Gelfand, I. M.

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function. By taking the Fourier transform, this system of equations is transformed into the system of ordinary differential equations

$$(2) \qquad \frac{dv(s,t)}{dt} = P(s,t)v(s,t), \quad (v(s,t) = \widetilde{u(x,t)}),$$

where the matrix $P(s,t)$ has elements which are polynomials in S multiplied by continuous functions of t, and the initial condition is $v(s,0) = v_0(s) = \tilde{u}_0$.

The basic theorems are the following. Let $Q(s,t_0,t)$ be the matrix of the normal fundamental solution of the system (2): $Q(s,t_0,t_0) = E$. I. If the elements of $Q(s,0,t)$ are multipliers in Φ for all $t \geqq 0$, the system (2) has a solution with arbitrary initial generalized vector-function $v_0(s) \in T^{(m)}(\Phi)$. II. If the elements of $Q(s,t,t_0)$ are multipliers in Φ for all t, $0 \leqq t \leqq t_0$, then (2) has a unique solution in the class $T^{(m)}(\Phi)$. For every system (2), let p_0 be the greatest order of the entire functions of s entering in $Q(s,t_0,t)$. III. Then the elements of $Q(s,t_0,t)$ are multipliers in $Z_r{}'$ for all $r > p_0$. IV. If the vector-function $v_0(x)$ satisfies the inequality

$$|v_0(x)| \leqq C_1 \exp\{C|x|^{p_0'-\epsilon}\}, \quad \epsilon > 0,$$

then the system (1) has a solution in generalized vector functions belonging to $T(z_r{}')$, where $r = p_0' - \delta$, $\delta > 0$. This solution is unique. A number of other theorems are given.

E. Hewitt (Seattle, Wash.).

GEL'FAND, I.M.

(4) Math

Mathematical Reviews
Vol. 15 No. 3
March 1954
Algebra

6-23-54
LL

Gel'fand, I. M., and Graev, M. I. **Unitary representations of the real unimodular group (principal nondegenerate series).** Izvestiya Akad. Nauk SSSR. Ser. Mat. 17, 189–248 (1953). (Russian)

The authors present a series of continuous unitary representations on Hilbert space of the real unimodular group (=group of space matrices of determinant unity) of arbitrary order, and prove their irreducibility. In the case of the 2×2 group, all continuous unitary irreducible representations were obtained by Bargmann [Ann. of Math. (2) 48, 568–640 (1947); these Rev. 9, 133] by infinitesimal methods. The present series of representations, called the principal non-degenerate series, is obtained by global methods parallel to those used by Gelfand and Neumark in determining all the continuous unitary irreducible representations of the complex unimodular group [Trudy Mat. Inst. Steklov. 36 (1950); these Rev. 13, 722]. In particular, all the present representations are of multiplier type, and they do not exhaust the representations (of the stated type) of the real unimodular groups.

I. E. Segal.

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GEL'FAND, I. M.

PA 249T41

USSR/Mathematics - Differential Operator 1 Feb 53

"Certain Simple Identity for Eigenvalues of a Differential Operator of Second Order." I.M. Gel'fand and B. M. Levitan

DAN SSSR, Vol 88, No 4, pp 593-596

The authors' purpose is to calculate the sum $(L_1-M_1) + (L_2-M_2) + \ldots$ where the L's are the eigenvalues of $y'' = [q(x)-L]\, y$ $(y'(o) = hy(o);\ y'(1) = -Hy(1)$, and the M's are the eigenvalues of $y'' = -My$ (same boundary conditions). This sum is found by simple operations to equal $\frac{1}{4}[q(o) + q(1)] + hH$. Presented by Acad A. N. Kolmogorov 28 Nov 52

249T41

GEL'FAND, I. M.

USSR/Mathematics - Group Theory 11 Sep 53

"A General Method for Expanding the Regular Representation of a Lie Group Into Irreducible Representations," I. M. Gel'fand and M. I. Grayev

DAN SSSR, Vol 92, No 2, pp 221-224

Note that in the theory of representations of groups the problem of expanding a regular representation of a group into irreducible ones is analogous to expanding a function in a Fourier integral (the analogy of Plancherel's formula). Find the integrals of an arbitrary fixed function x(g) in a Lie group G which

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are given over an arbitrary class of "general position" of adjoint elements e of G. Also express by means of this integral the value of x(g) in unit e of G. Presented by Acad M. V. Kel'dysh 3 Jul 53.

GEL'FAND, I. M.

USSR/Mathematics - Group Theory, Semisimple — 21 Sep 53

"An Analog of Plancherel's Formula for Real Semi-simple Lie Groups," I.M. Gel'fand and M.I. Grayev

DAN SSSR, Vol 92, No 3, pp 461-464

In the preceding issue (DAN 92, No 2 (1953)) the authors proposed a method which enables one to obtain very simply an analog of Plancherel's formula for complex semisimple Lie groups. Here they derive an analog of Plancherel's formula for real Lie group, for definiteness, only groups of real unimodular matrices. Note that the analog of Plancherel's

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formula for the case n=2 was in essence obtained first by V. Bargmann (Ann of Math. 48 (1947)) and later by Harish-Chandra (Proc Nat Acad Sci. 48 (1952)). Presented by Acad M.V. Keldysh 3 Jul 53.

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GEL'FAND, I. M.

The Committee on Stalin Prizes (of the Council of Ministers USSR) in the fields of science and inventions announces that the following scientific works, popular scientific books, and textbooks have been submitted for competition for Stalin Prizes for the years 1952 and 1953. (Sovetskaya Kultura, Moscow, No. 22-40, 20 Feb - 3 Apr 1954)

Name	Title of Work	Nominated by
Gel'fand, I. M.	"Lectures on Linear Algebra" (2d edition, textbook)	Moscow Mathematical Society

SO: W-30604, 7 July 1954

GEL'FAND, I. M.

USSR/Mathematics - Quantum - Physics

Card 1/1

Authors : Gel'fand, I. M., Memb. Corres. of Acad. of Sc. USSR. and Minlos, R. A.

Title : Solution of equations of quantized fields

Periodical : Dokl. AN SSSR, 97, Ed. 2, 209 - 212, July 1954

Abstract : A solution of equations of quantized fields. A somewhat new mathematical method, generally outlined in the article, has been applied to the solution of the quantized field equations. Two references.

Institution : ...

Submitted : April 28, 1954

APPROVED FOR RELEASE: 08/23/2000 CIA-RDP86-00513R000514620008-5"

CARD 1/2 PG - 141

SUBJECT USSR/MATHEMATICS/Topology
AUTHOR GEL'FAND I.M. and GRAEV M.I.
TITLE An analogue of the Plancherel formula for the classical groups.
PERIODICAL Trudy Moskovsk.mat.Obšč. 4, 375-404 (1955)
reviewed 7/1956

The generalization of the Plancherel formula to simi-simple Lie groups has mainly the following difficulty: The values of the integrals over classes of conjugate elements of the "general situation" of a sufficiently often differentiable function $x(g)$ $(g \in G)$ which vanishes outside of a small neighborhood of the unity is known; then the value of the function in the unit element shall be obtained. In their investigations on the representation of classical groups, I.M.Gel'fand and M.A.Nejmark have found a complicated solution of this problem for the case of the complex unimodular group (compare I.M.Gel'fand, M.A.Nejmark: Unitary representations of the classical groups, Moscow-Leningrad 1950). This process is now replaced by a more general and clear one and for the classical group it is carried out in detail. The result has the following form: $x(e) = LI_{\delta}|_{\delta=e}$, where I_{δ} is the integral of $x(g)$ over the class of all elements being conjugate to the diagonal matrix δ with different eigenvalues, and L is a certain linear homogeneous differential operator. For the proof results of M.Riesz (Acta. Math. 81, 1-221 (1949)) are used in a somewhat generalized form. Let $f(x)$ be a sufficiently often differentiable, outside of a compact set vanishing function in an Euclidean

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space of odd number of dimension and $\omega(x)$ a non-degenerated and non-definite quadratic form with an odd number of positive squares. Then the function $R(\lambda)$ being defined by the integral

$$\int_{\omega(x)>0} f(x)\,|\omega(x)|^{\frac{\lambda}{2}}\,dx,$$

Re $\lambda > 0$ and by the derivative of which, respectively, has simple poles at the places $\lambda = -m-2k$ and we have $\operatorname{Res}_{\lambda=-m-2k} R(\lambda) = c_k(\Delta^k f)(0)$ $(k=1,2,\ldots)$.

Here the c_k are certain constants and $\Delta = \sum_{p,q} c_{pq} \frac{\partial^2}{\partial x_p \partial x_q}$, where $\| c_{pq} \|$ is the inverse of the matrix of the quadratic form $\omega(x)$. An other derivative of the Plancherel formula for complex semi-simple groups was given by Harish-Chandra (Proc.Nat.Acad.Sci. USA 37, 813-818 (1951); Trans.Amer.Math.Soc. 76, 485-528 (1954)).

GEL'FAND, I.M.

Gel'fand, I. M., and Lidskii, V. B. On the structure of the regions of stability of linear canonical systems of differential equations with periodic coefficients. Uspehi Mat. Nauk (N.S.) 10 (1955), no. 1(63), 3–40. (Russian)

The system of 2k equations, is equivalent to the matrix differential equation $dY/dt = IH(t)Y$, where $H(t)$ is real, symmetric, piecewise continuous and of period ω in t ($-\infty < t < \infty$), and I has the form $\begin{pmatrix} 0 & E_k \\ -E_k & 0 \end{pmatrix}$, E_k being the kth order unit matrix. The system is to be "strongly stable" in the sense that all solutions of $dY/dt = I\tilde{H}(t)Y$ are bounded as $t \to \infty$ for any $\tilde{H}(t)$ of the same form subject to $\|\tilde{H}(t) - H(t)\| < \epsilon$ for some $\epsilon > 0$. The problem is to enumerate the connected regions formed by the set of such $H(t)$, within one of which any such $H(t)$ can be continuously deformed into any other. Fixing $Y(t)$ by $Y(0) = E_{2k}$, $dY/dt = IH(t)Y$, it is observed that $Y(t)$ ($0 \le t \le \omega$) traces a curve from E_{2k} to $Y(\omega)$ within the group of real symplectic matrices, and that any such curve, if suitably smooth, corresponds to an $H(t)$; furthermore, the strong stability depends only on $Y(\omega)$. This

Gel'fand, and Lidskii, V. B.

reduces the problem to (i) the enumeration of the connected regions formed by those $Y(\omega)$ which satisfy the strong stability conditions, and (ii) the determination of the connectivity of the group of real symplectic matrices. The solution of (i) is based on results announced by M. G. Krein [Dokl. Akad. Nauk SSSR (N.S.) 73 (1950), 445–448; MR 12, 100]; proofs being now supplied; there are 2^k regions for $Y(\omega)$, according to whether the k characteristic numbers of $Y(\omega)$ which lie on the upper half of the unit circle are of the first or second kinds, i.e. move into or outside the unit circle when $dY/dt = IH(t)Y$ is perturbed to $dY/dt = I(H(t) + \lambda Q(t))Y$, where λ is small, [illegible] $\lambda > 0$, and $Q(t)$ is periodic, real, symmetric and positive-definite. Concerning (ii), it is proved that the group of real symplectic matrices is homeomorphic to the topological product of the circumference of a circle and a simply-connected topological space; this is deduced from the fact that the set of unitary unimodular matrices form such a space [see, e.g., H. Weyl, The classical groups, Princeton, 1939, p. 268; MR 1, 42]. The final result is that there are 2^k infinite families of regions [illegible] each member of a family being [illegible] number of [illegible] section on the case $k = 1$, already treated [illegible] hovič [see the second following review]. F. V. Atkinson

Gel'fand, I. M. and Lidskii, V. B.

Lidskii, V. B. Oscillation theorems for canonical systems of differential equations. Dokl. Akad. Nauk SSSR (N.S.) 102 (1955), 877–880. (Russian)

Let the 2kth-order matrices

$$I=\begin{pmatrix}0 & E_k\\ -E_k & 0\end{pmatrix},\quad Y(t)=\begin{pmatrix}y_1(t) & y_2(t)\\ y_3(t) & y_4(t)\end{pmatrix},\quad H(t)=\begin{pmatrix}h_1(t) & h_2(t)\\ h_3(t) & h_4(t)\end{pmatrix},$$

where $H(t)$ is real and symmetric, E_k is the unit kth order matrix, and $y_r(t)$, $h_r(t)$ are kth order matrices satisfying the differential equation $dY/dt=IH(t)Y$, with $Y(0)=E_{2k}$. The "oscillation theorems" deal with the characteristic values $\rho_1(t), \cdots, \rho_k(t)$ of the (unitary and symmetric) matrix $(y_1(t)-iy_2(t))^{-1}(y_1(t)+iy_2(t))$. If $h_1(t)$ is positive definite, then as t increases the $\rho_r(t)$ move round the unit circle in the positive sense. Among other results is a comparison theorem for $dY_1/dt=IH_1(t)Y_1$, $dY_2/dt=IH_2(t)Y_2$ when $H_1(t)>H_2(t)$. With an appropriate numeration we have $\arg\rho_r^{(1)}(t)>\arg\rho_r^{(2)}(t)$ for all $t>0$. Separation theorems for the zeros of the determinants of $y_1(t)$, $y_2(t)$ are also mentioned. Slightly less general results had been given by M. G. Krein [see the preceding review], and M. Morse [Math. Ann. 103 (1930), 52–69].

F. V. Atkinson (Canberra).

Gel'fand, I.M. and Lidskii V.B.

Yakubovič, V. A. Questions of the stability of solutions of a system of two linear differential equations of canonical form with periodic coefficients. Mat. Sb. N.S. 37(79) (1955), 21-68. (Russian)

The author gives proofs and a detailed account of results already announced by him [Dokl. Akad. Nauk SSSR (N.S.) 78 (1951), 221-224; MR 13, 237]. The first half of the paper deals essentially with the case $k=1$ of the problem considered by Gel'fand and Lidskii (second preceding review, which see for notation). The present treatment uses different methods, and is considerably more detailed; also Lebesgue integrability is required of $H(t)$ in place of piecewise continuity. A long footnote (pp. 42-43) suggests a method of extending the argument beyond the case $k=1$. The author next finds conditions on $H(t)$ in order that the system should belong to a particular region of stability or instability, criteria are deduced regarding the stability of $y''+p(t)y=0$, where $p(t)$ is periodic. A section deals with a "comparison theorem", similar to that of Lidskii (see preceding review), and its consequences. For example, of matrix functions

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Gel'fand, I. M., and L[illegible]

$H_1(t)$, $H_2(t)$ give systems belonging to the [illegible] region, then so does any $H(t)$ with $H_1(t)$ [illegible] geometrical interpretations are given, and further stability criteria for $y'' + p(t)y = 0$. After a section apparently intended for numerical application, the author concludes with a section on the "oscillation theorem". He gives conditions, too involved to reproduce, for the stability and instability zones on the real λ-axis arising from the parametric case $H(t) = H_\lambda(t)$ to be separated by the eigen-values of the periodic and semi-periodic boundary conditions, occurring in a certain order. A particular case is $H_\lambda(t) = \lambda H_1(t) + H_2(t)$, where $H_1(t) \geq 0$ and $H_1(t) > 0$ on a set of positive measure.

F. V. A[illegible]

GEL'FAND, I.M.

Gel'fand, I. M., and Šapiro, Z. Ya. Homogeneous functions and their extensions. Uspehi Mat. Nauk (N.S.) 10 (1955), no. 3(65), 3-70. (Russian) 1-F/W

Cet article utilise la théorie des distributions de L. Schwartz [t. I et II, Hermann, Paris, 1950, 1951; MR 12, 31, 833]. Les résultats essentiels sont relatifs à des parties finies d'intégrales divergentes [cf. Hadamard, Le problème de Cauchy, Hermann, Paris, 1930; Bureau notamment Comm. Pure Appl. Math. 8 (1955), 143-202; MR 16, 826]; les résultats sont obtenus par la méthode de M. Riesz de prolongement analytique [Acta. Math. 81 (1949), 1-223; MR 10, 713].

(1) 1/6 3 9 8

(OWek)

§ 1. Soit $(x^\lambda)_{x>0}$ la fonction $=x^\lambda$ si $x>0$, $=0$ si $x<0$, Re $\lambda>-1$. L'application $\lambda\to(x^\lambda)_{x>0}$ du demi plan Re $\lambda>-1$ dans l'espace $\mathcal{D}'$ des distributions sur R, est holomorphe et se prolonge analytiquement au plan entier en une fonction méromorphe, avec des poles simples aux points $-1, -2, \cdots$. Le résidu au point $-k$ est

$$(-1)^{k-1}/(k-1)!\,\delta^{(k-1)}$$

($\delta^{(p)}$ = dérivée d'ordre p de la masse de Dirac δ). Résultats analogues pour $(|x|^\lambda)_{x<0}$, $|x|^\lambda$, etc.

§ 2. Soit dans R^n une fonction continue F, homogène de degré 1, >0 en dehors de l'origine (exemple: $|x|=r$); pour Re $\lambda>-n$, $x\to F^\lambda(x)$ définit une distribution F_λ; $\lambda\to F_\lambda$ est holomorphe dans Re $\lambda>-n$, à valeurs dans $\mathcal{D}'$; elle se prolonge analytiquement en une fonction méromorphe avec poles simples aux points $-n, -n-1, \cdots$. Etude des distributions homogènes de degré λ (i.e. $\langle T_x, \varphi(x/\alpha)\rangle = \alpha^{\lambda+n}\langle T,\varphi\rangle$ pour toute $\varphi\in\mathcal{D}$ = espace des fonctions indéfiniment différentiables à support compact). Exemple: soit Ω_n l'aire de la sphère unité dans R^n, le pole au point $-n$ de $2r^\lambda/\Omega_n\Gamma((\lambda+n)/2)$ est δ.

§ 3. **Applications. a)** Décomposition de δ. On désigne par Ω la sphère unité, $\omega=(\omega_1, \cdots, \omega_n)\in\Omega$, $d\omega$=élément d'aire superficielle. Si $f\in\mathcal{D}(R)$, soit f^* définie dans R^n par 1-F/W

$$f^*(x)=f(\omega_1x_1+\cdots+\omega_nx_n);$$

l'application $f\to f^*$ se prolonge par continuité en une application $T\to T^*$ de $\mathcal{D}'(R)$ dans $\mathcal{D}'(R^n)$. On désigne par $\delta^{(p)}(\omega_1x_1+\cdots+\omega_nx_n)$ l'image de $\delta^{(p)}$ dans cette application. Si n est impair on a:

$$\delta=\frac{(-1)^{(n-1)/2}}{2(2\pi)^{n-1}}\int_\Omega \delta^{(n-1)}(\omega_1x_1+\cdots+\omega_nx_n)\,d\omega,$$

formule due à A. A. Hačaturov [Uspehi Mat. Nauk (N.S.) 9 (1954), no. 3(61), 205-212; MR 16, 229]. Formule analogue si n est impair. Ces formules découlent facilement des §§ précédents et de la formule

$$(*)\quad \frac{1}{\pi^{(n-1)/2}\Gamma((\lambda+1)/2)}\int_\Omega |\omega_1x_1+\cdots+\omega_nx_n|^\lambda d\omega = 2r^\lambda\Gamma((\lambda+n)/2).$$

b) Solutions élémentaires d'opérateurs différentiels elliptiques L à coefficients constants. On utilise (*); en cherche v_ω ne dépendant que de $\omega_1 x_1 + \cdots + \omega_n x_n$, solution de

$$Lv_\omega = |\omega_1 x_1 + \cdots + \omega_n x_n|^\lambda / \pi^{(n-1)/2} \Gamma((\lambda+1)/2)$$

(donc problème à une variable). On fait ensuite tendre λ vers $-n$.

c) Problèmes de Cauchy. Par des méthodes analogues on se ramène à des problèmes à deux variables. Dans le cas où l'opérateur différentiel est homogène, les A. retrouvent les formules de Herglotz-Petrowsky [pour une étude plus générale, cf. Leray, Hyperbolic differential equations, Inst. Advanced Study, Princeton, 1953; MR 16, 139].

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§ 4. Une fonction g définie sur R^n ou sur une sphère est dite équivalente à une fonction homogène au voisinage d'un point s'il existe un système de coordonnées locales dans lequel g est homogène. Soit G une fonction indéfiniment différentiable sur R^n. On définit la notion de point réductible M de la variété $G=0$ par récurrence sur la dimension; le point M est dit réductible si 1) G est équivalente à une fonction homogène au voisinage de M, 2) l'intersection de $G=0$ avec une sphère assez petite de centre M est composée de points réductibles sur la sphère (si $n=1$, on garde seulement 1)). Si l'on peut choisir le système de coordonnées locales de façon que G soit homogène de degré m et ne dépende que de k variables, M est dit d'ordre k et de degré m. On considère la distribution G^λ définie par $\int_{G>0} G^\lambda(x)\varphi(x)dx$, Re λ assez grand.

Si la variété $G=0$ est bornée, formée de points réductibles d'ordre 1, l'application $\lambda \to G^\lambda$ se prolonge en une fonction méromorphe. A chaque composante connexe de $G=0$ composée de points de degré m correspond la suite de poles simples $-1/m, -2/m, \cdots$. Pour le calcul des résidus les A. utilisent un résultat de Leray [C. R. Acad. Sci. Paris 234 (1952), 1112-1115; MR 14, 477]. Dans le cas général, les composantes de $G=0$ formées de points d'ordre r et de degré m donnent des poles (multiples) aux points $-r/m, -(r+1)/m, \cdots$.

§ 5. Transformation de Fourier de fonctions et distributions homogènes. On se ramène à une intégrale sur Ω par passage en coordonnées polaires. Tableau de formules.

J. L. Lions (Nancy).

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GEL'FAND, I.M.

Identities for eigenvalues of a second-order differential operator.
Usp.mat.nauk 11 no.1:191-198 Ja-F '55. (MLRA 9:6)
(Differential equations, Partial)(Eigenvalues)(Operators(Mathematics))

GEL'FAND, I. M.

L

USSR/ Mathematics - Theory of probabilities

Card 1/1 Pub. 22 - 3/49

Authors : Gel'fand, I. M., Mem. Corresp. Acad. of Scs. USSR

Title : Generalized casual processes

Periodical : Dok. AN SSSR 100/5, 853-856, Feb 11, 1955

Abstract : A description of casual processes which may cover practically all important cases is presented. If the x(t) is a casual process, then any linear device will give a distribution of probabilities not for the quantity $x(t_0)$, but for the quantity $F(\varphi) = \int \varphi(t)x(t)dt$ where $\varphi(t)$ is a certain function characterizing the device. (Provided the device does not switch in and out instantly). A definition of a generalized casual process and the conditions which it may satisfy are given. Eleven references: 1 French, 8 USSR and 2 Italian (1930-1954).

Institution : .....

Submitted : .....

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GEL'FAND, I. M.

USSR/Mathematics

Card 1/1 Pub. 22 - 1/47

Authors : Gel'fand, I. M., Memb. Corresp. of Acad. of Sc. USSR.; and Graev, M. I.

Title : Traces of unitary representations of a real unimodular group

Periodical : Dok. AN SSSR, 100/6, 1037-1040, Feb 21, 1955

Abstract : The role of the trace (or character) of a non-reducible representation which is defined as a trace of a representation matrix in the theory of finite matrix groups is explained. Mathematical formulas are given for the trace of basic series of non-reducible unitary representations of a substantial unimodular matrix group. Direct transfer of the trace concept into the theory of infinite unitary representations is not obvious because the representation operators being unitary may not have any signs of the ordinary sense. Some examples supporting the trace concept are shown. Four references: 3 USSR and 1 USA (1950-1954).

Institution : The M. V. Lomonosov State University, Moscow

Submitted : December 1, 1954

Gel'fand, I. M.

Gel'fand, I. M., and Šilov, G. E. On a new method in uniqueness theorems for solution of Cauchy's problem for systems linear partial differential equations. Dokl. Akad. Nauk SSSR (N.S.) 102 (1955), 1065–1068. (Russian)

Préliminaires: soit $S(\alpha, \beta; A, B)$ l'espace des fonctions φ indéfiniment différentiables sur R (α, β, A, B positifs) telles que pour tout ε, $\delta > 0$, il existe $N_{\varepsilon,\delta}(\varphi) < \infty$ avec

$$|x^k \varphi^{(n)}(x)| \leq N_{\varepsilon,\delta}(\varphi)(A+\delta)^k k^{k\alpha}(B+\varepsilon)^n n^{n\beta},$$

pour tout x, k et n; topologie naturelle. Lemme: soit $f(s) = \sum_{n \geq 0} a_n s^n$, une fonction entière d'ordre $\leq 1/\beta$, de type $\leq \beta/B^{1/\beta}e^{\beta}$; alors l'opérateur différentiel infini $f(D)$, $D = d/dx$, est un opérateur linéaire continu de $S(\alpha, \beta; A, B)$ dans $S(\alpha, \beta; A, Be^{\beta})$. Généralisation des espaces et du lemme à N variables.

Application: on considère le problème de Cauchy pour le système d'évolution

$$(*) \qquad \frac{\partial}{\partial t} = P\left(\frac{1}{i}\frac{\partial}{\partial x}, t\right),$$

P étant une matrice carrée (m, m), dont les coefficients sont des opérateurs différentiels linéaires d'ordre q sur R^N, $x \in R^N$, à coefficients indépendants de x, continus en t. On associe à $(*)$ le système $d/dt = P(s, t)$, de matrice fon-

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0

(1)

damentale $Q(s, t_0, t)$, fonction entière de s, d'ordre $q_0 \leq q$ [cf. Gel'fand et Šilov, Uspehi Mat. Nauk (N.S.) 8 (1953), no. 6 (58), 3-54; MR 15, 867] Le lemme (dans R^N) et les raisonnements usuels [Schwartz, Ann. Inst. Fourier, Grenoble 2 (1951), 19-49; MR 13, 242] donnent l'existence et l'unicité du problème de Cauchy dans le dual de $S_\alpha^\beta = \bigcup_{A,B} S(\alpha, \beta; A, B)$ $(\alpha \geq 1 - 1/q_0)$.

J. L. Lions.

Gel'fand, I. M.

1 - F/W

Gel'fand, I. M., and Kostyučenko, A. G. Expansion in ~~eigenfunctions~~ of ~~differential and~~ other operators. Dokl. Akad. Nauk SSSR (N.S.) 103 (1955), 347-352. (Russian)

Notations: Φ=espace de Fréchet de fonctions indéfiniment différentiables sur R^n; Φ'=dual de Φ; on suppose $\Phi \subset \Phi'$. On donne sur Φ une forme sesquilinéaire (φ, ψ) définie positive continue; soit $\bar{\Phi}$ le complété de Φ pour cette structure; on a: $\Phi \subset \bar{\Phi} \subset \Phi'$. Soit A opérateur linéaire continu de Φ dans Φ avec $(A\varphi, \psi) = (\varphi, A\psi)$ pour tout φ, $\psi \in \Phi$. On suppose qu'il existe $\bar{A}$, hypermaximal dans $\bar{\Phi}$, d'ensemble de définition contenant Φ, avec $\bar{A}\varphi = A\varphi$ pour tout $\varphi \in \Phi$. Soit A' le transposé de A dans Φ'. Un élément T de Φ' est dit "fonction propre généralisée" si $A'T = \lambda T$, $\lambda \in C$. Théorème: Sous les hypothèses ci dessus, il existe un système complet de fonctions propres généralisées.

(1)

Les A. en déduisent un résultat de „diagonalisation" de
A [cf. des résultats plus précis dans les travaux de Maut-
ner et Gårding; cf. notamment Gårding, Applications of
the theory of direct integrals of Hilbert spaces to some
integral and differential operators, Univ. of Maryland,
1954 [MR 17, 159] où l'on trouvera d'autres indications
bibliographiques]. Exemples: A est un opérateur diffé-
rentiel sur R ou sur R^n, elliptique ou non; certains
espaces Φ sont construits à partir des propriétés de crois-
sance des coefficients de A. Généralisation à $A+\lambda B$,
$B>0$.

J. L. Lions (Nancy).

Gel'fand, I. M.

USSR/Theoretical Physics - General Problems B-1

Abst Journal : Referat Zhur - Fizika, No 12, 1956, 33707

Author : Gel'fand, I. M., Yaglom, A. M.

Institution : None

Title : Methods of the Theory of Random Processes in Quantum Physics

Original Periodical : Vestn. Leningr. Un-ta, 1956, No 1, 33-34

Abstract : Brief Discussion of the possible utilization of methods of the theory of integration in functional spaces in problems of quantum mechanics.

Card 1/1

Gel'fand, I. M.

1-F/W

Berezin, F. A.; and Gel'fand, I. M. Some remarks on the theory of spherical functions on symmetric Riemannian manifolds. Trudy Moskov. Mat. Obšč. 5 (1956), 311–351. (Russian)

Let G be a semi-simple Lie group, and let $x \to x^*$ be an involutary anti-automorphism of G such that the subgroup G_0 of all x with $x^* = x^{-1}$ is compact and such that every x in G may be written in the form uh, where $u \in G_0$ and $h^* = h$. Then every $G_0:G_0$ double coset is invariant under $*$, and it follows that those members of the group algebra $L^1(G)$ which are constant on the $G_0:G_0$ double cosets form a commutative subalgebra R of $L^1(G)$. Now let L be an arbitrary irreducible unitary representation of G whose restriction to G_0 contains the identity as a direct summand. It follows from the commutativity of R that this restriction contains the identity just once. Thus if ϕ is a unit vector in the space of L such that $L_\xi(\phi) = \phi$ for all $\xi \in G_0$, then the function $x \to (L_x(\phi), \phi)$ is uniquely determined by L. It is a continuous function, constant on the $G_0:G_0$ double cosets, which we shall denote by ϕ_L. The functions of the form ϕ_L are called zonal spherical functions. If ϕ_L is a zonal spherical function, then the map $f \to \int \phi_L(x) f(x) dx$ is a homomorphism of R onto the complex numbers, and every such homomorphism may be so obtained.

BEREZIN, F.A.; and GEL'FAND, I.M.

1-FW

3

"These known facts and notions and others related to them are recalled in the introductory section of the paper. In the second section the law of multiplication in the ring R is studied. Considering first the case in which G is the direct product with itself of the group of all $n\times n$ unitary unimodular matrices and G_0 is the diagonal subgroup, the members of R are considered as functions on the set D of $G_0:G_0$ double cosets and the multiplication law thrown into the form $(f_1f_2)(t)=\iint f_1(t_1)f_2(t_2)a(t_1, t_2, t)dt_1dt_2$, where t, t_1, and t_2 are in D and a is an explicitly described function on $D\times D\times D$. There are relations between the function a, the zonal spherical functions and certain differential operators, and these are explored in detail. In this case (and in general when G is the direct product with itself of a compact group and G_0 is the diagonal subgroup) the zonal spherical functions are in a natural one-to-one correspondence with the characters of the original compact group. Next it is briefly indicated how these results may be generalized to the case in which the unitary unimodular group is replaced by an arbitrary compact semi-simple Lie group. Further indications then treat the closely parallel facts which hold in the case in which G is a complex semi-simple Lie group and G_0 is a maximal

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...., F. A., and Gel'fand, I. M.

compact subgroup of G. By way of application a theorem on the proper values of sums and products of matrices is proved. (For a statement of this theorem see the review of a paper of Lidskiĭ [Dokl. Akad. Nauk SSSR (N.S.) 75 (1950), 769-772; MR 12, 591] in which another proof is given.)

In the third section it is shown that the zonal spherical functions satisfy the functional equation $\phi_L(x_1)\phi_L(x_2) = \int_{G_0} \phi_L(x_1 u x_2) du$ and that the right-hand side may be interpreted as the average value of ϕ_L over a "sphere" of "radius" $G_0 x_1 G_0$ with center a "distance" $G_0 x_2 G_0$ from the origin G_0. The $G_0 : G_0$ double cosets are in natural one-to-one correspondence with the equivalence classes of pairs of points in the homogeneous space G/G_0 under the action of G and in this sense can be regarded as generalized distances. This functional equation is examined in detail in the special case in which G/G_0 is the n-dimensional sphere.

The results of section 4 are also presented in terms of generalized distances. Let $V_{t,x}$ denote the operator which takes a function f on G/G_0 into its mean value on a sphere with center at x and "radius" t. Let $\Delta^1, \Delta^2, \cdots, \Delta^n$ denote the generalized Laplace operators for G/G_0 intro-

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APPROVED FOR RELEASE: 08/23/2000 CIA-RDP86-00513R000514620008-5"

Berezin, F.A.; and Gel'fand, I.M.

1-FW

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duced in earlier articles [Gel'fand, ibid. 70 (1950), 5–8; MR 11, 498; Gel'fand and Cetlin, ibid. 71 (1950), 825–828; MR 12, 9]. Then if $\psi(t, x) = V_{t,x}f$ for some function f, we have $\Delta_2^k\psi(t, x) = \Delta_1^k\psi(t, x)$, where on the left side t is held

fixed and on the right side x is held fixed and ψ as a function of t is regarded as a function of a point in G/G_0 a distance t from G_0. In addition it is shown that $V_{t,x}$ may be expressed as a function of the operators $\Delta^1, \Delta^2, \cdots, \Delta^n$, the function being constructed from the zonal spherical functions.

Let G be a compact semi-simple Lie group and let Ξ denote the algebra of all complex-valued functions ξ on the equivalence [illegible] that [illegible] $\sum_L |\xi(L)| [illegible] (L) < \infty$ and where

$$\xi_1\xi_2(L) = \sum_{L_1, L_2} \xi_1(L_1)\xi_2(L_2)G(L_1, L_2, L).$$

Here $G(L_1, L_2, L)$ is the number of times that L is contained in the reduction of the Kronecker product of L_1 and L_2. Section five is chiefly devoted to the structure and properties of this algebra (defined in a different but equivalent manner) and especially to the analogies that exist between it and the algebra R of section two. Here

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Berezin, F. A.; and Gel'fand, I. M. 1-FW 3

the function G plays the role of the function a in section two and has many analogous properties. Again a special case is treated in detail and generalizations more briefly indicated. At the end of the section some of the results of section 2 on complex semi-simple groups are generalized to the real case and to certain non-semi-simple groups. *G. W. Mackey* (Cambridge, Mass.). 5/5

Gelfand, I.M.

CARD 1/1 PG - 991

SUBJECT USSR/MATHEMATICS/Topology
AUTHOR BERESIN F.A., GELFAND I.M.
TITLE Some remarks to the theory of spherical functions on symmetric Riemannian manifolds.
PERIODICAL Uspechi mat.Nauk 11, 3, 211-218 (1956)
reviewed 7/1957

To a symmetric Riemannian manifold G/G_0 with the group G the authors consider spherical functions; if $g \to T_g$ is a unitary representation such that for a certain $\xi_0 \neq 0$ holds $(T_g)\xi_0 = \xi_0$ for all $g \in G_0$, then the functions $(\xi,(T_g)n)$ are called spherical functions; they are called zonal if they are constant on the "spheres" with the "center" x_0. Especially for the manifolds of the semi-simple groups (G consists of the $x \to a^{-1} x b$) and for the cases "G = complex semi-simple group, G_0 = maximal compact group" the law of multiplication of the zonal functions is given. Further the Laplace-operators (i.e. exchangeable with the $f(x) \to f(gx)$, $g \in G$) are brought in relation with the establishment of mean values, and finally a duality between the ring of the class functions of the compact group G and the algebra of the representations of G is explained. No proofs are given.

GELFAND, I.M.

Distr: 4E1f/4F1

Gel'fand, I. M. On some problems of functional analysis. Uspehi Mat. Nauk (N.S.) 11 (1956), no. 6(72), 3-12. (Russian)

The author points out the widening domain of functional analysis and points to the applications of functional analysis to numerical analysis, theory of probability, theory of differential equations, and information theory. He believes that hydrodynamics (the flow of viscous fluids, compressible gases, theory of turbulence), and theoretical physics (quantum theory and theory of elementary particles) will have a strong influence on the future course of development of functional analysis. Problems in the following fields are indicated: (1) linear topological spaces, (2) linear and quasi-linear partial differential equations, (3) measure theory, (4) rings of type P_1, and (5) hypergeometric functions. No specific references are given. *J. P. LaSalle* (Notre Dame, Ind.).

BEREZIN, F.A.; GEL'FAND, I.M.; GRAYEV, M.I.; NAYMARK, M.A.

Representation of groups. Usp.mat.nauk 11 no.6:13-40 N-D '56. (MIRA 10:3)

(Groups, Theory of)

GEL'FAND, I. M.

517.392

3891. ON THE NUMERICAL CALCULATION OF CONTINUOUS INTEGRALS. I.M. Gel'fand and N.N. Chentsov. Zh. eksper. teor. Fiz. Vol. 31, No. 6(12), 1106-7 (1956). In Russian.

Continuous integrals are expressed as Stieltjes integrals of sufficiently high multiplicity. An example is given in which a continuous integral is replaced by 190- and 280-fold integrals, summed (or calculated) by a Monte Carlo method. O.A. Chikal

Gelfand, I.M.

CARD 1/2 PG - 995

SUBJECT USSR/MATHEMATICS/Theory of probability
AUTHOR GELFAND I.M., KOLMOGOROV A.N., YAGLOM A.M.
TITLE On a general definition of an amount of information.
PERIODICAL Doklady Akad.Nauk 111, 745-748 (1956)
reviewed 7/1957

Let $\mathfrak{T}$ be a Boolean algebra and let P denote a probability on $\mathfrak{T}$. If $\mathfrak{A}$ and $\mathfrak{L}$ are two finite subalgebras of $\mathfrak{T}$, then the expression

$$I(\mathfrak{A},\mathfrak{L}) = \sum_{i,j} P(A_i B_j) \log \frac{P(A_i B_j)}{P(A_i)P(B_j)}$$

is by definition "the amount of information contained in the results of the experiment $\mathfrak{A}$ relative to the results of the experiment $\mathfrak{L}$" ($I(\mathfrak{A},\mathfrak{L}) = I(\mathfrak{L},\mathfrak{A})$). Generally,

$$I(\mathfrak{A},\mathfrak{L}) = \sup_{\mathfrak{A}_1 \subseteq \mathfrak{A},\ \mathfrak{L}_1 \subseteq \mathfrak{L}} I(\mathfrak{A}_1,\mathfrak{L}_1), \tag{1}$$

where $\mathfrak{A}_1$ and $\mathfrak{L}_1$ are finite subalgebras; symbolically (1) can be written

Doklady Akad.Nauk 111, 745-748 (1956) CARD 2/2 PC - 995

$$I(\mathfrak{A},\mathfrak{L}) = \iint_{\mathfrak{A}\,\mathfrak{L}} P(d\mathfrak{A}\,d\mathfrak{L}) \log \frac{P(d\mathfrak{A}\,d\mathfrak{L})}{P(d\mathfrak{A})P(d\mathfrak{L})}.$$

Suppose now that $\mathfrak{T}$ is a Boolean σ-algebra, P a σ-additive probability on $\mathfrak{T}$ and (X, S_X) (S_X a σ-algebra) a measurable space. A random element of the space X is a homomorphism $\xi^*(A) = B$ of S_X into $\mathfrak{T}$. The expression

$$I(\xi,\eta) = I(\mathfrak{T}_\xi, \mathfrak{T}_\eta)$$

is taken as definition of the amount of information, $\mathfrak{T}_\xi = \xi^*(S_X)$. A condition under which $I(\xi,\eta)$ is finite is given. Finally some properties of this expression are discussed.

GEL'FAND, I. M.

Distr: 4E3d

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3446

ON QUANTITIES WITH ANOMALOUS PARITY AND ON A POSSIBLE EXPLANATION OF PARITY DEGENERACY OF K-MESONS. I. M. Gel'fand and M. L. Tsetlin. Soviet Phys. JETP 4, 947-9(1957) July.

RMZ

4-RML

GEL FAND, I.M.

SUBJECT USSR/MATHEMATICS/Functional analysis CARD 1/1 PG -679
AUTHOR GEL'FAND, J.M., JAGLOM A.M.
TITLE The integration in function spaces and its applications in quantum physics.
PERIODICAL Uspechi mat.Nauk 11, 1, 77-114 (1956)
reviewed 4/1957

In the present paper the authors give a detailed establishment of Feynman's method of the path integrals (Rev.modern Phys. 20, 367-387 (1948)) by aid of the Wiener integrals (Acta math. Uppsala 55, 111-258 (1930)). After an introducing section in which the Wiener integrals are introduced, in the second section the application to quantum mechanics is treated in detail. The application of similar methods in the quantum field theory is treated very short. The third section brings the treatment of some fundamental properties and some useful computing rules for the Wiener integrals. The last section contains some special problems the treatment of which is very simple with the methods developed here. A translation of this interesting paper will be published soon in the "Fortschritte der Physik".

GELFAND I.M.

SUBJECT USSR/MATHEMATICS/Theory of probability CARD 1/4 PG - 731

AUTHOR GELFAND I.M., JAGLOM A.M.

TITLE The computation of the set of communication about a random function contained in an other random function.

PERIODICAL Uspechi mat.Nauk 12, 1, 3-52 (1957)
reviewed 5/1957

The first chapter in essential corresponds to the appendix 7 of Shannon and Weaver "A mathematical theory of communication" but it contains some new results. Let ξ and η be discrete random terms which can attain the values x_i $(i=1,2,\dots,n)$ and y_k $(k=1,2,\dots,m)$ with the probabilities $P_\xi(i)$ and $P_\eta(k)$. Let $P_{\xi\eta}(i,k)$ be the probability that at the same time ξ attains the value x_i and η the value y_k. The set of communication about η wontained in ξ then reads

$$I(\xi,\eta)=\sum_{i=1}^{n}\sum_{k=1}^{m}P_{\xi\eta}(i,k)\log\frac{P_{\xi\eta}(i,k)}{P_\xi(i)\,P_\eta(k)}.$$

For arbitrary (not discrete) ξ and η the set of communication is defined as

$$I(\xi,\eta)=\sup I\left[\xi(\Delta_1,\Delta_2,\dots,\Delta_n),\eta(\Delta_1',\Delta_2',\dots\Delta_m')\right],$$

Uspechi mat. Nauk 12, 1, 3-52 (1957) CARD 2/4 PG - 731

where the sup has to be taken over all possible subdivisions of the ranges of values of ξ and η into finite numbers of free of common points intervals Δ_i and Δ_k', respectively.

It is stated that in general the dependence of the set of communication $I(\xi, \eta)$ on the probability distribution of the pair of vectors (ξ, η) is discontinuous, but that always

$$I(\xi, \eta) \leq \underline{\lim}_{n \to \infty} I(\xi_n, \eta_n)$$

if the sequence (ξ_n, η_n) converges to (ξ, η) with respect to the probability distribution.

A further new result is contained in the theorem: In order that $I(\xi, \eta)$ is finite, it is necessary and sufficient that the probability distribution $P_{\xi\eta}$ is absolutely continuous with respect to the distribution $P_\xi \cdot P_\eta$. Then

$$I(\xi, \eta) = \int \alpha(x,y) \log \alpha(x,y) \, dP_\xi(x) \, dP_\eta(y), \text{ where}$$

$$(x,y) = \frac{dP_{\xi\eta}(x,y)}{dP_\xi(x) dP_\eta(y)} .$$

[illegible] use a formula which follows from (1) by putting

$$C_1 = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} \qquad C_1^{-1} = \begin{pmatrix} A^{-1} & 0 \\ 0 & B^{-1} \end{pmatrix}.$$

Then

$$\frac{\det A \det B}{\det C} = \left\{\det(CC_1^{-1})\right\}^{-1} = \begin{vmatrix} E & DB^{-1} \\ D^1A^{-1} & E \end{vmatrix}^{-1}$$

and

$$I(\xi, \eta) = -\frac{1}{2} \log \det (E - DB^{-1}D^1A^{-1}).$$

On the basis of these preparations several examples for the computation of the set of communication are computed.

GELFAND, I. M.

SUBJECT USSR/MATHEMATICS/Algebra CARD 1/2 PG - 815
AUTHOR GEL'FAND I.M.
TITLE On subrings of the ring of continuous functions.
PERIODICAL Uspechi mat.Nauk 12, 1, (1957) 249-251
reviewed 6 (1957)

The present communication is a completion to the preceding note of Šilov. The author puts seven questions:

1. Does there exist an antisymmetric ring with an one-dimensional carrier ?
2. Is it possible in the case of an arbitrary antisymmetric ring with a two-dimensional carrier to transform the carrier homeomophically such that the ring coincides with a ring of the type A(G) ? (A(G) is the totality of the functions which are analytic in the plane domain G and continuous in $\bar{G}$).
3. Is the ring A(G) the maximal antisymmetric ring with the carrier $\bar{G}$?
4. Does there exist antisymmetric rings the carriers of which are a two-dimensional sphere or a torus ?
5. Does there exist an antisymmetric ring with a carrier being homeomorphic to the three-dimensional cube ?
6. In four dimensions antisymmetric rings are well-known, which consist of analytic functions of two variables in the closed domain $\bar{G}$ of the two-dimensional complex space. Does there exist further antisymmetric rings with four-dimensional carriers ? Are the mentioned rings the maximal antisymmetric rings with the carrier $\bar{G}$?
7. Are the notions antisymmetric and analytic identical for a ring with uniform convergence ?

Gel'fand, I.M.

AUTHOR: KULAKOV,A.M., GEL'FAND,I.M., Engineers, PA - 2385
Metallurgical Combine of Magnitogorsk.

TITLE: Performance Practice of Automation of the Rolling Mills Heating Installations. (Opyt ekspluatatsii avtomatiki nagrevatel'nykh ustroystv prokatnykh stanov, Russian).

PERIODICAL: Stal', 1957, Vol 17, Nr 1, pp 80 - 83 (U.S.S.R.).
Received: 5 / 1957 Reviewed: 5 / 1957

ABSTRACT: About 140 vertical ingot heating furnaces are available for the rolling mill train of Magnitogorsk. Saving of 1 % fuel amounts to 800.000 roubles per annum. First the attempt at thermal control and automation in connection with the regeneration ingot heating furnaces of the blooming mill train Nr 3 is described. Waste caused by heating these furnaces consists to 85 - 90 % of the ingots of quiet steel. The best means of reducing waste caused by heating is the automation of the heating process of the ingots. It consists in regulating the temperature in the ingot heating furnaces, control of the consumption ratio of air and gas, pressure regulation in the working chamber of the furnace, automatic swithing of valves and automatic switching off of gas and air. For an optimum control and regulation thermoelements and radiation pyrometers are used simultaneously. In the second part of this paper the thermal control and automation of the three-zonal methodical furnaces LPTs-1 of the sheet rolling mill Nr 1 are described. It

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GEL'FAND, I.M., Prof., USSR Academy of Sciences.

"In the Field of Functional Analysis," paper submitted for Seventh Int'l Congress of Mathematicians, Edinburgh, Scotland,' 14-21 Aug 58.

BRAGINSKIY, S. I., GEL'FAND, I. M. and FEDORENKO, R. P.

"The Theory of the Compression and Pulsation of a Plasma Column in a Strong Pulse Discharge," (Work carried out 1957-1958); pp. 201-221.

"The Physics of Plasmas; Problems of Controlled Thermonuclear Reactions." VOl. IV. 1958, published by Inst. Atomic Energy, Acad. Sci. USSR.
resp. ed. M. A. Leontovich, editorial work V. I. Kogan.

Available in Library.

16(1) PHASE I BOOK EXPLOITATION SOV/1325

Gel'fand, Izrail' Moiseyevich and Georgiy Yevgen'yevich Shilov

Nekotoryye voprosy teorii differentsial'nykh uravneniy (Some Problems of the Theory of Differential Equations) Moscow, Fizmatgiz, 1958. 274 p. (Series: Obobshchennyye funktsii, vyp. 3) 8,000 copies printed.

Eds.: Agranovich, M.S. and Stebakova, L.A.; Tech. Ed.: Kryuchkova, V.N.

PURPOSE: This book is the third of a series of five monographs on functional analysis and is intended for mathematicians and for specialists in allied sciences. To read the book it is necessary to have a good background in mathematics and a knowledge of the results presented in the second book of the series.

COVERAGE: The book deals with the application of the theory of generalized functions to two classical problems of mathematical

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analysis: the problem of expansion of differential operators in eigenfunctions and the Cauchy problem for partial differential equations with constant coefficients or with coefficients dependent only on time. The theory of fundamental spaces of type W which is needed to study the Cauchy problem, is also presented. The authors thank those who participated in the Moscow State University seminar on generalized functions and differential equations, where many sections of this book were discussed. Special gratitude is expressed to V.M. Borok, Ya. I. Zhitomirskiy, G.N. Zolotarev, and A.G. Kostyuchenko. There are 64 references, of which 39 are Soviet, 13 English, 6 French, and 6 German.

TABLE OF CONTENTS:

16(1)

PHASE I BOOK EXPLOITATION SOV/1218

Gel'fand, Izrail' Moiseyevich and Shilov, Georgiy Yevgen'yevich

Prostranstva osnovnykh i obobshchennykh funktsiy (Spaces of Fundamental and Generalized Functions) Moscow, Fizmatgiz, 1958. 307 p. (Series: Obobshchennyye funktsii, vyp. 2) 7,000 copies printed.

Eds.: Agranovich, M.S. and Stebakova, L.A.; Tech. Ed.: Gavrilov, S.S.

PURPOSE: This book is the second of a series of five monographs on functional analysis. primarily intended for mathematicians, although it may be useful to others having a good mathematical background and a knowledge of the fundamentals of functional analysis.

COVERAGE: This book is devoted to the further development of the theory of generalized functions presented in the first book

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Spaces of Fundamental and Generalized Functions SOV/1218

of the series. In particular the book deals with the transfer of the technique of operation with generalized functions studied in the first book to more extensive classes of spaces. The basis of the theory of generalized functions is the theory of countable normal spaces, the presentation of which makes up the greater part of the book. The class of all countable normed spaces in many problems is too extensive for the theory of generalized functions. For this reason certain special types of countable normed spaces are introduced and studied. The spaces studied in this book are to be used in the third book of the series, which is to be devoted to certain applications of the theory of generalized functions to differential equations. The authors thank D.A. Raykov, B. Ya. Levin, G.N. Zolotarev, N. Ya. Vilenkina, and M.S. Agranovich for assistance in preparing the book. There are 39 references, of which 10 are Soviet, 7 English, 17 French, and 5 German.

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PHASE I BOOK EXPLOITATION SOV/2242

16(1)

Gel'fand, Izrail' Moiseyevich, Robert Adol'fovich Minlos, and Zorya Yakovlezna Shapiro

Predstavleniya gruppy vrashcheniy i gruppy Lorentsa, ikh primeniya (Rotation Group and Lorentz Group Representations and Their Applications) Moscow, Fizmatgiz, 1958. 368 p. 7,000 copies printed.

Eds.: F. A. Berezin and L. A. Stebakova; Tech. Ed.: S. S. Gavrilov.

PURPOSE: This book is intended for mathematicians and physicists and for students of mathematics and physics.

COVERAGE: This book is devoted to a detailed study of the representations of rotation groups in 3-dimensional space and to the Lorentz group. For the benefit of physicists and physics students the authors have included in the book all basic material on representation theory which is applicable to quantum mechanics. Mathematicians and mathematics students who are studying representation of Lie groups may use the book as an introduction to the general theory of representations. In addition the material included

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in the book renders sufficiently clear the connection between representation theory and other branches of mathematics, such as spherical functions, tensors, differential equations, etc., which had not previously been analyzed in the general case. I. M. Gel'fand and Z. Ya. Shapiro wrote the first part of the book on rotation groups. K. A. Minlos wrote the second part on representations of the Lorentz group and relativistic-invariant equations. This part was based mainly on the work of I. M. Gel'fand and A. M. Yaglom "General Relativistic-invariant Equations and Infinite Dimensional Representations of a Lorentz Group" (Zhurnal eksperimental'noy i teoreticheskoy fiziki, Vol 18, No 8, 1948). The authors thank F. A. Berezin, editor of the book, for his assistance. There are 25 references: 23 Soviet, 1 German, and 1 English.

TABLE OF CONTENTS:

PHASE I BOOK EXPLOITATION 629

Gel'fand, Izrail' Moiseyevich and Shilov, Georgiy Yevgen'yevich

Obobshchennyye funktsii i deystviya nad nimi (Generalized Functions and Operations With Them) Moscow, Gos. izd-vo fiziko-matematicheskoy lit-ry, 1958. 439 p. (Series: Obobshchennyye funktsii, vyp. 1) 8,000 copies printed.

Eds.: Agranovich, M. S. and Ryvkin, A. Z., Tech. Ed.: Brudno, K. F.

PURPOSE: This book is the first of a series of five monographs on functional analysis intended for scientific workers, graduate students and senior university students in mathematics, physics and allied sciences. It can also be useful for enegineers.

COVERAGE: The basic concepts and definitions of generalized functions (distributions) are introduced, their properties described and operations with them demonstrated. Fourier transformations of generalized functions of one and of several variables, and Fourier transformations in connection

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with certain differential equations are analyzed. Generalized functions on surfaces and fundamental solutions of differential equations with constant coefficients are studied. The general theory of homogeneous generalized functions is presented. In the preface, Soviet mathematicians S. L. Sobolev, Z. Ya.Shapiro, G. Ye. Shilov and N. Ya Vilenkin are mentioned in connection with publications on generalized functions. The authors thank their co-workers, in particular V. A. Borovikov, N. Ya. Vilenkin, M. I. Grayev, M. S. Agranovich and Z. Ya. Shapiro for their assistance in preparing the book. There are 22 references, 6 of which are Soviet, 5 English, 8 French and 3 German.

TABLE OF CONTENTS:

BABENKO, K.I.; GEL'FAND, I.M.

Several remarks on hyperbolic systems. Nauch. dokl. vys. skoly; fiz.-mat. nauki no.1:12-18 '58. (MIRA 12:3)

1. Moskovskiy gosudarstvennyy universitet im. M.V. Lomonosova.
(Differential equations, Partial)

AUTHORS: Gel'fand, I.M., Frolov, A.S. and Chentsov, N.N. SOV/140-58-5-4/14

TITLE: Calculation of Continuous Integrals With the Monte-Carlo Method (Vychisleniye kontinual'nykh integralov metodom Monte-Karlo)

PERIODICAL: Izvestiya vysshikh uchebnykh zavedeniy. Matematika, 1958, Nr 5, pp 32-45 (USSR)

ABSTRACT: This is a survey consisting of 10 paragraphs and a summary. The application of the Monte-Carlo method for the calculation of integrals of high (even of denumerable) number of variables is discussed in many aspects. The Soviet contributions (Bakhvalov, Korobov, the authors, Kolmogorov, Sobol') as well as the western contributions in this new direction are appreciated. The authors present some interesting examples (diminution of dispersion, determination of the trajectory for the Brownian motion etc.). In the text 4 Soviet and 7 American papers are mentioned.

ASSOCIATION: Matematicheskiy institut imeni V.A.Steklova AN SSSR (Mathematical Institute imeni V.A.Steklov AS USSR)

SUBMITTED: December 6, 1957 (Date of Lecture, Leningrad)

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GEL'FAND, Israil' Moiseyevich; SHILOV, Georgiy Yevgen'yevich; RYVKIN, A.Z., red.; BRUDNO, K.F., tekhn.red.

[Generalized functions and operations on them] Obshchennye funktsii i deistviia nad nimi. Moskva, Gos.izd-vo fiziko-matematicheskoi lit-ry, 1959. 470 p. (Obshchennye funktsii, no.1). (MIRA 13:4)

(Functional analysis)

GEL'FAND, I.M.

16(0) PHASE I BOOK EXPLOITATION SOV/3460

Moskovskoye matematicheskoye obshchestvo

Trudy, t. 8 (Transactions of the Moscow Mathematical Society, Vol 8) Moscow, Fizmatgiz, 1959. 518 p. Errata slip inserted. 2,050 copies printed.

Ed.: A.P. Lapko; Tech. Ed.: S.S. Gavrilov; Editorial Board: P.S. Aleksandrov, I.M. Gel'fand, and O.N. Golovin.

PURPOSE: This book is intended for mathematicians and theoretical physicists.

COVERAGE: This book contains a collection of articles by leading Soviet mathematicians on problems in pure and applied mathematics. All articles were written in 1957 and 1958. Among the topics discussed are: analytic - operator functions, function spaces, nonstationary plane flow of a viscous non-compressible liquid, root spaces, products of groups representations, ordinary and partial differential equations, 3rd and 4th order linear equations, homogeneous spaces, spectral theory of operators, and generalized random processes. References accompany each article.

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GEL'FAND, I. M.

21(4) PHASE I BOOK EXPLOITATION SOV/2583

International Conference on the Peaceful Uses of Atomic Engergy. 2nd, Geneva, 1958.

Doklady sovetskikh uchenykh; yadernyye reaktory i yadernaya energetika. (Reports of Soviet Scientists; Nuclear Reactors and Nuclear Power) Moscow, Atomizdat, 1959. 707 p. (Series: Its: Trudy, vol. 2) Errata slip inserted. 8,000 copies printed.

General Eds.: N.A. Dollezhal, Corresponding Member, USSR Academy of Sciences, A.K. Krasin, Doctor of Physical and Mathematical Sciences, A.I. Leypunskiy, Member, Ukrainian SSR Academy of Sciences, and V.S. Novikov, Corresponding Member, USSR Academy of Sciences, and V.S. Fursov, Doctor of Physical and Mathematical Sciences; Ed.: A.P. Alyab'yev; Tech. Ed.: Ye. I. Mazel'.

PURPOSE: This book is intended for scientists and engineers engaged in reactor designing, as well as for professors and students of higher technical schools where reactor design is taught.

COVERAGE: This is the second volume of a six-volume collection on the peaceful use of atomic energy. The six volumes contain the reports presented by Soviet scientists at the Second International Conference on Peaceful Uses of Atomic Energy, held from September 1 to 13, 1958 in Geneva. Volume 2 consists of three parts. The first is devoted to atomic power plants under construction in the Soviet Union; the second to experimental and research reactors, the experiments carried out on them, and the work to improve them; and the third, which is predominantly theoretical, to problems of nuclear reactor physics and construction engineering. Yu. I. Koryakin is the science editor of this volume. See SOV/2081 for titles of all volumes of the set. References appear at the end of the articles.

8 (2) SOV/119-59-4-4/18

AUTHOR: Gel'fand, I. M., Engineer

TITLE: An Automatic Controller of the Air-fuel Ratio Under Simultaneous Supply of Three Kinds of Fuel for Burning (Regulyator sootnosheniya toplivo-vozdukh pri odnovremennoy podache dlya goreniya topliva trekh vidov)

PERIODICAL: Priborostroyeniye, 1959, Nr 4, pp 9 - 10 (USSR)

ABSTRACT: The automatic controller investigated in this paper belongs to the double-zone soaking furnace of the Magnitogorskiy metallurgicheskiy kombinat (Magnitogorsk Metallurgical Kombinat). It is supplied with a gas mixture (blast furnace gas + coke oven gas, coke oven gas and mazut). The air necessary for the combustion is provided by a medium-pressure centrifugal fan. The unit used for the automatic control is composed of instruments and controllers coming from series production. In figure 2 the principal circuit diagram of the air-fuel mixture control is presented. The principle of operation of the individual elements is briefly discussed. The actual air-fuel mixture controller is based upon an electronic zero relay for the astatic part of an isodrome relay of the type IR 130 with a

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voltage amplification by means of two cascades. The controller discussed permits to reduce the number of secondary recording instruments and the size of the pertaining switchboard panel. The rate of consumption of each of the three fuels can be recorded on one single diagram. This controller exhibits a simple design and is reliable in operation. There are 3 figures, and 2 Soviet references.

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16(1) SOV/42-14-2-2/19

AUTHOR: Gel'fand, I.M.

TITLE: Some Problems of Theory of Quasilinear Equations

PERIODICAL: Uspekhi matematicheskikh nauk, 1959, Vol 14, Nr 2, pp 87-158 (USSR)

ABSTRACT: The present paper originated in lectures given by the author 1957-1958 at the Moscow State University and which were written down by K.V.Brushlinskiy and V.F.D'yachenko. The author does not aim at giving an exact representation of final results but he describes single, not finally solved statements of the theory which shall be developed. The author hopes that his paper will incite to solve the given problems. The author thanks K.I.Babenko, S.K.Godunov, and V.F.D'yachenko for the discussion of several questions, and G.I.Barenblatt and O.A.Oleynik for writing some parts of the manuscript. Contents: Introduction; §1. Evolution systems of equations; §2. Quasilinear system of equations of first order. Discontinuous solutions; §3. Fundamental relations of the thermodynamics; §4. Equations of the hydrodynamics of ideal fluids; §5. Motion equations for the presence of tenacity; §6. System of Lagrange coordinates; §7. Characteristics; §8. Stability of discontinuous solutions; §9. Decay of an arbitrary discontinuity; §10. On uniqueness and existence theorems; §11. Two-dimensional evolution systems; §12. Linear equations with

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discontinuous coefficients; §13. Equations of magnetic hydrodynamics; §14. Characteristic cone in the equations of magnetic hydrodynamics; §15. Problem of the thermal self-ignition; §16. Problem of becoming stationary of chemical processes; §17. Normal flame propagation. Additions.

The author mentions N.N.Semenov, Ya.B.Zel'dovich, and I.G. Petrovskiy.

There are 20 figures, and 17 references, 16 of which are Soviet, and 1 American.

SUBMITTED: November 17, 1958

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16(1) SOV/42-14-2-5/19

AUTHORS: Gel'fand, I.M., and Pyatetskiy-Shapiro, I.I.

TITLE: Theory of the Representations and Theory of Automorphic Functions

PERIODICAL: Uspekhi matematicheskikh nauk, 1959, Vol 14, Nr 2, pp 171-194 (USSR)

ABSTRACT: In the paper of the authors and S.V.Fomin [Ref 4] firstly the connection between the theory of infinite-dimensional representations of Lie groups and the theory of automorphic functions was pointed out. In the present paper this connection is investigated furthermore. It is shown that many questions of the theory of automorphic functions can be treated uniformly with the aid of infinite-dimensional representations. The calculation of the dimension of the space of automorphic forms is reduced to the determination of the multiplicity with which the corresponding irreducible representation appears in the decomposition of a certain representation. In a simple manner the authors introduce nonanalytic automorphic functions. The trace formula of [Ref 4] is generalized to the case of an arbitrary non-compact Lie group. The trace formula of Selberg

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[Ref 2] is a special case. The duality theorem of Cartan is transferred to the considered case.
The authors mention A.G.Kostyuchenko and M.A.Naymark.
There are 11 references, 5 of which are Soviet, 4 American, 1 German, and 1 French.

SUBMITTED: December 2, 1958

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16(1) SOV/42-14-3-1/22

AUTHOR: Gel'fand, I.M.

TITLE: Some Questions of Analysis and Differential Equations

PERIODICAL: Uspekhi matematicheskikh nauk, 1959, Vol 14, Nr 3, pp 3-20 (USSR)

ABSTRACT: The author formulates a large number of interesting unsolved problems from the theory of partial differential equations and from the analysis ; e.g. :
1. Into which connected components decomposes the set of weakly elliptic systems

$$\sum A_{ik}^{pq} \frac{\partial^2 u^{(p)}}{\partial x_i \partial x_k} = f_q \qquad (p,q = 1,2,\ldots,n) \quad ?$$

2. If the system belongs to the same component as $\Delta u_i = 0$, then it is to be decided whether the Dirichlet problem is uniquely solvable for this system.
3. All the boundary value problems for elliptic systems are to be given, the solution of which is unique with the exception of one finitely dimensional subspace and correctness.
4. The boundary conditions are to be given under which

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$\frac{\partial u}{\partial t} = L\left(\frac{\partial}{\partial x_i}, x\right) u$ admits a correct Cauchy problem with a unique solution.

5. Is it true that a boundary value problem is correct, if and only if it is correct in every point ?

6. Is it possible to obtain every theorem of existence for hyperbolic equations with the aid of energy integrals, if arbitrary correct boundary conditions are given ?

7. The existence of the vacuum measure for the Tirring equation

$$\Box \psi + k^2 \psi = \lambda \psi^3$$

is to be proved.

8. What kind of general methods can be developed in quantum theory, if the existence of the vacuum measure of interacting fields is assumed?

The following authors are mentioned: Mityagin, Gorin, M.S. Livshits, M.V. Keldysh, V.B. Lidskiy, I.G. Petrovskiy, Ye.M. Landis, Bitsadze, I.N. Vekua, M.I. Vishik.

Some Questions of Analysis and Differential Equations SOV/42-14-3-1/22

The author thanks S.V. Fomin for valuable discussions and assistance in writing the final text.
There are 10 references, 8 of which are Soviet, 1 American, and 1 French.

SUBMITTED: February 23, 1959

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16(1) SOV/20-124-1-3/69

AUTHOR: Gel'fand, I.M. Corresponding Member, Academy of Sciences, USSR

TITLE: On the Structure of the Ring of Rapidly Decreasing Functions on a Lie Group (O strukture kol'tsa bystro ubyvayushchikh funktsiy na gruppe Li)

PERIODICAL: Doklady Akademii nauk SSSR, 1959, Vol 124, Nr 1, pp 19-21 (USSR)

ABSTRACT: Let G be the group of the complex matrices $g = \begin{Vmatrix} \alpha & \beta \\ \gamma & \delta \end{Vmatrix}$, $\alpha\delta - \beta\gamma = 1$. The irreducible representations are given by

(1) $T_g f(z) = f\left(\frac{\alpha z + \gamma}{\beta z + \delta}\right)(\beta z + \delta)^{n_1 - 1}\overline{(\beta z + \delta)}^{n_2 - 1}$, $n_1 - n_2 = n$ integer. A function is called quickly decreasing if for all n it holds: $|x(g)| = 0(\| g \|^{-n})$. The expressions $P(D)x(g)$ are called derivatives of $x(g)$, where P is a polynomial in the Lie operator D. Let Γ be the set of all those infinitely differentiable $x(g)$, the derivatives of which are quickly decreasing. If in Γ the multiplication is defined as con -

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volution, and if a natural topology is introduced, then the fundamental group ring of G arises. If $x(g) \in \Gamma$, then the kernel $K(z_1, z_2; n_1, n_2)$ of the operator $\int x(g) T_g dg$ is denoted as Fourier transform of $x(g)$. The ring Γ transforms into a ring of kernels with usually defined multiplication. The author formulates necessary and sufficient conditions which are to be satisfied by a function $K(z_1, z_2, n_1, n_2)$ in order to be a Fourier transform of $x(g)$; e.g. 1.)-K has to be infinitely differentiable with respect to z_1 and z_2. 2.) There hold the relations

$$\int K(z_1, z_2 - z; n_1, n_2) z^{-n_1-1} \bar{z}^{-n_2-1} dz\, d\bar{z} =$$

$$= \int K(z_1 - z, z_2; -n_1, -n_2) z^{-n_1-1} \bar{z}^{-n_2-1} dz\, d\bar{z}$$

$$\frac{\partial^{n_1}}{\partial_{z_1}^{n_1}} K(z_1, z_2; n_1, n_2) = (-1)^{n_1} \frac{\partial^{n_1}}{\partial_{z_2}^{n_1}} K(z_1, z_2; -n_1, n_2)$$

n_2 integer $\quad n_1 = 1, 2, \ldots$

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etc.
The integrals $\int x(g)a(g)dg$ are denoted as moments of $x(g)$, where $a(g)$ is a matrix element of a finite-dimensional representation of G. The author directs to the importance of the results for the theory of representations.
There are 4 references, 2 of which are Soviet and 2 American.

SUBMITTED: September 24, 1958

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16(1)

AUTHORS: Gel'fand, I.M., Corresponding Member of the AS USSR, and Grayev, M.I.

SOV/20-127-2-4/70

TITLE: Resolution of Lorentz Group Representations Into Irreducible Representations in Spaces of Functions Defined on Symmetrical Spaces

PERIODICAL: Doklady Akademii nauk SSSR, 1959, Vol 127, Nr 2, pp 250-253 (USSR)

ABSTRACT: Let G be a Lorentz group, i.e. the group of complex matrices of second order $g = \left\| \begin{matrix} \alpha & \beta \\ \gamma & \delta \end{matrix} \right\|$ with the determinant 1. Let X be a symmetrical space with the motion group G. In the space of functions f(x) on X let to every $g \in G$ correspond a translation operator T_g: $T_g f(x) = f(xg)$. The obtained representation of G shall be decomposed into irreducible representations. The authors [Ref 2] have solved this problem if X is a Lobachevskiy space. In the present paper the same problem is treated in an other X. As a model of this space there may serve e.g. the exterior of the sphere (the "absolute") in the real projective

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space. Since in the present case the subgroup of the revolutions is not compact (this was used essentially in [Ref 2] for the case of the Lobachevskiy space) the authors propose a different method which in essential bases on a certain decomposition for the δ-function.
There are 3 references, 2 of which are Soviet, and 1 German.

SUBMITTED: May 5, 1959

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16(1) SOV/20-127-3-2/71

AUTHORS: Gel'fand, I.M., Corresponding Member, Academy of Sciences, USSR, Pyatetskiy-Shapiro, I.I.

TITLE: On a Poincaré Theorem

PERIODICAL: Doklady Akademii nauk SSSR, 1959, Vol 127, Nr 3, pp 490-493 (USSR)

ABSTRACT: The authors generalize a well-known result of Poincaré on the mean number of revolutions to ergodic dynamic systems with a summable directional field.
Let M be a compact differentiable manifold; $\gamma_1, \gamma_2, \ldots, \gamma_p$ the base of the group of integral homologies of M. Let x be a point of M. Let a dynamic system, i.e. a one-parameter group $x \to x_t$ with the invariant measure μ be defined on M. A trajectory rising in x is assumed to return infinitely often into a neighborhood U of x, and this may happen for the parameter values $t_1 < t_2 < \ldots$ $(t_k \to \infty)$. If the points x and x_{t_k} are connected by a trajectory lying in U, then the homology class of the obtained closed cycle $\gamma(t_k)$ does not depend on the

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choice of the trajectory. Let $\gamma(t_k) = m_1(t_k)\gamma_1 + \dots + m_p(t_k)\gamma_p$.

$$\lim_{k\to\infty} \frac{m_i(t_k)}{t_k} = \lambda_i$$ is denoted as mean frequency of rotation.

It is shown that this limit exists and is equal to the same number for almost all trajectories. Two examples are given : 1. M the torus 2. $M = G/\Gamma$, where G is the group of real matrices of second order and Γ a discreet subgroup of G without elements of finite order.
S.V. Fomin is mentioned.
There are 4 references, 2 of which are Soviet, 1 American, and 1 French.

SUBMITTED: May 14, 1959

PHASE I BOOK EXPLOITATION SOV/4328

Gel'fand, Izrail' Moiseyevich, Dmitriy Abramovich Raykov, and Georgiy Yevgen'yevich Shilov

Kommutativnyye normirovannyye kol'tsa (Commutative Normed Rings) Moscow, Fizmatgiz, 1960. 316 p. (Series: Sovremennyye problemy matematiki) 5,000 copies printed.

Ed.: S. A. Vilenkina; Tech. Ed.: S. S. Gavrilov.

PURPOSE: This book is intended for mathematicians (students in advanced courses, aspirants, and scientific workers) engaged in functional analysis and its applications.

COVERAGE: The book deals with the theory of commutative normed rings and its applications to analysis and topology. A report by I. M. Gel'fand and M. A. Naymark entitled, "Normed Involution Rings and Their Representations", which may serve as an introduction to the theory of noncommutative normed involution rings, is presented as an appendix. The following personalities are mentioned: L. A. Lyusternik, V. I. Sobolev,

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Commutative Normed Rings

A. N. Kolmogorov, S. V. Fomin, A. Zigmund, P. S. Aleksandrov, B. A. Fuks, P. Khalmosh, L. S. Pontryagin, [redacted], and F. Khausdorf. There are 87 references: 38 Soviet, 23 English, 13 French, and 13 German.

TABLE OF CONTENTS:

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16.4600 [illegible]-1-9/27

AUTHORS: Gel'fand, I. M., and Sia-Do-shing (Sya-Do-shin)

TITLE: On Positive Definite Distributions

PERIODICAL: Uspekhi matematicheskikh nauk, 1960, Nr 1, pp 185-190 (USSR)

ABSTRACT: Let f(x) be an even continuous function of the real variable x. M. G. Kreyn ⌈On a General Method of Decomposing Positive Definite Kernels Into Elementary Products. DAN Nr 1 (1946) 3-6⌋ showed that if $f(x + t) + f(x - t)$ is a positive definite kernel, i.e., for arbitrary $x_1, x_2, \ldots, x_m$ and $\xi_1, \xi_2, \ldots, \xi_m$

$$\sum [f(x_\mu + x_\nu) + f(x_\mu - x_\nu)] \xi_\mu \bar{\xi}_\nu \geq 0, \tag{1.1}$$

then there exist positive measures $\mu(\lambda)$ and $\sigma(\lambda)$ such that

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$$f(x) = \int_0^\infty \cos x\lambda \, d\mu(\lambda) + \int_0^\infty \operatorname{ch} x\lambda \, d\nu(\lambda).$$

(1.1) is equivalent to the following for arbitrary even continuous and finite function $\varphi(t)$

$$\int_0^\infty \int_0^\infty [f(x+t) + f(x-t)] \varphi(x) \varphi(t) \, dx \, dt \geq 0. \qquad (1.2)$$

Let K be the space of all finite, infinitely differentiable functions, and J be a subspace of K consisting of all even functions $\varphi(x) \in K$. Let the linear functional for f(x) over K be

$$(f, \varphi) = \int_{-\infty}^{\infty} f(x) \varphi(x) \, dx.$$

Then (1.2) is written as

$$(f, \varphi * \varphi^*) = 2 \int_0^\infty \int_0^\infty [f(x+t) + f(x-t)] \varphi(x) \varphi(t) \, dx \, dt \geq 0, \qquad (1.3)$$

Card 2/5 where $\varphi \in J$, $\varphi^*(x) = \varphi(-x)$ and $\varphi^* \varphi^*$ - convolution of φ and φ^*. In this note the authors give a general form of the functional over K which satisfies (1.3), i.e., a general form of an even distribution f(x) for which f(x + t) + f(x - t) is positive definite.

Theorem 1. Let T be an even linear functional over K, (i.e., (T, φ(x)) + (T, φ(-x)) for $\varphi \in$ K). If T is a positive definite functional over the subspace J, i.e., for arbitrary $\varphi \in$ J

$$(T, \varphi * \varphi^*) > 0, \tag{1.4}$$

then there exist positive measures $\mu(\lambda)$ and $\sigma(\lambda)$ such that

$$T = \int_{-\infty}^{\infty} \cos x\lambda \, d\mu(\lambda) + \int_{0}^{\infty} \operatorname{ch} x\lambda \, d\sigma(\lambda), \tag{1.5}$$

in other words

$$(T, \varphi) = \int_{0}^{\infty} \int_{-\infty}^{\infty} \cos x\lambda \varphi(x) \, dx \, d\mu(\lambda) + \int_{0}^{\infty} \int_{-\infty}^{\infty} \operatorname{ch} x\lambda \varphi(x) \, dx \, d\sigma(\lambda). \tag{1.6}$$

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For a given T in the above theorem the measures are not necessarily unique. Let Z_2^2 be the space of all entire functions $\varphi(z)$ of second order satisfying

$$|\varphi(x)| < C_1 e^{-C_2|x|^2}$$

on the real axis, where C_1, C_2 are constants. Let J_2 be the subspace of Z_2^2 consisting of all even functions $\varphi(z) \in Z_2^2$. Replacing in Theorem 1 K and J by Z_2^2 and J_2, respectively, gives an analogous result, where in this case the measures are unique. Theorem 2. Let T be an even linear functional over Z_2^2 and positive definite over the subspace J_2. Then there exist measures $\mu(\lambda)$ and $\sigma(\lambda)$, uniquely defined, such that for an arbitrary function $\varphi \in Z_2^2$ the decomposition (1.5) holds. The author proves Theorem 1 and give an example showing nonuniqueness of the measures $\mu(\lambda)$

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On Positive Definite Distributions

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and $\sigma(\lambda)$. In the proof of Theorem 2, the following lemma is used. Lemma. If $f(z)$ is an entire function of the exponential type and $f(x) \geqslant 0$ on the real axis then there exists an entire function $\varphi(z)$ of the exponential type such that

$$f(z) = \varphi(z)\bar{\varphi}(z).$$

There are 6 Soviet references.

SUBMITTED: July 26, 1958

Card 5/5

GEL'FAND, I.M.; GRAYEV, M.I.

Correction to the article "Geometry of homogeneous spaces, group representation in homogeneous spaces, and problems of integral geometry connected with them. Part 1." Trudy Mosk.mat.ob.va 9:562 '60. (MIRA 13:9)

(Groups, Theory of)

GEL'FAND, I.M.

Integral geometry and its relation to the theory of representations. Usp. mat. nauk 15 no.2:155-164 Mr-Ap '60. (MIRA 13:9)

(Geometry)

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16.3500

AUTHOR: Gel'fand, I.M.

TITLE: On Elliptic Equations

PERIODICAL: Uspekhi matematicheskikh nauk, 1960, Vol.15, No.3, pp.121-132

TEXT: The present paper is a completing to (Ref.1).
The author considers the class of systems

(1) $$A(x,\frac{\partial}{\partial x})u(x) = f(x),$$

defined in a domain Ω, where the number of variables, the number of unknown functions and the order of the highest derivative are given, while the coefficients are arbitrary continuous functions (A is a quadratic matrix the elements of which are polynomials in $\frac{\partial}{\partial x_j}$ with coefficients depending on x; u and f are vectors). This class is called a class of given structure. If the ellipticity is demanded, then a number of systems is separated, while the set of the other systems decomposes into components with respect to their connection. Fixing in the matrix B of the boundary condition

(2) $$B(x,\frac{\partial}{\partial x})u(x)\Big|_{\Gamma} = g(x)$$

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On Elliptic Equations

the order of the highest derivative, while the coefficients may be arbitrary continuous functions, then the elliptic systems (1) with a given structure and the boundary conditions (2) form a class of problems of given structure. Demanding further regularity, then again a number of problems is separated; the remainder decomposes into components. Two problems are called homotopically equivalent if they can be transferred into one another by a continuous change of the matrices A and B, where ellipticity and regularity remain always preserved. The experiences until now confirm the point of view that the essential non-local properties of elliptic systems are homotopically invariant. In this connection the author gives two problems. I. Determination of all homotopic invariants; II. Explanation of the meaning of these invariants in the classical sense. Especially it can be conjectured that the index of the problem is an invariant.
Then the problem of the homotopic invariants is still formulated in another manner. The author considers the boundary problem arising from (1)-(2) by putting $y = \lambda(x-x^0)$ and then taking $\lambda \to \infty$. If x^0 lies on the boundary Γ of Ω, then in the limit case one obtains a halfspace instead of Ω and in this halfspace (1) changes to a system with constant coefficients of the type

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(7) $A^{o}(x^{o}, \frac{\partial}{\partial x})u(x) = 0$

(again x instead of y etc). Let S be the space of limit problems for $x_0 \in \Gamma$. It is stated that the homotopy problem leads to the investigation of a part of the group $\pi_{n-1}(S)$ of S. As an example the elliptic problem

(5) $A(x)\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} = f(x,y)$,

(6) $B(x)u(x)|_{\Gamma} = 0$

is investigated with the aid of these methods, where E is the unit matrix and $\det\{A-\lambda E\} \neq 0$ for real λ.
The author mentions I.G.Petrovskiy, Ya.B.Lopatinskiy, B.V.Boyarskiy, I.N. Vekua, F.D.Gakhov and A.I.Vol'pert.He thanks M.S.Agranovich and Z.Ya. Shapiro for assistance. There are 14 references: 10 Soviet, 2 American, 1 Swedish and 1 Italian.

SUBMITTED: January 14, 1960

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AUTHORS: Gel'fand, J. M., Corresponding Member of the Academy of Sciences USSR, and Tsetlin, M. L.

TITLE: Continual Models of Controlling Systems 16

PERIODICAL: Doklady Akademii nauk SSSR, 1960, Vol. 131, No. 6, pp. 1242-1245

TEXT: Structurally and timely discrete models are little effective for the description of complicated (biological) systems. Therefore the authors propose to replace the discrete models by continuous ones. In the present (preliminary) publication the authors consider as the simplest model an "active tissue" possessing the following properties: 1. Each point of the medium is instantly excitable, where the intervals between two consecutive excitations of the same point possess a lower bound R different from zero. 2. The excitation can propagate in the medium, where the speed of propagation is variable. 3. A certain time T after the last excitation there takes place a new spontaneous excitation of the point. The authors consider three examples of processes which can take place in a medium with above-mentioned properties.

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Continual Models of Controlling Systems

The authors thank the participants of the seminary, the physiologists J. S. Balakhovskiy, V. S. Gurfinkel', V. B. Malkin and M. L. Shik for valuable discussion.
There are 3 figures, and 3 references: 2 Soviet and 1 Mexican.

ASSOCIATION: Matematicheskiy institut imeni V. A. Steklova AN SSSR (Mathematical Institute imeni V. A. Steklov AS USSR)

SUBMITTED: January, 13, 1960

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16.4400

AUTHORS: Gel'fand, I. M., Corresponding Member of the Academy of Sciences USSR, Grayev, M. J.

TITLE: Integrals Over Hyperplanes of Fundamental and Generalized Functions

PERIODICAL: Doklady Akademii nauk SSSR, 1960, Vol. 135, No. 6, pp. 1307-1310

TEXT: The Radon transformation of a function $f(z)$, $z = (z_1, \ldots, z_n)$ in the n-dimensional complex affine space is defined by

$$(2)\quad \check{f}(\xi;s) = \int f(z)\,\delta(s-(\xi,z))\,dz\,d\bar{z},$$

where $\xi = (\xi_1, \ldots, \xi_n)$, $(\xi,z) = \xi_1 z_1 + \ldots + \xi_n z_n$, $dz = dz_1 \ldots dz_n$, $\delta(s)$ is a generalized function of the complex variable s which is defined by $(\delta(s), \varphi(s)) = \varphi(0)$. The functions f are assumed to be infinitely often differentiable with respect to $z_1, \ldots, z_n, \bar{z}_1, \ldots, \bar{z}_n$ and to be quickly decreasing together with the derivatives (quickly decreasing means that, for $|z| \to \infty$ and arbitrary $k > 0$ it holds $|f(z)| = o(|z|^{-k})$. The inverse formula

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is

$$(3)\ f(z) = \frac{(-1)^{n-1}}{(2\pi)^{2n-2}} \int_{\Xi} \Psi(\xi ; (\xi, z))\, \omega_{\xi} \quad \text{where}$$

$$\Psi(\xi, s) = \frac{\partial^{2n-2}\check{f}(\xi ; s)}{\partial s^{n-1}\, \partial \bar{s}^{n-1}}$$

The integral is extended over an arbitrary surface Ξ with the real dimension 2n - 2 in the space ξ, which has exactly one point of intersection with almost every straight line through $\xi = 0$. The differential form $\frac{1}{2n}\omega_{\xi}$ is the volume of the cone, the apex of which is $\xi = 0$ and the base of which is the surface element.

Theorem 1: In order that $\Psi(\xi ; s)$ is the Radon transform of an infinitely often differentiable function in the real space and vanishes quickly together with the derivatives, it is necessary and

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sufficient that 1.) $\varphi(\alpha\xi ; \alpha s) = |\alpha|^{-1} \varphi(\xi, s)$ for every $\alpha \neq 0$; 2.) $\varphi(\xi, s)$ is infinitely often differentiable with respect to $\xi_1, \ldots, \xi_n$ and s for $(\xi_1, \ldots, \xi_n) \neq 0$; 3.) for every derivative $D\varphi$ of φ with respect to ξ, s and every $m > 0$ for $|s| \to \infty$ it holds uniformly in ξ:

$$D\varphi(\xi ; s) = o(|s|^{-m})$$

(here ξ varies in a compact domain of the space with the point $\xi = 0$ slackened; 4.) the integral

$$\int_{-\infty}^{\infty} \varphi(\xi ; s)\, s^k ds$$

is a homogeneous polynomial in ξ of the degree k $(k = 0, 1 \ldots)$.

Theorem 2 contains the analogous statement for the complex case.

The Radon transformation of a generalized function in the complex space is defined so that the usual definition is obtained for the fundamental functions. The formula

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Integrals Over Hyperplanes of Fundamental and Generalized Functions

$$(5)\quad \int F(z)f(z)dz d\bar{z} = \frac{(-1)^{n-1}}{(2\pi)^{2n-2}} \int \check{F}(\xi; s)\, \check{f}_{\xi}^{(n-1,n-1)}(\xi; s)\, ds\, d\bar{s}$$

is the starting point. (5) is briefly written as $(F,f) = (\check{F}, \check{f}_{\xi}^{(n-1,\ n-1)})$. As Radon transform of a generalized function F the authors denote the functional $\check{F}$ which is defined by the equation $(\check{F}, \check{f}_{\xi}^{(n-1,\ n-1)}) = (F,\ f)$ on the set of the functions $\check{f}_{\xi}^{(n-1,\ n-1)}$, where $\check{f}$ runs through the Radon transforms of the fundamental functions. Thereby $\check{F}$ is chiefly defined in the subspace of the fundamental functions which satisfy certain additional relations. $\check{F}$ can be continued to the whole space of the fundamental functions in different ways. The authors give 10 examples of Radon transformations.

There are 5 references: 3 Soviet and 2 German.

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