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GEL'FOND; A.O.

Gel'fond, A. O., and [illegible] On functions whose derivatives are zero at two points. Izvestiya Akad. Nauk SSSR. Ser. Mat. 11, 547-560 (1947). (Russian)

The authors discuss several cases in which $f^{(\nu_n)}(1)=0$ for a sequence $\{\nu_n\}$ and $f^{(k)}(0)=0$ for $k\neq\nu_n$ imply that $f(z)\equiv 0$. Let

$$\sigma_n^{\nu_n}=\frac{\nu_n!}{(\nu_n-\nu_{n-1})!\cdots(\nu_1-\nu_0)!\,\nu_0!}.$$

Then $f(z)\equiv 0$ in the following cases: (A) $f(z)$ is regular in $|z|\leq R$, where $R>\sigma=\liminf\sigma_n$; (B) $f(z)=\sum_0^\infty a_k z^k$ is an entire function, $\lim\sigma_n=\infty$ and $\lambda_n=\min_{k\geq n}\sigma_k$, $|a_k|<(\delta_k/\nu_k)\lambda_k^{-\nu_k}$ and $\sum\delta_k<\infty$; (C) $f(z)$ is an entire function of finite order $\rho>1$ and $\lim\log\sigma_n/\log\nu_n=1/\rho$.

If $\{\nu_s\}$ is an arithmetic progression, $\nu_s=ps-1$, $f(z)\equiv 0$ if it is entire and of exponential type $a<p/e$; moreover, p/e cannot be replaced by a smaller number for sufficiently large p.

R. P. Boas, Jr. (Providence, R. I.).

Source: Mathematical Reviews, Vol 1 No. 7

GEL'FOND, A. O.

Gel'fond, A. O. "Approximation of algebraic irrationals and their logarithms," Vestnik Mosk. un-ta, 1948, No. 9, p. 3-25 - Bibliog: 7 items

SO: U-2888, Letopis Zhurnal'nykh Statey, No. 1, 1949

GEL'FOND, A.O.

Gel'fond, A. O., and Linnik, Yu. V. On Thue's method in the problem of effectiveness in quadratic fields. Doklady Akad. Nauk SSSR (N.S.) 61, 773-776 (1948). (Russian)

Thue's method in the theory of Diophantine equations leads to upper bounds for the number of solutions, but in general not to bounds for these solutions themselves. The implications of this fact in the problem of obtaining all imaginary quadratic fields of class number 1 are discussed. Further, a generalized form of the Thue-Siegel theorem is given without proof; it gives the same exponent as Dyson's recent improvement of this theorem [Acta Math. 79, 225-240 (1947); these Rev. 9, 412]. *K. Mahler.*

Source: Mathematical Reviews, Vol 10 No. 6

GEL'FOND, A. O.

Gel'fond, A. O. The approximation of algebraic numbers by algebraic numbers and the theory of transcendental numbers. Uspehi Matem. Nauk (N.S.) 4, no. 4(32), 19–49 (1949). (Russian)

The author reports on the history and recent progress of the theory of transcendental numbers, laying particular stress on the work by Russian mathematicians beginning with Euler. After an introductory paragraph, the following subjects are discussed. [2] Liouville's theorem on the approximation of algebraic numbers, and its improvement by Thue and Siegel. [3] The author's recent further improvement of the Thue-Siegel theorem [similar to, but more general than, Dyson's work in Acta Math. 79, 225–240 (1947); these Rev. 9, 412; see also T. Schneider, Arch. Math. 1, 288–295 (1949); these Rev. 10, 592] in a paper unavailable outside the USSR, and its connection with the problem of effectiveness, i.e., of finding bounds for the solutions and not only for their number. The Thue-Siegel method does not have this property, but more recent work by N. Fel'dman and the author [also unavailable] is said to be promising in this direction. [4] Transcendency of e and π. Sketch of a proof of Lindemann's theorem without use of Hermite's explicit formulae. Siegel's work on Bessel functions. Measures of transcendency of e and π, in particular the new results by Fel'dman [see the second following review]. [5] Transcendency of α^β and $(\log \alpha)/(\log \beta)$ for algebraic α, β. The author's earlier result for e^π, and the final complete solution by him and Schneider. Sketch of his proof. Schneider's work on elliptic functions. The author's work on measures of transcendency of α^β and $(\log \alpha)/(\log \beta)$; its connection with the problem of effectiveness [C. R. (Doklady) Acad. Sci. URSS (N.S.) 7 (1935 II), 177–182; Bull. Acad. Sci. URSS. Sér. Math. [Izvestiya Akad. Nauk SSSR] 1939, 509–518; Rec. Math. [Mat. Sbornik] N.S. 7(49), 7–25 (1940); these Rev. 1, 295, 292]. Sketch of a new method of the author by which he proves that if $a \neq 0, \neq 1$ is algebraic, and α is a cubic irrational, then a^α and a^{α^2} are algebraically independent over the rational field. Outlook on further applications of this method, e.g., to improve still more the measure of transcendency of $(\log \alpha)/(\log \beta)$. [6] Expressions defined by series or products, and especially the work by Morduchay-Boltovskoy. The paper ends with a bibliography.

K. Mahler (Manchester).

Source: Mathematical Reviews, Vol 11 No. 4

GEL'FOND, A.O.

Card 1/3

Source: Mathematical Reviews, Vol 11 No 4

Gel'fond, A. O. On the algebraic independence of transcendental numbers of certain classes. Uspehi Matem. Nauk (N.S.) 4, no. 5(33), 14–48 (1949). (Russian)

By means of a method similar to one used by him already previously [Doklady Akad. Nauk SSSR (N.S.) 64, 277–280 (1949); these Rev. 10, 682], the author proves the following three theorems. (1) Let $\eta_0, \eta_1, \eta_2, \alpha_1, \alpha_2$ be real or complex numbers such that η_0, η_1, η_2, as well as $1, \alpha_1, \alpha_2$, are linearly independent over the rational field. Let there be two positive constants τ, x' such that $|x_0\eta_0+x_1\eta_1+x_2\eta_2|>\exp(-\tau x\log x)$ for all rational integers x_0, x_1, x_2 satisfying

$$x=\max(|x_0|, |x_1|, |x_2|)>x'.$$

Then at least two of the 11 numbers

(a) $\alpha_1, \alpha_2, \exp(\eta_i\alpha_k), \quad i=0, 1, 2;\ k=0, 1, 2;\ \alpha_0=1,$

are algebraically independent over the rational field.

(2) Let $\eta_0, \eta_1, \alpha_1, \alpha_2$ be real or complex numbers such that $\eta_0\neq 0$, η_1/η_0 is irrational, and $1, \alpha_1, \alpha_2$ are linearly independent over the rational field. Let there be two positive constants τ, x' such that $|x_0+x_1\eta_1/\eta_0|>\exp(-\tau x^2\log x)$ for all rational integers x_0, x_1 satisfying $x=\max(|x_0|, |x_1|)>x'$. Then at least two of the 10 numbers

$$\eta_0, \eta_1, \alpha_1, \alpha_2, \exp(\eta_i\alpha_k), \quad i=0, 1;\ k=0, 1, 2;\ \alpha_0=1,$$

are algebraically independent over the rational field.

(3) Let a, b, α, β be algebraic real or complex numbers such that a is different from 0 and 1, and that b and $(\log\alpha)/(\log\beta)$ are irrational. Then there exists to every $\epsilon>0$ a number H_0 with the following properties. If $P(x)\neq 0$ is a polynomial of degree at most s and with rational integral coefficients of absolute value at most H, then

$$|P(a^b)|>\exp\left\{-\frac{s^2}{\log^2 s}(s+\log H)\log^{2+\epsilon}(s+\log H)\right\},$$

$$\left|P\left(\frac{\log\alpha}{\log\beta}\right)\right|>\exp\{-s^2(s+\log H)^{2+\epsilon}\}.$$

if $H > H_0$.

On specializing the numbers η_i and α_k, (1) and (2) give, e.g., the following corollaries. (4) If α and $a \neq 0$, $\neq 1$ are algebraic and α is of higher than the third degree, then at least two of the numbers a^α, a^{α^2}, a^{α^3}, a^{α^4} are algebraically independent. If α is of the third degree, then already a^α and a^{α^2} are algebraically independent. [See the paper cited at the beginning of this review.] (5) If $a \neq 0$, $\neq 1$ is algebraic, and $\eta \neq 0$ is rational, then at least two of the numbers $a^{\exp(k\eta)}$, $k = 1, 2, 3, 4$, are algebraically independent. (6) If $\nu \neq 0$ is a rational number, then at least one of the numbers $\exp(e^{k\nu})$, $k = 1, 2, 3$, is algebraically independent of e, thus transcendental; and at least one of the numbers $\exp(\pi^{k\nu+1})$, $k = 1, 2, 3$, is transcendental. Theorem 3 is also of interest because, e.g., the Thue-Siegel theorem on algebraic numbers implies only the weaker inequality $|P\{(\log \alpha)/(\log \beta)\}| > e^{-\epsilon H}$.

The proofs of (1)–(3) are rather similar; they are based on a number of lemmas which are of interest in themselves. For instance, (1) is proved as follows. Assume that every two of the 11 numbers (a) are algebraically dependent. Then there exist two transcendental numbers ω and ω_1, where $\omega_1^\nu + c_1(\omega)\omega_1^{\nu-1} + \cdots + c_\nu(\omega) = 0$ and $c_1(\omega), c_2(\omega), \cdots, c_\nu(\omega)$ are polynomials in ω with integral coefficients, such that the numbers (a) are of the form S_i/T_i ($i = 1, \cdots, 11$), the S_i and T_i being polynomials in ω of arbitrary degree and in ω_1 of degree at most $\nu - 1$ with integral coefficients. Denote by N a sufficiently large positive integer, and put $T = T_1 T_2 \cdots T_{11}$, and $p = [N^3 \log^{-1} N] + 1$, $p_1 = [N^3/\log^{\frac{1}{2}} N]$, $\lambda = 3 + r$, $\mu = [11\lambda N^2 \log^{-1} N]$. In the function

$$f(z) = \sum_{k_0=0}^{p-1} \sum_{k_1=0}^{N} \sum_{k_2=0}^{N} \sum_{k_3=0}^{N} A_{k_0 k_1 k_2 k_3} z^{k_0} \exp\{(k_1\eta_1 + k_2\eta_2 + k_3\eta_3)z\}$$

let the A's be of the form

$$A_{k_0 \cdots k_3} = \sum_{\lambda_1=0}^{p_1} C_{k_0 \cdots k_3 \lambda_1} \omega^{\lambda_1},$$

where the C's are integers not all zero. Then the numbers

$$f_{t k_0 k_1 k_2} = T^\mu f^{(t)}(k_0 + k_1\alpha_1 + k_2\alpha_2),$$

where $0 \leq k_i \leq \lambda N^2 \log^{-1} N$, $i = 0, 1, 2$, are polynomials in ω and ω_1 with integral coefficients, and at least one of them is not zero. Denote by $\lambda_2, \lambda_3, \cdots$ positive constants independent of N. By means of Dirichlet's principle, integral values

Source: Mathematical Reviews, Vol. 11 No. 4

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of the C's may be chosen such that

$$|C_{k_0 \cdots k_2}| < \exp(\lambda_3 N^2 \log^{-1} N),$$

$f_{k_0 k_1 k_2} = 0$, i.e., $f(k_0 + k_1\alpha_1 + k_2\alpha_2) = 0$, for

$$0 \leq k_i \leq [\lambda_2 N^2 \log^{-1} N] = 9,$$

$i = 0, 1, 2$. If Γ is the circle $|\zeta| = 2N^{3/2}$, therefore,

$$f(z) = \frac{1}{2\pi i}\int_\Gamma \prod_{k_0=0}^{9} \prod_{k_1=0}^{9} \prod_{k_2=0}^{9} \left\{\frac{z - k_0 - k_1\alpha_1 - k_2\alpha_2}{\zeta - k_0 - k_1\alpha_1 - k_2\alpha_2}\right\} \frac{f(\zeta)d\zeta}{\zeta - z},$$

and so

$$|f(z)| < \exp(-\lambda_1 N^3 \log^{-1} N), \qquad |z| \leq N,$$

whence

$$|f_{k_0 k_1 k_2}| < \exp(-\tfrac{1}{2}\lambda_2 N^6 \log^{-1} N),$$
$$0 \leq k_i \leq \lambda N^2 \log^{-1} N;\ i = 0, 1, 2.$$

Select now k_0, k_1, k_2 according to these inequalities such that $f_{k_0 k_1 k_2} \neq 0$. Then $f_{k_0 k_1 k_2} = P(\omega, \omega_1)$ is a polynomial in ω and ω_1 with integral coefficients. Let H be its height (i.e., the maximum of the absolute values of the coefficients), and n its degree in ω; it may be assumed of degree $\nu - 1$ in ω_1. Then one shows that $n + \log H < \lambda_1 N^3 \log^{-1} N$. Denote by $\omega_1, \omega_2, \cdots, \omega_\nu$ the conjugates of ω_1 over the field generated by ω. The norm $P^*(\omega) = \prod_{i=1}^{\nu} P(\omega, \omega_i)$ does not vanish and is a polynomial in ω with integral coefficients, say of degree n_0 and height H_0. One shows that

$$|P^*(\omega)| < \exp(-\lambda_5 N^6 \log^{-1} N),$$
$$\max(n_0, \log H_0) < \lambda_6 N^3 \log^{-1} N.$$

By a new test of transcendency given in the paper, ω must then be algebraic contrary to hypothesis. *K. Mahler.*

Source: Mathematical Reviews, Vol 11 No. 4

GEL'FOND, A. O.

Gel'fond, A. O. **On the algebraic independence of algebraic powers of algebraic numbers.** Doklady Akad. Nauk SSSR (N.S.) 64, 277–280 (1949). (Russian)

Let α be a root of an irreducible algebraic equation of degree 3 and $a \neq 0, 1$ any algebraic number. The author shows that, for any fixed interpretation of $\log a$, the numbers $a^{\alpha} = e^{\alpha \log a}$ and $a^{\alpha^2} = e^{\alpha^2 \log a}$ are algebraically independent in the field of the rational numbers. The proof, which is not given in full detail, makes use of results said to have been proved by the author in another paper inaccessible to the reviewer. It is stated that the method can be applied without appreciable change to prove that if $\omega_1, \omega_2, \cdots, \omega_n$ (ω_1 rational) form a basis for the ring of algebraic integers of an algebraic field K, then no relation $P(a^{\omega_i}, a^{\omega_j}) = 0$ ($i \neq j \neq 1$) is possible where $P(x, y)$ is a polynomial in x, y with rational coefficients; an improvement of this result, which is to appear elsewhere, is announced. [Note. The reviewer suspects that in lemma 3 the case $k_1 = k_2 = k_3 = 0$ is meant to be excluded. Otherwise the lemma becomes trivial since B_s is then a multiple of $f^{(s)}(0)$ which is zero by hypothesis.] *R. A. Rankin* (Cambridge, England).

Source: Mathematical Reviews, Vol 10, No. 10

Gel'fond, A. O.

Gel'fond, A. O. On the algebraic independence of transcendental numbers of certain classes. Doklady Akad. Nauk SSSR (N.S.) 67, 13-14 (1949). (Russian)

Three general theorems are stated which, it is claimed, can be deduced by a variation of the methods used in an earlier paper [same Doklady (N.S.) 64, 277-280 (1949); these Rev. 10, 682]. These theorems are too complicated to state here, but the following consequences may be mentioned. (I) If a and α are algebraic numbers and $a \neq 0, 1$, and if the degree of α exceeds 2, then it is not possible to express each of the four numbers a^{α}, a^{α^2}, a^{α^3}, a^{α^4} algebraically in terms of one of them. When α is a cubic irrational it follows that a^{α} and a^{α^2} are algebraically independent in the field of the rational numbers, a result already proved in the paper referred to. (II) If a is as before and ν is rational and not zero, then it is not possible to express each of the four numbers a^{ν}, [illegible], [illegible], [illegible] algebraically in terms of one of them and, in particular, at least one of them is transcendental. A similar result holds for the first three of the four numbers.

R. A. Rankin (Cambridge, England).

Source: Mathematical Reviews, 1950 Vol 11 No. 2

Gel'fond, A.O.

Gel'fond, A. O. The approximation of algebraic numbers by algebraic numbers and the theory of transcendental numbers. Acta Math. Acad. Sci. Hungar. 1, 229-260 (1950). (Russian)

This paper was already printed under the same title in Uspehi Matem. Nauk (N.S.) 4, no. 4 (32), 19-49 (1949); these Rev. 11, 231. *K. Mahler* (Manchester).

Source: Mathematical Reviews, Vol 13 No.7

GELFOND, A.O.

Gel'fond, A. O. On the generalized polynomials of S. N. Bernštein. Izvestiya Akad. Nauk SSSR. Ser. Mat. 14, 413–420 (1950). (Russian)

Let $0=\alpha_0<\alpha_1\leq\alpha_2\leq\cdots$, $\alpha_n\to\infty$, $\sum\alpha_n^{-1}=\infty$. If

$$r_{kn}=[(1-\alpha_1/\alpha_{k+1})\cdots(1-\alpha_1/\alpha_n)]^{1/\alpha_1}$$

and

$$q_{kn}=(-1)^{n-k}\frac{\alpha_{k+1}\cdots\alpha_n}{2\pi i}\int_{|z|=1+\alpha_n}\frac{x^z\,dz}{(z-\alpha_k)\cdots(z-\alpha_n)}$$

$$=(-1)^{n-k}\alpha_{k+1}\cdots\alpha_n[\alpha_k\cdots\alpha_n],\qquad 0\leqq k\leqq n,$$

where $[\alpha_k\cdots\alpha_n]$ are the divided differences of the function x^t, then the "polynomial" $B_n(f,x)=\sum_{k=0}^{n}f(r_{kn})q_{kn}(x)$ converges uniformly towards $f(x)$ for any $f(x)$ continuous on $[0,1]$. The $q_{kn}(x)$ are linear aggregates of functions $x^{\alpha_k}\log^\nu x$, where ν is a nonnegative integer less than the multiplicity μ_k of α_k. In case $\mu_k=1$, $k=1,2,\cdots$, this has been given by Hirschman and Widder [Duke Math. J. 16, 433–438 (1949); these Rev. 11, 29]. The author uses a more convenient technique which enables him to estimate the degree of approximation. Two inaccuracies on pp. 417–418 are easily corrected. *G. Lorentz* (Toronto, Ont.).

Source: Mathematical Reviews, Vol 12 No. 5 1

Gel'fond, A. O.

Gel'fond, A. O., and Fel'dman, N. I. On the measure of relative transcendentality of certain numbers. Izvestiya Akad. Nauk SSSR. Ser. Mat. 14, 493–500 (1950). (Russian)

The authors improve an earlier result due to Gel'fond [Doklady Akad. Nauk SSSR (N.S.) 64, 277–280 (1949); these Rev. 10, 682]. Let α be a root of an irreducible equation of degree 3 and $a \neq 0, 1$ an algebraic number. Let $P(x, y)$ be any polynomial in x and y having integral coefficients each of modulus not exceeding H and of degrees n_1 and n_2 in x and y. Then, for any $\epsilon > 0$,

$$|P(a^{\alpha}, a^{\alpha^2})| \geq e^{-\sigma^{4+\epsilon}}$$

provided that $\sigma = \max(n_1+n_2, \log H) > \sigma_0$. The proof makes use of ideas similar to those introduced in the paper referred to above. *R. A. Rankin* (Cambridge, England).

Source: Mathematical Reviews, Vol 12 No. 1.

GEL'FOND A. O.,

USSR/Mathematics - Approximations Jan/Feb 51

"Quasipolynomials That Deviate the Least From Zero on the Interval (0, 1)," A. O. Gel'fond

"Iz Ak Nauk SSSR, Ser Matemat" Vol XV, No 1, pp 9-16

Following S. N. Bernshteyn's methods, Gel'fond finds lower and upper bounds of divergence of quasipolynomial that deviates least from zero. Submitted 11 Oct 50.

[redacted] 170T51

Gel'fond, A. O.

Gel'fond, A. O., and Leont'ev, A. F. On a generalization of Fourier series. Mat. Sbornik N.S. 29(71), 477–500 (1951). (Russian)

Let $f(z)=\sum_0^\infty a_n z^n$ ($a_n \neq 0$, $n=0, 1, \cdots$) be an entire function of order ρ and finite, non-zero type σ, with (a) $\lim_{n\to\infty} n^{1/\rho}|a_n|^{1/n}=(\sigma e\rho)^{1/\rho}$; and let $F(z)=\sum_0^\infty b_n z^n$ be an arbitrary analytic function, regular in $|z|<R$ ($R\leqq\infty$). Define the operator (b) $D^n F=D^n(F, f)=\sum_{k=n}^\infty b_k(a_{k-n}/a_k)z^{k-n}$. This series for $D^n F$ converges in $|z|<R$. (The authors remark that condition (a) can be weakened, but that it is assumed for simplicity in formulas, etc.) The operator $D^n F$ is a generalization of differentiation in the sense that for the particular choice $f(z)=e^z$ we have $D^n(F, e^z)=d^n F(z)/dz^n$. Let $\varphi(t)=\sum_0^\infty c_n t^n$ be an entire function of order $\leqq\rho$ and finite type σ_1 (where if the order is less than ρ then take $\sigma_1=0$). The principal aim of the paper is to consider the equation of infinite order in the generalized derivative:

$$\text{(c)} \qquad L[F]=\sum_0^\infty c_n D^n F=0.$$

Since $L[f(\lambda z)]=\sum_0^\infty c_n\lambda^n f(\lambda z)=\varphi(\lambda)f(\lambda z)$ (where f was given above) the function $\varphi(\lambda)$ is called the characteristic function of equation (c). [The reference to Fourier series in the title is explained as follows: The analytic function $F(z)$ is expressible in a Fourier series (in multiples of $2\pi z$) if it is periodic with a period 1; i.e., if $F(z+1)-F(z)=0$. Now this last relation is representable as an equation of form (c) if we take $f(z)=e^z$, $\varphi(t)=e^t-1$.]

To determine the convergence properties of series (c), set (d) $\varphi(\lambda)f(\lambda z)=\sum_0^\infty B_n(z)\lambda^n$, (e) $A_n(z)=B_n(z)/a_n$, $n=0, 1, \cdots$, and (f) $\psi(z, t)=\sum_0^\infty A_n(z)/t^{n+1}$. It is shown that for $r>0$, series (f) converges in $|z|<r$, $|t|>\mu(r)=(\sigma_1/\sigma+r^\rho)^{1/\rho}$, so that ψ is regular for z, t in this region; and that for $F(z)$ regular in $|z|<R=\mu(r)$, then series (c) converges in $|z|<r$, and in every circle $|z|<r-\epsilon$ ($\epsilon>0$), $L[F]$ has the representation (g) $L[F]=(2\pi i)^{-1}\int_C\psi(z, t)F(t)dt$. (Here C is the circle $|t|=R_1$ with $\mu(r-\epsilon)<R_1<R$.) If λ is a zero of $\varphi(\lambda)$, and is of multiplicity p, then the p functions $z^m f^{(m)}(\lambda z)$ ($m=0, 1, \cdots, p-1$) are particular solutions of $L[F]=0$.

Now take a function $f_1(z)=\sum_0^\infty \alpha_n z^n$ satisfying the same conditions as $f(z)$, and, keeping the same characteristic

Source: Mathematical Reviews, Vol 13 No. 7

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Gel'fond, A.O.

function $\varphi(t)$, form an operator L_1: $L_1[F]=\sum_0^\infty c_n D^n(F, f_1)$. The functions $\eta(z)=\sum_0^\infty (a_n/\alpha_n)z^n$, $\eta_1(z)=\sum_0^\infty (\alpha_n/a_n)z^n$ are analytic in $|z|<1$. Form with these the operators

(h)
$$M[F]=\sum_0^\infty \frac{a_n}{\alpha_n} b_n z^n = \frac{1}{2\pi i}\int_C \eta\left(\frac{z}{t}\right)F(t)\frac{dt}{t},$$
$$M_1[F]=\sum_0^\infty \frac{\alpha_n}{a_n} b_n z^n = \frac{1}{2\pi i}\int_C \eta_1\left(\frac{z}{t}\right)F(t)\frac{dt}{t},$$

where $F(z)=\sum_0^\infty b_n z^n$ is regular in $|z|<R$ and C is the circle $|t|=R_1$ with $|z|<R_1<R$. Then $M[F]$ and $M_1[F]$ are regular functions in $|z|<R$; and $M_1[F]$ is inverse to $M[F]$: (i) $MM_1[F]=M_1M[F]=F(z)$. Also,

(j)
$$D^n(F, f_1)=M^{-1}\{D^n(M[F], f)\}.$$

We thus have (k) $L_1[F]=M^{-1}LM[F]$; so that if F satisfies $L_1[y]=\Phi(z)$ then $M[F]$ satisfies $L[y]=M[\Phi]$, and if F_1 satisfies $L[y]=\Phi_1(z)$ then $M^{-1}[F_1]$ satisfies $L_1[y]=M^{-1}[\Phi_1]$. As seen earlier, if λ_n $(n=1, 2, \cdots)$ are the zeros of $\varphi(t)$, and p_n the corresponding multiplicities, then $L[y]=0$ has as particular solutions $y_j=z^m f^{(m)}(\lambda_n z)$, with $m=0, 1, \cdots, p_n-1$; $n=1, 2, \cdots$; $j=p_1+p_2+\cdots+p_{n-1}+m+1$; and similarly, $\bar{y}_j=z^m f_1^{(m)}(\lambda_n z)$ are solutions of $L_1[y]=0$. It then follows that if $F(z)$, satisfying equation (c) in $|z|<R$, has there the uniformly convergent representation

$$F(z)=\lim_{k\to\infty}\sum_{j=1}^{m_k}\beta_j y_j(z)$$

then the solution $M^{-1}[F]$ of $L_1[y]=0$ will have in $|z|<R$ the representation $M^{-1}[F]=\lim_{k\to\infty}\sum_{j=1}^{m_k}\beta_j\bar{y}_j(z)$.

Consider the equation (l) $L[F]=0$ where $L[F]$ is now given in the integral form (g). Moreover, take

$$f(z)=\sum_0^\infty a_n z^n=\sum_0^\infty [\sigma^{n/\rho}/\Gamma(n/\rho+1)]z^n,$$

so that $1/a_n=\int_0^\infty x^n d\tau(x)$, $\tau(x)=-e^{-\sigma x^\rho}$. (From what was stated earlier about operators L and L_1, it suffices to discuss the case of one representative $f(z)$.) The function $\psi(z, t)$ of (g) is now shown to have the form

(m)
$$\psi(z, t)=\frac{1}{t}\int_0^\infty \varphi\left(\frac{x}{t}\right)f\left(\frac{zx}{t}\right)d\tau(x).$$

Moreover, if $F(z)$ is regular in $|z|<\mu(r)$ and satisfies equation (l), then for $|z|<r-\epsilon$ $(\epsilon>0)$ the relation

Source: Mathematical Reviews, Vol 13 No. 7

2/3 MOG, AFL

(n) $$\int_{|t|=\mu_1} \frac{F(t)}{t}dt \int_{|u|=q} \frac{du}{\varphi(u)} \int_0^{\infty} \left\{ \varphi\left(\frac{x}{t}\right) - \varphi(u) \right\} \times \frac{f(zx) - f(zu)}{\frac{x}{t} - u} d\tau(x) = 0$$

holds, where $\mu(r-\epsilon) < \mu_1 < \mu(r)$, and $\varphi(u) \neq 0$ on $|u| = q$. Also for such F, if we set

(o) $$Q(z, q) = \frac{1}{(2\pi i)^2} \int_{|t|=\mu_1} \frac{F(t)}{t} dt \int_{|u|=q} \frac{f(zu)}{\varphi(u)} du \times \int_0^{\infty} \frac{\varphi(x, t) - \varphi(u)}{\frac{x}{t} - u} d\tau(x),$$

then $Q(z, q)$ is a linear combination of the solutions $y_j(z)$ corresponding to those zeros of $\varphi(u)$ lying in $|u| < q$: $Q(z, q) = \sum_{j=1}^{s} \beta_j y_j(z)$. The coefficients β_j are independent of q,

so $Q(z, q)$ represents a partial sum of order s of the formal series (p) $\sum_{j=1}^{\infty} \beta_j y_j(z)$. It is then shown that the remainder term $R(z, q) = F(z) - Q(z, q)$ is given by

(q) $$R(z, q) = -\frac{1}{(2\pi i)^2} \int_{|t|=\mu_1} \frac{F(t)}{t^{n+1}} dt \int_{|u|=q} \frac{du}{u^n \varphi(u)} \times \int_0^{\infty} \frac{\varphi(x, t) - \varphi(u)}{\frac{x}{t} - u} f\left(\frac{zx}{t}\right) d\tau(x)$$

in $|z| < r - \epsilon$ (n being an arbitrary non-negative integer).

The basic theorem of the paper is now established: Let there exist an infinite sequence of circles $|u| = q_m$ ($m = 1, 2, \cdots$) on which

(r) $$|\varphi(u)| > e^{-(\theta+\epsilon)|u|^{\rho}}$$

($\theta \geqq 0$) for $m > N(\epsilon)$, $\epsilon > 0$ arbitrary. If $F(z)$ is regular and satisfies equation (l) in $|z| < R$, then

(s) $$|F(z) - Q(z, q_m)| < \exp\{(\theta + \sigma_1 + \sigma r^{\rho} - \sigma R^{\rho} + \epsilon) q_m^{\rho}\}$$

for $|z| < r$ and $m > N_1(\epsilon)$. As a corollary, if $\varphi(u)$ satisfies the conditions just stated, and if $F(z)$ is regular and satisfies (l) in $|z| < R = \mu_1(r) = [(\theta + \sigma_1)/\sigma + r^{\rho}]^{1/\rho}$, then for $|z| < r$, $F(z)$ is given by $F(z) = \lim_{m\to\infty} Q(z, q_m)$, uniformly in $|z| < r - \epsilon$ ($\epsilon > 0$ arbitrary). It is shown moreover that $\varphi(z)$, being of order ρ and type σ_1, a possible choice of θ in (r) is $\theta = [2\rho]\sigma_1$ where $[2\rho]$ = greatest integer not exceeding 2ρ.

I. M. Sheffer (State College, Pa.)

Source: Mathematical Reviews, Vol 13 No. 7

3/3 NOG, AFL

Gel'fond, A. O.

Gel'fond, A. O. Linear differential equations of infinite order with constant coefficients and asymptotic periods of entire functions. Trudy Mat. Inst. Steklov., v. 38, pp. 42-67. Izdat. Akad. Nauk SSSR, Moscow, 1951. (Russian) 20 rubles.

The author considers in the complex domain equations of the form $L[F]=\sum_{n=0}^{\infty}a_nF^{(n)}(z)=\Phi(z)$, where $\phi(t)=\sum a_nt^n$ is an entire function of exponential type σ, and obtains several novel results. If α is a root of $\phi(t)=0$ of multiplicity $s+1$ (or more), then $z^se^{\alpha z}$ is a solution of $L[F]=0$, and the author shows first that any solution $F(z)$ of the homogeneous equation, regular in a circle $|z|\leqq\sigma_0$, $\sigma_0>\sigma$, is representable by a uniformly convergent series of these functions in some neighborhood of the origin. In fact,

$$(2\pi i)^{-2}\int_{|\xi|=\sigma_1+2\rho}\frac{F(\xi)}{\xi-z}\int_{|t|=R}\frac{dt}{\phi(t)}\int_0^{\infty}\frac{\phi(x/(\xi-z))-\phi(t)}{x/(\xi-z)-t}e^{-x}dx\,d\xi,$$

$\sigma_0>\sigma_1>\sigma$, is a linear combination of functions $z^se^{\alpha z}$, and the difference between this and $F(z)$ can be estimated. If $F(z)$ is an entire function, something can be said about the rapidity of convergence of the approximating sums.

The author next considers the inhomogeneous equation $L[F]=\Phi(z)$, with the object of showing that there is a solution of roughly the same growth as $\Phi(z)$ when $\Phi(z)$ is an entire function of greater than exponential type. His result is expressed in terms of a regularized maximum modulus $\bar{M}(r)$, defined for $r=|z|>1$ by

$$\log\bar{M}(r)=r\max_{1\leq t\leq r}t^{-1}\log M(t),$$

where $M(t)$ is the maximum modulus. Then if $\Phi(z)$ is of greater than exponential type, $\epsilon>0$ and $\theta>1$, the equation $L[F]=\Phi(z)$ has a solution $F_0(z)$ such that

$$|F_0(z)|<c(\epsilon,\theta)[\bar{M}(\theta\rho)]^{(1+\epsilon)/\log\theta},\quad |z|\leqq\rho.$$

A function F with period ω satisfies the equation

$$L[F]=F(z+\omega)-F(z)=0,$$

and the corresponding $\phi(z)$ is $e^{\omega z}-1$. Hence a generalization of the fact that a nonconstant entire function cannot have two independent periods is the theorem, which the author proves, that if $F(z)$ satisfies $L_1[F]=L_2[F]=0$, with $\phi_1(t)$ and $\phi_2(t)$ having no zeros in common, except perhaps the origin, then $F(z)$ must be a polynomial, of degree not exceeding one less than the multiplicity of the zero at the origin which has smaller multiplicity.

Source: Mathematical Reviews, Vol 13 No. 10

1/2

Finally the author generalizes Whittaker's definition of an asymptotic period [Interpolatory function theory, Cambridge, 1935] by requiring that $F(s+\omega)-F(s)$ is of lower order than $F(s)$ in a generalized sense. He calls $F(s)$ of greater order than $F_1(s)$ if there are positive increasing functions u and w, such that $u(2x)/u(x)\to 1$ and for some $\theta>1$

$$\limsup u[\log \mathfrak{M}(r)]/w(r) > \limsup u[\log \mathfrak{M}_1(\theta r)]/w(r),$$

$\mathfrak{M}$ being defined as above. (If $u(x)$ and $w(x)$ are both $\log x$ we have the usual definition of order and asymptotic period.) Then if an entire function has any asymptotic periods in the generalized sense, they lie on a single straight line, they form a set of measure zero (these properties were proved by Whittaker with the old definition), and the ratio of two asymptotic periods is either a rational or a transcendental number.

R. P. Boas, Jr. (Evanston, Ill.).

Source: Mathematical Reviews, Vol 13 No. 10

Gel'fond, A. O. On integral valuedness of analytic functions. Doklady Akad. Nauk SSSR (N.S.) 81, 341–344 (1951). (Russian)

Let E. $E_1=\{\alpha_1, \alpha_2, \cdots\}$, and $E_2=\{\beta_1, \beta_2, \cdots\}$ be three enumerable sets of complex numbers with ∞ as only limit point and such that E consists of all sums $\alpha_i+\beta_j$. Write

$$N_1(r)=\sum_{|\alpha_i|\leq r} 1, \quad N_2(r)=\sum_{|\beta_j|\leq r} 1, \quad N(r)=\min(N_1(r), N_2(r)).$$

Denote by $f(z)$ an integral function, by $M(r)$ the maximum of $|f(z)|$ on $|z|=r$, and by K an algebraic field of finite degree ν. Assume that $f(\gamma)$, for all $\gamma \epsilon E$, is an integer in K with the following property: There exists to every $\delta>0$ a $C_0(\delta)>0$ independent of γ such that $f(\gamma)$ and all its conjugates with respect to K are $<C_0M(|\gamma|)^{1+\delta}$ in absolute value. The author proves that then two constants θ and λ can be given such that, if

$$\log M(\theta r)<\lambda N(r),$$

then $f(z)$ satisfies a functional equation

$$\sum_{k=1}^{m} A_k f(z+\beta_k)=0, \qquad m>1,$$

where $A_1, \cdots, A_m$ are non-zero rational integers. (It suffices to take $\theta>4$, $\lambda>4(\nu+1)\log(\frac{1}{2}\theta-1)$.)

K. Mahler.

Source: Mathematical Reviews, Vol 13 No. 5

GEL'FOND, A. O.

Transcedental and algebric numbers. Moskva, Gos. izd-vo tekhniko-teoret. lit-ry, 1952. 224 p. (54-17551)

GEL'FOND, A.O.

*Gel'fond, A. O. Rešenie uravnenii v celyh čislah. [The solution of equations in whole numbers.] Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow-Leningrad, 1952. 63 pp. 85 kopecks.
A booklet in the series, Popular Lectures in Mathematics.

Source: Mathematical Reviews, Vol 13 No.10

GEL'FOND, A.O.

Gel'fond, A. O. The distribution of fractional parts and convergence of functional series with gaps. Moskov. Gos. Univ. Učenye Zapiski **148**, Matematika **4**, 60–68 (1951). (Russian)

Let L be an increasing sequence of real numbers $y_1, y_2, \cdots$ and M a sequence of points $\tau_k = (\beta_{1k}, \beta_{2k}, \cdots, \beta_{\nu k})$ in the unit cube in ν-dimensional space, so that $0 \leqq \beta_{ik} < 1$ for $1 \leqq i \leqq \nu$, $k = 1, 2, \cdots$. The fractional parts $\{f_i(y)\}$ of ν functions $f_i(y)$ are said to be (φ, M) distributed in ν-dimensional space if the set of inequalities

$$|\{f_i(y)\} - \beta_{ik}| \leqq \varphi(y) \quad (i = 1, 2, \cdots, \nu)$$

[in the paper the braces { } are omitted] possess for every τ_k of M an infinity of solutions in values of y belonging to L. Here $\varphi(y)$ is any given decreasing function satisfying $0 \leqq \varphi(y) \leqq 1$ for $y > 0$ and $\lim_{y\to\infty} \varphi(y) = 0$. Two theorems which are similar to the two theorems of an earlier paper of the author [Doklady Akad. Nauk SSSR (N.S.) **64**, 437–440 (1949); these Rev. **10**, 682] are proved and from them results concerning gap-series and the construction of analytic functions with prescribed fractional parts at the points $z = 1, 2, 3, \cdots$ are proved. The final result concerns the Dirichlet series $f(z) = \sum_0^\infty C_n e^{-\lambda_n z}$ where $\{\lambda_n\}$ is an increasing sequence such that $\lim_{n\to\infty} n/\lambda_n = 0$, and $U(z)$ is defined by $U(z) = \prod_1^\infty (1 - z^2/\lambda_n^2)$. The author proves that if the series for $f(z)$ converges in the half-plane $\Re z > 0$ and if $\limsup_{n\to\infty} |C_n U'(\lambda_n)|^{1/\lambda_n} = 1$, then $f(z)$ cannot be continued analytically onto the imaginary axis.

The proof of the first theorem needs slight modification since, in contrast to the earlier paper, $|z_{ik}| \geqq 1$, so that the inequality (8) need not be true.

R. A. Rankin.

SCI MATHEMATICAL REVIEW (unclassified)
Vol XIV, No 4, April 1953, pp 341-438

GEL'FOND, A.O.

Mathematical Reviews
Vol. 14 No. 8
Sept. 1953
Analysis

*Gel'fond, A. O. Isčislenie konečnyh raznostel. [The calculus of finite differences.] Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow-Leningrad, 1952. 479 pp. 10.20 rubles.

This book is based on the author's book of the same title [ONTI, Moscow-Leningrad, 1936], which he has revised and supplemented with material on calculus of finite differences for complex variables. All of the material presented has been previously published. Chapter I, after disposing of elementary questions, treats the general interpolation process for a triangular table, devoting considerable attention to representation and approximation problems. (A minor error—formula (83), used in the proof of the theorem on p. 67, is "unfounded"—is corrected in the paper reviewed below.) Chapter II is an extensive treatment of convergence and regularity properties for the Newton series with interpolation points $1, 2, \cdots$; the chapter concludes with a treatment, largely based on previous published work of the author, of general interpolation points. Chapter III is concerned with the general problem of determining an entire function having given elements. A typical problem of this type is: find all entire functions $F(z)$ satisfying $F^{(n)}(n)=0$, $n=1, 2, \cdots$. A number of such problems are treated in detail. There is a discussion of the connection of problems of this type with certain moment problems in the complex domain, and finally applications of the theory of infinite linear differential equations to problems escaping the mo-

(0010)

ment technique. The fourth chapter is an elementary discussion of summation problems, Bernoulli numbers and polynomials, Euler's formula, etc. Most of Chapter V is concerned with a fairly standard treatment of finite difference equations. The last section of Chapter V, on differential equations of finite order, is based on Gelfond's paper in Trudy Mat. Inst. Steklov. 38, 42–67 (1951) [these Rev. 13, 929]. This book is very much in the spirit of the modern Russian school concerned with the so-called constructive theory of functions, approximative methods for the solution of differential equations, and so forth. The book is a valuable collection of results in these directions. The exposition is excellent. *J. M. Danskin* (Washington, D. C.).

DELONE, B. N.; KUROSH, A. G.; KOLMOGOROV, A. N.; MARKOV, A. A.; GELFOND, A.O.;
MEYMAN, N. N.; VILENKIN, N. Ya.

Algebra

Development of algebra. Usp.mat.nauk 7 No. 3, 1952.

9. Monthly List of Russian Accessions, Library of Congress, November 1952, Uncl.

GEL'FOND, A. O.

USSR/Mathematics - Interpolational Problem 21 May 52

"An Interpolational Problem," A. O. Gel'fond, Corr Mem, Acad Sci USSR

"Dok Ak Nauk SSSR" Vol LXXXIV, No 3, pp 429-432

In the author's book "Calculus of Finite Differences," 1951, in Section 5 of Chapter I, he discusses the general interpolational process for the case of a triangular table of interpolation nodes. In the current article he generalizes and refines the theorems of this section of the book. Submitted 5 Apr 52.

225T46

GEL'FOND, A. O.

USSR/Mathematics - Number Theory Feb 53

"An Elementary Approach to Certain Problems on the Distribution of Primes," A. O. Gel'fond, Chair of Number Theory

Vest Moskov U, Ser Fiz-Mat i Yest Nauk, No 1, pp 21-26

Remarks that in recent years, since the works of A. Selberg, who gave elementary demonstrations of the law governing the distribution of primes in a natural series and the Dirichlet theory on progressions, renewed interest has been shown in elementary approaches to problems of analytical number theory and their

269T84

solutions, which do not rely on an investigation of the analytical properties of the xi function and Dirichlet's L-series. Shows that many consequences of a number character can be drawn with sufficient accuracy from the behavior of the function $x^y/(1-x^y)$, for $/x/< 1$ and $y > 0$. Presented 27 Oct 52.

USSR/Mathematics - Finite Differences Mar/Apr 53

"Perron's Theorem in the Theorem of Difference Equations," A. O. Gel'fond and I. M. Kubenskaya

"Iz Ak Nauk SSSR, Ser Matemat" Vol 17, No 2, pp 83-86

Obtain appreciable results that supplement Perron's theorem concerning the behavior of the solutions of linear difference equations with limit-constant coefficients (i.e., $a_m(x) = a_m$ for $x \to \infty$ and $a_k \neq 0$ for any x, where a_k is coefficient of $f(x)$). Submitted 13 Nov 52.

244T91

GEL'FOND, A. O.

The Committee on Stalin Prizes (of the Council of Ministers USSR) in the fields of science and inventions announces that the following scientific works, popular scientific books, and textbooks have been submitted for competition for Stalin Prizes for the years 1952 and 1953. (Sovetskaya Kultura, Moscow, No. 22-40, 20 Feb - 3 Apr 1954)

Name	Title of Work	Nominated by
Gel'fond, A. O.	"Transcendental and Algebraic Numbers" "Calculus of Finite Differences"	Mathematical Institute imeni V. A. Steklov, Academy of Sciences USSR

SO: W-30604, 7 July 1954

~~GEL'FOND, A. O.~~

USSR/Mathematics - Number theory FD-629

Card 1/1 : Pub. 47-1/5

Author : Gel'fond, A. O.

Title : Beating the natural series against the classes of a group of linear substitutions

Periodical : Izv. AN SSSR, Ser. mat., 18, 297-306, Jul/Aug 1954

Abstract : Discusses certain properties of sequences of whole numbers against which the natural series of a group of linear substitutions is made to beat. Generalizes a Tauber theorem of Hardy. No references.

Institution : --

Submitted : March 1, 1954

GEL'FOND, A.

USSR/Mathematics - Polynomials

Card : 1/1

Authors : Gel'fond, A., Memb. Corres. of Acad. of Sc. USSR.

Title : Polynomials, least deviating from zero together with their derivatives

Periodical : Dokl. AN SSSR, 96, Ed. 4, 689 - 691, June 1954

Abstract : The problem of finding polynomials of the n degree with real coefficients, when the coefficient at the highest term is unity, is described. This problem can be reduced to a problem on polynomials the least deviating from the zero (0), as has been solved by P. L. Chebyshov. The author offers an asymtotically accurate evaluation of numbers, $\sigma_{m, n}$, which go into the polynomials. A solution for a particular case of the problem is given.

Institution : ...

Submitted : March 31, 1954

CHEBYSHEV, P.L.; VINOGRADOV, I.M., akademik, redaktor; GEL'FOND, A.O.; VAVILOV, S.I., akademik, redaktor; PETROVSKIY, I.G., redaktor; BYKOV, K.M., akademik, redaktor; KAZANSKIY, B.A., akademik, redaktor; GERMOGENOV, A.V., redaktor; SHMIDT, O.Yu., akademik, redaktor; ANDREYEV, N.N., akademik, redaktor; SHCHERBAKOV, D.I., akademik, redaktor; YUDIN, P.F., akademik, redaktor; DELONE, B.N., redaktor; KOSHTOYANTS, Kh.S., redaktor; SAMARIN, A.M., redaktor; LEBEDEV, D.M., professor, redaktor; FIGUROVSKIY, N.A., professor, redaktor; KUZNETSOV, I.V., kandidat filosofskikh nauk, redaktor; AUZAN, N.P., tekhnicheskiy redaktor.

[Selected works] Izbrannye trudy. Otvetstvennyi redaktor I.M.Vinogradov. Redaktor-sostavitel' A.O.Gel'fond. Moskva, Izd-vo Akademii nauk SSSR, 1955. 926 p. (MLRA 8:4)

1. Chlen-korrespondent Akademii nauk SSSR (for Delone, Koshtoyants) (Mathematics)

GEL'FOND, A. O.

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Gel'fond, A. O. On uniform approximations by polynomials with integral rational coefficients. Uspehi Mat. Nauk (N.S.) 10, no. 1(63), 41–65 (1955). (Russian)

1 - F/W

Dans l'introduction sont exposés les principaux résultats concernant la possibilité d'approcher les fonctions définies sur un segment par des polynômes à coefficients entiers: l'approximation uniforme d'une fonction f continue sur un segment de longueur l est toujours possible si $l<1$, impossible si $l>4$ (à part les cas triviaux), et elle nécessite certaines conditions arithmétiques si $1<l<4$ [pour des conditions nécessaires et suffisantes, cf. Fekete, C. R. Acad. Sci. Paris **239**, 1337–1339, 1455–1457 (1954); MR **16**, 694]; l'approximation dans L_λ ($\lambda>0$) est possible, sans conditions arithmétiques, si $l<4$ [pour $\lambda=2$, l'auteur cite E. Aparisio, Dissertation, Moscow, 1954]. Les paragraphes suivants sont consacrés à des théorèmes liant la rapidité de l'approximation uniforme sur le segment [0, 1] aux propriétés locales de la fonction f approchie: divisabilité d'ordre m, analyticité, etc.; moyennant les hypothèses naturelles sur f aux points 0 et 1, l'auteur montre que les énoncés de Jackson et S. Bernstein (relatifs à l'approximation par des polynômes quelconques) valent encore, quand il s'agit d'approximation par des polynômes à coefficients entiers.

J. P. Kahane (Montpellier).

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Gel'fond, A. O.

Transactions of the Third All-union Mathematical Congress (Cont.) Moscow, Jun-Jul '56, Trudy '56, V. 1, Sect. Rpts., Izdatel'stvo AN SSSR, Moscow, 1956, 237 pp.

Card 16/80

CARD 1/2 PG - 405

SUBJECT USSR/MATHEMATICS/Number theory
AUTHOR GEL'FOND A.O.
TITLE On an arithmetic equivalent to the analyticity of Dirichlet's L-series on the straight line Re s = 1.
PERIODICAL Izvestija Akad. Nauk 20, 145-166 (1956)
reviewed 11/1956

The author considers some inequations which correspond to the analyticity of the function $L(s,\chi)$ on the straight line Re s = 1. From these inequations the author obtains estimations of finite segments of the series

$$\frac{L'(s,\chi)}{L(s,\chi)} \quad \text{and} \quad \frac{1}{L(s,\chi)}$$

for Re s = 1.
Let $\Lambda(n)$ be Mangold's function

$$\Lambda(n) = \ln p, \quad n = p^k; \qquad \Lambda(n) = 0, \quad n \neq p^k,$$

χ be a character mod m. If $\chi \neq \chi_0$ or $\tau \neq 0$, then the relation

$$(1) \qquad \sum_{n \leq x} \frac{\chi(n)\Lambda(n)}{n^{1+i\tau}} = q \ln x + O(1)$$

can be established with elementary means. Here q is either 0 or -1 and the remainder term for every x is bounded by a constant being independent of x. This relation corresponds to the fact that

$$\frac{L'(s,\chi)}{L(s,\chi)} = -\sum \frac{\chi(n)\Lambda(n)}{n^s}$$

as a logarithmic derivative of an analytic function for $s = 1+i\tau$ is either singular or possesses a simple pole in this point. Therefore the author denotes the elementary proved relation (1) as an arithmetic equivalent of the analyticity of $L(s,\chi)$ on Re s = 1.
Starting from (1) in a somewhat precised form the author obtains the estimations

$$\sum_{n\leq x} \frac{\chi(n)\Lambda(n)-q}{n^{1+i\tau}} = O\left[\ln^7|\tau|\right], \quad \sum_{n\leq x} \frac{\chi(n)\mu(n)}{n^{1+i\tau}} = O\left[\ln^7|\tau|\right]$$

$$q = \begin{cases} 1, & \chi = \chi_0 \\ 0, & \chi \neq \chi_0 \end{cases} \qquad |\tau| \geq 0$$

for arbitrary χ and τ. Therefrom there results an estimation for the increase of $L(s,\chi)$ for $s = 1+i\tau$.

CARD 1/4 PG - 458

SUBJECT USSR/MATHEMATICS/Integral equations
AUTHOR GEL'FOND A.O.
TITLE On estimations of some determinants and the application of these estimations for the investigation of the distribution of eigen-values.
PERIODICAL Mat.Sbornik, n. Ser. 39, 3-22 (1956)
reviewed 12/1956

The author gives some estimations of the generalized Van der Mond determinant

$$\Delta_{n+1} = \begin{vmatrix} x_0^{\alpha_0} & \cdots\cdots & x_0^{\alpha_n} \\ \vdots & & \\ x_n^{\alpha_0} & \cdots\cdots & x_n^{\alpha_n} \end{vmatrix} ,$$

e.g.:
1. If $\delta > 0,\ \alpha_0 = 0,\ 0 < \alpha_1 < \dots < \alpha_n$, then the inequation

$$D_{n+1} = \int_0^1 \dots \int_0^1 \prod_{s=0}^{n} x_s^{\delta-1} \Delta_{n+1}^2 \, dx_0 \dots dx_n < \frac{1}{\delta}(\delta + 2\alpha_1)^{-n} \frac{\alpha_1^2}{(\alpha_1+\delta)^2} e^{-\frac{2\sqrt{2}}{3\sqrt{3}}\pi(n+1)^{3/2} - \gamma_3 n \ln}$$

is valid, where γ_3 does not depend on $\delta, \alpha_1, \ldots, \alpha_n$.

2. Let $m \geq 1$ be integral, $\alpha_k = e^{\pi\sqrt{k}} + \varphi(k)$, $\varphi(k) \geq 0$, $\alpha_k - \alpha_{k-1} \geq m+1$, $\varphi(k) = O(k^{-4} e^{\pi\sqrt{k}})$, $0 \leq k \leq n$. Then

$$\int_0^1 \cdots \int_0^1 \prod_{s=0}^{n} (1-x_s)^{2m} \Delta_{n+1}^2 \, dx_0 \ldots dx_n > e^{-\frac{2}{3}(3m+2)\pi n^{3/2} - \gamma_5 n \ln n},$$

γ_5 - constant.

From the last estimation there follows the theorem: Let

$$K(x,y) = \frac{(1-x)^m (1-y)^m}{1-xy}$$

and let be given the integral equation

$$f(x) = \lambda \int_0^1 K(x,y) f(y) dy.$$

Then

$$D(-r) > e^{\frac{1}{\pi m} \ln^3 r}, \quad r > r_o, \quad r_o \text{ at most depending on } m,$$

where

$$D(\lambda) = \sum_{n=o}^{\infty} \frac{(-\lambda)^n}{n!} \int_o^1 \ldots \int_o^1 \begin{vmatrix} K(x_1,x_1) & \ldots K(x_1,x_n) \\ K(x_n,x_1) & \ldots K(x_n,x_n) \end{vmatrix} dx_1 \ldots dx_n.$$

From this and further estimations there follows: Let the kernel $K(x,y)$ of the integral equation

$$f(x) = \lambda \int_0^1 K(x,y) f(y) dy$$

satisfy the following conditions:

1) $|K(x,y)| < \frac{e^{\gamma_o}}{[xy(1-x)(1-y)]^{1/2-\delta}}, \frac{1}{2} \geq \delta > 0, \quad \gamma_o > 0$ for $0 \leq x \leq 1, \quad 0 \leq y \leq 1$;

2) $K(x,y)$ is analytic and regular with respect to z inside of

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$$\left| z - \frac{1}{2} \pm \frac{1}{2\ \mathrm{tg}\ \frac{\pi}{2q}} \right| \leq \frac{1}{2\sin\frac{\pi}{2q}}, \quad q \geq 1, \quad q \text{ - given.}$$

3)
$$|K(x,y)| < \frac{e^{\gamma_0}}{[x(1-x)]^{\frac{1}{2}-\delta}\ \varepsilon^{\frac{p}{q}}}, \quad p \geq 0, \quad \text{(p arbitrarily fixed)}$$

for $\varepsilon > 0$, $\varepsilon \leq \frac{1}{2}$ and z inside of the contour

$$z = \frac{[1+(1-2\varepsilon)e^{i\varphi}]^{1/q}}{[1+(1-2\varepsilon)e^{i\varphi}]^{1/q} + [1-(1-2\varepsilon)e^{i\varphi}]^{1/q}} \qquad 0 \leq \varphi \leq 2\pi.$$

If these conditions are satisfied, then

$$D(\lambda) < c\ e^{\eta \ln^3 (r+2)}, \quad \eta = \frac{20^2}{28\pi^2}\ \frac{q(p+\frac{q}{2}+\delta)^3}{\delta^3}, \quad c > 0.$$

INSTITUTION: Moscow.

GRADSHTEYN, I.S. (Moscow) ROYZE-BEKETOV, F.S. (Khar'kov); MINLOS, R.A. (Moscow) SKOPETS, Z.A. (Yaroslavl'); GEL'FOND, A.O. (Moscow); YAGLOM, A.M. (Moscow); ROBINSON, R.M. (SShA); DUBNOV, Ya.S. (Moscow); STECHKIN, S.B. (Moscow)

Problems of higher mathematics. Mat. pros. no.1:224-227 '57. (MIRA 11:7)

(Mathematics--Problems, exercises, etc.)

GEL'FOND, A.O. (Moscow)

Approximating algebraic numbers by rational numbers. Mat. pros. no.2:35-50 '57. (MIRA 11:7)

(Numbers, Theory of)

GEL'FOND, O.A.

Some characterising features of L. Euler's ideas in the field of mathematical analysis and his "Introduction into the infinitesimal analysis." Usp.mat.nauk 12 no.4:29-39 Jl-Ag '57. (MIRA 10:10)
(Euler, Leonhard, 1707-1783) (Functional analysis)

AUTHORS: Gel'fond, A.O., Leont'yev, A.F. and Shabat, B.V. SOV/42-13-6-28/33

TITLE: Aleksey Ivanovich Markushevich (on the Occasion of his 50th Birthday) (Aleksey Ivanovich Markushevich (K pyatidesyatiletiyu so dnya rozhdeniya))

PERIODICAL: Uspekhi matematicheskikh nauk, 1958, Vol 13, Nr 6, pp 213-220 (USSR)

ABSTRACT: This is a brief account of the life of A.I.Markushevich: born in 1908 at Petrozavodsk, studied till 1930 under Romanovskiy at Tashkent; aspirant under Lavrent'yev at Moscow. Candidate dissertation on polynomial approximation of analytic functions in 1934. Since 1938 docent at the Moscow State University. Doctor dissertation in 1944 on approximations and expansions of functions in series. 1950 vice-president of the Academy of Pedagogical Sciences. 1958 first deputy of the minister of education of the RSFSR (Russian Soviet Federated Socialist Republic). His pupils are: N.A.Davydov, G.Ts.Tumarkin, S.Ya. Khavinson. There follows a list of 83 publications (1928-1957) and a photo of Markushevich.

Card 1/1

GEL'FOND, Aleksandr Osipovich; SOLOV'YEVA, L.A., red.; KRYUCHKOVA, V.N., tekhn.red.

[Calculation of finite differences] Ischislenie konechnykh raznostei. Izd.2., dop. Moskva, Gos.izd-vo fiziko-matem.lit-ry, 1959. 400 p. (MIRA 13:5)

(Difference equations)

PHASE I BOOK EXPLOITATION SOV/4070

Gel'fond, Aleksandr Osipovich

Ischisleniye konechnykh raznostey (Calculus of Finite Differences). 2d ed., enl. Moscow, Fizmatgiz, 1959. 400 p. 8,000 copies printed.

Ed.: L.A. Solov'yeva; Tech. Ed.: V.N. Kryuchkova.

PURPOSE: This is a textbook for university mathematics students.

COVERAGE: The book discusses the general theory of finite differences, the interpolation problem, Newton's series, entire functions, construction of entire functions with given elements, summing of functions, Bernoulli numbers and Bernoulli polynomials, and finite differences equations. The book has been published in East Germany, Red China, Czechoslovakia, Rumania and other countries. There are 20 references: 16 Soviet, 2 English, 1 German, and 1 French. The author mentions textbooks by I.P. Natanson, A. Markov, D. Selivanov, and N. Nerlund.

Card 1/10

Gel'fond, A. O.

16(0) PHASE I BOOK EXPLOITATION SOV/3177

Matematika v SSSR za sorok let, 1917-1957. tom 1: Obzornyye stat'i (Mathematics in the USSR for Forty Years, 1917-1957) Vol 1: Review Articles) Moscow, Fizmatgiz, 1959. 1002 p. 5,500 copies printed.

Eds: A. G. Kurosh, (Chief Ed.), V. I. Bityutskov, V. G. Boltyanskiy, Ye. B. Dynkin, G. Ye. Shilova, and A. P. Yushkevich; Ed. (Inside book): A. P. Lapko; Tech. Ed.: S. N. Akhlamov.

PURPOSE: This book is intended for mathematicians and historians of mathematics interested in Soviet contributions to the field.

COVERAGE: This book is Volume I of a major 2-volume work on the history of Soviet mathematics. Volume I surveys the chief contributions made by Soviet mathematicians during the period 1947-1957; Volume II will contain a bibliography of major works since 1917 and biographic sketches of some of the leading mathematicians. This work follows the tradition set by two earlier works: Matematika v SSSR za pyatnadtsat' let (Mathematics in the USSR for 15 Years) and Matematika v SSSR za tridtsat' let (Mathematics in the USSR for 30 Years). The book is divided into the major divisions of the field, i.e., algebra, topology, theory of probabilities, functional analysis, etc., and contributions and outstanding problems in each discussed. A listing of some 1400 Soviet mathematicians is included with references to their contributions in the field.

16(2) 16.1000

AUTHOR: Gel'fond, A.O. SOV/38-23-6-1/11

TITLE: On a Common Property of Computation Systems

PERIODICAL: Izvestiya Akademii nauk SSSR, Seriya matematicheskaya, 1959, Vol 23, Nr 6, pp 809 - 814 (USSR)

ABSTRACT: Let $\theta > 1$ be real. Each number α, $0 < \alpha \leq 1$ is uniquely representable by

$$(1)\ \alpha = \sum_1^{\infty} \frac{\lambda_n}{\theta^n} = \sum_1^n \frac{\lambda_k}{\theta^k} + \frac{x_{n+1}}{\theta^n}, \quad 0 \leq x_n < 1, \quad x_n = x_n(\alpha)$$

where all λ_n are integer, $0 \leq \lambda_k < \theta$ and

$$(2)\ x_1 = \alpha,\ x_2 = \{\theta x_1\},\ \ldots,\ x_{n+1} = \{\theta x_n\},\ \ldots,$$
$$\lambda_1 = [\alpha\theta],\ \ldots,\ \lambda_n = [\theta x_n],\ \ldots.$$

where $\{x\}$ and $[x]$ denote the fractional and the integral part of x. Let $\psi(x) = 1$ for $0 \leq x \leq 1$ and $= 0$ for $0 > x$, $x > 1$. If for the sequence $x_n(\alpha)$ there exists the limit value

$$(4) \quad \lim_{N\to\infty} \frac{1}{N} \sum_{1}^{N} \psi(1 - t + x_n) = \sigma(\psi)$$

and if $\sigma(t)$ is equal for almost all α, then $\sigma(t)$ is denoted as normal law of distribution. For the determination of the normal law of distribution of the numbers $x_n(\alpha)$ for arbitrary non-integer θ the author defines the number t_k for $\alpha = 1$ according to (2) and puts

$$(5) \quad \sigma_0(t) = \frac{1}{\tau} \sum_{1}^{\infty} \frac{\psi(1-t_k + t)}{\theta^{k-1}}, \quad \tau = \sum_{1}^{\infty} \frac{t^k}{\theta^{k-1}}, \quad 1 = \sum_{1}^{n} \frac{\lambda_n}{\theta^k} + \frac{t_{n+1}}{\theta^n}$$

Theorem I : If $\theta > 1$ is a non-integer number, then for almost all α there holds the relation

$$(6) \quad \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \psi(1-t+x_k) = \sigma(t) = \int_0^t \sigma_0(x)dx = \frac{1}{\tau} \sum_{k=1}^{\infty} \frac{\min(t,t_k)}{\theta^{k-1}}, \quad t_1 = 1$$

On a Common Property of Computation Systems SOV/38-23-6-1/11

Theorem II : If Θ is an integral algebraic number with the property that all the conjugate numbers of it are <1 according to the absolute value, then $\sigma_0(x)$ has finitely many points of discontinuity.

There are 2 non-Soviet references, 1 of which is Swedish, and 1 Italian.

SUBMITTED: May 14, 1959

S/038/60/024/03/01/008

AUTHORS: Gel'fond, A.O., and Sarmanov, O.B.

TITLE: On the Occasion of the 80th Birthday of Sergey Natanovich Bernshteyn

PERIODICAL: Izvestiya Akademii nauk SSSR, Seriya matematicheskaya, 1960, Vol. 24, No. 3, pp. 309-314

TEXT: This is a survey on the papers of S.N. Bernshteyn on the theory of differential equations, the theory of approximations of functions and probability calculus, and a list of Bernshteyn's publications during the years 1950 - 1959 with 20 titles.
The authors mention P.L. Chebyshev, Ye.I. Zolotarev, A.A. Markov and N.I. Akhiyezer.

✓

Card 1/1

TROST, Ernst; FEL'DMAN, N.I. [translator]; GEL'FOND, A.O., red.

[Prime numbers] Prostye chisla. Moskva, Gos.izd-vo fiziko-matem.lit-ry, 1959. 135 p. Translated from the German. (MIRA 14:2)

(Numbers, Prime)

88326

S/038/60/024/004/002/010 XX
C 111/ C 333

16.3900

AUTHOR: Gel'fond, A. O.

TITLE: On Some Functional Equations Which are Deductions of Equations of Riemannian Type

PERIODICAL: Izvestiya Akademii nauk SSSR, Seriya matematicheskaya, 1960, Vol. 24, No. 4, pp. 469-474

TEXT: A function f(s) is said to belong to the class R, if it has the form

$$f(s) = \zeta^{p_0}(s) \prod_1^k L^{p_n}(s, \chi_n), \quad \sum_0^k p_n = p$$

(1)

$$\zeta(s) = \sum_1^\infty n^{-s}, \quad L(s, \chi_n) = \sum_{m=1}^\infty \chi_n(m) m^{-s}$$

where all χ_n are primitive non-principal characters of certain moduli D_n, and all p_i are nonnegative integers. The function f(s) satisfies the equation

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$$(2)\quad \begin{aligned} \bar{f}(s) &= e^{-i\gamma} 2^{p}(2\pi)^{-ps} D^{p(s-1/2)}\, \Gamma^{p}(s) \prod_{1}^{p} \cos\frac{\pi}{2}(\theta_k - s) f(s) \\ \bar{f}(s) &= \zeta^{p_0}(s) \prod_{k} L^{p_\nu}(s, \bar{\chi}_\nu) \end{aligned}$$

where $\theta_k = 0, 1, \gamma$ and $D \geq 1$ are real and $p \geq 1$ integer. Furthermore, let $\psi_q(x)$ be defined by

$$(3)\quad \psi_q(x) = \frac{1}{2\pi i} \int_{\sigma} \left(\frac{e^{z} - e^{-z}}{2z} \right)^{q} e^{xz} \frac{dz}{z}$$

where the integral on the straight line Re $z = \sigma > 0$ is taken in positive direction, $q \geq 0$ is integer and x real. Let the function u(s) be defined by

$$(7)\quad u(s) = 2^{p}(2\pi)^{-ps} D^{p(s-1/2)}\, \Gamma^{p}(s) \prod_{1}^{p} \cos\frac{\pi}{2}(\theta_k - s)$$

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where $\theta_k = 0,1$.

Theorem I: If f(s) belongs to the class R, where (2) is satisfied and $q > \frac{p}{4} - 1$, then for $\alpha > 0$, $m_1 \geq 0$, $m_2 \geq 0$ in $\frac{1}{2} \leq \mathrm{Re}\, s \leq 1$ it holds the representation

$$f(s) = \frac{1}{2\pi i}\int_{(0)} \left(\frac{e^{\alpha z} - e^{-\alpha z}}{2\alpha z}\right)^q \left[e^{-m_2 z}\frac{u(z+s)}{u(s)} - e^{m_1 z}\right] f(z+s)\frac{dz}{z} +$$
$$+ \sum_1^{n_1} \frac{a_n}{n^s}\psi_q\left(\frac{m_2 - \ln n}{\alpha}\right) + \frac{e^{i\gamma}}{u(s)}\sum_1^{n_2}\frac{\bar{a}_n}{n^s}\psi_q\left(\frac{m_1 - \ln n}{\alpha}\right) - R_1(s) - R_2(s),$$
$$n_1 = [\exp(m_2 + \alpha q)], \quad n_2 = [\exp(m_1 + \alpha q)], \tag{10}$$

where the integral is taken on the straight line Re z = 0, $\psi_q(x)$ is defined by (3), the numbers a_n and $\bar{a}_n$ are the coefficients of the expansions

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$$f(s) = \sum_{1}^{\infty} \frac{a_n}{n^s}, \quad \overline{f}(s) = \sum_{1}^{\infty} \frac{\overline{a}_n}{n^s}$$

and finally

(11)

$$R_1(s) = \frac{1}{(p_0-1)!} \frac{d^{p_0-1}}{dz^{p_0-1}} \left[\frac{(z+s-1)^{p_0} f(z+s)}{z} \left(\frac{e^{\alpha z} - e^{-\alpha z}}{2\alpha z} \right)^q \right]_{z=-s}$$

$$R_2(s) = \frac{\gamma_1}{u(s)(p_0-1)!} \frac{d^{p_0-1}}{dz^{p_0-1}} \left[\frac{(z+s)^{p_0} f(1-z-s)}{z} \left(\frac{e^{\alpha z} - e^{-\alpha z}}{2\alpha z} \right)^q \right]_{z=-s}$$

From theorem I there can be obtained approximation equations for the L-series:

Theorem II: If $p \geq 1$, $q > \frac{p}{6} + \mu + 1$, $\mu \geq 0$ are integers,

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$$1 \geq \alpha > \tau^{-1/2}, \quad m_1 = m_2 = m = \frac{1}{2}\lambda'(\tau), \quad s = \frac{1}{2} + i\tau ,$$

$$\lambda'(\tau) = \frac{u'(s)}{u(s)}, \quad u(s) = \left[2(2\pi)^{-s}\Gamma(s) \cos \frac{\pi s}{2} \right]^p$$

then it holds the approxiamtion equation

$$\zeta^p(s) = \sum_{n < \exp(m - aq)} \left[\nu_p(n) n^{-s} + \frac{\nu_p(n)}{u(s)} n^{s-1} \right] +$$
$$+ \sum_{\exp(m-aq) < n < \exp(m+aq)} \left[\frac{\nu_p(n)}{n^s} \psi_q\left(\frac{m - \ln n}{\alpha}\right) + \frac{\nu_p(n)}{u(s)\, n} \psi_q\left(\frac{m - \ln n}{\alpha}\right) \right] +$$
$$+ \sum_{\exp(m-aq) < n < \exp(m+aq)} \frac{1}{n^s} \sum_{1}^{\mu} \frac{c_k}{\alpha^k \tau^{\frac{k}{2}}} \psi_q^{(k)}\left(\frac{m - \ln n}{\alpha}\right) + O\left((\alpha\sqrt{\tau})^{-\mu-1} \tau^{\frac{p}{6}}\right), \quad (13)$$

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On Some Functional Equations Which are Deductions of Equations of Riemannian Type

where $\nu_p(n)$ is the number of the solutions of the equation $x_1 \ldots \ldots x_p \leq n$ in x_k, $\psi_q(x)$ defined by (3), and $c_k = O(1)$ in τ and do not depend on $\varkappa$.

Let $\chi(n)$ denote the primitive character of the module D and let

$$u(s) = 2^p D^{p(s-1/2)} \Gamma^p(s) \cos^p \frac{\pi}{2} (\theta - s), \tag{14}$$

$$m = \frac{1}{2} \frac{u'(s)}{u(s)}, \quad s = \frac{1}{2} + i\tau, \quad \theta = \frac{1}{2} \left[1 - \chi(-1) \right].$$

Theorem III: If $p \geq 1$, $q \geq \frac{p}{4} + \mu + 1$, $\mu \geq 0$ are integers, $1 \geq \alpha > \tau^{-1/4}$, $m_1 = m_2 = m$, then it holds approximatively

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On Some Functional Equations Which are Deductions of Equations of Riemannian Type

$$L^p(s,\chi) = \sum_{n<\exp(m-aq)} \left[\frac{\chi(n)\,\nu_p(n)}{n^s} + \frac{e^{\gamma i}}{u(s)} \frac{\overline{\chi}(n)\,\nu_p(n)}{n^{1-s}} \right] +$$

$$+ \sum_{\exp(m-aq)<n<\exp(m+aq)} \left[\frac{\chi(n)\nu_p(n)}{n^s} + \frac{e^{\gamma i}}{n(s)} \frac{\overline{\chi}(n)\,\nu_p(n)}{n^{1-s}} \right] \psi_q\left(\frac{m-\ln n}{\alpha}\right) +$$

$$+ \sum_{\exp(m-aq)<n<\exp(m+aq)} \frac{1}{n^s} \sum_{s}^{\mu} \frac{c_k}{\alpha^k \tau^{\frac{k}{2}}} \psi^{(k)}\left(\frac{m-\ln n}{\alpha}\right) + O\left((\alpha\sqrt{\tau})^{-\mu-1} (D\tau)^{\frac{p}{4}}\right), \quad (15)$$

where $\nu_p(n)$ and c_k are as before, while γ depends on $\chi(n)$ in a certain way, $\tau \geq 1$.

Theorem IV: It holds the estimation

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$$\varphi(t)=\frac{1}{\pi}\int_{-\sqrt{\ln t}}^{\sqrt{\ln t}}\left(\frac{\sin \alpha x}{\alpha x}\right)^{q}\sin\frac{x}{2}\,\ln\frac{t}{2\pi}\cdot\varphi(t+x)\,dx+O(e^{\sqrt{\ln t}}), \qquad (18)$$

$$\alpha=2(\ln t)^{-\frac{1}{2}},\quad q=\ln t,\quad \varphi(x)=\zeta(s)\left[2(2\pi)^{-s}\Gamma(s)\cos\frac{\pi s}{2}\right]^{\frac{1}{2}},$$

$$s=\frac{1}{2}+ix,\quad x>1,\quad t>1.$$

There are 7 references: 1 Soviet, 1 American, 2 Japanese, 2 German and 1 English.

SUBMITTED: January 27, 1960

Card 8/8

ZALGALLER, V.A. (Leningrad); RUDENKO, N. (Moskva); DAVYDOV, U. (Gomel'); RABINOVICH, V. (Petropavlovsk-Kazakhstanskiy); BESKIN, L.N. (Moskva); TAMATAR, I.Ya. (Moskva); SKOPETS, Z.A. (Yaroslavl'); DUBNOV, Ya.S. (Moskva); GEL'FOND, A.O. (Moskva); ROBINSON, R.M. (SShA); BALK, M.B. (Smolensk); SHUB-SIZONENKO, Yu.A. (Moskva)

Solutions to the problems. Mat. pros. no.5:261-274 '60. (MIRA 13:12)

(Mathematics--Problems, exercises, etc.)

KASSELS, V.S. [Cassels, John William Scott]; POLOSUYEV, A.M. [translator]; GEL'FOND, A.O., red.

[Introduction to diophantine approximation] Vvedenie v teoriiu diofantovykh priblizhenii. Pod red. i s dop. A.O. Gel'fona. Translated from the English. Moskva, Izd-vo inostr. lit-ry, 1961. 212 p. (MIRA 15:2)
(Diophantine analysis)

GEL'FOND, A.O.; SARMANOV, O.V.

Sergei Natanovich Bernshtein; on his 80th birthday. Izv.AN SSSR Ser.mat. 24 no.3:309-314 My-Je '61. (MIRA 14:4)

(Bernshtein, Sergei Natanovich, 1880-)

GEL'FOND, A.O.; LINNIK, Yu.V.; CHUDAKOV, N.G.; YAKUBOVICH, V.A.; LINNIK, IU.V.; CHUDAKOV, N.G.; IAKUBOVICH, V.A.

An incorrect work of N.I.Gavrilov. Usp.mat.nauk 17 no.1:265-267 Ja-F '62. (MIRA 15:3)

(Functions, Zeta)
(Gavrilov, N.I.)

GEL'FOND, Aleksandr Osipovich; LINNIK, Yuriy Vladimirovich. Prinimali uchastiye: VINOGRADOV, A.I.; MANIN, Yu.I.; KARATSUBA, A.A., red.; AKSEL'ROD, I.Sh., tekhn. red.

[Elementary methods in the analytical theory of numbers] Elementarnye metody v analiticheskoi teorii chisel. Moskva, Fizmatgiz, 1962. 269 p. (MIRA 16:3)
(Numbers, Theory of)

ACC NR: AP7011373

SOURCE CODE: UR/0039/66/071/003/0289/0296

AUTHOR: Gel'fond, A. O. (Moscow)

ORG: none

TITLE: Evaluation of the imaginary parts of the roots of polynomials with the bounded derivatives of their logarithms on the real axis

SOURCE: Matematicheskiy sbornik, v. 71, no. 3, 1966, 289-296

TOPIC TAGS: root calculation, polynomial

SUB CODE: 12

ABSTRACT: The n'th degree polynomial $P_N(x)$ satisfies the inequality

$$\left|\frac{P_n'(x)}{P_n(x)}\right| = \left|\sum_{k=1}^{n}\frac{1}{x-z_k}\right| < M, \quad |x| < \infty, \quad z_k = x_k + iy_k.$$

It has been shown by previous authors that $|y_k| > \gamma_n(M) > 0.$ and

$\gamma_n(M) = M^{-1}\gamma_n, \ \gamma_n > \frac{1}{2}q_0^n$ for which $q = \sqrt{2} - 1 = 0{,}414 \ldots, \ q = 0{,}649 \ldots$

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UDC: 512.31

0931 1765

ACC NR: AP7011373

In this article the author obtains further results in this direction by means of 4 lemmas. In particular, he proves the inequality

$$\gamma_n > \frac{1}{17 \ln n}, \quad n > n_0.$$

Orig. art. has: 31 formulas. [JPRS: 40,393]

Card 2/2

GEL'FOND, A.S.

Local electric cementation with solid carburizing agents. A. S. Gel'fond. Russ. 54,224, Nov. 11, 1938. Local cementation is effected by using electrodes of different cross sections so as to obtain a current of greater density in the section to be cemented.

ASM-SLA METALLURGICAL LITERATURE CLASSIFICATION *ca.*

Accelerating cementation. A. S. Gel'fond. Russ 54,177, Nov. 30, 1938. Cementation is accelerated and the elec. current is distributed more uniformly in the carburizing agent by continuous introduction of heat from the outside into the cementation chamber.

ASB-SLA METALLURGICAL LITERATURE CLASSIFICATION

32-7-20/49

AUTHOR: Gel'fond, A.S.

TITLE: The Metallographical Method of Investigating Plastic Deformation (Metallograficheskiy metod issledovaniya plasticheskoy deformatsii)

PERIODICAL: Zavodskaya Laboratoriya, 1957, Vol. 23, Nr 7, pp. 826 - 827 (USSR)

ABSTRACT: Here the method is recommended for the investigation of plastic deformation on samples of steel containing a low percentage of carbon, with a clearly marked structure of striped perlite, which undergoes modifications after pressure has been exercised. For this purpose the original distance between two perlite stripes is measured and compared with that found in the deformed state. This experiment is carried out with a perlite shaving produced on a lathe, which is then sawed off parallel to the stripe and then ground. Micro-grinding is carried out vertical to the stripes. Investigation of the microstructure makes the main process of the production of shavings appear to be one of the displacement of the sawn-off layer. On the micro-picture it is then possible to measure the angle of displacement as well as the angle of orientation of the structure with respect to the direction of cutting. There are 2 tables.

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32-7-20/49

The Metallographical Method of Investigation Plastic Deformation

ASSOCIATION: Semiluki Works for Fireproof Material
(Semilukskiy ogneupornyy zavod)

AVAILABLE: Library of Congress

Card 2/2

Gel'fond, A.S.

AUTHOR: Gel'fond, A. S. 131-2-8/10

TITLE: The Rebuilding of RP M-2 Rotor Excavators (Rekonstruktsiya rotornykh ekskavatorov RP M-2).

PERIODICAL: Ogneupory, 1958, Nr 2, pp. 88-95 (USSR)

ABSTRACT: The excavators were constructed by the Zuyev factories of the Ministry for Electric Power Plants of the USSR for the purpose of coal loading, they were, however, unsuited for this purpose. They also proved to be not strong enough for clay excavation. They were reinforced and rebuilt at comparatively low costs. The advantage of the original machine consisted of the fact, that it was joined together from independent joint mechanisms with individual drive, which facilitates maintenance work. The low specific ground pressure (0,64 kg/cm^2), its small outer dimensions and the weight of these machines comply with the conditions of pit operation. At present, five of such rebuilt excavators operate in the pits of the Semilukek works for refractory materials. The principal deficiencies of the old excavators and the method of their removal are subsequently specified. The old rotor is shown in figure 1, which was replaced by the new one

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(figure 2). In figures 3 and 4 the new model is described. Figure 5 shows the rebuilt drive of the rotor, which is subsequently described. In figure 8 the rebuilding of the bearing of the drive shaft is illustrated and in figure 9 the same is done for the transporter drive. Figure 10 shows an total view of the rebuilt rotor excavator, the table containing its technical characteristic previous to and after rebuilding.
There are 10 figures, 1 table.

ASSOCIATION: Semiluki Plant of Refractory Materials (Semilukskiy ogneupornyy zavod).

AVAILABLE: Library of Congress

Card 2/2

GEL'FOND A

AUTHORS: Gel'fond, A., Karandeyev, K., Chistyakov, N., Shumilovskiy, N., Levin, M., Yermakov, V., Kobrinskiy, N., and others 105-58-4-35/37

TITLE: V. N. Mil'shteyn (Deceased)

PERIODICAL: Elektrichestvo, 1958, Nr 4, pp. 94-94 (USSR)

ABSTRACT: Obituary notice. On January 9, 1958 Professor Viktor Naumovich Mil'shteyn, Dr. of Technical Sciences died at the age of 44. After he finished the Moskau Institute for Power Engineering he worked in industry and as pedogogue. In 1938 he became Candidate and in 1945 Dr. of Technical Sciences. Since then he was Director of the Chair for Electric and Automatic Apparatus at the Moskau Institute for Aviation imeni Ordzhonikidze. In 1949 he changed over to the Scientific Research Institutes for Systems at the Committee for Standards, Measures and Measuring Apparatus. At the same time he worked as pedogogue at the Penza Institute for Industry and then at the Moskau Electrotechnical Institute for Telecommunications. He wrote many

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V. N. Mil'shteyn (Deceased) 105-58-4-35/37

publications and many inventions were made by him. His scientific work included the field of theoretical electrical engineering and radio engineering as well as the problems on the theory and the calculation of measuring instruments, automation elements and electromagnetic mechanisms. Before his death he had his monography "The Energetic Relations in Electrical Measuring Instruments" printed. There are 1 figure.

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1. Obituary

Card 2/2

AUTHOR: Gel'fond, A. S. 131-58-6-6/14

TITLE: Rotor Excavator for Exposing a Layer of Refractory Clay (Rotornyy ekskavator dlya zachistki plasta ognoupornoy gliny)

PERIODICAL: Ognoupory, 1958, Nr 6, pp. 264-267 (USSR)

ABSTRACT: At the Latnenskoye deposit clay of first quality is located in the top layers. It is therefore necessary to carry out its exposure with the least possible losses. In 1956 an excavator was constructed for its exposure according to a proposal by S. I. Konyayev, however, it turned out to be insufficient as regards its construction. The author of this article together with I. G. Pashchenko, V. A. Korobkin and M. S. Gribanov developed and introduced a rotor construction of the excavator for the exposure work; it can be seen in figure 1. In the rotor there is a short belt conveyor which transports the rocks to the receiving transporter. The drive is arranged by a motor and a reducing gear (figure 2). Then the formulae of calculation

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Rotor Excavator for Exposing a Layer of Refractory Clay

131-58-6-6/14

for the simultaneously operating number of buckets and the span are mentioned, which amounts to 67 mm, and then the construction of the excavator is described in detail. In figure 3 the scheme of the cutting process is shown. At the "Strelitsa" deposit this excavator was introduced and hand work was abolished. The excavator is operated by an engineer and an assistant. Thus the expenses of exposure works decreased abruptly. The excavator, however, is still to be altered and modernized. In figure 4 the scheme of this excavator is shown. Among other the following technical data of the excavator are mentioned: rotor diameter 3,3 m; bucket number 24; bucket content 25 liters; rotor revolutions 6 per minute; number of excavations per minute 144; cutting velocity 1,03 m/second; cutting power - 1120 kg; motor - 14 kW; theoretical capacity of excavator 120 m^2/hour; total weight of the excavator 30 t. There are 4 figures.

ASSOCIATION: Semilukskiy ogneupornyy zavod (Semiluki Works of Refractories)

Card 2/2

1. Earth moving equipment--Design 2. Earth moving equipment --Performance 3. Refractory materials--Production

SOV-127-58-8-11/27

AUTHOR: Gel'fond, A.S., Engineer

TITLE: The Construction of Buckets for Rotor Excavators (O konstruktsii ispolnitel'nogo organa rotornykh ekskavatorov)

PERIODICAL: Gornyy zhurnal, 1958, Nr 8, pp 50-54 (USSR).

ABSTRACT: After the war, rotor excavators were used for extraction of fire-resistant clay at various deposits of the Union. The author describes their use at the Semilukkskiy ogneupornyy zavod (The Semiluki Fire-Resistant Plant) and Chelayabinskoye Rudoupravleniye (Chelabinsk Mining Administration). The author is concerned with finding the most practical model for work both in frozen and non-frozen ground. Practice showed that the ten bucket excavator was the most appropriate type. The use of these excavators with specially constructed cogs increased the production of clay by 25% in 1957. There is 1 graph, 5 diagrams and 2 tables.

ASSOCIATION: Semilukskiy ogneupornyy zavod (Semiluki Plant of Refractory Materials)

1. Clays 2. Earth moving equipment

Card 1/1

AUTHOR: Gel'fond, A. S. SOV/131-58-8-7/12

TITLE: The Repair of Individual Parts of the Electrical Equipment of the Excavator SE - 3 (Restavratsiya detaley elektrooborudovaniya ekskavatora SE - 3)

PERIODICAL: Ogneupory, 1958, Nr 8, pp 377-379 (USSR)

ABSTRACT: These examples of repair work were taken from the practice of the mining department of the **Semiluki** factory of refractory materials. Figure 1 shows a centering device for mounting the motor and the generator, which is described. The repair of bearing seats is carried out in three different ways: by placing a sleeve on to the shaft journal (Fig 2), which can be recommended only for small electromotors; by mounting the bearing by means of an adhesive if wear is not considerable; by restoring the previous dimensions by metallization (Fig 3). A metal-spraying device is shown by figure 4. Re-turning of the collectors is usually carried out on the excavator without removing the motor and generator, the rotor of the generator being driven by the mains-electromotor. There are 4 figures.

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The Repair of Individual Parts of the Electrical Equipment of the Excavator SE - 3 SOV/131-58-8-7/12

ASSOCIATION: Semilukskiy ogneupornyy zavod (Semiluki Factory of Refractories)

Card 2/2

25(2) SOV/118-59-2-10/26

AUTHOR: Gel'fond, A.S., Engineer

TITLE: Loading Machines of Continuous Operation (Pogruzochnyye mashiny nepreryvnogo deystviya)

PERIODICAL: Mekhanizatsiya i avtomatizatsiya proizvodstva, 1959, Nr 2, pp 32-34 (USSR)

ABSTRACT: The article deals with multi-bucket and rotary excavators with a bucket capacity ranging from 100 to 3,600 liters, mainly utilized in coal and iron ore open-cut mines and in extracting fire clay. The Gornoye upravleniye Semilukskogo ogneupornogo zavoda (Mining Administration of the Semiluki Refractory Plant) confirms the expediency of using excavators of continuous operation for various loading activities. The multi-bucket excavator (produced by the Mechanical Repair Shop of the Semiluki Refractory Plant) is serviced by 1 operator and eliminates the need for 10 workers. Its productivity is

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SOV/118-59-2-10/26

Loading Machines of Continuous Operation

55 tons per hour, its bucket capacity is 35 liters. There are 9 buckets. The number of scoopings is 1,042 per hour. The length of the bucket frame is 4.15 m. The tractive effort of the bucket chain is 2,700 kg. Its scooping height is 2.5 m. The excavator is driven by two motors with a total capacity of 17.7 kw. The Mining Management has also designed, and is producing, a rotary excavator with a bucket capacity of 20 liters to be used in coal stores of the Strelitsa and Bakhcheyevo mines for the fueling of locomotives, storing of coal and loading of trucks. The following technical data are given: diameter of the rotor - 2.2 m, number of buckets - 6, bucket capacity - 20 liters, bucket width - 0.35 m, number of rotations - 10 per minute, number of pourings - 3,600 per hour and productivity - 72 cu m per hour. The total capacity of the motors is - 48.8 kw. There are 5 diagrams, and 2 photographs.

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14 (5)

AUTHOR: Gel'fond, A. S. SOV/131-59-5-5/12

TITLE: Two-bracket Conveyer Belt Reloader (Dvukhkonsol'nyy lentochnyy peregruzhatel')

PERIODICAL: Ogneupory, 1959, Nr 5, pp 212-215 (USSR)

ABSTRACT: In the Mining Administration of the Semiluki Works, a self-propelled reloader 35 m long was designed and manufactured on the basis of the dredge RPM-2 as can be seen from figure 1. It consists of a distributing conveyer belt 20 m long, and a taking-over conveyer belt 15 m long, which are mounted on a turntable. Both the conveyer belts have the form of chutes with no side walls. The new design of reloader (Fig 2) permits a permanent unloading point to be maintained, and the excavation depth and the unloading height to be increased. Table 1 shows the total output of the electric motors while table 2 gives a technical description. Conclusion: The use of a two-bracket reloader permits the shifting of transport tracks to be reduced, thus decreasing the number of operators and facilitating their work. There are 3 figures, 2 tables, and 2 Soviet references.

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Two-bracket Conveyer Belt Reloader SOV/131-59-5-5/12

ASSOCIATION: Semilukskiy ogneupornyy zavod (Semiluki Works of Refractories)

Card 2/2

GEL'FOND, A.S., inzh.

Improving the wheel of the excavator for working frozen ground.
Stroi. i dor. mash. 7 no.9:12-13 S '62. (MIRA 15:10)
(Excavating machinery) (Frozen ground)

ACC NR: AP7003850 (A) SOURCE CODE: UR/0122/67/000/001/0068/0070

AUTHORS: Zotova, L. K. (Candidate of technical sciences, Docent); Shashkin, A. S. (Candidate of technical sciences); Gel'fond, A. S. (Engineer)

ORG: none

TITLE: New chip-breaking mechanisms for turning lathes

SOURCE: Vestnik mashinostroyeniya, no. 1, 1967, 68-70

TOPIC TAGS: lathe, cutting tool, high speed metal cutting, alloy, steel/ T5K10 alloy, 1K62 lathe, 1Kh18N9T steel, 2Kh13 steel, 40Kh steel, 30KhGSNA steel

ABSTRACT: Gear-and-lever chip-breaking mechanisms for lathes are described. The breaking method used here was described earlier by L. K. Zotova, A. S. Shashkin, and A. S. Gel'fond (Universal'nyye struzhkolomatel'nyye mekhanizmy dlya tokarnykh stankov. M., GOSINTI, No. 21-64-763/8, 1964). The mechanisms provide diverse and controllable rules for cutting-tool feed (see Fig. 1). Control tests were performed with T5K10 hard alloy and with 1Kh18N9T, 2Kh13, 40Kh, and 30KhGSNA steels. It was found that intermittent cutting ensured reliable and stable chip breaking over a fairly wide range of machinable materials, cutting conditions, and tool geometry. High-speed turning with the use of the described mechanisms provides safer operating conditions, since the chips are obtained in the form of short, coiled spirals. The mechanism permits the lathe to be switched from intermittent to continuous cutting. Application

 UDC: 621.941.014.8

ACC NR: AP7003850

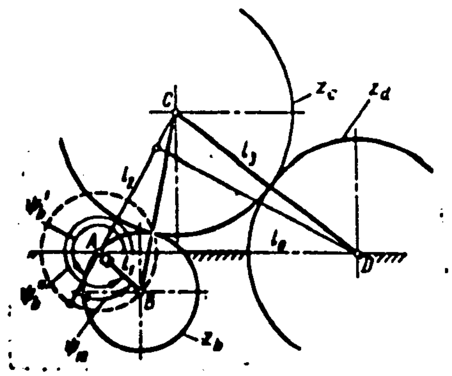

Fig. 1.

of these mechanisms does not interfere with the general-purpose operation of a lathe when properly designed. Orig. art. has: 2 formulas, 2 diagrams, and 3 graphs.

SUB CODE: 13/ SUBM DATE: none/ ORIG REF: 005

Card 2/2

GEL'FOND, E.I. (Mamadysh)

A case of dissecting aortic aneurysm. Kaz. med. zhur. 4:61 Jl-Ag'63 (MIRA 17:2)

GELFOND G.M.

LEVCHENKO, G.I., admiral, otvetstvennyy red.; DEMIN, L.A., dots., kand. geogr. nauk, inzh.-kontr-admiral, glavnyy red.; FRUMKIN, N.S., polkovnik, zamestitel' otvetstvennogo red.; ABAN'KIN, P.S., admiral, red.; ALAFUZOV, V.A., prof., kand. voenno-morskikh nauk, admiral, red.; ANAN'ICH, V.Ye., kontr admiral zapasa, red.; ACHKASOV, V.I., kand. istor. nauk, kapitan 1 ranga, red.; BARANOV, A.N., red.; BELLI, V.A., prof., kontr-admiral v otstavke, red.; BESKROVNYY, L.G., prof., doktor istor. nauk, polkovnik zapasa, red.; BOLTIN, Ye.A., kand. voen. nauk, general-mayor, red.; VERSHININ, D.A., kapitan 1 ranga, red.; VITVER, I.A., prof., doktor geogr. nauk, red.; GEL'FOND, G.M., dots., kand. voenno-morskikh nauk, kapitan 1 ranga, red., GLINKOV, Ye.G., inzh.-kontr-admiral v otstavke, red.; YELISEYEV, I.D., vitse-admiral, red.; ZOZULYA, F.V., admiral, red.; ISAKOV, I.S., prof., Admiral Flota Sovetskogo Soyuza, red.; KAVRAYSKIY, V.V. [deceased], prof., doktor fiz.-mat. nauk, inzh.-kontr-admiral v otstavke, red.; KALESNIK, S.V., red.; KOZLOV, I.A., dots. kand. voenno-morskikh nauk, kapitan 1 ranga, red.; KOMAROV, A.V., vitse-admiral, red.; KUDRYAVTSEV, M.K., general leytenant tekhnicheskikh voysk, red.; LYUSHKOVSKIY, M.V., dots., kand. istor. nauk, polkovnik, red.; MAKSIMOV, S.N., dots., kand. voenno-morskikh nauk, kapitan 1 ranga, red.; OKUN', S.B., prof., doktor istor. nauk, red.; ORLOV, B.P., prof., doktor geogr. nauk, red.; PAVLOVICH, N.B., prof., kontr-admiral v otstavke, red.; PANTELEYEV, Yu.A., admiral, red.; PITERSKIY, N.A., kand. voenno-morskikh nauk, kontr-admiral, red.; PLATONOV, S.P., general-leytenant, red.; POZNYAK, V.G., dots., general leytenant, red.; SALISHCHEV, K.A., prof., doktor tekhn. nauk,

(Continued on next card)

LEVCHENKO, G.I.---(continued) Card 2.

red.; SIDOROV, A.L., prof., doktor istor. nauk., red.; SKORODUMOV, L.A., kontr-admiral, red.; SNEZHINSKIY, V.A., prof., doktor voenno-morskikh nauk, inzh.-kapitan 1 ranga, red.; SOLOV'YEV, I.N., dots., kand. voenno-morskikh nauk, kapitan 1 ranga, red.; STALBO, K.A., kontr-admiral, red.; STEPANOV, G.A. [deceased], dots., vitse-admiral, red.; TOMASHEVICH, A.V., prof., doktor voenno-morskikh nauk, kontr-admiral v otstavke, red.; TRIBUTS, V.F., kand. voenno-morskikh nauk, admiral, red.; CHERNYSHOV, F.I., kontr-admiral, red.; SHVEDE, Ye.Ye., prof. doktor voenno-morskikh nauk, kontr-admiral, red.; CHURBAKOV, A.I., tekhn. red.; VASIL'YEVA, Z.P., tekhn. red.; VIZIROVA, G.N., tekhn. red.; GOROKHOV, V.I., tekhn. red.; GRIN'KO, A.M., tekhn. red.; KUBLIKOVA, M.M., tekhn. red.; MALINKO, V.I., tekhn. red.; SVIDERSKAYA, G.V., tekhn. red.; CHERNOGOROVA, L.P., tekhn. red.; GUREVICH, I.V., tekhn. red.; BUKHANOVA, N.I., tekhn. red.; NIKOLAYEVA, I.N., tekhn. red.; RADOVIL'SKAYA, E.O., tekhn. red.; TIKHOMIROVA, A.S., tekhn. red.; BELOCHKIN, P.D., tekhn. red.; LOYKO, V.I., tekhn. red.; ROMANYUK, I.G., tekhn. red.; YAROSHEVICH, K.Ye., tekhn. red.

[Sea atlas] Morskoi atlas. Otv. red. G.I. Levchenko. Glav. red. L.A. Demin. [Moskva] Izd. Glav. shtaba Voenno-morskogo flota. Vol.3. [Military and historical. Pt.1. Pages 1-45] Voenno-istoricheskii. Zamestitel' otv. red. po III tomu N.S. Frumkin. Pt.1. Listy 1-45. 1958. _____ [Military and historical maps, pages 46-52]

(Continued on next card)

LEVCHENKO, G.I.----(continued) Card 3.
Voenno-istoricheskie karty, listy 46-52. 1957. (MIRA 11:10)

1. Russia (1923- U.S.S.R.) Ministerstvo oborony. 2. Nachal'nik Glavnogo upravleniya geodezii i kartografii Ministerstva vnutrennikh del SSSR (for Baranov). 3. Chlen-korrespondent Akademii nauk SSSR (for Kalesnik). 4. Deystvitel'nyy chlen Akademii pedagogicheskikh nauk RSFSR (for Orlov).

(Ocean--Maps)

1. LISIN, G. G. - GELFOND, M. G., ENGS.

2. USSR (600)

4. Concrete Blocks

7. Redesigning machine SM-185 for producing slit-hollow, slag-concrete blocks. Biul.stroi.tekh. 9 no. 21, 1952

9. Monthly List of Russian Accessions, Library of Congress, March 1953, Unclassified.

GEL'FOND, M., inzh.; PLAVIN, B., inzh.

Vibrating forms for prefabricating stairs. Stroitel' no.5:18 My '60. (MIRA 13:9)
(Stair building) (Vibrators)

GEL'FOND, S. (g.Odessa); SHIGANOV, A. (g.Chernigov); SMETANINA, Z., pryadil'shchitsa, udarnik kommunisticheskogo truda; DIL'DIN, M., rabochiy; SKRIPKIN, P. (g.Ulan-Ude); FILIPPOV, A. (g.Petropavlovsk); CHERNYKH, Vl. (g.Kursk)

From letters to the editors. Sov. profsoiuzy 16 no.21:54-57 N '60. (MIRA 13:10)

1. Fabrika imeni Balashova, g.Ivanovo (for Smetanina). 2. Sovkhoz "Teplichnyy", Moskovskaya obl. (for Dil'din).
(Trade unions)

TOMKEVICH, I., inzh. (Leningrad); SMOL'YANOV, I. (Novosibirsk); GOLOPEROV, I.; SANUCHIN, T. (g. Sterlitamak); NIKIFOROV, N., kranovshchik (g. Aktyubinsk); GEL'FOND, S. (Odessa)

Do more today than you did yesterday. Sov. profsoiuzy 18 no.19:19 O '62. (MIRA 15:9)

1. Predsedatel' Donetskogo oblastnogo komiteta professional'nogo soyuza rabochikh neftyanoy i khimicheskoy promyshlennosti, g. Donetsk (for Goloperov).

(Socialist competition) (Technological innovations)