^2}}\right] = \tag{8.12}$$

$$= \frac{1}{2}\rho v_0^2\gamma^2\ln\frac{(\sqrt{z^2+r^2}+z)(\sqrt{z_0^2+r_0^2}-z_0)}{r^2}\cdot\frac{\xi_2(M-1)+v_0 t - Mz}{\xi_1(M-1)+v_0 t - Mz}.$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

In the vicinity of the body $r\to 0$

$$\xi_2 = v_0\frac{at+z}{a+v_0},\ \xi_1 = \frac{1}{M^2-1}[M^2 z\cos 2\vartheta - v_0 t + \\ + M\sqrt{v_0 t - z\cos 2\vartheta - (M^2-1)z^2\sin 2\vartheta}].$$

Substituting ξ_1, ξ_2 and the equation of the body $r=\gamma(v_0t-z)$ in (8.12), we obtain the pressure on the body, and by integration of the pressure, the force F. If θ is close to $\pi/2$, then expanding ξ_1 in the vicinity of $\theta=\pi/2$ in a series, we obtain on the body:

$$p_1 = \frac{1}{2}\rho v_0^2\gamma^2 \ln\left\{\frac{4z^2}{\gamma^2(v_0^2t^2-z^2)}\times \right.\\ \left.\times\frac{1-\cos 2\vartheta}{2}\left[1+\frac{z(1+\cos 2\vartheta)}{(v_0t+z)}+\frac{(M^2-1)z^2\sin^2 2\vartheta}{4(v_0t+z)^2}\right)\right\}.$$

This pressure differs from its value for vertical entry (7.21) by the amount Δp:

$$\Delta p = \frac{1}{2}\rho v_0^2\gamma^2\left[\ln\frac{1-\cos 2\vartheta}{2}+\frac{z(1+\cos 2\vartheta)}{(v_0t+z)}+\frac{M^2-1}{4}\frac{z^2\sin^2 2\vartheta}{(v_0t+z)^2}\right].$$

The axial force of resistance by comparison with the value for $\theta=\pi/2$ changes by an amount ΔF which for subsonic movement is equal to

$$\Delta F = \pi\rho v_0^4\gamma^4 t^2\left[\frac{1}{2}\ln\frac{1-\cos 2\vartheta}{2}+\left(\frac{3}{2}-2\ln 2\right)(1+\cos 2\vartheta)+\right.\\ \left.+\frac{M^2-1}{4}\left(\frac{7}{2}-5\ln 2\right)\sin 2\vartheta\right], \tag{8.13}$$

for supersonic movement,

$$\Delta F = \pi\rho v_0^4\gamma^4 t^2\left[\frac{2M-1}{2M^2}\ln\frac{1-\cos 2\vartheta}{2}+\left(\frac{4M-1}{2M^2}-\right.\right.\\ \left.-2\ln\frac{1+M}{M}\right)(1+\cos 2\vartheta)+\frac{M^2-1}{4}\left(\frac{6M-1}{2M^2}+\right.\\ \left.\left.+\frac{2}{M+1}-5\ln\frac{1+M}{M}\right)\sin^2 2\vartheta\right]. \tag{8.14}$$

2. Definition of ϕ_2. The asymmetric current field caused by the presence of the angle $\alpha(t)$ and the transverse displacements $\lambda(t)$ in the cylindrical system $Ozr\theta$ satisfies the equation

$$\frac{\partial^2\varphi_2}{\partial z^2}+\frac{\partial^2\varphi_2}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi_2}{\partial r}+\frac{1}{r^2}\frac{\partial^2\varphi_2}{\partial\theta^2}=\frac{1}{a^2}\frac{\partial^2\varphi_2}{\partial t^2}. \tag{8.15}$$

The boundary condition (8.6) states that the solution to the problem can be found in the form $\phi_2=\Phi(z, r)\cos\theta$, where Φ satisfies the equation

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$\frac{\partial^2\Phi}{\partial z^2}+\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r}\frac{\partial\Phi}{\partial r}+\frac{1}{r^2}\Phi=\frac{1}{a^2}\frac{\partial^2\Phi}{\partial t^2}, \quad (8.16)$$

the condition $\Phi=0$ at the free boundary and the condition

$$\frac{\partial\Phi}{\partial r}=[-\dot{H}\alpha+\dot{\lambda}+(z-H)\dot{\alpha}]\equiv C(z,t) \quad (8.17)$$

on the surface of the body.

The general form of the solution of the equation (8.16) is easily written, giving attention to the following fact: if $\phi(z,r)$ satisfies equation (7.1), then $\Phi=\partial\phi/\partial r$ satisfies equation (8.16), which is proved with the help of differentiation. Thus, the solution to equation (8.16) is the function

$$\Phi=-\frac{1}{4\pi}\frac{\partial}{\partial r}\int_{-\infty}^{\infty}\frac{1}{R}q(\xi,t')\,d\xi, \quad (8.18)$$

where it is necessary to select q so as to satisfy the conditions (8.17) on the body and the condition on the free surface. From the condition on the body we have the integral differential equation for q:

$$-\frac{1}{4\pi}\frac{\partial^2}{\partial r^2}\int_{-\infty}^{\infty}\frac{1}{R}q(\xi,t')\,d\xi=C(z,t),\quad 0<z<H(t),$$

which we shall solve approximately.

Just as when obtaining formula (7.8), we find that for $r\to 0$, Φ and $\partial\Phi/\partial r$ are asymptotically equal:

$$\Phi=\frac{1}{2\pi}\frac{q(z,t)}{r}+o(q),$$
$$\frac{\partial\Phi}{\partial r}=-\frac{1}{2\pi}\frac{q(z,t)}{r^2}+o\left(\frac{q}{r}\right), \quad (8.19)$$

from which in the segment $0<z<H(t)$ we obtain

$$q(z,t)=-2\pi f^2[H(t)-z]\,C(z,t). \quad (8.20)$$

For $z>H(t)$ we set $q=0$.

From (8.19) and (8.20) it follows that q is on the order of $o(\delta^2C)$ and with the help of the asymptotic formula (8.19) it is defined with an accuracy $o(\delta^3C)$. In order to satisfy the condition on the free boundary, just as before, it is necessary to mirror map the body and place the sources on the corresponding segment of the axis of the "reflected body":

$$q_0(z_0,t)=2\pi f^2[H(t)-z_0]\,C(z_0,t).$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

The solution to the problem will be

$$\Phi = \Phi_1 + \Phi_2,$$

$$\Phi_1 = -\frac{1}{4\pi}\frac{\partial}{\partial r}\int_0^{\xi_1}\frac{1}{R}q\left(\xi, t-\frac{1}{a}R\right)d\xi,$$

$$\Phi_2 = -\frac{1}{4\pi}\frac{\partial}{\partial r}\int_0^{\xi_1}\frac{1}{R_0}q_0\left(\xi, t-\frac{1}{a}R_0\right)d\xi,$$

where ξ_2 and ξ_1 are determined from (8.10). However, in the vicinity of the body $\Phi_1=(1/2\pi)(q/r)+o(\delta^2C)$ and $\Phi_2=o(\delta^2C)$; therefore with an accuracy to the terms $o(\delta^2C)$ the potential ϕ_2 near the surface of the body will be

$$\varphi_2 = \frac{1}{r}f^2[H(t)-z][\dot{H}\alpha-\dot{\lambda}-(z-H)\dot{\alpha}]\cos\theta.$$

As is obvious, ϕ_2 does not depend on the compressibility of the liquid or on the presence of the free surface. Additional pressure on the body caused by transverse and rotational motion is

$$p_2 = -\rho\frac{\partial\varphi_2}{\partial t} = \rho\left\{2\dot{H}f'[H(t)-z][\dot{H}\alpha-\dot{\lambda}-(z-H)\dot{\alpha}] + f[H(t)-z]\frac{\partial}{\partial t}[\dot{H}\alpha-\dot{\lambda}-(z-H)\dot{\alpha}]\right\}\cos\theta. \quad (8.21)$$

For the axial force of resistance F_x, the size and force F_y and the moment L with respect to the tip we obtain:

$$F_x = 0,$$

$$F_y = -\int_0^{H(t)} f[H(t)-z]dz\int_0^{2\pi}p_2\cos\theta d\theta =$$

$$= \rho\{(\dot{H}^2\alpha-\ddot{H}\dot{\lambda}+H\dot{H}\dot{\alpha})S+(\dot{H}\dot{\alpha}+\ddot{H}\alpha-\ddot{\lambda})Q_0+\ddot{\alpha}Q_1\}, \quad (8.22)$$

$$L = -\int_0^{H(t)}[H(t)-z]f[H(t)-z]dz\int_0^{2\pi}p_2\cos\theta d\theta =$$

$$= \rho\{H\dot{H}(\dot{H}\alpha-\dot{\lambda}+H\dot{\alpha})S-\dot{H}(\dot{H}\alpha-\dot{\lambda})Q_0+(\ddot{H}\alpha-\ddot{\lambda})Q_1+\ddot{\alpha}Q_2\},$$

where

$$S = \pi f^2[H(t)], \quad Q_0 = \pi\int_0^{H(t)}f^2(\xi)\,d\xi, \quad (8.23)$$

$$Q_1 = \pi\int_0^{H(t)}\xi f^2(\xi)\,d\xi, \quad Q_2 = \pi\int_0^{H(t)}\xi^2 f^2(\xi)\,d\xi.$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

Thus, the normal force and moment caused by the presence of transverse and rotational motion of the body do not depend on the compressibility of the medium, the presence of a free surface, and they are completely determined by the geometry of the body and its law of motion.

In the special case the motion of the cone at a constant angle α, v_0=const, λ=0 will be obtained

$$F_y = \pi\rho v_0^4 t^2 \gamma^2 \alpha; \quad L = \frac{2}{3}\pi\rho v_0^5 t^3 \gamma^2 \alpha,$$

so that the center of pressure of the submerged part is at a distance $z^* = (2/3)v_0 t$ from the apex of the cone.

§9. Penetration of a Compressible Liquid by Thin Flat Bodies. Penetration by a Wedge

Let us consider the vertical submersion of a thin profile (Figure 1.19). The stationary cartesian system will be bound to the point of contact, the Oz axis will be directed vertically downward, the Oy axis, along the free boundary. The law of subversion of the tip will be denoted by z=H(t), the equations of the right and left generatrices of the profile, by y=f[H(t)-z] and y=g[H(t)-z] [14, 15]. The potential satisfies the equation

$$\frac{\partial^2\varphi}{\partial z^2} + \frac{\partial^2\varphi}{\partial y^2} = \frac{1}{a^2}\frac{\partial^2\varphi}{\partial t^2}, \tag{9.1}$$

the condition ϕ=0 or the free boundary z=0 and the boundary condition

$$\frac{\partial\varphi}{\partial y} = \dot{H}(t) f'[H(t)-z], \quad \frac{\partial\varphi}{\partial y} = \dot{H}(t) g'[H(t)-z] \tag{9.2}$$

respectively on the right and left generatrices of the profile.

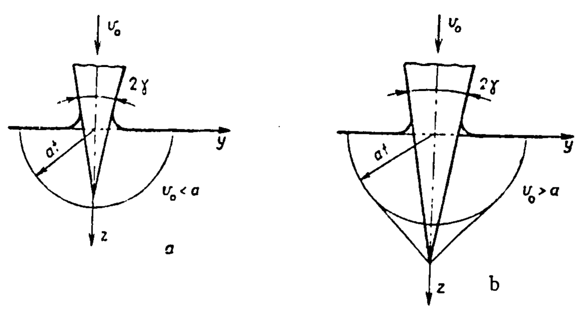

Figure 1.19

In the two-dimensional case the potential in the vicinity of y=0 is limited to the opposite of the spatial case, where for r→0 it has the logarithmic

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

singularity (7.8), and therefore the boundary conditions (9.2) can be taken from the profile surface to the plane y=0.

As usual, we satisfy the condition on the free boundary by continuing ϕ unevenly to the halfspace z<0.

Let us represent the right and left generatrices of the profile in the form

$$y_в = f_2 + f_1, \quad y_н = f_2 - f_1, \tag{9.3}$$

where

$$f_1 = \frac{f-g}{2}, \; f_2 = \frac{f+g}{2}.$$

Then on the basis of linearity, the problem can be broken down into two problems $\phi=\phi_1+\phi_2$, where ϕ_1 corresponds to flow around a symmetric profile $y=\pm f_1(z)$ with the boundary condition

$$\frac{\partial \varphi_1}{\partial y} = \pm \dot{H} f_1'[H(t) - z] \tag{9.4}$$

for $y=\pm 0$ and $0<z<H(t)$, and ϕ_2 corresponds to streamline flow around a distorted body of 0 thickness (the midline of the initial profile) with the equation $y=f_2(z)$ and the boundary condition

$$\frac{\partial \varphi_2}{\partial y} = \dot{H} f_2'[H(t) - z], \; y = \pm 0, \; 0 < z < H(t). \tag{9.5}$$

From the conditions (9.4) and (9.5) it follows that ϕ_1 is even, and ϕ_2 is odd with respect to y; therefore it is sufficient to find these functions for $y \geqslant 0$.

1. Penetration by a symmetric profile. From the property of evenness of ϕ_1 it follows that $\partial\phi_1/\partial y=0$ in the segment of the z-axis not occupied by the profile, z>H(t). Using the continuation of ϕ_1 to the halfplane z<0, we find that $\partial\phi_1/\partial y$ is known on the entire axis y=0:

$$\frac{\partial \varphi_1}{\partial y} = \begin{cases} \dot{H}(t) f_1'[H(t) - z], & 0 < z < H(t), \\ -\dot{H}(t) f_1'[H(t) + z], & -H(t) < z < 0, \\ 0, & |z| > H(t). \end{cases}$$

According to the theory of the delaying potential the solution of the investigated problem is given by the formula

$$\varphi_1(z, y, t) = -\frac{a}{\pi} \iint\limits_S \frac{\partial \varphi_1}{\partial y}(\xi, 0, \tau) \frac{d\xi d\tau}{\sqrt{a^2(t-\tau)^2 - (z-\xi)^2 - y^2}}, \tag{9.6}$$

where S is the region of the plane (ξ, τ) in which the expression under the square root sign is positive, and $\tau>0$. This region is the interior included between the hyperbola

$$\tau = t - \frac{1}{a}\sqrt{(z-\xi)^2 + y^2}$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

and the trajectory of the leading edge of the profile $\xi=H(t)$ and its mirror mapping $\xi=-H(t)$.

Being given the law of motion H(t), by formula (9.6) it is possible to calculate ϕ_1, and then the pressure $p=-\rho\partial\phi/\partial t$ and the remaining characteristics. Let us present some results for the wedge with a half-apex angle γ [14] moving with constant velocity v_0. For subsonic motion $v_0<a$ the pressure on the wedge (the y-axis=0) is

$$p=\frac{\rho v_0^2\gamma}{\pi\sqrt{1-M^2}}\ln\frac{M\sqrt{1-\xi^2}+\xi\sqrt{1-M^2}}{M\sqrt{1-\xi^2}-\xi\sqrt{1-M^2}},\quad \xi=\frac{z}{at}. \tag{9.7}$$

For the force of resistance on the wedge we have

$$F=2\gamma\int_0^{v_0 t} p\,dz=\frac{4\rho v_0^3\gamma^2 t}{\pi\sqrt{1-M^2}}\ln\left(1+\sqrt{1-M^2}\right). \tag{9-8}$$

Completing the limiting transition for $a\to\infty$, we obtain the pressure distribution and the force of resistance on penetration into an incompressible liquid:

$$p=\frac{\rho v_0^2\gamma}{\pi}\ln\frac{1+\frac{z}{v_0 t}}{1-\frac{z}{v_0 t}},$$

$$F=\frac{4}{\pi}\rho v_0^3\gamma^2 t\ln 2,$$

which corresponds to formula (2.10).

The pressure and the force of resistance for sonic penetration will be found by the limiting transition for $M\to 1$

$$p=\frac{\rho v_0^2\gamma}{\pi}\,\frac{2\xi}{\sqrt{1-\xi^2}},\qquad F=\frac{4}{\pi}\rho v_0^3\gamma^2 t.$$

For supersonic motion on a section of the wedge $0<z<at$ subjected to the effect of the free boundary, the pressure is

$$p=\frac{2\rho v_0^2\gamma}{\pi\sqrt{M^2-1}}\operatorname{arctg}\left(\frac{\xi}{M}\sqrt{\frac{M^2-1}{1-\xi^2}}\right),\qquad (0<\xi<1). \tag{9.9}$$

In the segment $at<z<v_0 t$ where the effect of the free boundary is not filled, the pressure is constant

$$p=\frac{\rho v_0^2\gamma}{\sqrt{M^2-1}}. \tag{9.10}$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

The force of resistance in the sections $0<z<at$ and $at<z<v_0t$, respectively, is

$$F_1 = \frac{2\rho v_0^3 \gamma^2}{\pi\sqrt{M^2-1}}\, t\left[\pi\left(\frac{1}{M}-1\right)+2\,\mathrm{arc\,tg}\sqrt{M^2-1}\right], \tag{9.11}$$

$$F_2 = \frac{2\rho v_0^3 \gamma^2}{\sqrt{M^2-1}}\, t\left(1-\frac{1}{M}\right) \tag{9.12}$$

and the total force of resistance to penetration by the wedge is expressed by the formula

$$F = \frac{4\rho v_0^3 \gamma^2}{\pi\sqrt{M^2-1}}\, t\,\mathrm{arc\,tg}\sqrt{M^2-1}. \tag{9.13}$$

Let us also note that in the section of the wedge $0<z<at$ for subsonic and supersonic motion the pressure is expressed by different formulas (9.7) and (9.9) and depends on the M number at the same time as on penetration by the cone in the corresponding section of the generatrix the pressure does not depend on the M number.

2. Penetration of an Asymmetric Profile. For the function ϕ_2 the boundary condition is known in the region occupied by the body $0<z<H(t)$ and its reflection $-H(t)<z<0$. If the motion is supersonic, then disturbance ahead of the body is absent and, consequently, $\partial\phi_2/\partial y=0$ for $|z|>H(t)$, $y=0$. In this case $\partial\phi_2/\partial y$ is known on the entire axis $y=0$, and the formula (9.6) again gives the effective solution.

For subsonic motion the disturbance outside the circle $z^2+y^2=a^2t^2$ and, consequently, $\partial\phi_2/\partial y=0$ for $|z|>at$, $y=0$. However, in the segments $H(t)<z<at$, $-at<z<H(t)$ the value of $\partial\phi_2/\partial y$ is unknown, for on the basis of unevenness of ϕ_2 in this section $\phi_2=0$. Therefore, before using formula (9.6), it is necessary to determine $\partial\phi_2/\partial y$ in the indicated section from the condition $\phi_2=0$. From (9.6) it follows that for determination of $\partial\phi_2/\partial y$ in the region $H(t)<z<at$, an integral equation is obtained. Solving this equation with respect to formula (9.6), we obtain the solution of the initial problem.

Let, for example, a plate be submerged with constant velocity $v_0>a$ and angle of attack α=const. Then the pressure on the windward side of the plate will be expressed by the formulas (9.9) and (9.10). On the leeward side, on the basis of unevenness, the pressure will be expressed by the same formula, but with a minus sign. Hence, for the force of resistance we obtain the previous expressions (9.11)-(9.13). For the lateral force and moment with respect to the tip from the sections $0<z<at$ and $at<z<v_0t$ we obtain:

$$Y_1 = \frac{2\rho v_0^3 \gamma}{\pi\sqrt{M^2-1}}\, t\left[\pi\left(\frac{1}{M}-1\right)+2\,\mathrm{arc\,tg}\sqrt{M^2-1}\right],$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$L_1 = \frac{2\rho v_0^4 \gamma}{\pi \sqrt{M^2-1}} t^2 \left[\pi\left(\frac{1}{M}-1\right) + 2\,\mathrm{arc\,tg}\sqrt{M^2-1} + \right.$$
$$\left. + \frac{\pi}{2M^2}\left(M^2-1-M\sqrt{M^2-1}\right)\right],$$
$$Y_2 = \frac{2\rho v_0^3 \gamma}{\sqrt{M^2-1}} t\left(1-\frac{1}{M}\right),$$
$$L_2 = \frac{2\rho v_0^4 \gamma t^2}{\sqrt{M^2-1}}\left[1-\frac{1}{M}-\frac{1}{2M^2}(M^2-1)\right].$$

The total lateral force and its moment acting on the plate are

$$Y = \frac{4\rho v_0^3 \gamma}{\pi\sqrt{M^2-1}} t\,\mathrm{arctg}\sqrt{M^2-1},$$
$$L = \frac{2\rho v_0^4 \gamma}{\pi\sqrt{M^2-1}} t^2\left(2\,\mathrm{arctg}\sqrt{M^2-1} - \frac{\pi}{2}\frac{\sqrt{M^2-1}}{M}\right).$$

The pressure center is located at a distance from the leading edge of the plate

$$l = v_0 t\left(1 - \frac{\pi}{4}\frac{\sqrt{M^2-1}}{M\,\mathrm{arctg}\sqrt{M^2-1}}\right).$$

For $M\to\infty$, $\ell=(1/2)v_0t$, that is, the pressure center is in the middle of the submerged part of the plate. For $M\to 1$ the pressure center is at the following distance from the leading edge of the plate:

$$l = v_0 t\left(1-\frac{\pi}{4}\right) = 0{,}2146 v_0 t.$$

It must be noted that the position of the center of pressure of the plate depends on the M-number at the same time as for a coné with any M the pressure center is at a distance of 2/3 of the submerged part.

Penetration of a Compressible Liquid by Blunt Bodies

§10. Penetration of a Compressible Liquid by a Slightly Distorted Outline

Let a cylindrical body make contact with the free surface of a compressible liquid at rest at the time t=0 (Figure 1.20).

The generatrices of the cylinder are parallel to the surface of the undisturbed liquid occupying the lower halfspace, and its velocity v_0 at the time of impact is perpendicular to the surface of the liquid. In this statement the movement of the liquid will be plane parallel, and the parameters of motion will depend

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

on the cartesian coordinates $x_0 y_0$ and the time t. The origin of the coordinates is selected on the free surface at the point of encounter of the body with the surface of the liquid. The Ox_0 axis is directed to the right with respect to the free surface, the Oy_0 axis into the liquid, downward. It is proposed that the velocity v_0 is much less than the speed of sound a in the liquid: $v_0/a \ll 1$. During the process of penetration, the rate of expansion of the boundary of the contact area of the investigated blunt outline in the plane $x_0 y_0$ is on the order of $V = v_0/\mathrm{tg}\beta$, where β is the angle of inclination of the outline to the Ox_0 axis. If V is greater than the speed of sound in the liquid, the body acts on the liquid with respect to the surface which expands with supersonic velocity. As was demonstrated below, the compressibility must be considered also for high subsonic values of the velocity V. The condition $v_0/a \ll 1$ and the assumption that V/a is on the order of one can be satisfied if $\beta \ll 1$. Considering insignificant variation of the liquid density, by comparison with the initial density, it is possible to linearize the Euler equations of motion. As a result, for the velocity potential $\bar{\phi}(x_0, y_0, t)$, the velocity components v_x, y_y and pressure p in the liquid we obtain a wave equation. Let us write it for the velocity potential $\bar{\phi}$ [16]:

$$\frac{\partial^2 \bar{\varphi}}{\partial x_0^2} + \frac{\partial^2 \bar{\varphi}}{\partial y_0^2} = \frac{1}{a^2} \frac{\partial^2 \bar{\varphi}}{\partial t^2}. \tag{10.1}$$

The pressure is determined from the linearized Cauchy-Lagrange integral

$$p = -\rho \frac{\partial \bar{\varphi}}{\partial t}. \tag{10.2}$$

The restrictions placed above permit linearization of the boundary conditions also. The boundary conditions are carried over to the $y_0=0$ axis. Since on the free surface outside the contact area with the liquid the pressure during the entire time of motion is constant and equal to p_0, from (10.2) it follows that on this part of the liquid boundary $\bar{\phi}(x_0, y_0, t)=0$. As a result, we obtain the following initial and boundary conditions:

$$t = 0, \quad \bar{\varphi} = 0, \quad \frac{\partial \bar{\varphi}}{\partial t} = 0; \tag{10.3}$$

$$t > 0, \quad y_0 = 0, \quad \frac{\partial \bar{\varphi}}{\partial y_0} = v_0 \quad \text{in segment L};$$

$$t > 0, \quad y_0 = 0, \quad \bar{\varphi} = 0 \quad \text{in segment L}', \tag{10.4}$$

where v_0 is the component of the penetration rate of the body with respect to the y_0 axis. It will be a function of x_0, t for an elastic body and a function of only time t for a solid state. L is the projection of the wetted surface of the body on the Ox_0 axis, L' is the entire Ox_0 axis with the exception of the L part. If we add the equations describing the motion and deformation of the penetrating body to equations (10.1)-(10.4), we obtain a closed system for determining the motion as a whole.

Let a thin slightly bent wing of finite span move at a low local angle of attack β with supersonic velocity U. The origin of the moving coordinate system $Ox_1y_1z_1$ bound to the moving wing will be taken at the forward point of the wing; the Oz_1 axis will be directed opposite to the direction of the flight speed U,

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

the Ox_1 axis will be perpendicular to Oz_1 in the plane of the wing, Oy_1 will be perpendicular to the Ox_1z_1 plane. In the moving coordinate system the motion of the liquid will be a steady state. In the linearized situation the velocity potential $\phi_1(x_1, y_1, z_1)$ satisfies the wave equation [17]:

$$\frac{\partial^2\varphi_1}{\partial x_1^2}+\frac{\partial^2\varphi_1}{\partial y_1^2}=\mu^2\frac{\partial^2\varphi_1}{\partial z_1^2}, \qquad (10.5)$$

where $\mu^2=U^2/a^2-1$, $U/a>1$. The boundary conditions of this problem will be

$$\begin{aligned} &y_1=0, \quad \frac{\partial\varphi_1}{\partial y_1}=U\beta(x_1, z_1) \quad \text{in the section } L_1;\\ &y_1=0, \quad \varphi_1(x_1, y_1)=0 \quad \text{in the section } L_1';\\ &z_1=0, \quad \varphi_1=\frac{\partial\varphi}{\partial z_1}=0, \end{aligned} \qquad (10.6)$$

where L_1 is the projection of the wing on the plane $y_1=0$, L'_1 is the rest of the plane $y_1=0$. Let us note that the vortex sheet behind the body is not considered, that is, the wing is incident in the direction of the z_1 axis. From a comparison of equations (10.5) and (10.6) with equations (10.1), (10.3) and (10.4) it is obvious that for

$$\begin{aligned} &\varphi_1\to\bar{\varphi}, \quad x_1\to x_0, \quad y_1\to y_0, \quad z_1\to t;\\ &\mu^2\to\frac{1}{a^2}, \quad L_1\to L, \quad L_1'\to L'. \end{aligned} \qquad (10.7)$$

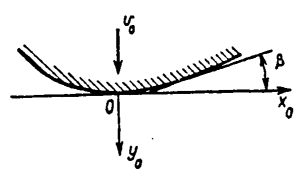

Figure 1.20

The problem of supersonic movement of a slightly bent wing is identical to the problem of penetration of a compressible liquid halfspace by a slightly distorted outline. Consequently, the solution of any specific problem of penetration can be obtained from the solution of a corresponding problem of supersonic movement of the wing. The velocity potential of the disturbed movement of a gas during supersonic movement of a thin wing of finite span is represented in the form [17]:

$$\varphi_1(x_1, y_1, z_1)=-\frac{1}{\pi}\iint\limits_S\frac{\varphi_{y_1}(\xi_1, \zeta)\,d\xi_1 d\zeta}{\sqrt{(z-\zeta)^2-\mu^2[(x_1-\xi_1)^2+y_1^2]}},$$

where

$$\varphi_{y_1}(x_1, z_1)=\left.\frac{\partial\varphi_1}{\partial y_1}\right|_{y_1=0}.$$

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Using the analogy of (10.7) for the velocity potential of disturbed motion of the liquid in the penetration problem, we obtain:

$$\bar{\varphi}(x_0, y_0, t) = -\frac{a}{\pi}\iint\limits_{\sigma} \frac{\bar{\varphi}_{y_0}(\xi, \tau)\, d\xi d\tau}{\sqrt{a^2(t-\tau)^2 - (x_0-\xi)^2 - y_0^2}},$$

$$\bar{\varphi}_{y_0}(x_0, t) = \left.\frac{\partial\bar{\varphi}}{\partial y_0}\right|_{y_0=0},$$

where σ is the part of the plane ξ, τ cut off by the hyperbola

$$a\tau = at - \sqrt{(x_0-\xi)^2 + y_0^2}.$$

Let us consider a blunt rigid wedge with apex angle 2γ which moves in the direction of the Oy_0 axis with constant velocity v_0 (Figure 1.21).

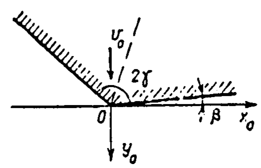

Figure 1.21

Let penetration of the liquid by the wedge begin at the time t=0. For the wedge boundary conditions (10.4) are written in the form

$$\begin{aligned} &\frac{\partial\bar{\varphi}}{\partial y_0} = v_0 \qquad v_0 \operatorname{ctg}(2\gamma + \beta)\, t < x_0 < v_0 \operatorname{ctg}\beta \cdot t, \\ &\bar{\varphi}(x_0, 0, t) = 0, \qquad v_0 \operatorname{ctg}\beta \cdot t < x_0, \qquad x_0 < v_0 \operatorname{ctg}(2\gamma + \beta) \cdot t. \end{aligned} \tag{10.8}$$

The speeds of the points of intersection of the face of the wedge with the free surface right and left will be denoted, respectively, by V_1 and V_2. It is obvious that

$$V_1 = v_0 \operatorname{ctg}\beta, \qquad V_2 = v_0 \operatorname{ctg}(2\gamma + \beta). \tag{10.9}$$

Let us denote

$$M_1 = V_1/a, \qquad M_2 = V_2/a. \tag{10.10}$$

These Mach numbers will be called supersonic and subsonic depending on whether they are greater or less than one. Three cases can occur:

a) Both Mach numbers are greater than one,

b) One of the Mach numbers is greater than one and the other is less than one,

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c) Both Mach numbers are less than one.

The problem of penetration of the wedge as a constant velocity, as has already been noted, does not contain the characteristic linear dimension, and it is self-similar. In accordance with §1 of this chapter let us introduce the new values of x, y, ϕ by the formulas:

$$x = \frac{x_0}{at}, \quad y = \frac{y_0}{at}, \quad \bar{\varphi}(x_0, y_0, t) = a^2 t \varphi(x, y). \tag{10.11}$$

In these coordinates the pressure and any velocity component of the liquid satisfy the equation (1.40). Let us write it for the component v_y:

$$(1-x^2)\frac{\partial^2 v_y}{\partial x^2} - 2xy\frac{\partial^2 v_y}{\partial x \partial y} + (1-y^2)\frac{\partial^2 v_y}{\partial y^2} - 2x\frac{\partial v_y}{\partial x} - 2y\frac{\partial v_y}{\partial y} = 0. \tag{10.12}$$

The equation (10.12) for $x^2+y^2<1$ is of the elliptic type. By the Chaplygin substitution (1.45) in this region it becomes the Laplace equation (1.48)

$$\varepsilon\frac{\partial}{\partial \varepsilon}\left(\varepsilon\frac{\partial v_y}{\partial \varepsilon}\right) + \frac{\partial^2 v_y}{\partial \theta^2} = 0. \tag{10.13}$$

Thus, the velocity components v_x, v_y and the pressure p in the plane ε, θ can be considered as a real part of some analytical function in the complex plane $\tau=\varepsilon e^{i\theta}$. For example, according to (1.49), v_y can be represented as the real part of the function $\xi(\tau)$:

$$\xi(\tau) = v_y(\varepsilon, \theta) + if(\varepsilon, \theta). \tag{10.14}$$

Here the differentials of the velocity components and the imaginary part of the function (10.14) turn out to be related by the expressions (1.53) and (1.54):

$$dv_x = \frac{1}{1-x^2}\left(xydv_y + \frac{1-\varepsilon^2}{1+\varepsilon^2}df\right), \tag{10.15}$$

$$dv_y = \frac{1}{1-y^2}\left(xydv_x - \frac{1-\varepsilon^2}{1+\varepsilon^2}df\right). \tag{10.16}$$

At points of the Ox and Oy axes, the first terms in the righthand side of formula (10.15) obviously disappear. This greatly facilitates the solution of the specific problems.

For supersonic motion of a delta wing, if the coordinate axes are selected so that the equation (10.5) exists, and the origin of the coordinates will be at the apex of the wing, the movement of the liquid that arises will be conical [16]. This means that the velocity components will be functions of the ratios

$$\xi = \frac{x_1}{z_1}, \quad \eta = \frac{y_1}{z_1}. \tag{10.17}$$

Equation (10.5) will be satisfied by any velocity component of the liquid and also the pressure in it. If in equation (10.5) we proceed to the new coordinates

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ξ, η and then use the substitution of variables (the Chaplygin conversion):

$$r = \sqrt{\xi^2 + \eta^2}, \quad r = \frac{2\varepsilon}{1+\varepsilon^2}, \quad \theta = \operatorname{arctg} \xi/\eta,$$

then in the plane ε, θ we obtain the Laplace equation (10.13) for the region inside the circle.

If one of the velocity components is defined inside the circle by the method of analytical representation (10.14), the other two components are determined by the quadrature using the expressions analogous to (10.15), (10.16) [16, 18].

Thus, the solution of the problem of penetration by a wedge and supersonic movement of a delta wing in the elliptic region are reduced to the same boundary problem.

Let us proceed with the solution of the problem of penetration of a compressible liquid halfspace by a blunt wedge for the above-indicated three possible combinations of Mach numbers M_1 and M_2.

§11. Penetration of a Compressible Liquid by a Blunt Wedge

a) Both Mach Numbers on the Faces of the Wedge Supersonic. In the plane of the coordinates Oxy, the picture of the motion is demonstrated in Figure 1.22. In the investigated case the region of disturbed movement will be enclosed by the Mach lines AC and A_1C_1 originating at the points A and A1 of intersection of the surface of the liquid with the places of the wedge and the arc CC_1. The Mach lines are tangent at the points C and C_1 to the circle of unit radius with center at the origin of the coordinates. In the regions ABC and $A_1B_1C_1$ where the equation (10.12) is of the hyperbolic type, the flow is defined just as for supersonic movement of a wedge with Mach numbers M_1 and M_2 and normal velocity component on the wedge $v_n=v_0$. Therefore in these regions the parameters of motion are constant and are defined, correspondingly by the formula

$$v_x = \frac{v_0}{\sqrt{M_1^2-1}}, \quad v_y = v_0, \quad p = \frac{\rho a M_1 v_0}{\sqrt{M_1^2-1}}, \quad 1 < x \leqslant M_1,$$

$$v_x = -\frac{v_0}{\sqrt{M_2^2-1}}, \quad v_y = v_0, \quad p = \frac{\rho a M_2 v_0}{\sqrt{M_2^2-1}}, \quad -M_2 < x < -1. \tag{11.1}$$

The values of the velocity component and the pressure defined by formula (11.1) are valid to points of the arc BC and B_1C_1 of a unit circle, and they define the boundary values on these segments the boundary of the elliptical region of equation (10.12).

For determination of the parameters of motion inside a halfcircle of unit radius, let us return to the plane $\tau=\varepsilon e^{i\theta}$ where, for example, the velocity component v_y satisfies the Laplace equation (10.13). In this plane the region of motion again will be a halfcircle of unit radius with center at the origin of the coordinates (Figure 1.23).

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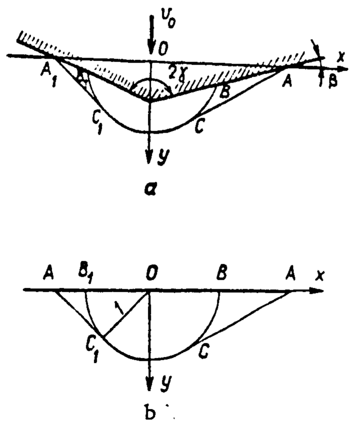

Figure 1.22

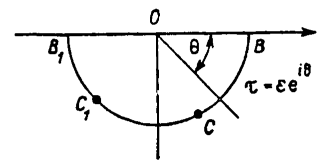

Figure 1.23

The boundary conditions for v_y are the following. On the segment of the horizontal axis BB_1 and the segments BC and B_1C_1 the arcs of the halfcircle $v_y=v_0$. In the segment of the arc CC_1: $v_y=0$, $v_x=0$. Let us note that the analogous boundary conditions on the boundary of the halfcircle (inside the Mach cone emitting from the apex of the wing) are obtained for supersonic motion of a delta wing where the sides of the angle emerge beyond the Mach cone beginning at the apex of the wing [18]. Let us represent the harmonic (in the plane $\tau=\varepsilon e^{i\theta}$) function v_y as the real part of a complex function (10.14), and let us map the halfcircle on the halfplane by the complex variable z with respect to the formula

$$z=\left(\frac{\tau+1}{\tau-1}\right)^2. \tag{11.2}$$

Here the outline of the halfcircle goes to the real axis of the plane z (Figure 1.24).

The solution of the problem in the plane z is given by the formula [14, 19]

$$v_y+if=v_0+\frac{v_0}{\pi i}\ln\frac{\lambda_1+z}{\lambda_2+z}, \tag{11.3}$$

where the following notation is introduced

$$\lambda_1=\frac{M_1+1}{M_1-1},\quad \lambda_2=\frac{M_2-1}{M_2+1}. \tag{11.4}$$

Using (10.15) and the linearized Cauchy-Lagrange integral, the solution (11.3) will permit determination of all of the pressure parameters.

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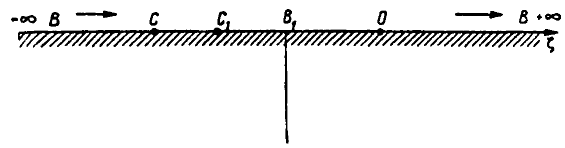

Figure 1.24

The excess pressure p and velocity v_x along the wedge in the section $-1\leq x\leq 1$ in the plane of the dimensionless coordinates x, y are defined by the following expressions [14, 19]:

$$p=\frac{2\rho a v_0}{\pi}\left\{\frac{M_2}{\sqrt{M_2^2-1}}\operatorname{arctg}\sqrt{\frac{(M_2-1)(1-x)}{(M_2+1)(1+x)}}+\right.$$
$$\left.+\frac{M_1}{\sqrt{M_1^2-1}}\operatorname{arctg}\sqrt{\frac{(M_1-1)(1+x)}{(M_1+1)(1-x)}}\right\}, \tag{11.5}$$

$$v_x=\frac{2v_0}{\pi}\left\{\frac{1}{\sqrt{M_1^2-1}}\operatorname{arctg}\sqrt{\frac{(M_1-1)(1+x)}{(M_1+1)(1-x)}}-\right.$$
$$\left.-\frac{1}{\sqrt{M_2^2-1}}\operatorname{arctg}\sqrt{\frac{(M_2-1)(1-x)}{(M_2+1)(1+x)}}\right\}. \tag{11.6}$$

The solution to the problem where one of the numbers M_1 and M_2 is equal to one will be obtained from the solution (11.5) and (11.6) if in them the corresponding number M approaches one.

For symmetric penetration at the wedge $M_1=M_2=M$ in the disturbed region of motion, outside the unit circle in the plane x, y (the regions ABC and $A_1B_1C_1$ in Figure 1.22), according to (11.1), we obtain

$$v_x=\pm\frac{v_0}{\sqrt{M^2-1}};\quad v_y=v_0,\quad p=\frac{\rho M v_0}{\sqrt{M^2-1}},$$

$$M=\frac{v_0}{a}\operatorname{ctg}\beta,\ 1\leqslant x\leqslant M,\quad -M\leqslant x\leqslant -1.$$

In this case the formulas (11.5) and (11.6) assume the form:

$$p=\frac{2}{\pi}\frac{\rho a v_0 M}{\sqrt{M^2-1}}\operatorname{arctg}\sqrt{\frac{M^2-1}{1-x^2}},$$

$$v_x=\frac{2v_0}{\pi}\frac{1}{\sqrt{M^2-1}}\operatorname{arctg}\left(x\sqrt{\frac{M^2-1}{1-x^2}}\right),\qquad -1\leqslant x\leqslant 1. \tag{11.7}$$

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For $M \to 1$ these formulas give

$$p = \frac{2\rho a v_0}{\pi} \frac{1}{1 - x^2}, \quad v_x = \frac{2v_0}{\pi} \frac{x}{\sqrt{1 - x^2}}.$$

The force acting on the wedge is defined by integration of the excess pressure on the section of the wetted part of the wedge. As a result, for this force $F(t)$ we obtain

$$F(t) = \rho a v_0^2 t \{\operatorname{ctg} \beta + |\operatorname{ctg}(\beta + 2\gamma)|\}. \tag{11.8}$$

For symmetric penetration this formula gives

$$F(t) = 2\rho v_0^2 a t \operatorname{ctg} \beta. \tag{11.9}$$

In formulas (11.8) and (11.9) the expressions

$$v_0 \{\operatorname{ctg} \beta + |\operatorname{ctg}(\beta + 2\gamma)|\} t,$$
$$2v_0 \operatorname{ctg} \beta \cdot t$$

denote the length of the wetted surface of the wedge. Thus, the total force acting on the wedge at the time t is exactly equal to the instantaneous force for normal impact of the plate with a velocity v_0, the width of which is equal to the width of the wetted part of the wedge at this time. In reference [20] it is demonstrated that this situation is valid in the more general case. In the sections on axisymmetric penetration, this problem is discussed in more detail. However, it is necessary to note that the pressure distribution along the face of the wedge is essentially not constant.

b) Let the Number M_1 Be Greater than One, the Number M_2 Less Than One (Mixed Problem). The picture of the flow in the plane of the dimensionless coordinates x, y in this case will be shown in Figure 1.25, a.

For the complex function (10.14) in the investigated case in the plane ε, θ (Figure 1.25, b) we obtain the following conditions on the boundary of the halfcircle. On the section of the wedge A_1B and on the arc BC of the halfcircle $v_y=v_0$; on the remaining part CB_1 of the arc of the halfcircle $v_y=0$, $v_x=0$; in the section B_1A_1 of the free surface the velocity potential and, consequently, the velocity component v_x are equal to zero. Then from (10.15) it follows that in this section df=0, and since the desired parameters of motion at the point B_1 are continuous, it is possible to assume that on B_1A_1 the function f=0. Thus, for the desired function (10.14) we obtain the mixed problem, the solution of which will be constructed by the known method of [6].

Analogous boundary conditions are obtained for supersonic movement of a delta wing where one side is inside and the other, outside the Mach cone emerging from the apex [18]. After mapping the halfcircle onto the halfplane of the variable z using the function (11.2), the solution to the problem is represented in the form [19]

$$v_y + if = v_0 - \frac{v_0}{\pi i} \ln \frac{\sqrt{1 + \lambda_2/\lambda_1} + \sqrt{1 - \lambda_2/z}}{\sqrt{1 + \lambda_2/\lambda_1} - \sqrt{1 - \lambda_2/z}} + Ai \sqrt{\frac{z}{z - \lambda_2}}.$$

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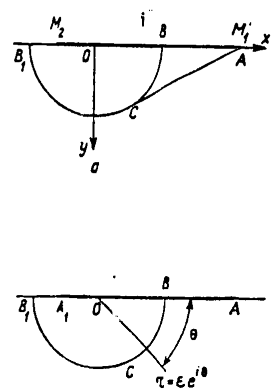

b

Figure 1.25

According to formula (10.15) of the preceding section, since

$$1-x^2=\left(\frac{1-e^2}{1+e^2}\right)^2, \qquad \frac{1-e^2}{1+e^2}=\frac{2\sqrt{z}}{1+z},$$

at the point z=0 we have a singularity. However, z=0 corresponds to the point of intersection of the leading wave with the free surface, and in it the solution must be regular. For this purpose at the point z=0 the following must be satisfied: $(d/dz)(v_y+if)=0$. This condition defines the constant A in the above-presented solution:

$$A=-\frac{2v_0}{\pi}\sqrt{\frac{\lambda_1+\lambda_2}{\lambda_1}}.$$

The final solution is expressed as:

$$v_y+if=v_0-\frac{v_0}{\pi i}\ln\frac{\sqrt{1+\lambda_2/\lambda_1}+\sqrt{1-\lambda_2/z}}{\sqrt{1+\lambda_2/\lambda_1}-\sqrt{1-\lambda_2/z}}+ \\ +\frac{2v_0}{\pi i}\sqrt{\frac{\lambda_1+\lambda_2}{\lambda_1}}\sqrt{\frac{z}{z-\lambda_2}}. \qquad (11.10)$$

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On the real axis ξ of the plane z for the imaginary part of the desired function, from (11.10) we obtain

$$f=\frac{v_0}{\pi}\ln\frac{\sqrt{1+\lambda_2/\lambda_1}+\sqrt{1-\lambda_2/\xi}}{\sqrt{1+\lambda_2/\lambda_1}-\sqrt{1-\lambda_2/\xi}}-\frac{2v_0}{\pi}\sqrt{\frac{\lambda_1+\lambda_2}{\lambda_1}}\cdot\sqrt{\frac{\xi}{\xi-\lambda_2}}. \qquad (11.11)$$

In these formulas the following notation is introduced:

$$\lambda_1=\frac{M_1+1}{M_1-1},\quad \lambda_2=\frac{1-M_2}{1+M_2},\quad \xi=\frac{1+x}{1-x}. \qquad (11.12)$$

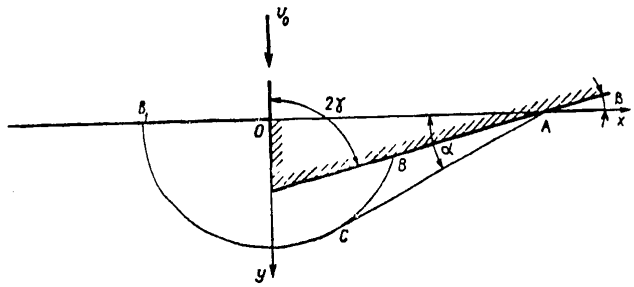

Figure 1.26

Using (10.15) it is easy to calculate the value of the velocity component v_x along the wetted part of the wedge. In the dimensionless plane x, y along the x axis we shall have

$$v_x=\frac{v_0}{\pi}\left[\frac{2}{\sqrt{M_1^2-1}}\operatorname{arctg}\left(\sqrt{\frac{\frac{1+x}{1-x}-\lambda_2}{\lambda_1+\lambda_2}}\right)-\right.$$
$$\left.-\sqrt{1+\lambda_2/\lambda_1}\cdot\frac{\lambda_2+1}{\sqrt{\frac{1+x}{1-x}-\lambda_2}}\right]. \qquad (11.13)$$

The pressure along the wetted part is defined by the formula:

$$p=\frac{2\rho a v_0}{\pi}\left[\frac{M_2\sqrt{M_1+M_2}}{(1+M_2)\sqrt{M_1+1}}\sqrt{\frac{1-x}{M_2+x}}+\right.$$
$$\left.+\frac{M_1}{\sqrt{M_1^2-1}}\operatorname{arctg}\sqrt{\frac{(M_1-1)(M_2+x)}{(M_1+M_2)(1-x)}}\right],\quad -M_2\leqslant x\leqslant 1. \qquad (11.14)$$

In the region $1\leq x\leq M_1$ obviously we have

$$p=\frac{\rho a v_0 M_1}{\sqrt{M_1^2-1}},\quad v_x=\frac{v_0}{\sqrt{M_1^2-1}}.$$

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From the solutions (11.13) and (11.14) it is possible to obtain the pressure and velocity distribution for the case where one face of the wedge is perpendicular to the free surface of the liquid. In this case $M_2=0$, $\lambda_2=1$; then

$$p = \frac{2\rho a v_0 M_1}{\sqrt{M_1^2-1}} \operatorname{arctg} \sqrt{\frac{(M_1-1)x}{M_1(1-x)}},$$

$$v_x = \frac{v_0}{\pi}\left[\frac{2}{\sqrt{M_1^2-1}} \operatorname{arctg} \sqrt{\frac{2x}{1-x}\cdot\frac{1}{\lambda_1+1}} - 2\sqrt{\frac{M_1}{M_1+1}}\sqrt{\frac{1-x}{\lambda}}\right]. \quad (11.15)$$

The investigated case of penetration is illustrated in Figure 1.26. The solution of this problem in the general case where the velocity v_0 makes an acute angle with the Ox axis is presented in references [14, 19]. In this study it is proposed that the speed of the liquid on the left edge is finite, that is, there is smooth streamlined flow around the trailing edge (see §12).

c) Both Mach Numbers Are Subsonic ($M_1<1$, $M_2<1$). The picture of the flow in the plane ε, θ in this case is illustrated in Figure 1.27.

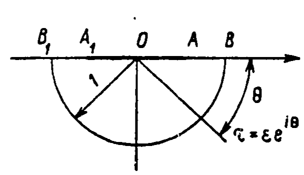

Figure 1.27

In order to use the analogy with the problem of streamlined flow around a supersonic delta wing, it is more convenient instead of the complex function (10.14) to introduce the following function into the investigation

$$W = v_x(\varepsilon,\ \theta) + if(\varepsilon,\ \theta). \quad (11.16)$$

Here the relation between the velocity components and the function f(ε, θ) is presented in the following form [19]:

$$dv_y = \frac{1}{1-y^2}\left[xydv_x - \frac{1-\varepsilon^2}{1+\varepsilon^2}\,df\right]. \quad (11.17$$

The function (11.16) has the following boundary conditions. The velocity v_y in the section AA_1 (the wetted part of the wedge) is constant and equal to v_0; then, according to (11.17), in this section the imaginary part of the function (11.16), that is, f(ε,θ) is constant. In the sections A_1B_1, AB $v_x=0$, and on the arc of unit radius BB_1 the velocity components v_x, v_y are equal to zero.

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The obtained boundary problem completely coincides with the problem of determining the velocity components of the gas in the direction of supersonic movement of a delta wing, the edges of which are inside the Mach cone emerging from its apex. This mixed boundary problem for determining the function (11.16) is solved by the Keldysh-Sedov method[1] [6].

The results of the calculations along the wetted part of the wedge, for example, for excess pressure, give the following expression [20]:

$$p = \frac{\rho a v_0}{2E - (1-k^2)K} \cdot \frac{1}{\sqrt{(1+M_1)(1+M_2)}} \times$$
$$\times \left[M_1 \sqrt{\frac{M_2 + x}{M_1 - x}} + M_2 \sqrt{\frac{M_1 - x}{M_2 + x}} \right], \qquad -M_2 \leqslant x \leqslant M_1. \tag{11.18}$$

In formula (11.18) K and E denote the complete first and second order elliptic integrals, respectively, with the modulus k where

$$k = \frac{(1-M_1)(1-M_2)}{(1+M_1)(1+M_2)}. \tag{11.19}$$

The solution of (11.18) remains in force when one of the numbers M_1 and M_2 is equal to one. When one of the faces of the wedge is perpendicular to the free boundary of the liquid, for example, the left boundary, the corresponding M_2 is equal to zero. For such penetration from (11.18) we obtain:

$$p = \frac{\rho a v_0}{2E - (1-k^2)K} \cdot \frac{M_1}{\sqrt{1+M_1}} \sqrt{\frac{x}{(-x+M_1)}},$$
$$k = \frac{1-M_1}{1+M_1}. \tag{11.20}$$

For symmetric penetration $M_1=M_2=M<1$, formula (11.18) gives:

$$p = \frac{\rho a v_0}{E_1} \frac{M}{\sqrt{1 - \frac{x^2}{M^2}}}, \tag{11.21}$$

where E_1 is the complete second type elliptic integral with modulus k_1 and

$$k_1^2 = 1 - \left(\frac{1-k}{1+k}\right)^2;$$

for the symmetric wedge

$$k_1^2 = 1 - M^2.$$

---

[1]For this purpose it is necessary to map the region of disturbed motion onto the halfplane of the auxiliary complex plane.

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The solution of (11.21) remains valid for M=1. The total force acting on the wetted surface of the wedge is obtained by integrating the pressure (11.18) in this section:

$$F(t) = \rho a v_0 \frac{2\pi M_1 M_2}{\sqrt{(1+M_1)(1+M_2)} + \sqrt{(1-M_1)(1-M_2)}} \frac{at}{E_1}. \quad (11.22)$$

In the case of symmetric penetration by a wedge

$$F(t) = \rho a v_0 \frac{\pi M}{2E_1} \cdot 2v_0 t \operatorname{ctg} \beta. \quad (11.23)$$

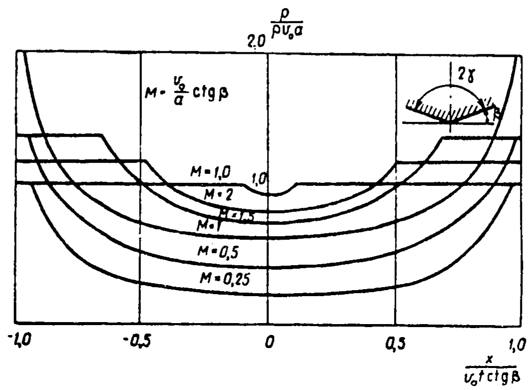

Figure 1.28

It is possible to show that $\pi M/2E_1<1$, and, consequently, for subsonic values of the numbers M_1 and M_2 the total force of resistance to penetration is less than the force of the hydraulic impact of the plate of the same width at the time t=0 [20]. For $a\to\infty$ at the limit from formula (11.23) the formula for an incompressible liquid is obtained for a force of resistance with symmetric penetration of a compressible liquid. From the results of investigating the symmetric penetration of an incompressible liquid by a blunt wedge it is known that the excess pressure on the surface of the wedge is given by the formula (2.25):

$$p = \frac{\rho v_0^2 \operatorname{ctg} \beta}{\sqrt{1 - \left(\frac{x}{v_0 t} \operatorname{tg} \beta\right)^2}}. \quad (11.24)$$

and the force acting on the wedge, by formula (2.26):

$$F(t) = \pi \rho v_0^3 t \operatorname{ctg}^2 \beta. \quad (11.25)$$

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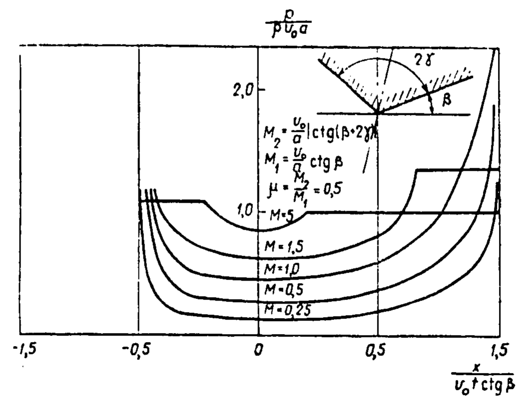

Figure 1.29

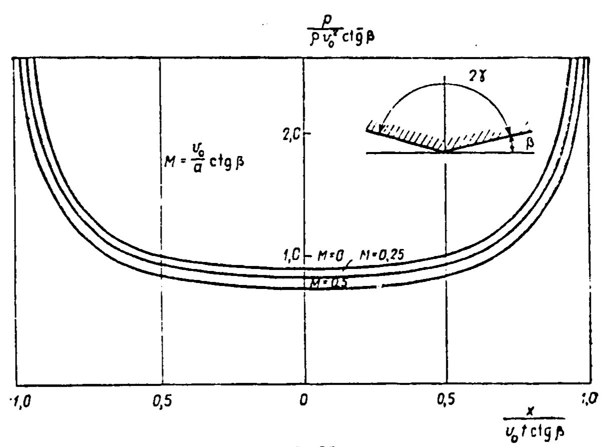

Figure 1.30

Let us note that for values of M_1 and M_2 larger than one and the condition $v_0 << a$, from the equality

$$\sin\alpha = \frac{1}{M} = \frac{a\,tg\,\beta}{}$$

it follows that the Mach angle α of the characteristics beginning with the points of intersection of the supersonic face of the wedge with the free surface is much greater than the angle β formed by the face with this surface: $\alpha >> \beta$.

The numerical results for the case of entry of a symmetric wedge are presented in Figure 1.28, and for the asymmetric case, in Figure 1.29 [20]. Figure 1.30 shows the pressure distributions for the Mach numbers M=0.5, 0.25 and M=0,

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which corresponds to an incompressible liquid. The graphs in this figure show that for M numbers less than 0.25, the effect of the compressibility is insignificant.

§12. Penetration of a Compressible Liquid by a Flat Plate

a) Incidence of the Plate at an Angle to the Surface in the Presence of an Angle of Attack. Let a rigid flat plate fall with a constant velocity v_0 directed at the angle α to the undisturbed free surface on the free surface of an ideal compressible liquid occupying the lower halfspace. At the time of hitting the water the plane of the plate will be at an angle β to the free surface. It is required that the force and the rotational moment acting on the plate from the liquid will be determined. The diagram of the incidence of the plate is shown in Figure 1.31. The problem is considered planar on the basis of the fact that the plate has infinite length. The problem is solved under the assumption that as the plate submerges, the point E, which is the point of intersection of the plane of the plate with the free surface of the liquid, moves in the positive direction of the Ox axis (to the right) with a velocity equal to the speed of sound a in the liquid. The given case is possible where the angle β is small. Considering $v_0 \sin \alpha \ll a$, let us solve the problem in the linear statement [21]. The mass forces are not taken into account. The period of submersion of the plate from the time of contact of the trailing edge with the liquid to the time the leading edge of the plate is submerged in the liquid is considered. With this statement the problem will be self-similar.

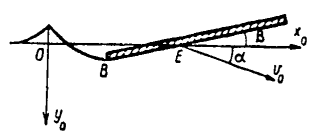

Figure 1.31

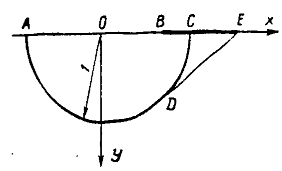

Figure 1.32

The origin of the coordinates is placed at the point of contact of the trailing edge of the plate with the free surface. The Ox_0 axis is directed along the free surface to the right, the Oy_0 axis is vertically downward. Since the angle β is small, the boundary conditions on the plate are carried over to the horizontal surface. It is proposed that the free surface changes little, the trailing edge has streamlined flow smoothly around it, and there is no leakage of the liquid to the upper surface of the plate. In this case the boundary conditions on the free surface are also carried over to the horizontal surface. Here it must be noted that in connection with the fact that the rate of displacement of the point E horizontally is higher than the speed of sound, the free surface to the right of the point E remains undisturbed as the plate submerges. During the time t after contact the trailing edge of the plate moves horizontally and vertically, respectively, by the amount

$$x_{OB} = v_0 t \cos\alpha, \quad y_{OB} = v_0 t \sin\alpha.$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

The projection of the wetted part of the plate on the Ox_0 axis will be defined by the formula

$$BE=\frac{v_0 t\sin\alpha}{\operatorname{tg}\beta}.$$

The coordinate of the point of intersection E of the plate with the free surface will be

$$x_{OE}=x_{OB}+BE=v_0 t\left(\cos\alpha+\frac{\sin\alpha}{\operatorname{tg}\beta}\right).$$

In the adopted linear statement the region of disturbed motion of the liquid in the Oxy plane is shown in Figure 1.32. The segment BE of the Ox axis corresponds to the wetted part of the surface of the plate. Inside the region DCE, the effect of the free surface is not felt. In this region the parameters of the liquid are determined by the known formulas for steady-state supersonic streamline flow around a thin plate. For determination of the flow inside the halfcircle of unit radius, we use the fact that in the plane $\tau=\varepsilon e^{i\theta}$ the velocity component v_y satisfies the Laplace equation, and it can be represented in the form of the real part of the complex function $\xi(\tau)$:

$$\zeta(\tau)=v_y(\varepsilon,\ \theta)+if(\varepsilon,\ \theta). \tag{12.1}$$

Another component v_x, as was pointed out many times, is calculated by formula (1.53). For determination of the function (12.1) we have the following boundary conditions. On the basis of constancy of the pressure in the section AB corresponding to the free surface, v_x=C, but then from (1.53) it follows that df=0, and since the desired functions are continuous at the point A, in the section AB it is possible to set f=0. In the section BC of the wetted surface of the plate and on the arc CD we have the boundary condition $v_y=v_0\sin\alpha$. On the remaining part of the halfcircle (the arc AD) the velocity component v_y=0, v_x=0. For determination of the complex function (12.1), the mixed problem was obtained. This problem can easily be solved using the Keldysh-Sedov formula [22] if we map the region on the upper halfplane of the variable z using the function

$$z=\left(\frac{\tau+1}{\tau-1}\right)^2.$$

As a result, for $\xi(\tau)$ we find the expression

$$\zeta(\tau)=v_y+if=v_0\sin\alpha\left[1-\frac{1}{\pi i}\ln\frac{\sqrt{1+\frac{\mu}{\lambda}}+\sqrt{1-\frac{\mu}{z}}}{\sqrt{1+\frac{\mu}{\lambda}}-\sqrt{1-\frac{\mu}{z}}}\right],$$

$$\lambda=\frac{M+1}{M-1},\quad \mu=\frac{1+\nu}{1-\nu},\quad \nu=\frac{v_0}{a}\cdot\cos\alpha, \tag{12.2}$$

$$M=\frac{v_0}{a}\left(\cos\alpha+\frac{\sin\alpha}{\beta}\right),\quad \operatorname{tg}\beta\to\beta.$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

Using formula (12.2) all of the parameters of motion are calculated by simple quadratures. In the section of the wetted surface the excess pressure can be defined using the Lagrange integral

$$p = \frac{1}{\pi}\rho a v_0 \sin\alpha \sqrt{\frac{\lambda+\mu}{\lambda}}\left[\frac{\lambda+1}{\sqrt{\lambda+\mu}}\operatorname{arctg}\sqrt{\frac{(1+x)(1-x)-\mu}{\lambda+\mu}} - \right.$$
$$\left. - \frac{1}{\sqrt{\mu}}\operatorname{arctg}\sqrt{\frac{(1+x)(1-x)-\mu}{\mu}} + \frac{\pi}{2\sqrt{\mu}}\right]. \quad (12.3)$$

According to formula (12.3) the pressure on the plate from the liquid increases from the trailing edge ($x = v_0 \cos\alpha/a$) to the point C ($x=1$). In the section CE the pressure is constant and equal to

$$p = \frac{\rho a v_0 M}{\sqrt{M^2-1}}\frac{\sin\alpha}{\beta}. \quad (12.4)$$

The point E has the coordinate

$$x_E = \frac{v_0}{a}\left(\cos\alpha + \frac{\sin\alpha}{\beta}\right).$$

The total force acting on the plate from the liquid

$$F = \rho a^3 \beta t (M-\nu)\sqrt{\frac{M-\nu}{M+1}}\left(M+1-\sqrt{\frac{1+\nu}{2}}\right). \quad (12.5)$$

The moment L_0 of the pressure force of the liquid on the plate with respect to the origin of the coordinate is expressed by the formula

$$L_0 = \frac{1}{2}\rho a^3 v_0 t^2 \sin\alpha\sqrt{\frac{M-\nu}{M+1}}\left[(M+1)\left(M-\frac{1-\nu}{2}\right) + \right.$$
$$\left. + (1-\nu)\sqrt{\frac{1+\nu}{2}}\right]. \quad (12.6)$$

The coordinate of the application of the equivalent force is defined by the formula

$$x_1 = \left(\frac{at}{2}\right)\frac{(M+1)\left(M-\frac{1-\nu}{2}\right)+(1-\nu)\sqrt{\frac{1+\nu}{2}}}{M+1-\sqrt{\frac{1+\nu}{2}}}. \quad (12.7)$$

Here x_1 is the coordinate of application of the equivalent force in the coordinate $x_0 0 y_0$. For submersion of the plate in the liquid, where the point E (the boundary of the wetted width of the plate surface) shifts along the surface with the speed of sound in the liquid ($M=1$), equations (12.5) and (12.7) assume the form

$$F = \rho a^3 \beta t (1-\nu)\sqrt{\frac{1-\nu}{2}}\left(2-\sqrt{\frac{1+\nu}{2}}\right),$$
$$x_1 = \frac{at}{2}\frac{\sqrt{2}(1+\nu)+(1-\nu)\sqrt{1+\nu}}{2\sqrt{2}-\sqrt{1+\nu}}. \quad (12.8)$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

If the submersion velocity of the plate is directed vertically downward ($\alpha=\pi/2$), then for M>1 the force will be

$$F=\rho a^2\beta tM\sqrt{\frac{M}{M+1}}\left(1+M-\frac{\sqrt{2}}{2}\right), \tag{12.9}$$

and the point of its application is

$$x_1=\frac{at}{2}\,\frac{(M+1)(2M-1)+\sqrt{2}}{2M+2-\sqrt{2}}. \tag{12.10}$$

Here

$$M=\frac{v_0}{a\beta}.$$

If introduction of the plate takes place at an angle $\alpha=\pi/2$ and M=1, then from formulas (12.9) and (12.10) we obtain

$$F=\frac{1}{2}\rho a^2\beta t(2\sqrt{2}-1),\quad x_1=\frac{at(\sqrt{2}+1)}{2(2\sqrt{2}-1)}.$$

For $\alpha=\pi/2$ from formula (12.3) we obtain

$$p=\frac{1}{\pi}\rho av_0\sqrt{\frac{\lambda+1}{\lambda}}\left[\frac{\lambda+1}{\sqrt{\lambda+1}}\operatorname{arctg}\sqrt{\frac{(1+x)/(1-x)-1}{\lambda+1}}-\right.$$
$$\left.-\operatorname{arctg}\sqrt{\frac{1+x}{1-x}-1}+\frac{\pi}{2}\right].$$

This pressure does not coincide with the pressure expressed by formula (11.15). The reason for this consists in the fact that in the problem of this section the trailing edge (point B) has streamline flow of the liquid smoothly around it, which led to the requirements of finiteness of the velocity at this point. If it is not required that the velocity be finite (which would be improper in this problem), then the formula in the plane z along the plate that follows would be obtained for the pressure:

$$p=\frac{\rho av_0}{\pi}\sqrt{\frac{\lambda+\mu}{\lambda}}\left[\frac{\lambda+1}{\sqrt{\lambda+\mu}}\operatorname{arctg}\sqrt{\frac{z-\mu}{\lambda+\mu}}+\right.$$
$$\left.+(1+\mu)\frac{1}{\sqrt{z-\mu}}\right]. \tag{12.11}$$

Here $\alpha=\pi/2$ after transition to the plane Oxy and replacement of λ by M according to formula (12.2) from formula (12.11) along the plate we obtain

$$p=\frac{2\rho av_0M}{\sqrt{M^2-1}}\operatorname{arctg}\sqrt{\frac{(M-1)x}{M(1-x)}},$$

which coincides with formula (11.15).

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b) Vertical Incidence of a Plate. Let at the time t=0 an absolutely rigid plate of width 2c with constant velocity v_0 directed vertically to the free surface, begin to penetrate an ideal compressible liquid occupying the lower halfspace. The penetration diagram is shown in Figure 1.33. For solution of the problem it is proposed that there is no leakage of the liquid to the upper surface of the plate. Accordingly, the problem of penetration by the plate is equivalent to introduction of a solid state with leading flat tip in the form of a rectangle of width 2c and length $\ell \gg 2c$. The Ox axis of the stationary coordinate system is directed along the horizontal free undisturbed surface, and the Oy axis, vertically downward. It is assumed that $v_0 \ll a$; consequently, the equations of motion of the liquid are linearized, and we obtain the wave equation for the potential. For linearization of the boundary conditions it is necessary to satisfy the inequality $v_0 t \ll 2c$ which will be valid if

$$0 < t < \frac{2c}{a}. \qquad (12.12)$$

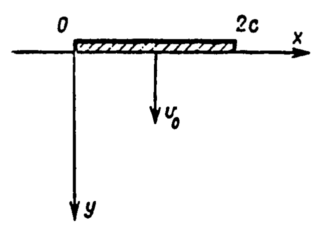

Figure 1.33

Thus, according to the inequality (12.12), we shall consider the time interval during which the boundary conditions on the plate and the free surface can be carried over to the horizontal plane. Since on the free surface of the liquid the excess pressure p=0, and the liquid is in a state of rest before entry ($\phi=\partial\phi/\partial t=0$), from the linearized Lagrange integral

$$p = -\rho \frac{\partial \varphi}{\partial t}$$

it follows that $\phi=0$ on the free surface. Let us introduce the dimensionless variables in the form

$$p' = \frac{p}{\rho a v_0}, \quad \varphi' = \frac{\varphi}{v_0 c}, \quad x' = \frac{x}{c},$$

$$y' = \frac{y}{c}, \quad t' = \frac{at}{c}$$

and in the following arguments let us stipulate omission of the strokes on the variables. Then for $0<t\leq 2$ the solution of the problem will be described by the equation

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = \frac{\partial^2 \varphi}{\partial t^2} \qquad (y>0) \qquad (12.13)$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

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with the following boundary and initial conditions:

$$\left.\frac{\partial\varphi}{\partial y}\right|_{y=0}=1 \quad \text{for } 0<x\leqslant 2, \tag{12.14}$$

$$\varphi|_{y=0}=0 \quad \text{for} \quad x<0 \text{ and } x>2,$$

$$\varphi=\frac{\partial\varphi}{\partial t}=0 \quad \text{for } t=0. \tag{12.15}$$

The Lagrange integral assumes the form

$$p=-\rho\frac{\partial\varphi}{\partial t}. \tag{12.16}$$

The system (12.13)-(12.15) for $t\leq 2$ can be solved by the above-presented method of self-similar coordinates and the Chaplygin transformation. Here the problem of determining the pressure directly will be solved. Obviously the pressure also satisfies the wave equation (12.2). Using equality (12.16), on the basis of conditions (12.14) and (12.15) we obtain the following boundary and initial conditions for the excess pressure:

$$y=0,\quad \frac{\partial p}{\partial y}=-\delta(t) \quad \text{for} \quad 0<x\leqslant 2,$$

$$p=0 \quad \text{for } x<0 \text{ and } x>2, \tag{12.17}$$

$$t=0,\quad p=\frac{\partial p}{\partial t}=0.$$

Here $\delta(t)$ is the Dirac delta function. The solution of this problem is presented in reference [23]. The pressure on the plate from the liquid in the time interval $0<t\leq 2$ is given by the formula

$$p=1-\frac{2}{\pi}\operatorname{arctg}\sqrt{\frac{t}{x}-1}\,H(t-x)-$$

$$-\frac{2}{\pi}\operatorname{arctg}\sqrt{\frac{t}{2-x}-1}\,H(t-2+x). \tag{12.18}$$

Here H is the unit Heviside function.

For the dimensionless force F_0 acting per unit length of the plate for $t\leq 2$, we obtain

$$\frac{F}{2c\rho a v_0}=F_0=1-\frac{t}{2}. \tag{12.19}$$

The formulas (12.18) and (12.19) are generalized to the case of penetration by a plate with variable velocity v(t), the initial value of which is v_0:

$$t=0,\quad v(0)=v_0.$$

The value of the pressure $\bar{p}$ for variable penetration velocity is expressed in terms of the pressure (12.18) using the Duamel integral

$$\bar{p}=\frac{d}{dt}\int_0^t p(t-\tau,\ x)V(\tau)\,d\tau,$$

here

$$V(t)=\frac{v(t)}{v_0}.$$

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

Differentiating the righthand side, we obtain

$$\bar{p} = p(t, x) + \int_0^t p(t-\tau, x) \frac{dV}{d\tau} d\tau.$$

Analogously, the force $\bar{F}$ with variable rate of submersion of the plate is defined in terms of the force of (12.19) using the Duamel integral by the formula

$$\frac{\bar{F}}{2c\rho a v_0} = \bar{F}_0 = F_0(t) + \int_0 F_0(t-\tau) \frac{dV}{d\tau} d\tau.$$

Let a plate of unit length have a mass equal to m. Then the equation of motion of the plate, the initial velocity of which is v_0, will be written as follows in dimensionless coordinates:

$$v_0 m \frac{a}{c} \frac{dV}{dt} = -2c\rho a v_0 \bar{F}_0.$$

Using the expression for the force $\bar{F}_0$, we obtain

$$\frac{dV}{dt} = -\varepsilon \left[F_0(t) + \int_0^t F_0(t-\tau) \frac{dV}{d\tau} d\tau \right]. \quad (12.20)$$

Here

$$\varepsilon = \frac{2\rho c^2}{m}.$$

If we introduce the new function $S = \int_0^t V(x)dx$, then from (12.20) we obtain the second order differential equation

$$\frac{d^2S}{dt^2} + \varepsilon \frac{dS}{dt} - \frac{\varepsilon}{2} S = 0 \quad (12.21)$$

with the initial condition

$$S = 0, \quad \frac{dS}{dt} = 1 \quad \text{for} \quad t = 0. \quad (12.22)$$

Solving this equation, for the velocity and acceleration of the plate we obtain the following expressions:

$$\frac{dS}{dt} = \frac{\varepsilon}{\lambda_2 - \lambda_1} \left[\left(1 - \frac{1}{2\lambda_1}\right) e^{\lambda_1 t} - \left(1 - \frac{1}{2\lambda_2}\right) e^{\lambda_2 t} \right], \quad (12.23)$$

$$\frac{d^2S}{dt^2} = \frac{\varepsilon}{\lambda_2 - \lambda_1} \left[\left(\lambda_1 - \frac{1}{2}\right) e^{\lambda_1 t} - \left(\lambda_2 - \frac{1}{2}\right) e^{\lambda_2 t} \right], \quad (12.24)$$

$$\lambda_{1,2} = -\frac{1}{2}\left(\varepsilon \mp \sqrt{\varepsilon^2 + 2\varepsilon}\right).$$

Investigating (12.24), we find that $d^2S/dt^2 = 0$ at the time

$$t_0 = \frac{2}{\sqrt{\varepsilon^2 + 2\varepsilon}} \ln\left(\sqrt{\varepsilon^2 + 2\varepsilon} + \varepsilon + 1\right),$$

$$t_0 < 2, \quad \frac{dt_0}{d\varepsilon} < 0.$$

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

Hence, it follows that before the diffraction wave from one edge reaches the other, the acceleration of the plate changes sign. Consequently, a negative pressure appears in the liquid on the surface of the plate at some time $t_1 < t_0$, the smooth streamlined flow is disturbed, and separation of the liquid can occur in this case.

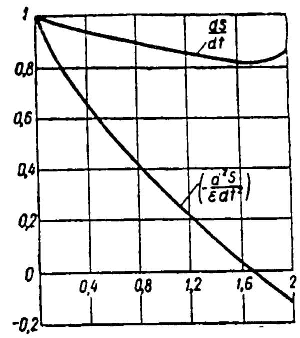

Figure 1.34

In Figure 1.34 the graph is presented for the variation of the force acting on the plate from the liquid and the variation of its velocity in dimensionless variables calculated by formulas (12.23), (12.24) [24] for $\varepsilon=0.2$. Let us note that in the case of $\varepsilon \to 0$ (a heavy plate) from (12.20) we obtain the acceleration $dV/dt \to 0$ and, consequently, the velocity $V \to 1$ -- penetration with constant velocity, that is, the previously investigated case.

In conclusion, let us note that the problem of incidence of a plate along the normal to the free surface was investigated in references [25, 26].

§13. Consideration of the Lift of the Free Surface on Penetration of a Compressible Liquid by a Blunt Wedge

Let us consider penetration of a compressible liquid by a blunt wedge considering the lift of the free surface. The conditions of the problem are the same as in §11. In the case where on penetration the Mach number on the faces of the wedge is subsonic, as was noted above, the free surface of the liquid is in motion. In this case the disturbed free surface on both sides of the wedge influences the movement of the liquid which occurs now inside the halfcircle of radius at.

Let us give the approximate method of considering the free surface on penetration by a blunt wedge with Mach numbers M_1 and M_2 less than one. The penetration rate is constant. As has already been noted above, the conditions and the restrictions given in §11 remain in force. The requirements justifying the transfer of the boundary conditions to the horizontal surface of the liquid initially at rest are also satisfied. During the penetration process the pressure on the free surface remains constant, equal to the initial pressure in the liquid. At the same time the pressure of the liquid in the vicinity of the apex of the penetrating wedge increases, that is, the pressure gradient increases in the direction of the free boundary. Since the gravitational force is not considered, under the effect of

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the pressure gradient with time the free surface will rise upward, also increasing the wetted part of the wedge. The movement is self-similar. The picture of the flow in the plane of the self-similar coordinate

$$x = \frac{x_0}{at}, \quad y = \frac{y_0}{at}$$

is illustrated in Figure 1.35.

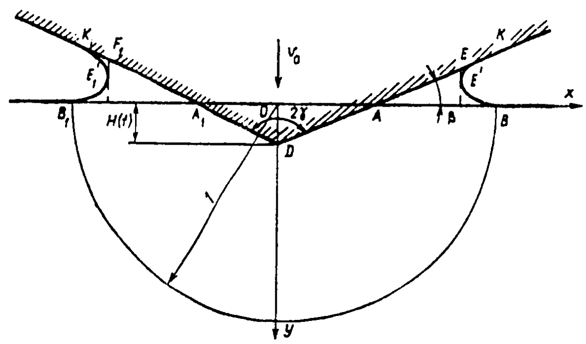

Figure 1.35

The sections E'EK and $E'_1E_1K_1$ of the region of motion of the liquid are called "spray." For the small values of the angles β and π-(2γ+β) investigated here the thickness of the spray with respect to the normal to the face of the wedge is on the order of Hβ, where $H=v_0t$ is the depth of penetration. In reference [2] it is demonstrated that when solving the problem of penetration it is possible to neglect the effect of the spray; then, approximately, the deflection point of the free surface E' coincides with the point E on the face of the wedge. The same thing pertains to the points E'_1 and E_1 on the left side of the pattern of motion (Figure 1.35). In this approximation the wetted sections of the wedge will be the sections DE and DE_1, but not the sections DA and DA_1 as assumed in §11. On the basis of self-similarity of the problem the displacement rates of the points E and E_1 along the horizontal surface of the liquid at rest (along the x-axis) will be constant. Let us denote these velocities by $\dot{c}$ and $\dot{c}_1$, respectively. Then in the plane of the self-similar coordinates xOy (Fig 1.36) the distances O_1E, O_2E_1 will be

$$O_1E = \frac{\dot{c}t}{at} = \frac{\dot{c}}{a} = M_1^0, \quad O_2E_1 = \frac{\dot{c}_1t}{at} = \frac{\dot{c}_1}{a} = M_2^0. \tag{13.1}$$

After determining the numbers M_1^0 and M_2^0 the problem is solved analogously to the solution of the problem presented in §11, c. The total force is calculated by integrating the pressure with respect to x in the interval $-\dot{c}_1t \leq x \leq \dot{c}t$. Thus, the problem is reduced to determining the velocity $\dot{c}$ and $\dot{c}_1$. Let us note that if the case of mixed penetration occurs, for example, $M_1>1$, $M_2<1$, the corresponding solution of the problem considering the effect of the free surface will be obtained from the solution of §11, b, replacing the Mach number M_2 in this solution by M_2^0 (the Mach number $M_1>1$ is left unchanged).

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Thus, the solution obtained according to (13.1) will contain two constants c and $\dot{c}_1$ as parameters subject to definition. In particular, the rate of lifting the particles of the free surface to the right of the point A of the face of the wedge -- v_y (just as the speed of the particles of the liquid boundary to the left of the point A_1) -- is a known function which depends on the parameters

$$v_y = v_y(x,\ M_1^0,\ M_2^0),\quad M_1^0 \leqslant x \leqslant 1,\quad y=0. \tag{13.2}$$

The coordinate of the particle e on the free boundary before the beginning of penetration which at the time t comes in contact with the wedge at the point E, is equal to $\dot{c}t$. That is, in the Oxy plane this point has the coordinates: $\dot{c}/a$, 0. Analogously, the point e_1 in this plane has the coordinates $-\dot{c}_1/a$, 0. The height of lifting of the particle e at the time it arrives at the point E of the face of the wedge is h, where

$$h = eE = \int_{t_0}^{t} v_y d\tau.$$

Here t_0 is the time of arrival of the disturbance at the point $e=at_0=\dot{c}t$. Therefore the preceding expression can be written as follows:

$$h = \int_{\frac{\dot{c}t}{a}}^{t} v_y\left(\frac{\dot{c}t}{a\tau},\ M_1^0,\ M_2^0\right) d\tau. \tag{13.3}$$

Making the substitution of the variables in the integral

$$x = \frac{\dot{c}t}{a\tau},$$

we obtain

$$h = \frac{\dot{c}t}{a}\int_{\frac{\dot{c}}{a}}^{1} v_y(x,\ M_1^0,\ M_2^0)\frac{dx}{x^2}. \tag{13.4}$$

The velocity of the particles of the free surface left of the point A_1 of the face of the wedge analogously to (13.2) is written in the form

$$v_y = v_y(x,\ M_1^0,\ M_2^0),\qquad -1 \leqslant x \leqslant -M_2^0.$$

The lift h_1 of the particle e_1 of the free boundary (Figure 1.36) is defined by the formula

$$h_1 = \int_{t_0}^{t} v_y d\tau,$$

where t_0 is the time of arrival of the disturbances of the point e_1 determined from the equality $at_0=\dot{c}_1t$. Thus, the lift h_1 can be represented by the integral

$$h_1 = \int_{\frac{\dot{c}_1}{a}t}^{t} v_y\left(-\frac{\dot{c}_1 t}{a\tau},\ M_1^0,\ M_2^0\right) d\tau. \tag{13.5}$$

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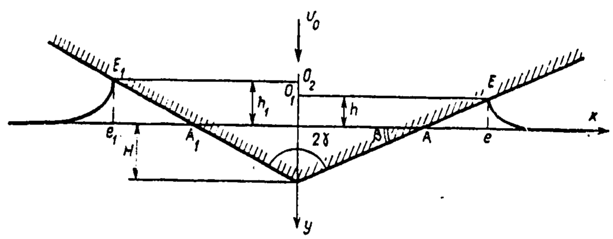

Figure 1.36

By substitution of the integration variable τ

$$x = \frac{\dot{c}_1 t}{a\tau}$$

the integral (13.5) is reduced to the form:

$$h_1 = \frac{\dot{c}_1 t}{a} \int\limits_{\frac{\dot{c}_1}{a}}^{1} v_y(-x,\ M_1^0,\ M_2^0)\,\frac{dx}{x^2}. \tag{13.6}$$

Obviously, the integrals in the righthand sides of (13.4) and (13.6) have positive real values.

From Figure 1.36 it is easy to be convinced of the correctness of the equalities:

$$v_0 t + h = \dot{c} t\,\mathrm{tg}\,\beta,$$
$$v_0 t + h_1 = \dot{c}_1 t\,\mathrm{tg}\,(2\gamma + \beta).$$

Hence, we obtain two equations for determining the constants $\dot{c}$ and $\dot{c}_1$:

$$v_0 + M_1^0 \int\limits_{M_1^0}^{1} v_y(x,\ M_1^0,\ M_2^0)\,\frac{dx}{x^2} = \dot{c}\,\mathrm{tg}\,\beta,$$
$$v_0 + M_2^0 \int\limits_{M_2^0}^{1} v_y(x,\ M_1^0,\ M_2^0)\,\frac{dx}{x^2} = \dot{c}_1\,\mathrm{tg}\,(2\gamma + \beta). \tag{13.7}$$

In the case of symmetric penetration

$$M_0 = M_1^0 = M_2^0;\quad \gamma = \left(\frac{\pi}{2} - \beta\right),\quad \dot{c} = \dot{c}_1$$

and system (13.7) is reduced to one equation for determining the velocity[1]

[1]In this case it is possible to limit ourselves only to the right half of the region of motion of the liquid.

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$v_0 + M_0 \int_{M_0}^{1} v_y(x, M_0) \frac{dx}{x^2} = \dot{c}\,\mathrm{tg}\,\beta,$$

$$M_0 = \frac{\dot{c}}{a}. \qquad (13.8)$$

For the given v_0, γ and β the system of equations (13.7) defines the velocities $\dot{c}$ and $\dot{c}_1$, and by formulas (13.1), the numbers M_1^0, M_2^0. For symmetric penetration at the wedge, the velocity of the liquid on the free surface is represented in the form

$$v_x = 0; \quad v_y = C \int_{1}^{\sqrt{\frac{\xi-1}{\xi-\lambda}}} \sqrt{\frac{\eta^2-1}{\lambda\eta^2-1}}\,d\eta, \quad \lambda < \xi \leqslant \infty. \qquad (13.9)$$

The value of ξ at the upper bound of the integral (13.9) is related to the dimensionless self-similar coordinate x by the equality

$$\xi = \frac{1}{1-x^2},$$

and the constants λ and C are defined as follows:

$$\lambda = \frac{1}{1-M_0^2} = \frac{1}{k_1^2},$$

$$C = -\frac{v_0}{k_1 E_1(k_1)}, \qquad (13.10)$$

where E_1 is the total second type elliptic integral with modulus k_1. Using (13.9) equation (13.8) establishes the following relation for determination of the velocity c:

$$\pi M = 2 M_0 E_1(k_1),$$

$$M = \frac{v_0}{a}\,\mathrm{ctg}\,\beta, \quad M_0 = \frac{\dot{c}}{a}. \qquad (13.11)$$

The function $E_1(k_1)$ is tabulated. Some values of the ratio

$$\frac{\pi}{2}\frac{v_0}{\dot{c}}\,\mathrm{ctg}\,\beta \cdot E_1(k_1)$$

and the numbers M_0 and M are presented in Table 1.1 for different values of k_1^2. In the case of $k_1^2=1$ (M=0) corresponding to an incompressible liquid, for the velocity $\dot{c}$ we have

$$\dot{c} = \frac{\pi}{2} v_0\,\mathrm{ctg}\,\beta,$$

which coincides with the result of §2. For $k_1=0$ (M=1) velocity $\dot{c}$ coincides with the velocity $V=v_0$ ctg β and it is equal to the speed of sound in the liquid:

$$\dot{c} = v_0\,\mathrm{ctg}\,\beta = a.$$

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Table 1.1

k_1^2	E_1	M_0	M
0,00	$\frac{\pi}{2}$	1	1
0,10	1,523	0,949	0,921
0,20	1,489	0,894	0,848
0,30	1,445	0,837	0,770
0,40	1,399	0,775	0,691
0,50	1,351	0,707	0,608
0,60	1,298	0,632	0,522
0,70	1,230	0,548	0,429
0,80	1,178	0,447	0,336
0,90	1,105	0,316	0,222
1,00	1	0,00	0

Considering the lift of the free surface of the liquid the formulas for excess pressure p_1 on the surface of the wedge and the force of resistance F_1 during symmetric penetration assume the form

$$p_1 = \frac{2}{\pi}\frac{\rho \dot{c}^2 \operatorname{tg}\beta M_0}{\sqrt{M_0^2 - x^2}}; \quad |x| < M_0; \tag{13.12}$$

$$F_1 = 2\rho(\dot{c})^2 \operatorname{tg}\beta t.$$

The ratio of the force F calculated without considering the effect of the lift of the free surface by formula (11.23) to the force F_1 is given by the equality

$$\frac{F}{F_1} = 4\left(\frac{E_1}{\pi}\right)^2. \tag{13.13}$$

For the limiting case of an incompressible liquid (M_0=0) and for the case where the value of $\dot{c}$ is equal to the speed of sound (M_0=1), this ratio is, respectively,

$$\frac{F}{F_1} = \frac{4}{\pi^2}, \quad \frac{F}{F_1} = 1. \tag{13.14}$$

§14. Penetration of a Compressible Liquid by a Blunt Wedge in the Nonlinear Statement

Just as in the problems of penetration by a blunt wedge in the linear statement, it is assumed here that the penetration rate is constant, less than the speed of sound in the liquid and directed vertically downward, that is, perpendicular to the free surface of the liquid at rest before beginning of penetration. A study is made of symmetric penetration where the faces of the wedge form an identical angle β with the free surface [19]. Let the rate V of displacement of the point A and C of the intersection of the faces of the wedge with the free surface be greater than the speed of sound in the undisturbed liquid:

$$V = v_0 \operatorname{ctg}\beta > a. \tag{14.1}$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

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Then on penetration of the wedge shock waves occur.

Let us consider the case where these waves are attached to points A and C. The pattern of the motion in this problem is shown in Figure 1.37.

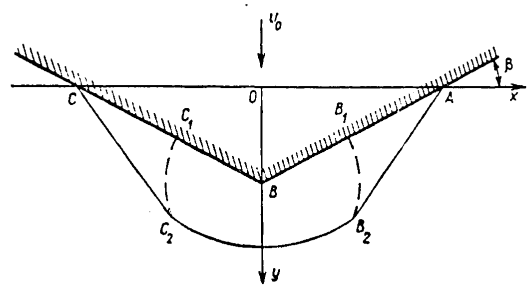

Figure 1.37

In this figure the line AB_2C_2C is the front of the formed shock wave. The dotted lines B_1B_2, C_1C_2 outline the region of diffraction at the apex of the wedge B behind the shock wave from the regions of disturbed motion behind the sections AB_2, CC_2 of the shock wave. The formation of shock waves at the points A and C during the penetration process takes place by the same law as for the wedge during its supersonic motion. Therefore the sections AB_2 and CC_2 of this shock wave which are influenced by the region of diffraction will be rectilinear. Then the values of the liquid parameters in the regions AB_1B_2A and CC_1C_2C are constant and are determined from the conditions on the shock wave and its continuity at the points A and C. These values define the boundary conditions in the sections B_1B_2 and C_1C_2 of the boundary of the region of diffraction. In the section of the wedge B_1BC_1 which is also part of the boundary of the diffraction region, the component of the liquid velocity normal to the surface of the wedge is given. Finally, the remaining section of the diffraction boundary B_2C_2 is part of the formed shock wave. In this previously unknown section of the boundary, the parameters of the liquid are related by the conditions at the shock front. This curvilinear section B_2C_2 of the shock wave must be defined during the solution of the problem. Obviously the motion that arises behind the shock wave will be self-similar. The origin of the coordinates is placed at the point of contact between the apex of the wedge and the free surface, the Oy axis is directed vertically with respect to the direction of penetration, the Ox axis, along the free surface to the right.

As was established in the first section of this chapter, the motion is described by the equation, the region of the imaginary characteristics of which is defined by the condition:

$$(v_x-\xi)^2+(v_y-\eta)^2<a^2, \tag{14.2}$$

where ξ, η are defined by the formulas $\xi=x/t$, $\eta=y/t$. The pattern of the motion in the ξ, η plane is illustrated in Figure 1.38. The region where the

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inequality (14.2) is satisfied coincides with the region of diffraction of the wedge angle [14, 19]. Let us denote the velocity components behind the rectilinear sections of the shock wave by z_{1x} and v_{1y}. On the basis of continuity of motion behind the wave, the equation of the lines B_1B_2 and C_1C_2 according to (14.2) will be written:

$$(v_{1x}, -\xi)^2 + (v_{1y} - \eta)^2 = a^2. \qquad (14.3)$$

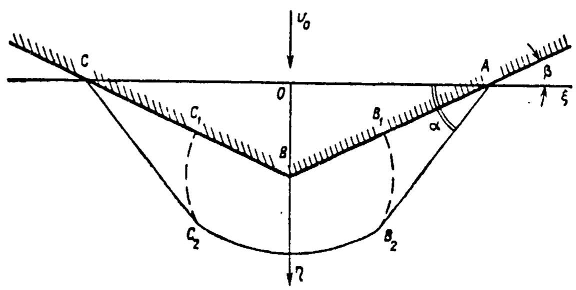

Figure 1.38

Since v_{1x}, v_{1y} are constants, according to (14.3) the lines C_1C_2, B_1B_2 are arcs of the corresponding circles. The problem stated here is equivalent to the problem of diffraction of a shock wave from the apex of a solid wedge investigated by many authors [14, 19, 23, 27, 28]. The results of the studies of the problem of penetration by blunt wedges in the linear statement performed above under the condition (14.1) and also the results of the studies of the related nonlinear problem of reflection of a shock wave from the apex of the wedge in the diffraction region lead to the conclusion: in the sections B_1B and C_1B of the faces of the wedge where the effect of diffraction is felt, there is a continuous decrease in the velocity and pressure from points B_1 and C_1 to the apex B. This phenomenon is similar to the motion of a compressible liquid behind a piston in a cylindrical tube having a bottom. Therefore we shall naturally try approximately to define the parameters of the flow in these sections of the wedge by the theory of uniform motion of a liquid [19]. The results of this approximation turned out to agree well with the results of the analytical and numerical calculations performed in reference [27], where an estimate of the method proposed here is presented.

On the basis of symmetry of the problem we shall limit ourselves to the study of the righthand half of the region of motion behind the wave. The parameters of the liquid behind the rectilinear section AB_2 of the shock wave are calculated simply. However, in order to obtain the approximate solution along the face of the wedge in the region of effect of the diffraction it is more convenient to vary the statement of the problem somewhat. Let us communicate a velocity V to the liquid-wedge system equal with respect to magnitude and opposite with respect to direction to the rate of displacement of the point A along the free surface. As a result, the shock wave AB_2 becomes stationary, and the wedge moves in the direction of its edge AB with a velocity

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$$v_2 = \frac{v_0}{\sin\beta} > V. \tag{14.4}$$

At some point in time the wetted part of the face of the wedge is v_2t. With this handling of the problem the speed of the liquid particle at the apex of the wedge is v_2.

Let α be the least angle of the oblique discontinuity with horizontal ρ_0, p_0 are the density and pressure in the undisturbed liquid. The parameters of motion behind the discontinuity will be indicated by the subscript "1." The equation of state of the liquid is considered to be known. Let us take it in the generally accepted form for water:

$$p - p_0 = \frac{k}{n}\left[\left(\frac{\rho}{\rho_0}\right)^n - 1\right], \tag{14.5}$$

where k and n are the experimental constants.

Behind the rectilinear part of the shock wave, outside the region of diffraction, the parameters are constant and are determined from conditions on the shock front (V is the velocity of the oncoming uniform flow):

$$\begin{gathered} \rho_0 V \sin\alpha = \rho_1 v_1 \sin(\alpha - \beta), \\ \rho_0 V \sin\alpha\,[V\sin\alpha - v_1 \sin(\alpha-\beta)] = p_1 - p_0, \\ V\cos\alpha = v_1\cos(\alpha-\beta), \\ p_1 - p_0 = \frac{k}{n}\left[\left(\frac{\rho_1}{\rho_0}\right)^n - 1\right]. \end{gathered} \tag{14.6}$$

From these relations and formula (14.1) we obtain

$$\begin{gathered} v_1^2 = \frac{v_0^2}{\operatorname{tg}^2\beta} - \frac{p_1}{\rho_0}\frac{\lambda+1}{\lambda}, \qquad \lambda = \frac{\rho_1}{\rho_0}, \\ \sin^2\alpha = \frac{\lambda}{\lambda-1}\,\frac{2\lambda\sin^2\beta}{2\lambda\sin^2\beta + (\lambda-1)\cos^2\beta \pm \cos\beta\sqrt{(\lambda-1)^2\cos^2\beta - 4\lambda\sin^2\beta}}, \\ p_1 - p_0 = \frac{\rho_0 v_0^2 2\lambda\cos^2\beta}{2\lambda\sin^2\beta + (\lambda-1)\cos^2\beta \pm \cos\beta\sqrt{(\lambda-1)^2\cos^2\beta - 4\lambda\sin^2\beta}}, \\ p_1 - p_0 = \frac{k}{n}[\lambda^n - 1]. \end{gathered} \tag{14.7}$$

The sign in front of the square root sign is taken from the condition that the angle of the shock wave is the least angle. The system of equations (14.7) defines the constant parameters behind the rectilinear shock wave to the boundary B_1B_2 of the region of diffraction.

Let us proceed to an approximate determination of the parameters of motion of the liquid with respect to the uniform theory along the face of the wedge in the region of diffraction [19].

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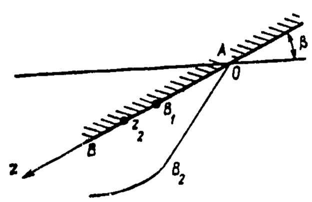

Figure 1.39

In the inverse problem the liquid particles approach the shock wave with a velocity V and then move along the face of the wedge at a velocity v_1 to the boundary of the region of diffraction B_1 on the wedge. Since at the apex of the wedge the velocity v_2 is greater than v_1, in the section B_1B rarefaction occurs. It is proposed that this rarefaction be described along the wall using plane uniform motion. Let us place the origin of the coordinate axis Oz at the point A of the free surface and direct it along the walls from A to B (Fig 1.39). Let us use the Riemann solution which in our notation is written as follows:

$$\frac{z}{t} = v - a,$$

$$v + \frac{2a}{n-1} = v_1 + \frac{2a_1}{n-1}, \qquad (14.8)$$

where a is the speed of sound in the disturbed liquid. On the rarefaction wave front

$$v = v_1; \quad a = a_1.$$

Therefore the boundary of the rarefaction front is defined by the equation

$$z_{B_1} = (v_1 - a_1) t = z_1. \qquad (14.9)$$

The cross section z_2 where the speed of the liquid reaches v_2 defined by formula (14.4) is found from the Riemann equations (14.8) which give

$$a_2 = a_1 + \frac{n-1}{2}(v_1 - v_2), \quad z_2 = \left[v_2 - a_2 - \frac{n-1}{2}(v_1 - v_2)\right] t. \qquad (14.10)$$

Let us note that the cross section z_2 cannot coincide with the apex of the wedge, for according to the formulas (14.10) this would mean that the speed of sound a_2 at the point z_2 is equal to zero, which is physicallyimpossible. Consequently, in the approximate uniform statement investigated here the medium expands in the interval from z_1 to z_2. From the cross section z_2 to the apex of the wedge $z_B = v_2 t$ the liquid parameters remain constant, equal to the value of these parameters at the boundary z_2. In the section from z_1 to z_2 the expansion process is defined by the equation:

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$$p - p_0 = \frac{k}{n}\left[\left(\frac{a}{a_0}\right)^{\frac{2n}{n-1}} - 1\right]; \quad \frac{z}{t} = v - a;$$
$$v + \frac{2a}{n-1} = v_1 + \frac{2a_1}{n-1}, \tag{14.11}$$

where a_0 is the speed of sound in the undisturbed liquid. Hence, for pressure in this region we have

$$p - p_0 = \frac{k}{n}\left[\left(\frac{n-1}{n+1}\cdot\frac{1}{a_0}\right)^{\frac{2n}{n-1}}\left(v_1 + \frac{2a_1}{n-1} - \frac{z}{t}\right)^{\frac{2n}{n-1}} - 1\right]. \tag{14.12}$$

For pressure in the cross section z_2 we obtain

$$p_{z_2} - p_0 = \frac{k}{n}\left[\left(\frac{a_2}{a_0}\right)^{\frac{2n}{n-1}} - 1\right], \quad a_2 = a_1 + \frac{n-1}{2}(v_1 - v_2). \tag{14.13}$$

Thus, the pressure in the section from the point A adjacent to the free surface to the boundary of the rarefaction region $z_1=(v_1-a_1)t$ is constant, equal to p_1 and determined from the system of equations (14.7). In the region between the boundary z_1 and $z_2=(v_2-a_2)t$ the pressure is defined by the formula (14.12), finally, from the boundary z_2 to the apex of the wedge $z_B=v_2t$, the pressure is again constant and defined by the formula (14.13). Let us note that the more exact, but complex calculations have confirmed good approximation which gives the calculation of the pressure along the face of the wedge by the simple formulas presented here [23, 27]. With respect to pressure it is easy to calculate the force acting on the wedge in the vertical direction for any point in time. This force F is

$$F = 2\sin\beta(\mathcal{F} + \mathcal{F}_1 + \mathcal{F}_2), \tag{14.14}$$

where the force $\mathcal{F}$ acts on the section $0z_1$ and is defined by the formula

$$\mathcal{F} = p_1(v_1 - a_1)t.$$

$\mathcal{F}_1$ is the force acting on the section z_1z_2 given in the form

$$\mathcal{F}_1 = \frac{kt}{n}\,\frac{n+1}{n-1}\,a_1\left\{\frac{n-1}{3n-1}\left[\left(\frac{a_1}{a_0}\right)^{\frac{2n}{n-1}} - \left(\frac{a_2}{a_0}\right)^{\frac{2n}{n-1}}\cdot\frac{a_2}{a_1}\right] + \frac{a_2}{a_1} - 1\right\}.$$

Finally, the force $\mathcal{F}_2$ in the section z_2z_B is

$$\mathcal{F}_2 = \frac{kt}{n}\left[\left(\frac{a_2}{a_0}\right)^{\frac{2n}{n-1}} - 1\right]a_2.$$

In these expressions it is possible to proceed to the densities by the formulas

$$\frac{a_2}{a_0} = (\rho_2/\rho_0)^{\frac{n-1}{2}}, \quad \frac{a_2}{a_1} = (\rho_2/\rho_1)^{\frac{n-1}{2}}, \quad \frac{a_1}{a_0} = (\rho_1/\rho_0)^{\frac{n-1}{2}}.$$

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

§15. Penetration of a Compressible Liquid by Blunt Three-Dimensional Bodies

Let us consider the three-dimensional problem of the penetration of a compressible liquid with a free surface by a rigid or elastic body. The initial velocity of the body v_0 is perpendicular to the plane boundary of the liquid, and penetration occurs in the direction of this velocity. Let the penetration velocity be much less than the speed of sound in the liquid and the ratio of the density ρ of the disturbed motion of the liquid to its initial density ρ_0 be insignificant:

$$\frac{v_0}{a} \ll 1, \quad \frac{\rho}{\rho_0} - 1 \ll 1. \tag{15.1}$$

Below, the initial liquid density is written without a subscript. Let V be the displacement rate of the point of intersection of the surface of the body with the surface of the liquid with respect to the free boundary

$$V = \frac{v_0}{\operatorname{tg} \beta} = v_0 \operatorname{ctg} \beta, \tag{15.2}$$

where β is the angle of inclination of the surface of the body to the free horizontal boundary of the liquid (Figure 1.40). When the velocity V is greater than the speed of sound a, that is,

$$M = \frac{V}{a} = \frac{v_0}{a} \operatorname{ctg} \beta > 1, \tag{15.3}$$

the contact area expands with respect to the free surface of the liquid at supersonic velocity, and consideration of the compressibility in the penetration problem is necessary. For example, the impact of a cylindrical body with flat front tip against the liquid surface can be considered as impact at $V \to \infty$. In order to obtain reliable results in this problem consideration of the compressibility of the liquid is mandatory. When determining the force effect on the penetrating body, the influence of the compressibility can be significant even for a ratio of the velocity V to the speed of sound in the liquid on the order of one [14, 19]. This requirement and the condition for the penetration velocity in formula (15.1) lead to the conclusion: the compressibility must be considered in the case where the angle β has an order much less than one ($\beta \ll 1$). Consequently, the depth of penetration of a blunt body will be small, and the conditions (15.1) justify the solution of the problem on the basis of the linearized equations of hydrodynamics.

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Here the boundary conditions are also linearized. Let us take the origin of the rectangular cartesian coordinate system at the point of contact of the body surface with the liquid boundary at the beginning of penetration; let us direct the Oz axis vertically downward, into the liquid, and let us place the Ox and Oy axes on the boundary plane of the liquid.

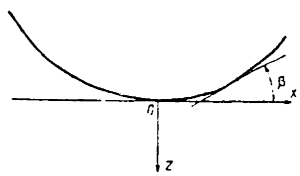

Figure 1.40.

As a result of linearization, the boundary conditions at the contact surface between the solid state and the liquid and the boundary conditions at the free surface of the liquid are carried over to the plane z = 0. On the surface z = 0, in the contact area, the liquid velocity component with respect to the z axis is taken equal to the velocity component of the body with respect to this axis. When solving the problem, the liquid is considered nonviscous, and the motion that arises is considered potential. The velocity potential ϕ satisfies the wave equation

$$\Delta\varphi = \frac{1}{a^2}\frac{\partial^2\varphi}{\partial t^2}. \tag{15.4}$$

The pressure in the liquid is defined by the linearized Cauchy-Lagrange equation

$$p = -\rho\frac{\partial\varphi}{\partial t}. \tag{15.5}$$

Here and hereafter p denotes the difference between the pressure of the disturbed motion of the liquid and the initial pressure p_0. The initial conditions in the liquid are written as follows (before beginning of penetration the liquid is at rest):

$$t = 0,\quad \varphi = 0,\quad \frac{\partial\varphi}{\partial t} = 0. \tag{15.6}$$

The boundary conditions on the plane z = 0 have the form

$$t > 0,\quad \frac{\partial\varphi}{\partial z} = v \text{ on } S,$$

$$z = 0\quad \varphi = 0 \text{ on } S_0. \tag{15.7}$$

Here v is the penetration velocity, S is the contact area on the plane z = 0, S_0 is the area of the plane z = 0 after subtracting the area S from it. In the general case, on introduction of a deformable body, the penetration velocity v will

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be a function of x, y, t. For a rigid body v is only a function of time t. In the case of penetration of a deformable body, it is necessary to add equations that describe the deformed and stressed states of the penetrating body to equations (15.4-15.7), and then we obtain the closed system for the solution of the problem. The velocity potential satisfying equation (15.4) and the conditions (15.6) and (15.7) in terms of the velocity component along the z axis on the surface z = 0 is written in the form [16, 20, 23]

$$\varphi(x, y, z, t) = -\frac{1}{2\pi}\iint \frac{\varphi_z\left(\xi, \eta, 0, t-\frac{R}{a}\right) d\xi\, d\eta}{R}, \qquad (15.8)$$

where

$$\varphi_z = \left(\frac{\partial\varphi}{\partial z}\right)_{z=0}, \quad R = \sqrt{(x-\xi)^2 + (y-\eta)^2 + z^2}. \qquad (15.9)$$

The velocity potential on the plane z = 0 for the entire time of motion is equal to zero everywhere except the points of the contact area S. Outside the contact area the excess pressure on this plane is zero. Since the derivative of ϕ_z (x, y, 0, t) is equal to zero for t < 0, the integration limits in the right-hand side of formula (15.8) can be extended to the entire plane ξ, η. Then the pressure in the liquid can be written as follows:

$$p = -\rho\frac{\partial\varphi}{\partial t} = \frac{\rho}{2\pi}\iint \frac{\varphi_z'\left(\xi, \eta, 0, t-\frac{R}{a}\right) d\xi\, d\eta}{R}, \qquad (15.10)$$

where the stroke denotes differentiation of ϕ_z with respect to the argument $\tau = t - (R/a)$. The force F(t) with which the body acts on the liquid is defined as follows:

$$F(t) = \iint_S p(x, y, 0, t)\, dx\, dy =$$
$$= \frac{\rho}{2\pi}\iint dx\, dy \iint \frac{\varphi_z'\left(\xi, \eta, 0, t-\frac{r}{a}\right) d\xi\, d\eta}{r}, \qquad (15.11)$$

where now $r = \sqrt{(x-\xi)^2 + (y-\eta)^2}$.

Independently of whether the boundary of the contact area expands with supersonic or subsonic velocity or, according to formula (15.3), independently of the inequalities $M \lesssim 1$, the pressure outside the area S on the plane z = 0 is zero. This means the integration with respect to S in formula (15.11) can be extended to the entire plane z = 0. Then it is possible to change the order of integration in formula (15.11), which gives

$$F(t) = \frac{\rho}{2\pi}\iint d\xi\, d\eta \iint \frac{\varphi_z'\left(\xi, \eta, 0, t-\frac{r}{a}\right) dx\, dy}{r}. \qquad (15.12)$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

The integration variables with respect to x and y will be replaced by integration with respect to r and θ, when

$$\theta = \operatorname{arctg}\frac{y-\eta}{x-\xi}. \qquad (15.13)$$

However, the expression under the integral sign in (15.12) does not depend on the angle θ. Using the equality dxdy = rdrdθ and the condition

$$\varphi_z\left(x, y, 0, t-\frac{r}{a}\right) = 0 \quad \text{for} \quad t-\frac{r}{a}<0, \qquad (15.14)$$

after integration with respect to θ, from (15.12) we obtain

$$F(t) = \rho \iint d\xi d\eta \int_0^{at} \varphi_z'\left(\xi, \eta, 0, t-\frac{r}{a}\right) dr. \qquad (15.15)$$

In the internal integral of the formula (15.15) let us proceed from integration with respect to r to integration with respect to τ = t - (r/a), and let us consider that the stroke of the function ϕ_z denotes differentiation with respect to τ. Then it is possible to calculate the internal integral. As a result, we have

$$F(t) = \rho a \iint \varphi_z(\xi, \eta, 0, t)\, d\xi d\eta, \qquad (15.16)$$

where integration extends to the entire plane z = 0. Formula (15.16) confirms that the force F(t) is expressed in terms of the instantaneous values of the velocity at the points of the surface z = 0 at the investigated point in time t. Here the pressure distribution at an arbitrary point in time depends on the velocities of the points on the surface z = 0 during the preceding points in time. When the number M defined by formula (15.3) is greater than 1 (M > 1) the disturbances in the liquid are limited to waves beginning with the supersonic edges of the penetrating body. The free boundary of the liquid outside the area S is at rest in this case, and, consequently, the expression under the integral sign in (15.16) outside the area S is equal to zero. The integral (15.16) for this case can be written in the form [20]

$$F(t) = \rho a S \bar{v}, \qquad (15.17)$$

where $\bar{v}$ is the average velocity of the body with respect to the contact area S. For a rigid body the velocity $\bar{v}$ is equal to the instantaneous velocity of the body v(t) at the time t. Let us note that the pressure distribution at the points of the contact surface is far from identical, but its mean value is equal to $\rho a\bar{v}$. For the subsonic value of the number M(M < 1) the disturbances in the liquid overtake the outline of the contact area S, and part of the free boundary of the liquid outside the area S turns out to be in a state of disturbed motion. Since the investigated disturbances are propagated with the speed of sound in a liquid at rest, the area S_1 of the disturbed region of motion of the free boundary of the liquid outside the contact area is

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$S_1 = \pi a^2 t^2 - S. \tag{15.18}$$

In this case (M < 1) outside the circle of radius at with center at the origin of the coordinates in the plane z = 0 the liquid boundary is in a state of rest, and the integral (15.16) can be written as follows:

$$F(t) = \rho a \iint_S \varphi_z(\xi, \eta, 0, t)\, d\xi\, d\eta + \\ + \rho a \iint_{S_1} \varphi_z(\xi, \eta, 0, t)\, d\xi\, d\eta. \tag{15.19}$$

Let us denote by $\bar{v}$ and $\bar{v}_1$ the mean values of the velocities in the areas S and S_1 of the plane z = 0; then from formula (15.19) we obtain

$$F = \rho a S \bar{v} + \rho a S_1 \bar{v}_1. \tag{15.20}$$

Under ordinary conditions $\bar{v}_1$ is negative, and the second term in the right-hand side of formula (15.20) expresses the effect of a decrease in the force of the effect of the liquid on the penetrating body caused by the effect of the free surface. For supersonic motion of the edges of the body over the surface of the liquid (M > 1) this effect is equal to zero: the free boundary outside the contact area has no influence on the penetration process.

The following sections discuss the solutions of specific problems.

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

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§16. Penetration of a Compressible Liquid by a Blunt Cone Considering the Lift of the Free Surface

A study was made of the problem of penetration of an ideal compressible liquid in a state of rest occupying the half-space $z \geq 0$ by a rigid cone at constant velocity v_0 with apex angle 2γ (see Figure 1.41) [29]. The velocity of the cone v_0 is directed along its axis perpendicular to the plane $z = 0$ and $v_0 \ll a$. In addition, it is assumed that $\beta \ll 1$ (a blunt cone), and the case where $v_0 \operatorname{ctg} \beta < a$ is considered. Obviously, the given problem will be axisymmetric and self-similar. As is easy to imagine from the physical picture of the flow, the free surface in the vicinity of the cone will be raised with the course of time, further increasing the wetted surface of the cone. For a correct statement of the problem it is necessary to consider the lift of the liquid, as a result of which, from smallness of the angle β, the wetted surface of the cone can increase significantly. Let us denote the unknown radius of this wetted surface by c. Let us neglect the effect of the spray, for with small β the vertical component of the momentum carried away by the spray will be small, where $\dot{c}$ = const, as follows from self-similarity of the problem. Here $\dot{c}$ is the expansion rate of the periphery of the wetted surface. When solving the hydrodynamic problem, we shall still consider the velocity $\dot{c}$ to be given. Then, linearizing the equations of motion of the liquid and the boundary conditions [7], for definition of the velocity vector $\bar{v} = \{v_r(t, r, z), v_z(t, r, z)\}$ and the pressure $p(t, r, z)$ we obtain the following system of equations of the initial and the boundary conditions:

$$\Delta\bar{v} = \frac{\partial^2 \bar{v}}{a^2 \partial t^2}, \quad \Delta p = \frac{\partial p^2}{a^2 \partial t^2} \quad \text{for} \quad z = 0,\ t > 0;$$

$$\bar{v} = \frac{\partial \bar{v}}{\partial t} = 0, \quad p = \frac{\partial p}{\partial t} = 0 \quad \text{for} \quad t = 0; \tag{16.1}$$

$$v_z = v_0 \quad \text{for} \quad z = 0,\ 0 \leqslant r < \dot{c}t; \quad p = 0 \quad \text{for} \quad z = 0,\ \dot{c}t < r,$$

where $\bar{v}$ and p are related by the linearized Euler equations. In addition, let us require that $p \to 0$ and $\bar{v} \to 0$ for $r^2 + r^2 \to \infty$, and let us require that p and $\bar{v}$ will be integral in the vicinity of the edge of the wetted surface of the cone $z = 0$, $r = \dot{c}t$ and that the apex of the cone $z = 0$, $r = 0$, which is necessary for uniqueness of the solution.

The solution of system (16.1) includes the parameter $\dot{c}$ which must be defined by the Wagner method [1] from the following kinematic expression which relates to

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

motion of a particle of the free surface of the liquid (r* in Figure 1.41) to the motion of the cone:

$$-\int_0^t v_z(\tau, \dot{c}t, 0)\, d\tau + v_0 t = \dot{c}t \operatorname{tg} \beta. \quad (16.2)$$

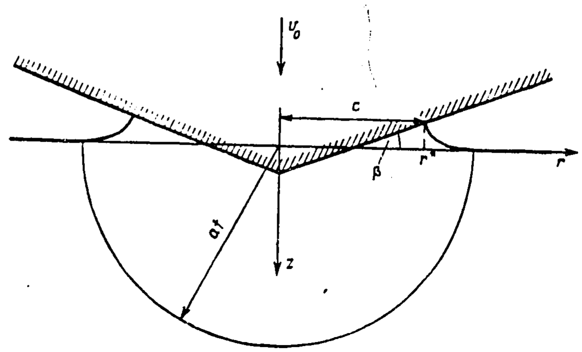

Figure 1.41.

In [30] a method is proposed for reducing the axisymmetric problems for the wave equation to two-dimensional problems, namely, representation of the solution of the axisymmetric problem in the form of a superposition of the solutions of the two-dimensional problems. Let us introduce the system of cartesian coordinates ξ, η, z rotated relative to the x, y, z system around the z axis by the angle ω:

$$\xi = x\cos\omega + y\sin\omega = r\cos(\varphi - \omega),$$
$$\eta = -x\sin\omega + y\cos\omega = r\sin(\varphi - \omega),$$

where r, ϕ, z are the cylindrical coordinate system related to the x, y, z system by the formulas

$$x = r\cos\varphi, \quad y = r\sin\varphi, \quad z = z.$$

In the coordinate system ξ, η, z let us consider the two-dimensional solution of the linearized equations of motion of an ideal liquid, that is, such that the velocity vector $\overline{v}_1$ and the pressure p_1 do not depend on η, satisfy the wave equation

$$\frac{\partial^2 f}{\partial \xi^2} + \frac{\partial^2 f}{\partial z^2} = \frac{\partial^2 f}{a^2 \partial t^2};$$

and the velocity vector $\overline{v}_1$ lies in the plane ξz: $\overline{v}_1 = \{v_{1\xi}, v_{1z}\}$. Then the expression

$$\overline{v} = \int_{-\pi}^{\pi} \overline{v}_1(t, \xi, z)\, d\omega \quad \text{and} \quad p = \int_{-\pi}^{\pi} p_1(t, \xi, z)\, d\omega \quad (16.3)$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

make up the solution of a three-dimensional problem for linearized equations of motion of an ideal liquid, for they are a superposition of the solutions of the equations of motion. Here the functions $\overline{v}$ and p will satisfy the wave equation with three spatial variables x, y, z. For the velocity components v_r, v_z, v_ϕ and for the pressure p we obtain the following expressions (making the substitution $\phi - \omega = \Omega$ and considering periodicity of the functions under the integral sign in (16.3) with respect to Ω):

$$\begin{aligned} v_r &= 2\int_0^\pi v_{1\xi}(t, r\cos\Omega, z)\cos\Omega\, d\Omega, \\ v_z &= 2\int_0^\pi v_{1z}(t, r\cos\Omega, z)\, d\Omega, \\ v_\varphi &= 0, \quad p = 2\int_0^\pi p_1(t, r\cos\Omega, z)\, d\Omega. \end{aligned} \tag{16.4}$$

Consequently, the velocity v and the pressure p do not depend on ϕ, that is, they are solutions of an axisymmetric problem for the linearized equations of motion of an ideal liquid and they satisfy the wave equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = \frac{\partial^2 f}{a^2\,\partial t^2}.$$

It is possible to show that the expressions (16.4) establish one-to-one correspondence between the solutions of the planar and the axisymmetric problems [31]. Since in the self-similar problem $\overline{v}$ and p are uniform functions of the coordinates and times of zero measurement, the two-dimensional solutions $\overline{v}_1$ and p_1 also must be uniform functions of the zero measurement and, consequently, by the method of functionally invariant solutions [30], they can be represented in the form

$$\begin{aligned} v_{1\xi}(t, \xi, z) &= \operatorname{Re} V(\theta), \quad v_{1z}(t, \xi, z) = \operatorname{Re} W(\theta), \\ p_1(t, \xi, z) &= \operatorname{Re} U(\theta), \end{aligned} \tag{16.5}$$

where $U(\theta)$, $V(\theta)$ and $W(\theta)$ are analytical functions in the region $\operatorname{Im}\theta > 0$, and the complex variable θ is defined explicitly from the equation

$$\delta \equiv t - \theta\xi - z\sqrt{a^{-2} - \theta^2} = 0, \tag{16.6}$$

where the branch of the radical $(a^{-2} - \theta^2)^{1/2}$ is fixed as follows: a section is made in the plane θ along the intervals of the real axis $(-\infty, a^{-1}]$ and $[a^{-1}, +\infty)$, and the value of the radical is considered positive for $\theta = 0$. Here the equation (16.6) maps the upper half-circle $z > 0$, $z^2 + \xi^2 < a^2t^2$ of the real plane ξ, z onto the upper half-plane of the complex variable θ, the half-circle $z^2 + \xi^2 = a^2t^2$ onto the segment of the real axis $[-a^{-1}, a^{-1}]$, and the segment $[-at, at]$ onto the remaining part of the real axis. The two-dimensional solution obtained inside the half-circle, according to [30], is continuously continued through the arc of the circle $z^2 + \xi^2 = a^2t^2$ to the outside of the half-circle along the half-tangents

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

defined by the equation (16.6), as a result of the fact that θ as a solution of (16.6) maintains a constant value along them (for $z > at$ we assume the two-dimensional solution to be identically equal to zero). Thus, the constructed two-dimensional solution is the generalized solution of the wave equation [30]. Substituting expressions (16.5) in (16.4), we obtain the following formulas for the velocity components v_r, v_z and the pressure p in the axisymmetric self-similar problem:

$$v_r = 2\mathrm{Re}\int_0^\pi V(\theta)\cos\Omega\, d\Omega, \quad v_z = 2\mathrm{Re}\int_0^\pi W(\theta)\, d\Omega,$$
$$p = 2\,\mathrm{Re}\int_0^\pi U(\theta)\, d\Omega, \tag{16.7}$$

where θ is explicitly defined from (16.6) for $\xi = r\cos\Omega$ or, according to the selected branch of the radical $(a^{-2} - \theta^2)^{1/2}$ and the choice of the half-tangents explicitly by the formulas:

$$\theta = \frac{atr\cos\Omega + iz\sqrt{a^2t^2 - z^2 - r^2\cos^2\Omega}}{a(r^2\cos^2\Omega + z^2)} \quad \text{for} \quad a^2t^2 > r^2\cos^2\Omega + z^2,$$
$$\theta = \frac{atr\cos\Omega \pm z\sqrt{z^2 + r^2\cos^2\Omega - a^2t}}{a(r^2\cos^2\Omega + z^2)} \quad \text{for} \quad a^2t^2 < r^2\cos^2\Omega + z^2,\ z < at, \tag{16.8}$$

where the + and - signs in the lower formula of (16.8) are taken, correspondingly, for $\cos\Omega < 0$ and $\cos\Omega > 0$, and the radicals in both formulas are considered arithmetic (for $z > at$, setting $\bar{v}_1 \equiv 0$, $p_1 \equiv 0$, in formulas (16.4), we obtain $v \equiv 0$, $p \equiv 0$). Here, since $\bar{v}_1$ and p_1 satisfy the linearized Euler equations

$$\frac{\partial v_{1z}}{\partial t} = -\frac{\partial p_1}{\rho\,\partial z}, \quad \frac{\partial v_{1\xi}}{\partial t} = -\frac{\partial p_1}{\rho\,\partial \xi},$$

the functions $U(\theta)$, $V(\theta)$, $W(\theta)$ are not independent. It is easy to see that the Euler equations will be satisfied if

$$V'(\theta) = U'(\theta)\frac{\theta}{\rho}, \quad W'(\theta) = U'(\theta)\sqrt{a^{-2} - \theta^2}/\rho. \tag{16.9}$$

Thus, the solution of the axisymmetric self-similar problem for $\bar{v}$ and p is found in the form of (16.7) where the functions $U(\theta)$, $V(\theta)$, $W(\theta)$ are regular in the region $\mathrm{Im}\theta > 0$, and they are related by the expressions (16.9), and θ is determined from (16.6) or (16.8).

When solving the investigated axisymmetric problem basically we shall follow the procedure proposed in [32] when solving similar elastic problems.

Differentiating the equations (16.7) for v_z and p with respect to t, for $z = 0$ we obtain:

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$\frac{\partial p}{\partial t} = 2\mathrm{Re}\int_0^\pi \frac{U'(\theta)}{r\cos\Omega}\,d\Omega, \quad \frac{\partial v_z}{\partial t} = 2\mathrm{Re}\int_0^\pi \frac{W'(\theta)}{r\cos\Omega}\,d\Omega. \qquad (16.10)$$

In the equations of (16.10), we introduce a new complex variable ν by the formula $\theta = \nu^{1/2}$, performing the section in the plane ν along the positive half-axis $[0, +\infty)$. Then the half-plane $\mathrm{Im}\theta > 0$ is mapped onto the plane ν with the section $[0, +\infty)$. Considering equation (16.9) relating $U'(\theta)$ and $W'(\theta)$, from (16.10) we obtain:

$$\mathrm{Re}\int_l \frac{F'(\nu)\,d\nu}{\sqrt{\nu-\nu_0}} = \frac{r}{2}\,\frac{\partial p}{\partial t}, \quad \mathrm{Re}\int_l \frac{F'(\nu)\sqrt{a^{-2}-\nu}}{\sqrt{\nu-\nu_0}}\,d\nu = \frac{r\rho}{2}\,\frac{\partial v_z}{\partial t}, \qquad (16.11)$$

where

$$\nu_0 = \frac{t^2}{r^2}, \quad U(\theta) = U(\nu^{\frac{1}{2}}) \equiv F(\nu)$$

and, consequently, $F(\nu)$ must be regular in the plane ν outside the section $[0, +\infty)$; for the branch of the radical $(a^{-2} - \nu)^{1/2}$, the section was taken along the interval of the positive half-axis $[a^{-2}, +\infty)$, and the radical is considered positive for $\nu = 0$; for isolation of a single-valued branch of the radical $(\nu - \nu_0)^{1/2}$ the section $[\nu_0, +\infty)$ was taken, and for $\nu = 0$ the argument of the radical is considered equal to $\pi/2$.

The outline ℓ (Figure 1.42) was obtained as follows. In Figure 1.43 the outline ℓ_0 is shown in the plane θ to which the integration path $[0, \pi]$ crosses from the formulas (16.7) for $z > 0$. For $z = 0$ ($z \to +0$) the ends of the outline ℓ_0 (the points L and K) will lie on the segment $[-a^{-1}, a^{-1}]$ symmetrically with respect to the origin $\theta = 0$ and, consequently, on replacement of $\theta = \nu^{1/2}$, the outline ℓ_0 becomes the outline ℓ which can be represented in the form of a circle of arbitrary radius R (on the basis of the analytical nature of $F(\nu)$ outside the section $[0, +\infty)$) and two sections of identical length KT' and LT, passed, as is indicated in Figure 1.42, along the lower and upper sides of the section $[0, +\infty)$, respectively, where the points K and L hit the same point $\nu = \nu_0$.

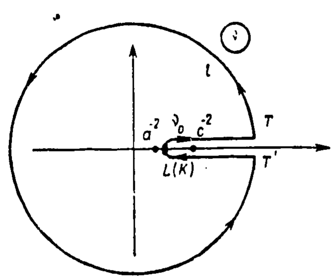

Figure 1.42.

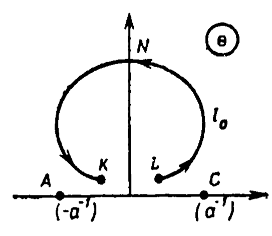

Figure 1.43.

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For satisfaction of the initial conditions in equations (16.11) it is necessary that it be possible for $\nu_0 < a^{-2}$ to contract the integration loop ℓ to one point, that is, the function $F'(\nu)$ must be analytical outside the section $[a^{-2}, +\infty)$. Since according to the boundary conditions, the expression for $\partial p/\partial t$ from (16.11) must vanish for $\nu_0 < \dot{c}^{-2}$, $F'(\nu)$ must be regular outside the section $[\dot{c}^{-2}, +\infty)$, and since $\partial v_z/\partial t$ from (16.11) must disappear for $\nu_0 > \dot{c}^{-2}$, accordingly, the integral function in this expression must be analytical for $\mathrm{Re}\,\nu > \nu_0 > \dot{c}^{-2}$, and it decreases at infinity just as $o(\nu^{-1})$ so that the integral with respect to the circle disappears at $R \to \infty$. Then it is possible to set

$$F'(\nu) = \frac{A(\nu)}{(\dot{c}^{-2} - \nu)^n},$$

where n is an integer, $A(\nu)$ is an integral analytical function which does not disappear at $\nu = \dot{c}^{-2}$. From the condition of integralness, the pressure on the edge of the wetted surface of the cone must be $n \leq 2$. Hence, it is easy to see that $A(\nu)$ must be limited: $A(\nu) = A = \text{const}$ and $n = 2$. Thus, we obtain

$$F'(\nu) = A(\dot{c}^{-2} - \nu)^{-2} \quad \text{and} \quad F(\nu) = A\nu\dot{c}^{-2}(\dot{c}^{-2} - \nu)^{-1} + C_1.$$

For $z = 0$ analogously to the expressions (16.11), we obtain:

$$p = \sqrt{\nu_0}\,\mathrm{Re}\int_l \frac{F(\nu)\,d\nu}{\nu\sqrt{\nu - \nu_0}};$$
$$v_z = \frac{\sqrt{\nu_0}}{\rho}\,\mathrm{Re}\int_l \frac{1}{\nu\sqrt{\nu - \nu_0}}\left[\int_0^{\nu} F'(\mu)\sqrt{a^{-2} - \mu}\,d\mu + C_2\right]d\nu. \qquad (16.12)$$

In (16.12) the integration with respect to μ is carried out by the loop lying to the same side of the real axis as the point $\mu = \nu$.

From (16.12) it follows that for satisfaction of the initial conditions it is necessary that the functions under the integral sign in (16.12) be analytical at the point $\nu = 0$, that is $C_1 = C_2 = 0$ and, consequently,

$$F(\nu) = A\nu\dot{c}^2(\dot{c}^{-2} - \nu)^{-1},$$

and the formulas (16.12) assume the form

$$p = \dot{c}^2\sqrt{\nu_0}\,\mathrm{Re}\int_l \frac{A\,d\nu}{(\dot{c}^2 - \nu)\sqrt{\nu - \nu_0}},$$
$$v_z = \frac{\sqrt{\nu_0}}{\rho}\,\mathrm{Re}\int_l \frac{A\,d\nu}{\nu\sqrt{\nu - \nu_0}}\int_0^{\nu}\frac{\sqrt{a^{-2} - \mu}}{(\dot{c}^{-2} - \mu)^2}\,d\mu. \qquad (16.13)$$

Now from (16.13) it is obvious that p actually vanishes for $\nu_0 < \dot{c}^{-2}$. Let us define the constant A from the boundary condition $v_z = v_0$ for $\nu_0 > \dot{c}^{-2}$, $z = 0$:

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$\frac{\sqrt{\nu_0}}{\rho}\,\mathrm{Re}\int_l \frac{A\,d\nu}{\nu\sqrt{\nu-\nu_0}}\cdot\int_0^{\nu}\frac{\sqrt{a^{-2}-\mu}}{(\dot{c}^{-2}-\mu)^2}\,d\mu = v_0,$$

where the integral with respect to μ can be represented in the form

$$\int_0^{\nu}\frac{\sqrt{a^{-2}-\mu}}{(\dot{c}^{-2}-\mu)^2}\,d\mu = \int_0^{-\infty}\frac{\sqrt{a^{-2}-\mu}}{(\dot{c}^{-2}-\mu)^2}\,d\mu + \int_{\infty}^{\nu}\frac{\sqrt{a^{-2}-\mu}}{(\dot{c}^{-2}-\mu)^2}\,d\mu = B + F_0(\nu).$$

Here in the second integral the integration is carried out along the ray $\arg\mu = \arg\nu$ and B is given by the expression

$$B = -\dot{c}\left[\gamma^{\frac{1}{2}} + (1-\gamma)^{-\frac{1}{2}}\arccos\left(\gamma^{\frac{1}{2}}\right)\right],\quad \gamma = \dot{c}^2 a^{-2}.$$

Since $F_0(\nu)$ changes sign on going through the section $[\dot{c}^{-2}, +\infty)$, then

$$\int_l \frac{F_0(\nu)\,d\nu}{\nu\sqrt{\nu-\nu_0}} = 0 \quad \text{for} \quad \nu_0 > \dot{c}^{-2},$$

and we obtain $v_0 = 2\pi AB\rho^{-1}$. Hence,

$$A = -\frac{v_0\rho\sqrt{1-\gamma}}{2\pi\dot{c}\left[\sqrt{\gamma(1-\gamma)} + \arccos\sqrt{\gamma}\right]}. \tag{16.14}$$

For the pressure distribution p on the wetted surface of the cone ($\nu_0 > \dot{c}^{-2}$) we obtain

$$p = \frac{v_0\rho\,\dot{c}^2 t\sqrt{1-\gamma}}{\sqrt{\dot{c}^2 t - r^2}\,\left[\sqrt{\gamma(1-\gamma)} + \arccos\sqrt{\gamma}\right]}, \tag{16.15}$$

where p has the integral singularity at $r = \dot{c}t$.

In order to obtain the pressure at any point of the half-space $z \geq 0$, we make the substitution of variable $\mu = (a^{-2} - \theta^2)^{1/2}$ in formula (16.7) for p and, considering the branch of the radical $(a^{-2} - \theta^2)^{1/2}$, we obtain for $a^2t^2 > r^2 + z^2$:

$$p = 2A\dot{c}^2\,\mathrm{Re}\int_{l_1}\frac{(za^{-2} - t\mu)\,d\mu}{(\dot{c}^{-2} - a^{-2} + \mu^2)\sqrt{r^2a^{-2} - t^2 + 2tz\mu - \mu^2(r^2+z^2)}}. \tag{16.16}$$

where the loop ℓ_1 in the region of $\mathrm{Re}\,\mu > 0$ is illustrated in Figure 1.44 and A is given by the formula (16.14). Here, at the points K and L we have: $\mu = \mu_1$ $\mu = \mu_2$, respectively, where μ_1 and μ_2 are the roots of the quadratic trinomial under the sign of the radical in (16.16)

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$\mu_{1,2} = \frac{atz \pm ri\sqrt{a^2t^2 - r^2 - z^2}}{a(r^2 + z^2)}. \tag{16.17}$$

For isolation of the single-valued branch of the radical in (16.16), a section is made in the plane μ from the point L to K (the dotted line) and the branch of the radical is taken such that its argument will be equal to π/2 for real values of μ greater than a^{-1}. Then considering that on different sides of the section LK the expression under the integral sign in (16.16) assumes opposite values with respect to sign but equal with respect to absolute magnitude and has two simple poles at the points

$$\mu = \pm i(\dot{c}^{-2} - a^{-2})^{\frac{1}{2}},$$

using the remainder theorem, from (16.16) we finally obtain:

$$p = -2\pi A\dot{c}^2 \left[t\sqrt{\frac{\sqrt{A_0^2 + B_0^2} - A_0}{2(A_0^2 + B_0^2)}} - \frac{z}{a^2\sqrt{\dot{c}^{-2} - a^{-2}}}\sqrt{\frac{\sqrt{A_0^2 + B_0^2} + A_0}{2(A_0^2 + B_0^2)}} \right]. \tag{16.18}$$

where

$$r^2 + z^2 < a^2t^2, \quad A_0 = z^2(\dot{c}^{-2} - a^{-2}) + \dot{c}^{-2}r^2 - t^2,$$
$$B_0 = 2tz(\dot{c}^{-2} - a^{-2})^{\frac{1}{2}},$$

and the radicals are considered arithmetic. In particular, for $z \to 0$ and $r > \dot{c}t$ the expression (16.18) vanishes, and for $z \to 0$ and $r < \dot{c}t$ it coincides with the expression (16.15). Let us note that the expression (16.18) disappears for $r^2 + z^2 \to a^2t^2$, and for $\dot{c} \to a$ from (16.14) we find $A \to -v_0\rho/4\pi a$, and expression (16.18) gives at the limit:

$$p = \frac{v_0\rho a^2 t}{2} \frac{a^2t^2 - r^2 - z^2}{(a^2t^2 - r^2)^{\frac{3}{2}}}. \tag{16.19}$$

Formula (16.19) coincides with the analogous limit formula obtained from solution of the problem of penetration of a compressible liquid by a blunt cone at $\dot{c} = v_0 \operatorname{ctg} \beta > a$, when $v_0 \operatorname{ctg} \beta \to a$ (see the following section).

Analogously, it is possible to calculate the velocity components v_z and v_r in the region $r^2 + z^2 < a^2t^2$. Let us note that for $r^2 + z^2 > a^2t^2$ ($z < at$) all the functions v_z, v_r and p vanish, for the points K and L of the ends of the integration loop ℓ_1 in the plane μ are incident at the same point lying on a segment of the real axis $[0, a^{-1}]$ in the plane μ (since in the plane θ they lie on the segment

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$[-a^{-1}, a^{-1}]$ symmetrically with respect to the point $\theta = 0$) and, consequently, by the Cauchy theorem the integrals over the closed loop ℓ_1 vanish.

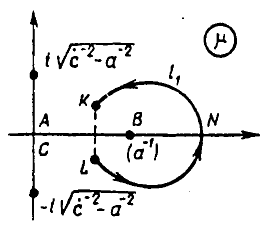

Figure 1.44.

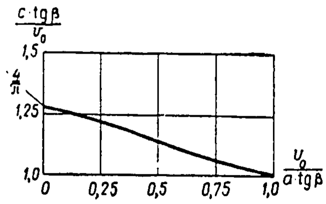

Figure 1.45.

Now let us define the radius of the wetted surface $c = \dot{c}t$, substituting the expression for v_z from (16.13) in equation (16.2). Then after some transformations equation (16.2) assumes the form

$$-\frac{A\dot{c}}{\rho_\cdot}\int_\gamma^1 d\tau\int_\gamma^\tau \frac{d\mu}{\mu\sqrt{\tau-\mu}}\int_\gamma^\mu \frac{\sqrt{\nu-\gamma}\,d\nu}{(1-\nu)^2}+v_0=\dot{c}\,\mathrm{tg}\,\beta, \qquad (16.20)$$

where the radicals are considered to be arithmetic. After integration we finally obtain the following expression relating $v_0 a^{-1}$ ctg β and γ:

$$\gamma+\frac{\sqrt{\gamma}\arccos\sqrt{\gamma}}{\sqrt{1-\gamma}}=\frac{2v_0}{a}\,\mathrm{ctg}\,\beta, \qquad (16.21)$$

where $\gamma = \dot{c}^2/a^2$.

The numerical solution of equation (16.21) is presented in Figure 1.45. It is obvious that $\dot{c}$ depends on a for a constant value of v_0 ctg β and $\dot{c} \to 4v_0(\mathrm{ctg}\ \beta)/\pi$ for $v_0(\mathrm{ctg}\ \beta)/a \to 0$ (the case of an incompressible liquid) and when $v_0(\mathrm{ctg}\ \beta)/a \to 1$, then $\dot{c} \to v_0$ ctg $\beta \to a$, and the free surface of the liquid outside the cone remains undisturbed, which corresponds to the physical picture of the flow.

Returning to formula (16.15), we note that if we consider the parameter $\dot{c}$ given, independent of a, then the pressure according to formula (16.15) depends on a in contrast to the other limiting case--penetration of a compressible liquid by a thin cone with subsonic velocity where the pressure of the cone does not depend on a [23].

In the case of an incompressible liquid the formula for the pressure on the surface of the blunt cone (§3) is obtained from (16.15) for $a \to \infty$:

$$p=\frac{2v_0\rho\dot{c}}{\pi}\left[1-\left(\frac{r}{\dot{c}t}\right)^2\right]^{-\frac{1}{2}}, \qquad (16.22)$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

where $\dot{c}t$ just as in (16.15), is the radius of the wetted surface of the cone. Formula (16.15) differs from (16.22) by the presence of the factor

$$2^{-1}\pi[\gamma^{\frac{1}{2}}+(1-\gamma)^{-\frac{1}{2}}\arccos(\gamma^{\frac{1}{2}})]^{-1},$$

which varies within the limits from $\pi/4$ to 1 and, consequently, for the same values of v_0, r, t and $\dot{c}$ the pressure in the compressible liquid is less than the pressure in the incompressible liquid.

If in the formula (16.15) $\dot{c}$ is given by equation (16.21), for the pressure on the surface of the blunt cone we obtain the formula:

$$p=\frac{\rho\dot{c}^2\,\mathrm{tg}\,\beta}{2}\left[1-\left(\frac{r}{\dot{c}t}\right)^2\right]^{-\frac{1}{2}}. \qquad (16.23)$$

If we do not consider the lift of the free surface in the compressible liquid, then $\dot{c}=v_0$ ctg β, and the formula (16.15) assumes the form

$$p=\frac{v_0^2\rho}{\mathrm{tg}\,\beta}\left[1-\left(\frac{r\,\mathrm{tg}\,\beta}{v_0t}\right)^2\right]^{-\frac{1}{2}}\times$$
$$\times\left\{\frac{v_0}{a\,\mathrm{tg}\,\beta}+\left[1-\left(\frac{v_0}{a\,\mathrm{tg}\,\beta}\right)^2\right]^{-\frac{1}{2}}\arccos\frac{v_0}{a\,\mathrm{tg}\,\beta}\right\}^{-1}. \qquad (16.24)$$

For the case of an incompressible liquid considering lift of the free surface, that is, according to (3.36) for $\dot{c}=4v_0(\mathrm{ctg}\,\beta)/\pi$, the pressure distribution on the cone is given by the expression

$$p=\frac{8v_0^2\rho}{\pi^2\,\mathrm{tg}\,\beta}\left[1-\left(\frac{\pi r\,\mathrm{tg}\,\beta}{4v_0t}\right)^2\right]^{-\frac{1}{2}}, \qquad (16.25)$$

and without considering the lift of the liquid, by formula (16.22), where $\dot{c}=v_0$ ctg β:

$$p=\frac{2v_0^2\rho}{\pi\,\mathrm{tg}\,\beta}\left[1-\left(\frac{r\,\mathrm{tg}\,\beta}{v_0t}\right)^2\right]^{-\frac{1}{2}}. \qquad (16.26)$$

Using formulas (16.23)-(16.26), the forces acting on the cone will be defined by the following relations:

For a compressible liquid considering the lift of the surface

$$F_1=\pi\rho(\mathrm{tg}\,\beta)\dot{c}^4t^2, \qquad (16.27)$$

For a compressible liquid without considering the surface lift

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$F_2 = \frac{2\pi v_0^4 \rho t^2}{\operatorname{tg}^3\beta \left[\frac{v_0}{a \operatorname{tg}\beta} + \frac{\arccos\left(\frac{v_0}{a \operatorname{tg}\beta}\right)}{\sqrt{1-\left(\frac{v_0}{a \operatorname{tg}\beta}\right)^2}} \right]}, \tag{16.28}$$

For incompressible liquid considering the lift of the surface (see formula 3.38)

$$F_3 = \pi \operatorname{tg}\beta \left(\frac{4v_0}{\pi \operatorname{tg}\beta}\right)^4 \rho t^2, \tag{16.29}$$

For incompressible liquid without considering the surface lift (see formula 3.29)

$$F_4 = 4v_0^4 \rho t^2 \operatorname{ctg}^3\beta. \tag{16.30}$$

Then, referring to F_4, we have

$$\text{I)}\ \frac{F_1}{F_4} = \frac{\pi}{4}\left(\frac{\dot{c} \operatorname{tg}\beta}{v_0}\right)^4,$$

$$\text{II)}\ \frac{F_2}{F_4} = \frac{\pi}{2}\left\{\frac{v_0}{a \operatorname{tg}\beta} + \left[1-\left(\frac{v_0}{a \operatorname{tg}\beta}\right)^2\right]^{-\frac{1}{2}} \arccos\frac{v_0}{a \operatorname{tg}\beta}\right\}^{-1},$$

$$\text{III)}\ \frac{F_3}{F_4} = \left(\frac{4}{\pi}\right)^3, \quad \text{IV)}\ \frac{F_4}{F_4} = 1.$$

The graphs of the functions (I-IV) of $v_0(\operatorname{ctg}\beta)/a$ are presented in Figure 1.46.

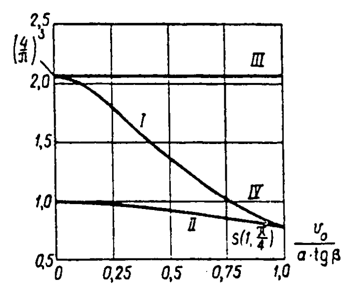

Figure 1.46.

It is necessary to note that the difference in the hydrodynamic forces (I-IV) acting on the surface of the penetrating cone in four different cases arises not only from the difference in pressures, but also the difference in areas of wetted surfaces. From these graphs it is obvious that the behavior of the force is described most exactly and physically correctly by curve (I), which for $v_0(\operatorname{ctg}\beta)/a \to 0$ gives the case of an incompressible liquid considering lift of the free surface (III), and for $v_0(\operatorname{ctg}\beta)/a \to 1$, when $\dot{c} \to a$, it gives a result which coincides with the analogous limiting result obtained for the case $\dot{c} = v_0 \operatorname{ctg}\beta > a$

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for v_0 ctg $\beta \to a$ (the point S(1, $\pi/4$)). In the last, limiting case the theory of an incompressible liquid considering lift of the liquid (graph III) gives an increase in the indicated limiting result by approximately 2.6 times. Consequently, for a more exact description of the hydrodynamic forces for not very small values of the parameter $v_0(\text{ctg } \beta)/a$ by comparison with one, it is necessary to use the curve (I) which is suitable for the entire range of variation of the parameter $v_0(\text{ctg } \beta)/a$ from zero to one.

Obviously, the expressions obtained for the pressure and the forces acting on the penetrating cone according to formulas (16.27)-(16.30) will also be valid for a cone of finite height to the time t* when the free surface of the liquid reaches its base. Let us denote the radius of the base of the cone of finite height in terms of r_0. Then the time t = t* can be determined from the equality $\dot{c}t^* = r_0$. In this case for a cone of finite height penetrating a compressible liquid, considering the lift of the free surface, the magnitude of the force from (16.27) after transformations will be

$$F_1 = 2\left(\frac{\dot{c}}{v_0}\right)^2 \text{tg}\,\beta \frac{\rho v_0^2}{2} \pi r_0^2. \tag{16.31}$$

For an incompressible liquid considering the lift of the free surface it is possible to write formula (3.36) of the present chapter:

$$\dot{c}t^* = \frac{4}{\pi} v_0 t^* \text{ctg}\,\beta = r_0,$$

then

$$t^* = \frac{\pi}{4} \frac{\text{tg}\,\beta}{v_0} r_0.$$

Thus, for t = t* from (16.29) the value of the force is

$$F_3 = \frac{32}{\pi^2 \text{tg}\,\beta} \frac{\rho v_0^2}{2} \pi r_0^2. \tag{16.32}$$

Let us introduce the resistance coefficients C_1 and C_3: for compressible liquid

$$C_1 = 2\left(\frac{\dot{c}}{v_0}\right)^2 \text{tg}\,\beta$$

and incompressible liquid

$$C_3 = \frac{32}{\pi^2 \text{tg}\,\beta}.$$

In Figure 1.47 values are presented for the parameter C_1 tg β and C_3 tg β as a function of the M number:

$$M = \frac{v_0}{a\,\text{tg}\,\beta}.$$

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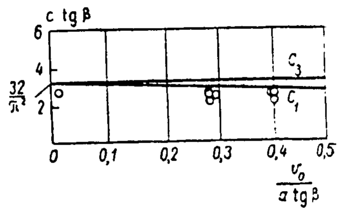

Figure 1.47.

For the number M = 0.5, the ratio $C_1/C_3 = 0.8$; for M = 1 the ratio $C_1/C_3 = 0.61$.

The dots on this curve denote the experimental data for the cone $\beta = 10°$ for $t \approx t^*$.

The experimental results, as is obvious from the figure, are closer to the theoretical curve--the model of a compressible liquid. The deviation will be up to 15 percent.

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

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§17. Penetration of a Compressible Liquid by a Blunt Cone for V > a

A study was made of the problem of the penetration of an ideal compressible liquid at rest occupying the entire lower half-space by a very blunt cone of circular cross section [7, 23]. The velocity of the cone v_0 is assumed to be constant, less than the speed of sound in the liquid $v_0 < a$ and directed downward, perpendicular to the free horizontal surface. It is proposed that the cone is so blunt that as it is introduced the periphery of the intersection of the cone with the free surface shifts over the surface at a velocity greater than the speed of sound in the liquid. Under this condition the shock wave arises which cuts off the region of disturbed motion from the liquid at rest. It is proposed that the shock wave does not depart from the line of intersection of the surface of the cone with the free undisturbed surface of the liquid. The shock wave front will be an axisymmetric surface. Since the penetration pattern has axial symmetry, hereafter we shall limit ourselves to investigation of the motion in the meridional plane Ox_0y_0 (Figure 1.48).

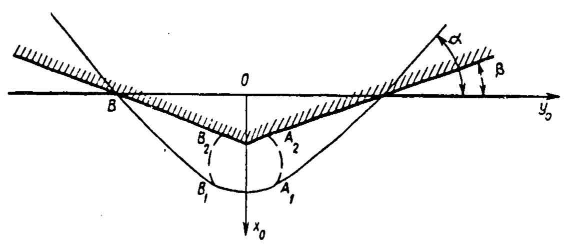

Figure 1.48.

The origin of the cartesian coordinate system Ox_0y_0 is taken at the point of contact of the apex of the cone with the free surface of the liquid at the time t = 0 (the beginning of penetration). In Figure 1.48 the line AA_1B_1B corresponds to the front of the formed shock wave. The horizontal free surface of the liquid to the left of the point A and to the right of the point B for the given statement of the problem has no influence on the region of disturbed motion of the liquid. The points A and B are shifted along the free surface at a velocity $V = v_0 \operatorname{ctg} \beta$. The angle of inclination of the shock wave front α to horizontal at the point A is determined from the law of formation of a plane shock wave. Indeed, the shock wave front beginning with the points A and B will not be rectilinear, for penetration

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

of an axisymmetric configuration takes place. Here a study is made of the problem in the linear statement. The penetration of the cone in the nonlinear statement of the problem will be considered below. It is proposed that the penetration takes place under the condition

$$v_0 \ll a, \tag{17.1}$$

$$V = v_0 \operatorname{ctg} \beta > a. \tag{17.2}$$

The region of disturbed motion on penetration by this cone will be cut off from the liquid at rest by the line of the weak wave consisting of the straight lines AA_1 and BB_1 tangent to the circle of radius at with its center at the origin of the coordinates and the arc A_1B_1 of this circle. In the plane of self-similar coordinates $x = x_0/at$, $y = y_0/at$, this region is found in Figure 1.49. The indicated circle is the boundary of the region where diffraction from the apex of the cone is felt. From the conditions (17.1) and (17.2) we find that $\beta \ll 1$ and, consequently, it is possible to carry over the boundary conditions to the plane $x_0 = 0$. Then the solution of the problem by definition of the potential of the disturbed motion of the liquid in the cylindrical coordinates x_0, y_0, θ reduces to solution of the equation

$$\frac{\partial^2\varphi}{\partial x_0^2} + \frac{\partial^2\varphi}{\partial y_0^2} + \frac{1}{y_0}\frac{\partial\varphi}{\partial y_0} = \frac{1}{a^2}\frac{\partial^2\varphi}{\partial t^2} \tag{17.3}$$

under the following boundary and initial conditions:

$$v_n = \frac{\partial\varphi}{\partial n} \approx \frac{\partial\varphi}{\partial x_0}\bigg|_{x=0} \approx v_0 \quad \text{for} \quad 0 \leqslant y_0 \leqslant y_0^*,\ 0 \leqslant \theta \leqslant 2\pi, \tag{17.4}$$

$$v_n = \frac{\partial\varphi}{\partial x_0}\bigg|_{x=0} = 0 \quad \text{for} \quad y_0 > y_0^*,\ 0 \leqslant \theta \leqslant 2\pi,$$

$$\varphi = \frac{\partial\varphi}{\partial t} = 0 \quad \text{for} \quad t = 0, \tag{17.5}$$

here $y_0^*(t) = V \cdot t \simeq v_0 t/\beta$, for $\operatorname{tg} \beta \simeq \beta$.

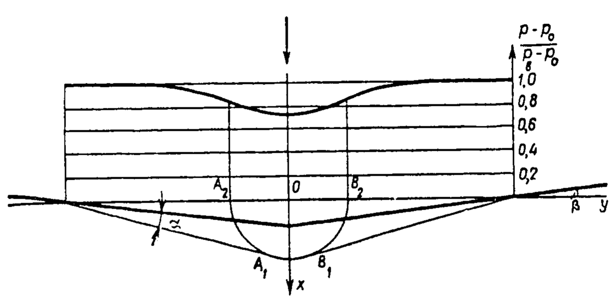

Figure 1.49.

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

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The solution of the system (17.3)-(17.5) by the method of the delaying potential in the investigated case is written in the form

$$\varphi(x_0, y_0, t) = -\frac{1}{2\pi}\int_0^{2\pi} d\theta \int_0^{y^*(t)} \frac{v_n(t', \zeta)\zeta\, d\zeta}{\sqrt{x_0^2 + y_0^2 + \zeta^2 - 2y_0\zeta\cos\theta}}, \tag{17.6}$$

where

$$t' = t - \frac{1}{a}\sqrt{x_0^2 + y_0^2 + \zeta^2 - 2y_0\zeta\cos\theta}.$$

Since $v_n = v_0$, then

$$\frac{\partial\varphi}{\partial t} = -\frac{v_0}{2\pi}\int_0^{2\pi} \frac{y_0^* \frac{dy_0^*}{dt}\, d\theta}{\sqrt{x_0^2 + y_0^2 + (y_0^*)^2 - 2y_0 y_0^* \cos\theta}}, \tag{17.7}$$

and y_0^* is determined from the formula

$$y_0^* = \frac{v_0}{\beta}\left[t - \frac{1}{a}\sqrt{x_0^2 + y_0^2 + (y_0^*)^2 - 2y_0 y_0^* \cos\theta}\right].$$

Hence,

$$y_0^* = -\frac{M}{M^2-1}\Big[at - My_0\cos\theta \pm \pm\sqrt{(at - My_0\cos\theta)^2 + (a^2t^2 - x_0^2 - y_0^2)(M^2-1)}\Big], \tag{17.8}$$

where $M = (v_0/a\beta) > 1$.

Differentiating (17.8) with respect to time, we obtain

$$\frac{dy_0^*}{dt} = -\frac{Ma}{M^2-1}\left[1 \pm \frac{M(Mat - y_0\cos\theta)}{\sqrt{(at - My_0\cos\theta)^2 + (a^2t^2 - x_0^2 - y_0^2)(M^2-1)}}\right]. \tag{17.9}$$

In the dimensionless variables $x = x_0/at$ and $y = y_0/at$ these formulas assume the form

$$\frac{y_0^*}{at} = -\frac{M}{M^2-1}[1 - My\cos\theta \pm R], \tag{17.10}$$

$$\frac{dy_0^*}{a\,dt} = -\frac{M}{M^2-1}\left[1 \pm \frac{M(M - y\cos\theta)}{R}\right], \tag{17.11}$$

where

$$R = \sqrt{(1 - My\cos\theta)^2 + (1 - x^2 - y^2)(M^2-1)}.$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

Let us introduce the following two integrals into the investigation:

$$Q_1 = \int_0^{\pi} \frac{1 - My\cos\theta}{R}\, d\theta, \quad Q_2 = 2\int_0^{\gamma} \frac{1 - My\cos\theta}{R}\, d\theta,$$

where

$$\gamma = \arccos \frac{1 + \sqrt{(1 - x^2 - y^2)(1 - M^2)}}{My}.$$

A simple investigation of the solution of (17.6) shows that in the region of disturbed flow the (excess) pressure $p = -\rho\partial\phi/\partial t$ is expressed by the formulas:

$$p = \rho\frac{M^2 a v_0}{\pi(M^2 - 1)}(\pi - Q_1) \quad \text{for} \quad x^2 + y^2 < 1,$$

$$p = -\rho\frac{M^2 a v_0}{\pi(M^2 - 1)} Q_2 \quad \text{for} \quad x^2 + y^2 > 1. \qquad (17.12)$$

At the point B (x = 0, y = M) behind the reflected wave the pressure is defined as

$$p_B = \frac{\rho a v_0 M}{\sqrt{M^2 - 1}}.$$

Taking this into account, it is possible to represent formula (17.12) in the following form:

$$\frac{p}{p_B} = \frac{M}{\pi\sqrt{M^2 - 1}}(\pi - Q_1) \quad \text{for} \quad x^2 + y^2 < 1,$$

$$\frac{p}{p_B} = -\frac{M}{\pi\sqrt{M^2 - 1}} Q_2 \quad \text{for} \quad x^2 + y^2 > 1. \qquad (17.13)$$

In Figure 1.49, the pressure distribution along the generatrix of the cone calculated by formulas (17.13) is presented for M = 3.73. According to §15, formula (15.17), the force acting on the cone in the vertical direction is

$$F = \pi\rho a v_0^3 (\mathrm{ctg}^2\beta) t^2.$$

On the leading wave the equation of which has the form

$$y = -x\,\mathrm{ctg}\,\alpha + \frac{1}{\sin\alpha}; \quad \left(\sin\alpha = \frac{1}{M}\right), \qquad (17.14)$$

with the help of formulas (17.6) and (17.12) we obtain

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$$v_x = v_0 \sqrt{2\frac{(\cos\alpha - x)}{y\sin 2\alpha}}, \quad v_y = v_x \cdot \mathrm{tg}\,\alpha,$$
$$p = \frac{\rho a v_0}{\cos\alpha}\sqrt{2\frac{(\cos\alpha - x)}{y\sin 2\alpha}}. \qquad (17.15)$$

Here let us point out that by analogy with the two-dimensional case (see §11) from the formula

$$\sin\alpha = \frac{1}{M} = \frac{a\beta}{v_0}$$

and from the condition (17.1) it follows directly that $\alpha \gg \beta$.

Thus, in the given linear statement of the penetration problem the Mach angle α is large by comparison with the angle β. In the special case $\alpha = \pi/2$ ($M = 1$), formula (17.12) gives

$$p = \frac{\rho a v_0 (1 - x^2 - y^2)}{2(1 - y^2)^{\frac{3}{2}}} \quad \text{for} \quad x^2 + y^2 < 1, \; p = 0 \quad \text{for} \quad x^2 + y^2 > 1.$$

Let us note that formula (17.6) defines the velocity potential also in the case where the penetrating axisymmetric body has a deformed line deviating little from the Oy_0 axis as the generatrix. The penetration velocity here also can be a variable that satisfies the condition (17.2).

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§18. Nonlinear Problem of Penetration of a Compressible Liquid by a Blunt Cone

Here a study is made of the vertical symmetric entry of a blunt cone with constant subsonic velocity v_0 into a compressible liquid half-space. The expansion rate of the radius of the circle, which is the line of intersection of the cone with the free surface, is greater than the speed of sound a_0 in the undisturbed liquid. Under these conditions, as was noted in §17, a shock wave is formed in the liquid which cuts off the region of the disturbed motion from the liquid at rest. It is proposed that the shock wave does not depart from the line of intersection of the cone with the free boundary of the liquid. On the basis of studying the corresponding linearized problem, it is possible to propose that the shock wave in the meridional cross section is a curve consisting of two lines AA_1 and BB_1 differing little from straight lines and joined by the curve A_1B_1 under the apex of the cone (Figure 1.48).

In the meridional plane x_0Oy_0 the origin of the coordinates is placed at the point of contact of the apex of the cone with the free surface of the liquid at the beginning of penetration. The Ox_0 axis is directed vertically downward in the direction of the velocity v_0; the Oy_0 axis is directed along the surface of the liquid at rest, to the right.

In the investigated axisymmetric problem the motion of the liquid is self-similar. In the plane of the self-similar coordinates $\xi = x_0/t$, $\eta = y_0/t$, the picture of the motion is illustrated in Figure 1.50. The lines A_1A_2, B_1B_2 in this figure denote sections of the boundaries of the region of effect of the diffraction of the apex of the cone. The motion of the liquid is determined from the system of quasilinear equations (1.9) under the corresponding boundary conditions. This system of equations, just as the second-order partial differential equation equivalent to it (1.10) is of the elliptical type inside the region where the following condition is satisfied (v_x, v_y are the velocity components with respect to the x_0, y_0 axes):

$$(v_x-\xi)^2+(v_y-\eta)^2<a^2. \qquad (18.1)$$

This region coincides with the region of effect of the apex of the cone [14].

When the left-hand side of the inequality in formula (18.1) is larger than a^2, the indicated equations of the hyperbolic type and the corresponding characteristics are real. These characteristics are represented by the formulas (1.14) and (1.15). The problem was investigated in this statement in [19, 14, 23]. The condition that

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

the equations are hyperbolic is satisfied after the sections AA_1 and BB_1 of the shock wave. As was noted above, these sections are almost rectilinear, and, consequently, the flow behind them can be considered potential. From the results of the first section of this chapter it follows that in the case of potential motion in the region where the condition

$$(v_x - \xi)^2 + (v_y - \eta)^2 > a^2, \qquad (18.2)$$

is satisfied, the characteristics of the equation of motion of the liquid are represented in the form

$$\left(\frac{d\eta}{d\xi}\right)_{1,2} = \eta'_{1,2} = \frac{UV \pm a\sqrt{W^2 - a^2}}{a^2 - U^2}, \qquad (18.3)$$

$$dU + \eta'_{2,1}\, dV + \frac{d\xi}{a^2 - U^2}\left[\frac{a^2 V}{\eta} - W^2 + 3a^2\right] = 0. \qquad (18.4)$$

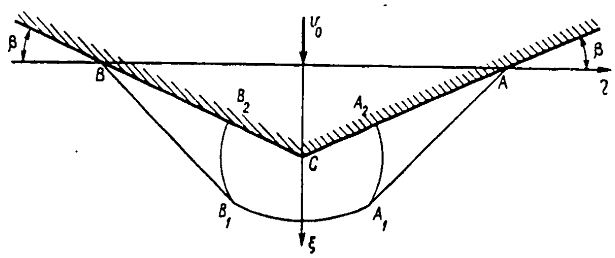

Figure 1.50.

In these equations the following notation is introduced:

$$U = v_x - \xi,\quad V = v_y - \eta,\quad W^2 = U^2 + V^2. \qquad (18.5)$$

Let us note that the problem investigated here is analogous to the problem of shock wave diffraction at the apex of the cone [14, 19, 23]. On the basis of symmetry of the problem let us consider the region of real characteristics to the right of the 0ξ axis (Figure 1.50). Let α be the angle of the shock wave AA_1 with the horizontal. This angle along the shock wave is a variable. In the region AA_1A_2A where the effect of diffraction on the apex of the cone is not felt, using the characteristics of (18.3) and the conditions on the shock wave it is possible to construct the flow parameters and the section AA_1 of the shock wave. Let us denote by D the velocity of the shock wave. From formula (1.23) of the first section it follows that at any point of the shock wave in the plane ξ, η the followine equality exists

$$D = \xi\cos\alpha + \eta\sin\alpha. \qquad (18.6)$$

The equation of state of the liquid will be taken in the form already used previously in the section on the nonlinear problem of penetration by a wedge. The conditions on the shock front at the point A are written as follows (Figure 1.50):

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$p_1 - p_0 = \frac{\rho_0 a_0^2}{n}\left[\left(\frac{\rho_1}{\rho_0}\right)^n - 1\right], \quad p_1 - p_0 = \rho_0 D v_1,$$
$$\rho_0 D = \rho_1 (D - v_1), \qquad (18.7)$$
$$v_1 = v_0 \frac{\cos\beta}{\cos(\alpha - \beta)},$$
$$D = v_0 \frac{\sin\alpha}{\operatorname{tg}\beta}.$$

Here β is the angle of the generatrix of the penetrating cone with the free surface of the liquid at rest, p_1, ρ_1, v_1 are the pressure, density and velocity of the liquid behind the shock wave at the point A; p_0, ρ_0, a_0 are the pressure, density and speed of sound in the liquid at rest. The last equality in the system (18.7) follows from formula (18.6). Five unknowns at the point A are defined from the five equations in formula (18.7) (α is the least angle):

α, D, ρ_1, p_1, v_1.

Let us note that at the point A the parameters of the liquid, the velocity and angle of the shock wave are determined from the same equations from which these parameters are determined at the convergence point, on penetration of a compressible liquid by a blunt wedge with the same apex angle. Thus, for the wedge and the cone of identical apex angle at the point A the liquid parameters coincide. Furthermore, the flow is defined using the characteristics just as for supersonic steady-state motion of a liquid. The method of constructing the solution coincides completely with the method of solving the problem of reflection of the shock wave from the apex of the cone [14, 19, 23]. From the point A the element of the shock wave AM_1 is plotted at an angle α defined from the system (18.7), and the values of the parameters of the point A are carried over to the point M_1. From the point M_1, the characteristic (18.3) (its element) is drawn, which intersects the generatrix of the cone closer to the point A. Let us denote this point by M_2 (Figure 1.51). The speed of the liquid at the point M_2 is found using the boundary condition on the generatrix (the normal component of the velocity of the liquid on the generatrix of the cone is equal to the normal velocity component of the cone) and the corresponding equation of the characteristic (18.4) in which the differentials are replaced by finite differences. The speed of sound at the point M_2 is determined from the Lagrange integral (1.27). Let us introduce the speed of sound a:

$$a^2 = \left(\frac{dp}{d\rho}\right) = \frac{\rho_0 a_0^2}{\rho}\left(\frac{\rho}{\rho_0}\right)^n. \qquad (18.8)$$

Then this integral in finite differences for any two close points 1 and 2 will be written as follows [14, 19]:

$$\frac{a_2^2 - a_1^2}{n-1} = \frac{v_2^2 - v_1^2}{2} + \xi_1 (v_{2x} - v_{1x}) + \eta_1 (v_{2y} - v_{1y}). \qquad (18.9)$$

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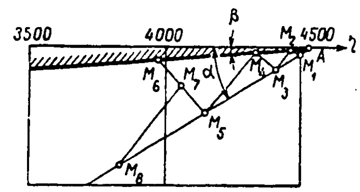

Figure 1.51.

Formulas (18.8) and (18.9) define the speed of sound and density at the point M_2 if the parameters of motion at the point M_1 are known. The pressure at the point M_2 is determined from the equation of state of the liquid

$$p-p_0=\frac{\rho_0 a_0^2}{n}\left[\left(\frac{\rho}{\rho_0}\right)^n-1\right]. \tag{18.10}$$

From the point M_2, an element of the characteristic (18.3) is drawn to the intersection with the continuation of the segment of the shock wave AM_1 at the point M_3. The first three equations in formula (18.7), the equation (18.6) and the condition along the characteristic M_2M_3 which we obtain from (18.4) replacing the differentials by finite differences are valid at the point M_3 at the shock wave. The five obtained equations define five unknown parameters on the shock wave, including the angle α_3. For further solution of the problem from the point M_3 the shock wave α is plotted at an angle α_3. During the course of constructing the solution, another problem of determining the parameters at the intersection point of two characteristics emerging from two close points at which the liquid parameters are already known, is encountered. This problem is solved by the usual method. Thus, the flow behind the shock wave is defined "step by step" in the region of real characteristics.

As an example, the parameters of motion of the liquid are defined by the indicated method for penetration by a blunt cone at eight points shown in Figure 1.51.

The calculations were performed for the following data. The penetration velocity v_0 = 397 m/sec, the angle β = 5°.

$n = 7.15$, ρ_0 = 101.94 kG-sec^2/m^4, a_0 = 1,515 m/sec.

The results of the calculation are presented in Table 1.2. From the table, the slow variation of the parameters of motion on going away from the point A is obvious. The angle of inclination α of the shock wave varies insignificantly so that the shock wave remains almost rectilinear. This is in agreement with the results of the solution of the problem in the linear statement.

By the method of characteristics it is possible to calculate the parameters of the disturbed motion to some boundary close to the line which is defined by the equation

$$(v_x-\xi)^2+(v_y-\eta)^2=a^2.$$

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

Table 1.2

		A	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
ξ	m/sec	0	22,8	5,3	75,5	17,3	229,6	49,7	139,6	425,2
η	m/sec	4538	4500	4477	4412	4340	4155	3970	4063	3830
v_x	m/sec	377	377	377	375	377	371	345	358	357
v_y	m/sec	227	227	226	225	224	221	218	219	208
a	m/sec	2877	2877	2873	2867	2866	2845	2816	2831	2776
D	m/sec	2338	2338	—	2333	—	2322	—	—	2285
ρ/ρ_0		1,232	1,232	1,231	1,230	1,230	1,227	1,223	1,225	1,218
p	kg/cm^2	10490	10490	10440	10370	10370	10140	9830	9990	9330
$\alpha°$		31°01	31°01	—	30°56	—	30°46	—	—	30°15

In the diffraction region the parameters of motion of the liquid and the corresponding section A_1B_1 of the shock wave remain unknown (Figure 1.50). The definition of the motion in this region encounters difficulties of the same nature as when determining the motion between a blunt body and the departing shock wave in the supersonic steady-state flow. In the case of penetration investigated here (just as in the problem of reflection of the shock wave from the apex of the cone, the problem is still more complicated by the fact that the solution obtained in the diffraction region must fit with the solution obtained by the method of characteristics at a finite distance.

The effect of diffraction leads to the fact that beginning with the points A_2 and B_2 along the generatrices of the cone in the direction of its apex there will be further, more intense continuous decrease in pressure. Its minimum is reached at the apex of the cone.

In [27] the approximate solution was obtained for the problem of the diffraction of a shock wave of constant intensity from the apex of a blunt cone. It is demonstrated that in the self-similar plane ξ, η in the diffraction region the pressure on the cone in the section A_2C (Figure 1.50) decreases by a parabolic law as a function of the distance from the apex of the cone. In view of the complete analogy of the mathematical statement of the problem of reflection of a shock wave from the apex of the cone and the problem of penetration of a compressible liquid by a blunt cone investigated here, it is possible to expect that the pressure will vary similarly also in this problem in the diffraction region along the generatrix.

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

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§19. Impact of a Rigid Cylinder With the Surface of a Compressible Liquid

Let an impact take place at the time t = 0 with a velocity $v_0 \ll a$ between an absolutely rigid circular cylinder of radius r_0 with flat front tip and the free surface of an ideal compressible liquid occupying the lower half-space z > 0 (Figure 1.52) [23, 33]. Here a is the speed of sound of the undisturbed liquid. Under this condition, as is easy to demonstrate, for the initial time interval $\Delta t \sim r_0/a$ where the compressibility of the liquid is significant, the problem will be linear, and it is described in the cylindrical system of dimensionless coordinates $r_1 z_1$ by the following equation and conditions:

$$\frac{\partial^2\varphi}{\partial r_1^2} + \frac{\partial\varphi}{r_1\,\partial r_1} + \frac{\partial^2\varphi}{\partial z_1^2} = \frac{\partial^2\varphi}{\partial\tau^2};\ \varphi = 0 \quad \text{for} \quad r_1 > 1,\ z_1 = 0;$$

$$\frac{\partial\varphi}{\partial z_1} = v(\tau)\,r_0 \quad \text{for} \quad 0 \leqslant r_1 < 1,\ z_1 = 0; \qquad (19.1)$$

$$\varphi = \frac{\partial\varphi}{\partial\tau} = 0 \quad \text{for} \quad \tau = 0;$$

$$\tau = \frac{at}{r_0},\quad r_1 = \frac{r}{r_0} \quad z_1 = \frac{z}{r_0}.$$

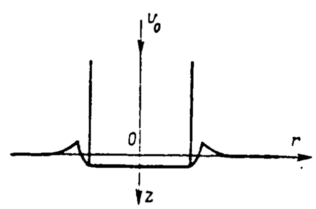

Figure 1.52.

Here $\phi(r_1, z_1, \tau)$ is the potential of the disturbed motion, $v(\tau)$ is the penetration velocity. Hereafter, the subscript 1 will be omitted on the dimensionless independent variables. This linear statement gives a proper solution at all points except the small region at the edge of the disk where as a result of discontinuity of the direction of the velocities, the latter must have a singularity.

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In the problem the force F(t) is found which acts on the disk at $0 \le \tau < 1$.

The Laplace transformation [34] with respect to τ is applied to the system (19.1):

$$\frac{\partial^2\Phi}{\partial r^2} + \frac{\partial\Phi}{r\,\partial r} + \frac{\partial^2\Phi}{\partial z^2} = p^2\Phi \ (\mathrm{Re}\, p > 0),$$
$$\Phi_{z=0} = 0 \quad \text{for} \quad r > 1, \tag{19.2}$$
$$\left[\frac{\partial\Phi}{\partial z}\right]_{z=0} = V(p)\, r_0 \quad \text{for} \quad 0 \leqslant r < 1.$$

Then, applying the Hankel transformation to equation (19.2) [34], with respect to r, it is easy to obtain for z = 0:

$$\int_0^1 \Phi_{z=0} J_0(r\,x)\, r\, dr = -\frac{1}{\sqrt{p^2 + x^2}} \times$$
$$\times \left[\int_1^\infty \left(\frac{\partial\Phi}{\partial z}\right)_{z=0} J_0(r\,x)\, r\, dr + V(p)\, r_0 \int_0^1 J_0(r\,x)\, r\, dr\right]. \tag{19.3}$$

In (19.2) and (19.3) the following notation is adopted:

$J_n(r\,x)$ is an n-th order Bessel function,

$V(p) \doteqdot v(\tau), \quad \Phi(r, z, p) \doteqdot \varphi(r, z, \tau).$

The reverse Hankel transformation for (19.3) with replacement of r by ρ for $\rho > 1$ gives

$$\int_1^\infty \left(\frac{\partial\Phi}{\partial z}\right)_{z=0} r\, dr \int_0^\infty J_0(rx)\, J_0(\rho x) \frac{x\, dx}{\sqrt{p^2 + x^2}} =$$
$$= -V(p)\, r_0 \int_0^\infty J_1(x)\, J_0(x\rho) \frac{dx}{\sqrt{p^2 + x^2}}. \tag{19.4}$$

It is easy to demonstrate that

$$\int_1^\infty J_0(rx)\, J_0(x\rho) \frac{x\, dx}{\sqrt{p^2 + x^2}} = \frac{1}{\pi i} \int_{b-i\infty}^{b+i\infty} \frac{\alpha(s)\, s\, ds}{\sqrt{p^2 - s^2}},$$
$$\int_0^\infty J_1(x)\, J_0(x\rho) \frac{dx}{\sqrt{p^2 + x^2}} = \frac{1}{\pi i} \int_{b-i\infty}^{b+i\infty} K_0(s\rho)\, I_1(s) \frac{ds}{\sqrt{p^2 - s^2}}. \tag{19.5}$$

Here $0 < b < \mathrm{Re}\,p$; $K_n(x)$ and $I_n(x)$ are the MacDonald and Bessel functions of an imaginary argument, respectively, of n-th order. The branch $y\sqrt{p^2 - s^2}$ is selected so that

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$\mathrm{Re}\sqrt{p^2-s^2}>0$ for $0<\mathrm{Re}\,s<\mathrm{Re}\,p.$

$$\alpha(s)=\begin{cases}K_0(sr)\,I_0(s\rho) & \text{for } r>\rho,\\ K_0(s\rho)\,I_0(sr) & \text{for } \rho>r.\end{cases}$$

Substituting the expressions (19.5) in (19.4) and making the substitution of variable $s_1 = s/p$, we obtain

$$\int_1^{\infty}\left(\frac{\partial\Phi}{\partial z}\right)_{z=0} r\,dr\int_L \frac{\alpha(ps)\,s\,ds}{\sqrt{1-s^2}}= \\ =-\frac{r_0}{p}V(p)\int_L K_0(sp\rho)\,I_1(sp)\frac{ds}{\sqrt{1-s^2}}. \tag{19.6}$$

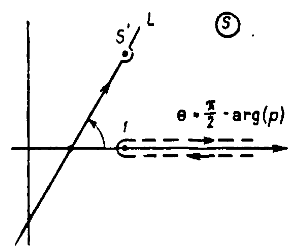

Figure 1.53.

Here the subscript 1 on s_1 is omitted. The loop L is shown in Figure 1.53. Now it is easy to note that the integration loop in (19.6) can be deformed along the section of the branch $\sqrt{1-s^2}$, as shown in Figure 1.53. Then it is possible to apply the asymptotic expansions of the cylindrical functions for large values of the arguments, making one significant simplification.

Let us imagine

$$I_n(q)=\frac{1}{2}(-i)^n\left[H_n^{(2)}(qi)+H_n^{(1)}(qi)\right]\ (q=sp,\ spr,\ sp\rho).$$

Here $H_n^{(j)}(qi)$ is the j-th type, n-th order Hankel function. It is easy to note that the function $H_n^{(1)}(qi)$ gives the delay factor exp (-2q) by comparison with $H_n^{(2)}(qi)$. From the shape of the deformed loop it is obvious that this factor satisfies the inequality $|\exp(-2q)| \le \exp(-2\mathrm{Re}\,p)$. Consequently, the function $H_n^{(1)}(qi)$ makes a contribution to the solution of (19.6) in the form of secondary, tertiary, and other diffraction waves.

For the inverse Laplace transformation these terms appear only for $\tau > 2$, which is excluded by the condition of the problem. Consequently, instead of $I_n(q)$ it is

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necessary to substitute the function $1/2\ (-i)^n H_n^{(2)}(qi)$ in (19.6) and use its asymptotic form. Then it is easy to see that for the first approximation with respect to $1/p$, we obtain the following, if we deform the loop to the former position after application of the asymptotic form):

$$\int_1^\infty \left(\frac{\partial\Phi}{\partial z}\right)_{z=0} \sqrt{r}\, dr \int_L \frac{e^{spr-sp\rho}}{\sqrt{1-s^2}}\, ds = -\frac{r_0}{p} V(p) \int_L \frac{e^{sp-sp\rho}}{s\sqrt{1-s^2}}\, ds. \tag{19.7}$$

Let us assume that

$$\left(\frac{\partial\Phi}{\partial z}\right)_{z=0} \sim M_0(\rho-1)^\beta \quad \text{for} \quad \rho\to 1,\ \beta > -1.$$

Then [35]

$$\Psi(s, p)e^{-sp} \sim M_1 s^{-1-\beta} \quad \text{for} \quad s\to\infty,$$

$$\Psi(s, p) = \int_1^\infty \left(\frac{\partial\Phi}{\partial z}\right)_{z=0} e^{spr}\sqrt{r}\, dr.$$

Here M_0 and M_1 are constants. Applying the Wiener-Hopf method to equation (19.7), we obtain (Figure 1.53)

$$\int_L \left[\frac{\Psi(s, p)}{\sqrt{1-s}} e^{-sp} + \frac{V(p)\, r_0}{ps\sqrt{1-s}}\right] \frac{ds}{s-s'} = 0. \tag{19.8}$$

Using the fact that the first term in brackets of the expression under the integral sign (19.8) is an analytical function in the half-plane Res < 1, from (19.8) it is easy to obtain

$$\Psi(s, p) = V(p)\frac{r_0}{ps}\left(\sqrt{1-s}-1\right)e^{sp}. \tag{19.9}$$

If we find the second approximation of $\Psi(s, p)$ with respect to $1/p$, then instead of (19.7) it is necessary to take

$$\int_1^\infty \left(\frac{\partial\Phi}{\partial z}\right)_{z=0} \sqrt{r}\, dr \int_L \frac{e^{spr-sp\rho}}{\sqrt{1-s^2}}\, ds = -\frac{r_0}{p} V(p) \int_L \frac{e^{sp-sp\rho}}{s\sqrt{1-s^2}} \left(1-\frac{3}{8ps}-\frac{1}{8p s\rho}\right) ds. \tag{19.10}$$

It must be noted that in the expression under the integral sign of the left-hand side of (19.10) in the term $1 + \gamma$, the value of γ has been omitted

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$\gamma = \frac{1}{8psr} - \frac{1}{8ps\rho}.$$

This is done for the following reason. In order to find the term of the second approximation it is necessary to multiply both sides of the complete equation (19.10) by exp $(s'p\rho)$, integrate with respect to ρ from 1 to infinity and determine the second term in the asymptotic expansion of the left-hand side with respect to powers of $1/p$. It turns out that the value of γ has no influence on this term; it is related to the terms of higher order. This is easy to demonstrate if we substitute $[\partial\Phi/\partial z]_{z=0}$ found from (19.9) in the left-hand side of the complete equation (19.10). Then the result of integration of the terms related to γ will give a value of the third order in the expansion with respect to $1/p$.

Thus, after repetition of all the arguments analogous to the case of the first approximation for (19.10), we obtain

$$\Psi(s, p) = e^{sp} r_0 \frac{V(p)}{ps} \left[\sqrt{1-s} - 1 + \frac{1}{2sp}(1 - \sqrt{1-s}) - \right.$$
$$\left. - \frac{1}{4p} \sqrt{1-s} \right]. \qquad (19.11)$$

From (19.11) it is easy to obtain

$$\int_1^\infty \left(\frac{\partial \Phi}{\partial z}\right)_{z=0} r\,dr = \frac{r_0}{p} V(p) \left[-\frac{1}{2} + \frac{1}{8p} + o(p^{-2}) \right]. \qquad (19.12)$$

Then from (19.3) and (19.12) we obtain:

$$\int_0^1 \Phi_{z=0}\, r\,dr = \frac{r_0}{p} V(p) \left[-\frac{1}{2} + \frac{1}{2p} - \frac{1}{8p^2} + o(p^{-3}) \right],$$

$$F(\tau) = -2\pi r_0 \rho a \int_0^1 \left(\frac{\partial \varphi}{\partial \tau}\right)_{z=0} r\,dr \approx$$

$$\approx \pi r_0^2 \rho a \left[v(\tau) - \int_0^\tau v(x)\,dx + \frac{1}{4}\int_0^\tau v(x)(\tau - x)\,dx \right].$$

If the cylinder has large mass, it is possible for $0 \le \tau < 1$ to set $v(\tau) = v_0$.

Then with accuracy to $O(\tau^3)$ the following is obtained

$$F(\tau) = \pi r_0^2 \rho a v_0 \left(1 - \tau + \frac{1}{8}\tau^2\right).$$

Analogously, considering the following terms of the asymptotic expansion, we obtain

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APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$F(\tau) = \pi r_0^2 \rho a v_0 \left[1 - \tau + \frac{1}{8}\tau^2 + \frac{\tau^3}{48} + \frac{\tau^4}{384} + O(\tau^5)\right]. \tag{19.13}$$

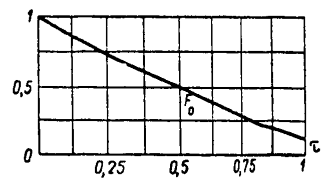

Figure 1.54.

The function (19.13) is represented in Figure 1.54, where

$$F_0 = \frac{F(\tau)}{\pi r_0^2 \rho a v_0}.$$

In [36] a study was made of the problem of impact of a cylindrical body against the surface of a compressible liquid in the approximate statement considering the nonlinear effect of the liquid medium and elastic properties of the penetrating cylinder. As a result, the following relation was obtained for the maximum pressure at the time of impact:

$$p = \frac{\rho v_0 D}{1 + \dfrac{\rho D}{\sqrt{\dfrac{\rho_1 E (1-\sigma)}{(1+\sigma)(1-2\sigma)}}}},$$

where D is the velocity of the shock wave in the liquid, ρ_1, E, σ are the density, the modulus of elasticity and the Poisson coefficient of the cylinder material.

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

§20. Impact of a Cylindrical Elastic Shell With a Liquid Filler Against the Surface of a Compressible Liquid

A study is made of the impact of an elastic thin-walled shell of circular cross section, radius R, thickness h, with flat leading tip and liquid filler against the horizontal surface of a compressible liquid at rest occupying the half-space. The velocity of the impact is directed along the axis of the shell perpendicularly to the surface of the liquid.

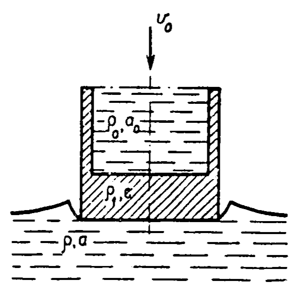

Figure 1.55.

At the time t = 0 normal impact of the leading tip of the shell, which is a rigid disk of radius R and mass m against the surface of the liquid (Figure 1.55). It is required that the stressed state of the shell be determined for small values of the time considering the filler--an ideal compressible liquid. The investigation of the problem of impact of the cylinder against the surface of the compressible liquid was made in the preceding section. If the depth of penetration of the disk $U_1(t)$ and its velocity $\dot{U}_1(t)$ are represented in the form

$$U_1(t) = -v_0 t + U(t), \quad \dot{U}_1(t) = -v_0 + \dot{U}(t), \tag{20.1}$$

for t the asymptotic value of the force of the resistance f of the liquid can be written as follows:

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$f(t) = \pi R^2 \rho a \left\{ v_0 - \dot{U}(t) - \frac{a}{R}\left(v_0 t - \int_0^t \dot{U}\, d\tau\right) + \right.$$
$$\left. + \frac{a^2}{4R^2}\left[\frac{v_0 t^2}{2} - \int_0^t \dot{U}(t-\tau)\, d\tau\right]\right\}. \quad (20.2)$$

In formulas (20.1), (20.2), v_0 is the initial velocity of the impact, ρ and a are the density and the speed of sound in the liquid at rest into which the shell penetrates. The additional mixing of U(t) occurring as a result of the forces of resistance to penetration, the elastic forces of the shell at the front tip and the pressure of the liquid filler is subject to definition during the course of solution of the problem.

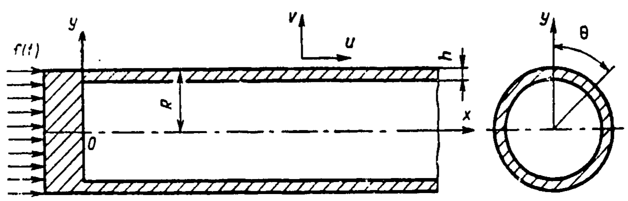

Figure 1.56.

Now it is possible to formulate the problem as follows. An external force given by formula (20.2) begins to act on the bottom of the shell moving with constant velocity v_0, from the point in time t = 0. It is required that the stressed state of the shell be determined. In the meridional plane the origin of the cylindrical coordinate axes xOy connected with the body will be taken at the center of the disk; the Ox axis is directed along the axis of the shell, the Oy axis, perpendicular to the Ox axis (Figure 1.56). In agreement with formula (20.1), the displacement of the points of the shell in the axial direction $u_1(x, y, t)$ will be written in the form

$$u_1(x, y, t) = -v_0 t + u(x, y, t). \quad (20.3)$$

Let v(x, y, t) be the transverse shift of the shell walls. Since the bottom is rigid, in the cross section x = 0 there is no transverse shift, and on the basis of formula (20.1) we have

$$x = 0,\ u(0, y, t) = u(t),\ v = 0.$$

From the equality (20.3) we obtain the velocity and deformation of the shell along its axis:

$$\frac{\partial u_1}{\partial t} = -v_0 + \frac{\partial u}{\partial t},\quad \frac{\partial u_1}{\partial x} = \frac{\partial u}{\partial x},$$
$$\frac{\partial u_1}{\partial y} = \frac{\partial u}{\partial y}.$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

The equations of motion of the shell in the adopted coordinated system have the form

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\sigma_y - \sigma_\theta}{y} = \rho_1 \frac{\partial^2 v}{\partial t^2},$$
$$\frac{\partial \sigma_x}{\partial x} + \frac{1}{y}\frac{\partial}{\partial y}(y\tau_{xy}) = \rho_1 \frac{\partial^2 u}{\partial t^2}, \tag{20.4}$$

where σ_x, σ_y, σ_θ, τ_{xy} are the stresses in the generally accepted notation, ρ_1 is the density of the shell material. The stresses are related to the deformations linearly by Hook's law.

We shall consider that the longitudinal shift u of the shell does not depend on the coordinate y and, as is usually assumed in shell theory, the normal transverse stress σ_y is negligibly small by comparison with the normal annular and axial stresses σ_θ and σ_x. Let us integrate the equations of motion of the shell and the relations expressing Hook's law with respect to small thickness of its wall h([h/R] << 1). Then, introducing the stresses and shifts averaged with respect to this thickness, using the theorem of the mean and the assumptions made above, we obtain (the average values of the stresses and strains are denoted by the same letters as denoted the corresponding nonaveraged values)*:

$$\frac{\partial \tau}{\partial x} - \frac{\sigma_\theta}{R} + \frac{p(x,t)}{R} = \rho_1 \frac{\partial^2 v}{\partial t^2}, \quad \frac{\partial \sigma_x}{\partial x} = \rho_1 \frac{\partial^2 u}{\partial t^2},$$
$$\sigma_x = \rho_1\left(a_1^2 \frac{\partial u}{\partial x} + k^2 \frac{v}{R}\right), \tag{20.5}$$
$$\sigma_\theta = \rho_1\left(k^2 \frac{\partial u}{\partial x} + a_1^2 \frac{v}{R}\right), \quad \tau = \rho_1 a_2^2 \frac{\partial v}{\partial x}.$$

$$a_1^2 = \frac{4\mu(\lambda+\mu)}{\rho_1(\lambda+2\mu)} = \frac{1}{\rho_1}\frac{E}{(1-\sigma^2)}, \quad k^2 = \sigma a_1^2, \quad a_2^2 = \frac{\mu}{\rho_1}. \tag{20.6}$$

In these equations σ_x, σ_θ, τ are the axial, annular and tangential transverse averaged stresses, u, v are the average longitudinal and transverse shifts, λ, μ are the Lame coefficients, E, σ are the Young's modulus and the Poisson coefficient, p(x, t) is the difference between the liquid pressure and its initial pressure, p_0 equal to the outside pressure. The system of equations (20.5) is reduced to two equations in the shifts:

* If we do not consider the equality

$$\frac{\partial \tau_{xy}}{\partial y} + \frac{\tau_{xy}}{y} = \frac{\partial (y\tau_{xy})}{\partial y}$$

and the averaging is carried out term by term, then instead of the second equation of system (20.5), we obtain

$$\frac{\partial \sigma_x}{\partial x} + \frac{\tau}{R} = \rho_1 \frac{\partial^2 u}{\partial t^2}.$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$a_2^2 \frac{\partial^2 v}{\partial x^2} - \frac{k^2}{R} \frac{\partial u}{\partial x} - a_1^2 \frac{v}{R^2} + \frac{p(x, t)}{\rho_1 h} = \frac{\partial^2 v}{\partial t^2},$$
$$a_1^2 \frac{\partial^2 u}{\partial x^2} + \frac{k^2}{R} \frac{\partial v}{\partial x} = \frac{\partial^2 u}{\partial t^2}. \quad (20.7)$$

The initial and the boundary conditions of equations (20.7) will be

$$t = 0,\ x \geqslant 0,\ u = v = \frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = 0,\ p(x, 0) = 0;$$
$$t > 0,\ x = 0,\ m \frac{\partial^2 u}{\partial t^2} = f + 2\pi R h \sigma_x (0, t) - \pi R^2 p(0, t); \quad (20.8)$$
$$v(0, t) = 0.$$

At infinity ($x = \infty$) during the time of the motion the shell and the liquid in it are at rest.

After the investigated small time interval, the disturbances arising at the front end of the shell, as a rule, do not reach the other end of the shell. Therefore a semi-infinite shell is considered. The investigation of the shell of finite length does not introduce any theoretical difficulties.

The solution of the systems (20.7) is represented in the form of the sum of the solutions of the equations

$$a_2^2 \frac{\partial^2 v}{\partial x^2} - \frac{k^2}{R} \frac{\partial u}{\partial x} - a_1^2 \frac{v}{R^2} = \frac{\partial^2 v}{\partial t^2},$$
$$a_1^2 \frac{\partial^2 u}{\partial x^2} + \frac{k^2}{R} \frac{\partial v}{\partial x} = \frac{\partial^2 u}{\partial t^2} \quad (20.9)$$

with the initial and boundary conditions

$$t = 0,\ x \geqslant 0,\ u = v = \frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = 0,$$
$$t > 0,\ x = 0,\ m \frac{\partial^2 u}{\partial t^2} = f - 2\pi R h \sigma_x (0, t) - \pi R^2 p(0, t),\ v(0, t) = 0 \quad (20.10)$$

and the solutions of the equations

$$a_2^2 \frac{\partial^2 v}{\partial x^2} - \frac{k^2}{R} \frac{\partial u}{\partial x} - a_1^2 \frac{v}{R^2} + \frac{p(x, t)}{\rho_1 h} = \frac{\partial^2 v}{\partial t^2},$$
$$a_1^2 \frac{\partial^2 u}{\partial x^2} + \frac{k^2}{R} \frac{\partial v}{\partial x} = \frac{\partial^2 u}{\partial t^2} \quad (20.11)$$

with the initial and boundary conditions

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$t=0,\ x>0,\ u=v=\frac{\partial u}{\partial t}=\frac{\partial v}{\partial t}=0,$$
$$t>0,\ x=0,\ m\frac{\partial^2 u}{\partial t^2}=2\pi Rh\sigma_x(0,t). \qquad (20.12)$$

On the basis of the boundary conditions of (20.12) of equations (20.11) it is possible to assume that the axial stress σ_x arising out of the effect of the pressure of the liquid on the sidewall of the shell is negligibly small by comparison with the stresses σ_θ and τ. That is, in this problem along with σ_y we assume σ_x equal to zero. Then instead of the equations (20.11) with the conditions (20.12) from the system (20.5) we obtain one equation

$$a_2^2\frac{\partial^2 v}{\partial x^2}-c^2\frac{v}{R^2}+\frac{p(x,t)}{h\rho_1}=\frac{\partial^2 v}{\partial t^2},\quad c^2=\frac{E}{\rho_1} \qquad (20.13)$$

with the initial and boundary conditions

$$t=0,\ x\geqslant 0,\ v=\frac{\partial v}{\partial t}=0,$$
$$t>0,\ x=0,\ v(0,t)=0. \qquad (20.14)$$

It is necessary to add the equations of motion of a liquid in the shell which are considered in the one-dimensional acoustic approximation to the equations (20.9), (20.13):

$$\frac{\partial w}{\partial t}=-\frac{1}{\rho_0}\frac{\partial p}{\partial x},$$
$$\frac{\partial w}{\partial x}+\frac{1}{\rho_0 a_0^2}\frac{\partial p}{\partial t}+\frac{2}{R}\frac{\partial v}{\partial t}=0, \qquad (20.15)$$

where ρ_0, w are the density and velocity of the liquid, a_0 is the speed of sound in it. For (20.15), we have the condition:

$$t=0,\ x\geqslant 0,\ w=\frac{\partial w}{\partial t}=0,$$
$$t>0,\ x=0,\ w=w_0=\frac{\partial u(0,t)}{\partial t}=\dot{u}(t). \qquad (20.16)$$

The basic part of the transverse shift occurring as a result of the pressure of the liquid on the sidewall of the shell is taken into account further in the second equation of system (20.15). Using equation (20.13), the system (20.15) is reduced to one fourth-order equation for determining the velocity of the liquid (or the pressure in it):

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$\frac{\partial^2}{\partial t^2}\left(\frac{1}{a_0^2}\frac{\partial^2 w}{\partial t^2}-\frac{\partial^2 w}{\partial x^2}\right)-a_2^2\frac{\partial^2}{\partial x^2}\left(\frac{1}{a_0^2}\frac{\partial^2 w}{\partial t^2}-\frac{\partial^2 w}{\partial x^2}\right)+$$
$$+\frac{c^2}{R^2}\left(\frac{1}{\lambda_0^2}\frac{\partial^2 w}{\partial t^2}-\frac{\partial^2 w}{\partial x^2}\right)=0, \quad (20.17)$$
$$\lambda_0^2=\left(\frac{1}{a_0^2}+\frac{2\rho_0 R}{Eh}\right)^{-1}.$$

If we consider the inertia of the shell walls and the transverse tangential stress, then the equation (20.17) becomes the known second-order equation describing the propagation of the "hydraulic impact" with a velocity λ_0.

Equation (20.17) has two characteristic velocities

$$\frac{dx}{dt}=\pm a,\ \frac{dx}{dt}=\pm a_2. \quad (20.18)$$

The study of the asymptotic form of the solution of equation (20.17) for small values of the time demonstrates that the wave with the velocity a_2 does not introduce disturbances into the liquid. Accordingly (the initial period of motion is investigated) in equation (20.17) we drop the terms taking into account the transverse stress. As a result, we obtain:

$$\frac{\partial^2}{\partial t^2}\left(\frac{1}{a_0^2}\frac{\partial^2 w}{\partial t^2}-\frac{\partial^2 w}{\partial x^2}\right)+\frac{c^2}{R^2}\left(\frac{1}{\lambda_0^2}\frac{\partial^2 w}{\partial t^2}-\frac{\partial^2 w}{\partial x^2}\right)=0. \quad (20.19)$$

The solution of equations (20.9), (20.13) and (20.19) and the first equation of the system (20.15) will be found using the Laplace transformation. Let us introduce the following correspondences between the originals and the transforms:

$$u(x,t)\div U(x,s),\ v(x,t)\div V(x,s),\ v_1(x,t)\div V_1(x,s),$$
$$w(x,t)\div W(x,s),\ p(x,t)\div Q(x,s). \quad (20.20)$$

Here v_1 and V_1 denote the solutions of equation (20.13) and its transform, s is the complex parameter of the Laplace transformation. After solution of the corresponding ordinary differential equations the transforms (20.20) are represented in the following form:

$$U=C_1\left\{\left(a_2^2\gamma_1^2-s^2-\frac{a_1^2}{R^2}\right)\exp(x\gamma_1)-\frac{\gamma_1}{\gamma_2}\left(a_2^2\gamma_2^2-s^2-\frac{a_1^2}{R^2}\right)\exp(x\gamma_2)\right\},$$
$$V=C_1\gamma_1\frac{k^2}{R}\{\exp(x\gamma_1)-\exp(x\gamma_2)\},$$
$$\gamma_{1,2}=\frac{1}{a_1a_2\sqrt{2}}\sqrt{A\mp\sqrt{A^2-B}}, \quad (20.21)$$
$$A=a_2^2s^2+a_1^2\left(s^2+\frac{a_1^2}{R^2}\right)-\frac{k^4}{R^2},\ B=4a_1^2a_2^2s^2\left(s^2+\frac{a_1^2}{R^2}\right).$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$W = W_0 \exp\left[-\frac{xs}{a_0}\sqrt{\frac{s^2+\omega^2}{s^2+\nu^2}}\right], \quad Q = \rho_0 a_0 \sqrt{\frac{s^2+\nu^2}{s^2+\omega^2}}\, W, \tag{20.22}$$

$$w_0 \div W_0 = sU(0, s), \quad \omega^2 = \frac{2\rho_0 a_0^2}{\rho_1 hR} + \nu^2, \quad \nu^2 = \frac{c^2}{R^2},$$

$$V_1 = \frac{k}{N_1^2 - N_2^2}\{\exp(-xN_2) - \exp(-xN_1)\},$$

$$k = \frac{a_0}{a_2^2}\,\frac{\rho_0 sU(0, s)}{\rho_1 h}\sqrt{\frac{s^2+\nu^2}{s^2+\omega^2}}, \tag{20.23}$$

$$N_1^2 = \frac{s^2+\nu^2}{a_2^2}, \quad N_2 = \frac{s}{a_0}\sqrt{\frac{s^2+\omega^2}{s^2+\nu^2}}.$$

The value of C_1 in formulas (20.21) is determined from the boundary condition (20.10) written in the transforms

$$x = 0, \quad ms^2U(0, s) = F \div 2\pi Rh\rho_1 a_1^2 \frac{dU}{dx} - \pi R^2 \rho_0 a_0 \sqrt{\frac{s^2+\nu^2}{s^2+\omega^2}}\, sU(0, s). \tag{20.24}$$

Here F is the transform of the external force f. In the general case finding the originals of the transforms (20.21)-(20.23) is a difficult problem. However, for small values of the time it is possible to obtain an effective asymptotic representation of the originals. It is easy to establish that for $s \to \infty$ in the rough and more exact approximations the following asymptotic representations occur*:

$$\gamma_1 = \left\{-\frac{s}{a_1};\ -\frac{\sqrt{s^2+e_1^2}}{a_1} \approx -\frac{s}{a_1}\left(1 + \frac{n_1^2}{s^2}\right)\right.,$$

$$n_1^2 = \frac{e_1^2}{2} = \frac{a_1^2}{R^2}\sigma^2(1-\sigma),$$

$$\gamma_2 = \left\{-\frac{s}{a_2};\ -\frac{\sqrt{s^2+e_2^2}}{a_2} \approx -\frac{s}{a_2}\left(1 + \frac{n_2}{s^2}\right)\right., \quad n_2^2 = \frac{e_2^2}{2} = \frac{a_1^2}{2R^2} - n_1, \tag{20.25}$$

$$\frac{s}{a_0}\sqrt{\frac{s^2-\omega^2}{s^2+\nu^2}} = \left\{\frac{s}{a_0},\ \frac{s}{a_0}\left(1 + \frac{k}{s^2}\right)\right., \quad k = \frac{1}{2}(\omega^2 - \nu^2).$$

According to formula (20.2) the transform F of the force f acting on the end of the shell is defined in the form

$$F = \pi R^2 \rho a v_0\left(\frac{1}{s} - \frac{a}{R}\frac{1}{s^2} + \frac{a^2}{4R^2}\frac{1}{s^3}\right) - \pi R^2 \rho a\left(sU - \frac{aU}{R} - \frac{a^2U}{4R^2 s}\right). \tag{20.26}$$

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* For the averaging indicated in the first footnote of this section we have

$$n_1^2 = \frac{a_1^2\sigma}{R^2 2}(1 - 2\sigma^2).$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

From the first equation of formula (20.21) for the mapping of U and its derivative with respect to x at x = 0 we obtain:

$$U(0, s) = C_1 \frac{\gamma_1 - \gamma_2}{\gamma_2} \left(a_2^2 \gamma_1 \gamma_2 + s^2 + \frac{a_1^2}{R^2} \right), \quad \frac{dU}{dx} = C_1 a_2^2 \gamma_1 (\gamma_1^2 - \gamma_2^2). \qquad (20.27)$$

Substituting these mappings and the mapping F from (20.26) in the boundary condition (20.24), we obtain the linear equation defining the integration constant $C_1(s)$. After determining this value it is easy to see that for the asymptotic values of (20.25) the originals of the mappings (20.21)-(20.23) are represented by quadratures [37, 38]. We shall limit ourselves to the investigation of the simplest asymptotic representation where the terms of order $1/s^2$ and less by comparison with values on the order of one are neglected in the mappings. In this case, we have ($s \to \infty$)

$$\gamma_1 = -\frac{s}{a_1}, \ \gamma_2 = -\frac{s}{a_2}, \ \sqrt{\frac{s^2 + \omega^2}{s^2 + \nu^2}} = 1, \ N_1 = \frac{s}{a_2}, \ N_2 = \frac{s}{a_0}. \qquad (20.28)$$

Here the mappings introduced above are simplified and assume the form

$$U = \frac{\pi R^2 \rho a v_0}{m} \frac{1}{s^3} \frac{s - \alpha}{s + \beta} e^{-\frac{x}{a_1} s},$$
$$V = \frac{\pi R a \rho v_0 k^2 a_1}{m (a_1^2 - a_2^2)} \frac{1}{s^4} \frac{s - \alpha}{s + \beta} [e^{-\frac{x}{a_1} s} - e^{-\frac{x}{a_2} s}], \qquad (20.29)$$
$$V_1 = \frac{\rho_0 a_0^3 a_2}{\rho_1 h (a_0^2 - a_2^2)} \frac{U(0, s)}{s} [e^{-\frac{x}{a_0} s} - e^{-\frac{x}{a_2} s}].$$

The following notation has been introduced

$$\alpha = \frac{a}{R}, \ \beta = \frac{\pi}{m} (2Rh\rho_1 a_1 + R^2 \rho_0 a_0). \qquad (20.30)$$

When the shell does not contain a liquid, the mapping V_1 is equal to zero, and in the formula (20.30) the last term in the expression for β is absent. As is obvious from the first equality (20.29) in the above-indicated approximation the mapping U for longitudinal shift does not depend on the wave propagated at a velocity of a_2. When keeping the largest term considering this wave, the mapping U has the form

$$U(x, s) = \frac{\pi R^2 \rho a v_0}{m} \left\{ \frac{1}{s^3} \frac{s - \alpha}{s + \beta} e^{-\frac{x}{a_1} s} - \frac{a_1^3 a_2}{R^2 (a_1^2 - a_2^2)} \frac{1}{s^4} \frac{s - \alpha}{s + \beta} e^{-\frac{x}{a_2} s} \right\}.$$

The mappings of the deformations and the shift rate of the shell are obtained by differentiation of the corresponding mappings in (20.29):

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$\frac{\partial u}{\partial x} \div \frac{dU}{dx}, \quad \frac{\partial u}{\partial t} \div sU, \quad \frac{\partial v}{\partial x} \div \frac{dV}{dx}, \quad \frac{\partial v_1}{\partial x} \div \frac{dV_1}{dx}. \tag{20.31}$$

Let us introduce two functions into the investigation:

$$\begin{gathered} \varphi(t) = (\alpha+\beta)\, e^{-\beta t} - \frac{\alpha\beta^2}{2} t^2 + \beta(\alpha+\beta)\, t - (\alpha+\beta), \\ f(t) = (\alpha+\beta)\, e^{-\beta t} + \alpha\beta t - (\alpha+\beta). \end{gathered} \tag{20.32}$$

Then, going from the mappings of (20.29), (20.31) to their originals, for the longitudinal shifts, deformations and velocities we obtain:

$$\begin{aligned} u(x, t) &= \frac{\pi R^2 \rho a v_0}{m\beta^3} H\left(t - \frac{x}{a_1}\right) \varphi\left(t - \frac{x}{a_1}\right), \\ \frac{\partial u}{\partial x} &= \frac{\pi R^2 \rho a v_0}{m a_1 \beta^2} H\left(t - \frac{x}{a_1}\right) f\left(t - \frac{x}{a_1}\right), \\ \frac{\partial u}{\partial t} &= -\frac{\pi R^2 \rho a v_0}{m\beta^2} H\left(t - \frac{x}{a_1}\right) f\left(t - \frac{x}{a_1}\right). \end{aligned} \tag{20.33}$$

For the transverse deformations

$$\begin{aligned} \frac{\partial v}{\partial x} &= -\frac{\pi R \rho a v_0 k^2}{m\beta^3 (a_1^2 - a_2^2)} H\left(t - \frac{x}{a_1}\right) \varphi\left(t - \frac{x}{a_1}\right) + \\ &\quad + \frac{\pi R \rho a v_0 k^2 a_1}{m a_2 \beta^3 (a_1^2 - a_2^2)} H\left(t - \frac{x}{a_2}\right) \varphi\left(t - \frac{x}{a_2}\right), \\ \frac{\partial v_1}{\partial x} &= -\frac{\pi R^2 \rho a v_0 \rho_0 a_0^2}{m \rho_1 h \beta^3 (a_0^2 - a_2^2)} H\left(t - \frac{x}{a_0}\right) \varphi\left(t - \frac{x}{a_0}\right) + \\ &\quad + \frac{\pi R^2 \rho a v_0 \rho_0 a_0^3}{m \rho_1 h a_2 \beta^3 (a_0^2 - a_2^2)} H\left(t - \frac{x}{a_2}\right) \varphi\left(t - \frac{x}{a_2}\right). \end{aligned} \tag{20.34}$$

In the formulas (20.33), (20.34), H(z) denotes the unit Heviside function. The transverse shifts v and v_1 are expressed by the formulas:

$$\begin{aligned} v(x, t) &= \frac{\pi R \rho a v_0 k^2 a_1}{m\beta^4 (a_1^2 - a_2^2)} \left\{ H\left(t - \frac{x}{a_1}\right) \Phi\left(t - \frac{x}{a_1}\right) - \right. \\ &\qquad \left. - H\left(t - \frac{x}{a_2}\right) \Phi\left(t - \frac{x}{a_2}\right) \right\}, \\ v_1(x, t) &= \frac{\pi R^2 \rho a v_0 \rho_0 a_0^3}{m\beta^4 (a_0^2 - a_2^2)\, \rho_1 h} \left\{ H\left(t - \frac{x}{a_0}\right) \Phi\left(t - \frac{x}{a_0}\right) - \right. \\ &\qquad \left. - H\left(t - \frac{x}{a_2}\right) \Phi\left(t - \frac{x}{a_2}\right) \right\}. \end{aligned} \tag{20.35}$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

In these formulas the function $\Phi(t)$ has the form

$$\Phi(t) = -(\alpha+\beta)e^{-\beta t} - \frac{\alpha\beta^3}{6}t^3 + \beta^2\frac{(\alpha+\beta)}{2}t^2 - \beta(\alpha-\beta)t + (\alpha+\beta). \tag{20.36}$$

According to (20.5) the average stresses in the shell are defined as follows:

$$\tau = \rho_1 a_2^2\left(\frac{\partial v}{\partial x} + \frac{\partial v_1}{\partial x}\right),\quad \sigma_x = \rho_1\left(a_1^2\frac{\partial u}{\partial x} + k^2\frac{v+v_1}{R}\right),$$

$$\sigma_\theta = \rho_1\left(k^2\frac{\partial u}{\partial x} + a_1^2\frac{v+v_1}{R}\right).$$

The pressure of the liquid inside the shell is represented in the form

$$p = -\frac{\pi R^2\rho a v_0}{m\beta^2}\rho_0 a_0 H\left(t-\frac{x}{a_0}\right)f\left(t-\frac{x}{a_0}\right).$$

The penetration rate of the rigid bottom of the shell is defined by the formula (20.1)

$$\dot{u}_1 = -\left[v_0 + \frac{\pi R^2\rho a v_0}{m\beta^2}f(t)\right].$$

Hence, for acceleration of the rigid bottom of mass m, we obtain

$$\frac{\partial^2 u_1}{\partial t^2} = \frac{\partial^2 u}{\partial t^2} = \frac{\pi R^2\rho a v_0}{m}\left[\left(1+\frac{\alpha}{\beta}\right)e^{-\beta t} - \frac{\alpha}{\beta}\right].$$

Now let the penetrating cylindrical body be a continuous elastic rod of semi-infinite length of radius R. In the one-dimensional statement for the longitudinal shift u(x, t) we obtain the equation

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}. \tag{20.37}$$

Here the wave velocity is denoted by c

$$c^2 = \frac{E}{\rho_1}.$$

The initial and the boundary conditions of the problem will be

$$t = 0,\ u = \frac{\partial u}{\partial t} = 0, \tag{20.38}$$

$$t > 0,\ x = 0,\ -\pi R^2\sigma_x = f(t).$$

f(t) in the boundary condition (20.38) is defined by the formula (20.2). In the investigated problem the mapping U(x, s) of the shift u(x, t) has the form

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$U(x,s)=\frac{kv_0}{s^2}\,\frac{s^2-\frac{a}{R}+\frac{a^2}{4R^2}}{s^2-k\left(\frac{a}{R}s-\frac{a^2}{4R^2}\right)},\tag{20.39}$$

$$k=\frac{M}{M+\frac{v_0}{c}},\quad M=\frac{\rho a v_0}{\rho_1 c^2}.$$

The mappings of the deformation and the shift rate are defined by the correspondences:

$$\frac{\partial u}{\partial x}\div\frac{dU}{dx},\quad \frac{\partial u}{\partial t}\div sU.$$

Formula (20.39) shows that these mappings are simple, and their originals are defined without additional restrictions. As a result, for deformations and shift rate we have

$$\begin{aligned}\frac{\partial u}{\partial x}=&-\frac{kv_0}{c\omega}e^{N\left(t-\frac{x}{c}\right)}\left\{\sin N\omega\left(t-\frac{x}{c}\right)+\omega\cos N\omega\left(t-\frac{x}{c}\right)+\right.\\&+\frac{b^2}{4}\,\frac{1}{N^2(1+\omega^2)}\left[\sin N\omega\left(t-\frac{x}{c}\right)-\omega\cos N\omega\left(t-\frac{x}{c}\right)\right]-\\&\left.-\frac{b}{N}\sin N\omega\left(t-\frac{x}{c}\right)\right\}H\left(t-\frac{x}{c}\right)-\\&-\frac{kv_0}{c}\,\frac{b^2}{4}\,\frac{1}{N^2(1+\omega^2)}H\left(t-\frac{x}{c}\right),\quad \frac{\partial u}{\partial t}=-c\frac{\partial u}{\partial x},\\&b=\frac{a}{R},\quad \omega^2=\frac{\rho_1 c}{\rho a},\quad N=\frac{kb}{2}.\end{aligned}\tag{20.40}$$

The stress σ_x and the total velocity $\dot{u}_1$ of the particles of the elastic rod are defined as follows:

$$\sigma_x=E\frac{\partial u}{\partial x},\quad \dot{u}_1=-v_0+\frac{\partial u}{\partial t}.$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

§21. Estimating the Effect of Viscosity on Penetration of a Liquid by Solid States

In order to estimate the forces of viscosity on penetration of a liquid by solid states, let us use the well-developed boundary layer theory [39, 40]. First let us consider the problem of penetration by a semi-infinite plate with constant velocity v_0.

Let us select the origin of the moving system of coordinates at the edge of the plate, let us direct the ξ axis along the plate, the y axis along the normal to it (Figure 1.57).

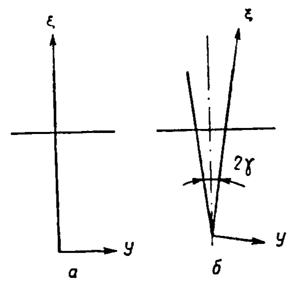

Figure 1.57.

In the moving system of coordinates the equations of motion and continuity in the boundary layer of a barotropic liquid without pressure gradient are written in the form

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial \xi} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}, \tag{21.1}$$

$$\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial y} = 0. \tag{21.2}$$

Here u, v are components of the relative velocity of the liquid with respect to the ξ and y axes, respectively; ν is the coefficient of kinematic viscosity of the liquid which is assumed to be constant. Let us proceed in equations (21.1)-(21.2) to the new variables:

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$u(\xi, y, t) = v_0\bar{u}(s, \eta),\ v(\xi, y, t) = \sqrt{\frac{\nu}{t}}\,\bar{v}(s, \eta),$$
$$s = \frac{\xi}{v_0 t},\ \eta = \frac{y}{\sqrt{\nu t}},$$

where $\bar{u}$ and $\bar{v}$ are the dimensionless components of the velocity.

Omitting the bar over the dimensionless variables, we obtain:

$$-s\frac{\partial u}{\partial s} - \frac{\eta}{2}\frac{\partial u}{\partial \eta} + u\frac{\partial u}{\partial s} + v\frac{\partial u}{\partial \eta} = \frac{\partial^2 u}{\partial \eta^2}, \tag{21.3}$$

$$\frac{\partial u}{\partial s} + \frac{\partial v}{\partial \eta} = 0,\ 0 < s \leqslant 1,\ 0 < \eta < \infty. \tag{21.4}$$

The boundary conditions for the equations (21.3)-(21.4) will be

$$u(s, 0) = 0 \quad \text{for} \quad \eta = 0,$$
$$u(s, \infty) = 0 \quad \text{for} \quad \eta = \infty.$$

For equations (21.3)-(21.4), it is possible to introduce the current function according to the equalities:

$$u = \frac{\partial \psi}{\partial \eta},\ v = -\frac{\partial \psi}{\partial s}. \tag{21.5}$$

Here the continuity equation is satisfied identically, and the substitution of expressions (21.5) in the equation of motion (21.3) leads to an equation for the current function:

$$-s\frac{\partial^2 \psi}{\partial s \partial \eta} - \frac{\eta}{2}\frac{\partial^2 \psi}{\partial \eta^2} + \frac{\partial \psi}{\partial \eta}\frac{\partial^2 \psi}{\partial \eta \partial s} - \frac{\partial \psi}{\partial s}\frac{\partial^2 \psi}{\partial \eta^2} = \frac{\partial^3 \psi}{\partial \eta^3} \tag{21.6}$$

with the boundary conditions

$$\psi(s, 0) = 0,\ \frac{\partial \psi}{\partial \eta} = 0 \quad \text{for} \quad \eta = 0,$$
$$\frac{\partial \psi}{\partial \eta} = 1 \quad \text{for} \quad \eta = \infty.$$

Let us represent the current function in the form

$$\psi(s, \eta) = \sqrt{s}\, f\left(\frac{\eta}{\sqrt{s}}\right). \tag{21.7}$$

Then for the longitudinal component of the velocity we obtain the expression

$$u = \frac{\partial \psi}{\partial \eta} = \frac{\partial \psi}{\partial \bar{y}}\frac{\partial \bar{y}}{\partial \eta} = f'(\bar{y}),\ \bar{y} = \frac{\eta}{\sqrt{s}}. \tag{21.8}$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

Substituting the expression for Ψ from the expression (21.7) in equation (21.6), for determination of the dimensionless current function f we obtain the Blasius equation:

$$2f''' + ff'' = 0. \tag{21.9}$$

The boundary conditions for the third-order ordinary differential equation (21.9) will be

$$\begin{aligned} f = 0,\ f' = 0 \quad &\text{for} \quad \bar{y} = 0, \\ f' = 1 \quad &\text{for} \quad \bar{y} = \infty. \end{aligned} \tag{21.10}$$

The equation (21.9) was integrated numerically, by the Runge-Kutta method, by many authors, and its solution has been tabulated [39].

By the tangential frictional stress on the plate we obtain the known expression

$$\tau = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\mu v_0}{\sqrt{\nu t}}\left(\frac{\partial^2 \psi}{\partial \eta^2}\right)_{\eta=0} = \frac{\mu v_0}{\sqrt{\nu t}}\,\frac{f''(0)}{\sqrt{s}}, \tag{21.11}$$

where μ is the coefficient of dynamic viscosity of the liquid which is assumed to be constant.

The force of frictional resistance is

$$F_\tau(t) = 2f''(0)\,\rho v_0^2 \sqrt{\nu t}\int_0^1 \frac{ds}{\sqrt{s}} = 4f''(0)\,\rho v_0^2 \sqrt{\nu t}, \qquad f''(0) = 0{,}33206. \tag{21.12}$$

Let us define the force of the resistance on penetration of a viscous liquid by a thin wedge. Neglecting the pressure gradient, for small half-apex angles of the wedge, the force of the frictional resistance F_τ is close to the force of resistance to penetration of the plate, and it is possible to consider that it is defined by the formula (21.12). In this approximation the total force of resistance to penetration of the wedge is

$$F(t) = F_n(t) + F_\tau(t). \tag{21.13}$$

Here F_n is the force of resistance on penetration caused by the component p_{yy} of the viscous stress tensor. It is possible to show that the value of $F_n(t)$ coincides with high accuracy with the force of resistance to penetration of an ideal liquid by a thin wedge. Actually, the expression for the component of the stress tensor p_{yy} has the form:

$$p_{yy} = -p_1(\xi, t) - \frac{2}{3}\mu\frac{\partial u}{\partial \xi} + \frac{4}{3}\mu\frac{\partial v}{\partial y}, \tag{21.14}$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

where $p_1(\xi, t)$ is the pressure in the boundary layer, u, v are the longitudinal and the transverse components of the velocity, respectively. Let us write the continuity equation for the problem of penetration of a compressible liquid by a thin wedge in the linear statement

$$\frac{1}{a^2}\frac{\partial p_1}{\partial t} + \rho\frac{\partial u}{\partial \xi} + \rho\frac{\partial v}{\partial y} = 0. \tag{21.15}$$

Substituting $\partial v/\partial y$ from equation (21.15) in equation (21.14), we obtain the expression for the stress p_{yy} for $y = 0$:

$$(p_{yy})_{y=0} = -p_1(\xi, t) - \frac{4}{3}\frac{\nu}{a^2}\frac{\partial p_1}{\partial t}. \tag{21.16}$$

The values of p_1 in the stationary coordinate system for incompressible and compressible liquids, respectively, are presented in §2 and §9. On substitution of p_1 in expression (21.16) it is first necessary to proceed to the stationary coordinate system.

The magnitude of the resistance force $F_n(t)$ is calculated by the formula

$$F_n(t) = 2\int_0^{v_0 t} (p_{yy})_{y=0} d\xi.$$

Using equality (21.16), after calculating the integral we shall have

$$F_n(t) = \frac{4}{\pi}\varphi(M)\rho v_0^2\gamma^2\left(v_0 t + \frac{2}{3}\frac{\nu M}{a}\right),\ t > 0,$$

$$\varphi(M) = \frac{\ln(1+\sqrt{1-M^2})}{\sqrt{1-M^2}},\quad 0 < M < 1, \tag{21.17}$$

$$\varphi(M) = \frac{\operatorname{arctg}(\sqrt{M^2-1})}{\sqrt{M^2-1}},\quad M \geqslant 1.$$

As is obvious, the second term in the parentheses in the right-hand side of formula (21.17), considering the effect of viscosity, is constant. For an incompressible liquid ($M = 0$), the force $F_n(t)$ does not depend on the viscosity. Introducing the depth of penetration $H = v_0 t$, we rewrite formula (21.17) in the form

$$F_n(t) = \frac{4}{\pi}\varphi(M)\rho v_0^2\gamma^2 H\left(1 + \frac{2}{3}\frac{\nu M}{aH}\right). \tag{21.18}$$

The depth H at which the second term in parentheses is 0.01 for water ($\nu = 10^{-6}$ m^2/sec, a = 1,500 m/sec, $M \approx 1$), is on the order of 10^{-8} m, that is, in reality when determining the component F_n of the resistance force the viscosity effect can be neglected. On penetration of an incompressible liquid by a wedge, from (21.17) we have

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$F_n(t) = \frac{4}{\pi} \ln 2\rho v_0^3 \gamma^2 t, \quad \varphi(0) = \ln 2. \tag{21.19}$$

For this case let us compile the ratio

$$\Phi(t) = \frac{F_n}{F_\tau} = \frac{\ln 2}{\pi f''(0)} \gamma^2 v_0 \sqrt{\frac{t}{\nu}}, \tag{21.20}$$

or, introducing the depth of penetration $H = v_0 t$,

$$\Phi(t) = \frac{F_n}{F_\tau} = \frac{\ln 2}{\pi f''(0)} \gamma^2 \sqrt{\frac{v_0 H}{\nu}}.$$

The depth of submersion at which F_τ is 10 percent of F_n is defined by the formula

$$H = \frac{100\pi^2 \nu [f''(0)]^2}{\gamma^4 v_0 \ln^2 2},$$

for a thin wedge with half-apex angle $\gamma = 10°$ penetrating water ($\nu = 10^{-6}$ m^2/sec) with a velocity 100 m/sec, this depth is $2.5 \cdot 10^{-3}$ m. For a depth of penetration $H = 2.5 \cdot 10^{-1}$ m the frictional force F_τ will be 1 percent of the resistance force F_n. The estimate of the effect of the pressure gradient on the frictional force on penetration by a thin wedge can be made by the Karman-Polhausen integral method [39, 40].

Let us introduce the thickness of the boundary layer δ into the investigation and let us proceed to the dimensionless variables by the formulas:

$$u = v_0 \bar{u}(s, \eta), \quad p_1 = \rho a v_0 \bar{p}_1(s), \tag{21.21}$$
$$\delta = \sqrt{\nu t}\, \bar{\delta}(s), \; s = \frac{\xi}{at}, \; \eta = \frac{y}{\sqrt{\nu t}}.$$

In these equalities the dimensionless parameters are noted by a bar at the top. The velocities of the external flow with respect to the wedge are expressed in terms of the absolute velocity of the disturbed motion of the liquid u_1 (see §2, 9) by the formula

$$u_\infty = v_0 \bar{u}_\infty = v_0 - u_1. \tag{21.22}$$

Let us select the distribution of the dimensionless velocity by the thickness of the boundary layer in the form

$$\bar{u}(s, \eta) = \frac{2\bar{u}_\infty(s)}{\bar{\delta}(s)} \eta - \frac{\bar{u}_\infty(s)}{\bar{\delta}^2(s)} \eta^2, \; 0 \leqslant \eta \leqslant \bar{\delta}(s), \tag{21.23}$$

then for the tangential stress on the wedge we obtain the formula

$$\tau = \frac{2\mu v_0}{\sqrt{\nu t}} \cdot \frac{u_\infty(s)}{\delta(s)}. \tag{21.24}$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

In formula (21.24) and below the bar over the dimensionless parameters is omitted.

In accordance with the Karman-Polhausen method for determination of the dimensionless thickness of the boundary layer $\delta(s)$, we obtain the equation

$$\left(\frac{s}{3}-\frac{2}{15}Mu_\infty\right)u_\infty\frac{d\delta}{ds}+\left(\frac{s}{3}\frac{du_\infty}{ds}-\frac{u_\infty}{6}+\right.$$
$$\left.+\frac{1}{3}Mu_\infty\frac{dp_1}{ds}+\frac{2}{5}Mu_\infty\frac{du_\infty}{ds}\right)\delta=-\frac{2u_\infty}{\delta}. \quad (21.25)$$

This ordinary first-order differential equation is solved under the condition

$$\delta(0)=0,\quad s=0. \quad (21.26)$$

On integration of equation (21.25) usually the new desired function $\lambda = \delta^2$ is introduced.

The force of the resistance F_τ^* is defined by the formula

$$F_\tau^*=2\int_0^{v_0t}\tau d\xi=4\rho v_0^2\sqrt{\nu t}\,(I_1+I_2). \quad (21.27)$$

Here in the case of a compressible liquid I_1, I_2 denote the integrals:

$$I_1(M,\gamma)=\frac{1}{M}\int_0^M\frac{ds}{\delta(s)},\quad I_2(M,\gamma)=-\frac{1}{M}\int_0^M\frac{u_1(s)\,ds}{\delta(s)},$$
$$s=\frac{\xi}{at}. \quad (21.28)$$

In the case of penetration into an incompressible liquid these integrals assume the form

$$I_1(\gamma)=\int_0^1\frac{ds}{\delta(s)},\quad I_2(\gamma)=-\int_0^1\frac{u_1(s)\,ds}{\delta(s)},\quad s=\frac{\xi}{v_0t}. \quad (21.29)$$

The dimensionless thickness of the boundary layer which enters into the term under the integrals (21.29) is defined from the differential equation

$$\left(\frac{s}{3}-\frac{2}{15}u_\infty\right)u_\infty\frac{d\delta}{ds}+\left(\frac{s}{3}\frac{du_\infty}{ds}-\frac{u_\infty}{6}+\frac{2}{5}u_\infty\frac{du_\infty}{ds}\right)\delta=$$
$$=-\frac{2u_\infty}{\delta},\quad s=\frac{\xi}{v_0t}. \quad (21.30)$$

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

In the absence of a pressure gradient the integrals I_2 are equal to zero. The equations (21.25) and (21.30) were integrated numerically by the Runge-Kutta method. In Table 1.3 with an apex angle of $\gamma = 10°$ the values of I_1 and I_2 are presented for different Mach numbers (M = 0 corresponds to the case of the incompressible liquid).

Table 1.3

M	$I_1(M, 10°)$	$I_2(M, 10°)$	F_τ^*/F_τ
0	0,3591	0,0711	1,2955
0,01	0,4074	0,0037	1,2408
0,3	0,3258	−0,0048	0,9666
0,6	0,3109	−0,0176	0,8832
0,9	0,2867	−0,0241	0,7910
1,0	0,2401	−0,0248	0,6484
1,2	0,2906	−0,0281	0,7905
1,5	0,2844	−0,0272	0,7745

In the last row of the table, the ratio of the force F_τ^* calculated by the formulas (21.27) to the force of resistance of the plate F_τ calculated by the formula (21.12) is presented. Table 1.3 shows that consideration of the pressure gradient leads to some variation of the force of frictional resistance by comparison with the plate. It is easy to show that this difference decreases with a decrease in the apex angle of the wedge. However, the variation of the frictional force caused by the pressure gradient does not change the order of the ratio of the forces (21.20).

The viscosity effect on penetration of a compressible liquid by blunt bodies can be estimated using the generalized Newton's law. In §12 the asymptotic solution was obtained for the problem of impact entry of a rigid plate (half-band) of width 2c into a liquid for the initial period of time (t < c/a, a is the speed of sound in the liquid). In the initial stage of submersion, the component p_{yy} of the stress tensor has primary influence on the resistance force, and the role of the tangential frictional forces is insignificant.

The expression for p_{yy} has the form [39]

$$p_{yy} = -p_1 - \frac{2}{3}\mu\frac{\partial u}{\partial \xi} + \frac{4}{3}\mu\frac{\partial v}{\partial y}. \tag{21.31}$$

it is assumed that $p_1(\xi, t)$ is the pressure known from the solution of the external problem (§12).

The continuity equation has the form

$$\frac{1}{a^2}\frac{\partial p_1}{\partial t} + \rho\frac{\partial u}{\partial \xi} + \rho\frac{\partial v}{\partial y} = 0. \tag{21.32}$$

Substituting the value of $\partial v/\partial y$ from equation (21.32) in expression (21.31), we obtain

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$(p_{yy})_{y=0} = -p_1 - \frac{4}{3}\frac{\nu}{a^2}\frac{\partial p_1}{\partial t}. \tag{21.33}$$

The magnitude of the resistance force for vertical penetration of a plate of width 2c with constant velocity v_0 is

$$F_n(t) = \int_0^{2c} p_1 d\xi + \frac{8}{3}\frac{\nu}{a^2}\int_0^{at}\frac{\partial p_1}{\partial t}d\xi = 2\rho a v_0 c\left(1 - \frac{at}{2c}\right) + \frac{4}{3}\mu v_0, \quad 0 < t < \frac{c}{a}. \tag{21.34}$$

It is possible to assign the following form to the formula (21.34):

$$F_n(t) = 2\rho a v_0 c\left[\left(1 - \frac{at}{2c}\right) + \frac{2}{3}\frac{\nu}{ac}\right], \quad 0 < t < \frac{c}{a}. \tag{21.35}$$

For t = c/a, from (21.35) we obtain:

$$F_n(t) = 2\rho a v_0 c\left[\frac{1}{2} + \frac{2}{3}\frac{\nu}{ac}\right].$$

For a plate with a width on the order of one ($2c \approx 1$) penetrating water, the value in brackets in formula (21.35) has the order $[(1/2) + 10^{-8}]$, that is, it is possible to neglect the forces of viscosity and set F_n equal to

$$F_n(t) = 2\rho a v_0 c\left(1 - \frac{at}{2c}\right), \quad 0 < t < \frac{c}{a}. \tag{21.36}$$

Let us note that in the problem of normal impact of a cylinder against a liquid surface (§19) the relative effect of the viscosity on the resistance force has the same order as in the two-dimensional problem of penetration of the plate, and it can be neglected.

The solution of the problem of vertical penetration by a thin cone obtained in §7 indicates that the cone introduces a weaker disturbance into the liquid than a thin wedge with the same apex angle and penetration velocity. The estimates analogous to the estimates made in the two-dimensional case indicate that the magnitude of the force $F_n(t)$ coincides with great accuracy with the magnitude of the force of resistance to penetration of an ideal liquid by a thin cone.

For determination of the tangential stress τ and the frictional force F_τ it is necessary to solve the problem of the boundary layer with the external flow velocity u_∞ equal to

$$u_\infty = v_0(1 - u_1), \tag{21.37}$$

where u_1 is the dimensionless velocity of the disturbed motion of an ideal liquid on the generatrix of a cone on the order of $o(\gamma^2)$ (§7). If we neglect the small

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

addition in expression (21.37) by comparison with one, we obtain the problem of the boundary layer with the velocity of the external flow equal to the penetration velocity of the cone v_0. In the meridional plane the origin of the moving coordinate system is placed at the apex of the cone, the ξ axis is directed along its generatrix, and the y axis is perpendicular to it (Figure 1.57b). Then for the assumption made above, the basic equations of axisymmetric boundary layer in the plane $yO\xi$ are written in the form

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial \xi}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}, \tag{21.38}$$

$$\frac{\partial (ru)}{\partial \xi}+\frac{\partial (rv)}{\partial y}=0. \tag{21.39}$$

Here u, v are components of the relative velocity of the liquid with respect to the ξ and y axes, respectively; $r(\xi)$ is the equation of the generatrix of the cone.

Let us introduce dimensionless parameters and independent variables by the following formulas into equations (21.38)-(21.39):

$$u(\xi, y, t)=v_0\bar{u}(s,\eta),\ v(\xi, y, t)=\sqrt{\frac{\nu}{t}}\,\bar{v}(s,\eta),$$

$$s=\frac{\xi}{v_0 t},\ \eta=\frac{y}{\sqrt{\nu t}},\ r(\xi)=\gamma\xi=\gamma v_0 ts.$$

Omitting the bar over the dimensionless variables, we have

$$-s\frac{\partial u}{\partial s}-\frac{\eta}{2}\frac{\partial u}{\partial \eta}+u\frac{\partial u}{\partial s}+v\frac{\partial u}{\partial \eta}=\frac{\partial^2 u}{\partial \eta^2}, \tag{21.40}$$

$$\frac{\partial (su)}{\partial s}+\frac{\partial (sv)}{\partial \eta}=0,\quad 0\leqslant s\leqslant 1,\quad 0\leqslant \eta\leqslant\infty. \tag{21.41}$$

The boundary conditions for the equations (21.40)-(21.41) will be

$u(s, 0)=0$ for $\eta=0$,

$u(s, \infty)=1$ for $\eta=\infty$.

Let us introduce the current function according to the equality

$$su=\frac{\partial \Psi}{\partial \eta},\ sv=-\frac{\partial \Psi}{\partial s}. \tag{21.42}$$

Substitution of expressions (21.42) in equation (21.40) leads to the equation for the current function

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$s^2 \frac{\partial \Psi}{\partial \eta} - s^3 \frac{\partial^2 \Psi}{\partial \eta \partial s} - \frac{\eta s^2}{2} \cdot \frac{\partial^2 \Psi}{\partial \eta^2} - \left(\frac{\partial \Psi}{\partial \eta}\right)^2 + s \frac{\partial \Psi}{\partial \eta} \cdot \frac{\partial^2 \Psi}{\partial \eta \partial s} - s \frac{\partial \Psi}{\partial s} \cdot \frac{\partial^2 \Psi}{\partial \eta^2} = s^2 \frac{\partial^3 \Psi}{\partial \eta^3} \quad (21.43)$$

with the boundary conditions

$$\Psi = \frac{\partial \Psi}{\partial \eta} = 0 \quad \text{for} \quad \eta = 0,$$

$$\frac{1}{s} \frac{\partial \Psi}{\partial \eta} = 1 \quad \text{for} \quad \eta = \infty.$$

Let us assume that the current function can be represented by the equality

$$\Psi(s, \eta) = s \sqrt{\frac{s}{3}}\, f\left(\sqrt{3} \frac{\eta}{\sqrt{s}}\right), \quad (21.44)$$

then for the dimensionless current function $f(z)$, $z = \sqrt{3}\ (\eta/\sqrt{s})$ we again have the Blasius equation (21.9) with the boundary conditions (21.10).

The tangential frictional stress on the surface of a cone is

$$\tau = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\mu v_0}{\sqrt{\nu t}} \cdot \frac{1}{s} \left(\frac{\partial^2 \Psi}{\partial \eta^2}\right)_{\eta=0} = \frac{\sqrt{3}\mu v_0}{\sqrt{\nu t}} \cdot \frac{f''(0)}{\sqrt{s}}. \quad (21.45)$$

For the force of frictional resistance on penetration of a viscous liquid by a thin cone we obtain:

$$F_\tau(t) = 2\pi\gamma \int_0^{v_0 t} \tau \xi d\xi = \frac{4}{\sqrt{3}} \pi f''(0) \rho v_0^3 \gamma t \sqrt{\nu t}, \qquad f''(0) = 0{,}33206. \quad (21.46)$$

The ratio of the force of resistance $F_n(t)$ during subsonic penetration of an ideal liquid by a thin cone (formula (7.22)) to the force F_τ is $(0 \le M < 1)$

$$\Phi(t) = \frac{F_n(t)}{F_\tau(t)} = \frac{\sqrt{3}\,\gamma^2 v_0}{4 f''(0)} \left(\ln \frac{1}{2\gamma}\right) \sqrt{\frac{t}{\nu}}. \quad (21.47)$$

For $v_0 = 100$ m/sec, $\gamma = 10°$, the inverse of this ratio is 10 percent at a depth of $H = v_0 t = 1.9 \cdot 10^{-2}$ m and 1 percent for $H = 1.9$ m. These estimates are retained also for supersonic penetration of the thin cone investigated in §7. Consideration of the pressure gradient by the Karman-Polhausen method leads to the following equation for the thickness of the boundary layer:

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

$$\left(\frac{s}{3}-\frac{6}{15}Mu_\infty\right)u_\infty\frac{d\delta}{ds}+\left(\frac{s}{3}\frac{du_\infty}{ds}-\frac{u_\infty}{6}+\frac{1}{3}Mu_\infty\frac{dp_1}{ds}-\right.$$
$$\left.-\frac{2}{15}\frac{Mu_\infty^2}{s}+\frac{2}{5}Mu_\infty\frac{du_\infty}{ds}\right)\delta=-\frac{2u_\infty}{\delta},\quad s=\frac{\xi}{at}. \tag{21.48}$$

Here the dimensionless variables δ, u_∞, p_1 are defined by the formulas (21.21). The equation (21.48) is solved under the condition $\delta(0) = 0$. The frictional stress on the generatrix of the cone is defined by the formula (21.24), where $\delta(s)$ is the thickness of the boundary layer on the generatrix of the cone. For the frictional resistance force we obtain the expression

$$F_\tau^* = 4\pi\rho v_0^3 t\sqrt{\nu t}\,(I_1 + I_2), \tag{21.49}$$

where I_1 and I_2 denote the integrals

$$I_1=\frac{1}{M}\int_0^M\frac{sds}{\delta(s)},\quad I_2=-\frac{1}{M}\int_0^M\frac{u_1(s)\,sds}{\delta(s)}. \tag{21.50}$$

For subsonic penetration of a compressible liquid by a cone ($M < 1$) as demonstrated in §7, the external solution does not depend on the Mach number, and it coincides with the corresponding solution of the problem of penetration of an incompressible liquid by a thin cone. In this case the thickness of the boundary layer is determined from the equation

$$\left(\frac{s}{3}-\frac{2}{15}u_\infty\right)u_\infty\frac{d\delta}{ds}+\left(\frac{s}{3}\frac{du_\infty}{ds}-\frac{u_\infty}{6}-\frac{2}{15}\frac{u_\infty^2}{s}+\right.$$
$$\left.+\frac{2}{5}u_\infty\frac{du_\infty}{ds}\right)\delta=-\frac{2u_\infty}{\delta},\quad s=\frac{\xi}{v_0 t}, \tag{21.51}$$

and the integrals I_1 and I_2 in formula (21.50) assume the form

$$I_1=\int_0^1\frac{sds}{\delta(s)},\quad I_2=-\int_0^1\frac{u_1(s)\,sds}{\delta(s)}. \tag{21.52}$$

The equation (21.51) was numerically integrated by the Runge-Kutta method. As a result of the calculations, the following values were obtained for the integrals (21.52) and the ratio of the force F_τ^* to F_τ calculated by the formulas (21.49) and (21.46), respectively:

$$I_1 = 0{,}1628,\; I_2 = -0{,}0038,\; \frac{F_\tau^*}{F_\tau} = 0{,}8294. \tag{21.53}$$

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10845
CSO: 8144/0496

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

ORIGINAL TABLE OF CONTENTS

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

APPROVED FOR RELEASE: 2007/02/08: CIA-RDP82-00850R000300100006-1

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* The remainder of the book was not translated.

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