RENTSCH, W.

"The Application of Modern Electronic Measuring Methods in Mining."

Nachrichtentechnik, 3, 218-21 (May, 1953)

SO: SCIENCE ABSTRACTS, Section B, Electrical Engineering Abstracts. (September 1953), Unclass.

93280

S/194/62/000/005/150/157
D271/D308

AUTHOR: Rentsch, W.

TITLE: Electronic apparatus for storing and re-transmitting pulse sequences

PERIODICAL: Referativnyy zhurnal. Avtomatika i radioelektronika, no. 5, 1962, abstract 5-7-275 f (East German patent specification, class 21a5, 32/01, no. 21151, 14.4.1961)

TEXT: A store is patented in which the initial instant of information retrieved, pulse repetition frequency and intervals between pulse groups can be varied within wide limits. One cold cathode tube is used for storing each pulse sequence; control of pulse storing and retrieval is performed by a circuit with cold cathode tubes. The start of pulse sending is determined by the controlled ignition instant of the tube which governs the pulse retrieval. The interval between pulse groups which are sent out is determined by a time circuit, and pulse repetition frequency is determined by the frequency of the pulse generator. A re-set tube is provided which returns the store to its initial condition. The circuit is shown for the

Card 1/2

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D271/D308

Electronic apparatus for storing ...

control of two sent-out pulse groups, with arbitrary repetition frequency and interval between groups; this circuit is intended for long-distance telephony. Dekatrons are used as memory elements; they are not shown in the circuit. [Abstractor's note: Complete translation].

Card 2/2

JINDRA, A.; HENTZ, J.

Ion exchange chromatography in the determination of sympathomimetic amines. Cesk. farm. 1 no. 11-12:625-630 1952. (CIML 24:1)

1. Of the Institute of Pharmaceutical Chemistry of Charles University, Prague.

17

CA

Determination of local anesthetics by ion exchange. A. Jindra and J. Rentz (Charles Univ., Prague). *J. Pharm. Pharmacol.* 4, 646-7(1952).—The acid component of local anesthetics (20-50 mg.) is adsorbed on a column of 8-10 g. of Amberlite IRA-400 from soln. in 20 ml. of a mixt. of H_2O 5, and 95% EtOH 15 ml., and the bases are eluted with 30 ml. of hot EtOH(96%) and titrated with 0.1 *N* HCl. The method is applied successfully to procaine, larocaine, tutocaine, nupercaine, amylocaine, amethocaine, and diocaine. Most deviations from the theoretical content were less than 0.5%. Procaine-HCl ointment is extd. with hot H_2O:EtOH(1:1) and the ext. passed through the column as above. The results with 2-g. samples (40 mg. of compd.) were 95.1 and 99.0% of the theoretical concn. The presence of an electrolyte is a disturbing factor as the H_2O-EtOH soln. is more basic when the anion has been removed. Cf. Jindra and Pohorsky, *C.A.* 45, 7299*b*.

S. W. Goldstein

Renyi, Alfred

Rényi, Alfred. On a Tauberian theorem of O. Szász. Acta Univ. Szeged. Sect. Sci. Math. 11, 119–123 (1946).

The author observes that the Szász condition that $n^{-1}\sum_1^n k^p|a_k|^p$ is bounded for all n is a sufficient Tauberian condition, together with the Abel summability of the series $\sum a_k$, to ensure its convergence provided that $p>1$, but not if $p=1$. In the latter case, he shows that a sufficient Tauberian condition is that $\sigma_n^r-\sigma_n^{r+1}$ tends to a limit as $n\to\infty$, where σ_n^r is the Cesàro mean of order r of $\sum|a_k|$. In particular, a sufficient condition is that $n^{-1}\sum_1^n k|a_k|$ tends to a limit as $n\to\infty$.

H. R. Pitt (Belfast).

Source: Mathematical Reviews, Vol 8, No. 3

RENYI, ALFRED

Rényi, Alfréd. Integral formulae in the theory of convex curves. Acta Univ. Szeged. Sect. Sci. Math. 11, 158–166 (1947).

Given a convex plane closed curve C, suppose its supporting lines shifted inwards by the same distance μ to positions l_μ. The common part of the negative half-planes of the lines l_μ is termed the internal parallel curve $C(\mu)$ at distance μ [see B. v. Sz. Nagy, same Acta 9, 253–257 (1939); these Rev. 1, 264]. The curve $C(\mu)$ shrinks to a point, or to a segment, when $\mu=\rho$, where ρ is the radius of the largest inscribed circle of C. Write $A(\mu)$ for the area enclosed by $C(\mu)$, $P(\mu)$ for its length, $\kappa(\mu)$ for its "characteristic." In the case in which C is a polygon, $C(\mu)$ is a polygon also and $\kappa(\mu)$ is defined as $2\sum \tan \frac{1}{2}\beta_k$, where the β_k denote the external angles of $C(\mu)$. As the author observes, $\kappa(\mu)$ is then an increasing function of μ, and $\kappa(\mu)\geqq 2\pi$, $\kappa(\mu)=-P'(\mu)$, $P(\mu)=-A'(\mu)$, where P', A' are derivatives of P, A almost everywhere. From these facts, which he extends from convex polygons to convex curves in general by passage to the limit, the author derives integral representations of P and A, from which he obtains in a few lines inequalities connecting A, P, κ, ρ. Some of these are well known, for instance Bonnesen's form of isoperimetric inequality; others, such as (1) $A(0)\leqq \rho P(\frac{1}{2}\rho)$, are new. The author generalizes his method by defining the internal parallel curve $C_1(\mu)$ of a convex curve C_1 relative to a convex curve C_2 and the inequalities then concern mixed areas and characteristics.

L. C. Young (Princeton, N. J.).

Source: Mathematical Reviews, 1948, Vol 9, No. 3

RÉNYI, A.

Rényi, A. On the minimal number of terms of the square of a polynomial. Hungarica Acta Math. 1, 30–34 (1947).

Denote by $Q(n)$ the minimum number of terms of the square of a polynomial containing n terms. Put $g(n)=Q(n)/n$. Rédei raised the question of investigating $Q(n)$. Kalmár, Rédei and the author proved that $\liminf g(n)=0$. The author proves that $n^{-1}\{g(1)+\cdots+g(n)\}\to 0$. The proof follows easily from the following facts: $g(n\cdot m)\leqq g(n)\cdot g(m)$; $g(n)<3/2$; $g(4n+1)<28/29$. The author conjectures that $\lim g(n)=0$.

P. Erdös (Syracuse, N. Y.).

Source: Mathematical Reviews, 1948, Vol 9, No. 4

RENYI, A

Alfred

2

Rényi, Alfréd. On the measure of equidistribution of point sets. Acta Univ. Szeged. Sect. Sci. Math. 13, 77–92 (1949).

Let E be a set on $(0, 1)$, of measure $|E|$, and characteristic function $F(x)$; $F(x)$ is extended so as to be periodic with period 1. Let $G(t)=\int_0^1 F(x)F(x+t)dx$, $m(E)=\min G(t)$, $\mu(E)=m(E)/|E|^2$, so that $0\leqq\mu(E)<1$ if $0<|E|<1$. The author calls $\mu(E)$ the measure of equidistribution of E; it actually measures the equidistribution of the set of distances between points of E. The main purpose of the paper is to construct a set E with assigned (arbitrarily small) positive measure and $\mu(E)$ arbitrarily close to 1. Such a set always intersects all its translates in a set whose measure is bounded from zero. The author then introduces a measure of k-fold equidistribution and proves a weaker theorem for $k>1$ which, however, implies a generalization of this intersection property. Finally he makes a connection with Singer's "difference bases" in number theory and proves a theorem about them. *R. P. Boas, Jr.* (Evanston, Ill.).

Source: Mathematical Reviews, Vol 11 No. 9

RENYI, ALFRED

Rényi, Alfréd. Some remarks on independent random variables. Hungarica Acta Math. 1, no. 4, 17–20(1949).

Let $X_1, X_2, \cdots, X_n$ be independent bounded random variables all having the same distribution function $F(x)$. Assume furthermore that

$$\int_{-\infty}^{\infty} x\,dF(x)=0, \quad \int_{-\infty}^{\infty} x^2\,dF(x)=\sigma^2.$$

Let M_n be the greatest lower bound of those M for which $\Pr\{|X_1+\cdots+X_n|<M\}=1$. The principal result of this paper is the inequality $M_n \geqq \frac{1}{2}n\sigma^2/\Phi(0)$, where

$$\Phi(0)=\int_{-\infty}^{0} F(x)\,dx.$$

The result is best possible as can be seen by considering Bernoulli variables. *M. Kac* (Ithaca, N. Y.).

Source: Mathematical Reviews, 1950 Vol 11 No. 8

Rényi, Alfred

Rényi, Alfréd. Probability methods in number theory. Publ. Math. Collectae Budapest 1, no. 21, 9 pp. (1949).

This paper is the author's inaugural lecture at the University of Budapest. First a brief summary of some applications of probabilistic methods to number theory are given. These also include incorrect applications which the author analyzes and criticizes. The major part of the paper is devoted to the exposition of the author's approach to the "large sieve." The details of this approach have been published elsewhere [J. Math. Pures Appl. (9) 28, 137–149 (1949); Compositio Math. 8, 68–75 (1950); these Rev. 11, 161, 581].

M. Kac (Ithaca, N. Y.).

Source: Mathematical Reviews, Vol 12, No. 3.

Rényi, Alfréd

Rényi, Alfréd. On the coefficients of schlicht functions. Publ. Math. Debrecen 1, 18–23 (1949).

The author shows that the Bieberbach conjecture $|a_n| \leqq n$, $n=2, 3, \cdots$, for functions (1) $f(z)=z+a_2z^2+\cdots+a_nz^n+\cdots$, regular and schlicht for $|z|<1$, holds for a special class of schlicht functions. This class, first investigated by V. Paatero [see Ann. Acad. Sci. Fennicae. Ser. A. 37, no. 9 (1933)], consists of those schlicht functions (1) which map the unit circle on a domain G of boundary rotation $\alpha \leqq 4\pi$, i.e., for $0 \leqq |z| < 1$

$$(2) \qquad \int_0^{2\pi} |\Re\{1+zf''(z)/f'(z)\}| d(\arg z) \leqq \alpha \leqq 4\pi.$$

This end is accomplished by showing with the aid of a lemma of S. Banach [see Fund. Math. 7, 225–236 (1925)] that the domain G is the limit of domains convex in one direction. For the class of functions convex in one direction the Bieberbach conjecture was earlier shown to be true [see M. S. Robertson, Amer. J. Math. 58, 465–472 (1936)].

In case $\alpha \leqq 3\pi$ a sharper estimate for $|a_n|$ is obtained:

$$(3) \qquad |a_n| \leqq \prod_{k=2}^{n} (1+(\alpha-2\pi)/k\pi) \leqq n^{(\alpha-2\pi)/\pi}, \qquad n=2, 3, \cdots,$$

which is sharp for $\alpha=2\pi$ (convex functions) but not for $\alpha=3\pi$. The reviewer believes that inequality (3) of the paper should read

$$\prod_{k=2}^{n}\left(1+\frac{\alpha-2\pi}{k\pi}\right) \leqq \exp\left\{\left(\frac{\alpha-2\pi}{\pi}\right)\cdot\sum_{k=2}^{n} k^{-1}\right\} \leqq n^{(\alpha-2\pi)/\pi}.$$

M. S. Robertson (New Brunswick, N. J.).

Source: Mathematical Reviews, 1950 Vol 11 No. 2

RENYI, ALFRED

2000

Rényi, Alfréd. Die prinzipiellen Fragen der Wahrscheinlichkeitsrechnung im Lichte des dialektischen Materialismus. Philosophisches Jahrbuch, 1952, Zusammenfassung, pp. 7–8. Akadémiai Kiadó, Budapest, 1954.
Abstract of a paper in Mat. Lapok 1 (1949), 27–64; MR 11, 374.

1-F/W

Basic Problems of the Probability Calculations, Investigated from the Point of View of Dialectic Materialism

Renyi, Alfred,

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Rényi, Alfréd. On the geometry of conformal mapping. Acta Sci. Math. Szeged 12, Leopoldo Fejér et Frederico Riesz LXX annos natis dedicatus, Pars B, 215–222 (1950).

Let $f(z)=z+a_2z^2+\cdots$ be univalent in the unit circle and let $C(r)$ denote the conformal image of the circumference $|z|=r$ $(0<r<1)$ which is yielded by the mapping $z\rightarrow f(z)$. By suitable utilization of known results on univalent functions in the unit circle, the author obtains additional geometric information regarding the curves $C(r)$. For instance, he proves that, for small r, $C(r)$ is contained between two circles of radii $r+O(r^3)$ and $r-O(r^3)$, respectively.

Z. Nehari (St. Louis, Mo.).

Source: Mathematical Reviews, Vol 11 No. 9

Renyi, Alfred

Rényi, Alfréd. 30 years of mathematics in the Soviet Union. II. New lines of research in probability theory. I. Mat. Lapok 1, 91–137 (1950). (Hungarian. Russian and English summaries)

Survey of achievements in probability theory by mathematicians working in the USSR during the last 30 years. Two more parts are to follow.

Source: Mathematical Reviews, Vol 11 No. 10

RENYI, ALFRED

Rényi, Alfréd. Contributions to the theory of independent random variables. Acta Math. Acad. Sci. Hungar. 1, 99–108 (1950). (Russian. English summary)

A sequence of real-valued functions defined, for instance, on the unit interval is called maximal if it separates almost all points of the interval. Suppose that $\{f_n\}$ is a maximal sequence of independent random variables, such that each f_n assumes only a finite number of different values, that the variance of $\sum_{j=1}^{n} f_j$ tends to infinity with n, and that A is a measurable set of positive measure. Let $\{g_n\}$ be the sequence of partial sums of the f's, normalized in the usual manner, so that the mean and the variance of each g_n are 0 and 1, respectively. The author's main result (theorem 1) is that if the distribution functions of the g's converge to the (necessarily normalized) Gaussian distribution, then the conditional distributions of the g's, relative to A, do the same thing. Theorems 2, 3, and 4 are easy consequences of theorem 1, and are all of the same form: From an assumed convergence of distribution functions one concludes to another such convergence.

P. R. Halmos.

Source: Mathematical Reviews, Vol 12 No.8

RÉNYI, A.

Jánossy, L., Rényi, A., and Aczél, J. **On composed Poisson distributions. I.** Acta Math. Acad. Sci. Hungar. 1, 209–224 (1950). (English. Russian summary)

Let $X(t)$ be a stochastic process such that (1) $X(t)$ assumes only non-negative integral values, (2) the increments of $X(t)$ over a set of non-overlapping t-intervals are mutually independent, (3) the distribution of $X(t+h)-X(t)$ is independent of t. Under these circumstances the distribution of $X(t)$ is given by an infinitely divisible law, and is therefore a "composed Poisson distribution." [Cf. P. Lévy, Théorie de l'addition des variables aléatoires, Gauthier-Villars, Paris, 1937, chapter 7.] The authors are apparently unaware of the pertaining literature and prove this result directly without using the advantages resulting from the use of generating functions. In the next section they characterize their processes as those described by infinitely divisible distributions. This is essentially a repetition of the argument in terms of generating functions, except that the authors introduce the superfluous condition that $E(X(t))=t$ (whereas it is not necessary to assume the existence of $E(X(t))$). Finally the authors check that certain distributions are infinitely divisible, although this property is usually implied in their definition.

W. Feller (Princeton, N. J.).

Source: Mathematical Reviews, Vol 13 No. 7

Rényi, Alfréd

Rényi, Alfréd. On the summability of Cauchy-Fourier series. Publ. Math. Debrecen 1, 162–164 (1950).

Let $tf(t)\epsilon L(0, \pi)$, where $f(t)$ is odd and of period 2π, and let $b_n = (2/\pi)\int_0^\pi f(t) \sin nt\, dt$. Titchmarsh [Proc. London Math. Soc. (2) 23, xli–xliii (1924)] remarked that $b_n = o(n)$ and that, for $t \not\equiv 0 \pmod{2\pi}$, the principal value Fourier series (*) $\sum b_n \sin nt$ is summable $(C, 1)$ if and only if $f(t)$ satisfies a local $(C, 1)$ summability condition. Here the author (a) shows that, for $t \not\equiv \pi$, (*) is summable $(C, 1)$ if and only if the Fourier series of $f(t) \sin t$ is summable $(C, 1)$ and (b) observes that (*) is the derived Fourier series of an integral of $f(t)$ for $t \not\equiv 2\pi$, so that Titchmarsh's statement follows from W. H. Young's theory of restricted Fourier series [ibid. (2) 17, 195–236 (1918)]. The author's formula (7) seems to require some modification.

L. S. Bosanquet (London).

Source: Mathematical Reviews, Vol 12, No. 3.

RENYI, A.

"The Basic Theory of Probability." p.227 (GODISHNIK, MATEMATIKA I FIZIKA, Vol. 47, no. 1, 1950/51-1951/52, Sofiya.)

SO: Monthly List of ~~Russian~~ East European Accessions, Library of Congress, Vol. 3, No. 3, March ~~1955~~ 1954, Uncl.

RÉNYI, ALFRED

Rényi, Alfred. On a theorem of the theory of probability and its application in number theory. Časopis Pěst. Mat. Fys. 74 (1949), 167–175 (1950). (Russian. Czech summary)

The theorem of the theory of probability is the same as one given previously [J. Math. Pures Appl. (9) 28, 137–149 (1949); Compositio Math. 8, 68–75 (1950); these Rev. 11, 161, 581]. It is applied here to prove the following result: Let $\Lambda(n)$ be the Mangoldt function and set $\psi(N)=\sum_{n=1}^{N}\Lambda(n)$, $\psi(N,p,r)=\sum_{n\leq N,\, n\equiv r \pmod p}\Lambda(n)$. Let $\frac{1}{3}\leq\alpha<\frac{1}{2}$ and β,γ,δ positive numbers subject to the condition $\beta+\gamma+\delta<\alpha^{-1}-2$. Then, for all primes $p<N^{\alpha}$, except at most $N^{\alpha(1-\delta)}$, and all $r=1,2,\cdots,p-1$, except at most $p^{1-\gamma}$, one has $\psi(N,p,r)=\psi(N)/p+\theta\psi(N)/p^{1+\beta}$, where $|\theta|\leq 1$.

M. Kac (Ithaca, N. Y.).

Source: Mathematical Reviews, Vol 12 No. 8

RÉNYI, ALFRÉD

Rényi, Alfréd. On problems connected with the Poisson distribution. Magyar Tud. Akad. Mat. Fiz. Oszt. Közleményei 1, 202–212 (1951). (Hungarian)

This is an expository lecture on the Poisson distribution. Applications of this distribution in telephone engineering are discussed in detail, other applications are briefly mentioned. *E. Lukacs* (Washington, D. C.).

Source: Mathematical Reviews, Vol 13 No. 10

RÉNYI, A.

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Jánossy, L., Rényi, A., and Aczél, J. On compound Poisson distributions. I. Magyar Tud. Akad. Mat. Fiz. Oszt. Közleményei 1, 315–328 (1951). (Hungarian)

This is the Hungarian version of a paper by the same authors [Acta Math. Acad. Sci. Hungar. 1, 209–224 (1950); these Rev. 13, 663]. *E. Lukacs* (Washington, D. C.).

Source: Mathematical Reviews, Vol 13 No.10

RENYI, ALFRED

Rényi, Alfréd. On compound Poisson distributions. II. Magyar Tud. Akad. Mat. Fiz. Oszt. Közleményei 1, 329–341 (1951). (Hungarian)

This is the Hungarian version of a paper by the same author [Acta Math. Acad. Sci. Hungar. 2, 83–98 (1951); these Rev. 13, 663]. *E. Lukacs* (Washington, D. C.).

ource: Mathematical Reviews, Vol 13 No. 10

Rényi, Alfréd

Rényi, Alfréd, and Turán, Pál. Two proofs of a theorem of L. Jánossy. Magyar Tud. Akad. Mat. Fiz. Oszt. Közleményei 1, 369–370 (1951). (Hungarian)

In the paper reviewed above Jánossy proved the following theorem: "Let $G(z)=\sum_{k=0}^{\infty} p_k z^k$ and assume that $p_0>0$, $p_k \geqq 0$ $(k=1, 2, \cdots)$ and $G(1)=1$, then the equation $G(z)=z$ has at most one root in the interior of the unit circle. This root can only be real and positive." The first author gives an elementary algebraic proof while the proof of the second author uses Rouché's theorem. The theorem is however not new, it may be found in Feller's book [An introduction to probability theory . . . , Wiley, New York, 1951, p. 226; these Rev. 12, 424] where it occurs (as in Jánossy's paper) in connection with branching process. *E. Lukacs.*

Source: Mathematical Reviews, Vol 13 No. 10

RÉNYI, ALFRÉD

Rényi, Alfréd. On composed Poisson distributions. II. ~~Acta Math.~~ Acad. Sci. Hungar. 2, 83-98 (1951). (English. Russian summary)

The arguments of the preceding review are generalized to the time-inhomogeneous case. The author introduces unnecessary uniformity conditions which are not used in P. Lévy's approach. The paper follows the same pattern except that (following an oral suggestion of Kolmogorov) the author refers to the theory of infinitely divisible distributions without, however, noticing that the latter contains the developed theory. *W. Feller* (Princeton, N. J.).

Source: Mathematical Reviews, Vol 13 No. 7

Renyi, Alfred

Rényi, Alfréd. On some problems concerning Poisson processes. Publ. Math. Debrecen 2, 66–73 (1951).

The Poisson process is derived from postulates of the usual type. It is shown that if every event in a Poisson process starting at time 0 is the birth of an individual whose life is a random variable (with distribution which may depend on the moment of birth), then the number of individuals alive at time t has a Poisson distribution.

J. L. Doob (Urbana, Ill.).

Source: Mathematical Reviews, Vol 13 No. 1

RENYI, A.

RENYI, A.,
RENYI, C.,
SURANYI, J.: On the Independance of the Simple Fields in the Euclidean Space With n Dimensions

Math

Rényi, A., Rényi, C., et Surányi, J. Sur l'indépendance des domaines simples dans l'espace Euclidien à n dimensions. Colloquium Math. 2, 130–135 (1951).

The sets $E_1, \cdots, E_n$ are said to be independent if no intersection $F_1, \cdots, F_n$ is empty, where each F_j is either E_j or its complement. The following theorems are proved. (A) For each $n \geqq 1$ the maximum number of open n-dimensional intervals (sides parallel to the coordinate axes) in Euclidean n-space $E^{(n)}$ is $2n$. (B) For each $n \geqq 1$ the maximum number of n-dimensional independent spheres in $E^{(n)}$ is $n+1$. (C) If $N(k)$ is the maximum number of polygonal open convex independent domains in the plane, each polygon having $\leqq k$ sides, then $\lim_{k\to\infty} N(k)/\log k = 1/\log 2$.

J. L. Doob (Urbana, Ill.).

Source: Mathematical Reviews, Vol. 13 No. 2

RÉNYI, ALFRED

2000

Rényi, Alfréd. Remarks concerning the zeros of certain integral functions. C. R. Acad. Bulgare Sci. 3, no. 2-3 (1950), 9–10 (1951). (English. Russian summary)

Let $F(z)=\int_0^1 f(t)\cos zt\,dt$. If $f(t)$ is real, $n+m$ is odd, $f^{(k)}(1)=0$ $(k=0, 1, \cdots, n-1)$, $f^{(2k+1)}(0)=0$ $(1\leqq 2k+1\leqq n)$, and $g(t)=f^{(n)}(t)/t^m$ is integrable, nonnegative and nondecreasing, then $F(z)$ has only real roots; the same is true when $G(z)=\int_0^1 g(t)\sin zt\,dt$ has only real roots. This result is obtained very simply by integrating repeatedly by parts in $G(z)$ and appealing to Pólya's theorem [Math. Z. 2, 352–383 (1918)] that $F(z)$ has only real roots when $f(t)$ is nonnegative and nondecreasing. Other similar results are proved similarly. These theorems generalize some of Ilieff [same C. R. 2, no. 1, 17–20 (1949); these Rev. 11, 236].

R. P. Boas, Jr. (Evanston, Ill.).

Source: Mathematical Reviews, Vol 13 No.7

RENYI, Alfred

(3)

Mathematical Reviews
Vol. 15 No. 2
Feb/ 1954
Numerical and Graphical
Methods

Rényi, Alfréd. Remarques concernant un traité de P. Gombás et R. Gáspár. Magyar Tud. Akad. Alkalm. Mat. Int. Közl. 1 (1952), 393–397 (1953). (Hungarian. Russian and French summaries)

Le travail contient quelques remarques mathématiques en connexion avec le travail, Acta Phys. Acad. Sci. Hungaricae 1, 66–74 (1951) [ces Rev. 13, 993].

Résumé de l'auteur.

Renyi, Alfred

Rényi, Alfréd. On the mathematical work of Károly I - F/W
Jordan. Mat. Lapok 3, 111-121 (1952). (Hungarian)
A list of Jordan's scientific papers is included.

(1)

RENYI, A.

Mathematical Reviews
Vol. 14 No. 8
Sept. 1953
Analysis

Rényi, A. On projections of probability distributions. Acta Math. Sci. Hungar. 3, 131–142 (1952). (Russian summary)

The following theorem is due to Radon: Let D be a bounded domain, and $f(x, y)$ a continuous function defined in it. Assume that the integral of $f(x, y)$ vanishes along every chord of $f(x, y)$; then $f(x, y)$ is identically 0. The author first of all points out that Radon's theorem follows from the following result due to Cramér and Wold: Let $F(x, y)$ be a distribution function. Then $F(x, y)$ is uniquely determined by the values of its projections on all the straight lines through the origin, i.e., by the values of the integrals.

$$F_{l_\varphi}(p) = \iint_{x \cos \varphi + y \sin \varphi \leq p} dF(x, y),$$

where φ denotes the angle between the line l_φ and the x-axis.

Renyi, A.

The author then gives Hajós's proof for his conjecture: A discrete mass distribution consisting of n distinct mass points is completely determined if its projections on $n+1$ arbitrary different straight lines through the origin are given. The author points out that this theorem is best possible, i.e., $n+1$ cannot be replaced by n; further, he discusses three-dimensional generalizations.

The author further raises the following problem: Put $f(x, y) = 1/\pi(1-x^2-y^2)^{1/2}$ for $x^2+y^2<1$ and $f(x, y)=0$ for $x^2+y^2 \geqq 1$. Then clearly the integral of $f(x, y)$ on every chord of the unit circle is 1. Is there any other curve of constant breadth for which there exists a function $f(x, y)$ whose integral is constant on every chord? *P. Erdős.*

8-16-54 LL

RENYI, ALFRED

1 - F/W

Rényi, Alfréd. János Bolyai, a great revolutionist of science. Mat. Lapok 3, 173–178 (1952). (Hungarian)

(1)

RENYI, ALFRED

Mathematical Reviews
Vol. 15 No. 3
March 1954
Analysis

(3)

Rényi, Alfréd, et Takács, Lajos. Sur les processus d'événements dérivés par un processus de Poisson et sur leurs applications techniques et physiques. Magyar Tud. Akad. Alkalm. Mat. Int. Közl. 1 (1952), 139–146 (1953). (Hungarian. Russian and French summaries)

The first author proved earlier [Publ. Math. Debrecen 2, 66–73 (1951); Magyar Tud. Akad. Mat. Fiz. Oszt. Közleményei 1, 202–212 (1951); these Rev. 13, 51, 958] that the number of happenings inaugurated by the events of a Poisson process forms also a Poisson process. A new proof of this theorem is given which is based on a limit theorem for Poisson convergence of sequences of sums of random variables. *E. Lukacs* (Washington, D. C.).

7-8-54 LL

RENYI, Alfred

Mathematical Reviews
Vol. 14 No. 11
Dec. 1953
Analysis

8-10-54
LL

Math
3

Rényi, Alfred. On a conjecture of H. Steinhaus. Ann. Soc. Polon. Math. 25 (1952), 279-287 (1953).

A countable family $\{f_n\}$ of bounded measurable functions on the unit interval is called maximal whenever there exists a measurable set Z of measure zero such that if x and y do not belong to Z and if $f_n(x) = f_n(y)$ for all n, then $x = y$. The principal theorem of the paper asserts that if $\{f_n\}$ is maximal, then (*) the set of all finite products of positive integral powers of the f_n's is complete in L^2. The family $\{f_n\}$ is called saturated (with respect to stochastic independence) if it is stochastically independent but is not properly contained in any other stochastically independent set. The conjecture of Steinhaus (referred to in the title) was that if $\{f_n\}$ is saturated, then (*) holds. This is disproved by a family of one term: $f_1(x) = x$ or 1 according as $0 \leq x \leq \frac{1}{2}$ or $\frac{1}{2} < x \leq 1$. The connection between Steinhaus' insufficient condition and the author's sufficient one is as follows: if $\{f_n\}$ is independent and maximal, then it is saturated.

P. R. Halmos (Chicago, Ill.).

RENYI, ALFRED

RÉNYI, ALFRED

A new Method in the Theory of Orderly Random Tests

Math

GERM

*Rényi, Alfred. Eine neue Methode in der Theorie der geordneten Stichproben. Bericht über die Mathematiker-Tagung in Berlin, Januar, 1953, pp. 203–212. Deutscher Verlag der Wissenschaften, Berlin, 1953. DM 27.80.

A statement of some of the results in Acta Math. Acad. Sci. Hungar. 4, 191–231 (1953); MR 15, 885.

J. Wolfowitz (Ithaca, N. Y.).

1 - F/W

"H. Steinhaus and his intuition."
Kozlemenyei, Budapest, Vol 3, No 1, 1953, p. 37

SO: Eastern European Accessions List, Vol 3, No 10, Oct 1954, Lib. of Congress

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Mathematical Reviews
Vol. 15 No. 2
Feb. 1954
Analysis

7-13-54

Rényi, Alfréd. On projections of probability distributions. Magyar Tud. Akad. Mat. Fiz. Oszt. Közleményei 3, 59–69 (1953). (Hungarian)

The author considers the following theorem of Radon: if K is a bounded domain in the plane and if f is a continuous function on K such that the integral of f vanishes on every chord of K, then f is identically zero. He reformulates and generalizes the theorem and shows thereby that it can be easily derived from the theorem of Cramér and Wold: Every planar probability distribution is uniquely determined by its linear projections. The paper concludes with some analogous combinatorial results concerning the unique determination of mass distributions concentrated at a finite set.

P. R. Halmos (Chicago, Ill.).

RENYI, Alfred

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Mathematical Reviews
May 1954
History

10-4-54 LL

Rényi, Alfréd. The significance of the viewpoint of the geometry of Bolyai-Lobačevskiĭ. Magyar Tud. Akad. Mat. Fiz. Oszt. Közleményei 3, 253–273 (1953). (Hungarian)

A. RENYI.

"On projections of probability distributions" p. 131 (ACTA MATHEMATICA ACADEMIAE SCIENTIARUM HUNGARICAE, Vol. 3, no. 3, 1953, Budapest, Hungary)

SO: Monthly List of East European Accessions, L.C., Vol. 2 No. 7, July 1953, Uncl.

[illegible]. [illegible].

"Report on the work of the 3d Section of the Hungarian Academy of Sciences."
Kozlemenyei, Budapest, Vol 3, No 3, 1953, p. 295

SO: Eastern European Accessions List, Vol 3, No 10, Oct 1954, Lib. of Congress

[illegible], [illegible].

"Work of the Institute of Applied Mathematics in industrial application of calculus of probabilities; with commentaries by P. Kovacs and others."
Kozlemenyei, Budapest, Vol 3, No 3, 1953, p. 363

SO: Eastern European Accessions List, Vol 3, No 10, Oct 1954, Lib. of Congress

RENYI, Alfred

(2)

Rényi, Alfréd. On the theory of ordered samples. Magyar Tud. Akad. Mat. Fiz. Oszt. Közleményei **3**, 467–503 (1953). (Hungarian)

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The author's method for studying order statistics is based on the following lemma: Let X_k^* be the kth order statistic of a sample of n observations drawn from an exponential population. Then $X_k^* = \sum_{j=1}^{k} Y_j/(n-j+1)$ where the Y_j are independently and identically distributed random variables which obey the exponential law of the parent population. This property of the exponential distribution was already noticed by B. Epstein and M. Sobel [J. Amer. Statist. Assoc. 48, 486–502 (1953); these Rev. 15, 143] in connection with their work on life testing but was not used for a systematic study of order statistics. With the aid of the probability integral transformation, this lemma becomes a convenient tool for the study of the order statistics of a sample drawn from an arbitrary population with continuous cumulative distribution function. A number of known results on order statistics are derived by this method. The method is also used to obtain four theorems which lead to test criteria analogous to the Kolmogorov and Smirnov tests. Let $F(x)$ be the distribution function of the population and $F_n(x)$ the empirical distribution derived from a sample of n. The author's theorems involve the relative deviations $[F_n(x) - F(x)]/F(x)$. As an example, we list two of the simpler results:

Mathematical Review.
June 1954
Analysis

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Renyi, Alfred

(a) $$\lim_{n\to\infty} P\left\{n^{\frac{1}{2}} \sup_{a\leq F(x)\leq 1} \left[\frac{F_n(x)-F(x)}{F(x)}\right]<y\right\} = \begin{cases}\Phi(y[a/(1-a)]^{\frac{1}{2}})-\frac{1}{2} & \text{if } y>0,\\ 0 & \text{if } y\leq 0.\end{cases}$$

(b) $$\lim_{n\to\infty} P\left\{n^{\frac{1}{2}} \sup_{a\leq F(x)\leq 1} \left|\frac{F_n(x)-F(x)}{F(x)}\right|<y\right\} = \begin{cases}L(y[a/(1-a)]^{\frac{1}{2}}) & \text{if } y>0,\\ 0 & \text{if } y\leq 0.\end{cases}$$

Here $a>0$ is an arbitrary positive constant while

$$\Phi(z)=(2\pi)^{-\frac{1}{2}}\int_{-\infty}^{z} \exp\left[-\tfrac{1}{2}t^2\right]dt,$$

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$$L(z)=\frac{4}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k}{2k+1}\exp\left[\frac{-(2k+1)^2\pi^2}{8z^2}\right].$$

Tables of $L(y[a/(1-a)]^{\frac{1}{2}})$ are given for $y=0.1$, 0.5(0.5)17 and $a=0.01(0.01)0.1(0.1)0.5$. Asymptotic probabilities for the suprema of relative deviations over an interval (a, b) with $0<a<b<1$ are also derived. An essential tool in establishing these results are four probability limit theorems which are also of independent interest. These limit theorems are generalizations of results obtained by P. Erdös and M. Kac [Bull. Amer. Math. Soc. 52, 292–302 (1946); these Rev. 7, 459]. E. Lukacs (Washington, D. C.).

RENYI, ALFRED

Rényi, Alfréd. On the theory of order statistics. Acta Math. Acad. Sci. Hungar. 4, 191–231 (1953). (Russian summary)

English version of the author's paper in Magyar Tud. Akad. Mat. Fiz. Oszt. Közleményei 3, 467–503 (1953); these Rev. 15, 544.

RÉNYI, Alfred, direktor.

Strengthen the ties between theoretical and applied mathematics. Priroda 42 no.11:69-73 N '53. (MLRA 6:11)

1. Institut prikladnoy matematiki Akademii nauk Vengerskoy Narodnoy Respubliki.
(Hungary--Mathematics) (Mathematics--Hungary)

RENYI, A.

HAJOS, G.; RENYI, A. "Elementary proofs of some basic facts concerning order statistics." In English.
Acta Mathematica, Budapest, Vol 5, No 1/2, 1954, p. 1

SO: Eastern European Accessions List, Vol 3, No 10, Oct 1954, Lib. of Congress

Renyi, A.

I-F/W

Ren'i, Al'fred [Rényi, Alfréd]. The ideological significance of the geometry of Bolyai-Lobačevskiĭ. Acta Math. Acad. Sci. Hungar. 5, supplementum, 21–42 (1954). (Russian)

Russian version of a paper originally published in Hungarian [Magyar Tud. Akad. Mat. Fiz. Oszt. Közl. 3, 253–273 (1953); MR 15, 383]. SMW

RENYI, ALFRED

*Rényi, Alfréd. Valószínűségszámítás. [The calculus of probabilities.] Tankönyvkiadó, Budapest, 1954. 746 pp. 130 Ft.

Probability theory and its applications had been neglected in the curriculum of Hungarian universities until very recently when the author started to lecture regularly on these topics. He had therefore to prepare mimeographed notes for his students which lead, after repeated revisions, to the publication of the present book. It is thus the first modern Hungarian text book on probability theory and offers an excellent introduction into this field. The book is organized into three parts. The first part (chapters 1–5, pages 1–175) covers the elementary theory; it starts with a brief discussion of the Boolean algebra of events and adopts Kolmogorov's approach [Grundbegriffe der Wahrscheinlichkeitsrechnung, Springer, Berlin, 1933] for the foundations of the theory. The second part (chapters 6–11, pages 176–439) is intended to guide the reader into the higher parts of the theory and it culminates in a discussion of the laws of large numbers. To indicate the scope of this part we mention that it includes Kolmogorov's necessary and

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Renyi, Alfred

sufficient condition for the validity of the strong law of large numbers as well as the law of the iterated logarithm for independently and identically distributed bounded random variables. The third part (chapters 12–16, pages 440–653) deals with a variety of more advanced topics such as characteristic functions, and the central-limit theorem in its classical form. It also gives a brief introduction into several areas, for instance stochastic processes, in which active research is still going on at present.

The book contains two chapters on mathematical statistics: chapter 10 surveys briefly the elements, while chapter 15 deals with the theory of order statistics. The author presents in chapter 15 the approach to order statistics which the developed in a series of recent papers [Magyar Tud. Akad. Mat. Fiz. Oszt. Közl. 3, 467–503 (1953); Berichte über die Mathematiker-Tagung in Berlin, Deutscher Verlag der Wissenschaften, Berlin, 1953, pp. 203–212; Hajós and Rényi, Acta Math. Acad. Sci. Hungar. 5, 1–6 (1954); MR 15, 544, 885; 16, 603; 15, 972; 16, 729]. The theorems of Kolmogorov and Smirnov on the limiting distribution of the maximum discrepancy between the theoretical and the empirical distribution function as well as some recent results of the author [loc. cit.] are given without proof, but the significance and

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also applications of these results are discussed in some detail. Fairly recent investigations of B. V. Gnedenko and V. S. Korolyuk [Dokl. Akad. Nauk SSSR (N.S.) 80, 525-528 (1951); MR 13, 570] dealt with the exact distribution of the maximum deviation (or absolute value of the maximum deviation) between two empirical distribution functions based on two samples of equal size drawn from the same population. These results are derived and the corresponding limiting distributions are determined.

The book is in several respects similar to B. V. Gnedenko's book [Course in the theory of probability, Gostehizdat, Moscow-Leningrad, 1950; MR 13, 565]. Both books are text books rather than monographs, they are written at the same level and are very much aware of the needs of students. Moreover, the two authors have the same general outlook as evidenced also by occasional philosophical and political remarks and references to Marxist literature. There is however a big difference in the style of these two books. Unlike Gnedenko, the present author does not aim at a concise and terse presentation. He gives a detailed and interesting exposition and wishes to demonstrate that a mathematically sound foundation of the theory is important not only to the mathematician but also to the student of applied probability theory. The book is pedagogically very well organized; in this reviewer's opinion it will lead the student to the proper

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Rényi, Alfréd

appreciation of the value of a rigorous mathematical argument even if he should be oriented primarily towards some specific applications. A valuable feature of the book is the large number of illustrative examples which are taken from a great variety of fields. These are discussed in detail and are dispersed throughout the text. They are often used to give an additional motivation for the theoretical developments. At the end of each chapter there is a section with many problems. Some of these are just exercises while others are used to add supplementary material. A few problems contain even unpublished results. The book has four appendices. The first two provide tools from Boolean algebra, set theory and measure theory. Appendix 3 presents briefly the author's new axiomatic foundation for probability theory which was published recently [Magyar Tud. Akad. Mat. Fiz. Oszt. Közl. 4, 369–427 (1954); MR 16, 599]. Appendix 4 contains a survey of the history of probability theory which was also published earlier [ibid. 4, 447–466 (1954); MR 16, 659]. It is regrettable that this book is useful only to Hungarian students; it would deserve to be added to the foreign language publications of the Hungarian Academy of Sciences. *E. Lukacs* (Washington, D.C.)

Renyi, Alfred

Czipszer, János, and Rényi, Alfréd. On the completeness of certain trigonometric systems. Magyar Tud. Akad. Mat. Fiz. Oszt. Közl. 5 (1955), 391–410. (Hungarian)

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The authors give a complete account of the complex values of τ for which the sets

(1) $\{\cos(k+\tau)x\}$, (2) $\{1, \cos(k+\tau)x\}$, (3) $\{\sin(k+\tau)x\}$ $(k=0, 1, 2, \cdots)$

are complete in $[0, \pi]$. The completeness is meant in the space L^p, $1\leqq p<\infty$, as well as in C and C^0, respectively. [C^0 is the space of all functions in C vanishing at $x=0$.] For instance, in the case (1) the necessary and sufficient condition is:

$$\operatorname{Re}\tau\leqq\tfrac{1}{2}+1/(2p) \text{ in } L^p\ (p>1),$$
$$\operatorname{Re}\tau<1 \text{ in } L,$$
$$\operatorname{Re}\tau\leqq\tfrac{1}{2},\ \tau\neq\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{3}{2}, \cdots \text{ in } C.$$

The set (1) is never complete in C^0. In the proof of the non-completeness the formula

$$\frac{1}{\pi}\int_0^\pi (2\cos\tfrac{1}{2}x)^{a-1}\cos\tfrac{1}{2}bx\,dx=\frac{\Gamma(a)}{\Gamma(\frac{1}{2}(a+b+1))\Gamma(\frac{1}{2}(a-b+1))};\ \operatorname{Re} a>0,$$

and similar formulas are used. Two other trigonometric sets are also considered, refining a result of K. Šaldukov [Uspehi Mat. Nauk (N.S.) 8 (1953), no. 6(58), 143–153; MR 16, 241].

G. Szegő (Stanford, Calif.).

Renyi, Alfred

Rényi, Alfréd. On a combinatorial problem. Mat. Lapok 6 (1955), 151–164. (Hungarian. Russian and English summaries)

The following problem arose in connection with the crossing of types of alfalfa: let $M(n, r)$ denote the number of elements of the shortest sequence formed from the integers $1, \cdots, n$ that is such that each pair of integers (i, j), $1 \leq i < j \leq n$, occurs at least once in positions in which they are separated by not more than $r-1$ elements of the sequence. The paper contains some inequalities concerning $M(n, r)$ and some constructions. It is clear that $M(n, 1) = \binom{n}{2} + 1$, if n is odd, and $M(n, 1) = \binom{n}{2} + \frac{1}{2}n$, if n is even.

It is shown that

$$\frac{1}{2r} \leq \liminf_{n\to\infty} \frac{M(n, r)}{n^2} \leq \limsup_{n\to\infty} \frac{M(n, r)}{n^2} \leq \frac{1}{r+1}.$$

It is known that $\lim M(n, 2)/n^2 = \frac{1}{4}$ (N. G. de Bruijn, unpublished); however, the existence of the limit for $r = 3, 4, \cdots$ is an open question. *H. W. Kuhn.*

Math

Generalization of an enequality of Kolmogorov. in English. p. 281
Vol. 6, No. 3/4, 1955.

SO: EAST EUROPEAN ACCESSIONS LIST Vol. 5 5, No. 7, July 1956.

RENYI, ALFRED

1-FW

Rényi, Alfréd. On a new axiomatic theory of probability. Acta Math. Acad. Sci. Hungar. 6 (1955), 285–335. (Russian summary)

It is proposed to define conditional probability as the primary concept. Let S be an arbitrary set, $\mathfrak{A}$ a σ-algebra of subsets of S, $\mathfrak{B}$ a non-empty subset of $\mathfrak{A}$. A function $P(A|B)$ of two set variables is defined for $A \in \mathfrak{A}$ and $B \in \mathfrak{B}$ satisfying the axioms (I) $P(A|B) \geq 0$, $P(B|B)=1$; (II) for each B, $P(\cdot|B)$ is a measure;

(III) $$P(A|BC)P(B|C)=P(AB|C)$$

if $A \in \mathfrak{A}$, B, C and $BC \in \mathfrak{B}$. If so the collection $[S, \mathfrak{A}, \mathfrak{B}, P(\cdot|\cdot)]$ is called a conditional probability space. Various simple consequences of the axioms and alternative forms of (III) are given. A sufficient condition is given so that $P(A|B)=Q(AB)/Q(B)$, where Q is a measure on $\mathfrak{A}$ and $Q(B)>0$ (cf. the next review). Special cases of conditional probability spaces are discussed; for example in spaces with ordered dimensions like Euclidean spaces $P(A|B)$ may reduce to the quotient form based on a measure in the proper dimension of B. This includes the "Cavalieri spaces" in solid geometry where the conditioning is the sectioning by hyperplanes and the Cavalieri principle of comparing volumes is satisfied. Many illustrations are

Renyi, Alfred

taken from "ordinary probability theory". The typical situation is when the conditional probability can be expressed as the limit of a quotient leading to indeterminate forms of the type 0/0 or ∞/∞. A typical example is the result of Erdös and the reviewer that if S_n is the sum of n independent, identically distributed, integer-valued random variables of span one and mean zero, then

$$P(S_n=a)/P(S_n=b)$$

tends to one as $n\to\infty$ for any two integers a and b. In the present terminology S_n tends to the uniform conditional probability distribution on the set of all integers. Similar phenomena have been discovered for Markov chains. A conditional law of large numbers is proved leading to an extension of Borel's normal numbers to Cantor's series. {Despite the interest of the paper it remains to be seen if these notions furnish more than a convenient viewpoint. The existence of the limit in (52) corresponding to an extension of the aforementioned result to a general Markov chain, has been negated by an example due to F. J. Dyson and the reviewer (unpublished).}

K. L. Chung (Chicago, Ill.).

A, Renyi.

1-F W

Rényi, A. On conditional probability spaces generated by a dimensionally ordered set of measures. Teor. Veroyatnost. i Primenen. 1 (1956), 61–71. (Russian summary)

The main result is a weaker (but more easily proved) version of a theorem of A. Czásár [Acta Math. Acad. Sci. Hungar. 6 (1955), 337–361; MR 18, 340]. The paper is semi-expository. *D. A. Darling* (Ann Arbor, Mich.)

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Rényi, A.

3

Erdös, P.; and Rényi, A. On some combinatorical problems. Publ. Math. Debrecen 4 (1956), 398–405.

Let $C_k(n)$ denote the least number of combinations of k out of n elements ($k, n=2, 3, \cdots$) such that every pair of elements occurs at least once in a combination. Clearly $C_k(n) \geq n(n-1)/[k(k-1)$ and in case of equality the combinations form a balanced incomplete block design with $\lambda=1$.

The authors investigate the asymptotic behaviour of $C_k(n)$. Putting $C_k(n)k(k-1)/[n(n-1)]=c_k(n)$ the authors prove 1) $\lim_{n\to\infty} c_k(n)$ exists for every k. 2) $\lim_{n\to\infty} c_k(n)=1$ if k is a prime-power. 3) $\lim_{k\to\infty} \lim_{n\to\infty} c_k(n)=1$.

Let $D_k(n)$ denote the length of the shortest sequence formed from $1, 2, \cdots, n$ in which any two distinct digist are at least once in such a position that they are separated by at most k numbers. Using the preceding results and a lemma of Szele [Mat. Fiz. Lapok 50 (1943), 223–256; MR 8, 284 the authors prove that $\lim_{n\to\infty} D_k(n)/n^2$ exists.

H. B. Mann (Columbus, Ohio).

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AUTHORS: Palásti, Ilona, Rényi, Alfréd

TITLE: Monte-Carlo methods as minimax strategies

PERIODICAL: Referativnyy zhurnal, Matematika, no. 11, 1962, 61, abstract 11V316 (Magyar tud. akad. Mat. kutató int. közl., 1956 (1957), v. 1, no. 4, 529 - 545; Hungarian; summaries in Russian, English)

TEXT: Monte-Carlo methods are considered from the viewpoint of the theory of games. As an example the authors consider the numerical approximation of the interval $I = \int^1 f(x)\,dx$ of a continuous function f (x) by the sum $S = n^{-1} \sum_{k=1}^{n} f(x_k)$. The pure strategy of player B consists in the choice of a function f (x). It is assumed that the set Φ of admissible functions f (x) consists of all continuous functions satisfying the condition

$$\int_0^1 [f(x) - \int_0^1 f(t)\,dt]^2\,dx = s^2 ;$$

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where $s>0$ is a specified constant. The pure strategy of the player A consists in the selection of a system of points $(x_1, x_2, \ldots, x_n)$ of the interval (0, 1). A's loss is defined by the quantity $\Delta = (S - I)^2$. It is demonstrated that one of the minimax strategies for player A is the mixed strategy defined by usual Lebesgue measure specified on measurable subsets of an n-dimensional cube K_n. The same holds true also in the case of an r-dimensional integral $(r = 2, 3, \ldots)$. The mean error is equal to $s/\sqrt{n}$, independent of r. The authors investigate the analogous problem of estimating the sum $Y = \sum_{k=1}^{N} y_k$ by the quantity $\eta = Nn^{-1}\sum_{j=1}^{n} y_{k_j}$, where $k = (k_1, \ldots, k_n)$ is some subset of the set (1, 2, ..., N). It is emphasized that, if the set of admissible functions or sums changes, the minimax strategy also changes. A number of experiments carried out by the Monte-Carlo method is described.

From the Authors' summary

[Abstracter's note: Complete translation]

RENYI, A.

3

Rényi, A.; and Zergényi, E. An inequality for uncorrelated random-variables. Czechoslovak Math. J. 6(81) (1956), 415–419. (Russian summary)

Let $\xi_1, \xi_2, \cdots, \xi_k, \cdots$ be a sequence of uncorrelated random variables with mean values 0 and variances D_k^2. Let c_k denote a non-increasing sequence of positive numbers, satisfying the inequality $1 < c \leqq c_k/c_{2k} \leqq C$ ($k=1, 2, \cdots$). The authors prove that

$$E\left(\sup_{n \leqq k} c_k^2 \left|\sum_{j=1}^{k} \xi_j\right|^2\right) \leqq K\left(c_n^2 \sum_{j=1}^{n} D_j^2 + \sum_{j=n+1}^{\infty} D_j^2 c_j^2 \log^2 j\right)$$

for $n=1, 2, \cdots$, where the constant K depends only on the constants c and C. This inequality simplifies the proof of the strong law of large numbers for uncorrelated random variables. *J. Wolfowitz* (Ithaca, N.Y.).

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RÉNYI, A.

Prékopa, A.; Rényi, A.; and Urbanik, K. On the limiting distribution of sums of independent random variables in bicompact commutative topological groups. Acta Math. Acad. Sci. Hungar. 7 (1956), 11–16. (Russian. English summary)

The main result is the following: If ξ_n ($n=1, 2, \cdots$) are independent equidistributed random variables assuming values in a compact commutative topological group G and if $P(\xi_n \in U)>0$ for every open non-empty subset U of G, then the sequence of measures

$$\mu_n(E)=P(\xi_1+\cdots+\xi_n \in E),$$

defined for all Borel subsets of G, converges weakly to the Haar probability measure on G. The proof is based on reducing the theorem, through consideration of the character group, to the special cases when G is either the additive group of reals mod 1 or a finite cyclic group, which were proved by P. Lévy [Bull. Soc. Math. France 67 (1939), 1–41; MR 1, 62] and [among others, see below] A. Dvoretzky and J. Wolfowitz [Duke Math. J. 18 (1951), 501–507; MR 12, 639].

{Remark by reviewer: It seems to be little known that

Prekopa, A.; Renyi, A.; Urbanik, K.

the above result even for non-commutative compact groups, as well as many related results, were proved by Y. Kawada and K. Itô [Proc. Phys.-Math. Soc. Japan (3) 22 (1940), 977-998; MR 2, 223] by straightforward use of the theory of unitary representations. Kawada and Itô make the additional assumption that the group is separable, but developments since 1940 have made it well-known that this condition is superfluous.}

A. Dvoretzky (New York, N.Y.).

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Renyi, Alfred

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1-F\W

Rényi, Alfréd. On the distribution of the digits in Cantor's series. Mat. Lapok 7 (1956), 77–100. (Hungarian. Russian and English summaries)

In this paper the author proves that the well-known theorem of Borel on the frequency of the digits of the q-ary expansions can be generalised for Cantor series if $\sum 1/q_n = \infty$. In a previous paper [Acta Math. Acad. Sci. Hungar. 6 (1955), 285–335; MR 18, 339] the author proved a slightly weaker theorem, but the present proof is simpler. The case $\sum_{n=1}^{\infty} 1/q_n < \infty$ is also considered. [Cf. #6406, #6407 below]. *P. Erdős* (Birmingham)

RENYI, A.

Erdös, P.; and Rényi, A. On the number of zeros of successive derivatives of analytic functions. Acta Math. Acad. Sci. Hungar. 7 (1956), 125–144. (Russian summary)

Math

Let $N_k(r)$ denote the number of zeros of $f^{(k)}(z)$ in $|z| \leq r$. The authors prove several theorems on the asymptotic behavior of $N_k(r)$, generalizing previous results of Pólya [Bull. Amer. Math. Soc. 49 (1943), 178–191; MR 4, 192] and Evgrafov [Interpolyacionnaya zadača Abelya-Gončarova, Gostehizdat, Moscow, 1954; MR 16, 1104]. They also discuss r_k, the modulus of the zero of $f^{(k)}(z)$ that is closest to the origin. Theorem 1. If $f(z)$ is regular in $|z|<1$, then $\liminf k^{-1}N_k(r) \leq K(r)$, where $K=K(r)$ is the positive root of $r=K(1+K)^{-1-1/K}$. More precisely, this is true when k runs through the values for which $f^{(k)}(0) \neq 0$, and in this form it is best possible. For an entire function, Theorem 1 gives $\liminf k^{-1}N_k(1)=0$. If the growth of the entire function is restricted, more can be said. Theorem 2. Let $g(r) \uparrow \infty$ and let h be its inverse function. If

$$\liminf \{\log M(r)\}/g(r) < 1,$$

then $\liminf k^{-1}h(k)N_k(1) \leq e^2$. This is essentially best possible. Let f be any entire function and let H be the inverse of $\log M(r)$. Then $\liminf k^{-1}H(k)N_k(1) \leq e^2$ and

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Erdös, P.; Rényi, A.

(Theorem 3) lim inf $k^{-1}H(k)/r_k \leq e/\log 2$. Theorem 3 generalizes a theorem of Alander's [Uppsala thesis, 1914]. Theorem 4. If f is regular in $|z|<R$ and not a polynomial, then lim sup $kr_k \geq R \log 2$. The authors remark that $\sum r_k$ always diverges (except for polynomials) and hence [conjectured by Erwe, Arch. Math. 7 (1956), 55–58; MR 17, 835] if $f^{(n)}(z_n)=0$ with $|z_{n+1}| \leq \frac{1}{2}|z_n|$ then f is a polynomial. The authors conjecture that for entire functions lim sup$(r_1+\cdots+r_k)/\log k=\infty$.

R. P. Boas, Jr.

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Gmco

RENYI, A.

On the independence in the limit of sums depending on the same sequence of independent random variables. In English. p.319.
(Acta Mathematica, Vol. 7, no. 3/4, 1956, Budapest, Hungary)

SO: Monthly List of East European Accessions (EEAL) LC. Vol. 6, No. 9, Sept. 1957. Uncl.

Rényi, Alfréd. A characterization of Poisson processes. Magyar Tud. Akad. Mat. Kutató Int. Közl. 1 (1956), 519–527 (1957). (Hungarian. Russian and English summaries)

A recurrent process is formed by a sequence of events which occur at random times $0=t_0<t_1<\cdots<t_n<\cdots$ and where the time intervals t_n-t_{n-1} between consecutive occurences are independently and identically distributed random variables with common distribution function $F(x)$. The author considers a transformation T_q of this process which consists of a contraction (=replacement of the time point t_n by qt_n with $0<q<1$) followed by a rarefaction (=cancellation of events, independently of each other, with probability $1-q$). The application of T_q transforms the given recurrent process into a new recurrent process; the distribution function of the time intervals between consecutive occurrences in the transformed process is denoted by $T_q[F(x)]$. The author's principal result is a characterization of the Poisson process (i.e., the process with $F(x)=1-e^{-\beta x}$): the Poisson process is the only process which is invariant under the transformation T_q. It is also shown that $\lim_{q\to0} T_q[F(x)] = 1-e^{-\beta x}$ provided that (i) $F(0)=0$ and (ii) the first moment of $F(x)$ is finite. Moreover, if (i) and (ii) are satisfied and if the sequence q_j is such that $0<q_j<1$, $\lim_{n\to\infty} q_1q_2\cdots q_n=0$, then the recurrent process obtained by applying consecutively the transformations $T_{q_1}, T_{q_2}, \cdots, T_{q_n}$ approaches a Poisson process.

E. Lukacs (Washington, D.C.)

Rényi, Alfréd. On the distribution function, $L(z)$. Magyar Tud. Akad. Mat. Kutató Int. Közl. 2 (1957), 43–50. (Hungarian. Russian and English summaries)

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1-F\W

The author publishes three proofs for the fact that the distribution function

$$L(z)=\frac{4}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k}{2k+1}\exp\left(-\frac{(2k+1)^2\pi^2}{8z^2}\right)\ (z>0)$$

can be expressed as follows:

$$L(z)=\sum_{k=-\infty}^{\infty}(-1)^k[\Phi((2k+1)z)-\Phi((2k-1)z)].$$

{It is to be remarked that this paper is antedated.}

L. Takács (London)

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RB

RENYI, A.

D-3

BULGARIA/Atomic and Molecular Physics - Statistical Physics. Thermodynamics.

Abs Jour : Ref Zhur - Fizika, No 11, 1958, No 24930

Author : Renyi A.
Inst : Budapest, Hungary
Title : A New Deduction of Maxwell's law of Velocity Distribution

Orig Pub : Izv. Matem. in-t. B"lg. AN, 1957, 2, 45-55

Abstract : A new axiomatic theory of probability (whichis a natural generalization of the Kolmogorov theory of probability) is proposed. This theory leads to a concept of distribution of probabilities for an unlimited measure. By way of an example of application of the theory to statistical mechanics the author gives a simple derivation of the Maxwell distribution, formulated in the form of the following theorem: if the components of velocities of each of N particles of an ideal gas are independent of each other andof the components of the remaining particles, and if one assumes that these random quantities have a regular uniform distribution over the entire real axis, then the distribution of the probabilities of the

Card : 1/2

RENYI, A.

Algorithms for producing real numbers

p. 265 (Magyar Tudomanyos Akademia. Matematikai es Fizikai Osztaly. Kozlemenyei. Vol. 7, no. 3/4, 1957. Budapest, Hungary)

Monthly Index of East European Accessions (EEAI) LC. Vol.7, no. 2, February 1958

RENYI, A., Arato, M.

Probabilistic proof of a theorem on the approximation of continous functions by means of generalized Bernstein polynomials. In English. p.91
(ACTA MATHEMATICA. Vol. 8, no. 1/2, 1957, Hungary)

SO: Monthly List of East European Accessions (EEAI) LC. Vol. 6, no. 12, Dec. 1957. Uncl.

RENYI, A.

On the asymptotic distribution of the sum of a random number of independent random variables. In English. p. 193.
(ACTA MATHEMATICA. Vol. 8, no. 1/2, 1957, Hungary)

SO: Monthly List of East European Accessions (EEAL) LC. Vol. 6, no. 12, Dec. 1957. Uncl.

RENYI, A.

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Erdös, P.; and Rényi, A. On the number of zeros of successive derivatives of entire functions of finite order. Acta Math. Acad. Sci. Hungar. 8 (1957), 223–225. 1-FW 76

The authors sharpen one of their results [same Acta 7 (1956), 125–144; MR 18, 201] for entire functions of finite order at least 1. The theorem reads: if f is an entire function of finite order $\rho \geq 1$, if $M(r)$ is its maximum modulus, and $H(y)$ is the inverse of $y=\log M(x)$; and if N_k is the number of zeros of $f^{(k)}(z)$ in $|z|<1$; then

$$\liminf k^{-1} N_k H(k) < e^{2-1/\rho}.$$

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R. P. Boas, Jr. (Evanston, Ill.).

4523:

Rényi, A. On mixing sequences of sets. Acta Math. Acad. Sci. Hungar. 9 (1958), 215-228.

A sequence of measurable sets $\{A_n\}$ in a measure space is called by the author a strongly mixing sequence if $\lim_{n\to\infty} \mu(A_n B) = \alpha\mu(B)$ for any measurable set B satisfying $\mu(B) < \infty$. If the measure space is a probability space $[\Omega, A, P]$ the defining relation for strong mixing becomes, with the usual notation for conditional probability, $\lim_{n\to\infty} P(A_n|B) = \alpha$.

The author proves several theorems showing that the property of strong mixing of a sequence of sets depends only on the internal relation of the sets in the sequence and is independent of the underlying probability measure. This independence is also enjoyed, under certain conditions, by limiting distributions of sequences of random variables (theorem 4). More precisely, theorems 1 and 2 state: A necessary and sufficient condition that a sequence $\{A_n\}$ of measurable sets be a strongly mixing sequence is that $\lim_{n\to\infty} P(A_n|A_k) = \alpha$ for $k = 0, 1, 2, \cdots$. In this case $\lim Q(A_n|B) = \alpha$ for every probability measure Q equivalent to P.

Similar theorems are proved for weak mixing and several applications are made. *Y. N. Dowker* (London)

RENYI, A.; ERDOS, P.

On singular radii of power series. In English. p. 159.

MAGYAR TUDOMANYOS AKADEMIA METEMATIKAI KUTATO INTEZETENEK KOZLEMENYEI. PUBLICATIONS OF THE MATHEMATICAL INSTITUTE OF THE HUNGARIAN ACADEMY OF SCIENCES. Budapest, Hungary. Vol. 3, no. 3/4, 1958.

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Uncl.

RENYI, A.

On the probabilistic generalization of the large sieve of Linnik. In English. p. 199

MAGYAR TUDOMANYOS AKADEMIA MATEMATIKAI KUTATO INTEZETENEK KOZLEMENYEI. PUBLICATIONS OF THE MATHEMATICAL INSTITUTE OF THE HUNGARIAN ACADEMY OF SCIENCES. Budapest, Hungary. Vol. 3. no. 3/4, 1958

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On mixing sequences of random variables. In English. p. 389.

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Uncl.

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Monthly List of East European Accessions (EEAI). LC, Vol. 9, no. 1, Jan., 1960.

Uncl.

APPROVED FOR RELEASE: Tuesday, August 01, 2000 CIA-RDP86-00513R0014446

RENYI, Alfred (Budapest)

On measures of dependence. In English. Acta mat. Hung. 10 no.3/4: 441-451 '59. (EEAI 9:5)

1. Member of the Hungarian Academy of Sciences.
(Hilbert space) (Correlation (Statistics)) (Probabilities)

RENYI, A.

Summation methods and probability theory. Mat kut kozl MTA 4 no.3/4: 389-399 '59. (EEAI 9:9)

(Probabilities)
(Distribution (Probability theory))

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On connected graphs. I. Mat kut kozl MTA 4 no.3/4:385-388 '59.
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(Topology)

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A contribution on the serial and parallel coupling of autoclaves and the theory of mixing. Mat kut kozl MTA 4 no.2:155-165 '59 (EEAI 9:3)

1. A Magyar Tudomanyos Akademia Matematikai Kutato Intezete. Szerkeszto, A Magyar Tudomanyos Akademia Matematikai Kutato Intezetenek Kozlemenyei

(Autoclaves) (Mixing) (Probabilities) (Bauxite)

RENYI, A.

Some remarks on the theory of trees. In English. p. 73.

MAGYAR TUDOMANYOS AKADEMIA MATEMATIKAI KUTATO INTEZETENEK KOZLEMENYEI. PUBLICATIONS OF THE MATHEMATICAL INSTITUTE OF THE HUNGARIAN ACADEMY OF SCIENCES. Budapest, Hung ary. Vol. 4, no. 1, 1959

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Uncl.

RENYI, A.

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(Probabilities) (Lattice theory)

ERDOS, P. (Budapest); RENYI, A., acad. (Budapest)

On the strength of connectedness of a random graph. Acta may Hung 12 no.1/2:261-267 '61. (EEAI 10:9)

1. Corresponding member, Hungarian Academy of Sciences (for Erdos).
2. Hungarian Academy of Sciences (for Renyi).

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