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Translation

TROOP CONTROL THROUGH PERT METHODS

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TROOP CONTROL THROUGH PERT METHODS

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[Book by Pavel Grigor'yevich Skachko, Vladimir Maksimovich Kulikov, and Georgiy Kitovich Volkov entitled "Troop Control With the Aid of Program Evaluation and Review Technique Methods," Voyenizdat, 13,000 copies, 143 pages; annotation, foreword, conclusion, and table of contents translated and published in JPRS 63371, 5 Nov 74; UDC: 358: 119:355.5(07)]

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[III - USSR - 4 FOUO]

FOREWORD TO THE SECOND EDITION*

In the comparatively short period of time which has passed since practical incorporation of initial experience with the program evaluation and review technique (PERT) method, its effectiveness has been graphically revealed in the most diversified areas of military activity, both in planning small processes pertaining to servicing and readying combat equipment for utilization, and in planning overall combat operations of subunits and units.

Experience has shown that two conditions must be observed for effective application of the PERT method in any program:

1) the basic elements of the PERT method (including preliminary project analysis, construction of a network schedule, determination of time and other indices) should be thoroughly studied;

2) the PERT method can be practically adopted only following thorough training of personnel, with a full comprehension of control objectives and complexity of the processes being planned; at the same time it is essential to have a precise idea of means and available time when drawing up network schedule plans.

Experience has confirmed that a superficial attitude toward PERT method invariably leads to discredit and project failure. Systematized and sequential study and dissemination of the PERT method, as well as thoroughly thought-through organization of work with network schedules is a guarantee of success in working with application of program evaluation and review technique.

Amassed experience in application of a PERT system indicates that strict observance of the following conditions is essential from an organizational-practical standpoint for successful mastery of this system:

thorough study, by commanders of all echelons, of the fundamentals of planning with the employment of critical-path methods;

* The First Edition was published in 1968, under the title "Planirovaniye boyevykh deystviy i upravleniye voyskami s pomoshch'yu setevykh grafikov" [Planning Combat Operations and Troop Control With the Aid of Network Schedules].

ability of staff officers to construct network schedules applicable to the specific conditions of military life and activities;

precise execution of all measures specified by network schedules.

Organization and implementation of a program evaluation and review technique system. As experience shows, the entire PERT work cycle breaks down into two stages:

construction of a network schedule plan and its analysis for the purpose of determining available reserves of time and resources (as a rule this job contains several sequential stages, at each of which a better plan is formulated than the plan produced at the previous stage);

regular comparison of the plan with the actual course of progress of the process, since following final adoption it becomes a concrete project calendar network schedule and is utilized for improving activities pertaining to direction and monitoring of process execution; bottlenecks in the project process are revealed in the course of such an analysis, resources are redistributed, and the schedule is revised up to the end of the process; execution of these cycles is of fundamental importance.

The first edition of this volume was useful for initial adoption of program evaluation and review technique methods into the practical activities of troops, staffs, and military establishments.

In this present edition the fundamentals of program evaluation and review technique are presented in a simple and easily understandable form, and recommendations are given on its application with a number of practical examples.

As follows from the very essence of the PERT system, the authors emphasize that it is an aggregate of computation methods, organizational measures and monitoring techniques, taking into account the complex processes of combat activities of troops and staffs. Following are the end objectives of employment of a PERT system:

elucidation and mobilization of reserves of time and resources hidden in efficient organization of combat operations;

exercise of control of combat operation processes according to the principle of "leading element," with prediction and prevention of potential malfunctions in the course of execution of combat operations, and improvement of the combat indices of planned processes connected with the conduct of combat operations;

improvement in the effectiveness and efficiency of control as a whole, with clear-cut distribution of responsibility among commanders of different levels and their staffs.

One of the specific features of a PERT system is utilization of a new, highly sophisticated form of plan representation. It is proposed that the entire process of conduct of combat operations be depicted in a single network flowchart, which not only substantially facilitates perception of substance of the process but also greatly facilitates the entire subsequent process of direction of combat operations during their execution. A network model of combat operations provides much more information than the presently employed combat operations planning documents.

A high degree of planning and control objectivity, considerable flexibility and efficiency, and creation of conditions for rapid and efficient management are the principal characteristic features of a program evaluation and review technique system. A high degree of objectivity is achieved due to precise plan revision and adjustment in the course of combat operations, which makes it possible to examine adopted decisions taking account of the actual state of affairs, to obtain forecasts of the future, to provide for the subsequent development of events, and to foresee possible departures from the schedule and influence of these deviations from execution of subsequent operations and for a finite period of time.

A program evaluation and review technique system makes it possible quantitatively to measure the degree of uncertainty inherent in any process connected with the conduct of combat operations. In controlling combat operations, the commander as a rule encounters a large number of surprises which are difficult to foresee in advance. PERT methods make it possible to determine the potential nature of actions by the enemy and enable one to be prepared to localize these unexpected occurrences.

A program evaluation and review technique system compels the commander to concentrate his attention and efforts in those areas which are a bottleneck at a given moment and threaten failure to meet the timetable of execution of the combat mission and demand immediate correction. At the same time other operations, although not diverting the commander's attention, do not get out from under his observation and control.

A PERT system helps the commander separate the main from secondary matters and precisely to specify those tasks which are performed at each level of leadership.

Control in a PERT system is based on a specially created flow of current information, which continuously or periodically is received by the commander.

A network model forms the basis of a PERT system -- a graphic representation of a combat operations plan, which in the literature is called a network schedule or network chart. It is precisely the type of employed model which determined the very name of the system. Theory of linear complexes constitutes the mathematical foundation of program evaluation and review technique [literally: network planning and management (control)] method. The network method of planning and management, in contrast to other methods of investigation, does not require a special mathematical structure, and the terms and concepts which one encounters in employing this method are perceived almost intuitively. Because of this, the PERT method is easily mastered.

In this, second edition the authors have expanded the basic terms and concepts of critical-path, program evaluation and review technique method, have made the material more easily understandable, have added additional terms and definitions, have simplified a number of methods, and have employed a number of new examples for demonstrating the range of application of PERT methods in military affairs, particularly in the area of direct planning and control of combat operations.

In this volume the authors present examples applying to the battalion level. It did not make sense from a methodological standpoint to cite examples of a larger scale, since this would sharply increase the amount of labor required and would make it somewhat difficult to understand the substance of the matter. The principles of application of the PERT method are the same for any echelon, and therefore the reader, after gaining an understanding of the method, will be able to apply it in planning any complex process and scientific management and control of that process.

In this second edition the authors have succeeded in stressing the fundamental idea of program evaluation and review technique -- the constructed network model should be adequate to the system being simulated. They have drawn the reader's attention to the fact that construction of a network model is one of the most difficult and critical tasks of simulation and modeling.

The authors have elaborated and presented in a very comprehensible form methods of manual calculation of network schedules, which is extremely important when working in field conditions.

One should bear in mind that the more complex the planned process, in which large numbers of personnel take part, the greater the results produced by the PERT method. The method of constructing scale network schedules proposed by the authors furnishes commanders and staffs precisely with that tool which enables them to construct a flawless system of interaction and coordination of the personnel and resources taking part in combat operations.

Further serious and thorough study of this scientific method, which enables one to reach optimal solutions in many areas of troop combat activities, and extensive adoption of this method in combat activities and daily troop routine will unquestionably produce considerable results.

Lt Gen Tank Trps Prof G. T. Zavizion,
Doctor of Military Sciences

Chapter I. GENERAL CONCEPTS AND ELEMENTS OF A PROGRAM EVALUATION AND REVIEW TECHNIQUE SYSTEM. PROCEDURE OF CONSTRUCTING NETWORK SCHEDULES

1. General Method Concepts

1. Description of Program Evaluation and Review Technique Method

At the present time, in light of the resolutions of the historic 24th CPSU Congress, there is taking place in the nation's economy rapid development of a new direction in organization of the complex processes connected with expenditure of manpower, material, energy and other resources. Critical-path method or program evaluation and review technique method is experiencing the greatest dissemination among the most important new forms and methods of improving management. The successful and rapid spread of this method literally in almost all areas of human activity is due first and foremost to its great advantages over planning methods based on strip or linear charts.

These advantages consist in the following.

1. Employment of the program evaluation and review technique on the basis of better, logically and mathematically substantiated work organization, as practical experience has shown, generates significant economy of manpower, time and resources, ensures planning and monitoring of complex projects* simultaneously in several directions, makes it possible to eliminate from the area of intensified monitoring these activities which do not influence prompt and timely accomplishment of the task as a whole, and promotes finding bottlenecks and overcoming them in a prompt and timely manner.

In military affairs employment of program evaluation and review technique, on the basis of precise calculations and analysis of logical and process relations, also makes it possible to gain time and to economize in manpower and resources in solving problems pertaining to many matters of logistic support and troop combat training.

* The term project in program and evaluation review technique method (PERT) is defined as a specific, prior established aggregate of all operations which must be performed in order to achieve the stated goal. We shall also encompass with this term the planning of any combat operations at all echelons.

2. Network schedules furnish a graphic and easy-to-perceive representation of a plan, elaborated both as a whole and part by part. They make it possible to revise decisions taking into account the actual state of affairs, to obtain forecasts, and to predict possible departures from the plan and their consequences which may affect the plan execution timetable. Network schedules will make it possible quantitatively to measure the uncertainty characteristic of any plan. A program evaluation and review technique system (PERT*) makes it possible extensively and efficiently to employ electronic computer hardware.

3. Program evaluation and review technique, utilizing network schedules as project models, makes it possible precisely to represent the volume and extent of the problem; to elucidate with any degree of detail the operations involved in a project; to establish the interrelationship between these operations; to determine the events occurrence of which is essential in order to achieve the stated partial and end objectives; precisely to distribute responsibilities among project participants; to exclude the possibility of omitting operations which are objectively necessary in order to achieve project goals.

4. Program evaluation and review technique makes it possible more extensively to utilize in planning activities the experience of the most competent and highly trained project personnel, as well as practically verified statistical data in order to achieve the most realistic estimate of requirements in manpower and resources required for performance of operations; in advance, in the course of project model analysis, to locate hidden reserve potential and to specify ways to utilize this potential and, in particular, ways to utilize noncritical project resources, channeling them toward acceleration of critical operations, achieving accomplishment of the entire project in a shorter period of time and with less expenditure of manpower and material resources.

5. Program evaluation and review technique makes it possible to employ a simple method of introducing changes, refinements and additions to project plans, which leads to flexibility and continuity of planning and ensures simplification of information and the system of record keeping, as well as rapid project enlistment of new management personnel and uninterrupted control when replacing project managers. When employing electronic computers, program evaluation and review technique ensures rapid computation of a large number of project plan variants, from which the optimal variant is selected.

Thus program evaluation and review technique makes it possible to obtain scientifically substantiated answers to the most important questions which arise during planning and coordination of many interrelated projects.

Concluding this general description of program evaluation and review technique, we must note that it can be employed regardless of the scale and complexity of the project. Program evaluation and review technique produces the greatest effect in complex projects, in complex dynamic controlled systems, where extensive employment of electronic computers is possible. But in addition, program evaluation and review technique also produces substantial positive effect in executing extremely

* Henceforth the term program evaluation and review technique will sometimes be abbreviated to PERT for the sake of brevity.

small-scale projects which include only several dozen events. For example, program evaluation and review technique can be successfully applied in elaborating not only an entire system of troop combat training or combat readiness measures but also individual system elements.

2. Division of a Program Evaluation and Review Technique System

Program evaluation and review technique is directly related to cybernetics as the science of optimal control of complex dynamic systems.

We know that any control, both in the national economy and military affairs, presupposes the existence of:

a controlled object (controlled system);

environment (situation conditions);

a system or device acting upon the controlled object (controlling system).

The object of control can be machines, persons, and military subunits (establishments).

The most characteristic objects of control under present-day conditions are objects with a substantial number of collective bodies participating in an operation (project) and with a large number of operations, that is, complex dynamic controlled systems. The complexity of a controlled system is characterized by the number of elements, nature and number of relations between them, as well as number of various possible states of the system. The dynamic nature of a system is manifested in constant change of states, environment, as well as change in parameters.

Environment (situation conditions) is defined as external influences on the object of control, the conditions in which a controlled system is operating and will be operating in the process of achieving the designated goal. The environment for military troops is the concrete situation in which they are operating, that is, the degree of hostile activity, conditions of terrain and season, weather, logistic support, etc. System parameters change together with a change in external influences, as a consequence of which attainment of the ultimate goal is made more difficult or facilitated.

Control system is defined as a control agency (headquarters staff, establishment, factory management, etc) which possesses an appropriately developed network of subordinate control agencies (headquarters staffs, establishments, shops, sections) or individual executors (commanders). In other words, it is a ramified management (administrative) edifice which possesses the requisite resources and equipment for executing development management functions (troop actions, design activities, or production process). A control system, in conformity with the principles of modern cybernetics, should possess the same degree of ramification as the controlled system in order to provide for the control process. The operating effectiveness of a control system depends first and foremost on the quality of operation of the information service.

Control process research investigations conducted both here and abroad indicate that approximately 40 percent of the time expended by the manager and his support edifice goes for acquisition, study and analysis of information, 12 percent on decision-making, 30 percent on giving instructions and orders and communicating assignments to executing personnel, and 18 percent on verifying execution.

Thus collecting and processing information take up the greatest amount of time in control and management. This fact has influenced the classification and division of control systems.

As practical experience has indicated, control systems can be classified by purpose, control parameters, technical level, and by types of models. Such a division also occurs in a program evaluation and review technique system.

Program evaluation and review technique systems are subdivided by purpose into single-function and multipurpose.

A single-function system is characterized by an aggregate of actions aimed at achieving one specific goal, although many individual executants or subunits may take part in this aggregate of actions; but they have a single goal, such as to ensure a high degree of subunit combat readiness, to plan and execute a battalion exercise in a prompt and timely manner, etc.

A multipurpose system is employed when it is necessary to control the activities of a number of subunits (units) of different arms, which are pursuing different goals within a unified aggregate of missions.

Program evaluation and review technique systems are subdivided by control parameters into systems based only on time parameters or various parameters in combination: time, cost, resources, and technical-economic indicators.

The time parameter is considered in all cases.

The following eight versions of control system can be listed, in relation to the combination of various parameters:

1) PERT -- time;

2) PERT -- time-cost;

3) PERT -- time-resources;

4) PERT -- time-technical and economic indicators;

5) PERT -- time-cost-resources;

6) PERT -- time-cost-technical and economic indicators;

7) PERT -- time-resources-technical and economic indicators;

8) PERT -- time-cost-resources-technical and economic indicators.

In practice PERT systems with the time parameter are most common.

All PERT systems can be characterized, on the basis of technical level, as machine systems, based on extensive employment of various means of mechanization, computer hardware, communications, and television.

Five levels of a PERT control system are distinguished.

1. Zero level, which is characterized by employment of only the simplest means of mechanization of management labor and the simplest computer hardware. At this level there is no control system in the broad meaning of the term.

2. First level -- this represents a qualitative leap forward in development of a planning and management system, although externally this level differs little from the zero level in scale of application of means of mechanization and computer hardware.

The first level is characterized by regular arrival of input data and their processing according to a specified program, by regular utilization of output information by management, and by uniformity of input and output information, as a rule only in one parameter.

3. The second level is characterized by employment of high-powered electronic computers with highly developed machine memory systems. Information based on diversified parameters is utilized in second-level control systems.

4. The third level is characterized by the creation of special integrated systems with simulation, as well as with managerial training. Control systems of this level are distinguished by exceptionally high information service performance quality.

5. The fourth level constitutes development of third-level systems. Fourth-level control systems provide for modeling/simulation, playing through a process, managerial training, and automation of part of the decision-making process.

Control systems are broken down on the basis of types of models into systems employing linear models and systems employing network models. Model in this context is defined as a plan of development prepared in such a manner that it most fully reflects the entire course of events pertaining to achieving the stated goal under given conditions.

Graphic methods of modeling are the most common in practice. They are more versatile and visually graspable. Graphic methods of modeling (planning) can also be extensively employed in practical troop activities.

Examples of such modeling include various calendar schedules of events, combat training schedules, plans for conduct of field exercises and other documents drawn up on maps or in the form of diagrams and charts with the necessary explanatory notes. One version of graphic modeling of combat operations employed for organization of coordination, training of officers and for other purposes is sandbox simulation of a possible combat situation.

In the past, before the appearance of network schedules, various versions of linear charts (Figure 1) were in widespread use and continue to be employed today.

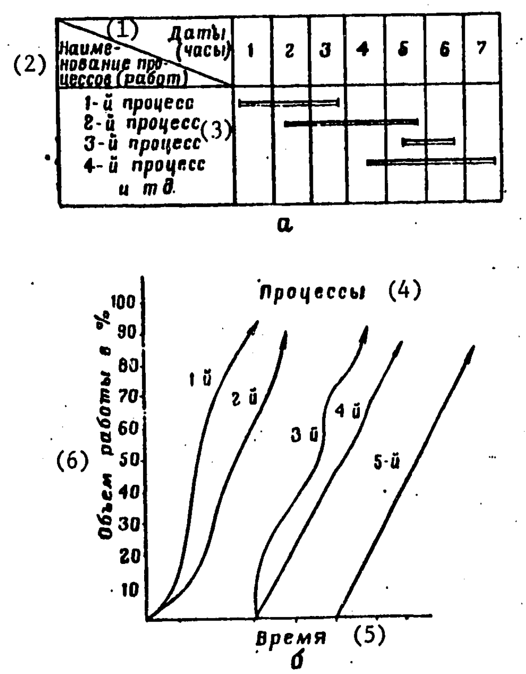

Figure 1. a -- linear; b -- cyclogram

Key:

1. Dates (hours)
2. Designation of process (operations)
3. 1st process, 2nd process, ... etc
4. Processes
5. Time
6. Volume of work, as percentage

Alongside the positive aspects of linear charts and cyclograms (graphic expression of assigned tasks, sequence and timetable of execution), however, they contain serious drawbacks. Following are the principal deficiencies:

inadequate reflection of the links between processes and operations and their interdependence;

a particularly static approach to their preparation, since the solutions embodied in them take on a congealed form; the schedule is disrupted and becomes unrealistic;

limited possibilities of predicting and monitoring the course of project execution;

the composition of operations and the timetable for their execution are designated unambiguously, which inevitably leads subsequently to revision of the plan model, which is very difficult to do on linear charts;

linear models do not reflect that uncertainty which is inherent in the processes and project as a whole.

Therefore it became necessary to construct a model which would make it possible to carry out with great effectiveness planning, monitoring, plan correction and revision, control, and extensively to employ electronic computers.

Network schedules and a program evaluation and review technique system, the virtues and advantages of which were discussed above, are fully in conformity with these requirements. We should emphasize here, however, that network schedules, while providing a high degree of objectivity in planning and forecasting the course of project development, as well as a high degree of flexibility and efficiency of management, do not exclude in control and management employment of linear charts, cyclograms, and particularly graphic schedules of troop combat operations worked out on maps or diagrams.

2. Fundamentals of Construction of Network Models

A program evaluation and review technique system is based on graphic representation of the project plan in the form of an arrow logic diagram (network schedule), in which the entire aggregate of operations is broken down into separate, clearly defined operations. The network schedule presents the logical interrelationship and interconditionality of all operations and the sequence of their execution, from the beginning to the final goal of the project.

3. Elements of Network Schedules

Let us examine the basic concepts, definitions and terms required to master program evaluation and review technique.

In constructing network schedules, one proceeds from three basic concepts: work, event, and path.

Work is any labor process or action accompanied by expenditures of time and resources. For example, loading ammunition at a military supply facility, march by a subunit (unit) into a concentration area, etc.

The term "work" also includes waiting (a passive process), which requires neither expenditure of labor nor resources. For example, day or night halt for personnel during execution of a march, waiting one's turn to cross a water obstacle on a ferry or bridge in conformity with the crossing schedule, etc.

The term "work" is also defined as a simple relationship between two or more measures (processes). This will be fictitious, or empty work, which requires neither the expenditure of time, labor, nor resources.

The duration of work, just as any process, can be measured quantitatively in units of time: minutes, hours, days, etc. Work can also have other quantitative evaluations, such as laboriousness/labor intensiveness, cost, expenditure of material resources, etc. In addition, each job should have a designation, which reveals its content, such as "Mission Briefing," "Situation Estimate," "Alert-Warning Subunits," "Loading Ammunition on Alert," "Unit March From Point A to Point B," "Unit (Subunit) Halt During March," etc.

An event is the result of a given process, the intermediate or end result of performance of one or several preceding work operations, which makes it possible to proceed to subsequent operations. For example, assembly and formation of columns of subunits on a combat alert at permanent basing locations are completed, which makes it possible to commence their movement to the designated concentration area.

Thus an event, in contrast to a work operation, is not a process, has no duration, and is not accompanied by any expenditures (time, resources).

An event on network schedules is usually represented by circles, work by solid arrows, and empty work (dependence) by dashed arrows (Figure 2).

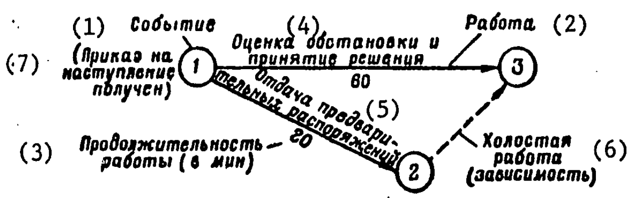

Figure 2. Representation of Events and Work Operations in a Network Chart

Key:

1. Event
2. Work
3. Duration of work (in minutes)
4. Situation estimate and decision-making
5. Issuing of warning orders
6. Empty work (dependence)
7. Attack order received

The arrows are not vectors. They can be of any length and direction.

Any work operation (arrow) on a network chart joins only two events and represents the process of transition of one event to another. The event from which arrows originate is called initial (or preceding); an event at which arrows end is called terminal (or succeeding) for the given work operations. One and the same event may alternately be preceding and succeeding. The initial event for an entire network is called the starting event, while the terminal event of an entire network is called the concluding event.

Work operations in a network are usually coded by the numbers of the events between which they run. For example, the work operation "Mission Briefing" (Figure 3) will be coded (1, 2), while operation "Time Calculation" may be assigned code (2, 3).

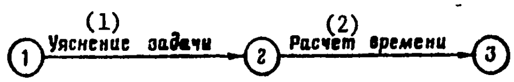

Figure 3. Coding Work Operations by Events

Key:
1. Mission Briefing
2. Time Calculation

In Figure 3 event (2) is the succeeding event for operation (1, 2) and the preceding event for operation (2, 3).

One and the same event may terminate several work operations or only one operation, while one or several operations may also originate from it. Therefore occurrence of an event may depend on completion of one or several work operations respectively.

It is evident from Figure 4 that operations (d and e) can begin only if operations (a, b and c) are completed.

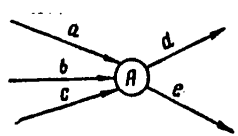

Figure 4. Dependence of Operations

All work operations in a network chart differ in character and are of differing duration.

Thus an event is considered accomplished only when the most protracted of all operations terminating at that event is completed.

The internal relationship between operations in a network schedule is determined by observance of a fundamental rule: in a network schedule all operations are interlinked -- the commencement of a succeeding operation is dependent upon completion of a preceding operation. It follows from this rule that in a network schedule there cannot be a single operation which is not connected at its initiation and termination to other operations through events. In other words, there can be no events in a network schedule the commencement of which would not signify the termination of at least one operation and simultaneously the commencement of another.

Starting and concluding events are an exception. A starting event does not have a preceding operation and is initial for the entire program (for example, "March Order Received," "Signal to Sound Unit Combat Alert Received"), while a concluding event signifies a conclusion to the entire aggregate of project measures (for example, "Regiment Ready to Carry Out Combat Mission").

Each operation contained in the schedule should possess quite specific content. One should not, for example, designate an operation as "Commencement of Ammunition Loading" or "Conclusion of Allocation of Missions to Subunits," for subjective notions about the terms "Commencement" and "Conclusion" are possible here.

In some cases accomplishment of part of some work operation is a condition for commencement of one or several other operations. In this instance the operation can be broken down into two or several independent segments, concretely specified by time (resources). Each segment of the divided operation should be viewed as an independent operation.

It is just as important precisely to formulate the designation of events. It is incorrect, for example, to designate an event "Conclude Situation Estimate." Such an event should be designated "Situation Estimate Concluded." A precise formulation of events concentrates the attention of schedule preparers and persons supervising and managing the project not only on what operations should be performed to attain the project final objective but also on what should be the result of each operation or group of operations preceding a given event and in what concrete form an operation (operations) should be concluded so that the following operation can commence.

In a network schedule there should be no closed loops, that is, circular couplings, since such a coupling can be logically absurd. The linkage of operations shown in Figure 5, for example, cannot occur in practice, since situation estimate performed following time calculation cannot precede mission briefing.

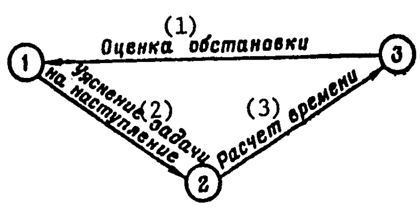

Figure 5. Closed Loop

Key:
1. Situation estimate
2. Attack mission briefing
3. Time calculation

A path is any uninterrupted logical (technological) sequence of work operations (chain of operations) from the starting (first) to the concluding event, that is, from commencement of plan elaboration to the final goal. Here one should bear in mind that no path can pass twice through the same event; any path may run along an empty operation; several paths can pass through one and the same event.

Path length is determined by the sum total duration of operations lying on that path. There may be many paths from the starting to the concluding event. As a result of preparation and analysis of a network schedule, one reveals that path the total duration of operations on which will be maximum. This path is called critical. It determines the requisite time of execution of all operations contained in the network schedule.

All operations lying on the critical path are also critical operations. The duration of these operations determines duration of the critical path. Shortening critical operations or extending the timetable of operations will correspondingly lead to a shortening or lengthening of the path. Thus all critical operations are potentially a plan bottleneck.

A network may contain several critical paths. Paths close in time to critical are called subcritical. All other paths, and these are the majority, differ downward significantly in duration from critical and subcritical paths. Such paths are called unstressed (or noncritical) paths.

Operations lying on a critical path are designated by heavy lines or a double (colored) line, as is shown in Figure 6. Designation on the chart of operations which are on a critical path makes it possible to present in graphic form that sequence of operations which determines the total task accomplishment time. This is especially important when analyzing complex plans, execution of which involves the participation of a large number of executants.

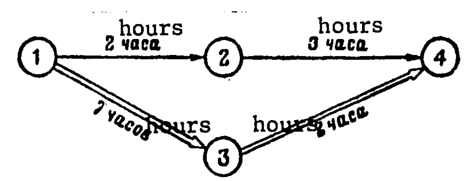

Figure 6. Representation of a Critical Path (path 1, 3, 4 is critical)

Six complete paths can be found on the chart (Figure 7), depicting an aggregate of principal measures pertaining to readying a subunit for an attack. In order to determine which of these paths is critical, their duration must be compared:

$$L_1 = (0, 1, 3, 7, 9);\quad t(L_1) = (5 + 5 + 60 + 20) = 90 \text{ min};$$
$$L_2 = (0, 1, 3, 5, 7, 9);\quad t(L_2) = (5 + 5 + 0 + 60 + 20) = 90 \text{ min};$$
$$L_3 = (0, 1, 2, 5, 7, 9);\quad t(L_3) = (5 + 15 + 60 + 60 + 20) = 160 \text{ min};$$
$$L_4 = (0, 1, 2, 6, 8, 9);\quad t(L_4) = (5 + 15 + 15 + 30 + 15) = 80 \text{ min};$$
$$L_5 = (0, 1, 4, 6, 8, 9);\quad t(L_5) = (5 + 5 + 0 + 30 + 15) = 55 \text{ min};$$
$$L_6 = (0, 1, 4, 8, 9);\quad t(L_6) = (5 + 5 + 30 + 15) = 55 \text{ min}.$$

The third of these paths is the longest, and it will be the critical path. The first and second paths are subcritical. The remaining paths in this network schedule will be unstressed (noncritical).

In addition to complete paths, the following are also distinguished in a network schedule:

path preceding or following a given event; the path from an initial event to a given event is called preceding, and a path linking this event with the concluding event -- path following the given event;

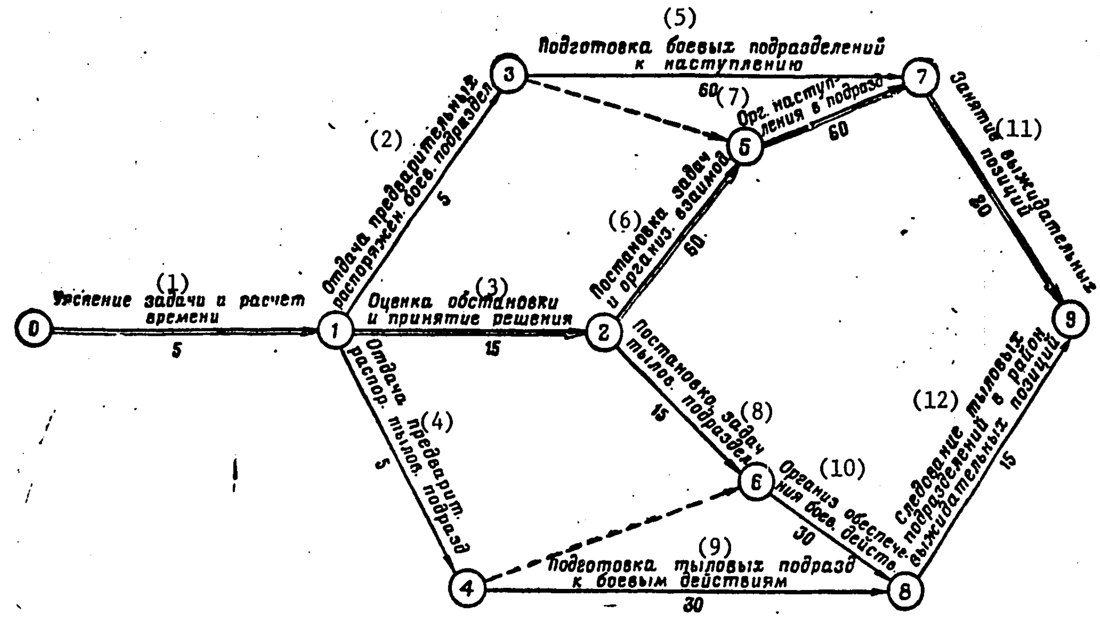

Figure 7. Paths in a Network Schedule

Key:

1. Mission briefing and time calculation
2. Issuing of warning order to combat subunits
3. Situation estimate and decision-making
4. Issuing of warning order to rear services subunits
5. Readying of combat subunits for attack
6. Allocation of tasks and organization of cooperation and coordination
7. Organization for attack in the subunits
8. Allocation of tasks to rear services subunits
9. Readying of rear services subunits for combat operations
10. Organization of support of combat operations
11. Take up position in assembly area
12. Movement by rear services subunits to assembly area

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a path between any two events in the chart, neither of which is a starting or concluding event.

In the chart (Figure 7), for example, path (0, 1, 2) will be preceding for event (2); paths (2, 5, 7, 9) and (2, 6, 8, 9) will be succeeding for this event. Paths (1, 3, 5) and (1, 2, 5) are paths between two events, such as events (1 and 5).

Any noncritical path has a time reserve which is equal to the difference between the duration of the critical path and the duration of the noncritical path.

Operations lying on a noncritical path also possess a time reserve, that is, they allow for changes in their execution timetables.

The existence of time reserves in noncritical operations makes it possible freely to maneuver internal resources by increasing the execution time of certain noncritical operations (within the limits of time reserve) and thus to speed up execution of critical and subcritical operations. It is precisely this which is the main element in program evaluation and review technique.

In Figure 7 we took a simple network in which it is not difficult to find the critical and subcritical paths. For more complex networks with a large number of events, calculation must be performed according to a method specially developed for this purpose. Small networks (schedules), containing up to 150-200 events, are usually calculated manually. Networks with from 200 to 1000 events or more are as a rule calculated on electronic computers. One should bear in mind, however, that large networks (more than 1000 events) are visually poorly graspable. In connection with this, in large projects it is advisable to construct several network schedules.

In constructing network schedules, it is recommended that increasing empty linkages (operations) be avoided. They should be employed in a network only if they are essential.

Detailing of a network schedule is determined by the extent and complexity of the project, computer capabilities, as well as the structure (level) of control. The higher the control level, the fewer details in the network.

For a program with up to 200 events, usually a single general network schedule is constructed. Several networks are constructed in planning large-scale operations, in the accomplishment of which many executing entities participate. In such cases there usually can be three degrees of network detailing.

Networks of the first degree of detailing are constructed in consolidated form, for elaboration of the overall structure and course of operations -- these are networks for the top level of leadership (summary networks). In such networks operations (arrows) represent entire aggregates of measures on a large scale.

Networks of the second degree of detailing are constructed for the middle leadership echelon. Such networks are particular or local networks, which are worked out in greater detail, although each arrow in these networks can also represent several operations (aggregate of operations).

Primary network schedules are designated for the lower leadership echelon. These schedules can be detailed to the level defined by the boundaries of responsibility of executing personnel and agencies (for example, operations under the authority of a single executant). Primary networks contain a larger number of events.

Detailed networks, in connection with the necessity of reflecting a large number of interrelationships and dependences, contain a substantially larger number of empty operations.

Since there are several versions of networks based on degree of detailing, networks are constructed both in a descending and ascending direction, most frequently ascending. In this case there arises the task of unifying primary networks into particular (local) ones, and subsequently into summary or consolidated networks. So-called joining of networks is performed.

The list of general concepts should also include the question of modification of graphic representation. This also can include the following three variants:

first -- networks with orientation on operations; above the arrows operations are designated, while only a number is assigned to events;

second -- networks with orientation on events; in these networks not operations but events are designated on the charts, while arrows indicate only the linkage between events;

third -- combined orientation (both operations and events are designated).

Networks of the second and third variants are employed in large-scale plans in preparing synthesized (consolidated) schedules.

Networks of the first variant are more advisable for small and medium networks. These are most frequently a technological plan (project) model. It more graphically shows the sequence of operations and their interlinkage. Networks of the first variant are recommended for beginning study and assimilation of program evaluation and review technique.

Finally, we should mention division of network schedules into two variants connected with the elements of uncertainty in given programs. Two types of network models are distinguished on the basis of this attribute: determined and stochastic (probability).

Determined models are defined as models in which there are no uncertainties; everything in them is known.

Stochastic models are models in which there are elements of uncertainty. For example, in scientific research and experimental design projects it is impossible precisely to establish expenditures of time and resources, and the number of personnel required for performance of given particular operations. In these cases one employs the probability method of estimating possible expenditures of time, resources, etc.

4. Procedure of Constructing Network Schedules

General Principles

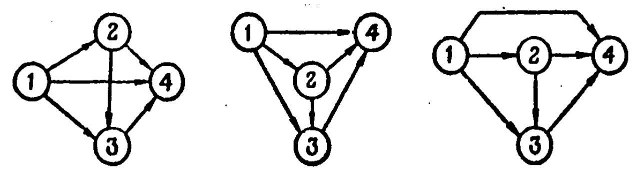
There exist several general procedures of constructing network schedules:

from beginning to end (from starting to concluding event);

from middle to end and beginning;

from end to beginning.

We shall follow the most widespread method -- from beginning to end, from left to right. Each event with a larger serial number is positioned somewhat to the right of the preceding number. Arrows may be of any length and direction, but mandatorily running from left to right. It is essential to avoid if possible mutual intersecting of arrows (Figure 8).



Undesirable   Better

Figure 8. Construction of a Network

To achieve this, it is better to shift various events on the diagram or to represent the arrow in the form of a broken line.

In proceeding to construct a network, it is essential:

to establish what operations should be completed before a given operation begins (what operations should precede a given operation);

to determine what operations can be commenced after completion of a given operation;

to determine what operations can be performed simultaneously with a given operation.

In constructing a network, a rough version is always first prepared (Figure 9). As a rule the external appearance of the network is ignored. The rough network usually appears very complex. Principal attention is devoted to a logically correct determination and mutual sequence of events. After the network has been constructed and the logical linkages checked and verified, it can be placed into better order (Figure 10).

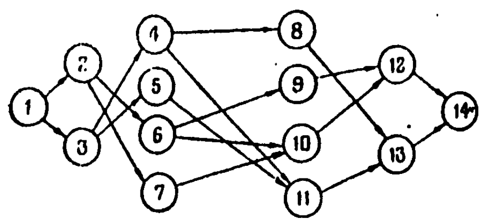


Figure 9. Preliminary, Rough Network Schedule

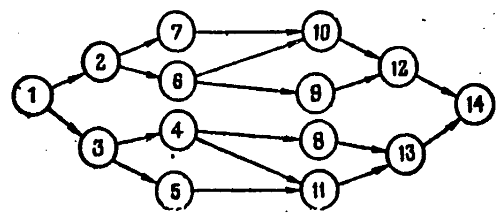


Figure 10. Ordered Version of Network Schedule

Even after a network has been put into order, however, intersecting operations (arrows) may remain. This is undesirable from the standpoint of a chart's visual appearance and ease of working with it, but it is entirely allowable and sometimes unavoidable by virtue of the logical linkages and interdependence of events in the program.

Frequently it is necessary to add so-called "forgotten" operations in the process of putting a network in order.

Numbering of Events

As stated above, events are numbered in such a manner that a larger serial number is positioned somewhat to the right of the preceding number, according to the following rule: the number of a preceding event of an operation cannot be larger than the number of a succeeding event of this same operation.

In constructing complex networks, however, one encounters certain difficulties in numbering events, as a result of which errors occur, which subsequently can lead to errors in computations. In order to eliminate errors and facilitate the numbering of events in networks, the so-called arrow deletion technique is employed. This technique consists essentially in the following.

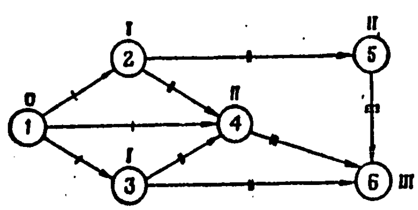


Figure 11. Numbering Events by the Arrow Deletion Method

First one finds an event on the chart (Figure 11) which does not have incoming operations (the first, that is, starting event of a schedule is always such an event), and a zero rank is assigned to this event; we mark the zero rank number above event (1).

We then mark through (with one line) all arrows proceeding from this event and among succeeding events we look for those which do not have incoming arrows (other than deleted), and assign them first rank (I).

We then mark (with two lines or a different-colored pencil) all arrows proceeding from events of first rank (I), and among succeeding events look for those which do not have incoming arrows, and assign them second rank (II).

We continue in this sequence up to the end of the chart, assigning subsequent events which do not have incoming arrows, third (III), fourth (IV) ranks, etc. Events are numbered by rank in ascending order, beginning with the starting event.

Representation of Parallel Operations

In constructing network schedules one very frequently encounters complex linkages where two or more operations have common initial and terminal events. These operations are carried out in parallel (jointly), but they are of differing duration. Parallel performance of operations should be depicted in such a manner that any operation can be joined only with two events. In this case it is necessary to depict the interlinkage of operations, introducing an additional event and an empty link (Figure 12).

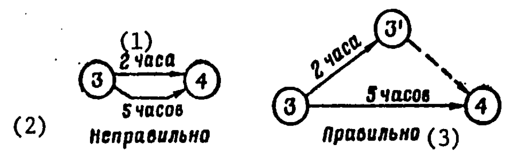


Figure 12. Graphic Representation of Parallel Operations

Key:
1. Hours
2. Incorrect
3. Correct

Representation of Differentiated-Dependent Operations

In constructing network schedules one can encounter conditions where in order to perform an operation, such as operation (5, 6) in Figure 13a, it is necessary first to perform several operations: in Figure 13a -- operations (2, 5), (3, 5) and (4, 5), while for another operation (5, 7), proceeding from common event (5), performance only of one of the preceding operations (4, 5) is a preliminary condition. In this case the interlinkage and interdependence of operations cannot be portrayed as is indicated in Figure 13a, for with such a representation the commencement of operation (5, 7) depends on accomplishment of all three preceding operations (2, 5), (3, 5), (4, 5), and this is not in conformity with the program conditions.

Here, just as in representation of parallel operations, one should introduce an additional event (4') and an empty link into the network (Figure 13b).

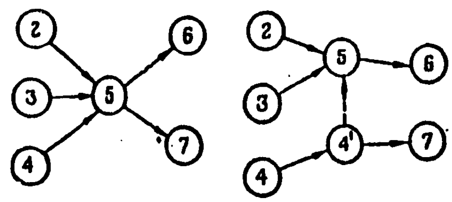


Figure 13. Construction of a Network Taking Into Account the Relationship of Operations

As already stated, in some cases accomplishment of a portion of any operation is a condition for commencement of one or several operations (see operation (5, 6) in Figure 14a), operation (4, 5) in Figure 14a). Therefore it would be incorrect to represent on the chart (Figure 14a) the preceding operation entirely, and then to delete from the terminal event (5) of this operation the succeeding operation (5, 6), the commencement of which depends on completion of only part of the preceding operation. In this case, for a correct representation of the interlinkage and sequence of performance of operations, we must divide operation (4, 5) in the chart into component parts which are concretely time-determined, and introduce an additional event (4') -- Figure 14b.

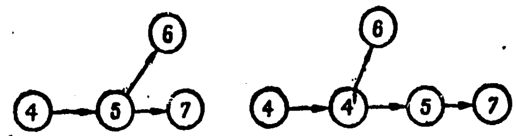


Figure 14. Construction of a Network Allowing for the Fact That Subsequent Operations Can Begin Following Completion of a Portion of the Preceding Operation

External Insertions in Schedules

In order to reflect in a draft plan the time and place of arrival in the unit (subunit) of additional supplies, personnel replacements, technical documentation (for example, for performance of design activities) and other information, so-called insertions are placed on network schedules. External insertions, in contrast to operations and events, are customarily depicted separately: by a double circle with a zero (Figure 15).

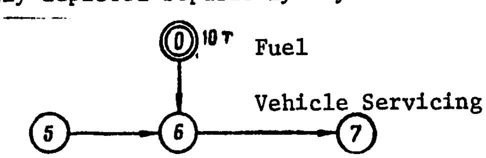


Figure 15. Representation of External Insertions on a Chart

If there are two or more operations proceeding from an event with which it is necessary to link an insertion, the latter is depicted in a linkage with an event additionally introduced via a fictitious operation (Figure 16).

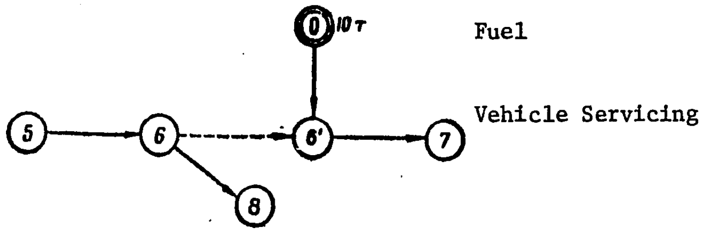


Figure 16. Representation of External Insertions on a Chart With Two or More Operations Proceeding From an Event

Procedure of Representation of Organizational Linkages

In constructing schedules one must represent not only technological links and relations but also organizational linkages, such as the sequential movement of teams of repair personnel and the requisite equipment for organizing repair or servicing of motor transport and armored vehicles in field conditions. Let us assume that there are three work operations: A, B, and C (A -- repair of tracks and suspension; B -- repair of engine; C -- repair and zeroing of weapons), which must be performed on two tanks. For performance of repairs there are three teams of maintenance specialists for each type of work operation, furnished the required equipment and tools, whereby only one team -- for tracks and suspension maintenance -- is available at the commencement of repairs.

These operations can be performed at once on two tanks in parallel, sequentially shifting maintenance specialist teams and their equipment from one (I) damaged tank to the other (II). The sequence of performance of these operations can be represented in a twofold manner in a network schedule (figures 17 and 18).

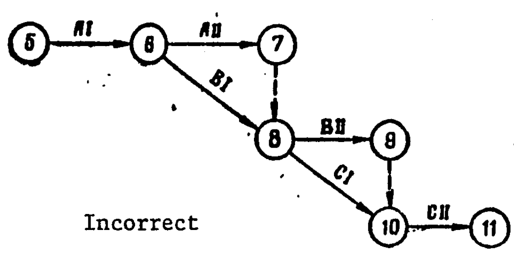


Figure 17. Incorrect Representation of Sequence of Performance of Technological and Organizational Linkages

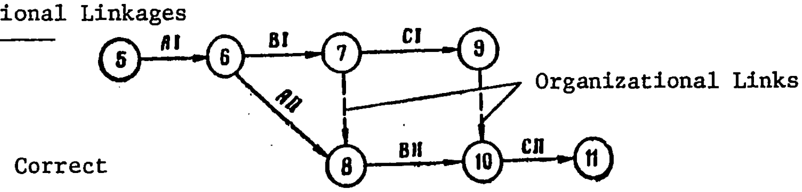


Figure 18. Correct Representation of Sequence of Technological and Organizational Linkages

Upon examining the chart in Figure 17, we can establish that organizational and technological links in this chart are shown incorrectly. This is due to the fact that in this segment of a schedule of tank repair in field conditions, the dashed arrows indicate not organizational links but the technological dependence of commencement of work on repairing the engine of tank (II) on completion of repair of the tracks and suspension of this same tank or commencement of repair of weapons on completion of engine repair. In addition, job commencement (CI) has entirely unwarrantedly been placed in a dependence on completion of work operation (AII). In this case all organizational and technological links must be represented as shown in Figure 18.

Two-Way Linkages

In addition to organizational linkages, charts can also reflect two-way linkages (relations) which, just as in other cases, are represented by introduction of fictitious operations (dashed arrows). For example, there are three processes: A, B, C. Completion of process C depends on the results of processes A and B. In this instance there arise two-way relations which can be depicted as shown in Figure 19.

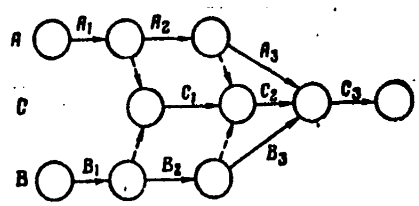


Figure 19. Representation of Dependent Processes

When introducing fictitious operations into a network, however, one should bear in mind that their quantity and the direction of dashed-line arrows (linkages) may reflect on the critical path (Figure 20).

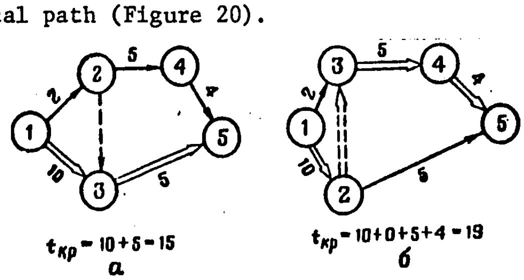


Figure 20. Influence of Direction of Fictitious Operations on Length of Critical Path

As is evident from Figure 20a, fictitious operation (2, 3) does not affect the critical path. In the other case (Figure 20b), with a change in the direction of dependence in the same program, the fictitious operation led to an increased duration of the critical path. Therefore when introducing fictitious (empty) operations into a network, in each instance it is essential rigorously to analyze the necessity of a given linkage (dependence) in the program.

Simplification of Linkages

When constructing network schedules it is sometimes advisable to consolidate operations by replacing a number of operations with one "aggregate" operation if some group of operations has a single initial and a single terminal event (Figure 21). This figure contains the simplest example of consolidation of operations.

In practice, however, there more frequently occur cases where a group of operations possessing a single initial and terminal event cannot be replaced by a single "aggregate" operation due to the linkages of individual operations within the group via intermediate events with other parts of the network schedule. In this case one usually does not go beyond simplification of the network of the group of operations in question (Figure 22).

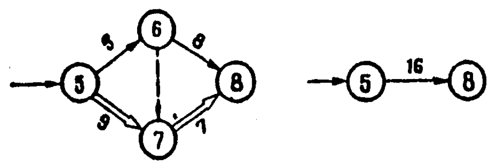


Figure 21. Replacement of a Group of Operations by a Single "Aggregate" Operation

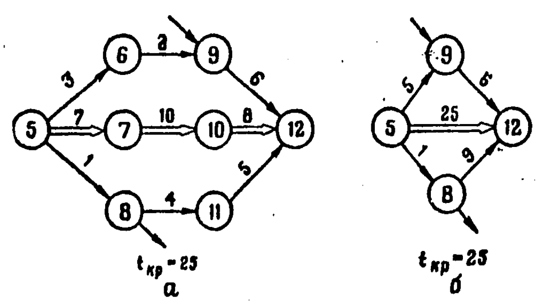


Figure 22. Replacement of a Group of Operations in Networks With Events Possessing Intermediate Linkages

It is evident in Figure 22a that one cannot replace the original network with a single "aggregate" operation as is done in the first instance, since it is linked through events (8 and 9) with other parts of the network schedule of the entire program. We have simplified the network, but we have left in it events (8 and 9), without disrupting the linkages of the process in question with other parts of the network schedule (Figure 22b).

Such a simplification of networks significantly improves the visual appearance of the overall network schedule and simplifies its calculation and optimization.

Verification of Correctness of Construction of a Network

A network schedule constructed according to the procedures described above is usually checked. During this verification one determines whether the chart contains operations bearing identical codes (especially for parallel operations). If such operations are discovered, additional events and fictitious operations should be added.

One then checks to determine whether there are dead-end events in the chart, that is, events from which no operation begins, other than the concluding event of the schedule. If such events are discovered, the operations which enter such events and the events themselves must be eliminated from the schedule. It is also necessary to eliminate events which are not preceded by an event, other than the starting event.

Finally, one checks the order of presentation of differentiated-dependent operations, organizational linkages, and checks to make sure there are no closed loops in the chart (Figure 5).

In complex programs, following verification of primary and local network schedules, they are joined into a consolidated network schedule of the overall program.

Joining Networks

It was stated above that network schedules in complex programs are subdivided, by degree of detailing and function, into consolidated networks, particular (local), and primary networks. Such a division makes it possible to determine the level of detailing of network schedules and creates more favorable conditions for monitoring the course of their development and control, and also makes it possible, when constructing the initial program schedule as a whole, to enlist the efforts of specialists or commanders (executants) of subordinate units (subunits) or sections who are well acquainted with the sequence, extent and qualitative evaluations of operations at their echelon.

In connection with such a division of network schedules by degree of detailing, when constructing consolidated overall program schedules there arises the necessity of unifying (joining) particular (local) schedules into a general (consolidated) network. The process of connecting networks is accompanied by discovering and correcting mismatches, various discrepancies, and by simplification of local schedules.

For ease of joining networks and in order to eliminate the repetition of operation codes, in a consolidated schedule a specific quantity of numbers is given to each executant (subunit, section) for numbering events, and input or output boundary events of the networks they are constructing are determined (matched). For the sake of clarity of representation, each subunit (section) can be assigned its own event symbol (Figure 23). More than enough numbers are assigned to the sections (subunits). For example, from No 121 to 140 to the first subunit, from No 140 to No 160 to the second, etc.

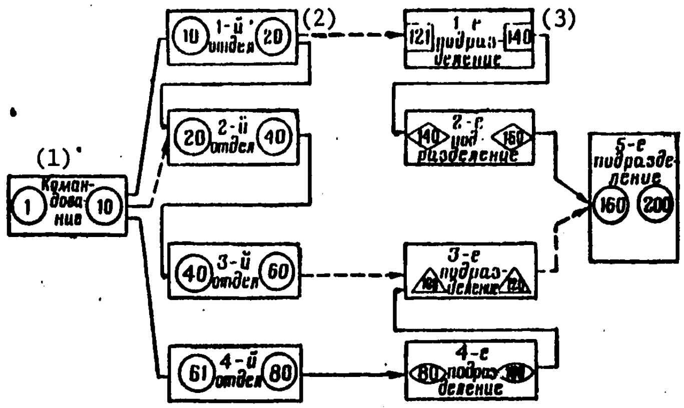


Figure 23. Schematic Diagram of Network Joining

Key:
1. Command
2. Section
3. Subunit

Joining of networks can be performed in detail, when particular networks are being connected for the most part in the form in which they are submitted to higher headquarters, or in less detail when a consolidated network is constructed chiefly of "aggregate" operations which include an entire aggregate of measures (operations) performed by a given section (subunit).

Networks are more frequently connected in an ascending direction, from lower to higher echelons, by boundary events. Boundary event is the term employed for events which are linked by operations to other responsible executants of subunits (sections) and are thus common to two and more networks. Boundary events may be input (in Figure 23 -- 10, 20, and 40 respectively for the first, second, and third sections) and output (in Figure 23 -- 10, 20, and 40 respectively for the command, the first and second section).

Input events indicate what group of operation results must be received by a given executing agency from other executing agencies. The output event for each executing agency in turn will be those operation results which it is to transmit to another executant. For example, event (5) "Tank Engine Repair Completed" (Figure 24) will be the output boundary event of a particular network schedule of one maintenance subunit and the input boundary event for the schedule of another subunit, which installs tank equipment.

Figure 24 shows a method of joining networks. As we see, networks are connected by a transition event, after which all other elements of the particular schedule

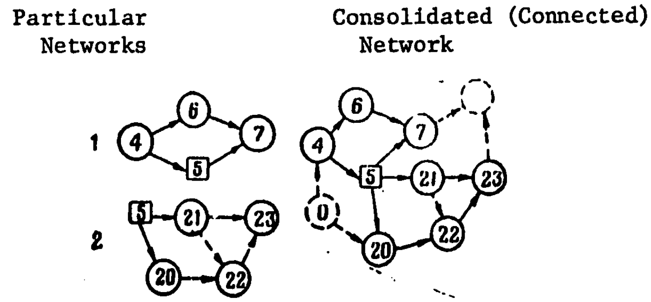


Figure 24. Method of Joining Networks

are eliminated (if they are not simplified or consolidated), and conditional starting and concluding events are additionally introduced (indicated by dashed-line circles).

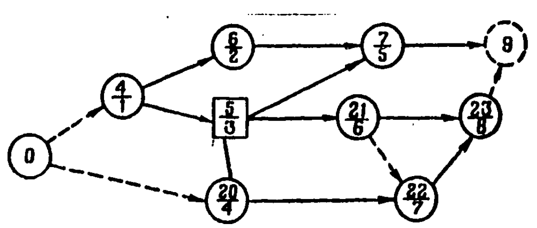


Figure 25. Possible Numbering of Events in a Matched and Joined Network

Introduction of conditional starting and concluding events is of importance for subsequent calculations on a consolidated network schedule. A numbering of events which is continuous for the entire schedule is then performed on the matched and joined network, while also leaving the old number in the circle indicating each event (Figure 25), so that in the consolidated network it is easy to determine what subunit (section) bears responsibility for performance of a given operation.

In complex programs, when matching and joining primary schedules and particular schedules, a certain number of operations and events, responsibility for which is assigned to individual subunits, can be omitted and replaced by "aggregate" operations, since the details of individual particular tasks are of no great significance for a higher level of control. For example, in designing and building new models of equipment, or at a repair plant, the designing (assembly) of specific machine components (assemblies), consisting of a number of individual operations, is included in the consolidated network as one independent operation (Figure 26).

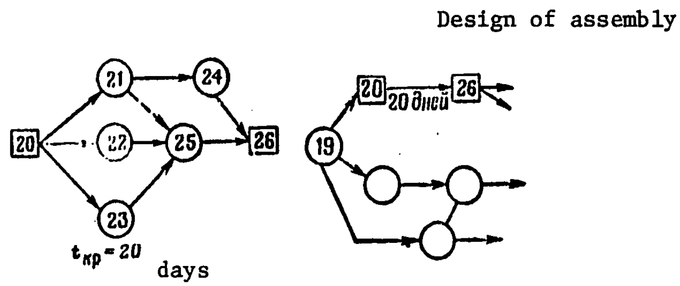


Figure 26. Replacement of a Bundle of Operations by a Single Operation in Constructing a Consolidated Schedule

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We have presented the basic rules and procedures for constructing network schedules, which should be observed when proceeding with program evaluation and review or analysis of any processes (programs) in military affairs.

Sequence of Operations in Constructing Network Schedules

Construction of network schedules should be based first and foremost on a comprehensive analysis of the basic and intermediate program objectives.

The first stage of construction of a network model of a given process in military affairs is formulation of the task which determines the ultimate objective of the program. In addition to the principal (ultimate) objective, however, the program should also specify intermediate goals, which should be interlinked both in sequence and in attainment results.

Intermediate goals determine the level of program execution and constitute particular tasks, which must be accomplished in order to achieve the main goal. For example, the end objective for a program modeling a unit march to an exercise area will be concentration of the unit by the designated time, in area A, B, C, let us say, in a state of readiness to carry out the combat mission, while the following will correspondingly be the intermediate goals of this program:

organization of the unit's march;

execution of the march and concentration of subunits in area A, B, C;

making the subunits combat ready following the march in the new concentration area.

The second stage of the operation, preceding construction of a network schedule, consists in preparing a block diagram of the program, or a so-called "program tree," which should graphically show the extent and stages of operations.

For construction of a block diagram ("program tree"), the entire program (system) is divided into subsystems, the subsystems in turn are divided into groups, and the groups into individual elements.

It is recommended that a block diagram be developed primarily in complex plans, when there are several intermediate goals, while they in turn can have their own subgoals. One feature of such a complex structure is the presence at each level of a number of independent terminal or intermediate units (subunits) which perform their tasks independently of one another. The number of levels in the structure depends chiefly on the complexity of the program and can differ for individual branches of the structure. One branch develops in a fair amount of detail and possesses many levels, while another ends at the second or third level and does not divide further.

Preliminary development of a block diagram improves the visual clarity of the program, establishes a more clear-cut interrelationship between its individual parts, and simplifies management within the boundaries of each level.

The higher commander (higher headquarters) should be responsible for preparing the block diagram; the higher commander determines the degree of breakdown of the program development (lower level), designates persons and agencies responsible for operations in the component parts of the program (other than staff), and furnishes executing persons and agencies input data for program evaluation and review.

Figure 27 contains a sample version of a program block diagram ("development tree"), the goal of which is design and construction of a new machine (we shall conditionally designate it "Object T-100").

In those cases where preparation of a block diagram is acknowledged to be inadvisable, the overall volume of operations can be broken up by another method -- by constructing a consolidated network schedule (Figure 28).

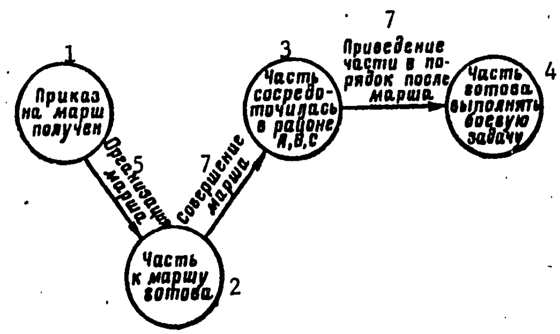

Figure 28. Consolidated Network Schedule

Key:

1. March order received
2. Unit ready to march
3. Unit concentrated in area A, B, C
4. Unit ready to execute combat mission
5. Organization of march
6. Execution of march
7. Making unit ready after march

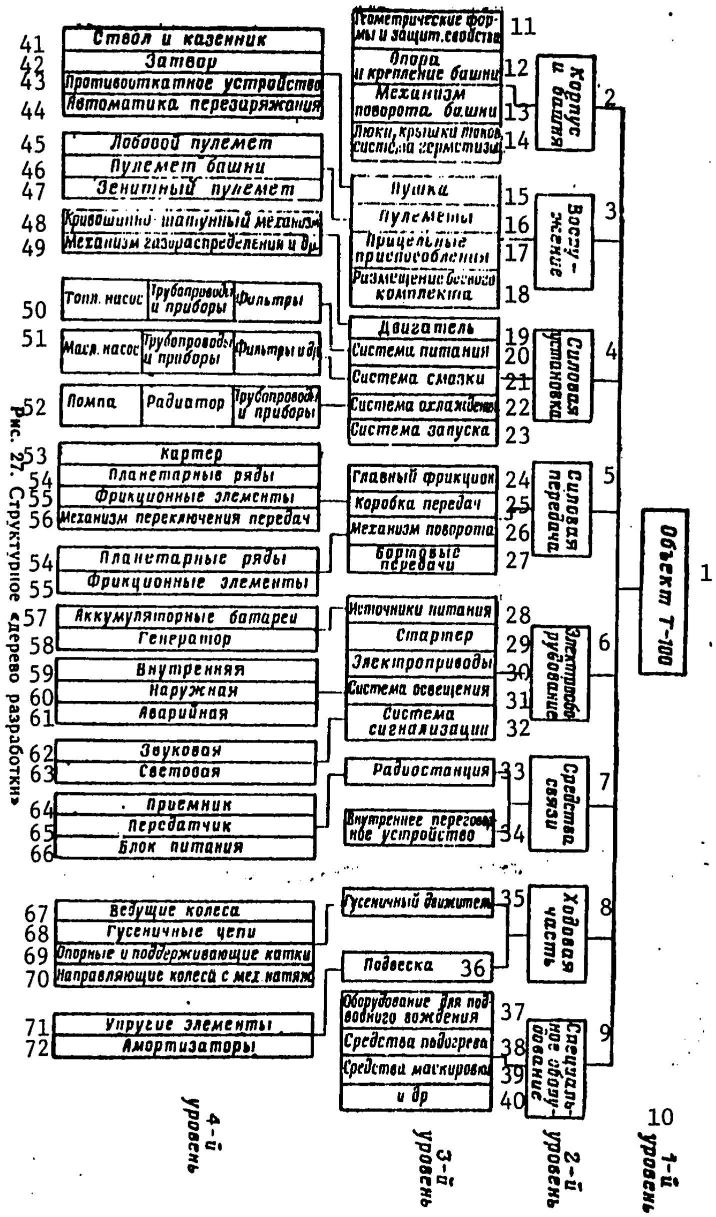

Figure 27. Structural "Development Tree"

Key:

1. Object T-100
2. Hull and turret
3. Armament
4. Powerplant
5. Transmission
6. Electrical equipment
7. Communications gear
8. Tracks and suspension
9. Special equipment
10. Level

(Key to Figure 27 on preceding page, cont'd)

11. Geometric shapes and protective properties
12. Turrent ring and securement
13. Turrent traverse mechanism
14. Hatches, hatch covers, sealing system
15. Gun
16. Machineguns
17. Aiming devices
18. Ammo stowage
19. Engine
20. Fuel system
21. Lubrication system
22. Cooling system
23. Starter system
24. Master clutch
25. Gearbox
26. Steering mechanism
27. Final drive
28. Power supply
29. Starter
30. Electric drives
31. Lights
32. Signaling system
33. Radio
34. Intercom
35. Track drive
36. Suspension
37. Equipment for operation submerged
38. Heaters
39. Means of camouflage and concealment
40. Etc
41. Barrel and breech ring
42. Breech mechanism
43. Recoil system
44. Automatic loading system
45. Hull machinegun
46. Turret mounted machinegun
47. Antiaircraft machinegun
48. Crankshaft and connecting rod assembly
49. Valve gear, etc
50. Fuel pump, feed lines and instruments, filters
51. Oil pump, feed lines and instruments, filters, etc
52. Pump, radiator, feed lines and instruments
53. Gearbox
54. Epicyclic gears
55. Clutch elements
56. Gear shifting mechanism
57. Storage batteries
58. Generator
59. Interior
60. Exterior
61. Emergency
62. Horn
63. Light
64. Receiver
65. Transmitter
66. Power supply
67. Driving sprockets
68. Tracks
69. Road wheels and top rollers
70. Idler wheels with track tensioning mechanism
71. Elastic elements
72. Shock absorbers

The third stage of operations in constructing network schedules is the listing of operations at each echelon of program development and determination of time estimates.

The list of operations can be reduced to a table, an example of which is contained in Table 1.

Table 1.

№ по пор.	4 Наименование работ	5 Код работ	Продолжительность работ, мин 6
1	Уяснение задачи	(1, 2)	10
2	Предварительные распоряжения подразделениям	(2, 3)	30
3	Оценка обстановки и т. д.	(2, 4)	60

Key:
1. Mission briefing
2. Warning orders to subunits
3. Situation estimate, etc
4. Operations
5. Code of operations
6. Duration of operations, minutes

The "Code of Operations" column in the table can also be filled in after preparation of the initial chart. As regards making operation time estimates, for frequently repeating operations, for which specific standard times are available, duration of operations shall be specified in conformity with these standards and taking concrete situation conditions into account.

In those cases where there are no objectively substantiated time standards for duration of operations (in scientific research, experimental design and other activities), one employs the probability method of determining their duration. A method of formulating time estimates with this technique is presented in Chapter II.

Then an initial network schedule is constructed at each echelon proceeding from the list of operations, it is calculated and analyzed, and on the basis of this one determines the degree to which the initial schedule corresponds to the mission assigned the unit. If the initial network schedule of performance of an aggregate of operations does not provide for prompt task execution, schedule optimization (improvement) is effected, in the interests of prompt and timely achievement of the program goal at the given echelon. Optimized schedules of subunits (sections) are submitted to higher headquarters, where a consolidated network schedule is drawn up.

Sometimes, however, desirable or necessary calendar timetables for execution of assigned operations will not be indicated in research programs for subunit commanders (executing agencies) at the stage of drafting the initial plan when assigning the task of analysis of a model of an aggregate of various measures, for the purpose of achieving greater objectivity of estimating a project timetable. In this case executants will determine only the duration of individual operations, the sequence of their execution and logical linkages of operations, independent of the calendar timetable of their execution, which can be determined in a substantiated manner only as a result of network calculations.

A consolidated network schedule of the program as a whole, following verification and elimination of all disagreements and discrepancies, is calculated, analyzed and, when necessary, reoptimized.

Such a schedule, depending on the quantity of operations and events encompassed by it, may be fully represented or constitute a consolidated network with fragments of more important details.

Chapter II. METHODS OF CALCULATING NETWORK SCHEDULES

1. Determining Duration of Operations

Network schedules which reflect troop combat operations are as a rule constructed on a time line. They can, however, also be constructed not on the basis of time but other parameters, such as based on consumption or supply of resources or on the basis of monetary expenditures. In these cases network calculation should also be performed according to the corresponding parameter on the basis of which the network is constructed. In this study we shall examine construction and optimization of network schedules, that is, their improvement on a time parameter.

The duration of each operation in time is usually written down in the process of constructing the network. In constructing a network, as soon as two events and the operation linking these events have been specified, the operation name or designation is written above it, and below it -- its time of duration in seconds, minutes, hours, days, or weeks (depending on the nature of the process and convenience of working with the network schedule).

One should bear in mind the fact that correctness of statement of time characteristics is of paramount importance. The quality of a chart and the efficiency of process management on the basis of a given chart will depend on the correctness of the designated time estimates. If, for example, operation execution times are understated, that is, are less than actually required, this will lead to haste in preparation for doing a given job and the entire operation as a whole. Such haste, as indicated by the experience of the last war, can lead to failure of combat operations and consequently to unwarranted and useless casualties. Obviously the objective of the combat engagement will not be achieved under such conditions.

On the other hand, overstating the time for carrying out individual operations can lead to delay in the timetable for commencement of combat operations, that is, to purposeless and unwarranted loss of time, of which the enemy can take advantage and strengthen his defense or prepare for and mount a counterstroke.

Where should one obtain time estimates of operations which are closest to reality?

There exist two possibilities for this.

For work actions or operations on which a fair amount of experience has been amassed both in peacetime and in time of war, estimates can be adopted on the basis of this experience, that is, such estimates can be taken from standard figures derived in the course of combat training or in the course of combat operations.

The correctness of unambiguous selection of such time indices is grounded in probability theory.

In military affairs one is usually dealing with random variables. We know from probability theory that with a large number of tests, the arithmetic mean value of a random variable remains practically constant (stops changing). In other words, with a large number of independent tests, the arithmetic mean of obtained values of random variable $M^*[X]$ converges in probability with its mathematical expectation $M[X]$. This relationship between the arithmetic mean and the mathematical expectation of a random variable comprises the content of one of the forms of the law of large numbers. This is exceptionally important for practical activities in the respect that with a large number of experiments one takes the statistical (experimental) value of a given variable, considering it as differing little from the actual value. Therefore utilization of statistical, that is, experimental data is a scientifically substantiated approach to solving all types of problems connected with random processes.

An important conclusion proceeds from this: it is essential to amass experimental data on all operations connected with troop combat activities and to summarize them into standard reference tables and catalogues. This also applies to expenditures of resources (manpower, material, monetary, energy). If one has such standard catalogues available, it will be easy to place time and other parameters in network schedules.

Thus employment of experimentally obtained statistical data is one of the possibilities for determination of time and other characteristics necessary for estimating the duration of work operations or expenditure of resources and their placement in network schedules.

There is also another possibility of obtaining the characteristics indicated above. It is based on probability methods of determining random variables. The fact is that in random or stochastic processes on which there is insufficient experience, or in totally new processes, for which there is no experience whatsoever, there is also a possibility of determining a quantitative estimate of random phenomena. This quantitative estimate can be determined on the basis of the probability of occurrence of a given event. For all practical purposes in this case it is advisable to proceed in this manner. Three time estimates: optimistic, pessimistic, and most probable -- are given by experienced commanders or military engineers (schedule executants) for operations for which there are no time estimates.

Optimistic estimate -- the shortest operation duration of those possible, that is, time during which an operation can be performed with the most favorable confluence of circumstances. We shall denote this estimate by t_{min}. As we know from experience, in most cases the probability of accomplishing the job in this time is approximately 0.01.

Pessimistic estimate -- the greatest possible work operation duration from the experience of the executant, that is, time to do the job with an extremely infelicitous confluence of circumstances. We shall designate this estimate with t_{max}. The probability of accomplishing the job in this time is also approximately 0.01.

Most probable estimate -- the possible time of accomplishment of a given job under the condition that no unexpected difficulties arise. We shall designate this estimate with $t_{N.B}$. Probability of accomplishing the job in this time will be maximum.

If we require of operation executants only one estimate in place of three, the possible result is that the estimate may prove to be overstated or understated, that is, unrealistic. An unrealistic operation time or unrealistic material expenditures indicated in the schedule can lead to the same miscalculations which are frequently encountered in planning processes with the aid of linear charts, that is, with the aid of presently existing traditional methods.

The probability of completing a work operation in a time less than the optimistic estimate will be very small. The probability of completing a work operation in a time exceeding the pessimistic estimate will similarly be small. The probability of completing a work operation in the realistic estimate time, that is, in the time of the most probable time estimate, will be the greatest. Proceeding from this, at first glance it may seem that one must employ the most probable time estimate. However, if everything is weighed well, this should not be the procedure.

Let us examine this with a specific example.

We know that the number of target points scored at the firing range is a random variable which is characterized by a certain distribution of probabilities for each person on the firing line. In carrying out his firing assignment, each individual may score a certain minimum, maximum and most probable number of points. The presumable estimate of number of points scored for each individual on each round fired is quite similar to the tentative estimate of the time required to accomplish each work operation in our case. We shall now assume that the number of points scored by the rifleman is characterized by the statistical series presented in Table 2.

Table 2.

(1) Вероятность (p)	0,2	0,6	0,1	0,05	0,03	0,02	0,00
(2) Случайная величина (выбитые при стрельбе очки) . .	10	9	8	7	6	5	1

Key:
1. Probability
2. Random variable (target points scored)

We see from this series that a hit in the nine ring is most probable (p=0.6). If we take the most probable score as our criterion, we can assume that a given soldier will score 900 points with 100 rounds fired. But this will be incorrect. We know from theory of probability that in order correctly to determine the presumable number of points which will be scored by an individual, it is necessary to find some average quantity. For this we must multiply the value of the random variable by its corresponding probabilities and add the obtained results. For our example the arithmetic mean value differs from the most probable, which is equal to 9. Thus the number of points scored by an individual with 100 rounds will be close to 883.

Consequently, if in determining time estimates we take only one realistic or most probable estimate, we shall be making the same mistake as with the soldier on the firing line. There will be no such error if three time estimates are employed to calculate the expected time for accomplishing each work operation.

In a theoretical respect, solving such a problem involves certain mathematical difficulties, since this would require knowing the distribution function of the probabilities which characterize the duration of work operations.

When a commander gives three time estimates on the duration of performance of some operation, that is, gives an optimistic, most probable, and pessimistic estimates, he is defining a certain probability distribution analogous to the distributions presented in Figure 29.

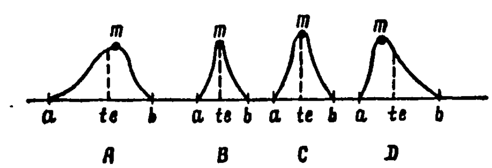

Figure 29. Possible Probability Distributions

A -- optimistic estimate of duration of work operation; m -- most probable estimate of duration of work operation; B -- pessimistic estimate of duration of work operation; te -- average duration of work operation if the given operation were multiply repeated

---

Quantity t_e for each of the distributions contained in Figure 29 constitutes the mathematical expectation or statistical average value of three work operation duration estimates. In other words, quantity t_e is the average duration of a given work operation in case of multiple repetition.

The relative positions of quantities a, m, and b (Figure 29) depend on the numerical values of these quantities, specified by the commander. Their relative positions in turn determine the value or position of quantity t_e.

The value of quantity t_e based on three estimates is determined with the formula

$$t_e = \frac{a + 4m + b}{6}. \qquad (1)$$

We shall examine below the derivation of this formula. A special analysis and experience in employing PERT methods indicate that this formula is an intelligent compromise between potential result accuracy and awkwardness of the computation process.

However, in order to be certain of the value of the anticipated work operation performance time, which nevertheless remains a random variable, it is necessary to know what error we make in our estimate, that is, by what amount the actual work operation performance time may deviate from the expected values. To estimate the degree of possible deviations from the expected value, one customarily employs the sums of the products of the squares of the differences of the random variables and their mathematical expectations by the value of the probabilities of these random variables. The quantity obtained in this manner, characterizing possible variance of the random variable relative to its expected value, is called standard deviation.

Random variables are continuous and discontinuous, and therefore standard deviations for these variables are also respectively determined with the following formulas:

for discontinuous variables:

$$D[X] = \sum_{i=1}^{n} (x_i - m_x)^2 p_i; \qquad (2)$$

for continuous variables:

$$D[X] = \int_{-\infty}^{\infty} (x_i - m_x)^2 fx\,dx, \qquad (3)$$

where D[X] is standard deviation; x_i -- value of random variable; m_x -- mathematical expectation of random variable; P_i -- probability of obtaining the value of a random variable.

Of importance for program evaluation and review is that the overall distribution variance of the sum of a set of mutually independent random variables is equal to the sum of the values of the distribution variances for each random variable separately.

We recommend that a reader who is unfamiliar with probability theory, for practical determination of standard deviation, employ the following formula:

$$\sigma^2 = \left(\frac{b-a}{6}\right)^2, \qquad (4)$$

where σ^2 is standard deviation; b -- pessimistic estimate; a -- optimistic estimate.

This formula and the curves in Figure 29 show that the further apart the optimistic and pessimistic estimates, that is, the greater the span of distribution, the greater will be the uncertainty connected with the operation in question and, on the other hand, the less the variance, the more accurate the operation duration estimate will be, and consequently the optimistic and pessimistic estimates lie closer to one another.

For example, let us say that two commanders have determined the duration of some one operation, the estimate of which is contained in Table 3.

Table 3.

(1) Кто определяет	(2) Оценка		
	(3) оптимистическая (a)	(4) наиболее вероятная (t_c)	(5) пессимистическая (b)
(6) Первый командир	4	6	8
(7) Второй командир	10	12	13

Key:
1. Who determines
2. Estimate
3. Optimistic
4. Most probable
5. Pessimistic
6. First commander
7. Second commander

In order to determine which commander more correctly determined duration of the operation, we must compute the variance for each of them:

$$\sigma^2 = \left(\frac{8-4}{6}\right)^2 = \left(\frac{4}{6}\right)^2 = (0{,}67)^2 = 0{,}45;$$

$$\sigma^2 = \left(\frac{13-10}{6}\right)^2 = \left(\frac{3}{6}\right)^2 = (0{,}5)^2 \doteq 0{,}25.$$

Our calculation indicates that variance is less with the second commander, and consequently his estimates are more correct. It follows from this that these estimates should be used for our calculations.

Thus in order to determine who is less confident of his estimates, we must calculate the standard deviation of each set of estimates and compare the obtained standard deviations.

Let us examine the derivation of formulas (1) and (4). These formulas are connected with β-distribution.

β-distribution is a distribution of random variable t, which changes in the interval [A, B], where $A > 0$, $B > 0$, the density of probability of which is determined by the formula

$$f(t) = \begin{cases} 0 & -\infty < t < A \\ \dfrac{(t-A)^{\alpha}(B-t)^{\nu}}{(B-A)^{\alpha+\nu+1}\beta(\alpha+1, \nu+1)} & A \leqslant t \leqslant B \\ 0 & B < t < \infty, \end{cases} \qquad (5)$$

where Euler's function of the first type

$$\beta(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}; \qquad (6)$$

Euler's function of the second type

$$\Gamma(r)=\int_0^\infty x^{r-1}e^{-x}\,dx.$$

Figure 30 shows the appearance of this distribution for stipulated values of parameters.

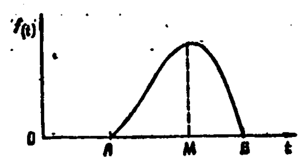

Figure 30. Distribution of a Random Variable Which Changes Within Interval (A, B)

We shall also examine the β-distribution of a normalized random variable determined by the linear transformation

$$t=A+(B-A)u. \qquad (7)$$

The transformed function of density is reduced to normalized form:

$$\varphi(u)=\begin{cases}0 & -\infty<u<0\\ \dfrac{u^{\alpha}(1-u)^{\nu}}{\beta(\alpha+1,\ \nu+1)} & 0\leqslant u\leqslant 1\\ 0 & 1<u<\infty.\end{cases} \qquad (8)$$

For normalized β-distribution we have:

$$\bar{u}=E(u)=\frac{\alpha+1}{\alpha+\nu+2}; \qquad (9)$$

$$\sigma_u^2=\frac{(\alpha+1)(\nu+1)}{(\alpha+\nu+3)(\alpha+\nu+2)^2}. \qquad (10)$$

For nonnormalized β-distribution respectively:

$$\bar{t}=E(t)=A+(B-A)\frac{\alpha+1}{\alpha+\nu+2}$$

and

$$\sigma_t^2=\frac{(B-A)^2(\alpha+1)(\nu+1)}{(\alpha+\nu+3)(\alpha+\nu+2)^2}.$$

A mode of nonnormalized distribution corresponding to f'(t)=0 is equal to

$$M=\frac{A\nu+B\alpha}{\alpha+\nu}. \qquad (11)$$

This enables us to write $\bar{t}$ in the form

$$\bar{t}=E(t)=\frac{A+B+(\alpha+\nu)M}{\alpha+\nu+2}. \qquad (12)$$

In existing methods, and in the PERT method in particular, one selects:

$$\alpha = 2 + \sqrt{2}; \tag{13}$$
$$\nu = 2 - \sqrt{2} \tag{14}$$

or

$$\alpha = 2 - \sqrt{2}; \tag{15}$$
$$\nu = 2 + \sqrt{2}. \tag{16}$$

This gives:

$$t = E(t) = \frac{A + B + (\alpha + \nu) M}{\alpha + \nu + 2} = \frac{A + B + 4M}{6}; \tag{17}$$

$$\sigma_t^2 = \frac{(B - A)^2 (\alpha + 1)(\nu - 1)}{(\alpha + \nu + 3)(\alpha + \nu + 2)^2} = \frac{(B - A)^2}{36} = \left(\frac{B - A}{6}\right)^2. \tag{18}$$

If we replace upper-case with lower-case letters, we shall obtain an exact expression of formulas (1) and (4), namely:

$$t_e = \frac{a + 4m + b}{6}$$

and

$$\sigma^2 = \left(\frac{b - a}{6}\right)^2.$$

Thus we have established how expressions (1) and (4) were obtained.

Calculation of expected work operation performance time for the three estimates is complex and laborious. Expected work operation performance time can also be determined on the basis of two estimates.

Experience indicates that determination of the most probable time estimate always causes difficulty for operation executants, while determination of optimistic and pessimistic estimates does not cause such difficulties.

In view of this fact, Soviet scientists D. I. Golenko and V. S. Mikhel'son proposed determining expected time on the basis of two estimates. The equations they have proposed are as follows:

for computing expected work operation duration time:

$$t_e = \frac{3a + 2b}{5}; \tag{19}$$

for computing standard deviation

$$\sigma^2 = 0{,}04\,(b - a)^2, \tag{20}$$

where a -- optimistic estimate; b -- pessimistic estimate.

At the present time many prefer to employ formula (19) for determining expected job duration time, for it has been proven by special studies that the difference in result in comparison with the time obtained with formula (1) can be not more than 1 percent. Such a difference is not of any great practical significance.

Summarizing what has been stated in this section, we can conclude that time estimates for determining duration of execution of work operations connected with military affairs must be taken from standard catalogues, manuals, regulations, and summary tables.

In connection with this, military experts have the task of working up standards and preparing standard reference handbooks for all work operations which can take place in the course of combat actions. This will require the conduct of a large number of tests and experiments in order to determine the time required to accomplish each work operation. For operations the execution time of which cannot be established experimentally, it is expedient to employ the probability method of determining their duration. All time estimates obtained in this way must be tabulated in reference standards catalogues and distributed to the troops. Commanders and military engineers should take work operation time estimates from these standards manuals.

In those instances where reference lists do not contain time estimates and experience on certain kinds of operations is not amassed, it is necessary to employ formula (1) or (19) to determine duration of work operations.

2. Calculating Network Schedule Parameters

When a network has been constructed, matched, joined and checked, one proceeds with calculation of its parameters.

Calculation of the parameters of a network schedule consists in determining:

early and late times of event occurrence;

time of early and late commencement and ending of operations;

critical path;

all kinds of work operation reserves (complete and partial reserves of the first and second types).

In addition to the above, the following enter into the calculation in stochastic networks:

determination of standard deviations of work operations;

determination of probability of occurrence of key events or the entire process at the calculated time.

Before proceeding with calculation of network parameters, we shall become acquainted with the calculation diagram and principal conventional symbols. For this we shall take a number of operations performed by the commander in organizing for an attack (Figure 31).

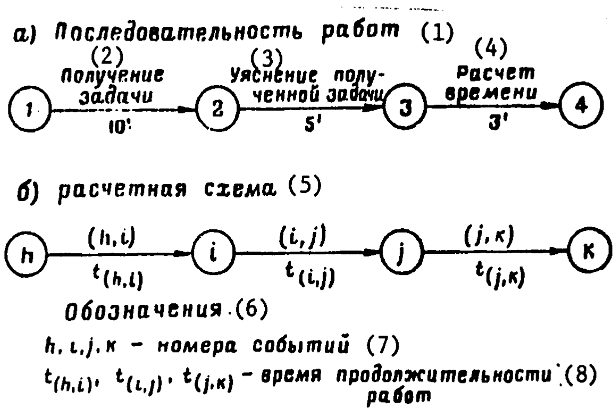

Figure 31. Basic Layout of Calculation Diagram

Key:

1. Sequence of work operations
2. Assignment of mission
3. Mission briefing
4. Time calculation
5. Calculation diagram
6. Symbols
7. Numbers of events
8. Time of duration of work operations

The calculation diagram shown in Figure 31 constitutes a chain of events and operations of a specific sequence, where:

h -- the event from which those operations commence which directly link to the preceding event; i -- initial or preceding event of operation (i, j); j -- terminal or succeeding event of operation (i, j); k -- event into which are linked those operations which proceed from the succeeding event; (h, i) -- preceing operation; (i, j) -- operation in question; (j, k) -- succeeding operation.

A calculation diagram facilitates understanding the physical significance of a network schedule.

There exist three methods of calculating networks: analytic, tabular, and graphic.

5. Analytic Method of Calculation

Formulas are employed with the analytic method, and therefore we shall acquaint ourselves with the symbols employed in calculations:

$t_{(i,\ j)}$ -- duration of operation; $t_{(h,\ i)}$ -- duration of preceding operation (h, i); $t_{(j,\ k)}$ -- duration of succeeding operation (j, k); $t_{p(i)}$ -- time of early occurrence of event (i); $t_{\text{п}(i)}$ -- time of late occurrence of event (i); $t_{p.H(i,\ j)}$ -- time of early commencement of operation (i, j); $t_{\text{п}.H(i,\ j)}$ -- time of late commencement of operation (i, j); $t_{p.o(i,\ j)}$ -- time of early completion of operation (i, j); $t_{n.o(i, j)}$ -- time of late completion of operation (i, j); $\bar{L}_{1(i)}$ -- maximum path preceding event (i); $\bar{L}_{2(i)}$ -- maximum path succeeding event (i); $t_{(L)}$ -- duration of any path; t_{kp} -- duration of critical path; $P_{(L)}$ -- full time reserve of path (L); $P_{(i)}$ -- time reserve of event (i); $P_{\text{п}(i, j)}$ -- full time reserve of operation (i, j); $P_{c(i, j)}$ -- free time reserve of operation; $P'_{\text{ч}(i, j)}$ -- partial time reserve of first type of operation (i, j); $P''_{n(i, j)}$ -- partial time reserve of second type of operation (i, j); $K_{H(i, j)}$ -- coefficient of intensity of operation (i, j); $K_{c(i, j)}$ -- coefficient of freedom of operation (i, j).

Having become acquainted with the principal symbols and terms employed in calculating network schedules, we shall proceed directly to calculation of network schedule parameters.

Determination of Early and Late Event Occurrence Times

In order to calculate the parameters of network schedules, it is necessary to know the earliest possible event occurrence time $t_{(p)i}$ and the latest allowable event occurrence time $t_{\text{п}(i)}$.

As an example we shall examine a network schedule for readying a tank battalion for an offensive action in conditions where the battalion commander is on the terrain in the sector of the forthcoming operation, has received his orders, and his deputies and company commanders have reported to him to receive their orders. The battalion is located in a concentration area 3 kilometers from the assembly area assigned to the battalion (Figure 32). The battalion will be ready to initiate the offensive action 2 hours and 10 minutes after receiving orders.

We shall determine early and late event occurrence times. For this we shall construct a network schedule of readying the tank battalion for an attack (Figure 33).

In the schedule (Figure 33) the zero event (0) designates "Attack Order Received," while the 14th event (14) designates "Tank Battalion Ready for Attack." The other events are intermediate and constitute the result of execution of corresponding work operations. For example, event (1) is the result of execution of operation (0, 1) and is designated "Tank battalion commander offensive mission briefing," and event (2) -- "Time Calculation Performed," event (4) -- "Warning Orders Issued to Combat Subunits," etc.

In this schedule operation designations are indicated above the arrows, and their duration in minutes below each arrow.

We know from the preceding material that events indicated in a network schedule do not possess any duration. Each event is a result of the preceding work operation (or several operations) and occurs instantaneously, as it were, as soon as the preceding operation is completed, or the longest of the preceding operations if there are several of them, such as operation (6, 9), which precedes event (9). Since events do not possess duration, completion time of a preceding operation is the commencement time of the succeeding operation.

Since the battalion commander will commence organization for the attack after receiving the attack order, we shall assume the time of the earliest occurrence of

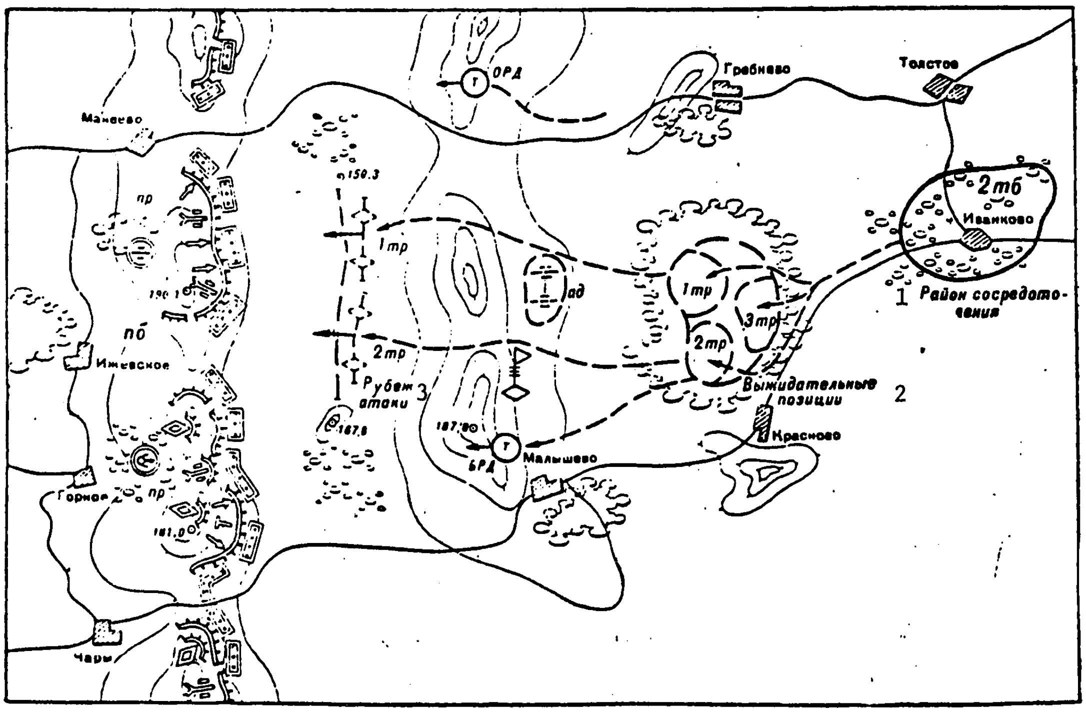

Figure 32. Position of Opposing Sides During Organization for an Offensive Action by the 2d Tank Battalion

Key:

1.	Concentration area	пр.	Infantry company
2.	Assembly area	тр.	Tank company
3.	Line of departure	тб.	Tank battalion
пб.	Infantry battalion	ad.	Artillery battalion
орд.	Separate reconnaissance patrol	прд.	Combat reconnaissance patrol

the starting (zero) event to be zero. The zero event is the commencement of the battalion commander's work.

As is evident from the chart, operation (0, 1) runs 5 minutes, since event (1) occurs as soon as operation (0, 1) ends and under no circumstances earlier, a time of 5 minutes will be the earliest possible time of occurrence of event (1).

Event (2) will take place immediately upon completion of operation (1, 2). In view of the fact that events do not possess duration, the earliest time of occurrence of event (2) will be the total duration time of the two operations preceding event (2), namely $t_{(0, 1)}+t_{(1, 2)}=5+5=10$ minutes. This is how we determine the earliest times of occurrence of events preceded by one operation.

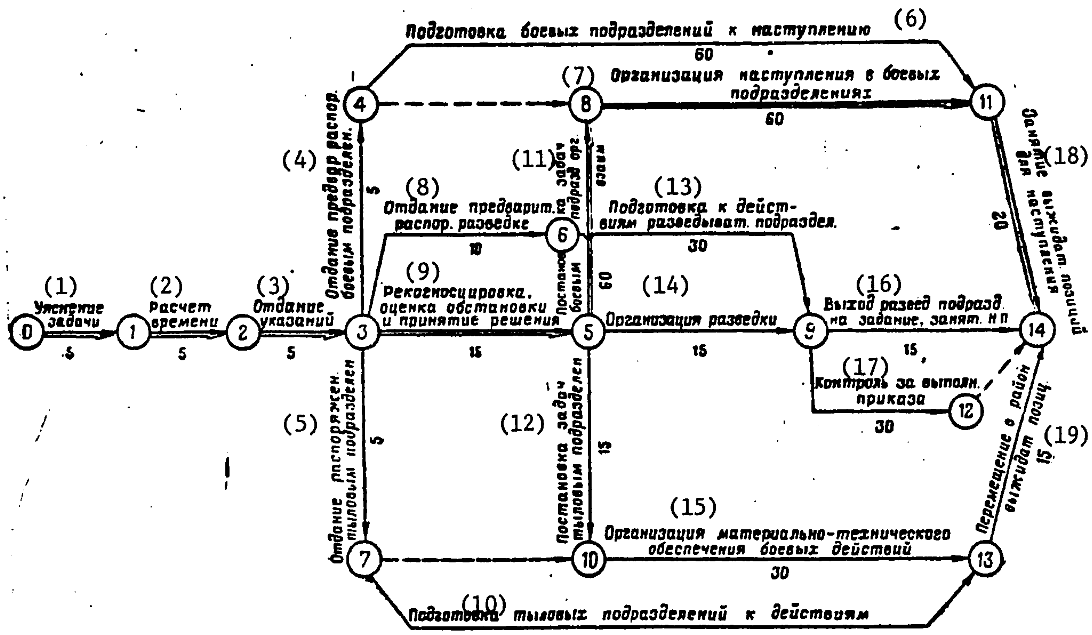

Figure 33. Network Schedule of Readying Tank Battalion for Attack (example)

Key:

1. Mission briefing
2. Time calculation
3. Issuing instructions
4. Issuing warning order to combat subunits
5. Issuing warning order to rear services subunit
6. Readying combat subunits for attack
7. Organization for attack in combat subunits
8. Issuing warning order to reconnaissance
9. Commander's reconnaissance, situation estimate, and decision-making
10. Readying rear services subunits for action
11. Allocation of tasks to combat subunits
12. Allocation of tasks to rear services subunits
13. Readying reconnaissance subunits for action
14. Organization of reconnaissance
15. Organization of combat service support
16. Reconnaissance subunits proceed to execute mission, establish observation posts
17. Verification of execution of order
18. Occupy assembly area for offensive operation
19. Movement to assembly area

If an event in the network is preceded by several operations, the earliest event occurrence time will come only when the greatest-duration operations preceding the event occur. For example, we shall determine the earliest possible time of occurrence of event (8) in Figure 33. Event (8) is preceded by two sequences of operations, namely: (0,1), (1,2), (2,3), (3,4), (4,8) and (0,1), (1,2), (2,3), (3,5), (5,8). Event (8) will occur only when all operations preceding this event have occurred. But these sequences of operations (we shall henceforth call them paths) are of differing duration. Let us calculate duration of these paths:

first path:

$$t_{(0,1)}+t_{(1,2)}+t_{(2,3)}+t_{(3,4)}+t_{(4,8)}=5+5+5+5+0=20 \quad \text{minutes;}$$

second path:

$$t_{(0,1)}+t_{(1,2)}+t_{(2,3)}+t_{(3,5)}+t_{(5,8)}=5+5+5+15+60=90 \quad \text{minutes.}$$

If all operations lying on the first path and preceding event (8) are accomplished, this event will not yet occur, for operations (3,5) and (5,8), which lie on the second path, will not yet be completed. Event (8) will occur as soon as these operations are finished. Consequently the earliest possible time of occurrence of event (8) will be a time 90 minutes after commencement of the process.

Synthesizing the above, we can draw the following conclusion: the time of early occurrence of any event in a network is equal to the duration of the maximum path preceding the given event.

If we designate with $\bar{L}_1$ the maximum-duration path preceding event (i), the above rule can be expressed analytically:

$$t_{p(i)}=t(\bar{L}_{1(i)}). \qquad (21)$$

Considering that early completion time of operation (i,j) is the time of earliest occurrence of event (j), then $t_{p(j)}$ can be expressed by $t_{p(i)}$ and $t(i,j)$. In this case we shall have

$$t_{p(j)}=\max_{j}(t_{p(i)}+t_{(i,j)}). \qquad (22)$$

Formula (22) can be read as follows: the earliest time of occurrence of event (j) is equal to the largest of the sums of the earliest occurrences of events (i) and the durations of corresponding operations preceding event (j).

It is quite obvious that in order to utilize formula (22), it is essential to know the time of occurrence of the starting event. In most cases it is convenient to assume that it is equal to zero, although in principle a non-zero value can also be determined for it.

Summarizing the above, we can conclude that the minimally requisite time between the occurrence of the starting and given events is that time which corresponds to advance of the process (operation) along the greatest-duration path of all paths joining the starting and given event. This time is measured by the sum of operations contained in the path and determines the earliest date of occurrence of the given event.

As is evident from Figure 33, a network model of a process (network) includes a number of sequences of work operations. We have called these sequences paths.

Let us examine these paths in the network schedule shown in Figure 33, and summarize them in Table 4.

Table 4.

(1) Номер пути (последовательность работ)	(2) Работы, образующие путь (последовательность работ)
1	(0,1), (1,2), (2,3), (3,4), (4,11), (11,14)
2	(0,1), (1,2), (2,3), (3,4), (4,8), (8,11), (11,14)
3	(0,1), (1,2), (2,3), (3,5), (5,8), (8,11), (11,14)
4	(0,1), (1,2), (2,3), (3,5), (5,9), (9,12), (12,14)
5	(0,1), (1,2), (2,3), (3,5), (5,9), (9,14)
6	(0,1), (1,2), (2,3), (3,5), (5,10), (10,13), (13,14)
7	(0,1), (1,2), (2,3), (3,6), (6,9), (9,12), (12,14)
8	(0,1), (1,2), (2,3), (3,6), (6,9), (9,11)
9	(0,1), (1,2), (2,3), (3,7), (7,10), (10,13), (13,14)
10	(0,1), (1,2), (2,3), (3,7), (7,13), (13,14)

Key:
1. Path number (sequence of work operations)
2. Operations forming path (sequence of operations)

The interlinkages between operations are such that each of these sequences should mandatorily be observed. Although intuitively this is entirely understandable, let us assume that in our network sequence No 10 is not observed, which does not permit execution of operations (3,7), (7,13), and (13,14), as a result of which event (14) will not occur, that is, completion of the entire complex will not occur. One should bear in mind that noncompletion of one of the jobs contained in a network leads to nonaccomplishment of the entire operation.

Knowing the duration of each work operation separately, one can easily determine the duration of execution of each sequence in the chart by simply adding up the times of duration of the appropriate work operations. Thus one can determine the length of each path in the schedule. Let us compute the duration of paths for our example (Table 5).

It was previously stated that observance of each sequence of operations in our example is obligatory. Therefore the entire aggregate of work operations by the battalion commander and his staff pertaining to organization of the attack cannot be completed sooner than 170 minutes from the moment of commencement of this work. Since work operations in the schedule were calculated in minutes, consequently the entire aggregate of work operations in the network will take 170 minutes. This will be the earliest completion time for the entire aggregate of operations.

Table 5.

(1) Номер пути (последовательность работ)	(2) Общая продолжительность пути (последовательности работ), мин
1	(0,1) + (1,2) + (2,3) + (3,4) + (4,11) + (11,14) = 5 + 5 + 5 + 5 + 60 + 20 = 100
2	(0,1) + (1,2) + (2,3) + (3,4) + (4,8) + (8,11) + (11,14) = 5 + 5 + 5 + 5 + 0 + 60 + 20 = 100
3	(0,1) + (1,2) + (2,3) + (3,5) + (5,8) + (8,11) + (11,14) = 5 + 5 + 5 + 15 + 60 + 60 + 20 = 170
4	(0,1) + (1,2) + (2,3) + (3,5) + (5,9) + (9,12) + (12,14) = 5 + 5 + 5 + 15 + 15 + 30 + 0 = 75
5	(0,1) + (1,2) + (2,3) + (3,5) + (5,9) + (9,14) = 5 + 5 + 5 + 15 + 15 + 15 = 60
6	(0,1) + (1,2) + (2,3) + (3,5) + (5,10) + (10,13) + (13,14) = 5 + 5 + 5 + 15 + 15 + 30 + 15 = 90
7	(0,1) + (1,2) + (2,3) + (3,6) + (6,9) + (9,12) + (12,14) = 5 + 5 + 5 + 10 + 30 + 30 + 0 = 85
8	(0,1) + (1,2) + (2,3) + (3,6) + (6,9) + (9,14) = 5 + 5 + 5 + 10 + 30 + 15 = 70
9	(0,1) + (1,2) + (2,3) + (3,7) + (7,10) + (10,13) + (13,14) = 5 + 5 + 5 + 5 + 0 + 30 + 15 = 65
10	(0,1) + (1,2) + (2,3) + (3,7) + (7,13) + (13,14) = 5 + 5 + 5 + 5 + 30 + 15 = 65

Key:
1. Path number (sequence of operations)
2. Total duration of path (sequence of operations), minutes

Each of the 10 sequences of work operations listed in the network schedule (Figure 33), from the starting to the concluding event, is a path. The longest-duration path is called the critical path. Consequently the ending time of the process planned in the schedule is determined by the length of the critical path. We shall discuss paths in more detail below.

In our example, finding the earliest time of occurrence of concluding event (14), one can determine the length of the critical path and consequently also determine the minimum time required to ready the tank battalion for the attack with the adopted sequence and duration of work operations.

Summarizing the above, we can draw the following conclusion. When computing the earliest event occurrence time, one must calculate time following the maximum path from the starting event to the event in question. Determining sequentially the early time of occurrence of events, we reach the concluding event. The earliest time of occurrence of the concluding event determines the earliest possible completion time for the entire aggregate of work operations.

To gain a better understanding, we shall once again examine with a concrete example the procedure of determining the earliest times of occurrence of events, as well as the time required to complete the entire program. Figure 34 contains a conditional schedule. Above the arrows we have placed numbers which indicate the

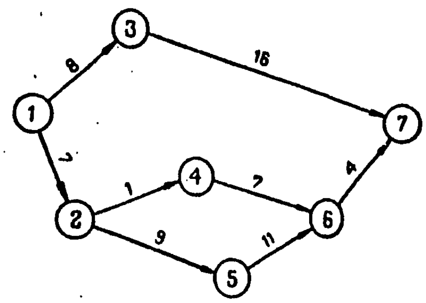

Figure 34. Conditional Network Schedule (example for determining earliest event occurrence times)

duration of work operations in units of time, let us say in hours. Proceeding in the same manner as when examining the chart in Figure 33, we shall determine early event occurrence times, which will look as follows:

$$t_{p(2)} = t_{p(1)} + t_{(1,2)} = 0 + 7 = 7;$$
$$t_{p(3)} = t_{p(1)} + t_{(1,3)} = 0 + 8 = 8;$$
$$t_{p(4)} = t_{p(2)} + t_{(2,4)} = 7 + 1 = 8;$$
$$t_{p(5)} = t_{p(2)} + t_{(2,5)} = 7 + 9 = 16;$$
$$t_{p(6)} = t_{p(5)} + t_{(5,6)} = 16 + 11 = 27;$$
$$t_{p(7)} = t_{p(6)} + t_{(6,7)} = 27 + 4 = 31.$$

Then, using formula (22), we find the time of earliest occurrence of event (6), when it is preceded by two paths, and we take the greater of the sums of early occurrences of events (i) and durations of corresponding work operations $t_{(i,j)}$. In our example the earliest time of completion of the entire program (set of work operations) will be equal to 31 units of time, that is, 31 hours.

We shall now examine another example (Figure 35), in which we must determine the earliest times of occurrence of events and the time of the entire program.

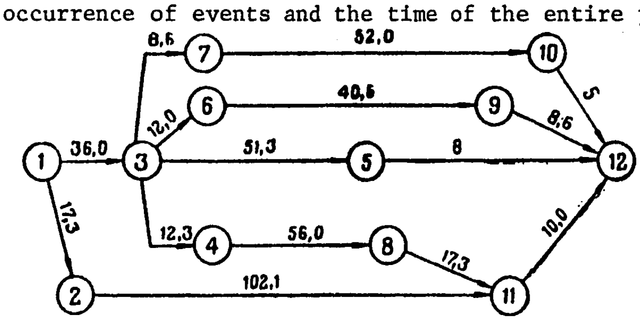

Figure 35. Network Model of Set of Work Operations

Figure 35 contains a network model of a set of work operations and indicates the duration of each operation. Reasoning the same as in the preceding examples, we shall determine the earlier event occurrence times and the time required for the entire program (the entire set of work operations), indicated in Figure 36. It is evident from the chart that the entire program time totals 131.6 units of time.

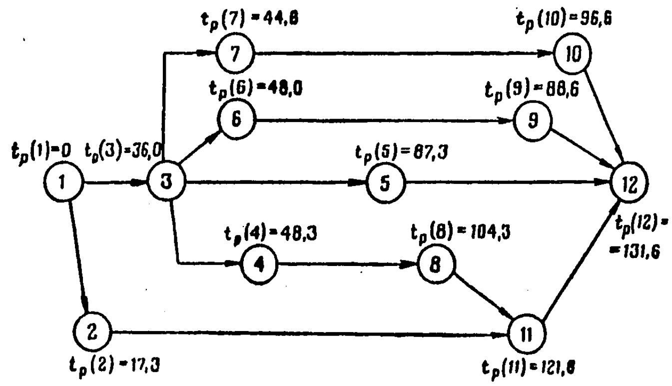

Figure 36. Network Model of a Set of Work Operations, With Indication of the Earliest Event Occurrence Time and Time of Completion of the Entire Program

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Having become acquainted with methods of determining early event occurrence times, we shall examine a method of determining the latest possible $t_{п(i)}$ event occurrence time. Each event in the network should occur at a time whereby sufficient time remains for accomplishment of all work operations following these events.

Let us return once again to the chart in Figure 33 and examine event (9). This event should take place in such a manner that after its occurrence there remains time for accomplishment of operations (9, 12) and (12, 14). In order to find this time we must subtract from the time of occurrence of the concluding event, which is characterized by the length of the critical path, the maximum path from the paths following the given event (i).

Path (9, 12, 14) will be the maximum subsequent path for event (9). Consequently,

$$t_{п(9)} = t_{кр} - (t_{(9,\,12)} + t_{(12,\,14)}) = 170 - (30 + 0) = 140 \quad \text{minutes.}$$

This means that the latest time of occurrence of event (9) should not exceed 140 minutes after the start of the process. If this event takes place later, the entire process will not be completed by the prior calculated time, that is, in 170 minutes; the time will increase, by the amount by which event (9) occurs late.

If we designate with $\bar{L}_{2(i)}$ the maximum path following event (i), this rule can be expressed with the following formula:

$$t_{п(i)} = t_{кр} - t(\bar{L}_{2(i)}). \qquad (23)$$

Thus an event late occurrence time is calculated as the difference between the duration of the critical path and the duration of the maximum path of those paths following event (i).

One proceeds in the reverse manner in computing the time of the latest allowable event occurrence time, that is, not as in determining the earliest event occurrence time, but as follows: calculation begins with the concluding event and proceeds to the starting event.

One should bear in mind, however, that for events lying on the critical path, event early occurrence time $t_{p(i)}$ is equal to event late occurrence time $t_{п(i)}$, that is $t_{p(i)}=t_{п(i)}$.

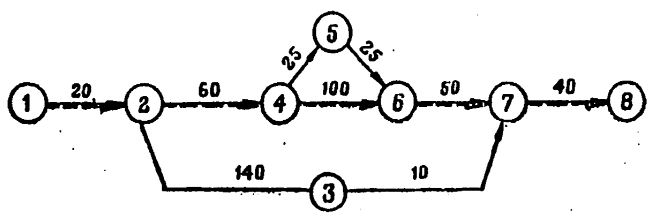

Figure 37. Network Schedule (Example) for Determining Event Late Occurrence Time

Now we shall determine with a concrete example (Figure 37) late occurrence times for events (8), (7), (6), and (3). The numbers above the arrows indicate days.

We shall first determine the length of critical path in days:

$$t_{кр}=t_{(1,2)}+t_{(2,4)}+t_{(4,6)}+t_{(6,7)}+t_{(7,8)}=$$
$$=20+60+100+50+40=270.$$

Consequently, the critical path is equal to 270 days. We then determine late occurrence time of concluding event (8). It is determined by the critical path. We know that the earliest occurrence time for a concluding event $t_{п(8)}$ is 270 days. But we also know that the earliest occurrence time for the concluding event is equal to concluding event late occurrence time $t_{п(8)}$, that is,

$$t_{p(8)}=t_{п(8)}=270.$$

Then we determine in the same manner the late occurrence time for event (7):

$$t_{п(7)}=t_{кр}-t_{(7,8)}=270-40=230.$$

After this we determine late occurrence time for event (6):

$$t_{п(6)}=t_{кр}-(t_{(7,8)}+t_{(6,7)})=270-(40+50)=270-90=180.$$

And finally, we determine late occurrence time for event (3):

$$t_{п(3)}=t_{кр}-(t_{(7,8)}+t_{(3,7)})=270-(40+10)=270-50=220.$$

Thus in our example the late occurrence time (times) of events (8), (7), (6), and (3) are 270, 230, 180, and 220 days respectively.

Determination of Early Start and Finish Time of Work Operations

Each event in a network schedule is simultaneously a terminal event for certain operations and an initial event for others. In the chart depicted in Figure 33, for example, event (3) is the end of operation (2, 3) and the beginning of operations (3, 4), (3, 5), (3, 6) and (3, 7). Therefore, in addition to an early date of occurrence of a given event, the time of possible early beginning and ending of work operations is also determined for network schedule calculations.

Work operation early commencement time ($t_{p.H(i,\,j)}$) is determined as follows. It was noted above that events do not possess duration. As soon as a work operation has ended, one can assume that the event has occurred. On the basis of this one can state that the early commencement time of a work operation is equal to event early occurrence time, that is,

$$t_{p.H(i,\,j)} = t_{p(i)} \qquad (24)$$

or, substituting $t_{p(i)} = \max\,(t_{p.H(h,\,i)} + t_{(h,\,i)})$, we obtain

$$t_{p.H(i,\,j)} = \max\,(t_{p.H(h,\,i)} + t_{(h,\,i)}). \qquad (25)$$

In Figure 33 an early beginning, of work operation (2, 3), for example, will be equal to the early occurrence time of event (2), that is, $t_{p.H(2,3)}=t_{p(2)}$:

$$t_{p.H(2)} = t_{p(1)} + t_{(1,\,2)} = 5 + 5 = 10 \text{ minutes.}$$

Since $t_{p.H(2,3)}=t_{p.(2)}$, then $t_{p.H(2,3)}=10$ minutes. This means that the earliest commencement time for operation (2, 3) will be 10 minutes later than commencement of the process.

Work operation early completion time (($t_{p.o(i,\,j)}$) is defined as the sum of the early occurrence time of event (i) and duration of operation (i, j). This can be expressed by the following formula:

$$t_{p.o(i,\,j)} = t_{p(i)} + t_{(i,\,j)}.$$

Let us examine early completion time of operation (2, 3) in Figure 33. Since the early occurrence time of event (2) was previously determined (($t_{p(2)}=10$ minutes), we shall take the time of operation (2, 3) from the chart. It is equal to 5 minutes. Consequently, early completion time of operation (2, 3) will be

$$t_{p.o(2,\,3)} = 10 + 5 = 15 \text{ minutes.}$$

This means that early completion time of operation (2, 3) can occur 15 minutes after commencement of the process.

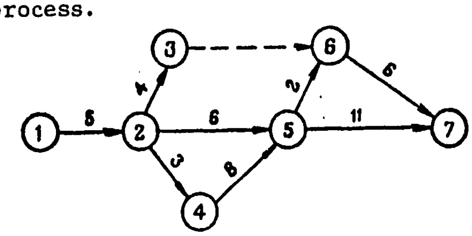

Figure 38. Network Model (Example for Determining Early Start and Early Completion of Work Operations)

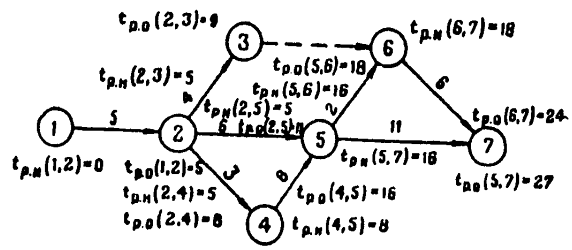

Figure 39. Solution of the Problem Presented in Figure 38, With Indication of Early Start and Early Completion of Operations

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Now we shall follow with another example the procedure of determining work operation early start and finish time. Figure 38 contains a schedule of a simple operation, indicating individual work operations, their sequence and duration. Determine early commencement and completion time for each work operation and compare your solution with the results given in Figure 39, where these times are indicated above each work operation.

At the beginning of each work operation is placed the time of its early start, and at the end -- work operation early completion time.

Determination of Critical Path

We discussed above the term path. We shall recall that in any network schedule each sequence from the starting to the concluding event is a path. There can be many such paths, and they can be of differing duration. A special place in program evaluation and review technique, however, is assigned to the so-called critical path.

Critical path is that path in a network schedule, running from the starting to concluding event, which is the longest in duration of time. When we say "the longest path in the network," this should be precisely construed as to mean that this is the minimally requisite time objectively needed to accomplish the entire group of work operations contained in the network. The entire aggregate of work operations cannot objectively be accomplished in less time than the critical path time.

Why is it called critical? There are two reasons for this. First of all, precisely this path determines, by its character in the network schedule, the duration of accomplishment of the entire aggregate of work operations. Secondly, the term "critical path" should draw attention to this group, to this chain of work operations. It is precisely the sequence of work operations lying on this path which determines the course of the business at hand (process, aggregate of work operations, program). While delay in other operations not lying on the critical path may in principle not affect accomplishment of the entire process as a whole, any delay in critical-path operations increases its duration, and therefore increases the time required to accomplish the entire process.

Consequently calculation of the schedule and determination of the critical path are essential in order to concentrate attention precisely on the critical chain of events. The network schedule not only requires this but also provides the possibility of seeing such a path. The critical path is precisely that link in the overall network of complex interlinkages, by seizing which one can pull the entire chain. The positive properties of the critical path consist in practical application of the idea of a master link for the control of complex processes.

There exists a well known principle of concentration of attention and resources on decisive axes and sectors during critical periods of performance of combat missions. In traditional troop control practices, however, selection of such sectors as well as proper time is accomplished by the intuition and experience of the command authorities. No process model is examined.

Network schedules constitute a complex operation model, carried out under quite specific conditions and requiring specific types of resources and time. Such a model depicts specific features of the entire operation (program) in an interdependence and interconditionality. The idea of master link in an overall network of events and operations, which was stated above, receives here a quite sharp, well-defined graphic description.

Determination of the critical path and consequently elucidation of the events and operations lying on this path makes it possible to concentrate attention primarily on precisely those critical operations and events which determine the duration of the overall operation (process), rather than scattering attention on the entire mass of events and operations comprising the network. Indeed, we can see in Figure 33 that of the total of 14 events, only 8 events lie on the critical path (0, 1, 2, 3, 5, 8, 11, and 14), while only 7 of the total of 22 operations lie on the critical path: (0, 1), (1, 2), (2, 3), (3, 5), (5, 8), (8, 11), and (11, 14).

It is quite understandable that in this instance it is easier to follow just the events and operations lying on the critical path. This is especially important when there are very many operations in the program being planned.

Table 6 shows the percentage share of critical operations in programs of varying complexity.

Table 6.

Общее количество работ в сети (1)	Количество критических работ (2)	Удельный вес критических работ, % (3)
10	3—4	30—40
100	12—15	12—15
1000	70—80	7—8
5000	150—160	3—1

Key:

1. Total number of operations in network
2. Number of critical operations
3. Percentage share of critical operations

The critical path is found as a result of calculation of the entire network and determination of the earliest possible event occurrence and operation completion times. As soon as the earliest completion time for the last operation is found, as well as that of the concluding event in the network, the earliest completion time for the entire operation (program) will also be found, that is, the program critical time will be determined.

If we turn to the chart (Figure 33), we shall see 10 sequences of operations. Each of these sequences is a path. The durations of all paths were calculated. Path 3 proved to be the longest (see Table 5). Its duration is 170 minutes. This is the critical path of this network. This time for readying the battalion for an attack, however, is not satisfactory, for according to the specified conditions the battalion is to be ready to attack in 2 hours and 10 minutes, that is, in 130 minutes. Consequently it is necessary to alter the work sequence to ready the battalion for action, in order to shorten the critical path by 40 minutes. The method and procedure of improving the network will be examined in the next chapter.

One must bear in mind that there may be several critical paths in a single network. Sometimes the critical path in networks may branch out into several paths and once again converge into a single path.

Thus a critical path is distinguished by two characteristics, which play an important role in analyzing the aggregate of work operations:

the greatest amount of time is expended on passage from the starting to the concluding event along the critical path;

delay in the occurrence of any event lying on the critical path causes precisely the same delay in occurrence of the concluding event.

All complete paths other than critical paths are called unstressed paths. These unstressed paths in turn are divided into subcritical paths and paths possessing a considerable time reserve.

Paths close in duration to critical are called subcritical. Subcritical paths are characterized by the fact that in carrying out a number of measures which can shorten critical paths, subcritical paths may become critical.

Experience in employing program evaluation and review technique indicates that one should also always closely moniter subcritical paths. If only critical paths are closely watched and subcritical paths ignored, there is greater probability of unwarrantedly large extension of operation execution times outside the critical path. Extending operations time on paths which differ little from critical paths can alter the situation, and a subcritical path on which delay of work operations has occurred may become a critical path.

Concentration of attention on critical and subcritical paths will make it possible to foresee and predict the occurrence of bottlenecks and breakdowns in operations in the course of a process.

One should bear in mind that subcritical paths possess only a small time reserve.

Unstressed paths, which possess a large time reserve, are characterized by the fact that resources can be removed from these paths (equipment, for example) and transferred to critical paths in order to speed up operations on these paths and thus shorten critical path time and consequently time of the entire process as a whole. Precisely this is the principal element in program evaluation and review technique, since it enables one to maneuver internal reserves and resources and thus to speed up accomplishment of the process (operation) without resorting to utilization of outside manpower and resources.

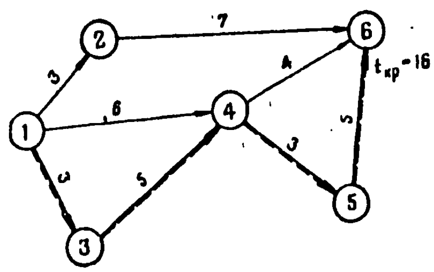

Figure 40. Example for Determining Critical Path on a Schedule

Now we shall examine with a concrete example the procedure of determining critical path. Figure 40 contains a network schedule of a group of operations, which shows the operations, their duration and sequence, as well as five paths. Analyzing the chart, we can determine the duration of each path, namely:

for path № 1 $t_{(1,2)} + t_{(2,6)} = 3 + 7 = 10;$

for path № 2 $t_{(1,4)} + t_{(4,6)} = 6 + 4 = 10;$

for path № 3 $t_{(1,3)} + t_{(3,4)} + t_{(4,6)} = 3 + 5 + 4 = 12;$

for path № 4 $t_{(1,4)} + t_{(4,5)} + t_{(5,6)} = 6 + 3 + 5 = 14;$

for path № 5 $t_{(1,3)} + t_{(3,4)} + t_{(4,5)} + t_{(5,6)} = 3 + 5 + 3 + 5 = 16.$

The critical path is the path of longest duration in time. Consequently, in our example the critical path (t_{kp}) is equal to 16 and runs through events 1, 3, 4, 5, and 6 (Figure 41).

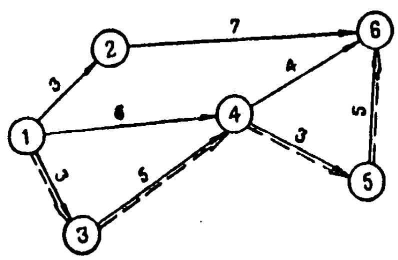

Figure 41. Solution of the Problem Presented in Figure 40

We shall endeavor once again to find the critical and subcritical paths in the schedule contained in Figure 42, on which there are a total of five paths.

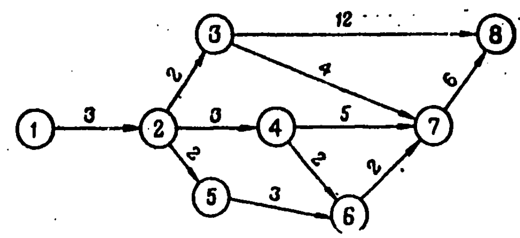

Figure 42. Example for Determining Critical and Subcritical Paths

Analyzing the schedule and determining the duration of sequences of operations, we obtain the following:

for path № 1 $t_{(1,2)} + t_{(2,3)} + t_{(3,8)} = 3 + 2 + 12 = 17$;

for path № 2 $t_{(1,2)} + t_{(2,3)} + t_{(3,7)} + t_{(7,8)} = 3+2+4+6 = 15$;

for path № 3 $t_{(1,2)} + t_{(2,4)} + t_{(4,7)} + t_{(7,8)} = 3+3+5+6=17$;

for path № 4 $t_{(1,2)} + t_{(2,4)} + t_{(4,6)} + t_{(6,7)} + t_{(7,8)} = 3 + 3 +$
$+ 2 + 2 + 6 = 16$;

for path № 5 $t_{(1,2)} + t_{(2,5)} + t_{(5,6)} + t_{(6,7)} + t_{(7,8)} = 3 + 2 +$
$+ 3 + 2 + 6 = 16$.

As is evident from the calculation, there are two critical and two subcritical paths on the chart (Figure 43).

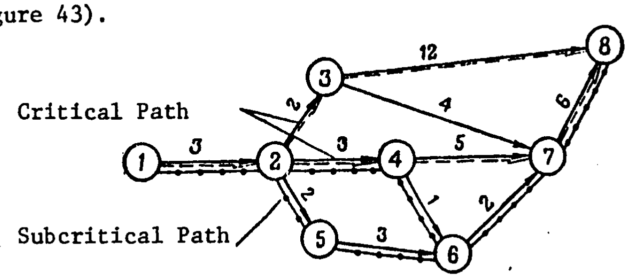

Figure 43. Solution of Problem Contained in Figure 42

Critical paths:

path No 1, running through events (1), (2), (3), and (8);

path No 3, running through events (1), (2), (4), (7), and (8).

Subcritical paths:

path No 4, passing through events (1), (2), (4), (6), (7), and (8);

path No 5, passing through events (1), (2), (5), (6), (7), and (8).

Thus in solving the problem in the chart (Figure 42), we obtain two critical and two subcritical paths. It follows from this that there may be several critical and subcritical paths in a single schedule.

Determination of Latest Allowable Operation Start and Completion Times

We shall recall that in the network schedule (Figure 33), the earliest possible operation commencement times were determined sequentially from the starting (zero) event to the concluding (14th) event. We utilized formulas (27) and (28).

Having determined the early commencement time for the concluding (14th) event, we shall establish the critical path and completion time for the entire process. All other paths are shorter than critical. If we remove resources from these paths and shift them to the critical path, we can shorten its duration. Of course this will lengthen the chains of operations from which resources were removed.

One should bear in mind, however, that the paths from which resources have been removed should not be excessively lengthened, and no other path should become longer than the initially calculated critical path. Of course it is necessary to maintain some limit to path extension. For this we employ the terms and indicator of the latest allowable operation start and completion time. This indicator is designated by the symbol $t_n(i)$, where i is the number of the event for which the indicator is determined. This indicator is to designate the time beyond which it is impermissible to delay the performance of work operations which converge on a specific event. If a work operation is extended beyond $t_n(i)$, this means delay of completion of the entire process.

It is quite understandable that $t_n(i)$ is linked with events, and particularly with the concluding events in the network. After early completion time for the concluding event has been determined, it becomes obvious that early and late completion time for the concluding event should be equal, that is, $t_n(i)=t_p(i)$. This assumption is logical, for there is no sense in immediately establishing that operations will extend beyond the time of the concluding event.

Knowledge of the earliest and latest allowable event occurrence time makes it possible to specify those which lie on the critical path. Events the earliest and latest time of occurrence of which coincide lie on the critical path. The arrows which link these events correspond to the critical sequence of work operations.

It has already been established that each work operation has an early start and an early completion. In like manner, each operation can have a late start and late completion.

Thus one can state that the early start and late completion times of an operation are determined by moving in reverse from the concluding to the initial event, that is, from right to left.

Operation late commencement times ($t_{п.H(i, j)}$) are defined as the difference between late completion ($t_{п.o(i, j)}$) and duration of the operation proper ($t_{(i, j)}$):

$$t_{п.н(i, j)} = t_{п.о(i, j)} - t_{(i, j)}. \quad (26)$$

Considering that late completion of a given operation constitutes late commencement of the succeeding operation, that is, $t_{п.o(i, j)} = t_{п.B(i, k)}$, formula (26) can be presented in the following form:

$$t_{п.н(i, j)} = t_{п.н(j, k)} - t_{(i, j)}. \quad (27)$$

We shall take the schedule (Figure 33) and determine late commencement, for example, of operation (11, 14). We know that operation time for occurrence of the concluding event and all events lying on the critical path is equal to the late occurrence time of these events, that is, $t_{p(i)} = t_{H(i)}$. Early commencement and late completion times of event (14) will occur as soon as operation (11, 14), which lies on the critical path, ends. But as we know, this operation will end 170 minutes after the process begins. Consequently, the latest completion time for operation (11, 14) will be 170 minutes after commencement of work operations. Duration of operation (11, 14) is 20 minutes. Substituting 20 minutes into formula (26), we obtain

$$t_{п.н(11, 14)} = t_{п.о(11, 14)} - t_{(11, 14)} = 170 - 20 = 150 \text{ minutes}.$$

This means that operation (11, 14) should begin not later than 150 minutes after commencement of the process. If it begins later, the entire process will not fit within the calculated time, that is, it will exceed 170 minutes.

Operation late completion times ($t_{п.o(i, j)}$) are determined by the late commencement time of the subsequent operation, that is, they are equal to them:

$$t_{п.о(i, j)} = t_{п.н(j, k)}. \quad (28)$$

Let us take, for example, operation (8, 11) and determine its late completion time (Figure 33). The late commencement time of operation (11, 14) was determined above. It is equal to $t_{п.H(11, 14)} = 150$ minutes. But since operation (11, 14) is a subsequent operation in respect to operation (8, 11), that is, is operation (j, k), late completion of the operation preceding it, that is, operation (8, 11), is equal to late commencement of operation (11, 14). Consequently,

$$t_{п.о(8, 11)} = t_{п.н(11, 14)} = 150 \text{ minutes}.$$

If one determines an operation late completion time, after which several operations occur, the late completion time of such operation will be equal to the minimum value of all subsequent operations:

$$t_{п.о(i, j)} = \min t_{п.н(j, k)}. \quad (29)$$

For example, we shall take the schedule (Figure 33) and determine the late completion time of operation (6, 9). For this we shall determine the late commencement times of the operations following event (9), that is, operations (9, 12) and (9, 14) in minutes:

$$t_{п.н(9,12)} = t_{п.о(9,12)} - t_{(9,12)} = 170 - 30 = 140;$$

$$t_{п.н(9,14)} = t_{п.о(9,14)} - t_{(9,14)} = 170 - 15 = 155.$$

Since the minimum late commencement time value for operations following operation (6, 9) is 140 minutes, we shall take this minimum value.

Substituting into formula (29) values $t_{п.H(9,\ 12)}$, we shall take the minimum of these:

$$t_{п.о(6,9)} = \min(t_{п.н(9,12)};\ t_{п.н(9,14)}) = \min(140;\ 155) = 140.$$

Thus operation (6, 9) should terminate not later than 140 minutes after commencement of the process, for otherwise it can cause extension of the entire operation in question.

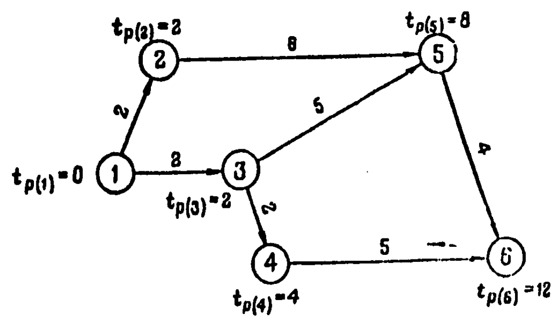

Figure 44. Network Schedule of a Group of Operations (Example for Determining Operation Late Start and Completion Times)

Knowing the method of determining operation late start and completion times, we shall find these parameters in the network schedule in Figure 44.

Utilizing formulas (26) and (28), we shall determine the late start and late completion time of each operation indicated in the schedule (Figure 44). As a result we shall obtain the figures indicated on the chart in Figure 45.

Computing Time Reserves of Events, Work Operations, and Paths

In any network schedule all unstressed paths, events and work operations lying on noncritical paths possess time reserves. Elucidation and utilization of these reserves constitutes the main idea of program evaluation and review technique, since manpower and material resources can be removed from work operations and paths possessing time reserves and directed to perform operations lying on critical paths. With this one can shorten the execution time of critical work operations and consequently of the overall operation without resorting to enlisting outside resources, utilizing only internal reserves and resources and maneuvering them.

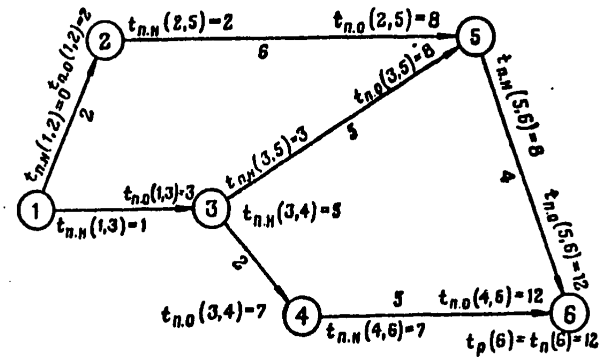

Figure 45. Schedule With Solution Results for the Problem Contained in Figure 44

To gain a better understandanding of the business of determining reserves of paths, events and operations, we shall take the initial network (Figure 33) and, applying the knowledge acquired as a result of studying the preceding material, we shall represent it with already computed early and late event occurrence times, early and late operation start and completion times. We shall indicate early and late event occurrence times alongside the circles which designate the events; we shall indicate duration of operations below, in the middle of each arrow; above the arrows we shall indicate: at the beginning -- early and late commencement times, at the end -- early and late operation completion times.

Figure 46 contains an initial network schedule for readying a tank battalion to attack from a position of close contact with the enemy, with a calculated timetable. Here one can examine our network schedule with computed time estimates of events and work operations.

Determination of Path Full Time Reserve

It was noted above that the length of a critical path is greater than the length of any other path in a network schedule. The difference between the length of the critical path t_{kp} and length of any other path $t_{(L)}$ is called the full time reserve $P_{(L)}$ of this path and is determined with the formula

$$P_{(L)} = t_{\kappa p} - t_{(L)}, \qquad (30)$$

where $P_{(L)}$ is path full time reserve; $t_{(L)}$ -- length of any path.

We shall determine the full time reserve of path 0, 1, 2, 3, 7, 10, 13, and 14 in the schedule (Figure 46). For this we calculate the length of this path in minutes

$$t_{(0,1)} + t_{(1,2)} + t_{(2,3)} + t_{(3,7)} + t_{(7,10)} + t_{(10,13)} + t_{(13,14)} = 5 + 5 + 5 + 5 + 0 + 30 + 15 = 65.$$

Length t_{kp}=170, consequently,

$$P_{(L)} = t_{\kappa p} - t_{(L)} = 170 - 65 = 105.$$

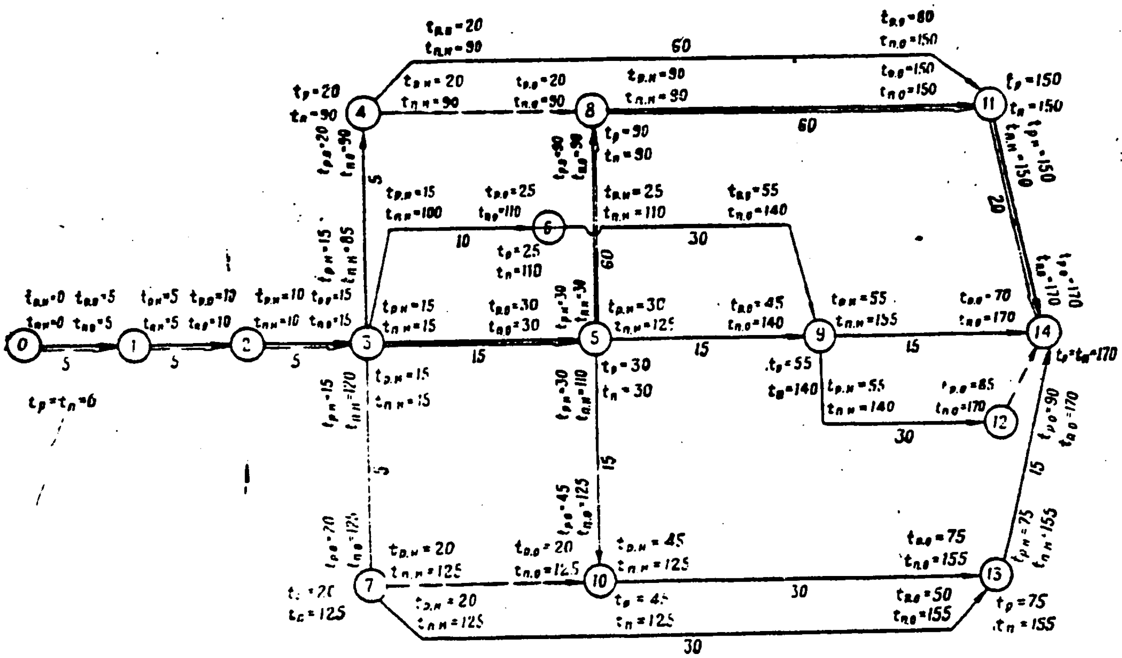

Figure 46. Calculated Parameters of Initial Network Schedule for Readying Tank Battalion for an Attack

The shorter any path is in comparison with the critical path, the larger its full time reserve.

A path's full time reserve indicates the extent to which the duration of all operations applying to path L can be increased without greatly influencing overall process time. In other words, the full time reserve of a given path $P_{(L)}$ indicates the maximum allowable increase in the length of this path $t_{(L)}$. If one further increases $t_{(L)}$, the critical path will shift to that path.

A path's full time reserve is common for all work operations of the corresponding segment. This should be defined as follows: if the longest noncritical path does not intersect at any point with the critical path (other than the starting and concluding events), it determines the reserve for the events being analyzed. The amount of reserve is common to all these events and is equal to the difference between the critical and noncritical paths.

If the path in question intersects the critical path, then the path's full time reserve is determined from the individual noncritical segments. A grouping of events on the basis of their common reserve is possible only whereby these events are arranged in sectors enclosed between a pair of critical-path events. Then common reserve is determined by the longest segment passing along them. This longest path will consist of a segment of the critical path preceding the given segment, the given segment and the subsequent critical-path segment.

Thus we shall make a generalized statement: a path's full time reserve can be defined as reserve applying to a noncritical segment of a path enclosed between two events of the critical path and common to the operations of this segment.

Determination of Time Reserves of Events

Event time reserve $P_{(i)}$ is defined as the difference between the late occurrence time and early occurrence time of this event:

$$P_{(i)} = t_{п(i)} - t_{р(i)}. \tag{31}$$

As is evident from the chart (Figure 46), the time values of early ($t_{p(i)}$) and late completion time ($t_{п(i)}$) of events lying on a critical path are equal. For example: for event (1) $t_{p(1)}=t_{п(1)}=5$; for event (3) $t_{p(3)}=t_{п(3)}=15$; for event (5) $t_{p(5)}=t_{п(5)}=30$; for event (14) $t_{p(14)}=t_{п(14)}=170$.

This indicates that critical-path events have no time reserve. Equality $t_{p(i)}=t_{п(i)}$ is one of the features of a critical path. In addition, early and late start and early and late completion of work operations lying on a critical path coincide. This means that these operations also have no time reserves.

As regards events lying on noncritical paths, with them there is a certain difference between early and late occurrence time, that is, between $t_{p(i)}$ and $t_{п(i)}$. For example, for event (9) $t_{p(9)}=55$ minutes, while $t_{п(9)}=140$ minutes. As we see, there is a difference here: 140-55=85 minutes. This difference indicates that even event (9) can occur 85 minutes later than early occurrence of this event $t_{p(6)}$, and this will by no means influence completion of the process of readying the tank battalion for an offensive action.

If we consider that the actual time of occurrence of an event with any arbitrary number (i) is designated by $t_{д(i)}$, then obviously relation $t_{p(i)} \leq t_{д(i)} \leq t_{п(i)}$, should be observed, while for events lying on critical paths equality $t_{p(i)}=t_{д(i)}=t_{п(i)}$ should be observed. If this condition is not met, that is, if it occurs that $t_{p(i)}=t_{п(i)} < t_{д(i)}$, this will lead to delay of work operations and to failure to meet the process timetable as a whole.

Determination of Time Reserves of Work Operations

Previously it was established that path time reserve $P_{(L)}$ can be utilized to increase the duration of operations lying on this path. Therefore one can state that any operation of path L possesses a time reserve on that segment which does not coincide with the critical path. An operation's time reserve, determined with the aid of the time reserve of the path on which it is located, is called operation full time reserve (i, j) and is designated by $P_{п}(i, j)$.

It is evident from the chart (Figure 46) that operation (3, 5) can apply to several paths. This means that several different paths can run through operation (3, 5). Since the lengths of these paths are as a rule unequal, consequently the time reserves of these paths will also differ.

The full time reserve of a work operation is determined as the time reserve of the maximum path passing through this operation. The time reserve of any path may be distributed among the individual work operations located on the given path only within the limits of the full time reserves of these operations.

Quantity $P_{\text{п}(i,\ j)}$ indicates by how much time duration $t_{(i,\ j)}$ of individual work operation $(i,\ j)$ can be increased so that the length of the maximum path $t_{L_{max}}$ passing through this work operation will not exceed critical-path length.

Work operations lying on critical paths do n o t have a time reserve, that is, $P_{\text{п}}(i,\ j)=0$.

We shall examine the formation of work operation reserves with a concrete example.

It is evident in Figure 46 that work operation early and late completion time is connected with events, and at the same time the onset of early and late operation completion time is connected with the course proper of work operations which begin and end at the corresponding event. It follows from this that there is a certain time reserve wherever it is possible to shift some event without shifting work operation completion time.

Let us examine this in greater detail, for which we shall construct a table, in which we shall show the numerical value of the difference between $t_{\text{п}(i)}$ and $t_{p(i)}$ of each event (Table 7).

Table 7.

Time Время	Event Событие														
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$t_{\text{п}(i)}$	0	5	10	15	90	30	110	125	90	140	125	150	170	155	170
$t_{p(i)}$	0	5	10	15	20	30	25	20	90	55	45	150	85	80	170
$t_{\text{п}(i)} - t_{p(i)}$	0	0	0	0	70	0	85	105	0	85	80	0	85	75	0

In our example 7 events have a difference between $t_{\text{п}(i)}$ and $t_{p(i)}$. This signifies that only these events can be shifted without affecting the process completion timetable. All these events, however, if we examine the diagram, are of a complex nature: several work operations converge and diverge in each of these events. In view of this fact, we shall determine how the difference $t_{\text{п}}(i)$ and $t_{p(i)}$ makes it possible to shift or extend operation completion times. For this we shall examine several groups of work operations contained in the network shown in Figure 46.

The first group contains operations lying on the critical path. These operations are indicated on the chart with double-line arrows. We shall list them: (0, 1), (1, 2), (2, 3), (3, 5), (5, 8), (8, 11), (11, 14).

Let us examine operation (3, 5), the content of which consists of situation estimate and decision-making. It contains the initial (3) and terminal event (5). If for this operation we analyze early and late start and finish times, we shall see that early start time is equal to operation late start time, that is, $t_{p.\pi}(3, 5)=t_{\pi.H}(3, 5)=15$, and early completion time is equal to operation late completion time, that is, $t_{p.o}(3, 5)=t_{\pi.o}(3, 5)=30$. Consequently, a shift cannot be made in this work operation, because there is no operation time reserve.

Thus critical operations, just as critical events, do not contain time reserves.

These work operations are based on specified resources and the work operation time values $t_{(i, j)}$ which are based on this distribution. For critical operations analysis of shift and duration change is clear: it is desirable to shorten these operations in order to shorten the entire process as a whole. Critical-path events can be shifted only by enlisting additional outside resources or by enlisting resources from other operations in the same network. This can be done either at the planning stage or during accomplishment of the work operations process.

The second group includes work operations in the network for which neither the initial nor terminal events are critical-path events. There are five such operations in our network (Figure 46): (6, 9), (9, 12), (7, 10), (7, 13), and (10, 13). Analysis of these operations makes it possible to establish the existence of work operations available time reserve (in the literature it is sometimes called work operations independent time reserve).

Operation available time reserve $P_{c(i, j)}$ is formed in those cases where the duration of an operation is less than the difference between the earliest of the possible terminal event occurrence times and the latest of the allowable completion times for the initial event of the given work operation. This is shown schematically in Figure 47.

As is evident from Figure 47, available reserve is calculated with the formula

$$P_{c(i, j)} = t_{p(j)} - t_{\pi(i)} - t_{(i, j)}. \qquad (32)$$

The conditions for occurrence of this reserve are as follows: $t_{p(i)}-t_{\pi(i)} > t_{(i, j)}$, that is, under the condition $t_{p(i)}-t_{\pi(i)} \leqslant t_{(i, j)}$ there is no avialable reserve.

Work operation available reserve indicates that one can increase the duration of an operation by the amount of its available reserve, without affecting the occurrence times of its inital and terminal events and without affecting the amount of time reserves of all other work operations in the network.

One should bear in mind that when computing available reserve by formula (32), the results may prove negative. This will signify a shortage of time. Negative quantities do not have practical significance, and therefore available time reserve should be calculated only for those operations which actually have a reserve. This as a rule is accomplished by selecting values between zero and a positive quantity, in other words by calculating max 0, $t_{p(j)}-t_{\pi(i)}-t_{i, j)}$.

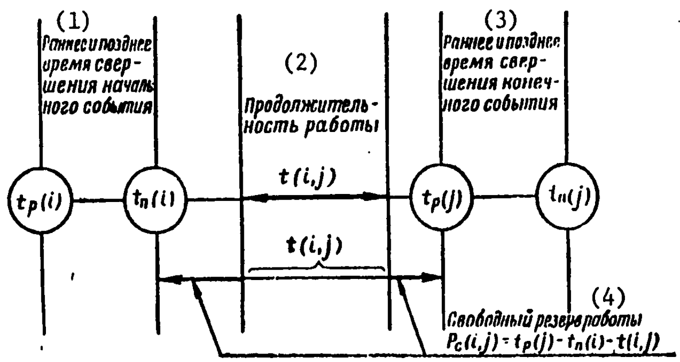

Figure 47. Schematic Representation of Work Operation Available Reserve

Key:
1. Early and late occurrence time of initial event
2. Duration of work operation
3. Early and late occurrence time of terminal event
4. Work operation available reserve

The third group contains work operations which are linked by their end or beginning with the critical path. These operations make it possible to elucidate work operation partial time reserves. Work operation partial reserves are formed at the points of intersection of paths of differing duration and occur in operations involving paths of less duration.

Partial reserves are of two types. We shall exmine this with the aid of the diagram in Figure 48.

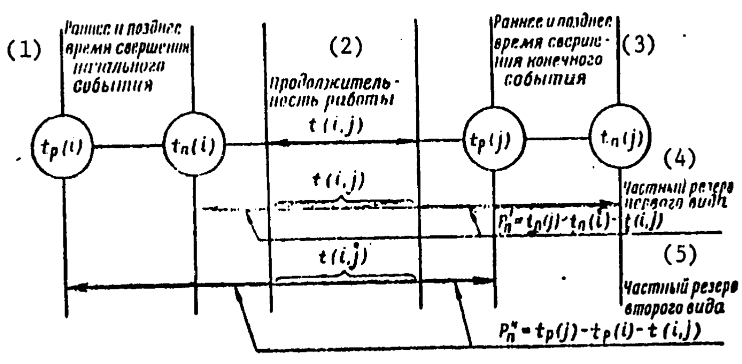

Figure 48. Schematic Diagram of Formation of Partial Reserves of Work Operations

Key to Figure 48 on preceding page:

1. Initial event early and late occurrence time
2. Duration of work operation
3. Terminal event early and late occurrence time
4. Partial reserve of first type
5. Partial reserve of second type

Partial reserve of the first type $P'_{n(i, j)}$ is formed in work operations which follow events from which paths of differing duration diverge (Figure 49). This reserve indicates what portion of the work operation full time reserve can be used to increase the duration of the given operation and other subsequent operations lying on the path segment between the two closest points of its intersection with paths of greater length, under the condition that this increase will not reduce time reserves of any of the preceding work operations.

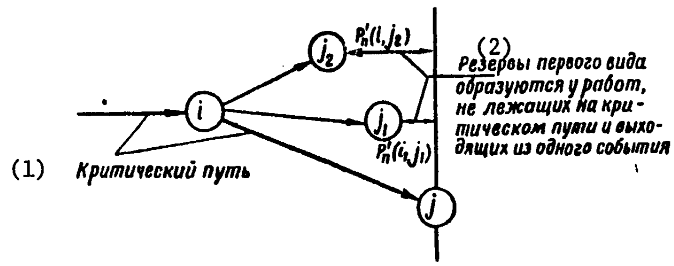

Figure 49. Schematic Diagram of Formation of Reserves of the First Type

Key:

1. Critical path
2. Reserves of the first type are formed in operations not lying on the critical path and proceeding from one event

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In military affairs partial reserve of the first type $P'_{n(i, j)}$ can also be viewed as an indicator of the time by which a given operation can be initiated earlier than the scheduled time, whereby this will not affect the final completion time for the entire operation. Partial reserve of the first type, as is evident from figures 48 and 49, is calculated with the formula

$$P'_{\text{п}(i,j)} = t_{\text{п}(j)} - t_{\text{п}(i)} - t_{(i,j)} \qquad (33)$$

or

$$P'_{\text{п}(i,j)} = P_{\text{п}(i,j)} - P_{(i)}. \qquad (34)$$

Partial reserve of the second type $P''_{n(i, j)}$ is formed in operations directly preceding an event at which paths of different length intersect (Figure 50). This reserve shows what part of a work operation's full time reserve can be used to increase the duration of that operation and operations preceding it, lying on the path segment between the two closest points of its intersection with paths of greater length, under the condition that this increase will not shorten time reserves of any subsequent operation.

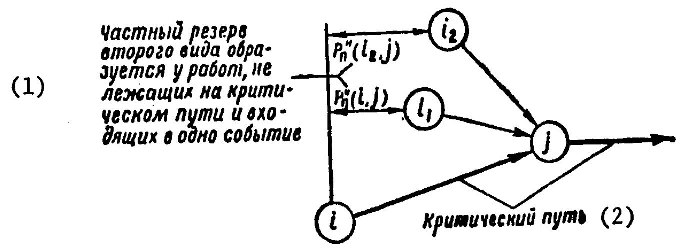

Figure 50. Schematic Diagram of Formation of Reserves of the Second Type

Key:
1. Partial reserve of the second type is formed in operations not lying on the critical path and entering a single event
2. Critical path

In military affairs partial reserve of the second type $P''_n(i, j)$ can also be viewed as an indicator of the time by which a given operation can commence later than the scheduled time, whereby this does not affect the final completion time of the overall operation.

As is evident from figures 48 and 50, partial reserve of the second type is determined with the formula

$$P''_{n(i,j)} = t_{p(j)} - t_{p(i)} - t_{(i,j)} \qquad (35)$$

or

$$P''_{n(i,j)} = P_{n(i,j)} - P_{(j)}. \qquad (36)$$

Thus determination of time reserves of paths, events and work operations is essential in order to improve initial plans, to check and monitor work operations and to forecast operations, proceeding from their actual status and likely changes.

In order to synthesize the material on work operation time reserves, we shall prepare a general diagram of all types of work operation reserves, so that they can be viewed together. This will make it possible better to remember schemes of formation of work operation reserves. Thus for a work operation with initial event (i) and terminal event (j), work operation time reserves are determined according to the scheme contained in Figure 51.

Determination of Coefficient of Intensity of Work Operations

In calculating network schedules it is important to know the characteristics of intensity of work operation performance timetables. The coefficient of intensity of work operations constitutes this characteristic: it is equal to the relation of the duration of path segments which are noncoincident and linked between the same events, one of which is a maximum-duration path passing through this work operation, while the other is a critical path.

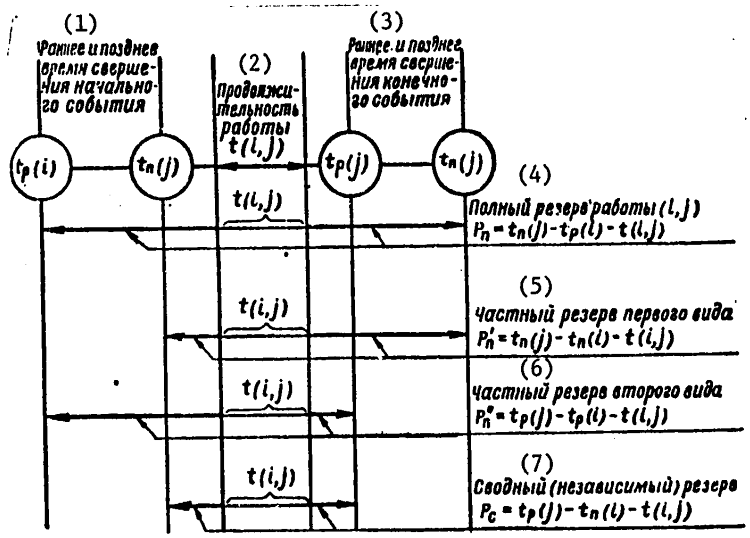

Figure 51. Summary Schematic Diagram of Formation of Work Operation Reserves

Key:

1. Initial event early and late occurrence time
2. Duration of work operations
3. Terminal event early and late occurrence time
4. Full reserve of work operation
5. Partial reserve of first type
6. Partial reserve of second type
7. Combined (independent) reserve

If duration of the critical path segment is designated by $t'_{kp(L)}$, coinciding with path $L_{(i)max}$, and the length of the maximum path -- via $t_{(L)max}$, which passes through the given work operation, the coefficient of intensity can be expressed with the formula

$$K_{н(i,j)} = \frac{t_{(L)\max} - t'_{кр(L)}}{t_{кр} - t'_{кр(L)}}. \qquad (37)$$

We shall take a network schedule as an example (Figure 46) and determine the coefficient of intensity of operations $K_{H(5,\ 9)}$ and $K_{H(10,\ 13)}$.

First of all we shall determine $t_{(L)max}$ for work operations (5, 9) and (10, 13):

$$1)\ t_{(L)\max(5,9)} = t_{(0,1)} + t_{(1,2)} + t_{(2,3)} + t_{(3,5)} + t_{(5,9)} + t_{(9,12)} + t_{(12,14)} = 5 + 5 + 5 + 15 + 15 + 30 + 0 = 75;$$

$$2)\ t_{(L)\max(10,13)} = t_{(0,1)} + t_{(1,2)} + t_{(2,3)} + t_{(3,7)} + t_{(7,10)} + t_{(10,13)} + t_{(13,14)} = 5 + 5 + 5 + 5 + 0 + 30 + 15 = 65.$$

We then determine the duration of critical path segments $t'_{kp(L)}$, coincident with $t_{(L)max(5,\ 9)}$ and $t_{(L)max(10,\ 13)}$:

$$1)\ t_{\text{кр}(L)(5,9)} = t_{(0,1)} + t_{(1,2)} + t_{(2,3)} + t_{(2,3)} + t_{(3,5)} = 5+5+5+ + 15 = 30;$$

$$2)\ t_{\text{кр}(L)(10,13)} = t_{(0,1)} + t_{(1,2)} + t_{(2,3)} = 5+5+5 = 15.$$

Possessing $t_{(L)\max}$ and $t'_{\text{кр}}$, we determine coefficients of intensity for operations (5, 9), and (10, 13):

$$1)\ K_{\text{н}(5,9)} = \frac{t_{(L)\max(5,9)} - t'_{\text{кр}(L)(5,9)}}{t_{\text{кр}} - t'_{\text{кр}(L)(5,9)}} = \frac{75-30}{170-30} = \frac{45}{140} = 0{,}32;$$

$$2)\ K_{\text{н}(10,13)} = \frac{t_{(L)\max(10,13)} - t'_{\text{кр}(L)(10,13)}}{t_{\text{кр}} - t'_{\text{кр}(L)(10,13)}} = \frac{65-15}{170-15} = \frac{50}{155} = 0{,}32.$$

The absolute values of the coefficient of intensity of work operations are equal. Consequently, equal attention must be devoted to both operations. Execution timetables are tougher for work operations with a high coefficient of intensity, and considerable attention must be devoted to work operations on this path.

Determination of Coefficient of Availability

In order to describe the relative magnitude of work operation available time reserve $P_{c(i,\ j)}$, we determine coefficient of availability $K_{c(i,\ j)}$ with the formula

$$K_{c(i,j)} = \frac{t_{\text{р}(j)} - t_{\text{п}(i)}}{t_{(i,j)}}. \qquad (38)$$

This coefficient indicates the degree to which one can increase the duration of a work operation without affecting the occurrence times of all events and all other work operations in the network.

If a work operation has an available time reserve, then

$$K_{c(i,\ j)} > 1.$$

If $K_{c(i,\ j)} < 1$, this indicates that this work operation lacks an available time reserve.

Determination of Probability of Occurrence of a Concluding or any Reference Event on Schedule

In determined networks in which unambiguous work operation execution time estimates are given (on the basis of experience, standard figures, regulations or manuals), the probability of occurrence of any event is equal to 1.

In stochastic networks, that is, in networks where time estimates are stated on the basis of three estimates (a, m, and b), in which there are a large number of events and the normal law of distribution is in operation, the probability of occurrence of an event at the calculated time can be determined with the formula

$$P_{(j)} = \Phi\left(\frac{t_{\text{эд}} - t_{p(j)}}{\sqrt{\sum \sigma^2}}\right), \tag{39}$$

where $P_{(j)}$ -- probability of occurrence of event in question; Φ -- Laplace function; $t_{\text{эд}}$ -- directive time; $t_{p(j)}$ -- early occurrence time of event (j); $\Sigma\sigma^2$ -- sum of standard deviations of work operations, which were utilized in calculating the earliest time of occurrence of the corresponding event.

With formula (39) one can determine the probability of occurrence of any event in the network. This probability, however, is determined only for important reference events and for the concluding event in the network. Having determined the probability of occurrence of the concluding event in the network, we establish the probability of accomplishment of the scheduled overall operation (process).

Determination of the probability of accomplishment of the overall operation (process) on the scheduled timetable makes it possible to determine the quality of process planning.

Until such time as a special study is made, evidently in estimating probability of process (event) completion, one can adopt in military affairs the same criteria employed in the civilian economy, namely the probability of occurrence of the concluding or any reference event should be within

$$0.35 < P_{(j)} < 0.65 \tag{40}$$

Such an inequality signifies that the planned process will be accomplished on schedule, and there is no need for further optimization (improvement) of the network which is examined in the following chapter.

If the probability of accomplishment of an operation (event) is less than 0.35, this means that the odds of fitting the planned process into the calculated time frame are poor. Inequality $P_{(j)} > 0.65$ will signify that there is considerable unutilized reserve potential in the network. In both cases it is essential to optimize the network and to achieve realization of the above inequality (40).

Network schedules can be calculated manually (by the manual algorithm method) and on digital computers.

In this volume we shall examine in the greatest detail methods of calculating network schedules for the manual algorithm method, for small networks (up to 200 events) can be calculated manually in almost the same time as with an electronic computer. This is due to the fact that entry of input data, processing of results and putting these results into a form convenient for analysis with existing information input and display devices require considerable time. Experience has been amassed, however, in manual calculation also of more complex networks, containing from 1500 to 2500 events.

In conditions of combat operations, in many cases manual calculation of networks will be essential, although on the whole one should bear in mind that wherever computers are available, it is expedient to use this equipment in calculating networks.

We have previously stated that network schedules can be calculated by analytic, tabular, and graphic methods.

The analytic method, examined in Subsection 5, consists in calculating network schedule parameters on the basis of formulas. This method of calculation constitutes the basis in calculating schedules with tabular and graphic methods. The results obtained by this method, however, require systematization, on which one must expend additional time.

The tabular and graphic methods of calculation provide for systematization of calculations, which provides convenience in network analysis. These two methods of computation are examined in Subsections 6 and 7.

6. Tabular Method of Calculation

The tabular method of calculation consists in the following: a calculation table is filled in on the basis of the constructed schedule; on the basis of this table, employing a special algorithm (sequence of operations), one calculates all the parameters of the network schedules. This is the most convenient method, which makes it possible more fully to calculate the princ:pal parameters of a network schedule.

Table 8 shows the tabular method of calculating the initial network schedule contained in Figure 33. All calculated parameters of the network schedule are indicated in Table 8.

Let us examine the procedure of calculating network schedule parameters as shown in the table.

Determination of Early Start and Early Completion of Work Operations

To perform calculation of these parameters, we proceed as follows.

1. We prepare the calculation table form (see Table 8).

2. In columns 1 and 2 of Table 8, we enter from the network schedule all work operations in the network. Operations are entered with the numbers of the events bounding each work operation, such as (0, 1), (1, 2), (2, 3). It is important that work operations be entered in a very definite sequence, in ascending order of the initial numbers of the work operations. In Table 8, for example, we begin with operation (0, 1), followed by work operations in ascending order of initial event numbers: (1, 2), (2, 3), (3, 4), etc.

One should bear in mind that if several operations proceed from a single event, we also enter all of them in the table in ascending order, but only as regards the terminal event. One and the same number is placed in column 1. In the chart (Figure 33), for example, four operations proceed from event (3). We enter in column 1 the number of the initial event four times (in four rows), that is, event (3), while we enter in column 2 the numbers of the terminal events of the operations in ascending order, that is, we write the numbers 4, 5, 6, 7. In this manner we enter all operations proceeding from the third event, that is, operations (3, 4), (3, 5), (3, 6) and (3, 7).

Table 8. Tabular Method of Network Schedule Calculation (Example of Calculation of the Network Schedule Shown in Figure 33)

(7) Работы		Раннее начало работы	Продолжительность работы	Раннее окончание работы
Начальное событие	Конечное событие			
(1)	(2)	(3)	(4)	(5)
0	1	0	5	5
1	2	5	5	10
2	3	10	5	15
3	4	15	5	20
3	5	15	15	30
3	6	15	10	25
3	7	15	5	20
4	8	20	0	20
4	11	20	60	80
5	8	30	60	90
5	9	30	15	45
5	10	30	15	45
6	9	25	30	55
7	10	20	0	20
7	13	20	30	50
8	11	90	60	150
9	12	55	30	85
9	14	55	15	70
10	13	45	30	75
11	14	150	20	170
12	14	85	0	85
13	14	75	15	90

Позднее начало работы	Продолжительность работы	Позднее окончание работы	Полный резерв времени работы	Частный резерв времени первого вида	Частный резерв времени второго вида
(6)	(4) 7	(8)	(9)	(10)	(11)
0	5	5	0	0	0
5	5	10	0	0	0
10	5	15	0	0	0
85	5	90	70	70	0
15	15	30	0	0	0
100	10	110	85	85	0
120	5	125	105	105	0
90	0	90	70	0	70
90	60	150	70	0	70
30	60	90	0	0	0
125	15	140	95	95	10
110	15	125	80	80	0
110	30	140	85	0	0
125	0	125	105	0	25
125	30	155	105	0	25
90	60	150	0	0	0
140	30	170	85	0	0
155	15	170	100	15	100
125	30	155	80	0	0
150	20	170	0	0	0
170	0	170	85	0	85
155	15	170	80	0	80

Key:

1. Initial event
2. Terminal event
3. Early start of operation
4. Duration of operation
5. Early completion of operation
6. Late start of operation
7. Work operations
8. Late completion of operation
9. Work operation full time reserve
10. Partial time reserve of first type
11. Partial time reserve of second type

We proceed in like manner until all work operations in the network are entered.

3. Then we enter in columns 4 and 7 opposite each operation their duration in common units of measure (minutes, hours, days, weeks). Since the duration of each work operation is a concrete quantity, the same numbers for each work operation will appear in each row of both columns, that is, in columns 4 and 7. It is convenient for computation to enter the duration of operations in two columns: in column 4 -- for determination of early completion of a work operation, and in column 7 -- for determination of late commencement of a work operation. These are two columns with the duration of the same work operations.

4. After this we determine on Table 8 early start and early completion of operations, that is, we perform computation and fill in columns 3 and 5. It is recommended that both columns be filled in simultaneously row by row, that is, first we fill in the row in column 3, and then in column 5, only after this proceeding to the next lower row. We fill in the rows in the columns downward.

We fill in columns 3 and 5 as follows:

first we enter in the first row of column 3 the early commencement time of the operation which begins from the initial (zero for us) event; the count begins from the initial event, and therefore zero is considered the beginning of the first operation (0, 1), and it is entered in the first row of column 3;

then we add to the early start of work operation (0, 1) the duration of this operation, thus obtaining early completion of operation (0, 1), and we enter this time in the first row of column 5; we utilize the following formula:

$$t_{р.о(0,1)} = t_{р.н(0,1)} + t_{(0,1)} = 0 + 5 = 5.$$

The number 5 does in fact stand in the first row of column 5.

Subsequent filling in of the table is based on the rule that early completion of a preceding operation is the early start of a succeeding operation, that is, $t_{p.o(i, j)}=t_{p.n(j, k)}$. In order to enter the early commencement of operation (1, 2), we take early completion of the preceding operation, that is, operation (0, 1), which is 5 (first row of column 5), and we enter this time in the second row of column 3 opposite operation (1, 2). This will also be the early start of the following operation in respect to operation (0, 1).

The algorithm of operations with manual calculation is in this instance indicated in Table 8 by arrows with triangles in the first, second and third rows from the top.

If among the preceding operations there are operations entering a single event and consequently ending on the same number, we take for the early start of the following operation the maximum early completion value of the operations entering a single event, that is, terminating with the same number. In Table 8, for example, there are two operations terminating in event (8), namely operations (4, 8) and (5, 8), the early completions of which are equal to 20 and 90 units of time respectively. We must take the maximum value for early commencement of following operation (8, 11), which begins at the event which ends the preceding operations, that is, early completion of operation (5, 8) equal to 90 units of time.

An algorithm of operations with manual calculation is indicated in Table 8 in this instance by arrows with circles in the 8th, 9th and 16th rows from the top.

In this sequence, working downward to the end of the table, we determine early start and early completion of work operations.

Thus, having discussed the procedure of determining operation early start and early completion, we can state the following generalization: to find the early start time of a given work operation, one examines all work operations entering the initial event of the given operation. From the column for early completion of entering work operations $t_{p.o(h, i)}$ we select the maximum operation completion time, which is carried over to the column for entering work operation early completion $t_{p.H(i, j)}$. Calculation is performed downward: $t_{p.o(i, j)}$ -- early completion time of operation (i, j) is equal to the early commencement time of this operation plus its duration, that is, $t_{p.o(i, j)}=t_{p.n(i, j)}+t_{(i, j)}$.

Determination of Late Start and Late Completion of Work Operations

Early start ($t_{п.H(i, j)}$) and late completion ($t_{п.o(i, j)}$) of work operations are determined in reverse order, from bottom to top. We also fill in columns 6 and 8 (see Table 8) simultaneously, row by row: first we fill in a row in column 8, and then the same row in column 6. Calculation begins with determination of the late completion time of the last operation in the calculation table. In Table 8 this will be operation (13, 14).

We know from the previously presented material that the last event in a network always lies on the critical path. And the early completion time of the last operation lying on the critical path and ending with the concluding critical event is equal to its late completion time. Proceeding from this, we determine the last critical operation in Table 8, take its early completion time and enter this number in column 8 in the same row in which this operation is entered. This same number is entered in column 8 in the rows of all those operations with the network concluding event as terminal event. In Table 8, for example, operation (11, 14) is the last critical event, ending with concluding event (14). We find in column 5 the early completion time of this operation, which was previously calculated. This time is equal to 170 units of time. We enter the number 170 in column 8 opposite operation (11, 14), as well as opposite all operations ending with the concluding event in the network, that is, event (14). Thus the number 170 is entered in column 8 opposite operations (9, 14), (11, 14), (12, 14) and (13, 14). The table is then calculated, beginning from the last row and moving upward. We take work operation late completion time ($t_{п.o(i, j)}$), subtract from it the time of operation (i, j) proper, and enter the remainder in column 6 in that same row. We utilize the formula

$$t_{п.н(i, j)}=t_{п.о(i, j)}-t_{(i, j)}.$$

In calculation table 8, operation (13, 14) is the last operation. In column 8 we find its late completion time, 170, from which we subtract the time of operation

(13, 14), which is 15 minutes, and enter the remainder (170-15=155) in column 6 in the row in which operation (13, 14) is entered. Then, moving upward in column 2, we note the terminal events of the work operations. If the terminal event in a higher row is the same as in a lower row (for example, as in our illustration event (14) in the first, second, third, and fifth lines from the bottom), the operation is repeated. In our case we find terminal event (14) in the second row from the bottom in column 2, that is, the same event as in the bottom row. The late completion time of all operations ending with event (14) has already been determined. Therefore from 170 we subtract $t_{(12, 14)}$, which is equal to zero, and we enter 170 in the second row from the bottom in column 6, etc.

If the figure in the upper row of column 2 does not coincide with the figure in the lower row of that same column, we proceed as follows: we remember the figure and find that same figure, but in column 1 and below. Finding the required figure in column 1, we proceed to the right in the row in which the figure was entered, proceeding to column 6, and remember the number entered in that column. Then this number is transferred to column 8 and entered in the row in which a figure was found in column 2 which differs from the figure in the preceding row.

The algorithm of this rule is indicated in Table 8 by arrows and squares. For example, proceeding up the table, we determine the $t_{n.o}$ of operations (13, 14), (12, 14), (11, 14) and approach operation (10, 13), the terminal event of which (13) differs from the terminal event of the operation entered one row lower, that is, operation (11, 14). Remembering the number 13, we note the figures to the left and below this operation in column 1. In fact, the number 13 will be entered in the last row of column 1 -- this is the initial event of operation (13, 14). Proceeding to the right down this row to column 6, in which we find the number 155, we remember this number and carry it over to column 8, into the row in which operation (10, 13) is indicated, that is, where the number 13 is entered, from which the search began.

If we analyze these operations, they consist essentially in the following: we needed to determine late completion of operation (10, 13). If this is expressed in a diagram, it will look like the illustration in the Figure 52.

Figure 52. Diagram of Dependence of Operations

It is evident in Figure 52 that operation (10, 13) is the operation in question, while operation (13, 14) is the succeeding one. Event (13) is the boundary event between these operations. Operation (10, 13) ends with event (13), and the next operation (13, 14) commences with this same event (13). As soon as operation (10, 13) ends, operation (13, 14) immediately begins, that is, late commencement of operation (13, 14) is late completion of operation (10, 13). We proceeded precisely in this manner: we found the late commencement of operation (13, 14), which is 155 minutes, and placed it as late completion of the operation in question (10, 13).

If column 1 contains several of the same figures, and this means that several operations proceed from a single event, we take the minimum late commencement value of all the operations beginning with one and the same number and proceed just as in the first instance. For example, we wish to determine late completion of operation (6, 9). The terminal event of this operation is event (9). Remembering this number, we turn to column 1 and note there two numbers 9, with which operations (9, 12) and (9, 14) begin. We find the late commencement of these operations $t_{n.H(9, 12)}=140$ and $t_{n.H(9, 14)}=155$, take the smaller value, that is, $t_{n.H(9, 12)}=140$, and enter this figure as late completion of operation (6, 9). Since in our example there is one other operation which terminates with event (9) -- operation (5, 9) -- we also enter for this operation late completion time $t_{n.o(5. 9)}=140$.

In this case the algorithm of operations is indicated in Table 8 by the arrows with rectangular boxes.

Following this procedure, we complete our calculation (up to the top of the table).

Examining the procedure of determining late completion and late commencement of work operations, we can conclude the following:

to determine the late completion time of a given work operation, one examines all operations proceeding from the terminal event of the operation in question; one selects from column 6, late commencement of outgoing operations $(t_{n.o(i)})$, the smallest late commencement time, which is transferred to column 8, late completion of the operation in question $((t_{n.o(i, j)})$; calculation is performed from bottom to top;

the late commencement time of the operation in question is equal to the late completion time of this operation minus its duration, that is,

$$t_{n.H(i, j)} = t_{n.o(i, j)} - t_{(i, j)}.$$

Determination of Full and Partial Time Reserves of Work Operations

The full time reserve of a work operation is determined as the difference between operation late completion time $(t_{n.o(i, j)})$ and operation early completion time $(t_{p,o(i, j)})$, or as the difference between operation late commencement time $(t_{n.H(i, j)})$ and operation early commencement $(t_{p.n(i, j)})$, that is, $t_{n.o(i, j)}-t_{p.o(i, j)}$ or $t_{n.H(i, j)}-t_{p.H(i, j)}$.

Using the table, this reserve is determined as the difference of the figures contained in columns 8 and 5 or in columns 6 and 3. For example, let us determine the reserve of operation (3, 6). For this we take operation (3, 6) and find the number 110 opposite it in column 8. We subtract from it the number entered in column 5 opposite this operation, the number 25. The difference between the figures in column 8 and 5 for this operation, 85, is entered in column 9 opposite this operation.

To determine partial time reserve of the first type P'_n we examine operations with the same initial events. We select from column 6 of these operations minimum time $t_{n.H}$, which is subtracted from time $t_{n.H}$ of the operation in question. If only one operation proceeds from an event, the partial reserve of the first type P'_n of this operation is equal to zero -- in Table 8, for example, $P'_{n(1, 2)}=0$.

In Table 8 we determine the partial reserve of the first type for operation (3, 4). In column 1 we find operations which begin with event (3). There are four such operations: (3, 4), (3, 5), (3, 6), (3, 7). In column 6 we find the minimum value of these operations. The least such value is 15 minutes, for operation (3, 5). Then we subtract from $t_{n.H}$ of each of these four operations which begin with event (3) the minimum value of operation (3, 5), that is, 15 minutes. We enter the difference in column 10 opposite each operation respectively. We calculate the partial reserve of the first type for the following operations:

$$P'_{n(3,4)} = t_{n.H(3,4)} - t_{n.H(3,5)} = 85 - 15 = 70;$$
$$P'_{n(3,5)} = t_{n.H(3,5)} - t_{n.H(3,5)} = 15 - 15 = 0;$$
$$P'_{n(3,6)} = t_{n.H(3,6)} - t_{n.H(3,5)} = 100 - 15 = 85;$$
$$P'_{n(3,7)} = t_{n.H(3,7)} - t_{n.H(3,5)} = 120 - 15 = 105.$$

In fact, the determined figures are entered in column 10 opposite these operations in Table 8.

To determine partial reserve of the second type P''_n of a given operation, we examine operations possessing identical terminal events. We select from column 5 of these operations the maximum operation completion time, from which we subtract $t_{p.o}$ of the operation in question.

If only one operation terminates at an event, the partial reserve of the second type P''_n of this operation is equal to zero. With Table 8 we determine the partial reserve of the second type of operation (5, 9). In column 2 we find operations possessing identical events. There are two such operations: (5, 9) and (6, 9). We turn to column 5, where the $t_{p.o}$ of these operations has been calculated. For operation (5, 9) $t_{p.o(5, 9)}=45$, while for operation (6, 9) $t_{p.o(6.9)}=55$. We take the larger figure, 55. From this quantity we subtract $t_{p.o}$ of each operation.

Consequently $\quad P''_{n(5,9)} = 55 - 45 = 10; \quad P''_{n(6,9)} = 55 - 55 = 0.$

Indeed, in column 11 of Table 8 we shall see the number 10 opposite operation (5, 9), and the figure 0 opposite operation (6, 9).

For operation (3, 6), only one operation enters the terminal event, and consequently $P''_{n(3, 6)}=0$.

7. Graphic Method of Calculation

The graphic method is employed as a rule in calculating small networks. One feature of this method is the fact that all calculated parameters are determined directly on the chart.

There are two variants of the graphic method of calculation: calculation with recording of calculated parameters and with change in the graphic configuration of the network, and calculation with recording of calculated parameters without reconfiguring the network.

The latter method of calculation is subdivided in turn into multisector and four-sector.

In military affairs the four-sector graphic method of calculation is most acceptable, since it is more compact and convenient for rapid computation of small networks. This is valuable when employing critical-path method in conditions of a rapidly changing situation during combat. With this method the network circle which designates an event is divided into four parts (sectors). The symbols adopted for the four-sector method of network calculation are shown in Figure 53.

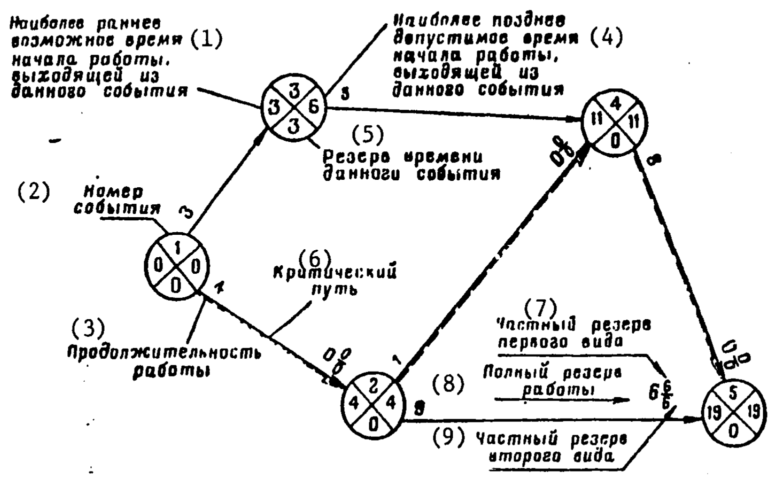

Figure 53. Four-Sector Method of Network Calculation

Key:

1. Earliest possible start time for work operation proceeding from a given event
2. Event number
3. Duration of operation
4. Latest allowable starting time for an operation proceeding from a given event
5. Time reserve of given event
6. Critical path
7. Partial reserve of first type
8. Full reserve of work operation
9. Partial reserve of second type

Proceeding from Figure 53, one can include the following: the event number is placed in the upper sector; we place in the left-hand sector the earliest possible starting time of an operation proceeding from the event in question; we place in the right-hand sector the latest allowable starting time of an operation proceeding from the event in question; we place in the lower sector the time reserve of the event in question.

The numbers above the arrows indicate the following: the number at the beginning of the arrow -- duration of the work operation; the mixed number at the end of the arrow -- operation time reserves, with the integer indicating full reserve of the work operation, the numerator of the fraction -- partial reserve of the first type, and the denominator of the fraction -- partial reserve of the second type of the operation in question.

Let us examine the procedure of network computation by the graphic method. For this we shall take our initial network -- readying a tank battalion for attack from a position of close contact with the enemy (Figure 33). Figure 54 contains our initial network schedule calculated by the graphic method.

We shall describe below the network computation procedure.

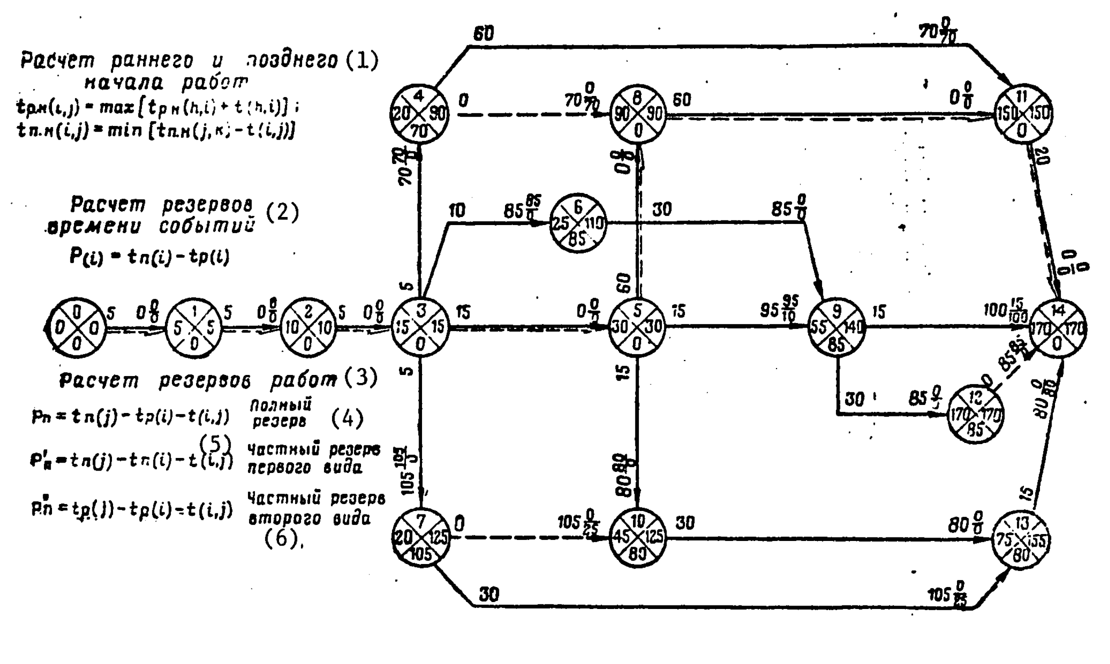

Figure 54. Four-Sector Graphic Calculation of Initial Network Schedule

Key:

1. Calculation of early and late start of operations
2. Calculation of time reserves of events
3. Calculation of reserves of work operations
4. Full reserve
5. Partial reserve of first type
6. Partial reserve of second type

Calculation of Earliest Possible Commencement Time of Work Operations

The earliest possible time of commencement of work operations is calculated from left to right, beginning with the starting event and ending with the concluding event. We take as the earliest work operation commencement the greatest accumulated time figure for all paths which lead to the event in question. Calculation is performed in the following sequence using the algorithm, formula (25).

Early beginning is equal to zero for the starting event, and consequently also for work operations proceeding from the starting event.

We shall formulate in words the algorithm represented by formula (25): early beginning of the operation in question (i, j) is equal to the maximum value of the sum of the early start and duration of the operations preceding the operation in question.

If the operation in question is preceded by only one operation, its early start is equal to the sum of the early start and duration proper of the preceding operation. For example, we shall determine the early start of operations (1, 2) and (2, 3):

$$t_{p.н(1,2)} = t_{p.н(0,1)} + t_{(0,1)} = 0 + 5 = 5;$$
$$t_{p.н(2,3)} = t_{p.н(1,2)} + t_{(1,2)} = 5 + 5 = 10.$$

We proceed as follows in determining the early start of operation (1, 2) with the chart: we find event (0), take in its left-hand sector the starting time of operation (0, 1), which is equal to zero, and add to it the time of operation (0, 1) proper, which is equal to 5 minutes. We enter the resulting sum in the left-hand sector of event (1). The number 5, entered in the left-hand sector of event (1), indicates the early starting time of operation (1, 2).

To determine $t_{p.H(2,\ 3)}$ we find event (2), take the starting time of operation (2, 3) in its left-hand sector, which is 10, and add to it the time of the operation proper, which is equal to 5 minutes. We enter the resulting sum in the left-hand sector of event (3). This time will be the early starting time for all operations proceeding from event (3), that is, operations (3, 4), (3, 5), (3, 6), (3, 7).

If the operation in question is preceded by several operations, the early commencement of this operation is determined from the maximum sum of starting time and duration of all preceding operations. In our illustration, for example, in determining the early start of operation (8, 11) we examine all preceding operations, namely operations (4, 8) and (5, 8). We determine $t_{p.H(8,\ 11)}$ for these incoming operations:

$$t_{p.н(8,11)} = t_{p.н(4,8)} + t_{(4,8)} = 20 + 0 = 20;$$
$$t_{p.н(8,11)} = t_{p.н(5,8)} + t_{(5,8)} = 30 + 60 = 90.$$

We take the maximum sum, that is, $t_{p.H(8,\ 11)}=90$, and enter it in the left-hand sector of event (8). This is the early start of operation (8, 11).

We employ these procedures to determine the early start of all operations and reach the concluding event in the network.

Calculation of Latest Allowable Commencement Time of Work Operations

The latest starting time of an operation is figured in reverse order, that is, proceeding from right to left, beginning with the concluding event and ending with the starting event.

For the concluding event in the network the early start of subsequent operations, of which this event has none, as we know, denotes at the same time the late completion of all preceding operations. Therefore the early beginning of an operation is transferred from the left-hand sector of the concluding event to the right-hand sector of the event in question, and we begin calculating in reverse order (from right to left) back to the starting event. We enter in the right-hand sector the minimum values of the difference between the late start of a subsequent operation and the duration of the operation in question.

We perform our calculation with the algorithm, formula (27).

We shall calculate for our schedule the late start of operations (11, 14), (8, 11) and (5, 8):

$$t_{\text{п.н}(11,14)} = t_{\text{п.н}(14,k)} - t_{(11,14)} = 170 - 20 = 150;$$
$$t_{\text{п.н}(8,11)} = t_{\text{п.н}(11,14)} - t_{(8,11)} = 150 - 60 = 90.$$

We proceed as follows in our network schedule calculations: we subtract from the number 170, entered in the right-hand sector of event (14), the duration of operation (11, 14), which is 20, and obtain 150. We enter the obtained figure in the right-hand sector of event (11). We proceed in like manner in determining $t_{n.H(8,\ 11)}$: we subtract 60 from 150 and enter the result, 90, in the right-hand sector of event (8).

If several operations proceed from an event, such as from event (5), we proceed as follows: we calculate the late start of all operations proceeding from the event in question, and take the least of the obtained figures.

In our example:

$$t_{\text{п.н}(5,8)} = t_{\text{п.н}(8,11)} - t_{(5,8)} = 90 - 60 = 30;$$
$$t_{\text{п.н}(5,9)} = t_{\text{п.н}(9,12)} - t_{(5,9)} = 140 - 15 = 125;$$
$$t_{\text{п.н}(5,10)} = t_{\text{п.н}(10,13)} - t_{(5,10)} = 125 - 15 = 110.$$

The least value of the three is 30. Therefore we enter 30 in the right-hand sector of event (5). We shall recall that the minimum time in these conditions is taken in order to ensure the requsite time for accomplishing the longest-duration operations, without affecting the calculated completion time for the overall operation (process). We continue to the starting event, following this procedure in our calculations.

Calculation of Time Reserves

The time reserve of each event is defined as the difference between the late and early completion time of each event: $P_{(i)}=t_{n(i)}-t_{p(i)}$.

Calculating for the network schedule, we proceed as follows: we subtract from the quantity in the right-hand sector of the event the quantity in the left-hand sector of the same event. We enter the result in the bottom sector of the event. Let us compute, for example, the time reserve for event (3), (4), and (8):

$$P_{(3)} = t_{n(3)} - t_{p(3)} = 15 - 15 = 0;$$
$$P_{(4)} = t_{n(4)} - t_{p(4)} = 90 - 20 = 70;$$
$$P_{(8)} = t_{n(8)} - t_{p(8)} = 90 - 90 = 0.$$

Work operation full time reserve (P_n) is computed as the difference between the late completion time of the event terminating an operation, the early completion time of the event with which the operation in question begins, and the time (duration) of the operation proper:

$$P_n = t_{n(j)} - t_{p(i)} - t_{(i,j)}.$$

For example, the full reserve of operation (9, 14) is

$$P_{n(9,14)} = t_{n(14)} - t_{p(9)} - t_{(9,14)} = 170 - 55 - 15 = 100.$$

We write the obtained quantity at the end of the arrow as the integer of the mixed number which indicates time reserve.

Determining total work operation reserve on the schedule, we proceed as follows: we subtract from the number in the right-hand sector of event (14) the number in the left-hand sector of event (9) and the duration of the operation proper:

$$P_{n(9,14)} = 170 - 55 - 15 = 100.$$

Partial reserve of the first type ($P'_{n(i,\ j)}$) is computed as follows:

$$P'_{n(i,j)} = t_{n(j)} - t_{n(i)} - t_{(i,j)}.$$

For operation (9, 14)

$$P'_{n(9,14)} = t_{n(14)} - t_{n(9)} - t_{(9,14)} = 170 - 140 - 15 = 15.$$

This reserve is determined as follows on the network schedule: we subtract from the contents of the right-hand sector of event (14) the amount in the right-hand sector of event (9) and the duration of operation (9, 14):

$$P'_{n(9,14)} = 170 - 140 - 15 = 15.$$

We enter the obtained result as the numerator of the fraction placed above the arrow at the end of operation (9, 14).

Partial reserve of the second type ($P''_{n(i,\ j)}$) is determined as the difference:

$$P''_{n(i,j)} = t_{p(j)} - t_{p(i)} - t_{(i,j)}.$$

For operation (9, 14)

$$P''_{n(9,14)} = t_{p(14)} - t_{p(9)} - t_{(9,14)} = 170 - 55 - 15 = 100.$$

On the network schedule this reserve is determined as follows: we subtract from the number in the left-hand sector of event (14) the number in the left-hand sector of event (9) and the duration of work operation (9, 14):

$$P'_{\text{п}(9,\,14)} = 170 - 55 - 15 = 100.$$

We enter the resulting number as the denominator of the fraction placed above the arrow at the end of operation (9, 14).

Reserves are determined in like manner for all operations.

Chapter III. OPTIMIZATION OF NETWORK SCHEDULES AND THEIR UTILIZATION IN THE COURSE OF PLAN (PROJECT) EXECUTION

1. Methods of Optimizing Networks

In the preceding chapter we examined the methods and techniques of constructing and analyzing an initial network schedule. The calculations shown in figures 46 and 54 and in Table 8 convincingly show that the duration of the critical path (0, 1, 2, 3, 5, 8, 11, 14) exceeds the determined time. At the same time many events contain reserves, such as events 7, 9, and 12.

This state of affairs may also be typical of other practical situations, where alongside intensity in some sectors, there are zones of unutilized possibilities. Our computed initial network schedule proved unsatisfactory, since its figures exceed the requirement -- to complete readying the battalion for an offensive action within 2 hours and 10 minutes.

The plan may also contain a hidden defect: the plan is satisfactory from the standpoint of meeting the directive timetable, but it is far from optimal from the standpoint of mobilization of all time reserves and resources.

Let us return to our example. We need to reduce by 40 minutes the contemplated initial plan.

How can this be achieved?

Within the framework of traditional planning methods, in such cases one would usually be limited either to volitional actions based on intuition or to issuing instructions to a subordinate in the following form: "Shall be completed on schedule."

In carrying out simple operations (processes), intuition based on experience may prove adequate. In carrying out complex processes, however, not intuition is needed but precise calculations. Program evaluation and review technique is precisely that tool which enables one to calculate whether under given specific conditions a planned process can be shortened by the required amount of time and thus fit within the mandated timetable.

A network model is simply indispensable for analyzing complex processes (programs), for it provides the opportunity to reject disadvantageous process planning variants and makes it possible to see how it can be sped up.

What recommendations can a network model give us?

First of all it will suggest that process accomplishment time must be reduced by shortening the critical path, not by shortening all paths. If, following the first reduction, it turns out that the new critical path, although shorter than the preceding one, exceeds the directive timetable, it too must be shortened. And we continue to proceed in this manner until the time specified by the higher commander is met. This process is called network optimization (improvement).

As already noted, network optimization can be performed on the basis of various criteria: time, resources, cost, and flow.

8. Network Optimization by Time

It is expedient to optimize a schedule by time in the following manner.

1. Check the correctness of entered time estimates of critical-zone work operations. If they are unwarrantedly overstated, they should be brought into conformity with established standard figures for performance of work operations. One should strive for the minimum allowable duration of critical-zone work operations.

But if time estimates are not overstated, they should not be arbitrarily revised in order to obtain the specified process (program) plan completion time.

2. Analyze the possibility of intensification of performance of critical work operations by utilizing the resources of noncritical-zone operations which possess time reserves. Reserves, however, must be utilized within reasonable limits. An endeavor to reduce reserves in order to shorten the overall work operations timetable can lead to an increase in the number of critical paths and ultimately to an attempt to redistribute resources in such a manner that all paths become critical. But this will complicate process management and control in the course of performance of work operations, since there will not be sufficient reserves for eliminating possible breakdowns.

3. Analyze the possibility of maximum combining of critical operations by breaking down work operations and carrying them out in parallel. We shall note that time-combining critical work operations produces maximum effect.

4. Alter the process (sequence) of performing work operations in order to shorten total duration, that is, alter the topology of the network. This path is the most difficult, and it is desirable to employ it in those cases when other techniques do not lead to the desired results.

5. Shorten the timetable by enlisting additional resources.

In our example we shall utilize the third technique to shorten the time required to ready a tank battalion for an attack.

Experience indicates that shortening a critical path must begin with long processes. Usually, all other conditions being equal, the probability of reducing them is greater, and consequently the anticipated effect is also greater. Here there are three such processes on the critical path (Figure 33) -- operations (5, 8), (8, 11) and (11, 14). Obviously their overall duration can be reduced by breaking down operations (5, 8) and (8, 11) and their parallel execution, as well as by simultaneous performance of operations (11, 14) and (8, 11).

Let the battalion commander perform allocation of tasks and organization of coordination in the 1st and 2d Tank companies, with his second in command doing the same job in the 3d Tank Company. Then operation (5, 8) will be represented by two work operations in our new, optimized schedule (Figure 55), namely operations (5, 8) and (5, 9). Their duration will be 40 and 20 minutes respectively.

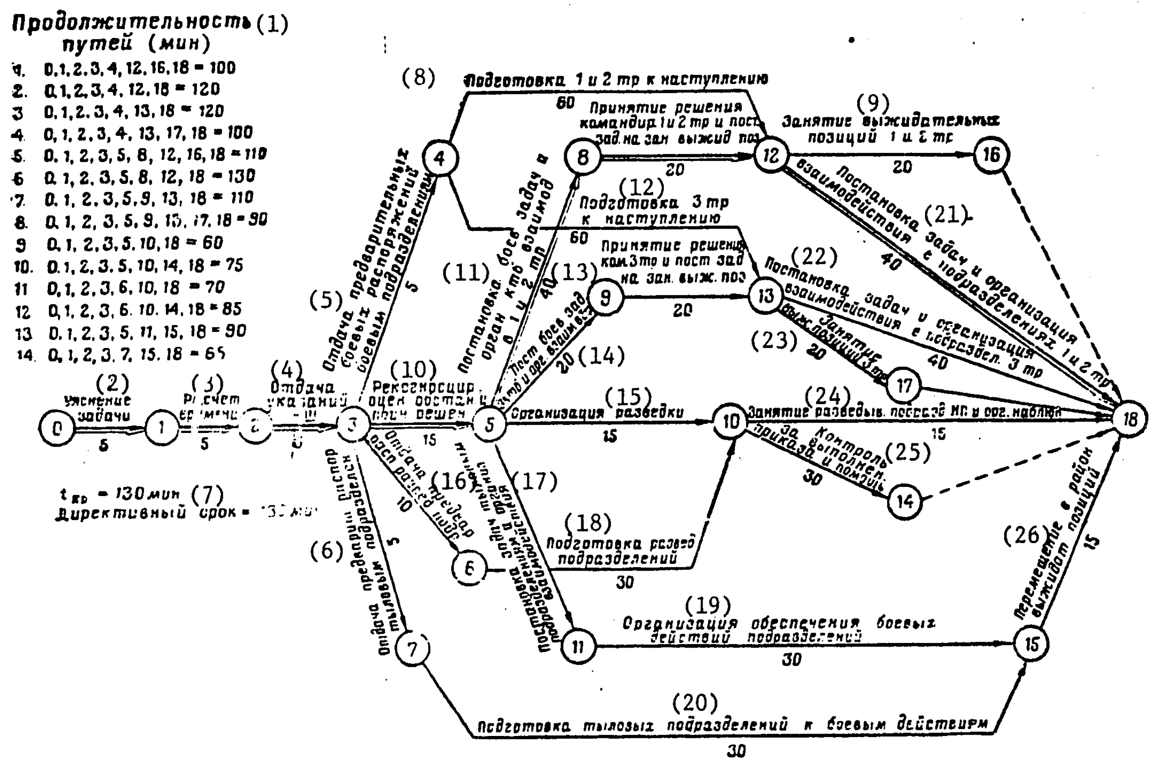

Figure 55. Optimized Initial Network Schedule for Readying a Tank Battalion for an Attack from a Position of Close Contact With the Enemy

Key:

1. Duration of paths (minutes)
2. Mission briefing
3. Time calculation
4. Issuing instructions to executive officer
5. Issuing warning orders to combat subunits
6. Issuing warning orders to rear services subunits
7. Directive time

(Key to Figure 55 cont'd)

8. Ready 1st and 2d Tank companies for attack
9. 1st and 2d Tank companies occupy assembly area
10. Commander's reconnaissance, situation estimate, and commander's plan
11. Allocation of combat missions and organization by tank battalion commander of coordination in 1st and 2d Tank companies
12. Ready 3d Tank Company for attack
13. Commander of 3d Tank Company formulates plan and allocates tasks for proceeding to assembly area
14. Allocation of combat tasks by tank battalion deputy commander and organization of coordination in 3d Tank Company
15. Organization of reconnaissance
16. Issuing of warning orders to reconnaissance subunits
17. Allocation of tasks to rear services subunits and organization of coordination
18. Readying of reconnaissance subunits
19. Organization of support of combat actions of subunits
20. Readying rear services subunits for combat operations
21. Allocation of tasks and organization of coordination in subunits of 1st and 2d Tank companies
22. Allocation of tasks and organization of coordination in subunits of 3d Tank Company
23. 3d Tank Company proceeds to assembly area
24. Reconnaissance subunits set up observation posts and organize surveillance
25. Monitoring execution of order and assistance
26. Proceed to assembly area

In order for operation (11, 14) to be carried out simultaneously with operation (8, 11) in Figure 33, it is necessary that operation (8, 11) be partially accomplished, that is, that the company commanders assign movement tasks to their subunits. In these conditions tanks will be able to proceed to the assembly area during allocation of tasks and organization of coordination by the company commanders. This is indicated in the optimized schedule (Figure 55): operations (8, 12), (12, 16) and (12, 18).

Shortening of the critical path is achieved in the optimized schedule by breaking down critical operations and executing them in parallel. Work operation time estimates remain unchanged: for example, if in the original schedule 60 minutes was expended on operation (5, 8) (based on 20 minutes for each company), 20 minutes is also spent on each company in the optimized schedule in operations (5, 8) and (5, 9).

Such a breakdown of operations and their parallel execution has made it possible to shorten the critical path by 40 minutes and thus to meet the ordered timetable. Since the goal has been achieved, no further optimization is performed on the schedule.

Let us examine another example of time optimization of a network, but in this instance in the area of planning combat training. There is no doubt that the overall combat training plan will consist of a large number of events. To understand the essence of the process of network optimization, however, it suffices to examine a portion of the overall plan.

We shall assume a certain particular network schedule, which reflects subunit combat training planning at a training center (Figure 56). As is evident from the schedule, the duration of the critical path is 18 training days. Our task is to optimize this schedule in such a manner that the time required to accomplish the scheduled training topics and exercises does not exceed 12 training days.

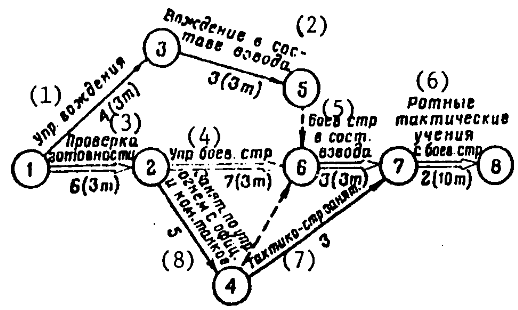

Figure 56. Network Model of Organization of Subunit Combat Training at a Training Center

Key:

1. Driving exercise
2. Driving as a platoon unit
3. Checking readiness
4. Live fire exercise
5. Live fire exercise as platoon unit
6. Company tactical exercises with live fire
7. Battle drill
8. Fire control drill with officers and tank commanders

In preparing the schedule it was assumed that each day three tanks must be assigned for working on firing and driving drills.

Our task is to shorten the duration of operations lying on the critical path by a total of 6 days.

Initially we shall attempt to accomplish this by enlisting additional resources. Indeed, operations (1, 2) and (2, 6) can be sped up if we assign six rather than three tanks for testing the readiness of subunits to fire the standard round and for the firing drill proper. In this case the duration of operations (1, 2) and (2, 6) will be cut in half. Thus we can shorten these work operations by six days by additional enlistment of equipment. It would seem that the problem is solved and that we have met the indicated timetable. But this is not so. We must bear in mind that when revising event occurrence times there may develop a new critical path, which will now determine the completion time of all work operations.

In our example the paths which pass through events (1), (2), (4), (6), (7) and (8) and (1), (2), (4), (7) and (8) have become new critical paths. The duration of each of them, as we can easily determine from the above, is 13 days. This means that we must shorten the duration of the new critical paths by a minimum of 24 hours. We shall attempt to achieve this by parallel performance of operations (1, 2), and (2, 4).

In our example partial accomplishment of operation (1, 2) will be an adequate condition for starting operation (2, 4). After checking at least one company to determine whether it is ready to fire the standard round, it will be possible to drill this company's officers and tank commanders in fire control. This makes it possible precisely to define the parameters of this condition, to break up operation (1, 2) into two independent operations, and to revise the schedule as shown in Figure 57.

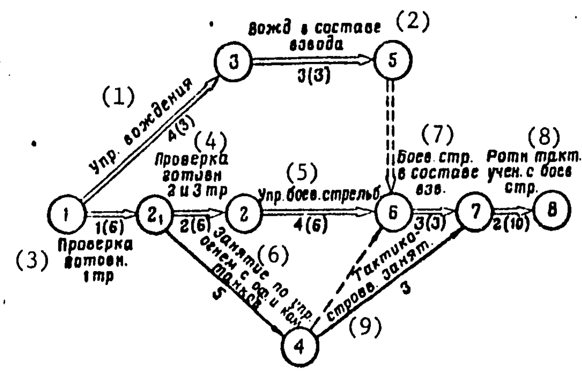

Figure 57. Time-Optimized Network Schedule of Organization of Combat Training of Subunit at Training Center

Key:

1. Driving drill
2. Driving as platoon unit
3. Checking readiness of 1st Tank Company
4. Checking readiness of 2d and 3d Tank companies
5. Live fire exercise
6. Fire control drill with officers and tank commanders
7. Live fire as platoon unit
8. Company tactical exercise with live fire
9. Battle drills

Operation (1, 2) is presented in the new, optimized schedule as two operations: $(1, 2_1)$, one day in duration, and $(2_1, 2)$, two days in duration. Accomplishment of operation $(1, 2_1)$ is a sufficient condition for beginning operation $(2_1, 4)$.

We should bear in mind that after each change in duration of the critical path, we must once again calculate all time parameters for the revised schedule. Calculating the network, we see that the path which in the initial chart (Figure 56) passes through events (1), (2), (4), (7), and (8) and which had a duration of 16 days, was shortened by five days and has a duration of 11 days in the optimized schedule (Figure 57), that is, does not exceed the duration of the critical path in the optimized schedule.

Thus without changing the duration of each separate operation, as a result of parallel performance of operations we have succeeded in shortening by six training days the entire process of battalion training.

Of course the new critical path should be subjected to further analysis. This process must be continued until we achieve the desired result, that is, until the execution time for the entire aggregate of work operations is equal to or less than the specified time.

It is possible that two or more critical paths may form in the course of such a sequential reduction. Our example illustrates this point. Two critical paths are formed in the chart in Figure 57: one of them comprises sequence of operations (1, 2_1), (2_1, 2), (2, 6), (6, 7) and (7, 8), and the other -- sequence of operations (1, 3), (3, 5), (5, 6), (6, 7), (7, 8). Each is equal to 12 days in duration. If this schedule version is not optimal, both these paths should be analyzed, and first of all those operations which are common to both paths. In our example these are operations (6, 7) and (7, 8).

In this case there is no need to continue reducing the critical path, since the newly constructed and calculated network schedule (Figure 57) provides a length of critical paths (1, 2_1, 6, 7, 8) and (1, 3, 5, 6, 7, 8) equal to 12 training days, which corresponds to the specified directive time.

Improvement of a network schedule terminates when possibilities of shortening the critical path are exhausted.

In this connection we should point out another advantage of employing PERT methods in comparison with existing planning methods.

It is no secret that sometimes a project completion date is designated without considering the possibilities and concrete conditions of plan execution. In our example the directive time was possibly limited to 12 days because it was necessary more quickly to free the range and tank training area at the training center. A network model will also prove highly useful in this instance. If a task cannot be achieved, it will exhaustively show the reasons for this. In addition, with the aid of a network model it is possible to determine the shortest time within which the targeted process can be completed.

We have examined the procedure of time-optimizing a network with only two examples. But this by no means exhausts the area of application of network models. It is extremely useful to employ this method of time-optimizing a network in determining ways to shorten the timetable of various tasks performed by troops, which can include the following: troop response to a combat alert, execution of a march, river-crossing operations, setting up river crossing sites, decontamination, field fortification of a defensive area, occupation of a defensive position, preparation and delivery of an airborne assault force, preparation for and conduct of tactical exercises, construction of training facilities, preparation for and holding of competitions and sports holidays, and preparation for and holding of inspection parades. Stated more concisely, in all processes where a time savings must be achieved, a network model will prove to be a valid tool for developing optimal action variants.

Finally, utilizing optimization methods, it is possible not only to predict planned processes but also to verify the effectiveness and correctness of already executed operations and programs and impartially to determine those deficiencies and omissions which have occurred in these processes. This will help avoid similar miscalculations in the future. This can prove highly useful in preparing critiques of all types of exercises. A network model will help precisely determine what subunits were late in accomplishing their mission, through whose fault, and on what items attention should be focused in order to correct problems revealed at exercises.

Correct utilization of the techniques of network schedule time optimization will enable commanders and staff officers in a substantiated manner to find ways to shorten the execution timetables of any complex processes.

We previously examined methods of optimization of network schedules with time checking. At the same time we frequently encounter in practice a situation where expenditures of manpower and resources, or in other words material capabilities, prove decisive in execution of an aggregate of operations. In these cases there arises the need for network optimization by resources.

9. Network Optimization by Resources

Success in the execution of a mission depends not only on precise organization of operations but also on how correctly manpower and weapons, as well as material resources are distributed. Of great importance in planning operations is economical expenditure of manpower and resources, training time, etc. This is ensured by precise calculation of figures characterizing the dimensions of material, manpower and monetary expenditures and their efficient distribution. Network planning methods make it possible to achieve a correspondence between specified program completion times and the resources allocated for them.

A need for optimizing a network by resources may arise in solving a broad range of problems, in particular the following:

distribution of personnel and weapons in carrying out combat missions;

construction of defensive fortifications and establishment of river crossing sites;

logistic support of troop combat operations;

planning combat training;

establishment of training facilities, etc.

We shall examine the procedure of network schedule optimization by resources with a concrete example.

Let there be a particular network schedule of performance of flight training activities by cadet pilots (Figure 58). We must improve this network schedule so that no more than 20 aircraft will be utilized daily. On the chart the figures in

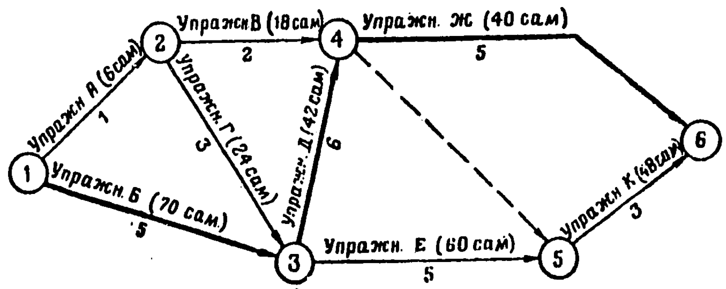

Figure 58. Initial Network Schedule of Performance of Flight Training Activities by Cadet Pilots

Key:
Упражн. Exercise
сам. Aircraft

parentheses above the arrows indicate the required number of aircraft, while the figures below the arrows indicate the number of days during which flight training activities are to be performed.

We shall optimize the schedule step by step.

First step. We calculate the initial schedule. In our example the calculation is performed in tabular fashion (Table 9).

Table 9.

Количество предшествующих работ	Код работ	Продолжительность работ	(6) Наиболее раннее время		(7) Наиболее позднее время		Общий резерв времени	Частный резерв времени
			начала	окончания	начала	окончания		
(1)	(2)	(3)	(4)	(5)	6	7	(8)	(9)
0	1—2	1	0	1	1	2	1	0
0	1—3	5	0	5	0	5	0	0
1	2—3	3	1	4	2	5	1	1
1	2—4	2	1	3	9	11	8	8
2	3—4	6	5	11	5	11	0	0
2	3—5	5	5	10	8	13	3	1
2	4—5	0	11	11	13	13	2	0
2	4—6	5	11	16	11	16	0	0
2	5—6	3	11	14	13	16	2	2

Key:
1. Number of preceding operations
2. Code of operations
3. Duration of operations
4. Beginning
5. End
6. Earliest time
7. Latest time
8. Total time reserve
9. Partial time reserve

Second step. A linear chart (Table 10) is set up on the basis of the network schedule. The chart is constructed as follows: the work operation codes are entered in column 2 of Table 10, duration of work operations in days in column 3, and in column 4 -- partial reserve of the second type, which indicates how much later a given work operation can begin. In our example second-type reserve of each work operation is indicated in column 9 (see Table 9). These figures are also entered in column 4 opposite each work operation (see Table 10).

Table 10. Linear Chart

№ по пор.	Код (1) работы	$t_{(i,j)}$	$P_{(i,j)}$	Дни 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1-2	1	0	6															
2	1-3	5	0	14	14	14	14	14											
3	2-3	3	1		8 / 6	8 / 6	8 / 6	6											
4	2-4	2	8		9	9			3	3	3	3	3	3					
5	3-4	6	0						7	7	7	7	7	7					
6	3-5	5	1						12 / 10	12 / 10	12 / 10	12 / 10	12 / 10	10					
7	4-5	0	0																
8	4-6	5	0												8	8	8	8	8
9	5-6	3	2												16 / 12	16 / 12	16 / 12	12	
10	Общее количество самолетов в день до оптимизации (2)			20	31	31	22	14	19	19	19	19	19	7	24	24	24	8	8
11	Количество самолетов в день после оптимизации (3)			20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	8

Key:
1. Operation code
2. Total number of aircraft per day prior to optimization
3. Number of aircraft per day after optimization

Let us stipulate that in constructing the linear chart the scheduled duration of operation ($t_{(i, j)}$) is indicated by a solid line, work operation partial time reserve by dots, and duration of work operation after optimization -- by a dashed line. We shall enter operation (1, 2), one day in duration, with a solid line in column 5, and we shall indicate above the line the required number of aircraft -- 6 aircraft.

Operation (1, 3), which begins simultaneously with operation (1, 2) and continues for 5 days, occupies colums 5-9 on the chart. Indicated in each column above the solid line is the number of aircraft required for each day (70:5=14 aircraft).

All other operations are depicted on the chart in the same manner; there is strict adherence to the principle that each succeeding operation is entered after completion of the preceding operation. For example, operation (2, 3) is indicated beginning on the second day, since preceding operation (1, 2) terminated at the end of the first day.

The next step in working on the linear chart is determination of the total number of aircraft required each day for performance of training activities, and a comparison with the capabilities of the training subunit (20 aircraft per day). Corresponding overall figures are entered in the second row of each day.

As is evident from the chart, aircraft requirements vary: they exceed 20 aircraft on the 2d, 3d, 4th, 12th, 13th, and 14th days, and are less than 20 aircraft on the 5th, 6th, 7th, 8th, 9th, 10th, 11th, 15th, and 16th days. Requirements for the first day are 20 aircraft.

Third step. Operations which possess partial reserves are examined. This is necessary in order appropriately to extend the time of flight training activities and to adjust the number of aircraft available on each day. Operation (1, 2) does not possess a partial reserve, while operation (1, 3) is critical, and therefore the operation execution time is not subject to extension.

Operation (2, 3) has a reserve of one day (5th day), that is, the time of this operation can be increased from 3 to 4 days without risk of increasing the training program overall execution time. Then six instead 8 aircraft can be assigned each day for performance of exercise D [Г]. The new distribution of aircraft by days is indicated in the third row by the dashed line.

Operation (2, 4) possesses a reserve of 8 days. One can easily see that performance of exercise C [В] (operation 2, 4) should be shifted, since the aircraft limit on the 2d, 3d, 4th and 5th days has been expended. Before determining specifically in what period of time this exercise should be accomplished and how many aircraft per day should be scheduled, it is necessary to consider how many aircraft per day will be required to perform exercises E [Д] and F [E], that is, for performance of critical (3, 4) and subcritical (3, 5) operations.

Operation (3, 4) will take 6 days with a daily allocation of 7 aircraft. We shall note that this operation does not possess a reserve.

Operation (3, 5) possesses a one-day partial reserve. Consequently the performance time for this operation can be increased to 6 days, with an expenditure of 10 aircraft per day. The new distribution of aircraft is emphasized by the dashed line. Thus 17 aircraft will be allocated each day for operations (2, 4) and (3, 5). In order not to exceed the aircraft limit (20 aircraft), three aircraft per day for a period of 6 days, beginning with the 6th day, can be scheduled for performance of operation (2, 4).

Finally we analyze operation (5, 6) in order to search for possibilities of increasing the execution time of exercise H [K] without exceeding the established standard aircraft allocation for each day. Operation (5, 6) possesses a 2-day partial reserve. In order for our plan to meet the specified requirements, however, it is sufficient to extend operation (5, 6) by only one day. Then exercise H will be accomplished over the course of 4 days, with an allocation of 12 aircraft per day. Following optimization, the number of aircraft per day is figured, and the result is entered in row 11.

Thus having improved the network schedule, we can achieve a situation whereby no more than 20 aircraft will be allocated each day for the performance of flight training activities.

For a better understanding of the procedure of network schedule optimization, we shall now examine with a concrete example another optimization problem. We shall assume that performance of a certain group of work operations has been assigned to a subunit. The process sequence, the linkage and interdependence of these operations are depicted in a network schedule (Figure 59).

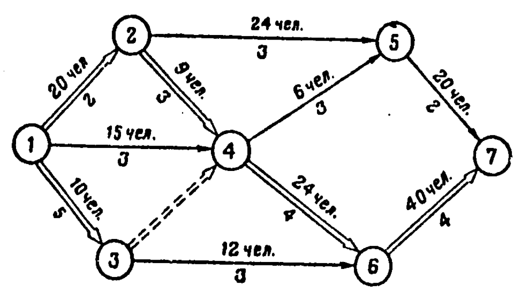

Figure 59. Network Model of an Aggregate of Subunit Work Operations

Key:
    чел. Men

We know the time required for performance of each operation (see numbers under arrows) and the required number of personnel detailed to these work operations (see numbers above arrows). We also know that the subunit can detail for work operations not more than 15 men per day. We must find the optimal variant of distribution of personnel among work operations, which will ensure execution of the aggregate of work operations on schedule with the available manpower resources.

Table 11 is drawn up on analogy with Table 10, after which we perform optimization of the network schedule represented in Figure 59.

As a result of optimization, we obtain an optimal variant of distribution of subunit personnel among work operations (Table 11), under the condition of completion of the entire aggregate of work operations by the specified time.

Table 11.

(1) Код работы	$t(i,j)$	$P''_{п}$	1	2	3	4	5	6	7	8	9	10	11	12	13
1-2	2	0	10	10											
1-3	5	0	2	2	2	2	2								
1-4	·3	2	5 3	5 3	5 3	3	3								
2-4	3	0			3	3	3								
2-5	3	3			8 4	8 4	8 4	4	4	4					
3-4	0	0													
3-6	3	1				4 3	4 3	4 3	3						
4-5	3	0						2	2	2					
4-6	4	0						6	6	6	6				
5-7	2	3									10 5	10 5	5	5	
6-7	4	0										10	10	10	10
(2) Распределение личного состава по работам до оптимизации			17	17	18	17	16	12	8	8	16	20	10	10	10
(3) Распределение личного состава по работам после оптимизации			15	15	12	15	15	15	15	12	11	15	15	15	10

Key:
1. Operation code
2. Distribution of personnel among work operations prior to optimization
3. Distribution of personnel among work operations following optimization

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In conclusion we should note that optimization of an execution program by resources furnishes valuable information for organization of a smooth, rhythmic work process. With limited resources, optimization becomes absolutely essential. It is particularly helpful in choosing an optimal variant of actions in performing the following tasks:

fortification and improvement of defensive areas, routes of movement, deployment lines, and deployment areas for various weapons;

construction of bridges and equipping of river-crossing sites;

performance of emergency rescue and recovery activities;

performance of decontamination of personnel and radiological decontamination of equipment;

servicing, repair and recovery of combat equipment;

equipping of training facilities;

planning combat training;

organization of logistic support of combat operations, etc.

10. Optimization of Network Schedules by Flow

Work operations must be organized in such a manner that maximum effectiveness is achieved in utilization of equipment (manpower resources). In other words, there should be no personnel and equipment idle time due to inefficient organization of work operations.

It is very difficult to determine the optimal sequence of process-similar work operations by conventional methods. At the same time such a sequence can be found comparatively easily on the basis of flow analysis of network schedules.

We shall examine how this is done with the following simple example. Upon returning from a field exercise, four tank subunits of unidentical composition and equipped with vehicles of various makes are to perform TO No 2 servicing operations, including flushing out filters. In addition to vehicle crews, maintenance shop personnel will be enlisted for these activities, and equipment and accessories available at the technical servicing station (PTO) will also be utilized.

The technological sequence of performance of processes and their performance times for each tank subunit are indicated in Table 12.

Table 12.

(6) Последовательность процессов	(7) Процессы работы	(8) Время выполнения процессов работы для подразделений, ч			
		1	2	3	4
(1)	Заправка танков горючим . .	0,8	0,5	0,5	1
(2)	Внутренняя очистка машин	0,7	0,3	0,5	1
(3)	Мойка	2,5	1	1,5	3,5
(4)	Промывка фильтров	2	1	1,5	3
(5)	Регулировочные работы . . .	3	1,5	2	4

Key:
1. Fueling tanks
2. Cleaning vehicle interiors
3. Washing
4. Flushing out filters
5. Tuning and adjustment procedures
6. Sequence of processes
7. Work processes
8. Work process execution time for subunits, hours

It is necessary to determine efficient organization of tank servicing at the PTO. Tank servicing must be scheduled in such a manner that maintenance shop personnel and equipment (accessories) available at the PTO are utilized efficiently, without idle time, in a flow-line mode.

Problems of this kind are also solved sequentially, step by step.

First step. We construct equipment servicing network schedules for each subunit separately, whereby we assume that all processes are carried out in parallel, simultaneously and independently of one another, without resources limitation.

Two additional events are introduced: starting event and concluding event -- which are linked to the schedules by fictitious linkages. This results in a single network schedule (Figure 60).

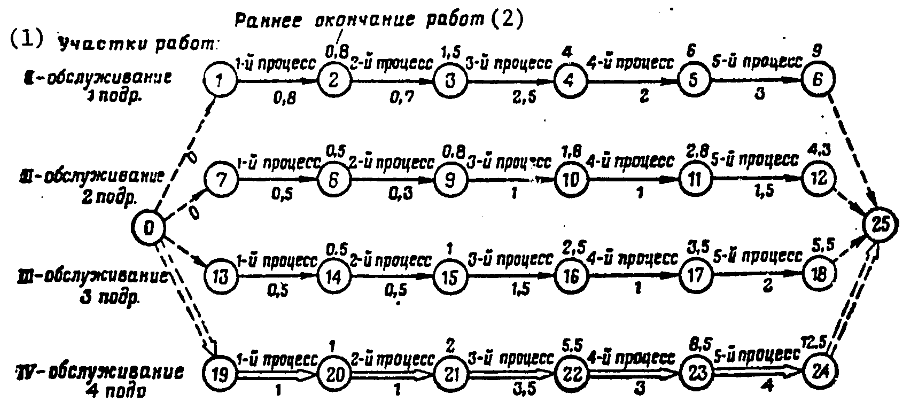


Figure 60. Consolidated Network Schedule of Equipment Servicing by Four Tank Subunits Upon Returning From a Field Exercise

Key:
1. Work operation sectors
2. Early completion of work operations

обслуживание. Servicing
подр. Subunit
-й процесс. -th process

Second step. Work operation early completion times are determined. For this we employ the formula

$$t_{р.о(i,j)} = t_{р.н(i,j)} + t_{(i,j)}.$$

Then work operation early completion times for the first process will be as follows:

$$t_{р.о(1,2)} = 0 + 0{,}8 = 0{,}8;$$
$$t_{р.о(7,8)} = 0 + 0{,}5 = 0{,}5;$$
$$t_{р.о(13,14)} = 0 + 0{,}5 = 0{,}5;$$
$$t_{р.о(19,20)} = 0 + 1 = 1.$$

We determine in the same manner work operation early completion times for the remaining processes and enter the obtained results above the corresponding event (Figure 60).

We add up duration by work operation sectors and establish that the critical path passes through work operation sector IV (along work operations involving servicing the 4th subunit) and is equal to 12.5 hours.

The other paths are shorter:

$$t_{(L_1)} = 9; \quad t_{(L_2)} = 4{,}3; \quad t_{(L_3)} = 5{,}5.$$

Third step. Sequence for work operation sectors is determined, that is, we determine in what sequence it is expedient and advisable to service the subunits. For this purpose we prepare an auxiliary table, in which work operation early completion times are placed by processes in ascending order of their absolute magnitude (Table 13).

Table 13.

Процессы работы (1)	(2) Ранние сроки окончания работ
1-й процесс	0,5 (II участок); 0,5 (III участок); 0,8 (I участок); 1 (IV участок)
2-й процесс	0,8 (II участок); 1 (III участок); 1,5 (I участок); 2 (IV участок)
3-й процесс	1,8 (II участок); 2,5 (III участок); 4 (I участок); 5,5 (IV участок)
4-й процесс	2,8 (II участок); 3,5 (III участок); 6 (I участок); 8,5 (IV участок)
5-й процесс	4,3 (II участок); 5,5 (III участок); 9 (I участок); 12,5 (IV участок)

Key:

1. Work operation processes
2. Work operation early completion

-й процесс. -th process
участок. Sector

Analyzing the figures in Table 13, one can establish that in the majority of processes the sequence of servicing is as follows: sector II, sector III, sector I, sector IV. We shall adopt this as a common sequence for all processes. Then the maintenance shop teams set up to perform the specified processes should first service the combat vehicles of the 2d subunit, then the 3d and 1st, and finally the 4th subunit.

Fourth step. We proceed to shape the process flows and construct a new network schedule. We take account of the specified equipment servicing sequence.

In constructing a chart, we must follow the logic of construction, which consists in the following (Figure 61).

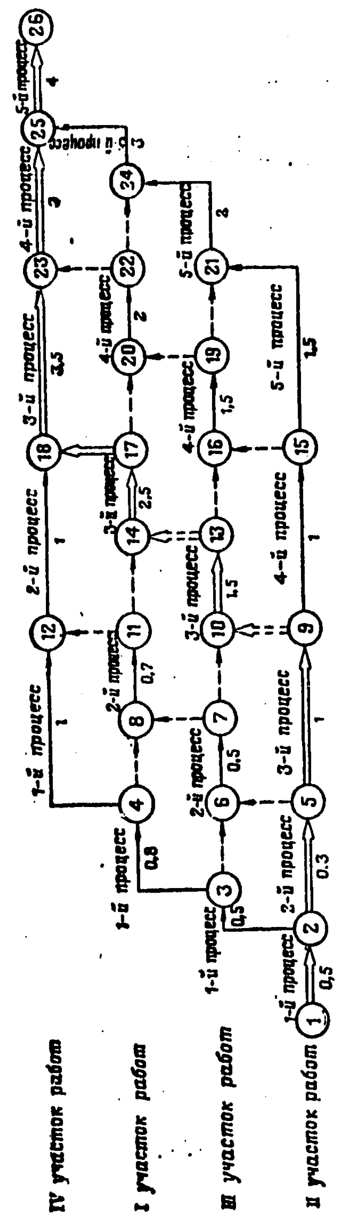


Figure 61. Network Schedule Constructed on the Basis of Process Flows

Key:
участок работ. Work operation sector -й процесс. -th process

The 1st process in sector III can begin following its completion in sector II, while the start of this process in sector I will be after completion in sector III, etc. Therefore on the chart work operations (1, 2), (2, 3), (3, 4), (4, 12) are represented as sequentially performed, while the 2d process in sector II can commence following completion of the 1st process. Consequently event (2) is the initial event for operation (2, 5), which reflects this process. The 2d process in sector III is determined not only by completion of the 2d process in sector II (cleaning of tank interiors for the 2d subunit), but also by termination of the 1st process in sector III (fueling of the tanks of the 3d subunit).

In other words cleaning the interiors of the tanks of the 3d subunit with the aid of equipment available at the PTO interior cleaning station can commence following fueling of the tanks of the 3d subunit and after the 2d subunit frees the appropiate equipment.

It would seem that for representation of these conditions it suffices to link events (5) and (3) with a fictitious linkage and designate the 2d process in sector III by work operation (3, 7), but the result of this will be that performance of the 1st process in sector I (fueling the tanks of the 1st subunit) depends on completion of the 2d process in sector II (cleaning the interior of the tanks of the 2d subunit). But this is not in conformity with reality. Therefore additional event (6), which is joined by fictitious linkages with events (3) and (5), is introduced for a correct representation of the mutual linkage of work operations of this bundle and their actual sequence.

The relationship for events (8), (10), (14), (16), (20) and for the commencement of work operations (8, 11), (10, 13), (14, 17), (20, 22) is indicated in like manner.

The result is the network schedule contained in Figure 61.

Fifth step. One calculates the parameters of the network wchedule. Network schedule parameters can be calculated graphically or by the tabular mode, following the described method.

Table 14 contains the results of network schedule calculation by the tabular method.

It is evident from Table 14 that the critical path passes through events (1), (2), (5), (9), (10), (13), (14), (17), (18), (23), (25), (26) and t_{kp}=16.3 hours.

Sixth step. We check to see whether there are any discontinuities in the processes. For this we construct, on the basis of the network schedule, a linear chart of work operation early start (Table 15).

Analyzing the chart, we see that there are short-duration organizational gaps in the 2d process between operations (2, 5) and (6, 7); (6, 7) and (8, 11); (8, 11) and (12, 18), gaps of 0.2, 0.3 and 0.3 hours respectively. We shall designate these gaps with the subscripts R_o II, III, R_o III, I, R_o I, IV, with the Roman numerals designating the numbers of the sectors between which a gap exists.

There are gaps of greater duration in the last two processes. In the 4th process there are three gaps of 0.5, 1, and 1.5 hours duration, and in the 5th process

Table 14.

Количество предшествующих работ	Код работ	Продолжительность работ	(6) Наиболее раннее время		(7) Наиболее позднее время		Общий резерв времени	Частный резерв времени второго вида
			начала	окончания	начала	окончания		
(1)	(2)	(3)	(4)	(5)	6	7	(8)	(9)
0	1—2	0,5	0	0,5	0	0,5	0	0
1	2—3	0,5	0,5	1	0,8	1,3	0,3	0
1	2—5	0,3	0,5	0,8	0,5	0,8	0	0
1	3—4	0,8	1	1,8	1,8	2,6	0,8	0
1	3—6	0	1	1	1,3	1,3	0,3	0
1	4—8	0	1,8	1,8	2,6	2,6	0,8	0
1	4—12	1	1,8	2,8	3,8	4,8	2	0
1	5—6	0	0,8	0,8	1,3	1,3	0,5	0,2
1	5—9	1	0,8	1,8	0,8	1,8	0	0
2	6—7	0,5	1	1,5	1,3	1,8	0,3	0
1	7—8	0	1,5	1,5	2,6	2,6	1,1	0,3
1	7—10	0	1,5	1,5	1,8	1,8	0,3	0,3
2	8—11	0,7	1,8	2,5	2,6	3,3	0,8	0
1	9—10	0	1,8	1,8	1,8	1,8	0	0
1	9—15	1	1,8	2,8	4,8	5,8	3	0
2	10—13	1,5	1,8	3,3	1,8	3,3	0	0
1	11—12	0	2,5	2,5	4,8	4,8	2,3	0,3
1	11—14	0	2,5	2,5	3,3	3,3	0,8	0,8
2	12—18	1	2,8	3,8	4,8	5,8	2	2
1	13—14	0	3,3	3,3	3,3	3,3	0	0
1	13—16	0	3,3	3,3	6,3	6,3	3	0
2	14—17	2,5	3,3	5,8	3,8	5,8	0	0
1	15—16	0	2,8	2,8	5,8	5,8	3,5	0,5
1	15—21	1,5	2,8	4,3	5,8	7,3	3	0,5
2	16—19	1,5	3,3	4,8	5,8	7,3	2,5	0
1	17—18	0	5,8	5,8	5,8	5,8	0	0
1	17—20	0	5,8	5,8	7,3	7,3	1,5	0
2	18—23	3,5	5,8	9,3	5,8	9,3	0	0
1	19—20	0	4,8	4,8	7,3	7,3	2,5	1
1	19—21	0	4,8	4,8	7,3	7,3	2,5	0
2	20—22	2	5,8	7,8	7,3	9,3	1,5	0
2	21—24	2	4,8	6,8	7,3	9,3	2,5	1,0
1	22—23	0	7,8	7,8	9,3	9,3	1,5	1,5
1	22—24	0	7,8	7,8	9,3	9,3	1,5	0
2	23—25	3	9,3	12,3	9,3	12,3	0	0
2	24—25	3	7,8	10,8	9,3	12,3	1,5	1,5
2	25—26	4	12,3	16,3	12,3	16,3	0	0
	26—к		16,3		16,3			

Key to Table 14 on preceding page:

1. Number of preceding work operations
2. Code of work operations
3. Duration of work operations
4. Start
5. Completion
6. Earliest time
7. Latest time
8. General time reserve
9. Partial time reserve, second type

there are three gaps of 0.5, 1.0, and 1.5 hours duration. These gaps indicate that maintenance shop personnel, and PTO equipment which facilitate vehicle servicing are being utilized inefficiently. They are not being utilized continuously but with interruptions, that is, after completing servicing one subunit they stand idle waiting for the opportunity to proceed with servicing another subunit. This is particularly evident in the 4th process. The maintenance shop team which flushes out filters on equipped benches, after completing work operation (9, 15) -- flushing out filters for the 2d subunit -- is forced to wait 0.5 hour before it can proceed with flushing out filters for the 3d subunit. After completing work operation (16, 19), the team is once again forced to take a 1-hour break. An organizational break of equal duration is observed between operations (20, 22) and (23, 25).

Such periods of idle time in utilization of personnel and complex equipment attest to poor organization of labor.

What must be done to improve organization of labor in these processes and to achieve a situation whereby maintenance shop personnel and PTO equipment would be utilized in an uninterrupted flow mode?

Further work on the chart will help correct these deficiencies. We shall take the following steps.

Seventh step. A comparative analysis is made of the quantitative indices of organizational gaps with total and partial time reserves of the preceding work operations. These indices are characterized by the following figures.

In the 2d process:

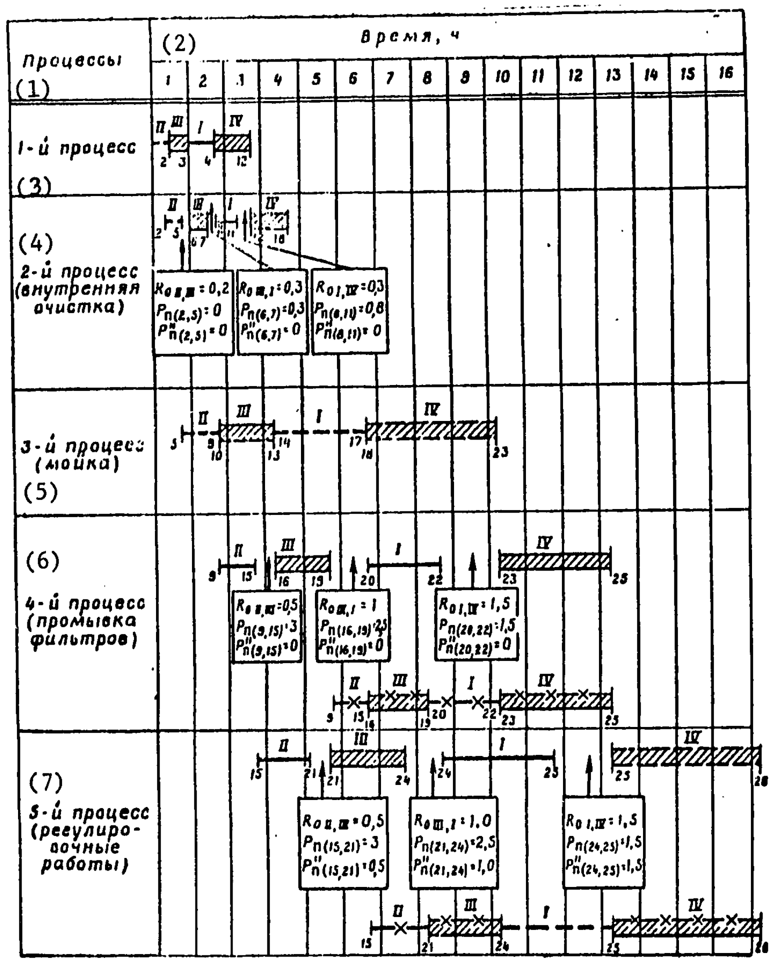
first gap between operations (2, 5) and (6, 7):

$$R_{o\,II,\,III} = 0{,}2;$$
$$P_{n\,(2,\,5)} = 0;$$
$$P^{*}_{n\,(2,\,5)} = 0;$$

second gap between operations (6, 7) and (8, 11):

$$R_{o\,III,\,I} = 0{,}3;$$
$$P_{n\,(6,\,7)} = 0{,}3;$$
$$P^{*}_{n\,(6,\,7)} = 0;$$

Table 15.



Legend:

- R_0 Organizational gap in processes
- $R_{0\,II,III}$ Organizational gap between sectors II and III
- Work operation (1, 2)
- $P_{n(2,5)}$ Full reserve of work operation (2, 5)
- $P''_{n(2,5)}$ Partial reserve of second type of work operation (2, 5)
- Work operations lying on critical path
- Optimized work operations

Key:

1. Processes
2. Time, hours
3. 1st process
4. 2d process (interior cleaning)
5. 3d process (washing)
6. 4th process (flushing out filters)
7. 5th process (tuning and adjustment procedures

third gap between operations (8, 11) and (12, 18):

$$R_{o\,I,\,IV} = 0{,}3;$$
$$P_{п\,(8,\,11)} = 0{,}8;$$
$$P^{*}_{п\,(8,\,11)} = 0.$$

In the 4th process:

first gap between operations (9, 15) and (16, 19):

$$R_{o\,II,\,III} = 0{,}5;$$
$$P_{п(9,\,15)} = 3;$$
$$P^{*}_{п(9,\,15)} = 0;$$

second gap between operations (16, 19) and (20, 22):

$$R_{o\,III,\,I} = 1{,}0;$$
$$P'_{п\,(16,\,19)} = 2{,}5;$$
$$P^{*}_{п(16,\,19)} = 0;$$

third gap between operations (20, 22) and (23, 25):

$$R_{o\,I,\,IV} = 1{,}5;$$
$$P_{п\,(20,\,22)} = 1{,}5;$$
$$P^{*}_{п\,(20,\,22)} = 0.$$

In the 5th process:

first gap between operations (15, 21) and (21, 24):

$$R_{o\,II,\,III} = 0{,}5;$$
$$P_{п\,(15,\,21)} = 3;$$
$$P^{*}_{п(15,\,21)} = 0{,}5;$$

second gap between operations (21, 24) and (24, 25):

$$R_{o\,III,\,I} = 1{,}0;$$
$$P_{п\,(21,\,24)} = 2{,}5;$$
$$P^{*}_{п\,(21,\,24)} = 1{,}0;$$

third gap between operations (24, 25) and (25, 26):

$$R_{o\,I,\,IV} = 1{,}5;$$
$$P_{п\,(24,\,25)} = 1{,}5;$$
$$P^{*}_{п\,(24,\,25)} = 1{,}5.$$

An organizational gap (R_o) can be determined from a linear chart as well as from a network schedule parameters calculation table, with the formula

$$R_o = t_{р.\,н\,(j,\,k)} - t_{р.\,о\,(i,\,j)}. \tag{41}$$

For example, we must determine the organizational gap between operations (9, 15) and (16, 19) with the aid of tables 14 and 15:

$$R_{o\,II,\,III} = t_{р.\,н\,(16,\,19)} - t_{р.\,о\,(9,\,15)} = 3{,}3 - 2{,}8 = 0{,}5.$$

All obtained indices are recorded on the linear chart as indicated in Table 15.

After performing this operation, we proceed with improving the chart in order to obtain a flow. First we determine the possibility of process flow optimization. Such a possibility is established with the following relations:

$$R_0 = P_{n(i,j)};$$
$$R_0 = P'_{n(i,j)};$$
$$P_{n(i,j)} > R_0 > P'_{n(i,j)},$$

that is, we can optimize with work operation time reserves (total or partial).

If an organizational gap exceeds in magnitude work operation total time reserve, it is impossible to flow-optimize the process without extending the total execution time of the entire aggregate of work operations. The first gap in the 2d process graphically illustrates this case.

In fact, the organizational gap between operations (2, 5) and (6, 7) is equal to: $R_{o\ II,\ III}$ -- 0.2, while total and partial time reserves of operation (2, 5) are equal to 0, that is, this operation lies on the critical path. Consequently, there is no possibility of transitioning from early to late work operation start and completion times (early and late operation start and completion times are equal for a critical work operation).

It follows from this that the 2d process does not require flow optimization.

If one nevertheless attempts to make the 2d process a flow process, the time of execution of the entire aggregate of operations will be extended by 0.8 hours ($R_{o\ II,\ III}+R_{o\ III,\ I}+R_{o\ I,\ IV}$=0.2+0.3+0.3).

Let us examine the 4th process with the aim of flow optimization. It is evident from Table 15 that this can be made a flow process, since:

$$P_{n(9,15)} > R_{o\ II,\ III} > P'_{n(9,15)} (3 > 0{,}5 > 0);$$
$$P_{n(16,19)} > R_{o\ III,\ I} > P'_{n(16,19)} (2{,}5 > 1{,}0 > 0);$$
$$P_{n(20,22)} = R_{o\ I,\ IV} > P'_{n(20,22)} (1{,}5 = 1{,}5 > 0).$$

In order to achieve continuous, uninterrupted work in the 4th process, we shall refine the schedule, beginning with the end.

The third gap is equal to 1.5 hours.

Since operation (20, 22) has a total reserve of $P_{n(20,\ 22)}$=1.5, we can shift its commencement and completion time by the amount of the gap, without risking extending the onset time of the concluding event.

Reasoning in similar fashion, we shift the execution times of operations (16, 19) and (9, 15), achieving continuity in the work of the filter flushing team.

We organize flow in the 5th process as well with this same method, by shifting from early to late commencement and completion times for operations (24, 25), (21, 24), and (15, 21).

Thus we flow-optimize the network schedule and achieve efficient organization of utilization of maintenance shop personnel as well as PTO gear and equipment for servicing vehicles.

With the first schedule (before optimization), the maintenance shop filter flushing team commenced work 1.8 hours after vehicle servicing began, while with the new schedule this team can commence work in 4.8 hours. And this will have no effect on the time specified for completion of the aggregate of work operations pertaining to servicing all vehicles.

In addition, according to the original plan the maintenance team was compelled to lose 3 hours on idle time in the course of performing filter flushing operations for all tank subunits. Now the maintenance shop personnel can utilize this time for other work operations.

The above example of flow optimization of a network schedule clearly shows that the dscribed method is an effective tool for achieving efficient organization of a complex aggregate of work operations, during execution of which idle time for skilled personnel, vehicles and equipment is eliminated.

The above-discussed techniques of network flow optimization can also be utilized in solving other operational-tactical and technical problems.

Let us examine one more example, in order to gain a better understanding of the question of network schedule optimization.

A subunit headquarters staff was assigned the task of organizing an exercise to demonstrate weapons and combat equipment for four groups of trainees of various occupational specialties. Each group can inspect equipment simultaneously only at one training station. The sequence of arrival of groups at the first training station, specified by the higher commander, is known. The groups should not waste time waiting their turn to inspect equipment.

Equipment inspection time at each training station differs for each group and is indicated in Table 16.

Table 16.

(1) Группа	(2) Время осмотра техники, находящейся на учебных местах, ч		
	№ 1	№ 2	№ 3
1-я	2	1	1
2-я	0,5	1	2
3-я	0,5	2	0,5
4-я	0,5	1	0,5

Key to Table 16 on preceding page:

1. Group
2. Time for inspecting equipment at training stations, hours

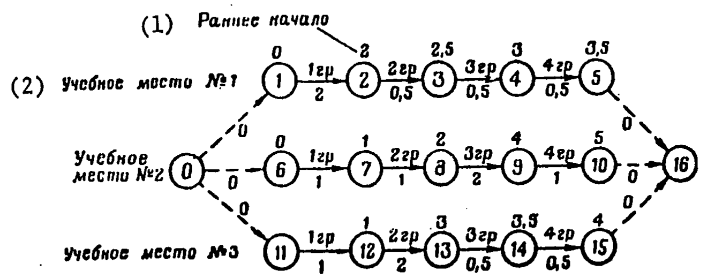


Figure 62. Initial Network Schedule of Demonstration of Weapons and Combat Equipment

Key:

1. Early start
2. Training station...

гр. Group

The task is to determine an optimal variant of combat equipment and weapons inspection by all groups, whereby the least amount of time would be expended. Analyzing the process of equipment inspection at three training stations, we shall obtain the schedule shown in Figure 62. Optimizing this schedule for the purpose of improving the inspection flow, we shall obtain a new network schedule (Figure 63).

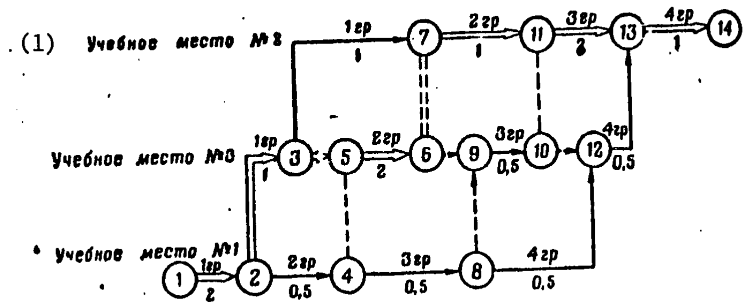


Figure 63. Optimized Network Schedule of Demonstration of Weapons and Combat Equipment by Process Flow

Key:

1. Training station...

гр. Group

Table 17 contains network schedule parameter calculation figures.

Proceeding from the ascending order of early completion of work operations, the equipment inspection sequence will be as follows: training station 1, training station 3, and training station 2.

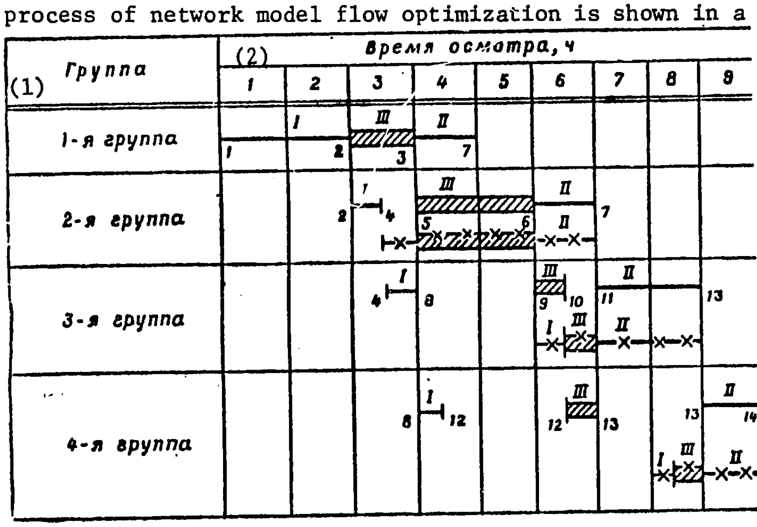
Table 17.

(1) Количество предшествующих работ	(2) Код работ	$t(i,j)$	$t_{р.н}(i,j)$	$t_{р.о}(i,j)$	$t_{п.н}(i,j)$	$t_{п.о}(i,j)$	$P_{п}(i,j)$	$P^{*}_{н}(i,j)$
0	1—2	2	0	2	0	2	0	0
1	2—3	1	2	3	2	3	0	0
1	2—4	0,5	2	2,5	2,5	3	0,5	0
1	3—5	0	3	3	3	3	0	0
1	3—7	1	3	4	4	5	1	1
1	4—5	0	2,5	2,5	3	3	0,5	0,5
1	4—8	0,5	2,5	3	5	5,5	2,5	0
2	5—6	2	3	5	3	5	0	0
1	6—7	0	5	5	5	5	0	0
1	6—9	0	5	5	5,5	5,5	0,5	0
2	7—11	1	5	6	5	6	0	0
1	8—9	0	3	3	5,5	5,5	2,5	2
1	8—12	0,5	3	3,5	7	7,5	4	2
2	9—10	0,5	5	5,5	5,5	6	0,5	0
1	10—11	0	5,5	5,5	6	6	0,5	0,5
1	10—12	0	5,5	5,5	7,5	7,5	2	0
2	11—13	2	6	8	6	8	0	0
2	12—13	0,5	5,5	6	7,5	8	2	2
2	13—14	1	8	9	8	9	0	0

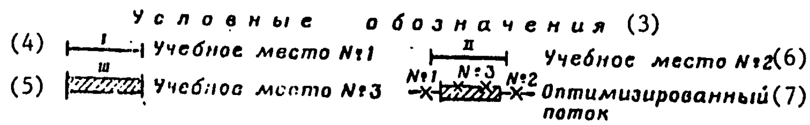
Key:
1. Number of preceding operations
2. Code of work operations

The process of network model flow optimization is shown in a linear chart (Table 18).

Table 18.



(Legend and key to table on following page)



Key:

1. Group
2. Inspection time, hours
3. Legend
4. Training station 1
5. Training station 3
6. Training station 2
7. Optimized flow

Thus we have studied methods of optimizing network schedules, with the aid of which we have changed the duration of operations, distributed resources or determined flows, which in complex processes can provide a reliable guarantee of observance of specified timetables.

2. Tying in Network Schedules to Calendar Timetables and Constructing Scale Network Schedules

Since network schedules are a control and management tool, following calculation and optimization, schedules are usually tied in to calendar timetables.

Schedules are tied in to calendar timetables proceeding from the concrete conditions of planning and consideration of the conditions in which the process will be carried out. Two methods of tying in network schedules to calendar timetables can be employed: time-linking with the aid of time scales, and tying in with the aid of constructing scale network schedules. Both these methods can be successfully employed in military affairs. Let us examine them.

Tying in schedules to calendar timetables with the aid of time scales is usually employed when planning prolonged processes (combat operations), such as when planning construction of large military installations and structures, the training process, the process of manufacture, repair and renovation of combat equipment, large-scale projects, etc.

In these cases network schedules are tied in to calendar timetables in the following sequence. First of all two time scales are plotted -- one above the other. One places on the upper time scale a natural series of numbers from 1 to the number signifying the end of the scheduled process. This upper series is nothing other than the number of units of time required for completion of the entire process. One places in the lower series calendar times (calendar days of months, minus days off and holiday), if the process is taking place in peacetime.

Let us take, for example, the process of major overhaul of a group of combat vehicles, scheduled by network method, figured for 21 days. The process of overhauling this group begins on 1 November 1972.

Table 19 contains a tie-in of the calculated schedule to calendar dates.

Table 19.

	(1) Дни производства работ																				
(2) Расчетные сроки в днях	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
(3) Календарные даты	1.11	2	3	4	9	10	13	14	15	16	17	20	21	22	23	24	27	28	29	30	1.12

Key:
1. Days of performance of work operations
2. Calendar times in days
3. Calendar dates

Calculated times are placed in the upper scale of Table 19, and in the lower -- calendar dates minus days off and holidays.

Tying in to calendar times with the aid of a scale network schedule is accomplished as a rule in processes connected with planning troop combat activities, as well as all types of combat, combat service and logistic support. Such tying in essentially constitutes constructing a scale network schedule and makes it possible to employ a scale network schedule as an operating document for allocating tasks to troops, for organization of coordination and all types of support, as well as for directing the planned process in the course of combat actions.

Let us examine a scale network schedule and the principles of its construction. As an example we shall take an optimized initial network schedule, as shown in Figure 55, and transpose it to a scale (Figure 64).

A scale network schedule (Figure 64) is a definitively calculated and optimized network schedule transposed to a time scale. Construction of such a schedule is a very useful stage in the overall process, and in many cases an essential stage, since a scale network schedule gives a clear picture of the course of a process on a time axis. With the aid of such a schedule one can see the entire planned operation (process) on a time axis in that process sequence in which the given operation (process) should develop. Essentially a scale network schedule reflects the process of coordination of the personnel and equipment participating in the operation (process) as regards missions, objectives and time, and therefore constitutes, as it were, a coordination schedule table.

In addition, a scale network schedule can serve as an operation document, on the basis of which one can assign fully substantiated and consequently objective tasks to subunits and units participating in an engagement (process). At the same time, possessing a scale network schedule, one can control a process, utilizing internal resources and time reserves, since a scale network schedule enables one to see the entire planned engagement (process), to see available internal resources and reserves, and thus efficiently to control the course of an engagement (process). Knowing the entire course of combat actions (process) as a whole and possessing time and resource reserves, one can maneuver these resources and reserves, achieving completion of all events in the network on schedule.

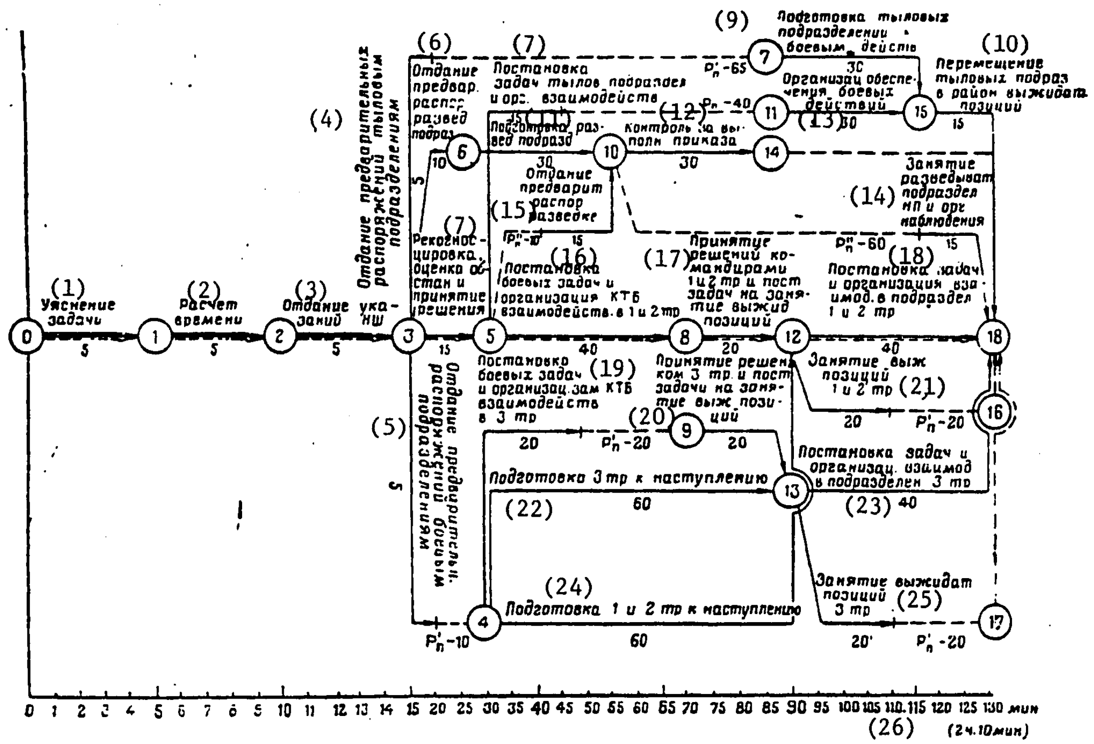


Figure 64. Scale Network Schedule of Preparation of a Tank Battalion for an Offensive Action

Key:

1. Mission briefing
2. Time calculation
3. Issuing instructions to executive officer
4. Issuing warning orders to rear services subunits
5. Issuing warning orders to combat subunits
6. Issuing warning orders to reconnaissance subunits
7. Commander's reconnaissance, situation estimate, and decision-making/planning
8. Allocation of missions to rear services subunits and organization of coordination
9. Preparation of rear services subunits for combat actions
10. Movement of rear services subunits into assembly area
11. Preparation of reconnaissance subunits
12. Verification of execution of order
13. Organization of support of combat actions
14. Reconnaissance subunits occupy observation posts and organize surveillance
15. Issuing warning orders to reconnaissance
16. Allocation of combat tasks and organization by tank battalion commander of coordination in 1st and 2d Tank companies
17. Decision-making/planning by commanders of 1st and 2d Tank companies and allocation of tasks to proceed to and occupy assembly area

(Key to Figure 64, cont'd)

18. Allocation of tasks and organization of coordination in subunits of 1st and 2d Tank companies
19. Allocation of combat missions and organization by tank battalion second in command of coordination in 3d Tank Company
20. Decision-making/planning by commander of 3d Tank Company and allocation of tasks for proceeding to and occupying assembly area
21. Proceeding to and occupying assembly area by 1st and 2d Tank companies
22. Preparation of 3d Tank Company for the attack
23. Allocation of tasks and organization of coordination in subunits of 3d Tank Company
24. Preparation of 1st and 2d Tank companies for attack
25. 3d Tank Company proceeds to and occupies assembly area
26. 2 hours and 10 minutes

A scale network schedule provides the possibility of forecasting the course of combat actions (process), of predicting possible deviations long before they occur, and of taking prompt measures to prevent them. A scale network schedule makes it possible to make optimal decisions in each concrete situation.

Thus a scale network schedule is a reliable instrument of troop control during execution of diversified tasks.

A scale network schedule should be constructed in the following sequence. A time scale is plotted on millimeter graph paper or on a ruled sheet of paper. Seconds, minutes, hours, days, weeks, or months can be taken as unit of time. This is decided specifically on each occasion, depending on conditions, and one takes those units of measurement which are convenient for process control and management. Minutes are taken as unit of measurement in Figure 64.

We shall note that for convenience it is more advisable to vary the time scale. This is necessary in order graphically to portray brief-duration operations and to accommodate the designation of a work operation above the arrow. This has been done in our example (Figure 64). From 0 to 15, time on the scale is indicated every minute, while from this point to the end of the scale time is indicated every 5 minutes.

After the scale is plotted with the aid of an optimized schedule and calculation table, the scale schedule is plotted. Construction begins with the critical path. Critical-path work operations in the schedule are extended into a single line, which also determines the length of the schedule.

The starting event (the zero event in our schedule) is placed on the zero ordinate at the center of the chart. The entire network is plotted at scale from this event. Each work operation is laid out in scale, based on the duration of each operation. Each work operation terminates with the event which follows it.

The centers of events are placed on vertical lines running from their completion time marks on the scale. Operation (0, 1), for example, terminates with event

(1). The duration of this work operation is 5 minutes. We find on the time scale the mark with the number 5 and run from this mark a line perpendicular to the time scale. On this perpendicular line we place opposite the zero event event (1), the center of which we place on the perpendicular line. We join the zero and first events with an arrow, above which we write the designation of the work operation, and below the arrow we indicate its duration in minutes.

All other work operations are plotted in scale in like manner. In those cases where a work operation proceeds at an angle to the scale, its duration is measured as a projection onto the scale. The events between which a work operation lies should be plotted onto the scale network schedule according to the optimized schedule calculation table, taking into account partial reserves of work operations. We shall utilize Table 20, which contains this calculation, to construct our scale network schedule.

Table 20. Optimized Network Schedule Calculation Table

Events (События) i	j	$t_{р.н(i,j)}$	$t_{(i,j)}$	$t_{р.о(i,j)}$	$t_{п.н(i,j)}$	$t_{(i,j)}$	$t_{п.о(i,j)}$	$P_п$	$P'_п$	$P''_п$
0	1	0	5	5	0	5	5	0	0	0
1	2	5	5	10	5	5	10	0	0	0
2	3	10	5	15	10	5	15	0	0	0
3	4	15	5	20	25	5	30	10	10	0
3	5	15	15	30	15	15	30	0	0	0
3	6	15	10	25	60	10	70	45	45	0
3	7	15	5	20	80	5	85	65	65	0
4	12	20	60	80	30	60	90	10	0	10
4	13	20	60	80	30	60	90	10	0	0
5	8	30	40	70	30	40	70	0	0	0
5	9	30	20	50	50	20	70	20	20	0
5	10	30	15	45	85	15	100	55	55	10
5	11	30	15	45	70	15	85	40	40	0
6	10	25	30	55	70	30	100	45	0	0
7	15	20	30	50	85	30	115	65	0	25
8	12	70	20	90	70	20	90	0	0	0
9	13	50	20	70	70	20	90	20	0	10
10	14	55	30	85	100	30	130	45	15	0
10	18	55	15	70	115	15	130	60	0	60
11	15	45	30	75	85	30	115	40	0	0
12	16	90	20	110	110	20	130	20	20	0
12	18	90	40	130	90	40	130	0	0	0
13	17	80	20	100	110	20	130	30	20	0
13	18	80	40	120	90	40	130	10	0	10
14	18	85	0	85	130	0	130	45	0	45
15	18	75	15	90	115	15	130	40	0	40
16	18	110	0	110	130	0	130	20	0	20
17	18	100	0	100	130	0	130	30	0	30

Note: In Table 20, 22, and 23, critical work operations are set off by heavy lines

If a work operation possesses a time reserve of the first type, the operation should be plotted in scale, beginning with the initial event. The end of a work operation should be marked with a vertical stroke, from which the first-type time reserve of this operation is laid out by a dashed line to the terminal event. The value of this reserve is indicated under the dashed line. For example, let us plot operation (5, 11) onto the scale network schedule (Figure 64). We find this work operation in Table 20 and see that the given operation has a first-type partial time reserve of 40 minutes. This work operation has a duration of 15 minutes. We plot this operation in scale from event (5). Since the work operation began 30 minutes after work commencement by the battalion commander, it will terminate at 45 minutes. We run an arrow (work operation) to this time and terminate it with a vertical stroke. We then run a dashed line equal to 40 minutes, indicating the partial reserve of this work operation, and plot the terminal event (11) of this operation, the late completion time of which is 85 minutes. Under this dashed line we indicate the amount of this reserve P'_n, equal to 40 minutes.

If a work operation has a second-type partial time reserve, the operation is indicated beginning from the terminal event, plotted according to late completion time. For example, let us plot operation (10, 18) on the chart. It is evident from Table 20 that the work operation has a second-type partial time reserve of 60 minutes. We plot the terminal event (18) on the basis of late completion time, which is equal to (according to Table 19) 130 minutes. From event (18) we lay out in scale operation (10, 18), equal to 15 minutes, and mark the start of this operation with a vertical stroke. This operation will begin 15 minutes after the battalion commander's work commences and will end after 130 minutes, terminating with event (18). We then run a dashed line in scale, equal to 60 minutes, from the start of this operation to event (10), indicating under the dashed line the amount of second-type time reserve P''_n.

If a work operation does not possess partial time reserves, it is plotted at the calculated early start and completion time. For example, operation (6, 10) does not possess partial time reserves, and therefore we plot it at scale on the scale network schedule by early start and completion time.

Possessing a scale network schedule, one can see the entire process as a whole, can see work operations and their available reserves, as well as mutual process and logical linkages on a time axis. For a given concrete time one can see process progress, what work operations should commence later, and which ones should begin earlier than the calculated time. One can establish from the schedule from what work operation one should begin, what resources should be utilized and for what time in order to speed up the entire process as a whole.

The availability of the scale network schedule will help distribute work operations among process executants. For example, in our schedule (Figure 64) all operations lying on the critical path up to event (8) will be performed by the battalion commander. Beginning with event (5), however, principal work operations pertaining to organization of combat actions and coordination are performed in parallel. Operation (5, 8), for example, is performed by the battalion commander, operation (5, 9) by his second in command, and operations (5, 10) and (5, 11) by the battalion executive officer. The battalion executive officer will begin performing work operation (5, 11) upon completion of operation (5, 10), utilizing the reserve of operation (5, 11), which is equal to 40 minutes (Table 20).

The network schedule is also a reliable instrument in process control and management, which will be examined below.

3. Utilization of PERT Methods in Process Management/Control

In the preceding sections we have examined in detail the methods of constructing and optimizing network schedules. The reader has been able to see that as a result of detailed analysis of a network model one can find a practical and expedient solution and specify an optimal plan for execution of an aggregate of work operations.

This does not yet signify, however, that input data and an optimized network schedule are absolutely accurate and that there will be no deviations from the specified plan in the process of execution of work operations. The probability of occurrence of such deviations is due to the presence of random factors which affect the performance of work operations but cannot always be taken precisely into account. Therefore it is essential to manage and control a planned process in a flexible and efficient manner. It will be necessary to introduce corrections in the course of execution of the original plan.

How does process control/management take place with the existing traditional method of planning?

In the course of accomplishment of work operations a situation may develop where everything is proceeding according to plan at one place and a delay occurs elsewhere. Steps are taken to correct this delay, and the plan is reworked. In these conditions, however, it is impossible to know what effect the delay will have on the end result of combat actions (project). Work operations are suitably adjusted by adopted measures at one point, but then a delay occurs at a third location, etc.

Network methods are effective in that they enable one to investigate and grasp the picture in the course of combat operations or a process. After receiving information on the status of work operations, the entire network is analyzed: the length of the new critical path is determined, reserves are calculated, occurring obstacles are determined, and their effect on other work operations is established. Methods and means to eliminate them are determined proceeding from the overall situation. This makes it possible to direct a process with foresight and to forecast a process in a prompt and timely manner.

While in projects where program evaluation and review technique is not employed, observation of the process is conducted as if through a narrow slit, enabling one to see only a small area of work operations, with PERT methods one can scan the entire complex aggregate of measures down to the tiniest details. A network model enables one scientifically to analyze the course of a work operation and to make the most expedient decisions.

Critical-path method possesses the advantage that it enables one to employ special means and methods which provide full-valued information on the actual status of the program and its completion prospects relative to assigned tasks, and also enables one to elucidate critical areas in the course of process execution on the basis of various parameters (time, resources, etc). Such information enables one to make prompt and effective decisions on control/management of the process.

The stage of process control and management begins following final network correction and adjustment. It begins at the moment the network is activated and ends simultaneously with completion of the aggregate of work operations. The purpose of this stage is to achieve a situation whereby the terminal event of the network schedule will occur at the predetermined time and within the limits of allocated resources, in spite of possible unforeseen difficulties and deviations in the completion times of individual schedule work operations.

In connection with this, planning tasks in the course of performance of work operations include monitoring the actual status of operations, determination and analysis of occurring changes, plan correction and adjustment, redistribution of resources, and on each occasion preparation of a new forecast chart.

Critical-path method specifies a number of measures for successful accomplishment of these tasks, providing for continuous surveillance of the progress of work operations, thanks to a well organized information service and periodic reports received from executants on progress in accomplishment of work operations on the critical and subcritical paths, and less frequently on other work operations as well.

Information volume and content should be differentiated in conformity with different levels of leadership, and the volume of received information should be maximally precise and concise. Information periodicity depends on concrete conditions and is determined by the project supervisor.

Information transmitted in an upward direction should contain not only data on the status of work operations in progress but also on all likely changes. Reports may contain information on new work operations and events, on possible changes in the linkages and topology of networks, on revision of time estimates, on change in times of allocation of manpower, supplies and other resources, and on full or partial completion of given work operations.

Information on these questions should be presented in encoded form. It is best to employ a single-digit code to designate the status of work operations. For example:

0 -- work operation removed, eliminated;

1 -- additional work operation, newly adopted;

2 -- work operation proceeding with delay (not being carried out);

3 -- work operation proceeding on schedule;

4 -- work operation proceeding ahead of schedule;

5 -- completed work operation.

A two-digit system of codes can be employed to communicate the reasons for non-accomplishment of work operations. Applied to control and management of combat training, this system of codes could be as follows:

11 -- inadequate motor transport availability;

22 -- personnel diverted to other activities;

33 -- training facilities not in a state of readiness;

44 -- weather factor;

88 -- other reasons.

Three-digit codes can be adopted to communicate an evaluation of the quality of a completed work operation:

101 -- work operation performed with mark of excellent;

102 -- work operation performed with mark of good;

103 -- work operation performed with mark of satisfactory;

104 -- work operation unsatisfactory.

Each received report should be submitted by the executing agencies to the higher agency by the rigorously specified time, in the form of a card, such as that contained in Table 21.

Table 21.

Шифр раздела боевой подготовки (1)	Шифр части, подразделения, отдела (2)	(3) Шифр ответственного исполнителя	(4) № информации	(5) Дата информации
07	09	023	10	15.6

Код работы (6)		(9) Состояние работы	(10) Время работы в сутках	(11) Окончание работы (плановое)	(12) Прогноз		(15) Резерв времени (частный)		Примечание (18)
начало (7)	конец (8)				(13) продолжительность	дата (14)	по плану (16)	оставшийся (17)	
1	2	3	4	5	6	7	8	9	10
85	87	5—103	7	14.6	—	—	—	—	—
49	50	2—22	6	10.6	10	14.6	8	4	—

Key:

1. Combat training section code
2. Code of unit, subunit, section
3. Code of executing agency
4. Report number
5. Date of report
6. Work operation code
7. Start
8. End

(Key to Table 21, cont'd)

9. Status of work operation
10. Time of work operation in days
11. Completion of work operation (scheduled)
12. Forecast
13. Duration
14. Date
15. Time reserve (partial)
16. According to plan
17. Remaining
18. Comments

This card is essentially a standard document which contains a large quantity of information. At the same time this document can be prepared very quickly and transmitted to the command authority by any communication channels.

In Table 21 the codes signify the following: 07 -- the report pertains to weapon training; 09 -- 3d Tank Battalion; 023 -- tank battalion commander. Let operation (85, 87) in our example signify performance of a training exercise with a subcaliber tube; the numbers 5-103 in the first row of column 3 denote that the exercise performed by this subunit received a mark of satisfactory.

Work operation (49, 50) signifies live-fire activities by tank platoons.

The number 2 in the second row of column 3 indicates that the work operation is proceeding with delay, while the number 22 in that same column indicates the reason for the delay (personnel diverted to other activities). The duration of this work operation, according to the original schedule, was 6 days (column 4), while its completion was scheduled for 10 June (column 5). Adjustment had to be made, however. The executant, performing calculations taking account of the new conditions, determined that duration of the work operation would increase to 10 days (column 6), while its completion time would have to be extended to 14 June (column 7).

According to the original schedule, operation (49, 50) had a partial reserve of 8 days (column 8). In the new version of the local network schedule, this reserve was reduced to 4 days.

On the one hand, strict order in submitting reports creates a precise work rhythm. Deliberation of the work operations schedule and estimating their duration disciplines executants and prevents them from arbitrarily changing the completion times of work operations or cancelling them. On the other hand the project supervisor is kept continuously informed on the situation and sees those work areas which are threatened by potential difficulties, which enables him promptly to take the requisite steps to correct them.

After gathering information on the course of work operations, one proceeds to update the network schedule. The initial network schedule is revised by the immediate executing agencies. Local schedules are refined by middle-echelon leaders, while the consolidated network schedule is refined by the highest control echelon. Processing of data does not differ from the analogous procedure employed in constructing the original network: calculation of critical path, determination of work operation completion times and reserves, drafting of recommendations, etc, that is, the already familiar process of network optimization is repeated.

A new network schedule is constructed as a result of updating a number of initial plan estimates. Thus it is a collective forecast of the course of work operations at a given moment in time. Analysis of the new schedule makes it possible, just as in the original plan, to determine the critical path, to reestimate time reserves for events of unstressed paths, and to estimate the probability of event completion on schedule.

One should bear in mind that a new network schedule may differ greatly from the initial optimized schedule.

Recommendations on decision-making are presented on the basis of data obtained with the aid of the new network schedule.

Network schedule analysis data at each stage (following each report) should be sent to the executing agencies in order that they be informed of their status in the overall course of work operations. This element is new in comparison with existing methods of control and management of aggregates of work operations.

In the absence of electronic computers for collecting, processing and analyzing information, it will evidently be necessary to expend a comparatively large amount of time on decision-making. At the present time this circumstance limits the area of application of critical-path methods in the course of control and management of processes which are of short duration. Program evaluation and review technique provides an opportunity to forecast short-lived processes as well, altering the network model in conformity with the projected conditions which may occur in the couise of a short-lived process. And this makes it possible in the planning stage to provide for requisite measures if delays occur in the course of the process.

We shall clarify this with the following example. Let us assume that the battalion commander and his staff have the task of determining the optimal variant of subunit response to a combat alert. The solution can be found by means of repeated practical response to a combat alert. But this method is impractical. It requires large expenditures of material and manpower resources and time. Therefore it will hardly be advantageous to perform such experiments. By employing PERT methods, however, one can utilize another way to solve the assigned problem -- to find an optimal variant by modeling the process of response by subunits to a combat alert. To achieve this end, possible action variants are played out on a constructed and optimized network model, by feeding in the most probable delays, breakdowns, and deviations from the designated plan.

Playing out the process of subunit response to a combat alert and employing PERT methods, one calculates the critical path, determines time reserves, and specifies recommendations for eliminating bottlenecks. These recommendations are grounded on concrete data obtained on the basis of scientific analysis.

Determining the expediency of applying PERT methods, one must take into account the fact that the groundwork of successful accomplishment of control tasks is laid during the period of organization of actions, during the period of planning,

Planning for combat operations (process) with employment of critical-path methods, the commander thoroughly scrutinizes the forthcoming process, more precisely estimates the extent and duration of forthcoming work operations, gains an understanding of the complex interlinkage, logical and process sequence of work operations, and determines weak areas. This makes it possible to select from the entire aggregate of work operations the principal ones which determine the total time for accomplishing the mission, that is, the critical-path work operations.

Thus program evaluation and review technique enables the commander knowledgeably to direct a process, even if he does not possess considerable practical experience.

Scale network schedules greatly facilitate control of a scheduled process. Having such a schedule at his disposal, a commander, with knowledge of work operation time reserves and delay time, will immediately be able to respond to disruption of the course of work operations.

4. Some Recommendations on Adoption of PERT Methods in Line Units, Military Educational Institutions and Establishments

In the era of rapid scientific and technological advance, one of the most pressing problems is the problem of improving control of complex processes. The question of improving control has always been at the center of attention of our Communist Party and Soviet Government. But it has never been so acute as at the present time. The 24th CPSU Congress stated the task of continuing the policy of improving management of the national economy and improving planning.

This congress decision also fully applies to the system of military management and control. Indeed, in the era of rapid advances in military technology substantial changes have taken place in the structure of the Armed Forces, in weaponry, military equipment, and views on the character of combat operations have changed. These factors advance the problem of scientific troop control to the forefront of military-scientific problems. PERT methods can lend certain assistance in solving this problem. These methods contain considerable possibilities for improving efficiency and effectiveness of control. realization of these possibilities depends to a significant degree on how correctly is executed the measure to adopt a PERT system into the practical activities of staffs, military educational institutions, and establishments.

Appropriate training of personnel is a decisive condition for successful adoption of new methods of planning and management. Experience indicates that suitable training of personnel in methods of constructing, processing, analyzing and optimizing network schedules constitutes a base for adoption of a PERT system. Only trained personnel can utilize in full measure all the advantages offered by the new methods of planning and management. A superficial acquaintance with program evaluation and review technique methods and a nonsystemic approach to things leads to project discrediting and failure.

In training cadres, one can combine officer independent study of the appropriate literature with regular seminars. As experience has shown, however, brief courses of instruction are the most effective form of teaching PERT methods.

Training should involve not only the immediate executing personnel but also leader personnel. Success in adoption of a PERT system depends in large measure on the attitude of leaders toward the new methods, and this is understandable, for decisions pertaining to practical possibilities of applying PERT measures are made by top-echelon leaders. In addition, just as any new method, program evaluation and review technique requires greater attention and support by leader personnel than existing planning methods.

In the process of training, considerable time should be devoted to practical training sessions, at which officers should acquire initial skills in working with schedules and acquire the ability to calculate parameters and optimize networks.

It is also essential to convince officers that critical-path method also promotes more precise coordination of the actions of individual executing agencies, improves organization of work operations, eliminates idle time and reduces the number of work stoppages. It is important to demonstrate to the trainees that PERT methods make it possible to establish a precise interlinkage among work operations, which gives an integral picture of prospects for plan execution and helps each executant more clearly see and understand his role in the overall program. It is also useful to convince officers that the new and advanced methods make it possible more fully to consider the actual capabilities of subunits and correctly to assign tasks to them, as well as to give initiative to commanders locally in elaborating and implementing measures pertaining to executing the designated program.

More rapid mastery of PERT methods also depends in large measure on correct organization of work of a psychological nature. Psychological preparation should be directed primarily toward overcoming conservatism and certain resistance on the part of some individuals. One must consider the fact that for an extended period of time officers were indoctrinated in a spirit of traditional planning methods. They are well acquainted with this field and successfully apply it in their activities. Now new methods are appearing, which make it possible quickly to discover lack of initiative and excessive caution on the part of executing personnel and which demand of leader personnel thorough knowledge of the technology of the controlled process and the ability to make clear and concrete decisions, rather than issuing general instructions, and force one to depart from the customary work style.

The new methods should be put into practice when utilizing existing methods, and their results should be compared. Adoption measures should enable officers to become convinced through their own experience that the new methods are superior and accessible.

For a beginning it is necessary to select a fairly simple problem (program), in order that overcoming difficulties proceeds in parallel with amassing experience. This should be an interesting problem, so as to gain the interest not only of officers and noncommissioned officers working on a given project but also the other members of the collective as well. It is also desirable that a project be typical, so that subsequently experience and know-how will be more easily disseminated. In addition, it is necessary to endeavor to ensure that the program is sufficiently important so that in the process of optimization it would be more advantageous to enlist specialists from other subunits.

We shall note, however, that one should not go to the opposite extreme and from the very outset take up a very difficult project. First one must master PERT methods, become accustomed to their peculiarities, and then adopt PERT into one's daily activities. For example, one can recommend that network schedules be constructed pertaining to preparing subunits for inspection parade, preparation for firing exercises, for holding athletic competitions, on preparing for conducting demonstration drills, etc. The effect of adoption of new methods may stand out more clearly in these small and uncomplicated examples.

The full effect of application of critical-path method should be expected when PERT is employed in planning large and complex processes. The more complex the aggregate of designated work operations, the greater the effect which will be obtained from application of critical-path method, since in this case it is much more difficult to find a correct organizational solution due to the large number of linkages between a program's individual parts and areas. Therefore, after officers master the techniques of working with small network schedules, one can proceed to the next stage of adoption, to utilization of critical-path method for planning complex practical tasks, such as for planning combat training by large headquarters staffs, elaboration of plans for making troops combat ready, etc.

It may be more advisable to establish 2-4 man PERT terms for effective utilization of the new planning methods by headquarters staffs. The officers appointed to these teams should possess a high degree of training and preparation, should thoroughly know the sequence of performance of designated work operations, and should possess total mastery of the method of constructing and analyzing network schedules.

The duties of this team are determined by the stages of adoption.

At the stage of preparation to adopt PERT methods, the team may be assigned the following duties: study of program evaluation and review technique methods by the officers, preparation and duplication of documents on program evaluation and review technique (standard schedules, blanks for transmitting information, etc).

At the planning stage it is advisable to assign the team the following: preparation of a block diagram; collection of primary or partial network schedules from executing individuals; preparation of a consolidated network schedule and its optimization; issuing of requisite data to leader personnel for optimal decision-making; communication of the ratified network schedule to all personnel concerned.

At the control stage the team should perform the following: monitor prompt presentation of information from executing personnel; analyze, correct and update the network schedule; prepare proposals for improving the plan; specify measures aimed at preventing failure to meet the timetable for performance of individual work operations; inform headquarters sections and services on progress in performance of the aggregate of work operations; prepare reports for higher headquarters;

collect and analyze statistical data for creating a standard data base; develop methods and instruction materials.

In our opinion establishment of a PERT team by no means diminishes the role and responsibility of the respective chiefs for plan execution. On the contrary, the PERT team should work in close contact with all services, regularly consult with them, and receive from them the requisite input data and recommendations. The PERT team in turn can facilitate the job of the various services and sections, since it can furnish at any time requisite information on progress in program execution.

With the adoption of critical-path method, the commander (chief) will be able to make decisions on the basis of analysis of objective information received from the network schedule, and consequently will be able to make scientifically substantiated decisions.

CONCLUSION

Summing up, we should once again emphasize that the program evaluation and review technique system originated and is rapidly developing by virtue of objective practical demands connected with the problems of improving methods of planning, analysis of complex processes and controlling them in production, science and military affairs.

Growth in equipment of the armed forces with new weapons and various complex combat and special eqipment is accompanied by change not only in the conditions but also the character of combat operations. At the same time the interrelationships between line organizations of the various military services, arms and special troops are growing and becoming more complex to an equal degree, and the conditions of troop control are becoming increasingly more complex.

In today's battle headquarters at the various echelons will be receiving an enormous flow of the most diversified information. From this flow it is essential to select and analyze in rapid fashion the most important and necessary information, to evaluate the most important interlinkages, to perform numerous calculations and to prepare possible solutions. Therefore it is for all practical purposes impossible to accomplish this work in a prompt and timely manner without a specially developed system of planning, analysis and control, tested and assimilated in advance, based on extensive utilization of electronic computers. Of considerable interest in connection with this is a PERT system, which is an effective planning, analysis and control instrument. This is the most versatile of all control systems and can be employed in the most diversified areas of practical activity.

Adoption of PERT into practical management and control can be viewed as a qualitatively new leap forward, as a new stage in the development of control, which makes it possible sharply to improve control and management.

A PERT system first appeared comparatively recently and is presently in the stage of development and improvement. Newness and insufficient experience in application of the methods of this system are at the present time somewhat limiting their application for solving practical problems in military affairs.

Program evaluation and review technique methods are continuing to be improved. They are not yet completely formulated at the present stage of their development and practical adoption: subjective evaluations have not been eliminated, and mathematically ideal calculations are not yet ensured. But even at the present stage PERT makes it possible substantially to improve planning of various complex processes in military affairs and troop control as a whole.

In view of successes in system improvement and adoption, as well as future prospects for equipping with electronic computers, we can state that in the near future the majority of planning and control tasks, research and design tasks in military affairs will be handled with the aid of critical-path methods and that these methods will form the foundation of automating a number of processes in troop control.

One can also expect that in the course of further assimilation and improvement of the PERT method and the further evolution of computer hardware, the process of construction, analysis and optimization of a project (program) network, which today still requires a large expenditure of time and resources, will be automated.

In order effectively to utilize the superiority and capabilities of program evaluation and review technique, it is advisable to train a broad group of staff officers and commanders in matters of methodology and principles of PERT. It is very important that the level of qualifications of officers directly involved in PERT be extremely high. These officers should possess a consummate knowledge of the various processes of combat training or troop combat activities which are analyzed with the aid of network schedules.

If program evaluation and review technique is performed in production or in maintenance subunits, PERT team officers should possess excellent knowledge of the production technology or maintenance process. These officers should also possess a thorough mastery of the methods of construction and analysis of network schedules.

Successful practical adoption of program evaluation and review technique methods and the effectiveness of their application will also depend in decisive fashion on knowledge of the general principles of the PERT system and the commitment of officers bearing responsibility and directing various processes in each instance.

Program evaluation and review technique methods offer extensive opportunities in matters of objective analysis of processes and in matters of improving troop control. This does not signify, however, that PERT methods totally reject other techniques and methods of calculation and analysis which presently exist and are employed in the military. On the contrary, they will be used most effectively in connection with and in combination with traditional methods which have been put to a practical test. One should remember that a PERT system does not free the commander from situation estimate and decision-making. The system merely helps, on the basis of objective situation analysis and forecasting, in making the most appropriate decisions.

APPENDIX

EXAMPLE OF SOLVING THE PROBLEM 'PLANNING A TANK BATTALION MARCH' BY PERT

Conditions of the problem. The task is to plan a 125 km tank battalion march. Three first-class roads can be utilized for accomplishing the march. In order to avoid running up mileage on the tanks, the latter will be transported on heavy-load trailers. The march will terminate when the tanks reach the assembly area. Nine hours are allocated for the march (Figure 65).

Procedure of solving the problem. This problem is solved in the following sequence:

an initial network schedule is constructed;

correctness of construction of the initial network schedule is checked;

events in the network schedule are numbered;

parameters of the initial network schedule are calculated and compared with directive parameters;

the initial network schedule is optimized;

a scale network schedule is constructed.

1. Construction of Initial Network Schedule

Knowing the process technology, we construct the initial network schedule (Figure 66) from left to right. We write the work operation designation above each arrow (work operation) and its duration in minutes under the arrow.

We do not number the events. If events are numbered in the initial schedule, this is done only after checking the schedule for correctness of construction. Event numbering is performed at this time, as indicated below.

2. Checking Correctness of Construction of Initial Network Schedule

The schedule is considered correctly constructed if it contains no closed loops or dead-end events. As is evident from Figure 66, the schedule contains neither dead-end events not closed loops. Consequently the schedule is constructed correctly.

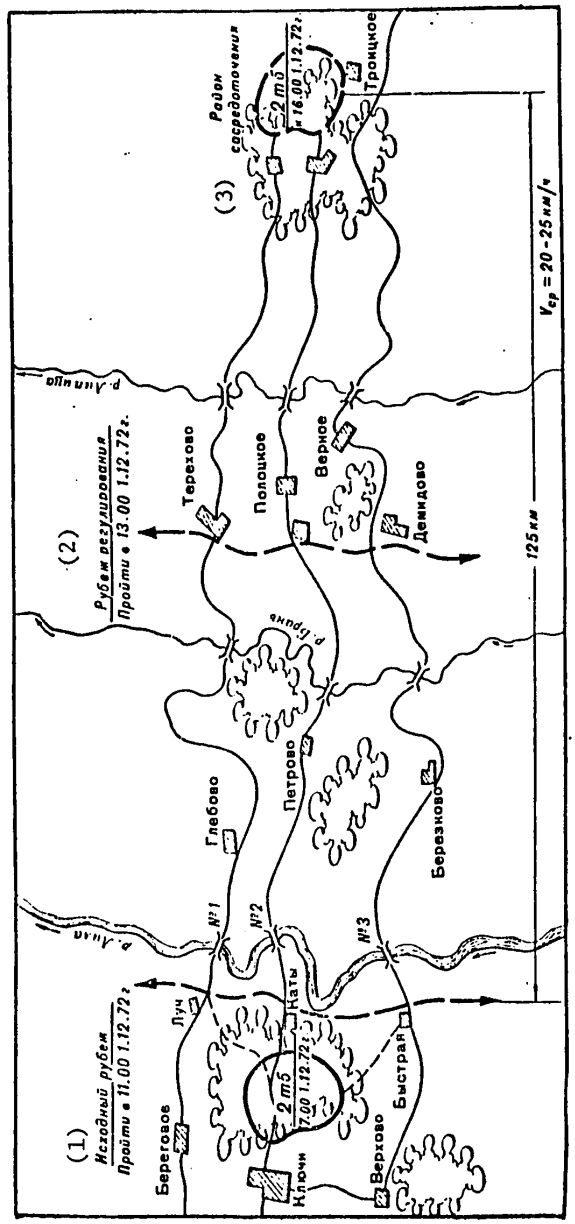


Figure 65. Start Point for 2d Tank Battalion and March Task

Key:

1. Pass start point at 1100 hours on 1 December 1972
2. Pass traffic control point at 1300 hours on 1 December 1972
3. Concentration area

2mb. 2d Tank Battalion

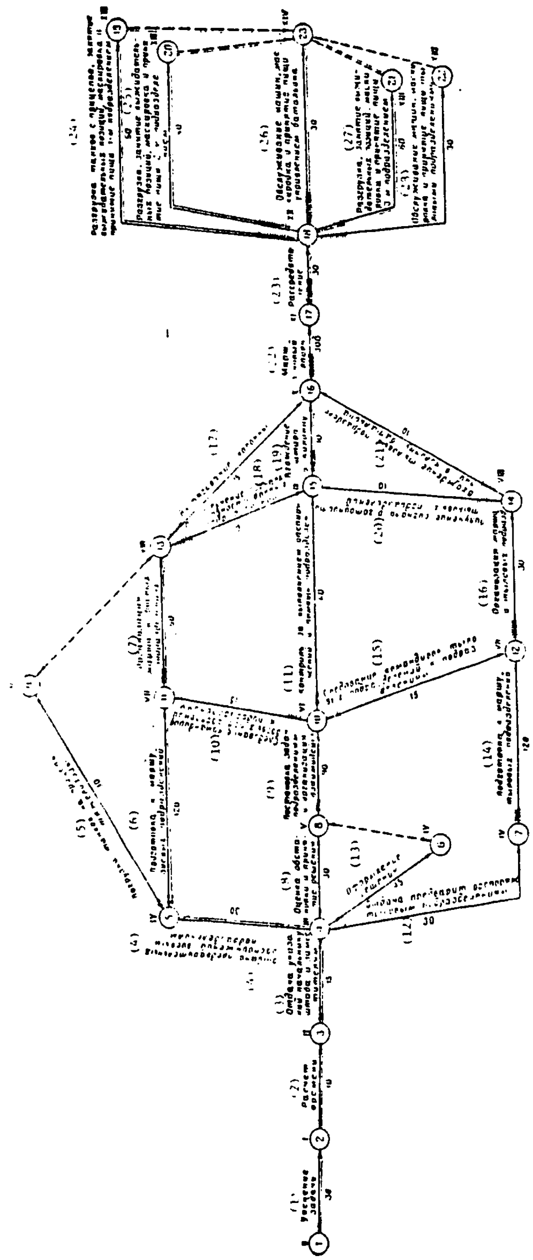


Figure 66. Initial Network Schedule of Independent Tank Battalion March

Key to Figure 66:

1. Mission briefing
2. Time calculation
3. Issuing instructions to executive officer and deputies
4. Issuing warning orders to combat subunits
5. Loading tanks on heavy-load trailers
6. Readying combat subunits for march
7. Organization for march in combat subunits
8. Situation estimate and decision-making
9. Allocation of tasks to subunits and organization of coordination
10. Commanders of combat subunits proceed to their subunits
11. Verification of execution of instructions and assistance to subunits
12. Issuing warning orders to rear services subunits
13. Formulating commander's plan
14. Readying rear services subunits for march
15. Commanders of rear services subunits proceed to their subunits
16. Organization for march in rear services subunits
17. Column moves out
18. Signal indicating readiness of combat subunits received
19. Headquarters enters column
20. Signal indicating readiness of rear services subunits received
21. Rear services subunits enter battalion column
22. March to new area
23. Dispersion
24. Tanks unloaded from trailers, take up assembly area positions, concealment and camouflage, meal taken by 1st subunit
25. Unloading, take up assembly area positions, concealment and camouflage, meal taken by 2d subunit
26. Battalion headquarters services vehicles, concealment and camouflage, takes meal
27. Unloading, take assembly area positions, concealment and camouflage, meal taken by 3d subunit
28. Rear services subunits service vehicles, concealment and camouflage, meal

3. Numbering Events in Network Schedule

Events in the initial schedule are numbered if they were not already numbered, or are renumbered if they were initially numbered arbitrarily. Numbering of events is performed by the method of cross-stroking arrows (work operations), beginning with the starting event (Figure 66). We proceed as follows: cross-stroke all operations proceeding from the starting event, and assign it zero rank; mark the rank above this event; assign the first number to the zero-rank events and enter it within the starting event circle; first rank is assigned to all events proceeding after the starting event, into which no arrow enters after cross-stroking arrows proceeding from the zero-rank event. In our example only one work operation proceeds from the zero-rank event. We strike it out with one perpendicular cross stroke and consider it cancelled. Then no operation enters the following event. We assign it first rank and place the Roman number I above this event. We assign to the first-rank event the next serial number 2 and mark it with an Arabic numeral inside the circle denoting the first-rank event.

We further proceed in like manner, moving from left to right and sequentially marking work operations with two, three, four, five, etc cross strokes. We place rank numbers above the events with Roman numerals.

If there are several events of the same rank in the network, these events are equal in status and can be numbered sequentially in any order. In Figure 66, for example, there are three events of rank IV, which are assigned the numbers (5), (6), and (7). The numbers of these events can be mutually exchanged. Nothing will be changed from this.

If events are renumbered according to their rank, old ones (arbitrarily assigned numbers) are erased and new numbers written in their place.

4. Calculation of Parameters of Initial Network Schedule and Comparing Them With Directive Parameters

Parameters are calculated by the tabular method (Table 22). Calculating the parameters of the initial network schedule, we see that the length of the critical path is 675 minutes, that is, 11 hours and 15 minutes. This means that with our adopted initial plan of action, the battalion will not meet the directive timetable, that is, will not accomplish its assigned tasks within the allocated 9 hours. Consequently, in order to fit within the directive timetable it is necessary to alter the original plan of action, that is, to improve the initial plan. Toward this end we shall optimize the initial schedule.

Table 22. Calculation of Initial Network Schedule of Independent Tank Battalion March

(1) Работа										
(2) начальное событие	(3) конечное событие	$t_{р.н}(i,j)$	$t(i,j)$	$t_{р.о}(i,j)$	$t_{п.н}(i,j)$	$t(i,j)$	$t_{п.о}(i,j)$	$R_п$	$R'_п$	${}^{\bullet}R_п$
1	2	3	4	5	6	7	8	9	10	11
1	2	0	30	30	0	30	30	0	0	0
2	3	30	10	40	30	10	40	0	0	0
3	4	40	15	55	40	15	55	0	0	0
4	5	55	30	85	55	30	85	0	0	0
4	6	55	45	100	55	45	100	0	0	0
4	7	55	30	85	80	30	110	25	25	0
4	8	55	30	85	70	30	100	25	15	0
5	9	85	30	115	235	30	265	150	150	0
5	11	85	120	205	85	120	205	0	0	0
6	8	85	0	85	100	0	100	15	0	0
7	12	85	120	205	110	120	230	25	0	0
8	10	85	90	175	100	90	190	15	0	0
9	13	115	0	115	265	0	265	150	0	150
10	11	175	15	190	190	15	205	15	0	0
10	12	175	15	190	215	15	230	40	25	15
10	15	175	60	235	215	60	275	40	25	40
11	13	205	60	265	205	60	265	0	0	0
12	14	205	30	235	230	30	265	30	0	0
13	15	265	10	275	265	10	275	0	0	0
13	16	265	15	280	270	15	285	5	5	5
14	15	235	10	245	265	10	275	30	0	30
14	16	235	10	245	275	10	285	40	10	40
15	16	275	10	285	275	10	285	0	0	0
16	17	285	300	585	285	300	585	0	0	0
17	18	585	30	615	585	30	615	0	0	0
18	19	615	60	675	615	60	675	0	0	0
18	20	615	60	675	615	60	675	0	0	0
18	21	615	60	675	615	60	675	0	0	0
18	22	615	30	645	615	30	675	30	30	0
18	23	615	30	645	645	30	675	30	30	0
19	23	675	0	675	675	0	675	0	0	0
20	23	675	0	675	675	0	675	0	0	0
21	23	675	0	675	675	0	675	0	0	0
22	23	645	0	645	675	0	675	30	0	30

Key:
1. Work operation
2. Initial event
3. Terminal event

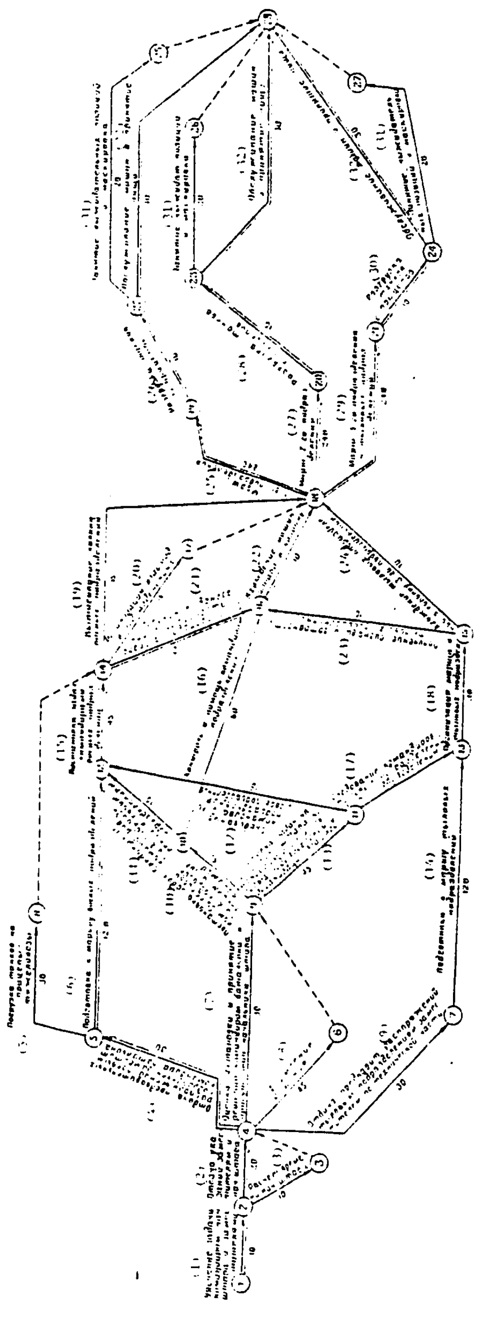


Figure 67. Optimized Schedule of Independent Tank Battalion March

5. Optimization of Initial Network Schedule

We shall optimize the network schedule by shoftening the critical path. In our case the network is improved by changing the process technology, by separation and parallel execution of certain work operations. Figure 67 contains an optimized schedule:

1. Briefing of commander, executive officer and deputies
2. Issuing of instructions to deputies and executive officer
3. Time calculation by executive officer
4. Issuing of warning orders to combat subunits by battalion second in command
5. Loading tanks on heavy-load trailers
6. Readying combat subunits for march
7. Situation estimate and decision-making by battalion commander in the presence of his executive officer
8. Formulation of commander's plan
9. Issuing of warning orders to rear services subunits by deputy commander for technical affairs
10. Allocation of tasks by the battalion commander to 1st and 2d subunits and organization of coordination
11. Commanders of 1st and 2d subunits proceed to their subunits
12. Commander of 3d subunit proceeds to his subunit
13. Allocation of tasks by executive officer to commanders of 3d subunit and rear services subunits and organization of coordination
14. Readying rear services subunits for march
15. Allocation of tasks by commanders of combat subunits
16. Verification and assistance to commanders of subunits
17. Commanders of rear services subunits proceed to their subunits
18. Organization for march in rear services subunits
19. Columns of combat subunits move out
20. Organization of coordination in subunits
21. Reports received on readiness of combat subunits
22. Headquarters vehicles enter column
23. Signal received on readiness of rear services subunits
24. Rear services subunits enter column of 3d subunit
25. March of 1st subunit
26. Tanks unloaded from trailers
27. March of 2d subunit
28. Tanks unloaded from trailers
29. March of 3d subunit and rear services subunits
30. Tanks unloaded from trailers
31. Occupation of assembly area positions, concealment and camouflage
32. Vehicle servicing and meal

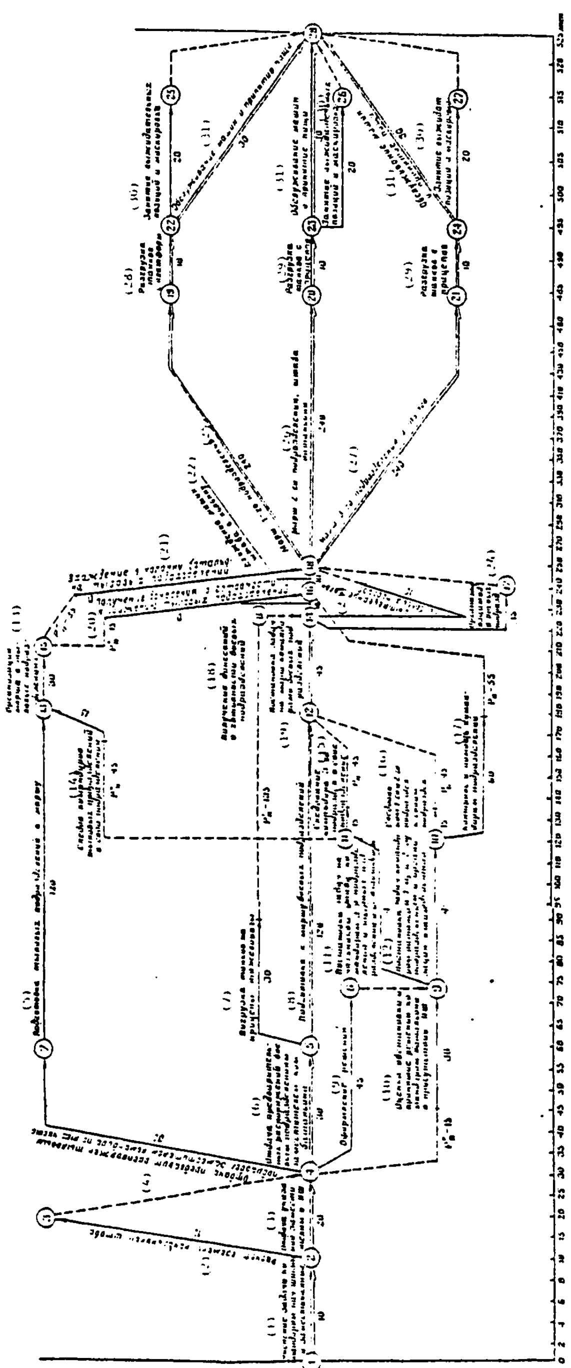


Figure 68. Scale Network of Independent Tank Battalion March

Key to Figure 68:

1. Mission briefing of commander, executive officer and deputies
2. Time calculation by executive officer
3. Issue of instructions to deputies and executive officer
4. Issue of warning orders to rear services subunits by deputy commander for technical affairs
5. Readying rear services subunits for march
6. Issuing of warning orders to combat subunits by battalion second in command
7. Loading tanks on heavy-load trailers
8. Readying subunits for march
9. Formulation of commander's plan
10. Situation estimate and battalion commander decision-making in the presence of executive officer
11. Allocation of tasks by executive officer to commanders of 3d subunit and rear services subunits and organization of coordination
12. Allocation of tasks by battalion commander to 1st and 2d subunits and organization of coordination
13. Organization for march in rear services subunits
14. Commanders of rear services subunits proceed to their subunits
15. Commander of 3d subunit proceeds to his subunit
16. Commanders of 1st and 2d subunits proceed to their subunit
17. Verification and assistance to subunit commanders
18. Receipt of reports on readiness of combat subunits
19. Allocation of march tasks by commanders of combat subunits
20. Receipt of report on readiness of rear services subunits to move out
21. Rear services subunits enter battalion column
22. Headquarters vehicles enter column
23. Columns of combat subunits move out
24. Organization of coordination in combat subunits
25. March of 1st subunit
26. March of 2d subunit, battalion headquarters
27. March of 3d subunit and trains
28. Unloading tanks from flatcars
29. Unloading tanks from trailers
30. Taking up assembly positions, concealment and camouflage
31. Vehicle servicing and meal

Table 23. Calculation of Optimized Schedule of Independent Tank Battalion March

(1) Работа										
(2) начальное событие	(3) конечное событие	$t_{р.н(i,j)}$	$t_{(i,j)}$	$t_{р.о(i,j)}$	$t_{п.н(i,j)}$	$t_{(i,j)}$	$t_{п.о(i,j)}$	$R_п$	$R'_п$	$R''_п$
1	2	3	4	5	6	7	8	9	10	11
1	2	0	10	10	0	10	10	0	0	0
2	3	10	10	20	20	10	30	10	10	0
2	4	10	20	30	10	20	30	0	0	0
3	4	20	0	20	30	0	30	10	0	10
4	5	30	30	60	30	30	60	0	0	0
4	6	30	45	75	75	45	120	45	45	0
4	7	30	30	60	90	30	120	60	60	0
4	9	30	30	60	90	30	120	60	60	15
5	8	60	30	90	195	30	225	135	135	0
5	12	60	120	180	60	120	180	0	0	0
6	9	75	0	75	120	0	120	45	0	0
7	13	60	120	180	85	120	205	25	0	0
8	14	90	0	90	225	0	225	135	0	135
9	10	75	45	120	120	45	165	45	0	0
9	11	75	45	120	120	45	165	45	0	0
10	12	120	15	135	165	45	180	45	0	0
10	16	120	60	150	175	60	235	55	10	55
11	12	120	15	135	165	15	180	45	25	0
11	13	120	15	135	190	15	205	70	0	45
12	14	180	45	225	180	45	225	0	0	0
13	15	180	30	210	205	30	235	25	0	0
11	16	225	10	235	225	10	235	0	0	0
11	17	225	15	240	230	15	245	5	5	0
11	18	225	15	240	230	15	245	5	5	5
15	16	210	10	220	225	10	235	15	10	15
15	18	210	10	220	235	10	245	25	0	25
16	19	235	10	245	235	10	245	0	0	0
17	18	240	0	240	245	0	245	5	0	5
18	19	245	240	485	245	240	485	0	0	0
18	20	245	240	485	245	240	485	0	0	0
18	21	245	240	485	245	240	485	0	0	0
19	22	485	10	495	485	10	495	0	0	0
20	23	485	10	495	485	10	495	0	0	0
21	24	485	10	495	485	10	495	0	0	0
22	25	495	20	515	505	20	525	10	10	0
22	28	495	30	525	495	30	525	0	0	0
23	26	495	20	515	505	20	525	10	10	0
23	28	495	30	525	495	30	525	0	0	0
24	27	495	20	515	505	20	525	10	10	0
24	28	495	30	525	495	30	525	0	0	0
25	28	515	0	515	525	0	525	10	0	10
26	28	515	0	515	525	0	525	10	0	10
27	28	515	0	515	525	0	525	10	0	10

Key: 1. Work operation
2. Initial event
3. Terminal event

From a comparison of the initial (Figure 66) and optimized (Figure 67) network schedules we can easily determine the changes which have occurred in the initial (original) plan of action. For example, operation (1, 2) in the initial schedule is divided into two and in the optimized schedule is represented by two operations -- (1, 2) and (2, 4). Operation (2, 3) is carried forward and in the optimized schedule begins 10 minutes after receipt of the march task, which led to a shortening of the critical path by 25 minutes. Operation (3, 4) in the initial schedule is shortened, since in the optimized schedule the battalion commander's deputies and executive officer are briefed together with the battalion commander, and therefore it is sufficient to tell them who is to issue orders to whom. The work operation with the code (11, 13) has been separated and in the optimized schedule is represented by two operations -- (12, 14) and (14, 17). The latter operation is performed in parallel with operation (14, 16), as a result of which the critical path is shortened by 15 minutes. At the same time operation (8, 10) in the initial schedule is replaced by two operations -- (9, 10) and (9, 11), which in the optimized schedule are performed in parallel. The total duration of these two operations is equal to the duration of operation (8, 10) in the initial network schedule.

Those operations with the code (16, 17) and (17, 18) in the initial schedule are separated and represented in the optimized schedule by three operations -- (18, 19), (18, 20), and (18, 21). The new schedule provides for each subunit to proceed by its own route, which makes it possible to increase speed and consequently to shorten the critical path by 90 minutes.

Finally, those operations which in the initial schedule bear the codes (18, 19), (18, 20), and (18, 21) are separated and performed in parallel; in the optimized schedule they are represented by operations [(19, 22), (22, 25), (22, 28)], [(20, 23), (23, 26), (23, 28)], [(21, 24), (23, 24), (24, 28)]. This shortens the critical path by an additional 20 minutes.

After optimizing the schedule, we shall calculate its parameters. Table 23 contains calculation of network schedule parameters. It is evident from Table 23 that the critical path of the optimized schedule is equal to 525 minutes (8.75 hours). The directive timetable has been achieved, and therefore there is no further schedule optimization.

Thus by changing the process technology and parallel performance of work operations, we have succeeded in shortening march preparation and execution time by 150 minutes, fitting it into the directive timetable.

6. Construction of a Scale Network Schedule

Since the optimization process has been completed, on the basis of Table 23 and the optimized network schedule (Figure 67) we construct a scale network schedule, which is shown in Figure 68. In this schedule the path from event (18) to the concluding event separates into three branches. The dashed lines with arrowheads denote logical links or empty operations. The dashed lines which begin solid-line arrows or are their continuation designate partial or overall work operation reserves respectively. Types of time reserves of work operations and their magnitude are indicated below the dashed lines and are designated as follows:

$$P_n'-5;\ P_n''-45 \text{ and } P_n-45,$$

where P_n' is partial reserve of the first type; P_n'' -- partial reserve of the second type; P_n -- total work operation reserve.

The numbers after the letters indicate the amount of reserve.

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